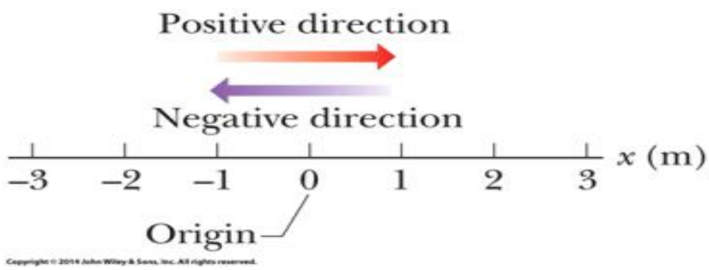
Chapter 2: Motion along a Straight Line

- Position is measured relative to a reference point (the origin, or zero point, of an axis)



- Distance is a scalar quantity

- Displacement = final position – initial position →→ DISPLACEMENT is a vector quantity

$$\Delta x = x_{final} - x_{initial}$$

- Average speed:

$$S_{avg} = \frac{total\ distance}{\Delta t}$$

- Average velocity:

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

- Instantaneous velocity:

$$v_{Ins} = \frac{dx}{dt}$$

- Average acceleration:

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

- Instantaneous acceleration:

$$a_{Ins} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

x(t) → v(t) → a(t) **Differentiation**

x(t) ← v(t) ← a(t) **Integration**

- Constant Acceleration:

Equation	Missing Quantity
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
$x - x_0 = vt - \frac{1}{2}at^2$	v_0

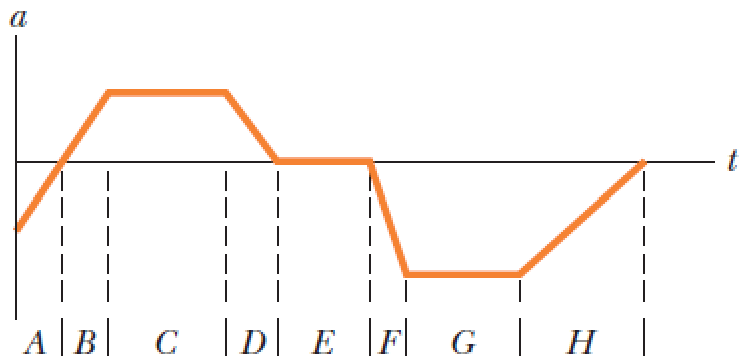
- Free- Fall Acceleration: Straight-Line motion with constant acceleration

An object rising or falling freely near Earth's surface.

+ y →→ vertically upward

$a = g = -9.8 \text{ m/s}^2$

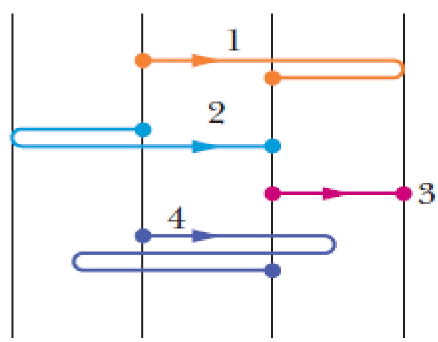
Q-2) The below figure gives the acceleration $a(t)$ of a Chihuahua as it chases a German shepherd along an axis. In which of the time periods indicated does the Chihuahua move at constant speed?



E where is the Chihuahua has a zero acceleration

Constant speed means zero acceleration, $\frac{d(constant)}{dt} = zero \rightarrow\rightarrow a = \frac{dv}{dt}$

Q-3) The below figure shows four paths along which objects move from a starting point to a final point, all in the same time interval. The paths pass over a grid of equally spaced straight lines. Rank the paths according to (a) the average velocity of the objects and (b) the average speed of the objects, greatest first.



a) All tie (The four paths have the same displacement; the difference between the final and initial position)

b) 4, tie of 1 and 2, then 3

- Average velocity: $v_{avg} = \frac{\Delta x}{\Delta t}$
- Average speed: $S_{avg} = \frac{total\ distance}{\Delta t}$

Q-8) The following equations give the velocity v(t) of a particle in four situations: (a) v = 3; (b) v =$4t^2$ +2t - 6; (c) v = 3t- 4; (d) v =$5t^2$ -3. To which of these situations do the equations of Table 2-1 apply?

