COMP2421—DATA STRUCTURES AND ALGORITHMS

Splay Trees

Dr. Radi Jarrar
Department of Computer Science
Birzeit University

STUDENTS-HUB.com Uploaded By: Jibreel Bornat

Splay Tree

- Splay tree is a binary search tree.
- It has one interesting difference: whenever an element is looked up in the tree, the splay tree reorganizes to **move** that element to the root of the tree, without breaking the binary search tree invariant.
- Meaning, it brings the recently accessed item to the root of the tree.

STUDENTS-HUB.com Uploaded By: Jibreel Bornat

Splay Tree (2)

- Splay trees provide excellent locality. Frequently accessed items are easy to find. Infrequent items are far away from the root.
- The performance of the splay tree is O(log n). For a sequence of M operations performed on an n-node, splay tree takes O(M log n) time.

STUDENTS-HUB.com Uploaded By: Jibreel Bornat

Splay Tree (3)

- The 90–10 rule states that 90% of the accesses are to 10% of the data items. However, balanced search trees do not take advantage of this rule.
- A cache stores in main memory the contents of some of the disk blocks. The goal is that when a disk access is requested, the block can be found in the main memory cache and thus save the cost of an expensive disk access.
- Browsers make use of the same idea: a cache stores locally the previously visited Web pages.

STUDENTS-HUB.com Uploaded By: Jibreel Bornat

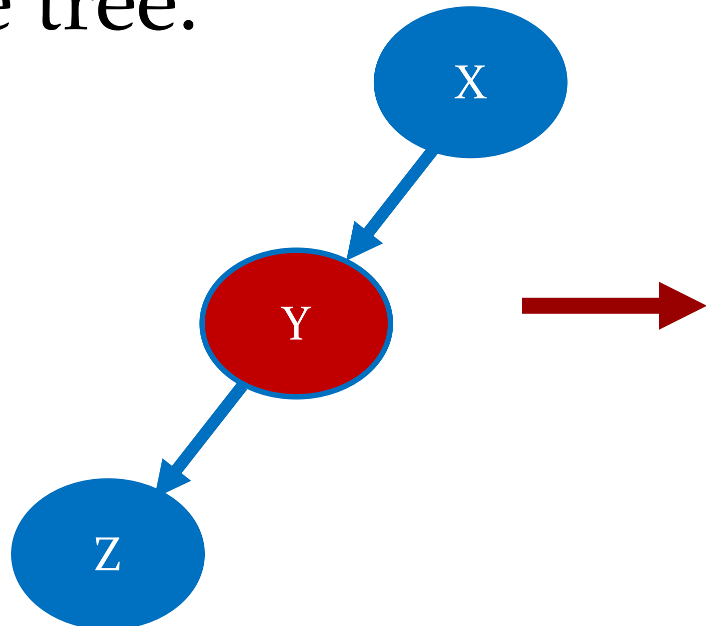
Splay Rotations

- When an element is accessed in Splay tree, tree rotations are used to move that element to the top of the tree.
- Type of rotations:
  1. Simple rotation
  2. Zig-Zig & Zag-Zag rotation
  3. Zig-Zag rotation

STUDENTS-HUB.com Uploaded By: Jibreel Bornat

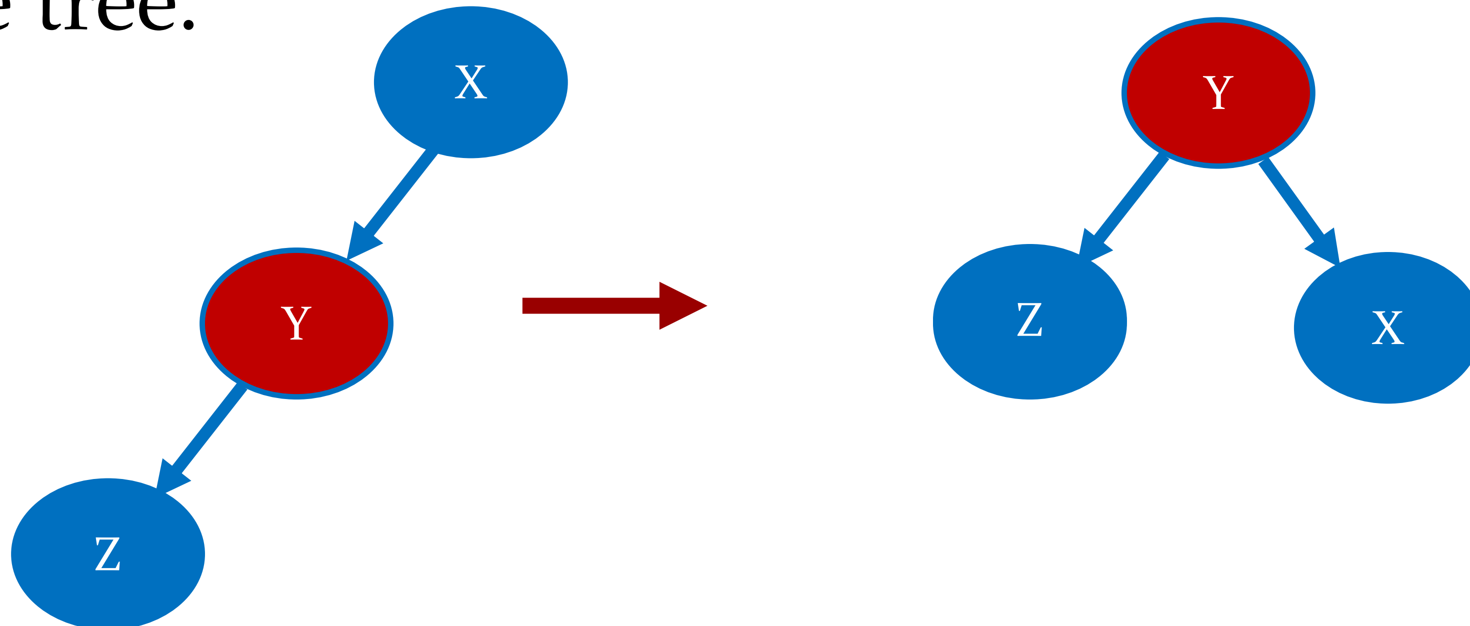
Simple Rotation

- This is similar to the tree rotation used in AVL tree.
- In this case it is applied to the root of the splay tree, moving the splayed node Y up to become the new root of the tree.



Simple Rotation

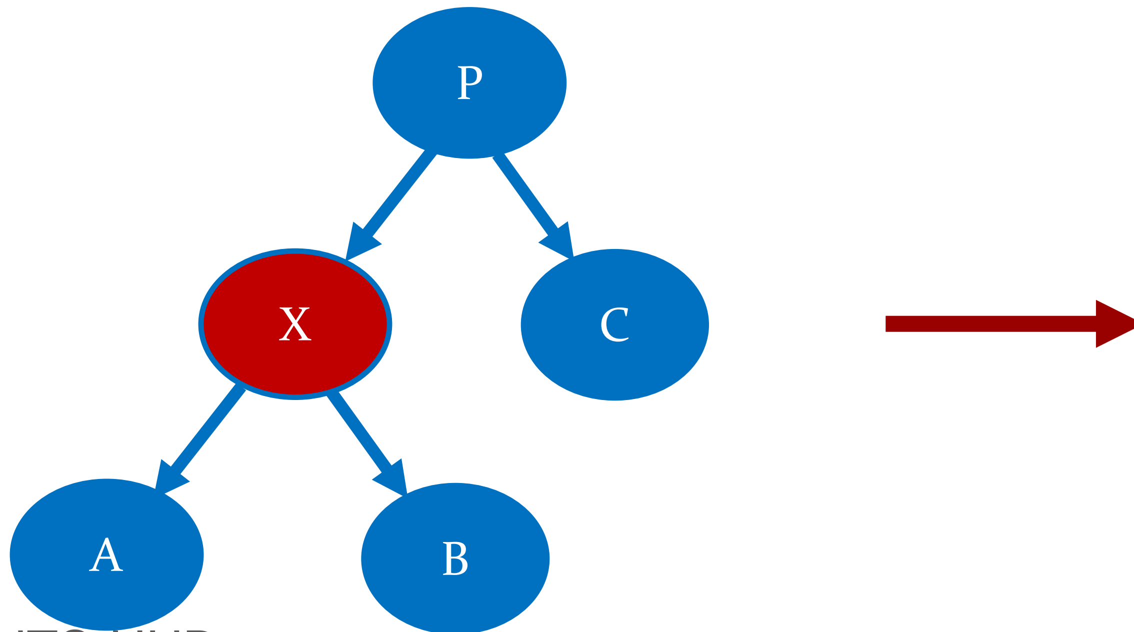
- This is similar to the tree rotation used in AVL tree.
- In this case it is applied to the root of the splay tree, moving the splayed node X up to become the new root of the tree.



