10:1 Part 1

Monday, March 1, 2021 9:41 AM

Sequen: finite / infinite

Converge / Diverge

Exp(1) $a_n = \sqrt{n}$, $n = 1, 2, 3, \dots$

$n=1 \Rightarrow a_1 = \sqrt{1} = 1$ $\quad$ 1^{st} term

$n=2 \Rightarrow a_2 = \sqrt{2}$ $\quad$ 2^{nd} term

$n=3 \Rightarrow a_3 = \sqrt{3}$ $\quad$ 3^{rd} term

$\vdots$

$a_n = \sqrt{n}$ $\quad$ n^{th} term

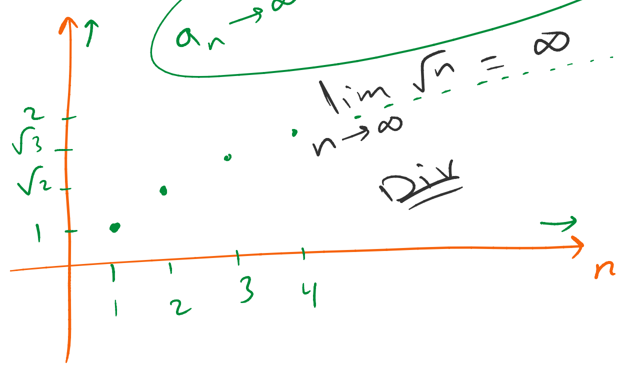
$n = 100 \Rightarrow a_{100} = \sqrt{100} = 10$

∞

$a_n \to \infty$ as $n \to \infty$

$\lim_{n\to\infty} \sqrt{n} = \infty$

Div



$\lim_{n\to\infty} \sqrt{n} = \infty \Rightarrow a_n = \sqrt{n}$ div

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(2) $b_n = \frac{1}{n}$, $n = 1, 2, 3, \dots$

$n=1 \Rightarrow b_1 = \frac{1}{1} = 1$

$n=2 \Rightarrow b_2 = \frac{1}{2}$

$n=3 \Rightarrow b_3 = \frac{1}{3}$

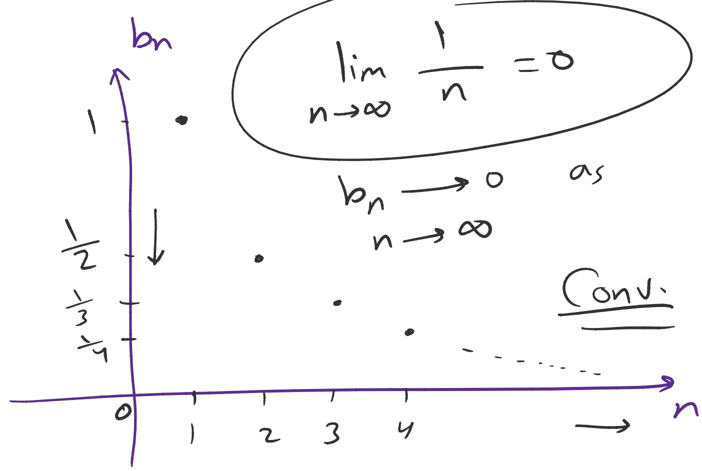
$\vdots$

$b_n \to 0$

$\lim_{n\to\infty} \frac{1}{n} = 0$

$b_n \to 0$ as $n \to \infty$

Conv.



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(3) $c_n = (-1)^n \frac{1}{n}$ $\quad$ Alternating sequence

