Ch.1 | Digital Systems and binary Numbers

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Popular Number Systems

1. Binary = Radix 2
   - each digit called bit
   - only two digit 0's and 1's.

2. OCtal = Radix 8
   - only eight digit : 0 to 7
   - Digits 8 and 9 not used.

3. Decimal = Radix 10
   - 10 digits, 0 → 9

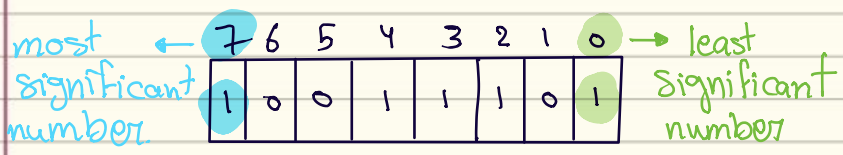
4. Hexa Decimal = Radix 16
   - 16 digits, 0 → 9 and A → F
   - A = 10, B = 11, ..., F = 15.

memorise table.

Decimal Radix 10	Binary Radix 2	Octal Radix 8	Hex Radix 16
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Bit numbering :

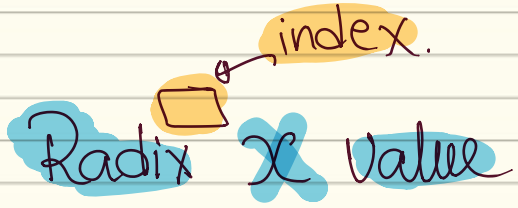
- least Significant Bit [LSB] is Right most. (bit 0)
- Most Significant Bit [MSB] is left most.



Decimal Value of Binary Number

- each bit represent a power of 2.
- every binary number is represent a power of 2.

to convert from any system to decimal



Binary to Decimal :-

ex:- $(1\,0\,0\,1\,1\,1\,0\,1)_2$ ← value

index = 7 6 5 4 3 2 1 0

radix = 2

$= 2^0 \times 1 + 2^1 \times 0 + 2^2 \times 1 + 2^3 \times 1 + 2^4 \times 1 + 2^5 \times 0 + 2^6 \times 0 + 2^7 \times 1$

$= (157)_{10}$

Octal to Decimal

ex:- $(2107)_8 = (\quad)_{10}$

3 2 1 0

$8^0 \times 7 + 8^1 \times 0 + 8^2 \times 1 + 8^3 \times 2 = (1095)_{10}$

ex. $(2051)_4$ to decimal

invalid.

Radix = 4, so possible numbers are between 0 → 3

ex. $(3BA4)_{16}$ to decimal $16^0 \times 4 + 16^1 \times 10 + 16^2 \times 11 + 16^4 \times 3$

$= 15268$

NOTE:-

possible number for radix r

is 0 → r-1

ex, r = 8, 0 → 7

r = 2, (0,1)

r = 4, 0 → 3

⋮

Convert from decimal to any system

- Repeatedly divide the decimal integer by the radix of the Sys
- each remainder is a digit in the translated value.
- Stop when quotient is zero.

ex. Convert $(37)_{10}$ to Binary

radix = 2

37	2
18	1 → LSB
9	0
4	1
2	0
1	0
0	1 → MSB

$(37)_{10} = (100101)_2$

ex. Convert $(422)_{10} \rightarrow (\quad)_{16}$

422	16
26	6 → LSB
1	10
0	1 → MSB

$(422)_{10} = (1A6)_{16}$

من للقدس إلى أنت

* اختيار نقسم على Radix حتى يطلع الرقم 0

* Converting octal and hexa decimal to Binary is easy.

→ $8 = 2^3$ → so each 3 digits equals 1 bit in Binary and vice versa.

→ $16 = 2^4$ → so each 4 digits in hexa equals 1 bit in Binary and vice versa.

7	5	3	0	5	5	2	3	6	2	4	Octal
1 1 1 0 1 0 1 1	0 0 0 1 0 1 1 0	1 0 1 0 0 1 1 1	1 0 0 0 1 0 1 0 0								32-bit binary
E	B	1	6	A	7	9	4				Hexadecimal

* Starting from [LSB] group each 4 Bits into a Hex digit or each 3 bits into one octal digit.

