جامعة بيرزيت
BIRZEIT UNIVERSITY

Computer Science Department

Worksheet #1 , COMP233

1- Prove the following logical equivalence:

$$\sim ((-p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$$

2- Construct truth table for the following statement.

$$P \wedge \sim R \leftrightarrow Q \vee R$$

3- Does the following argument form is Valid or Invalid?

$p \vee q$

$p \rightarrow \sim q$

$p \rightarrow r$

$\therefore r$

4- Let *Q(n)* be the predicate "$n^2 \leq 30$."

a. Write $Q(2)$, $Q(-2)$, $Q(7)$, and $Q(-7)$, and indicate which of these statements are true and which are false.

b. Find the truth set of $Q(n)$ if the domain of n is $\mathbf{Z}$.

c. If the domain is the set $\mathbf{Z}^+$ of all positive integers, what is the truth set of $Q(n)$?

5. True or false

$\forall$ Positive integers m and n, $m \cdot n \geq m + n$.

6. Consider the following statement:

$\forall$ basketball players x, x is tall.

Which of the following are equivalent ways of expressing this statement?

a. Every basketball player is tall.

b. Among all the basketball players, some are tall.

c. Some of all the tall people are basketball players.

d. Anyone who is tall is a basketball player.

e. All people who are basketball players are tall.

f. Anyone who is a basketball player is a tall person.

7- Consider the statement

"All integers are rational numbers but some rational numbers are not integers."

a. formalize the statement in the form "$\forall x$, if then , but $\exists$ x such that ."

b. Let **Rational*(x)*** be "***x* is a rational number**" and **Int*(x)*** be "***x* is an integer**." Write the given statement formally using only the symbols: Rational(x), Int(x), $\forall$, $\exists$, $\wedge$, $\vee$, $\sim$, and $\rightarrow$.

8- Consider the statement:

$$\forall x \in \mathbb{R}, \exists \text{ a real number } y \text{ such that } x + y = 0.$$

a. Verbalize the statement.

b. Write a negation for the statement in **verbal form** and **formal form**.

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Worksheet #2 , COMP233

1- Assume that *k* is a particular integer.

a. Is -17 an odd integer?

b. Is 0 an even integer?

c. Is $2k - 1$ odd?

2. Assume that *m* and *n* are particular integers.

a. Is $6m + 8n$ even?

b. Is $10mn + 7$ odd?

3. Prove or disprove? For all integers n, if n is odd then $\frac{n-1}{2}$ is odd.

4. Prove that the product of an even integer and an odd integer is even.

5. Write the following rational numbers as a ratio of two integers.

a. 4.6037

b. $320.5\overline{492492492}$

6. Consider the statement: The square of any rational number is a rational number.

a. Write the statement formally using a quantifier and a variable.

b. Determine whether the statement is true or false and justify your answer.

7 . Is $6m(2m + 10)$ divisible by 4?

8. If $n = 4k + 1$, does 8 divide $n^2 - 1$?

9. For all integers *a, b,* and *c,* if $a \mid b$ and $a \mid c$ then $a \mid (b + c)$.

10. Use the unique factorization theorem to write the following integers in standard factored form.

1,176

11. a. If *a* and *b* are integers and $12a = 25b$, does $12 \mid b$? does $25 \mid a$? Explain.

b. If *x* and *y* are integers and $10x = 9y$, does $10 \mid y$? does $9 \mid x$? Explain.

12. Prove that for all integers *n,* if $n \bmod 5 = 3$ then $n^2 \bmod 5 = 4$.

13. Prove if *n* is an integer , then $n(n+1)$ even number.

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Q35. The fourth power of any integer has the form 8m or 8m+1 for some integer m

Q35: Solution:

Suppose: n is any integer number which particular but arbitrarily chosen.

