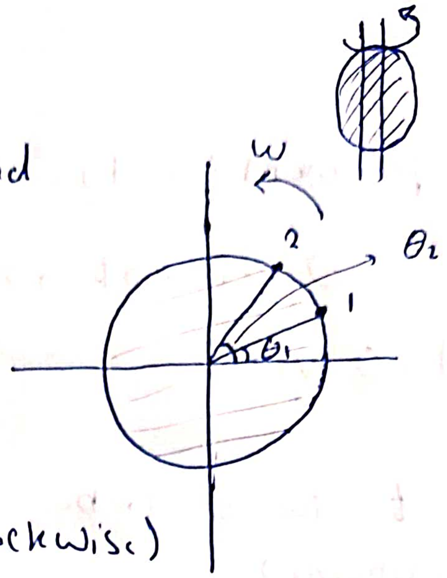
Ch 10: Rotation (الحركة الدورانية)

Rotational varibles:

- Initial angular position = θ_1 rad
- final angular position = θ_2 rad



⇒ Angular Displacment:

$$\Delta\theta = \theta_2 - \theta_1 \text{ rad}$$

$\Delta\theta \rightarrow$ positive (Counter clockwise)

$\Delta\theta \rightarrow$ negative (clockwise)

- Average angular velocity:

$$w_{avg} = \frac{\Delta\theta}{\Delta t} \text{ rad/s}$$

- instantanous angular velocity:

$$w_{inst} = \frac{d\theta}{dt}$$

- ~~Angul~~ Average angular accleration:

$$\alpha_{avg} = \frac{\Delta w}{\Delta t}$$

- Instantanous angular accleration:

$$\alpha_{inst} = \frac{dw}{dt}$$

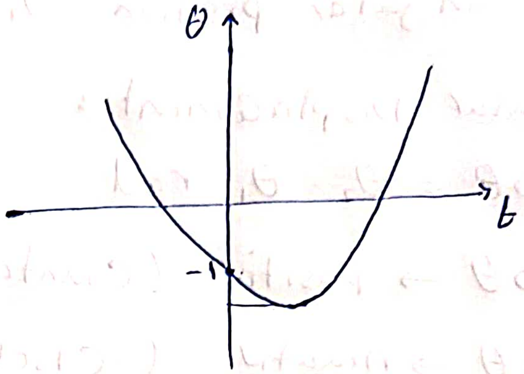
$\alpha\ (\theta, \Delta\theta, \omega, \alpha) \rightarrow$ ⊕ counter clockwise

$\quad\quad\quad\quad\quad\quad\quad \hookrightarrow$ ⊖ clockwise

Sample problem (10.01)

Disk , $\theta = -1 - 0.6t + 0.25t^2$

① plot θ vs. t



② Find t for θ to be minimum?

θ is minimum $\frac{d\theta}{dt} = 0$

$-0.6 + 0.5t = 0$

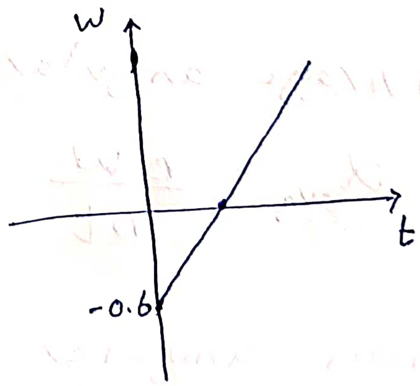
$t = 1.2$ sec

$\theta_{min} = -1 - 0.6(1.2) + 0.25(1.2)^2$

$= -1.36$ rad $= -1.36$ rad $\left(\frac{180}{\pi}\right)$

$=$

3) $\omega = \frac{d\theta}{dt} = -0.6 + 0.5t$ rad/s



4) $\alpha = \frac{d\omega}{dt} = 0.5$ rad/s^2

"the disk rotate clockwise then stops momentarily at $t = 1.2$ & reverse its rotation (counter clockwise)" sample problem 10-2

$\alpha = 5t^3 - 4t \quad rad/s^2$

at $t=0$, $\theta_0 = 2$ rad

$\omega_0 = 5$ rad/s

a) $\omega(t)$?!

$\alpha = \frac{d\omega}{dt} \Rightarrow \int d\omega = \int \alpha \, dt$

$\omega = \int 5t^3 - 4t \, dt$

$\omega = \frac{5}{4} t^4 - \frac{4}{2} t^2 + c$

$\omega(0) = 5 = \frac{5}{4}(0)^4 - \frac{4}{2}(0)^2 + c$

$\Rightarrow \boxed{c = 5}$

$\omega(t) = \frac{5}{4} t^4 - 2t^2 + 5$

b) $\theta(t)$?

