FUNDAMENTALS OF

PHYSICS

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INSTRUCTOR SOLUTIONS MANUAL

EXTENDED

WILEY

Chapter 1

1. **THINK** In this problem we're given the radius of Earth, and asked to compute its circumference, surface area and volume.

EXPRESS Assuming Earth to be a sphere of radius

$$R_E = \left(6.37 \times 10^6 \text{ m}\right)\left(10^{-3} \text{ km/m}\right) = 6.37 \times 10^3 \text{ km},$$

the corresponding circumference, surface area and volume are:

$$C = 2\pi R_E, \quad A = 4\pi R_E^2, \quad V = \frac{4\pi}{3}R_E^3.$$

The geometric formulas are given in Appendix E.

ANALYZE (a) Using the formulas given above, we find the circumference to be

$$C = 2\pi R_E = 2\pi(6.37 \times 10^3 \text{ km}) = 4.00 \times 10^4 \text{ km}.$$

(b) Similarly, the surface area of Earth is

$$A = 4\pi R_E^2 = 4\pi\left(6.37 \times 10^3 \text{ km}\right)^2 = 5.10 \times 10^8 \text{ km}^2,$$

(c) and its volume is

$$V = \frac{4\pi}{3}R_E^3 = \frac{4\pi}{3}\left(6.37 \times 10^3 \text{ km}\right)^3 = 1.08 \times 10^{12} \text{ km}^3.$$

LEARN From the formulas given, we see that $C \sim R_E$, $A \sim R_E^2$, and $V \sim R_E^3$. The ratios of volume to surface area, and surface area to circumference are $V/A = R_E/3$ and $A/C = 2R_E$.

2. The conversion factors are: $1 \text{ gry} = 1/10 \text{ line}$, $1 \text{ line} = 1/12 \text{ inch}$ and 1 point = 1/72 inch. The factors imply that

$$1 \text{ gry} = (1/10)(1/12)(72 \text{ points}) = 0.60 \text{ point}.$$

Thus, 1 gry^2 = (0.60 point)^2 = 0.36 point^2, which means that $0.50 \text{ gry}^2 = 0.18 \text{ point}^2$.

3. The metric prefixes (micro, pico, nano, …) are given for ready reference on the inside front cover of the textbook (see also Table 1–2).

(a) Since 1 km = 1 × 10^3 m and 1 m = 1 × 10^6 μm,

$$1\,\text{km} = 10^3\ \text{m} = \left(10^3\ \text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 10^9\ \mu\text{m}.$$

The given measurement is 1.0 km (two significant figures), which implies our result should be written as 1.0 × 10^9 μm.

(b) We calculate the number of microns in 1 centimeter. Since 1 cm = 10^{-2} m,

$$1\,\text{cm} = 10^{-2}\ \text{m} = \left(10^{-2}\text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 10^4\ \mu\text{m}.$$

We conclude that the fraction of one centimeter equal to 1.0 μm is 1.0 × 10^{-4}.

(c) Since 1 yd = (3 ft)(0.3048 m/ft) = 0.9144 m,

$$1.0\,\text{yd} = \left(0.91\text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 9.1 \times 10^5\ \mu\text{m}.$$

4. (a) Using the conversion factors 1 inch = 2.54 cm exactly and 6 picas = 1 inch, we obtain

$$0.80\ \text{cm} = \left(0.80\ \text{cm}\right)\left(\frac{1\ \text{inch}}{2.54\ \text{cm}}\right)\left(\frac{6\ \text{picas}}{1\ \text{inch}}\right) \approx 1.9\ \text{picas}.$$

(b) With 12 points = 1 pica, we have

$$0.80\ \text{cm} = \left(0.80\ \text{cm}\right)\left(\frac{1\ \text{inch}}{2.54\ \text{cm}}\right)\left(\frac{6\ \text{picas}}{1\ \text{inch}}\right)\left(\frac{12\ \text{points}}{1\ \text{pica}}\right) \approx 23\ \text{points}.$$

5. **THINK** This problem deals with conversion of furlongs to rods and chains, all of which are units for distance.

EXPRESS Given that 1 furlong = 201.168 m, $1\,\text{rod} = 5.0292\ \text{m}$ and $1\,\text{chain} = 20.117\ \text{m}$, the relevant conversion factors are

$$1.0\ \text{furlong} = 201.168\ \text{m} = (201.168\ \cancel{\text{m}})\frac{1\ \text{rod}}{5.0292\ \cancel{\text{m}}} = 40\ \text{rods},$$

and

$$1.0\ \text{furlong} = 201.168\ \text{m} = (201.168\ \cancel{\text{m}})\frac{1\ \text{chain}}{20.117\ \cancel{\text{m}}} = 10\ \text{chains}.$$

Note the cancellation of m (meters), the unwanted unit.

ANALYZE Using the above conversion factors, we find

(a) the distance d in *rods* to be $d = 4.0\ \text{furlongs} = \left(4.0\ \text{fu\cancel{r}longs}\right)\dfrac{40\ \text{rods}}{1\ \text{fu\cancel{r}long}} = 160\ \text{rods},$

(b) and in *chains* to be $d = 4.0 \text{ furlongs} = (4.0 \text{ fu}\not{\text{r}}\text{longs})\dfrac{10 \text{ chains}}{1 \text{ fu}\not{\text{r}}\text{long}} = 40 \text{ chains}.$

LEARN Since 4 furlongs is about 800 m, this distance is approximately equal to 160 rods ($1 \text{ rod} \approx 5 \text{ m}$) and 40 chains ($1 \text{ chain} \approx 20 \text{ m}$). So our results make sense.

6. We make use of Table 1-6.

(a) We look at the first ("cahiz") column: 1 fanega is equivalent to what amount of cahiz? We note from the already completed part of the table that 1 cahiz equals a dozen fanega. Thus, 1 fanega = $\frac{1}{12}$ cahiz, or 8.33×10^{-2} cahiz. Similarly, "1 cahiz = 48 cuartilla" (in the already completed part) implies that 1 cuartilla = $\frac{1}{48}$ cahiz, or 2.08×10^{-2} cahiz. Continuing in this way, the remaining entries in the first column are 6.94×10^{-3} and 3.47×10^{-3}.

(b) In the second ("fanega") column, we find 0.250, 8.33×10^{-2}, and 4.17×10^{-2} for the last three entries.

(c) In the third ("cuartilla") column, we obtain 0.333 and 0.167 for the last two entries.

(d) Finally, in the fourth ("almude") column, we get $\frac{1}{2}$ = 0.500 for the last entry.

(e) Since the conversion table indicates that 1 almude is equivalent to 2 medios, our amount of 7.00 almudes must be equal to 14.0 medios.

(f) Using the value (1 almude = 6.94×10^{-3} cahiz) found in part (a), we conclude that 7.00 almudes is equivalent to 4.86×10^{-2} cahiz.

(g) Since each decimeter is 0.1 meter, then 55.501 cubic decimeters is equal to 0.055501 m^3 or 55501 cm^3. Thus, 7.00 almudes = $\frac{7.00}{12}$ fanega = $\frac{7.00}{12}$(55501 cm^3) = 3.24×10^4 cm^3.

7. We use the conversion factors found in Appendix D.

$$1 \text{ acre} \cdot \text{ft} = (43{,}560 \text{ ft}^2) \cdot \text{ft} = 43{,}560 \text{ ft}^3$$

Since 2 in. = (1/6) ft, the volume of water that fell during the storm is

$$V = (26 \text{ km}^2)(1/6 \text{ ft}) = (26 \text{ km}^2)(3281 \text{ft/km})^2(1/6 \text{ ft}) = 4.66\times10^7 \text{ ft}^3.$$

Thus,

$$V = \frac{4.66 \times 10^7 \text{ ft}^3}{4.3560 \times 10^4 \text{ ft}^3/\text{acre} \cdot \text{ft}} = 1.1 \times 10^3 \text{ acre} \cdot \text{ft}.$$

8. From Fig. 1-4, we see that 212 S is equivalent to 258 W and 212 – 32 = 180 S is equivalent to 216 – 60 = 156 Z. The information allows us to convert S to W or Z.

(a) In units of W, we have

$$50.0 \text{ S} = (50.0 \text{ S})\left(\frac{258 \text{ W}}{212 \text{ S}}\right) = 60.8 \text{ W}$$

(b) In units of Z, we have

$$50.0 \text{ S} = (50.0 \text{ S})\left(\frac{156 \text{ Z}}{180 \text{ S}}\right) = 43.3 \text{ Z}$$

9. The volume of ice is given by the product of the semicircular surface area and the thickness. The area of the semicircle is $A = \pi r^2/2$, where r is the radius. Therefore, the volume is

$$V = \frac{\pi}{2} r^2 z$$

where z is the ice thickness. Since there are 10^3 m in 1 km and 10^2 cm in 1 m, we have

$$r = (2000 \text{km})\left(\frac{10^3 \text{ m}}{1 \text{km}}\right)\left(\frac{10^2 \text{ cm}}{1 \text{m}}\right) = 2000 \times 10^5 \text{ cm}.$$

In these units, the thickness becomes

$$z = 3000 \text{m} = (3000 \text{m})\left(\frac{10^2 \text{ cm}}{1 \text{m}}\right) = 3000 \times 10^2 \text{ cm}$$

which yields $V = \frac{\pi}{2}\left(2000 \times 10^5 \text{ cm}\right)^2 \left(3000 \times 10^2 \text{ cm}\right) = 1.9 \times 10^{22} \text{ cm}^3.$

10. Since a change of longitude equal to $360°$ corresponds to a 24 hour change, then one expects to change longitude by $360°/24 = 15°$ before resetting one's watch by 1.0 h.

11. (a) Presuming that a French decimal day is equivalent to a regular day, then the ratio of weeks is simply 10/7 or (to 3 significant figures) 1.43.

(b) In a regular day, there are 86400 seconds, but in the French system described in the problem, there would be 10^5 seconds. The ratio is therefore 0.864.

12. A day is equivalent to 86400 seconds and a meter is equivalent to a million micrometers, so

$$\frac{(3.7\text{ m})(10^6\ \mu\text{m/m})}{(14\text{ day})(86400\text{ s/day})} = 3.1\ \mu\text{m/s}.$$

13. The time on any of these clocks is a straight-line function of that on another, with slopes $\neq$ 1 and y-intercepts $\neq$ 0. From the data in the figure we deduce

$$t_C = \frac{2}{7}t_B + \frac{594}{7}, \quad t_B = \frac{33}{40}t_A - \frac{662}{5}.$$

These are used in obtaining the following results.

(a) We find

$$t'_B - t_B = \frac{33}{40}(t'_A - t_A) = 495\text{ s}$$

when $t'_A - t_A = 600$ s.

(b) We obtain $t'_C - t_C = \frac{2}{7}(t'_B - t_B) = \frac{2}{7}(495) = 141$ s.

(c) Clock B reads $t_B = (33/40)(400) - (662/5) \approx 198$ s when clock A reads $t_A = 400$ s.

(d) From $t_C = 15 = (2/7)t_B + (594/7)$, we get $t_B \approx -245$ s.

14. The metric prefixes (micro (μ), pico, nano, …) are given for ready reference on the inside front cover of the textbook (also Table 1–2).

(a) $1\ \mu\text{century} = (10^{-6}\ \text{century})\left(\frac{100\text{ y}}{1\text{ century}}\right)\left(\frac{365\text{ day}}{1\text{ y}}\right)\left(\frac{24\text{ h}}{1\text{ day}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = 52.6\text{ min}.$

(b) The percent difference is therefore

$$\frac{52.6\text{ min} - 50\text{ min}}{52.6\text{ min}} = 4.9\%.$$

15. A week is 7 days, each of which has 24 hours, and an hour is equivalent to 3600 seconds. Thus, two weeks (a fortnight) is 1209600 s. By definition of the micro prefix, this is roughly $1.21 \times 10^{12}\ \mu$s.

16. We denote the pulsar rotation rate f (for frequency).

$$f = \frac{1\text{ rotation}}{1.55780644887275 \times 10^{-3}\text{ s}}$$

(a) Multiplying f by the time-interval t = 7.00 days (which is equivalent to 604800 s, if we ignore *significant figure* considerations for a moment), we obtain the number of rotations:

$$N = \left(\frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}}\right)(604800 \text{ s}) = 388238218.4$$

which should now be rounded to 3.88×10^8 rotations since the time-interval was specified in the problem to three significant figures.

(b) We note that the problem specifies the *exact* number of pulsar revolutions (one million). In this case, our unknown is t, and an equation similar to the one we set up in part (a) takes the form $N = ft$, or

$$1 \times 10^6 = \left(\frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}}\right) t$$

which yields the result t = 1557.80644887275 s (though students who do this calculation on their calculator might not obtain those last several digits).

(c) Careful reading of the problem shows that the time-uncertainty *per revolution* is $\pm 3\times10^{-17}$s . We therefore expect that as a result of one million revolutions, the uncertainty should be $(\pm 3\times10^{-17})(1\times10^6) = \pm 3\times10^{-11}$ s .

17. **THINK** In this problem we are asked to rank 5 clocks, based on their performance as timekeepers.

EXPRESS We first note that none of the clocks advance by exactly 24 h in a 24-h period but this is not the most important criterion for judging their quality for measuring time intervals. What is important here is that the clock advance by the same (or nearly the same) amount in each 24-h period. The clock reading can then easily be adjusted to give the correct interval.

ANALYZE The chart below gives the corrections (in seconds) that must be applied to the reading on each clock for each 24-h period. The entries were determined by subtracting the clock reading at the end of the interval from the clock reading at the beginning.

Clocks C and D are both good timekeepers in the sense that each is consistent in its daily drift (relative to WWF time); thus, C and D are easily made “perfect” with simple and predictable corrections. The correction for clock C is less than the correction for clock D, so we judge clock C to be the best and clock D to be the next best. The correction that must be applied to clock A is in the range from 15 s to 17s. For clock B it is the range from −5 s to +10 s, for clock E it is in the range from −70 s to −2 s. After C and D, A has the smallest range of correction, B has the next smallest range, and E has the greatest range. From best to worst, the ranking of the clocks is C, D, A, B, E.

CLOCK	Sun. -Mon.	Mon. -Tues.	Tues. -Wed.	Wed. -Thurs.	Thurs. -Fri.	Fri. -Sat.
A	–16	–16	–15	–17	–15	–15
B	–3	+5	–10	+5	+6	–7
C	–58	–58	–58	–58	–58	–58
D	+67	+67	+67	+67	+67	+67
E	+70	+55	+2	+20	+10	+10

LEARN Of the five clocks, the readings in clocks A, B and E jump around from one 24-h period to another, making it difficult to correct them.

18. The last day of the 20 centuries is longer than the first day by

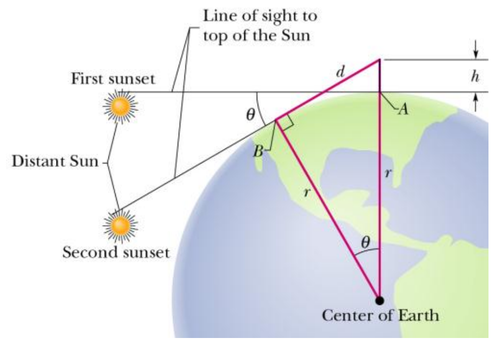
$$(20\text{ century})\,(0.001\text{ s/century}) = 0.02\text{ s.}$$

The average day during the 20 centuries is (0 + 0.02)/2 = 0.01 s longer than the first day. Since the increase occurs uniformly, the cumulative effect T is

$$\begin{aligned} T &= (\text{average increase in length of a day})(\text{number of days}) \\ &= \left(\frac{0.01\text{ s}}{\text{day}}\right)\left(\frac{365.25\text{ day}}{\text{y}}\right)(2000\text{ y}) \\ &= 7305\text{ s} \end{aligned}$$

or roughly two hours.

19. When the Sun first disappears while lying down, your line of sight to the top of the Sun is tangent to the Earth's surface at point A shown in the figure. As you stand, elevating your eyes by a height h, the line of sight to the Sun is tangent to the Earth's surface at point B.



Let d be the distance from point B to your eyes. From the Pythagorean theorem, we have

$$d^2 + r^2 = (r+h)^2 = r^2 + 2rh + h^2$$

or $d^2 = 2rh + h^2$, where r is the radius of the Earth. Since $r \gg h$, the second term can be dropped, leading to $d^2 \approx 2rh$. Now the angle between the two radii to the two tangent points A and B is θ, which is also the angle through which the Sun moves about Earth during the time interval t = 11.1 s. The value of θ can be obtained by using

$$\frac{\theta}{360°} = \frac{t}{24\text{ h}}.$$

This yields

$$\theta = \frac{(360°)(11.1\text{ s})}{(24\text{ h})(60\text{ min/h})(60\text{ s/min})} = 0.04625°.$$

Using $d = r\tan\theta$, we have $d^2 = r^2\tan^2\theta = 2rh$, or

$$r = \frac{2h}{\tan^2\theta}$$

Using the above value for θ and h = 1.7 m, we have $r = 5.2\times10^6$ m.

20. (a) We find the volume in cubic centimeters

$$193\text{ gal} = (193\text{ gal})\left(\frac{231\text{ in}^3}{1\text{ gal}}\right)\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)^3 = 7.31\times10^5\text{ cm}^3$$

and subtract this from 1×10^6 cm^3 to obtain 2.69×10^5 cm^3. The conversion gal → in^3 is given in Appendix D (immediately below the table of Volume conversions).

(b) The volume found in part (a) is converted (by dividing by (100 cm/m)^3) to 0.731 m^3, which corresponds to a mass of

$$\left(1000\ \text{kg/m}^3\right)\left(0.731\text{ m}^2\right) = 731\text{ kg}$$

using the density given in the problem statement. At a rate of 0.0018 kg/min, this can be filled in

$$\frac{731\text{kg}}{0.0018\text{ kg/min}} = 4.06\times10^5\text{ min} = 0.77\text{ y}$$

after dividing by the number of minutes in a year (365 days)(24 h/day) (60 min/h).

21. If M_E is the mass of Earth, m is the average mass of an atom in Earth, and N is the number of atoms, then $M_E = Nm$ or $N = M_E/m$. We convert mass m to kilograms using Appendix D (1 u = 1.661×10^{-27} kg). Thus,

$$N = \frac{M_E}{m} = \frac{5.98 \times 10^{24} \text{ kg}}{(40 \text{ u})(1.661 \times 10^{-27} \text{ kg/u})} = 9.0 \times 10^{49}.$$

22. The density of gold is

$$\rho = \frac{m}{V} = \frac{19.32 \text{ g}}{1 \text{ cm}^3} = 19.32 \text{ g/cm}^3.$$

(a) We take the volume of the leaf to be its area A multiplied by its thickness z. With density $\rho = 19.32$ g/cm^3 and mass $m = 27.63$ g, the volume of the leaf is found to be

$$V = \frac{m}{\rho} = 1.430 \text{ cm}^3.$$

We convert the volume to SI units:

$$V = (1.430 \text{ cm}^3)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 = 1.430 \times 10^{-6} \text{ m}^3.$$

Since $V = Az$ with $z = 1 \times 10^{-6}$ m (metric prefixes can be found in Table 1–2), we obtain

$$A = \frac{1.430 \times 10^{-6} \text{ m}^3}{1 \times 10^{-6} \text{ m}} = 1.430 \text{ m}^2.$$

(b) The volume of a cylinder of length ℓ is $V = A\ell$ where the cross-section area is that of a circle: $A = \pi r^2$. Therefore, with $r = 2.500 \times 10^{-6}$ m and $V = 1.430 \times 10^{-6}$ m^3, we obtain

$$\ell = \frac{V}{\pi r^2} = 7.284 \times 10^4 \text{ m} = 72.84 \text{ km}.$$

23. **THINK** This problem consists of two parts: in the first part, we are asked to find the mass of water, given its volume and density; the second part deals with the mass flow rate of water, which is expressed as kg/s in SI units.

EXPRESS From the definition of density: $\rho = m/V$, we see that mass can be calculated as $m = \rho V$, the product of the volume of water and its density. With 1 g = 1×10^{-3} kg and 1 cm^3 = $(1 \times 10^{-2}\text{m})^3 = 1 \times 10^{-6}\text{m}^3$, the density of water in SI units (kg/m^3) is

$$\rho = 1 \text{ g/cm}^3 = \left(\frac{1 \text{ g}}{\text{cm}^3}\right)\left(\frac{10^{-3} \text{ kg}}{\text{g}}\right)\left(\frac{\text{cm}^3}{10^{-6} \text{ m}^3}\right) = 1 \times 10^3 \text{ kg/m}^3.$$

To obtain the flow rate, we simply divide the total mass of the water by the time taken to drain it.

ANALYZE (a) Using $m = \rho V$, the mass of a cubic meter of water is

$$m = \rho V = (1 \times 10^3 \text{ kg/m}^3)(1 \text{ m}^3) = 1000 \text{ kg}.$$

(b) The total mass of water in the container is

$$M = \rho V = (1 \times 10^3 \text{ kg/m}^3)(5700 \text{ m}^3) = 5.70 \times 10^6 \text{ kg},$$

and the time elapsed is t = (10 h)(3600 s/h) = 3.6×10^4 s. Thus, the mass flow rate R is

$$R = \frac{M}{t} = \frac{5.70 \times 10^6 \text{ kg}}{3.6 \times 10^4 \text{ s}} = 158 \text{ kg/s}.$$

LEARN In terms of volume, the drain rate can be expressed as

$$R' = \frac{V}{t} = \frac{5700 \text{ m}^3}{3.6 \times 10^4 \text{ s}} = 0.158 \text{ m}^3\text{/s} \approx 42 \text{ gal/s}.$$

The greater the flow rate, the less time required to drain a given amount of water.

24. The metric prefixes (micro (μ), pico, nano, …) are given for ready reference on the inside front cover of the textbook (see also Table 1–2). The surface area A of each grain of sand of radius $r = 50\ \mu\text{m} = 50 \times 10^{-6}$ m is given by $A = 4\pi(50 \times 10^{-6})^2 = 3.14 \times 10^{-8}$ m^2 (Appendix E contains a variety of geometry formulas). We introduce the notion of density, $\rho = m/V$, so that the mass can be found from $m = \rho V$, where $\rho = 2600$ kg/m^3. Thus, using $V = 4\pi r^3/3$, the mass of each grain is

$$m = \rho V = \rho\left(\frac{4\pi r^3}{3}\right) = \left(2600 \frac{\text{kg}}{\text{m}^3}\right)\frac{4\pi\left(50 \times 10^{-6} \text{ m}\right)^3}{3} = 1.36 \times 10^{-9} \text{ kg}.$$

We observe that (because a cube has six equal faces) the indicated surface area is 6 m^2. The number of spheres (the grains of sand) N that have a total surface area of 6 m^2 is given by

$$N = \frac{6 \text{ m}^2}{3.14 \times 10^{-8} \text{ m}^2} = 1.91 \times 10^8.$$

Therefore, the total mass M is $M = Nm = \left(1.91 \times 10^8\right)\left(1.36 \times 10^{-9} \text{ kg}\right) = 0.260$ kg.

25. The volume of the section is (2500 m)(800 m)(2.0 m) = 4.0×10^6 m^3. Letting "d" stand for the thickness of the mud after it has (uniformly) distributed in the valley, then its volume there would be (400 m)(400 m)d. Requiring these two volumes to be equal, we can solve for d. Thus, d = 25 m. The volume of a small part of the mud over a patch of area of 4.0 m^2 is (4.0)d = 100 m^3. Since each cubic meter corresponds to a mass of

1900 kg (stated in the problem), then the mass of that small part of the mud is 1.9×10^5 kg.

26. (a) The volume of the cloud is (3000 m)π(1000 m)^2 = 9.4 × 10^9 m^3. Since each cubic meter of the cloud contains from 50 × 10^6 to 500 × 10^6 water drops, then we conclude that the entire cloud contains from 4.7 × 10^{18} to 4.7 × 10^{19} drops. Since the volume of each drop is $\frac{4}{3}\pi(10\times10^{-6}\text{ m})^3 = 4.2\times10^{-15}$ m^3, then the total volume of water in a cloud is from 2×10^3 to 2×10^4 m^3.

(b) Using the fact that $1\text{ L} = 1\times10^3\text{cm}^3 = 1\times10^{-3}\text{m}^3$, the amount of water estimated in part (a) would fill from 2×10^6 to 2×10^7 bottles.

(c) At 1000 kg for every cubic meter, the mass of water is from 2×10^6 to 2×10^7 kg. The coincidence in numbers between the results of parts (b) and (c) of this problem is due to the fact that each liter has a mass of one kilogram when water is at its normal density (under standard conditions).

27. We introduce the notion of density, $\rho = m/V$, and convert to SI units: 1000 g = 1 kg, and 100 cm = 1 m.

(a) The density ρ of a sample of iron is

$$\rho = \left(7.87\text{ g/cm}^3\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3 = 7870\text{ kg/m}^3.$$

If we ignore the empty spaces between the close-packed spheres, then the density of an individual iron atom will be the same as the density of any iron sample. That is, if M is the mass and V is the volume of an atom, then

$$V = \frac{M}{\rho} = \frac{9.27\times10^{-26}\text{ kg}}{7.87\times10^3\text{ kg/m}^3} = 1.18\times10^{-29}\text{ m}^3.$$

(b) We set $V = 4\pi R^3/3$, where R is the radius of an atom (Appendix E contains several geometry formulas). Solving for R, we find

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3\left(1.18\times10^{-29}\text{ m}^3\right)}{4\pi}\right)^{1/3} = 1.41\times10^{-10}\text{ m}.$$

The center-to-center distance between atoms is twice the radius, or 2.82 × 10^{-10} m.

28. If we estimate the "typical" large domestic cat mass as 10 kg, and the "typical" atom (in the cat) as 10 u ≈ 2×10^{-26} kg, then there are roughly (10 kg)/(2×10^{-26} kg) ≈ 5×10^{26} atoms. This is close to being a factor of a thousand greater than Avogadro's number. Thus this is roughly a kilomole of atoms.

29. The mass in kilograms is

$$(28.9 \text{ piculs})\left(\frac{100\,\text{gin}}{1\,\text{picul}}\right)\left(\frac{16\,\text{tahil}}{1\,\text{gin}}\right)\left(\frac{10\,\text{chee}}{1\,\text{tahil}}\right)\left(\frac{10\text{ hoon}}{1\text{ chee}}\right)\left(\frac{0.3779\,\text{g}}{1\,\text{hoon}}\right)$$

which yields 1.747×10^6 g or roughly 1.75×10^3 kg.

30. To solve the problem, we note that the first derivative of the function with respect to time gives the rate. Setting the rate to zero gives the time at which an extreme value of the variable mass occurs; here that extreme value is a maximum.

(a) Differentiating $m(t) = 5.00t^{0.8} - 3.00t + 20.00$ with respect to t gives

$$\frac{dm}{dt} = 4.00t^{-0.2} - 3.00.$$

The water mass is the greatest when $dm/dt = 0,$ or at $t = (4.00/3.00)^{1/0.2} = 4.21\,\text{s}.$

(b) At $t = 4.21\,\text{s},$ the water mass is

$$m(t = 4.21\,\text{s}) = 5.00(4.21)^{0.8} - 3.00(4.21) + 20.00 = 23.2\text{ g}.$$

(c) The rate of mass change at $t = 2.00\text{ s}$ is

$$\left.\frac{dm}{dt}\right|_{t=2.00\,\text{s}} = \left[4.00(2.00)^{-0.2} - 3.00\right]\text{g/s} = 0.48\text{ g/s} = 0.48\frac{\text{g}}{\text{s}}\cdot\frac{1\,\text{kg}}{1000\text{ g}}\cdot\frac{60\text{ s}}{1\text{ min}}$$
$$= 2.89\times10^{-2}\text{ kg/min}.$$

(d) Similarly, the rate of mass change at $t = 5.00\text{ s}$ is

$$\left.\frac{dm}{dt}\right|_{t=2.00\,\text{s}} = \left[4.00(5.00)^{-0.2} - 3.00\right]\text{g/s} = -0.101\,\text{g/s} = -0.101\frac{\text{g}}{\text{s}}\cdot\frac{1\,\text{kg}}{1000\text{ g}}\cdot\frac{60\text{ s}}{1\text{ min}}$$
$$= -6.05\times10^{-3}\text{ kg/min}.$$

31. The mass density of the candy is

$$\rho = \frac{m}{V} = \frac{0.0200 \text{ g}}{50.0 \text{ mm}^3} = 4.00\times10^{-4} \text{ g/mm}^3 = 4.00\times10^{-4} \text{ kg/cm}^3.$$

If we neglect the volume of the empty spaces between the candies, then the total mass of the candies in the container when filled to height h is $M = \rho Ah$, where $A = (14.0 \text{ cm})(17.0 \text{ cm}) = 238 \text{ cm}^2$ is the base area of the container that remains unchanged. Thus, the rate of mass change is given by

$$\begin{aligned}\frac{dM}{dt} &= \frac{d(\rho Ah)}{dt} = \rho A\frac{dh}{dt} = (4.00\times10^{-4} \text{ kg/cm}^3)(238 \text{ cm}^2)(0.250 \text{ cm/s}) \\ &= 0.0238 \text{ kg/s} = 1.43 \text{ kg/min.}\end{aligned}$$

32. The total volume V of the real house is that of a triangular prism (of height $h = 3.0$ m and base area $A = 20 \times 12 = 240 \text{ m}^2$) in addition to a rectangular box (height $h' = 6.0$ m and same base). Therefore,

$$V = \frac{1}{2}hA + h'A = \left(\frac{h}{2} + h'\right)A = 1800 \text{ m}^3.$$

(a) Each dimension is reduced by a factor of 1/12, and we find

$$V_{\text{doll}} = \left(1800 \text{ m}^3\right)\left(\frac{1}{12}\right)^3 \approx 1.0 \text{ m}^3.$$

(b) In this case, each dimension (relative to the real house) is reduced by a factor of 1/144. Therefore,

$$V_{\text{miniature}} = \left(1800 \text{ m}^3\right)\left(\frac{1}{144}\right)^3 \approx 6.0 \times 10^{-4} \text{ m}^3.$$

33. **THINK** In this problem we are asked to differentiate between three types of tons: *displacement* ton, *freight* ton and *register* ton, all of which are units of volume.

EXPRESS The three different tons are defined in terms of *barrel bulk*, with $1 \text{ barrel bulk} = 0.1415 \text{ m}^3 = 4.0155 \text{ U.S. bushels}$ (using $1 \text{ m}^3 = 28.378 \text{ U.S. bushels}$). Thus, in terms of U.S. bushels, we have

$$1 \text{ displacement ton} = (7 \text{ barrels bulk})\times\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 28.108 \text{ U.S. bushels}$$

$$1 \text{ freight ton} = (8 \text{ barrels bulk})\times\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 32.124 \text{ U.S. bushels}$$

$$1 \text{ register ton} = (20 \text{ barrels bulk})\times\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 80.31 \text{ U.S. bushels}$$

ANALYZE (a) The difference between 73 “freight” tons and 73 “displacement” tons is

$$\Delta V = 73(\text{freight tons} - \text{displacement tons}) = 73(32.124 \text{ U.S. bushels} - 28.108 \text{ U.S. bushels})$$
$$= 293.168 \text{ U.S. bushels} \approx 293 \text{ U.S. bushels}$$

(b) Similarly, the difference between 73 “register” tons and 73 “displacement” tons is

$$\Delta V = 73(\text{register tons} - \text{displacement tons}) = 73(80.31 \text{ U.S. bushels} - 28.108 \text{ U.S. bushels})$$
$$= 3810.746 \text{ U.S. bushels} \approx 3.81\times 10^3 \text{ U.S. bushels}$$

LEARN With $1 \text{ register ton} > 1 \text{ freight ton} > 1 \text{ displacement ton},$ we expect the difference found in (b) to be greater than that in (a). This is indeed the case.

34. The customer expects a volume $V_1 = 20 \times 7056 \text{ in}^3$ and receives $V_2 = 20 \times 5826 \text{ in.}^3$, the difference being $\Delta V = V_1 - V_2 = 24600 \text{ in.}^3$, or

$$\Delta V = \left(24600 \text{ in.}^3\right)\left(\frac{2.54\,\text{cm}}{1 \text{ inch}}\right)^3\left(\frac{1\,\text{L}}{1000 \text{ cm}^3}\right) = 403\,\text{L}$$

where Appendix D has been used.

35. The first two conversions are easy enough that a *formal* conversion is not especially called for, but in the interest of *practice makes perfect* we go ahead and proceed formally:

(a) $11 \text{ tuffets} = \left(11 \text{ tuffets}\right)\left(\frac{2 \text{ peck}}{1 \text{ tuffet}}\right) = 22 \text{ pecks}$.

(b) $11 \text{ tuffets} = \left(11 \text{ tuffets}\right)\left(\frac{0.50 \text{ Imperial bushel}}{1 \text{ tuffet}}\right) = 5.5 \text{ Imperial bushels}$.

(c) $11 \text{ tuffets} = \left(5.5 \text{ Imperial bushel}\right)\left(\frac{36.3687 \text{ L}}{1 \text{ Imperial bushel}}\right) \approx 200 \text{ L}$.

36. Table 7 can be completed as follows:

(a) It should be clear that the first column (under “wey”) is the reciprocal of the first row – so that $\frac{9}{10} = 0.900$, $\frac{3}{40} = 7.50 \times 10^{-2}$, and so forth. Thus, 1 pottle = 1.56×10^{-3} wey and 1 gill = 8.32×10^{-6} wey are the last two entries in the first column.

(b) In the second column (under “chaldron”), clearly we have 1 chaldron = 1 chaldron (that is, the entries along the “diagonal” in the table must be 1’s). To find out how many chaldron are equal to one bag, we note that 1 wey = 10/9 chaldron = 40/3 bag so that $\frac{1}{12}$ chaldron = 1 bag. Thus, the next entry in that second column is $\frac{1}{12}$ = 8.33 × 10^{-2}. Similarly, 1 pottle = 1.74 × 10^{-3} chaldron and 1 gill = 9.24 × 10^{-6} chaldron.

(c) In the third column (under "bag"), we have 1 chaldron = 12.0 bag, 1 bag = 1 bag, 1 pottle = 2.08 × 10^{-2} bag, and 1 gill = 1.11 × 10^{-4} bag.

(d) In the fourth column (under "pottle"), we find 1 chaldron = 576 pottle, 1 bag = 48 pottle, 1 pottle = 1 pottle, and 1 gill = 5.32 × 10^{-3} pottle.

(e) In the last column (under "gill"), we obtain 1 chaldron = 1.08 × 10^5 gill, 1 bag = 9.02 × 10^3 gill, 1 pottle = 188 gill, and, of course, 1 gill = 1 gill.

(f) Using the information from part (c), 1.5 chaldron = (1.5)(12.0) = 18.0 bag. And since each bag is 0.1091 m^3 we conclude 1.5 chaldron = (18.0)(0.1091) = 1.96 m^3.

37. The volume of one unit is 1 cm^3 = 1 × 10^{-6} m^3, so the volume of a mole of them is 6.02 × 10^{23} cm^3 = 6.02 × 10^{17} m^3. The cube root of this number gives the edge length: $8.4\times10^5\ \text{m}^3$. This is equivalent to roughly 8 × 10^2 km.

38. (a) Using the fact that the area *A* of a rectangle is (width) × (length), we find

$$\begin{aligned} A_{\text{total}} &= (3.00\,\text{acre}) + (25.0\,\text{perch})(4.00\,\text{perch}) \\ &= (3.00\ \text{acre})\left(\frac{(40\ \text{perch})(4\ \text{perch})}{1\,\text{acre}}\right) + 100\,\text{perch}^2 \\ &= 580\ \text{perch}^2. \end{aligned}$$

We multiply this by the $\text{perch}^2 \rightarrow$ rood conversion factor (1 rood/40 perch^2) to obtain the answer: A_{total} = 14.5 roods.

(b) We convert our intermediate result in part (a):

$$A_{\text{total}} = (580\ \text{perch}^2)\left(\frac{16.5\,\text{ft}}{1\,\text{perch}}\right)^2 = 1.58\times10^5\ \text{ft}^2.$$

Now, we use the feet → meters conversion given in Appendix D to obtain

$$A_{\text{total}} = (1.58\times10^5\ \text{ft}^2)\left(\frac{1\,\text{m}}{3.281\,\text{ft}}\right)^2 = 1.47\times10^4\ \text{m}^2.$$

39. **THINK** This problem compares the U.K. gallon with U.S. gallon, two non-SI units for volume. The interpretation of the type of gallons, whether U.K. or U.S., affects the amount of gasoline one calculates for traveling a given distance.

EXPRESS If the fuel consumption rate is R (in miles/gallon), then the amount of gasoline (in gallons) needed for a trip of distance d (in miles) would be

$$V(\text{gallon}) = \frac{d\ \ (\text{miles})}{R\ (\text{miles/gallon})}$$

Since the car was manufactured in U.K., the fuel consumption rate is calibrated based on U.K. gallon, and the correct interpretation should be "40 miles per U.K. gallon." In U.K., one would think of gallon as U.K. gallon; however, in the U.S., the word "gallon" would naturally be interpreted as U.S. gallon. Note also that since $1\text{ U.K. gallon} = 4.5460900\text{ L}$ and $1\text{ U.S. gallon} = 3.7854118\text{ L}$, the relationship between the two is

$$1\text{ U.K. gallon} = (4.5460900\text{ L})\left(\frac{1\text{ U.S. gallon}}{3.7854118\text{ L}}\right) = 1.20095\text{ U.S. gallons}$$

ANALYZE (a) The amount of gasoline actually required is

$$V' = \frac{750\text{ miles}}{40\text{ miles/U.K. gallon}} = 18.75\text{ U.K. gallons} \approx 18.8\text{ U.K. gallons}$$

This means that the driver mistakenly believes that the car should need 18.8 U.S. gallons.

(b) Using the conversion factor found above, this is equivalent to

$$V' = (18.75\text{ U.K. gallons}) \times \left(\frac{1.20095\text{ U.S. gallons}}{1\text{ U.K. gallon}}\right) \approx 22.5\text{ U.S. gallons}$$

LEARN One U.K. gallon is greater than one U.S gallon by roughly a factor of 1.2 in volume. Therefore, 40 mi/U.K. gallon is less fuel-efficient than 40 mi/U.S. gallon.

40. Equation 1-9 gives (to very high precision!) the conversion from atomic mass units to kilograms. Since this problem deals with the ratio of total mass (1.0 kg) divided by the mass of one atom (1.0 u, but converted to kilograms), then the computation reduces to simply taking the reciprocal of the number given in Eq. 1-9 and rounding off appropriately. Thus, the answer is 6.0×10^{26}.

41. **THINK** This problem involves converting *cord*, a non-SI unit for volume, to SI unit.

EXPRESS Using the (exact) conversion 1 in. = 2.54 cm = 0.0254 m for length, we have

$$1\text{ ft} = 12\text{ in} = (12\text{ in.})\times\left(\frac{0.0254\text{ m}}{1\text{ in}}\right) = 0.3048\text{ m}.$$

Thus, $1\text{ ft}^3 = (0.3048\text{ m})^3 = 0.0283\text{ m}^3$ for volume (these results also can be found in Appendix D).

