key

$\frac{}{5}$

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

Section.......14D.......

• Name (Arabic)........................... Number..................

Q1) Choose the most correct answer. $f' < 0$ $f'' > 0$
If f is a function such that f' is negative and incresing on an interval I, then f is

1. Increasing and concave down on I
2. Decreasing and concave up on I (circled)
3. Decreasing and concave down on I
4. Increasing and concave up on I

Q2) Let $f(x) = x^2 - 2x, x \in [1, 2]$, find the value of c in the conclusion of the Mean value theorem .

$f'(x) = 2x - 2$

$f'(c) = \frac{f(2) - f(1)}{2 - 1}$

$2c - 2 = \frac{(2)^2 - 2(2) - [(1)^2 - 2(1)]}{1}$

$2c - 2 = 1 \Rightarrow 2c = 3 \Rightarrow \boxed{c = \frac{3}{2}}$

Q3) Let $f(x) = \frac{x^2}{x+1}$ where $f' = \frac{x(x+2)}{(x+1)^2}$ and $f'' = \frac{2}{(x+1)^3}$.
Find the intervals in which $f(x)$ is concave up and the intervals in which $f(x)$ is concave down ,
then find the inflection points(if any).

$D: (-\infty, \infty) / \{-1\}$

$f'' = \frac{2}{(x+1)^3}$

sign of f'': $- - -$ on $(-\infty, -1)$ ($\cap$), $+ +$ on $(-1, \infty)$ ($\cup$)

- Concave up on $(-1, \infty)$
- Concave down on $(-\infty, -1)$
- no inflection points

Key

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

• Name (Arabic)..... Number..... Section.....14D.....

Q1) If f is a function such that f' is positive and incresing on an interval I, then f is

$f'>0$ $f''>0$

1. Increasing and concave down on I
2. Decreasing and concave up on I
3. Decreasing and concave down on I
4. Increasing and concave up on I

Q2) Let $f(x) = x^2 - x, x \in [0, 2]$, find the value of c in the conclusion of the Mean value theorem

$f'(c) = \frac{f(2) - f(0)}{2 - 0}$ $\quad f' = 2x - 2$

$2c - 2 = \frac{(2)^2 - 2 - (0^2 - 0)}{2}$

$2c - 2 = 0 \Rightarrow 2c = 2 \Rightarrow \boxed{c = 1}$

Q3) Let $f(x) = \frac{-x^2}{x+1}$ where $f' = \frac{-x(x+2)}{(x+1)^2}$ and $f'' = \frac{-2}{(x+1)^3}$.
Find the intervals in which $f(x)$ is concave up and the intervals in which $f(x)$ is concave down , then find the inflection points(if any).

$D : (-\infty, \infty) / \{-1\}$

$f'' = \frac{-2}{(x+1)^3}$

$+ + + \; p \; - - -$ sign of f''
$\quad -1$
$\cup \quad \cap$

Concave up on $(-\infty, -1)$

Concave down on $(-1, \infty)$

no inflection points

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

• Name (Arabic)..... Number..... Section.....16D.....

Q1) Choose the most correct answer.
If the radius of a circle changes from 2 to 2.1, then the area of the circle changes approximately by

1. 0.4π
2. 0.2π
3. 0.3π
4. 0.6π

$A = r^2\pi \longrightarrow dA = 2r\pi\, dr$

$= 2(2)(\pi)(0.1)$

$= 0.4\pi$ (1)

Q2) Find the linearization of the function $y = x + \sin x$ at the point (π, π).

$L(x) = f(a) + f'(a)(x - a)$

$L(x) = f(\pi) + f'(\pi)(x - \pi)$

$= \pi + 0(x - \pi)$

$\boxed{L(x) = \pi}$ (1)

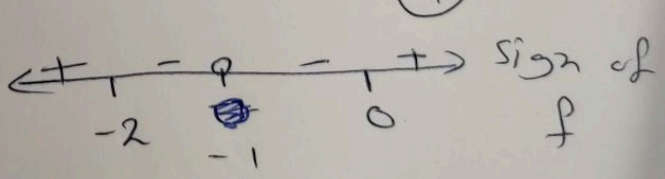
$f'(x) = 1 + \cos x$

$f'(\pi) = 1 + \cos\pi = 0$

Q3) Let $f(x) = \frac{x^2}{x+1}$ where $f' = \frac{x(x+2)}{(x+1)^2}$.
Find the intervals in which $f(x)$ is increasing and the intervals in which $f(x)$ is decreasing , then find the extreme values.