$a = \frac{dv}{dt} = constant$, the equations of Table 2-1 apply only for constant acceleration so just for a and c.

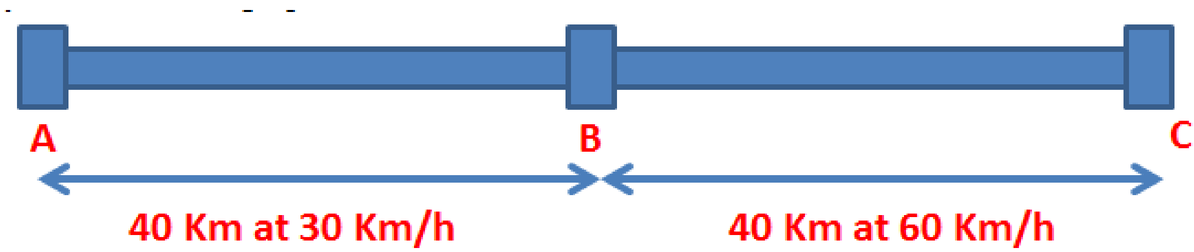
a) $v = 3, a = \frac{dv}{dt} = \frac{d(3)}{dt} = 0$; constant acceleration

b) $v = 4t^2 + 2t - 6\,, a = \frac{dv}{dt} = \frac{d(4t^2+2t-6)}{dt} = 8\,t + 2$

c) $v = 3t - 4\,, a = \frac{dv}{dt} = \frac{d(3t-4)}{dt} = 3$; constant acceleration

d) $v = 5t^2 - 3\,, a = \frac{dv}{dt} = \frac{d(5t^2-3)}{dt} = 10t$

P-3) An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during the full 80 km trip? (Assume that it moves in the positive *x* direction.) (b) What is the average speed? (c) Graph *x* versus *t* and indicate how the average velocity is found on the graph.



a) Trip: A → B, then B → C

$$t_{\,A\rightarrow B} = \frac{distance}{velocity} = \frac{40\ Km}{30\ Km/h} = 1.33\ h$$

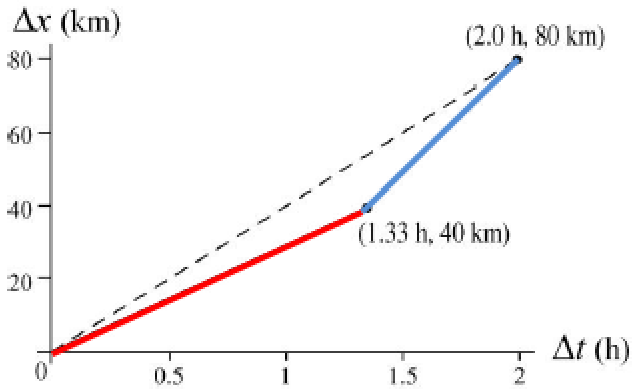
$$t_{\,B\rightarrow C} = \frac{distance}{velocity} = \frac{40\ Km}{60\ Km/h} = 0.67\ h$$

$$v_{avg,\ A\rightarrow C} = \frac{\Delta x}{\Delta t} = \frac{(x_{A\rightarrow C} - 0)}{((t_{\,A\rightarrow B} + t_{\,B\rightarrow C}) - 0)} = \frac{(80\ Km - 0)}{(2\ h - 0)} = +40 Km/h$$

b) Average speed:

$$S_{avg} = \frac{total\ distance}{\Delta t} = 40 Km/h$$

c)



The slope of the dashed line drawn from the origin to the final point $(\Delta t, \Delta x)$ represents the average velocity.

$$slope\ of\ the\ dashed\ line = \frac{\Delta\, x}{\Delta\, t} = \frac{(80\ Km - 0)}{(2\ h - 0)} = 40 Km/h$$

The slope of the first line segment is 30 Km/h and the slope of the second one is 60 Km/h.