ANALYZE The volume of a cord of wood is $V = (8\text{ ft})\times(4\text{ ft})\times(4\text{ ft}) = 128\text{ ft}^3$. Using the conversion factor found above, we obtain

$$V = 1\text{ cord} = 128\text{ ft}^3 = (128\text{ ft}^3)\times\left(\frac{0.0283\text{ m}^3}{1\text{ ft}^3}\right) = 3.625\text{ m}^3$$

which implies that $1\text{ m}^3 = \left(\frac{1}{3.625}\right)\text{cord} = 0.276\text{ cord} \approx 0.3\text{ cord}$.

LEARN The unwanted units ft^3 all cancel out, as they should. In conversions, units obey the same algebraic rules as variables and numbers.

42. (a) In atomic mass units, the mass of one molecule is (16 + 1 + 1)u = 18 u. Using Eq. 1-9, we find

$$18\text{u} = (18\text{u})\left(\frac{1.6605402\times10^{-27}\text{ kg}}{1\text{u}}\right) = 3.0\times10^{-26}\text{ kg}.$$

(b) We divide the total mass by the mass of each molecule and obtain the (approximate) number of water molecules:

$$N \approx \frac{1.4\times10^{21}}{3.0\times10^{-26}} \approx 5\times10^{46}.$$

43. A million milligrams comprise a kilogram, so 2.3 kg/week is 2.3×10^6 mg/week. Figuring 7 days a week, 24 hours per day, 3600 second per hour, we find 604800 seconds are equivalent to one week. Thus, (2.3×10^6 mg/week)/(604800 s/week) = 3.8 mg/s.

44. The volume of the water that fell is

$$\begin{aligned} V &= (26\text{ km}^2)(2.0\text{ in.}) = (26\text{ km}^2)\left(\frac{1000\text{ m}}{1\text{ km}}\right)^2(2.0\text{ in.})\left(\frac{0.0254\text{ m}}{1\text{ in.}}\right) \\ &= (26\times10^6\text{ m}^2)(0.0508\text{ m}) \\ &= 1.3\times10^6\text{ m}^3. \end{aligned}$$

We write the mass-per-unit-volume (density) of the water as: $\rho = \frac{m}{V} = 1\times10^3\text{ kg/m}^3$.

The mass of the water that fell is therefore given by $m = \rho V$:

$$m = (1\times10^3\text{ kg/m}^3)(1.3\times10^6\text{ m}^3) = 1.3\times10^9\text{ kg}.$$

45. The number of seconds in a year is 3.156 × 10^7. This is listed in Appendix D and results from the product

$$(365.25 \text{ day/y})\ (24 \text{ h/day})\ (60 \text{ min/h})\ (60 \text{ s/min}).$$

(a) The number of shakes in a second is 10^8; therefore, there are indeed more shakes per second than there are seconds per year.

(b) Denoting the age of the universe as 1 u-day (or 86400 u-sec), then the time during which humans have existed is given by

$$\frac{10^6}{10^{10}} = 10^{-4} \text{ u-day},$$

which may also be expressed as $\left(10^{-4} \text{ u-day}\right)\left(\dfrac{86400 \text{ u-sec}}{1 \text{ u-day}}\right) = 8.6 \text{ u-sec}.$

46. The volume removed in one year is $V = (75 \times 10^4 \text{ m}^2)(26 \text{ m}) \approx 2 \times 10^7 \text{ m}^3$, which we convert to cubic kilometers: $V = \left(2 \times 10^7 \text{ m}^3\right)\left(\dfrac{1 \text{ km}}{1000 \text{ m}}\right)^3 = 0.020 \text{ km}^3.$

47. **THINK** This problem involves expressing the speed of light in astronomical units per minute.

EXPRESS We first convert meters to astronomical units (AU), and seconds to minutes, using

$$1000 \text{ m} = 1 \text{ km}, \quad 1 \text{ AU} = 1.50 \times 10^8 \text{ km}, \quad 60 \text{ s} = 1 \text{ min}.$$

ANALYZE Using the conversion factors above, the speed of light can be rewritten as

$$c = 3.0 \times 10^8 \text{ m/s} = \left(\frac{3.0 \times 10^8 \text{ m}}{\text{s}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{\text{AU}}{1.50 \times 10^8 \text{ km}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 0.12 \text{ AU/min}.$$

LEARN When expressed the speed of light c in AU/min, we readily see that it takes about 8.3 (= 1/0.12) minutes for sunlight to reach the Earth (i.e., to travel a distance of 1 AU).

48. Since one atomic mass unit is $1 \text{ u} = 1.66 \times 10^{-24}$ g (see Appendix D), the mass of one mole of atoms is about $m = (1.66 \times 10^{-24} \text{ g})(6.02 \times 10^{23}) = 1 \text{ g}$. On the other hand, the mass of one mole of atoms in the common Eastern mole is

$$m' = \frac{75 \text{ g}}{7.5} = 10 \text{ g}$$

Therefore, in atomic mass units, the average mass of one atom in the common Eastern mole is

$$\frac{m'}{N_A} = \frac{10 \text{ g}}{6.02 \times 10^{23}} = 1.66 \times 10^{-23} \text{ g} = 10 \text{ u}.$$

49. (a) Squaring the relation 1 ken = 1.97 m, and setting up the ratio, we obtain

$$\frac{1 \text{ ken}^2}{1 \text{ m}^2} = \frac{1.97^2 \text{ m}^2}{1 \text{ m}^2} = 3.88.$$

(b) Similarly, we find

$$\frac{1 \text{ ken}^3}{1 \text{ m}^3} = \frac{1.97^3 \text{ m}^3}{1 \text{ m}^3} = 7.65.$$

(c) The volume of a cylinder is the circular area of its base multiplied by its height. Thus,

$$\pi r^2 h = \pi (3.00)^2 (5.50) = 156 \text{ ken}^3.$$

(d) If we multiply this by the result of part (b), we determine the volume in cubic meters: $(155.5)(7.65) = 1.19 \times 10^3 \text{ m}^3$.

50. According to Appendix D, a nautical mile is 1.852 km, so 24.5 nautical miles would be 45.374 km. Also, according to Appendix D, a mile is 1.609 km, so 24.5 miles is 39.4205 km. The difference is 5.95 km.

51. (a) For the minimum (43 cm) case, 9 cubits converts as follows:

$$9 \text{ cubits} = (9 \text{ cubits}) \left(\frac{0.43 \text{ m}}{1 \text{ cubit}} \right) = 3.9 \text{ m}.$$

And for the maximum (53 cm) case we have $9 \text{ cubits} = (9 \text{ cubits}) \left(\frac{0.53 \text{ m}}{1 \text{ cubit}} \right) = 4.8 \text{ m}.$

(b) Similarly, with 0.43 m $\rightarrow$ 430 mm and 0.53 m $\rightarrow$ 530 mm, we find 3.9×10^3 mm and 4.8×10^3 mm, respectively.

(c) We can convert length and diameter first and then compute the volume, or first compute the volume and then convert. We proceed using the latter approach (where d is diameter and ℓ is length).

$$V_{\text{cylinder, min}} = \frac{\pi}{4}\ell d^2 = 28\text{ cubit}^3 = \left(28\text{ cubit}^3\right)\left(\frac{0.43\,\text{m}}{1\text{ cubit}}\right)^3 = 2.2\text{ m}^3.$$

Similarly, with 0.43 m replaced by 0.53 m, we obtain $V_{\text{cylinder, max}} = 4.2\text{ m}^3$.

52. Abbreviating wapentake as "wp" and assuming a hide to be 110 acres, we set up the ratio 25 wp/11 barn along with appropriate conversion factors:

$$\frac{(25\text{ wp})\left(\frac{100\text{ hide}}{1\text{ wp}}\right)\left(\frac{110\text{ acre}}{1\text{ hide}}\right)\left(\frac{4047\text{ m}^2}{1\text{ acre}}\right)}{(11\text{ barn})\left(\frac{1\times10^{-28}\text{ m}^2}{1\text{ barn}}\right)} \approx 1\times10^{36}.$$

53. **THINK** The objective of this problem is to convert the Earth-Sun distance (1 AU) to parsecs and light-years.

EXPRESS To relate parsec (pc) to AU, we note that when θ is measured in radians, it is equal to the arc length s divided by the radius R. For a very large radius circle and small value of θ, the arc may be approximated as the straight line-segment of length 1 AU. Thus,

$$\theta = 1\text{ arcsec} = (1\text{ arcsec})\left(\frac{1\text{ arcmin}}{60\text{ arcsec}}\right)\left(\frac{1°}{60\text{ arcmin}}\right)\left(\frac{2\pi\text{ radian}}{360°}\right) = 4.85\times10^{-6}\text{ rad}.$$

Therefore, one parsec is

$$1\text{ pc} = \frac{s}{\theta} = \frac{1\text{ AU}}{4.85\times10^{-6}} = 2.06\times10^5\text{ AU}.$$

Next, we relate AU to light-year (ly). Since a year is about 3.16×10^7 s,

$$1\text{ly} = (186{,}000\,\text{mi/s})\left(3.16\times10^7\text{ s}\right) = 5.9\times10^{12}\text{ mi}.$$

ANALYZE (a) Since $1\text{ pc} = 2.06\times10^5\text{ AU}$, inverting the relation gives

$$1\text{ AU} = (1\text{ AU})\left(\frac{1\text{ pc}}{2.06\times10^5\text{ AU}}\right) = 4.9\times10^{-6}\text{ pc}.$$

(b) Given that $1\,\text{AU} = 92.9\times10^6\text{ mi}$ and $1\text{ ly} = 5.9\times10^{12}\text{ mi}$, the two expressions together lead to

$$1\,\text{AU} = 92.9\times10^6\text{ mi} = (92.9\times10^6\text{ mi})\left(\frac{1\,\text{ly}}{5.9\times10^{12}\text{ mi}}\right) = 1.57\times10^{-5}\text{ ly}.$$

LEARN Our results can be further combined to give $1\,\text{pc} = 3.2\,\text{ly}$. From the above expression, we readily see that it takes 1.57×10^{-5} y, or about 8.3 min, for Sunlight to travel a distance of 1 AU to reach the Earth.

54. (a) Using Appendix D, we have 1 ft = 0.3048 m, 1 gal = 231 in.^{3}, and 1 in.^{3} = 1.639 × 10^{-2} L. From the latter two items, we find that 1 gal = 3.79 L. Thus, the quantity 460 ft^2/gal becomes

$$460\ \text{ft}^2/\text{gal} = \left(\frac{460\ \text{ft}^2}{\text{gal}}\right)\left(\frac{1\ \text{m}}{3.28\ \text{ft}}\right)^2\left(\frac{1\ \text{gal}}{3.79\ \text{L}}\right) = 11.3\ \text{m}^2/\text{L}.$$

(b) Also, since 1 m^3 is equivalent to 1000 L, our result from part (a) becomes

$$11.3\ \text{m}^2/\text{L} = \left(\frac{11.3\ \text{m}^2}{\text{L}}\right)\left(\frac{1000\,\text{L}}{1\ \text{m}^3}\right) = 1.13\times10^4\ \text{m}^{-1}.$$

(c) The inverse of the original quantity is $(460\ \text{ft}^2/\text{gal})^{-1} = 2.17\times10^{-3}\ \text{gal/ft}^2$.

(d) The answer in (c) represents the volume of the paint (in gallons) needed to cover a square foot of area. From this, we could also figure the paint thickness [it turns out to be about a tenth of a millimeter, as one sees by taking the reciprocal of the answer in part (b)].

55. (a) The receptacle is a volume of (40 cm)(40 cm)(30 cm) = 48000 cm^3 = 48 L = (48)(16)/11.356 = 67.63 standard bottles, which is a little more than 3 nebuchadnezzars (the largest bottle indicated). The remainder, 7.63 standard bottles, is just a little less than 1 methuselah. Thus, the answer to part (a) is 3 nebuchadnezzars and 1 methuselah.

(b) Since 1 methuselah.= 8 standard bottles, then the extra amount is 8 − 7.63 = 0.37 standard bottle.

(c) Using the conversion factor 16 standard bottles = 11.356 L, we have

$$0.37\ \text{standard bottle} = (0.37\ \text{standard bottle})\left(\frac{11.356\ \text{L}}{16\ \text{standard bottles}}\right) = 0.26\ \text{L}.$$

56. The mass of the pig is 3.108 slugs, or (3.108)(14.59) = 45.346 kg. Referring now to the corn, a U.S. bushel is 35.238 liters. Thus, a value of 1 for the *corn-hog ratio* would be equivalent to 35.238/45.346 = 0.7766 in the indicated metric units. Therefore, a value of 5.7 for the *ratio* corresponds to 5.7(0.777) ≈ 4.4 in the indicated metric units.

57. Two jalapeño peppers have spiciness = 8000 SHU, and this amount multiplied by 400 (the number of people) is 3.2×10^6 SHU, which is roughly ten times the SHU value for a

single habanero pepper. More precisely, 10.7 habanero peppers will provide that total required SHU value.

58. In the simplest approach, we set up a ratio for the total increase in *horizontal depth x* (where Δx = 0.05 m is the increase in horizontal depth per step)

$$x = N_{\text{steps}}\,\Delta x = \left(\frac{4.57}{0.19}\right)(0.05\text{ m}) = 1.2\text{ m}.$$

However, we can approach this more carefully by noting that if there are $N = 4.57/.19 \approx 24$ rises then under normal circumstances we would expect $N - 1 = 23$ runs (horizontal pieces) in that staircase. This would yield (23)(0.05 m) = 1.15 m, which - to two significant figures - agrees with our first result.

59. The volume of the filled container is 24000 cm^3 = 24 liters, which (using the conversion given in the problem) is equivalent to 50.7 pints (U.S). The expected number is therefore in the range from 1317 to 1927 Atlantic oysters. Instead, the number received is in the range from 406 to 609 Pacific oysters. This represents a shortage in the range of roughly 700 to 1500 oysters (the answer to the problem). Note that the minimum value in our answer corresponds to the minimum Atlantic minus the maximum Pacific, and the maximum value corresponds to the maximum Atlantic minus the minimum Pacific.

60. (a) We reduce the stock amount to British teaspoons:

$$\begin{aligned}
1\text{ breakfastcup} &= 2\times 8\times 2\times 2 = 64\text{ teaspoons}\\
1\text{ teacup} &= 8\times 2\times 2 = 32\text{ teaspoons}\\
6\text{ tablespoons} &= 6\times 2\times 2 = 24\text{ teaspoons}\\
1\text{ dessertspoon} &= 2\text{ teaspoons}
\end{aligned}$$

which totals to 122 British teaspoons, or 122 U.S. teaspoons since liquid measure is being used. Now with one U.S cup equal to 48 teaspoons, upon dividing $122/48 \approx 2.54$, we find this amount corresponds to 2.5 U.S. cups plus a remainder of precisely 2 teaspoons. In other words,

$$122\text{ U.S. teaspoons} = 2.5\text{ U.S. cups} + 2\text{ U.S. teaspoons}.$$

(b) For the nettle tops, one-half quart is still one-half quart.

(c) For the rice, one British tablespoon is 4 British teaspoons which (since dry-goods measure is being used) corresponds to 2 U.S. teaspoons.

(d) A British saltspoon is $\frac{1}{2}$ British teaspoon which corresponds (since dry-goods measure is again being used) to 1 U.S. teaspoon.

Chapter 2

1. The speed (assumed constant) is v = (90 km/h)(1000 m/km) ⁄ (3600 s/h) = 25 m/s. Thus, in 0.50 s, the car travels a distance $d = vt$ = (25 m/s)(0.50 s) ≈ 13 m.

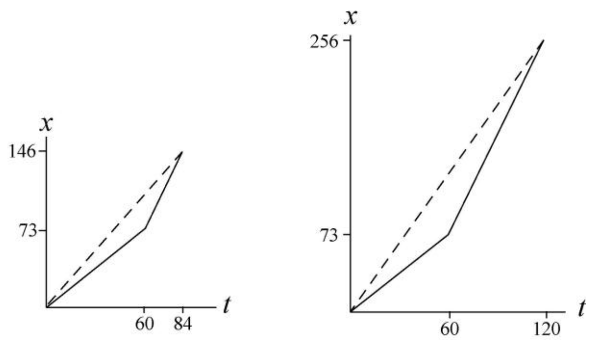
2. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

$$v_{\text{avg}} = \frac{73.2 \text{ m} + 73.2 \text{ m}}{\frac{73.2 \text{ m}}{1.22 \text{ m/s}} + \frac{73.2 \text{ m}}{3.05 \text{ m}}} = 1.74 \text{ m/s}.$$

(b) Using the fact that distance = vt while the velocity v is constant, we find

$$v_{\text{avg}} = \frac{(1.22 \text{ m/s})(60 \text{ s}) + (3.05 \text{ m/s})(60 \text{ s})}{120 \text{ s}} = 2.14 \text{ m/s}.$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before — the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.



3. **THINK** This one-dimensional kinematics problem consists of two parts, and we are asked to solve for the average velocity and average speed of the car.

EXPRESS Since the trip consists of two parts, let the displacements during first and second parts of the motion be Δx_1 and Δx_2, and the corresponding time intervals be Δt_1 and Δt_2, respectively. Now, because the problem is one-dimensional and both displacements are in the same direction, the total displacement is simply $\Delta x = \Delta x_1 + \Delta x_2$, and the total time for the trip is $\Delta t = \Delta t_1 + \Delta t_2$. Using the definition of average velocity given in Eq. 2-2, we have

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}.$$

To find the average speed, we note that during a time Δt if the velocity remains a positive constant, then the speed is equal to the magnitude of velocity, and the distance is equal to the magnitude of displacement, with $d = |\Delta x| = v\Delta t$.

ANALYZE

(a) During the first part of the motion, the displacement is Δx_1 = 40 km and the time taken is

$$t_1 = \frac{(40\text{ km})}{(30\text{ km/h})} = 1.33\text{ h}.$$

Similarly, during the second part of the trip the displacement is Δx_2 = 40 km and the time interval is

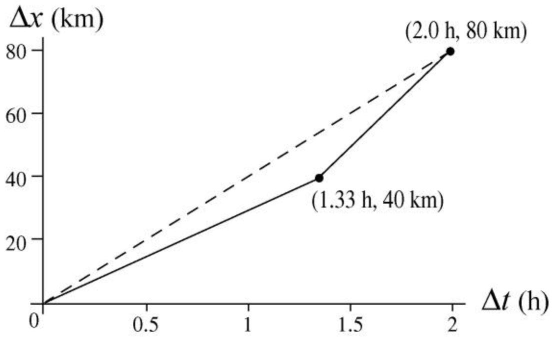
$$t_2 = \frac{(40\text{ km})}{(60\text{ km/h})} = 0.67\text{ h}.$$

The total displacement is $\Delta x = \Delta x_1 + \Delta x_2$ = 40 km + 40 km = 80 km, and the total time elapsed is $\Delta t = \Delta t_1 + \Delta t_2$ = 2.00 h. Consequently, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{(80\text{ km})}{(2.0\text{ h})} = 40\text{ km/h}.$$

(b) In this case, the average speed is the same as the magnitude of the average velocity: $s_{\text{avg}} = 40$ km/h.

(c) The graph of the entire trip, shown below, consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to $(\Delta t_1, \Delta x_1)$ = (1.33 h, 40 km) and the second having a slope of 60 km/h and connecting $(\Delta t_1, \Delta x_1)$ to $(\Delta t, \Delta x)$ = (2.00 h, 80 km).



From the graphical point of view, the slope of the dashed line drawn from the origin to $(\Delta t, \Delta x)$ represents the average velocity.

LEARN The average velocity is a vector quantity that depends only on the net displacement (also a vector) between the starting and ending points.

4. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the

motion) we have speed = D/t. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\dfrac{D}{v_{\text{up}}} + \dfrac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in v_{up} = 40 km/h and v_{down} = 60 km/h, yields 48 km/h for the average speed.

5. **THINK** In this one-dimensional kinematics problem, we're given the position function $x(t)$, and asked to calculate the position and velocity of the object at a later time.

EXPRESS The position function is given as $x(t) = (3\text{ m/s})t - (4\text{ m/s}^2)t^2 + (1\text{ m/s}^3)t^3$. The position of the object at some instant t_0 is simply given by $x(t_0)$. For the time interval $t_1 \le t \le t_2$, the displacement is $\Delta x = x(t_2) - x(t_1)$. Similarly, using Eq. 2-2, the average velocity for this time interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}.$$

ANALYZE (a) Plugging in t = 1 s into $x(t)$ yields

$$x(1\text{ s}) = (3\text{ m/s})(1\text{ s}) - (4\text{ m/s}^2)(1\text{ s})^2 + (1\text{ m/s}^3)(1\text{ s})^3 = 0.$$

(b) With t = 2 s we get $x(2\text{ s}) = (3\text{ m/s})(2\text{ s}) - (4\text{ m/s}^2)(2\text{ s})^2 + (1\text{ m/s}^3)(2\text{ s})^3 = -2$ m.

(c) With t = 3 s we have $x(3\text{ s}) = (3\text{ m/s})(3\text{ s}) - (4\text{ m/s}^2)(3\text{ s})^2 + (1\text{ m/s}^3)(3\text{ s})^3 = 0$ m.

(d) Similarly, plugging in t = 4 s gives

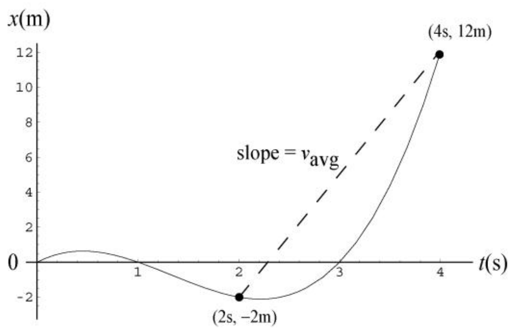
$$x(4\text{ s}) = (3\text{ m/s})(4\text{ s}) - (4\text{ m/s}^2)(4\text{ s})^2 + (1\text{ m/s}^3)(4\text{ s})^3 = 12\text{ m}.$$

(e) The position at t = 0 is x = 0. Thus, the displacement between t = 0 and t = 4 s is $\Delta x = x(4\text{ s}) - x(0) = 12\text{ m} - 0 = 12\text{ m}$.

(f) The position at t = 2 s is subtracted from the position at t = 4 s to give the displacement: $\Delta x = x(4\text{ s}) - x(2\text{ s}) = 12\text{ m} - (-2\text{ m}) = 14\text{ m}$. Thus, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14\text{ m}}{2\text{ s}} = 7\text{ m/s}.$$

(g) The position of the object for the interval $0 \le t \le 4$ is plotted below. The straight line drawn from the point at (t, x) = (2 s, –2 m) to (4 s, 12 m) would represent the average velocity, answer for part (f).



LEARN Our graphical representation illustrates once again that the average velocity for a time interval depends only on the net displacement between the starting and ending points.

6. Huber's speed is

$$v_0 = (200\text{ m})/(6.509\text{ s}) = 30.72\text{ m/s} = 110.6\text{ km/h},$$

where we have used the conversion factor 1 m/s = 3.6 km/h. Since Whittingham beat Huber by 19.0 km/h, his speed is v_1 = (110.6 km/h + 19.0 km/h) = 129.6 km/h, or 36 m/s (1 km/h = 0.2778 m/s). Thus, using Eq. 2-2, the time through a distance of 200 m for Whittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200\text{ m}}{36\text{ m/s}} = 5.554\text{ s}.$$

7. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time that elapses before they crash is t = (60 km)/(60 km/h) = 1.0 h. During this time, the bird travels a distance of $x = vt$ = (60 km/h)(1.0 h) = 60 km.

8. The amount of time it takes for each person to move a distance L with speed v_s is $\Delta t = L/v_s$. With each additional person, the depth increases by one body depth d

(a) The rate of increase of the layer of people is

$$R = \frac{d}{\Delta t} = \frac{d}{L/v_s} = \frac{dv_s}{L} = \frac{(0.25\text{ m})(3.50\text{ m/s})}{1.75\text{ m}} = 0.50\text{ m/s}$$

(b) The amount of time required to reach a depth of $D = 5.0$ m is

$$t = \frac{D}{R} = \frac{5.0\text{ m}}{0.50\text{ m/s}} = 10\text{ s}$$

9. Converting to seconds, the running times are t_1 = 147.95 s and t_2 = 148.15 s, respectively. If the runners were equally fast, then

$$s_{avg_1} = s_{avg_2} \quad \Rightarrow \quad \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

From this we obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1 \right) L_1 = \left(\frac{148.15}{147.95} - 1 \right) L_1 = 0.00135 L_1 \approx 1.4 \text{ m}$$

where we set $L_1 \approx 1000$ m in the last step. Thus, if L_1 and L_2 are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 by more than 1.4 m, then runner 2 would actually be faster.

10. Let v_w be the speed of the wind and v_c be the speed of the car.

(a) Suppose during time interval t_1, the car moves in the same direction as the wind. Then the effective speed of the car is given by $v_{eff,1} = v_c + v_w$, and the distance traveled is $d = v_{eff,1} t_1 = (v_c + v_w) t_1$. On the other hand, for the return trip during time interval t_2, the car moves in the opposite direction of the wind and the effective speed would be $v_{eff,2} = v_c - v_w$. The distance traveled is $d = v_{eff,2} t_2 = (v_c - v_w) t_2$. The two expressions can be rewritten as

$$v_c + v_w = \frac{d}{t_1} \quad \text{and} \quad v_c - v_w = \frac{d}{t_2}$$

Adding the two equations and dividing by two, we obtain $v_c = \frac{1}{2}\left(\frac{d}{t_1} + \frac{d}{t_2} \right)$. Thus, method 1 gives the car's speed v_c a in windless situation.

(b) If method 2 is used, the result would be

$$v_c' = \frac{d}{(t_1 + t_2)/2} = \frac{2d}{t_1 + t_2} = \frac{2d}{\dfrac{d}{v_c + v_w} + \dfrac{d}{v_c - v_w}} = \frac{v_c^2 - v_w^2}{v_c} = v_c \left[1 - \left(\frac{v_w}{v_c} \right)^2 \right].$$

The fractional difference is

$$\frac{v_c - v_c'}{v_c} = \left(\frac{v_w}{v_c} \right)^2 = (0.0240)^2 = 5.76 \times 10^{-4}.$$

11. The values used in the problem statement make it easy to see that the first part of the trip (at 100 km/h) takes 1 hour, and the second part (at 40 km/h) also takes 1 hour. Expressed in decimal form, the time left is 1.25 hour, and the distance that remains is 160 km. Thus, a speed v = (160 km)/(1.25 h) = 128 km/h is needed.

12. (a) Let the fast and the slow cars be separated by a distance d at $t = 0$. If during the time interval $t = L/v_s = (12.0 \text{ m})/(5.0 \text{ m/s}) = 2.40 \text{ s}$ in which the slow car has moved a distance of $L = 12.0$ m, the fast car moves a distance of $vt = d + L$ to join the line of slow cars, then the shock wave would remain stationary. The condition implies a separation of

$$d = vt - L = (25 \text{ m/s})(2.4 \text{ s}) - 12.0 \text{ m} = 48.0 \text{ m}.$$

(b) Let the initial separation at $t = 0$ be $d = 96.0$ m. At a later time t, the slow and the fast cars have traveled $x = v_s t$ and the fast car joins the line by moving a distance $d + x$. From

$$t = \frac{x}{v_s} = \frac{d+x}{v},$$

we get

$$x = \frac{v_s}{v - v_s} d = \frac{5.00 \text{ m/s}}{25.0 \text{ m/s} - 5.00 \text{ m/s}} (96.0 \text{ m}) = 24.0 \text{ m},$$

which in turn gives $t = (24.0 \text{ m})/(5.00 \text{ m/s}) = 4.80 \text{ s}$. Since the rear of the slow-car pack has moved a distance of $\Delta x = x - L = 24.0 \text{ m} - 12.0 \text{ m} = 12.0 \text{ m}$ downstream, the speed of the rear of the slow-car pack, or equivalently, the speed of the shock wave, is

$$v_{\text{shock}} = \frac{\Delta x}{t} = \frac{12.0 \text{ m}}{4.80 \text{ s}} = 2.50 \text{ m/s}.$$

(c) Since $x > L$, the direction of the shock wave is downstream.

13. (a) Denoting the travel time and distance from San Antonio to Houston as *T* and *D*, respectively, the average speed is

$$s_{\text{avg1}} = \frac{D}{T} = \frac{(55 \text{ km/h})(T/2) + (90 \text{ km/h})(T/2)}{T} = 72.5 \text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg2}} = \frac{D}{T} = \frac{D}{\frac{D/2}{55 \text{ km/h}} + \frac{D/2}{90 \text{ km/h}}} = 68.3 \text{ km/h}$$

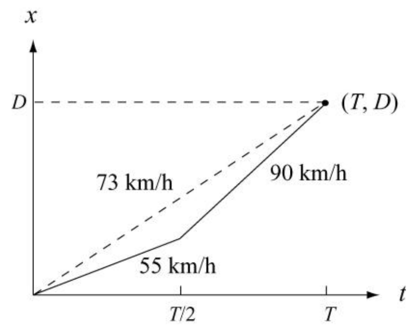
which should be rounded to 68 km/h.

(c) The total distance traveled (2*D*) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5 \text{ km/h}} + \frac{D}{68.3 \text{ km/h}}} = 70 \text{ km/h}.$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance *D* (the intent is *not* to make the student go to an atlas to look it up); the student can just as easily arbitrarily set *T* instead of *D*, as will be clear in the following discussion. We briefly describe the graph (with kilometers-per-hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to $(t_1, x_1) = (T/2, 55T/2)$ and the second having a slope of 90 and connecting (t_1, x_1) to (T, D) where $D = (55 + 90)T/2$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (T, D). The graph (not drawn to scale) is depicted below:



14. Using the general property $\frac{d}{dx}\exp(bx) = b\exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d(19t)}{dt}\right) \cdot e^{-t} + (19t) \cdot \left(\frac{de^{-t}}{dt}\right) .$$

If a concern develops about the appearance of an argument of the exponential ($-t$) apparently having units, then an explicit factor of $1/T$ where T = 1 second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1 - t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when t = 1 s. Now that we know *when* it stops, we find out *where* it stops by plugging our result t = 1 into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find x = 5.9 m.

15. We use Eq. 2-4 to solve the problem.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}\,(4 - 12t + 3t^2) = -12 + 6t.$$

Thus, at t = 1 s, the velocity is v = (–12 + (6)(1)) = –6 m/s.

(b) Since $v < 0$, it is moving in the $-x$ direction at t = 1 s.

(c) At t = 1 s, the *speed* is $|v|$ = 6 m/s.

(d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

(e) Yes, since v smoothly changes from negative values (consider the t = 1 result) to positive (note that as $t \to +\infty$, we have $v \to +\infty$). One can check that v = 0 when $t = 2$ s.

(f) No. In fact, from $v = -12 + 6t$, we know that $v > 0$ for $t > 2$ s.

16. We use the functional notation $x(t)$, $v(t)$, and $a(t)$ in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

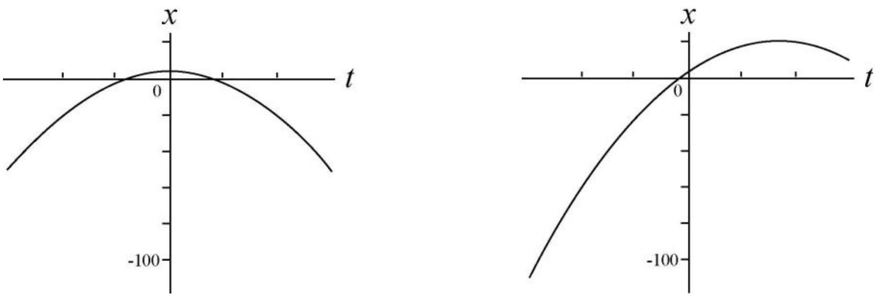
with SI units understood.

(a) From $v(t) = 0$ we find it is (momentarily) at rest at $t = 0$.

(b) We obtain $x(0) = 4.0$ m.

(c) and (d) Requiring $x(t) = 0$ in the expression $x(t) = 4.0 - 6.0t^2$ leads to $t = \pm 0.82$ s for the times when the particle can be found passing through the origin.

(e) We show both the asked-for graph (on the left) as well as the "shifted" graph that is relevant to part (f). In both cases, the time axis is given by $-3 \le t \le 3$ (SI units understood).



(f) We arrived at the graph on the right (shown above) by adding $20t$ to the $x(t)$ expression.

(g) Examining where the slopes of the graphs become zero, it is clear that the shift causes the $v = 0$ point to correspond to a larger value of x (the top of the second curve shown in part (e) is higher than that of the first).

17. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for $t = 2.00$ s and $t = 3.00$ s and obtain $x_2 = 21.75$ cm and $x_3 = 50.25$ cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25\text{ cm} - 21.75\text{ cm}}{3.00\text{ s} - 2.00\text{ s}}$$

which yields $v_{\text{avg}} = 28.5$ cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which, at time $t = 2.00$ s, yields $v = (4.5)(2.00)^2 = 18.0$ cm/s.

(c) At t = 3.00 s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ cm/s.

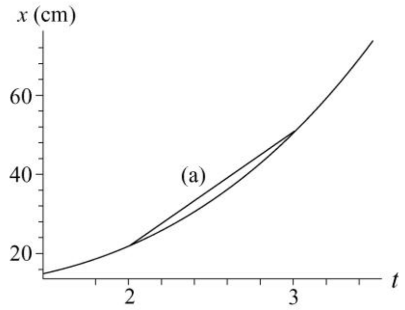
(d) At t = 2.50 s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \quad \Rightarrow \quad t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between t = 2 and t = 3 in this x-vs-t plot. The answers to parts (b), (c), (d), and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



18. (a) Taking derivatives of $x(t) = 12t^2 - 2t^3$ we obtain the velocity and the acceleration functions:

$$v(t) = 24t - 6t^2 \quad \text{and} \quad a(t) = 24 - 12t$$

with length in meters and time in seconds. Plugging in the value t = 3 yields $x(3) = 54$ m.

(b) Similarly, plugging in the value t = 3 yields $v(3)$ = 18 m/s.

(c) For t = 3, $a(3) = -12$ m/s^2.

(d) At the maximum x, we must have v = 0; eliminating the t = 0 root, the velocity equation reveals t = 24/6 = 4 s for the time of maximum x. Plugging t = 4 into the equation for x leads to x = 64 m for the largest x value reached by the particle.

(e) From (d), we see that the x reaches its maximum at t = 4.0 s.

(f) A maximum v requires a = 0, which occurs when t = 24/12 = 2.0 s. This, inserted into the velocity equation, gives v_{max} = 24 m/s.

(g) From (f), we see that the maximum of v occurs at t = 24/12 = 2.0 s.

(h) In part (e), the particle was (momentarily) motionless at t = 4 s. The acceleration at that time is readily found to be $24 - 12(4) = -24$ m/s^2.

(i) The *average velocity* is defined by Eq. 2-2, so we see that the values of x at $t = 0$ and $t = 3$ s are needed; these are, respectively, $x = 0$ and $x = 54$ m (found in part (a)). Thus,

$$v_{avg} = \frac{54-0}{3-0} = 18 \text{ m/s}.$$

19. **THINK** In this one-dimensional kinematics problem, we're given the speed of a particle at two instants and asked to calculate its average acceleration.

EXPRESS We represent the initial direction of motion as the $+x$ direction. The average acceleration over a time interval $t_1 \le t \le t_2$ is given by Eq. 2-7:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}.$$

ANALYZE Let $v_1 = +18$ m/s at $t_1 = 0$ and $v_2 = -30$ m/s at $t_2 = 2.4$ s. Using Eq. 2-7 we find

$$a_{avg} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{(-30 \text{ m/s}) - (+1 \text{ m/s})}{2.4 \text{ s} - 0} = -20 \text{ m/s}^2.$$

LEARN The average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity. This makes sense because the velocity of the particle is decreasing over the time interval. With $t_1 = 0$, the velocity of the particle as a function of time can be written as

$$v = v_0 + at = (18 \text{ m/s}) - (20 \text{ m/s}^2)t.$$

20. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

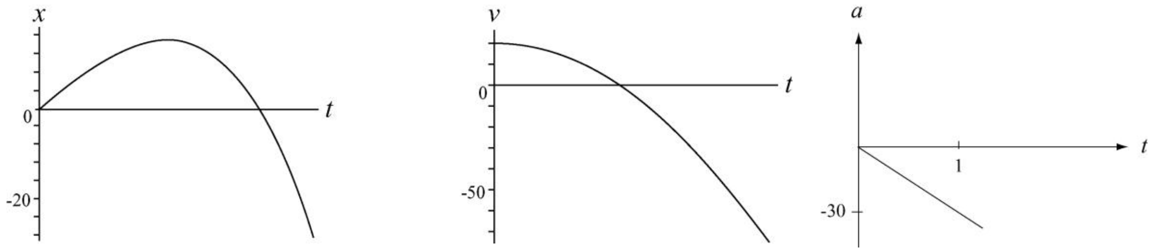
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From $0 = -30t$, we find $a(0) = 0$ (that is, it vanishes at $t = 0$).

(c) It is clear that $a(t) = -30t$ is negative for $t > 0$.

(d) The acceleration $a(t) = -30t$ is positive for $t < 0$.

(e) The graphs are shown below. SI units are understood.



21. We use Eq. 2-2 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during 5 min ≤ t ≤ 10 min is taken to be the positive x direction. We also use the fact that $\Delta x = v\Delta t'$ when the velocity is constant during a time interval $\Delta t'$.

(a) The entire interval considered is Δt = 8 – 2 = 6 min, which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t' = 8 - 5 = 3\,\text{min} = 180$ s. His position at t = 2 min is x = 0 and his position at t = 8 min is $x = v\Delta t' = (2.2)(180) = 396$ m. Therefore,

$$v_{\text{avg}} = \frac{396\text{ m} - 0}{360\text{ s}} = 1.10\text{ m/s}.$$

(b) The man is at rest at t = 2 min and has velocity v = +2.2 m/s at t = 8 min. Thus, keeping the answer to 3 significant figures,

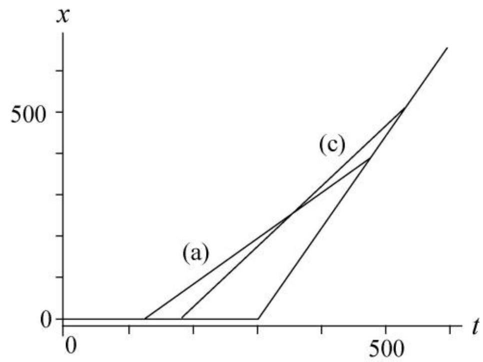
$$a_{\text{avg}} = \frac{2.2\text{ m/s} - 0}{360\text{ s}} = 0.00611\text{ m/s}^2 .$$

(c) Now, the entire interval considered is Δt = 9 – 3 = 6 min (360 s again), whereas the sub-interval in which he is moving is $\Delta t' = 9 - 5 = 4\,\text{min} = 240$ s). His position at $t = 3$ min is x = 0 and his position at t = 9 min is $x = v\Delta t' = (2.2)(240) = 528$ m. Therefore,

$$v_{\text{avg}} = \frac{528\text{ m} - 0}{360\text{ s}} = 1.47\text{ m/s}.$$

(d) The man is at rest at t = 3 min and has velocity v = +2.2 m/s at t = 9 min. Consequently, a_{avg} = 2.2/360 = 0.00611 m/s^2 just as in part (b).

