exercise 6.2:

Q9: consider the following data for two indep. random samples taken from 2 Normal pop.:

sample 1: 10, 7, 13, 7, 9, 8

sample 2: 8, 7, 8, 4, 6, 9.

م الآلة الحاسبة بنجيب هاي الأشياء..

sample 1	sample 2
$\bar{X}_1 = 9$	$\bar{X}_2 = 7$
$S_1^2 = 5.2$ ← الآلة بتطلع S_1 مش S_1^2 لازم تربيعها	$S_2^2 = 3.2$
$n_1 = 6$	$n_2 = 6$.

→

(1) point estimator for $\mu_1 - \mu_2$:

$$\bar{X}_1 - \bar{X}_2 = 2$$

(2) 90 % CI for $\mu_1 - \mu_2$: $\frac{\alpha}{2} = 0.05$.

$$df = \left\lfloor \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\left(\frac{1}{n_1 - 1}\right)\left(\frac{S_1^2}{n_1}\right)^2 + \left(\frac{1}{n_2 - 1}\right)\left(\frac{S_2^2}{n_2}\right)^2} \right\rfloor = \lfloor 9.46 \rfloor = 9$$

من الجدول

$$90\ \%\ IC = (9 - 7) \pm 1.833 \sqrt{\frac{5.2}{6} + \frac{3.2}{6}}$$

$$= 2 \pm 2.17$$

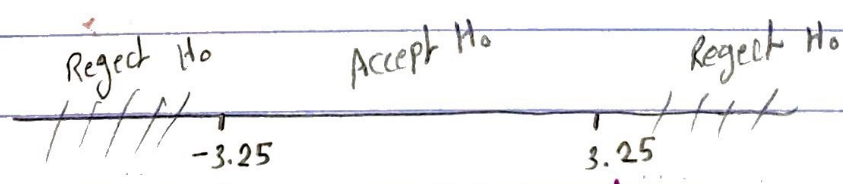
$$= (-0.17 ,\ 4.17) .$$

(3) perform $H_0 : \mu_1 - \mu_2 = 1$ — اذا ما اعطى α بستخدم α=0.05

$H_1 : \mu_1 - \mu_2 \neq 1$, use $\alpha = 0.01$.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \rightsquigarrow t = \frac{2 - 1}{\sqrt{1.4}} = 0.845 .$$

$t_{\frac{\alpha}{2}} = 3.25$, $df = 9$.



Don't Reject H_0 ($\alpha = 0.01$)

$\mu_1 - \mu_2 = 1$ ($\alpha = 0.01$).

critical value

Cont :

p-Value

df	0.2	0.1
9	↑ 0.883	1.383

our t

upper tail area = p-Value ☆

إذا كان upper أو lower

$\Rightarrow$ upper tail area > 0.2 → upper tail area = $\frac{1}{2}$ p-value ☆

إذا كان two tail test

p-value > 0.4

$\alpha = 0.01$

p-value $> \alpha$

$\therefore$ Don't Reject H_0 $(\alpha = 0.01)$.

$\mu_1 - \mu_2 = 1$ $(\alpha = 0.01)$.

Q7: Consider the following results for independent random samples taken from two populations:

sample 1	sample 2
$n_1 = 20$	$n_2 = 30$
$\bar{X}_1 = 22.5$	$\bar{X}_2 = 20.1$
$S_1 = 2.5$	$S_2 = 4.8$

a. What is the point estimater of the difference between the two pop. mean:

$P.E = \bar{X}_1 - \bar{X}_2 = 22.5 - 20.1 = 2.4$

b. What are the degrees of fredom for the t distribution?

$$df = \left\lfloor \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\left(\frac{1}{n_1-1}\right)\left(\frac{S_1^2}{n_1}\right)^2 + \left(\frac{1}{n_2-1}\right)\left(\frac{S_2^2}{n_2}\right)^2} \right\rfloor = \left\lfloor \frac{\left(\frac{2.5^2}{20} + \frac{4.8^2}{30}\right)^2}{\left(\frac{1}{19}\right)\left(\frac{2.5^2}{20}\right)^2 + \left(\frac{1}{29}\right)\left(\frac{4.8^2}{30}\right)^2} \right\rfloor = \lfloor 45.8 \rfloor$$