To convert from octal to hexa or vice verse

1. Convert from octal to binary.
2. Convert from binary to hexa.

important properties

* How many possible digits can we have in radix r ?

* what's the result of adding (1) to the largest digit in Radix r ?
ex. $(1)_2 + 1 = (10)_2$ $(7)_8 + 1 = (10)_8$

$(9)_{10} + 1 = (10)_{10}$ $(F)_{16} + 1 = (10)_{16}$

* what's the largest value using 3 digits in Radix r ?

$r^3 - 1$, ex:- In Binary $2^3 - 1 = 7 \rightarrow (111)_2$

ex. In OCtal $8 - 1$ $(777)_8$

⋮

In general :- the largest value using n digit in Radix r is ..

$(\text{radix} - 1)^n$ → number of digit

* How many possible values can be represented
- using n binary digits 2^n values , $0 \rightarrow 2^n - 1$
- using n octal digits 8^n values , $0 \rightarrow 8^n - 1$
- using n decimal digits 10^n values , $0 \rightarrow 10^n - 1$
- using n Hexa decimal digits 16^n values , $0 \rightarrow 16^n - 1$

In general :- using n digits in Radix

r^n values , 0 to $r^n - 1$.

representing fractions

- ✱ Integer values start with index 0.
- ✱ fraction values start with index -1.

Convert Decimal Fraction to Binary.

- by multiplying the fraction by 2 Repeatedly.
- collect integer bits.
- stop when the number on the most Right of the point equals 0.

ex. $(0.6875)_{10} = (.1011)_2$

0.6875 x2	1. 3750
0.3750 x2 =	0. 750
0.75 x2 =	1. 50
0.5 x2 =	1. 0

MSB (first bit) ↓ LSB (last bit)

* عكس الـ integers.

1.0 → stop when fraction equals zero.

✱ عند التحويل من Decimal إلى أي نظام آخر

- integers ⇓ Radix قسمة على
- fractions ↓ Radix ضرب في

ex. $(139.6875)_{10} = (\quad)_8$

2 1 0 -1 -2 -3 -4

139	8
17	3
2	1
0	2

$(213.54)_8$

0.6875 × 8 =	5.5000
0.5 × 8 =	4.0 stop.

* بنقسم السؤال إلى جزأين
integer جزء ✓
fractions جزء ✓
ثم نجمعهم مع بعض ✓

Converting Binary and Hexa to Binary so

integer: right to left — fraction: left to right

	7	2	6	1	3	.	2	4	7	4	5	2	Octal
0	1 1 1	0 1 0	1 1 0	0 0 1	0 1 1	.	0 1 0	1 0 0	1 1 1	1 0 0	1 0 1	0 1 0	Binary
	7	5	8	B	.	5	3	C	A	8			Hexadecima

important properties of fractions

* How many fractional values exist with m fractional Bits?

2^m In general. r^m value

radix

* whats the largest fraction value if m fraction digits are used in radix r?

$1 - r^{-m}$ → number of digits

→ Radix

ex. Convert $(299.8195)_{10} \rightarrow (20B.9A)_{12}$ at most two fractional digits, if necessary.

299	12
24	(11) = B
2	0
0	2

0.8195 × 12 =	9.834
0.834 × 12 =	10.008

arithmetic operation

Binary Addition :-

```
   1111
  00110110
  00011101 +
  --------
  01010011
```

(54) + (29) = (83)

Binary Subtraction :-

```
   0 2
  11011
  10110 -
  -----
  00101
```

* لما بنستلف بناخذ 2.

