

Name:..... Key

Number:.....

Question 1 Consider the curve $\mathbf{r}(t) = \cosh t \mathbf{i} - \sinh t \mathbf{j} + t \mathbf{k}$. Find the following(a) The velocity $\mathbf{v}(t)$

①
$$\vec{v}(t) = \sinh t \mathbf{i} - \cosh t \mathbf{j} + \mathbf{k}$$

(b) The speed $|\mathbf{v}(t)|$

①
$$|\vec{v}| = \sqrt{\cosh^2 t + \sinh^2 t + 1} = \sqrt{2} \cosh t$$

(c) The acceleration $\mathbf{a}(t)$

①
$$\vec{a}(t) = \cosh t \mathbf{i} - \sinh t \mathbf{j} + 0 \mathbf{k}$$

(d) $\mathbf{v}(t) \times \mathbf{a}(t)$

②
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sinh t & -\cosh t & 1 \\ \cosh t & -\sinh t & 0 \end{vmatrix}$$

$$= \sinh t \hat{\mathbf{i}} + \cosh t \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

(e) $|\mathbf{v}(t) \times \mathbf{a}(t)|$

①
$$|\vec{v} \times \vec{a}| = \sqrt{2} \cosh t$$

(f) The curvature $\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \frac{\sqrt{2} \cosh t}{2\sqrt{2} \cosh^3 t}$

①
$$= \frac{1}{2} \operatorname{sech}^2 t$$

(g) Find a_T and a_N .

③
$$a_T = \frac{d}{dt} |\vec{v}| = \sqrt{2} \sinh t$$

$$a_N = \kappa |\vec{v}|^2 = \frac{1}{2} \cosh^2 t = 1$$

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Question 1 Consider the function $f(x, y) = \ln(x^2 + y^2 - 1)$. Find the following

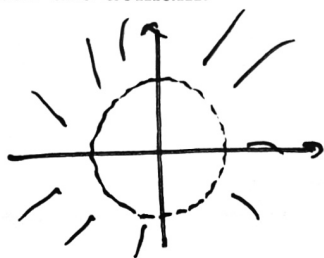
(2) (a) Domain of f .

$$D = \{(x, y) \mid x^2 + y^2 > 1\}$$

(2) (b) Range of f

$$(-\infty, \infty)$$

(1) (c) Sketch the domain.



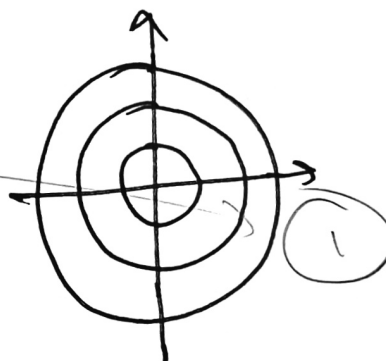
(1) (d) Find the boundary of the domain.

$$\{(x, y) \mid x^2 + y^2 = 1\}$$

(2) (e) Sketch some level curves of f .

$$\ln(x^2 + y^2 - 1) = c$$

$$\textcircled{1} x^2 + y^2 = e^c + 1 \quad \text{circles}$$



(1) (f) Is the domain open or closed.

open.

(1) (g) Is the domain bounded or unbounded.

unbounded.

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Question 1 Find the limit or show it does not exist

(a) $\lim_{(x,y) \rightarrow (1,1)} \frac{x^4 - y^4}{x - y}$

$$\textcircled{2} = \lim_{(x,y) \rightarrow (1,1)} \frac{(x-y)(x+y)(x^2+y^2)}{x-y} = 4$$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$

$$\textcircled{2} = \lim_{r \rightarrow 0} \frac{r^3 \cos^2 \theta \sin \theta}{r^2} = 0$$

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x-y}{x+y}$ $y = mx$

$$\textcircled{2} = \lim_{(x,y) \rightarrow (0,0)} \frac{x(1-m)}{x(1+m)} = \frac{1-m}{1+m} \text{ DNE}$$

Question 2 Let $f(x, y) = \tan(xy)$. Find $f_x, f_y, f_{xx}, f_{yy}, f_{xy}$.

$$\textcircled{1} f_x = y \sec^2(xy), f_y = x \sec^2(xy) \quad \textcircled{1}$$

$$\textcircled{1} f_{xx} = 2y^2 \sec^2(xy) \tan(xy)$$

$$\textcircled{1} f_{yy} = 2x^2 \sec^2(xy) \tan(xy)$$

$$\textcircled{1} f_{xy} = \sec^2(xy) + 2xy \sec^2(xy) \tan(xy).$$