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Simple Rotation (2)

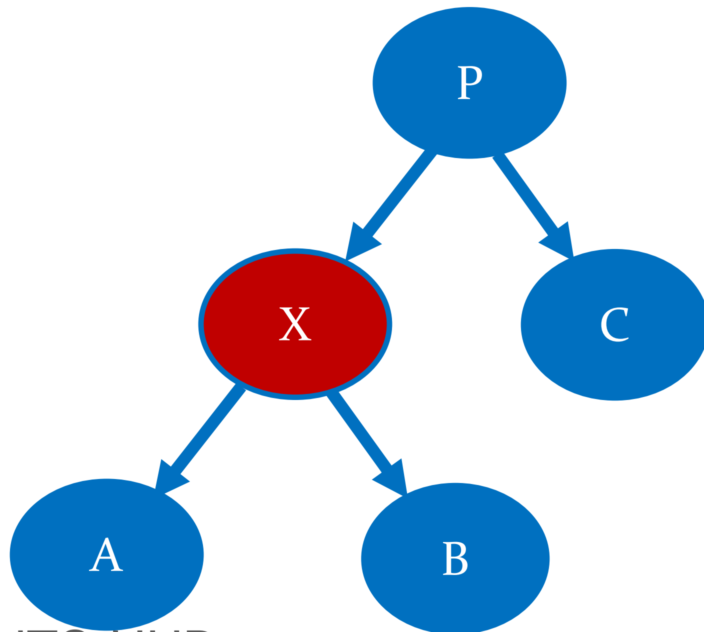
- The Zig-case and the Zag-case are the same as simple case
  - Let X be a non-root node on access path which we are rotating
  - If the parent of X is the root, then rotate X and the root

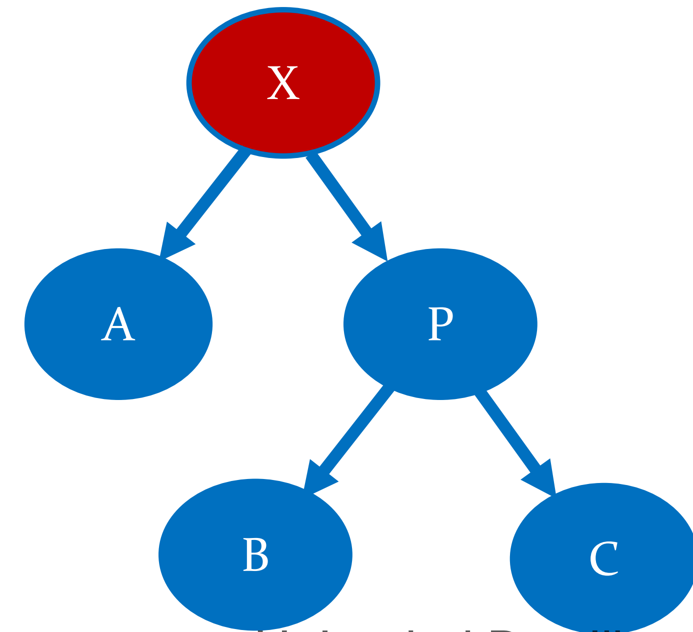


STUDENTS-HUB.com Uploaded By: Jibreel Bornat

Simple Rotation (2)

- The Zig-case and the Zag-case are the same as simple case
  - Let X be a non-root node on access path which we are rotating
  - If the parent of X is the root, then rotate X and the root



