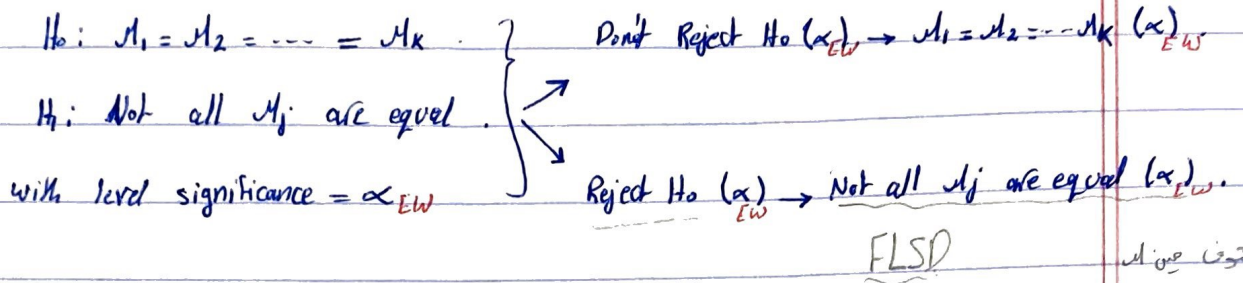


13.3: Multiple Comparison Procedures.



* FLSD Procedure

$H_0^{(i,j)}: \mu_i = \mu_j \quad i \neq j$
 $H_1^{(i,j)}: \mu_i \neq \mu_j \quad i, j \in \{1, \dots, k\}$

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notation: $(i,j) = i_j -$

* test statistic:

$t_{ij} = \frac{\bar{x}_i - \bar{x}_j}{\sqrt{MSE(\frac{1}{n_i} + \frac{1}{n_j})}}$ with $df = n_T - k$.

from ANOVA table

* level of significance = α_{cw}

* Number of comparisons = $\binom{k}{2}$

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* Rejection Rule.

- $|t_{ij}| \geq t_{\alpha_{cw}/2}$ with $df = n_T - k$.
 - p-value $\leq \alpha$
- we Reject $H_0 (\alpha_{cw})$.

- $\alpha_{EW} = 1 - (1 - \alpha_{CW})^{\binom{K}{2}}$

- Notation:

α_{CW} : Comparison wise Type I significance level

α_{EW} : Experiment wise Type I significance level.

- Bonferroni Adjustment:

- $\alpha_{EW} = \binom{K}{2} \alpha_{CW}$

- $\alpha_{CW} = \frac{\alpha_{EW}}{\binom{K}{2}}$

- Reject $H_0^{ij} (\alpha_{CW})$ if: $|\bar{x}_i - \bar{x}_j| \geq LSD^{ij}$

where $LSD^{ij} = t_{\frac{\alpha_{CW}}{2}}^{ij} \sqrt{MSE \left(\frac{1}{n_i} + \frac{1}{n_j} \right)}$

exp: Back to example of 13.1 and 13.2

$H_0: \mu_1 = \mu_2 = \mu_3$

H_1 : Not all μ_{ij} are equal.

$\bar{x}_1 = 79$

$\bar{x}_2 = 74$

$\bar{x}_3 = 66$

$S_1^2 = 34$

$S_2^2 = 20$

$S_3^2 = 32$

	df	SS	MS	F	$F_{0.05}$	p-value
TR	2	516	258	9	3.68	less than 0.01
E	15	430	28.67			
T	17	946				

↓
upper & lower bound

Cont exp:

Test 1

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

Find $\rightarrow |\bar{x}_1 - \bar{x}_2| = 5$
 $\rightarrow LSD^{12} = 6.59$
 $\left. \begin{array}{l} |\bar{x}_1 - \bar{x}_2| < LSD^{12} \\ \rightarrow \text{Don't Reject } H_0 \text{ (}\alpha=0.05\text{)} \\ \mu_1 = \mu_2 \text{ (}\alpha=0.05\text{)} \end{array} \right\}$

Test 2

$$H_0: \mu_1 = \mu_3$$

$$H_1: \mu_1 \neq \mu_3$$

$|\bar{x}_1 - \bar{x}_3| = 13$
 $LSD^{13} = 6.59$
 $\left. \begin{array}{l} |\bar{x}_1 - \bar{x}_3| > LSD \\ \rightarrow \text{Reject } H_0 \text{ (}\alpha=0.05\text{)} \\ \mu_1 \neq \mu_3 \text{ (}\alpha=0.05\text{)} \end{array} \right\}$

Test 3

$$H_0: \mu_2 = \mu_3$$

$$H_1: \mu_2 \neq \mu_3$$

$|\bar{x}_2 - \bar{x}_3| = 8$
 $LSD^{23} = 6.59$
 $\left. \begin{array}{l} |\bar{x}_2 - \bar{x}_3| > LSD \\ \rightarrow \text{Reject } H_0 \text{ (}\alpha=0.05\text{)} \\ \mu_2 \neq \mu_3 \text{ (}\alpha=0.05\text{)} \end{array} \right\}$

Find $LSD = \frac{2.13}{\downarrow \text{By t-table.}} \sqrt{28.67 \left(\frac{1}{6} + \frac{1}{6}\right)} = 6.59$

Note:

$(1-\alpha) CI$ for $\mu_i - \mu_j = (\bar{x}_i - \bar{x}_j) \pm LSD^{ij}$ where $LSD^{ij} = t_{\alpha/2} \sqrt{MSE \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$
 with $df = n_T - k$.

Note:

ANOVA	FISD
α	α
α_{EW}	α_{EW}

i (\in) observation no. : n_i
 j (\in) observation no. : n_j