$\omega = \frac{d\theta}{dt} \Rightarrow \int d\theta = \int \omega \, dt$

$\theta = \int \frac{5}{4} t^4 - 2t^2 + 5 \, dt$

$= \frac{t^5}{4} - \frac{2}{3} t^3 + 5t + c'$

$\theta(0) = 2 = 0 - 0 + 0 + c'$

$\boxed{c' = 2}$

$\theta(t) = \frac{t^5}{4} - \frac{2}{3} t^3 + 5t + 2$

* Rotation with constant angular acceleration

$\alpha = \text{Const.}$

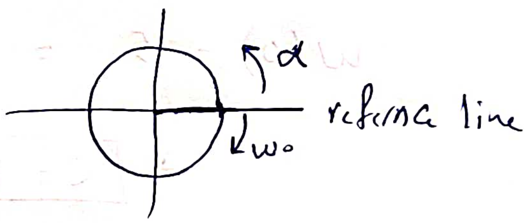
Translational motion	Angular motion
→ $V = V_0 + a t$	$\omega = \omega_0 + \alpha t$
→ $V^2 = V_0^2 + 2 a D$	$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$
→ $D = V_0 t + \frac{1}{2} a t^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

sample problem (10-3)

$\alpha = 0.35 \text{ rad/s}^2$

at $t = 0$, $\omega_0 = -4.6 \text{ rad/s}$

$\theta_0 = 0$



a) Find t when $\theta = 5$ rev. ?

دورة

$\boxed{1 \text{ rev} = 2\pi \text{ rad}}$

$\theta = 5 \text{ rev} \left(\frac{2\pi \text{ rad}}{\text{rev}} \right)$

$\theta = 10\pi \text{ rad}$, $\pi = 3.14$

$= 31.4 \text{ rad}$

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

$31.4 = -4.6 t + \frac{1}{2}(0.35) t^2$

$0.175 t^2 - 4.6 t - 31.4 = 0$

$\boxed{t = 32 \text{ sec}}$

b) Find t, when $\omega = 0$?

$\omega = \omega_0 + \alpha t$

$0 = -4.6 + 0.35\, t$

$t = 13$ sec

sampl problm (10-4)

$\omega_0 = 3.4$ rad/s after 20 rev :

$\Delta\theta = 20$ rev $\longrightarrow \omega = 20$ rad/s

Find t, α ?

$\Delta\theta = 20$ rev $= 20(2\pi)$ rad

$= 125.66$ rad

a) $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$(20)^2 = (3.4)^2 + 2\alpha(125.66)$

$\Rightarrow \alpha = -0.03$ rad/s^2

b) $\omega = \omega_0 + \alpha t$

$20 = 3.4 - 0.03\, t$

$t = 46.5$ sec.

problem 2

$\omega_0 = 30$ rev/s , $\omega = 0$, $\Delta t = 2$ min $= 2 \times 60$ sec

How many revolutions ??

$\Delta\theta$??

$\omega = \omega_0 + \alpha t$

$0 = 30 + \alpha(120)$

$\alpha = -0.25$ rev/s^2

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$0 = 30^2 + 2(-0.25)\Delta\theta$

$\boxed{\Delta\theta = 1800 \text{ rev}}$

problem 8

$\alpha = 6t^4 - 4t^2$ rad/s^2

at $t=0$ → $\omega_0 = 2.5$ rad/s

$\theta_0 = 1.5$ rad

a) Find $\omega(t)$

$\omega(t) = \int \alpha \, dt$

$= \int 6t^4 - 4t^2 \, dt$

$= \frac{6}{5}t^5 - \frac{4}{3}t^3 + c$

$\omega(0) = 0 - 0 + c = 2.5$

$c = 2.5$ ⇒ $\omega(t) = \frac{6}{5}t^5 - \frac{4}{3}t^3 + 2.5$ rad/s

b) $\theta(t)$?

$$\theta(t) = \int w \, dt$$

$$= \int \frac{6}{5} t^5 - \frac{4}{3} t^3 + 2.5 \, dt$$

$$= \frac{t^6}{5} - \frac{t^4}{3} + 2.5 t + c'$$

$$\theta(0) = c' = 1.5$$

$$\theta(t) = \frac{t^6}{5} - \frac{t^4}{3} + 2.5 t + 1.5 \text{ rad}$$

problem 16:

$\alpha = 1.2$ rad/s^2 , $w_0 = 0$

Find t to rotate through the first 2 rev ?