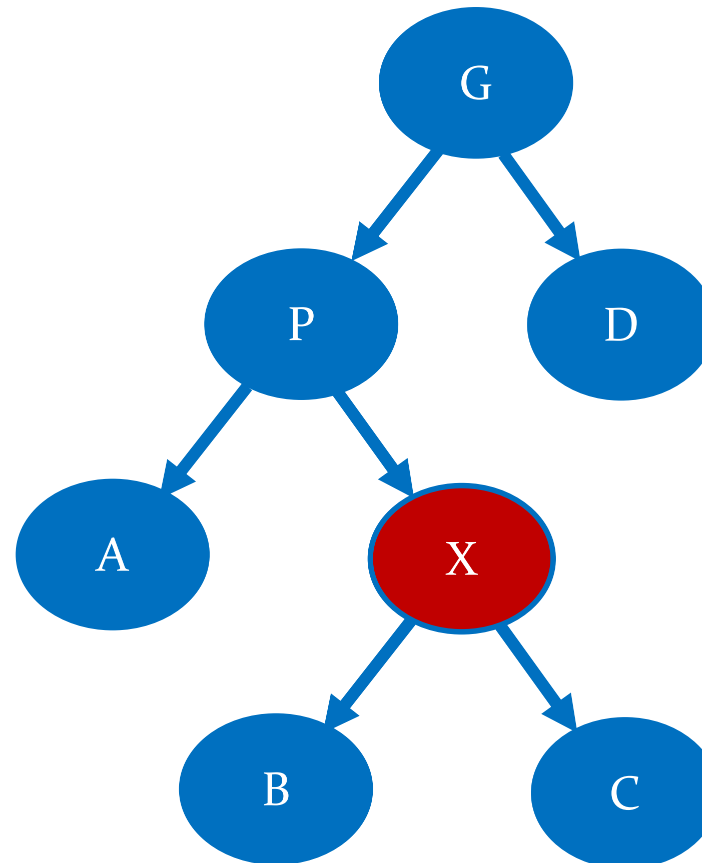


STUDENTS-HUB.com

Uploaded By: Jibreel Bornat

Zig-Zag case

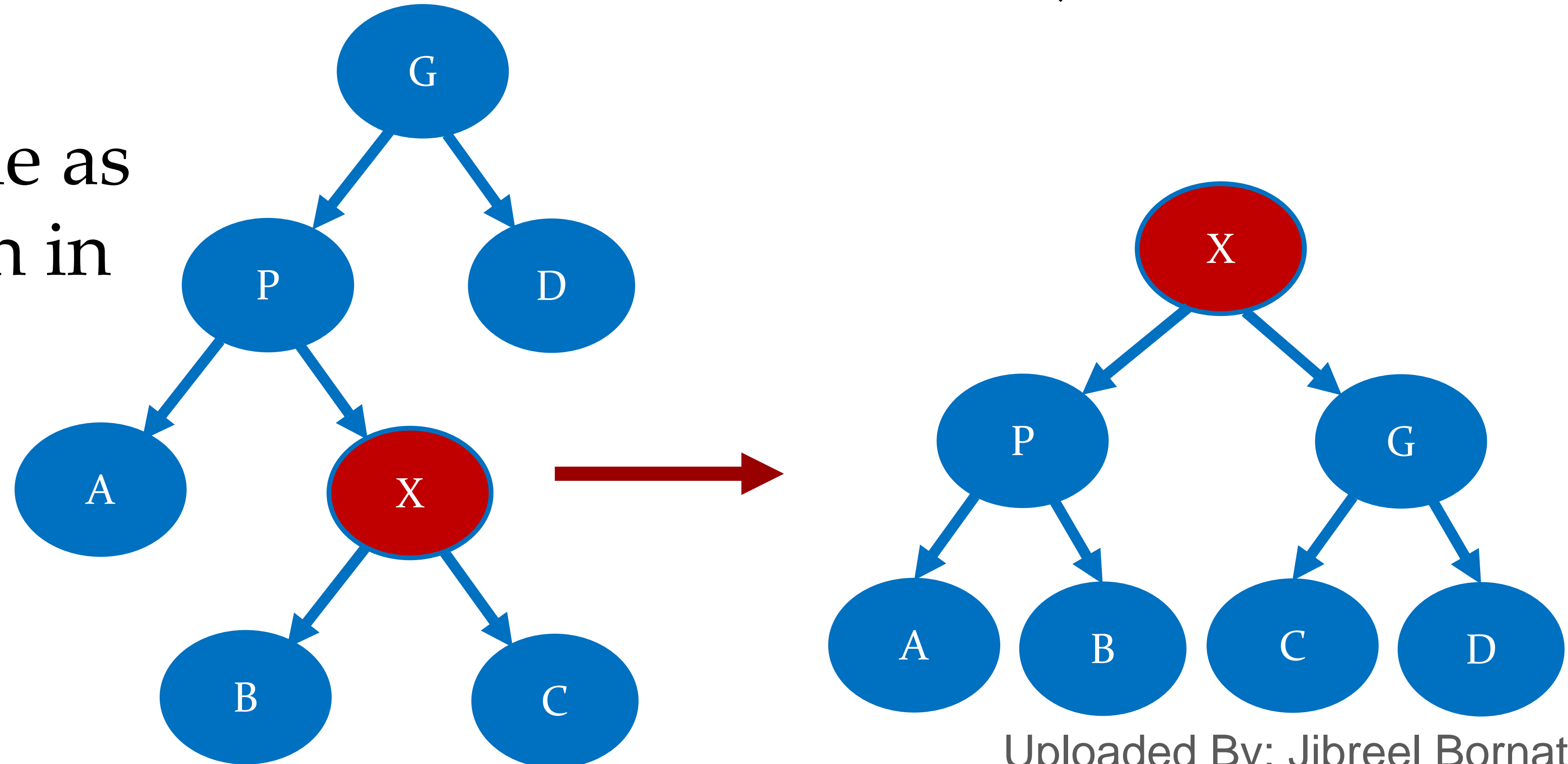
- In this case, X and both a parent P and a grandparent G, X is the right child of P and P is a left child of G (or vice versa).
- This is the same as double rotation in AVL tree



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Zig-Zag case

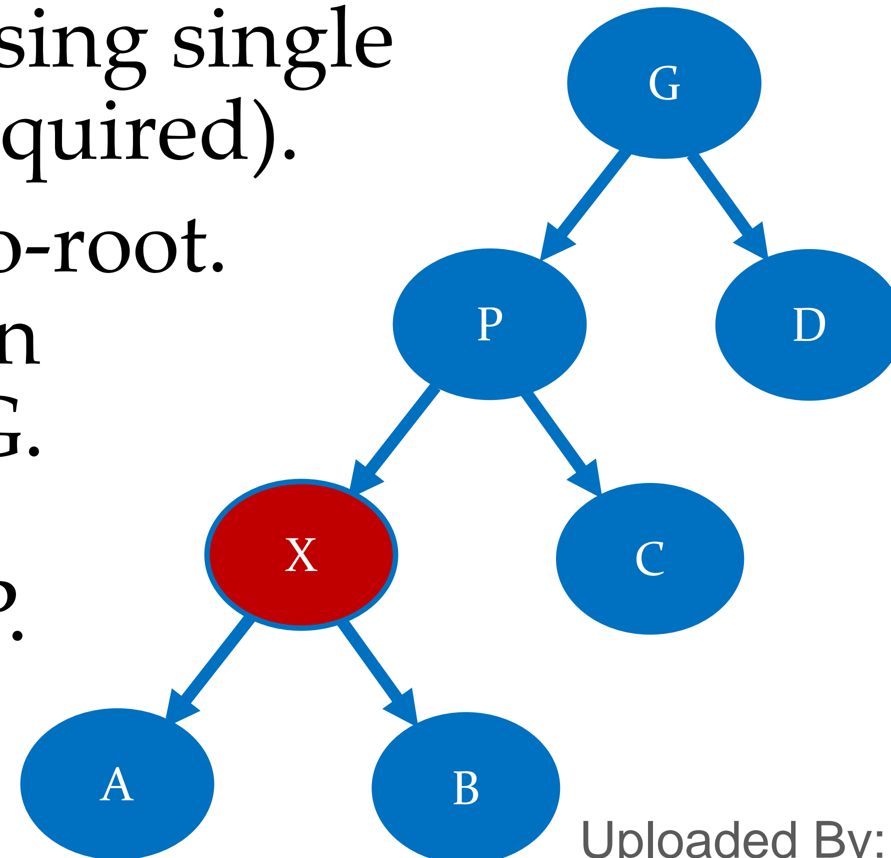
- In this case, X and both a parent P and a grandparent G, X is the right child of P and P is a left child of G (or vice versa).
- This is the same as double rotation in AVL tree



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Zig-Zig case

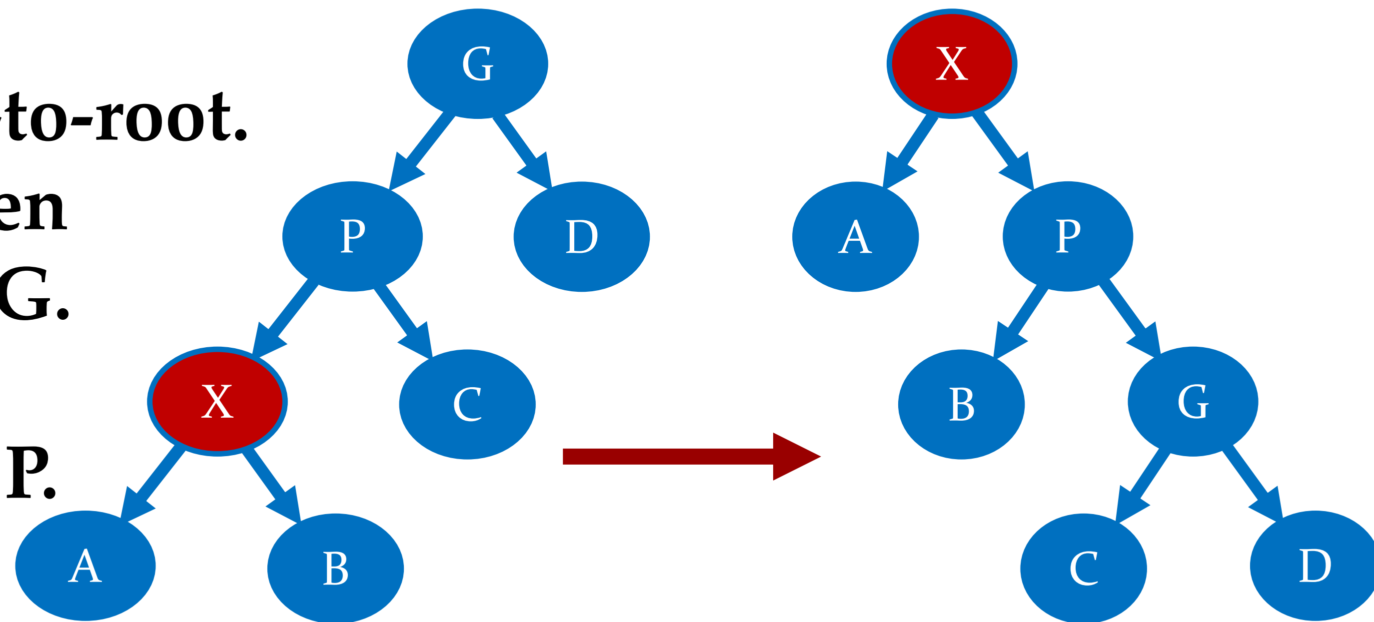
- Here X and P are either both left children or both right children.
- The transformation is done using single rotations (twice or more as required).
- This is different than rotate-to-root. Rotate-to-root rotates between X and P and between X and G. The zig-zig splay rotates between P and G and X and P.
- This case is not in AVL Trees.



