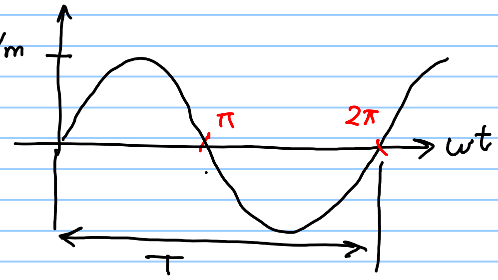
$$v_s(t) = V_m \sin(\omega t)$$

$\omega \rightarrow$ angular freq. (rad/sec)

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$ (Hz) $= \frac{1}{T}$

T : period in seconds



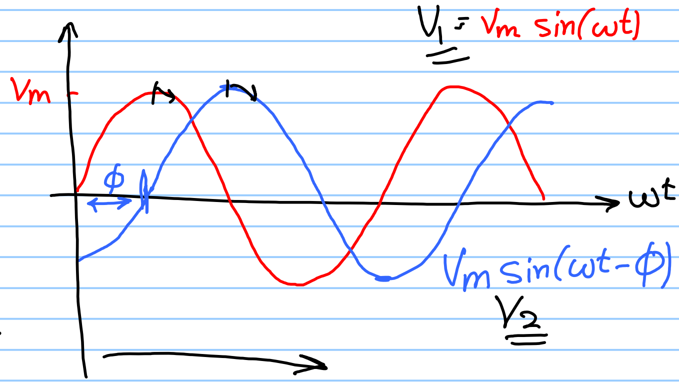
phase of sinusoids

lead   lag

→ V_1 leads V_2 by phase ϕ

V_2 lags V_1 〃 〃 〃

V_1 & V_2 are out of phase



$$A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$$

where $C = \sqrt{A^2+B^2}$ and $\theta = \tan^{-1}\frac{B}{A}$

⚠ ✱ phasors

Ex) $v_1(t) = 10\sin(\;\;t - 30^\circ)$, $v_2(t) = 15\sin(\;\;t + 10^\circ)$

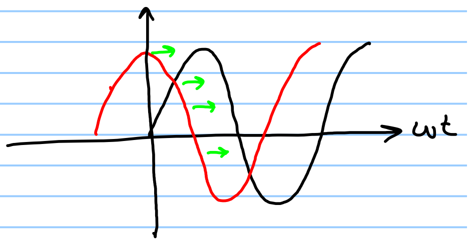
v_2 leads v_1 by 40°

Ex) $i_1(t) = 2\sin(377t + 45^\circ)$, $i_2(t) = 0.5\cos(377t + 10^\circ)$

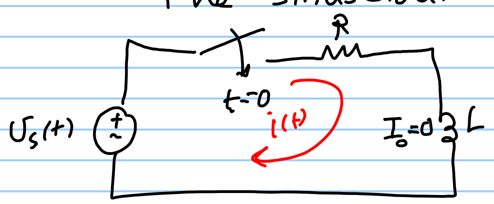
$i_1(t) = 2\cos(377t + 45^\circ - 90^\circ)$

$= 2\cos(377t - 45^\circ)$

$i_2(t)$ leads $i_1(t)$ by 55°



The sinusoidal Response



Find $i(t)$ for $t>0$

where $v_s(t) = V_m\cos(\omega t)$ V

KVL

$$-v_s(t) + R\,i(t) + L\frac{d}{dt}i(t) = 0$$

$$R\,i(t) + L\frac{d}{dt}i(t) = V_m\cos(\omega t)$$

$$i(t) = i_n(t) + i_p(t)$$

$$i(t) = A e^{-t/\tau} + i_f(t)$$

$\rightarrow i_f(t) = \underline{\underline{I_1}} \cos\omega t + \underline{\underline{I_2}} \sin \omega t$ (see the notes)

$$= \frac{R\,V_m}{R^2+(\omega L)^2} \cos\omega t + \frac{\omega L\, V_m}{R^2+(\omega L)^2} \sin\omega t$$

$$= C \cos(\omega t - \theta)$$

$$i_f(t) = \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$\hookrightarrow i(t) = \underbrace{A e^{-t/\tau}}_{\text{transient part}} + \underbrace{\frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cos\left(\quad t - \tan^{-1}\frac{\omega L}{R}\right)}_{\text{steady state component}}$$

find $i(t)$ for $t>0$

where $v_s(t) = V_m \cos(\quad t)$ V

The steady state sol. is a sinusoidal function with the same freq. as the source signal