P-17) The position of a particle moving along the x axis is given in centimeters by $x = 9.75 + 1.50t^3$, where t is in seconds. Calculate (a) the average velocity during the time interval t = 2.00 s to t = 3.00 s; (b) the instantaneous velocity at t = 2.00 s; (c) the instantaneous velocity at t =3.00 s; (d) the instantaneous velocity at t = 2.50 s; and (e) the instantaneous velocity when the particle is midway between its positions at t = 2.00 s and t = 3.00 s. (f) Graph x versus t and indicate your answers graphically.

a)

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(t = 3.0\ s) - x(t = 2.0\ s)}{(3.0 - 2.0)}$$

$$= \frac{(9.75 + 1.50(3)^3) - (9.75 + 1.50(2)^3)}{1.0} = 28.5\ cm/s$$

b) $v_{Ins} = \frac{dx}{dt} = 4.50\, t^2$

$$v(t = 2.00\ s) = 4.50\ (2.00)^2 = 18.0\ cm/s$$

c) $v(t = 3.00\ s) = 4.50\ (3.00)^2 = 40.5\ cm/s$

d) $v(t = 2.50\ s) = 4.50\ (2.50)^2 = 28.1\ cm/s$

e) $x(t = 3.0\ s) = (9.75 + 1.50(3)^3) = 50.25\ cm$

$x(t = 2.0\ s) = (9.75 + 1.50(2)^3) = 21.75\ cm$

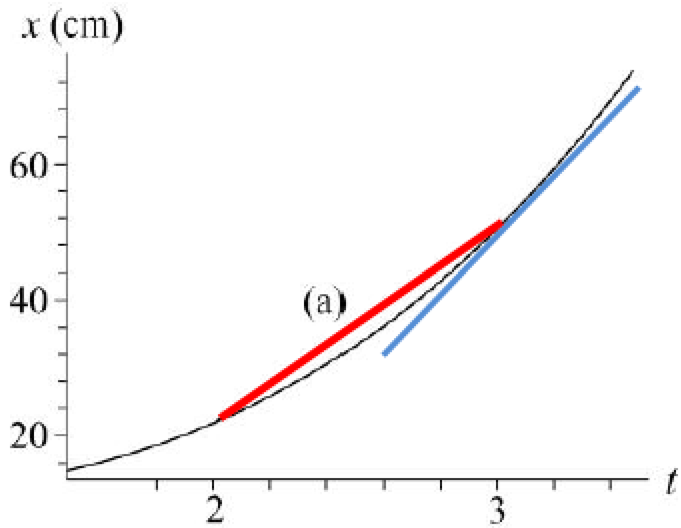
The position of the particle in the midway between t = 3.0 s and t= 2.0 s (x_{midway})

$$x_{midway} = \frac{50.25 + 21.75}{2} = 36\ cm$$

$$x_{midway} = 36\ cm = 9.75 + 1.50\, t_{midway}^3 \rightarrow\rightarrow\rightarrow t_{midway} = 2.6\ s$$

$$v\left(t_{midway} = 2.6\ s\right) = 4.50\ (2.6)^2 = 30.4\ cm/s$$

f) The slope of the red straight line between t = 2.0 s and t = 3.0 s is the answer of the part (a). The answers of the other parts corresponding to the slopes of the tangent lines to the curve at the specific time. For example, the slope of the blue tangent line at t = 3.0 s is the instantaneous at t = 3.0 s.



P-22) The position of a particle moving along the *x* axis depends on the time according to the equation $x = ct^2 - bt^3$, where *x* is in meters and *t* in seconds. What are the units of (a) constant *c* and (b) constant *b*? Let their numerical values be 3.0 and 2.0, respectively. (c) At what time does the particle reach its maximum positive *x* position? From *t* = 0.0 s to *t* = 4.0 s, (d) what distance does the particle move and (e) what is its displacement? Find its velocity at times (f) 1.0 s, (g) 2.0 s, (h) 3.0 s, and (i) 4.0 s. Find its acceleration at times (j) 1.0 s, (k) 2.0 s, (l) 3.0 s, and (m) 4.0 s.

a) $[c] = \frac{m}{s^2}$

b) $[b] = \frac{m}{s^3}$

c) The particle's velocity is ZERO when the particle reaches its maximum positive x position

$$x_{max} \; at \; v = ZERO$$

$$v = \frac{dx}{dt} = \frac{d(ct^2 - bt^3)}{dt} = 2ct - 3bt^2$$

Maximization of the position function: $v = 2ct - 3bt^2 = 0$

Take c = 3.0 and b= 2.0, $v = 6t - 6t^2 = 0 \rightarrow\rightarrow t - t^2 = 0$

$t(1 - t) = 0 \rightarrow\rightarrow t = 0.0\, s \; OR \; t = 1.0\, s.$

For t = 0.0 s, $x = 3t^2 - 2t^3 = 0\, m$ {rejected}

t = 1.0 s, $x = 3(1.0)^2 - 2(1.0)^3 = 1.0\, m$ {Accepted because we seek the maximum position}

