$\boxed{7.2}$ Natural Logarithms

$\boxed{2}$ Express the following logarithms in term of $\ln 5$ & $\ln 7$

(a) $\ln\left(\frac{1}{125}\right) = \ln\left(\frac{1}{5^3}\right) = \ln(1) - 3\ln 5$

$= -3\ln 5$

(b) $\ln 9.8 = \ln \frac{98}{10} = \ln \frac{49}{5} = \ln \frac{7^2}{5}$

$= 2\ln 7 - \ln 5$

$= 2\ln 7 - \ln 5$

(c) $\ln 7\sqrt{7} = \ln (7)(7)^{1/2} = \ln 7^{3/2} = \frac{3}{2}\ln 7$

(d) $\ln 1225 = \ln (7\times 5)^2 = 2\left[\ln 7 + \ln 5\right]$

(e) $\ln 0.056 = \ln \frac{56}{1000} = \ln \frac{8\cdot 7}{8\cdot 125} = \ln \frac{7}{125}$

$= \ln 7 - 3\ln 5$

(f) $\frac{\left(\ln 35 + \ln\left(\frac{1}{7}\right)\right)}{\ln 25} = \frac{\ln \frac{35}{7}}{\ln 25} = \frac{\ln 5}{\ln 5^2} = \frac{\ln 5}{2\ln 5}$

$= \boxed{\frac{1}{2}}$

$\boxed{4}$ Use the properties of Logarithms to simplify expression

$\boxed{a}$ $\ln \sec x + \ln \cos x$

$= \ln (\sec x)(\cos x)$

$= \ln 1 = 0$

$\boxed{b}$ $\ln (8x+4) - 2\ln 2$

$= \ln (8x+4) - \ln 2^2$

$= \ln \dfrac{8x+4}{4} = \ln (2x+1)$

ⓒ $3 \ln (t^2-1)^{1/3} - \ln (t+1)$

$= \ln (t^2-1) - \ln (t+1)$

$= \ln \dfrac{(t^2-1)}{(t+1)} = \ln \dfrac{(t-1)(t+1)}{t+1} = \ln (t-1)$

Find the derivative of y with respect to x, t or θ

$\boxed{21}$ $y = \dfrac{\ln x}{1+\ln x}$

$$y' = \frac{(1+\ln x)(\ln x)' - (\ln x)(1+\ln x)'}{(1+\ln x)^2}$$

$$= \frac{(1+\ln x)\cdot\frac{1}{x} - \ln x\left(\frac{1}{x}\right)}{(1+\ln x)^2}$$

$$= \frac{1}{x\,(1+\ln x)^2}$$

$\boxed{24}$ $y = \ln(\ln(\ln x))$

$$y' = \frac{1}{\ln(\ln x)} \cdot (\ln(\ln x))'$$

$$= \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x}(\ln x)'$$

$$= \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x\,\ln x\,\ln(\ln x)}$$

$\boxed{34}$ $y = \ln\left(\frac{(x+1)^5}{(x+2)^{10}}\right)^{\frac{1}{2}}$

$= \ln \frac{(x+1)^{\frac{5}{2}}}{(x+2)^{10}}$

$y = \frac{5}{2}\ln(x+1) - 10\ln(x+2)$

$y' = \frac{5}{2} \cdot \frac{1}{x+1} - 10\,\frac{1}{x+2}$

$y' = \frac{5}{2(x+1)} - \frac{10}{x+2}$

$\boxed{44}$ $\int_2^4 \frac{dx}{x\ln x}$

By substitution:-

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x = 2 \longrightarrow u = \ln 2$

$x = 4 \longrightarrow u = \ln 4$

$\int_{\ln 2}^{\ln 4} \frac{\cancel{x}\,du}{\cancel{x}\,u} = \int_{\ln 2}^{\ln 4} \frac{1}{u}\,du$

$= \ln|u|\Big]_{\ln 2}^{\ln 4}$

$= \ln\ln 4 - \ln\ln 2 \quad = \ln\left(\frac{\ln 4}{\ln 2}\right)$

$= \ln(2)$

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$\boxed{53}$ $\int \frac{dx}{2\sqrt{x}+2x}$

$= \int \frac{dx}{2\sqrt{x}\,(1+\sqrt{x})}$

By substitution:-

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$= \int \frac{\cancel{2\sqrt{x}}\,du}{\cancel{2\sqrt{x}}\,(u)}$

