Disk Method:

Def: Volume by Disks for Rotations about x-axis:

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\,(R(x))^2\,dx.$$

Note: The Cross-section is perpendicular to the axis of Revolution. (In Disk & Washer Method).

Def: Volume by Disks for Rotations about y-axis:

$$V = \int_c^d A(y)\,dy = \int_c^d \pi\,[R(y)]^2\,dy.$$

Washer Method:

Def: Volume by Washers for Rotations about x-axis:

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\left([R(x)]^2 - [r(x)]^2\right)dx$$

where $R[x]$: outer Radius , $r(x)$: Inner radius.

Def: Volume by Washers for Rotations about y-axis:

$$V = \int_c^d A(y)\,dy = \int_c^d \pi\left(R(y)^2 - r(y)^2\right)dy.$$

Shell Method:

Def: Volume by shell method for Rotation about a vertical line: $(x = L)$: or $(y\text{-axis})$:

$$V = \int_a^b 2\pi \begin{pmatrix} \text{shell} \\ \text{radius} \end{pmatrix} \begin{pmatrix} \text{shell} \\ \text{height} \end{pmatrix} dx$$

where: shell radius: Distance between cross section and the axis of Revolution.

shell height: Length of the cross-section.

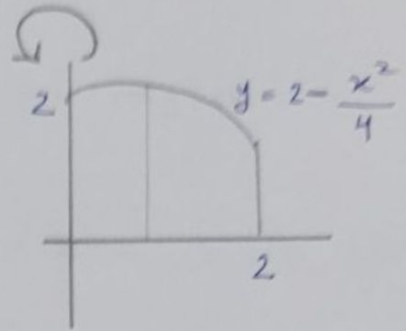
Note: The cross-section is parallel to the axis of Revolution.

Def: Volume by shell Method for Rotation about a Horizontal line $(x\text{-axis})$ or $(y = b)$:

$$V = \int_c^d 2\pi \begin{pmatrix} \text{shell} \\ \text{radius} \end{pmatrix} \begin{pmatrix} \text{shell} \\ \text{height} \end{pmatrix} dy.$$

6.2

(2) Use shell method : ...

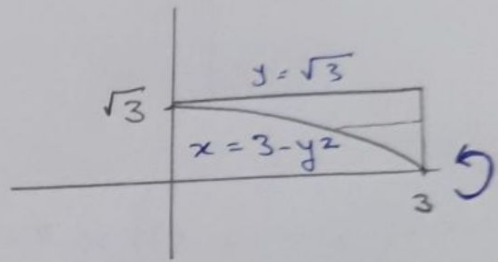


$$V = \int_0^2 2\pi \begin{pmatrix} \text{shell} \\ \text{radius} \end{pmatrix} \begin{pmatrix} \text{shell} \\ \text{height} \end{pmatrix} dx$$

$$= \int_0^2 2\pi \, (x)\left(2 - \frac{x^2}{4}\right) dx = \boxed{6\pi}$$

---

(4) Use shell method: ...



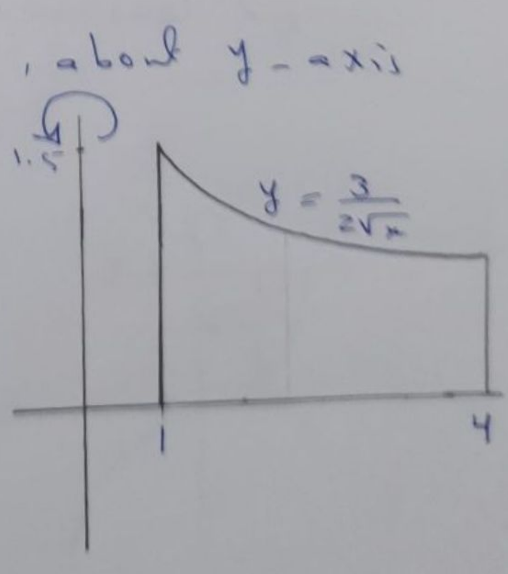
$$V = \int_0^{\sqrt{3}} 2\pi (y)(3 - (3 - y^2))\, dy$$

$$= 2\pi \int_0^{\sqrt{3}} y^3 \, dy = \boxed{\frac{9\pi}{2}}$$

---

(12) shell method: ...

