MATH 3331
QUIZ 3

Student Name: Answer key Student Number:

1. Use the definition to show that if $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences then $\{x_n + y_n\}$ is Cauchy.

Let $\epsilon > 0$.

$\exists N_1$ s.t. $|x_n - x_m| < \epsilon/2 \quad \forall n,m \geq N_1$

& $\exists N_2$ s.t. $|y_n - y_m| < \epsilon/2 \quad \forall n,m \geq N_2$

let $N = \max\{N_1, N_2\}$, then

$$|x_n + y_n - (x_m + y_m)| \leq |x_n - x_m| + |y_n - y_m|$$
$$< \epsilon/2 + \epsilon/2 = \epsilon \quad \forall n,m \geq N$$

2. Use the $\epsilon - \delta$ definition to show that $\lim_{x\to 2}(x^2 - x) = 2$.

Let $\epsilon > 0$.

$$|x^2 - x - 2| = |(x-2)(x+1)|$$

if $\delta = 1$, then $|x-2| < 1 \Rightarrow -1 < x-2 < 1 \Rightarrow 1 < x < 3$

$$\Rightarrow |x+1| \leq |x| + 1 < 4$$

let $\delta = \min\{1, \epsilon/4\}$,

$$|x-2| < \delta \Longrightarrow |x^2 - x - 2| = |x+1||x-2| < 4|x-2|$$
$$< 4\delta \leq \epsilon.$$