Octal addition :-

```
   1 1
   567
   243 +
  ----
  1032
```

7+3 = 10 > 8

10 = 1x8 + 2

carry ↲   carry ↲

$C_0 = A_0 \times 8 + B_0$

Carry ↲   ↳ sum

Octal Subtraction :-

```
   8+1 8+
  5 6 2 4
    2 6 5 -
  -------
    3 3 7
```

* بنستلف 8.

ex.

```
     7 7
   5 8 8 8
   6 0 0 0
     7 7 7 -
   -------
   5 0 0 1
```

Hexa Addition :-

ex.

```
   1 1
   ADD
   DAD +
  ----
  188A
```

A	B	C	D	E	F
10	11	12	13	14	15

D+D = 13+13 = 26 = 16x1 + 10

carry ↲   ↳ sum

A+D+1 = 24 = 16x1 + 8

carry ↲   ↳ sum

Hexa Subtraction :-

```
   16  16
   +6  +3  16+
  8 9 7 4 B
    5 8 7 C -
  ---------
    3 E A F
```

complement of numbers

"for unsigned numbers"

1 Radix Complement 8-

A- $r^n - N$

B- (r-1)'s Comp + 1

2 Diminished Radix Complement 8

(r-1)'s Comp.

$(r^n - 1) - N$

, r → radix
n → digits
N → the number

EX. Find the 9's Comp of (134795).

1	8
3	6
4	5
7	2
9	0
5	4

865204

* طريقة سريعة 8- معناها مكملة Comp ← فلطريقة سريعة، بنشوف كل رقم شو مكملته لـ 9.

أو حسب القانون

$r = 10, n = 6, N = 134795$ (1 2 3 4 5 6)

$= r^n - 1 - N$
$= 10^6 - 1 - 134795$
$= 999999 - 134795$
$= 865204$

n → 6 digits
N = → الرقم نفسه
r = radix

⇒ to find 10's Comp. 9's Comp +1.

Find the 10's Comp of (134795).

10's Comp = 9's Comp + 1
= 865204 + 1
= 865205

* كطريقة سريعة 8-
↳ بتترك كل الأصفار، وأول رقم بعد الأصفار بتكمله لـ (10)، والباقي بتكملهم لـ (9)

EX. Find 9's and 10's Comp of (546700).

9'sComp = 453299

10'sComp = 463300

546700
مكملة لـ 9 — مكملة لـ 10

453300

* to find the 1's comp 8- convert 0's to 1's and 1's to 0's

2's Comp = 1'sComp + 1.

* Remark 8-
The Comp of comp restors the original value ⇒ Comp(Comp(N)) = N

exampels

1. Consider a binary number : 1011001
   1's Complement :- 0100110
   2's complement :- 0100111 ⟲ +1

2. Consider a binary number :- 00100100
   1's complement :- 11011011
   2's complement :- 11011100 ⟲ +1

00100100

11011 100

flip same

+ طريقة ثانية :-
الأصفار وأول رقم بعدهم بضلهم ،،
واللي بعدهم بقلبهم [0←1 / 1←0]

Complement are used to simplify Subtraction operation in digital computers.

Advantages of simplification :-

1. it results in Simpler Circuits.
2. it results in low Cost.

which means (fewer and simpler hardware Components).

* Assume that we need to subtract x from y

then we have two Cases :- 1 $x \geq y$, 2 $x < y$

1 $x \geq y$

* the final answer will be positive

(r-1)'s Comp
x + (r-1)'s Comp
⇓
if their exist Carry add (1) to the result

r's Comp
x + r's comp.
⇓
if end Carry occurs will be discarded.

ex- X = 11101 , Y = 10111
find X-Y, using 1's complement.

X>Y , = X - Y
= X + 1's (Y)
= 11101 + (01000)
= 100110

1
11101
01000 +
100101
1 +
00110 → the final answer

end carry , add it to the result.

ex. X = 11101 , Y = 10111
find X-Y by 2's comp
= X + 2's (Y)
1
11101
01001 +
100110 → the final answer
will be discarded

NOTES-
Fraction إذا كان عندي +
بتعامل معه على انه رقم
صحيح وبعدها برجع
الـ point على نفس
مكانها.

ex:- 10110.00
1's Comp = 01001.11

ex:- 0101.101
2's Comp = 101.011

0101101
1010010 ← 1's comp
1010.011 +1

[2] X<Y → ① the final answer will be negative
② won't produce end carry
③ after taking the Comp of the final Carry, put the negative Sign.

ex. using 1's comp. to 3250 - 72532

= 03250 + (27468)

= 103250

27468 +

30718 → no end carry

1's (30718) = -69282

ex. using 9's comp. do 285.31 - 3459.20

= 285.31 + (6540.79)

1 1 1

285.31

6540.79 +

6826.10 → no end carry.

