1: Which of the following statements is true which is false

a) any element of a group has a unique inverse. T

b) if $G$ is a group then the equation $x^2 = e$ has a unique solution

c) In a group $G$, if $a, b, c \in G$ and $ab = ca$ then $b = c$. T

d) in a group $G$ if $a^2 = a$ then $a = e$.

Q2: G be a group s.t $(ab)^2 = a^2 b^2$ show that $ab = ba$.

suppose that $(ab)^2 = a^2 b^2$

$a^{-1}abab = a^{-1}aa\,bb$ multiply by $a^{-1}$ from left

$bab = a\,bb$

$babb^{-1} = a\,bbb^{-1}$ multiply by $b^{-1}$ from right.

$ba = ab$

So its Abelian

$Q_3$ Let $*$ be defined on $\mathbb{Q}^+$ by $a*b = 2ab$

Show that $(G, *)$ is a group.

Q3: let $*$ be defined on $Q^+$ by $a * b = 2ab$ show that $(G, *)$ is a group.

$G = (Q^+, *) \neq \emptyset$

① **Closure**: $\forall a,b \in G \rightarrow a*b \in G$.

$a*b = 2ab \in Q^+$ since $a,b \in Q^+$ ✓

② **Associative**: let $a,b,c \in G$

$(a*b)*c \overset{??}{=} a*(b*c)$

$\underbrace{2ab}_{} * c \overset{??}{=} a * 2bc$

$A * c \overset{?}{=} a * B$

$2Ac \overset{?}{=} 2aB$

$2(2ab)c \overset{??}{=} 2a(2bc)$

$4abc = 4abc$ so its Associtive.

③ **Identity**: $a*e = a$

$2ae = a \rightarrow \boxed{e = \frac{1}{2}}$

④ **Inverse**: $a*a^{-1} = \frac{1}{2} \rightarrow \left(2aa^{-1} = \frac{1}{2}\right) \times \frac{1}{2a}$

$\boxed{a^{-1} = \frac{1}{4a}}$