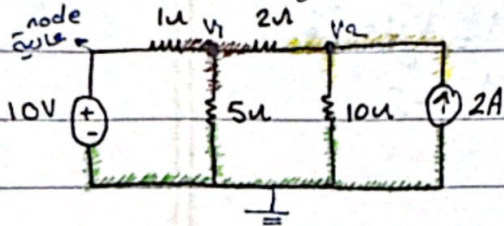


Chapter 4:

Techniques of circuit Analysis

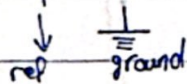
1. Node-voltage Method



Step 1: Determine the number of essential nodes, n_e .

$$n_e = 3$$

Step 2: Select one of these nodes as a reference node



يفضل اختيار النود الذي عليه أكثر تقاطعات

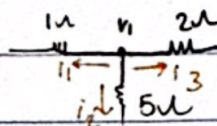
Step 3: Define the node voltages on the circuit

نقطتي تاء لباقي essential nodes

Step 4: Apply KCL at each node

KCL at node v_1 :

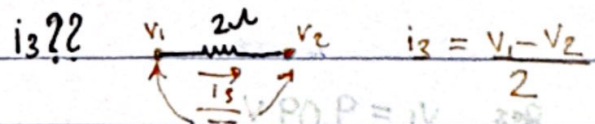
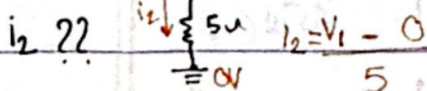
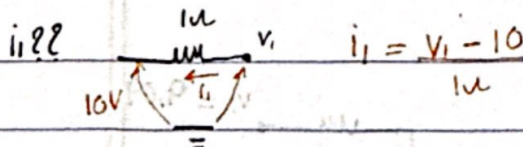
$$i_1 + i_2 + i_3 = 0 \quad (*)$$



بفرض التيارات

كالتي من النقطة عاين

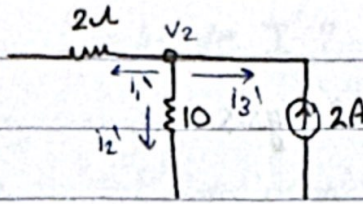
أدوم في



التيار في

$$\left(\frac{v_1 - 10}{1}\right) + \left(\frac{v_1 - 0}{5}\right) + \left(\frac{v_1 - v_2}{2}\right) = 0 \quad (1)$$

KCL at node V_2 \circ



$$i_1' + i_2' + i_3' = 0$$

$$i_1' = \frac{V_2 - V_1}{2}$$

$$i_2' = \frac{V_2 - 0}{10}$$

$$i_3' = V_2 \text{ فتدفق}$$

$$i_3' = -2$$

$$\left(\frac{V_2 - V_1}{2} \right) + \left(\frac{V_2 - 0}{10} \right) + (-2) = 0 \quad \text{--- ②}$$

$$\frac{V_1 - 10}{1} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{2} = 0$$

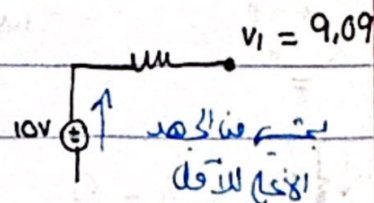
$$5(V_2 - V_1) + V_2 = 20 \quad \times (10)$$

$$6V_2 - 5V_1 = 20$$

$$10V_1 - 100 + 2V_1 + 5V_1 - 5V_2 = 0$$

$$15V_1 + 2V_1 - 5V_2 = 100$$

$$17V_1 - 5V_2 = 100$$

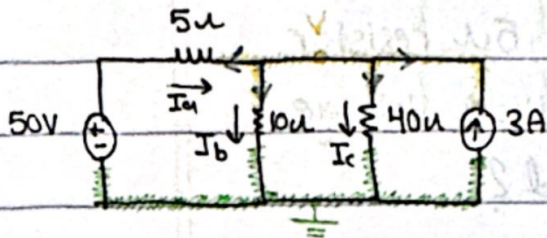


Res $V_1 = 9.09 \text{ V}$

$$V_2 = 10.91 \text{ V}$$

المصدر 10V يستخدم generator

Ex: Find I_a, I_b, I_c using Node - voltage method?



KCL at V :

$$\frac{V-50}{5} + \frac{V-0}{10} + \frac{V-0}{40} + (-3) = 0 \quad \times 40$$

$$8V - 400 + 4V + V = 120$$

$$13V = 520 \rightarrow V = 40 \text{ V}$$

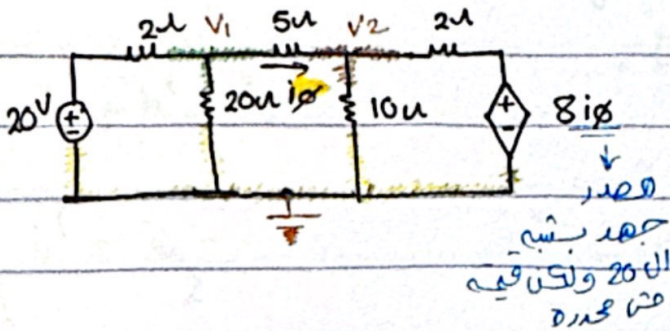
Currents

$$I_a = \frac{50-V}{5} = \frac{50-40}{5} = 2 \text{ A}$$

$$I_b = \frac{V-0}{10} = 4 \text{ A}$$

$$I_c = \frac{V-0}{40} = 1 \text{ A}$$

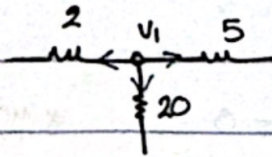
Ex 5



Find the power absorbed by the 5Ω resistor using Node Voltage method?

