Chapter #1

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Sum of Sinusoidal Signals

$\rightarrow$ Periodic if $\frac{\omega_i}{\omega_j} = \frac{N_i}{N_j}$ $\leftarrow$ rational

$$\omega_0 = GCD(\omega_1, \omega_2, \ldots, \omega_n)$$

$$f_0 = GCD(f_1, f_2, \ldots, f_n)$$

$$T_0 = LCM(T_1, T_2, \ldots, T_n)$$

Singularity functions:

① unit step function: $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

② Ramp function: $r(t) = \begin{cases} t & t > 0 \\ 0 & t < 0 \end{cases}$

$$u(t) = \frac{d\, r(t)}{dt}, \quad \int u(t)\,dt = r(t)$$

③ unit pulse function: $\Pi(t) = \begin{cases} 1 & -\frac{1}{2} < t < \frac{1}{2} \\ 0 & \text{o.w} \end{cases}$

④ unit impulse function: $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$

① $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$ ② $\delta(at) = \frac{1}{|a|}\delta(t)$

③ $\delta(t) = \delta(-t)$ $\Rightarrow$ Even function

④ Point Property : $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

⑤ Sampling Property : $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

⑥ Derivative of $\delta(x)$:

$$\int_{t_1}^{t_2} x(t)\,\delta^{(n)}(t-t_0) = \begin{cases} (-1)^n x^{(n)}(t_0) & t_1 < t_0 < t_2 \\ 0 & \text{o.w} \end{cases}$$

Even and odd Signals

Even $\Rightarrow$ $x(-t) = x(t)$

odd $\Rightarrow$ $x(-t) = -x(t)$

$$* \quad \underbrace{x(t)}_{\text{general signal}} = \underbrace{x_e(t)}_{\text{even signal}} + \underbrace{x_o(t)}_{\text{odd signal}}$$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

Energy and Power :

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt \longrightarrow \text{Periodic signal}$$

$$P = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \longrightarrow \text{Non-Periodic signal}$$

RMS Value :

$$RMS = \sqrt{\frac{1}{T} \int_T |x(t)|^2 dt}$$

$$RMS = \sqrt{P} \quad , \quad P = (RMS)^2$$

$$RMS = \frac{Peak}{\sqrt{2}}$$, for sinusodial functions

Orthogonal signals : The signals are orthogonal if they are mutually independent.

e.g. $$\int_{-\infty}^{\infty} x_1(t)\, x_2(t) = 0 \quad \text{Non-periodic}$$

$$\int_T x_1(t)\, x_2(t) = 0 \quad \text{periodic}$$

Notes: 1 two Harmonics with different frequencies are orthogonal

$$\sin(\omega_1 t + \theta_1) \; , \; \sin(\omega_2 t + \theta_2)$$

$$\omega_1 \neq \omega_2$$

② sin and cos functions with the same phase and the same frequency are orthogonal

③ DC Value and the sin function are orthogonal

④ $x_1(t)$, $x_2(t)$ are two orthogonal signals

$$y(t) = x_1(t) + x_2(t)$$

$$P_y = P_{x_1} + P_{x_2}$$

$$E_y = E_{x_1} + E_{x_2}$$

Chapter 2

Basic System Properties:

① Static and Dynamic systems

$$x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t)$$

Static: $y(t)$ depends on present Value of $x(t)$

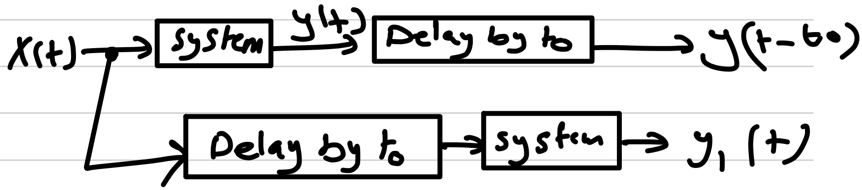
Dynamic: $y(t)$ depends on past or future Values of $x(t)$

② Causal and Non-causal Systems:

Causal: $y(t)$ is independent of future value of $x(t)$

Non-causal: $y(t)$ depends on future values of $x(t)$

③ TV and TIV Systems:



$y_1(t) = y(t-t_0)$ TIV System

$y_1(t) \neq y(t-t_0)$ TV system

④ Linear and Non-Linear Systems:

Linear if follows the principles of supperposition:

① Low of additivity LOA
② Low of Homogenity LOH

otherwise Non-Linear System.

LTI Systems:

$$x(t) \longrightarrow \boxed{\begin{matrix} LTI \\ h(t) \end{matrix}} \longrightarrow y(t)$$

$$y(t) = x(t) \underset{\substack{\uparrow \\ \text{convolution}}}{*} h(t)$$

Impulse Response for LTI:

When $x(t) = \delta(t)$, then $y(t) = h(t)$

By Laplace find $H(S)$, then Laplace inverse to find the impulse response $h(t)$

Frequency Responce: $H(\omega)$

$$H(\omega) = \int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

Steady state response:

Let $x(t) = X_m \cos(\omega_o t + \theta_x)$

ss response : $y(t) = |H(\omega)|\, X_m \cos(\omega_o t + \theta_x + \theta_{H(\omega)})$

Stability :

$$H(s) = \frac{b_0 + b_1 s + b_2 s^2 + \cdots}{a_0 + a_1 s + a_2 s^2 + \cdots}$$

$Q(s) \rightarrow$ (denominator)

$\Rightarrow$ BIBO or AS stable if all the poles (roots) of $Q(s)$ is real negative

$\Rightarrow$ Unstable if there is at least one pole (root) of $Q(s)$ real positive or repeated with zero real part root.

