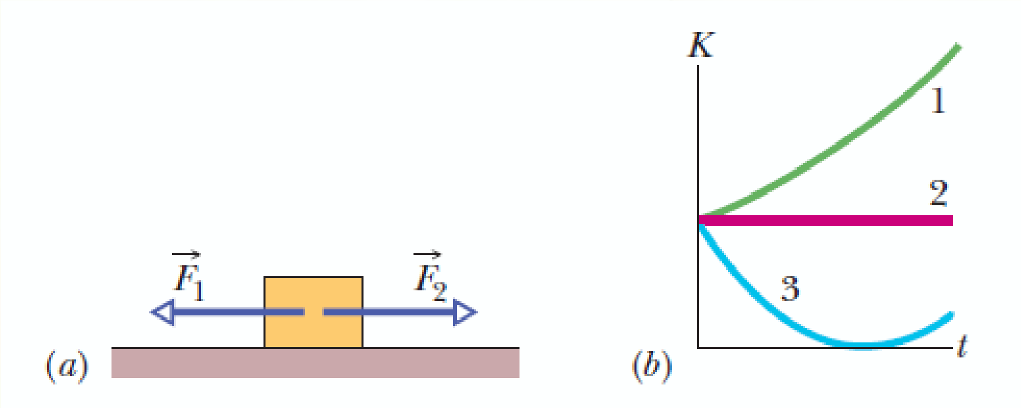
Ch. 7 Kinetic Energy and Work

Q-2 | The below figure shows two horizontal forces that act on a block that is sliding to the right across a frictionless floor. Figure (b) shows three plots of the block's kinetic energy K versus time t. which of the plots best corresponds to the following three situations:

a) $F_1 = F_2$, (b) $F_1 > F_2$, (c) $F_1 < F_2$?



$\Rightarrow W = \Delta K = \vec{F} \cdot \vec{d} = F d \cos\theta$

(a) $F_1 = F_2 \Rightarrow F_{net} = \text{Zero} \rightarrow W = \text{Zero} \rightarrow \Delta K = \text{Zero}$

No changing in the Kinetic energy $\Rightarrow K_f = K_i$

$\boxed{\text{The Red line (2)} \Rightarrow F_1 = F_2}$

(b) $F_1 > F_2$, F_{net} is to the Left; which is in the opposite direction of block's motion.

The block begins with $K = \frac{1}{2} m v_i^2$ and as it decelerates to $v = 0$, $K = \text{zero}$. The block then accelerates to the right with $v > 0$

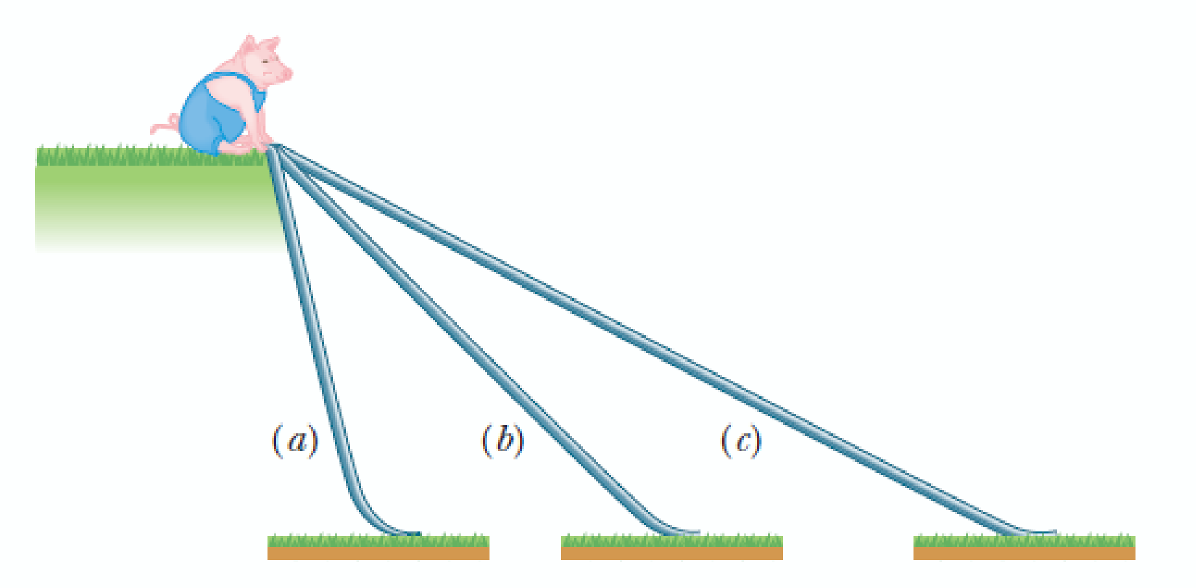
$\boxed{\text{the blue line (3)} \Rightarrow F_1 > F_2}$