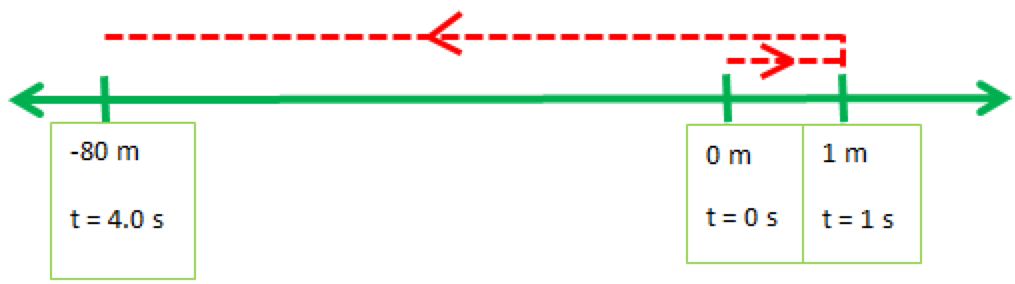
d) The particle moves from the origin to x = 1.0 m to the right, turns around, and goes back to 81 m to the left in the first 4 seconds.

t = 0.0 s, $x(t = 0.0\ s) = 0\ m$

t = 1.0 s, $x = 3(1.0)^2 - 2(1.0)^3 = \ 1.0\ m$, maximum position on the positive x axis.

t = 4.0 s, $x(t = 4.0\ s) = 3(4.0)^2 - 2(4.0)^3 = \ -80.0\ m$

Total Distance from t = 0 s to t = 4.0 s is 82 m [1m + 1m + 80 m = 82 m]



e) The displacement:

$\Delta x = x(t = 4.0\ s) - x(t = 0.0\ s) = \ -80.0\ m$

$$x = 3t^2 - 2t^3 \ \rightarrow\rightarrow\ v = \frac{dx}{dt} = 6t - 6t^2 \ \rightarrow\rightarrow\ a = \frac{dv}{dt} = 6 - 12t$$

f) $v\ (t = 1.0s\ ) = 0\ m/s$

g) $v\ (t = 2.0s\ ) = -12\ m/s$

h) $v\ (t = 3.0s\ ) = -36\ m/s$

i) $v\ (t = 4.0s\ ) = -72\ m/s$

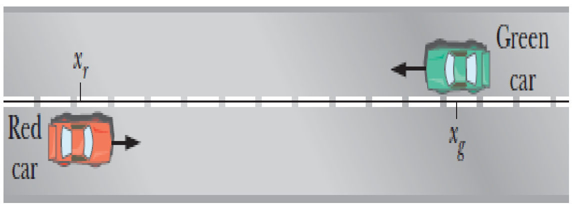
j) $a\ (t = 1.0s\ ) = -6\ m/s^2$

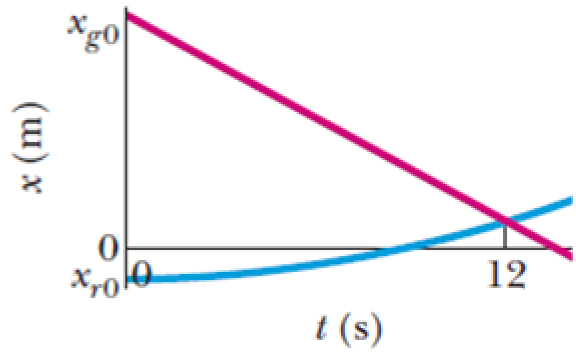
k) $a\ (t = 2.0s\ ) = -18\ m/s^2$

l) $a\ (t = 3.0s\ ) = -30\ m/s^2$

m) $a\ (t = 40s\ ) = -42\ m/s^2$

P-35) A red car and a green car that move toward each other. The below figure is a graph of their motion, showing the positions $x_{g0} = 270\, m$ and $x_{r0} = -35.0\, m$ at time t = 0. The green car has a constant speed of 20.0 m/s and the red car begins from rest. What is the acceleration magnitude of the red car?