$\Rightarrow$ marginally : if the poles not repeated and located on the imaginary axis.

Modeling By separate and integrate.

Chapter 3

Fourier series:

① Sinusoidal form

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

or $$x(t) = a_0 + \sum_{n=1}^{\infty} C_n (\cos(n\omega t + \theta_n))$$

$$C_n = \sqrt{a_n^2 + b_n^2} \qquad \theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$a_0 = \frac{1}{T}\int_T x(t)\, dt$$

$$a_n = \frac{2}{T}\int_T x(t) \cos(n\omega t)\, dt$$

$$b_n = \frac{2}{T}\int_T x(t) \sin(n\omega t)\, dt$$

② Complex form:

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega t}$$

$$X_n = \begin{cases} a_0 & , n=0 \\ \frac{a_n - jb_n}{2} & , n>0 \\ \frac{a_{-n} + jb_{-n}}{2} & , n<0 \end{cases}$$

or $X_n = \frac{1}{T}\int_T x(t)\, e^{-jn\omega t}\, dt$

* $X_{-n} = X_n^* \Rightarrow \begin{cases} |X_{-n}| = |X_n| & \text{even} \\ |\theta_{-n}| = -\theta_n & \text{odd} \end{cases}$

* $x(t)$ is even $\Rightarrow X_n$ real and even

$x(t)$ is odd $\Rightarrow X_n$ imaginary and odd

Notes:

1. $x(t)$ is even $\rightarrow b_n = 0$
2. $x(t)$ is odd $\rightarrow a_n = 0$
3. $x(t)$ is half-wave symmetry

$\rightarrow a_n = 0$, $b_n = 0$ for n even

Signal Power using spectral representation:

$$P = X_0^2 + 2\sum_{n=1}^{\infty} |X_n|^2$$

Steady state response of LTI system with FS representation:

$$x(t) \longrightarrow \boxed{\begin{matrix}\text{System}\\ \text{LTI}\end{matrix}} \longrightarrow y(t)$$

where $H(\omega) = |H(\omega)| \, e^{j\theta_H(\omega)}$

is the frequency response

if $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega t}$

SS response : $y(t) = \sum_{n=-\infty}^{\infty} |H(n\omega)| X_n e^{j(n\omega t + \theta_H(n\omega))}$

if $x(t) = \sum_{n=1}^{\infty} C_n \cos(n\omega t + \theta_n)$

SS response :

$$y(t) = \sum_{n=1}^{\infty} |H(n\omega)| C_n \cos(n\omega t + \theta_n + \theta_H(n\omega))$$

RMS calculation.

① $X_{RMS} = \sqrt{\frac{1}{T}\int_T x^2(t)\,dt}$

② $X_{RMS}^2 = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2$

$$X_{RMS} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2}$$

③ $X_{RMS} = \sqrt{a_0^2 + \sum_{n=1}^{\infty} C_{RMS,n}^2}$

$$C_{RMS,n} = \frac{C_n}{\sqrt{2}}$$

System and Signal Distortion

① $THD = \frac{\sqrt{C_{RMS,2}^2 + C_{RMS,3}^2 + \cdots}}{C_{RMS,1}}$

$$THD = \frac{\sqrt{X_{RMS}^2 - C_{RMS,1}^2}}{C_{RMS,1}}$$

$$DF = \frac{C_{RMS,1}}{X_{RMS}}, \quad DF = \sqrt{\frac{1}{1+(THD)^2}}$$

$$THD = \sqrt{\left(\frac{1}{DF}\right)^2 - 1}$$

Chapter 4

Fourier transform

$$FT[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

if $x(t) = \sum_{n=-\infty}^{\infty} X_n\, e^{jn2\pi f_0 t}$

then $X(f) = \sum_{n=-\infty}^{\infty} X_n\, \delta(f - nf_0)$

FT of Periodic signals:

$$x_p(t) = x(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = \sum_{n=-\infty}^{\infty} x(t - nT_0)$$

$$X_p(f) = \sum_{n=-\infty}^{\infty} f_0\, X(nf_0)\, \delta(f - nf_0)$$

$$X_n = f_0 \; X(nf_0)$$

Energy density function $G(f)$

$$x(t) \longrightarrow G_x(f) = |X(f)|^2$$

Frequency response

$$H(\omega) = H(s)\Big|_{s=j\omega}$$ $H(s)$: transfer function

Steady state response :

$$y(t) = \sum_{n=-\infty}^{\infty} |H(nf_0)| X_n \, e^{j(2\pi n f_0 t + \theta_{x_n} + \theta_H(nf_0))}$$

Hilbert transform :

$$\overset{H}{X}(f) = -j \, \mathrm{sgn}(f) \, X(f)$$

$$x_H(t) = x(t) * \frac{1}{\pi t}$$