(e) The horizontal line near the bottom of this x-vs-t graph represents the man standing at x = 0 for 0 ≤ t < 300 s and the linearly rising line for 300 ≤ t ≤ 600 s represents his constant-velocity motion. The lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of v-vs-t is not shown here, but would consist of two horizontal "steps" (one at v = 0 for 0 ≤ t < 300 s and the next at v = 2.2 m/s for 300 ≤

$t \le 600$ s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of a_{avg}).

22. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v(t)$ and $a(t)$ have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system.

(b) Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

(c) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(3.0\ \text{m/s}^2)}{3(2.0\ \text{m/s}^3)} = 1.0\ \text{s}\ .$$

For $t = 0$, $x = x_0 = 0$ and for $t = 1.0$ s, $x = 1.0$ m $> x_0$. Since we seek the maximum, we reject the first root ($t = 0$) and accept the second ($t = 1$s).

(d) In the first 4 s the particle moves from the origin to $x = 1.0$ m, turns around, and goes back to

$$x(4\ \text{s}) = (3.0\ \text{m/s}^2)(4.0\ \text{s})^2 - (2.0\ \text{m/s}^3)(4.0\ \text{s})^3 = -80\ \text{m}\ .$$

The total path length it travels is 1.0 m + 1.0 m + 80 m = 82 m.

(e) Its displacement is $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80$ m. Thus, $\Delta x = -80\ \text{m}$.

The velocity is given by $v = 2ct - 3bt^2 = (6.0\ \text{m/s}^2)t - (6.0\ \text{m/s}^3)t^2$.

(f) Plugging in $t = 1$ s, we obtain

$$v(1\ \text{s}) = (6.0\ \text{m/s}^2)(1.0\ \text{s}) - (6.0\ \text{m/s}^3)(1.0\ \text{s})^2 = 0.$$

(g) Similarly, $v(2\ \text{s}) = (6.0\ \text{m/s}^2)(2.0\ \text{s}) - (6.0\ \text{m/s}^3)(2.0\ \text{s})^2 = -12\text{m/s}$.

(h) $v(3\ \text{s}) = (6.0\ \text{m/s}^2)(3.0\ \text{s}) - (6.0\ \text{m/s}^3)(3.0\ \text{s})^2 = -36\ \text{m/s}$.

(i) $v(4\ \text{s}) = (6.0\ \text{m/s}^2)(4.0\ \text{s}) - (6.0\ \text{m/s}^3)(4.0\ \text{s})^2 = -72\ \text{m/s}$.

The acceleration is given by $a = dv/dt = 2c - 6b = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)t$.

(j) Plugging in $t = 1$ s, we obtain $a(1\ \text{s}) = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(1.0\ \text{s}) = -6.0\ \text{m/s}^2$.

(k) $a(2\ \text{s}) = 6.0\ \text{m/s}^2 - (12.0\ \text{m/s}^3)(2.0\ \text{s}) = -18\ \text{m/s}^2$.

(l) $a(3\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(3.0\text{ s}) = -30\text{ m/s}^2$.

(m) $a(4\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(4.0\text{ s}) = -42\text{ m/s}^2$.

23. **THINK** The electron undergoes a constant acceleration. Given the final speed of the electron and the distance it has traveled, we can calculate its acceleration.

EXPRESS Since the problem involves constant acceleration, the motion of the electron can be readily analyzed using the equations given in Table 2-1:

$$v = v_0 + at \qquad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (2-16)$$

The acceleration can be found by solving Eq. 2-16.

ANALYZE With $v_0 = 1.50\times10^5$ m/s, $v = 5.70\times10^6$ m/s, $x_0 = 0$ and $x = 0.010$ m, we find the average acceleration to be

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(5.7\times10^6\text{ m/s})^2 - (1.5\times10^5\text{ m/s})^2}{2(0.010\text{ m})} = 1.62\times10^{15}\text{ m/s}^2.$$

LEARN It is always a good idea to apply other equations in Table 2-1 not used for solving the problem as a consistency check. For example, since we now know the value of the acceleration, using Eq. 2-11, the time it takes for the electron to reach its final speed would be

$$t = \frac{v - v_0}{a} = \frac{5.70\times10^6\text{ m/s} - 1.5\times10^5\text{ m/s}}{1.62\times10^{15}\text{ m/s}^2} = 3.426\times10^{-9}\text{ s}$$

Substituting the value of t into Eq. 2-15, the distance the electron travels is

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + (1.5\times10^5\text{ m/s})(3.426\times10^{-9}\text{ s}) + \frac{1}{2}(1.62\times10^{15}\text{ m/s}^2)(3.426\times10^{-9}\text{ s})^2$$
$$= 0.01\text{ m}$$

This is what was given in the problem statement. So we know the problem has been solved correctly.

24. In this problem we are given the initial and final speeds, and the displacement, and are asked to find the acceleration. We use the constant-acceleration equation given in Eq. 2-16, $v^2 = v_0^2 + 2a(x - x_0)$.

(a) Given that $v_0 = 0$, $v = 1.6$ m/s, and $\Delta x = 5.0\,\mu$m, the acceleration of the spores during the launch is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(1.6\text{ m/s})^2}{2(5.0\times10^{-6}\text{ m})} = 2.56\times10^5\text{ m/s}^2 = 2.6\times10^4 g$$

(b) During the speed-reduction stage, the acceleration is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{0-(1.6\text{ m/s})^2}{2(1.0\times10^{-3}\text{ m})} = -1.28\times10^3\text{ m/s}^2 = -1.3\times10^2 g$$

The negative sign means that the spores are decelerating.

25. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given $v_0 = 0$; $v = 20$ m/s and $a = 2.0$ m/s^2. In part 2, the vehicle decelerates from its highest speed to a halt; we are given $v_0 = 20$ m/s; $v = 0$ and $a = -1.0$ m/s^2 (negative because the acceleration vector points opposite to the direction of motion).

(a) From Table 2-1, we find t_1 (the duration of part 1) from $v = v_0 + at$. In this way, $20 = 0 + 2.0t_1$ yields $t_1 = 10$ s. We obtain the duration t_2 of part 2 from the same equation. Thus, $0 = 20 + (-1.0)t_2$ leads to $t_2 = 20$ s, and the total is $t = t_1 + t_2 = 30$ s.

(b) For part 1, taking $x_0 = 0$, we use the equation $v^2 = v_0^2 + 2a(x - x_0)$ from Table 2-1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(20\text{ m/s})^2 - (0)^2}{2(2.0\text{ m/s}^2)} = 100\text{ m}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$x - 100\text{ m} = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (20\text{ m/s})^2}{2(-1.0\text{ m/s}^2)}.$$

Thus, the final position is $x = 300$ m. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

26. The constant-acceleration condition permits the use of Table 2-1.

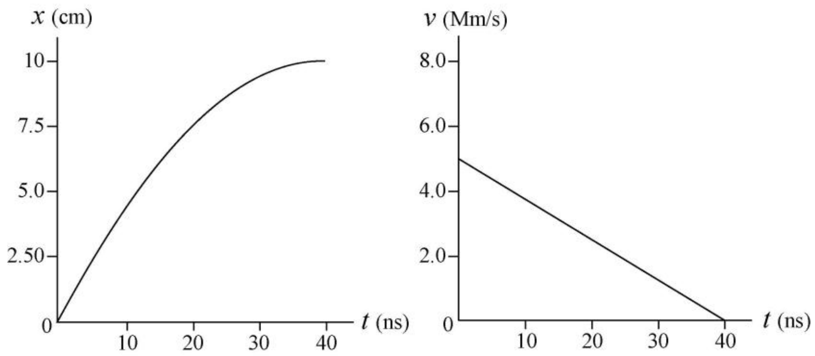
(a) Setting $v = 0$ and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\frac{(5.00\times10^6)^2}{-1.25\times10^{14}} = 0.100\text{ m}.$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t, so that other equations from Table 2-1 (such as $v = v_0 + at$ and

$x = v_0 t + \frac{1}{2}at^2$) are used in making these plots.



27. We use $v = v_0 + at$, with $t = 0$ as the instant when the velocity equals +9.6 m/s.

(a) Since we wish to calculate the velocity for a time *before* $t = 0$, we set $t = -2.5$ s. Thus, Eq. 2-11 gives

$$v = (9.6 \text{ m/s}) + \left(3.2 \text{ m/s}^2\right)(-2.5 \text{ s}) = 1.6 \text{ m/s}.$$

(b) Now, $t = +2.5$ s and we find $v = (9.6 \text{ m/s}) + \left(3.2 \text{ m/s}^2\right)(2.5 \text{ s}) = 18 \text{ m/s}.$

28. We take $+x$ in the direction of motion, so $v_0 = +24.6$ m/s and $a = -4.92$ m/s^2. We also take $x_0 = 0$.

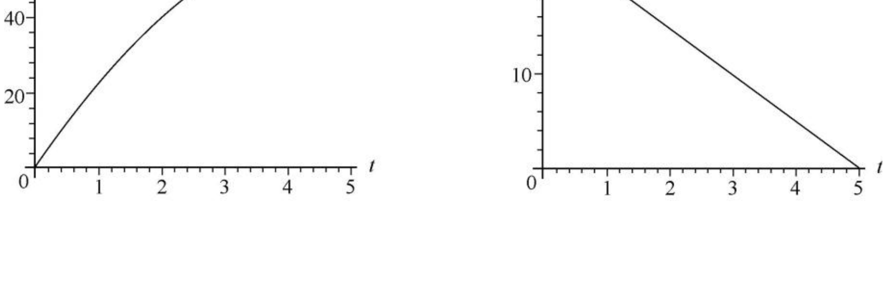
(a) The time to come to a halt is found using Eq. 2-11:

$$0 = v_0 + at \;\Rightarrow\; t = \frac{24.6 \text{ m/s}}{-4.92 \text{ m/s}^2} = 5.00 \text{ s}.$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \;\Rightarrow\; x = -\frac{(24.6 \text{ m/s})^2}{2\left(-4.92 \text{ m/s}^2\right)} = 61.5 \text{ m}.$$

(c) Using these results, we plot $v_0 t + \frac{1}{2}at^2$ (the x graph, shown next, on the left) and $v_0 + at$ (the v graph, on the right) over $0 \le t \le 5$ s, with SI units understood.



29. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a so that Table 2-1 can be used. Taking the direction of motion to be $+x$ then $a_1 = +1.22$ m/s^2 and $a_2 = -1.22$ m/s^2. We use SI units so the velocity at $t = t_1$ is $v = 305/60 = 5.08$ m/s.

(a) We denote Δx as the distance moved during t_1, and use Eq. 2-16:

$$v^2 = v_0^2 + 2a_1\Delta x \quad \Rightarrow \quad \Delta x = \frac{(5.08\text{ m/s})^2}{2(1.22\text{ m/s}^2)} = 10.59\text{ m} \approx 10.6\text{ m}.$$

(b) Using Eq. 2-11, we have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08\text{ m/s}}{1.22\text{ m/s}^2} = 4.17\text{ s}.$$

The deceleration time t_2 turns out to be the same so that $t_1 + t_2 = 8.33$ s. The distances traveled during t_1 and t_2 are the same so that they total to 2(10.59 m) = 21.18 m. This implies that for a distance of 190 m – 21.18 m = 168.82 m, the elevator is traveling at constant velocity. This time of constant velocity motion is

$$t_3 = \frac{168.82\text{ m}}{5.08\text{ m/s}} = 33.21\text{ s}.$$

Therefore, the total time is 8.33 s + 33.21 s ≈ 41.5 s.

30. We choose the positive direction to be that of the initial velocity of the car (implying that $a < 0$ since it is slowing down). We assume the acceleration is constant and use Table 2-1.

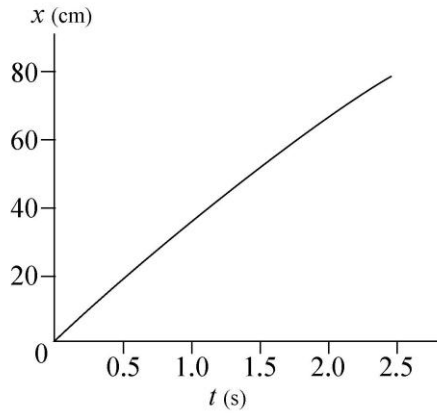
(a) Substituting v_0 = 137 km/h = 38.1 m/s, v = 90 km/h = 25 m/s, and $a = -5.2$ m/s^2 into $v = v_0 + at$, we obtain

$$t = \frac{25\text{ m/s} - 38\text{ m/s}}{-5.2\text{ m/s}^2} = 2.5\text{ s}.$$

(b) We take the car to be at $x = 0$ when the brakes are applied (at time $t = 0$). Thus, the coordinate of the car as a function of time is given by

$$x = (38\text{ m/s})t + \frac{1}{2}(-5.2\text{ m/s}^2)t^2$$

in SI units. This function is plotted from $t = 0$ to $t = 2.5$ s on the graph to the right. We have not shown the v-vs-t graph here; it is a descending straight line from v_0 to v.



31. **THINK** The rocket ship undergoes a constant acceleration from rest, and we want to know the time elapsed and the distance traveled when the rocket reaches a certain speed.

EXPRESS Since the problem involves constant acceleration, the motion of the rocket can be readily analyzed using the equations in Table 2-1:

$$v = v_0 + at \qquad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (2-16)$$

ANALYZE (a) Given that $a = 9.8\ \text{m/s}^2$, $v_0 = 0$ and $v = 0.1c = 3.0\times10^7\ \text{m/s}$, we can solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{3.0\times10^7\ \text{m/s} - 0}{9.8\ \text{m/s}^2} = 3.1\times10^6\ \text{s}$$

which is about 1.2 months. So it takes 1.2 months for the rocket to reach a speed of $0.1c$ starting from rest with a constant acceleration of 9.8 m/s^2.

(b) To calculate the distance traveled during this time interval, we evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$ and $v_0 = 0$. The result is

$$x = \frac{1}{2}\left(9.8\ \text{m/s}^2\right)(3.1\times10^6\text{s})^2 = 4.6\times10^{13}\ \text{m}.$$

LEARN In solving parts (a) and (b), we did not use Eq. (2-16): $v^2 = v_0^2 + 2a(x - x_0)$. This equation can be used to check our answers. The final velocity based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{0 + 2(9.8\ \text{m/s}^2)(4.6\times10^{13}\ \text{m} - 0)} = 3.0\times10^7\ \text{m/s},$$

which is what was given in the problem statement. So we know the problems have been solved correctly.

32. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020\ \text{km/h})\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)}{1.4\ \text{s}} = 202.4\ \text{m/s}^2\ .$$

In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = \left(\frac{202.4\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right)g = 21g\ .$$

33. **THINK** The car undergoes a constant negative acceleration to avoid impacting a barrier. Given its initial speed, we want to know the distance it has traveled and the time elapsed prior to the impact.

EXPRESS Since the problem involves constant acceleration, the motion of the car can be readily analyzed using the equations in Table 2-1:

$$v = v_0 + at \qquad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (2-16)$$

We take $x_0 = 0$ and $v_0 = 56.0$ km/h = 15.55 m/s to be the initial position and speed of the car. Solving Eq. 2-15 with $t = 2.00$ s gives the acceleration a. Once a is known, the speed of the car upon impact can be found by using Eq. 2-11.

ANALYZE (a) Using Eq. 2-15, we find the acceleration to be

$$a = \frac{2(x - v_0 t)}{t^2} = \frac{2\left[(24.0\text{ m}) - (15.55\text{ m/s})(2.00\text{ s})\right]}{(2.00\text{ s})^2} = -3.56\,\text{m/s}^2,$$

or $|a| = 3.56\text{ m/s}^2$. The negative sign indicates that the acceleration is opposite to the direction of motion of the car; the car is slowing down.

(b) The speed of the car at the instant of impact is

$$v = v_0 + at = 15.55\text{ m/s} + (-3.56\text{ m/s}^2)(2.00\text{ s}) = 8.43\text{ m/s}$$

which can also be converted to 30.3 km/h.

LEARN In solving parts (a) and (b), we did not use Eq. 1-16. This equation can be used as a consistency check. The final velocity based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{(15.55\text{ m/s})^2 + 2(-3.56\text{ m/s}^2)(24\text{ m} - 0)} = 8.43\text{ m/s},$$

which is what was calculated in (b). This indicates that the problems have been solved correctly.

34. Let d be the 220 m distance between the cars at $t = 0$, and v_1 be the 20 km/h = 50/9 m/s speed (corresponding to a passing point of $x_1 = 44.5$ m) and v_2 be the 40 km/h =100/9 m/s speed (corresponding to a passing point of $x_2 = 76.6$ m) of the red car. We have two equations (based on Eq. 2-17):

$$d - x_1 = v_o t_1 + \tfrac{1}{2}a t_1^2 \qquad \text{where } t_1 = x_1/v_1$$

$$d - x_2 = v_o t_2 + \tfrac{1}{2}a t_2^2 \qquad \text{where } t_2 = x_2/v_2$$

We simultaneously solve these equations and obtain the following results:

(a) The initial velocity of the green car is $v_o = -13.9$ m/s. or roughly -50 km/h (the negative sign means that it's along the $-x$ direction).

(b) The corresponding acceleration of the car is $a = -2.0$ m/s^2 (the negative sign means that it's along the $-x$ direction).

35. The positions of the cars as a function of time are given by

$$x_r(t) = x_{r0} + \frac{1}{2}a_r t^2 = (-35.0 \text{ m}) + \frac{1}{2}a_r t^2$$
$$x_g(t) = x_{g0} + v_g t = (270 \text{ m}) - (20 \text{ m/s})t$$

where we have substituted the velocity and not the speed for the green car. The two cars pass each other at $t = 12.0$ s when the graphed lines cross. This implies that

$$(270 \text{ m}) - (20 \text{ m/s})(12.0 \text{ s}) = 30 \text{ m} = (-35.0 \text{ m}) + \frac{1}{2}a_r(12.0 \text{ s})^2$$

which can be solved to give $a_r = 0.90$ m/s^2.

36. (a) Equation 2-15 is used for part 1 of the trip and Eq. 2-18 is used for part 2:

$\Delta x_1 = v_{o1}\,t_1 + \frac{1}{2}\,a_1\,t_1^2$ where $a_1 = 2.25$ m/s^2 and $\Delta x_1 = \frac{900}{4}$ m

$\Delta x_2 = v_2\,t_2 - \frac{1}{2}\,a_2\,t_2^2$ where $a_2 = -0.75$ m/s^2 and $\Delta x_2 = \frac{3(900)}{4}$ m

In addition, $v_{o1} = v_2 = 0$. Solving these equations for the times and adding the results gives $t = t_1 + t_2 = 56.6$ s.

(b) Equation 2-16 is used for part 1 of the trip:

$$v^2 = (v_{o1})^2 + 2a_1\Delta x_1 = 0 + 2(2.25)\left(\frac{900}{4}\right) = 1013 \text{ m}^2/\text{s}^2$$

which leads to $v = 31.8$ m/s for the maximum speed.

37. (a) From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

with $t = 1.0$ s, and then again with $t = 2.0$ s. This yields two equations for the two unknowns, v_0 and a:

$$0.0-(-2.0\text{ m})=v_0(1.0\text{ s})+\frac{1}{2}a(1.0\text{ s})^2$$
$$6.0\text{ m}-(-2.0\text{ m})=v_0\ (2.0\text{ s})+\frac{1}{2}a(2.0\text{ s})^2 .$$

Solving these simultaneous equations yields the results $v_0 = 0$ and $a = 4.0$ m/s^2.

(b) The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.

38. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34\text{ m/s}^2$ until the midway point and then decelerates at $a_2 = -1.34\text{ m/s}^2$ until it comes to a stop $(v_2 = 0)$ at the next station. The velocity at the midpoint is v_1, which occurs at $x_1 = 806/2 = 403$m.

(a) Equation 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \quad\Rightarrow\quad v_1 = \sqrt{2(1.34\text{ m/s}^2)(403\text{ m})} = 32.9\text{ m/s}.$$

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

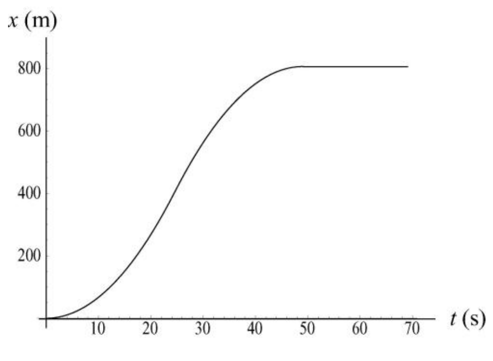
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \quad\Rightarrow\quad t_1 = \sqrt{\frac{2(403\text{ m})}{1.34\text{ m/s}^2}} = 24.53\text{ s}.$$

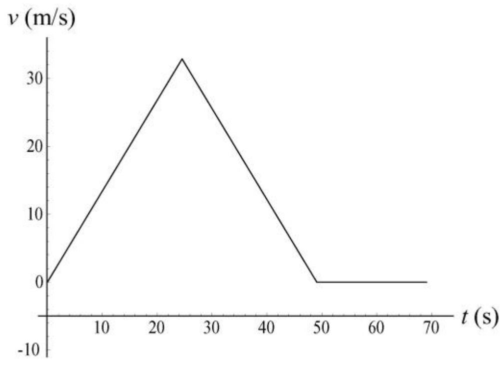
Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain $t = 49.1$ s for the travel time between stations.

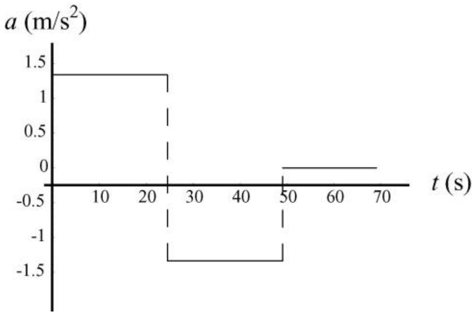
(c) With a "dead time" of 20 s, we have $T = t + 20 = 69.1$ s for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{806\text{ m}}{69.1\text{ s}} = 11.7\text{ m/s}.$$

(d) The graphs for x, v and a as a function of t are shown below. The third graph, $a(t)$, consists of three horizontal "steps" — one at 1.34 m/s^2 during $0 < t < 24.53$ s, and the next at -1.34 m/s^2 during $24.53\text{ s} < t < 49.1$ s and the last at zero during the "dead time" $49.1\text{ s} < t < 69.1$ s).







39. (a) We note that v_A = 12/6 = 2 m/s (with two significant figures understood). Therefore, with an initial x value of 20 m, car A will be at x = 28 m when t = 4 s. This must be the value of x for car B at that time; we use Eq. 2-15:

$$28 \text{ m} = (12 \text{ m/s})t + \tfrac{1}{2} a_B t^2 \qquad \text{where } t = 4.0 \text{ s}.$$

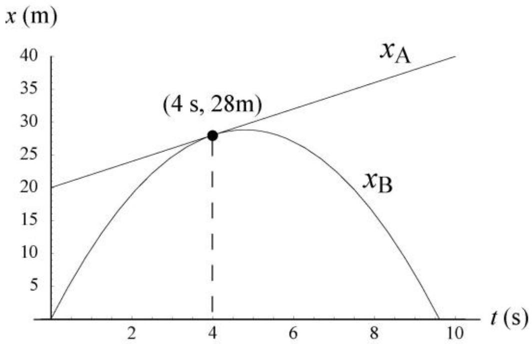
This yields $a_B = -2.5$ m/s^2.

(b) The question is: using the value obtained for a_B in part (a), are there other values of t (besides t = 4 s) such that $x_A = x_B$? The requirement is

$$20 + 2t = 12t + \tfrac{1}{2} a_B t^2$$

where $a_B = -5/2$. There are two distinct roots unless the discriminant $\sqrt{10^2 - 2(-20)(a_B)}$ is zero. In our case, it is zero – which means there is only one root. The cars are side by side only once at t = 4 s.

(c) A sketch is shown below. It consists of a straight line (x_A) tangent to a parabola (x_B) at t = 4.



(d) We only care about real roots, which means $10^2 - 2(-20)(a_B) \geq 0$. If $|a_B| > 5/2$ then there are no (real) solutions to the equation; the cars are never side by side.

(e) Here we have $10^2 - 2(-20)(a_B) > 0 \Rightarrow$ two real roots. The cars are side by side at two different times.

40. We take the direction of motion as $+x$, so $a = -5.18$ m/s^2, and we use SI units, so v_0 = 55(1000/3600) = 15.28 m/s.

(a) The velocity is constant during the reaction time T, so the distance traveled during it is

$$d_r = v_0 T - (15.28 \text{ m/s})\,(0.75 \text{ s}) = 11.46 \text{ m}.$$

We use Eq. 2-16 (with $v = 0$) to find the distance d_b traveled during braking:

$$v^2 = v_0^2 + 2ad_b \;\Rightarrow\; d_b = -\frac{(15.28 \text{ m/s})^2}{2\left(-5.18 \text{ m/s}^2\right)}$$

which yields $d_b = 22.53$ m. Thus, the total distance is $d_r + d_b = 34.0$ m, which means that the driver *is* able to stop in time. And if the driver were to continue at v_0, the car would enter the intersection in $t = (40 \text{ m})/(15.28 \text{ m/s}) = 2.6$ s, which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be 34 m) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is 32/15.28 = 2.1 s, which is too long (the light turns in 1.8 s). The driver is caught between a rock and a hard place.

41. The displacement (Δx) for each train is the "area" in the graph (since the displacement is the integral of the velocity). Each area is triangular, and the area of a triangle is 1/2(base) × (height). Thus, the (absolute value of the) displacement for one train (1/2)(40 m/s)(5 s) = 100 m, and that of the other train is (1/2)(30 m/s)(4 s) = 60 m. The initial "gap" between the trains was 200 m, and according to our displacement computations, the gap has narrowed by 160 m. Thus, the answer is 200 – 160 = 40 m.

42. (a) Note that 110 km/h is equivalent to 30.56 m/s. During a two-second interval, you travel 61.11 m. The decelerating police car travels (using Eq. 2-15) 51.11 m. In light of the fact that the initial "gap" between cars was 25 m, this means the gap has narrowed by 10.0 m – that is, to a distance of 15.0 m between cars.

(b) First, we add 0.4 s to the considerations of part (a). During a 2.4 s interval, you travel 73.33 m. The decelerating police car travels (using Eq. 2-15) 58.93 m during that time. The initial distance between cars of 25 m has therefore narrowed by 14.4 m. Thus, at the start of your braking (call it t_0) the gap between the cars is 10.6 m. The speed of the police car at t_0 is 30.56 – 5(2.4) = 18.56 m/s. Collision occurs at time t when $x_{\text{you}} = x_{\text{police}}$ (we choose coordinates such that your position is $x = 0$ and the police car's position is $x = 10.6$ m at t_0). Eq. 2-15 becomes, for each car:

$$x_{\text{police}} - 10.6 = 18.56(t - t_0) - \tfrac{1}{2}(5)(t - t_0)^2$$

$$x_{\text{you}} = 30.56(t - t_0) - \tfrac{1}{2}(5)(t - t_0)^2 \quad .$$

Subtracting equations, we find

$$10.6 = (30.56 - 18.56)(t - t_0) \quad \Rightarrow \quad 0.883 \text{ s} = t - t_0.$$

At that time your speed is $30.56 + a(t - t_0) = 30.56 - 5(0.883) \approx 26$ m/s (or 94 km/h).

43. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2-1 is permitted. We start with Eq. 2-17 and denote the train's initial velocity as v_t and the locomotive's velocity as v_ℓ (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance Δx consists of the original gap between them, D, as well as the forward distance traveled during this time by the locomotive $v_\ell t$. Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell .$$

We now use Eq. 2-11 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell$$

which leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell\right)\left(\frac{v_\ell - v_t}{D}\right) = -\frac{1}{2D}(v_\ell - v_t)^2 .$$

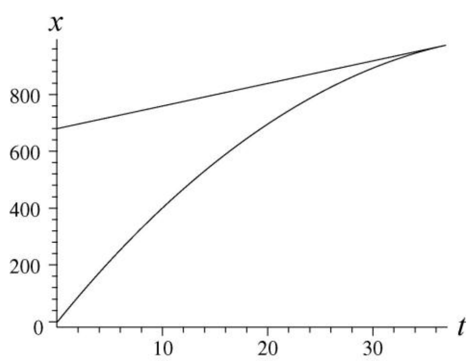
Hence,

$$a = -\frac{1}{2(0.676 \text{ km})}\left(29\frac{\text{km}}{\text{h}} - 161\frac{\text{km}}{\text{h}}\right)^2 = -12888 \text{ km/h}^2$$

which we convert as follows:

$$a = (-12888 \text{ km/h}^2)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2 = -0.994 \text{ m/s}^2$$

so that its *magnitude* is $|a| = 0.994$ m/s^2. A graph is shown here for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.



The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.

44. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) Using $y = v_0 t - \frac{1}{2}gt^2$, with $y = 0.544$ m and $t = 0.200$ s, we find

$$v_0 = \frac{y + gt^2/2}{t} = \frac{0.544\text{ m} + (9.8\text{ m/s}^2)(0.200\text{ s})^2/2}{0.200\text{ s}} = 3.70\text{ m/s}.$$

(b) The velocity at $y = 0.544$ m is

$$v = v_0 - gt = 3.70\text{ m/s} - (9.8\text{ m/s}^2)(0.200\text{ s}) = 1.74\text{ m/s}.$$

(c) Using $v^2 = v_0^2 - 2gy$ (with different values for y and v than before), we solve for the value of y corresponding to maximum height (where $v = 0$).

$$y = \frac{v_0^2}{2g} = \frac{(3.7\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 0.698\text{ m}.$$

Thus, the armadillo goes 0.698 – 0.544 = 0.154 m higher.

45. **THINK** As the ball travels vertically upward, its motion is under the influence of gravitational acceleration. The kinematics is one-dimensional.

EXPRESS We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g = -9.8$ m/s^2 (we take downward to be the $-y$ direction). We use the equations in Table 2-1 (with Δy replacing Δx) because this is a = constant motion:

$$v = v_0 - gt \qquad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2}gt^2 \qquad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \qquad (2-16)$$

We set $y_0 = 0$. Upon reaching the maximum height y, the speed of the ball is momentarily zero ($v = 0$). Therefore, we can relate its initial speed v_0 to y via the equation $0 = v^2 = v_0^2 - 2gy$. The time it takes for the ball to reach maximum height is given by $v = v_0 - gt = 0$, or $t = v_0/g$. Therefore, for the entire trip (from the time it leaves the ground until the time it returns to the ground), the total flight time is $T = 2t = 2v_0/g$.

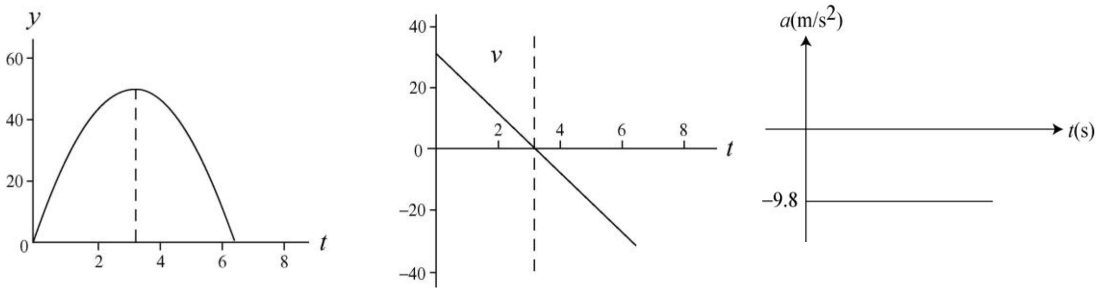
ANALYZE (a) At the highest point $v = 0$ and $v_0 = \sqrt{2gy}$. With $y = 50$ m, we find the initial speed of the ball to be

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8\text{ m/s}^2)(50\text{ m})} = 31.3\text{ m/s}.$$

(b) Using the result from (a) for v_0, the total flight time of the ball is

$$T = \frac{2v_0}{g} = \frac{2(31.3\text{ m/s})}{9.8\text{ m/s}^2} = 6.39\text{ s}$$

(c) The plots of *y*, *v* and *a* as a function of time are shown below. The acceleration graph is a horizontal line at –9.8 m/s^2. At *t* = 3.19 s, *y* = 50 m.



LEARN In calculating the total flight time of the ball, we could have used Eq. 2-15. At $t = T > 0$, the ball returns to its original position ($y = 0$). Therefore,

$$y = v_0 T - \frac{1}{2} g T^2 = 0 \;\Rightarrow\; T = \frac{2v_0}{g}$$

46. Neglect of air resistance justifies setting $a = -g = -9.8$ m/s^2 (where *down* is our $-y$ direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

(a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8\text{ m/s}^2)(-1700\text{ m})} = -183\text{ m/s}.$$

Its magnitude is therefore 183 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

47. **THINK** The wrench is in free fall with an acceleration $a = -g = -9.8$ m/s^2.

EXPRESS We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx):

$$v = v_0 - gt \qquad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \qquad (2-16)$$

Since the wrench had an initial speed $v_0 = 0$, knowing its speed of impact allows us to apply Eq. 2-16 to calculate the height from which it was dropped.

ANALYZE (a) Using $v^2 = v_0^2 + 2a\Delta y$, we find the initial height to be

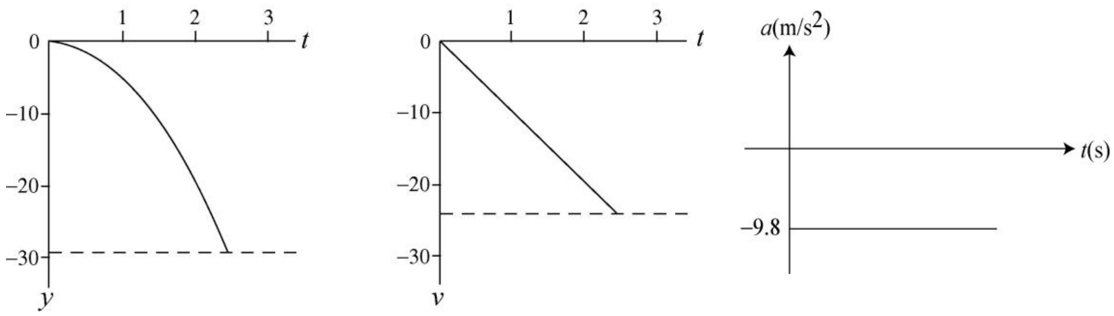
$$\Delta y = \frac{v_0^2 - v^2}{2a} = \frac{0-(-24\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 29.4\text{ m.}$$

So that it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we obtain a flight time of

$$t = \frac{v_0 - v}{g} = \frac{0-(-24\text{ m/s})}{9.8\text{ m/s}^2} = 2.45\text{ s.}$$

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. The acceleration graph is a horizontal line at –9.8 m/s^2.



LEARN As the wrench falls, with $a = -g < 0$, its speed increases but its velocity becomes more negative, as indicated by the second graph above.

48. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Noting that $\Delta y = y - y_0 = -30$ m, we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute t:

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with $v_0 = -12$ m/s since it is downward) leads, upon choosing the positive root (so that $t > 0$), to the result:

$$t = \frac{-12\text{ m/s} + \sqrt{(-12\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(-30\text{ m})}}{9.8\text{ m/s}^2} = 1.54\text{ s.}$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain v; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1\text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

49. **THINK** In this problem a package is dropped from a hot-air balloon which is ascending vertically upward. We analyze the motion of the package under the influence of gravity.

EXPRESS We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. This allows us to use Table 2-1 (with Δy replacing Δx):

$$v = v_0 - gt \qquad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \qquad (2-16)$$

We place the coordinate origin on the ground and note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12$ m/s and that its initial coordinate is $y_0 = +80$ m. The time it takes for the package to hit the ground can be found by solving Eq. 2-15 with $y = 0$.

ANALYZE (a) We solve $0 = y = y_0 + v_0 t - \frac{1}{2} g t^2$ for time using the quadratic formula (choosing the positive root to yield a positive value for t):

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 \text{ m/s} + \sqrt{(12 \text{ m/s})^2 + 2\left(9.8 \text{ m/s}^2\right)\left(80 \text{ m}\right)}}{9.8 \text{ m/s}^2} = 5.45 \text{ s} .$$

(b) The speed of the package when it hits the ground can be calculated using Eq. 2-11. The result is

$$v = v_0 - gt = 12 \text{ m/s} - (9.8 \text{ m/s}^2)(5.447 \text{ s}) = -41.38 \text{ m/s} .$$

Its final *speed* is 41.38 m/s.

LEARN Our answers can be readily verified by using Eq. 2-16 which was not used in either (a) or (b). The equation leads to

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{(12 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(0 - 80 \text{ m})} = -41.38 \text{ m/s}$$

which agrees with that calculated in (b).

50. The y coordinate of Apple 1 obeys $y - y_{o1} = -\frac{1}{2} g t^2$ where $y = 0$ when $t = 2.0$ s. This allows us to solve for y_{o1}, and we find $y_{o1} = 19.6$ m.

The graph for the coordinate of Apple 2 (which is thrown apparently at $t = 1.0$ s with velocity v_2) is

$$y - y_{o2} = v_2(t - 1.0) - \frac{1}{2} g (t - 1.0)^2$$

where $y_{o2} = y_{o1}$ = 19.6 m and where y = 0 when t = 2.25 s. Thus, we obtain $|v_2|$ = 9.6 m/s, approximately.

51. (a) With upward chosen as the $+y$ direction, we use Eq. 2-11 to find the initial velocity of the package:

$$v = v_o + at \quad \Rightarrow \quad 0 = v_o - (9.8 \text{ m/s}^2)(2.0 \text{ s})$$

which leads to v_o = 19.6 m/s. Now we use Eq. 2-15:

$$\Delta y = (19.6 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(2.0 \text{ s})^2 \approx 20 \text{ m} .$$

We note that the "2.0 s" in this second computation refers to the time interval $2 < t < 4$ in the graph (whereas the "2.0 s" in the first computation referred to the $0 < t < 2$ time interval shown in the graph).

(b) In our computation for part (b), the time interval ("6.0 s") refers to the $2 < t < 8$ portion of the graph:

$$\Delta y = (19.6 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(6.0 \text{ s})^2 \approx -59 \text{ m} ,$$

or $|\Delta y| = 59$ m.

52. The full extent of the bolt's fall is given by

$$y - y_0 = -\frac{1}{2} g t^2$$

where $y - y_0$ = –90 m (if upward is chosen as the positive y direction). Thus the time for the full fall is found to be t = 4.29 s. The first 80% of its free-fall distance is given by $-72 = -g\tau^2/2$, which requires time τ = 3.83 s.

(a) Thus, the final 20% of its fall takes $t - \tau$ = 0.45 s.

(b) We can find that speed using $v = -g\tau$. Therefore, $|v|$ = 38 m/s, approximately.

(c) Similarly, $v_{final} = -g t \quad \Rightarrow \quad |v_{final}|$ = 42 m/s.

53. **THINK** This problem involves two objects: a key dropped from a bridge, and a boat moving at a constant speed. We look for conditions such that the key will fall into the boat.

EXPRESS The speed of the boat is constant, given by $v_b = d/t$, where d is the distance of the boat from the bridge when the key is dropped (12 m) and t is the time the key takes in falling.

To calculate t, we take the time to be zero at the instant the key is dropped, we compute the time t when $y = 0$ using $y = y_0 + v_0 t - \frac{1}{2}gt^2$, with $y_0 = 45$ m. Once t is known, the speed of the boat can be readily calculated.