جواب بـ 20
الـ 20 ولت
منه 20

KCL at v_1



$$I_1 + I_2 + I_3 = 0$$

$$\frac{v_1 - 20}{2} + \frac{v_1 - 0}{20} + \frac{v_1 - v_2}{5} = 0 \quad \text{--- (1)}$$

KCL at v_2

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{10} + \frac{v_2 - 8i_x}{2} = 0 \quad \text{--- (2)}$$

بـ 20 كانه
عنه 20 ولت

$$i_x = \frac{v_1 - v_2}{5} \quad \text{dep source}$$

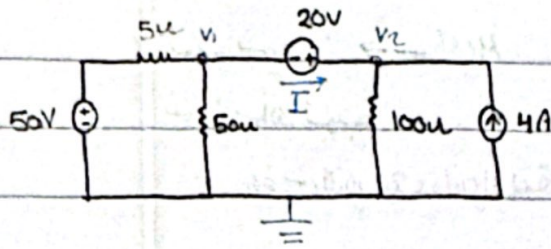
بـ 20 كانه
عنه 20 ولت

res after solving 1 & 2

$$v_1 = 16 \text{ V} \quad v_2 = 10 \text{ V} \quad i_x = 1.2 \text{ A}$$

$$P = I^2 R = i_x^2 (5) = (1.2)^2 \cdot 5 = 7.2 \text{ Watt}$$

Ex:



$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_1 - 50}{5} + \frac{V_1 - 0}{50} + I_3 = 0 \quad \text{--- (1)}$$

نظر في
I3

at node 2:

$$-I_3 + \frac{V_2 - 0}{100} + (-4) = 0 \quad \text{--- (2)}$$

مع الدائرة

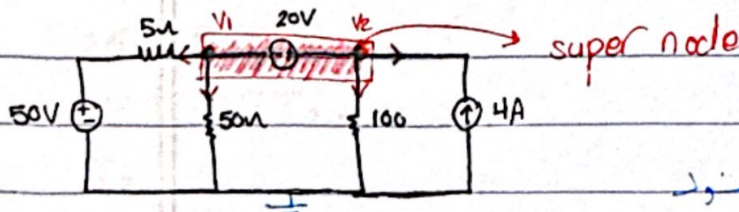
$$\frac{V_1}{5} - 10 + \frac{V_1}{50} + \frac{V_2}{100} = 4 \quad \text{--- (3) = 1+2}$$

$$V_2 - V_1 = 20 \quad \text{--- (4)}$$

بما ان V_2 و V_1 ليسا متباعدين
لذلك

after solving $V_1 = 60 \text{ V}$ $V_2 = 80 \text{ V}$

When a voltage source is between two essential nodes we can combine these nodes to form super-node.



بما اننا يمكننا ان نجعلها
نظر في

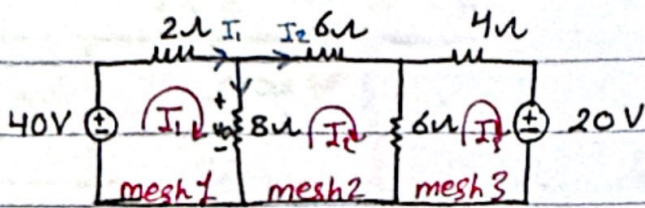
KCL at super-node:

$$V_2 - V_1 = 20$$

$$\frac{V_1 - 50}{5} + \frac{V_1 - 0}{50} + \frac{V_2 - 0}{100} + 4 = 0$$

مع الدائرة
نظر في

② Mesh - current method



Mesh → لوجبي

فتنجهها لوجبه

اجنوه في التيار مع عقارب الساعة

KVL in mesh 1:

$$-40 + 2I_1 + 8(I_1 - I_2) = 0 \quad \text{①}$$

التيار في عقارب الساعة - التيار في عقارب الساعة

KVL in mesh 2:

$$8(I_2 - I_1) + 6I_2 + 6(I_2 - I_3) = 0 \quad \text{②}$$

KVL in mesh 3:

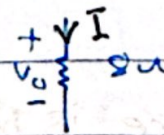
$$6(I_3 - I_2) + 4I_3 + 20 = 0 \quad \text{③}$$

by solving 1, 2, 3

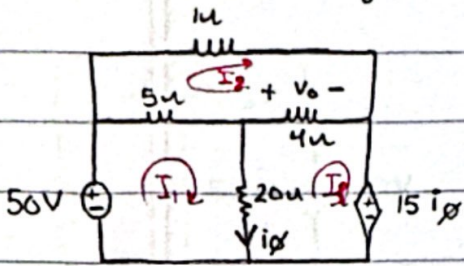
$$I_1 = 5.6A \quad I_2 = 2A \quad I_3 = -0.8A$$

Find V_o using mesh-current method

$$\begin{aligned} V_o &= 8(I_1 - I_2) \\ &= 28.8V \end{aligned}$$



Ex: Find V_o using mesh-current method;



mesh 1:

$$-50 + 5(I_1 - I_2) + 20(I_1 - I_3) = 0$$

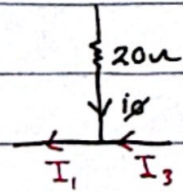
mesh 2:

$$5(I_2 - I_1) + I_2 + 4(I_2 - I_3) = 0$$

mesh 3:

$$20(I_3 - I_1) + 4(I_3 - I_2) + 15i_\phi = 0$$

$$i_\phi = I_1 - I_3$$



$$25I_1 - 5I_2 - 20I_3 = 50 \quad \text{--- (1)}$$

$$-5I_1 + 10I_2 - 4I_3 = 0 \quad \text{--- (2)}$$

$$-5I_1 - 4I_2 + 9I_3 = 0 \quad \text{--- (3)}$$

$$V_o = 4(I_3 - I_2) = 8V$$

$$2(1) + (2)$$

$$45I_1 - 44I_3 = 100 \quad \text{--- (4)}$$

$$I_3 = 28$$

$$I_1 = 29.6$$

$$I_2 = 26A$$

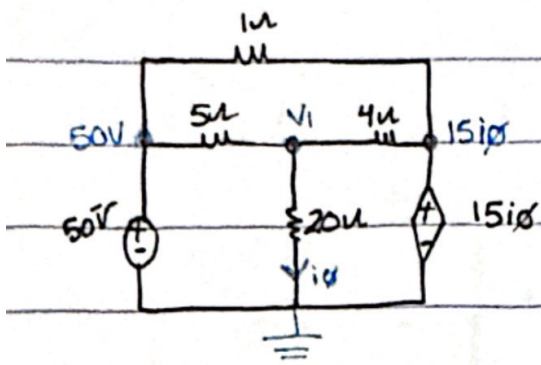
$$10(3) + 4(2)$$

$$-70I_1 + 74I_3 = 0$$

$$70I_1 = 74I_3$$

$$I_1 = 1.057I_3$$

(4) 3 2 2 2 2

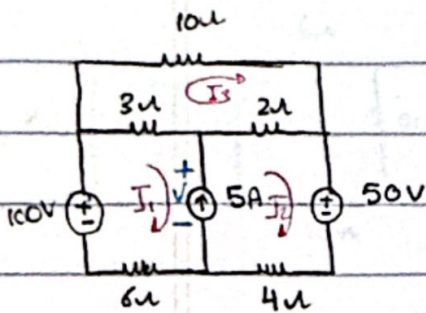


Node voltage method

$$\frac{V_1 - 50}{5} + \frac{V_1 - 0}{20} + \frac{V_1 - 15i_\phi}{4} = 0$$

$$i_\phi = \frac{V_1 - 0}{20} = \frac{V_1}{20}$$

Ex 8



$$-100 + 3(I_1 - I_3) - 5 + 6I_1$$

خطا صحت voltage مع تيار الكل voltage على

$$-100 + 3(I_1 - I_3) + V + 6I_1 = 0$$

$$-V + 2(I_2 - I_3) + 50 + 4I_2 = 0$$

$$\Rightarrow -100 + 3(I_1 - I_3) + 6I_1 + 2(I_2 - I_3) + 50 + 4I_2 = 0$$

$$10I_3 + 2(I_3 - I_2) + 3(I_3 - I_1) = 0 \quad \text{---(2)}$$

$$I_2 - I_1 = 5 \quad \text{---(3)}$$

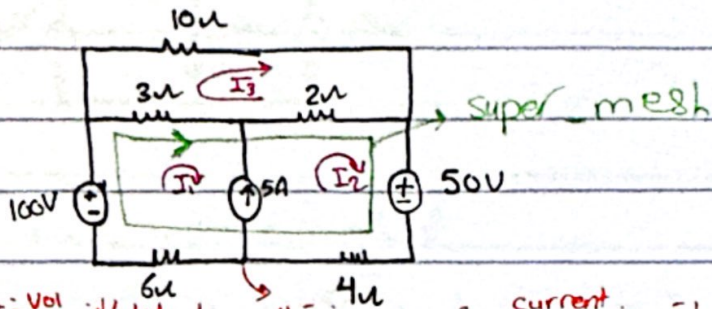
by solving

$$I_1 = 1.75 \text{ A}$$

$$I_2 = 6.75 \text{ A}$$

$$I_3 = 1.25 \text{ A}$$

The concept of super mesh:



من أجل حساب الجهد V_{xy} في 4Ω resistor \Rightarrow current source \Rightarrow super mesh

KVL on super mesh:

$$-100 + 3(I_1 - I_3) + 2(I_2 - I_3) + 50 + 4I_2 + 6I_1 = 0 \quad \text{--- (1)}$$

من معادلات \Rightarrow V_{xy} في 4Ω resistor

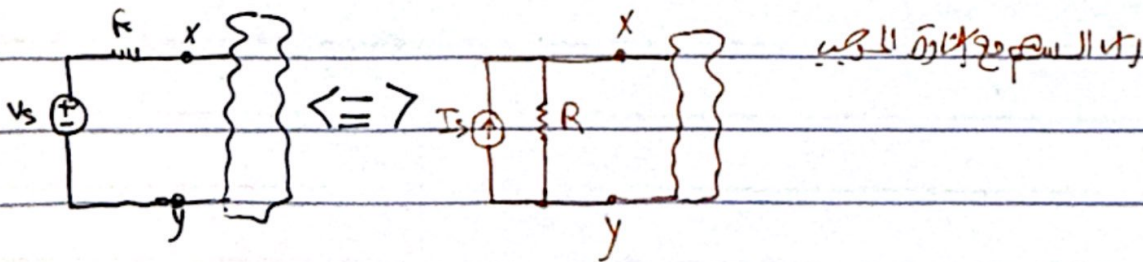
$$I_2 - I_1 = 5 \quad \text{--- (2)} \quad \text{super mesh (من $5A$ current source)}$$

KVL on mesh 3:

$$10I_3 + 2(I_3 - I_2) + 3(I_3 - I_1) = 0 \quad \text{--- (3)}$$

③ Source Transformation:

It allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor.



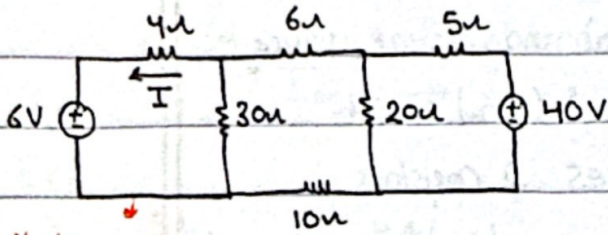
$$V_s = RI_R + V_{xy}$$

$$I_s = \frac{V_s}{R}$$

$$\frac{V_s}{R} = I_R + \frac{V_{xy}}{R}$$

$$I_s = I_R + \frac{V_{xy}}{R}$$

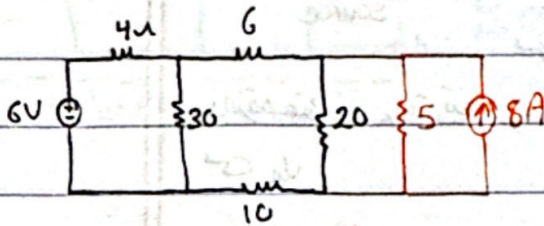
Ex :



Find the current I in the circuit using source transformation method.

تحويل المصادر

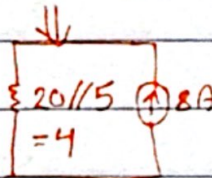
المصادر المتكافئة



$\frac{40}{5} = 8A$ عاكسة التيار

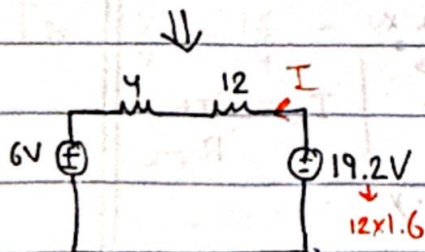
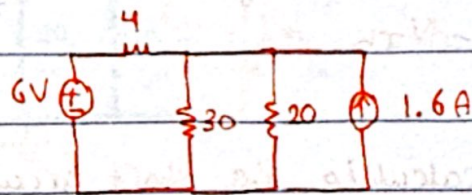
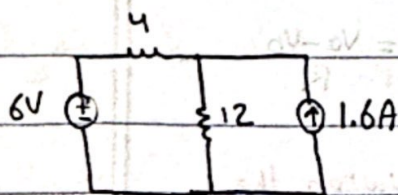
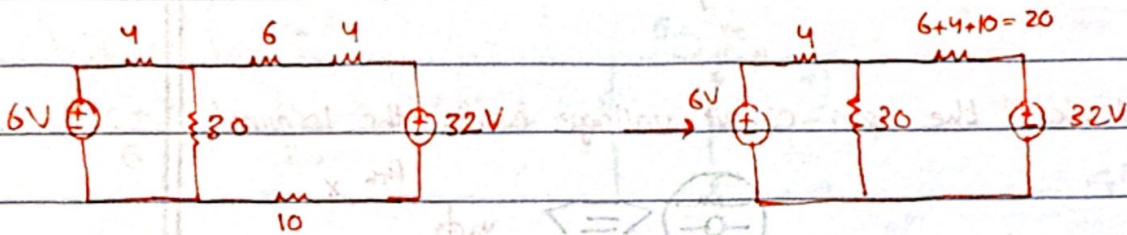
الاتجاه من الاصل الى اليمين

