

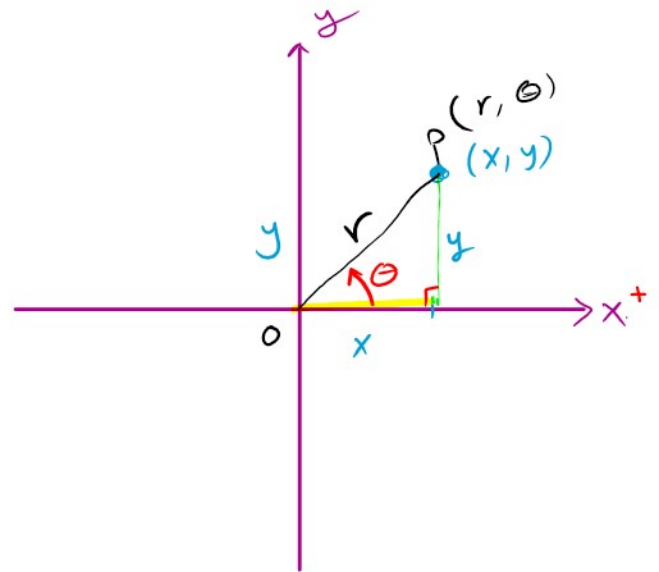
Coordinates

Cartesian Coordinates
(x, y)

are uniquely represented

Polar Coordinates
(r, θ)

is not unique

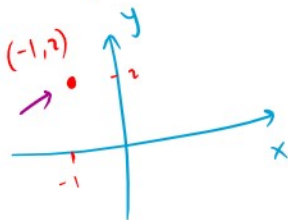
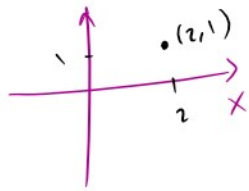


$$\cos \theta = \frac{x}{r} \Rightarrow x = r \cos \theta$$

$$\sin \theta = \frac{y}{r} \Rightarrow y = r \sin \theta$$

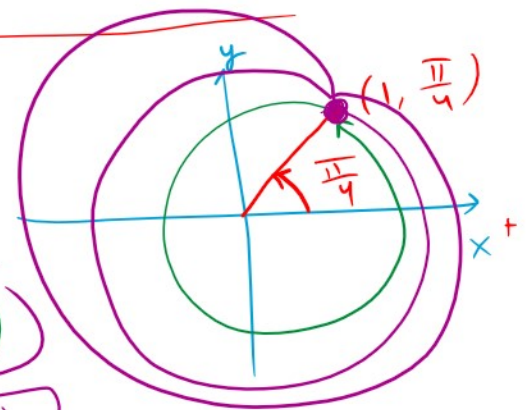
$$x^2 + y^2 = r^2$$

$$\tan \theta = \frac{y}{x}$$



Polar coordinates are unique

$$\begin{aligned} (r, \theta) &= \left(1, \frac{\pi}{4}\right) \\ &= \left(1, \frac{\pi}{4} + 2\pi\right) \\ &= \left(1, \frac{\pi}{4} - 4\pi\right) \end{aligned}$$



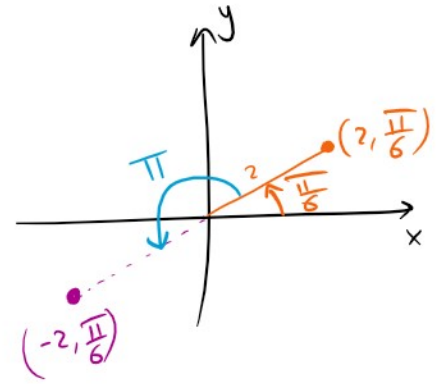
Polar coordinate

- $(r, \theta) = (r, \theta + \underline{2\pi} m)$ \rightarrow same direction
 $m = 0, \pm 1, \pm 2, \dots$