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Zig-Zig case

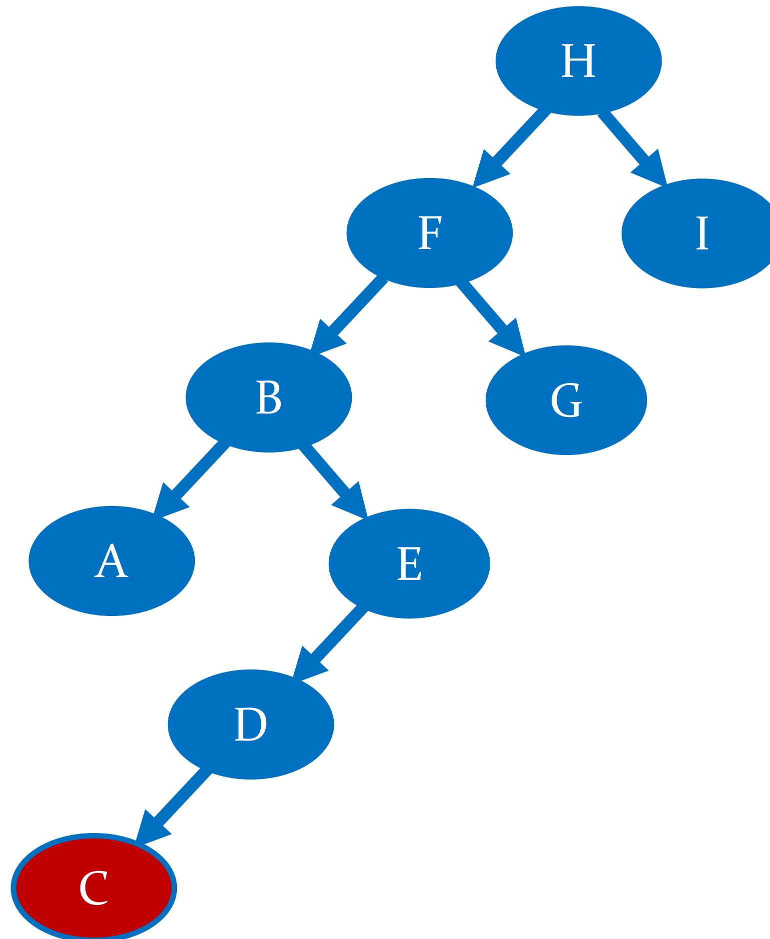
- Here X and P are either both left children or both right children.
- The transformation is done using single rotations (twice or more as required).
- **This is different than rotate-to-root. Rotate-to-root rotates between X and P and between X and G. The zig-zig splay rotates between P and G and X and P.**
- This case is not in AVL Trees.



Uploaded By: Jibreel Bornat

Example

- Splay the node C in the following tree.

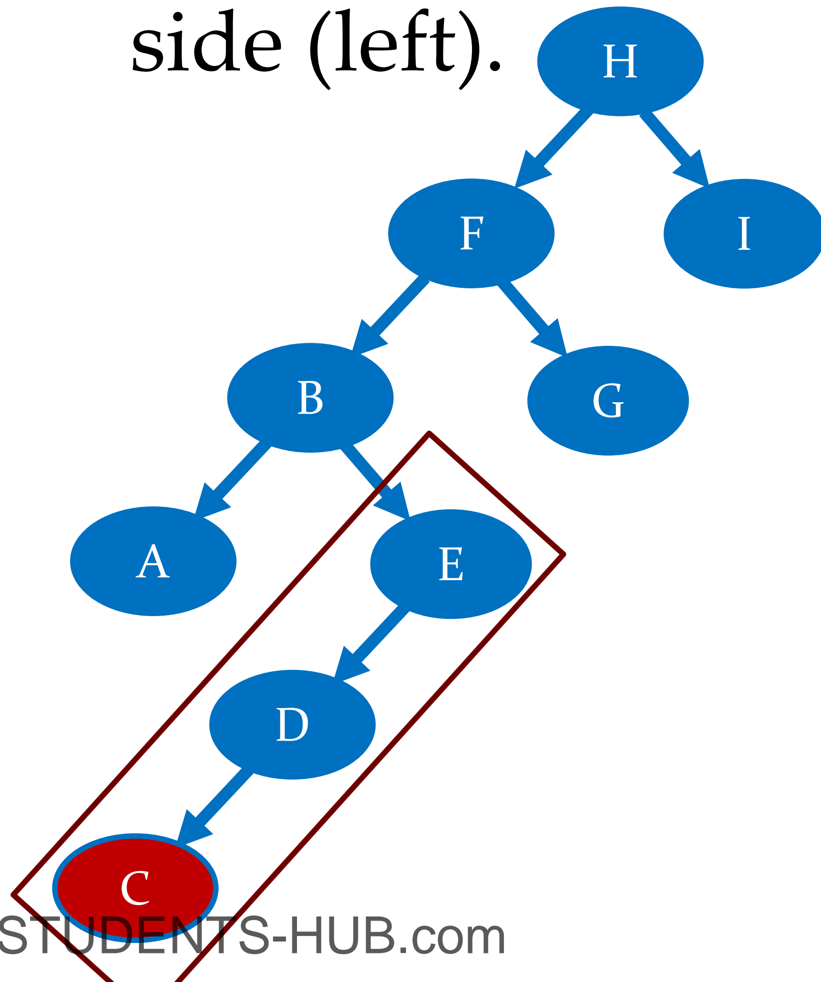


STUDENTS-HUB.com

Uploaded By: Jibreel Bornat

Example

- Step1: Zig-Zig because C and its ancestors are on the same side (left).

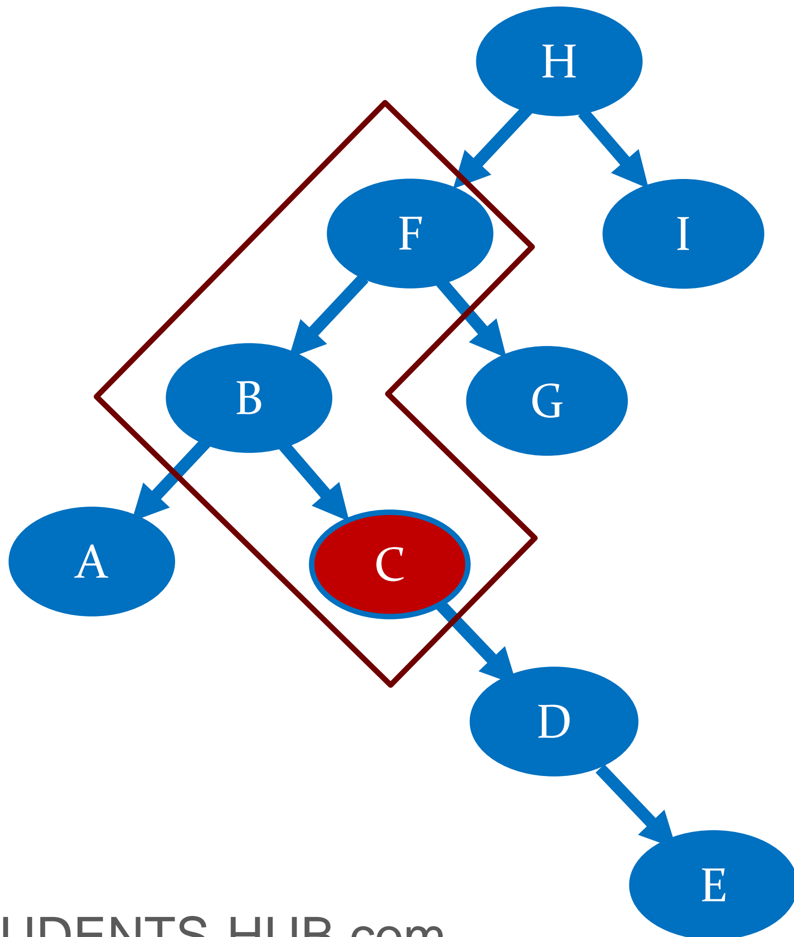


STUDENTS-HUB.com

Uploaded By: Jibreel Bornat

Example

- Step2: Zig-Zag because C is right to B and B is left to F.