توازى مع المقاومة



مخرجها $\frac{20 \parallel 5}{5} = 4$ Voltage
 $8 \times 4 = 32$

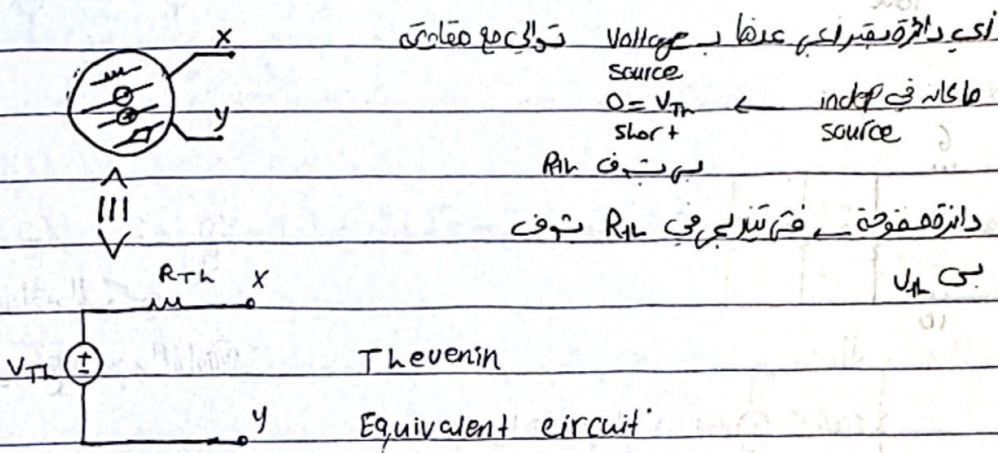
\Leftrightarrow



$I = \frac{19.2 - 6}{16} = 0.825A$

Ⓐ Thevenin Method

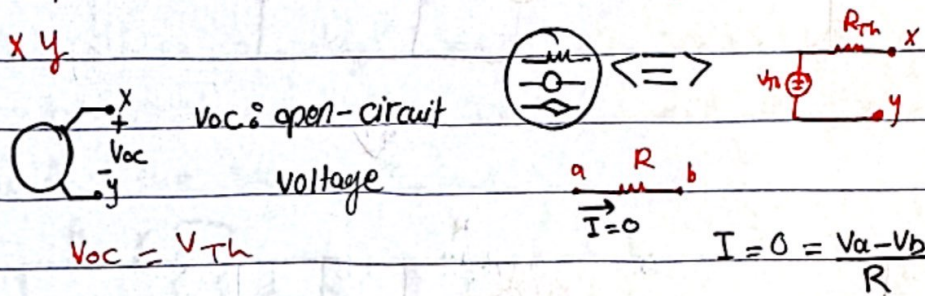
Thevenin Equivalent circuit: It is an independent voltage source (V_{Th}) in series with a resistor (R_{Th}) which replace an interconnection of sources & resistors



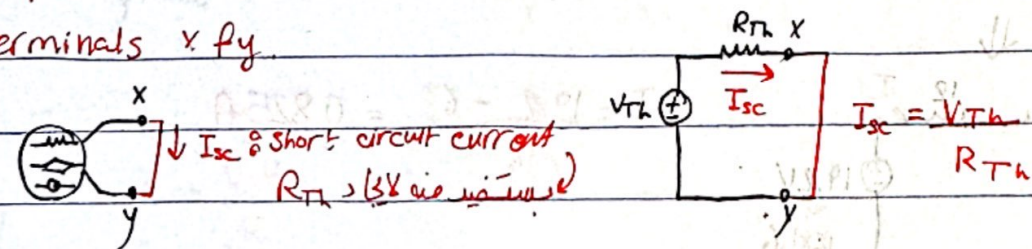
Methods to find Thevenin equivalent circuit.

Method 1

Step 1: calculate the open-circuit voltage across the terminals



Step 2: calculate the short circuit current between the terminals x & y.

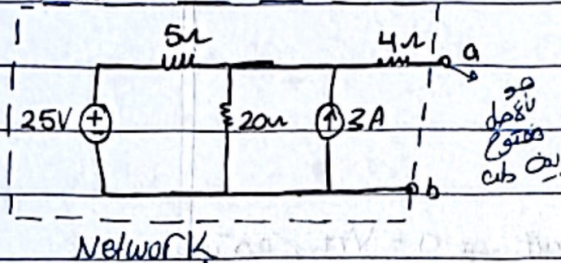


Step 3: Calculate R_{Th}

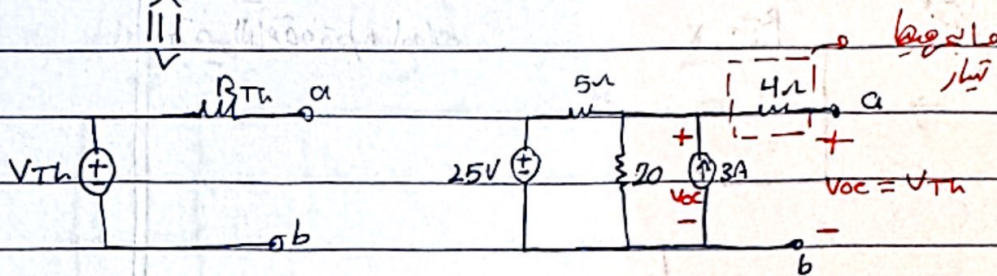
$$R_{Th} = \frac{V_{Th} = V_{oc}}{I_{sc}} \rightarrow \text{step 1}$$

$$I_{sc} \rightarrow \text{step 2}$$

Ex:

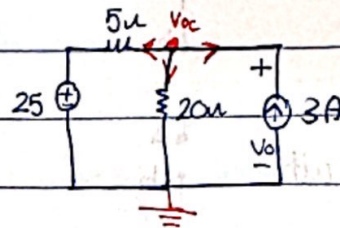


Find the Thevenin equivalent circuit across ab.