$= 45$

$1-\alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025$

c. At 95% confidence, what is the margen of error ?

$$E = t_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2.014 \sqrt{1.0805} = 2.093$$

df = 45
$\alpha = 0.025$

t distro

d. What is the 95% CI for the difference between two pop. means?

$$0.95\ CI = (\bar{x}_1 - \bar{x}_2) \mp E$$
$$= 2.4 \mp 2.093$$
$$= [0.307, 4.493]$$

8. Consider the following hypothisis test : $H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 - \mu_2 \neq 0$ Two tail test

The following results are from independent samples taken from two pop. :

sample 1	sample 2
$n_1 = 35$	$n_2 = 40$
$\bar{x}_1 = 13.6$	$\bar{x}_2 = 10.1$
$s_1 = 5.2$	$s_2 = 8.5$

a. What is the value of the test statistic ?

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{3.5 - 0}{1.6\ldots} = 2.18$$

b. What are the degrees of freedom for the distribution ?

$$df = \left\lfloor \frac{\left(\frac{5.2^2}{35} + \frac{8.5^2}{40}\right)^2}{\frac{1}{34}\left(\frac{5.2^2}{35}\right)^2 + \frac{1}{39}\left(\frac{8.5^2}{40}\right)^2} \right\rfloor = \lfloor 65.7 \rfloor = 65$$

c. What is the p-value ? $\alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025$, $t_{test} = 2.18$

df	0.025	0.01
65	1.997	2.385

t-test

upper tail area $\in (0.01, 0.025)$

p-value $\in (0.02, 0.05)$

$0.02 < \text{p-value} < 0.05$

d. At $\alpha = 0.05$, What is your Conclusion ?

p-value $< 0.05 = \alpha$ so we Reject H_0 .

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10. .... Construct a 95% CI estimate of the difference between the mean ... ?

Airport L : $n_1 = 50$ $\bar{X}_1 = 6.72$ $S_1 = 2.37$

الأول

الخامسة

Airport M : $n_2 = 50$ $\bar{X}_2 = 6.34$ $S_2 = 2.16$

$$0.95\ CI = (\bar{X}_1 - \bar{X}_2) \mp t_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$df = \left[\frac{\quad}{\quad} \right] = 79$$

$\rightarrow df = 79$, $\frac{\alpha}{2} = 0.025$

$\rightarrow t_{\frac{\alpha}{2}} = 1.990$.

$0.95\ CI = 0.38 \mp 0.902$

$= [-0.52 , 1.28]$.

Q11 :

$n_1 = 15$ $\quad\quad$ $\bar{X}_1 = 17.54$ $\quad\quad$ $S_1 = 2.24$.

$n_2 = 20$ $\quad\quad$ $\bar{X}_2 = 15.36$ $\quad\quad$ $S_2 = 1.99$.

a. point estimate of $\mu_1 - \mu_2$:

$$P.E = \bar{X}_1 - \bar{X}_2 = 2.18$$

b. 95% CI for $\mu_1 - \mu_2$:

$$df = \left\lfloor \frac{\left(\frac{2.24^2}{15} + \frac{1.99^2}{20}\right)^2}{\frac{1}{14}\left(\frac{2.24^2}{15}\right)^2 + \frac{1}{19}\left(\frac{1.99^2}{20}\right)^2} \right\rfloor = 28$$

$$df = 28 \ , \ \frac{\alpha}{2} = 0.025 \Rightarrow t_{\frac{\alpha}{2}} = 2.048$$

$$\Rightarrow 0.95 \ CI = 2.18 \mp 2.048 \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$= 2.18 \mp 1.49$$

$$= [0.69 \ , \ 3.67] .$$

c.

?

يلي 3 اسئلة خطأ

Q12:

a. $H_0: \mu_1 - \mu_2 \leq 0$ upper tail test.