$\Delta\theta = 2(2\pi)$ rad
$= 12.56$ rad

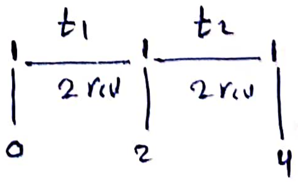
$$\Delta\theta = w_0 t + \frac{1}{2} \alpha t^2$$

$$12.56 = 0 + \frac{1}{2}(1.2) t^2$$

$$t_1 \simeq 4 \text{ sec}$$

b) Find t to rotate through the second 2 rev ?

$\Delta\theta = 4$ rev $= 8\pi$ rad



$$\Delta\theta = w_0 t + \frac{1}{2} \alpha t^2$$

$$8\pi = 0 + \frac{1}{2}(1.2) t^2 \Rightarrow t = 5.79 \text{ sec}$$

$t_2 = t - t_1$

$= 5.79 - 4$

$t_2 = 1.79$ sec

Ch 6 :

problem 33

f_k

$f_k = 70V$ N $\qquad m = 1000$ kg

$V_i = 100 km/h$

$V_f = 45 km/h \qquad$ Find t ??

$$\sum F_x = m a_x$$

$$-70V = m \frac{dV}{dt}$$

$$\int_0^t \frac{-70}{m} dt = \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\frac{-70}{m} t \Big|_0^t = \ln V \Big|_{V_i}^{V_f}$$

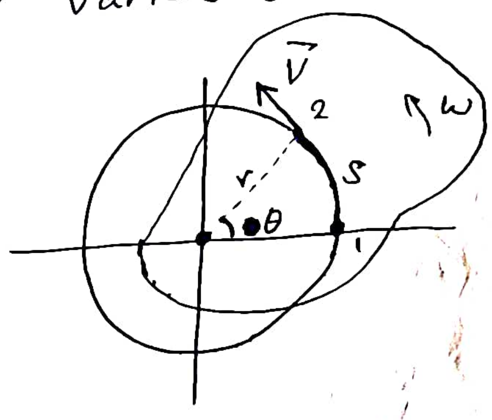
$$\frac{-70}{m} t = \ln V_f - \ln V_i$$

$$\frac{-70}{m} t = \ln \frac{V_f}{V_i}$$

$$t = \frac{m}{-70} \ln \frac{45}{100} = 11.4 \text{ sec}$$

Ch 10 : Lec 2

* Relating the Linear & Angular variables:



* arc (طول القوس) :

$$S = r\theta \quad (m)$$

* speed :

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

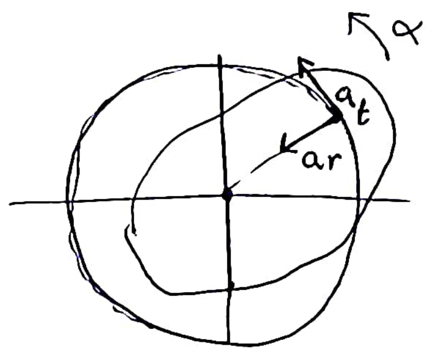
$$V = r\omega \quad (m/s)$$

* ω is ~~const.~~ the same for all the points on the rigid body.

$$\Rightarrow \quad V \propto r$$

* Acceleration:

$$\frac{dV}{dt} = r\frac{d\omega}{dt}$$



* $a_t = r\alpha \quad (m/s^2)$

[responsible for changes in the magnitude of the linear velocity $\vec{V}$]

* $a_r = \dfrac{V^2}{r} = \dfrac{(\omega r)^2}{r} = \omega^2 r$

[responsible for changes in the direction of the linear velocity $\vec{V}$]

* If $\alpha = 0 \Rightarrow a_t = \alpha r = 0$

but $a_t = \frac{dV}{dt} = 0 \Rightarrow V = \text{const.}$

if V is const $\Rightarrow$ uniform circular motion.

$V = \frac{2\pi r}{T} = \omega r \Rightarrow T = \frac{2\pi r}{V}$ (Period time)

$\Rightarrow \omega = \frac{2\pi}{T}$

Sample problem (10-5)

radius = 33.1 m

$\theta = ct^3$, $c = 6.39 \times 10^{-2}$ [from $t=0 \rightarrow t = 2.3$ sec]

at $t = 2.2$ sec find

a) ω ?