ANALYZE Since the initial velocity of the key is zero, the coordinate of the key is given by $y_0 = \frac{1}{2}gt^2$. Thus, the time it takes for the key to drop into the boat is

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(45\text{ m})}{9.8\text{ m/s}^2}} = 3.03\text{ s}.$$

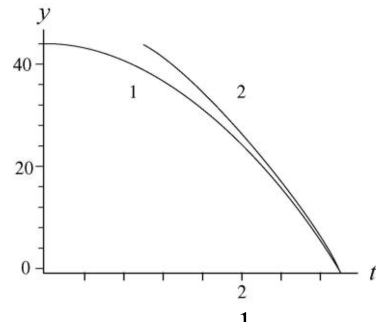
Therefore, the speed of the boat is $v_b = \dfrac{12\text{ m}}{3.03\text{ s}} = 4.0$ m/s.

LEARN From the general expression $v_b = \dfrac{d}{t} = \dfrac{d}{\sqrt{2y_0/g}} = d\sqrt{\dfrac{g}{2y_0}}$, we see that $v_b \sim 1/\sqrt{y_0}$. This agrees with our intuition that the lower the height from which the key is dropped, the greater the speed of the boat in order to catch it.

54. (a) We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with Δy replacing Δx) because this is constant acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity $-v_0$, so that v_0 is being used for the initial *speed*). SI units are used throughout.

$$\Delta y' = 0(t) - \frac{1}{2}gt^2$$
$$\Delta y = (-v_0)(t-1) - \frac{1}{2}g(t-1)^2$$

Since the problem indicates $\Delta y' = \Delta y = -43.9$ m, we solve the first equation for t (finding $t = 2.99$ s) and use this result to solve the second equation for the initial speed of the second stone:



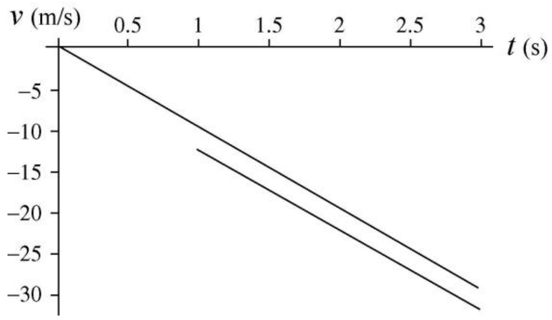
$$-43.9\text{ m} = (-v_0)(1.99\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(1.99\text{ s})^2$$

which leads to $v_0 = 12.3$ m/s.

(b) The velocity of the stones are given by

$$v_y' = \frac{d(\Delta y')}{dt} = -gt, \qquad v_y = \frac{d(\Delta y)}{dt} = -v_0 - g(t-1)$$

The plot is shown below:



55. **THINK** The free-falling moist-clay ball strikes the ground with a non-zero speed, and it undergoes deceleration before coming to rest.

EXPRESS During contact with the ground its average acceleration is given by $a_{\text{avg}} = \frac{\Delta v}{\Delta t}$, where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0\times10^{-3}$ s is the duration of contact. Thus, we must first find the velocity of the ball just before it hits the ground (y = 0).

ANALYZE (a) Now, to find the velocity just *before* contact, we take t = 0 to be when it is dropped. Using Eq. 2-16 with $y_0 = 15.0$ m, we obtain

$$v = -\sqrt{v_0^2 - 2g(y-y_0)} = -\sqrt{0-2(9.8\text{ m/s}^2)(0-15\text{ m})} = -17.15\text{ m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{0-(-17.1\text{ m/s})}{20.0\times10^{-3}\text{ s}} = 857\text{ m/s}^2.$$

(b) The fact that the result is positive indicates that this acceleration vector points upward.

LEARN Since Δt is very small, it is not surprising to have a very large acceleration to stop the motion of the ball. In later chapters, we shall see that the acceleration is directly related to the magnitude and direction of the force exerted by the ground on the ball during the course of collision.

56. We use Eq. 2-16,

$$v_B^2 = v_A^2 + 2a(y_B - y_A),$$

with a = –9.8 m/s^2, $y_B - y_A$ = 0.40 m, and $v_B = \frac{1}{3} v_A$. It is then straightforward to solve: v_A = 3.0 m/s, approximately.

57. The average acceleration during contact with the floor is $a_{\text{avg}} = (v_2 - v_1) / \Delta t$,

where v_1 is its velocity just before striking the floor, v_2 is its velocity just as it leaves the floor, and Δt is the duration of contact with the floor (12×10^{-3} s).

(a) Taking the y axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using $v_1^2 = v_0^2 - 2gy$. With $v_0 = 0$ and $y = -4.00$ m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8\text{ m/s}^2)(-4.00\text{ m})} = -8.85\text{ m/s}$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use $v^2 = v_2^2 - 2g(y - y_0)$ with $v = 0$, $y = -2.00$ m (it ends up two meters *below* its initial drop height), and $y_0 = -4.00$ m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8\text{ m/s}^2)(-2.00\text{ m} + 4.00\text{ m})} = 6.26\text{ m/s}.$$

Consequently, the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26\text{ m/s} - (-8.85\text{ m/s})}{12.0 \times 10^{-3}\text{ s}} = 1.26 \times 10^3\text{ m/s}^2.$$

(b) The positive nature of the result indicates that the acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

58. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as $t - t'$ where t is the value of time when it lands and t' is one second prior to that. The corresponding distance is $y - y' = 0.50h$, where y denotes the location of the ground. In these terms, y is the same as h, so we have $h - y' = 0.50h$ or $0.50h = y'$.

(a) We find t' and t from Eq. 2-15 (with $v_0 = 0$):

$$y' = \frac{1}{2}gt'^2 \Rightarrow t' = \sqrt{\frac{2y'}{g}}$$

$$y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}}.$$

Plugging in $y = h$ and $y' = 0.50h$, and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.50h)/g}{2h/g}} = \sqrt{0.50}.$$

Letting $t' = t - 1.00$ (SI units understood) and cross-multiplying, we find

$$t - 1.00 = t\sqrt{0.50} \Rightarrow t = \frac{1.00}{1 - \sqrt{0.50}}$$

which yields t = 3.41 s.

(b) Plugging this result into $y = \frac{1}{2}gt^2$ we find h = 57 m.

(c) In our approach, we did not use the quadratic formula, but we did "choose a root" when we assumed (in the last calculation in part (a)) that $\sqrt{0.50}$ = +0.707 instead of –0.707. If we had instead let $\sqrt{0.50}$ = –0.707 then our answer for t would have been roughly 0.6 s, which would imply that $t' = t - 1$ would equal a negative number (indicating a time *before* it was dropped), which certainly does not fit with the physical situation described in the problem.

59. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y-axis.

(a) The time drop 1 leaves the nozzle is taken as t = 0 and its time of landing on the floor t_1 can be computed from Eq. 2-15, with v_0 = 0 and y_1 = –2.00 m.

$$y_1 = -\frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00 \text{ m})}{9.8 \text{ m/s}^2}} = 0.639 \text{ s}.$$

At that moment, the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at t = 0.639/3 = 0.213 s and drop 3 leaves the nozzle at t = 2(0.213 s) = 0.426 s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is $t_2 = t_1 - 0.213$ s = 0.426 s. Its position at the moment drop 1 strikes the floor is

$$y_2 = -\frac{1}{2}gt_2^2 = -\frac{1}{2}(9.8 \text{ m/s}^2)(0.426 \text{ s})^2 = -0.889 \text{ m},$$

or about 89 cm below the nozzle.

(b) The time in free fall (up to the moment drop 1 lands) for drop 3 is $t_3 = t_1 - 0.426$ s = 0.213 s. Its position at the moment drop 1 strikes the floor is

$$y_3 = -\frac{1}{2}gt_3^2 = -\frac{1}{2}(9.8 \text{ m/s}^2)(0.213 \text{ s})^2 = -0.222 \text{ m},$$

or about 22 cm below the nozzle.

60. To find the "launch" velocity of the rock, we apply Eq. 2-11 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \quad \Rightarrow \quad 0 = v_0 - \left(9.8\ \text{m/s}^2\right)\left(2.5\ \text{s}\right)$$

so that v_0 = 24.5 m/s (with $+y$ up). Now we use Eq. 2-15 to find the height of the tower (taking y_0 = 0 at the ground level)

$$y - y_0 = v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y - 0 = \left(24.5\ \text{m/s}\right)\left(1.5\ \text{s}\right) - \frac{1}{2}\left(9.8\ \text{m/s}^2\right)\left(1.5\ \text{s}\right)^2.$$

Thus, we obtain y = 26 m.

61. We choose *down* as the $+y$ direction and place the coordinate origin at the top of the building (which has height H). During its fall, the ball passes (with velocity v_1) the top of the window (which is at y_1) at time t_1, and passes the bottom (which is at y_2) at time t_2. We are told $y_2 - y_1$ = 1.20 m and $t_2 - t_1$ = 0.125 s. Using Eq. 2-15 we have

$$y_2 - y_1 = v_1\left(t_2 - t_1\right) + \frac{1}{2}g\left(t_2 - t_1\right)^2$$

which immediately yields

$$v_1 = \frac{1.20\ \text{m} - \frac{1}{2}\left(9.8\ \text{m/s}^2\right)\left(0.125\ \text{s}\right)^2}{0.125\ \text{s}} = 8.99\ \text{m/s}.$$

From this, Eq. 2-16 (with v_0 = 0) reveals the value of y_1:

$$v_1^2 = 2gy_1 \quad \Rightarrow \quad y_1 = \frac{(8.99\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 4.12\ \text{m}.$$

It reaches the ground ($y_3 = H$) at t_3. Because of the symmetry expressed in the problem ("upward flight is a reverse of the fall'') we know that $t_3 - t_2$ = 2.00/2 = 1.00 s. And this means $t_3 - t_1$ = 1.00 s + 0.125 s = 1.125 s. Now Eq. 2-15 produces

$$y_3 - y_1 = v_1(t_3 - t_1) + \frac{1}{2}g(t_3 - t_1)^2$$

$$y_3 - 4.12\ \text{m} = (8.99\ \text{m/s})(1.125\ \text{s}) + \frac{1}{2}(9.8\ \text{m/s}^2)(1.125\ \text{s})^2$$

which yields $y_3 = H$ = 20.4 m.

62. The height reached by the player is y = 0.76 m (where we have taken the origin of the y axis at the floor and $+y$ to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8\ \text{m/s}^2)(0.76\ \text{m})} = 3.86\ \text{m/s}.$$

This is a consequence of Eq. 2-16 where velocity v vanishes. As the player reaches y_1

= 0.76 m – 0.15 m = 0.61 m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(0.61 \text{ m})} = 1.71 \text{ m/s} .$$

The time t_1 that the player spends *ascending* in the top Δy_1 = 0.15 m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2}(v_1 + v)t_1 \Rightarrow t_1 = \frac{2(0.15 \text{ m})}{1.71 \text{ m/s} + 0} = 0.175 \text{ s}$$

which means that the total time spent in that top 15 cm (both ascending and descending) is 2(0.175 s) = 0.35 s = 350 ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15 \text{ m} = v_0 t_2 - \frac{1}{2} g t_2^2 = (3.86 \text{ m/s}) t_2 - \frac{1}{2}(9.8 \text{ m/s}^2) t_2^2 ,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) t_2 = 0.041 s = 41 ms, which implies that the total time spent in that bottom 15 cm (both ascending and descending) is 2(41 ms) = 82 ms.

63. The time t the pot spends passing in front of the window of length L = 2.0 m is 0.25 s each way. We use v for its velocity as it passes the top of the window (going up). Then, with $a = -g = -9.8$ m/s^2 (taking *down* to be the $-y$ direction), Eq. 2-18 yields

$$L = vt - \frac{1}{2}gt^2 \quad \Rightarrow \quad v = \frac{L}{t} - \frac{1}{2}gt.$$

The distance H the pot goes above the top of the window is therefore (using Eq. 2-16 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{(L/t - gt/2)^2}{2g} = \frac{(2.00 \text{ m}/0.25 \text{ s} - (9.80 \text{ m/s}^2)(0.25 \text{ s})/2)^2}{2(9.80 \text{ m/s}^2)} = 2.34 \text{ m}.$$

64. The graph shows y = 25 m to be the highest point (where the speed momentarily vanishes). The neglect of "air friction" (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

(a) To find the acceleration due to gravity g_p on that planet, we use Eq. 2-15 (with $+y$ up)

$$y - y_0 = vt + \frac{1}{2}g_p t^2 \quad \Rightarrow \quad 25 \text{ m} - 0 = (0)(2.5 \text{ s}) + \frac{1}{2}g_p (2.5 \text{ s})^2$$

so that g_p = 8.0 m/s^2.

(b) That same (max) point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2}(v_0 + v)t \quad \Rightarrow \quad 25\text{ m} - 0 = \frac{1}{2}\,(v_0 + 0)(2.5\text{ s})$$

Therefore, v_0 = 20 m/s.

65. The key idea here is that the speed of the head (and the torso as well) at any given time can be calculated by finding the area on the graph of the head's acceleration versus time, as shown in Eq. 2-26:

$$v_1 - v_0 = \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

(a) From Fig. 2.15a, we see that the head begins to accelerate from rest (v_0 = 0) at t_0 = 110 ms and reaches a maximum value of 90 m/s^2 at t_1 = 160 ms. The area of this region is

$$\text{area} = \frac{1}{2}(160 - 110) \times 10^{-3}\text{s} \cdot \left(90\text{ m/s}^2\right) = 2.25\text{ m/s}$$

which is equal to v_1, the speed at t_1.

(b) To compute the speed of the torso at t_1=160 ms, we divide the area into 4 regions: From 0 to 40 ms, region A has zero area. From 40 ms to 100 ms, region B has the shape of a triangle with area

$$\text{area}_\text{B} = \frac{1}{2}(0.0600\text{ s})(50.0\text{ m/s}^2) = 1.50\text{ m/s}\,.$$

From 100 to 120 ms, region C has the shape of a rectangle with area

$$\text{area}_\text{C} = (0.0200\text{ s})\,(50.0\text{ m/s}^2) = 1.00\text{ m/s}.$$

From 110 to 160 ms, region D has the shape of a trapezoid with area

$$\text{area}_\text{D} = \frac{1}{2}(0.0400\text{ s})\,(50.0 + 20.0)\text{ m/s}^2 = 1.40\text{ m/s}.$$

Substituting these values into Eq. 2-26, with v_0 = 0 then gives

$$v_1 - 0 = 0 + 1.50\text{ m/s} + 1.00\text{ m/s} + 1.40\text{ m/s} = 3.90\text{ m/s},$$

or $v_1 = 3.90$ m/s.

66. The key idea here is that the position of an object at any given time can be calculated by finding the area on the graph of the object's velocity versus time, as shown in Eq. 2-30:

$$x_1 - x_0 = \begin{pmatrix} \text{area between the velocity curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}.$$

(a) To compute the position of the fist at t = 50 ms, we divide the area in Fig. 2-37 into two regions. From 0 to 10 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.010\text{ s})\ (2\text{ m/s}) = 0.01\text{ m}.$$

From 10 to 50 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.040\text{ s})\ (2 + 4)\text{ m/s} = 0.12\text{ m}.$$

Substituting these values into Eq. 2-30 with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01\text{ m} + 0.12\text{ m} = 0.13\text{ m},$$

or $x_1 = 0.13$ m.

(b) The speed of the fist reaches a maximum at t_1 = 120 ms. From 50 to 90 ms, region *C* has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.040\text{ s})\ (4 + 5)\text{ m/s} = 0.18\text{ m}.$$

From 90 to 120 ms, region D has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.030\text{ s})\ (5 + 7.5)\text{ m/s} = 0.19\text{ m}.$$

Substituting these values into Eq. 2-30, with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01\text{ m} + 0.12\text{ m} + 0.18\text{ m} + 0.19\text{ m} = 0.50\text{ m},$$

or $x_1 = 0.50$ m.

67. The problem is solved using Eq. 2-31:

$$v_1 - v_0 = \left(\begin{array}{c}\text{area between the acceleration curve}\\ \text{and the time axis, from } t_0 \text{ to } t_1\end{array}\right)$$

To compute the speed of the unhelmeted, bare head at t_1 = 7.0 ms, we divide the area under the *a* vs. *t* graph into 4 regions: From 0 to 2 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0020\text{ s})\ (120\text{ m/s}^2) = 0.12\text{ m/s}.$$

From 2 ms to 4 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.0020\text{ s})\ (120 + 140)\text{ m/s}^2 = 0.26\text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s}) (140 + 200) \text{ m/s}^2 = 0.34 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2}(0.0010 \text{ s}) (200 \text{ m/s}^2) = 0.10 \text{ m/s}.$$

Substituting these values into Eq. 2-31, with v_0=0 then gives

$$v_{\text{unhelmeted}} = 0.12 \text{ m/s} + 0.26 \text{ m/s} + 0.34 \text{ m/s} + 0.10 \text{ m/s} = 0.82 \text{ m/s}.$$

Carrying out similar calculations for the helmeted head, we have the following results: From 0 to 3 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0030 \text{ s}) (40 \text{ m/s}^2) = 0.060 \text{ m/s}.$$

From 3 ms to 4 ms, region B has the shape of a rectangle with area

$$\text{area}_B = (0.0010 \text{ s}) (40 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s}) (40 + 80) \text{ m/s}^2 = 0.12 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2}(0.0010 \text{ s}) (80 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

Substituting these values into Eq. 2-31, with $v_0 = 0$ then gives

$$v_{\text{helmeted}} = 0.060 \text{ m/s} + 0.040 \text{ m/s} + 0.12 \text{ m/s} + 0.040 \text{ m/s} = 0.26 \text{ m/s}.$$

Thus, the difference in the speed is

$$\Delta v = v_{\text{unhelmeted}} - v_{\text{helmeted}} = 0.82 \text{ m/s} - 0.26 \text{ m/s} = 0.56 \text{ m/s}.$$

68. This problem can be solved by noting that velocity can be determined by the graphical integration of acceleration versus time. The speed of the tongue of the salamander is simply equal to the area under the acceleration curve:

$$v = \text{area} = \frac{1}{2}(10^{-2} \text{ s})(100 \text{ m/s}^2) + \frac{1}{2}(10^{-2} \text{ s})(100 \text{ m/s}^2 + 400 \text{ m/s}^2) + \frac{1}{2}(10^{-2} \text{ s})(400 \text{ m/s}^2)$$
$$= 5.0 \text{ m/s}.$$

69. Since $v = dx/dt$ (Eq. 2-4), then $\Delta x = \int v\,dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base × height) and triangular $\left(\frac{1}{2}\text{ base} \times \text{height}\right)$ areas, we have

$$\begin{aligned} A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left((2)(4) + \frac{1}{2}(2)(4)\right) + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain Δx = 100 m.

70. To solve this problem, we note that velocity is equal to the time derivative of a position function, as well as the time integral of an acceleration function, with the integration constant being the initial velocity. Thus, the velocity of particle 1 can be written as

$$v_1 = \frac{dx_1}{dt} = \frac{d}{dt}\left(6.00t^2 + 3.00t + 2.00\right) = 12.0t + 3.00 .$$

Similarly, the velocity of particle 2 is

$$v_2 = v_{20} + \int a_2 dt = 20.0 + \int (-8.00t)dt = 20.0 - 4.00t^2.$$

The condition that $v_1 = v_2$ implies

$$12.0t + 3.00 = 20.0 - 4.00t^2 \quad \Rightarrow \quad 4.00t^2 + 12.0t - 17.0 = 0$$

which can be solved to give (taking positive root) $t = (-3 + \sqrt{26})/2 = 1.05$ s. Thus, the velocity at this time is $v_1 = v_2 = 12.0(1.05) + 3.00 = 15.6$ m/s.

71. (a) The derivative (with respect to time) of the given expression for x yields the "velocity" of the spot:

$$v(t) = 9 - \tfrac{9}{4}t^2$$

with 3 significant figures understood. It is easy to see that $v = 0$ when $t = 2.00$ s.

(b) At $t = 2$ s, $x = 9(2) - ¾(2)^3 = 12$. Thus, the location of the spot when $v = 0$ is 12.0 cm from left edge of screen.

(c) The derivative of the velocity is $a = -\tfrac{9}{2}t$, which gives an acceleration of -9.00 cm/m^2 (negative sign indicating leftward) when the spot is 12 cm from the left edge of screen.

(d) Since $v > 0$ for times less than $t = 2$ s, then the spot had been moving rightward.

(e) As implied by our answer to part (c), it moves leftward for times immediately after $t = 2$ s. In fact, the expression found in part (a) guarantees that for all $t > 2$, $v < 0$ (that is, until the clock is "reset" by reaching an edge).

(f) As the discussion in part (e) shows, the edge that it reaches at some $t > 2$ s cannot be the right edge; it is the left edge ($x = 0$). Solving the expression given in the problem statement (with $x = 0$) for positive t yields the answer: the spot reaches the left edge at $t = \sqrt{12}$ s ≈ 3.46 s.

72. We adopt the convention frequently used in the text: that "up" is the positive y direction.

(a) At the highest point in the trajectory $v = 0$. Thus, with $t = 1.60$ s, the equation $v = v_0 - gt$ yields $v_0 = 15.7$ m/s.

(b) One equation that is not dependent on our result from part (a) is $y - y_0 = vt + \frac{1}{2}gt^2$; this readily gives $y_{max} - y_0 = 12.5$ m for the highest ("max") point measured relative to where it started (the top of the building).

(c) Now we use our result from part (a) and plug into $y - y_0 = v_0t + \frac{1}{2}gt^2$ with $t = 6.00$ s and $y = 0$ (the ground level). Thus, we have

$$0 - y_0 = (15.68 \text{ m/s})(6.00 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(6.00 \text{ s})^2.$$

Therefore, y_0 (the height of the building) is equal to 82.3 m.

73. We denote the required time as t, assuming the light turns green when the clock reads zero. By this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as a and the (constant) speed of the truck as v then

$$\Delta x = \left(\frac{1}{2}at^2\right)_{\text{car}} = \left(vt\right)_{\text{truck}}$$

which leads to

$$t = \frac{2v}{a} = \frac{2(9.5 \text{ m/s})}{2.2 \text{ m/s}^2} = 8.6 \text{ s}.$$

Therefore,

$$\Delta x = vt = (9.5 \text{ m/s})(8.6 \text{ s}) = 82 \text{ m}.$$

(b) The speed of the car at that moment is

$$v_{\text{car}} = at = (2.2 \text{ m/s}^2)(8.6 \text{ s}) = 19 \text{ m/s}.$$

74. If the plane (with velocity v) maintains its present course, and if the terrain continues its upward slope of 4.3°, then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan\theta} = \frac{35 \text{ m}}{\tan 4.3°} = 465.5 \text{ m} \approx 0.465 \text{ km}.$$

This corresponds to a time of flight found from Eq. 2-2 (with $v = v_{avg}$ since it is constant)

$$t = \frac{\Delta x}{v} = \frac{0.465 \text{ km}}{1300 \text{ km/h}} = 0.000358 \text{ h} \approx 1.3 \text{ s}.$$

This, then, estimates the time available to the pilot to make his correction.

75. We denote t_r as the reaction time and t_b as the braking time. The motion during t_r is of the constant-velocity (call it v_0) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2} a t_b^2$$

where v_0 is the initial velocity and a is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied the velocity of the car is given by $v = v_0 + at_b$. Using this equation, with $v = 0$, we eliminate t_b from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2}\frac{v_0^2}{a} = v_0 t_r - \frac{1}{2}\frac{v_0^2}{a}.$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2}\frac{v_{01}^2}{a}, \qquad x_2 = v_{02} t_r - \frac{1}{2}\frac{v_{02}^2}{a}.$$

Solving these equations simultaneously for t_r and a we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02}\left(v_{02} - v_{01}\right)}$$

and

$$a = -\frac{1}{2}\frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2}.$$

(a) Substituting x_1 = 56.7 m, v_{01} = 80.5 km/h = 22.4 m/s, x_2 = 24.4 m and v_{02} = 48.3 km/h = 13.4 m/s, we find

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02}(v_{02} - v_{01})} = \frac{(13.4 \text{ m/s})^2 (56.7 \text{ m}) - (22.4 \text{ m/s})^2 (24.4 \text{ m})}{(22.4 \text{ m/s})(13.4 \text{ m/s})(13.4 \text{ m/s} - 22.4 \text{ m/s})}$$
$$= 0.74 \text{ s}.$$

(b) Similarly, substituting x_1 = 56.7 m, v_{01} = 80.5 km/h = 22.4 m/s, x_2 = 24.4 m, and

v_{02} = 48.3 km/h = 13.4 m/s gives

$$a = -\frac{1}{2}\frac{v_{02}v_{01}^2 - v_{01}v_{02}^2}{v_{02}x_1 - v_{01}x_2} = -\frac{1}{2}\frac{(13.4\text{ m/s})(22.4\text{ m/s})^2 - (22.4\text{ m/s})(13.4\text{ m/s})^2}{(13.4\text{ m/s})(56.7\text{ m}) - (22.4\text{ m/s})(24.4\text{ m})}$$
$$= -6.2\text{ m/s}^2.$$

The *magnitude* of the deceleration is therefore 6.2 m/s^2. Although rounded-off values are displayed in the above substitutions, what we have input into our calculators are the "exact" values (such as $v_{02} = \frac{161}{12}$ m/s).

76. (a) A constant velocity is equal to the ratio of displacement to elapsed time. Thus, for the vehicle to be traveling at a constant speed v_p over a distance D_{23}, the time delay should be $t = D_{23}/v_p$.

(b) The time required for the car to accelerate from rest to a cruising speed v_p is $t_0 = v_p/a$. During this time interval, the distance traveled is $\Delta x_0 = at_0^2/2 = v_p^2/2a$. The car then moves at a constant speed v_p over a distance $D_{12} - \Delta x_0 - d$ to reach intersection 2, and the time elapsed is $t_1 = (D_{12} - \Delta x_0 - d)/v_p$. Thus, the time delay at intersection 2 should be set to

$$t_{\text{total}} = t_r + t_0 + t_1 = t_r + \frac{v_p}{a} + \frac{D_{12} - \Delta x_0 - d}{v_p} = t_r + \frac{v_p}{a} + \frac{D_{12} - (v_p^2/2a) - d}{v_p}$$
$$= t_r + \frac{1}{2}\frac{v_p}{a} + \frac{D_{12} - d}{v_p}$$

77. **THINK** The speed of the rod changes due to a nonzero acceleration.

EXPRESS Since the problem involves constant acceleration, the motion of the rod can be readily analyzed using the equations given in Table 2-1. We take +*x* to be in the direction of motion, so

$$v = (60\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = +16.7\text{ m/s}$$

and $a > 0$. The location where the rod starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

ANALYZE (a) Using Eq. 2-7, we find the average acceleration to be

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{16.7\text{ m/s} - 0}{5.4\text{ s} - 0} = 3.09\text{ m/s}^2.$$

(b) Assuming constant acceleration $a = a_{\text{avg}} = 3.09\text{ m/s}^2$, the total distance traveled during the 5.4-s time interval is

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(3.09 \text{ m/s}^2)(5.4 \text{ s})^2 = 45 \text{ m}$$

(c) Using Eq. 2-15, the time required to travel a distance of $x = 250$ m is:

$$x = \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250 \text{ m})}{3.1 \text{ m/s}^2}} = 12.73 \text{ s}$$

LEARN The displacement of the rod as a function of time can be written as $x(t) = \frac{1}{2}(3.09 \text{ m/s}^2)t^2$. Note that we could have chosen Eq. 2-17 to solve for (b):

$$x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(16.7 \text{ m/s})(5.4 \text{ s}) = 45 \text{ m}.$$

78. We take the moment of applying brakes to be $t = 0$. The deceleration is constant so that Table 2-1 can be used. Our primed variables (such as $v_0' = 72 \text{ km/h} = 20 \text{ m/s}$) refer to one train (moving in the $+x$ direction and located at the origin when $t = 0$) and unprimed variables refer to the other (moving in the $–x$ direction and located at x_0 = +950 m when $t = 0$). We note that the acceleration vector of the unprimed train points in the *positive* direction, even though the train is slowing down; its initial velocity is v_0 = –144 km/h = –40 m/s. Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using Eq 2-16, it should stop (meaning $v' = 0$) at

$$x' = \frac{(v')^2 - (v_0')^2}{2a'} = \frac{0 - (20 \text{ m/s})^2}{-2 \text{ m/s}^2} = 200 \text{ m} .$$

The speed of the other train, when it reaches that location, is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{(-40 \text{ m/s})^2 + 2(1.0 \text{ m/s}^2)(200 \text{ m} - 950 \text{ m})}$$
$$= 10 \text{ m/s}$$

using Eq 2-16 again. Specifically, its velocity at that moment would be –10 m/s since it is still traveling in the $–x$ direction when it crashes. If the computation of v had failed (meaning that a negative number would have been inside the square root) then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the time it stops (Eq. 2-11 yields $t = 20$ s) and seeing where the unprimed train is at that moment (Eq. 2-18 yields $x = 350$ m, still a good distance away from contact).

79. The y coordinate of Piton 1 obeys $y - y_{01} = -\frac{1}{2}g t^2$ where $y = 0$ when $t = 3.0$ s. This allows us to solve for y_{o1}, and we find $y_{01} = 44.1$ m. The graph for the coordinate of Piton 2 (which is thrown apparently at $t = 1.0$ s with velocity v_1) is

$$y - y_{02} = v_1(t-1.0) - \tfrac{1}{2}\, g\, (t - 1.0)^2$$

where $y_{02} = y_{01} + 10 = 54.1$ m and where (again) $y = 0$ when $t = 3.0$ s. Thus we obtain $|v_1| = 17$ m/s, approximately.

80. We take $+x$ in the direction of motion. We use subscripts 1 and 2 for the data. Thus, $v_1 = +30$ m/s, $v_2 = +50$ m/s, and $x_2 - x_1 = +160$ m.

(a) Using these subscripts, Eq. 2-16 leads to

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{(50\text{ m/s})^2 - (30\text{ m/s})^2}{2(160\text{ m})} = 5.0\text{ m/s}^2 .$$

(b) We find the time interval corresponding to the displacement $x_2 - x_1$ using Eq. 2-17:

$$t_2 - t_1 = \frac{2(x_2 - x_1)}{v_1 + v_2} = \frac{2(160\text{ m})}{30\text{ m/s} + 50\text{ m/s}} = 4.0\text{ s} .$$

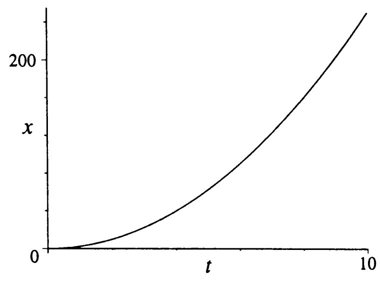
(c) Since the train is at rest ($v_0 = 0$) when the clock starts, we find the value of t_1 from Eq. 2-11:

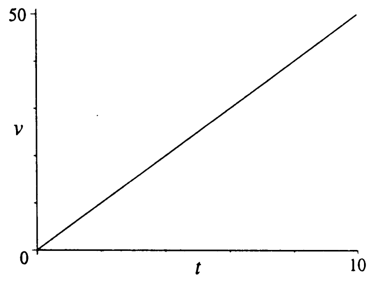
$$v_1 = v_0 + at_1 \;\Rightarrow\; t_1 = \frac{30\text{ m/s}}{5.0\text{ m/s}^2} = 6.0\text{ s} .$$

(d) The coordinate origin is taken to be the location at which the train was initially at rest (so $x_0 = 0$). Thus, we are asked to find the value of x_1. Although any of several equations could be used, we choose Eq. 2-17:

$$x_1 = \frac{1}{2}(v_0 + v_1)t_1 = \frac{1}{2}(30\text{ m/s})(6.0\text{ s}) = 90\text{ m} .$$

(e) The graphs are shown below, with SI units understood.





81. **THINK** The particle undergoes a *non-constant* acceleration along the $+x$-axis. An integration is required to calculate velocity.

EXPRESS With a non-constant acceleration $a(t) = dv/dt$, the velocity of the particle at time t_1 is given by Eq. 2-27: $v_1 = v_0 + \int_{t_0}^{t_1} a(t)dt$, where v_0 is the velocity at time t_0. In our situation, we have $a = 5.0t$. In addition, we also know that $v_0 = 17$ m/s at $t_0 = 2.0$ s.

ANALYZE Integrating (from t = 2 s to variable t = 4 s) the acceleration to get the velocity and using the values given in the problem, leads to

$$v = v_0 + \int_{t_0}^{t} a dt = v_0 + \int_{t_0}^{t} (5.0t)dt = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \tfrac{1}{2}(5.0)(4^2 - 2^2) = 47 \text{ m/s}.$$

LEARN The velocity of the particle as a function of t is

$$v(t) = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \frac{1}{2}(5.0)(t^2 - 4) = 7 + 2.5t^2$$

in SI units (m/s). Since the acceleration is linear in t, we expect the velocity to be quadratic in t, and the displacement to be cubic in t.

82. The velocity v at t = 6 (SI units and two significant figures understood) is $v_{\text{given}} + \int_{-2}^{6} a dt$. A quick way to implement this is to recall the area of a triangle ($\frac{1}{2}$ base × height). The result is v = 7 m/s + 32 m/s = 39 m/s.

83. The object, once it is dropped (v_0 = 0) is in free fall ($a = -g = -9.8$ m/s^2 if we take *down* as the $-y$ direction), and we use Eq. 2-15 repeatedly.

(a) The (positive) distance D from the lower dot to the mark corresponding to a certain reaction time t is given by $\Delta y = -D = -\frac{1}{2}gt^2$, or $D = gt^2/2$. Thus, for $t_1 = 50.0$ ms,

$$D_1 = \frac{(9.8 \text{ m/s}^2)(50.0 \times 10^{-3} \text{ s})^2}{2} = 0.0123 \text{ m} = 1.23 \text{ cm}.$$

(b) For t_2 = 100 ms, $D_2 = \dfrac{(9.8 \text{ m/s}^2)(100 \times 10^{-3} \text{ s})^2}{2} = 0.049 \text{ m} = 4D_1.$

(c) For t_3 = 150 ms, $D_3 = \dfrac{(9.8 \text{ m/s}^2)(150 \times 10^{-3} \text{ s})^2}{2} = 0.11 \text{ m} = 9D_1.$

(d) For t_4 = 200 ms, $D_4 = \dfrac{(9.8 \text{ m/s}^2)(200 \times 10^{-3} \text{ s})^2}{2} = 0.196 \text{ m} = 16D_1.$

(e) For t_4 = 250 ms, $D_5 = \dfrac{(9.8 \text{ m/s}^2)(250 \times 10^{-3} \text{ s})^2}{2} = 0.306 \text{ m} = 25D_1.$

84. We take the direction of motion as $+x$, take $x_0 = 0$ and use SI units, so $v = 1600(1000/3600) = 444$ m/s.

(a) Equation 2-11 gives $444 = a(1.8)$ or $a = 247\ \text{m/s}^2$. We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{247\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) g = 25g.$$

(b) Equation 2-17 readily yields

$$x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(444\ \text{m/s})(1.8\ \text{s}) = 400\ \text{m}.$$

85. Let D be the distance up the hill. Then

$$\text{average speed} = \frac{\text{total distance traveled}}{\text{total time of travel}} = \frac{2D}{\dfrac{D}{20\ \text{km/h}} + \dfrac{D}{35\ \text{km/h}}} \approx 25\ \text{km/h}\ .$$

86. We obtain the velocity by integration of the acceleration:

$$v - v_0 = \int_0^t (6.1 - 1.2t')dt'\ .$$

Lengths are in meters and times are in seconds. The student is encouraged to look at the discussion in Section 2-7 to better understand the manipulations here.

(a) The result of the above calculation is $v = v_0 + 6.1t - 0.6t^2$, where the problem states that $v_0 = 2.7$ m/s. The maximum of this function is found by knowing when its derivative (the acceleration) is zero ($a = 0$ when $t = 6.1/1.2 = 5.1$ s) and plugging that value of t into the velocity equation above. Thus, we find $v = 18$ m/s.

(b) We integrate again to find x as a function of t:

$$x - x_0 = \int_0^t v\,dt' = \int_0^t (v_0 + 6.1t' - 0.6t'^2)\,dt' = v_0 t + 3.05t^2 - 0.2t^3\ .$$

With $x_0 = 7.3$ m, we obtain $x = 83$ m for $t = 6$. This is the correct answer, but one has the right to worry that it might not be; after all, the problem asks for the total distance traveled (and $x - x_0$ is just the *displacement*). If the cyclist backtracked, then his total distance would be greater than his displacement. Thus, we might ask, "did he backtrack?" To do so would require that his velocity be (momentarily) zero at some point (as he reversed his direction of motion). We could solve the above quadratic equation for velocity, for a positive value of t where $v = 0$; if we did, we would find that at $t = 10.6$ s, a reversal does indeed happen. However, in the time interval we are concerned with in our problem ($0 \le t \le 6$ s), there is no reversal and the displacement is the same as the total distance traveled.

87. **THINK** In this problem we're given two different speeds, and asked to find the difference in their travel times.

EXPRESS The time is takes to travel a distance d with a speed v_1 is $t_1 = d/v_1$. Similarly, with a speed v_2 the time would be $t_2 = d/v_2$. The two speeds in this problem are

$$v_1 = 55 \text{ mi/h} = (55 \text{ mi/h})\frac{1609 \text{ m/mi}}{3600 \text{ s/h}} = 24.58 \text{ m/s}$$

$$v_2 = 65 \text{ mi/h} = (65 \text{ mi/h})\frac{1609 \text{ m/mi}}{3600 \text{ s/h}} = 29.05 \text{ m/s}$$

ANALYZE With $d = 700 \text{ km} = 7.0\times10^5 \text{ m}$, the time difference between the two is

$$\Delta t = t_1 - t_2 = d\left(\frac{1}{v_1} - \frac{1}{v_2}\right) = (7.0\times10^5 \text{ m})\left(\frac{1}{24.58 \text{ m/s}} - \frac{1}{29.05 \text{ m/s}}\right) = 4383 \text{ s}$$
$$= 73 \text{ min}$$

or about 1.2 h.

LEARN The travel time was reduced from 7.9 h to 6.9 h. Driving at higher speed (within the legal limit) reduces travel time.

88. The acceleration is constant and we may use the equations in Table 2-1.

(a) Taking the first point as coordinate origin and time to be zero when the car is there, we apply Eq. 2-17:

$$x = \frac{1}{2}\left(v + v_0\right)t = \frac{1}{2}\left(15.0 \text{ m/s} + v_0\right)\left(6.00 \text{ s}\right).$$

With x = 60.0 m (which takes the direction of motion as the $+x$ direction) we solve for the initial velocity: v_0 = 5.00 m/s.

(b) Substituting v = 15.0 m/s, v_0 = 5.00 m/s, and t = 6.00 s into $a = (v - v_0)/t$ (Eq. 2-11), we find a = 1.67 m/s^2.

