Exercises :

Q1 : Five observations were selected from each of three pop. The data obtained follow :

observation	sample 1	sample 2	sample 3
1	32	44	33
2	30	43	36
3	30	44	35
4	26	46	36
5	32	48	40
sample mean	30	45	36
sample variance (s_j^2)	6	4	6.50
sample standard deviation (s_j)	2.45	2	2.55

$n_T = 15$

$\rightarrow \bar{\bar{x}} = 37$

a. Compute the between treatments estimate of σ^2.

$$MSTR = \frac{\sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2}{K-1} = \frac{5(30-37)^2}{3-1} + \frac{5(45-37)^2}{3-1} + \frac{5(36-37)^2}{3-1}$$

$$= 122.5 + 160 + 2.5$$

245
320
5

$$MSTR = 285$$

b. Compute within-treatments estimate of σ^2 :

$$MSE = \frac{\sum_{j=1}^{k} (n_j - 1)S_j^2}{n_T - K} = \frac{(5-1)(6)}{15-3} + \frac{(5-1)(4)}{15-3} + \frac{(5-1)(6.5)}{15-3}$$

$$= 2 + 1.33 + 2.17$$

$$MSE = \underline{5.5}$$

c. At the $\alpha = 0.05$ level of significance, can we reject the null hypothesis that the means of the three populations are equal?

$$F = \frac{MSTR}{MSE} = \frac{285}{5.5} = 51.82$$

• p-value approach:

p-value < α

• critical-value approach:

$F_{0.05} = 3.89$

$F > F_\alpha$

so we reject H_0 ($\alpha = 0.05$)

d. set up the ANOVA table for this problem.

S.O.V	df	SS	ME	F
Treatments	2	570	285	51.82
Error	12	66	5.5	
Total	14	636		

Q2 : Four observations were selected from each of three populations.

observations	sample 1	sample 2	sample 3
1	165	174	169
2	149	164	154
3	156	180	161
4	142	158	148
$\bar{x}_j$	153	169	158
s_j^2	96.67	97.33	82.00

$\rightarrow \bar{\bar{x}} = 160$

a. Compute the between treatments estimate of σ^2.

$$MSTR = \frac{\sum n_j(\bar{x}_j - \bar{\bar{x}})^2}{k-1} = \frac{4(153-160)^2}{3-1} + \frac{4(169-160)^2}{3-1} + \frac{4(158-160)^2}{3-1}$$

98 162 8

$$= 268$$

b. Compute the within treatments estimate of σ^2 :

$$MSE = \frac{\sum_{j=1}^{n}(n_j-1)s_j^2}{n_T - k} = \frac{(4-1)(96.67)}{12-3} + \frac{(4-1)(97.33)}{12-3} + \frac{(4-1)(82)}{12-3}$$

32.22 32.44 27.33

$$= 92.$$

C. At $\alpha = 0.05$ level of significance can we reject the null hypothesis ?

$$F = \frac{MSTR}{MSE} = \frac{268}{92} = 2.91$$

- p-value approach :

  p-value > 0.1

  p-value > 0.05

  α

- critical approach :

  $F_\alpha = 4.26$

  $F \leq F_\alpha$

So we don't Reject the null hypothesis

d. set up the ANOVA table for this problem:

score of variation	df	sum of squares	mean square	F
Treatments	2	536	268	2.91
Error	9	828	92	
Total	11	1364		

Q3: samples were selected from three populations. The data obtained follow:

	Sample 1	sample 2	sample 3
1	93	77	88
2	98	87	75
3	107	84	73
4	102	95	84
5	85	75	82
$\bar{x}_j$	100	85	79
s_j^2	35.33	35.6	43.5

$\rightarrow \bar{\bar{x}} = 88$

a. between-treatments estimate of σ^2.

$$MSTR = \frac{\sum n_j(\bar{x}_i - \bar{\bar{x}})^2}{K-1} = \frac{\overset{360}{5(100-88)^2}}{3-1} + \frac{\overset{22.5}{5(85-88)^2}}{3-1} + \frac{\overset{202.5}{5(79-88)^2}}{3-1} = 585.$$

b. within treatments estimate of σ^2:

$$MSE = \frac{(n_j-1)\,s_j^2}{n_T - K} = \frac{4\,(35.33)}{15-3} + \frac{4(35.6)}{12} + \frac{4\,(43.5)}{12} = 38.14$$

c. $\alpha = 0.05$, can we reject the null hypothesis?