• $(r, \theta) = (r, \theta + 2\pi m)$ $m = \dots, -1, 0, 1, \dots$

Exp $(2, \frac{\pi}{6}) = (2, \frac{\pi}{6} - 2\pi) = (2, -\frac{11\pi}{6})$ ✓

$= (2, \frac{\pi}{6} + 2\pi)$ ✓



$(r, \theta) = (-r, \theta + \pi + 2\pi m)$

direction reverse

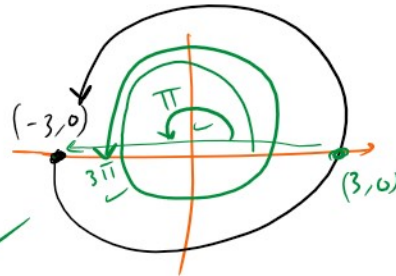
$m = 0, \pm 1, \pm 2, \dots$

$\theta + (2m+1)\pi$
مزری

Exp $P(3, 0) = (-3, \pi)$ ✓

مزری

$P(-3, 0) = (-3, 2\pi)$ ✓



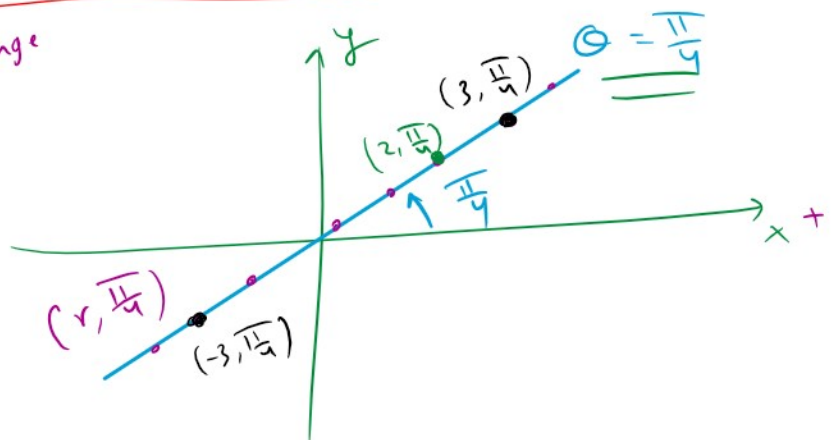
- $\pm \pi$
- $\pm 3\pi$
- $\pm 5\pi$
- \vdots

$P(2, \frac{2\pi}{3}) = (-2, \frac{2\pi}{3} - \pi) = (-2, -\frac{\pi}{3})$

$\theta = \theta_0$
 $\theta = \frac{\pi}{4}$ line

line makes angle of θ_0 with x^+

r change



of r_0

$r = r_0$

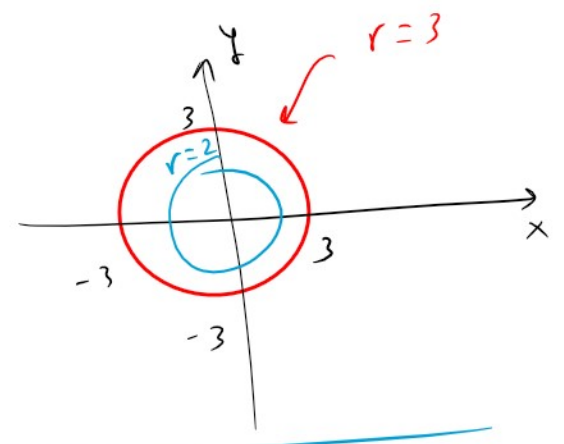
circle with radius $|r_0|$ and center origin