(c) Substituting v = 0 in $v^2 = v_0^2 + 2ax$ and solving for x, we obtain

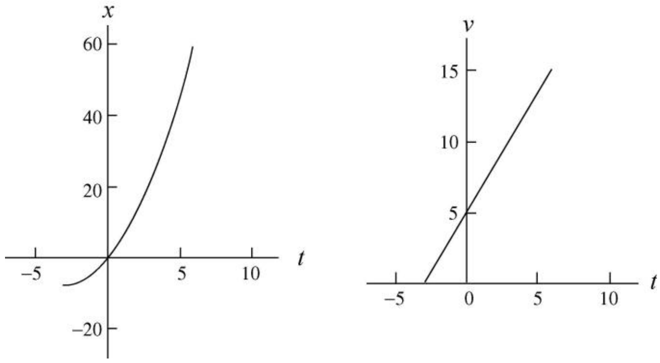
$$x = -\frac{v_0^2}{2a} = -\frac{(5.00 \text{ m/s})^2}{2\left(1.67 \text{ m/s}^2\right)} = -7.50 \text{m},$$

or $|x| = 7.50 \text{ m}$.

(d) The graphs require computing the time when v = 0, in which case, we use $v = v_0 + at' = 0$. Thus,

$$t' = \frac{-v_0}{a} = \frac{-5.00 \text{ m/s}}{1.67 \text{ m/s}^2} = -3.0 \text{s}$$

indicates the moment the car was at rest. SI units are understood.



89. **THINK** In this problem we explore the connection between the maximum height an object reaches under the influence of gravity and the total amount of time it stays in air.

EXPRESS Neglecting air resistance and setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion, we analyze the motion of the ball using Table 2-1 (with Δy replacing Δx). We set $y_0 = 0$. Upon reaching the maximum height H, the speed of the ball is momentarily zero ($v = 0$). Therefore, we can relate its initial speed v_0 to H via the equation

$$0 = v^2 = v_0^2 - 2gH \quad \Rightarrow \quad v_0 = \sqrt{2gH}\,.$$

The time it takes for the ball to reach maximum height is given by $v = v_0 - gt = 0$, or $t = v_0 / g = \sqrt{2H/g}$.

ANALYZE If we want the ball to spend twice as much time in air as before, i.e., $t' = 2t$, then the new maximum height H' it must reach is such that $t' = \sqrt{2H'/g}$. Solving for H' we obtain

$$H' = \frac{1}{2}gt'^2 = \frac{1}{2}g(2t)^2 = 4\left(\frac{1}{2}gt^2\right) = 4H\,.$$

LEARN Since $H \sim t^2$, doubling t means that H must increase fourfold. Note also that for $t' = 2t$, the initial speed must be twice the original speed: $v_0' = 2v_0$.

90. (a) Using the fact that the area of a triangle is $\frac{1}{2}$(base) (height) (and the fact that the integral corresponds to the area under the curve) we find, from $t = 0$ through $t = 5$ s, the integral of v with respect to t is 15 m. Since we are told that $x_0 = 0$ then we conclude that $x = 15$ m when $t = 5.0$ s.

(b) We see directly from the graph that $v = 2.0$ m/s when $t = 5.0$ s.

(c) Since $a = dv/dt$ = slope of the graph, we find that the acceleration during the interval $4 < t < 6$ is uniformly equal to -2.0 m/s^2.

(d) Thinking of $x(t)$ in terms of accumulated area (on the graph), we note that $x(1) = 1$ m; using this and the value found in part (a), Eq. 2-2 produces

$$v_{\text{avg}} = \frac{x(5) - x(1)}{5 - 1} = \frac{15 \text{ m} - 1 \text{ m}}{4 \text{ s}} = 3.5 \text{ m/s}.$$

(e) From Eq. 2-7 and the values $v(t)$ we read directly from the graph, we find

$$a_{\text{avg}} = \frac{v(5) - v(1)}{5 - 1} = \frac{2 \text{ m/s} - 2 \text{ m/s}}{4 \text{ s}} = 0.$$

91. Taking the $+y$ direction *downward* and $y_0 = 0$, we have $y = v_0 t + \frac{1}{2} g t^2$, which (with $v_0 = 0$) yields $t = \sqrt{2y/g}$.

(a) For this part of the motion, $y_1 = 50$ m so that $t_1 = \sqrt{\dfrac{2(50 \text{ m})}{9.8 \text{ m/s}^2}} = 3.2 \text{ s}$.

(b) For this next part of the motion, we note that the total displacement is $y_2 = 100$ m. Therefore, the total time is

$$t_2 = \sqrt{\frac{2(100 \text{ m})}{9.8 \text{ m/s}^2}} = 4.5 \text{ s}.$$

The difference between this and the answer to part (a) is the time required to fall through that second 50 m distance: $\Delta t = t_2 - t_1 = 4.5 \text{ s} - 3.2 \text{ s} = 1.3$ s.

92. Direction of $+x$ is implicit in the problem statement. The initial position (when the clock starts) is $x_0 = 0$ (where $v_0 = 0$), the end of the speeding-up motion occurs at $x_1 = 1100/2 = 550$ m, and the subway train comes to a halt ($v_2 = 0$) at $x_2 = 1100$ m.

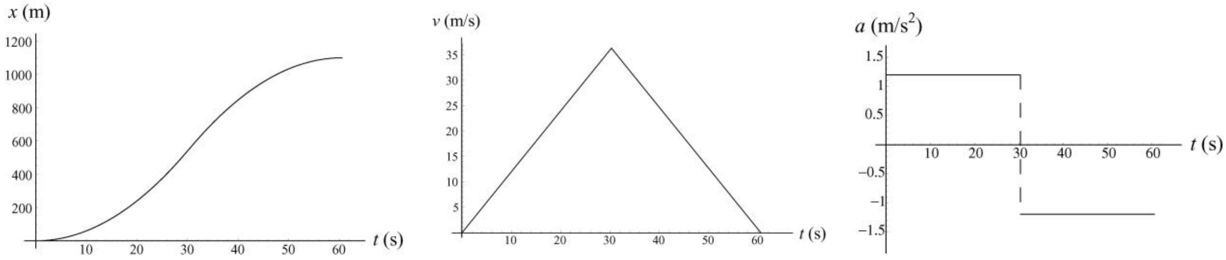
(a) Using Eq. 2-15, the subway train reaches x_1 at

$$t_1 = \sqrt{\frac{2x_1}{a_1}} = \sqrt{\frac{2(550 \text{ m})}{1.2 \text{ m/s}^2}} = 30.3 \text{ s}.$$

The time interval $t_2 - t_1$ turns out to be the same value (most easily seen using Eq. 2-18 so the total time is $t_2 = 2(30.3) = 60.6$ s.

(b) Its maximum speed occurs at t_1 and equals $v_1 = v_0 + a_1 t_1 = 36.3 \text{ m/s}$.

(c) The graphs are shown below:



93. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the stone's motion. We are allowed to use Table 2-1 (with Δx replaced by y) because the ball has constant acceleration motion (and we choose $y_0 = 0$).

(a) We apply Eq. 2-16 to both measurements, with SI units understood.

$$v_B^2 = v_0^2 - 2gy_B \;\Rightarrow\; \left(\frac{1}{2}v\right)^2 + 2g\left(y_A + 3\right) = v_0^2$$
$$v_A^2 = v_0^2 - 2gy_A \;\Rightarrow\; v^2 + 2gy_A = v_0^2$$

We equate the two expressions that each equal v_0^2 and obtain

$$\frac{1}{4}v^2 + 2gy_A + 2g(3) = v^2 + 2gy_A \;\Rightarrow\; 2g(3) = \frac{3}{4}v^2$$

which yields $v = \sqrt{2g(4)} = 8.85 \text{ m/s}$.

(b) An object moving upward at A with speed $v = 8.85$ m/s will reach a maximum height $y - y_A = v^2/2g = 4.00$ m above point A (this is again a consequence of Eq. 2-16, now with the "final" velocity set to zero to indicate the highest point). Thus, the top of its motion is 1.00 m above point B.

94. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y-axis. The total time of fall can be computed from Eq. 2-15 (using the quadratic formula).

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \;\Rightarrow\; t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root chosen. With $y = 0$, $v_0 = 0$, and $y_0 = h = 60$ m, we obtain

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5 \text{ s} .$$

Thus, "1.2 s earlier" means we are examining where the rock is at $t = 2.3$ s:

$$y - h = v_0(2.3\text{ s}) - \frac{1}{2}g(2.3\text{ s})^2 \;\Rightarrow\; y = 34 \text{ m}$$

where we again use the fact that $h = 60$ m and $v_0 = 0$.

95. **THINK** This problem involves analyzing a plot describing the position of an iceboat as function of time. The boat has a nonzero acceleration due to the wind.

EXPRESS Since we are told that the acceleration of the boat is constant, the equations of Table 2-1 can be applied. However, the challenge here is that v_0, v, and a are not explicitly given. Our strategy to deduce these values is to apply the kinematic equation $x - x_0 = v_0 t + \frac{1}{2}at^2$ to a variety of points on the graph and solve for the unknowns from the simultaneous equations.

ANALYZE (a) From the graph, we pick two points on the curve: $(t, x) = (2.0\text{ s}, 16\text{ m})$ and $(3.0\text{ s}, 27\text{ m})$. The corresponding simultaneous equations are

$$16\text{ m} - 0 = v_0(2.0\text{ s}) + \frac{1}{2}\, a(2.0\text{ s})^2$$

$$27\text{ m} - 0 = v_0(3.0\text{ s}) + \frac{1}{2}\, a(3.0\text{ s})^2$$

Solving the equations lead to the values $v_0 = 6.0$ m/s and $a = 2.0\text{ m/s}^2$.

(b) From Table 2-1,

$$x - x_0 = vt - \frac{1}{2}at^2 \quad \Rightarrow \quad 27\text{ m} - 0 = v(3.0\text{ s}) - \frac{1}{2}\,(2.0\text{ m/s}^2)(3.0\text{ s})^2$$

which leads to $v = 12$ m/s.

(c) Assuming the wind continues during $3.0 \le t \le 6.0$, we apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ to this interval (where $v_0 = 12.0$ m/s from part (b)) to obtain

$$\Delta x = (12.0\text{ m/s})(3.0\text{ s}) + \frac{1}{2}\,(2.0\text{ m/s}^2)(3.0\text{ s})^2 = 45\text{ m}.$$

LEARN By using the results obtained in (a), the position and velocity of the iceboat as a function of time can be written as

$$x(t) = (6.0\text{ m/s})t + \frac{1}{2}(2.0\text{ m/s}^2)t^2 \quad \text{and} \quad v(t) = (6.0\text{ m/s}) + (2.0\text{ m/s}^2)t.$$

One can readily verify that the same answers are obtained for (b) and (c) using the above expressions for $x(t)$ and $v(t)$.

96. (a) Let the height of the diving board be h. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). Thus, $y = h$ designates the location where the ball strikes the water. Let the depth of the lake be D, and the total time for the ball to descend be T. The speed of the ball as it reaches the surface of the lake is then $v = \sqrt{2gh}$ (from Eq. 2-16), and the time for the ball to fall from the board to the lake surface is $t_1 = \sqrt{2h/g}$ (from Eq. 2-15). Now, the time it spends descending in the lake (at constant velocity v) is

$$t_2 = \frac{D}{v} = \frac{D}{\sqrt{2gh}}\,.$$

Thus, $T = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{D}{\sqrt{2gh}}$, which gives

$$D = T\sqrt{2gh} - 2h = (4.80\text{ s})\sqrt{(2)(9.80\text{ m/s}^2)(5.20\text{ m})} - 2(5.20\text{ m}) = 38.1\text{ m}.$$

(b) Using Eq. 2-2, the magnitude of the average velocity is

$$v_{avg} = \frac{D+h}{T} = \frac{38.1\text{ m} + 5.20\text{ m}}{4.80\text{ s}} = 9.02\text{ m/s}$$

(c) In our coordinate choices, a positive sign for v_{avg} means that the ball is going downward. If, however, upward had been chosen as the positive direction, then this answer in (b) would turn out negative-valued.

(d) We find v_0 from $\Delta y = v_0 t + \frac{1}{2}gt^2$ with $t = T$ and $\Delta y = h + D$. Thus,

$$v_0 = \frac{h+D}{T} - \frac{gT}{2} = \frac{5.20\text{ m} + 38.1\text{ m}}{4.80\text{ s}} - \frac{(9.8\text{ m/s}^2)(4.80\text{ s})}{2} = 14.5\text{ m/s}$$

(e) Here in our coordinate choices the negative sign means that the ball is being thrown upward.

97. We choose *down* as the $+y$ direction and use the equations of Table 2-1 (replacing x with y) with $a = +g$, $v_0 = 0$, and $y_0 = 0$. We use subscript 2 for the elevator reaching the ground and 1 for the halfway point.

(a) Equation 2-16, $v_2^2 = v_0^2 + 2a(y_2 - y_0)$, leads to

$$v_2 = \sqrt{2gy_2} = \sqrt{2(9.8\text{ m/s}^2)(120\text{ m})} = 48.5\text{ m/s}.$$

(b) The time at which it strikes the ground is (using Eq. 2-15)

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2(120\text{ m})}{9.8\text{ m/s}^2}} = 4.95\text{ s}.$$

(c) Now Eq. 2-16, in the form $v_1^2 = v_0^2 + 2a(y_1 - y_0)$, leads to

$$v_1 = \sqrt{2gy_1} = \sqrt{2(9.8\text{ m/s}^2)(60\text{ m})} = 34.3\text{m/s}.$$

(d) The time at which it reaches the halfway point is (using Eq. 2-15)

$$t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(60 \text{ m})}{9.8 \text{ m/s}^2}} = 3.50 \text{ s} .$$

98. Taking $+y$ to be upward and placing the origin at the point from which the objects are dropped, then the location of diamond 1 is given by $y_1 = -\frac{1}{2}gt^2$ and the location of diamond 2 is given by $y_2 = -\frac{1}{2}g(t-1)^2$. We are starting the clock when the first object is dropped. We want the time for which $y_2 - y_1 = 10$ m. Therefore,

$$-\frac{1}{2}g(t-1)^2 + \frac{1}{2}gt^2 = 10 \quad \Rightarrow \quad t = (10/g) + 0.5 = 1.5 \text{ s}.$$

99. With $+y$ upward, we have $y_0 = 36.6$ m and $y = 12.2$ m. Therefore, using Eq. 2-18 (the last equation in Table 2-1), we find

$$y - y_0 = vt + \tfrac{1}{2}gt^2 \quad \Rightarrow \quad v = -22.0 \text{ m/s}$$

at $t = 2.00$ s. The term *speed* refers to the magnitude of the velocity vector, so the answer is $|v| = 22.0$ m/s.

100. During free fall, we ignore the air resistance and set $a = -g = -9.8$ m/s^2 where we are choosing *down* to be the $-y$ direction. The initial velocity is zero so that Eq. 2-15 becomes $\Delta y = -\frac{1}{2}gt^2$ where Δy represents the *negative* of the distance d she has fallen. Thus, we can write the equation as $d = \frac{1}{2}gt^2$ for simplicity.

(a) The time t_1 during which the parachutist is in free fall is (using Eq. 2-15) given by

$$d_1 = 50 \text{ m} = \frac{1}{2}gt_1^2 = \frac{1}{2}\left(9.80 \text{ m/s}^2\right)t_1^2$$

which yields $t_1 = 3.2$ s. The *speed* of the parachutist just before he opens the parachute is given by the positive root $v_1^2 = 2gd_1$, or

$$v_1 = \sqrt{2gh_1} = \sqrt{(2)\left(9.80 \text{ m/s}^2\right)(50 \text{ m})} = 31 \text{ m/s}.$$

If the final speed is v_2, then the time interval t_2 between the opening of the parachute and the arrival of the parachutist at the ground level is

$$t_2 = \frac{v_1 - v_2}{a} = \frac{31 \text{ m/s} - 3.0 \text{ m/s}}{2 \text{ m/s}^2} = 14 \text{ s}.$$

This is a result of Eq. 2-11 where *speeds* are used instead of the (negative-valued) velocities (so that final-velocity minus initial-velocity turns out to equal initial-speed minus final-speed); we also note that the acceleration vector for this part of the motion is positive since it points upward (opposite to the direction of motion — which makes it a deceleration). The total time of flight is therefore $t_1 + t_2 = 17$ s.

(b) The distance through which the parachutist falls after the parachute is opened is given by

$$d = \frac{v_1^2 - v_2^2}{2a} = \frac{(31\,\text{m/s})^2 - (3.0\,\text{m/s})^2}{(2)(2.0\,\text{m/s}^2)} \approx 240\,\text{m}.$$

In the computation, we have used Eq. 2-16 with both sides multiplied by –1 (which changes the negative-valued Δy into the positive d on the left-hand side, and switches the order of v_1 and v_2 on the right-hand side). Thus the fall begins at a height of $h = 50 + d \approx 290$ m.

101. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking down as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to $y = 0$.

(a) With $y_0 = h$ and v_0 replaced with $-v_0$, Eq. 2-16 leads to

$$v = \sqrt{(-v_0)^2 - 2g(y - y_0)} = \sqrt{v_0^2 + 2gh}\ .$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity).

(b) We use the quadratic formula to solve Eq. 2-15 for t, with v_0 replaced with $-v_0$,

$$\Delta y = -v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{-v_0 + \sqrt{(-v_0)^2 - 2g\Delta y}}{g}$$

where the positive root is chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, this becomes

$$t = \frac{\sqrt{v_0^2 + 2gh} - v_0}{g}.$$

(c) If it were thrown upward with that speed from height h then (in the absence of air friction) it would return to height h with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation).

(d) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having $+v_0$ in the equation where we had put in $-v_0$ in part (b). The details follow:

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root again chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, we obtain

$$t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g}.$$

102. We assume constant velocity motion and use Eq. 2-2 (with $v_{avg} = v > 0$). Therefore,

$$\Delta x = v\Delta t = \left(303\,\frac{\text{km}}{\text{h}}\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)\right)\left(100\times 10^{-3}\ \text{s}\right) = 8.4\ \text{m}.$$

103. Assuming the horizontal velocity of the ball is constant, the horizontal displacement is $\Delta x = v\Delta t$, where Δx is the horizontal distance traveled, Δt is the time, and v is the (horizontal) velocity. Converting v to meters per second, we have 160 km/h = 44.4 m/s. Thus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4\ \text{m}}{44.4\ \text{m/s}} = 0.414\ \text{s}.$$

The velocity-unit conversion implemented above can be figured "from basics" (1000 m = 1 km, 3600 s = 1 h) or found in Appendix D.

104. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. Thus, $x(t) = 50t + 10t^2$ with SI units (meters and seconds) understood.

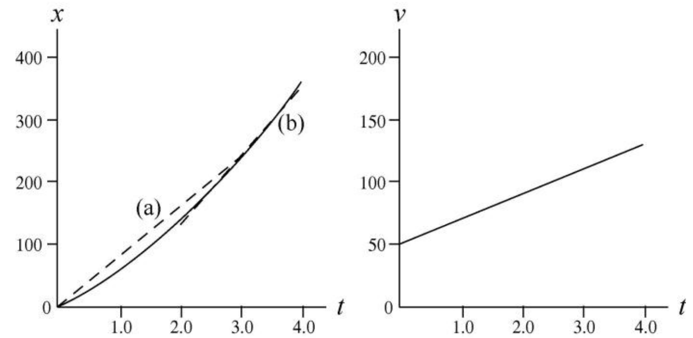
(a) The average velocity during the first 3 s is given by

$$v_{avg} = \frac{x(3) - x(0)}{\Delta t} = \frac{(50)(3) + (10)(3)^2 - 0}{3} = 80\ \text{m/s}.$$

(b) The instantaneous velocity at time t is given by $v = dx/dt = 50 + 20t$, in SI units. At $t = 3.0$ s, $v = 50 + (20)(3.0) = 110$ m/s.

(c) The instantaneous acceleration at time t is given by $a = dv/dt = 20\ \text{m/s}^2$. It is constant, so the acceleration at any time is $20\ \text{m/s}^2$.

(d) and (e) The graphs that follow show the coordinate x and velocity v as functions of time, with SI units understood. The dashed line marked (a) in the first graph runs from $t = 0$, $x = 0$ to $t = 3.0$s, $x = 240$ m. Its slope is the average velocity during the first 3s of motion. The dashed line marked (b) is tangent to the x curve at $t = 3.0$ s. Its slope is the instantaneous velocity at $t = 3.0$ s.



105. We take $+x$ in the direction of motion, so $v_0 = +30$ m/s, $v_1 = +15$ m/s and $a < 0$. The acceleration is found from Eq. 2-11: $a = (v_1 - v_0)/t_1$ where $t_1 = 3.0$ s. This gives $a = -5.0$ m/s^2. The displacement (which in this situation is the same as the distance traveled) to the point it stops ($v_2 = 0$) is, using Eq. 2-16,

$$v_2^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad \Delta x = -\frac{(30\text{ m/s})^2}{2(-5\text{ m/s}^2)} = 90\text{ m}.$$

106. The problem consists of two constant-acceleration parts: part 1 with $v_0 = 0$, $v = 6.0$ m/s, $x = 1.8$ m, and $x_0 = 0$ (if we take its original position to be the coordinate origin); and, part 2 with $v_0 = 6.0$ m/s, $v = 0$, and $a_2 = -2.5$ m/s^2 (negative because we are taking the positive direction to be the direction of motion).

(a) We can use Eq. 2-17 to find the time for the first part

$$x - x_0 = \frac{1}{2}(v_0 + v)\, t_1 \quad \Rightarrow \quad 1.8\text{ m} - 0 = \frac{1}{2}(0 + 6.0\text{ m/s})\, t_1$$

so that $t_1 = 0.6$ s. And Eq. 2-11 is used to obtain the time for the second part

$$v = v_0 + a_2 t_2 \quad \Rightarrow \quad 0 = 6.0\text{ m/s} + (-2.5\text{ m/s}^2)t_2$$

from which $t_2 = 2.4$ s is computed. Thus, the total time is $t_1 + t_2 = 3.0$ s.

(b) We already know the distance for part 1. We could find the distance for part 2 from several of the equations, but the one that makes no use of our part (a) results is Eq. 2-16

$$v^2 = v_0^2 + 2a_2\Delta x_2 \quad \Rightarrow \quad 0 = (6.0\text{ m/s})^2 + 2(-2.5\text{ m/s}^2)\Delta x_2$$

which leads to $\Delta x_2 = 7.2$ m. Therefore, the total distance traveled by the shuffleboard disk is (1.8 + 7.2) m = 9.0 m.

107. The time required is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7). First, we convert the velocity change to SI units:

$$\Delta v = (100\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = 27.8\text{ m/s}.$$

Thus, $\Delta t = \Delta v/a = 27.8/50 = 0.556$ s.

108. From Table 2-1, $v^2 - v_0^2 = 2a\Delta x$ is used to solve for a. Its minimum value is

$$a_{\min} = \frac{v_2 - v_0^2}{2\Delta x_{\max}} = \frac{(360\text{ km/h})^2}{2(1.80\text{ km})} = 36000\text{ km/h}^2$$

which converts to 2.78 m/s^2.

109. (a) For the automobile $\Delta v = 55 - 25 = 30$ km/h, which we convert to SI units:

$$a = \frac{\Delta v}{\Delta t} = \frac{(30\ \text{km/h})\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)}{(0.50\ \text{min})(60\ \text{s/min})} = 0.28\ \text{m/s}^2\ .$$

(b) The change of velocity for the bicycle, for the same time, is identical to that of the car, so its acceleration is also 0.28 m/s^2.

110. Converting to SI units, we have v = 3400(1000/3600) = 944 m/s (presumed constant) and Δt = 0.10 s. Thus, $\Delta x = v\Delta t$ = 94 m.

111. This problem consists of two parts: part 1 with constant acceleration (so that the equations in Table 2-1 apply), $v_0 = 0$, v = 11.0 m/s, x = 12.0 m, and $x_0 = 0$ (adopting the starting line as the coordinate origin); and, part 2 with constant velocity (so that $x - x_0 = vt$ applies) with v = 11.0 m/s, x_0 = 12.0, and x = 100.0 m.

(a) We obtain the time for part 1 from Eq. 2-17

$$x - x_0 = \frac{1}{2}\left(v_0 + v\right) t_1 \;\Rightarrow\; 12.0 - 0 = \frac{1}{2}\left(0 + 11.0\right)t_1$$

so that t_1 = 2.2 s, and we find the time for part 2 simply from 88.0 = (11.0)$t_2 \rightarrow t_2$ = 8.0 s. Therefore, the total time is $t_1 + t_2$ = 10.2 s.

(b) Here, the total time is required to be 10.0 s, and we are to locate the point x_p where the runner switches from accelerating to proceeding at constant speed. The equations for parts 1 and 2, used above, therefore become

$$x_p - 0 = \tfrac{1}{2}\left(0 + 11.0\ \text{m/s}\right)t_1$$
$$100.0\ \text{m} - x_p = \left(11.0\ \text{m/s}\right)\left(10.0\ \text{s} - t_1\right)$$

where in the latter equation, we use the fact that $t_2 = 10.0 - t_1$. Solving the equations for the two unknowns, we find that t_1 = 1.8 s and x_p = 10.0 m.

112. The bullet starts at rest (v_0 = 0) and after traveling the length of the barrel ($\Delta x = 1.2$ m) emerges with the given velocity (v = 640 m/s), where the direction of motion is the positive direction. Turning to the constant acceleration equations in Table 2-1, we use $\Delta x = \frac{1}{2}(v_0 + v)\, t$. Thus, we find t = 0.00375 s (or 3.75 ms).

113. There is no air resistance, which makes it quite accurate to set $a = -g = -9.8$ m/s^2 (where downward is the $-y$ direction) for the duration of the fall. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion; in fact, when the acceleration changes (during the process of catching the ball) we will again assume constant acceleration conditions; in this case, we have $a_2 = +25g$ = 245 m/s^2.

(a) The time of fall is given by Eq. 2-15 with v_0 = 0 and y = 0. Thus,

$$t=\sqrt{\frac{2y_0}{g}}=\sqrt{\frac{2(145\text{ m})}{9.8\text{ m/s}^2}}=5.44\text{ s}.$$

(b) The final velocity for its free-fall (which becomes the initial velocity during the catching process) is found from Eq. 2-16 (other equations can be used but they would use the result from part (a))

$$v=-\sqrt{v_0^2-2g(y-y_0)}=-\sqrt{2gy_0}=-53.3\text{ m/s}$$

where the negative root is chosen since this is a downward velocity. Thus, the speed is $|v|=53.3$ m/s.

(c) For the catching process, the answer to part (b) plays the role of an *initial* velocity (v_0 = –53.3 m/s) and the final velocity must become zero. Using Eq. 2-16, we find

$$\Delta y_2=\frac{v^2-v_0^2}{2a_2}=\frac{-(-53.3\text{ m/s})^2}{2(245\text{ m/s}^2)}=-5.80\text{ m},$$

or $|\Delta y_2|=5.80$ m. The negative value of Δy_2 signifies that the distance traveled while arresting its motion is downward.

114. During T_r the velocity v_0 is constant (in the direction we choose as $+x$) and obeys $v_0 = D_r/T_r$ where we note that in SI units the velocity is v_0 = 200(1000/3600) = 55.6 m/s. During T_b the acceleration is opposite to the direction of v_0 (hence, for us, $a < 0$) until the car is stopped ($v = 0$).

(a) Using Eq. 2-16 (with Δx_b = 170 m) we find

$$v^2=v_0^2+2a\Delta x_b \quad\Rightarrow\quad a=-\frac{v_0^2}{2\Delta x_b}$$

which yields $|a|$ = 9.08 m/s^2.

(b) We express this as a multiple of g by setting up a ratio:

$$a=\left(\frac{9.08\text{ m/s}^2}{9.8\text{ m/s}^2}\right)g=0.926g\ .$$

(c) We use Eq. 2-17 to obtain the braking time:

$$\Delta x_b=\frac{1}{2}\left(v_0+v\right)T_b \quad\Rightarrow\quad T_b=\frac{2\left(170\text{ m}\right)}{55.6\text{ m/s}}=6.12\text{ s}\ .$$

(d) We express our result for T_b as a multiple of the reaction time T_r by setting up a ratio:

$$T_b=\left(\frac{6.12\text{ s}}{400\times10^{-3}\text{ s}}\right)T_r=15.3T_r\ .$$

(e) Since $T_b > T_r$, most of the full time required to stop is spent in braking.

(f) We are only asked what the *increase* in distance D is, due to $\Delta T_r = 0.100$ s, so we simply have

$$\Delta D = v_0 \Delta T_r = (55.6 \text{ m/s})(0.100 \text{ s}) = 5.56 \text{ m} .$$

115. The total time elapsed is $\Delta t = 2 \text{ h } 41 \text{ min} = 161 \text{ min}$ and the center point is displaced by $\Delta x = 3.70 \text{ m} = 370 \text{ cm}$. Thus, the average velocity of the center point is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{370 \text{ cm}}{161 \text{ min}} = 2.30 \text{ cm/min}.$$

116. Using Eq. 2-11, $v = v_0 + at$, we find the initial speed to be

$$v_0 = v - at = 0 - (-3400)(9.8 \text{ m/s}^2)(6.5 \times 10^{-3} \text{ s}) = 216.6 \text{ m/s}$$

117. The total number of days walked is (including the first and the last day, and leap year)

$$N = 340 + 365 + 365 + 366 + 365 + 365 + 261 = 2427$$

Thus, the average speed of the walk is

$$s_{\text{avg}} = \frac{d}{\Delta t} = \frac{3.06 \times 10^7 \text{ m}}{(2427 \text{ days})(86400 \text{ s/day})} = 0.146 \text{ m/s}.$$

118. (a) Let d be the distance traveled. The average speed with and without wings set as sails are $v_s = d/t_s$ and $v_{ns} = d/t_{ns}$, respectively. Thus, the ratio of the two speeds is

$$\frac{v_s}{v_{ns}} = \frac{d/t_s}{d/t_{ns}} = \frac{t_{ns}}{t_s} = \frac{25.0 \text{ s}}{7.1 \text{ s}} = 3.52$$

(b) The difference in time expressed in terms of v_s is

$$\Delta t = t_{ns} - t_s = \frac{d}{v_{ns}} - \frac{d}{v_s} = \frac{d}{(v_s/3.52)} - \frac{d}{v_s} = 2.52\frac{d}{v_s} = 2.52\frac{(2.0 \text{ m})}{v_s} = \frac{5.04 \text{ m}}{v_s}$$

119. (a) Differentiating $y(t) = (2.0 \text{ cm})\sin(\pi t/4)$ with respect to t, we obtain

$$v_y(t) = \frac{dy}{dt} = \left(\frac{\pi}{2} \text{ cm/s}\right)\cos(\pi t/4)$$

The average velocity between $t = 0$ and $t = 2.0$ s is

$$v_{\text{avg}} = \frac{1}{(2.0\text{ s})}\int_0^2 v_y dt = \frac{1}{(2.0\text{ s})}\left(\frac{\pi}{2}\text{ cm/s}\right)\int_0^2 \cos\left(\frac{\pi t}{4}\right)dt$$
$$= \frac{1}{(2.0\text{ s})}\left(2\text{ cm}\right)\int_0^{\pi/2}\cos x\, dx = 1.0\text{ cm/s}$$

(b) The instantaneous velocities of the particle at t = 0, 1.0 s, and 2.0 s are, respectively,

$$v_y(0) = \left(\frac{\pi}{2}\text{ cm/s}\right)\cos(0) = \frac{\pi}{2}\text{ cm/s}$$
$$v_y(1.0\text{ s}) = \left(\frac{\pi}{2}\text{ cm/s}\right)\cos(\pi/4) = \frac{\pi\sqrt{2}}{4}\text{ cm/s}$$
$$v_y(2.0\text{ s}) = \left(\frac{\pi}{2}\text{ cm/s}\right)\cos(\pi/2) = 0$$

(c) Differentiating $v_y(t)$ with respect to t, we obtain the following expression for acceleration:

$$a_y(t) = \frac{dv_y}{dt} = \left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\sin(\pi t/4)$$

The average acceleration between t = 0 and t = 2.0 s is

$$a_{\text{avg}} = \frac{1}{(2.0\text{ s})}\int_0^2 a_y dt = \frac{1}{(2.0\text{ s})}\left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\int_0^2 \sin\left(\frac{\pi t}{4}\right)dt$$
$$= \frac{1}{(2.0\text{ s})}\left(-\frac{\pi}{2}\text{ cm/s}\right)\int_0^{\pi/2}\sin x\, dx = \frac{1}{(2.0\text{ s})}\left(-\frac{\pi}{2}\text{ cm/s}\right) = -\frac{\pi}{4}\text{ cm/s}^2$$

(d) The instantaneous accelerations of the particle at t = 0, 1.0 s, and 2.0 s are, respectively,

$$a_y(0) = \left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\sin(0) = 0$$
$$a_y(1.0\text{ s}) = \left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\sin(\pi/4) = -\frac{\pi^2\sqrt{2}}{16}\text{ cm/s}^2$$
$$a_y(2.0\text{ s}) = \left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\sin(\pi/2) = -\frac{\pi^2}{8}\text{ cm/s}^2$$

Chapter 3

1. **THINK** In this problem we're given the magnitude and direction of a vector in two dimensions, and asked to calculate its *x*- and *y*-components.

EXPRESS The *x*- and the *y*- components of a vector $\vec{a}$ lying in the *xy* plane are given by

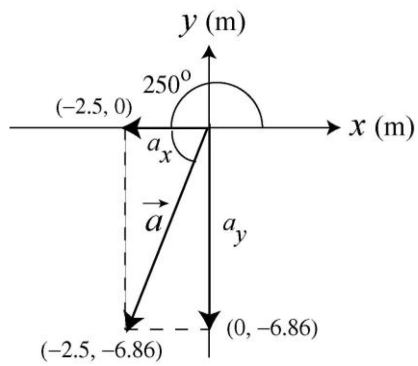
$$a_x = a\cos\theta, \qquad a_y = a\sin\theta$$

where $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$ is the magnitude and $\theta = \tan^{-1}(a_y / a_x)$ is the angle between $\vec{a}$ and the positive x axis. Given that $\theta = 250°$, we see that the vector is in the third quadrant, and we expect both the *x*- and the *y*-components of $\vec{a}$ to be negative.

ANALYZE (a) The *x* component of $\vec{a}$ is

$$a_x = a\cos\theta = (7.3 \text{ m})\cos 250° = -2.50 \text{ m},$$

(b) and the y component is $a_y = a\sin\theta = (7.3 \text{ m})\sin 250° = -6.86 \text{ m} \approx -6.9 \text{ m}$. The results are depicted in the figure below:



LEARN In considering the variety of ways to compute these, we note that the vector is 70° below the – *x* axis, so the components could also have been found from

$$a_x = -(7.3 \text{ m})\cos 70° = -2.50 \text{ m}, \quad a_y = -(7.3 \text{ m})\sin 70° = -6.86 \text{ m}.$$

Similarly, we note that the vector is 20° to the left from the – *y* axis, so one could also achieve the same results by using

$$a_x = -(7.3 \text{ m})\sin 20° = -2.50 \text{ m}, \quad a_y = -(7.3 \text{ m})\cos 20° = -6.86 \text{ m}.$$

As a consistency check, we note that $\sqrt{a_x^2 + a_y^2} = \sqrt{(-2.50\text{ m})^2 + (-6.86\text{ m})^2} = 7.3\text{ m}$ and $\tan^{-1}\left(a_y / a_x\right) = \tan^{-1}[(-6.86\text{ m})/(-2.50\text{ m})] = 250°$, which are indeed the values given in the problem statement.

2. (a) With r = 15 m and θ = 30°, the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = (15\text{ m})\cos 30° = 13\text{ m}.$$

(b) Similarly, the y component is given by $r_y = r\sin\theta = (15\text{ m})\sin 30° = 7.5\text{ m}$.

3. **THINK** In this problem we're given the x- and y-components a vector $\vec{A}$ in two dimensions, and asked to calculate its magnitude and direction.

EXPRESS A vector $\vec{A}$ can be represented in the *magnitude-angle* notation (A, θ), where

$$A = \sqrt{A_x^2 + A_y^2}$$

is the magnitude and

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

is the angle $\vec{A}$ makes with the positive x axis. Given that $A_x = -25.0$ m and $A_y = 40.0$ m, the above formulas can be readily used to calculate A and θ.

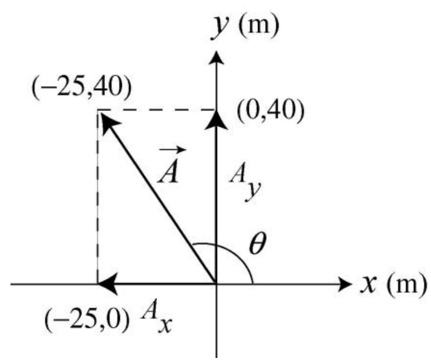
ANALYZE (a) The magnitude of the vector $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0\text{ m})^2 + (40.0\text{ m})^2} = 47.2\text{ m}$$

(b) Recalling that $\tan\theta = \tan(\theta + 180°)$,

$$\tan^{-1}[(40.0\text{ m})/(-25.0\text{ m})] = -58° \text{ or } 122°.$$

Noting that the vector is in the second quadrant (by the signs of its x and y components) we see that 122° is the correct answer. The results are depicted in the figure to the right.



LEARN We can check our answers by noting that the x- and the y- components of $\vec{A}$ can be written as

$$A_x = A\cos\theta, \qquad A_y = A\sin\theta.$$

Substituting the results calculated above, we obtain

$$A_x = (47.2 \text{ m})\cos 122° = -25.0 \text{ m}, \quad A_y = (47.2 \text{ m})\sin 122° = +40.0 \text{ m}$$

which indeed are the values given in the problem statement.

4. The angle described by a full circle is 360° = 2π rad, which is the basis of our conversion factor.

(a) $20.0° = (20.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.349 \text{ rad}$.

(b) $50.0° = (50.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.873 \text{ rad}$.

(c) $100° = (100°)\dfrac{2\pi \text{ rad}}{360°} = 1.75 \text{ rad}$.

(d) $0.330 \text{ rad} = (0.330 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 18.9°$.

(e) $2.10 \text{ rad} = (2.10 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 120°$.

(f) $7.70 \text{ rad} = (7.70 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 441°$.

5. The vector sum of the displacements $\vec{d}_{\text{storm}}$ and $\vec{d}_{\text{new}}$ must give the same result as its originally intended displacement $\vec{d}_{\text{o}} = (120 \text{ km})\hat{\text{j}}$ where east is $\hat{\text{i}}$, north is $\hat{\text{j}}$. Thus, we write

$$\vec{d}_{\text{storm}} = (100 \text{ km})\,\hat{\text{i}}, \quad \vec{d}_{\text{new}} = A\hat{\text{i}} + B\hat{\text{j}}.$$

(a) The equation $\vec{d}_{\text{storm}} + \vec{d}_{\text{new}} = \vec{d}_{\text{o}}$ readily yields A = –100 km and B = 120 km. The magnitude of $\vec{d}_{\text{new}}$ is therefore equal to $|\vec{d}_{\text{new}}| = \sqrt{A^2 + B^2} = 156 \text{ km}$.

(b) The direction is

$$\tan^{-1}(B/A) = -50.2° \text{ or } 180° + (-50.2°) = 129.8°.$$

We choose the latter value since it indicates a vector pointing in the second quadrant, which is what we expect here. The answer can be phrased several equivalent ways: 129.8° counterclockwise from east, or 39.8° west from north, or 50.2° north from west.