$D: (-\infty, \infty) / \{-1\}$

$f' = \frac{x(x+2)}{(x+1)^2}$ (1)



(1) increasing on $(-\infty, -2] \cup [0, \infty)$

decreasing on $[-2, 0] / \{-1\}$

max at $x = -2$, $f(-2) = \frac{(-2)^2}{-2+1} = \boxed{-4}$

min at $x = 0$, $f(0) = \boxed{0}$ (1)

Key

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

5

• Name (Arabic)..... Number..... Section.....16D.....

Q1) Choose the most correct answer.
If the radius of a circle changes from 3 to 3.1, then the area of the circle changes approximately by

$A = r^2\pi \qquad dA = 2r\pi\, dr$

$dA = 2(3)\pi(0.1)$

$= 0.6\pi$

1. 0.4π
2. 0.2π
3. 0.3π
4. 0.6π

Q2) Find the linearization of the function $y = x - \sin x$ at the point (π, π) (a)

$L(x) = f(\pi) + f'(\pi)(x - \pi)$

$f(\pi) = \pi$

$L(x) = \pi + 0(x - \pi)$

$f'(x) = 1 - \cos x$

$f'(\pi) = 1 - \cos(\pi)$

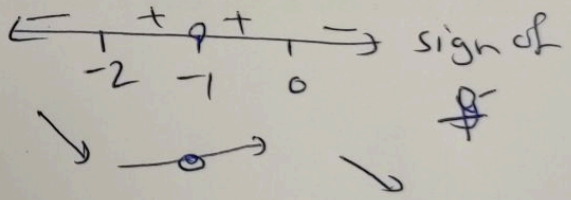
$= 0$

$\boxed{L(x) = \pi}$

Q3) Let $f(x) = \frac{-x^2}{x+1}$ where $f' = \frac{-x(x+2)}{(x+1)^2}$.
Find the intervals in which $f(x)$ is increasing and the intervals in which $f(x)$ is decreasing , then find the extreme values.

$D: (-\infty, \infty) / \{-1\}$

$f' = \frac{-x(x+2)}{(x+1)^2}$



increasing on $[-2, 0] / \{-1\}$

or $[-2, -1) \cup (-1, 0]$

decreasing on $(-\infty, -2] \cup [0, \infty)$

Local max at $x = 0 \rightarrow f(0) = \boxed{0}$

Local min at $x = -2 \rightarrow f(-2) = \frac{-(-2)^2}{-2+1} = \boxed{4}$

5

key

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

• Name (Arabic)..... Number..... Section.....20D.....

Q1) If the radius of a circle changes from 3 to 3.05, then the area of the circle changes approximately by

$A = r^2\pi \rightarrow dA = 2r\pi\, dr$

$dA = 2(3)(\pi)(0.05)$

$= 0.3\pi$

1. 0.6π
2. 0.2π
3. 0.3π (1)
4. 0.4π

Q2) Find the linearization of the function $y = x - \cos x$ at the point $(\frac{\pi}{2}, \frac{\pi}{2})$ (a)

$L(x) = f(\frac{\pi}{2}) + f'(\frac{\pi}{2})(x - \frac{\pi}{2})$

$f' = 1 - -\sin x$

$f'(\frac{\pi}{2}) = 1+1 = 2$

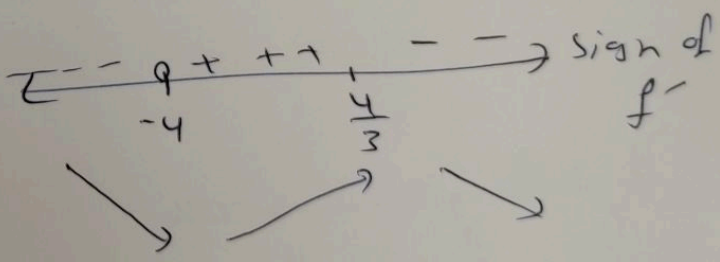
$= \frac{\pi}{2} + 2(x - \frac{\pi}{2})$

$= \frac{\pi}{2} + 2x - \frac{2\pi}{2} = \boxed{-\frac{\pi}{2} + 2x}$

Q3) Let $f(x) = \frac{-x^2+4x}{(x+4)^2}$ where $f' = \frac{4(4-3x)}{(x+4)^3}$ and $f'' = \frac{24(x-4)}{(x+4)^4}$ find the intervals in which $f(x)$ is increasing and the intervals in which $f(x)$ is decreasing , then find the extreme values.