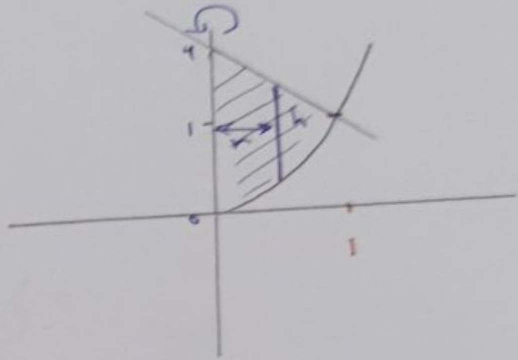
$y = \frac{3}{2\sqrt{x}}$, $y = 0$, $x = 1$, $x = 4$, about y-axis



$$V = \int_1^4 2\pi (x) \left(\frac{3}{2\sqrt{x}}\right) dx$$

$$= \int_1^4 3\pi \, x^{\frac{1}{2}} \, dx = \boxed{14\pi}$$

6.2 Q9) Use the shell method to find the volume of the solid generated by revolving the region bounded by the curves $y = x^2$, $y = 2 - x$, $x = 0$, $x \geq 0$ about the y-axis $(x = 0)$



$$x^2 = 2 - x \Rightarrow x^2 + x - 2 = 0$$
$$(x-1)(x+2) = 0$$

$\Rightarrow$ $x = 1$, $x = -2$ but $x \geq 0$

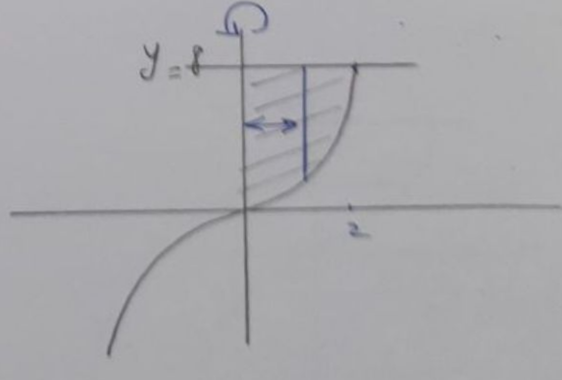
$\Rightarrow$ $x = 0$ & $x = 1$

$$V = \int_0^1 2\pi \begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{height}\end{pmatrix} dx = \int_0^1 2\pi x \left((2-x) - x^2\right) dx$$

$$= 2\pi \int_0^1 \left(2x - (x^2) - x^3\right) dx = 2\pi \left[x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{5\pi}{6}$$

Q24) Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = x^3$, $y = 8$, $x = 0$ about:

(a) y-axis

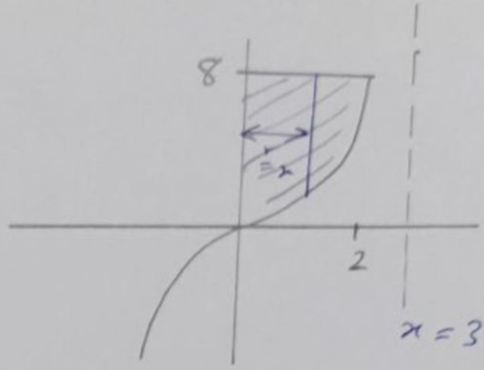


$$V = \int_0^2 2\pi \begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{height}\end{pmatrix} dx$$