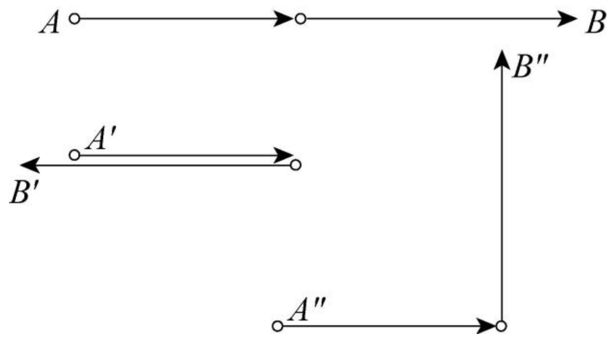
6. (a) The height is $h = d \sin\theta$, where d = 12.5 m and θ = 20.0°. Therefore, h = 4.28 m.

(b) The horizontal distance is $d \cos\theta$ = 11.7 m.

7. (a) The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below),

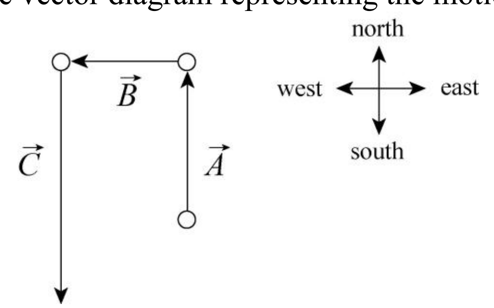
(b) anti-parallel (in opposite directions) to achieve a resultant 1 m long (primed case shown),
(c) and perpendicular to achieve a resultant $\sqrt{3^2+4^2}=5\,\text{m}$ long (the double-primed case shown).

In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by *A* (with or without primes, as the case may be) and the end is indicated by *B*.



8. We label the displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\text{i}}$ direction (+*x* direction) and *north* as the $\hat{\text{j}}$ direction (+*y* direction). All distances are understood to be in kilometers.

(a) The vector diagram representing the motion is shown next:



$$\vec{A} = (3.1\text{ km})\,\hat{\text{j}}$$
$$\vec{B} = (-2.4\text{ km})\,\hat{\text{i}}$$
$$\vec{C} = (-5.2\text{ km})\,\hat{\text{j}}$$

(b) The final point is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (-2.4\text{ km})\hat{\text{i}} + (-2.1\text{ km})\hat{\text{j}}$$

whose magnitude is

$$|\vec{r}| = \sqrt{(-2.4\text{ km})^2 + (-2.1\text{ km})^2} \approx 3.2\text{ km}\ .$$

(c) There are two possibilities for the angle:

$$\theta = \tan^{-1}\left(\frac{-2.1 \text{ km}}{-2.4 \text{ km}}\right) = 41°, \text{or } 221°.$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar ↔ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the +*x* axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

9. All distances in this solution are understood to be in meters.

(a) $\vec{a}+\vec{b} = [4.0+(-1.0)]\,\hat{\text{i}} + [(-3.0)+1.0]\,\hat{\text{j}} + (1.0+4.0)\hat{\text{k}} = (3.0\hat{\text{i}} - 2.0\hat{\text{j}} + 5.0\,\hat{\text{k}})$ m.

(b) $\vec{a}-\vec{b} = [4.0-(-1.0)]\hat{\text{i}} + [(-3.0)-1.0]\hat{\text{j}} + (1.0-4.0)\hat{\text{k}} = (5.0\,\hat{\text{i}} - 4.0\hat{\text{j}} - 3.0\,\hat{\text{k}})$ m.

(c) The requirement $\vec{a}-\vec{b}+\vec{c}=0$ leads to $\vec{c}=\vec{b}-\vec{a}$, which we note is the opposite of what we found in part (b). Thus, $\vec{c} = (-5.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}})$ m.

10. The *x, y*, and *z* components of $\vec{r}=\vec{c}+\vec{d}$ are, respectively,

(a) $r_x = c_x + d_x = 7.4\text{ m} + 4.4\text{ m} = 12\text{ m}$,

(b) $r_y = c_y + d_y = -3.8\text{ m} - 2.0\text{ m} = -5.8\text{ m}$, and

(c) $r_z = c_z + d_z = -6.1\text{ m} + 3.3\text{ m} = -2.8\text{ m}$.

11. **THINK** This problem involves the addition of two vectors $\vec{a}$ and $\vec{b}$. We want to find the magnitude and direction of the resulting vector.

EXPRESS In two dimensions, a vector $\vec{a}$ can be written as, in unit vector notation,

$$\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}}.$$

Similarly, a second vector $\vec{b}$ can be expressed as $\vec{b} = b_x\hat{\text{i}} + b_y\hat{\text{j}}$. Adding the two vectors gives

$$\vec{r} = \vec{a}+\vec{b} = (a_x + b_x)\hat{\text{i}} + (a_y + b_y)\hat{\text{j}} = r_x\hat{\text{i}} + r_y\hat{\text{j}}$$

ANALYZE (a) Given that $\vec{a} = (4.0\text{ m})\hat{\text{i}} + (3.0\text{ m})\hat{\text{j}}$ and $\vec{b} = (-13.0\text{ m})\hat{\text{i}} + (7.0\text{ m})\hat{\text{j}}$, we find the *x* and the *y* components of $\vec{r}$ to be

$$r_x = a_x + b_x = (4.0 \text{ m}) + (-13 \text{ m}) = -9.0 \text{ m}$$
$$r_y = a_y + b_y = (3.0 \text{ m}) + (7.0 \text{ m}) = 10.0 \text{ m}.$$

Thus $\vec{r} = (-9.0 \text{m})\,\hat{\text{i}} + (10 \text{m})\,\hat{\text{j}}$.

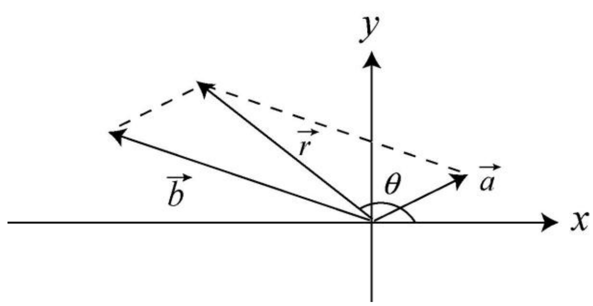
(b) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0 \text{ m})^2 + (10 \text{ m})^2} = 13 \text{ m}.$

(c) The angle between the resultant and the +x axis is given by

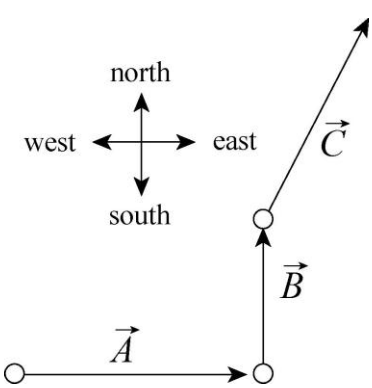
$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{10.0 \text{ m}}{-9.0 \text{ m}}\right) = -48° \text{ or } 132°.$$

Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from +x axis).

LEARN The addition of the two vectors is depicted in the figure below (not to scale). Indeed, since $r_x < 0$ and $r_y > 0$, we expect $\vec{r}$ to be in the second quadrant.



12. We label the displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\text{i}}$ direction (+x direction) and *north* as the $\hat{\text{j}}$ direction (+y direction). We note that the angle between $\vec{C}$ and the x axis is 60°. Thus,



$$\vec{A} = (50 \text{ km})\,\hat{\text{i}}$$
$$\vec{B} = (30 \text{ km})\,\hat{\text{j}}$$
$$\vec{C} = (25 \text{ km}) \cos(60°)\,\hat{\text{i}} + (25 \text{ km}) \sin(60°)\,\hat{\text{j}}$$

(a) The total displacement of the car from its initial position is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (62.5\ \text{km})\hat{\text{i}} + (51.7\ \text{km})\hat{\text{j}}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(62.5\,\text{km})^2 + (51.7\,\text{km})^2} = 81\ \text{km}.$$

(b) The angle (counterclockwise from +x axis) is $\tan^{-1}$ (51.7 km/62.5 km) = 40°, which is to say that it points 40° *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

13. We find the components and then add them (as scalars, not vectors). With d = 3.40 km and θ = 35.0° we find $d \cos\theta + d \sin\theta = 4.74$ km.

14. (a) Summing the x components, we have

$$20\ \text{m} + b_x - 20\ \text{m} - 60\ \text{m} = -140\ \text{m},$$

which gives $b_x = -80$ m.

(b) Summing the y components, we have

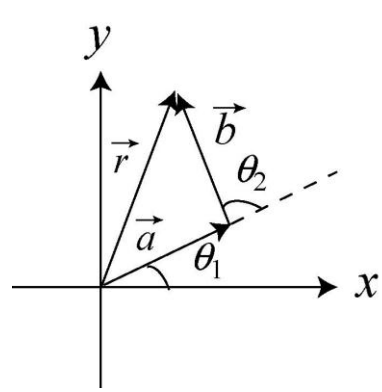
$$60\ \text{m} - 70\ \text{m} + c_y - 70\ \text{m} = 30\ \text{m},$$

which implies c_y =110 m.

(c) Using the Pythagorean theorem, the magnitude of the overall displacement is given by $\sqrt{(-140\ \text{m})^2 + (30\ \text{m})^2} \approx 143$ m.

(d) The angle is given by $\tan^{-1}(30/(-140)) = -12°$, (which would be 12° measured clockwise from the –x axis, or 168° measured counterclockwise from the +x axis).

15. **THINK** This problem involves the addition of two vectors $\vec{a}$ and $\vec{b}$ in two dimensions. We're asked to find the components, magnitude and direction of the resulting vector.



EXPRESS In two dimensions, a vector $\vec{a}$ can be written as, in unit vector notation,

$$\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}} = (a\cos\alpha)\hat{\text{i}} + (a\sin\alpha)\hat{\text{j}}.$$

Similarly, a second vector $\vec{b}$ can be expressed as $\vec{b} = b_x\hat{\text{i}} + b_y\hat{\text{j}} = (b\cos\beta)\hat{\text{i}} + (b\sin\beta)\hat{\text{j}}$. From the figure, we have, $\alpha = \theta_1$ and $\beta = \theta_1 + \theta_2$ (since the angles are measured from the +x-axis) and the resulting vector is

$$\vec{r} = \vec{a} + \vec{b} = [a\cos\theta_1 + b\cos(\theta_1 + \theta_2)]\hat{\text{i}} + [a\sin\theta_1 + b\sin(\theta_1 + \theta_2)]\hat{\text{j}} = r_x\hat{\text{i}} + r_y\hat{\text{j}}$$

ANALYZE (a) Given that $a = b = 10$ m, $\theta_1 = 30°$ and $\theta_2 = 105°$, the x component of $\vec{r}$ is

$$r_x = a\cos\theta_1 + b\cos(\theta_1 + \theta_2) = (10\text{ m})\cos 30° + (10\text{ m})\cos(30° + 105°) = 1.59\text{ m}$$

(b) Similarly, the y component of $\vec{r}$ is

$$r_y = a\sin\theta_1 + b\sin(\theta_1 + \theta_2) = (10\text{ m})\sin 30° + (10\text{ m})\sin(30° + 105°) = 12.1\text{ m}.$$

(c) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{(1.59\text{ m})^2 + (12.1\text{ m})^2} = 12.2$ m.

(d) The angle between $\vec{r}$ and the +x-axis is

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{12.1\text{ m}}{1.59\text{ m}}\right) = 82.5°.$$

LEARN As depicted in the figure, the resultant $\vec{r}$ lies in the first quadrant. This is what we expect. Note that the magnitude of $\vec{r}$ can also be calculated by using law of cosine ($\vec{a}, \vec{b}$ and $\vec{r}$ form an isosceles triangle):

$$r = \sqrt{a^2 + b^2 - 2ab\cos(180 - \theta_2)} = \sqrt{(10\text{ m})^2 + (10\text{ m})^2 - 2(10\text{ m})(10\text{ m})\cos 75°}$$
$$= 12.2\text{ m}.$$

16. (a) $\vec{a} + \vec{b} = (3.0\hat{\text{i}} + 4.0\hat{\text{j}})\text{ m} + (5.0\hat{\text{i}} - 2.0\hat{\text{j}})\text{ m} = (8.0\text{ m})\hat{\text{i}} + (2.0\text{ m})\hat{\text{j}}$.

(b) The magnitude of $\vec{a} + \vec{b}$ is

$$|\vec{a} + \vec{b}| = \sqrt{(8.0\text{ m})^2 + (2.0\text{ m})^2} = 8.2\text{ m}.$$

(c) The angle between this vector and the +x axis is

$$\tan^{-1}[(2.0\text{ m})/(8.0\text{ m})] = 14°.$$

(d) $\vec{b} - \vec{a} = (5.0\hat{\text{i}} - 2.0\hat{\text{j}})\text{ m} - (3.0\hat{\text{i}} + 4.0\hat{\text{j}})\text{ m} = (2.0\text{ m})\hat{\text{i}} - (6.0\text{ m})\hat{\text{j}}$.

(e) The magnitude of the difference vector $\vec{b} - \vec{a}$ is

$$|\vec{b} - \vec{a}| = \sqrt{(2.0\text{ m})^2 + (-6.0\text{ m})^2} = 6.3\text{ m}.$$

(f) The angle between this vector and the +*x* axis is $\tan^{-1}$[(–6.0 m)/(2.0 m)] = –72°. The vector is 72° *clockwise* from the axis defined by $\hat{\text{i}}$.

17. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

(a) Using unit-vector notation,

$$\vec{a} = (50\text{ m})\cos(30°)\hat{\text{i}} + (50\text{ m})\sin(30°)\hat{\text{j}}$$
$$\vec{b} = (50\text{ m})\cos(195°)\hat{\text{i}} + (50\text{ m})\sin(195°)\hat{\text{j}}$$
$$\vec{c} = (50\text{ m})\cos(315°)\hat{\text{i}} + (50\text{ m})\sin(315°)\hat{\text{j}}$$
$$\vec{a} + \vec{b} + \vec{c} = (30.4\text{ m})\hat{\text{i}} - (23.3\text{ m})\hat{\text{j}}.$$

The magnitude of this result is $\sqrt{(30.4\text{ m})^2 + (-23.3\text{ m})^2} = 38\text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the +*x* direction are $\tan^{-1}$[(–23.2 m)/(30.4 m)] = –37.5°, and 180° + (–37.5°) = 142.5°. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is –37.5°, which is to say that it is 37.5° *clockwise* from the +*x* axis. This is equivalent to 322.5° counterclockwise from +*x*.

(c) We find

$$\vec{a} - \vec{b} + \vec{c} = [43.3 - (-48.3) + 35.4]\hat{\text{i}} - [25 - (-12.9) + (-35.4)]\hat{\text{j}} = (127\hat{\text{i}} + 2.60\hat{\text{j}})\text{ m}$$

in unit-vector notation. The magnitude of this result is

$$|\vec{a} - \vec{b} + \vec{c}| = \sqrt{(127\text{ m})^2 + (2.6\text{ m})^2} \approx 1.30\times10^2\text{ m}.$$

(d) The angle between the vector described in part (c) and the +*x* axis is $\tan^{-1}(2.6\text{ m}/127\text{ m}) \approx 1.2°$.

(e) Using unit-vector notation, $\vec{d}$ is given by $\vec{d}=\vec{a}+\vec{b}-\vec{c}=(-40.4\ \hat{\text{i}}+47.4\ \hat{\text{j}})\ \text{m}$, which has a magnitude of $\sqrt{(-40.4\ \text{m})^2+(47.4\ \text{m})^2}=62$ m.

(f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the +x axis are $\tan^{-1}(47.4/(-40.4))=-50.0°$, and $180°+(-50.0°)=130°$. We choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second quadrant (indicated by the signs of its components).

18. If we wish to use Eq. 3-5 in an unmodified fashion, we should note that the angle between $\vec{C}$ and the +x axis is 180° + 20.0° = 200°.

(a) The x and y components of $\vec{B}$ are given by

$$B_x = C_x - A_x = (15.0\ \text{m}) \cos 200° - (12.0\ \text{m}) \cos 40° = -23.3\ \text{m},$$
$$B_y = C_y - A_y = (15.0\ \text{m}) \sin 200° - (12.0\ \text{m}) \sin 40° = -12.8\ \text{m}.$$

Consequently, its magnitude is $|\vec{B}|=\sqrt{(-23.3\ \text{m})^2+(-12.8\ \text{m})^2}=26.6\ \text{m}$.

(b) The two possibilities presented by a simple calculation for the angle between $\vec{B}$ and the +x axis are $\tan^{-1}$[(–12.8 m)/(–23.3 m)] = 28.9°, and 180° + 28.9° = 209°. We choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third quadrant (indicated by the signs of its components). We note, too, that the answer can be equivalently stated as $-151°$.

19. (a) With $\hat{\text{i}}$ directed forward and $\hat{\text{j}}$ directed leftward, the resultant is $(5.00\ \hat{\text{i}} + 2.00\ \hat{\text{j}})$ m . The magnitude is given by the Pythagorean theorem: $\sqrt{(5.00\ \text{m})^2+(2.00\ \text{m})^2}$ = 5.385 m ≈ 5.39 m.

(b) The angle is $\tan^{-1}(2.00/5.00) \approx 21.8°$ (left of forward).

20. The desired result is the displacement vector, in units of km, $\vec{A}$ = (5.6 km), 90º (measured counterclockwise from the +x axis), or $\vec{A}=(5.6\ \text{km})\hat{\text{j}}$, where $\hat{\text{j}}$ is the unit vector along the positive y axis (north). This consists of the sum of two displacements: during the whiteout, $\vec{B}=(7.8\ \text{km}),\ 50°$, or

$$\vec{B}=(7.8\ \text{km})(\cos 50°\hat{\text{i}}+\sin 50°\ \hat{\text{j}})=(5.01\ \text{km})\hat{\text{i}}+(5.98\ \text{km})\hat{\text{j}}$$

and the unknown $\vec{C}$. Thus, $\vec{A}=\vec{B}+\vec{C}$.

(a) The desired displacement is given by $\vec{C} = \vec{A} - \vec{B} = (-5.01 \text{ km})\ \hat{\text{i}} - (0.38 \text{ km})\ \hat{\text{j}}$. The magnitude is $\sqrt{(-5.01 \text{ km})^2 + (-0.38 \text{ km})^2} = 5.0$ km.

(b) The angle is $\tan^{-1}[(-0.38 \text{ km})/(-5.01 \text{ km})] = 4.3°$, south of due west.

21. Reading carefully, we see that the (*x, y*) specifications for each "dart" are to be interpreted as (Δx, Δy) descriptions of the corresponding displacement vectors. We combine the different parts of this problem into a single exposition.

(a) Along the *x* axis, we have (with the centimeter unit understood)

$$30.0 + b_x - 20.0 - 80.0 = -140,$$

which gives b_x = –70.0 cm.

(b) Along the *y* axis we have

$$40.0 - 70.0 + c_y - 70.0 = -20.0$$

which yields c_y = 80.0 cm.

(c) The magnitude of the final location (–140 , –20.0) is $\sqrt{(-140)^2 + (-20.0)^2} = 141$ cm.

(d) Since the displacement is in the third quadrant, the angle of the overall displacement is given by $\pi + \tan^{-1}[(-20.0)/(-140)]$ or 188° counterclockwise from the +*x* axis (or $-172°$ counterclockwise from the +*x* axis).

22. Angles are given in 'standard' fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Wherever the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\vec{E} = 3.73\ \hat{\text{i}} + 4.70\ \hat{\text{j}}$$
$$\vec{F} = 1.29\ \hat{\text{i}} - 4.83\ \hat{\text{j}}$$
$$\vec{G} = 1.45\ \hat{\text{i}} + 3.73\ \hat{\text{j}}$$
$$\vec{H} = -5.20\ \hat{\text{i}} + 3.00\ \hat{\text{j}}$$
$$\vec{E} + \vec{F} + \vec{G} + \vec{H} = 1.28\ \hat{\text{i}} + 6.60\ \hat{\text{j}}.$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{(1.28 \text{ m})^2 + (6.60 \text{ m})^2} = 6.72 \text{ m}$.

(c) Its angle measured counterclockwise from the +*x* axis is $\tan^{-1}(6.60/1.28) = 79.0°$.

(d) Using the conversion factor π rad = 180°, 79.0° = 1.38 rad.

23. The resultant (along the *y* axis, with the same magnitude as $\vec{C}$) forms (along with $\vec{C}$) a side of an isosceles triangle (with $\vec{B}$ forming the base). If the angle between $\vec{C}$ and the *y* axis is $\theta = \tan^{-1}(3/4) = 36.87°$, then it should be clear that (referring to the magnitudes of the vectors) $B = 2C\sin(\theta/2)$. Thus (since $C = 5.0$) we find $B = 3.2$.

24. As a vector addition problem, we express the situation (described in the problem statement) as $\vec{A} + \vec{B} = (3A)\hat{j}$, where $\vec{A} = A\hat{i}$ and $B = 7.0$ m. Since $\hat{i} \perp \hat{j}$ we may use the Pythagorean theorem to express *B* in terms of the magnitudes of the other two vectors:

$$B = \sqrt{(3A)^2 + A^2} \quad \Rightarrow \quad A = \frac{1}{\sqrt{10}} B = 2.2 \text{ m}.$$

25. The strategy is to find where the camel is ($\vec{C}$) by adding the two consecutive displacements described in the problem, and then finding the difference between that location and the oasis ($\vec{B}$). Using the magnitude-angle notation

$$\vec{C} = (24 \angle -15°) + (8.0 \angle 90°) = (23.25 \angle 4.41°)$$

so

$$\vec{B} - \vec{C} = (25 \angle 0°) - (23.25 \angle 4.41°) = (2.5 \angle -45°)$$

which is efficiently implemented using a vector-capable calculator in polar mode. The distance is therefore 2.6 km.

26. The vector equation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. Expressing $\vec{B}$ and $\vec{D}$ in unit-vector notation, we have $(1.69\hat{i} + 3.63\hat{j})$ m and $(-2.87\hat{i} + 4.10\hat{j})$ m, respectively. Where the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Adding corresponding components, we obtain $\vec{R} = (-3.18 \text{ m})\hat{i} + (4.72 \text{ m})\hat{j}$.

(b) Using Eq. 3-6, the magnitude is

$$|\vec{R}| = \sqrt{(-3.18 \text{ m})^2 + (4.72 \text{ m})^2} = 5.69 \text{ m}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{4.72 \text{ m}}{-3.18 \text{ m}}\right) = -56.0° \text{ (with } -x \text{ axis)}.$$

If measured counterclockwise from +*x*-axis, the angle is then $180° - 56.0° = 124°$. Thus, converting the result to polar coordinates, we obtain

$$(-3.18, 4.72) \rightarrow (5.69 \angle 124°)$$

27. Solving the simultaneous equations yields the answers:

(a) $\vec{d}_1 = 4\,\vec{d}_3 = 8\,\hat{\text{i}} + 16\,\hat{\text{j}}$, and

(b) $\vec{d}_2 = \vec{d}_3 = 2\,\hat{\text{i}} + 4\,\hat{\text{j}}$.

28. Let $\vec{A}$ represent the first part of Beetle 1's trip (0.50 m east or $0.5\,\hat{\text{i}}$) and $\vec{C}$ represent the first part of Beetle 2's trip intended voyage (1.6 m at 50º north of east). For their respective second parts: $\vec{B}$ is 0.80 m at 30º north of east and $\vec{D}$ is the unknown. The final position of Beetle 1 is

$$\vec{A} + \vec{B} = (0.5 \text{ m})\hat{\text{i}} + (0.8 \text{ m})(\cos 30°\,\hat{\text{i}} + \sin 30°\,\hat{\text{j}}) = (1.19 \text{ m})\,\hat{\text{i}} + (0.40 \text{ m})\,\hat{\text{j}}.$$

The equation relating these is $\vec{A} + \vec{B} = \vec{C} + \vec{D}$, where

$$\vec{C} = (1.60 \text{ m})(\cos 50.0°\hat{\text{i}} + \sin 50.0°\hat{\text{j}}) = (1.03 \text{ m})\hat{\text{i}} + (1.23 \text{ m})\hat{\text{j}}$$

(a) We find $\vec{D} = \vec{A} + \vec{B} - \vec{C} = (0.16 \text{ m})\hat{\text{i}} + (-0.83 \text{ m})\hat{\text{j}}$, and the magnitude is $D = 0.84$ m.

(b) The angle is $\tan^{-1}(-0.83/0.16) = -79°$, which is interpreted to mean 79º south of east (or 11º east of south).

29. Let $l_0 = 2.0 \text{ cm}$ be the length of each segment. The nest is located at the endpoint of segment *w*.

(a) Using unit-vector notation, the displacement vector for point *A* is

$$\begin{aligned}\vec{d}_A &= \vec{w} + \vec{v} + \vec{i} + \vec{h} = l_0(\cos 60°\hat{\text{i}} + \sin 60°\,\hat{\text{j}}) + \left(l_0\,\hat{\text{j}}\right) + l_0(\cos 120°\hat{\text{i}} + \sin 120°\,\hat{\text{j}}) + \left(l_0\,\hat{\text{j}}\right)\\ &= (2 + \sqrt{3})l_0\,\hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_A$ is $|\vec{d}_A| = (2 + \sqrt{3})(2.0 \text{ cm}) = 7.5 \text{ cm}$.

(b) The angle of $\vec{d}_A$ is $\theta = \tan^{-1}(d_{A,y}/d_{A,x}) = \tan^{-1}(\infty) = 90°$.

(c) Similarly, the displacement for point *B* is

$$\vec{d}_B = \vec{w}+\vec{v}+\vec{j}+\vec{p}+\vec{o}$$
$$= l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+\left(l_0\,\hat{\text{j}}\right)+l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+l_0(\cos 30°\hat{\text{i}}+\sin 30°\,\hat{\text{j}})+\left(l_0\,\hat{\text{i}}\right)$$
$$= (2+\sqrt{3}/2)l_0\,\hat{\text{i}}+(3/2+\sqrt{3})l_0\,\hat{\text{j}}.$$

Therefore, the magnitude of $\vec{d}_B$ is

$$|\vec{d}_B| = l_0\sqrt{(2+\sqrt{3}/2)^2+(3/2+\sqrt{3})^2} = (2.0\text{ cm})(4.3) = 8.6\text{ cm}.$$

(d) The direction of $\vec{d}_B$ is

$$\theta_B = \tan^{-1}\left(\frac{d_{B,y}}{d_{B,x}}\right) = \tan^{-1}\left(\frac{3/2+\sqrt{3}}{2+\sqrt{3}/2}\right) = \tan^{-1}(1.13) = 48°.$$

30. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6).

(a) The magnitude of $\vec{a}$ is $a = \sqrt{(4.0\text{ m})^2+(-3.0\text{ m})^2} = 5.0\text{ m}.$

(b) The angle between $\vec{a}$ and the $+x$ axis is $\tan^{-1}$ [(–3.0 m)/(4.0 m)] = –37°. The vector is 37° *clockwise* from the axis defined by $\hat{\text{i}}$.

(c) The magnitude of $\vec{b}$ is $b = \sqrt{(6.0\text{ m})^2+(8.0\text{ m})^2} = 10\text{ m}.$

(d) The angle between $\vec{b}$ and the $+x$ axis is $\tan^{-1}$[(8.0 m)/(6.0 m)] = 53°.

(e) $\vec{a}+\vec{b} = (4.0\text{ m}+6.0\text{ m})\,\hat{\text{i}}+[(-3.0\text{ m})+8.0\text{ m}]\hat{\text{j}} = (10\text{ m})\hat{\text{i}}+(5.0\text{ m})\hat{\text{j}}$. The magnitude of this vector is $|\vec{a}+\vec{b}| = \sqrt{(10\text{ m})^2+(5.0\text{ m})^2} = 11\text{ m}$; we round to two significant figures in our results.

(f) The angle between the vector described in part (e) and the $+x$ axis is $\tan^{-1}$[(5.0 m)/(10 m)] = 27°.

(g) $\vec{b}-\vec{a} = (6.0\text{ m}-4.0\text{ m})\,\hat{\text{i}}+[8.0\text{ m}-(-3.0\text{ m})]\,\hat{\text{j}} = (2.0\text{ m})\,\hat{\text{i}}+(11\text{ m})\,\hat{\text{j}}$. The magnitude of this vector is $|\vec{b}-\vec{a}| = \sqrt{(2.0\text{ m})^2+(11\text{ m})^2} = 11\text{ m}$, which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures) (this curious coincidence is made possible by the fact that $\vec{a}\perp\vec{b}$).

(h) The angle between the vector described in part (g) and the +x axis is $\tan^{-1}$[(11 m)/(2.0 m)] = 80°.

(i) $\vec{a}-\vec{b}=(4.0\text{ m}-6.0\text{ m})\,\hat{\text{i}}+[(-3.0\text{ m})-8.0\text{ m}]\,\hat{\text{j}}=(-2.0\text{ m})\,\hat{\text{i}}+(-11\text{ m})\,\hat{\text{j}}$. The magnitude of this vector is

$$|\vec{a}-\vec{b}|=\sqrt{(-2.0\text{ m})^2+(-11\text{ m})^2}=11\text{ m}.$$

(j) The two possibilities presented by a simple calculation for the angle between the vector described in part (i) and the +x direction are $\tan^{-1}$ [(–11 m)/(–2.0 m)] = 80°, and 180° + 80° = 260°. The latter possibility is the correct answer (see part (k) for a further observation related to this result).

(k) Since $\vec{a}-\vec{b}=(-1)(\vec{b}-\vec{a})$, they point in opposite (anti-parallel) directions; the angle between them is 180°.

31. (a) With a = 17.0 m and θ = 56.0° we find $a_x = a\cos\theta$ = 9.51 m.

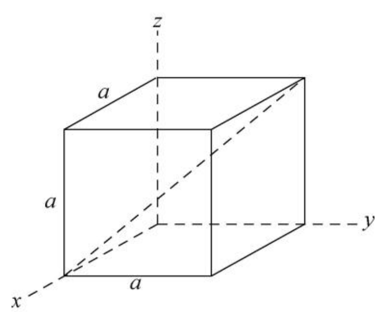
(b) Similarly, $a_y = a\sin\theta$ = 14.1 m.

(c) The angle relative to the new coordinate system is θ' = (56.0° – 18.0°) = 38.0°. Thus, $a'_x = a\cos\theta' = 13.4$ m.

(d) Similarly, $a'_y = a\sin\theta'$ = 10.5 m.

32. (a) As can be seen from Figure 3-30, the point diametrically opposite the origin (0,0,0) has position vector $a\,\hat{\text{i}} + a\,\hat{\text{j}} + a\,\hat{\text{k}}$ and this is the vector along the "body diagonal."

(b) From the point (a, 0, 0), which corresponds to the position vector $a\,\hat{\text{i}}$, the diametrically opposite point is (0, a, a) with the position vector $a\,\hat{\text{j}} + a\,\hat{\text{k}}$. Thus, the vector along the line is the difference $-a\hat{\text{i}}+a\hat{\text{j}}+a\hat{\text{k}}$.



(c) If the starting point is (0, a, 0) with the corresponding position vector $a\,\hat{\text{j}}$, the diametrically opposite point is (a, 0, a) with the position vector $a\,\hat{\text{i}} + a\,\hat{\text{k}}$. Thus, the vector along the line is the difference $a\,\hat{\text{i}}-a\,\hat{\text{j}}+a\,\hat{\text{k}}$.

(d) If the starting point is (a, a, 0) with the corresponding position vector $a\,\hat{\text{i}} + a\,\hat{\text{j}}$, the diametrically opposite point is (0, 0, a) with the position vector $a\,\hat{\text{k}}$. Thus, the vector along the line is the difference $-a\,\hat{\text{i}} - a\,\hat{\text{j}} + a\,\hat{\text{k}}$.

(e) Consider the vector from the back lower left corner to the front upper right corner. It is $a\,\hat{\text{i}} + a\,\hat{\text{j}} + a\,\hat{\text{k}}$. We may think of it as the sum of the vector $a\,\hat{\text{i}}$ parallel to the x axis and the vector $a\,\hat{\text{j}} + a\,\hat{\text{k}}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2 + a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus $\theta = 54.7°$. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(f) The length of any of the diagonals is given by $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$.

33. Examining the figure, we see that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $|\vec{a} \times \vec{b}| = (3.0)(4.0) = 12$ since the angle between them is 90º.

(b) Using the Right-Hand Rule, the vector $\vec{a} \times \vec{b}$ points in the $\hat{\text{i}} \times \hat{\text{j}} = \hat{\text{k}}$, or the $+z$ direction.

(c) $|\vec{a} \times \vec{c}| = |\vec{a} \times (-\vec{a} - \vec{b})| = |-(\vec{a} \times \vec{b})| = 12.$

(d) The vector $-\vec{a} \times \vec{b}$ points in the $-\hat{\text{i}} \times \hat{\text{j}} = -\hat{\text{k}}$, or the $-z$ direction.

(e) $|\vec{b} \times \vec{c}| = |\vec{b} \times (-\vec{a} - \vec{b})| = |-(\vec{b} \times \vec{a})| = |(\vec{a} \times \vec{b})| = 12.$

(f) The vector points in the $+z$ direction, as in part (a).

34. We apply Eq. 3-23 and Eq. 3-27.

(a) $\vec{a} \times \vec{b} = (a_x b_y - a_y b_x)\,\hat{\text{k}}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. Consequently, we obtain $[(3.0)(4.0) - (5.0)(2.0)]\hat{\text{k}} = 2.0\,\hat{\text{k}}$.

(b) $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$ yields (3.0)(2.0) + (5.0)(4.0) = 26.

(c) $\vec{a} + \vec{b} = (3.0 + 2.0)\,\hat{\text{i}} + (5.0 + 4.0)\,\hat{\text{j}} \Rightarrow (\vec{a} + \vec{b}) \cdot \vec{b} = (5.0)\,(2.0) + (9.0)\,(4.0) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and "dot" it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}}}{\sqrt{(2.0)^2 + (4.0)^2}}.$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(3.0)(2.0) + (5.0)(4.0)}{\sqrt{(2.0)^2 + (4.0)^2}} = 5.8.$$

35. (a) The scalar or dot product is (4.50)(7.30)cos(320º – 85.0º) = – 18.8 .

(b) The vector or cross product is in the $\hat{\text{k}}$ direction (by the right-hand rule) with magnitude |(4.50)(7.30) sin(320º – 85.0º)| = 26.9 .

36. First, we rewrite the given expression as 4($\vec{d}_{\text{plane}} \cdot \vec{d}_{\text{cross}}$) where $\vec{d}_{\text{plane}} = \vec{d}_1 + \vec{d}_2$ and in the plane of $\vec{d}_1$ and $\vec{d}_2$, and $\vec{d}_{\text{cross}} = \vec{d}_1 \times \vec{d}_2$. Noting that $\vec{d}_{\text{cross}}$ is perpendicular to the plane of $\vec{d}_1$ and $\vec{d}_2$, we see that the answer must be 0 (the scalar or dot product of perpendicular vectors is zero).

37. We apply Eq. 3-23 and Eq.3-27. If a vector-capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b}\times\vec{c} = -8.0\hat{\text{i}} + 5.0\hat{\text{j}} + 6.0\hat{\text{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3.0)(-8.0) + (3.0)(5.0) + (-2.0)(6.0) = -21.$$

(b) We note that $\vec{b} + \vec{c} = 1.0\hat{\text{i}} - 2.0\hat{\text{j}} + 3.0\hat{\text{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)(1.0) + (3.0)(-2.0) + (-2.0)(3.0) = -9.0.$$

(c) Finally,

$$\begin{aligned}\vec{a}\times(\vec{b}+\vec{c}) &= [(3.0)(3.0) - (-2.0)(-2.0)]\,\hat{\text{i}} + [(-2.0)(1.0) - (3.0)(3.0)]\,\hat{\text{j}} \\ &\quad + [(3.0)(-2.0) - (3.0)(1.0)]\,\hat{\text{k}} \\ &= 5\hat{\text{i}} - 11\hat{\text{j}} - 9\hat{\text{k}}\end{aligned}.$$

38. Using the fact that

$$\hat{\text{i}}\times\hat{\text{j}} = \hat{\text{k}},\quad \hat{\text{j}}\times\hat{\text{k}} = \hat{\text{i}},\quad \hat{\text{k}}\times\hat{\text{i}} = \hat{\text{j}}$$

we obtain

$$2\vec{A}\times\vec{B}=2\left(2.00\hat{i}+3.00\hat{j}-4.00\hat{k}\right)\times\left(-3.00\hat{i}+4.00\hat{j}+2.00\hat{k}\right)$$
$$=44.0\hat{i}+16.0\hat{j}+34.0\hat{k}.$$

Next, making use of

$$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$$
$$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$$

we have

$$3\vec{C}\cdot\left(2\vec{A}\times\vec{B}\right)=3\left(7.00\,\hat{i}-8.00\hat{j}\right)\cdot\left(44.0\,\hat{i}+16.0\hat{j}+34.0\hat{k}\right)$$
$$=3[(7.00)(44.0)+(-8.00)(16.0)+(0)(34.0)]=540.$$

39. From the definition of the dot product between $\vec{A}$ and $\vec{B}$, $\vec{A}\cdot\vec{B}=AB\cos\theta$, we have

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{AB}$$

With $A=6.00$, $B=7.00$ and $\vec{A}\cdot\vec{B}=14.0$, $\cos\theta=0.333$, or $\theta=70.5°$.

40. The displacement vectors can be written as (in meters)

$$\vec{d}_1=(4.50\text{ m})(\cos 63°\,\hat{j}+\sin 63°\,\hat{k})=(2.04\text{ m})\,\hat{j}+(4.01\text{ m})\,\hat{k}$$
$$\vec{d}_2=(1.40\text{ m})(\cos 30°\,\hat{i}+\sin 30°\,\hat{k})=(1.21\text{ m})\,\hat{i}+(0.70\text{ m})\,\hat{k}\,.$$

(a) The dot product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1\cdot\vec{d}_2=(2.04\,\hat{j}+4.01\hat{k})\cdot(1.21\hat{i}+0.70\hat{k})=(4.01\hat{k})\cdot(0.70\hat{k})=2.81\,\text{m}^2.$$

(b) The cross product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1\times\vec{d}_2=(2.04\,\hat{j}+4.01\hat{k})\times(1.21\hat{i}+0.70\hat{k})$$
$$=(2.04)(1.21)(-\hat{k})+(2.04)(0.70)\hat{i}+(4.01)(1.21)\hat{j}$$
$$=(1.43\,\hat{i}+4.86\,\hat{j}-2.48\,\hat{k})\,\text{m}^2.$$

(c) The magnitudes of $\vec{d}_1$ and $\vec{d}_2$ are

$$d_1=\sqrt{(2.04\text{ m})^2+(4.01\text{ m})^2}=4.50\text{ m}$$
$$d_2=\sqrt{(1.21\text{ m})^2+(0.70\text{ m})^2}=1.40\text{ m}.$$

Thus, the angle between the two vectors is

$$\theta = \cos^{-1}\left(\frac{\vec{d}_1 \cdot \vec{d}_2}{d_1 d_2}\right) = \cos^{-1}\left(\frac{2.81\ \text{m}^2}{(4.50\ \text{m})(1.40\ \text{m})}\right) = 63.5°.$$

41. **THINK** The angle between two vectors can be calculated using the definition of scalar product.