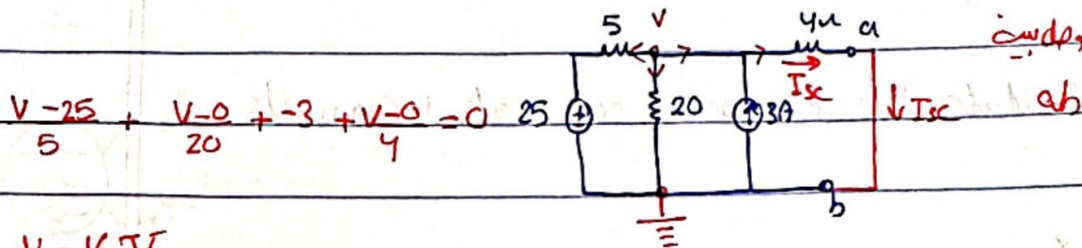
KCL at V_{oc} :

$$\frac{V_{oc} - 25}{5} + \frac{V_{oc} - 0}{20} + (-3) = 0$$



جوابه nodal بتقال
مع معادله واحدة

$$V_{oc} = V_{Th} = 32 \text{ V}$$

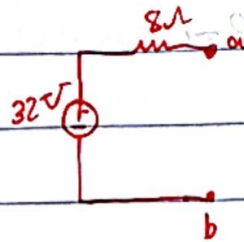


$$\frac{V - 25}{5} + \frac{V - 0}{20} + (-3) + \frac{V - 0}{4} = 0$$

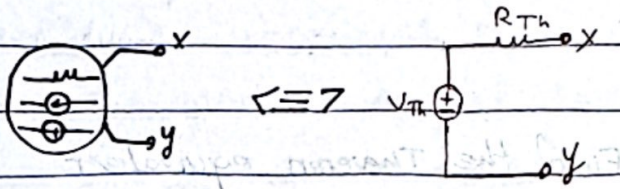
$$V = 16 \text{ V}$$

$$I_{sc} = \frac{V}{4} = \frac{16}{4} = 4 \text{ A}$$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{32}{4} = 8 \Omega$$

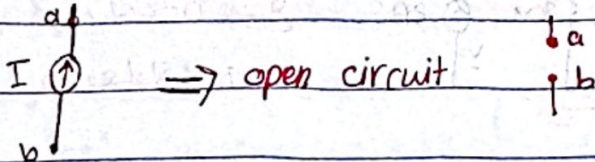
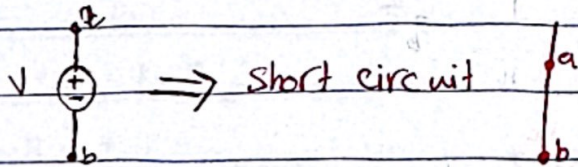
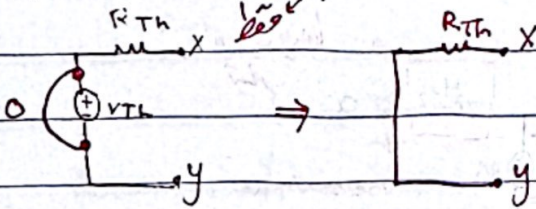


Method 2: It is useful if the network contains only indep sources & resistors.



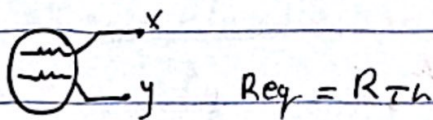
Step 1: V_{oc} - V_{th}

Step 2: Kill all indep sources in the circuit $\rightarrow 0 = V_{th}$

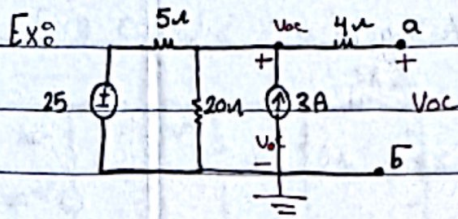


Step 3: calculate the resistance seen at terminals

x & y



يتم استخدام المبرهن حسب نظرية الدائرة كما هو موضح في النتيجة ولتكن V_{oc} ولتكن V_{oc} ولتكن V_{oc}



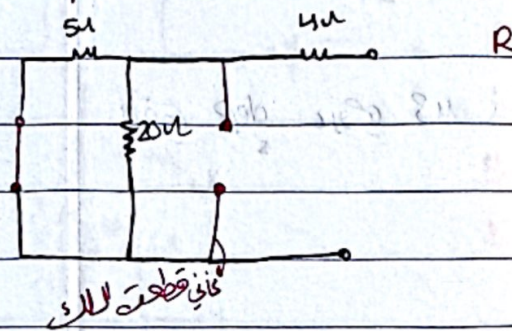
الدائرة بالأعلى

Find the Thevenin Equivalent circuit across the terminals a & b.

$$V_{oc} - 25 + \frac{V_{oc}}{5} + \frac{V_{oc}}{20} + (-3) + 0 = 0$$

$$V_{Th} = V_{oc} = 82V$$

2. Kill indep



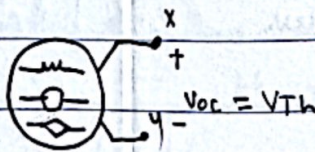
كأنني قطعته للأي

إزاف source dep ← (مربع على) method 2

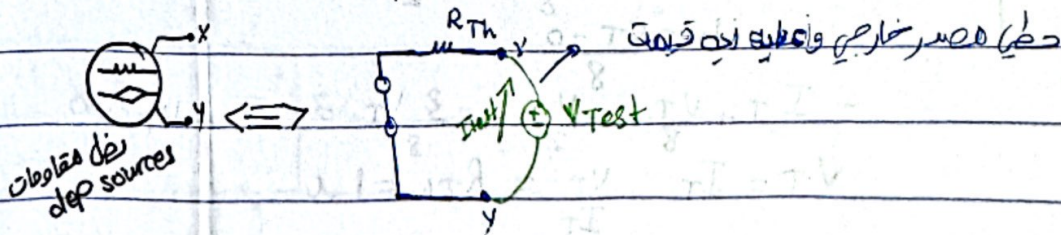
إزاف source dep ← (مربع على) method 1 or 3