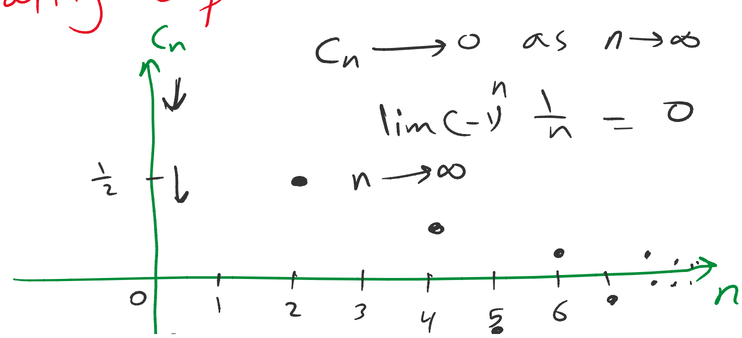
$n = 1, 2, 3, \dots$

$n=1 \Rightarrow c_1 = (-1)^1 \frac{1}{1} = -1$

$n=2 \Rightarrow c_2 = (-1)^2 \frac{1}{2} = \frac{1}{2}$

$c_n \to 0$ as $n \to \infty$

$\lim_{n\to\infty} (-1)^n \frac{1}{n} = 0$

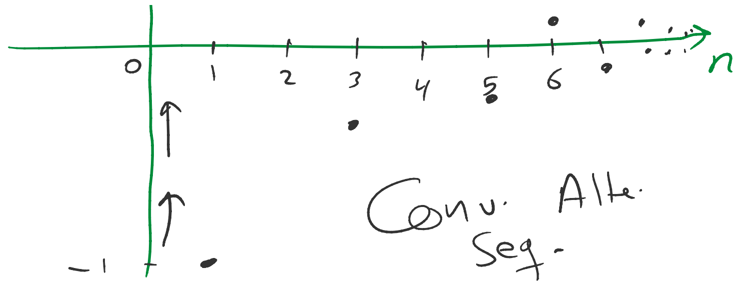


$n=2 \Rightarrow c_2 = (-1)\frac{1}{2} = \frac{1}{2}$

$n=3 \Rightarrow c_3 = -\frac{1}{3}$

$n=4 \Rightarrow c_4 = \frac{1}{4}$

$\vdots$



Conv. Alte. Seq.

Th $\lim\limits_{n\to\infty} a_n = A$, $\lim\limits_{n\to\infty} b_n = B$ **Then**

A number $a_n \to A$ as $n \to \infty$

B number $b_n \to B$ as $n\to\infty$

$\underbrace{a_n, b_n}_{\text{Conv.}}$

① $\lim\limits_{n\to\infty}(a_n + b_n) = A + B$ ✓

② $\lim\limits_{n\to\infty}(a_n - b_n) = A - B$ ✓

③ $\lim\limits_{n\to\infty} K a_n = KA$ ✓ K constant

④ $\lim\limits_{n\to\infty}(a_n b_n) = AB$ ✓

⑤ $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}$, $B \neq 0$ ✓

$\boxed{\lim\limits_{n\to\infty}\frac{1}{n} = 0}$

Exp ① $\lim\limits_{n\to\infty}\frac{-\sqrt{5}}{n} = -\sqrt{5}\lim\limits_{n\to\infty}\frac{1}{n} = -\sqrt{5}(0) = 0$

② $\lim\limits_{n\to\infty}\frac{7n-3}{5+14n} = \frac{7}{14} = \frac{1}{2}$

$\lim \frac{7-\frac{3}{n}}{\quad} = \frac{7-0}{\quad} = \frac{7}{14} = \frac{1}{2}$

$$\lim_{n\to\infty} \frac{7-\frac{3}{n}}{\frac{5}{n}+14} = \frac{7-0}{0+14} = \frac{7}{14} = \frac{1}{2}$$

(3) $$\lim_{n\to\infty} \frac{7n-3}{5+14n^2} = 0$$ ✓

$$\lim_{n\to\infty} \frac{\frac{7}{n}-\frac{3}{n^2}}{\frac{5}{n^2}+14} = \frac{0-0}{0+14} = \frac{0}{14} = 0$$

(4) $$\lim_{n\to\infty} \frac{7n^3-3}{5+14n} = \infty$$

$$\lim_{n\to\infty} \frac{7n^2-\frac{3}{n}}{\frac{5}{n}+14} = \frac{\infty-0}{0+14} = \frac{\infty}{14} = \infty$$

Sandwich Th

I need to know $\lim\limits_{n\to\infty} b_n$???

If $a_n \le b_n \le c_n$ for all n

and $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} c_n = L$, L finite

Then $\lim\limits_{n\to\infty} b_n = L$

$\lim \frac{\sin n}{}$

Exp Check Conv./Div. for ① $\lim_{n\to\infty} \frac{\sin n}{n}$

$$\frac{-1}{n} \le \frac{\sin n}{n} \le \frac{1}{n} \quad \forall n$$

for all

$$-\lim_{n\to\infty} \frac{1}{n} \le \lim_{n\to\infty} \frac{\sin n}{n} \le \lim_{n\to\infty} \frac{1}{n}$$

$\to 0$ $\quad\to 0\quad$ $\to 0$

by ST

$$-1 \le \sin n \le 1$$

$$0 \le (\sin n)^2 \le 1$$

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② $\lim_{n\to\infty} \frac{\sin^2 n}{2^n}$

$$\frac{0}{2^n} \le \frac{\sin^2 n}{2^n} \le \frac{1}{2^n}$$

$$\lim_{n\to\infty} 0 \le \lim_{n\to\infty} \frac{\sin^2 n}{2^n} \le \lim_{n\to\infty} \frac{1}{2^n}$$

