Power Series : are infinit sum of poly's

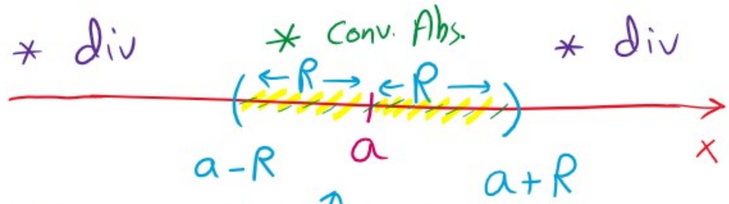
$$* = \sum_{n=0}^{\infty} a_n (x-a)^n = a_0 + a_1 (x-a) + a_2 (x-a)^2 + a_3 (x-a)^3 + \cdots$$

a_i : coefficients

a : center

R : Radius of Convergence

IC : Interval of Convergence



IC : $|x-a| < R$

$-R < x-a < R$

$\boxed{a - R < x < a + R}$

Note : To find R and IC $\Rightarrow$ we apply RT $\to$ Ratio Test

$\sum a_n (x-a)^n$

Exp ① $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$

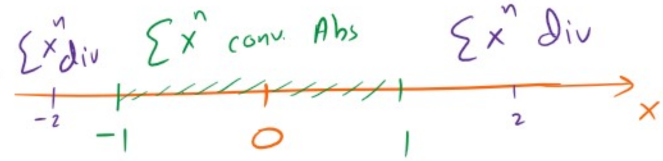
geometric since $r = x$

$\Downarrow$

$a = 0$ center

• If $\boxed{|x| < 1}$ $\Rightarrow \sum_{n=0}^{\infty} x^n$ converges to

$-1 < x < 1$



$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots = \frac{1}{1-x}$$

$x \in (-1,1)$

$IC = (-1,1)$

Take $x = \frac{1}{2} \Rightarrow \sum \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

Take $x = 2 \Rightarrow \sum 2^n$ div by n^{th} term test since $\lim_{n\to\infty} 2^n = \infty$

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$$f(x) = \frac{1}{1-x} \Rightarrow \text{we can approximate } f(x) \text{ by}$$

$|x| < 1$

$P_0(x) = 1$

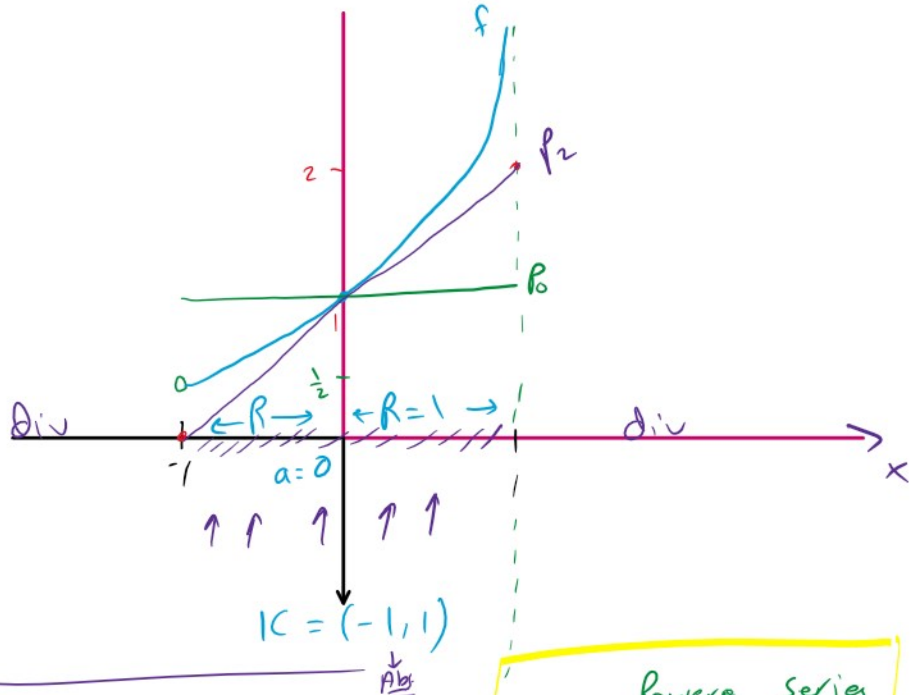
$P_1(x) = 1 + x$

$P_2(x) = 1 + x + x^2$

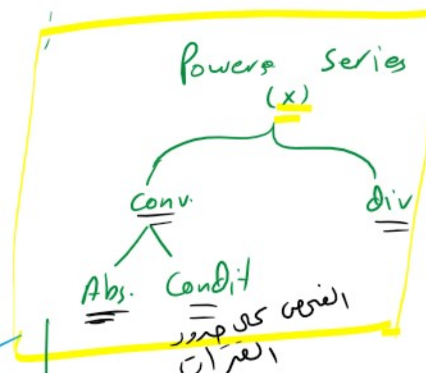
$P_3(x) = 1 + x + x^2 + x^3$

$f(-1) = \frac{1}{1--1} = \frac{1}{2}$

$f(0) = \frac{1}{1-0} = 1$



Exp Find R and IC for



(1) $$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} \longrightarrow a_n \qquad (a=0)$$