$\omega = \frac{d\theta}{dt} = 3ct^2 \Rightarrow \omega(2.2\text{ sec}) = 3c(2.2)^2$

$= 0.928$ rad/s

b) V ?

$V = \omega R$

$= 0.928 \times 33.1$

$= 30.7$ m/s

c) α ?

$\alpha = \frac{d\omega}{dt} = 6ct$

$\alpha(2.2) = 6c(2.2) = 0.843$ rad/s^2

d) $a_t = R\alpha$ (at $t = 2.2$ sec)

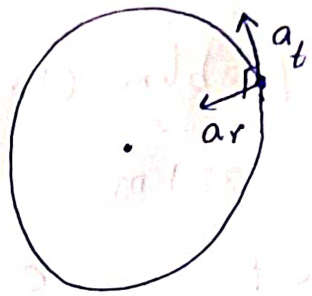
$= 33.1 \times 0.843$

$= 27.9$ m/s

e) $a_r = a_c = \dfrac{V^2}{R} = \omega^2 R$

$= \dfrac{(30.7)^2}{33.1} = 28.5\ m/s^2$

f) a_{net} ?

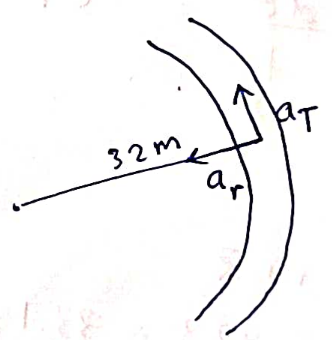


$a_{net} = \sqrt{a_r^2 + a_t^2}$

$= \sqrt{(27.9)^2 + (28.5)^2}$

$= 39.9\ m/s^2$

problem 32:



The car starts from rest.

Its speed increases at a Const. rate of $0.6\ m/s^2$ ($a_T = 0.6\ m/s^2$)

a) Find a_{net} at $t = 15$ sec ?

$a_T = \dfrac{dV}{dt} \Rightarrow V = \int a_T\, dt$

$V = \int 0.6\, dt$

$V = 0.6t + C$

but, $V(0) = 0 \Rightarrow C = 0$

$V = 0.6t \Rightarrow V(15 \text{ sec}) = 0.6(15)$

$= 9$ m/s

$$a_c \text{ (or } a_r) = \frac{v^2}{R} = \frac{(9)^2}{32} = 2.53 \ m/s^2$$

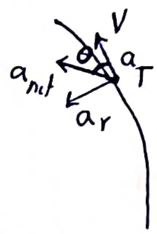
$$a_{net} = \sqrt{a_T^2 + a_r^2} = \sqrt{(0.6)^2 + (2.53)^2}$$

$$= 2.6 \ m/s^2$$

b) what angle does this net acceleration vector make with car velocity at this time?

$$\tan\theta = \frac{a_r}{a_T} \Rightarrow \theta = \tan^{-1}\left(\frac{a_r}{a_T}\right)$$

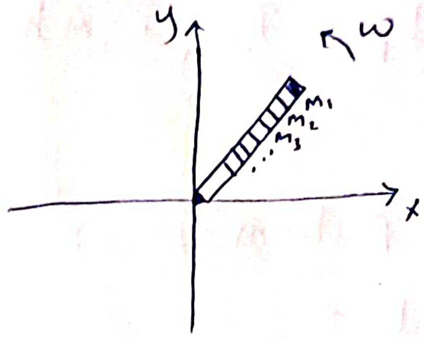
$$= 76.6^\circ$$



θ (bt. a_{net} & v) ; since v is in the direction of a_T.

* Kinetic Energy of Rotation :-

$K = \frac{1}{2} m v^2$, $v = \omega r$



$K = K_1 + K_2 + K_3 + \cdots$

$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \cdots$

$v_1 \neq v_2 \neq \cdots$

$v_1 > v_2$ $(v = \omega r)$

$r_1 > r_2$

$K = \frac{1}{2} m_1 (\omega r_1)^2 + \frac{1}{2} m_2 (\omega r_2)^2 + \cdots$

$= \frac{1}{2} \omega^2 \left[m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots \right]$

$= \frac{1}{2} \omega^2 \sum_i m_i r_i^2$

But $\sum_i m_i r_i^2 = I$ (rotational Inertia)
القصور الدوراني

For rigid body $I = \int r^2 dm$, $[I] = kg \cdot m^2$

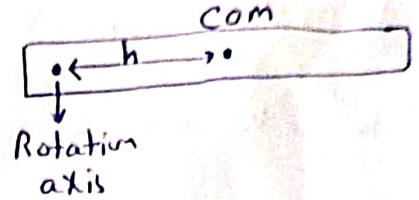
$\Rightarrow$ $\boxed{K = \frac{1}{2} I \omega^2}$

I (rotational Inertia) : Table 10-2 , Page 238
Rotation around Center of mass.