$$= \int_0^2 2\pi x \left(8 - x^3\right) dx = \frac{96\pi}{5}$$

(b) The Line $x = 3$

$$V = \int_0^2 2\pi\,(3-x)(8-x^3)\,dx = \frac{264}{5}\pi$$

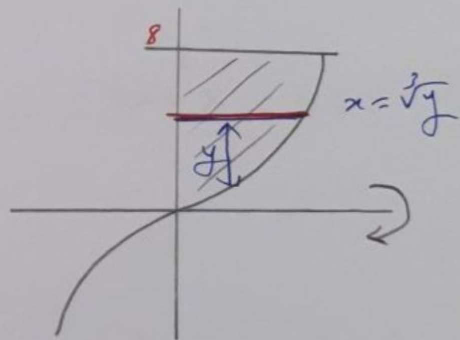


(c) The Line $x = -2$

$$V = \int_0^2 2\pi\,(x+2)(8-x^3)\,dx = \frac{336\pi}{5}.$$

(d) The x-axis $\Rightarrow dy$. (Horizontal)

$$V = \int_0^8 2\pi\, y\, y^{\frac{1}{3}}\,dy = \frac{768}{7}\pi$$

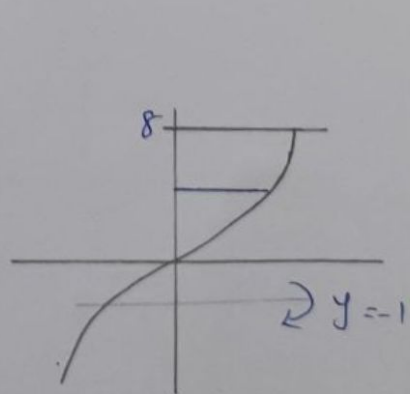


(e) The Line $y = 8$

$$V = \int_0^8 2\pi(8-y)\,y^{\frac{1}{3}}\,dy = \frac{576\pi}{7}$$

(f) The Line $y = -1$

$$V = \int_0^8 2\pi\,(y+1)\,y^{\frac{1}{3}}\,dy = \frac{936\pi}{7}$$



6.2

(14) $g(x) = \begin{cases} \dfrac{(\tan x)^2}{x} & , \quad 0 < x \leq \frac{\pi}{4} \\ 0 & , \quad x = 0 \end{cases}$

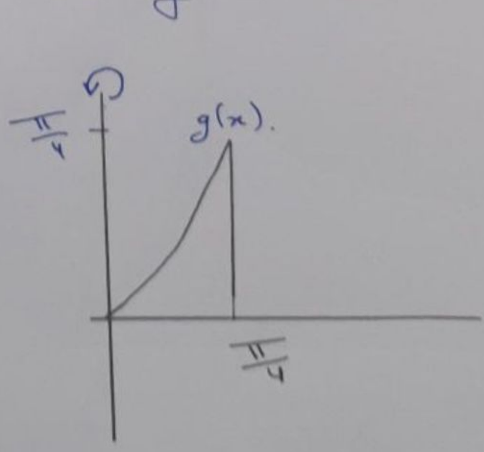
(a) show that $x\, g(x) = (\tan x)^2, \quad 0 \leq x \leq \frac{\pi}{4}$

$$x\, g(x) = \begin{cases} (\tan x)^2 & , \quad 0 < x \leq \frac{\pi}{4} \\ 0 & , \quad x = 0 \end{cases}$$

Now , since $\tan 0 = 0$, we have:

$$x\, g(x) = \begin{cases} (\tan x)^2 & , \quad 0 < x \leq \frac{\pi}{4} \\ (\tan \cancel{x})^2 & , \quad x = 0 \end{cases} = \tan^2 x \quad , 0 \leq x \leq \frac{\pi}{4}$$

(b) Find the volume of the Volume of the solid generated by revolving the region about the y-axis.