Complex Numbers (page 10-15) review

Rectangular form

$$Z = x + j y$$

Polar form

$$Z = |Z| \angle \theta$$

Exponential form

$$Z = |Z| e^{j\theta}$$

$$= |Z| (\cos\theta + j \sin\theta)$$

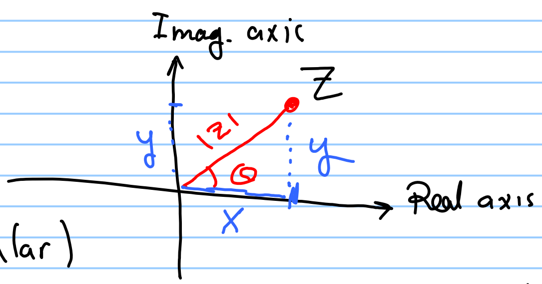
$$= \underline{|Z|\cos\theta} + j \underline{|Z| \sin\theta}$$

$$= X + j y$$

$\hookrightarrow Z = 10 \angle 35^\circ$ (polar)

$$= 10 \cos 35^\circ + j\, 10 \sin 35^\circ$$

$$= x + j y$$



$\hookrightarrow Z = 5 + j10$ (Rectangular)

$$= \sqrt{5^2+10^2} \angle \tan^{-1}\frac{10}{5} = \sqrt{x^2+y^2} \angle \tan^{-1}\frac{y}{x}$$

$$= |Z| \angle \theta$$

$\boxed{\text{Pol(}}$ $\boxed{\text{Rec(}}$

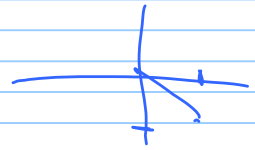
Pol(5, 10) = 11.1803 $\angle 63.43^\circ$ ✓ $\boxed{\text{RCL}} + \boxed{\text{E}}$

Ex1 $Z_1 = 4+j3 = 5\angle 36.9^\circ$

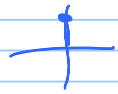
$Z_2 = 3+j4 = 5\angle 53.1^\circ$

$Z_1 + Z_1 = (4+j3)+(3+j4)$
$= 7+j7$

$Z_1 - Z_2 = (4+j3)-(3+j4)$
$= 1-j1$



$Z_1 \times Z_2 = (4+j3)(3+j4)$
$= 12+j16+j9-12$
$= j25 = 25\angle 90^\circ$



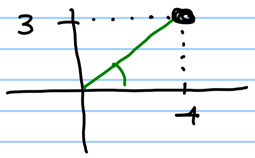
$= (5\angle 36.9^\circ)(5\angle 53.1^\circ)$
$= 25\angle 36.9^\circ + 53.1^\circ$
$= 25\angle 90^\circ$

$\frac{Z_1}{Z_2} = \frac{5\angle 36.9}{5\angle 53.1} = 1\angle 36.9^\circ - 53.1^\circ$
$= 1\angle -16.2^\circ$

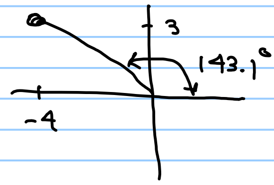
$= \frac{4+j3}{3+j4} \cdot \frac{3-j4}{3-j4}$ } try it

page 15

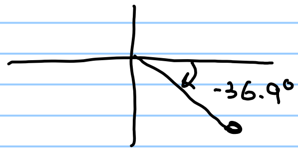
$Z_1 = 4+j3$
$= \sqrt{4^2+3^2}\angle \tan^{-1}\frac{3}{4}$
$= 5\angle 36.9^\circ$



$Z_2 = -4+j3$
$= 5\angle 143.1^\circ$



$Z_4 = 4-j3$
$= 5\angle -36.9^\circ$



phasors

input → circuit → output

$V_m \cos(\omega t+\theta)$ | $I_m \cos(\omega t+\phi)$

$V_m \sin(\omega t+\theta)$ | $I_m \sin(\omega t+\phi)$

$j V_m \sin(\omega t+\theta)$ | $j I_m \sin(\omega t+\phi)$

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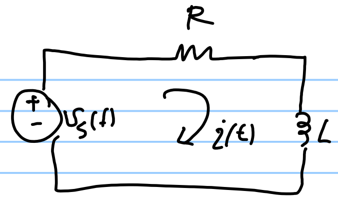


$v_s(t) = V_m \cos \omega t$

$$i(t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R}\right)$$

Phasor $v(t) = V_m \cos(\omega t + \theta_v)$

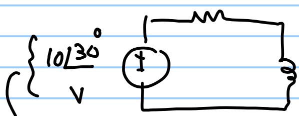
phasor Form $\vec{V} = V_m \angle \theta_v$

⚠ our reference is $\boxed{\text{cosine}}$

$i(t) = 6 \cos(50t - 40^\circ)$ A

phasor Form $\vec{I} = 6 \angle -40^\circ$ A

* any source, must be written in cos, if it was written in sin then → Convert to cos

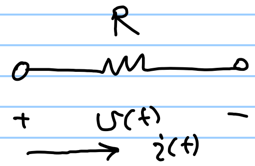


✓ $10 \cos(\omega t + 30^\circ)$

✗ $10 \sin(\omega t + 30^\circ)$?

