Exp (1) $\{1, 2, 3, 4, 5, \dots, n, \dots\}$ non decreasing monotonic $a_n = n$, $n = 1, 2, 3, \dots$

M ?? a_n is not bounded from above $\lim_{n\to\infty} a_n = \lim_{n\to\infty} n = \infty$

$m = 1, 0, -1, -2, -2.5, \dots$ lower boundes

1 → greatest lower bound

a_n is only bounded from below a_n Diverges

(2) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$ monotonic $b_n = \left(\frac{1}{2}\right)^n$, $n = 0, 1, 2, 3, \dots$ non increasing

$M = 1, 2, 3, 4.5, e, 100, \dots$ upper bounde $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$

1 → least upper bound

$m = 0, -1, -2, -3, \dots$ lower bounds b_n $\boxed{\text{converges to } 0}$

0 → greatest lower bound

b_n is bounded from below and bounded from above ⟹ b_n is bounded

(3) $3, 3, 3, 3, 3, \dots$ monotonic $C_n = 3$, $n = 1, 2, 3, \dots$ non increasing and non decreasing

$M = 3, 4, 5, 3.1, 9, 17, \dots$ upper boundes $\lim_{n\to\infty} C_n = \lim_{n\to\infty} 3 = 3$

3 → least upper bound

$m = 3, 2, 1, 0, -1, \dots$ lower boundes $\boxed{C_n \text{ converges to } 3}$

3 → greatest lower bound

C_n is bounded

Def • A sequence $\{a_n\}$ is bounded from above if $\exists$ a number M s.t

$a_n \leq M$ for all n (M ↓ upper bound)

• A sequence $\{a_n\}$ is bounded from below if $\exists$ a number m s.t

$a_n \geq m$ for all n (m ↓ lower bound)

• A sequence $\{a_n\}$ is bounded if it is bounded from above and is = = below

• A sequence $\{a_n\}$ is not bounded if it is not bounded from above and = = = below

Exp $\dots -2, -1, 0, 1, 2, 3, \dots$ not bounded sequence

Exp $\dots, -2, -1, 0, 1, 2, 3, \dots$ not bounded sequence

Def • A sequence $\{a_n\}$ is nondecreasing if $a_n \le a_{n+1}$ $\forall n$

$a_1 \le a_2 \le a_3 \le \dots$

• " " " " nonincreasing " $a_n \ge a_{n+1}$ $\forall n$

$a_1 \ge a_2 \ge a_3 \ge \dots$

• " " " " monotonic if it is either nondecreasing or non increasing

Exp $1, -1, 1, -1, 1, -1, \dots$ is not monotonic

Th If a sequence $\{a_n\}$ is both

1) bounded and
2) monotonic

Then $\{a_n\}$ converges.

Exp $a_n = \frac{1}{n}$, $n = 1, 2, 3, \dots$

① Find m, M

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

✓ $m = 0, -1, -2, \dots$ lower bounds

greatest lower bound

② Is a_n monotonic?

a_n is nonincreasing

Yes it is monotonic

③ Is a_n bounded?

... find m, M

✓ $m = \{0, -1, -2, \ldots\}$

greatest lower bound

✓ $M = \{1, 2, 3, \ldots\}$ upper bounds

lowest upper bound.

③ Is a_n bounded?

Yes since we found m, M

④ Does a_n converge?

Yes since $\{a_n\}$ monotonic and bounded

✓ $\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{1}{n} = 0$

