Disk Method:

Def: Volume by Disks for Rotations about x-axis:

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\,(R(x))^2\,dx.$$

Note: The Cross-section is perpendicular to the axis of Revolution. (For Disk & Washer Method).

Def: Volume by Disks for Rotations about y-axis:

$$V = \int_c^d A(y)\,dy = \int_c^d \pi\,[R(y)]^2\,dy.$$

Washer Method:

Def: Volume by Washers for Rotations about x-axis:

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\left([R(x)]^2 - [r(x)]^2\right)dx$$

where $R[x]$: outer Radius , $r(x)$: Inner radius.

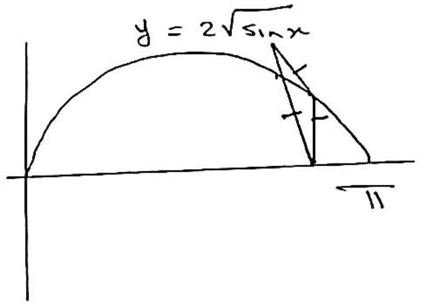
Def: Volume by Washers for Rotations about y-axis:

$$V = \int_c^d A(y)\,dy = \int_c^d \pi\left(R(y)^2 - r(y)^2\right)dy.$$

6.1 (5) Find the Volume of the Solid whose base is the region between the Curve $y = 2\sqrt{\sin x}$ and the Interval $[0, \pi]$ on the x-axis.

The Cross-sections perpendicular to the x-axis are: (dx)

(a) equilateral triangles with bases running from the x-axis to the Curve as shown:



$A(x)$ = Area of the Cross-section

$= \frac{1}{2}(\text{side})(\text{side}) \sin\frac{\pi}{3}$

$= \frac{1}{2}\left(2\sqrt{\sin x}\right)\left(2\sqrt{\sin x}\right)\frac{\sqrt{3}}{2} = \sqrt{3}\sin x.$

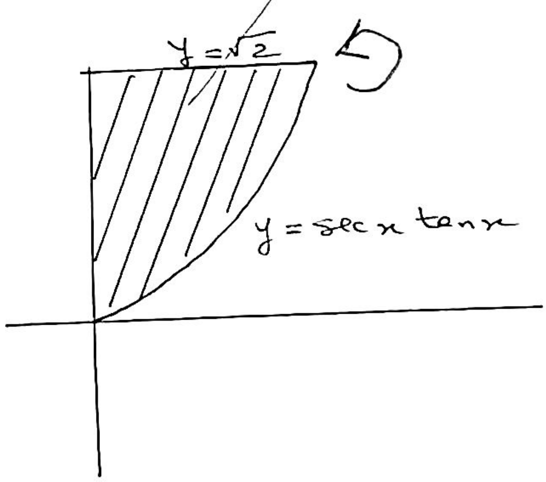
$V = \int_0^{\pi} \sqrt{3}\sin x \, dx = \sqrt{3}\left[-\cos x\right]_0^{\pi} = \sqrt{3} + \sqrt{3} = 2\sqrt{3}$

(b) squares with bases running from the x-axis to the Curve

$A(x) = (\text{side})^2 = \left(2\sqrt{\sin x}\right)^2 = 4\sin x$

$V = \int_0^{\pi} 4\sin x \, dx = 4\left[-\cos x\right]_0^{\pi} = 4 + 4 = 8$

(6.1) 25 Find the Volume of the Solid generated by Revolving the region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the Curve $y = \sec x \tan x$ and on the left by the y-axis, about the line $y = \sqrt{2}$.



Using Disk Method:

$$V = \int_a^b \pi \, [R(x)]^2 \, dx :$$

$$R[x] = \sqrt{2} - \sec x \tan x .$$

$$\Rightarrow \quad V = \int_0^{\frac{\pi}{4}} \pi \left[\sqrt{2} - \sec x \tan x\right]^2 dx$$

