

7.7 :- Hyperbolic Functions

[4] By using $\cosh^2 x - \sinh^2 x = 1$ find the values of the remaining five hyperbolic functions

• $\cosh x = \frac{13}{5}$, $x > 0 \rightarrow \sinh^2 x = \cosh^2 x - 1$

$$= \left(\frac{13}{5}\right)^2 - 1$$

$$\sinh^2 x = \frac{144}{25} \rightarrow \sinh x = \pm \frac{12}{5}$$

$\sinh x = -\frac{12}{5}$ reject	$\sinh x = \frac{12}{5}$ Accept $x > 0$
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• $\sinh x = \frac{12}{5}$

• $\tanh x = \frac{12}{13}$

• $\operatorname{csch} x = \frac{5}{12}$

• $\operatorname{sech} x = \frac{5}{13}$

• $\operatorname{coth} x = \frac{13}{12}$

[6] Rewrite the expression in terms of exponentials

$$\sinh(2 \ln x) = \frac{e^{2 \ln x} - e^{-2 \ln x}}{2}$$

$$= \frac{x^2 - x^{-2}}{2} = \frac{x^4 - 1}{2x^2}$$

$$\begin{aligned} \boxed{10} \quad & \ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) \\ &= \ln(e^x) + \ln(e^{-x}) \\ &= x - x = 0 \end{aligned}$$

Another solution

$$\begin{aligned} & \ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) \\ &= \ln\left[(\cosh x + \sinh x)(\cosh x - \sinh x)\right] \\ &= \ln\left[\cosh^2 x - \sinh^2 x\right] \\ &= \ln[1] = 0 \end{aligned}$$

Find the derivative of y with respect to x

$$\boxed{15} \quad y = 2\sqrt{x} \tanh \sqrt{x}$$

$$y' = \cancel{2\sqrt{x}} \cdot \operatorname{sech}^2(\sqrt{x}) \cdot \frac{1}{\cancel{2\sqrt{x}}} + \tanh \sqrt{x} \cdot \frac{\cancel{2} \cdot 1}{\cancel{2}\sqrt{x}}$$
$$= \operatorname{sech}^2(\sqrt{x}) + \frac{\tanh \sqrt{x}}{\sqrt{x}}$$

$$\boxed{18} \quad y = \ln(\cosh x)$$

$$y' = \frac{1}{\cosh x} \cdot \sinh x = \tanh x$$

$$\boxed{21} \quad y = \ln \cosh x - \frac{1}{2} \tanh^2 x$$

$$y' = \frac{1}{\cosh x} \cdot \sinh x - \frac{1}{2} \cdot 2 \tanh x \cdot \operatorname{sech}^2 x$$

$$y' = \tanh x - \tanh x \operatorname{sech}^2 x$$

$$= \tanh x [1 - \operatorname{sech}^2 x]$$

$$= \tanh^3 x$$

$$\boxed{42} \int \sinh \frac{x}{5} dx$$

$$\text{let } u = \frac{x}{5}$$

$$du = \frac{1}{5} dx$$

$$\int 5 \sinh u du$$

$$I = 5 \cosh u + C$$

$$I = 5 \cosh \left(\frac{x}{5} \right) + C$$

$$\boxed{46} \int \coth \frac{\theta}{\sqrt{3}} d\theta$$

$$\text{let } u = \frac{\theta}{\sqrt{3}}$$

$$du = \frac{1}{\sqrt{3}} d\theta$$

$$\sqrt{3} \int \coth u du$$

$$= \sqrt{3} \int \frac{\cosh u}{\sinh u} du$$

$$= \sqrt{3} \ln |\sinh u| + C$$

$$= \sqrt{3} \ln \left| \sinh \frac{\theta}{\sqrt{3}} \right| + C$$

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$$\int_0^{\ln 2} \tanh 2x \, dx$$

$$= \frac{1}{2} \int_0^{\ln 2} \frac{2 \sinh 2x}{\cosh 2x} \, dx$$

$$= \frac{1}{2} \left[\ln |\cosh 2x| \right]_0^{\ln 2}$$

$$= \frac{1}{2} \left[\ln \left| \frac{e^{2x} + e^{-2x}}{2} \right| \right]_0^{\ln 2}$$

$$= \frac{1}{2} \left[\ln \left| \frac{4 + 1/4}{2} \right| - \ln \left| \frac{2}{2} \right| \right]$$

$$= \frac{1}{2} \left[\ln \left(\frac{17}{8} \right) - 0 \right] = \frac{1}{2} \ln \left(\frac{17}{8} \right)$$

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$$\int_0^{\ln 2} 4 e^{-\theta} \sinh \theta \, d\theta$$

$$\int_0^{\ln 2} 4 e^{-\theta} \left(\frac{e^{\theta} - e^{-\theta}}{2} \right) d\theta$$

$$= 4 \int_0^{\ln 2} \frac{1 - e^{-2\theta}}{2} d\theta$$

$$= 2 \int_0^{\ln 2} \left(1 - \frac{e^{-2\theta}}{2} \right) d\theta$$

$$= 2 \left[\theta + \frac{e^{-2\theta}}{2} \right]_0^{\ln 2}$$

$$= 2 \left[\ln 2 + \frac{1}{8} - 0 - \frac{1}{2} \right]$$

$$= 2 \left[\ln 2 - \frac{3}{8} \right]$$

$$= \ln 4 - \frac{3}{4}$$

$$\boxed{57} \int_1^2 \frac{\cosh(\ln t)}{t} dt$$

$$= \int_0^{\ln 2} \cosh(u) du$$

$$= \sinh(u) \Big|_0^{\ln 2}$$

$$= \sinh(\ln 2) - \cancel{\sinh(0)}$$

$$= \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{2 - 1/2}{2} = \boxed{\frac{3}{4}}$$

let $u = \ln t$
 $du = \frac{1}{t} dt$

 $* t=1 \rightarrow u=0$
 $t=2 \rightarrow \ln 2$

$$\boxed{60} \int_0^{\ln 10} 4 \sinh^2\left(\frac{x}{2}\right) dx$$

Note.
 $\sinh^2 x = \frac{\cosh 2x - 1}{2}$

$$= \int_0^{\ln 10} 4 \left[\frac{\cosh x - 1}{2} \right] dx$$

$$= 2 \int_0^{\ln 10} \cosh x - 1 dx$$

$$= 2 \left[\sinh x - x \right]_0^{\ln 10}$$

$$= 2 \left[\sinh(\ln 10) - \ln 10 \right]$$

$$= 2 \left[\frac{e^{\ln 10} - e^{-\ln 10}}{2} - \ln 10 \right]$$

$$= 10 - \frac{1}{10} - 2 \ln 10$$

[Faint handwritten notes]

$$x_1 = \left(\frac{y}{x} \right)^2 \sin \theta$$

$$x_2 = \left[\frac{1}{2} x \sqrt{2} \right] \sin \theta$$

[Faint handwritten notes]

$$x = x \sin \theta$$

$$\left[\frac{1}{2} x \sqrt{2} \right] \sin \theta$$

$$\left[\frac{1}{2} x \sqrt{2} \right] \sin \theta$$

$$\frac{1}{2} x \sqrt{2} \sin \theta$$