Exercises :

11. $\bar{X}_1 = 30$, $\bar{X}_2 = 45$, $\bar{X}_3 = 36$, $MSE = 5.5$, $\alpha = 0.05$.

a. use FLSD procedure to test wether there is a significant difference between the means of pop.1 and pop.2 , pop.1 and 3 , pop. 2 and 3 .

	Test 1	Test 2	Test 3
	$H_0: \mu_1 = \mu_2$	$H_0: \mu_1 = \mu_3$	$H_0: \mu_2 = \mu_3$
	$H_1: \mu_1 \neq \mu_2$	$H_1: \mu_1 \neq \mu_3$	$H_1: \mu_2 \neq \mu_3$
Test statistic:	$\lvert \bar{X}_1 - \bar{X}_2 \rvert = 15$	$\lvert \bar{X}_1 - \bar{X}_3 \rvert = 6$	$\lvert \bar{X}_2 - \bar{X}_3 \rvert = 9$
Find LSD^{ij}:	$LSD^{12} = 3.23$	$LSD^{13} = 3.23$	$LSD^{23} = 3.23$
Rejection :	reject H_0 ($\alpha = 0.05$)	reject H_0 ($\alpha = 0.05$)	reject H_0 ($\alpha = 0.05$)

$t_{\frac{\alpha}{2}} = t_{0.025}$ with $df(E)$

$t_{0.025}$ with $df = 12$

$$LSD^{ij} = t_{\frac{\alpha}{2}} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = 2.179 \sqrt{5.5\left(\frac{1}{5} + \frac{1}{5}\right)} = 3.23$$

b. use FLSD procedure to develop a 95% confidance interval estimate of the difference between the means of pop. 1 and 2 . $1-\alpha = 0.95 \rightarrow \alpha = 0.05$.

$$0.95\ CI = (\bar{X}_1 - \bar{X}_2) \mp LSD$$
$$= 15 \mp 3.23$$
$$= (11.77 \, , \, 18.23) .$$

12. use $\alpha = 0.05$

	sample 1	sample 2	sample 3
1	63	82	69
2	47	72	54
3	54	88	61
4	40	66	48
$\bar{x}_j$	51	77	58
s_j^2	96.67	97.34	81.99
MSE	(290.01)	292.02	245.97

$\to \bar{\bar{x}} = 62$

$K = 3$

$n_T - K =$

$n_T = 12$

a. use Analysis of variance to test for a significant difference among the means of the three populations. F.

	df	SS	MS	F
Treatments	2	1448	724	7.87
Error	$\boxed{9}$	828	$\boxed{92}$ MSE	–
Total	11	2276	–	–

$SSTR = \sum n_j (\bar{x}_j - \bar{\bar{x}})^2 = 484 + 900 + 64$

$SSE = \sum (n_j - 1) s_j^2 = 828$

By F-table $F_\alpha = F_{0.05} = 4.26 \to F > F_\alpha$ so we reject H_0 $(\alpha = 0.05)$

(reject means of 3 pop. are equal) $(\alpha = 0.05)$

b. use FLSD procedure to see what means are different.

Test 1

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$|\bar{x}_1 - \bar{x}_2| = 26$

$LSD^{12} = 15.34$

$|\bar{x}_1 - \bar{x}_2| > LSD$

So reject H_0 $(\alpha = 0.05)$

$\mu_1 \neq \mu_2$ $(\alpha = 0.05)$

significant difference

Test 2

$H_0: \mu_1 = \mu_3$

$H_1: \mu_1 \neq \mu_3$

$|\bar{x}_1 - \bar{x}_3| = 7$

$LSD^{13} = 15.34$

$LSD > |\bar{x}_1 - \bar{x}_3|$

don't reject H_0 $(\alpha = 0.05)$

$\mu_1 = \mu_3$ $(\alpha = 0.05)$

No significant difference

Test 3

$H_0: \mu_2 = \mu_3$

$H_1: \mu_2 \neq \mu_3$.

$|\bar{x}_2 - \bar{x}_3| = 19$

$LSD^{23} = 15.34$

$|\bar{x}_2 - \bar{x}_3| > LSD$

reject H_0 $(\alpha = 0.05)$

$\mu_2 \neq \mu_3$ $(\alpha = 0.05)$

significant difference

$LSD = t_{\frac{\alpha}{2}} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$ df = 9

$= 2.262 \sqrt{12\left(\frac{1}{4} + \frac{1}{4}\right)}$

$\to 15.34$

13. Refer to Q6. At $\alpha = 0.05$ use FLSD procedure to test 1 and 3.

$\bar{X}_1 = 23 \qquad \bar{X}_3 = 21$

$SSTR = 104 \rightarrow MSTR = 52 \rightarrow K-1 = 2$

$SSE = 7.5 \rightarrow MSE = 4.89 \rightarrow n_T - K = 9$

$F = 10.63$

→ Test of 1 and 3:

$H_0: \mu_1 = \mu_3$

$H_1: \mu_1 \neq \mu_3$

$|\bar{x}_1 - \bar{x}_3| = 2$

$LSD^{13} = 3.54$

$LSD > |\bar{x}_1 - \bar{x}_3|$

so dont reject H_0 $(\alpha = 0.05) \Rightarrow \mu_1 = \mu_3$ $(\alpha = 0.03)$.

