ENEE2304

Circuit Analysis

Chapter 3 Analysis of Simple Resistive Circuits

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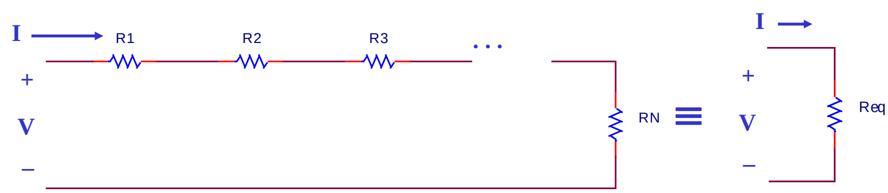
Reading Assignment: Chapter 3 in *Electric Circuits, 10th Edition* by Nilsson

Series Resistance

Two or more resistors in series can be replaced by a single equivalent resistance, R_{eq}, as shown below.



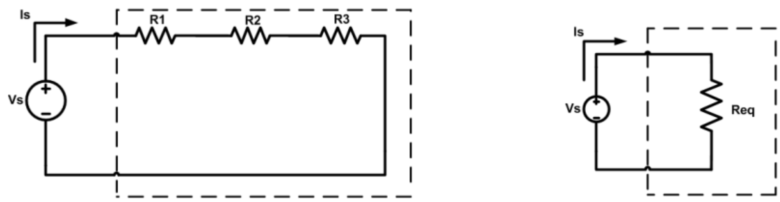
We use KVL to show that: $R_{eq} = R_1 + R_2 + R_3 + ... + R_N$

$$V = I R_1 + I R_2 + I R_3 + ... + I R_N$$

$$V = I (R_1 + R_2 + R_3 + ... + R_N)$$

$$V = I (R_{eq}); \text{ where } R_{eq} = R_1 + R_2 + R_3 + ... + R_N$$

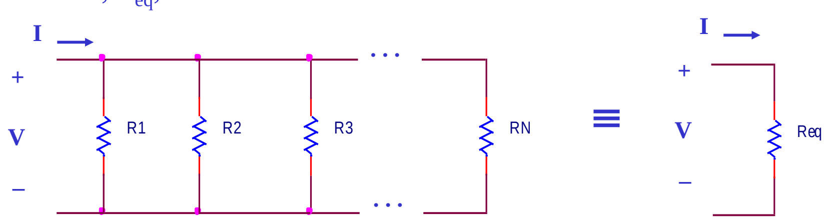
Resistors in series



$$Req = R1 + R2 + R3$$

Parallel Resistance

Two or more resistors in parallel can be replaced by a single equivalent resistance, R_{eq}, as shown below.



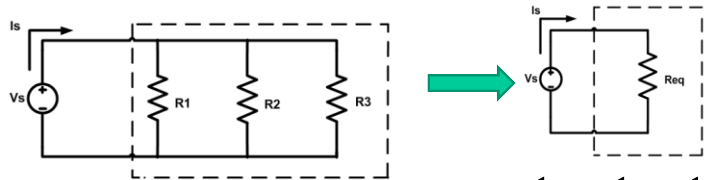
Use KCL to show that:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ldots\ldots + \frac{V}{R_N}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_N}\right) \quad \therefore R_{eq} = \frac{V}{I} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_N}\right)}$$

Resistors in parallel



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

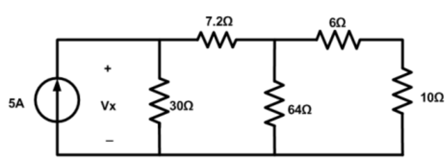
Two Resistors in parallel:

$$R_1 // R_2 = R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$= \frac{R_1 R_2}{R_1 + R_2}$$

$$R_1 // R_2 < \min(R_1, R_2)$$

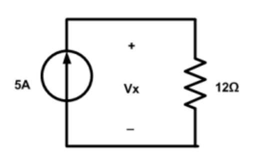
Find Vx?



$$16 // 64 = \frac{(16)(64)}{16 + 64}$$

$$= 12.8\,\Omega$$

$$12.8 + 7.2 = 20\,\Omega$$

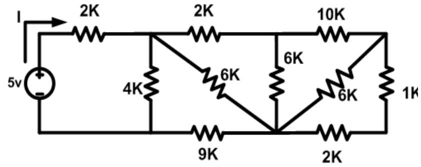


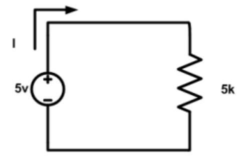
$$20 // 30 = \frac{(20)(30)}{20 + 30}$$

$$= 12\,\Omega$$

Vx = (5) (12) = 60 V

Find I?