$\to 0$ $\quad\quad\to 0$

$$a_n = \frac{1}{2^n} = \left(\frac{1}{2}\right)^n$$

$x \in (-1,1) \Rightarrow \boxed{\lim_{n\to\infty} x^n = 0 \quad , |x|<1}$

conv. to 0 by ST

$a_1 = \frac{1}{2}$

$a_2 = \frac{1}{2^2} = \frac{1}{4}$

$a_3 = \frac{1}{2^3} = \frac{1}{8}$

$a_4 = \frac{1}{16}$

$\vdots$ $\quad\to 0$

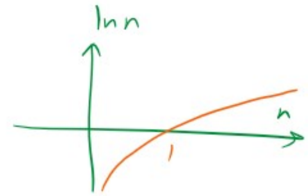
0

③ $\lim_{n\to\infty} (-1)^n \frac{1}{n} = 0$ Alternation

$$\frac{-1}{n} \le (-1)^n \frac{1}{n} \le \frac{1}{n}$$

$$-\lim_{n\to\infty} \frac{1}{n} \le \lim_{n\to\infty} (-1)^n \frac{1}{n} \le \lim_{n\to\infty} \frac{1}{n} \to 0$$

$\to 0$ by ST



Th5 / 10.1 // 6 cases

① $\lim_{n\to\infty} \frac{\ln n}{n} = 0$ $\quad \frac{\infty}{\infty}$ $\quad \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = \lim_{n\to\infty} \frac{1}{n} = 0$

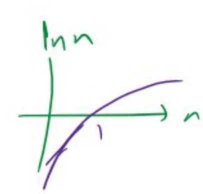
Exp $\lim_{n\to\infty} \frac{\ln n^3}{3n} = \lim_{n\to\infty} \frac{\cancel{3}\ln n}{\cancel{3}n} = \lim_{n\to\infty} \frac{\ln n}{n} = 0$

② $\lim_{n\to\infty} \sqrt[n]{n} = 1$ ✓ $\quad \lim_{n\to\infty} n^{\frac{1}{n}} = \infty^0$ indetermined power

$$e^{\ln f(x)} = f(x)$$

$$\lim_{n\to\infty} e^{\ln \sqrt[n]{n}} = \lim_{n\to\infty} e^{\frac{1}{n}\ln n} \qquad e^{\ln \sqrt[n]{n}} = \sqrt[n]{n}$$

$$= \lim_{n\to\infty} e^{\frac{\ln n}{n}} = e^{\lim_{n\to\infty} \frac{\ln n}{n}} = e^0 = 1$$



Exp 59 $\lim_{n\to\infty} \frac{\ln n}{\sqrt[n]{n}} = \frac{\lim_{n\to\infty} \ln n}{\lim_{n\to\infty} \sqrt[n]{n}} = \frac{\infty}{1} = \infty$

Exp (59) $\lim_{n\to\infty} \frac{\ln n}{\sqrt[n]{n}} = \frac{\lim_{n\to\infty} \ln n}{\lim_{n\to\infty} \sqrt[n]{n}} = \frac{\infty}{1} = \infty$

Exp $\lim_{n\to\infty} \sqrt[n]{n^3} = \lim_{n\to\infty} (n^3)^{\frac{1}{n}} = \lim_{n\to\infty} \left(n^{\frac{1}{n}}\right)^3 = 1^3 = 1$

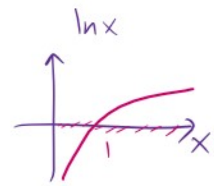
$(a^b)^c = a^{bc} = (a^c)^b$

Exp $\lim_{n\to\infty} \sqrt[n]{\pi n} = \lim_{n\to\infty} (\pi)^{\frac{1}{n}} \left(n^{\frac{1}{n}}\right) = \lim_{n\to\infty} \pi^{\frac{1}{n}} \lim_{n\to\infty} \sqrt[n]{n}$

$= (1)(1)$

$= 1$

(3) $\lim_{n\to\infty} x^{\frac{1}{n}} = 1$, $x > 0$



$\lim_{n\to\infty} x^{\frac{1}{n}} = \lim_{n\to\infty} e^{\ln x^{\frac{1}{n}}} = \lim_{n\to\infty} e^{\frac{\ln x}{n}} = e^{\ln x \lim_{n\to\infty} \frac{1}{n}} = e^{(\ln x)(0)} = e^0 = 1$

