Section 1

BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT
MATH1411 - QUIZ 5

Key

Name :....................... Student Number....................... Section 10/10

Question #1: Circle the correct answer: 3 Pt + 1 Bonus.

1) $\lim\limits_{x \longrightarrow 0} \frac{\sqrt{2+x}-\sqrt{2}}{\cos x - e^x}$

a) $\frac{-1}{\sqrt{2}}$

b) $\frac{-1}{2\sqrt{2}}$

c) $\frac{1}{\sqrt{2}}$

2) If $y = 10^{10^x}$, then $\frac{dy}{dx}$ is:

a) $10^{10^x} ln10$

b) $10^{10^x}(ln10)^2$

c) $10^{10^x} 10^x (ln10)^2$

3) If $y = xe^y$,then $\frac{dy}{dx}$ is

a) $\frac{y}{1-y}$

b) $\frac{y}{x(1-y)}$

c) $\frac{x}{y(1-y)}$

Question #2: Find the integral.

3pt. a) $\int_1^4 \frac{1}{\underset{u}{4x}\sqrt{\underset{u^2}{16x^2}-\underset{a^2}{5}}}dx = \frac{1}{4}\int \frac{du}{u\sqrt{u^2-(\sqrt{5})^2}}$

$u = 4x$
$du = 4\,dx$

$x=1 \longrightarrow u=4$
$x=4 \longrightarrow u=16$

$$= \frac{1}{4}\,\frac{1}{\sqrt{5}}\,\sec^{-1}\left(\left|\frac{u}{\sqrt{5}}\right|\right)\Bigg]_4^{16}$$

$$= \frac{1}{4\sqrt{5}}\,\sec^{-1}\frac{|u|}{\sqrt{5}}\Bigg]_4^{16}$$

$$= \frac{1}{4\sqrt{5}}\left[\sec^{-1}\frac{16}{\sqrt{5}} - \sec^{-1}\frac{4}{\sqrt{5}}\right]$$

3pt b) $\int \frac{12}{1+9x^2}dx$

let $u = 3x$
$du = 3\,dx$

$$\int \frac{\overset{4}{\cancel{12}}}{1+(u)^2}\,\frac{du}{\cancel{3}}$$

$$\int \frac{4}{1+u^2}\,du = 4\tan^{-1}(u) + C$$

$$= 4\tan^{-1}(3x) + C$$

Section 6

BIRZEIT UNIVERSITY
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Question #1: Circle the correct answer: 3pt. + 1 Bonus.

1) If $y = e^{\ln(x+\sqrt{x^2+16})}$, then $\frac{dy}{dx}$ is:

a) $e^{\ln(x+\sqrt{x^2+16})}$

b) $1 + \frac{x}{\sqrt{x^2+16}}$

c) $e^{\ln(x+\sqrt{x^2+16})}(1 + \frac{x}{\sqrt{x^2+16}})$

2) $2^{3\log_2 4} =$

a) 16

b) 8

c) 64

3) $\lim\limits_{x \longrightarrow 0^+} x^{\sqrt{x}} =$

a) 0

b) 1

c) e

Question #2: Find the integral.

let $U = x - 3$
$dU = dx$

3 Pt a) $\int \frac{1}{4+(x-3)^2}dx$

$$\int \frac{1}{4+U^2}\,dU = \int \frac{1}{(2)^2+(U)^2}\,dU$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{U}{2}\right) + C$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$$

let $U = x + 1$
$dU = dx$

3 pt. b) $\int \frac{x-2}{(x+1)^2+4}dx$

$$\int \frac{U-1-2}{U^2+4}\,dU$$

$$= \frac{1}{2}\int \frac{2U}{U^2+4}\,dU - \int \frac{3}{\underset{U^2+a^2}{U^2+4}}\,dU$$

$$\frac{1}{2}\ln|U^2+4| - 3\cdot\frac{1}{2}\tan^{-1}\left(\frac{U}{2}\right) + C$$

$$\frac{1}{2}\ln|(x+1)^2+4| - \frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$$

Section 21

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Key

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Question #1: Find the integral. + 1 Bonus

3 pt. a) $\int_0^1 \frac{1}{\sqrt{4-x^2}}dx$ $= \int_0^1 \frac{1}{\sqrt{(2)^2-(x)^2}}\,dx$

$= \sin^{-1}\left(\frac{x}{2}\right)\Big]_0^1$

$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$

$= \frac{\pi}{6} - 0$

$= \frac{\pi}{6}$

3 pt. b) $\int \frac{4x+3}{\sqrt{1-x^2}}dx$ $= \int \frac{4x}{\sqrt{1-x^2}}\,dx + \int \frac{3}{\sqrt{1-x^2}}\,dx$

$= -4(1-x^2)^{1/2} + 3\sin^{-1}(x) + C$

$\int \frac{4x}{\sqrt{1-x^2}}\,dx = \int \frac{-2\,du}{\sqrt{u}}$

$= -2\,\frac{u^{1/2}}{1/2} + C$

$= -4(1-x^2)^{1/2} + C$

Let $u = 1-x^2$
$du = -2x\,dx$

Question #2: Evaluate 3 pt.

a) $\lim_{x \longrightarrow 0^-} x \cot x.$ $0 \cdot -\infty$

$\lim_{x \to 0^-} \frac{x}{\tan x} = \frac{0}{0}$

$\lim_{x \to 0^-} \frac{1}{\sec^2 x} = \frac{1}{1} = \boxed{1}$

Section H

BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT
MATH1411 - QUIZ 5

Key

Name :...................... Student Number..................... Section ....10/10.

Question #1: Find the integral. + 1 Bonus.

3 Pt. a) $\int \frac{1}{4x\sqrt{16x^2-5}}dx$

let $u = 4x$
$du = 4dx$

$$\int \frac{1}{u\sqrt{u^2-(\sqrt{5})^2}} \cdot \frac{du}{4}$$

$$= \frac{1}{4}\int \frac{du}{u\sqrt{u^2-(\sqrt{5})^2}} = \frac{1}{4}\cdot\frac{1}{\sqrt{5}}\sec^{-1}\frac{|u|}{\sqrt{5}} + c$$

$$= \frac{1}{4\sqrt{5}}\sec^{-1}\frac{|4x|}{\sqrt{5}} + c$$

3 Pt. b) $\int_{\sqrt{3}}^{3} \frac{6}{9+x^2}dx$

$$\int_{\sqrt{3}}^{3} \frac{6}{(3)^2+x^2}dx$$

$$= 6\cdot\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\Big]_{\sqrt{3}}^{3} = 2\left[\tan^{-1}(1) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$$

$$= 2\left[\frac{\pi}{4} - \frac{\pi}{6}\right]$$

$$= \frac{\pi}{6}$$

Question #2: Evaluate

3pt. a) $\lim_{x \longrightarrow 0^+} x^{\sin x}$ $= 0^0$

let $f(x) = x^{\sin x}$

$\ln f(x) = \sin x \ \ln x$

$\lim_{x \to 0^+} \ln f(x) = \lim_{x \to 0^+} \sin x \ \ln x = 0 \cdot -\infty$

$= \lim_{x \to 0^+} \frac{\ln x}{\csc x} = \frac{-\infty}{\infty}$

$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\csc x \ \cot x}$

$= \lim_{x \to 0^+} \frac{-\sin^2 x}{x \cos x}$

$= \lim_{x \to 0^+} \frac{-2 \sin x \cos x}{x(-\sin x) + \cos x}$

$= 0$

Then $\lim_{x \to 0^+} x^{\sin x} = e^0 = 1$