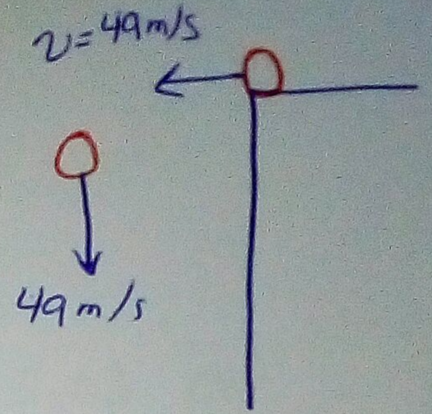
10-5 At time $t=0$, a rotating bicycle wheel is thrown horizontally from a rooftop with a speed of 49 m/s. By the time its vertical speed is also 49 m/s, it has completed 40 revolutions. What has been its average angular speed to that point in the fall?



$\Rightarrow v = v_0 + at$ "vertical motion"

$v_{0y} = 0$

$\Rightarrow 49 = gt$

$$\boxed{t = 5\ \text{sec}}$$

- In 5 seconds $\longrightarrow$ 40 revolutions

$\theta = 40 \times 2\pi = 80\pi$ rad , at $t_0 = 0$, $\theta_0 = 0$

$$\omega_{avg} = \frac{\theta - \theta_0}{t - t_0} = \frac{80\pi}{5} = 16\pi\ \frac{\text{rad}}{\text{sec}}$$

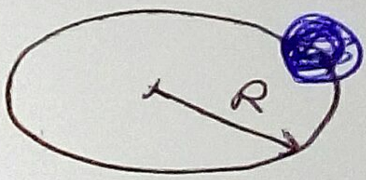
$$\boxed{\omega_{avg} = 50\ \frac{\text{rad}}{\text{sec}}}$$

10-9 | In 5.00 sec, a 2.00 kg stone moves in a horizontal circle of radius 2.00 m from rest to an angular speed of 4.00 rev/s. What are the stone's (a) average angular acceleration and (b) rotational inertia around the circle's center?

(a) • $\omega_0 = 0$

$$\omega = 4.00 \frac{rev}{sec} = 4.00 \times 2\pi \frac{rad}{sec}$$

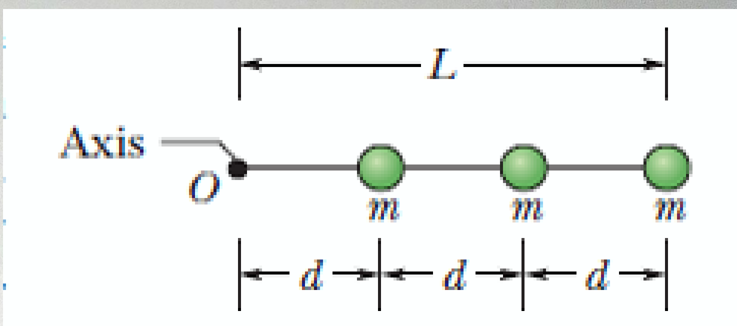
• $$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{4.00(2\pi) - 0}{5} = 5.03 \frac{rad}{sec^2}$$



(b) $$I = mr^2$$

$$= 2.0\,kg\,(2.0)^2 m^2 = 8\,kg \cdot m^2$$

10-38 The below figure shows three 0.010 kg particles that have been glued to a rod of length $L = 8.00$ cm and negligible mass. The assembly can rotate around a perpendicular axis through point O at the left end. If we remove one particle (that is, 33% of the mass), by what percentage does the rotational inertia of the assembly around the rotational axis decrease when that removed particle is (a) the inner most one (b) the outer most one?



$$\cdot\ I = \sum m_i r_i^2$$
$$= md^2 + m(2d)^2 + m(3d)^2$$
$$= md^2 + 4md^2 + 9md^2$$
$$\boxed{I = 14\,md^2}$$