* parallel axis theorem.

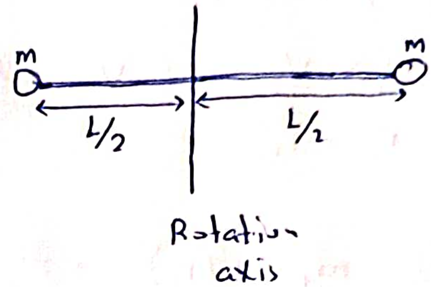
If the rotation is not around COM.

$$I_p = I_{com} + Mh^2$$



sample problem (10.6)

a) Find I_{com}



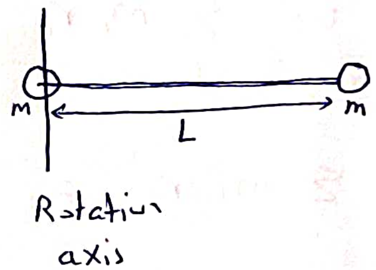
$$I = \sum_i m_i r_i^2$$

$$= m_1 r_1^2 + m_2 r_2^2$$

$$= m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{2} m L^2$$

b) $I = \sum m_i r_i^2$



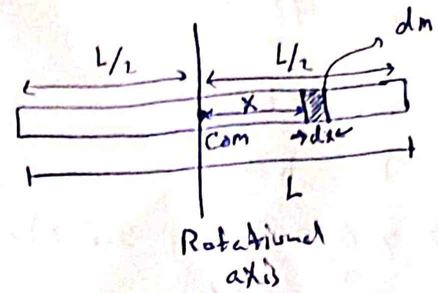
$$= m_1 r_1^2 + m_2 r_2^2$$

$$= m(0)^2 + m(L)^2$$

$$= mL^2$$

Sample problem 10.7

Uniform rod of mass M & length L.



a) Find I_{com} ?

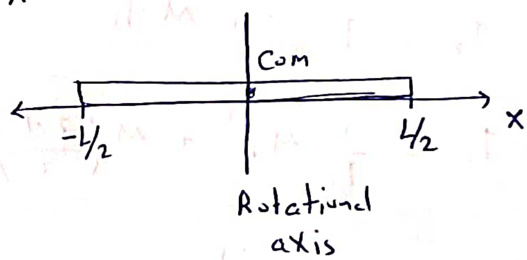
$$I = \int r^2 \, dm$$

Since the rod is uniform $\dfrac{dm}{dx} = \dfrac{M}{L}$

$$\Rightarrow dm = \frac{M}{L} dx$$

$I = \int x^2 \, dm$ (in one dim (x-dim))

$$= \int x^2 \left(\frac{M}{L} dx \right) = \frac{M}{L} \int_{-L/2}^{L/2} x^2 \, dx$$

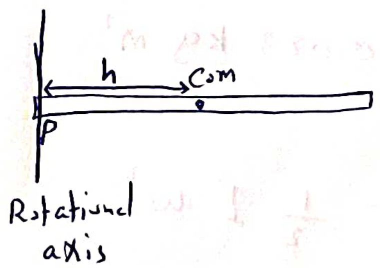


$$I = \frac{M}{L} \left. \frac{x^3}{3} \right|_{-L/2}^{L/2}$$

$$= \frac{M}{L} \left(\frac{(L/2)^3}{3} - \frac{(-L/2)^3}{3} \right)$$

$$\boxed{I_{\substack{rod \\ (Com)}} = \frac{1}{12} M L^2}$$

b) Find I_P ?