- $F = \frac{MSTR}{MSE} = \frac{585}{38.14} = 15.34$

- P-value approach:

  P-value < 0.01

  P-value < α

- critical approach:

  $F_\alpha = 3.89$

  $F > F_\alpha$

So we reject the null hypothesis.

d. set up the ANOVA table:

S.O.V	df	SS	MS	F
Treatments	2		MSTR = 585	F = 15.34
Error	12		MSE = 38.14	
Total	14			

Q4: A Random sample of 16 observation was selected from each of four populations. A portion of the ANOVA table follows: a. provide the missing entries for the table.

$k = 4$

$n_T = 4 \times 16 = 64$

source of variation	df	sum of squares	mean square	F
Treatments	$k-1 = 4-1 = 3$	1200	MSTR 400	$\frac{400}{5} = 80$
Error	$n_T - k = 60$	$1500 - 1200 = 300$	$\frac{300}{60} = 5$	
Total	63	1500		

$MSTR = \frac{SSTR}{df}$

$SSTR = MSTR \times df$

$= 400 \times 3$

$= 1200$

b. At $\alpha = 0.05$ can we reject the null hypothesis?

$F_{0.05}$ with $df_1 = 3$, $df_2 = 60$

$F_{0.05} = 2.76$ so $F > F_\alpha$ so reject the null hypothesis.

$K = 3$ $\quad n_T = 3(25) = 75$

Q5: Random sample of 25 observations were selected from each of three

For these data $SSTR = 120$ and $SSE = 216$.

a. set up the ANOVA table for this problem.

Sorce of variation	df	sum of squares	Mean square	F
Treatments	2	$SSTR = 120$	$MSTR = \frac{120}{2} = 60$	$F = \frac{60}{3} = 20$
Error	72	$SSE = 216$	$MSE = \frac{216}{72} = 3$	
Total	74	336		

b. At $\alpha = 0.05$ can we reject the null hypothesis.

$K - 1 = 2$

$n_T = 75$

$F =$

$df = 72$

• P-value approach

• critical value approach :

Q6:

	1	2	3
	20	28	20
	26	26	19
	24	31	23
	22	27	22
$\bar{X}_j$	23	28	21
S_j^2	6.67	4.67	3.33

observation $= n_j = 4$

Treatments $= 3$.

$\rightarrow \bar{\bar{X}} = 24$

By calculator

Q6 :

$$\rightarrow MSTR = \frac{\sum n_j(\bar{x}_j - \bar{\bar{x}})^2}{K-1} = \frac{4(23-24)^2}{3-1} + \frac{4(28-24)^2}{3-1} + \frac{4(21-24)^2}{3-1}$$

2 32 18

$$\boxed{MSTR = 52}$$

$$\rightarrow MSE = \frac{\sum (n_j-1)s_j^2}{n_T - K} = \frac{3(6.67)}{12-3} + \frac{3(4.67)}{9} + \frac{3(3.33)}{9} = \boxed{4.89}$$

$$\rightarrow F = \frac{MSTR}{MSE} = \frac{52}{4.89} = 10.63$$

- p-value approach :

  p-value < 0.01

  p-value $< 0.05\ \alpha$

- critical value approach

  $F_\alpha = 4.26$

  $F > F_\alpha$

so we can reject the null hypothesis.

Q7 : $n_j = 8$, treatments $= 3$, $n_T = 24$

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we find $\rightarrow$

	1	2	3
$\bar{x}_j$	5.75	5.5	5.25
s_j^2	1.64	2	1.93

$\rightarrow \bar{\bar{x}} = 5.5$

$$\rightarrow MSTR = \frac{\sum n_j(\bar{x}_j - \bar{\bar{x}})^2}{K-1} = 0.5$$

$$\rightarrow MSE = \frac{\sum (n_j-1)s_j^2}{n_T - K} = \frac{7(1.64)}{21} + \frac{7(2.55)}{21} + \frac{7(1.93)}{21} =$$

Q7: $F = \frac{MSTR}{MSE} = \frac{0.54}{2.04} = 0.26$

- p-value approach:

  p-value > 0.1

  p-value > α

- critical-value approach:

  $F_\alpha = 3.47$

  $F < F_\alpha$

So we cannot reject the null hypothesis.