EXPRESS Since the scalar product of two vectors $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\cdot\vec{b} = ab\cos\phi = a_x b_x + a_y b_y + a_z b_z\,,$$

the angle between them is given by

$$\cos\phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab} \Rightarrow \phi = \cos^{-1}\left(\frac{a_x b_x + a_y b_y + a_z b_z}{ab}\right).$$

Once the magnitudes and components of the vectors are known, the angle ϕ can be readily calculated.

ANALYZE Given that $\vec{a} = (3.0)\hat{\text{i}} + (3.0)\hat{\text{j}} + (3.0)\hat{\text{k}}$ and $\vec{b} = (2.0)\hat{\text{i}} + (1.0)\hat{\text{j}} + (3.0)\hat{\text{k}}$, the magnitudes of the vectors are

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2} = 5.20$$

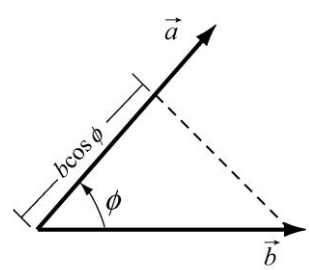
$$b = |\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2} = 3.74.$$

The angle between them is found to be

$$\cos\phi = \frac{(3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)}{(5.20)(3.74)} = 0.926,$$

or $\phi = 22°$.

LEARN As the name implies, the scalar product (or dot product) between two vectors is a scalar quantity. It can be regarded as the product between the magnitude of one of the vectors and the scalar component of the second vector along the direction of the first one, as illustrated below (see also in Fig. 3-18 of the text):



$$\vec{a}\cdot\vec{b} = ab\cos\phi = (a)(b\cos\phi)$$

42. The two vectors are written as, in unit of meters,

$$\vec{d}_1 = 4.0\,\hat{\mathrm{i}} + 5.0\,\hat{\mathrm{j}} = d_{1x}\hat{\mathrm{i}} + d_{1y}\hat{\mathrm{j}}, \quad \vec{d}_2 = -3.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}} = d_{2x}\hat{\mathrm{i}} + d_{2y}\hat{\mathrm{j}}$$

(a) The vector (cross) product gives

$$\vec{d}_1 \times \vec{d}_2 = (d_{1x}d_{2y} - d_{1y}d_{2x})\hat{\mathrm{k}} = [(4.0)(4.0) - (5.0)(-3.0)]\hat{\mathrm{k}} = 31\ \hat{\mathrm{k}}$$

(b) The scalar (dot) product gives

$$\vec{d}_1 \cdot \vec{d}_2 = d_{1x}d_{2x} + d_{1y}d_{2y} = (4.0)(-3.0) + (5.0)(4.0) = 8.0.$$

(c)

$$(\vec{d}_1 + \vec{d}_2) \cdot \vec{d}_2 = \vec{d}_1 \cdot \vec{d}_2 + d_2^2 = 8.0 + (-3.0)^2 + (4.0)^2 = 33.$$

(d) Note that the magnitude of the d_1 vector is $\sqrt{16+25}$ = 6.4. Now, the dot product is (6.4)(5.0)cosθ = 8. Dividing both sides by 32 and taking the inverse cosine yields θ = 75.5°. Therefore the component of the d_1 vector along the direction of the d_2 vector is 6.4cos$\theta \approx$ 1.6.

43. **THINK** In this problem we are given three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ on the *xy*-plane, and asked to calculate their components.

EXPRESS From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the +*x* axis is θ + 90°. In unit-vector notation, the three vectors can be written as

$$\vec{a} = a_x\hat{\mathrm{i}}$$
$$\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} = (b\cos\theta)\hat{\mathrm{i}} + (b\sin\theta)\hat{\mathrm{j}}$$
$$\vec{c} = c_x\hat{\mathrm{i}} + c_y\hat{\mathrm{j}} = [c\cos(\theta + 90°)]\hat{\mathrm{i}} + [c\sin(\theta + 90°)]\hat{\mathrm{j}}.$$

The above expressions allow us to evaluate the components of the vectors.

ANALYZE (a) The *x*-component of $\vec{a}$ is $a_x = a \cos 0° = a$ = 3.00 m.

(b) Similarly, the *y*-componnet of $\vec{a}$ is $a_y = a \sin 0° = 0$.

(c) The *x*-component of $\vec{b}$ is $b_x = b \cos 30°$ = (4.00 m) cos 30° = 3.46 m,

(d) and the *y*-component is $b_y = b \sin 30°$ = (4.00 m) sin 30° = 2.00 m.

(e) The *x*-component of $\vec{c}$ is $c_x = c \cos 120°$ = (10.0 m) cos 120° = –5.00 m,

(f) and the y-component is $c_y = c \sin 30° = (10.0 \text{ m}) \sin 120° = 8.66$ m.

(g) The fact that $\vec{c} = p\vec{a} + q\vec{b}$ implies

$$\vec{c} = c_x\hat{\text{i}} + c_y\hat{\text{j}} = p(a_x\hat{\text{i}}) + q(b_x\hat{\text{i}} + b_y\hat{\text{j}}) = (pa_x + qb_x)\hat{\text{i}} + qb_y\hat{\text{j}}$$

or

$$c_x = pa_x + qb_x, \qquad c_y = qb_y .$$

Substituting the values found above, we have

$$-5.00 \text{ m} = p\,(3.00 \text{ m}) + q\,(3.46 \text{ m})$$
$$8.66 \text{ m} = q\,(2.00 \text{ m}).$$

Solving these equations, we find $p = -6.67$.

(h) Similarly, $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.

LEARN This exercise shows that given two (non-parallel) vectors in two dimensions, the third vector can always be written as a linear combination of the first two.

44. Applying Eq. 3-23, $\vec{F} = q\vec{v} \times \vec{B}$ (where q is a scalar) becomes

$$F_x\,\hat{\text{i}} + F_y\,\hat{\text{j}} + F_z\,\hat{\text{k}} = q\left(v_yB_z - v_zB_y\right)\hat{\text{i}} + q\left(v_zB_x - v_xB_z\right)\hat{\text{j}} + q\left(v_xB_y - v_yB_x\right)\hat{\text{k}}$$

which — plugging in values — leads to three equalities:

$$4.0 = 2\,(4.0B_z - 6.0B_y)$$
$$-20 = 2\,(6.0B_x - 2.0B_z)$$
$$12 = 2\,(2.0B_y - 4.0B_x)$$

Since we are told that $B_x = B_y$, the third equation leads to $B_y = -3.0$. Inserting this value into the first equation, we find $B_z = -4.0$. Thus, our answer is

$$\vec{B} = -3.0\,\hat{\text{i}} - 3.0\,\hat{\text{j}} - 4.0\,\hat{\text{k}}.$$

45. The two vectors are given by

$$\vec{A} = 8.00(\cos 130°\,\hat{\text{i}} + \sin 130°\,\hat{\text{j}}) = -5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}}$$
$$\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}} = -7.72\,\hat{\text{i}} - 9.20\,\hat{\text{j}}.$$

(a) The dot product of $5\vec{A}\cdot\vec{B}$ is

$$5\vec{A}\cdot\vec{B}=5(-5.14\,\hat{\text{i}}+6.13\,\hat{\text{j}})\cdot(-7.72\,\hat{\text{i}}-9.20\,\hat{\text{j}})=5[(-5.14)(-7.72)+(6.13)(-9.20)]$$
$$=-83.4.$$

(b) In unit vector notation

$$4\vec{A}\times 3\vec{B}=12\vec{A}\times\vec{B}=12(-5.14\,\hat{\text{i}}+6.13\,\hat{\text{j}})\times(-7.72\,\hat{\text{i}}-9.20\,\hat{\text{j}})=12(94.6\,\hat{\text{k}})=1.14\times10^{3}\,\hat{\text{k}}$$

(c) We note that the azimuthal angle is undefined for a vector along the *z* axis. Thus, our result is "1.14×10^{3}, θ not defined, and $\phi = 0°$."

(d) Since $\vec{A}$ is in the *xy* plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90°.

(e) Clearly, $\vec{A} + 3.00\,\hat{\text{k}} = -5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}} + 3.00\,\hat{\text{k}}$.

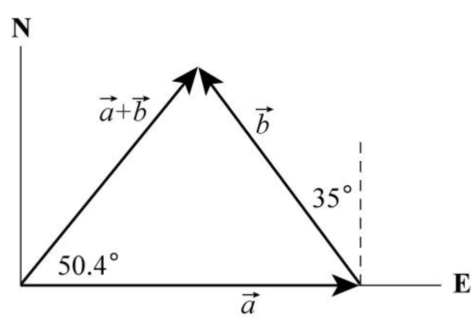
(f) The Pythagorean theorem yields magnitude $A=\sqrt{(5.14)^2+(6.13)^2+(3.00)^2}=8.54$.

The azimuthal angle is $\theta = 130°$, just as it was in the problem statement ($\vec{A}$ is the projection onto the *xy* plane of the new vector created in part (e)). The angle measured from the +*z* axis is

$$\phi = \cos^{-1}(3.00/8.54) = 69.4°.$$

46. The vectors are shown on the diagram. The *x* axis runs from west to east and the *y* axis runs from south to north. Then $a_x = 5.0$ m, $a_y = 0$,

$$b_x = -(4.0\text{ m})\sin 35° = -2.29\text{ m},\quad b_y = (4.0\text{ m})\cos 35° = 3.28\text{ m}.$$



(a) Let $\vec{c} = \vec{a} + \vec{b}$. Then $c_x = a_x + b_x = 5.00\text{ m} - 2.29\text{ m} = 2.71\text{ m}$ and $c_y = a_y + b_y = 0 + 3.28\text{ m} = 3.28\text{ m}$. The magnitude of *c* is

$$c=\sqrt{c_x^2+c_y^2}=\sqrt{(2.71\text{m})^2+(3.28\text{m})^2}=4.2\text{ m}.$$

(b) The angle θ that $\vec{c} = \vec{a} + \vec{b}$ makes with the +x axis is

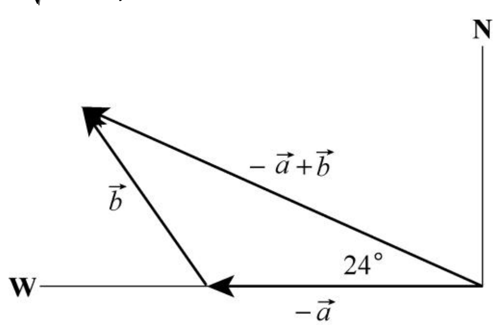
$$\theta = \tan^{-1}\left(\frac{c_y}{c_x}\right) = \tan^{-1}\left(\frac{3.28}{2.71}\right) = 50.5° \approx 50°.$$

The second possibility (θ = 50.4° + 180° = 230.4°) is rejected because it would point in a direction opposite to $\vec{c}$.

(c) The vector $\vec{b} - \vec{a}$ is found by adding $-\vec{a}$ to $\vec{b}$. The result is shown on the diagram to the right. Let $\vec{c} = \vec{b} - \vec{a}$. The components are

$$c_x = b_x - a_x = -2.29 \text{ m} - 5.00 \text{ m} = -7.29 \text{ m}$$
$$c_y = b_y - a_y = 3.28 \text{ m}.$$

The magnitude of $\vec{c}$ is $c = \sqrt{c_x^2 + c_y^2} = 8.0\,\text{m}$.



(d) The tangent of the angle θ that $\vec{c}$ makes with the +x axis (east) is

$$\tan\theta = \frac{c_y}{c_x} = \frac{3.28 \text{ m}}{-7.29 \text{ m}} = -4.50.$$

There are two solutions: –24.2° and 155.8°. As the diagram shows, the second solution is correct. The vector $\vec{c} = -\vec{a} + \vec{b}$ is 24° north of west.

47. Noting that the given 130° is measured counterclockwise from the +x axis, the two vectors can be written as

$$\vec{A} = 8.00(\cos 130°\,\hat{\text{i}} + \sin 130°\,\hat{\text{j}}) = -5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}}$$
$$\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}} = -7.72\,\hat{\text{i}} - 9.20\,\hat{\text{j}}.$$

(a) The angle between the negative direction of the y axis ($-\hat{\text{j}}$) and the direction of $\vec{A}$ is

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot(-\hat{\text{j}})}{A}\right) = \cos^{-1}\left(\frac{-6.13}{\sqrt{(-5.14)^2+(6.13)^2}}\right) = \cos^{-1}\left(\frac{-6.13}{8.00}\right) = 140°.$$

Alternatively, one may say that the $-y$ direction corresponds to an angle of 270°, and the answer is simply given by 270°–130° = 140°.

(b) Since the y axis is in the xy plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90.0°.

(c) The vector can be simplified as

$$\begin{aligned}\vec{A}\times(\vec{B}+3.00\,\hat{\text{k}}) &= (-5.14\,\hat{\text{i}}+6.13\,\hat{\text{j}})\times(-7.72\,\hat{\text{i}}-9.20\,\hat{\text{j}}+3.00\,\hat{\text{k}})\\ &= 18.39\,\hat{\text{i}}+15.42\,\hat{\text{j}}+94.61\,\hat{\text{k}}\end{aligned}$$

Its magnitude is $|\vec{A}\times(\vec{B}+3.00\hat{\text{k}})| = 97.6.$ The angle between the negative direction of the y axis ($-\hat{\text{j}}$) and the direction of the above vector is

$$\theta = \cos^{-1}\left(\frac{-15.42}{97.6}\right) = 99.1°.$$

48. Where the length unit is not displayed, the unit meter is understood.

(a) We first note that the magnitudes of the vectors are $a = |\vec{a}| = \sqrt{(3.2)^2+(1.6)^2} = 3.58$ and $b = |\vec{b}| = \sqrt{(0.50)^2+(4.5)^2} = 4.53$. Now,

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y = ab\cos\phi$$
$$(3.2)(0.50)+(1.6)(4.5) = (3.58)(4.53)\cos\phi$$

which leads to ϕ = 57° (the inverse cosine is double-valued as is the inverse tangent, but we know this is the right solution since both vectors are in the same quadrant).

(b) Since the angle (measured from +x) for $\vec{a}$ is $\tan^{-1}$(1.6/3.2) = 26.6°, we know the angle for $\vec{c}$ is 26.6° –90° = –63.4° (the other possibility, 26.6° + 90° would lead to a $c_x < 0$). Therefore,

$$c_x = c\cos(-63.4°) = (5.0)(0.45) = 2.2 \text{ m}.$$

(c) Also, $c_y = c\sin(-63.4°) = (5.0)(-0.89) = -4.5$ m.

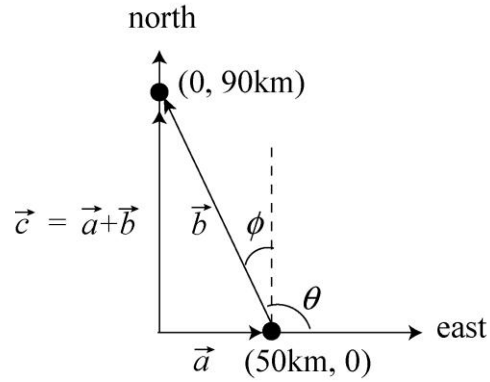
(d) And we know the angle for $\vec{d}$ to be 26.6° + 90° = 116.6°, which leads to

$$d_x = d\cos(116.6°) = (5.0)(-0.45) = -2.2 \text{ m}.$$

(e) Finally, $d_y = d \sin 116.6° = (5.0)(0.89) = 4.5$ m.

49. **THINK** This problem deals with the displacement of a sailboat. We want to find the displacement vector between two locations.

EXPRESS The situation is depicted in the figure below. Let $\vec{a}$ represent the first part of his actual voyage (50.0 km east) and $\vec{c}$ represent the intended voyage (90.0 km north). We look for a vector $\vec{b}$ such that $\vec{c} = \vec{a} + \vec{b}$.



ANALYZE (a) Using the Pythagorean theorem, the distance traveled by the sailboat is

$$b = \sqrt{(50.0 \text{ km})^2 + (90.0 \text{ km})^2} = 103 \text{ km}.$$

(b) The direction is

$$\phi = \tan^{-1}\left(\frac{50.0 \text{ km}}{90.0 \text{ km}}\right) = 29.1°$$

west of north (which is equivalent to 60.9° north of due west).

LEARN This problem could also be solved by first expressing the vectors in unit-vector notation: $\vec{a} = (50.0 \text{ km})\hat{\text{i}}, \vec{c} = (90.0 \text{ km})\hat{\text{j}}$. This gives

$$\vec{b} = \vec{c} - \vec{a} = -(50.0 \text{ km})\hat{\text{i}} + (90.0 \text{ km})\hat{\text{j}}.$$

The angle between $\vec{b}$ and the $+x$-axis is

$$\theta = \tan^{-1}\left(\frac{90.0 \text{ km}}{-50.0 \text{ km}}\right) = 119.1°.$$

The angle θ is related to ϕ by $\theta = 90° + \phi$.

50. The two vectors $\vec{d}_1$ and $\vec{d}_2$ are given by $\vec{d}_1 = -d_1\,\hat{\text{j}}$ and $\vec{d}_2 = d_2\,\hat{\text{i}}$.

(a) The vector $\vec{d}_2/4 = (d_2/4)\,\hat{\text{i}}$ points in the +x direction. The ¼ factor does not affect the result.

(b) The vector $\vec{d}_1/(-4) = (d_1/4)\hat{\text{j}}$ points in the +y direction. The minus sign (with the "–4") does affect the direction: −(–y) = + y.

(c) $\vec{d}_1 \cdot \vec{d}_2 = 0$ since $\hat{\text{i}} \cdot \hat{\text{j}} = 0$. The two vectors are perpendicular to each other.

(d) $\vec{d}_1 \cdot (\vec{d}_2/4) = (\vec{d}_1 \cdot \vec{d}_2)/4 = 0$, as in part (c).

(e) $\vec{d}_1 \times \vec{d}_2 = -d_1 d_2 (\hat{\text{j}} \times \hat{\text{i}}) = d_1 d_2\,\hat{\text{k}}$, in the +$z$-direction.

(f) $\vec{d}_2 \times \vec{d}_1 = -d_2 d_1 (\hat{\text{i}} \times \hat{\text{j}}) = -d_1 d_2\,\hat{\text{k}}$, in the −$z$-direction.

(g) The magnitude of the vector in (e) is $d_1 d_2$.

(h) The magnitude of the vector in (f) is $d_1 d_2$.

(i) Since $d_1 \times (\vec{d}_2/4) = (d_1 d_2/4)\hat{\text{k}}$, the magnitude is $d_1 d_2/4$.

(j) The direction of $\vec{d}_1 \times (\vec{d}_2/4) = (d_1 d_2/4)\hat{\text{k}}$ is in the +z-direction.

51. Although we think of this as a three-dimensional movement, it is rendered effectively two-dimensional by referring measurements to its well-defined plane of the fault.

(a) The magnitude of the net displacement is

$$|\vec{AB}| = \sqrt{|AD|^2 + |AC|^2} = \sqrt{(17.0\text{ m})^2 + (22.0\text{ m})^2} = 27.8\text{m}.$$

(b) The magnitude of the vertical component of $\vec{AB}$ is $|AD|$ sin 52.0° = 13.4 m.

52. The three vectors are

$$\vec{d}_1 = 4.0\,\hat{\text{i}} + 5.0\hat{\text{j}} - 6.0\,\hat{\text{k}}$$
$$\vec{d}_2 = -1.0\,\hat{\text{i}} + 2.0\hat{\text{j}} + 3.0\,\hat{\text{k}}$$
$$\vec{d}_3 = 4.0\,\hat{\text{i}} + 3.0\hat{\text{j}} + 2.0\,\hat{\text{k}}$$

(a) $\vec{r} = \vec{d}_1 - \vec{d}_2 + \vec{d}_3 = (9.0\text{ m})\hat{\text{i}} + (6.0\text{ m})\hat{\text{j}} + (-7.0\text{ m})\hat{\text{k}}$.

(b) The magnitude of $\vec{r}$ is $|\vec{r}| = \sqrt{(9.0\text{ m})^2 + (6.0\text{ m})^2 + (-7.0\text{ m})^2} = 12.9\text{ m}$. The angle between $\vec{r}$ and the z-axis is given by

$$\cos\theta = \frac{\vec{r}\cdot\hat{\text{k}}}{|\vec{r}|} = \frac{-7.0\text{ m}}{12.9\text{ m}} = -0.543$$

which implies $\theta = 123°$.

(c) The component of $\vec{d}_1$ along the direction of $\vec{d}_2$ is given by $d_{\parallel} = \vec{d}_1 \cdot \hat{\text{u}} = d_1\cos\varphi$ where φ is the angle between $\vec{d}_1$ and $\vec{d}_2$, and $\hat{\text{u}}$ is the unit vector in the direction of $\vec{d}_2$. Using the properties of the scalar (dot) product, we have

$$d_{\parallel} = d_1\left(\frac{\vec{d}_1\cdot\vec{d}_2}{d_1 d_2}\right) = \frac{\vec{d}_1\cdot\vec{d}_2}{d_2} = \frac{(4.0)(-1.0)+(5.0)(2.0)+(-6.0)(3.0)}{\sqrt{(-1.0)^2+(2.0)^2+(3.0)^2}} = \frac{-12}{\sqrt{14}} = -3.2\text{ m.}$$

(d) Now we are looking for $d_{\perp}$ such that $d_1^2 = (4.0)^2 + (5.0)^2 + (-6.0)^2 = 77 = d_{\parallel}^2 + d_{\perp}^2$. From (c), we have

$$d_{\perp} = \sqrt{77\text{ m}^2 - (-3.2\text{ m})^2} = 8.2\text{ m.}$$

This gives the magnitude of the perpendicular component (and is consistent with what one would get using Eq. 3-24), but if more information (such as the direction, or a full specification in terms of unit vectors) is sought then more computation is needed.

53. **THINK** This problem involves finding scalar and vector products between two vectors $\vec{a}$ and $\vec{b}$.

EXPRESS We apply Eqs. 3-20 and 3-24 to calculate the scalar and vector products between two vectors:

$$\vec{a}\cdot\vec{b} = ab\cos\phi$$
$$|\vec{a}\times\vec{b}| = ab\sin\phi.$$

ANALYZE (a) Given that $a = |\vec{a}| = 10$, $b = |\vec{b}| = 6.0$ and $\phi = 60°$, the scalar (dot) product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\cdot\vec{b} = ab\cos\phi = (10)\,(6.0)\cos 60° = 30.$$

(b) Similarly, the magnitude of the vector (cross) product of the two vectors is

$$|\vec{a}\times\vec{b}| = ab\sin\phi = (10)\,(6.0)\sin 60° = 52.$$

LEARN When two vectors $\vec{a}$ and $\vec{b}$ are parallel ($\phi = 0$), their scalar and vector products are $\vec{a}\cdot\vec{b} = ab\cos\phi = ab$ and $|\vec{a}\times\vec{b}| = ab\sin\phi = 0$, respectively. However, when they are perpendicular ($\phi = 90°$), we have $\vec{a}\cdot\vec{b} = ab\cos\phi = 0$ and $|\vec{a}\times\vec{b}| = ab\sin\phi = ab$.

54. From the figure, it is clear that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $\vec{a}\cdot\vec{b} = 0$ since the angle between them is 90º.

(b) $\vec{a}\cdot\vec{c} = \vec{a}\cdot(-\vec{a} - \vec{b}) = -|\vec{a}|^2 = -16$.

(c) Similarly, $\vec{b}\cdot\vec{c} = -9.0$.

55. We choose +x east and +y north and measure all angles in the "standard" way (positive ones are counterclockwise from +x). Thus, vector $\vec{d}_1$ has magnitude d_1 = 4.00 m (with the unit meter) and direction θ_1 = 225°. Also, $\vec{d}_2$ has magnitude d_2 = 5.00 m and direction θ_2 = 0°, and vector $\vec{d}_3$ has magnitude d_3 = 6.00 m and direction θ_3 = 60°.

(a) The x-component of $\vec{d}_1$ is $d_{1x} = d_1\cos\theta_1 = -2.83$ m.

(b) The y-component of $\vec{d}_1$ is $d_{1y} = d_1\sin\theta_1 = -2.83$ m.

(c) The x-component of $\vec{d}_2$ is $d_{2x} = d_2\cos\theta_2 = 5.00$ m.

(d) The y-component of $\vec{d}_2$ is $d_{2y} = d_2\sin\theta_2 = 0$.

(e) The x-component of $\vec{d}_3$ is $d_{3x} = d_3\cos\theta_3 = 3.00$ m.

(f) The y-component of $\vec{d}_3$ is $d_{3y} = d_3\sin\theta_3 = 5.20$ m.

(g) The sum of x-components is

$$d_x = d_{1x} + d_{2x} + d_{3x} = -2.83 \text{ m} + 5.00 \text{ m} + 3.00 \text{ m} = 5.17 \text{ m}.$$

(h) The sum of y-components is

$$d_y = d_{1y} + d_{2y} + d_{3y} = -2.83 \text{ m} + 0 + 5.20 \text{ m} = 2.37 \text{ m}.$$

(i) The magnitude of the resultant displacement is

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(5.17\text{ m})^2 + (2.37\text{ m})^2} = 5.69\text{ m}.$$

(j) And its angle is

$$\theta = \tan^{-1}(2.37/5.17) = 24.6°,$$

which (recalling our coordinate choices) means it points at about 25° north of east.

(k) and (l) This new displacement (the direct line home) when vectorially added to the previous (net) displacement must give zero. Thus, the new displacement is the negative, or opposite, of the previous (net) displacement. That is, it has the same magnitude (5.69 m) but points in the opposite direction (25° south of west).

56. If we wish to use Eq. 3-5 directly, we should note that the angles for $\vec{Q}, \vec{R}$, and $\vec{S}$ are 100°, 250°, and 310°, respectively, if they are measured counterclockwise from the $+x$ axis.

(a) Using unit-vector notation, with the unit meter understood, we have

$$\begin{aligned}\vec{P} &= 10.0\cos(25.0°)\,\hat{\text{i}} + 10.0\sin(25.0°)\,\hat{\text{j}}\\ \vec{Q} &= 12.0\cos(100°)\,\hat{\text{i}} + 12.0\sin(100°)\,\hat{\text{j}}\\ \vec{R} &= 8.00\cos(250°)\,\hat{\text{i}} + 8.00\sin(250°)\,\hat{\text{j}}\\ \vec{S} &= 9.00\cos(310°)\,\hat{\text{i}} + 9.00\sin(310°)\,\hat{\text{j}}\end{aligned}$$

$$\vec{P} + \vec{Q} + \vec{R} + \vec{S} = (10.0\text{ m})\hat{\text{i}} + (1.63\text{ m})\hat{\text{j}}$$

(b) The magnitude of the vector sum is $\sqrt{(10.0\text{ m})^2 + (1.63\text{ m})^2} = 10.2\text{ m}$.

(c) The angle is $\tan^{-1}(1.63\text{ m}/10.0\text{ m}) \approx 9.24°$ measured counterclockwise from the $+x$ axis.

57. **THINK** This problem deals with addition and subtraction of two vectors.

EXPRESS From the problem statement, we have

$$\vec{A} + \vec{B} = (6.0)\hat{\text{i}} + (1.0)\hat{\text{j}}, \qquad \vec{A} - \vec{B} = -(4.0)\hat{\text{i}} + (7.0)\hat{\text{j}}$$

Solving the simultaneous equations gives $\vec{A}$ and $\vec{B}$.

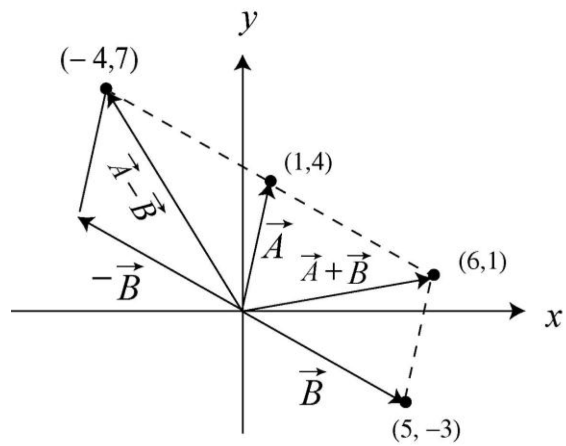
ANALYZE Adding the above equations and dividing by 2 leads to $\vec{A} = (1.0)\hat{\mathrm{i}} + (4.0)\hat{\mathrm{j}}$. The magnitude of $\vec{A}$ is

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(1.0)^2 + (4.0)^2} = 4.1$$

LEARN The vector $\vec{B}$ is $\vec{B} = (5.0)\hat{\mathrm{i}} + (-3.0)\hat{\mathrm{j}}$, and its magnitude is

$$B = |\vec{B}| = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.0)^2 + (-3.0)^2} = 5.8 .$$

The results are summarized in the figure to the right.



58. The vector can be written as $\vec{d} = (2.5\ \mathrm{m})\hat{\mathrm{j}}$, where we have taken $\hat{\mathrm{j}}$ to be the unit vector pointing north.

(a) The magnitude of the vector $\vec{a} = 4.0\,\vec{d}$ is (4.0)(2.5 m) = 10 m.

(b) The direction of the vector $\vec{a} = 4.0\vec{d}$ is the same as the direction of $\vec{d}$ (north).

(c) The magnitude of the vector $\vec{c} = -3.0\vec{d}$ is (3.0)(2.5 m) = 7.5 m.

(d) The direction of the vector $\vec{c} = -3.0\vec{d}$ is the opposite of the direction of $\vec{d}$. Thus, the direction of $\vec{c}$ is south.

59. Reference to Figure 3-18 (and the accompanying material in that section) is helpful. If we convert $\vec{B}$ to the magnitude-angle notation (as $\vec{A}$ already is) we have $\vec{B} = (14.4 \angle 33.7°)$ (appropriate notation especially if we are using a vector capable calculator in polar mode). Where the length unit is not displayed in the solution, the unit meter should be understood. In the magnitude-angle notation, rotating the axis by +20° amounts to subtracting that angle from the angles previously specified. Thus, $\vec{A} = (12.0 \angle 40.0°)'$ and $\vec{B} = (14.4 \angle 13.7°)'$, where the 'prime' notation indicates that the description is in terms of the new coordinates. Converting these results to (*x, y*) representations, we obtain

(a) $\vec{A} = (9.19\ \mathrm{m})\,\hat{\mathrm{i}}' + (7.71\ \mathrm{m})\,\hat{\mathrm{j}}'$.

(b) Similarly, $\vec{B} = (14.0\text{ m})\,\hat{\text{i}}' + (3.41\text{ m})\,\hat{\text{j}}'$.

60. The two vectors can be found be solving the simultaneous equations.

(a) If we add the equations, we obtain $2\vec{a} = 6\vec{c}$, which leads to $\vec{a} = 3\vec{c} = 9\,\hat{\text{i}} + 12\,\hat{\text{j}}$.

(b) Plugging this result back in, we find $\vec{b} = \vec{c} = 3\hat{\text{i}} + 4\hat{\text{j}}$.

61. The three vectors given are

$$\begin{aligned}\vec{a} &= \ \ 5.0\ \hat{\text{i}} + 4.0\ \hat{\text{j}} - 6.0\ \hat{\text{k}} \\ \vec{b} &= -2.0\ \hat{\text{i}} + 2.0\ \hat{\text{j}} + 3.0\ \hat{\text{k}} \\ \vec{c} &= \ \ 4.0\ \hat{\text{i}} + 3.0\ \hat{\text{j}} + 2.0\ \hat{\text{k}}\end{aligned}$$

(a) The vector equation $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ is

$$\begin{aligned}\vec{r} &= [5.0 - (-2.0) + 4.0]\hat{\text{i}} + (4.0 - 2.0 + 3.0)\hat{\text{j}} + (-6.0 - 3.0 + 2.0)\hat{\text{k}} \\ &= 11\,\hat{\text{i}} + 5.0\hat{\text{j}} - 7.0\hat{\text{k}}.\end{aligned}$$

(b) We find the angle from $+z$ by "dotting" (taking the scalar product) $\vec{r}$ with $\hat{\text{k}}$. Noting that

$$r = |\vec{r}\,| = \sqrt{(11.0)^2 + (5.0)^2 + (-7.0)^2} = 14,$$

Eq. 3-20 with Eq. 3-23 leads to

$$\vec{r} \cdot \vec{\text{k}} = -7.0 = (14)(1)\cos\phi \ \ \Rightarrow \ \ \phi = 120°.$$

(c) To find the component of a vector in a certain direction, it is efficient to "dot" it (take the scalar product of it) with a unit-vector in that direction. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}\,|} = \frac{-2.0\hat{\text{i}} + 2.0\hat{\text{j}} + 3.0\hat{\text{k}}}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}}.$$

We therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{(5.0)(-2.0) + (4.0)(2.0) + (-6.0)(3.0)}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}} = -4.9.$$

(d) One approach (if all we require is the magnitude) is to use the vector cross product, as the problem suggests; another (which supplies more information) is to subtract the result in part (c) (multiplied by $\hat{b}$) from $\vec{a}$. We briefly illustrate both methods. We note that if

a cos θ (where θ is the angle between $\vec{a}$ and $\vec{b}$) gives a_b (the component along $\hat{b}$) then we expect a sin θ to yield the orthogonal component:

$$a\sin\theta = \frac{|\vec{a}\times\vec{b}|}{b} = 7.3$$

(alternatively, one might compute θ form part (c) and proceed more directly). The second method proceeds as follows:

$$\begin{aligned}\vec{a} - a_b\hat{b} &= (5.0 - 2.35)\hat{\mathrm{i}} + (4.0 - (-2.35))\hat{\mathrm{j}} + ((-6.0) - (-3.53))\hat{\mathrm{k}} \\ &= 2.65\hat{\mathrm{i}} + 6.35\hat{\mathrm{j}} - 2.47\hat{\mathrm{k}}\end{aligned}$$

This describes the perpendicular part of $\vec{a}$ completely. To find the magnitude of this part, we compute

$$\sqrt{(2.65)^2 + (6.35)^2 + (-2.47)^2} = 7.3$$

which agrees with the first method.

62. We choose +x east and +y north and measure all angles in the “standard” way (positive ones counterclockwise from +x, negative ones clockwise). Thus, vector $\vec{d}_1$ has magnitude d_1 = 3.66 (with the unit meter and three significant figures assumed) and direction θ_1 = 90°. Also, $\vec{d}_2$ has magnitude d_2 = 1.83 and direction θ_2 = –45°, and vector $\vec{d}_3$ has magnitude d_3 = 0.91 and direction θ_3 = –135°. We add the x and y components, respectively:

$$\begin{aligned}x&:\ d_1\cos\theta_1 + d_2\cos\theta_2 + d_3\cos\theta_3 = 0.65\ \text{m} \\ y&:\ d_1\sin\theta_1 + d_2\sin\theta_2 + d_3\sin\theta_3 = 1.7\ \text{m}.\end{aligned}$$

(a) The magnitude of the direct displacement (the vector sum $\vec{d}_1 + \vec{d}_2 + \vec{d}_3$) is $\sqrt{(0.65\ \text{m})^2 + (1.7\ \text{m})^2} = 1.8$ m.

(b) The angle (understood in the sense described above) is $\tan^{-1}$ (1.7/0.65) = 69°. That is, the first putt must aim in the direction 69° north of east.

63. The three vectors are

$$\begin{aligned}\vec{d}_1 &= -3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}} \\ \vec{d}_2 &= -2.0\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}} \\ \vec{d}_3 &= 2.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 1.0\,\hat{\mathrm{k}}.\end{aligned}$$

(a) Since $\vec{d}_2 + \vec{d}_3 = 0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}$, we have

$$\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3) = (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \cdot (0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}) = 0 - 3.0 + 6.0 = 3.0 \text{ m}^2.$$

(b) Using Eq. 3-27, we obtain $\vec{d}_2 \times \vec{d}_3 = -10\,\hat{\mathrm{i}} + 6.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}$. Thus,

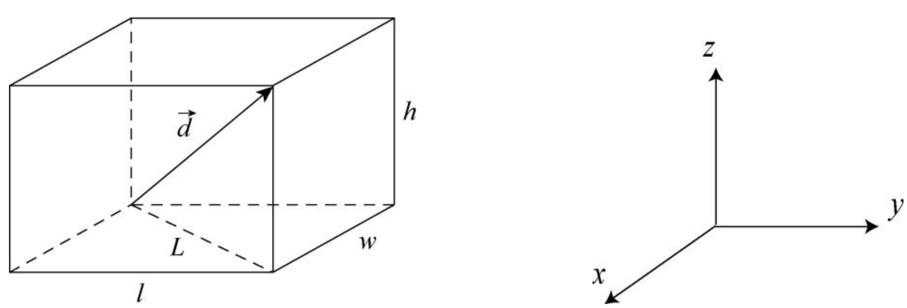
$$\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3) = (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \cdot (-10\,\hat{\mathrm{i}} + 6.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) = 30 + 18 + 4.0 = 52 \text{ m}^3.$$

(c) We found $\vec{d}_2 + \vec{d}_3$ in part (a). Use of Eq. 3-27 then leads to

$$\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3) = (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \times (0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}) = (11\,\hat{\mathrm{i}} + 9.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}) \text{ m}^2$$

64. **THINK** This problem deals with the displacement and distance traveled by a fly from one corner of a room to the diagonally opposite corner. The displacement vector is three-dimensional.

EXPRESS The displacement of the fly is illustrated in the figure below:



A coordinate system such as the one shown (above right) allows us to express the displacement as a three-dimensional vector.

ANALYZE (a) The magnitude of the displacement from one corner to the diagonally opposite corner is

$$d = |\vec{d}| = \sqrt{w^2 + l^2 + h^2}$$

Substituting the values given, we obtain

$$d = |\vec{d}| = \sqrt{w^2 + l^2 + h^2} = \sqrt{(3.70 \text{ m})^2 + (4.30 \text{ m})^2 + (3.00 \text{ m})^2} = 6.42 \text{ m}.$$

(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than *d*, the magnitude of the displacement.

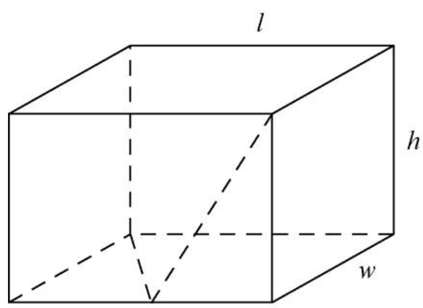
(c) The length of the path of the fly can be greater than *d*, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h = 11.0$ m.

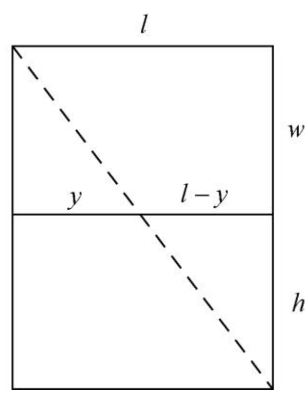
(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the *x* axis to be out of the page, the *y* axis to be to the right, and the *z* axis to be upward (as shown in the figure above). Then the *x* component of the displacement is w = 3.70 m, the *y* component of the displacement is 4.30 m, and the *z* component is 3.00 m. Thus, the displacement vector can be written as

$$\vec{d} = (3.70\text{ m})\hat{\text{i}} + (4.30\text{ m})\hat{\text{j}} + (3.00\text{ m})\hat{\text{k}}.$$

(f) Suppose the path of the fly is as shown by the dotted lines on the diagram (below left). Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown (above right).