$$V = \int_0^{\pi/4} 2\pi \underbrace{x\,(g(x))}_{\text{using part (a)}}\, dx$$

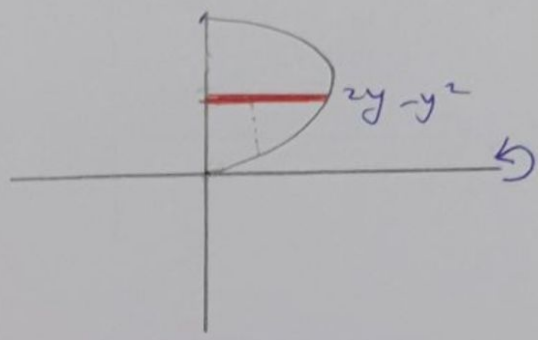
$$= \int_0^{\pi/4} 2\pi \tan^2 x \, dx$$

$$= \int_0^{\pi/4} 2\pi \,(\sec^2 x - 1)\, dx = 2\pi \Big[\tan x - x\Big]_0^{\pi/4} = \frac{4\pi - \pi^2}{2}.$$

6.2 17 Use shell method to find the volume for the solid generated by revolving $x = 2y - y^2$, $x = 0$ about the x-axis.

yes

$\Rightarrow 2y - y^2 = 0 \Rightarrow y = 0, 2$



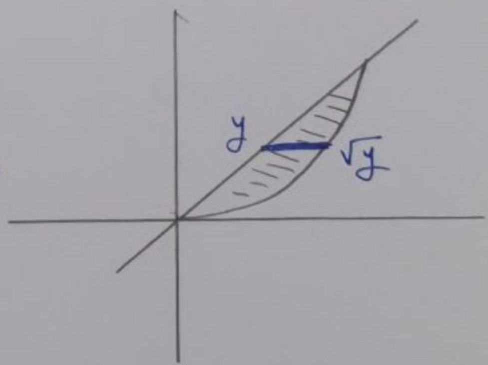
$$V = \int_0^2 2\pi \left(\begin{matrix}\text{shell}\\ \text{radius}\end{matrix}\right)\left(\begin{matrix}\text{shell}\\ \text{height}\end{matrix}\right) dy$$

$$= \int_0^2 2\pi\, y(2y - y^2)\, dy = \int_0^2 2\pi\,(2y^2 - y^3)\, dy = \boxed{\frac{8\pi}{3}}$$

yes

29 Compute the volume of the solid generated by revolving the region bounded by $y = x$ & $y = x^2$ about each coordinate axis using:

(a) Shell Method.



- about x-axis : $y = \sqrt{y} \Rightarrow y^2 - y = 0$

$$V = \int_0^1 2\pi\, y(\sqrt{y} - y)\, dy = \boxed{\frac{2\pi}{15}}$$

- about y-axis : $x = x^2 \Rightarrow x = 0, 1$

$$V = \int_0^1 2\pi\, x(x - x^2)\, dx = \boxed{\frac{\pi}{6}}$$

(b) Washer Method.

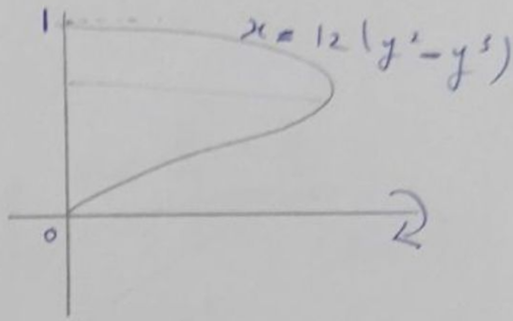
- x-axis : $V = \int_0^1 \pi\,[(x^2 - x^4)]\, dx = \boxed{\frac{2\pi}{15}}$

- y-axis : $V = \int_0^1 \pi\,[(\sqrt{y})^2 - y^2]\, dy = \boxed{\frac{\pi}{6}}$

6.2

(27) Use the shell method to find the Volume:

$x = 12(y^2 - y^3)$, $y = 0, 1$



(a) about the x-axis.

$$V = \int_0^1 2\pi\, y\,(12)(y^2 - y^3)\, dy = \frac{6\pi}{5}$$

(b) The Line $y = 1$

$$V = \int_0^1 2\pi\,(1-y)\left[12(y^2-y^3)\right] dy = \frac{4\pi}{5}$$

(c) The Line $y = \frac{8}{5}$

$$V = \int_0^1 2\pi\,\left(\frac{8}{5} - y\right)\left[12(y^2-y^3)\right] dy = 2\pi$$