(c) $F_1 < F_2$, F_{net} to the right as the displacement

ΔK is positive as the work $\Rightarrow K_f > K_i$

$\boxed{\text{the green line (1)} \Rightarrow F_1 < F_2}$

Q - 7 A greased pig has a choice of three frictionless slides along which to slide to the ground. Rank the slides according to how much work the gravitational force does on the pig during the descent, greatest first ?

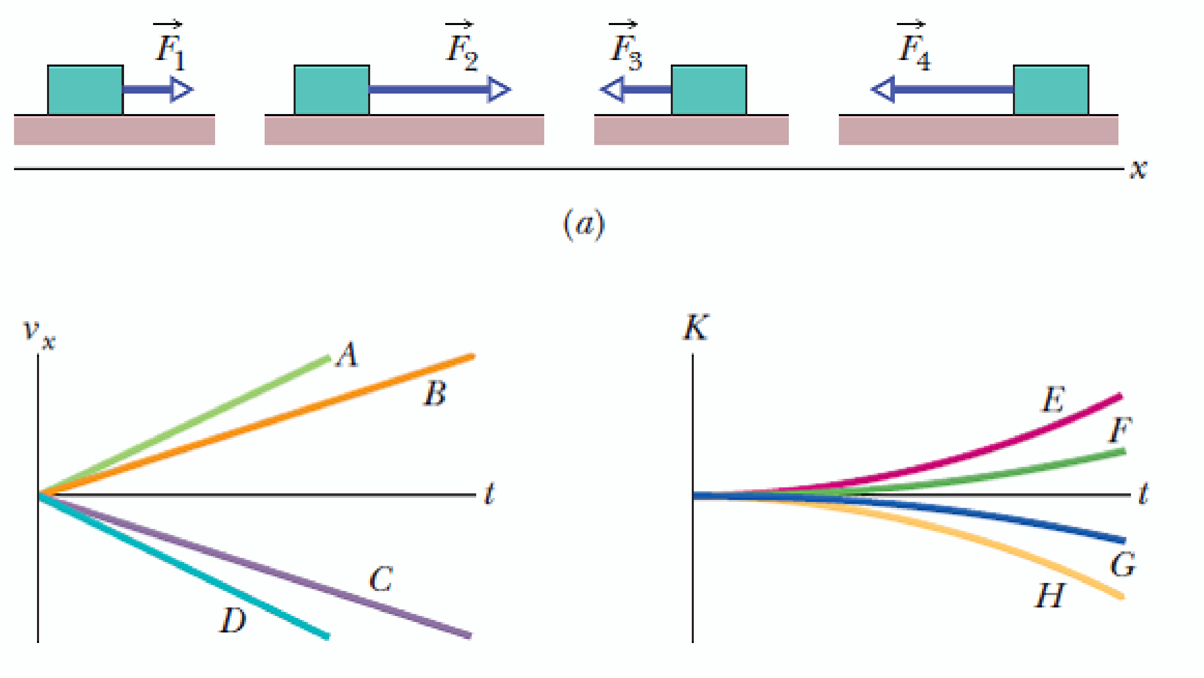


All are equal since the vertical displacement are all equal

$$W_g = \vec{F}_g \cdot \vec{d}$$

$$W_g = F_g \, d_{vertical} \cos(0^\circ)$$

Q-8 | Figure (a) shows four situations in which a horizontal force acts on the same block, which is initially at rest. The force magnitudes are $F_2 = F_4 = 2F_1 = 2F_3$. The horizontal component v_x of the block's velocity is shown in figure (b) for the four situations. (a) Which plot in figure (b) best corresponding to which force in figure (a)? (b) Which plot in figure (c) (for kinetic energy K versus time t) best corresponds to which plot in figure (b)?



