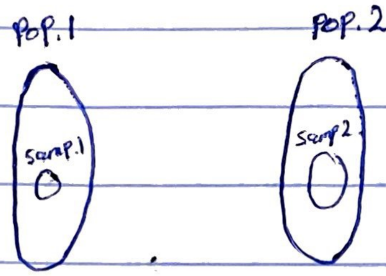
## 11.2 : Inferences about two population variances.




**Notation:**

$N_1$ : size of pop.1 | $N_2$ : size of pop 2. } ما بنعرفهم (قيمتهم)

$\sigma_1^2$ : variance of pop.1. | $\sigma_2^2$ : var. of pop 2.

$n_1$ : size of sample 1. | $n_2$ : size of sample 2.

$s_1^2$ : var. of sample 1. | $s_2^2$ : var. of sample 2.

★ sampling Dist. of $\frac{S_1^2}{S_2^2}$ when $\sigma_1^2 = \sigma_2^2$.

$$\frac{S_1^2}{S_2^2} \sim F \qquad df_1 = n_1 - 1$$
$$df_2 = n_2 - 1 .$$

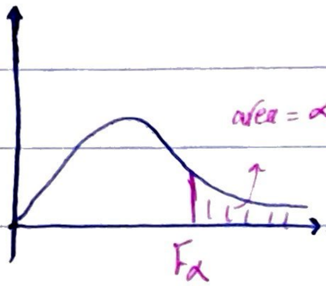
In words, $\frac{S_1^2}{S_2^2}$ has F distribution with $n_1 - 1$ degrees of freedom for the البسط numerator and $n_2 - 1$ degrees of freedom for the المقام denominator.

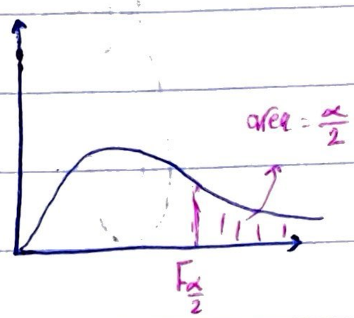
★ **Assuming :**

- sample 1 and sample 2 random
- sample 1 and sample 2 independent.
- sample 1 and sample 2 are from Normal population.
- $\sigma_1^2 = \sigma_2^2$.

★ F-dist ( $df_1$ and $df_2$ )




$H_0: \sigma_1^2 \le \sigma_2^2$

$H_1: \sigma_1^2 > \sigma_2^2$

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \ne \sigma_2^2$ .

LTT UTT

★ Test statistic : $F = \frac{S_1^2}{S_2^2}$ with $df_1 = n_1 - 1$ , $df_2 = n_2 - 1$ .

Note : Pop.1 is the population with higher sample variance .

★ Reject $H_0$ if $F \ge F_\alpha$ ( UTT ) .

Reject $H_0$ if $F \ge F_{\frac{\alpha}{2}}$ . ( TTT ) .

Reject $H_0$ if p-value $\le \alpha$ .

Q14 exp: Pop. 1 $n_1 = 16$ $s_1^2 = 5.8$

Pop. 2 $n_2 = 21$ $s_2^2 = 2.4$

$H_0: \sigma_1^2 \leq \sigma_2^2$ $\alpha = 0.05$

$H_1: \sigma_1^2 > \sigma^2$

(a) $F = \frac{s_1^2}{s_2^2} = \frac{5.8}{2.4} = 2.42$

| | $df_1 = 15$ | |
|---|---|---|
| $df_2 = 20$ | 0.10 | 1.84 |
| | 0.05 | 2.20 |
| | 0.025 | 2.57 |
| | 0.01 | 3.09 |

p-value $\in (0.25, 0.05)$

p-value $\leq \alpha$

→ Reject $H_0$ ($\alpha = 0.05$)

$\Rightarrow \sigma_1^2 > \sigma_2^2$ ($\alpha = 0.05$)

(b) $F = 2.42$ + The table ↑

critical value : $F_\alpha = F_{0.05} = 2.2$

→ $F \geq F_\alpha$

so we Reject $H_0$ ($\alpha = 0.05$)

$\sigma_1^2 > \sigma_2^2$ ($\alpha = 0.05$).