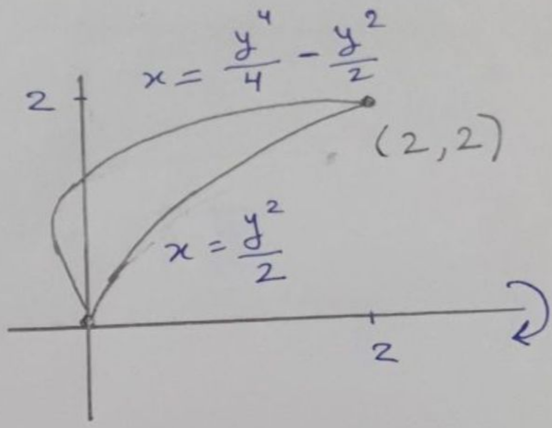
(d) The Line $y = -\frac{2}{5}$

$$V = \int_0^1 2\pi\,\left(y + \frac{2}{5}\right)\left[12(y^2-y^3)\right] dy = 2\pi$$

6.2 Use the shell method to find the volumes generated

(28) by revolving the shaded region about:

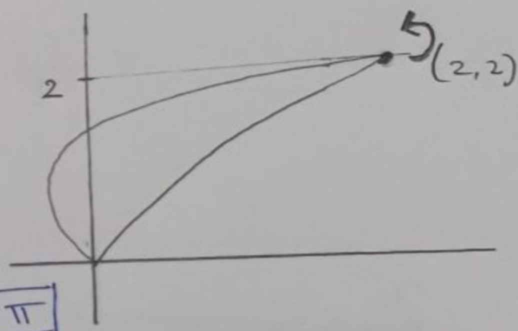
(a) x - axis:



$$V = \int_0^2 2\pi (y)\left(\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right) dy$$

$$= 2\pi \int_0^2 \left(y^3 - \frac{y^5}{4}\right) dy = \boxed{\frac{8\pi}{3}}$$

(b) The line $y = 2$.



$$V = \int_0^2 2\pi (2-y)\left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy$$

$$= 2\pi \int_0^2 \left(2y^2 - \frac{y^4}{2} - y^3 + \frac{y^5}{4}\right) dy = \boxed{\frac{8\pi}{5}}$$

(c) The line $y = 5$

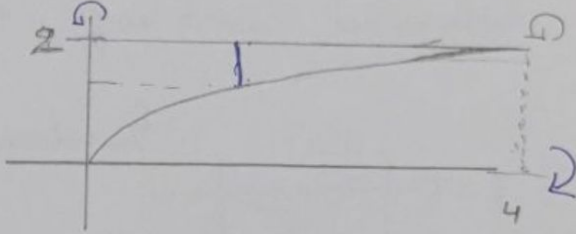
$$V = \int_0^2 2\pi (5-y)\left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \boxed{8\pi}$$

(d) The line $y = -\frac{5}{8}$

$$V = \int_0^2 2\pi \left(y + \frac{5}{8}\right)\left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] dy = \boxed{4\pi}$$

6.2 (32) $y = \sqrt{x}$, $y = 2$, $x = 0$ about

(a) The x-axis.