(a)

(b)

(c)

$$W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2) \; ; \; v_i = \text{Zero}$$ "initially at Rest"

$$W = \Delta K = \frac{1}{2} m v_x^2 = \vec{F} \cdot \vec{d} = F d \cos\theta$$

(a)
$A \rightarrow F_2$
$B \rightarrow F_1$
$C \rightarrow F_3$
$D \rightarrow F_4$

(b)
$E \leftrightarrow A$
$F \leftrightarrow B$
$G \leftrightarrow C$
$H \leftrightarrow D$

P-1 A proton (mass $m = 1.67 \times 10^{-27}$ kg) is being accelerated along a straight line at 3.6×10^{15} m/s² in a machine. If the proton has an initial speed of 2.4×10^{7} m/s and travels 3.5 cm, what then is (a) its speed and (b) the increase in it's kinetic energy?

• Final speed of the proton ⇒ $v_f^2 = v_i^2 + 2a\,\Delta x$

$$v_f = \sqrt{v_i^2 + 2a\,\Delta x}$$

$$v_f = \sqrt{(2.4 \times 10^{7})^2 + 2(3.6 \times 10^{15})(3.5 \times 10^{-2})}$$

$$\boxed{v_f = 2.9 \times 10^{7} \text{ m/s}}$$

• $\Delta K = K_f - K_i = \frac{1}{2} m_p v_f^2 - \frac{1}{2} m_p v_i^2$

$$\Delta K = \frac{1}{2} m_p (v_f^2 - v_i^2)$$

$$\Delta K = \frac{1}{2}(1.67 \times 10^{-27})\left((2.9 \times 10^{7})^2 - (2.4 \times 10^{7})^2\right)$$

$$\boxed{\Delta K = 2.2 \times 10^{-13} \text{ J}}$$

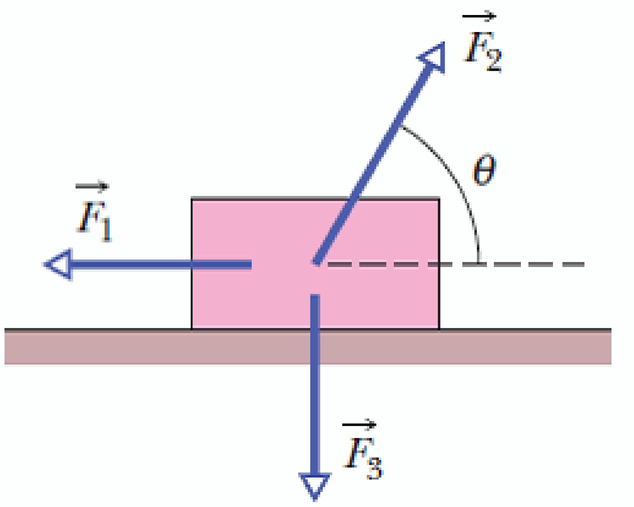
⇒ $\Delta K = \frac{1}{2} m (v_f^2 - v_i^2) = \frac{1}{2} m (2a\,\Delta x) = ma\,\Delta x$

$$\Delta K = F\,\Delta x = W$$

$$\Delta K = ma\,\Delta x = (1.67 \times 10^{-27})(3.6 \times 10^{15})(0.035)$$

$$\boxed{\Delta K = 2.1 \times 10^{-13} \text{ J}}$$

P-15 The below figure shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0°$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?



- The work done by a constant force:

$$W = \vec{F}\cdot\vec{d} = F d \cos\phi$$

$\Rightarrow W_1 = F_1 d \cos(0°) = (5)(3)(1)$

$$\boxed{W_1 = 15\ \text{J}}$$

$\Rightarrow W_2 = F_2 d \cos(180° - 60°)$

$= (9)(3)\cos(120°)$

$$\boxed{W_2 = -13.5\ \text{J}}$$

$\Rightarrow W_3 = F_3 d \cos(90°) = \text{Zero}$

$$\boxed{W_3 = \text{Zero}}$$

(a) The net work on the trunk

$$W = W_1 + W_2 + W_3$$

$$\boxed{W = +1.5\ \text{J}}$$

(b) $\Delta K = W = +1.5$ J

The kinetic energy of the trunk increases by 1.5 J.