$D = (-\infty, \infty) / \{-4\}$

$f' = \frac{4(4-3x)}{(x+4)^3}$



increasing on $(-4, \frac{4}{3}]$

decreasing on $(-\infty, -4) \cup [\frac{4}{3}, \infty)$

max at $\frac{4}{3}$

$f(\frac{4}{3}) = \frac{-(\frac{4}{3})^2 + 4(\frac{4}{3})}{(\frac{4}{3}+4)^2} = \frac{-\frac{16}{9} + \frac{16}{3}}{(\frac{16}{3})^2} = \frac{-\frac{16}{9} + \frac{48}{9}}{(\frac{16}{3})^2} = \frac{32}{(16)^2} = \frac{2}{16} = \boxed{\frac{1}{8}}$

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

• Name (Arabic)..... Number..... Section.....20D.....

Q1) Choose the most correct answer.
If the radius of a circle changes from 2 to 2.05, then the area of the circle changes approximately by

1. 0.2π
2. 0.6π
3. 0.3π
4. 0.4π

$A = r^2\pi \rightarrow dA = 2r\pi\, dr$
$= 2(2)(\pi)(0.05)$
$= 0.2\ \pi$

Q2) Find the linearization of the function $y = x + \cos x$ at the point $(\frac{\pi}{2}, \frac{\pi}{2})$

$L(x) = f(\frac{\pi}{2}) + f'(\frac{\pi}{2})(x - \frac{\pi}{2})$

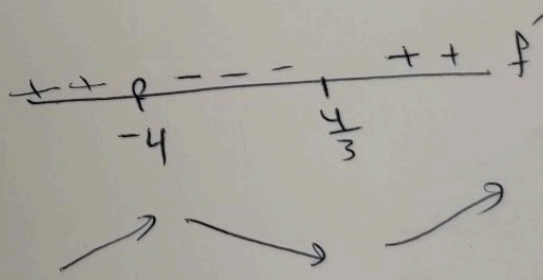
$= \frac{\pi}{2} + 0(x - \frac{\pi}{2})$

$L(x) = \frac{\pi}{2}$

$f' = 1 - \sin x$
$f'(\frac{\pi}{2}) = 1 - 1 = 0$

Q3) Let $f(x) = \frac{x^2-4x}{(x+4)^2}$ where $f' = \frac{4(3x-4)}{(x+4)^3}$ and $f'' = \frac{24(4-x)}{(x+4)^4}$ find the intervals in which $f(x)$ is increasing and the intervals in which $f(x)$ is decreasing , then find the extreme values.

$D: (-\infty, \infty) / \{-4\}$



f is increasing on $(-\infty, -4) \cup [\frac{4}{3}, \infty)$

f is decreasing on $(-4, \frac{4}{3}]$

f has local min at $x = \frac{4}{3}$

$f(\frac{4}{3}) = \frac{(\frac{4}{3})^2 - 4(\frac{4}{3})}{(\frac{4}{3}+4)^2} = \frac{\frac{16}{9} - \frac{16}{3}}{(\frac{16}{3})^2} = \frac{16-48}{\frac{9}{(16)^2}} = -\frac{32}{(16)^2} = -\frac{2}{16} = -\frac{1}{8}$

Q2) [24 %] Consider the function $f(x)$ and its first and second derivatives

$$f(x)=\frac{x^2-4x}{(x+4)^2},\quad f'(x)=\frac{4(3x-4)}{(x+4)^3},\quad f''(x)=\frac{24(4-x)}{(x+4)^4}$$

(1) The domain of f is $(-\infty,\infty)\setminus\{-4\}$

(2) $\lim_{x\to -4^+} f(x) = +\infty$

(3) $\lim_{x\to -4^-} f(x) = +\infty$

(4) $\lim_{x\to +\infty} f(x) =$

(5) $\lim_{x\to -\infty} f(x) =$

(6) Horizontal asymptote is $y=1$

(7) Vertical asymptote is $x=-4$

(8) the graph of f crosses the $x-$axis at the point(s) $(0,0),(4,0)$

(9) f is increasing on $(-\infty,-4)\cup[\frac{4}{3},\infty)$

(10) f is decreasing on $(-4,\frac{4}{3})$

(11) is $f(\frac{4}{3})$ an absolute maximum or minimum of $f(x)$? abs. min

(12) f is concave up on $(-\infty,-4)\cup(-4,4]$

(13) f is concave down on $[4,\infty)$

(14) f has inflection point(s) $(4,0)$

(15) the range of f is $(-\frac{1}{8},\infty)$

(16) Sketch the graph of f