(4) if $|x| < 1$ $\Rightarrow$ then $\lim_{n\to\infty} x^n = 0$

$-1 < x < 1$

$\lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$, $\lim_{n\to\infty} \left(-\frac{2}{3}\right)^n = 0$

$\lim 2^n = \infty$, $\lim \left(\frac{3}{2}\right)^n = \infty$ $\qquad \frac{1}{x^2} = x^{-2}$

$$\lim_{n\to\infty} 2^n = \infty \quad , \quad \lim_{n\to\infty} \left(\frac{3}{2}\right)^n = \infty \qquad \frac{1}{x^2} = x$$

Exp

$$\lim_{n\to\infty} \frac{\pi^{-n}}{e^{-n}} = \lim_{n\to\infty} \left(\frac{\pi}{e}\right)^{-n} = \lim_{n\to\infty} \left(\overset{<1}{\frac{e}{\pi}}\right)^n = 0$$

$e \approx 2.718$

$\pi \approx 3.14$

$$\lim_{n\to\infty} \left(\frac{\pi}{e}\right)^n = \infty$$

(5)

$$\lim_{n\to\infty} \left(1 + \frac{\overset{3}{x}}{n}\right)^n = e^{\overset{3}{x}}$$

$$\lim_{n\to\infty} e^{n \ln\left(1+\frac{x}{n}\right)} = \lim_{n\to\infty} e^{\frac{\ln\left(1+\frac{x}{n}\right)\to 0}{\frac{1}{n}\to 0}} \quad \frac{0}{0} \quad = \lim_{n\to\infty} e^{\frac{\frac{-x}{n^2}\Big/\left(1+\frac{x}{n}\right)}{\frac{-1}{n^2}}}$$

$$= \lim_{n\to\infty} e^{\frac{x}{1+\frac{x}{n}\to 0}} = e^{\frac{x}{1+0}} = e^x$$

Exp

$$\lim_{n\to\infty} \left(1 + \frac{3}{n}\right)^n = e^3$$

$$\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n = e^{-1} = \frac{1}{e}$$

(68)

$$\lim_{n\to\infty} \ln\left(1 + \frac{1}{n}\right)^n = \ln \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n \to e$$

$$= \ln e$$

$$= 1$$

$$= 1$$

$$\ln(n^2+n) \quad \frac{1}{n}$$

(84) $$\lim_{n\to\infty} \sqrt[n]{n^2+n} = \lim_{n\to\infty} e^{\ln(n^2+n)}$$

$$\sqrt[n]{n^2(1+\tfrac{1}{n})} \quad (n^2)^{\frac{1}{n}} \left(1+\tfrac{1}{n}\right)^{\frac{1}{n}} \quad \frac{\infty}{\infty} \quad \left(n^{\frac{1}{n}}\right)^2 \quad 1^2 \quad 1$$

$$= \lim_{n\to\infty} e^{\frac{\ln(n^2+n)}{n}} = e^{\lim_{n\to\infty} \frac{\ln(n^2+n)}{n}}$$

$$= e^{\lim_{n\to\infty} \frac{\frac{2n+1}{n^2+n}}{1}} = e^{\lim_{n\to\infty} \frac{2n+1}{n^2+n}} = e^0 = 1$$

(6) $$\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad \text{for any } x$$

"Taylor Series"

$$\lim_{n\to\infty} \frac{2^n}{n!} = 0 \quad , \quad \lim_{n\to\infty} \frac{(\frac{1}{2})^n}{n!} = 0$$

$$\lim_{n\to\infty} \frac{\pi^n}{n!} = 0$$

Exp $$\lim_{n\to\infty} \left(\frac{n+1}{n-1}\right)^n = \lim_{n\to\infty} \left(\frac{n-1+2}{n-1}\right)^n = \lim_{n\to\infty} \left(1+\frac{2}{n-1}\right)^n$$

$$u = n-1 \Rightarrow u+1 = n$$

$$= \lim_{u\to\infty} \left(1+\frac{2}{u}\right)^{u+1}$$

$u = n-1 \Rightarrow u+1 = n$

$n \to \infty \Rightarrow u \to \infty$

$= \lim_{u\to\infty} \left(1 + \frac{}{u}\right)$

$= \lim_{u\to\infty} \left(1 + \frac{2}{u}\right)\left(1 + \frac{2}{u}\right)^u$

$= \lim_{u\to\infty} \left(1 + \frac{2}{u}\right) \lim_{\cancel{u}\to\infty} \left(1 + \frac{2}{u}\right)^u$

$= (1+0)\left(e^2\right)$

$= e^2$