The two cars pass each other at t = 12.0 s when the two graphed lines cross as you can see in the figure.

$$x_{red} = x_{green}$$

The positions of the two cars are given by:

RED CAR:

$$x_{red}(t) = x_{r0} + v_{r0}\, t + \frac{1}{2}\, a_r\, t^2 ;[v_{r0} = 0; the\ red\ car\ begins\ from\ rest]$$

$$x_{red}(t) = x_{r0} + \frac{1}{2}\, a_r\, t^2$$

$$\boldsymbol{x_{red}(t) = -35.0\, m + \frac{1}{2}\, a_r\, t^2}$$

GREEN CAR:

$$x_{green}(t) = x_{g0} + v_{g0}\, t + \frac{1}{2}\, a_g\, t^2 ;[a_g = 0; the\ green\ car\ has\ aconstant\ speed]$$

$$x_{green}(t) = x_{g0} + v_{g0}\, t$$

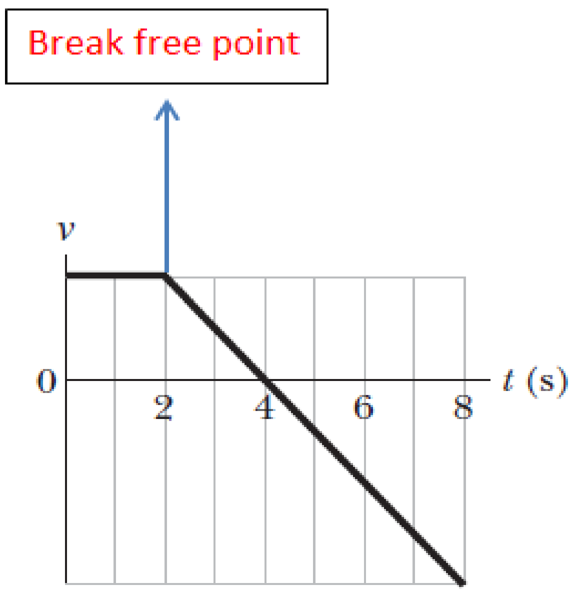
$$\boldsymbol{x_{green}(t) = 270\, m + (-20\, m/s)t}$$

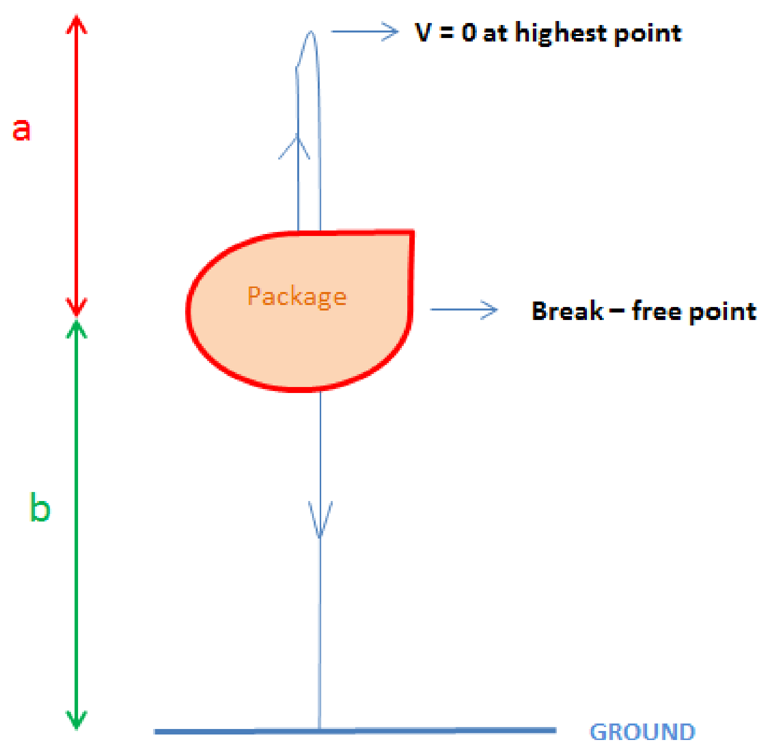
$$x_{red}(t = 12.0\, s) = x_{green}(t = 12.0\, s)$$

