

Exercises:

13. $X_i$ :	1	2	3	4	5
$Y_i$ :	3	7	5	11	14

$$A = 0.2$$

$$B = 2.6$$

$$S_x^2 = 2.5$$

$$S_y^2 = 20$$

a. Compute the MSE:

$$S^2 = \text{MSE} = \frac{\text{SSE}}{n-2} = \frac{12.4}{3} = 4.13$$

$$\text{SSE} = \text{SST} - \text{SSR} = 80 - 67.6 = 12.4$$

$$\text{SST} = (n-1)S_y^2 = 4(20) = 80$$

$$\text{SSR} = b_1^2(n-1)S_x^2 = (2.6)^2(4)(2.5) = 6.76 \times 4 \times 2.5 = 67.6$$

b. compute the standard error of the estimate:

$$S = \sqrt{\text{MSE}} = \sqrt{4.13} = 2.03$$

c. compute the estimated standard deviation of  $b_1$ :

$$S_{b_1} = \frac{S}{\sqrt{(n-1)S_x^2}} = \frac{2.03}{\sqrt{4(2.5)}} = 0.64$$

use the t test to test the  $H_0: \beta_1 = 0$  use  $\alpha = 0.05$ ,

$$H_1: \beta_1 \neq 0$$

$$t = \frac{b_1}{S_{b_1}} = \frac{2.6}{0.64} = 4.06$$

$$t_{\frac{\alpha}{2}} = t_{0.025} \text{ with } df = 3$$

$$t > t_{\frac{\alpha}{2}} \text{ so we Reject } H_0 (\alpha = 0.05)$$

$$\beta_1 \neq 0 (\alpha = 0.05)$$

← e.

$$A = 30.3$$

$$14. \quad x: \quad 2 \quad 3 \quad 5 \quad 1 \quad 8$$

$$B = -1.88$$

$$y: \quad 25 \quad 25 \quad 20 \quad 30 \quad 16$$

$$S_x^2 = 7.7$$

$$S_y^2 = 28.7$$

a. compute the MSE:

$$MSE = \frac{SSE}{n-2} = \frac{6.38}{3} = 2.13$$

$$SSR = b_1^2 (n-1) S_x^2 = 108.42$$

$$SST = (n-1) S_y^2 = 114.8$$

$$SSE = 6.38$$

b. compute the standard error:

$$S = \sqrt{MSE} = 1.46$$

c. compute the estimated standard deviation:

$$S_{b_1} = \frac{S}{\sqrt{(n-1) S_x^2}} = 0.26$$

d. use t-test to test hypotheses with  $\alpha = 0.05$ :

$$t = \frac{b_1}{S_{b_1}} = \frac{-1.88}{0.26} = -7.23$$

$$t_{\frac{\alpha}{2}} \text{ with } df = n-2$$

$$t_{\frac{\alpha}{2}} = 3.182$$

$|t| > t_{\frac{\alpha}{2}}$  so reject  $H_0$  ( $\alpha = 0.05$ ).

e. use F-test.

$$F = \frac{MSR}{MSE} = \frac{108.42}{2.13} = 50.9$$

$$F_{\alpha} \text{ with } df_1 = 1, df_2 = 3$$

$$F_{\alpha} = 10.13$$

$F > F_{\alpha}$  so Reject  $H_0$  ( $\alpha = 0.05$ )

$\beta_1 \neq 0$  ( $\alpha = 0.05$ ).