

diff: differentiable

$f'(x_0)$ :  $f$  prime at  $x_0$

$f''(x_0)$ :  $f$  double prime at  $x_0$

$f'''(x_0)$ :  $f$  triple prime at  $x_0$

$f^{(4)}(x_0) = f^{(4)}(x_0)$ :  $f$  super 4 at  $x_0$   $f, f', f'', f'''$

$$f^{(4)}(x_0) = [f'(x_0)]^4 = f'(x_0) \cdot f'(x_0) \cdot f'(x_0) \cdot f'(x_0)$$

(10)

$f^{(10)}(x_0)$ :  $f$  super 10 at  $x_0$  ...

Def The derivative of  $f(x)$  at point  $x_0$  is

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} \quad \text{"limit exist"}$$

$$\frac{(r)x_0 - (p+r)x_0}{p} \quad \begin{matrix} \sum_{i=1}^n (r)x_0 \\ \leftarrow p \end{matrix}$$

Ex  
Find derivative of  $f(x) = 2x - 1$  at  $x_0 = 2$

Find derivative of  $f(x) = 2x - 1$  at  $x_0 = 2$  using the definition of derivative

$$f'(x) = \underline{\underline{2}}$$

$$f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(2[2+h] - 1) - (2[2] - 1)}{h}$$


$$= \lim_{h \rightarrow 0} \frac{\cancel{4+2h-1} - \cancel{(4-1)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2h}}{\cancel{h}} = \boxed{2}$$

limit exists

$$f'(2) = 2$$

If  $f'(x_0)$  exists  $\Rightarrow$   $f$  is differentiable at  $x_0$

Q. when  $f$  is diff on  $[a, b]$  

A.  $\rightarrow$   $f$  must be diff on open interval  $(a, b)$

✓ "  $f'(x)$  exists for every  $x \in (a, b)$

$\rightarrow$  The right-hand derivative of  $f$  at  $a$  exists

$$n'. \quad 1. \quad f(a+h) - f(a)$$

$$f'_+(a) = \lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h}$$

→ The left-hand derivative of  $f$  at  $b$  exists

$$f'_-(b) = \lim_{h \rightarrow 0^-} \frac{f(b+h) - f(b)}{h}$$

Remark  $f$  is diff at  $c$  iff ( $\Leftrightarrow$ )  
if and only if

(1)  $f'_+(c)$  exists and

(2)  $f'_-(c)$  exists and

(3)  $f'_+(c) = f'_-(c)$

Exp (1) True / False

✓ If  $f$  is diff at  $c$  then  
 $f$  is cont. at  $c$  T

(2) If  $f$  is cont. at  $c$  then  
 $f$  is diff at  $c$  F

Exp Find cont. function at  $x=0$   
 but the function is not diff at  $x=0$

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$

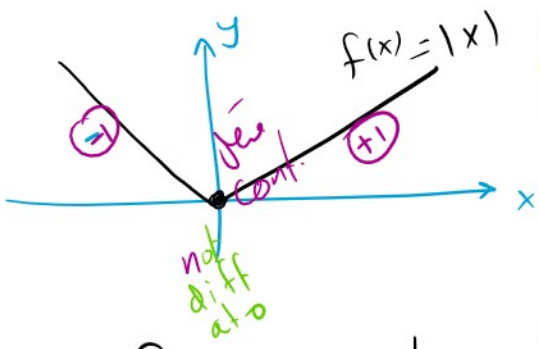


$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x = 0$$

$f$  cont. at  $x=0 \Rightarrow f(0) = \lim_{x \rightarrow 0} f(x)$

$0 = 0$

$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (-x) = 0$



But  $f$  is not diff at  $x=0$  since

$$f'_+(0) \neq f'_-(0)$$

$$\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h}$$

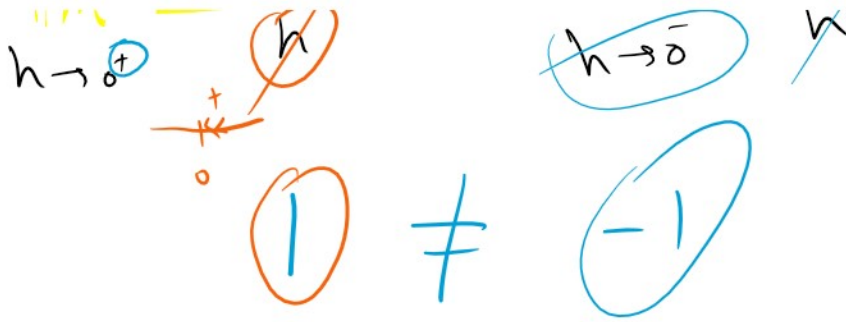
$$\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{h \rightarrow 0^+} \frac{f(h)}{h}$$