shell: $V = \int_0^2 2\pi y (y^2)\, dy = 8\pi$

washer: $V = \int_0^4 \pi \left[(2)^2 - (\sqrt{x})^2 \right] dx = 8\pi$

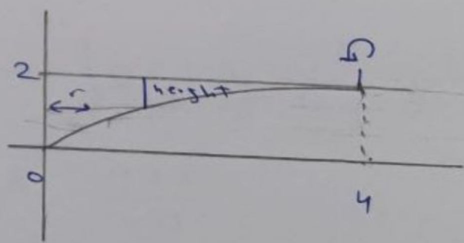
(b) The y-axis

shell : $V = \int_0^4 2\pi x \cancel{(2-\sqrt{x})} \, dx = \frac{32\pi}{5}$

$(2 - \sqrt{x})$

Disk: $V = \int_0^2 \pi (y^2)^2 \, dy = \frac{32}{5}\pi$

(c) The line $x = 4$



shell: $V = \int_0^4 2\pi (4 - x)(2 - \sqrt{x})\, dx$

$= \frac{224\pi}{15}$

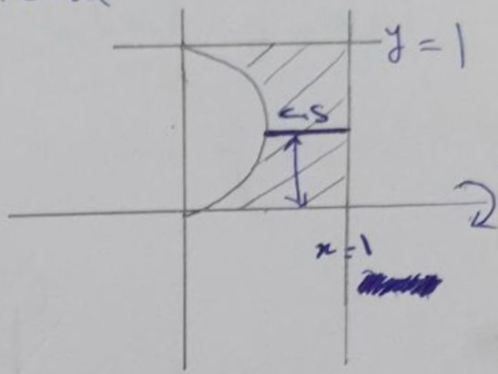
washer: $V = \int_0^2 \pi \left[4^2 - (4 - y^2)^2 \right] dy = \frac{224\pi}{15}$

(d) The line $y = 2$

shell: $V = \int_0^2 2\pi (2 - y)(y^2)\, dy = \frac{8\pi}{3}$

Disk: $V = \int_0^4 \pi (2 - \sqrt{x})^2 \, dx = \frac{8\pi}{3}$

Q34) (6.2) Find the volume of the solid generated by revolving the region in the first quadrant bounded by $x = y - y^3$, $x = 1$, $y = 1$ about:

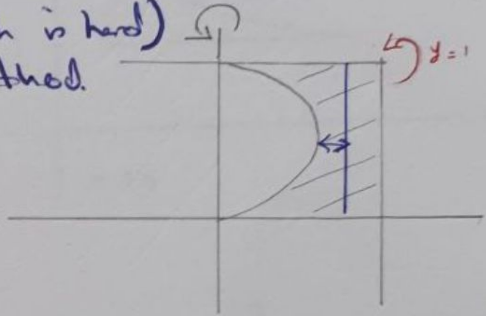


(a) The x-axis

$$V = \int_0^1 2\pi\, y\, [1 - (y - y^3)]\, dy$$

$$= \frac{11\pi}{15}$$

(Disk Method can't use, since we have to write $y = f(x)$ which is hard)
so, we use shell method.



(b) the y-axis

$$\cancel{V = \int_0^1 2\pi (1 - (\dots)) dx}$$

Use Washer method:

$$V = \int_0^1 \pi [1^2 - (y - y^3)^2]\, dy = \int_0^1 \pi (1 - y^2 - y^6 + 2y^4)\, dy$$

$$= \frac{97\pi}{105}$$

(c) The line $x = 1$

Use ~~Washer~~ Disk method: $V = \int_0^1 \pi [1 - (y - y^3)]^2\, dy = \frac{121\pi}{210}$

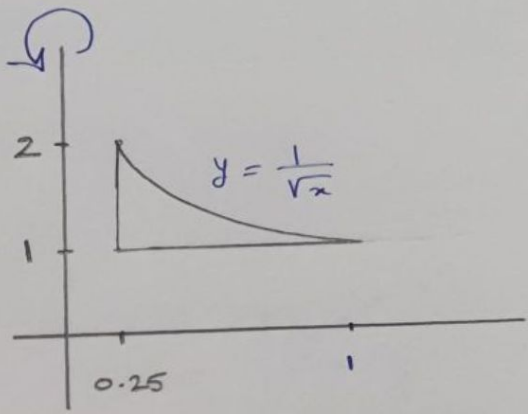
(d) The line $y = 1$: (shell method):

$$V = \int_0^1 2\pi (1 - y)[1 - (y - y^3)]\, dy = \frac{23\pi}{30}$$

6.2

(38) The region in the first quadrant that is bounded above by $y = \frac{1}{\sqrt{x}}$, on the left by $x = \frac{1}{4}$ and below by $y = 1$ is revolved about y-axis to a generate a solid. Find the Volume of the solid by:

