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Discrete Mathmatics

Quiz #2

Q1: Prove that if n is an integer which is not a multiple of 3, then $3 | n^2-1$

Solution :- 1. Restatement :

$$\forall n \in \mathbb{Z} \text{ if } 3 \nmid n \longrightarrow 3 | n^2-1$$

2. suppose that n is integer (Particular but arbitrarly chosen). and n is not a multiple of 3 $(3 \nmid n)$.

3. we need to show that $3 | n^2-1$

Rewrite n as a modulo of 3 by the quotient-remainde Theorem

the $n = 3q + r$, $0 \leq r < 3$
but $r \neq 0$ because $(3 \nmid n)$
then $0 < r < 3$

then $n = 3q + 1$ or $n = 3q + 2$

Case 1: $n = 3q + 1 \longrightarrow n^2 = (3q+1)^2 = 9q^2 + 6q + 1$

$n^2 - 1 = 9q^2 + 6q = 3(3q^2 + 2q)$

so, n^2-1 is a multiple of 3, Thus $3 | n^2-1$

Case 2: $n = 3q + 2 \longrightarrow n^2 = (3q+2)^2 = 9q^2 + 12q + 4$

$n^2-1 = 9q^2 + 12q + 3 = 3(3q^2 + 4q + 1)$

so, n^2-1 is a multiple of 3, Thus $3 | n^2-1$

Thus ; in the two cases $3 | n^2-1$ #