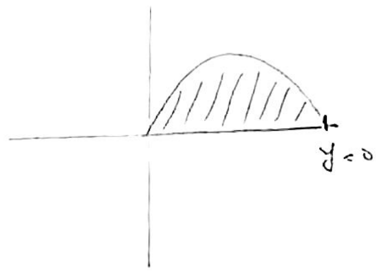
$$= \pi \int_0^{\frac{\pi}{4}} \Big[2 - 2\sqrt{2} \sec x \tan x + \underbrace{\sec^2 x \tan^2 x}_{\text{Let } u = \tan x} \Big] dx .$$

$$= \pi \left(\Big[2x\Big]_0^{\frac{\pi}{4}} - 2\sqrt{2} \, \Big| \sec x \Big]_0^{\frac{\pi}{4}} + \left[\frac{\tan^3 x}{3}\right]_0^{\frac{\pi}{4}} \right)$$

$$= \pi \left(\frac{\pi}{2} + 2\sqrt{2} - \frac{11}{3} \right) .$$

22) Find the Volume of the Solid generated by revolving the region bounded by $y = x - x^2$, $y = 0$ & the x-axis

$x - x^2 = 0 \Rightarrow x = 0, 1$

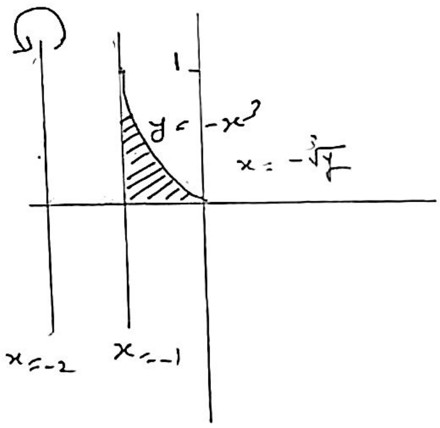


$$V = \int_0^1 \pi \left[x - x^2 \right]^2 dx =$$

$$= \pi \int_0^1 \left[x^2 - 2x^3 + x^4 \right] dx = \frac{\pi}{30}.$$

---

46) Find the Volume of the solid generated by revolving the region in the second quadrant bounded above by the Curve $y = -x^3$, below by the x-axis and on the Left by the Line $x = -1$ about the line $x = -2$.



$-y^{\frac{1}{3}} - 2$ $\quad$ $r(y) = -1 - 2$

$R(y) = 2 - y^{\frac{1}{3}}$ & $r(y) = 1$ distance

$$V = \int_0^1 \pi \left[(R(y))^2 - (r(y))^2 \right) dy$$

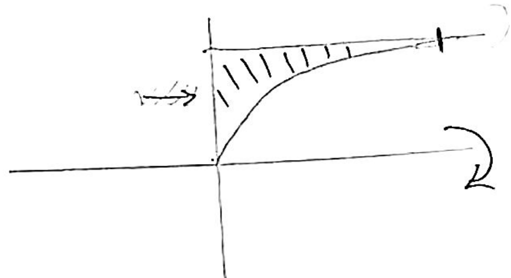
$$= \int_0^1 \pi \left[(2 - y^{\frac{1}{3}})^2 - 1 \right] dy$$

$$= \int_0^1 \pi \left[4 - 4y^{\frac{1}{3}} + y^{\frac{2}{3}} - 1 \right] dy = \pi \left[3y - 3y^{\frac{4}{3}} + \frac{3y^{5/3}}{5} \right]_0^1$$

$$= \boxed{\frac{3\pi}{5}}$$

* 6.1 (47) Find the Volume of the Solid generated by revolving the region bounded by $y=\sqrt{x}$ & $y=2$ & $x=0$ about: (a) The x-axis.

$$V = \int_0^4 \pi \left((2)^2 - (\sqrt{x})^2\right) dx = 8\pi$$

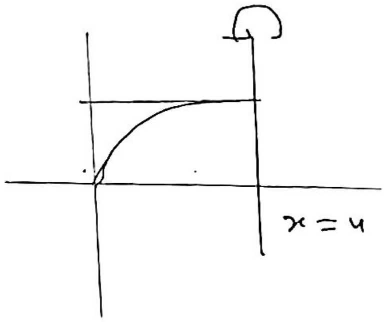


(b) y-axis: $x = y^2$.