9's (6826.10) = -3173.89

signed binary numbers

3 major techniques are used to represent signed numbers :-

1. Signed magnitude.
2. 1's Complement.
3. 2's complement.

[1] Signed magnitude :-

- left most bit is the sign bit :- 0 for (+)ve
  1 for (−)ve
- two represntation for Zero (NOT Good)!
- Range $-2^{(n-1)}-1 \rightarrow +(2^{n-1}-1)$.
- exampels :-

-1 —4 bits→ 1001
+6 —4 bit→ 0110 → positive
+15 —8 bit→ 00001111
-79 —8 bit→ 11001111 (1 = negative)

- the only way, that represent (+)ve and (−)ve.

[2] 1's complement

- 2 representation for 0.
- Rang $-2^{n-1}-1 \rightarrow +2^{n-1}-1$

لحل :- ① نكتب الرقم بالموجب
② نأخذ لها comp له.

ex. −16

+16 ① 010000
1's comp → 101111 (−) → −16

3 2's complement.

- only one representation for 0.
- Range $-2^{n-1} \rightarrow +2^{n-1}-1$
- 2's comp = 1's comp + 1

نرجع ناخس 2's comp عشان نجيب القيمة الأصلية.

1. يكتب الرقم بالموجب (آخر bit) بعدين
2. 1's comp
3. + 1

- only for negative number.
- 2'scomp(N) + N = 0

ex. -16

+16 = 0 1 0000

↳ 1's comp = 10 1111

+1 = 11 0000

* negative weight to the sign bit (MSB)

ex. 1011 0100

-128 64 32 16 8 4 2 1

4 + 16 + 32 + -128 = -76

* Range values for unsigned number $0 \rightarrow +(2^n - 1)$
* Range values for signed number $-2^{n-1} \rightarrow +2^{n-1}-1$ } according to 2's Comp.
* positive integers $0 \rightarrow 2^{n-1} - 1$
* negative integers $-2^{n-1} \rightarrow -1$

8 bits $-128 \rightarrow 127$

* Signed magnitude

$-2^{n-1}-1 \rightarrow +2^{n-1}-1$

ex. A = (0010 1100) = +44

↳ 2's (1101 0100) = -44

* Subtraction:

↳ Convert sub. into addition and get the 2's comp of the second with ignoring the carry.

* over flow occurs :-

1. adding two positive numbers and the Sum is neg.
2. adding two negative numbers and the Sum is pos.

ex. 00001111 + 11111000 = 00000111

1 Carry

1 00 1 1 1 0 1 0 + 1 0 0 1 1 1 0 1 = 0 1 1 1 0 1 1 1

جمعت (-) مع (-) أعطاني (+)

* بما إنه رقمين مختلفين يعني يوجد over flow

* إذا طلعوا متشابهين يعني ما يوجد over flow

* Carry is important when

- Adding or Sub, unsigend numbers.
- indicates that the unsigned Sum is out of range.

* اذا عندي رقم مخزن 2's comp وبدي أعرف قيمته

ex: 1101 → 2's comp.

- 0010 → 1's comp → +1 → 0011

= -3

ex. 00100100 = +36

11011011 → 1's comp

+1

(11011100) = -36

ex: 1001 → 2's comp

0110 → 1's comp → +1 → 0111 = -7

ex. (+5) → in 8bits

00000101 → 1's comp → 11111010 → +1 → 11111011

* positive numbers in signed magnitude and 2's comp. are the same.