$$-35.0\, m + \frac{1}{2}\, a_r\, (12.0\, s)^2 = 270\, m + (-20\, m/s)(12.0\, s)$$

$$a_r = 0.9\, m/s^2$$

p-51) As a runaway scientific balloon ascends at 19.6 m/s, one of its instrument packages breaks free of a harness and free-falls. The below figure gives the vertical velocity of the package versus time, from before it breaks free to when it reaches the ground. (a) What maximum height above the break-free point does it rise? (b) How high is the break-free point above the ground?





From the figure:

- At t = 2 sec, the package breaks free of the balloon.(its velocity starts decreasing)
- at t =4.0 s the package reaches its maximum height (v = 0) before it free – fall and it falls for 4.0 s until it reaches the ground.

At the maximum height: $v = 0 \ and \ it \ needs \ 2.0 \ s \ to \ reach \ it.$

$$v = v_0 + a\,t \ \rightarrow\rightarrow 0 = v_0 - (9.8\ m/s^2)\ 2.0\ s$$

$v_0 = 19.6\ m/s$, the package velocity when it breaks from the balloon

The maximum height above the break – free point :

$$\Delta y = v_0\,t + \frac{1}{2}\,g\,t^2$$

$$\Delta y = (19.6\ m/s)(2.0\ s) + \frac{1}{2}(-9.8\ m/s^2)(2.0\ s)^2 = 19.6\ m$$

The package is at its highest point and then falls for 4.0 s until it reaches the ground.

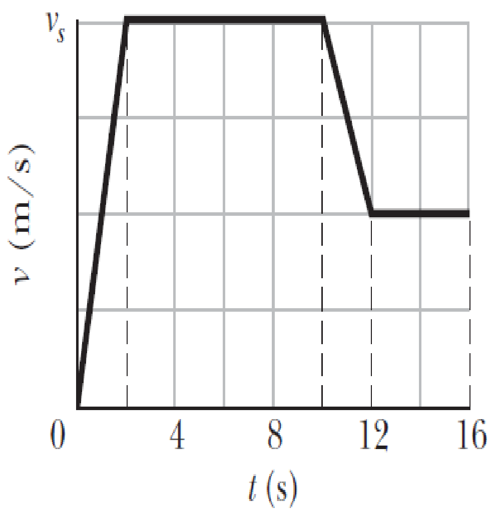
$$\Delta y = v_0\,t + \frac{1}{2}\,g\,t^2, v_0 = 0\ (maximum\ height)$$

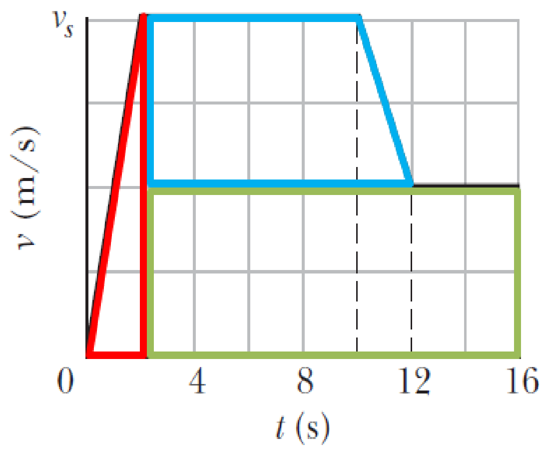
$$\Delta y = \frac{1}{2}\,g\,t^2 = \frac{1}{2}(9.8\ m/s^2)(4.0\ s)^2 = 78.4\ m$$

78.4 m is the total distance between the maximum point and the ground.

The high of the break-free point above the ground is 78.4 m – 19.6 m = 58.8 m

p-69) How far does the runner whose velocity–time graph is shown in the below figure travel in 16 s? The figure's vertical scaling is set by v_s = 8.0 m/s.





The runner displacement = the area under the velocity vs. time curve

$$\Delta x = \left(\frac{1}{2} \times 2 \times 8\right) + (14 \times 4) + \left(\frac{1}{2} \times (10 + 8) \times 4\right) = 100\ m$$