$r = 2 \Rightarrow r^2 = 4 \Rightarrow$

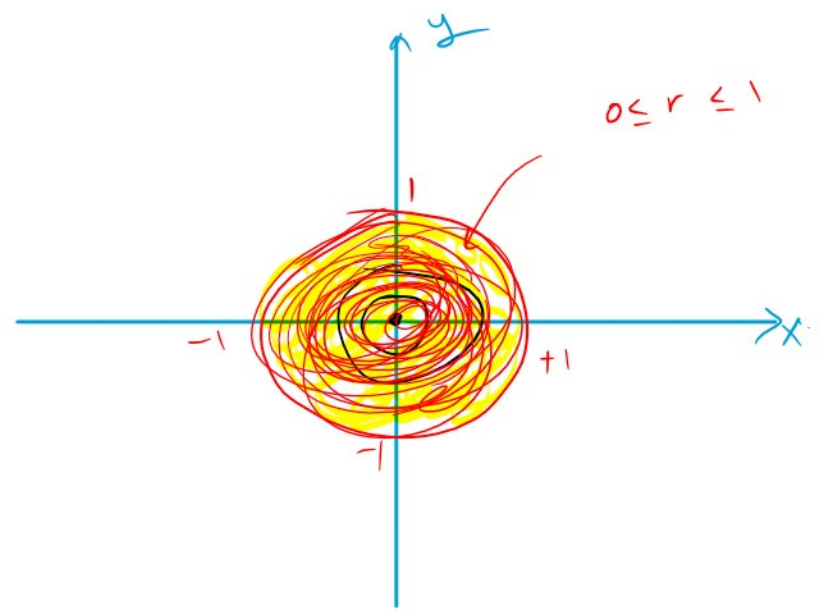
$x^2 + y^2 = 4$

circle with radius 2 and center origin

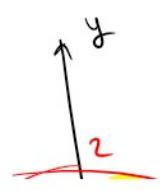
$r = 3$ $r = 2$

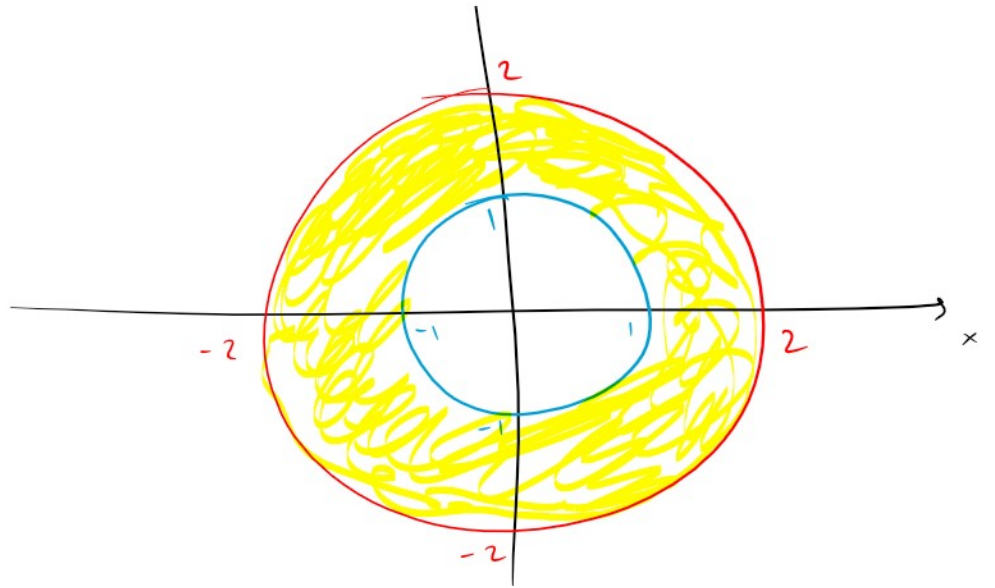


$0 \leq r \leq 1$

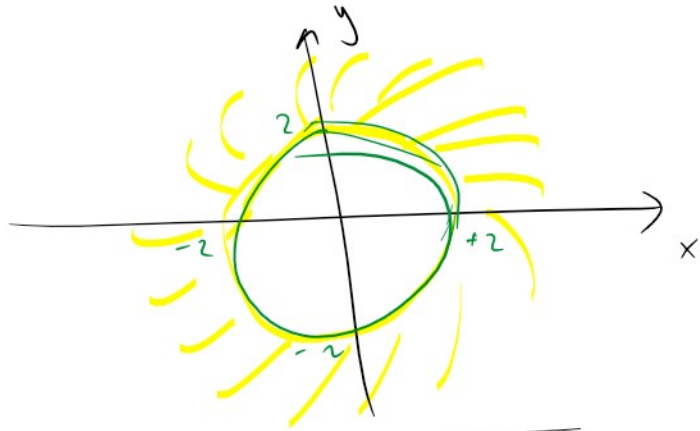


$1 \leq r \leq 2$