Show: that $n^4 = 8m$ or $n^4 = 8m+1$

Proof: using definition of any number which odd or even, then

$n = 2k$ even, odd numbers.
$n = 2k+1$

$\therefore$ 1) for case (n is even), then

$n^4 = (2k)^4 = 2^4 k^4 = 2^3(2k^4)$
$= 8(2k^4)$

Let $m = 2k^4 \Rightarrow \boxed{n^4 = 8m}$ #

2) (n is odd)

$n^4 = (2k+1)^4 = 16k^4 + 32k^3 + 24k^2 + 8k + 1$

$= 8\underbrace{(2k^4 + 4k^3 + 3k^2 + k)}_{m} + 1$

$\therefore \boxed{n^4 = 8m+1}$ #

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Q36. The product of any four consecutive integers is divisible by 8

Q36:

Suppose n is any integer (pbac)

Show: $n, n+1, n+2, n+3$ are consecutive integer divisible by 8

Proof: Since d is 4. [four consecuative]

by using quotient-Reminder theorm.

$m = qd + r$, $d = 4$

$\therefore m = 4q + r \quad 0 \leq r < 4$

for $r = 0, 1, 2, 3$

$\therefore m = 4q$, $m = 4q+1, 4q+2, 4q+3$

Find product: for $(r = 0)$

$n(n+1)(n+2)(n+3) = 4q(4q+1)(4q+2)(4q+3)\cancel{(4q+}$

$= 4q(4q+1)(2(2q+1))(4q+3)$

$= 8q(4q+1)(2q+1)(4q+3)$

$= 8\underbrace{(q(4q+1)(2q+1)(4q+3))}_{m}$

$= 8m$

So, this term is divisable by 8 $\boxed{8 \mid 8m}$

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Case 2:

for $(r = 1)$

$$n = 4q + r = 4q + 1$$

$\therefore$ Subsitute in (n)

$$n(n+1)(n+2)(n+3) = (4q+1)(4q+2)(4q+3)(4q+4)$$

$$= 8\underbrace{((4q+1)(2q+1)(4q+3)(q+1))}_{8m}$$

$$= 8m$$

$$\boxed{\therefore 8 \mid 8m} \ \#$$

Case 3: $(r = 2)$

$$n = 4q + 2$$

$$n(n+1)(n+2)(n+3) = (4q+2)(4q+3)(4q+4)(4q+5)$$

$$= 8\underbrace{((2q+1)(4q+3)(q+1)(4q+5))}_{m}$$

$$= 8m$$

$$\boxed{\therefore 8 \mid 8m} \ \#$$

and also for Case $r = 3$ (leaf for you)

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Q37. For any integer n, n^2 + 5 is not divisible by 4

Q37: Suppose n is any integer (pbac)

Show: n, n^2+5 is divisible by 4.

Case I any number means (odd, even)

even: $n = 2k$ for some k integer

$$(n^2+5) = (2k)^2+5$$
$$= 4k^2+5$$
$$= 4k^2+4+1$$
$$= 4\underbrace{(k^2+1)}_{m}+1$$
$$= 4m+1$$

Since multiplication and sum of any integer is integer

this is ⇒ odd

$\therefore$ $4 \nmid 4m+1$ (not divisible by 4.)

case II: odd

$$n = 2k+1$$
$$n^2+5 = (2k+1)^2+5$$
$$= 4k^2+4k+1+5$$
$$= 4\underbrace{(k^2+k+1)}_{q}+2$$

by theorem quotient-R

$$m = 4q+2$$

it's ⇒ not divisible by 4 Since the reminder r is 2

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Q39. Every prime number except 2 and 3 has the form 6q + 1 or 6q + 5 for some integer q

Q 39: Every prime number p

Show: $6q+1$ or $6q+5$ for some integer q is form of prime other than 2, 3

Proof: since it's consecutive, thus Use quotient + Reminder theorm.

$$\boxed{m = qd + r \qquad 0 \leq r < d}$$

$\therefore \quad P = 6q + r \qquad 0 \leq r < 6$

$r=0$, $P = 6q \Rightarrow \frac{P}{q} = 6$ Prime Not

$r=1$, $P = 6q+1$ ✓

$r=2$, $P = 6q+2 \Rightarrow P = 2(3q+1)$, 2 divide P

$r=3$, $P = 6q+3 \Rightarrow P = 3(2q+1)$ 3 divide P

$r=4$, $P = 6q+4 \Rightarrow P = 2(3q+2)$ 2 =

$r=5$, $P = 6q+5$ ✓

$\therefore$ for
$$\boxed{\begin{array}{ll} r=1 & P = 6q+1 \\ r=5 & P = 6q+5 \end{array}}$$