Method 3: It is useful if the circuit contains dep - sources

Step 1: calculate the open circuit voltage

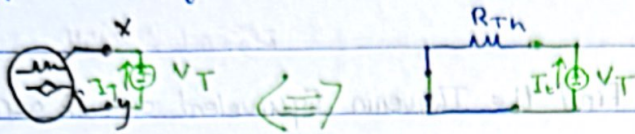


Step 2: Kill all indep sources in the circuit



حذف dep sources

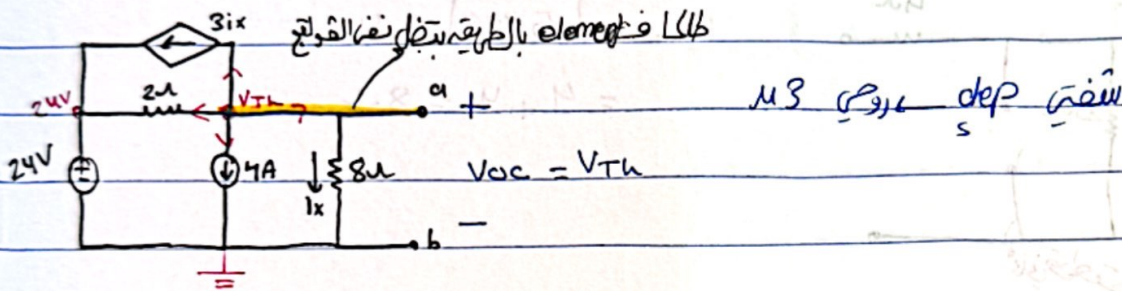
Step 3: Apply a test source on the terminals x, y.



Step 4: calculate R_{Th}

$$R_{Th} = \frac{V_T}{I_T}$$

Ex: Find the Thevenin equivalent circuit with respect to terminal ab.



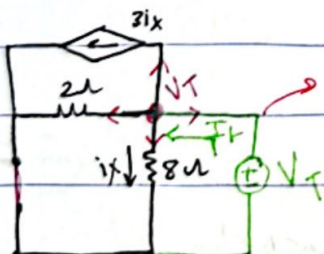
Kcl at V_{Th} :

$$3i_x + 4 + \frac{V_{Th}}{8} + \frac{V_{Th} - 24}{2} = 0$$

$$i_x = \frac{V_{Th}}{8}$$

$$\frac{3}{8}V_{Th} + 4 + \frac{V_{Th}}{8} + \frac{V_{Th} - 12}{2} = 0$$

$$V_{Th} = 8V$$



بقية عالية التردد
عند الوصل مع
es node

Kcl at V_T :

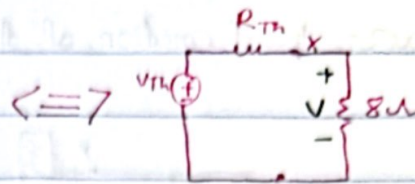
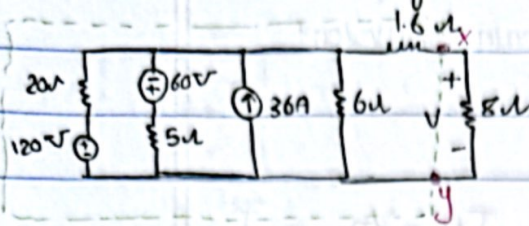
$$-I_T + \frac{V_T}{8} + \frac{V_T}{2} + 3i_x = 0$$

$$i_x = \frac{V_T - 0}{8}$$

$$-I_T + \frac{V_T}{8} + \frac{V_T}{2} + \frac{3V_T}{8} = -I_T \quad V_T = 0$$

$$V_T = I_T \quad \frac{V_T}{I_T} = R_{Th} = 1\Omega$$

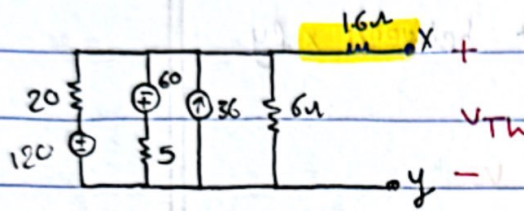
Ex: Calculate V using Thevenin Method



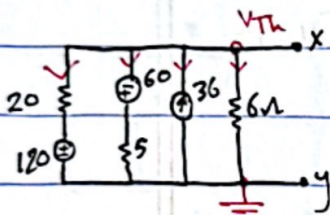
Thevenin Equivalent circuit

Voltage divider

$$V = \frac{8}{8 + R_{TH}} \cdot V_{TH}$$



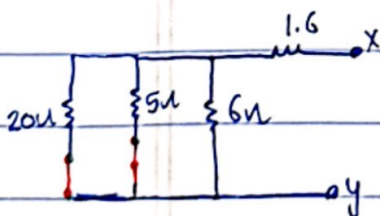
note: when x & y are open no current flows through the external load because there's no closed path for the current to circulate. As a result there's no current flowing through 1.6Ω



$$V_{TH} = \frac{V_{TH} - 36}{6} + \frac{(V_{TH} - 60)}{5} + \frac{(V_{TH} - 120)}{20}$$

Solve for V_{TH}

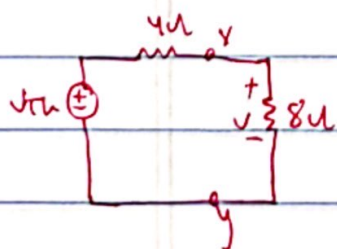
to find R_{TH} → Method 1 or 2 no need to use method 3 there is no indep