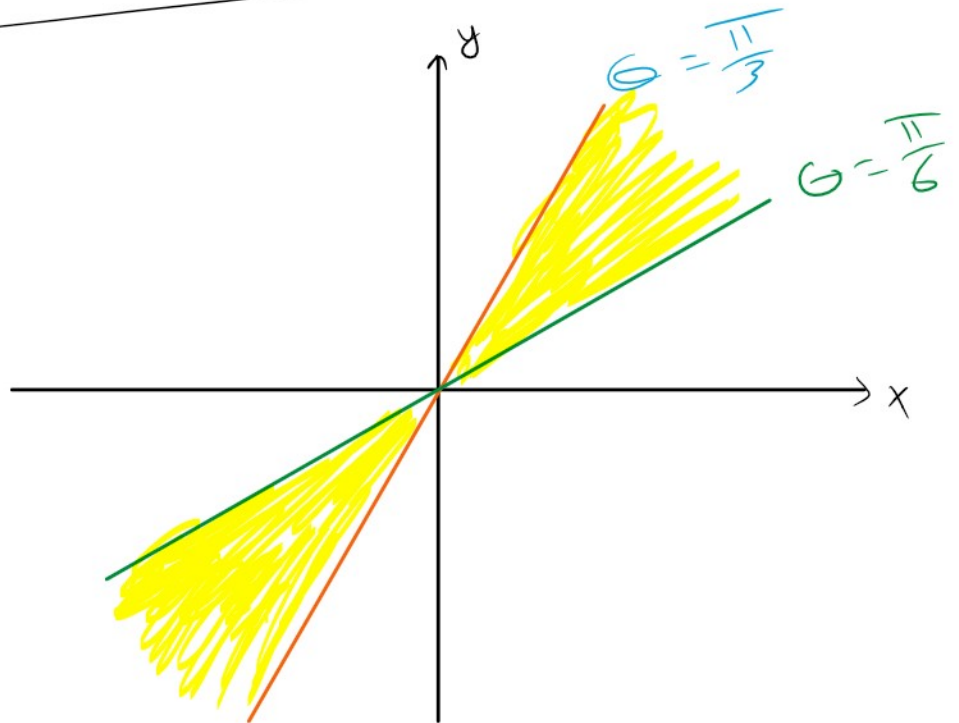




$$r > 2$$

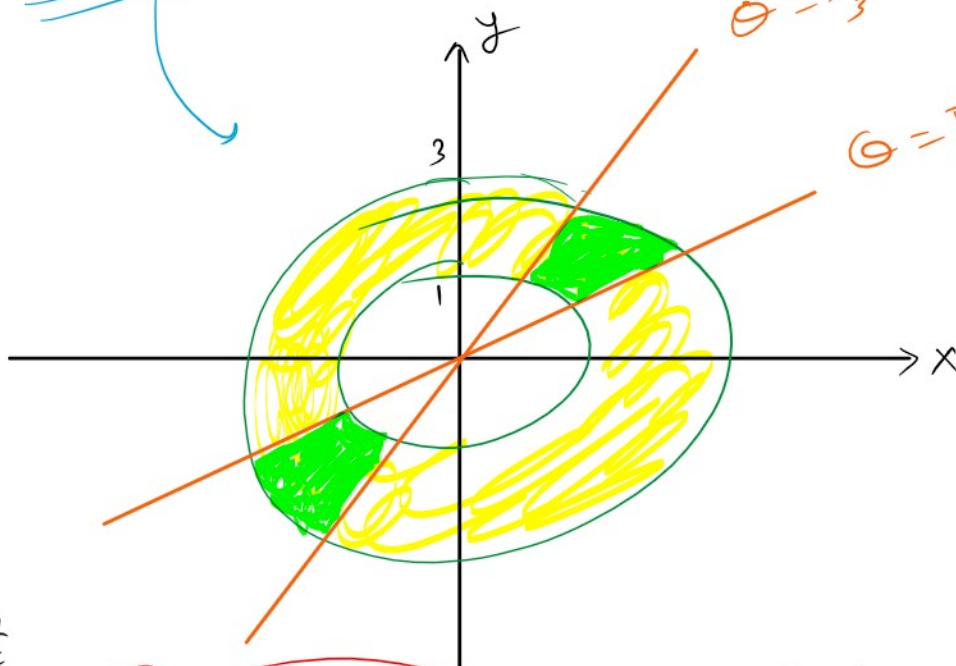


$$\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$$



$$1 \leq r \leq 3$$

$$\text{and } \frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$$



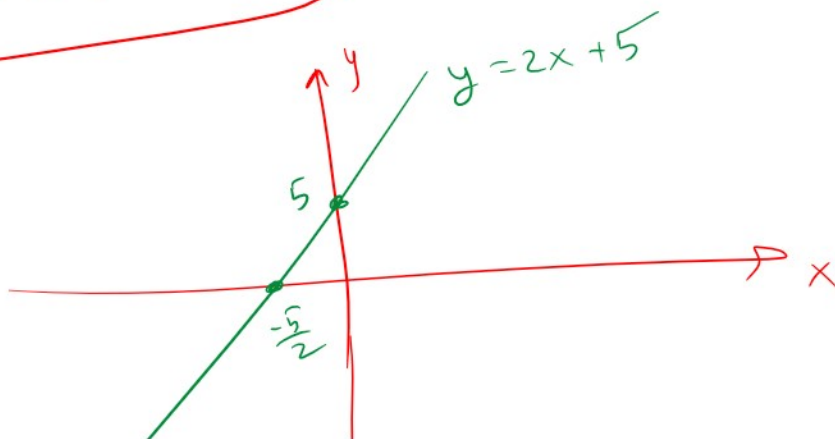
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Ex 1

$$r = \frac{5}{\sin \theta - 2 \cos \theta}$$

$$r \sin \theta - 2r \cos \theta = 5$$

$$y - 2x = 5$$

$$y = 2x + 5 \text{ line}$$



Find

Cartesian Eq

CE

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \\ x^2 + y^2 &= r^2 \\ \frac{y}{x} &= \tan \theta \end{aligned}$$