Q8: $\alpha = 0.05$, $n_j = 6$, $K = 3$, $n_T = 18$

	1	2	3
$\bar{x}_j$	5	4.5	6
S_j^2	0.8	0.3	0.4

$\rightarrow \bar{\bar{x}} = 5.17$

$\rightarrow MSTR = \frac{\sum n_j(\bar{x}_j - \bar{\bar{x}})^2}{K-1} = \frac{\underset{0.0867}{6(5-5.17)}}{2} + \frac{\underset{1.3467}{6(4.5-5.17)^2}}{2} + \frac{\underset{2.0667}{6(6-5.17)^2}}{2} = 3.5$

$\rightarrow MSE = \frac{\sum(n_j-1)S_j^2}{n_T - K} = \frac{5(0.8)}{15} + \frac{5(0.3)}{15} + \frac{5(0.4)}{15} = 0.5$

$\rightarrow F = \frac{MSTR}{MSE} = \frac{3.5}{0.5} = 7$

- p-value approach

  p-value < 0.01

  p-value < α

- critical-value approach:

  $F_\alpha = 3.68$

  $F > F_\alpha$

So we can reject the null hypothesis.

Q9 . use $\alpha = 0.05$ to test for any significant difference in job ... ?

$n_j = 15$, $K = 3$, $n_T = 45$

	1	2	3
$\bar{x}_j$	67.73	61.13	65.8
s_j^2	117.64	179.98	137.17

$\rightarrow \bar{\bar{x}} = 64.89$

$$\rightarrow MSTR = \frac{\sum n_j(\bar{x}_j - \bar{\bar{x}})^2}{K-1} = \frac{15(67.73-64.89)^2}{2} + \frac{15(61.13-64.89)^2}{2} + \frac{15(65.8-64.89)^2}{2}$$

$$= 60.492 \quad 66.032 \quad 6.21075$$

$$= 172.73$$

$$\rightarrow MSE = \frac{\sum (n_j - 1) s_j^2}{n_T - K} = \frac{14(117.64)}{42} + \frac{14(179.98)}{42} + \frac{14(137.17)}{42} =$$

$$39.21333333 + 59.99333333 + 45.72333333$$

$$= 144.93$$

$$\rightarrow F = \frac{MSTR}{MSE} = \frac{172.73}{144.93} = 1.19$$

- p-value approach :
- critical-value approach :

. df = 42 ما في بالجدول

:(

Q6. $n_j = 8$, $K = 3$, $n_T = 24$

	1	2	3
$\bar{x}_i$	92.24	89.65	84.8
s_j^2	39.33	24.73	5.13

$\rightarrow \bar{\bar{x}} = 88.90$

$$\rightarrow MSTR = \frac{\sum n_j(\bar{x}_j - \bar{\bar{x}})^2}{K-1} = \frac{8(92.24-88.9)^2}{2} + \frac{8(89.65-88.9)^2}{2} + \frac{8(84.8-88.9)^2}{2}$$

$$= 44.6224 + 2.25 + 67.24$$

$$= 114.11$$

$$\rightarrow MSE = \frac{\sum(n_j-1)s_j^2}{n_T - K} = \frac{7(39.33)}{21} + \frac{7(24.73)}{21} + \frac{7(5.13)}{21} = 23.06$$

$$\rightarrow F = \frac{MSTR}{MSE} = \frac{114.11}{23.06} = 4.95$$

- p-value approach:

  p-value $\in (0.01, 0.025)$

  p-value $< \alpha$

- critical-value approach:

  $F_\alpha = 3.47$

  $F > F_\alpha$

so we can Reject the null hypothesis

13.2 done ...