Birzeit University
MATH234
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Quiz 1(b)

Name:- **ID:-** **Sec.:-** **S.N.:-**

Q1. Consider the Linear system

$$x+y+z=1$$
$$2x+3y+3z=\alpha$$
$$3x+4y+\alpha z=\beta$$

Determine the values of α, β for which the linear system has no solutions, exactly one solution, or infinitely many solutions.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & \alpha \\ 3 & 4 & \alpha & \beta \end{bmatrix}$$ (1)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & \alpha-2 \\ 3 & 4 & \alpha & \beta \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & \alpha-2 \\ 0 & 1 & \alpha-3 & \beta-3 \end{bmatrix}$$ (2)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & \alpha-2 \\ 0 & 0 & \alpha-4 & \beta-1-\alpha \end{bmatrix}$$ (1)

Case $1: \alpha-4 \neq 0$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & \alpha-2 \\ 0 & 0 & 1 & \dfrac{\beta-1-\alpha}{\alpha-4} \end{bmatrix}$$

There is exactly one solution. (2)

Case $2: \alpha-4=0$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & \beta-5 \end{bmatrix}$$

Case 2.1: $\beta - 5 \neq 0$

There is no solution. 2

Case 2.2: $\beta - 5 = 0$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

There are infinitely many solutions. 2

Good Luck