Instead of making the Integration;

using parallel axis theorem:

$$I_P = I_{Com} + M h^2$$

$$= \frac{1}{12} M L^2 + M \left(\frac{L}{2} \right)^2 = \frac{1}{3} M L^2$$

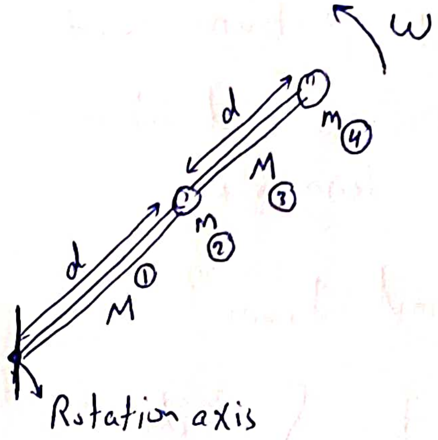
problem 41

$m = 0.85\ kg$

$M = 1.2\ kg$

$d = 5.6\ cm = 5.6 \times 10^{-2}\ m$

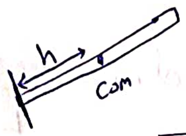
$w = 0.3\ rad/s$



a) Rotational Inertia (I)?

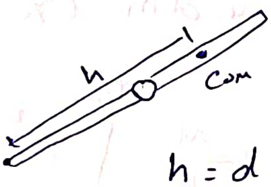
$$I = I_1 + I_2 + I_3 + I_4$$

$$I_1 = I_{com} + M h^2$$



$$\boxed{I_1 = \frac{1}{12} M L^2 + M\left(\frac{d}{2}\right)^2} \quad , \quad \boxed{L = d}$$

$$\boxed{I_2 = m d^2}$$



$$I_3 = I_{com} + M h^2 \qquad h = d + \frac{d}{2} = \frac{3d}{2}$$

$$\boxed{I_3 = \frac{1}{12} M L^2 + M\left(\frac{3d}{2}\right)^2}$$

$$\boxed{I_4 = m(2d)^2}$$

$$I = \frac{1}{12} M (d)^2 + \frac{1}{4} M d^2 + m d^2 + \frac{1}{12} M d^2 + \frac{9}{4} M d^2 + 4 m d^2$$

$$\boxed{I = \frac{8}{3} M d^2 + 5 m d^2}$$

$$I = 0.023\ kg \cdot m^2$$

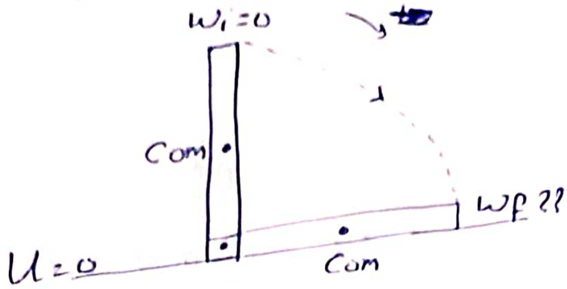
b) $K = \frac{1}{2} I w^2$

$$= \frac{1}{2}(0.023)(0.3)^2$$

$$\boxed{K = 1.1 \times 10^{-3}\ J}$$

Ch 10 : Lec 3

problem 63



$L = 1\,m$

* Find ω_f ??

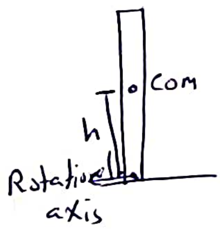
Using Conservation of E_{mech}:

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$0 + mg(y_{com}) = \frac{1}{2} I \omega_f^2 + 0$$

$$\boxed{mg(L/2) = \frac{1}{2} I \cdot \omega_f^2}$$



$h = \frac{L}{2}$

$$I = I_{com} + mh^2$$

$$= \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{3} mL^2$$

$$\boxed{\cancel{m} g \frac{\cancel{L}}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{1}{3}\cancel{m} L^2\right) \omega_f^2}$$

$$\omega_f = \sqrt{\frac{3g}{L}} = 5.42 \text{ rad/s}$$

* Find v_f at the end of the rod?

$$v = r\omega$$

$$v_f = 1 \times 5.42 = 5.42 \text{ m/s}$$

* find $V_{com(p)} = r_{com}\,\omega$

$= \frac{L}{2}\,\omega = \frac{1}{2}(5.42) = 2.7\ m/s$

* Torque : (العزم)

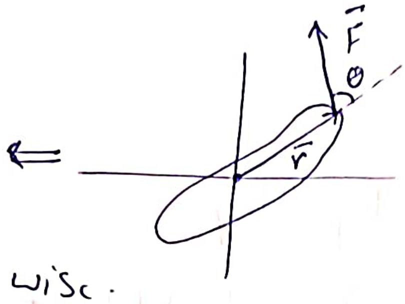
$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$, θ b/t $\vec{r}$ & $\vec{F}$