* Phasor relationships for circuit elements

Resistor



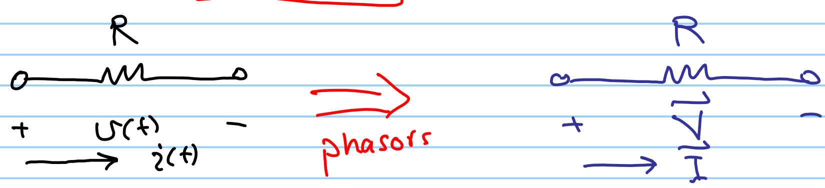
if $i(t) = I_m \cos(\omega t + \theta_i)$

then, $v(t) = R\, i(t)$

$= R I_m \cos(\omega t + \theta_i)$

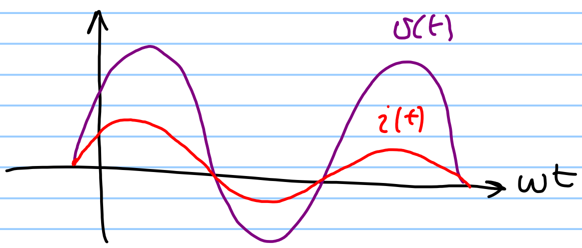
$\vec{V} = R I_m \angle \theta_i \longleftrightarrow \vec{I} = I_m \angle \theta_i$

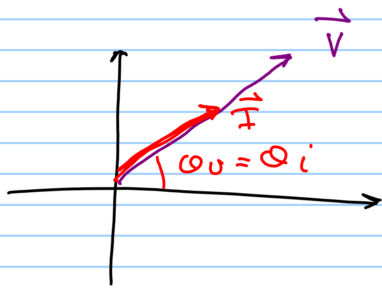
$$\boxed{\vec{V} = R\vec{I}}$$



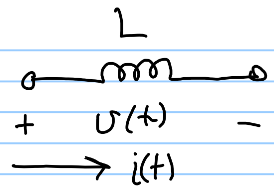
Note There is No phase shift between the current & voltage accross R !

$v(t)$ & $i(t)$ are in phase





V-I relationship for inductors



If $i(t) = I_m \cos(\omega t + \theta_i)$

then $v(t) = L \frac{d}{dt} i(t)$

$= L\, I_m \frac{d}{dt}\left[\cos(\omega t + \theta_i)\right]$

$= -\omega L\, I_m \sin(\omega t + \theta_i)$

But → our ref. is cosine

$\therefore v(t) = -\omega L\, I_m \cos(\omega t + \theta_i - 90^\circ)$

The phasor representation of the voltage is

$\vec{V} = -\omega L\, I_m\, e^{j(\theta_i - 90^\circ)}$

$= -\omega L\, I_m\, e^{j\theta_i}\, e^{-j90^\circ}$ $-j$

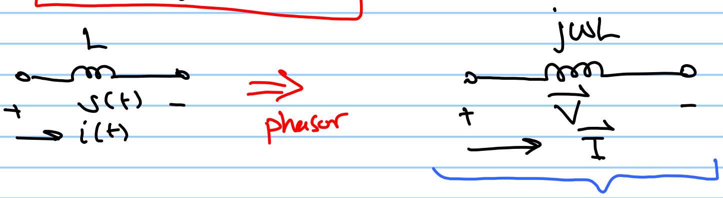
$= j\omega L\, \underbrace{I_m e^{j\theta_i}}$

$e^{-j90^\circ} = \cos(-90^\circ) + j\sin(-90^\circ)$

$= 0 - j$

$= -j$

$$\boxed{\vec{V} = j\omega L\, \vec{I}}$$

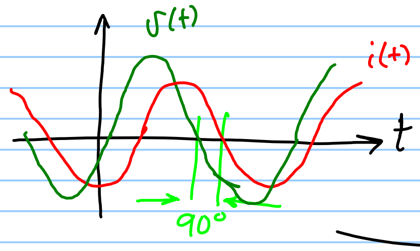


$\vec{V} = j\omega L\, \vec{I}$

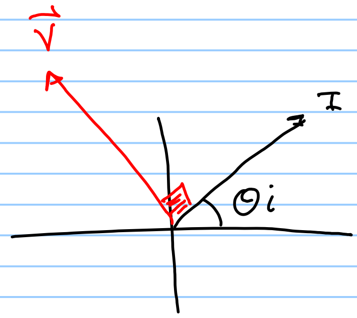
$= (\omega L \angle 90^\circ)(I_m \angle \theta_i)$