a) $I' \Rightarrow$ Inner most removed

$$I' = m(2d)^2 + m(3d)^2$$
$$I' = 13\,md^2$$

$\Rightarrow$ percentage of decreasing $= \dfrac{|I' - I|}{I} \times 100\%$

$$= \frac{|13 - 14|}{14} = 7.1\%$$

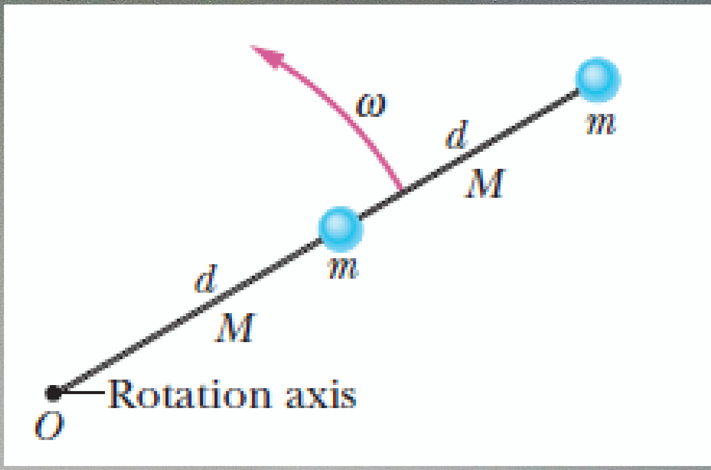
b) $I'' \Rightarrow$ outermost one is removed

$$I'' = md^2 + m(2d)^2$$
$$= 5\,md^2$$

$\Rightarrow$ percentage of decreasing $= \dfrac{|5md^2 - 14md^2|}{14\,md^2}$

$$= \frac{|5 - 14|}{14} = 64\%$$

10-41 Two particles, each with mass $m = 0.85$ kg, are fastened to each other, and to a rotation axis at O, by two thin rods, each with length $d = 5.6$ cm and mass $M = 1.2$ kg. The combination rotates around the rotation axis with the angular speed $\omega = 0.3$ rad/sec. Measured about O, what are the combination's (a) rotational inertia and (b) kinetic energy?



(a) ⇒ the first particle

$I_1 = md^2$

⇒ The second particle

$I_2 = m(2d)^2 = 4md^2$

⇒ The first rod [By parallel axis theorem]

$I_3 = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2$

$\boxed{I_3 = \frac{1}{3}Md^2}$

⇒ The second rod

$I_4 = \frac{1}{12}Md^2 + M\left(\frac{3d}{2}\right)^2$

$\boxed{I_4 = \frac{7}{3}Md^2}$

$I = I_1 + I_2 + I_3 + I_4$

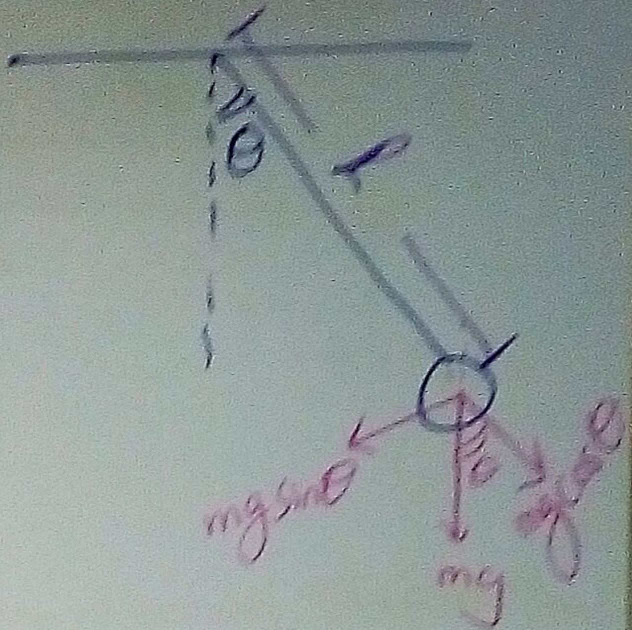
$= 5md^2 + \frac{8}{3}Md^2$

$= 5(0.85 \text{ Kg})(0.056 \text{ m})^2 + \frac{8}{3}(1.2 \text{ kg})(0.056 \text{ m})^2$

$\boxed{I = 0.023 \text{ kg}\cdot\text{m}^2}$

(b) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.023 \text{ kg}\cdot\text{m}^2)\left(0.3 \frac{\text{rad}}{\text{sec}}\right)^2 = 1.04\times10^{-3}$ J

10-47 A small ball of mass 0.75 kg is attached to one end of a 1.25 m Long massless rod, and the other end of the rod is hung from a pivot. When the resulting pendulum is 30° from the vertical, what is the magnitude of the gravitational torque calculated about the pivot?