(6) $xy = 2$ Find PE

$$r \cos \theta \cdot r \sin \theta = 2$$

$$r^2 \cos \theta \sin \theta = 2$$

$$r^2 \boxed{2 \cos \theta \sin \theta} = 4$$

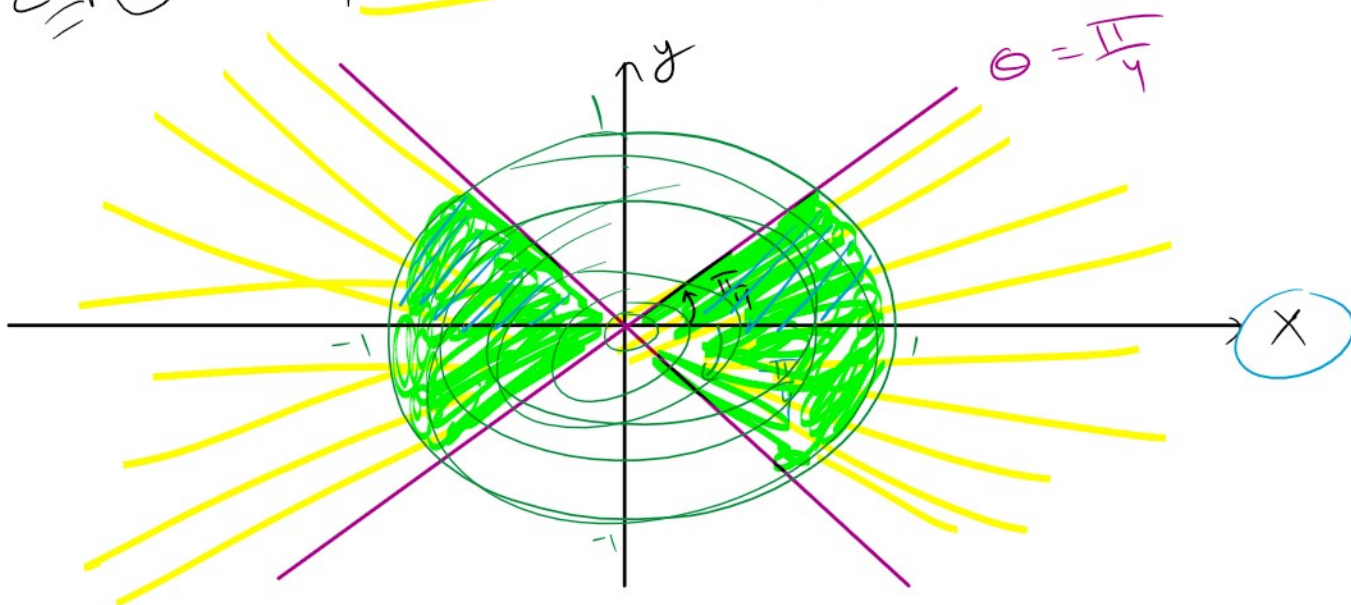
$$r^2 \sin 2\theta = 4$$

Exp (24)

$$-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

$$-1 \leq r \leq 1$$

Graph



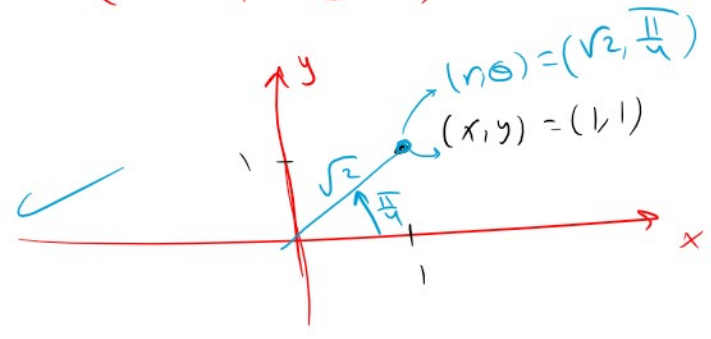
Exp (a) $(r, \theta) = (\sqrt{2}, \frac{\pi}{4})$ Find (x, y)

Ex 8 (a) $(r, \theta) = (\underline{\sqrt{2}}, \frac{\pi}{4})$

$$x = r \cos \theta = \sqrt{2} \cos \frac{\pi}{4} = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1$$

$$y = r \sin \theta = \sqrt{2} \sin \frac{\pi}{4} = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1$$

$$\underline{(r, \theta)} = \underline{(\sqrt{2}, \frac{\pi}{4})} = (x, y) = (1, 1)$$



(b) $(x, y) = (\underline{-3}, 0)$

Find $\underline{(r, \theta)}$

$$r^2 = x^2 + y^2 = (-3)^2 + (0)^2 = 9$$

$$\sqrt{r^2} = \sqrt{9} \Rightarrow |r| = 3 \Rightarrow \underline{r = 3}, r = -3 \text{ X}$$

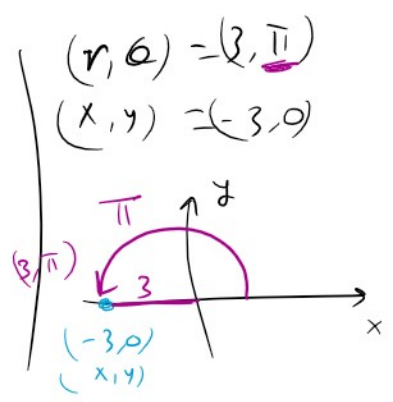
$0 \leq \theta < 2\pi$, $r \geq 0$

$$x = r \cos \theta \Rightarrow -3 = 3 \cos \theta \Rightarrow \underline{-1 = \cos \theta}$$

$$y = r \sin \theta \Rightarrow 0 = 3 \sin \theta \Rightarrow \underline{0 = \sin \theta}$$

