Math1411 Volumes (6.1 + 6.2)

Q1) Consider the region in the first quadrant enclosed between $y = x^3 + 3$, $y = 4$, and y-axis. Find the volume of the solid of revolution in the cases below. **(Do not evaluate the integrals)**

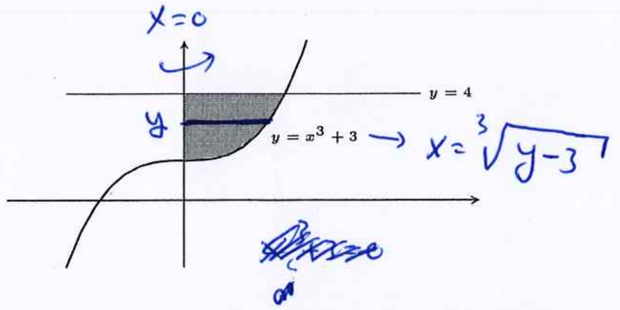
(a) The axis of revolution is the y-axis. Use the disk method.

$V = \int_c^d \pi R^2(y)\, dy$

$c = 3$

$d = 4$

$R(y) = \sqrt[3]{y-3}$

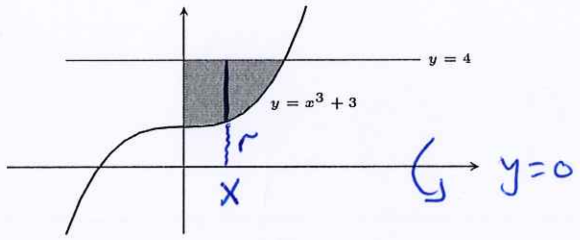


(b) The axis of revolution is the x-axis. Use the washer method.

$V = \pi \int_a^b R^2(x) - r^2(x)\, dx$

$a = 0$ $\quad r = x^3 + 3$

$b = 1$ $\quad R = 4$

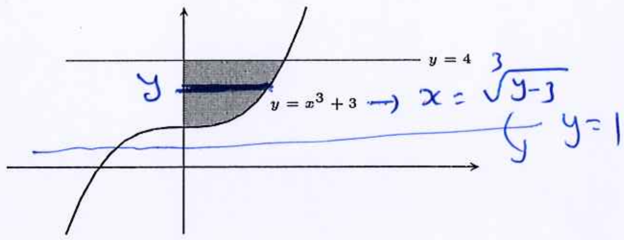


(c) The axis of revolution is the line $y = 1$. Use the shell method.

$V = 2\pi \int_c^d r\, h\, dy$

$c = 3$ $\quad r = y - 1$

$d = 4$ $\quad h = \sqrt[3]{y-3}$



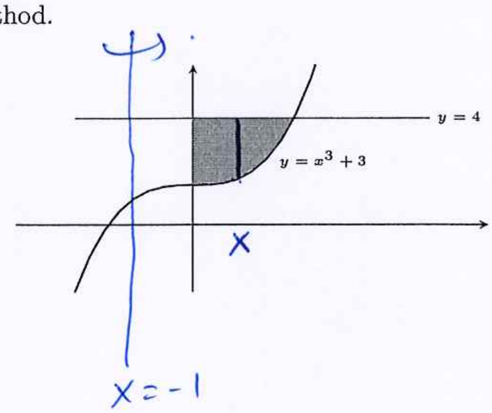
(d) The axis of revolution is the line $x = -1$. Use the shell method.

$V = 2\pi \int_a^b r\, h\, dx$

$a = 0$ $\quad r = x + 1$

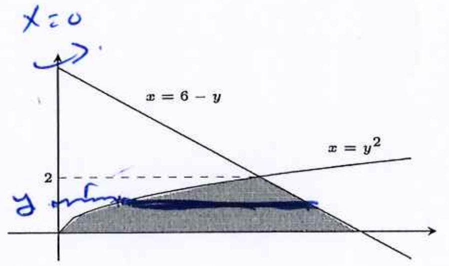
$b = 1$ $\quad h = 4 - (x^3 + 3)$

$= 1 - x^3$



(**Q2**) Consider the region in the first quadrant enclosed between $x = y^2$, $x = 6 - y$, and x-axis.
Find the volume of the solid of revolution in the cases below. (**Do not evaluate the integrals**)

(a) The axis of revolution is the y-axis. Use the washer method.

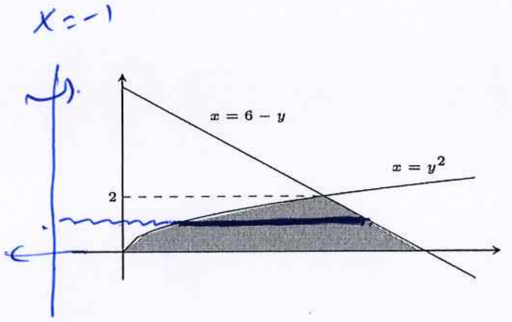


$V = \pi \int_c^d R^2(y) - r^2(y)\, dy$

$c = 0$ $\quad r = y^2$

$d = 2$ $\quad R = 6 - y$

(b) The axis of revolution is the line $x = -1$. Use the washer method.