Start at the Right

$$(1+2)//6=\frac{(3)(6)}{3+6}=2k\Omega$$

$$(2+10)//6=\frac{(12)(6)}{12+6}=4k\Omega$$

$$(2+4)//6=3k\Omega$$

$$(3+9)//4=3k\Omega$$

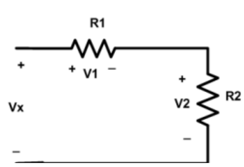
$$\therefore\ \text{Req}=(3+2)=5k\Omega$$

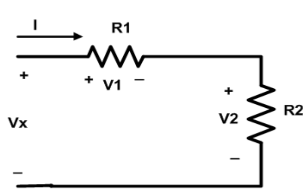
$$I=\frac{5\ v}{5\ k}=1mA$$

Voltage divider Rule

$$V_1=\frac{R_1}{R_1+R_2}V_X$$

$$V_2=\frac{R_2}{R_1+R_2}V_X$$





$$V_X=V_1+V_2$$

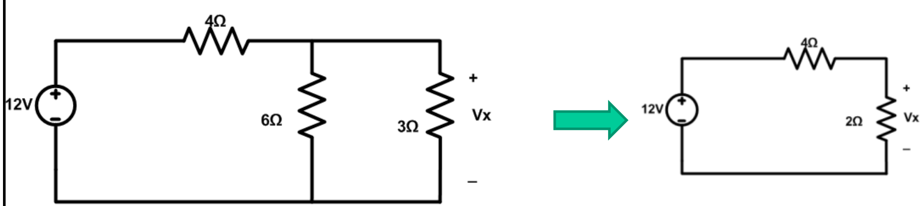
$$=I\ R_1+I\ R_2$$

$$\therefore I=\frac{V_X}{R_1+R_2}$$

$$V_1=R_1\ I=V_X\frac{R_1}{R_1+R_2}$$

$$V_2=R_2\ I=V_X\frac{R_2}{R_1+R_2}$$

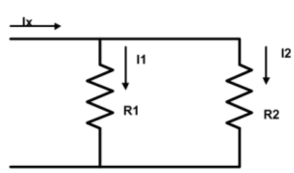
Find Vx?



$$3\Omega // 6\Omega = 2\Omega$$

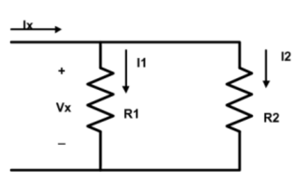
$$V_X = 12\frac{2}{2+4} = 4V$$

Current Divider Rule



$$I_1 = \frac{R_2}{R_1 + R_2} I_X$$

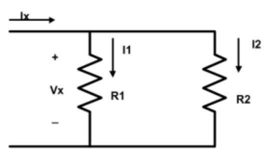
$$I_2 = \frac{R_1}{R_1 + R_2} I_X$$



$$I_X = I_1 + I_2$$

$$I_X = \frac{V_X}{R_1} + \frac{V_X}{R_2}$$

Proof

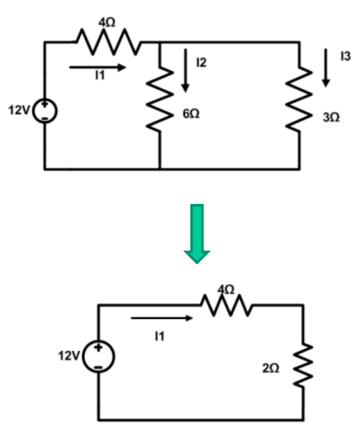


$$V_X = \frac{R_1 R_2}{R_1 + R_2} I_X$$

$$I_1 = \frac{V_X}{R_1} = \frac{R_2}{R_1 + R_2} I_X$$

$$I_2 = \frac{V_X}{R_2} = \frac{R_1}{R_1 + R_2} I_X$$

Find I3?

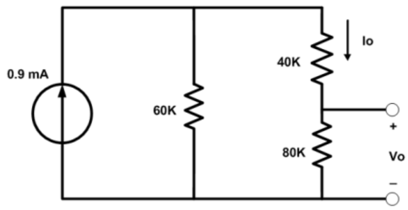


$$I_3 = \frac{6}{6+3} I_1$$

$$I_1 = \frac{12\ \text{V}}{(4+2)\Omega} = 2\ \text{A}$$

$$I_3 = \frac{6}{6+3} 2 = 1.33\text{A}$$

Find Vo?