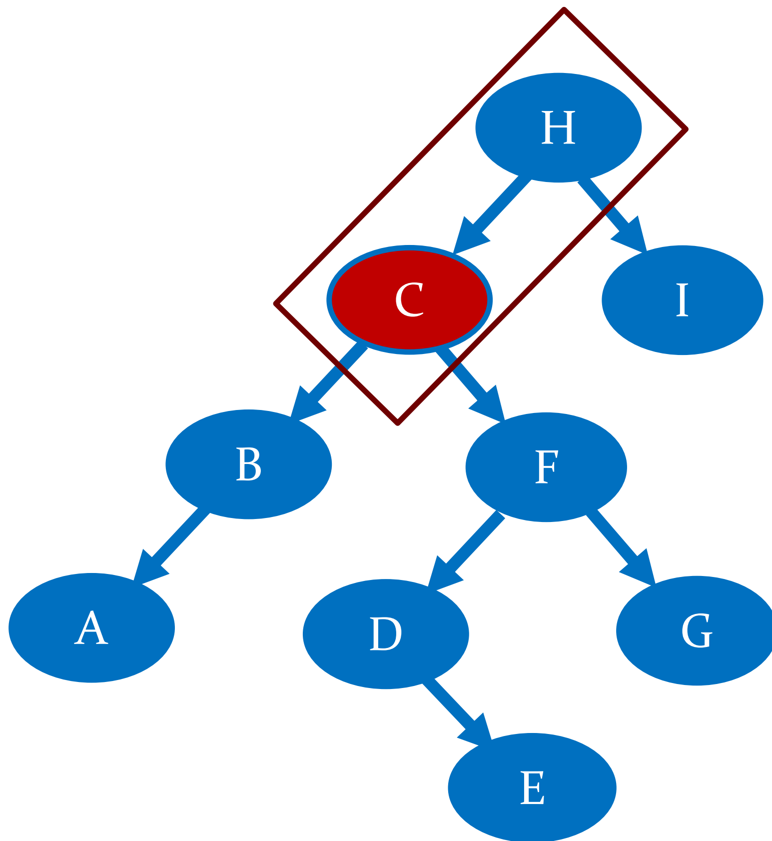


STUDENTS-HUB.com

Uploaded By: Jibreel Bornat

Example

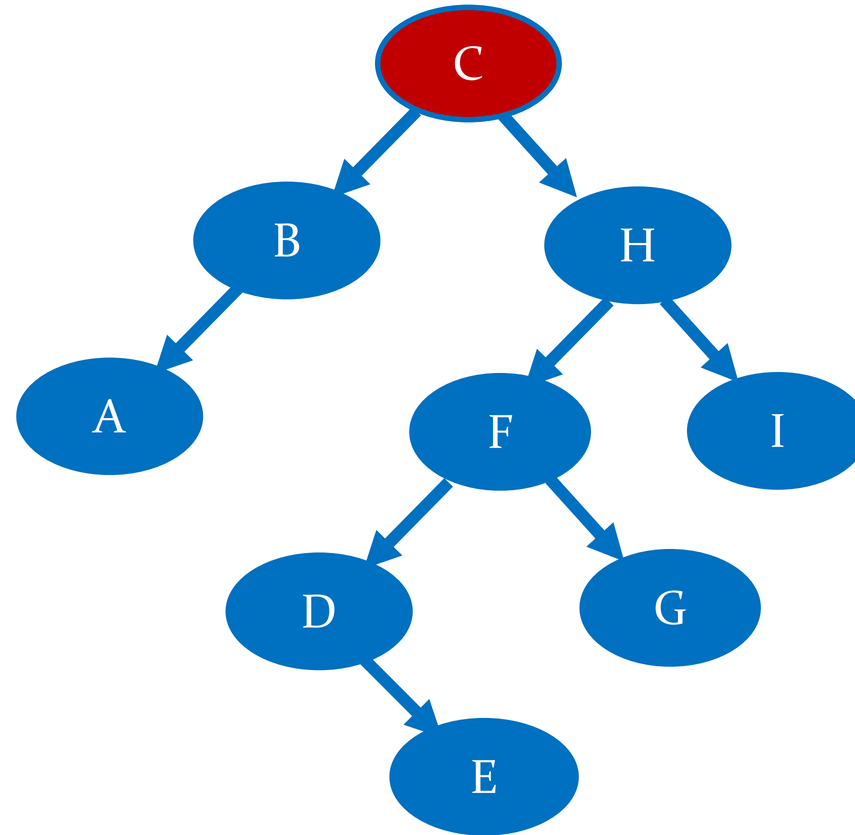
- Step3: Zig (single rotation) with the root H.



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Example

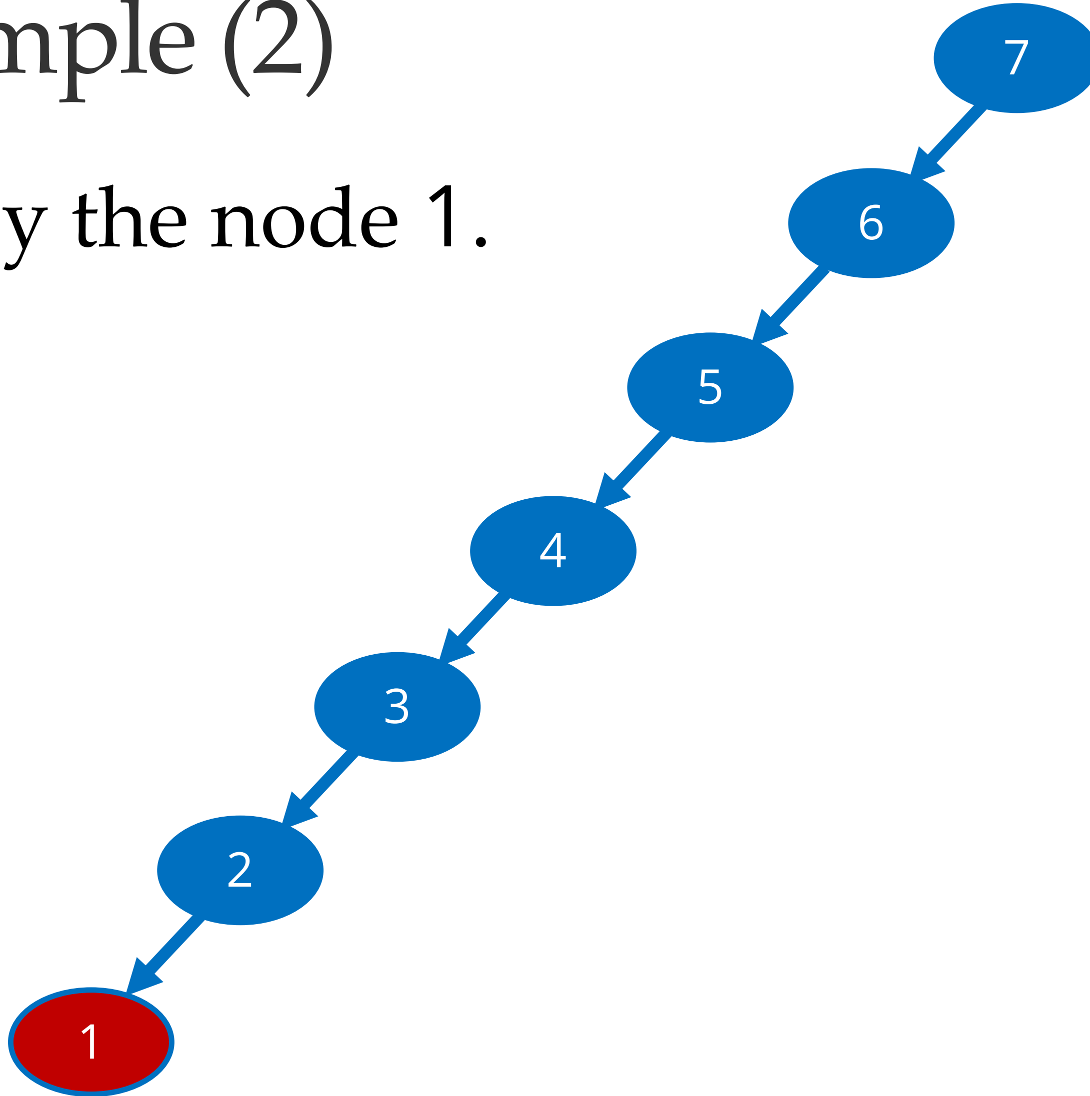
- Step3: Zig (single rotation) with the root H.