$$\lim_{h \rightarrow 0^-} \frac{f(h)}{h}$$

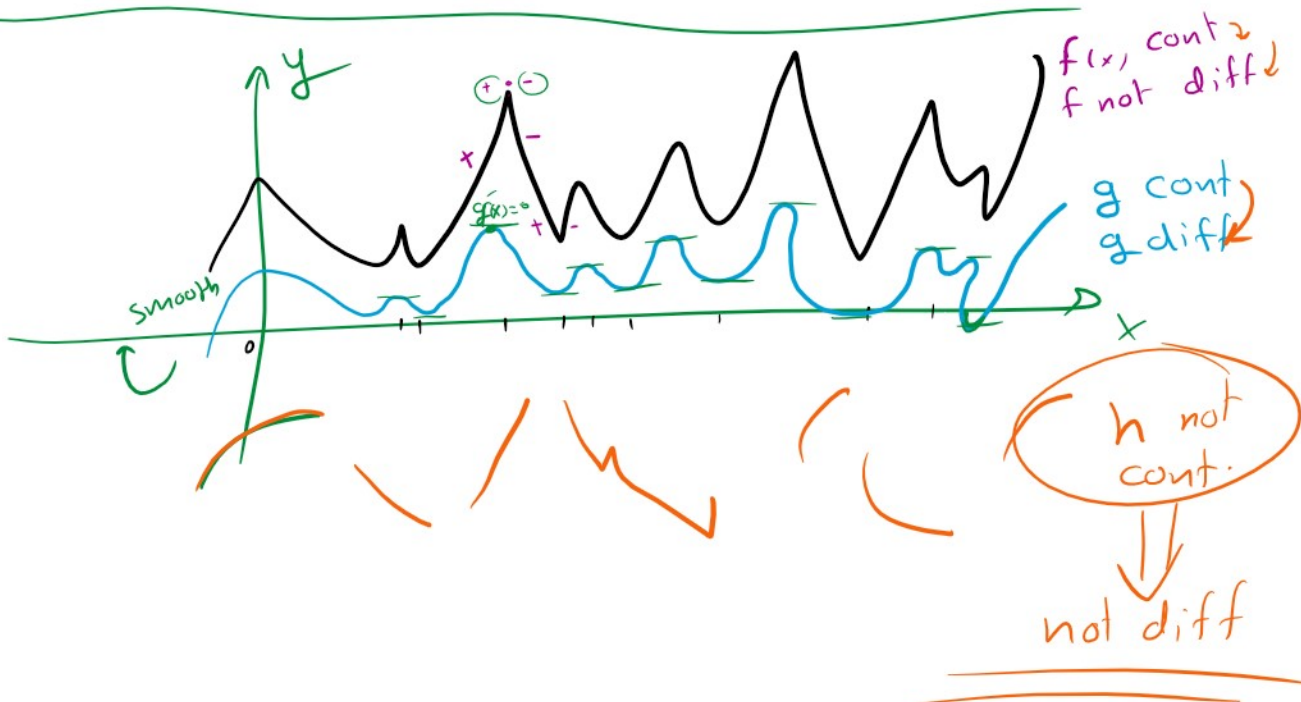






$f'(0)$  does not exist DNE

$f$  is diff at  $x_0$   $x_0$  is defined  $\forall$   $f$



Th (Differentiation Rules) قواعد الاشتقاق

Assume  $f$  and  $g$  diff at  $x$ . Then

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$\textcircled{1} \quad (f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$\textcircled{2} \quad (f(x) \cdot g(x))' = f(x)g'(x) + g(x)f'(x)$$

$$\textcircled{3} \quad \left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad , g \neq 0$$

$$\textcircled{4} \quad (f \circ g)'(x) = f'(g(x)) \cdot g'(x) \quad \text{Chain Rule}$$

$$(f \circ g)(x) = f(g(x))$$

Exp Find  $\frac{d}{dx} \left( \frac{5-x^2}{2x-4} \right) \Big|_{x=2}$

$$\begin{aligned} f'(x) &= \frac{df}{dx} \\ &= (f(x))' \\ &= Df \\ &= \frac{d}{dx} f \end{aligned}$$

$$\frac{(2x-4)(-2x) - (5-x^2)(2)}{(2x-4)^2} \Big|_{x=2}$$

$$\frac{(-4-4)(4) - (5-4)(2)}{(-4-4)^2}$$

3

-34

$$\frac{(-12)(4) - (1)(2)}{(-8)^2} = \frac{-48 - 2}{64} = \frac{-50}{64} = \frac{-34}{64}$$