$\theta = \pi, \pi \pm 2\pi, \pi \pm 4\pi, \dots$

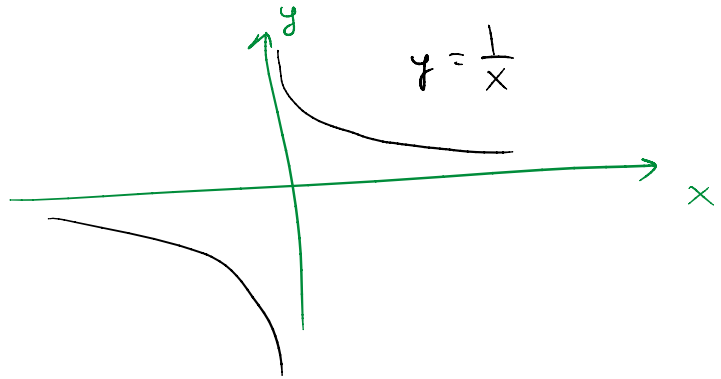
$(r, \theta) = (3, \pi)$



Exp (a) Find CE and sketch

$$r^2 \sin 2\theta = 2$$
$$r^2 / 2 \sin \theta \cos \theta = 2$$

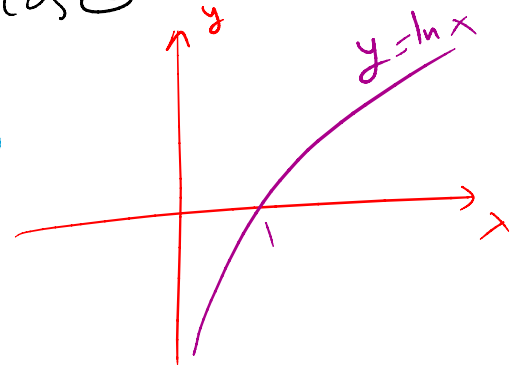
$$\frac{r \sin \theta}{y} \quad \frac{r \cos \theta}{x} = 1 \Rightarrow y = \frac{1}{x}$$



(b) $\underline{r \sin \theta} = \underline{\ln r} + \underline{\ln \cos \theta}$