(a) Washer Method.



$$V = \int_c^d \pi \left[(R(y))^2 - (r(y))^2 \right] dy$$

$$= \int_1^2 \pi \left[\left(\frac{1}{y^2}\right)^2 - \left(\frac{1}{4}\right)^2 \right] dy$$

$$= \int_1^2 \pi \left[\frac{1}{y^4} - \frac{1}{16} \right] dy = \pi \left[-\frac{1}{3} y^{-3} - \frac{y}{16} \right]_1^2 = \boxed{\frac{11\pi}{48}}$$

(b) Shell method:

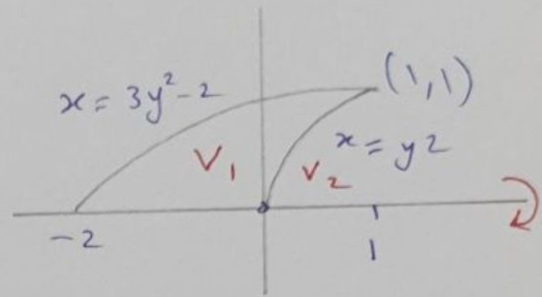
$$V = \int_{0.25}^{1} 2\pi (x) \left(\frac{1}{\sqrt{x}} - 1 \right) dx = \int_{0.25}^{1} 2\pi \left(x^{\frac{1}{2}} - x \right) dx$$

$$= 2\pi \left[\frac{2}{3} x^{3/2} - \frac{x^2}{2} \right]_{0.25}^{1} = \boxed{\frac{11\pi}{48}}$$

(39) The Region is Revolved about x-axis:

(6.2)

Disk:

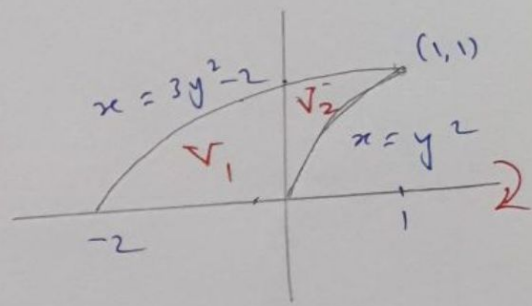


$$V = V_1 - V_2$$

$$V_1 = \int_{-2}^{1} \pi \left[\sqrt{\frac{x+2}{3}}\right]^2 dx = 1.5\pi$$

$$V_2 = \int_{0}^{1} \pi \left[\sqrt{x}\right]^2 dx = 0.5\pi$$

$$V = 1.5\pi - 0.5\pi = \pi$$



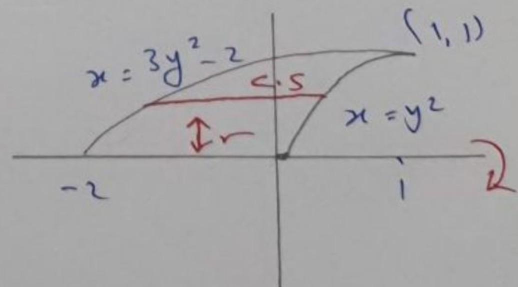
Washer:

$$V = V_1 + V_2$$

$$V_1 = \int_{-2}^{0} \pi \left(\left[\sqrt{\frac{x+2}{3}}\right]^2 - [0]^2 \right) dx = \frac{2}{3}\pi$$

$$V_2 = \int_{0}^{1} \pi \left(\left[\sqrt{\frac{x+2}{3}}\right]^2 - \left[\sqrt{x}\right]^2 \right) dx = \frac{1}{3}\pi$$

$$V_1 + V_2 = \frac{2}{3}\pi + \frac{1}{3}\pi = \pi$$



Shell

$$V = \int_{0}^{1} 2\pi (y) \left[y^2 - (3y^2 - 2) \right] dy = \pi$$