$\Rightarrow$ The net Work on the trunk

$$W = \vec{F}_{net} \cdot \vec{d} = F_{net}\, d \cos\theta$$

$$F_{net,x} = F_2 \cos(60^\circ) - F_1$$

$$\boxed{F_{net,x} = -0.5\ N}$$

$$F_{net,y} = F_2 \sin(60^\circ) - F_3$$

$$\boxed{F_{net,y} = 4.79\ N}$$

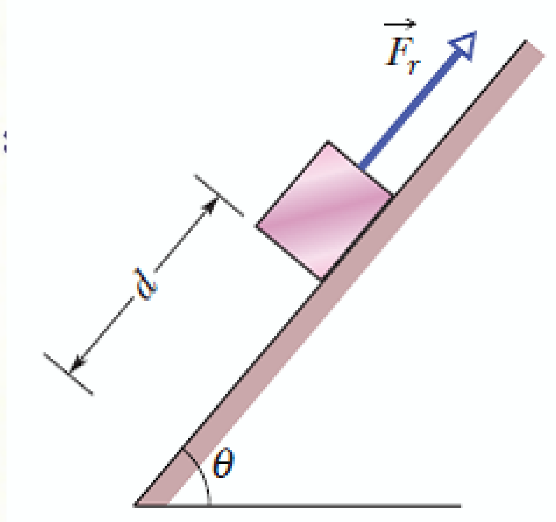
$\Rightarrow$ No Work done by $F_{net,y}$ on the trunk

$$W = F_{net,y}\, d \cos\phi \quad ; \quad \cos(90^\circ) = \text{Zero}$$

$$W = F_{net,x}\, d \cos\phi = \left(-\tfrac{1}{2}\right)(3)\cos(180^\circ)$$

$$\boxed{W = +1.5\ J}$$

P-19 | A block of ice slides down a frictionless ramp at angle $\theta = 50°$ while an ice worker pulls on the block (via a rope) with a force $\vec{F}_r$ that has a magnitude of 50N and is directed up the ramp. As the block slides through distance $d = 0.50$ m along the ramp, its kinetic energy increases by 80J. How much greater would its kinetic energy have been if the rope had not been attached to the block?



Work Done in Lifting and Lowering an Object:

$$\Delta K = K_f - K_i = W_a + W_g$$

$\Rightarrow W_a$ = Work done by the ice worker

$$W_a = \vec{F}_r \cdot \vec{d} = F_r\, d \cos\theta$$

[$\theta = 180°$, F_r up ramp and d is down ramp]

$$W_a = (50\,N)(0.50\,m)\cos(180°)$$

$$\boxed{W_a = -25\,J}$$

$$\cdot\ \Delta K = W_a + W_g$$

$$+80\,J = -25\,J + W_g \Rightarrow \boxed{W_g = 105\,J}$$

If the rope had not been attached to the block $\Rightarrow W_a = $ Zero

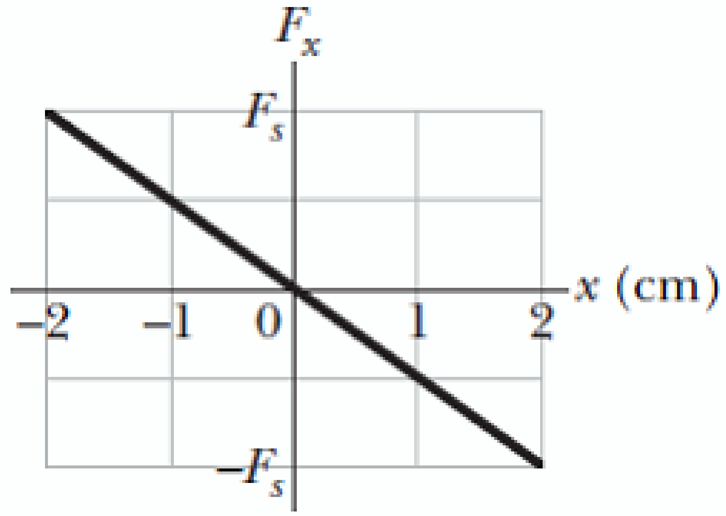
$$\Rightarrow \Delta K = W_g$$

$$\boxed{\Delta K = +105\,J}$$

$\Rightarrow$ The kinetic energy increases by 25J when the rope hadn't been attached

$$\Delta K = \begin{cases} 80\,J, & \text{with Rope} \\ 105\,J, & \text{No Rope} \end{cases}$$

P-32 The below figure gives spring force F_x versus position x for the spring-block arrangement. The scale is set by $F_s = 160.0$ N. We release the block at $x = 12$ cm. How much work does the spring do on the block when the block moves from $x_i = +8.0$ cm to (a) $x = +5.0$ cm, (b) $x = -5.0$ cm, (c) $x = -8.0$ cm, and (d) $x = -10.0$ cm?