$$V_o = I_o .(80\,k\Omega)$$

$$I_o = \frac{60\,k\Omega}{60\,k\Omega + (80\,k\Omega + 40\,k\Omega)} 0.9\,mA$$

$$I_o = 0.3\,mA$$

$$V_o = I_o .(80\,k\Omega) = 24\ \ V$$

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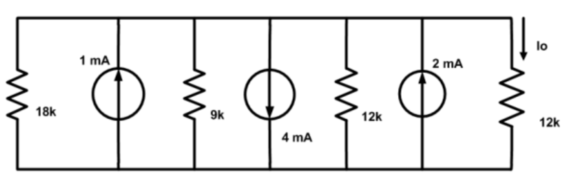
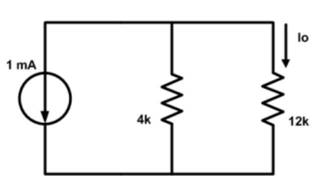
Find Io?



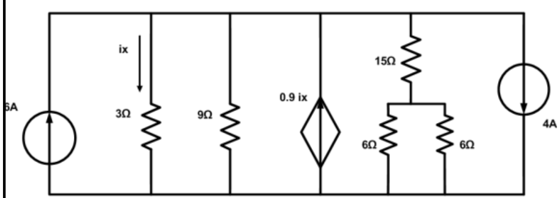
18k\\9k\\12k=4k



$$Io = -\frac{4k}{4k + 12k}\ 1mA$$

$$Io = -0.25mA$$

Find the power supplied by the 0.9 ix source?



- The 6A and 4A sources are combined into one source pointing up
- The two 6 Ω resistors are in parallel , and result is in series with 15 Ω
- The resulting 18 Ω is in parallel with 9 Ω yielding 6 Ω

Find the power supplied by the 0.9 ix source?

KCL :

$$2 + 0.9i_x = i_x + \frac{V_x}{6}$$

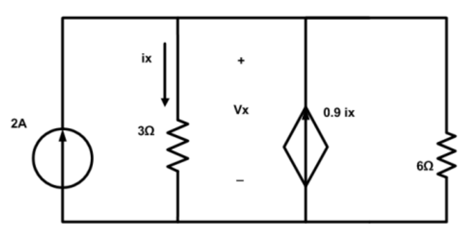
$$i_x = \frac{V_x}{3}$$

$$2 + 0.9i_x = i_x + \frac{3i_x}{6}$$

$$2 = 0.6i_x$$

$$i_x = \frac{2}{0.6} = \frac{10}{3} = 3.33\,A$$

$$V_x = 3i_x = 10\,V$$



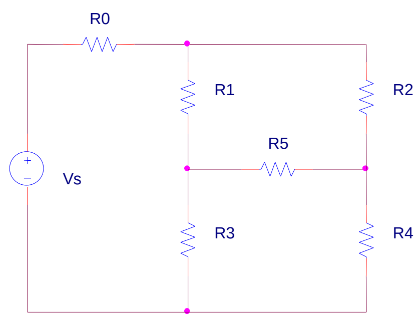
$$P_{(0.9ix)} = -(0.9ix)\,(Vx)$$

$$= -\left(0.9\left(\frac{10}{3}\right)\right)(10)$$

$$= -30\,W \text{ supplying}$$

Bridge Circuits

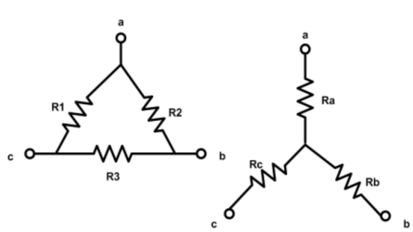
One type of resistive circuit that cannot be simplified through series and/or parallel combinations is the "bridge circuit." A bridge circuit is shown below (drawn twice). Study the circuit to verify that there are no series resistors and no parallel resistors.



Delta-to-Wye (Δ-Y) and Wye-to-Delta (Y-Δ) Transformations

Bridge circuits contain resistors that are connected in delta (Δ) and wye (Y) configurations. One way to analyze this circuit is to use a Δ-Y or a Y-Δ transformation.

Y and Δ connections of resistors are shown below:



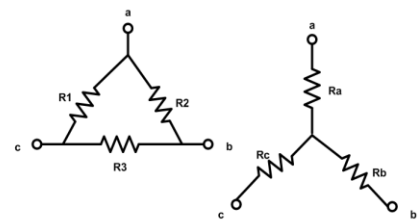
If the wye and delta circuits to be equivalent, then they should provide the same resistance between each pair of terminals (a-b, b-c, and c-a).