$\vec{\tau} \perp \vec{r}$ & $\vec{F}$

By right hand rule

$\vec{\tau}$ $\odot$ outward

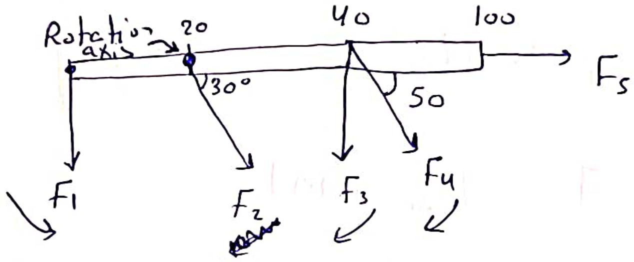
the rotation counter clock wise.



EX: Rotation axis at 20 cm

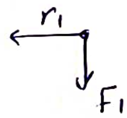
$F_1 = F_2 = F_3 = F_4 = F_5 = 15N$

Find $\vec{\tau}_{net}$?



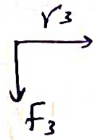
$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4 + \vec{\tau}_5$

$\vec{\tau}_1 = (0.2)(15)\sin 90 = 3\ N.m$ (outward)

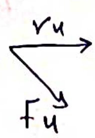


$\vec{\tau}_2 = (0)(15) = 0$

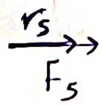
$\vec{\tau}_3 = (0.2)(15)\sin 90 = -3\ N.m$ (inward)



$\vec{\tau}_4 = (0.2)(15)\sin(50) = -2.3\ N\,m$ (inward)



$\vec{\tau}_5 = (0.8)(15)\sin 0 = 0$



$\vec{\tau}_{net} = 3 - 3 - 2.3 = -2.3\ N.m$ (inward)

the rotation clock wise.

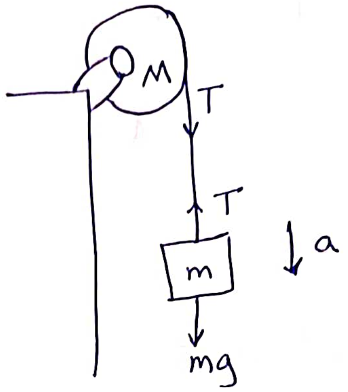
* Newtons second law in rotation:

$$\underbrace{\vec{F}_{net} = m\vec{a}}_{\text{linear motion}} \Rightarrow \underbrace{\vec{\tau}_{net} = I\vec{\alpha}}_{\text{rotational motion}}$$

sample problem 10-10:

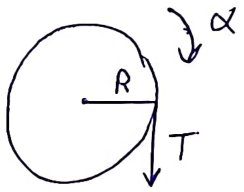
$M = 2.5$ kg , $R = 20$ cm (disk)

$m = 1.2$ kg



) Find a_m ? $\Sigma \vec{F} = m\vec{a}$

$$T - mg = m(-a) \quad — ①$$



For the disk: $\vec{\tau}_{net} = I\vec{\alpha}$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = RT \sin 90 = -RT \text{ (inward)}$$

$$I_{disk} = \frac{1}{2} M R^2$$

$$\Rightarrow \vec{\tau} = \left(\frac{1}{2} M R^2\right)(-\alpha)$$, The rotation clockwise $(-\alpha)$

$$\Rightarrow -RT = -\frac{1}{2} M R^2 \alpha$$

$$T = \frac{1}{2} M R \alpha$$, but $a_t = R\alpha$

$$\boxed{T = \frac{1}{2} M a} \quad — ②$$

sub ② in ① $\Rightarrow \frac{1}{2} M a - mg = -ma$

$$a = \frac{2m}{M+2m} g = \frac{2(1.2)}{2.5+2(1.2)}(9.8)$$

$$\boxed{a = 4.8 \ m/s^2}$$

② Find T ?

$$T = \frac{1}{2} M a = \frac{1}{2}(2.5)(4.8)$$
$$= 6 \ N$$

③ Find α ?

$$a = R\alpha \Rightarrow \alpha = \frac{a}{R} = \frac{4.8}{0.2} = 24 \ rad/s^2$$