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Example (2)

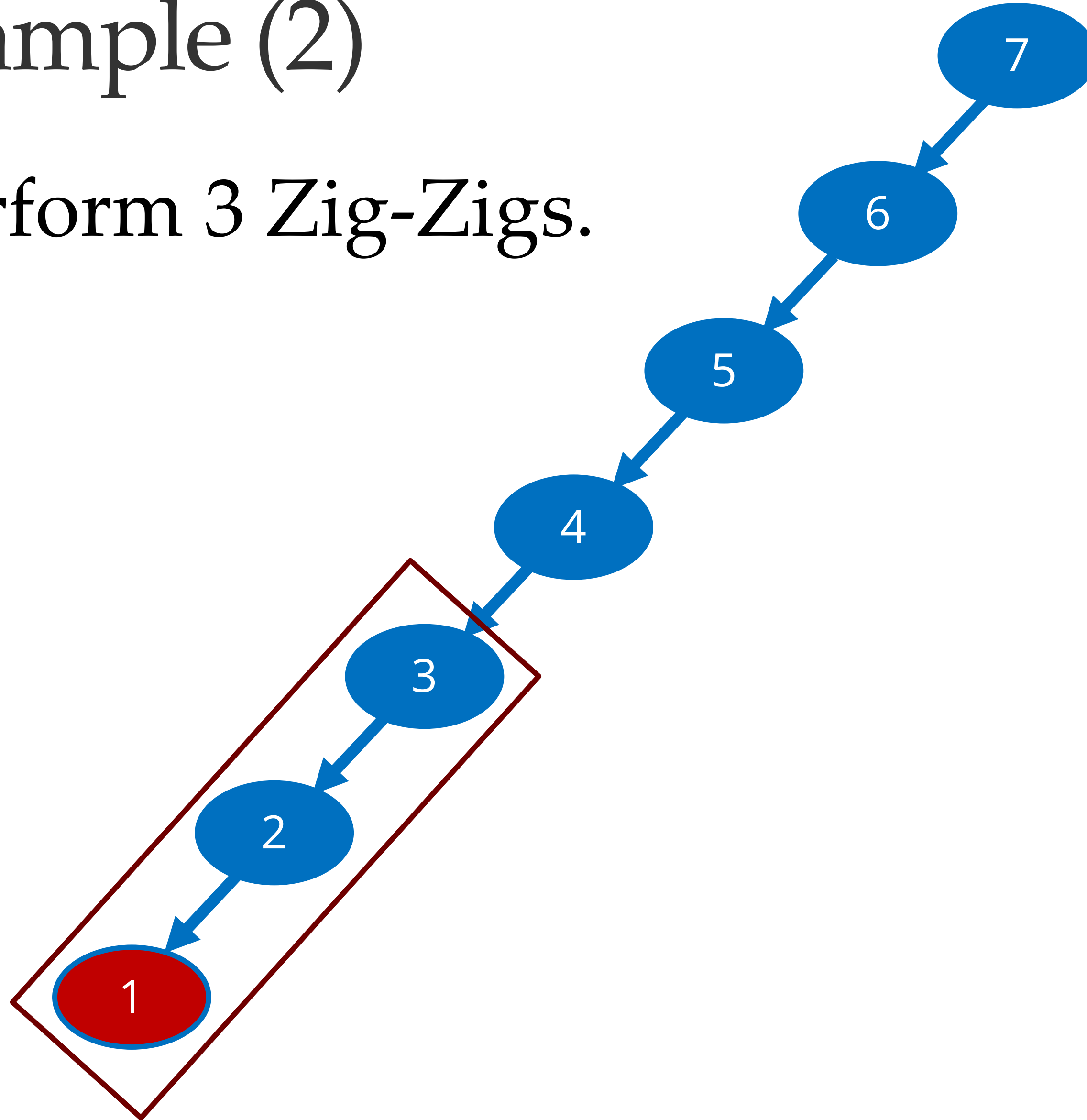
- Splay the node 1.



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Example (2)

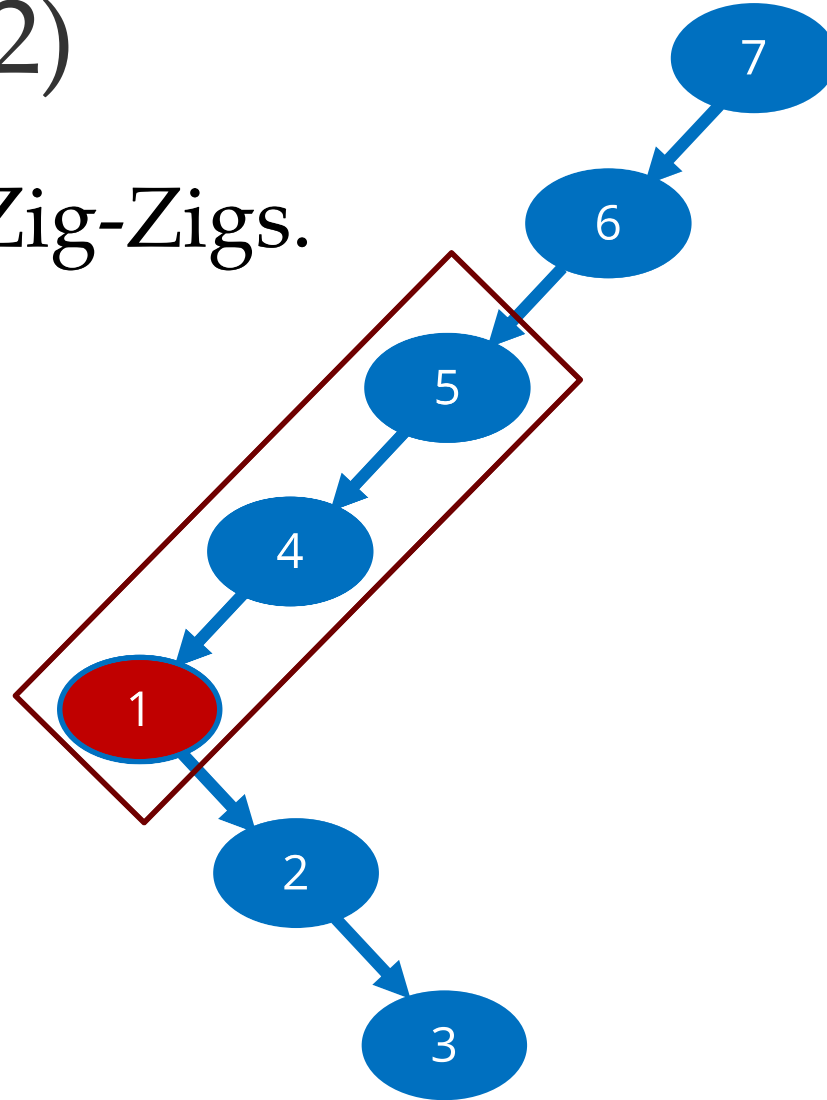
- Perform 3 Zig-Zigs.