Development: Determine the resistance seen at each set of terminals and equate them as follows:

$R_{a\text{-}b}$ (Delta) = $R_{a\text{-}b}$ (Wye)

$R_{b\text{-}c}$ (Delta) = $R_{b\text{-}c}$ (Wye)

$R_{c\text{-}a}$ (Delta) = $R_{c\text{-}a}$ (Wye)



$$Rab = Ra + Rb = \frac{R2(R1 + R3)}{R1 + R2 + R3}$$

$$Rbc = Rb + Rc = \frac{R3(R1 + R2)}{R1 + R2 + R3}$$

$$Rca = Rc + Ra = \frac{R1(R2 + R3)}{R1 + R2 + R3}$$

Solving this set of equations

Solving the equations on the previous page yields the following relationships:

$Y - \Delta$ *tranformation*

$$R1 = \frac{RaRb + RbRc + RcRa}{Rb}$$

$$R2 = \frac{RaRb + RbRc + RcRa}{Rc}$$

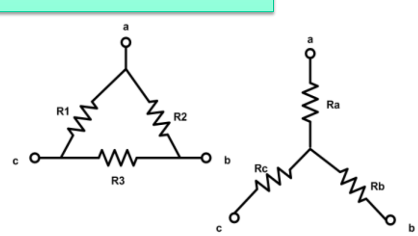
$$R3 = \frac{RaRb + RbRc + RcRa}{Ra}$$

$\Delta - Y$

$$Ra = \frac{R1\,R2}{R1 + R2 + R3}$$

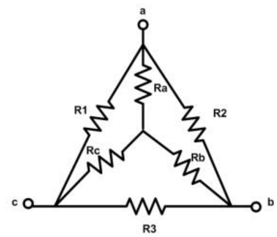
$$Rb = \frac{R2\,R3}{R1 + R2 + R3}$$

$$Rc = \frac{R3\,R1}{R1 + R2 + R3}$$



Note: The equations above must be used along with the circuit diagrams shown. The labeling of the resistors and nodes in the diagrams is critical.

Solving the Equations



$$Ra = \frac{R1\ R2}{R1 + R2 + R3}$$

$$Rb = \frac{R2\ R3}{R1 + R2 + R3}$$

$$Rc = \frac{R3\ R1}{R1 + R2 + R3}$$

- For a balanced case where:

$$Ra = Rb = Rc = Ry$$

$$R1 = R2 = R3 = R\Delta$$

$$R\Delta = 3\ Ry$$

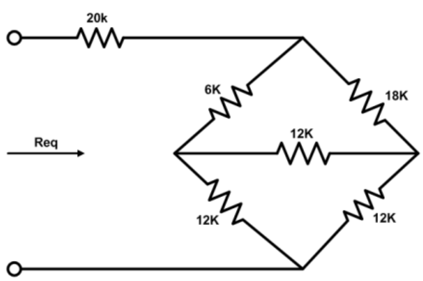
$$Ry = \frac{1}{3}\ R\Delta$$

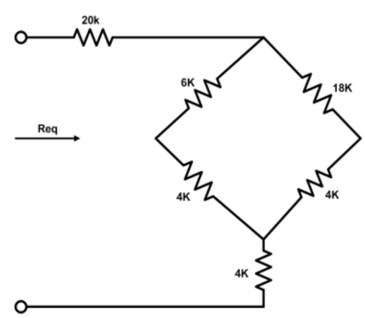
$$R1 = \frac{RaRb + RbRc + RcRa}{Rb}$$

$$R2 = \frac{RaRb + RbRc + RcRa}{Rc}$$

$$R3 = \frac{RaRb + RbRc + RcRa}{Ra}$$

Find Req?

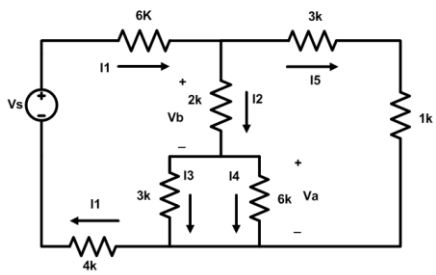




$$Req = 20k + 4k + (6k + 4k)\backslash\backslash(18k + 4k)$$

$$Req = 30.88k$$

Design: given $I4 = 0.5mA, find\ Vs$



$$Va = (6k\Omega)(0.5\ mA) = 3V$$

$$I3 = \frac{Va}{3k} = 1mA$$

$$I2 = I3 + I4 = 1.5mA$$

$$Vb = (2k\Omega)(1.5mA) = 3v$$

$$I5 = \frac{Va+Vb}{4k} = 1.5mA$$

$$I1 = I2 + I5 = 3mA$$

$$Vs = (10k\ \Omega)I1 + Vb + Va = 36v$$

Measuring Voltage and current

- Ammeter: designed to measure current
- Voltmeter : designed to measure voltage