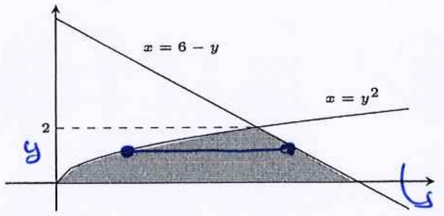


$V = \pi \int_c^d R^2(y) - r^2(y)\, dy$

$c = 0$ $\quad r = y^2 + 1$

$d = 2$ $\quad R = 6 - y + 1 = 7 - y$

(c) The axis of revolution is the x-axis. Use the shell method.

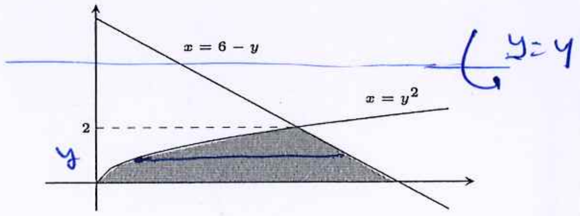


$V = 2\pi \int_c^d r\, h\, dy$

$c = 0$ $\quad r = y$

$d = 2$ $\quad h = 6 - y - y^2$

(d) The axis of revolution is the line $y = 4$. Use the shell method.

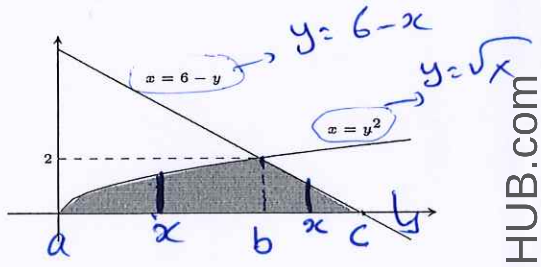


$V = 2\pi \int_c^d r\, h\, dy$

$c = 0$ $\quad r = 4 - y$

$d = 2$ $\quad h = 6 - y - y^2$

(e) The axis of revolution is the x-axis. Use the disk method.



$V = V_1 + V_2$

$V_1 = \int_a^b \pi R^2(x)\, dx$ $\quad\quad V_2 = \int_b^c \pi R^2(x)\, dx$

$a = 0$ $\quad\quad b = 4$

$b = 4$ $\quad\quad c = 6$

$R = \sqrt{x}$ $\quad\quad R = 6 - x$

Q3) **Find the volume of the solid of rotation for each case below.**
(Do not Evaluate the integral)

(a) Rotation about x-axis. Use the disk method.

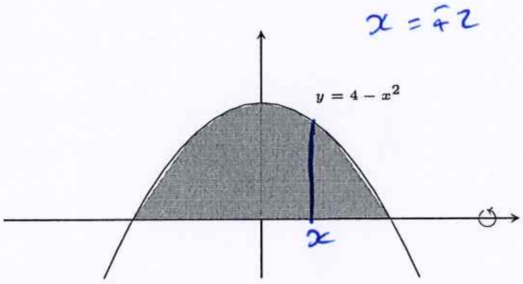
$4 - x^2 = 0$

$x = \mp 2$

$V = \int_a^b \pi R^2(x)\, dx$

$a = -2$ $\quad R = 4 - x^2$

$b = 2$



(b) Rotation about x-axis. Use the washer method.

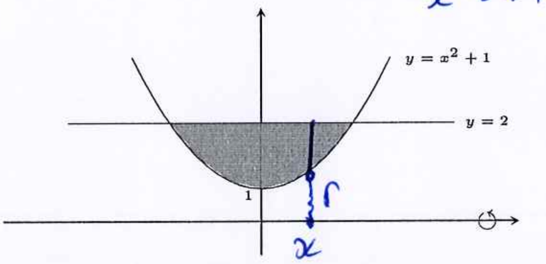
$x^2 + 1 = 2$

$x = \mp 1$

$V = \pi \int_a^b R^2(x) - r^2(x)\, dx$

$a = -1$ $\quad r = x^2 + 1$

$b = 1$ $\quad R = 2$

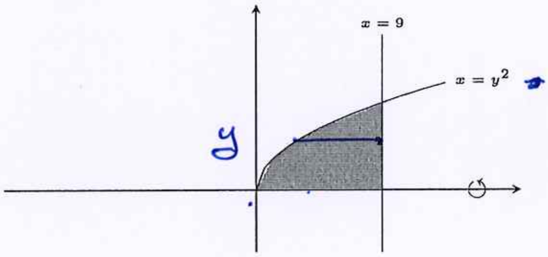


(c) Rotation about x-axis. Use the shell method.

$V = 2\pi \int_c^d r\, h\, dy$

$c = 0$ $\quad r = y$

$d = 3$ $\quad h = 9 - y^2$



(d) Rotation about $y = 2$. Use the shell method.

$V = 2\pi \int_c^d r\, h\, dy$

$c = 0$ $\quad r = 2 - y$

$d = 2$ $\quad h = 6 - y - y^2$