$$R_{TH} = (20 || 5) = 4$$

$$(4 || 6) = 2.4$$

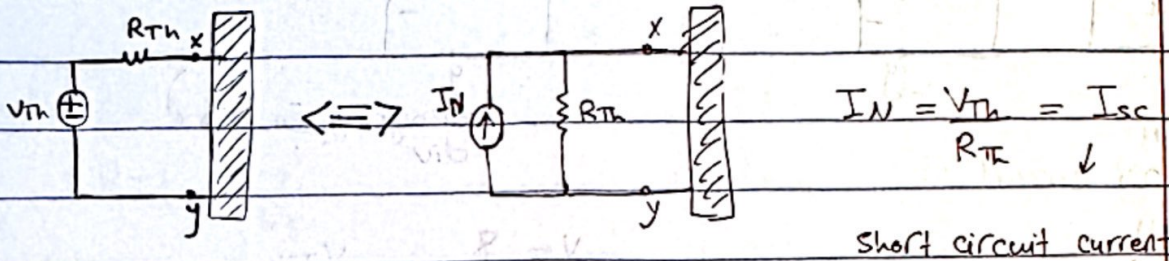
$$(2.4 + 1.6) = 4\Omega$$



$$V = \frac{8}{8 + 4} \cdot V_{TH} = 48V$$

Norton Equivalent circuit

It is the source transformation of thevenin equivalent circuit



Methods to find the Norton equivalent circuit between x & y

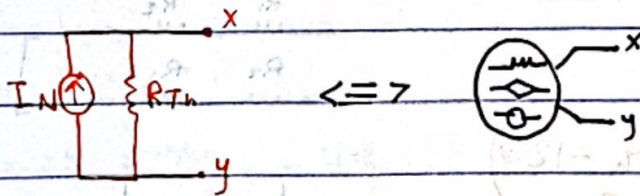
1]

Step 1: calculate the open circuit voltage, V_{oc}

$$V_{Th} = V_{oc}$$

Step 2: calculate the short circuit current, I_{sc} $I_N = I_{sc}$

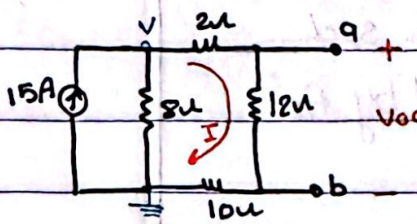
Step 3: calculate the $R_{Th} = \frac{V_{Th}}{I_N}$



2]

using the methods of thevenin equivalent circuit with source transformation

Ex: Find the Norton equivalent circuit with respect to the terminals a,b



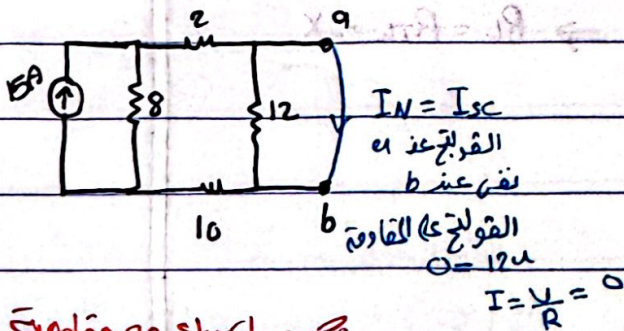
current divider $I = \frac{8}{8 + (2 + 12 + 10)} \cdot 15 = 3.75 \text{ A}$

$V_{Th} = V_{oc} = 12I = 45 \text{ V}$

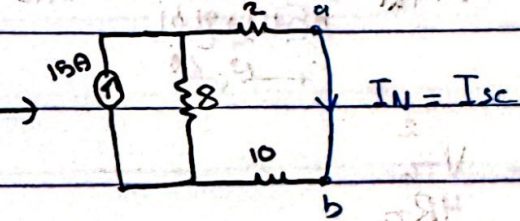
or by Node

$-15 + \frac{V}{8} + \frac{V}{24} = 0$

$\frac{4V}{24} = 15$
 $V = 90$



$V_{oc} = \frac{(12) \cdot 90}{(12) + 2 + 10} = 45 \text{ V}$

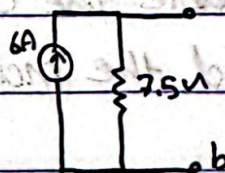


قصر الدارة short

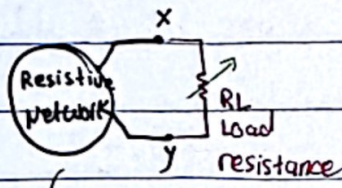
بالتي القارة

$R_{Th} = \frac{V_{Th}}{I_N} = \frac{45}{6} = 7.5 \Omega$

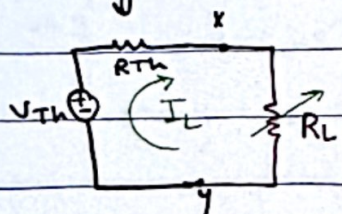
current $I_N = \frac{8}{8 + 12} \cdot 15 = 6 \text{ A}$



Maximum power Transfer



What is the value of R_L such that the power absorbed by R_L is maximum?



I_L is load current

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

The power absorbed by R_L :

$$P_L = I_L^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 \cdot R_L$$

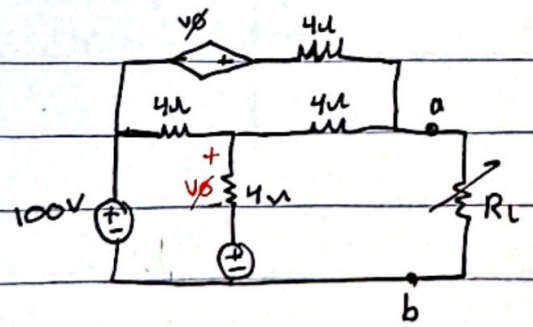
$$P(x) = \left(\frac{V_{TH}}{x + R_{TH}} \right)^2 \cdot x$$

بسته به x (dependent on x)
 اطلاق وین میکنیم (we release it)
 درجه یک (first degree)

$$\frac{dP}{dR_L} = 0 \Rightarrow R_L = R_{TH} = x$$

$$P_{L \max} = \frac{V_{TH}^2}{4R_{TH}}$$

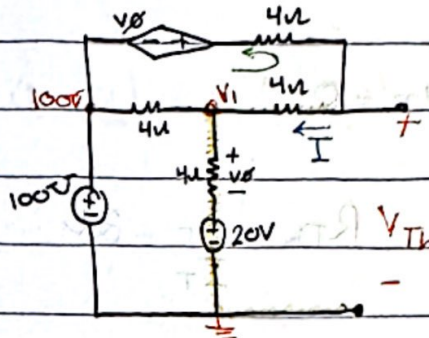
Ex: Find the value of R_L to enable the circuit to deliver maximum power and find the maximum power delivered to R_L .