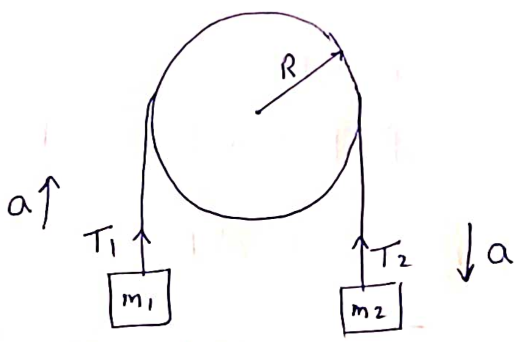
problem 51

$m_1 = 460 g = 0.46 kg$

$m_2 = 500 g = 0.5 kg$

$R = 5 \ cm = 5 \times 10^{-2} m$

when released from rest m_2 falls 75 cm in 5 sec.



a) Find a of the blocks ?

using the eq. of motion ($D = v_0 t + \frac{1}{2} a t^2$)

$$D = -75 \ cm = 0 + \frac{1}{2} a (5)^2$$

$$-0.75 = \frac{25}{2} a \Rightarrow a = 0.06 \ m/s^2$$

b) T_2 ? $\quad T_2 - m_2 g = m_2(-a) \quad [\Sigma F = ma]$

$$T_2 = mg - ma = 0.5(9.8 - 0.06)$$
$$= 4.87 \ N$$

c) T_1 ?? $\quad \Sigma F = ma$

$$T_1 - m_1 g = m_1 a$$

$$T_1 = m_1(g+a) = 0.46(9.8+0.06)$$

$$= 4.54\ N$$

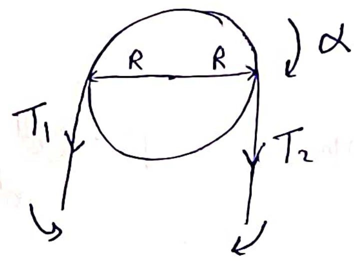
d) α of the pully ?

$a_t = R\alpha$, $\boxed{a_t = a}$ linear accleration.

$$\alpha = \frac{a}{R} = \frac{0.06}{5\times10^{-2}} = 1.2\ rad/s^2$$

e) Find I of the pully ?

$$\vec{\tau}_{net} = I\vec{\alpha}$$



But $\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2$

$$= \vec{R}\times\vec{T}_1 + \vec{R}\times\vec{T}_2$$

$$= RT_1 \sin 90 - RT_2 \sin 90$$

$$= R(T_1 - T_2)$$

$$= 5\times10^{-2}(4.54 - 4.87)$$

$$= -1.65\times10^{-2}\ N.m \quad \text{(inward)}$$

$$\tau_{net} = I\alpha$$

$$-1.65\times10^{-2} = I(-\cancel{0.06}\ 1.2)$$

$-\alpha$: clockwise.

$$\boxed{I = 0.0138\ kg.m^2}$$

* Work of the rotational motion:

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta \quad , \text{ for variable torque}$$

For const. torque: $W = \tau (\theta_f - \theta_i)$

* work - kinetic energy theorem:

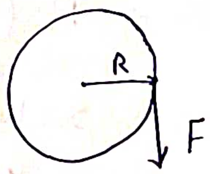
$$W_{net} = \Delta K = K_f - K_i$$

$$\boxed{W_{net} = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2}$$

* power: $P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau \omega$

problem 57

$I = 1 \times 10^{-3} \text{ kg.m}^2$, $R = 10 \text{ cm}$



$F = 0.5t + 0.3t^2$

at $t = 3$ sec : Find

a) α ?

$\tau = I\alpha$

$r \times F = I\alpha$

$R F \sin 90 = I\alpha$

$R(0.5t + 0.3t^2) = I\alpha$

$$\alpha = \frac{R}{I}(0.5t + 0.3t^2)$$

$$\alpha(3\,sec) = \frac{10\times10^{-2}}{10^{-3}}(0.5(3) + 0.3(3)^2)$$

$$= 4.2\times10^{2}\ rad/s^2$$

b) $w(3\,sec)$

$$\alpha = \frac{dw}{dt} = \frac{R}{I}(0.5t + 0.3t^2)$$

$$\frac{dw}{dt} = 100(0.5t + 0.3t^2)$$

$$\int dw = \int 50t + 30t^2\, dt$$

$$w = 25t^2 + 10t^3 + C$$

the pulley starts from rest $w(0) = 0 \Rightarrow C = 0$

$$w(3\,sec) = 25(3)^2 + 10(3)^3$$

$$= 4.95\times10^{2}\ rad/s$$