Uploaded By: Jibreel Bornat

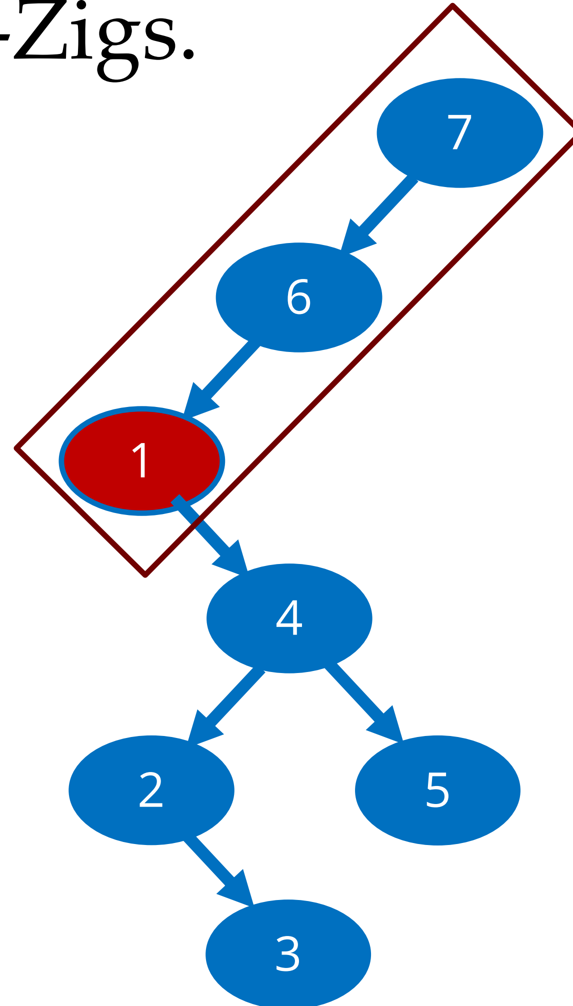
Example (2)

- Perform 3 Zig-Zigs.



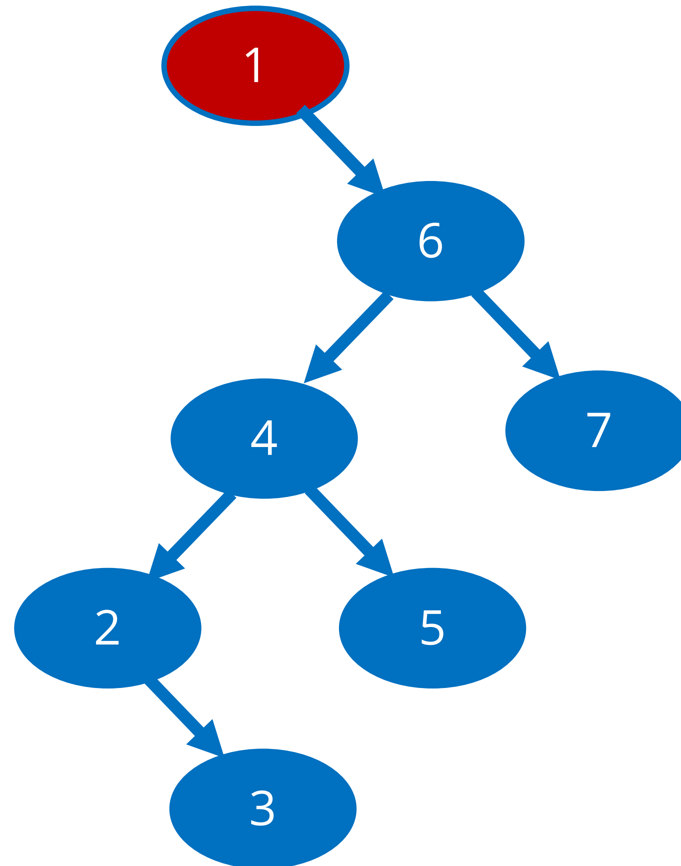
Example (2)

- Perform 3 Zig-Zigs.



Example (2)

- Perform 3 Zig-Zigs.



Uploaded By: Jibreel Bornat

Basic Operations on Splay Trees

- Insertion: when an element is inserted at the right position, a splay is performed. As a result, the newly inserted node becomes the root of the tree.
- Find: the last node accessed during the search is splayed:
  - If the search is successful, the node found becomes the root of the tree.
  - Unsuccessful search, the last node accessed prior to reaching NULL, is splayed to become the root of the tree.
- FindMin & FindMax: perform a splay after the access.

STUDENTS-HUB.com Uploaded By: Jibreel Bornat

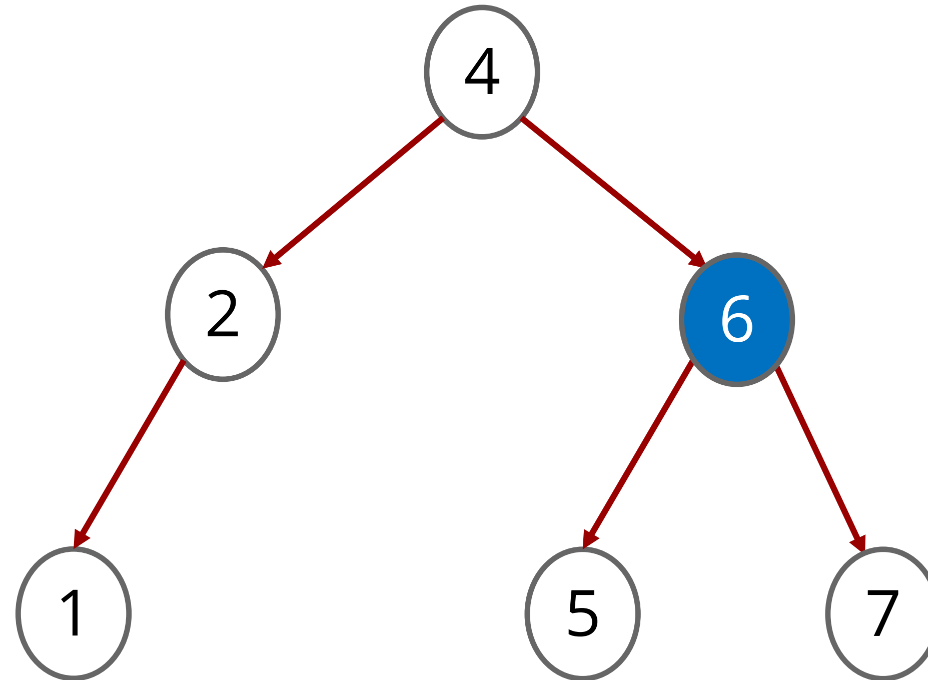
Basic Operations on Splay Trees (2)

- Delete operation:
  - Access the node to be deleted bringing it to the root.
  - Delete the root leaving two subtrees L (left) and R (right).
  - Find the largest element in L using FindMax operation, thus the root of L will have no right child.
  - Make R the right child of L's root.