## Derivatives of Trigonometric functions

$$\textcircled{1} (\sin x)' = \cos x$$

$$\textcircled{2} (\cos x)' = -\sin x$$

$$\textcircled{3} (\tan x)' = \sec^2 x$$

$$\textcircled{4} (\cot x)' = -\csc^2 x$$

$$f = \tan x = \frac{\sin x}{\cos x} \Rightarrow f' = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\textcircled{5} (\sec x)' = \sec x \tan x$$

$$\textcircled{6} (\csc x)' = -\csc x \cot x$$

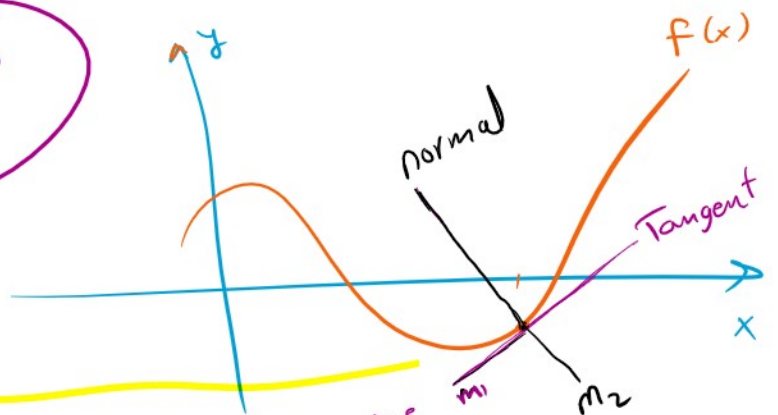
Ex  $f(x) = \tan \sqrt{x}$  Find tangent line at  $x = \pi^2$   $(x > 0)$

Tangent line

$$y - y_0 = m(x - x_0)$$

$\frac{dy}{dx}$

Point  $(x_0, y_0) = (\pi^2, f(\pi^2))$



Point  $(x_0, y_0) = (\pi^2, f(\pi^2))$

Exp

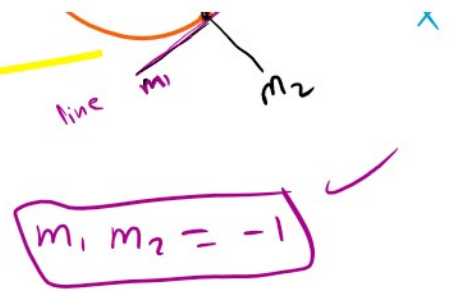
$$= (\pi^2, \tan \sqrt{\pi^2})$$

$$= (\pi^2, \tan \pi)$$

$$= (\pi^2, 0)$$

$x_0$     $y_0$

$$\sqrt{\pi^2} = |\pi|$$



$$m = f'(x_0) = f'(\pi^2)$$

$$= \sec^2 \sqrt{\pi^2} \cdot \frac{1}{2\sqrt{\pi^2}}$$

$$= \sec^2 \pi \cdot \frac{1}{2|\pi|}$$

$$= (1) \cdot \frac{1}{2\pi} = \frac{1}{2\pi}$$

$$f(x) = \tan \sqrt{x}$$

$$f'(x) = \sec^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$\sec^2 \pi = \frac{1}{\cos^2 \pi} = \frac{1}{(-1)^2} = 1$$