$x^n, (-1)^n \qquad x^{n+1}$

Apply $\underline{RT} \Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\cancel{x^{n+1}}\, x}{\cancel{(n+1)!}\,(n+1)} \cdot \frac{\cancel{n!}}{\cancel{x^n}} \right|$

$(n+1)! = (n+1)\, n!$

$$= |x| \boxed{\lim_{n\to\infty} \frac{1}{n+1}}$$

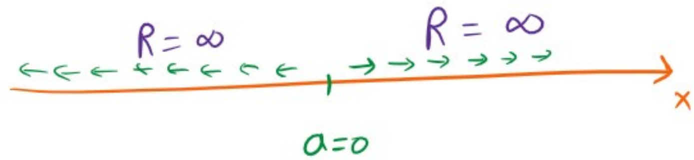
$\underline{RT}$

$\checkmark\ \rho < 1 \Rightarrow$ conv.

$\rho > 1 \Rightarrow$ div

$= |x| \ 0$

$= 0 \ \boxed{<} \ 1$



$a=0$

$R=\infty$
$IC=\mathbb{R}=(-\infty,\infty)$ $\Big\}\Rightarrow \sum \frac{(-1)^n x^n}{n!}$ conv. Abs. for every x

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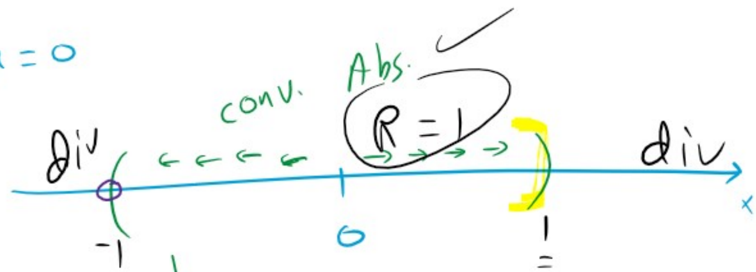
② $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \Rightarrow a=0$



Apply RT

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\cancel{x^{n+1}}}{n+1}\cdot\frac{n}{\cancel{x^n}}\right|$

$= |x| \boxed{\lim_{n\to\infty}\frac{n}{n+1}}$

$= |x| \; (1)$

$= |x| \boxed{<} 1$

$\boxed{-1 < x < 1}$

Conv. Abs.

To check conv. Condit
$\Rightarrow$ check endpoints

$x=-1 \Rightarrow \sum_{n=1}^{\infty} (-1)^{n-1}\frac{(-1)^n}{n} = \sum \frac{(-1)^{2n-1}}{n}$

$= \sum (-1)^{2n}(-1)^{-1}\frac{1}{n}$

$= \sum 1 \cdot \frac{1}{-1}\cdot\frac{1}{n} = -\sum\frac{1}{n}$ Harmonic div

$x=1 \Rightarrow \sum (-1)^{n-1}\frac{(1)^n}{n}$

$= \sum \frac{(-1)^{n-1}}{n}$ Alternating Harmonic series

conv

$x=1 \Rightarrow \sum\frac{(-1)^{n-1}}{n}$ is not conv. Abs

since $\sum|a_n| = \sum\frac{1}{n} \to$ div

$R=1$

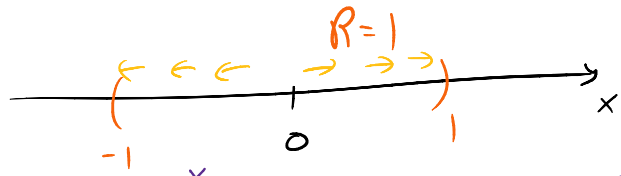
$IC=(-1,1]$

$x=1 \Rightarrow \sum (-1)^{n-1}\frac{x^n}{n}$ conv. but not Abs.

conv. conditionally

conv. Abs. only $(-1,1)$

(24) $\sum_{n=1}^{\infty} (\ln n)\, x^n \quad a=0$



Apply RT $\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\ln(n+1)\, \cancel{x^{n+1}}}{\ln n\, \cancel{x^n}} \right| = |x| \boxed{\lim_{n\to\infty} \frac{\ln(n+1)}{\ln n}} \quad \frac{\infty}{\infty}$

$= |x| \lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n}} = |x| \boxed{\lim_{n\to\infty} \frac{n}{n+1}} = |x|(1)$

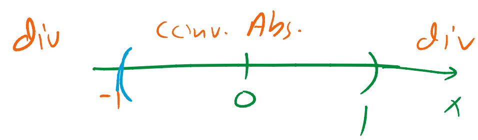
$= |x| \boxed{<} 1$

$-1 < x < 1$

Conv. Abs. on $(-1,1) \Rightarrow$ conv on $(-1,1)$

$R = 1$

$IC = (-1,1)$



$x = 1 \Rightarrow \sum \ln n\, x^n = \sum \ln n\, 1^n = \sum \ln n$ div by n^{th} term test (a_n)

$x = -1 \Rightarrow \sum \ln n\, x^n = \sum \ln n\, (-1)^n$ Alternating

(3) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \ln n = \infty \neq 0$

div

$\nexists\, x$ s.t $\sum \ln n\, (x)^n$ conv. Condit.