Work done by a spring force

$$W_s = \frac{1}{2} K x_i^2 - \frac{1}{2} K x_f^2$$

$$W_s = \frac{1}{2} K (x_i^2 - x_f^2)$$

⇒ Spring Force in x-direction

$$F_s = -K x \quad \text{Hooke's Law}$$

Spring constant equals the slope of F vs. x graph with negative sign.

$$K = 80 \text{ N/cm} = 8000 \text{ N/m}$$

$$\boxed{K = 8.0 \times 10^3 \text{ N/m}}$$

(a) $x_i = +8.0$ cm $\longrightarrow$ $x = +5.0$ cm

$$W_s = \frac{1}{2} K (x_i^2 - x_f^2)$$

$$= \frac{1}{2}(8.0 \times 10^3)(0.08^2 - 0.05^2) = 15.6 \text{ J}$$

(b) $x_i = +8.0$ cm $\longrightarrow$ $x = -5.0$ cm

$$W_s = \frac{1}{2}(8.0 \times 10^3)(0.08^2 - (-0.05)^2) = 15.6 \text{ J}$$

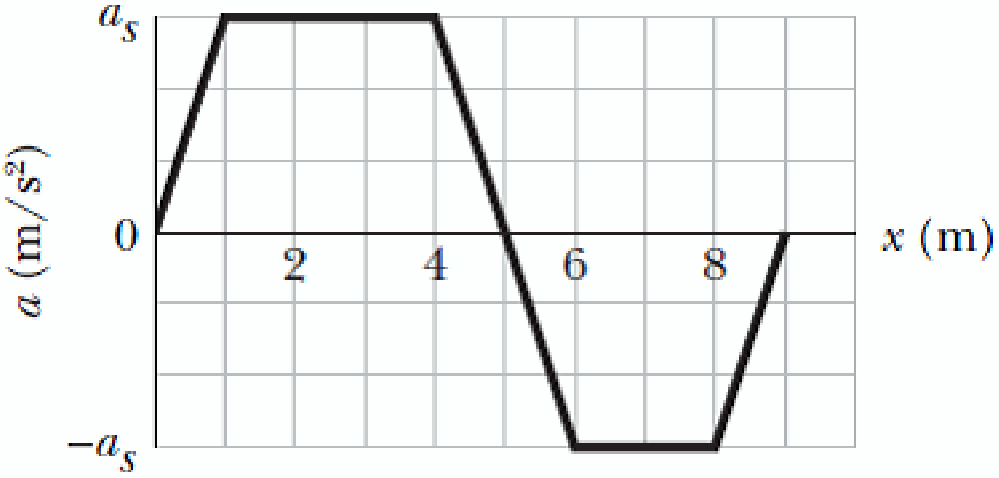
(c) $x_i = +8.0\text{ cm} \longrightarrow x = -8.0\text{ cm}$

$$W_s = \frac{1}{2} K \left(0.08^2 - (-0.08)^2\right) = \text{Zero J}$$

(d) $x_i = +8.0\text{ cm} \longrightarrow x = -10.0\text{ cm}$

$$W_s = \frac{1}{2} (8\times10^3)\left(0.08^2 - (-0.1)^2\right) = -14.4\text{ J}$$

P-37 The below figure gives the acceleration of a 2.00 Kg particle as an applied force $\vec{F}_a$ moves it from rest along an x axis from $x = 0$ to $x = 9.0\,m$. The scale of the figure's vertical axis is set by $a_s = 6.0\,m/s^2$. How much work has the force done on the particle when the particle reaches (a) $x = 4.0\,m$, (b) $x = 7.0\,m$, and (c) $x = 9.0\,m$? What is the particle's speed & direction of travel when it reaches (d) $x = 4.0\,m$, (e) $x = 7.0\,m$, and (f) $x = 9.0\,m$?