Tangent line

$$y - y_0 = m(x - x_0)$$

$$y - 0 = \frac{1}{2\pi} (x - \pi^2)$$

$$y = \frac{1}{2\pi} x - \frac{\pi}{2}$$

$$\dots - 7x^3 - 3x^2 - 12x + 20$$



Exp

Find points on curve where the tangent is parallel to x-axis

$$\frac{dy}{dx} = 6x^2 - 6x - 12 = 0$$

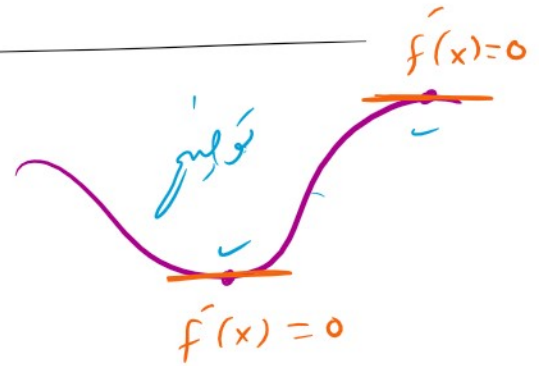
$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2, \quad x = -1 \Rightarrow y(-1) = 2(-1) - 3(1) + 12 + 20 = 27$$

$$y(2) = 2(8) - 3(4) - 12(2) + 20 = 0$$

$$(2, 0), (-1, 27)$$

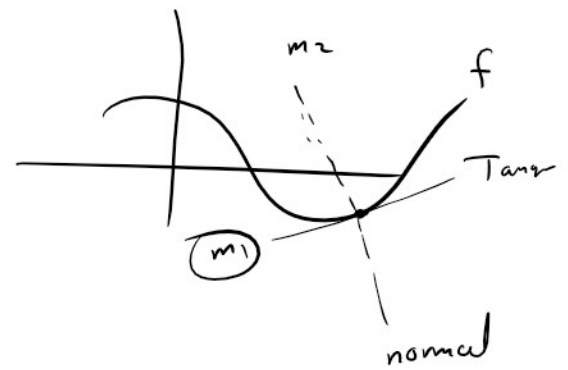


Exp Find normal line for  $f(x) = \sec x \tan x$  at  $x = 0$

$$f'(x) = \sec x \sec^2 x + \tan x \sec x \tan x$$

$$= \sec^3 x + \tan^2 x \sec x$$

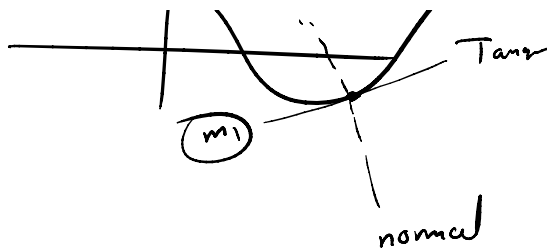
$$m_1 = f'(0) = \sec^3 0 + \tan^2 0 \sec 0$$



$$m_1, m_2 = -1$$

$$f'(x) = \sec x \sec^2 x + \tan x \sec x \tan x$$

$$= \sec^3 x + \tan^2 x \sec x$$



$$m_1 = f'(0) = \sec^3 0 + \tan^2 0 \sec 0$$

$$= 1 + (0)(1)$$

$$= 1 + 0$$

$$= 1$$

$$m_1 m_2 = -1$$

$$(1) m_2 = -1$$

$$\boxed{m_2 = -1}$$

normal eq.  $\Rightarrow y - y_0 = m_2 (x - x_0)$

$$y_0 = f(x_0) = f(0) = \sec 0 \tan 0 = (1)(0) = 0$$

$$y - 0 = -1(x - 0)$$

العمود  $\boxed{y = -x}$  normal line  
orthogonal line  
perpendicular line

Find tangent line

$$y - y_0 = m_1 (x - x_0)$$

$$y - 0 = (1)(x - 0)$$

$$\Rightarrow \boxed{y = x}$$

Tangent

Limit differential

# Implicit differentiation

Exp Find  $\frac{dy}{dx}$  if

①  $x^3 + y^2 = 7$

"Assuming  $y = f(x)$ "

$$3x^2 + 2y y' = 0$$

$$y' = -\frac{3x^2}{2y}$$

②

$$xy = \cos(xy)$$

$$x y' + y = -\sin(xy) [x y' + y]$$

$$= -x y' \sin(xy) - y \sin(xy)$$

$$x y' + x y' \sin(xy) = -y - y \sin(xy)$$

$$x y' [1 + \sin(xy)] = -y [1 + \sin(xy)]$$

$$-xy(1 + \sin(xy)) = -y(1 + \sin(xy))$$

$$\dot{y} = \frac{-y(1 + \sin(xy))}{x(1 + \sin(xy))} = -\frac{y}{x}$$

