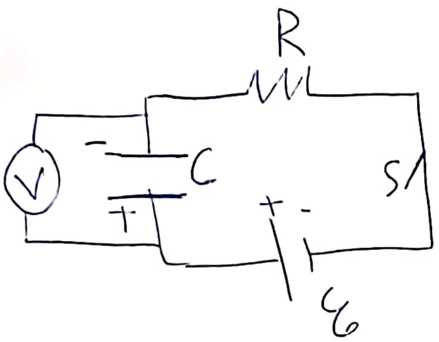
exp 9 : RC - Circuit

The aim :. is to find the time constant τ of an RC circuit and the value of its capacitor.

Theory



initially the capacitor is uncharged

$\therefore Q_0 = 0$ at $t = 0$

when the switch is closed charge will start accumulating on the capacitor.

$$V_c = \frac{Q}{C}$$

part A :

Charging

by Kirchhoff's second rule.

$$\sum V_j = 0$$

$$\therefore \varepsilon - IR - \frac{Q}{C} = 0$$

$$I = \frac{dQ}{dt}$$

$$\mathcal{E} - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} = \frac{\mathcal{E}C - Q}{RC}$$

$$\int_0^q \frac{dQ}{\mathcal{E}C - Q} = \int_0^t \frac{1}{RC}\, dt$$

let $u = \mathcal{E}C - Q$

$du = -dQ$

$$-\int \frac{du}{u} = \frac{1}{RC}\int_0^t dt$$

$$\ln\left[\frac{\mathcal{E}C - q}{\mathcal{E}C}\right] = e^{-t/RC}$$

$$\therefore \frac{q(t)}{C} = \frac{C\mathcal{E}}{\mathcal{E}}\left(1 - e^{-t/RC}\right)$$

Charging $\boxed{V(t) = \mathcal{E}\left(1 - e^{-t/\tau}\right)}$

$\boxed{\tau = RC}$

time constant.

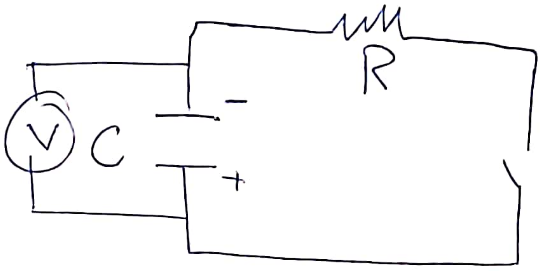
$V(\tau) = \mathcal{E}(1 - e^{-1}) = 0.63\mathcal{E}$

$\therefore$ time constant : is the time needed for the potential differences on the capacitor $V(t)$ to reach 0.63 of the max voltage $\mathcal{E}$

Part B : Discharging

the capacitor has an intial potential difference $\mathcal{E}$ and intial $Q = C\mathcal{E}$

By removing $\underset{\mathcal{E}}{|\,|}$ $\Rightarrow$



$$\sum V = 0$$

$$-IR - \frac{Q}{C} = 0 \Rightarrow I = \frac{dQ}{dt}$$

$$-\frac{dQ}{dt}R - \frac{Q}{C} = 0$$

divide by RQ

$$\int_{Q_0}^{Q(t)} \frac{dQ}{Q} = \int_0^t \frac{-dt}{RC}$$

$$Q(t) = Q_0 e^{-t/RC} = \mathcal{E}C e^{-t/RC}$$

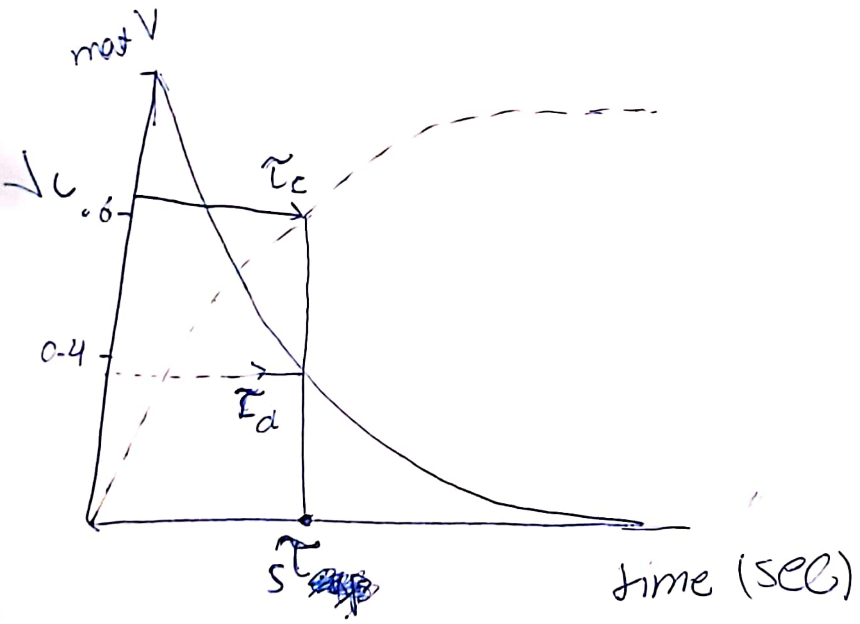
$$V(t) = \frac{Q(t)}{C} = \mathcal{E} e^{-t/RC}$$

time constant $t = \tau = RC$

$$V(\tau) = \mathcal{E} e^{-1}$$

$$\boxed{V(\tau) = 0.37\,\mathcal{E}}$$

Calculation



$$\tau_{exp} = \frac{\tau_d + \tau_c + \tau_s}{3}$$

$$\tau_{theo} = RC$$

semi-log graph papers

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time	$V_{charging}$	$V_{discharging}$
0		
5		
10		
15		
⋮		
60		
70		
80		
⋮ 120		
140		
⋮		

① $C = \dfrac{\bar{\tilde{\tau}}_{cap}}{R_{\text{color code}}}$

$$\frac{\Delta C}{C} = \frac{\overline{\Delta \tilde{\tau}}}{\bar{\tilde{\tau}}} + \frac{\Delta R}{R}$$

$\Delta \tilde{\tau} = \sigma_m(\tau)$ for the three values

② to find the slope of the line using this relation

$$\text{slope} = m = \frac{\ln(V_2) - \ln(V_1)}{t_2 - t_2}$$