$H_1: \mu_1 - \mu_2 > 0$.

b. $n_1 = 16$ $\bar{X}_1 = 525$ $S_1^2 = 59.42$

$n_2 = 12$ $\bar{X}_2 = 487$ $S_2^2 = 51.75$.

$P.E = \bar{X}_1 - \bar{X}_2 = \underline{38}$

C. P-value ?

$$t_{test} = \frac{(\bar{X}_1 - \bar{X}_2) - D_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{38 - 0}{21.06..} = 1.804$$

$df = \ldots = 25$

df	0.05	0.025
25	1.708	2.060

$\Rightarrow$ upper tail area $\in (0.05, 0.025)$

$\Rightarrow$ p-value $\in (0.05, 0.025)$.

d. At $\alpha = 0.05$, what is your conclusion? Regect H_0 if $P_{value} \leq \alpha$

p-value $< \alpha$

So we Regect H_0

Q14:

system A	system B	
$n_1 = 120$	$n_2 = 100$	$\alpha = 0.05$
$\bar{x}_1 = 4.1$	$\bar{x}_2 = 3.4$	
$S_1 = 2.2$	$S_2 = 1.5$	

$H_0 : \mu_1 - \mu_2 = 0$ Two tail test.

$H_1 : \mu_1 - \mu_2 \neq 0$

$$t = \frac{(4.1 - 3.4) - 0}{\sqrt{\frac{2.2^2}{120} + \frac{1.5^2}{100}}} = \frac{0.7}{0.25066578} = 2.79$$

p-value :

$p(Z < -2.79) = 1 - p(Z < 2.79)$

$= 1 - 0.9974$

$= 0.0026$

$$\text{p-value} = \underbrace{2}_{\text{Two tail}}(0.0026) = 0.0052$$

→ p-value $\leq \alpha$

$0.0052 \leq 0.05$

So Reject H_0.

system B having the lower mean checkout time.

Course $\qquad\qquad\qquad\qquad$ No course

Q16:

$n_1 = 35$ $\qquad\qquad$ $n_2 = 48$.

$\bar{x}_1 = 1058$ $\qquad\qquad$ $\bar{x}_2 = 983$.

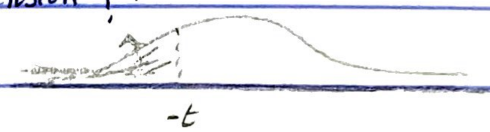
$S_1 = 90$ $\qquad\qquad$ $S_2 = 105$. $S_2^2 = 11025$

a. Formulate the hypotheses:

$H_0: \mu_1 - \mu_2 \geq 120$ } D_0 $\qquad$ lower tail test

$H_1: \mu_1 - \mu_2 < 120$.

b. use $\alpha = 0.05$ and data above, what is your conclusion? p-value



Find p-value

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{75 - 120}{\sqrt{461.116714}} = \frac{-45}{21.47361338}$$

$$\boxed{t = -2.09}$$

$$df = \left\lfloor \frac{(461.1160714)^2}{1575.270108 + 1122.475482} \right\rfloor = \left\lfloor \frac{212628.0313}{2697.74559} \right\rfloor = \lfloor 78.8 \rfloor = 78$$

df	0.025	0.01
78	1.991	2.375

p-value

p-value $\in (0.01, 0.025)$.

$\alpha >$ p-value $\qquad$ So we reject H_0.

c. what is the point estimate, ... provide a 95% CI.

point estimate = $\bar{x}_1 - \bar{x}_2 = 75$.

$1-\alpha = 0.95$

$\boxed{0.05 = \alpha}$

$95\% \ CI = (\bar{x}_1 - \bar{x}_2) \mp E$

$\rightarrow E = t_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = t_{0.025}(21.47361338)$

$= 1.991 \ (\ \ " \ \ )$

$= 42.75$

$95\% \ CI = 75 \mp 42.75$

$= (32.25 \ , \ 117.75)$.

d. مش مهم :)