$\rightarrow LSD^{13} = t_{\frac{\alpha}{2}} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)} = 2.262 \sqrt{4.89\left(\frac{1}{4} + \frac{1}{4}\right)} = 3.54$

df $= 9$

14: Refer to Q13, use FLSD procedure to develop a 95% CI estimate of the difference between the means of Pop 1 and 2. $1-\alpha = 0.95 \rightarrow \alpha = 0.05$

$\bar{X}_1 = 23 \quad , \quad \bar{X}_2 = 28$

$MSE = 4.89 \quad , \quad n_T - K = 9 \quad . \quad \rightarrow LSD = 3.54$.

$0.95\ CI = (\bar{x}_1 - \bar{x}_2) \mp LSD$

$= (23 - 28) \mp 3.54$

$= -5 \mp 3.54$

$= (-8.54, -1.46)$

15. Refer to Q8, use FLSD to determine where the difference occur. $\alpha = 0.05$.

$\bar{X}_1 = 5$ $\quad \bar{X}_2 = 4.5$ $\quad \bar{X}_3 = 6$

$MSE = 0.5$, $n_T - K = 15$ $\quad \frac{1}{n_i} = \frac{1}{n_j} = \frac{1}{6}$

$$LSD = t_{\frac{\alpha}{2}} \sqrt{MSE\left(\frac{1}{6} + \frac{1}{6}\right)} = 2.131 \sqrt{0.5\left(\frac{2}{6}\right)} = 0.87$$

Test 1	Test 2	Test 3
$H_0: \mu_1 = \mu_2$	$H_0: \mu_1 = \mu_3$	$H_0: \mu_2 = \mu_3$
$H_1: \mu_1 \neq \mu_2$	$H_1: \mu_1 \neq \mu_3$	$H_1: \mu_2 \neq \mu_3$
$\lvert \bar{X}_1 - \bar{X}_2 \rvert = 0.5$	$\lvert \bar{X}_1 - \bar{X}_3 \rvert = 1$	$\lvert \bar{X}_2 - \bar{X}_3 \rvert = 1.5$
$LSD^{12} = 0.87$	$LSD^{13} = 0.87$	$LSD^{23} = 0.87$
Don't reject H_0 ($\alpha = 0.05$)	reject H_0 ($\alpha = 0.05$)	reject H_0 ($\alpha = 0.05$)
$\mu_1 = \mu_2$ ($\alpha = 0.05$)	$\mu_1 \neq \mu_3$ ($\alpha = 0.05$)	$\mu_2 \neq \mu_3$ ($\alpha = 0.03$)
No difference	difference	difference

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16. $\bar{X}_1 = 7.1$ $\quad \bar{X}_2 = 9.1$ $\quad \bar{X}_3 = 9.9$ $\quad \bar{X}_4 = 11.4$ $\quad \bar{\bar{X}} = 9.38$

$S_1^2 = 1.21$ $\quad S_2^2 = 0.93$ $\quad S_3^2 = 0.7$ $\quad S_4^2 = 1.02$

	df	SS	MS	F
Treatments	3	57.77	19.26	19.86
Error	20	19.3	0.97	
Total	23			

$SSTR = \sum n_j(\bar{x}_j - \bar{\bar{x}})^2 = 31.1904 + 0.4704 + 1.6224 + 24.4824$

$SSE = (n_j - 1)S_j^2 = 6.05 + 4.65 + 3.5 + 5.1$

$=$

16. P-value approach

P-value < 0.01

P-value < α

so reject H_0 ($\alpha = 0.05$)

critical value approach.

$F_\alpha = 3.10$

$F > F_\alpha$

so reject H_0 ($\alpha = 0.05$)

b. use FLSD → Pop. 2 and 4, $\alpha = 0.05$.

$$LSD = t_{\frac{\alpha}{2}} \sqrt{0.97\left(\frac{1}{6} + \frac{1}{6}\right)} = 2.086 \sqrt{\quad} = 1.19$$

$H_0 : \mu_2 = \mu_4$

$H_1 : \mu_2 \neq \mu_4$

$|\bar{x}_2 - \bar{x}_4| = 2.3$

LSD = 1.19

reject H_0 ($\alpha = 0.05$)

$\mu_2 \neq \mu_4$ ($\alpha = 0.05$)

singificant difference.

17. Refer to Q16, use the Bonferroni Adjustment to test for a significance diffirance between all pairs of means. Assume that the maximum overall experementwise (α_{EW}) error rate of 0.05 is desired.

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$$\alpha_{CW} = \frac{\alpha_{EW}}{\binom{4}{2}} = \frac{0.05}{6} = 0.0083 \approx 0.01$$

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1 : Not all its equal.

P-value :

critical value

$F_\alpha = 4.94$