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Q40. If n is any odd integer, then n^4 mod 16 = 1

Q40 If n anyodd :

Suppose n is (pabc). is odd

Show: $n^4 \bmod 16 = 1$

proof: $n = 2k+1$ for some k

from defintion of odd.

$$\therefore n^4 = (2k+1)^4$$

$$= (2k+1)^2 (2k+1)^2$$

$$= (4k^2+4k+1)(4k^2+4k+1)$$

$$= 16k^4 + 32k^3 + 8k^2 + 8k + 1$$

$$= \cancel{16}(16k^4 + 32k^3 + 8k(k+1) + 1$$

$\Rightarrow$ since the product of even and odd is even then $8\underbrace{k(k+1)} = (2q) \times 8$

$$\therefore n^4 = 16k^4 + 32k^3 + 16q + 1$$

$$= 16\underbrace{(k^4 + 2k + q)}_{m} + 1$$

$\therefore n^4 = 16m + 1$ $\therefore$ this $\boxed{\text{done}}$

$\boxed{r=1}$

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Q49. If m, n, a, b, and d are integers, d > 0, and m mod d = a and n mod d = b, is (mn) mod d = ab? Prove your answers

Q49:

For all integers m, n, a, b, and $d > 0$

and $m \bmod d = a$ and $n \bmod d = b$

then $mn \bmod d = ab \bmod d$

by-qoutien-R theorm.

Since,

$m = dp + a$

$n = dq + b$

$\Rightarrow mn = (dp+a)(dq+b)$

$= (dp)(dq) + a(dq) + b(dp) + ab$

$= d^2q + d(aq+bp) + ab$

$= d(\underbrace{dq + aq + bp}_{\text{Intege}}) + ab$

$\therefore mn = dk + ab$

$\therefore mn \bmod d = k$

& $mn \operatorname{div} d = ab$

Counter example: for (a) #

$m = 4$ $d = 5$ $a = 4 \bmod 5 = 4$

$n = 3$ $b = 3 \bmod 5 = 3$ $\boxed{2 \neq ab}$

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Worksheet #3 , COMP233

1- Write an explicit formulas for the following sequences with the initial terms.

i – $\frac{1}{3}, \frac{4}{9}, \frac{9}{27}, \frac{16}{81}, \frac{25}{243}, \frac{36}{729}$

ii – $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}$

2- Compute the summations $\sum_{n=1}^{10}\left(\frac{1}{n}-\frac{1}{n+1}\right)$

3- Write the summation in expanded form $\sum_{j=1}^{n} j(j+1)$

4- Rewrite by separating off the final term. $\sum_{m=1}^{n+1} m(m+1)$

5- Write the following using product notation

$(1-t)\cdot(1-t^2)\cdot(1-t^3)\cdot(1-t^4)$

6- Transform the following by making the change of variable *i* = *k* + *1*.

$$\sum_{k=0}^{5} k(k-1)$$

$$\prod_{k=1}^{n} \frac{k}{k^2+4}$$

7- Write as a single summation or product. $\sum_{k=1}^{n}(2k-3)+\sum_{k=1}^{n}(4-5k)$

8- Prove that for all nonnegative integers *n* and *r* with *r* + 1 ≤ *n*,

$$\binom{n}{r+1}=\frac{n-r}{r+1}\binom{n}{r}$$

9- Prove by mathematical induction, *n*≥ *1*. $1^2+2^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$.

10 - Prove $3^{2n}-1$ is divisible by 8, for each integer $n \geq 0$.

11 – Prove $n^2 < 2^n$, for all integers $n \geq 5$.

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Worksheet #4 , COMP233

1- Let $C = \{n \in \mathbf{Z} \mid n = 6r - 5 \text{ for some integer } r\}$ and
$D = \{m \in \mathbf{Z} \mid m = 3s + 1 \text{ for some integer } s\}$.
Prove or disprove C=D.

