

Exercises:

Q2: suppose we have a multinomial population with four categories: A, B, C and D.

The null hypothesis is that the proportion of items is the same in every category;

$$H_0: \pi_A = \pi_B = \pi_C = \pi_D = 0.25 \quad K=4$$

A sample of size 300 yielded the following results. $K-1=3$ d.f.

$$f_A = 85 \quad f_B = 95 \quad f_C = 50 \quad f_D = 70$$

use $\alpha = 0.05$ to determine whether H_0 should be rejected. What is p-value?

$$\Rightarrow e_A = n(\pi_A) = 300(0.25) = 75 = e_B = e_C = e_D \quad \geq 5 \forall c$$

$$\rightarrow \chi^2 = \frac{(f_A - e_A)^2}{e_A} + \frac{(f_B - e_B)^2}{e_B} + \frac{(f_C - e_C)^2}{e_C} + \frac{(f_D - e_D)^2}{e_D}$$

$$= \frac{(85 - 75)^2}{75} + \frac{(95 - 75)^2}{75} + \frac{(50 - 75)^2}{75} + \frac{(70 - 75)^2}{75}$$

$$= \frac{100 + 400 + 625 + 25}{75} = 15.33 \dots$$

By chi-square: $\chi^2_{0.05, 3} = 7.879$ \downarrow p-value

3	12.838
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χ^2

p-value $\leq \alpha$

p-value ≤ 0.05

so we reject H_0 ($\alpha = 0.05$)

\downarrow

p-value less than 0.05

$$\alpha = 0.10$$

Q3:

Category	π_{i0}	f_i	$e_i = n\pi_{i0}$	$f_i - e_i$
Full service	$\frac{1}{3}$	264	249.33	14.67
Discount	$\frac{1}{3}$	255	249.33	5.67
Both	$\frac{1}{3}$	229	249.33	-20.33

$n = 748$

$$\chi^2 = \frac{(f_1 - e_1)^2}{e_1} + \frac{(f_2 - e_2)^2}{e_2} + \frac{(f_3 - e_3)^2}{e_3}$$

$$= \frac{660.6667}{249.33}$$

$k=3 \rightarrow df=2$

$$\chi^2 = 2.65$$

df \ α	0.90	0.10
2	0.211	4.605

$p\text{-value} \leq \alpha$
 $p\text{-value} \leq 0.10$
 $p\text{-value} \leq \alpha$
 so we reject H_0 ($\alpha = 0.1$)

Q4: $N=400$

π_{i0}	f_i	$e_i = n\pi_{i0}$	$(f_i - e_i)^2$
0.03	24	12	144
0.28	124	112	144
0.45	172	180	64
0.24	80	96	256

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$$\chi^2 = \frac{144}{12} + \frac{144}{112} + \frac{64}{180} + \frac{256}{96} = 16.3$$

Q4: χ^2 table

χ^2	0.1005
3	12.838

Annotations: \downarrow p-value, \uparrow χ^2

$p\text{-value} < \underline{0.1005} < 0.1$
 $\rightarrow p\text{-value} \leq \alpha$
 so we reject H_0 ($\alpha = 0.1$)

Q5: $n=200$, $\alpha=0.05$, $df=3$

π_{i0}	f_i	$e_i = n\pi_{i0}$	$(f_i - e_i)^2$
$\frac{1}{4}$	60	50	100
$\frac{1}{4}$	45	50	25
$\frac{1}{4}$	59	50	81
$\frac{1}{4}$	36	50	196

$$\chi^2 = \frac{100 + 25 + 81 + 196}{4} = \frac{402}{4} = 100.5$$

\leftarrow $\frac{df}{50}$

$p\text{-value} \leq \alpha$ so we reject H_0 ($\alpha = 0.05$)

Q6: $n=210$, $\alpha=0.05$

π_{i0}	f_i	e_i	$(f_i - e_i)^2$
$\frac{1}{7}$	20	30	100
$\frac{1}{7}$	30	30	0
$\frac{1}{7}$	30	30	0
$\frac{1}{7}$	25	30	25
$\frac{1}{7}$	35	30	25
$\frac{1}{7}$	20	30	100
$\frac{1}{7}$	50	30	400

Q7:

	π_{i0}	f_i	$e_i = n\pi_{i0}$	$(f_i - e_i)^2$	π
$\alpha = 0.05$	0.28	105	140	1225	
$n = \sum f_i = 500$	0.46	235	230	25	
	0.12	55	60	25	
	0.10	90	50	1600	
	0.04	15	20	25	

$$\chi^2 = \frac{1225}{140} + \frac{25}{230} + \frac{25}{60} + \frac{1600}{50} + \frac{25}{20}$$

$$= 42.5$$