$$= \ln (r \cos \theta)$$

$$y = \ln x$$



Exp $(x-5)^2 + y^2 = 25$

Find PE

$$x^2 - 10x + 25 + y^2 = 25$$

$$r^2 - 10r \cos \theta = 0$$

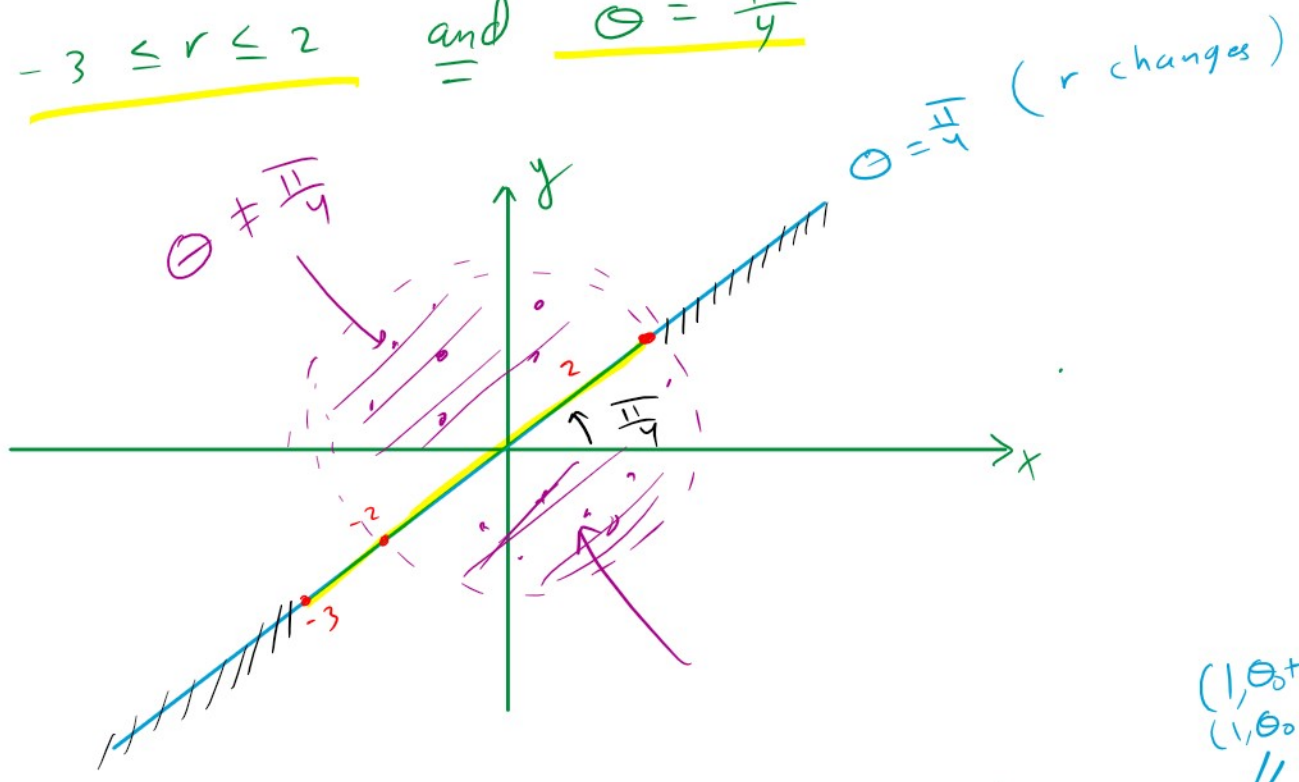
$$r^2 = 10r \cos \theta$$

$$r \neq 0$$

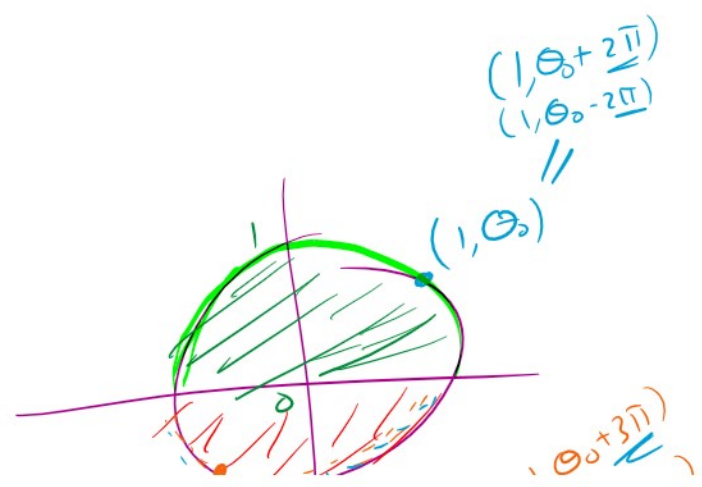
$$r = 10 \cos \theta$$

Exp sketch the set of points which satisfy

$$-3 \leq r \leq 2 \quad \text{and} \quad \theta = \frac{\pi}{4}$$



$$0 \leq r \leq 1$$



$$-1 \leq r \leq 1$$

$$|r| \leq 1$$



$$(-1, \theta_0 + \pi)$$

$$= (-1, \theta_0 + \pi + 2\pi) = (-1, \theta_0 + 3\pi)$$

$$= (-1, \theta_0 + \pi - 2\pi) = (-1, \theta_0 - \pi)$$

(16)

$$\theta = \frac{2\pi}{3}$$

$$r \leq -2$$