2- Let $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9\}$, and $C = \{2, 4, 6, 8\}$.
Find each of the following:

a. $A \cup B$ **b.** $A \cap B$ **c.** $A \cup C$ **d.** $A \cap C$

e. $A - B$ f. $B - A$ g. $B \cup C$ h. $B \cap C$

3- Let $C_i = \{i, -i\}$ for all nonnegative integers i. Find

a. $\bigcup_{i=0}^{4} C_i$

b. $\bigcap_{i=0}^{4} C_i$

c. Are $C0$, $C1$, $C2$, . . . mutually disjoint?

d. $\bigcup_{i=0}^{\infty} C_i$

e. $\bigcap_{i=0}^{\infty} C_i$

4- Use an ***element argument*** to prove the following statement

For all sets A and B, $(A \cap B) \cup (A \cap B^c) = A$

5- Prove for all sets A and B, $(A \cap B) \cap (A \cap B^c) = \emptyset$.

6- Prove or disprove for all sets A, B, and C, $(A \cap B) \cup C = A \cap (B \cup C)$.

7- Construct an ***algebraic proof*** for the given statement. Cite a property from Theorem 6.2.2 for every step. For all sets A, B, and C,

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

8- Use an element argument to prove the following statement:

For all sets A, B and C, $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(The solution of Q8 is in the next page)

Q8 /workSheet. Prove that $A\times(B\cup C)=(A\times B)\cup(A\times C)$

Suppose that A, B and C are a.c. sets.

To show that $A\times(B\cup C)=(A\times B)\cup(A\times C)$

we must show that $A\times(B\cup C)\subseteq(A\times B)\cup(A\times C)$

and we must show that $(A\times B)\cup(A\times C)\subseteq A\times(B\cup C)$

Part 1: $A\times(B\cup C)\subseteq(A\times B)\cup(A\times C)$

Suppose $(x,y)\in A\times(B\cup C)$

We must show that $(x,y)\in(A\times B)\cup(A\times C)$

$x\in A$ and $y\in B\cup C$

so, $y\in B$ or $y\in C$ by def. of union.

Case1: $y\in B$

since $x\in A$ and $y\in B$, then $(x,y)\in A\times B$

Hence, $(x,y)\in(A\times B)\cup(A\times C)$

Case2: $y\in C$

since $x\in A$ and $y\in C$; then $(x,y)\in A\times C$

Hence $(x,y)\in(A\times B)\cup(A\times C)$

therefore if $(x,y)\in A\times(B\cup C)$, then $(x,y)\in(A\times B)\cup(A\times C)$

$\therefore A\times(B\cup C)\subseteq(A\times B)\cup(A\times C)$

Part2: $(A\times B)\cup(A\times C)\subseteq A\times(B\cup C)$

suppose $(x,y)\in(A\times B)\cup(A\times C)$

we must show that $(x,y)\in A\times(B\cup C)$

Then, $(x,y)\in(A\times B)$ or $(x,y)\in(A\times C)$

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Case1: $(x,y)\in(A\times B)$

Then $x\in A$ and $y\in B$

Since $y\in B$ then $y\in B\cup C$ by def. of union.

Hence $x\in A$ and $y\in(B\cup C)$

$\therefore (x,y)\in A\times(B\cup C)$

Case2: $(x,y)\in(A\times C)$

Then $x\in A$ and $y\in C$

Since $y\in C$ then $y\in(B\cup C)$

Hence $x\in A$ and $y\in(B\cup C)$

$\therefore (x,y)\in A\times(B\cup C)$

in either cases $(A\times B)\cup(A\times C)\subseteq A\times(B\cup C)$

$\therefore$ Therefore $A\times(B\cup C)=(A\times B)\cup(A\times C)$

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Worksheet #5 , COMP233

Q#1
Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$ and define relations R and S from A to B as follows:
For all $(x, y) \in A \times B$,
$x \, R \, y \Leftrightarrow |x| = |y|$ and
$x \, S \, y \Leftrightarrow x - y$ is even.
State explicitly which ordered pairs are in $A \times B$, R, S.