* the 2's comp of N is the negative of N
* the sum of N and the 2's comp of N is Zero.

BINARY CODES

- do representation
- $\lceil \log_2 \square \rceil$

* لعشان 20 لون وبدي أمثلهم لون عشان الآخر يبعد البتات. كم bit بحتاج؟

$\lceil \log_2 20 \rceil = 5$

أو

$16 < 20 < 32$

$2^4 < 20 < 2^5$

the minimume number of bits.

ex. if you have 3 bits how many numbers can you do?

$2^3 = 8$

2^n → n = number of bits.

Decimal Codes:- $\lceil \log_2 10 \rceil = 4$

10 char عشان نمثلهم بحتاج 4bit *

Decimal	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
Unused	1010 ... 1111

Conversion And Coding

$(13)_{10} = (1101)_2$, this is Conversion. , 4 bit needed

$13 = (0001\ 0011)_{BCD}$, this is coding , 8 bit needed

* In general, Coding requires more bits than Conversion

* كل رقم في BCD بعبّر عنه لحاله

* أكبر عدد في BCD هو 9 (1001)

لكن

إذا طلع الجواب أكبر من 9

بضيف عليه 6

```
 8        BCD
 5 +   |  1000
----   |  0101 +
13     |  1101
       |  0110 +
      ----------
      0001 0011
        1    3
```

ex. 1897
    2905 +

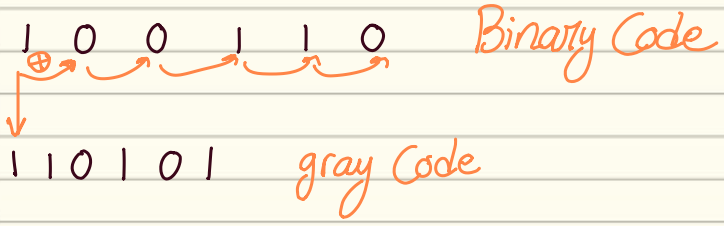
```
  0001  1000  1001  0111
  0010  1001  0000  0101  +
  ----------------------
  0100 10010  1010  1100
  0000  0110  0110  0110  +
  ----------------------
  0100  1000 10000 10010
  [ 4     8     0     2 ]
```

Gray Code

↳ successive values differ in only one bit

used :- sequence of binary numbers numbers generated by the hard ware that may cause an error or ambiguity during the transition from one number to the next.

from binary to gray



from gray to binary

1. The Gray code for the binary value (100110) is Answer 1 , and this code and the Gray code (110111) Answer 2 (are/are not) successive codes. (Fill in the blank)

Answer 1

Type your answer 110101

Answer 2

Type your answer yes

other decimal codes

Decimal	BCD 8421	5421 code	2421 code	8 4 -2 -1 code	Excess-3 code
0	0000	0000	0000	0000	0011
1	0001	0001	0001	0111	0100
2	0010	0010	0010	0110	0101
3	0011	0011	0011	0101	0110
4	0100	0100	0100	0100	0111
5	0101	1000	1011	1011	1000
6	0110	1001	1100	1010	1001
7	0111	1010	1101	1001	1010
8	1000	1011	1110	1000	1011
9	1001	1100	1111	1111	1100
Unused	...	...	...	...	...

* BCD 5421, 2421, 8 4 -2 -1 are wieghted codes

* Excess-3 is not weighted code.

EXCess-3

Decimal → BCD → EXCESS-3 (+ 0011)

1. The decimal number 17 can be represented in binary as Answer 1 and in Excess-3 as Answer 2. (Fill in the blank)

Answer 1

Type your answer: 10001

Answer 2

Type your answer: 0010 0000

17 BCD → 0001 0111

0001 0111 +
0011
0001 1010 +
0110
0010 0000

20

ex. $(27)_{10} + (39)_{10}$

0010 0111 +
0011
0010 1010 +
0110
0011 0000

30 +2 = 32

0011 1001 +
0011
0011 1100 +
0110
0100 0010

42

ربنا تقبل منا إنك أنت السميع العليم

روان فارس