STUDENTS-HUB.com Uploaded By: Jibreel Bornat

Basic Operations on Splay Trees (3)

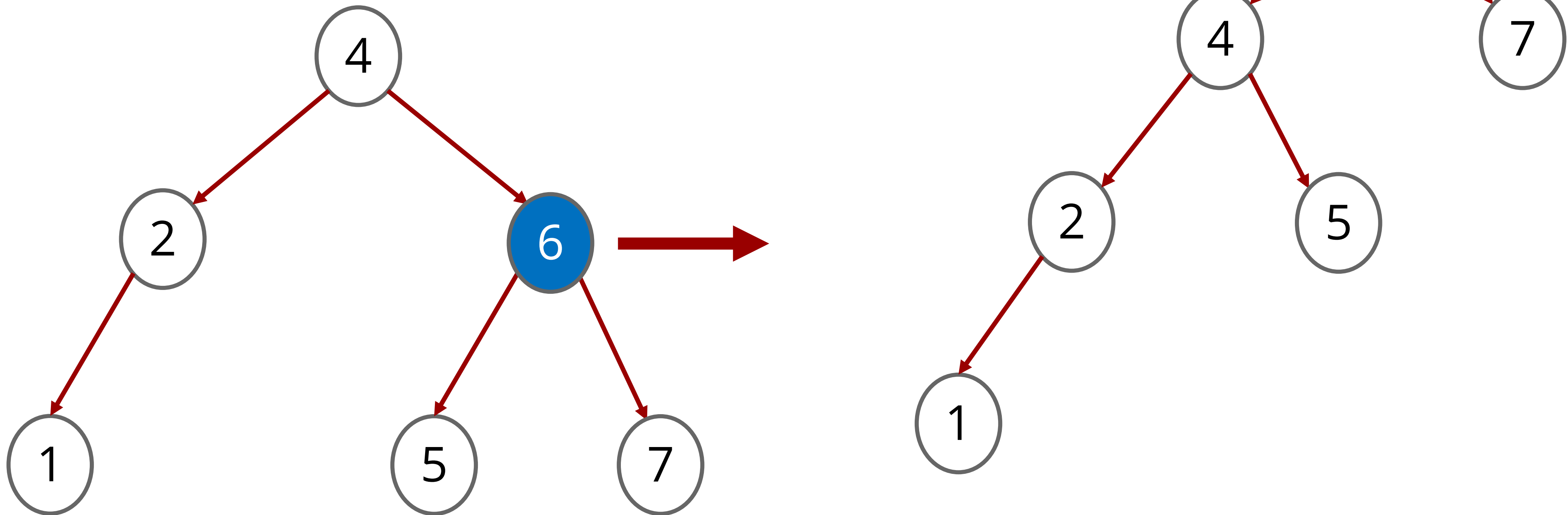
- Remove node 6.



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Basic Operations on Splay Trees (3)

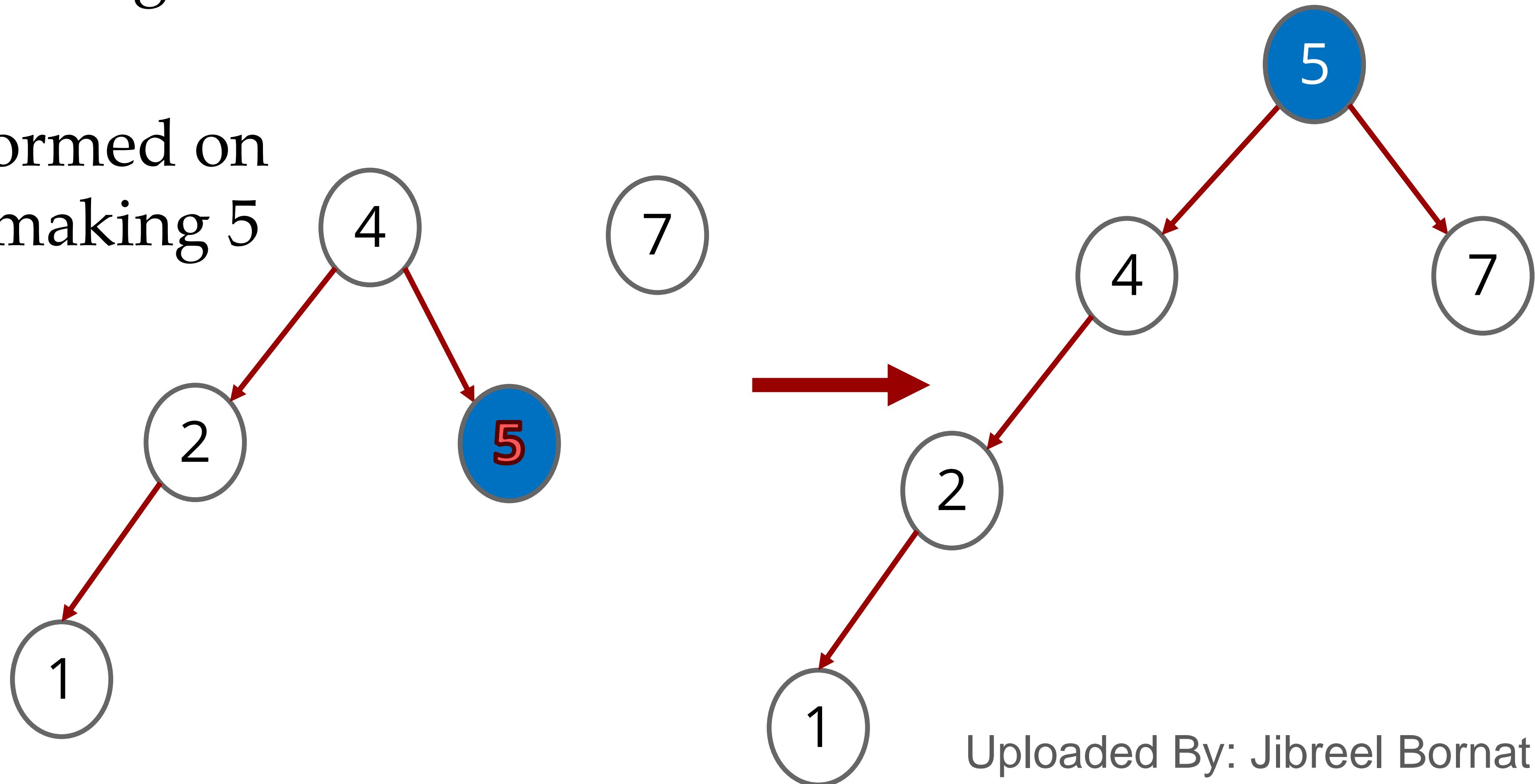
- Remove node 6.
  - Splay 6 to the root



 Uploaded By: Jibreel Bornat

Basic Operations on Splay Trees (3)

- Remove node 6.
  - Delete node 6, leaving two subtrees.
  - FindMax is performed on the left subtree, making 5 the new root.



STUDENTS-HUB.com
Uploaded By: Jibreel Bornat

Applications to Splay Tree

- Splay trees are the fastest balanced search trees.
- Network routers receiving packets & should decide where it should travel to. If an IP is used once, it is most likely to be used again. Thus, the look-up table is built using Splay tree.
- Implementation of cache, memory allocator, garbage collector, data compression, Intrusion Detection Systems.
- Unix *malloc*, GCC compiler, and sed string editor.

STUDENTS-HUB.com Uploaded By: Jibreel Bornat

Time Complexity

- Search: O(log n)
- Insert: O(log n)
- Delete: O(log n)

STUDENTS-HUB.com Uploaded By: Jibreel Bornat