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\theta$$

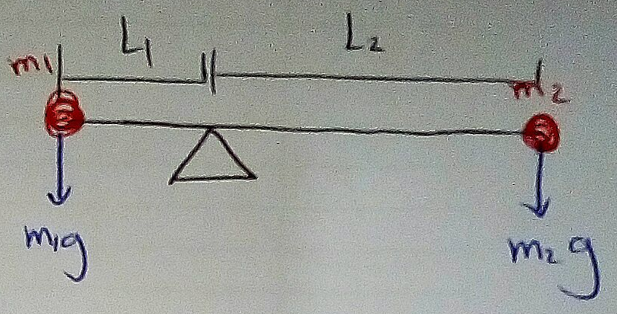
① $$\tau = L\,mg\sin\theta$$

$$\tau = mgL\sin\theta$$

$$= (0.75\,kg)(98\,\tfrac{m}{s^2})(1.25\,m)\sin 30^\circ$$

$$\boxed{\tau = 4.6\,N.m}$$

10 – 56 The below figure shows particles 1 and 2, each of mass m, fixed to the ends of a rigid massless rod of length $L_1 + L_2$, with $L_1 = 20$ cm and $L_2 = 80$ cm. The rod is held horizontally on the fulcrum and then released. What are the magnitudes of the initial accelerations of (a) particle 1 and (b) particle 2 ?



- $\tau = -m_2 g L_2 + m_1 g L_1$

$$= -mg(0.8) + mg(0.2)$$

$$= (-0.6\,mg)\ \text{N.m}$$ ⇒ The system returning clock wise

- $\tau = I\alpha$

$$I = \sum_{i=1}^{2} m_i r_i^2 = m L_1^2 + m L_2^2$$

$$= m(0.8)^2 + m(0.2)^2$$

$$\boxed{I = 0.68\,m\ \text{kg.m}^2}$$

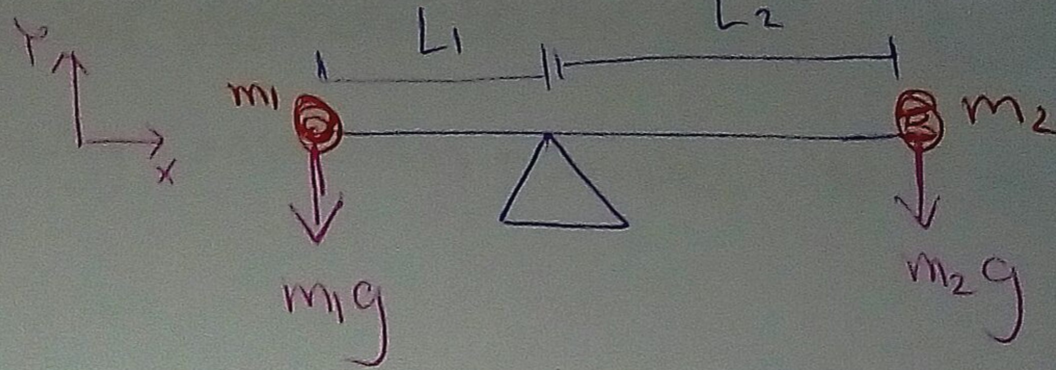
- $\tau = I\alpha$

$$\alpha = \frac{-0.6\,mg}{0.68\,m} = \frac{-0.6\,g}{0.68} = -8.65\ \frac{\text{rad}}{\text{sec}^2}$$

- $a_1 = \alpha L_1 = 8.65 \times 0.2 = 1.73\ \text{m/s}^2$
$a_2 = \alpha L_2 = 8.65 \times 0.8 = 6.92\ \text{m/s}^2$ } tangential acceleration

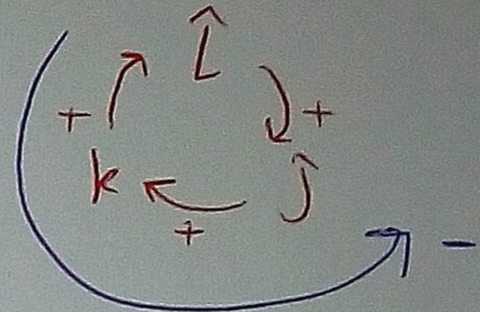
Initial acceleration ⇒ $v = 0$ at $t = 0$, $a_r = $ zero