$$V = \int_0^2 \pi \left[y^2\right]^2 dy = \frac{32\pi}{5}.$$

(c) Line $y = 2$

$$V = \int_0^4 \pi \left[(2-\sqrt{x})^2\right] dx = \frac{8\pi}{3}.$$



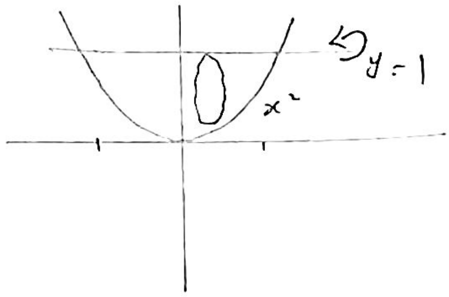
(d) Line $x = 4$.

$$V = \int_0^2 \pi \left[R(y)^2 - r^2(y)\right] dy$$

$$= \int_0^2 \pi \left[4^2 - (4-y^2)^2\right] dy = \frac{224}{15}\pi.$$

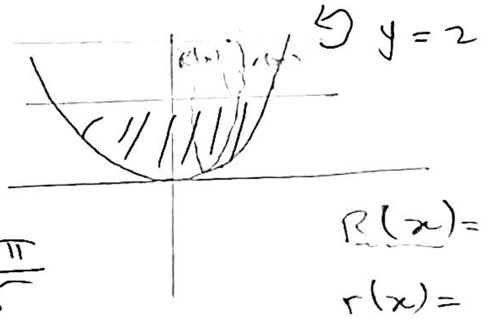
6.1 (49) Volume: $y = x^2$, $y = 1$: about:

(a) $y = 1$



$$V = \int_{-1}^{1} \pi \left[(1 - x^2)^2 \right] dx = \frac{16\pi}{15}$$

(b) $y = 2$

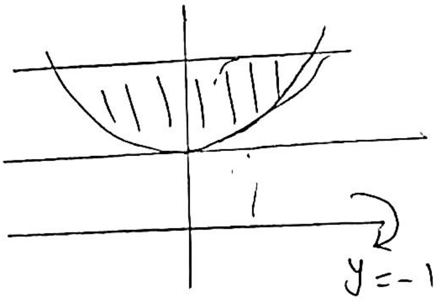


$$V = \int_{-1}^{1} \pi \left[(2 - x^2)^2 - 1^2 \right] dx = \frac{56\pi}{15}$$

$R(x) =$

$r(x) =$

(c) $y = -1$



$$V = \int_{-1}^{1} \pi \left[(2^2) - (1 + x^2)^2 \right] dx$$

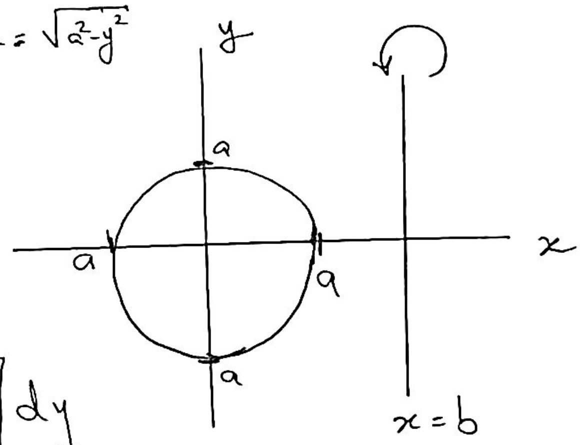
$$= \frac{64\pi}{15}$$

(6.1) 51 The Volume of a torus:

The Disk $x^2+y^2 \leq a^2$ is revolved about the line $x = b$ $(b>a)$ to generate a solid shaped like a doughnut and called a torus. Find its volume.

(Hint: $\int_{-a}^{a} \sqrt{a^2-y^2}\, dy = \frac{\pi a^2}{2}$).

Using Washer Method: $x^2+y^2=a^2 \Rightarrow x = \sqrt{a^2-y^2}$



$R(y) = b+\sqrt{a^2-y^2}$

$r(y) = b-\sqrt{a^2-y^2}$

$$V = \int_{-a}^{a} \pi \left[\left(b+\sqrt{a^2-y^2}\right)^2 - \left(b-\sqrt{a^2-y^2}\right)^2 \right] dy$$

$$= \pi \int_{-a}^{a} 4b\sqrt{a^2-y^2}\, dy$$

$$= 4b\pi \int_{-a}^{a} \sqrt{a^2-y^2}\, dy = 4b\pi \cdot \frac{\pi a^2}{2}$$

$$= 2\pi^2 a^2 b.$$