$= \omega L\, I_m \angle \theta_i + 90^\circ$

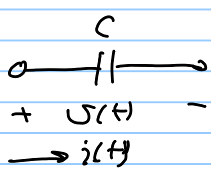
$v(t) = \omega L\, I_m \cos(\omega t + \theta_i + 90^\circ)$



$\boxed{i(t) \text{ lags } v(t) \text{ by } 90^\circ}$



V-I relationships for a capacitor



$i(t) = C \frac{d}{dt} v(t)$

assume $v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = C V_m\left[-\omega \sin(\omega t + \theta_v)\right]$

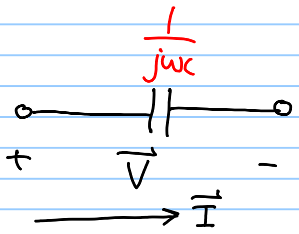
$= -\omega C\, V_m \cos(\omega t + \theta_v - 90^\circ)$

$\vec{I} = -\omega C\, V_m\, e^{j(\theta_v - 90^\circ)}$

$$\vec{I} = -\omega C V_m e^{j\theta_v} e^{-j90^\circ}$$

$$= j\omega C \underline{V_m e^{j\theta_v}}$$

$$\vec{I} = j\omega C \vec{V}$$



$$\left\{ \begin{aligned} & j\omega C = \frac{\vec{I}}{\vec{V}} \text{ !!} \\ & \Rightarrow \frac{\vec{V}}{\vec{I}} = \frac{1}{j\omega C} = -j\frac{1}{\omega C} \end{aligned} \right\}$$

$$\vec{V} = \frac{1}{j\omega C}\vec{I} = \left(-j\frac{1}{\omega C}\right)\left(I_m \angle \theta_i\right)$$

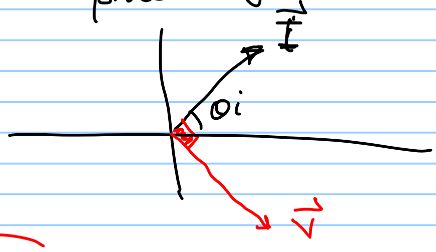
$$= \left(\frac{1}{\omega C} \angle -90^\circ\right)\left(I_m \angle \theta_i\right)$$

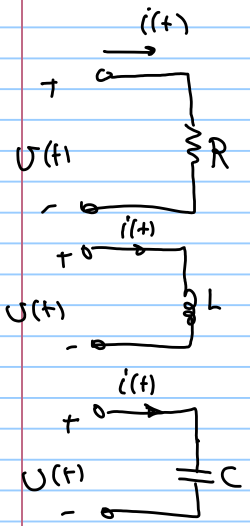
$$\vec{V} = \frac{I_m}{\omega C} \angle \theta_i - 90^\circ$$

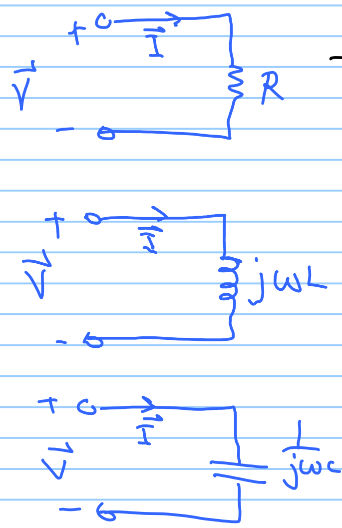
i leads v by 90°

v lags i by 90°

phasor diagram







$$\vec{V} = R\,\vec{I}$$

$$\vec{V} = j\omega L\,\vec{I}$$

$$\vec{V} = \frac{1}{j\omega C}\vec{I}$$

$$\vec{V} = Z(j\omega)\,\vec{I}$$

Impedance & admittance

$Z(j\omega) = \frac{\vec{V}}{\vec{I}}$ Impedance (Ω) $\qquad \vec{V} = Z\,\vec{I}$

$Y(j\omega) = \frac{\vec{I}}{\vec{V}}$ Admittance ($\mho$)

$Z = \frac{1}{Y}$