Q#2 determine whether the given relation is
reflexive, **symmetric**, **transitive**, or **none of these**.
Justify your answers.
For all positive integers m and n, $m \, D \, n \Leftrightarrow m \mid n$.

D is the "divides" relation on $\mathbf{Z}^+$:

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Worksheet #6 , COMP233

1- Suppose that on a true/false exam you have no idea at all about the answers to three questions. You choose answers randomly and therefore have a 50–50 chance of being correct on any one question. Let *CCW* indicate that you were correct on the first two questions and wrong on the third, let *WCW* indicate that you were wrong on the first and third questions and correct on the second, and so forth.
a. List the elements in the sample space whose outcomes are all possible sequences of correct and incorrect responses on your part.'
b. Write each of the following events as a set and find its probability:
(i) The event that exactly one answer is correct.
(ii) The event that at least two answers are correct.
(iii) The event that no answer is correct.

2- If the largest of 56 consecutive integers is 279, what is the smallest?

3- One box contains two black balls (labeled B1 and B2) and one white ball. A second box contains one black ball and two white balls (labeled W1 and W2). Suppose the following experiment is performed: One of the two boxes is chosen at random. Next, a ball is randomly chosen from the box. Then a second ball is chosen at random from the same box without replacing the first ball.

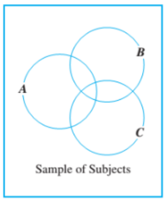
a. Construct the possibility tree showing all possible outcomes of this experiment.
b. What is the total number of outcomes of this experiment?
c. What is the probability that two black balls are chosen?
d. What is the probability that two balls of opposite color are chosen?

4- Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. For example, 9A2D16 and BC5416 are hexadecimal numbers.
a. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F, and are 5 digits long?
b. How many hexadecimal numbers begin with one of the digits 4 through D, end with one of the digits 2 through E, and are 6 digits long?

5- Prove that for all integers $n \geq 3$, $P(n + 1, 3) - P(n, 3) = 3P(n, 2)$.

6- Suppose that all license plates consist of from four to six symbols chosen from the 26 letters of the alphabet together with the ten digits 0–9.
a. How many license plates are possible if repetition of symbols is allowed?
b. How many license plates do not contain any repeated symbol?

7- A group of eight people are attending the movies together.
a. Two of the eight insist on sitting side-by-side. In how many ways can the eight be seated together in a row?
b. Two of the people do not like each other and do not want to sit side-by-side. Now how many ways can the eight be seated together in a row?
8- A study was done to determine the efficacy of three different drugs—*A*, *B*, and *C*—in relieving headache pain. Over the period covered by the study, 50 subjects were given the chance to use all three drugs. The following results were obtained:
21 reported relief from drug *A*.
21 reported relief from drug *B*.
31 reported relief from drug *C*.



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9 reported relief from both drugs *A* and *B*.
14 reported relief from both drugs *A* and *C*.
15 reported relief from both drugs *B* and *C*.
41 reported relief from at least one of the drugs.
Note that some of the 21 subjects who reported relief from drug *A* may also have reported relief from drugs *B* or *C*. A similar occurrence may be true for the other data.
a. How many people got relief from none of the drugs?
b. How many people got relief from all three drugs?
c. Let *A* be the set of all subjects who got relief from drug *A*, *B* the set of all subjects who got relief from drug *B*, and *C* the set of all subjects who got relief from drug *C*. Fill in the numbers for all eight regions of the diagram below.
d. How many subjects got relief from *A* only?

9- A student council consists of 15 students.

a. In *how* many ways can a committee of six be selected from the membership of the council?
b. Two council members have the same major and are not permitted to serve together on a committee. How many ways can a committee of six be selected from the membership of the council?
c. Two council members always insist on serving on committees together. If they can't serve together, they won't serve at all. How many ways can a committee of six be selected from the council membership
d. Suppose the council contains eight men and seven women.
(i) How many committees of six contain three men and three women?

10- Find how many solutions there are to the given equation that satisfy the given condition.

$x1 + x2 + x3 = 20$, each x_i is a positive integer.

$x1 + x2 + x3 = 20$, each x_i is a nonnegative integer.

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