- $\vec{\tau}_{m_1} = \vec{r} \times \vec{F}$



$$\vec{\tau}_{m_1} = -L_1(\hat{i}) \times m_1 g(-\hat{j})$$

$$\boxed{\vec{\tau}_{m_1} = L_1 m_1 g \, \hat{k}}$$



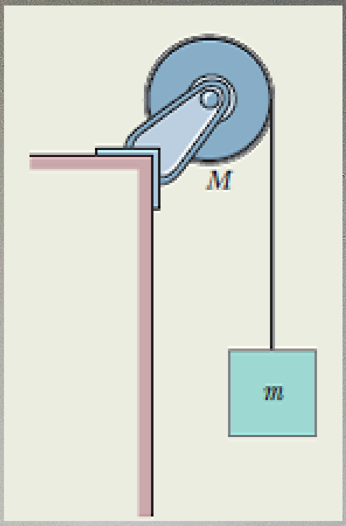
$$\vec{\tau}_{m_2} = L_2(\hat{i}) \times m_2 g(-\hat{j})$$

$$\boxed{\vec{\tau}_{m_2} = L_2 \, m_2 g \, (-\hat{k})}$$

$m_2 g \Rightarrow$ Clock wise rotation (negative)

$m_1 g \Rightarrow$ Counter-clock wise rotation (positive)

10-58 (a) If $R = 12$ cm, $M = 400$ g and $m = 50$ g in the below figure Find the speed of the block after it has descended 50 cm starting from rest. Solve the problem using energy conservation principles. (b) Repeat (a) with $R = 5.0$ cm ?



By conservation of mechanical Energy

$$\Delta E_{mec} = \Delta U + \Delta K = \text{Zero}$$

$$\Delta U = -\Delta K$$

$$-mg(50 \times 10^{-2}\,m) = -\left[\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2\right] \quad \text{—— (1)}$$

* Disk Rotational Inertia $I_{Disk} = \frac{1}{2} M R^2$

Angular speed $\omega = \frac{v}{R}$

$\Rightarrow$ Rotational Kinetic energy for the Disk $\frac{1}{2} I \omega^2$

$$= \frac{1}{2}\left[\frac{1}{2} M R^2\right]\left[\frac{v^2}{R^2}\right] = \frac{1}{4} M v^2$$

equation (1) $\Rightarrow$

$$mg(0.5\,m) = \frac{1}{2} m v^2 + \frac{1}{4} M v^2$$

$$mg(0.5\,m) = \frac{1}{4}[2m + M] v^2$$

$$v = \sqrt{\frac{4\,mg\,(0.5\,m)}{2m + M}} = \sqrt{\frac{4(0.05\,kg)(9.8\,\frac{m}{s^2})(0.5\,m)}{2(0.05\,kg) + (0.4\,kg)}} = 1.4\,\frac{m}{s}$$

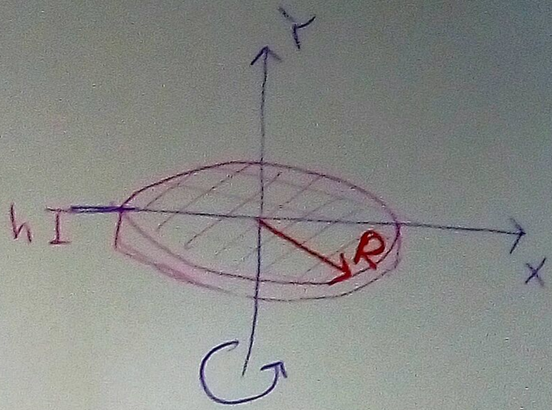
(b) $R = 5.0$ cm , $v = 1.4$ m/s ; the same

The Rotational kinetic energy for the disk is independent of its radius

The Disk moment of inertia $\Rrightarrow$

$$I = \int r^2 \, dm$$ "Continuous mass distribution"

use $\rho = \frac{\text{mass}}{\text{Volume}} = \frac{M}{V}$



$$M = \rho V$$

$$dm = \rho \, dV \Rrightarrow$$

$$V_{Disk} = \pi r^2 h$$

$$dV = 2\pi r h$$

$$I = \rho \int_0^R r^2 \, dV$$

$$= \frac{M}{V} \int_0^R 2\pi r^3 h \, dr$$

$$= \frac{M(2\pi h)}{\pi R^2 h} \int_0^R r^3 \, dr = \frac{2M}{R^2} \left[\frac{r^4}{4}\right]_0^R$$

$$I = \frac{2M}{R^2} \frac{R^4}{4} = \frac{1}{2} M R^2$$

$$\boxed{I_{Disk} = \frac{1}{2} M R^2}$$