$= \int \frac{1}{u}\,du = \ln|u| + C$

$= \ln|1+\sqrt{x}| + C$

$= \ln(1+\sqrt{x}) + C$ since $1+\sqrt{x} > 0$

(54) $\int \frac{\sec x \, dx}{\sqrt{\ln(\sec x + \tan x)}}$

Let $u = \ln(\sec x + \tan x)$

$du = \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}\,dx$

$du = \sec x \, dx$

$\int \frac{\sec x}{\sqrt{u}} \frac{du}{\sec x} = \int \frac{1}{\sqrt{u}}\,du$

$= \frac{u^{1/2}}{1/2} + C$

$= 2\left[\ln(\sec x + \tan x)\right]^{1/2} + C.$

$\boxed{65}$

$$y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$$

$$\ln y = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{2(x^2+1)}\cdot 2x - \frac{2}{3}\,\frac{1}{x+1}$$

$$y' = \left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)}\right]\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$$

$\boxed{62}$

$$y = \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$$

$$\ln y = \frac{1}{3}\left[\ln x + \ln(x+1) + \ln(x-2) - \ln(x^2+1) - \ln(2x+3)\right]$$

$$\frac{y'}{y} = \frac{1}{3}\left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{1}{x^2+1}(2x) - \frac{2}{2x+3}\right]$$

$$y' = \frac{1}{3}\left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3}\right]\left(\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}\right)^{1/3}$$

78 (a) Prove that $f(x) = x - \ln x$ is increasing for $x > 1$

(b) Using part (a), show that $\ln x < x$ if $x > 1$

(a) $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x} > 0$ for $x > 1$

$\longrightarrow$ since $f'(x) > 0$ on $(1, \infty)$

$\longrightarrow$ $f(x)$ is increasing on $(1, \infty)$

(b) f is increasing when $x > 1$ $[x_1 > x_2 \longrightarrow f(x_1) > f(x_2)]$

$x > 1$

Then $\longrightarrow$ $f(x) > f(1)$

$x - \ln x > 1 - \ln(1)$

$x - \ln x > 1 > 0$

$x - \ln x > 0 \longrightarrow$ $\boxed{x > \ln x \ , \text{if } x > 1}$

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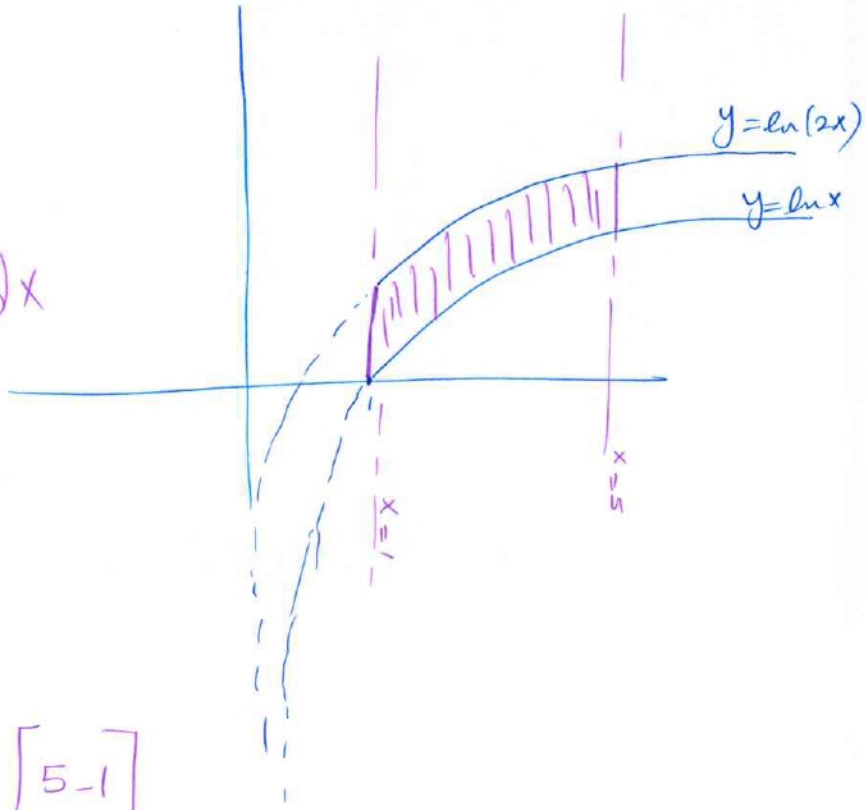
$\boxed{71}$ Find the area between the curve $y = \ln x$ & $y = \ln 2x$ from $x = 1$ – $x = 5$



$$\text{Area} = \int_1^5 \ln(2x) - \ln(x) \, dx$$

$$= \int_1^5 \ln\left(\frac{2x}{x}\right) dx$$

$$= \int_1^5 \ln(2) \, dx = \ln(2) [5 - 1]$$

$$= 4 \ln(2)$$

$$\text{Area} = \ln 16$$