• Work Done by a Variable Force:

$$W = \int_{x_i}^{x_f} F(x)\,dx \quad ; \text{ If } \vec{F} \text{ has only an } x\text{-Component}$$

$= $ Area under the F vs. x curve

Newton's 2^{nd} Law

$$\vec{F} = m\vec{a}$$

(a) $W_{\text{done on the particle } (x_i=0 \to x_f = 4.0m)}$

$$W = \frac{1}{2}[4+3](6)(2)$$

$$\boxed{W = 42\,J}$$

(b) $W(x_i = 0 \to x = 7.0m) = W_{0m \to 5m} + W_{5m \to 7m}$

$$= \left(\frac{1}{2}[5+3](12)\right) - \left(\frac{1}{2}[1+2](12)\right)$$

$$\boxed{W = 30\,J}$$

(c) $W(x_i = 0\,m \rightarrow x = 9.0\,m) = W_{0m \rightarrow 7m} + W_{7m \rightarrow 9m}$

$= 30\,J - \left[\frac{1}{2}(2+1)(12)\right]$

$$\boxed{W_{0m \rightarrow 9.0m} = 12\,J}$$

(d) $W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$; Starts from Rest $v_i = 0$

$\Rightarrow$ particle's speed at $x = 4.0\,m$

$W = \frac{1}{2} m v_f^2 = 42\,J$

$$\boxed{v_f = 6.5\ m/s}$$

(e) v_f at $x = 7.0\,m$

$W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$, $v_i = 0$

$W = \frac{1}{2} m v_f^2 = 30\,J$

$$\boxed{v_f = 5.5\ m/s}$$

(f) v_f at $x = 9.0\,m$

$W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$, $v_i = 0$

$12\,J = \frac{1}{2} m v_f^2$

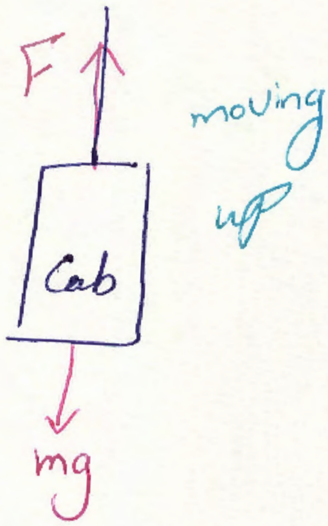
$$\boxed{v_f = 3.5\ m/s}$$

$\Rightarrow$ Velocity vector points to positive x-direction.

P-46 | The Loaded Cab of an elevator has a mass of 3.0×10^3 Kg and moves 210 m up the shaft in 23 s at constant speed. At What average rate does the force from the cable do work on the Cab?

The Average Rate of the Work Done by the cable on the Cab

$\Rightarrow$ Power : $P = \vec{F} \cdot \vec{v} = F v \cos\theta$



· moving up with a constant speed

$a = \text{zero} \longrightarrow F_{net} = \text{zero}$

$$\boxed{F = mg}$$

$\theta = 0°$ $\Rightarrow$ Elevator moves up ward and the force of the cable is upward.

$$\boxed{v = \frac{\Delta X}{\Delta t} = \frac{210\text{ m}}{23\text{ s}} = 9.13\text{ m/s}}$$

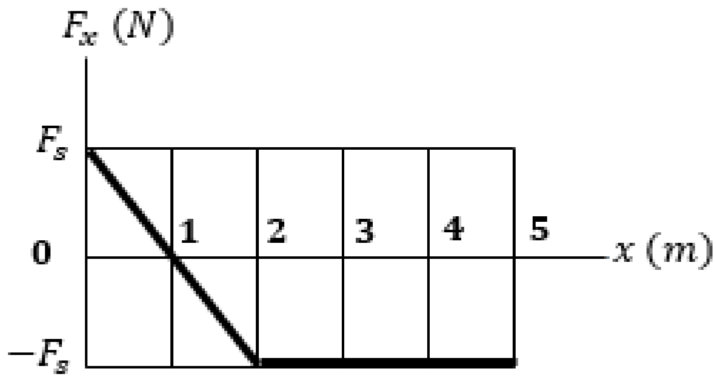
$$P = F v \cos\theta$$

$$P = mg \frac{\Delta X}{\Delta t} \cos(0°)$$

$$= 3.0 \times 10^3 (9.8)(9.13) = 268434.78 \text{ Watt}$$

$$\boxed{P = 2.7 \times 10^5 \text{ watt}}$$

P-54) The only force acting on a $2.0\ Kg$ body as the body moves along an x axis varies as shown in the figure. The scale of the figure's vertical axis is set by $F_s = 4.0\ N$. The velocity of the body at $x = 0$ is $4\ m/s$. (a) What is the kinetic energy of the body at $x = 3\ m$? (b) At what value of x will the body have a kinetic energy of $8.0\ J$? (c) What is the maximum kinetic energy of the body between $x = 0$ and $x = 5\ m$?