assignment problems

$$R_L = R_{Th}$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$



$$V_1 - 20 = V_x + 20$$

$$\frac{V_1 - 20}{4} + \frac{V_1 - (V_x + 100)}{8} + \frac{V_1 - 100}{4} = 0$$

نویس
= $V_x + 20$

$$\frac{V_x + 20 - 20}{4} + \frac{V_x + 20 - (V_x + 100)}{8} + \frac{V_x + 20 - 100}{4} = 0$$

$$= \frac{V_x}{4} + (-10) + \frac{V_x}{4} + (-20)$$

$$= \frac{V_x}{2} = 30$$

$$V_x = 60 \text{ V}$$

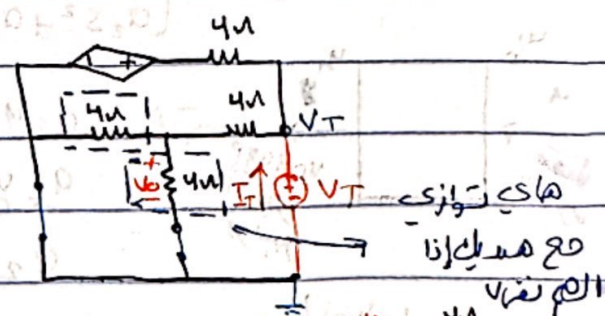
$$V_{Th} = 4I + V_x + 20 \quad (2) = 120 \text{ V}$$

$$I = \frac{V_x + 100 - (V_x + 20)}{8} = 10 \text{ A}$$

$$V_{Th} = 40 + 60 + 20 = 120 \text{ V}$$

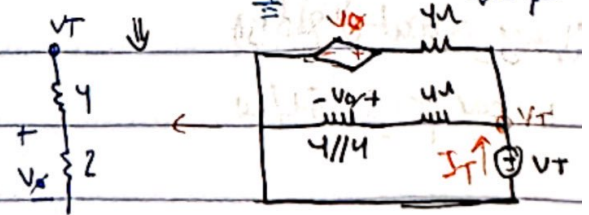
R_{Th} "Method 3"

روش دیگر برای V_{Th} و R_{Th}



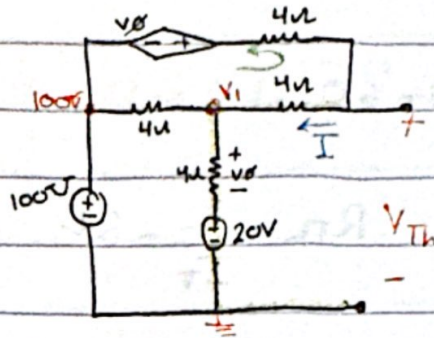
$$\frac{V_T - 0}{6} + (-I_T) + \frac{V_T - V_x}{4} = 0$$

$$V_x = \frac{2}{2+4} V_T = \frac{1}{3} V_T$$



$$R_L = R_{Th}$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$



$$V_1 - 20 = 4I$$

$$V_1 = V_{\alpha} + 20$$

$$\frac{V_1 - 20}{4} + \frac{V_1 - (V_{\alpha} + 100)}{8} + \frac{V_1 - 100}{4} = 0$$

نکته: $V_1 = V_{\alpha} + 20$

$$\frac{V_{\alpha} + 20 - 20}{4} + \frac{V_{\alpha} + 20 - (V_{\alpha} + 100)}{8} + \frac{V_{\alpha} + 20 - 100}{4} = 0$$

$$= \frac{V_{\alpha}}{4} + (-10) + \frac{V_{\alpha}}{4} + (-20)$$

$$= \frac{V_{\alpha}}{2} = 30$$

$$V_{\alpha} = 60 \text{ V}$$

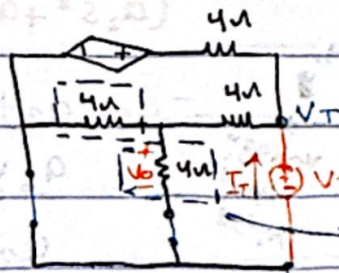
$$V_{Th} = 4I + V_{\alpha} + 20$$

$$I = \frac{(V_{\alpha} + 100) - (V_{\alpha} + 20)}{8} = 10 \text{ A}$$

$$V_{Th} = 40 + 60 + 20 = 120 \text{ V}$$

R_{Th} "Method 3"

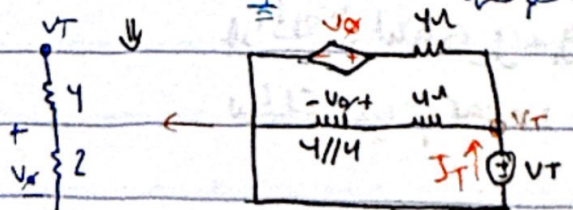
ببین V_T و V_{α} را موازی



ما یک موازی
مع هم داریم
التم تقابل

$$\frac{V_T - 0}{6} + (-I_T) + \frac{V_T - V_{\alpha}}{4} = 0$$

$$V_{\alpha} = \frac{2}{2+4} V_T = \frac{1}{3} V_T$$



$$-I_T + \frac{V_T}{6} + \frac{1}{6} V_T = 0$$

$$I_T = \frac{V_T}{3}$$

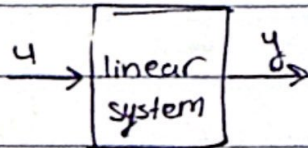
$$R_{Th} = \frac{V_T}{I_T} = 3\Omega$$

$$R_L = 3\Omega$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{120^2}{4(3)} = 1.2 \text{ KW}$$

⑤ Superposition :

For any linear system is driven by more than one indep source, the total response is the sum of the individual responses.



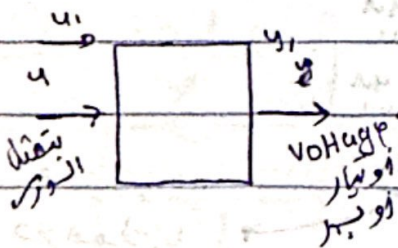
linear differential equation

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = u(t)$$

$$\frac{y(s)}{u(s)} = H(s)$$

$$(a_2 s^2 + a_1 s + a_0) y(s) = u(s)$$

$$y = \left(\frac{1}{a_2 s^2 + a_1 s + a_0} \right) u(s)$$



$$a_2 y'' + a_1 y' + a_0 = u$$

$$a_2 y_1'' + a_1 y_1' + a_0 = u_1$$

$$a_2 y_2'' + a_1 y_2' + a_0 = u_2$$

total output is $y_1 + y_2$

linear system