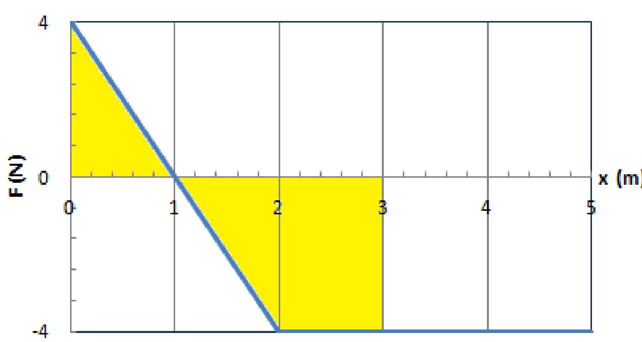
a) The kinetic energy of the body at $x = 3\ m$

$$\Delta K = W_{0\to 3}$$

$$K_3 - K_0 = W_{0\to 3}$$

$$K_3 = K_0 + W_{0\to 3}$$

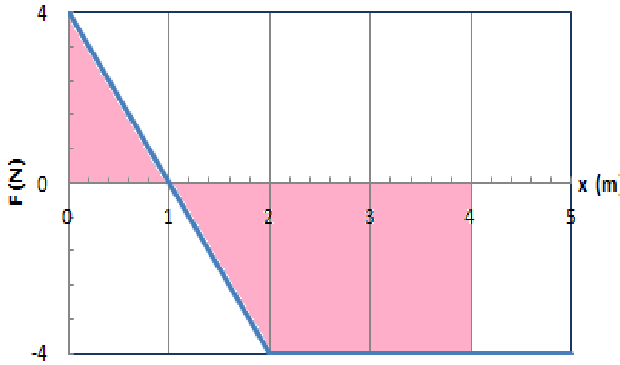
$$K_3 = K_0 + \int_0^3 F_x dx$$



$$K_3 = \frac{1}{2} m v_0^2 + area_{0\to 3}$$

$$K_3 = \frac{1}{2}(2)(4^2) - 4 = 12\ J$$

b) At what value of x the body have a kinetic energy of $8\,J$?



$$\Delta K = W_{0\to x}$$

$$K_x - K_0 = W_{0\to x}$$

$$8 - 16 = W_{0\to x}$$

$$-8 = \int_0^x F_x dx = area_{0\to x}$$

It is clear from the graph that

$x = 4\,m$

$$\Delta K = W_{0\to x}$$

$$K_x - K_0 = W_{0\to x}$$

$$8 - 16 = W_{0\to x}$$

$$-8 = W_{0\to 2} + W_{2\to d}$$

$$-8 = 0 + ((-4)(\mathrm{d})) \to\to d = 2$$

x = 4.0 m

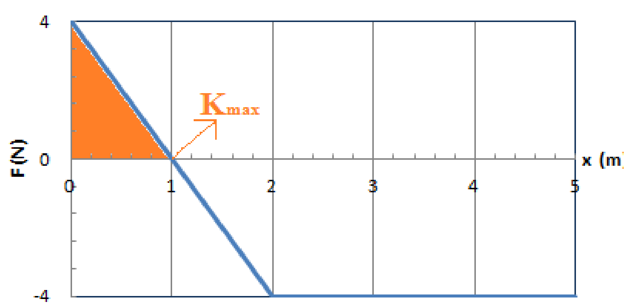
c) The maximum kinetic energy of the body between $x = 0$ and $x = 5\,m$.

$$\Delta K = W_{0\to x}$$

$$K_x = K_0 + W_{0\to x}$$

The kinetic energy is maximum when the work is maximum

thus, K_{max} at $x = 1\,m$



$$K_{max} = K_1 = K_0 + W_{0\to 1}$$

$$K_{max} = 16 + \int_0^1 F_x dx$$

$$K_{max} = 16 + area_{0\to 1} = 16 + 2 = 18\,J$$