The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$s_{\min} = \sqrt{(w+h)^2 + l^2} = \sqrt{(3.70\text{ m} + 3.00\text{ m})^2 + (4.30\text{ m})^2} = 7.96\text{ m}.$$

LEARN To show that the shortest path is indeed given by $s_{\min}$, we write the length of the path as

$$s = \sqrt{y^2 + w^2} + \sqrt{(l-y)^2 + h^2}\ .$$

The condition for minimum is given by

$$\frac{ds}{dy} = \frac{y}{\sqrt{y^2+w^2}} - \frac{l-y}{\sqrt{(l-y)^2+h^2}} = 0 .$$

A little algebra shows that the condition is satisfied when $y = lw/(w+h)$, which gives

$$s_{\min} = \sqrt{w^2\left(1+\frac{l^2}{(w+h)^2}\right)} + \sqrt{h^2\left(1+\frac{l^2}{(w+h)^2}\right)} = \sqrt{(w+h)^2+l^2} .$$

Any other path would be longer than 7.96 m.

65. (a) This is one example of an answer: $(-40\,\hat{\mathrm{i}} - 20\,\hat{\mathrm{j}} + 25\,\hat{\mathrm{k}})$ m, with $\hat{\mathrm{i}}$ directed anti-parallel to the first path, $\hat{\mathrm{j}}$ directed anti-parallel to the second path, and $\hat{\mathrm{k}}$ directed upward (in order to have a right-handed coordinate system). Other examples include $(40\,\hat{\mathrm{i}} + 20\,\hat{\mathrm{j}} + 25\,\hat{\mathrm{k}})$ m and $(40\hat{\mathrm{i}} - 20\,\hat{\mathrm{j}} - 25\,\hat{\mathrm{k}})$ m (with slightly different interpretations for the unit vectors). Note that the product of the components is positive in each example.

(b) Using the Pythagorean theorem, we have $\sqrt{(40\text{ m})^2+(20\text{ m})^2}$ = 44.7 m ≈ 45 m.

66. The vectors can be written as $\vec{a} = a\hat{\mathrm{i}}$ and $\vec{b} = b\hat{\mathrm{j}}$ where $a,\ b > 0$.

(a) We are asked to consider

$$\frac{\vec{b}}{d} = \left(\frac{b}{d}\right)\hat{\mathrm{j}}$$

in the case $d > 0$. Since the coefficient of $\hat{\mathrm{j}}$ is positive, then the vector points in the $+y$ direction.

(b) If, however, $d < 0$, then the coefficient is negative and the vector points in the $-y$ direction.

(c) Since cos 90° = 0, then $\vec{a} \cdot \vec{b} = 0$, using Eq. 3-20.

(d) Since $\vec{b}/d$ is along the y axis, then (by the same reasoning as in the previous part) $\vec{a}\cdot(\vec{b}/d) = 0$.

(e) By the right-hand rule, $\vec{a} \times \vec{b}$ points in the $+z$-direction.

(f) By the same rule, $\vec{b} \times \vec{a}$ points in the $-z$-direction. We note that $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$ is true in this case and quite generally.

(g) Since sin 90° = 1, Eq. 3-24 gives $|\vec{a}\times\vec{b}|=ab$ where a is the magnitude of $\vec{a}$.

(h) Also, $|\vec{a}\times\vec{b}|=|\vec{b}\times\vec{a}|=ab$.

(i) With $d > 0$, we find that $\vec{a}\times(\vec{b}/d)$ has magnitude ab/d.

(j) The vector $\vec{a}\times(\vec{b}/d)$ points in the $+z$ direction.

67. We note that the set of choices for unit vector directions has correct orientation (for a right-handed coordinate system). Students sometimes confuse "north" with "up", so it might be necessary to emphasize that these are being treated as the mutually perpendicular directions of our real world, not just some "on the paper" or "on the blackboard" representation of it. Once the terminology is clear, these questions are basic to the definitions of the scalar (dot) and vector (cross) products.

(a) $\hat{\mathrm{i}}\cdot\hat{\mathrm{k}}=0$ since $\hat{\mathrm{i}}\perp\hat{\mathrm{k}}$

(b) $(-\hat{\mathrm{k}})\cdot(-\hat{\mathrm{j}})=0$ since $\hat{\mathrm{k}}\perp\hat{\mathrm{j}}$.

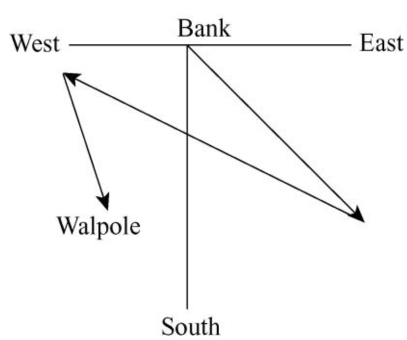
(c) $\hat{\mathrm{j}}\cdot(-\hat{\mathrm{j}})=-1$.

(d) $\hat{\mathrm{k}}\times\hat{\mathrm{j}}=-\hat{\mathrm{i}}$ (west).

(e) $(-\hat{\mathrm{i}})\times(-\hat{\mathrm{j}})=+\hat{\mathrm{k}}$ (upward).

(f) $(-\hat{\mathrm{k}})\times(-\hat{\mathrm{j}})=-\hat{\mathrm{i}}$ (west).

68. A sketch of the displacements is shown. The resultant (not shown) would be a straight line from start (Bank) to finish (Walpole). With a careful drawing, one should find that the resultant vector has length 29.5 km at 35° west of south.



69. The point P is displaced vertically by $2R$, where R is the radius of the wheel. It is displaced horizontally by half the circumference of the wheel, or πR. Since R = 0.450 m,

the horizontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. If the x axis is horizontal and the y axis is vertical, the vector displacement (in meters) is $\vec{r} = \left(1.414\ \hat{\text{i}} + 0.900\ \hat{\text{j}}\right)$. The displacement has a magnitude of

$$|\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4} = 1.68\,\text{m}$$

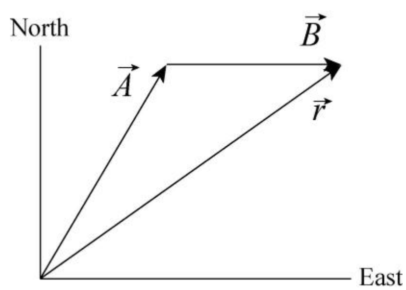
and an angle of

$$\tan^{-1}\left(\frac{2R}{\pi R}\right) = \tan^{-1}\left(\frac{2}{\pi}\right) = 32.5°$$

above the floor. In physics there are no "exact" measurements, yet that angle computation seemed to yield something *exact*. However, there has to be some uncertainty in the observation that the wheel rolled half of a revolution, which introduces some indefiniteness in our result.

70. The diagram shows the displacement vectors for the two segments of her walk, labeled $\vec{A}$ and $\vec{B}$, and the total ("final") displacement vector, labeled $\vec{r}$. We take east to be the $+x$ direction and north to be the $+y$ direction. We observe that the angle between $\vec{A}$ and the x axis is 60°. Where the units are not explicitly shown, the distances are understood to be in meters. Thus, the components of $\vec{A}$ are $A_x = 250 \cos 60° = 125$ and $A_y = 250 \sin 60° = 216.5$. The components of $\vec{B}$ are $B_x = 175$ and $B_y = 0$. The components of the total displacement are

$$r_x = A_x + B_x = 125 + 175 = 300$$
$$r_y = A_y + B_y = 216.5 + 0 = 216.5.$$



(a) The magnitude of the resultant displacement is

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(300\text{ m})^2 + (216.5\text{ m})^2} = 370\,\text{m}.$$

(b) The angle the resultant displacement makes with the $+x$ axis is

$$\tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{216.5\text{ m}}{300\text{ m}}\right) = 36°.$$

The direction is $36°$ north of due east.

(c) The total *distance* walked is d = 250 m + 175 m = 425 m.

(d) The total distance walked is greater than the magnitude of the resultant displacement. The diagram shows why: $\vec{A}$ and $\vec{B}$ are not collinear.

71. The vector $\vec{d}$ (measured in meters) can be represented as $\vec{d} = (3.0 \text{ m})(-\hat{\text{j}})$, where $-\hat{\text{j}}$ is the unit vector pointing south. Therefore, $5.0\vec{d} = 5.0(-3.0 \text{ m } \hat{\text{j}}) = (-15 \text{ m})\,\hat{\text{j}}$.

(a) The positive scalar factor (5.0) affects the magnitude but not the direction. The magnitude of $5.0\vec{d}$ is 15 m.

(b) The new direction of $5\vec{d}$ is the same as the old: south.

The vector $-2.0\vec{d}$ can be written as $-2.0\vec{d} = (6.0 \text{ m})\,\hat{\text{j}}$.

(c) The absolute value of the scalar factor (|–2.0| = 2.0) affects the magnitude. The new magnitude is 6.0 m.

(d) The minus sign carried by this scalar factor reverses the direction, so the new direction is $+\hat{\text{j}}$, or north.

72. The ant's trip consists of three displacements:

$$\vec{d}_1 = (0.40 \text{ m})(\cos 225°\,\hat{\text{i}} + \sin 225°\,\hat{\text{j}}) = (-0.28 \text{ m})\,\hat{\text{i}} + (-0.28 \text{ m})\,\hat{\text{j}}$$
$$\vec{d}_2 = (0.50 \text{ m})\,\hat{\text{i}}$$
$$\vec{d}_3 = (0.60 \text{ m})(\cos 60°\,\hat{\text{i}} + \sin 60°\,\hat{\text{j}}) = (0.30 \text{ m})\,\hat{\text{i}} + (0.52 \text{ m})\,\hat{\text{j}},$$

where the angle is measured with respect to the positive x axis. We have taken the positive x and y directions to correspond to east and north, respectively.

(a) The x component of $\vec{d}_1$ is $d_{1x} = (0.40 \text{ m})\cos 225° = -0.28 \text{ m}$.

(b) The y component of $\vec{d}_1$ is $d_{1y} = (0.40 \text{ m})\sin 225° = -0.28 \text{ m}$.

(c) The x component of $\vec{d}_2$ is $d_{2x} = 0.50 \text{ m}$.

(d) The y component of $\vec{d}_2$ is $d_{2y} = 0 \text{ m}$.

(e) The *x* component of $\vec{d}_3$ is $d_{3x} = (0.60\text{ m})\cos 60° = 0.30\text{ m}$.

(f) The *y* component of $\vec{d}_3$ is $d_{3y} = (0.60\text{ m})\sin 60° = 0.52\text{ m}$.

(g) The *x* component of the net displacement $\vec{d}_{\text{net}}$ is

$$d_{\text{net},x} = d_{1x} + d_{2x} + d_{3x} = (-0.28\text{ m}) + (0.50\text{ m}) + (0.30\text{ m}) = 0.52\text{ m}.$$

(h) The *y* component of the net displacement $\vec{d}_{\text{net}}$ is

$$d_{\text{net},y} = d_{1y} + d_{2y} + d_{3y} = (-0.28\text{ m}) + (0\text{ m}) + (0.52\text{ m}) = 0.24\text{ m}.$$

(i) The magnitude of the net displacement is

$$d_{\text{net}} = \sqrt{d_{\text{net},x}^2 + d_{\text{net},y}^2} = \sqrt{(0.52\text{ m})^2 + (0.24\text{ m})^2} = 0.57\text{ m}.$$

(j) The direction of the net displacement is

$$\theta = \tan^{-1}\left(\frac{d_{\text{net},y}}{d_{\text{net},x}}\right) = \tan^{-1}\left(\frac{0.24\text{ m}}{0.52\text{ m}}\right) = 25° \text{ (north of east)}$$

If the ant has to return directly to the starting point, the displacement would be $-\vec{d}_{\text{net}}$.
(k) The distance the ant has to travel is $|-\vec{d}_{\text{net}}| = 0.57\text{ m}.$

(l) The direction the ant has to travel is $25°\text{ (south of west)}$.

73. We apply Eq. 3-23 and Eq. 3-27.

(a) $\vec{a}\times\vec{b} = (a_x b_y - a_y b_x)\,\hat{\text{k}}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any *z* components. Consequently, we obtain $((3.0)(4.0) - (5.0)(2.0))\hat{\text{k}} = 2.0\hat{\text{k}}$.

(b) $\vec{a}\cdot b = a_x b_x + a_y b_y$ yields (3.0)(2.0) + (5.0)(4.0) = 26.

(c) $\vec{a} + \vec{b} = (3.0 + 2.0)\,\hat{\text{i}} + (5.0 + 4.0)\,\hat{\text{j}} \Rightarrow (\vec{a} + \vec{b})\cdot\vec{b} = (5.0)\,(2.0) + (9.0)\,(4.0) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and "dot" it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}}}{\sqrt{(2.0)^2 + (4.0)^2}}.$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(3.0)(2.0) + (5.0)(4.0)}{\sqrt{(2.0)^2 + (4.0)^2}} = 5.81.$$

74. The two vectors $\vec{a}$ and $\vec{b}$ are given by

$$\vec{a} = 3.20(\cos 63°\,\hat{\mathrm{j}} + \sin 63°\,\hat{\mathrm{k}}) = 1.45\,\hat{\mathrm{j}} + 2.85\,\hat{\mathrm{k}}$$

$$\vec{b} = 1.40(\cos 48°\,\hat{\mathrm{i}} + \sin 48°\,\hat{\mathrm{k}}) = 0.937\hat{\mathrm{i}} + 1.04\,\hat{\mathrm{k}}$$

The components of $\vec{a}$ are $a_x = 0$, $a_y = 3.20 \cos 63° = 1.45$, and $a_z = 3.20 \sin 63° = 2.85$. The components of $\vec{b}$ are $b_x = 1.40 \cos 48° = 0.937$, $b_y = 0$, and $b_z = 1.40 \sin 48° = 1.04$.

(a) The scalar (dot) product is therefore

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = (0)(0.937) + (1.45)(0) + (2.85)(1.04) = 2.97.$$

(b) The vector (cross) product is

$$\begin{aligned}\vec{a}\times\vec{b} &= \left(a_y b_z - a_z b_y\right)\hat{\mathrm{i}} + \left(a_z b_x - a_x b_z\right)\hat{\mathrm{j}} + \left(a_x b_y - a_y b_x\right)\hat{\mathrm{k}} \\ &= \left((1.45)(1.04) - 0\right)\hat{\mathrm{i}} + \left((2.85)(0.937) - 0\right)\hat{\mathrm{j}} + \left(0 - (1.45)(0.937)\right)\hat{\mathrm{k}} \\ &= 1.51\hat{\mathrm{i}} + 2.67\hat{\mathrm{j}} - 1.36\hat{\mathrm{k}}.\end{aligned}$$

(c) The angle θ between $\vec{a}$ and $\vec{b}$ is given by

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{ab}\right) = \cos^{-1}\left(\frac{2.97}{(3.20)(1.40)}\right) = 48.5°.$$

75. We orient $\hat{\mathrm{i}}$ eastward, $\hat{\mathrm{j}}$ northward, and $\hat{\mathrm{k}}$ upward, and use the following fundamental products:

$$\begin{aligned}\hat{\mathrm{i}}\times\hat{\mathrm{j}} &= -\hat{\mathrm{j}}\times\hat{\mathrm{i}} = \hat{\mathrm{k}} \\ \hat{\mathrm{j}}\times\hat{\mathrm{k}} &= -\hat{\mathrm{k}}\times\hat{\mathrm{j}} = \hat{\mathrm{i}} \\ \hat{\mathrm{k}}\times\hat{\mathrm{i}} &= -\hat{\mathrm{i}}\times\hat{\mathrm{k}} = \hat{\mathrm{j}}\end{aligned}$$

(a) "north cross west" = $\hat{\mathrm{j}}\times(-\hat{\mathrm{i}}) = \hat{\mathrm{k}}$ = "up."

(b) "down dot south" = $(-\hat{k})\cdot(-\hat{j}) = 0$.

(c) "east cross up" = $\hat{i}\times(\hat{k}) = -\hat{j}$ = "south."

(d) "west dot west" = $(-\hat{i})\cdot(-\hat{i}) = 1$.

(e) "south cross south" = $(-\hat{j})\times(-\hat{j}) = 0$.

76. Let A denote the magnitude of $\vec{A}$; similarly for the other vectors. The vector equation is $\vec{A} + \vec{B} = \vec{C}$ where B = 8.0 m and C = 2A. We are also told that the angle (measured in the 'standard' sense) for $\vec{A}$ is 0° and the angle for $\vec{C}$ is 90°, which makes this a right triangle (when drawn in a "head-to-tail" fashion) where B is the size of the hypotenuse. Using the Pythagorean theorem,

$$B = \sqrt{A^2 + C^2} \Rightarrow 8.0 = \sqrt{A^2 + 4A^2}$$

which leads to $A = 8/\sqrt{5} = 3.6$ m.

77. We orient $\hat{i}$ eastward, $\hat{j}$ northward, and $\hat{k}$ upward.

(a) The displacement is $\vec{d} = (1300\text{ m})\hat{i} + (2200\text{ m})\hat{j} + (-410\text{ m})\hat{k}$.

(b) The displacement for the return portion is $\vec{d}' = -(1300\text{ m})\hat{i} - (2200\text{ m})\hat{j}$ and the magnitude is $d' = \sqrt{(-1300\text{ m})^2 + (-2200\text{ m})^2} = 2.56\times 10^3\text{ m}$.

The net displacement is zero since his final position matches his initial position.

78. Let $\vec{c} = \vec{b}\times\vec{a}$. Then the magnitude of $\vec{c}$ is $c = ab \sin\phi$. Since $\vec{c}$ is perpendicular to $\vec{a}$ the magnitude of $\vec{a}\times\vec{c}$ is ac. The magnitude of $\vec{a}\times(\vec{b}\times\vec{a})$ is consequently

$$|\vec{a}\times(\vec{b}\times\vec{a})| = ac = a^2 b \sin\phi .$$

Substituting the values given, we obtain

$$|\vec{a}\times(\vec{b}\times\vec{a})| = a^2 b \sin\phi = (3.90)^2(2.70)\sin 63.0° = 36.6 .$$

79. The area of a triangle is half the product of its base and altitude. The base is the side formed by vector $\vec{a}$. Then the altitude is $b \sin\phi$ and the area is $A = \frac{1}{2}ab\sin\phi = \frac{1}{2}|\vec{a}\times\vec{b}|$. Substituting the values given, we have

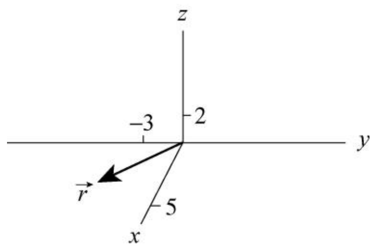
$$A = \frac{1}{2}ab\sin\phi = \frac{1}{2}(4.3)(5.4)\sin 46° \approx 8.4 .$$

Chapter 4

1. (a) The magnitude of $\vec{r}$ is

$$|\vec{r}| = \sqrt{(5.0\text{ m})^2 + (-3.0\text{ m})^2 + (2.0\text{ m})^2} = 6.2\text{ m}.$$

(b) A sketch is shown. The coordinate values are in meters.



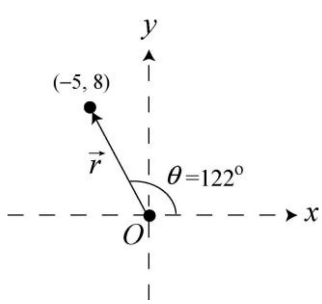
2. (a) The position vector, according to Eq. 4-1, is $\vec{r} = (-5.0\text{ m})\,\hat{\text{i}} + (8.0\text{ m})\hat{\text{j}}$.

(b) The magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-5.0\text{ m})^2 + (8.0\text{ m})^2 + (0\text{ m})^2} = 9.4\text{ m}.$

(c) Many calculators have polar ↔ rectangular conversion capabilities that make this computation more efficient than what is shown below. Noting that the vector lies in the *xy* plane and using Eq. 3-6, we obtain:

$$\theta = \tan^{-1}\left(\frac{8.0\text{ m}}{-5.0\text{ m}}\right) = -58° \text{ or } 122°$$

where the latter possibility (122° measured counterclockwise from the +*x* direction) is chosen since the signs of the components imply the vector is in the second quadrant.



(d) The sketch is shown to the right. The vector is 122° counterclockwise from the +*x* direction.

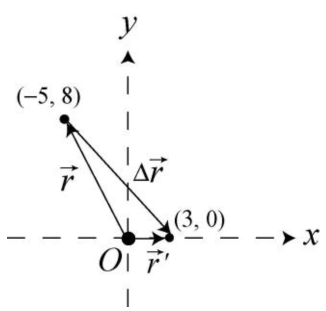
(e) The displacement is $\Delta\vec{r} = \vec{r}\,' - \vec{r}$ where $\vec{r}$ is given in part (a) and $\vec{r}\,' = (3.0\text{ m})\hat{\text{i}}$. Therefore, $\Delta\vec{r} = (8.0\text{ m})\hat{\text{i}} - (8.0\text{ m})\hat{\text{j}}$.

(f) The magnitude of the displacement is

$$|\Delta\vec{r}| = \sqrt{(8.0\text{ m})^2 + (-8.0\text{ m})^2} = 11\text{ m}.$$

(g) The angle for the displacement, using Eq. 3-6, is

$$\tan^{-1}\left(\frac{8.0\text{ m}}{-8.0\text{ m}}\right) = -45° \text{ or } 135°$$



where we choose the former possibility (–45°, or 45° measured *clockwise* from +*x*) since the signs of the components imply the vector is in the fourth quadrant. A sketch of $\Delta\vec{r}$ is shown on the right.

3. The initial position vector $\vec{r}_o$ satisfies $\vec{r} - \vec{r}_o = \Delta\vec{r}$, which results in

$$\vec{r}_o = \vec{r} - \Delta\vec{r} = (3.0\hat{\text{j}} - 4.0\hat{\text{k}})\text{m} - (2.0\hat{\text{i}} - 3.0\hat{\text{j}} + 6.0\hat{\text{k}})\text{m} = (-2.0\text{ m})\,\hat{\text{i}} + (6.0\text{ m})\,\hat{\text{j}} + (-10\text{ m})\,\hat{\text{k}}\,.$$

4. We choose a coordinate system with origin at the clock center and +*x* rightward (toward the "3:00" position) and +*y* upward (toward "12:00").

(a) In unit-vector notation, we have $\vec{r}_1 = (10\text{ cm})\hat{\text{i}}$ and $\vec{r}_2 = (-10\text{ cm})\hat{\text{j}}$. Thus, Eq. 4-2 gives

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-10\text{ cm})\hat{\text{i}} + (-10\text{ cm})\hat{\text{j}}.$$

The magnitude is given by $|\Delta\vec{r}| = \sqrt{(-10\text{ cm})^2 + (-10\text{ cm})^2} = 14\text{ cm}.$

(b) Using Eq. 3-6, the angle is

$$\theta = \tan^{-1}\left(\frac{-10\text{ cm}}{-10\text{ cm}}\right) = 45^\circ\text{ or }-135^\circ.$$

We choose -135° since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-10\text{ cm})\hat{\text{i}} + (-10\text{ cm})\hat{\text{j}} \rightarrow (14\text{cm}\angle -135^\circ).$$

(c) In this case, we have $\vec{r}_1 = (-10\text{ cm})\hat{\text{j}}$ and $\vec{r}_2 = (10\text{ cm})\hat{\text{j}}$, and $\Delta\vec{r} = (20\text{ cm})\hat{\text{j}}$. Thus, $|\Delta\vec{r}| = 20\text{ cm}.$

(d) Using Eq. 3-6, the angle is given by

$$\theta = \tan^{-1}\left(\frac{20\text{ cm}}{0\text{ cm}}\right) = 90^\circ.$$

(e) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

(f) The corresponding angle for a full-hour sweep is also zero.

5. **THINK** This problem deals with the motion of a train in two dimensions. The entire trip consists of three parts, and we're interested in the overall average velocity.

EXPRESS The average velocity of the entire trip is given by Eq. 4-8, $\vec{v}_{\rm avg} = \Delta\vec{r}/\Delta t$, where the total displacement $\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$ is the sum of three displacements (each result of a constant velocity during a given time), and $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ is the total amount of time for the trip. We use a coordinate system with +*x* for East and +*y* for North.

ANALYZE (a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0\ \frac{\rm km}{\rm h}\right)\left(\frac{40.0\ \rm min}{60\ \rm min/h}\right)\hat{\rm i} = (40.0\ {\rm km})\hat{\rm i}.$$

The second displacement has a magnitude of $(60.0\ \frac{\rm km}{\rm h})\cdot(\frac{20.0\ \rm min}{60\ \rm min/h}) = 20.0\ \rm km$, and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = (20.0\ {\rm km})\cos(40.0°)\,\hat{\rm i} + (20.0\ {\rm km})\sin(40.0°)\,\hat{\rm j} = (15.3\ {\rm km})\,\hat{\rm i} + (12.9\ {\rm km})\,\hat{\rm j}.$$

Similarly, the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0\ \frac{\rm km}{\rm h}\right)\left(\frac{50.0\ \rm min}{60\ \rm min/h}\right)\hat{\rm i} = (-50.0\ {\rm km})\,\hat{\rm i}.$$

Thus, the total displacement is

$$\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (40.0\ {\rm km})\hat{\rm i} + (15.3\ {\rm km})\,\hat{\rm i} + (12.9\ {\rm km})\,\hat{\rm j} - (50.0\ {\rm km})\,\hat{\rm i}$$
$$= (5.30\ {\rm km})\,\hat{\rm i} + (12.9\ {\rm km})\,\hat{\rm j}.$$

The time for the trip is $\Delta t = (40.0 + 20.0 + 50.0)$ min = 110 min, which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\rm avg} = \frac{(5.30\ {\rm km})\,\hat{\rm i} + (12.9\ {\rm km})\,\hat{\rm j}}{1.83\ \rm h} = (2.90\ {\rm km/h})\,\hat{\rm i} + (7.01\ {\rm km/h})\,\hat{\rm j}.$$

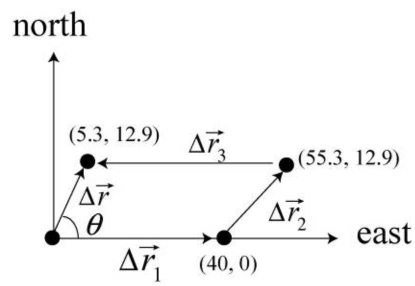
The magnitude of $\vec{v}_{\rm avg}$ is $|\vec{v}_{\rm avg}| = \sqrt{(2.90\ {\rm km/h})^2 + (7.01\ {\rm km/h})^2} = 7.59\ \rm km/h.$

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{v_{{\rm avg},y}}{v_{{\rm avg},x}}\right) = \tan^{-1}\left(\frac{7.01\ \rm km/h}{2.90\ \rm km/h}\right) = 67.5°\ \ \text{(north of east)},$$

or $22.5°$ east of due north.

LEARN The displacement of the train is depicted in the figure below:



Note that the net displacement $\Delta\vec{r}$ is found by adding $\Delta\vec{r}_1$, $\Delta\vec{r}_2$ and $\Delta\vec{r}_3$ vectorially.

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for dx/dt.

(a) Equation 4-10 leads to

$$v(t) = \frac{d}{dt}\,(3.00t\hat{\mathrm{i}} - 4.00t^2\hat{\mathrm{j}} + 2.00\hat{\mathrm{k}}) = (3.00\text{ m/s})\hat{\mathrm{i}} - (8.00\text{ m/s}^2)t\,\hat{\mathrm{j}}$$

(b) Evaluating this result at t = 2.00 s produces $\vec{v} = (3.00\hat{\mathrm{i}} - 16.0\hat{\mathrm{j}})$ m/s.

(c) The speed at t = 2.00 s is $v = |\vec{v}| = \sqrt{(3.00\text{ m/s})^2 + (-16.0\text{ m/s})^2} = 16.3$ m/s.

(d) The angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-16.0\text{ m/s}}{3.00\text{ m/s}}\right) = -79.4° \text{ or } 101°$$

where we choose the first possibility (79.4° measured *clockwise* from the +x direction, or 281° counterclockwise from +x) since the signs of the components imply the vector is in the fourth quadrant.

7. Using Eq. 4-3 and Eq. 4-8, we have

$$\vec{v}_{\text{avg}} = \frac{(-2.0\hat{\mathrm{i}} + 8.0\hat{\mathrm{j}} - 2.0\hat{\mathrm{k}})\text{ m} - (5.0\hat{\mathrm{i}} - 6.0\hat{\mathrm{j}} + 2.0\hat{\mathrm{k}})\text{ m}}{10\text{ s}} = (-0.70\hat{\mathrm{i}} + 1.40\hat{\mathrm{j}} - 0.40\hat{\mathrm{k}})\text{ m/s}.$$

8. Our coordinate system has $\hat{\mathrm{i}}$ pointed east and $\hat{\mathrm{j}}$ pointed north. The first displacement is $\vec{r}_{AB} = (483\text{ km})\hat{\mathrm{i}}$ and the second is $\vec{r}_{BC} = (-966\text{ km})\hat{\mathrm{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483\ \text{km})\hat{\text{i}} - (966\ \text{km})\hat{\text{j}}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483\ \text{km})^2 + (-966\ \text{km})^2} = 1.08\times 10^3\ \text{km}$.

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{-966\ \text{km}}{483\ \text{km}}\right) = -63.4°.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{(483\ \text{km})\hat{\text{i}} - (966\ \text{km})\hat{\text{j}}}{2.25\ \text{h}} = (215\ \text{km/h})\hat{\text{i}} - (429\ \text{km/h})\hat{\text{j}}$$

with a magnitude $|\vec{v}_{\text{avg}}| = \sqrt{(215\ \text{km/h})^2 + (-429\ \text{km/h})^2} = 480\ \text{km/h}$.

(d) The direction of $\vec{v}_{\text{avg}}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{\text{avg}} = (480\ \text{km/h}\ \angle\ -63.4°)$.

(e) Assuming the AB trip was a straight one, and similarly for the BC trip, then $|\vec{r}_{AB}|$ is the distance traveled during the AB trip, and $|\vec{r}_{BC}|$ is the distance traveled during the BC trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483\ \text{km} + 966\ \text{km}}{2.25\ \text{h}} = 644\ \text{km/h}.$$

9. The (x,y) coordinates (in meters) of the points are A = (15, −15), B = (30, −45), C = (20, −15), and D = (45, 45). The respective times are t_A = 0, t_B = 300 s, t_C = 600 s, and t_D = 900 s. Average velocity is defined by Eq. 4-8. Each displacement $\Delta\vec{r}$ is understood to originate at point A.

(a) The average velocity having the least magnitude (5.0 m/600 s) is for the displacement ending at point C: $|\vec{v}_{\text{avg}}| = 0.0083$ m/s.

(b) The direction of $\vec{v}_{\text{avg}}$ is 0° (measured counterclockwise from the +x axis).

(c) The average velocity having the greatest magnitude ($\sqrt{(15\text{ m})^2 + (30\text{ m})^2}\,/\,300\text{ s}$) is for the displacement ending at point *B*: $|\vec{v}_{avg}| = 0.11$ m/s.

(d) The direction of $\vec{v}_{avg}$ is 297° (counterclockwise from +*x*) or −63° (which is equivalent to measuring 63° *clockwise* from the +*x* axis).

10. We differentiate $\vec{r} = 5.00t\,\hat{\text{i}} + (et + ft^2)\,\hat{\text{j}}$.

(a) The particle's motion is indicated by the derivative of $\vec{r}: \vec{v} = 5.00\,\hat{\text{i}} + (e + 2ft)\,\hat{\text{j}}$. The angle of its direction of motion is consequently

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(e + 2ft)/5.00].$$

The graph indicates $\theta_o = 35.0°$, which determines the parameter *e*:

$$e = (5.00\text{ m/s})\tan(35.0°) = 3.50\text{ m/s}.$$

(b) We note (from the graph) that $\theta = 0$ when $t = 14.0$ s. Thus, $e + 2ft = 0$ at that time. This determines the parameter *f* :

$$f = \frac{-e}{2t} = \frac{-3.5\text{ m/s}}{2(14.0\text{ s})} = -0.125\text{ m/s}^2.$$

11. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

(a) Plugging into the given expression, we obtain

$$\vec{r}\Big|_{t=2.00} = [2.00(8) - 5.00(2)]\hat{\text{i}} + [6.00 - 7.00(16)]\hat{\text{j}} = (6.00\hat{\text{i}} - 106\hat{\text{j}})\text{ m}$$

(b) Taking the derivative of the given expression produces

$$\vec{v}(t) = (6.00t^2 - 5.00)\,\hat{\text{i}} - 28.0t^3\,\hat{\text{j}}$$

where we have written *v*(*t*) to emphasize its dependence on time. This becomes, at $t = 2.00$ s, $\vec{v} = (19.0\hat{\text{i}} - 224\hat{\text{j}})$ m/s.

(c) Differentiating the $\vec{v}(t)$ found above, with respect to *t* produces $12.0t\,\hat{\text{i}} - 84.0t^2\,\hat{\text{j}}$, which yields $\vec{a} = (24.0\hat{\text{i}} - 336\hat{\text{j}})\text{ m/s}^2$ at $t = 2.00$ s.

(d) The angle of $\vec{v}$, measured from +*x*, is either

$$\tan^{-1}\left(\frac{-224\text{ m/s}}{19.0\text{ m/s}}\right) = -85.2° \text{ or } 94.8°$$

where we settle on the first choice (–85.2°, which is equivalent to 275° measured counterclockwise from the +x axis) since the signs of its components imply that it is in the fourth quadrant.

12. We adopt a coordinate system with $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north; the coordinate origin is the flagpole. We "translate" the given information into unit-vector notation as follows:

$$\vec{r}_\text{o} = (40.0\text{ m})\hat{\text{i}} \quad \text{and} \quad \vec{v}_\text{o} = (-10.0\text{ m/s})\hat{\text{j}}$$
$$\vec{r} = (40.0\text{ m})\hat{\text{j}} \quad \text{and} \quad \vec{v} = (10.0\text{ m/s})\hat{\text{i}}.$$

(a) Using Eq. 4-2, the displacement $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r} - \vec{r}_\text{o} = (-40.0\text{ m})\hat{\text{i}} + (40.0\text{ m})\hat{\text{j}}$$

with a magnitude $|\Delta\vec{r}| = \sqrt{(-40.0\text{ m})^2 + (40.0\text{ m})^2} = 56.6\text{ m}.$

(b) The direction of $\Delta\vec{r}$ is

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{40.0\text{ m}}{-40.0\text{ m}}\right) = -45.0° \text{ or } 135°.$$

Since the desired angle is in the second quadrant, we pick $135°$ ($45°$ north of due west). Note that the displacement can be written as $\Delta\vec{r} = \vec{r} - \vec{r}_\text{o} = (56.6\angle 135°)$ in terms of the magnitude-angle notation.

(c) The magnitude of $\vec{v}_\text{avg}$ is simply the magnitude of the displacement divided by the time (Δt = 30.0 s). Thus, the average velocity has magnitude (56.6 m)/(30.0 s) = 1.89 m/s.

(d) Equation 4-8 shows that $\vec{v}_\text{avg}$ points in the same direction as $\Delta\vec{r}$, that is, $135°$ ($45°$ north of due west).

(e) Using Eq. 4-15, we have

$$\vec{a}_\text{avg} = \frac{\vec{v} - \vec{v}_\text{o}}{\Delta t} = (0.333\text{ m/s}^2)\hat{\text{i}} + (0.333\text{ m/s}^2)\hat{\text{j}}.$$

The magnitude of the average acceleration vector is therefore equal to $|\vec{a}_\text{avg}| = \sqrt{(0.333\text{ m/s}^2)^2 + (0.333\text{ m/s}^2)^2} = 0.471\text{ m/s}^2$.

(f) The direction of $\vec{a}_{\text{avg}}$ is

$$\theta = \tan^{-1}\left(\frac{0.333\ \text{m/s}^2}{0.333\ \text{m/s}^2}\right) = 45° \text{ or } -135°.$$

Since the desired angle is now in the first quadrant, we choose $45°$, and $\vec{a}_{\text{avg}}$ points north of due east.

13. **THINK** Knowing the position of a particle as function of time allows us to calculate its corresponding velocity and acceleration by taking time derivatives.

EXPRESS From the position vector $\vec{r}(t)$, the velocity and acceleration of the particle can be found by differentiating $\vec{r}(t)$ with respect to time:

$$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}.$$

ANALYZE (a) Taking the derivative of the position vector $\vec{r}(t) = \hat{\text{i}} + (4t^2)\hat{\text{j}} + t\hat{\text{k}}$ with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\text{i}} + 4t^2\,\hat{\text{j}} + t\,\hat{\text{k}}) = 8t\,\hat{\text{j}} + \hat{\text{k}}.$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}(8t\,\hat{\text{j}} + \hat{\text{k}}) = 8\,\hat{\text{j}}.$$

LEARN The particle undergoes constant acceleration in the +y-direction. This can be seen by noting that the y component of $\vec{r}(t)$ is $4t^2$, which is quadratic in t.

14. We use Eq. 4-15 with $\vec{v}_1$ designating the initial velocity and $\vec{v}_2$ designating the later one.

(a) The average acceleration during the $\Delta t = 4$ s interval is

$$\vec{a}_{\text{avg}} = \frac{(-2.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}} + 5.0\,\hat{\text{k}})\ \text{m/s} - (4.0\,\hat{\text{i}} - 22\,\hat{\text{j}} + 3.0\,\hat{\text{k}})\ \text{m/s}}{4\ \text{s}} = (-1.5\ \text{m/s}^2)\,\hat{\text{i}} + (0.5\,\text{m/s}^2)\,\hat{\text{k}}.$$

(b) The magnitude of $\vec{a}_{\text{avg}}$ is $\sqrt{(-1.5\ \text{m/s}^2)^2 + (0.5\ \text{m/s}^2)^2} = 1.6\,\text{m/s}^2$.

(c) Its angle in the xz plane (measured from the +x axis) is one of these possibilities:

$$\tan^{-1}\left(\frac{0.5\ \text{m/s}^2}{-1.5\ \text{m/s}^2}\right) = -18° \text{ or } 162°$$

where we settle on the second choice since the signs of its components imply that it is in the second quadrant.

15. **THINK** Given the initial velocity and acceleration of a particle, we're interested in finding its velocity and position at a later time.

EXPRESS Since the acceleration, $\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}} = (-1.0\ \text{m/s}^2)\hat{\text{i}} + (-0.50\ \text{m/s}^2)\hat{\text{j}}$, is constant in both x and y directions, we may use Table 2-1 for the motion along each direction. This can be handled individually (for x and y) or together with the unit-vector notation (for $\Delta\vec{r}$).

Since the particle started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. Along the x-direction, we have

$$x(t) = v_{0x}t + \frac{1}{2}a_x t^2, \qquad v_x(t) = v_{0x} + a_x t$$

Similarly, along the y-direction, we get

$$y(t) = v_{0y}t + \frac{1}{2}a_y t^2, \qquad v_y(t) = v_{0y} + a_y t\ .$$

Known: $v_{0x} = 3.0\ \text{m/s}, v_{0y} = 0,\ a_x = -1.0\ \text{m/s}^2,\ a_y = -0.5\ \text{m/s}^2$.

ANALYZE (a) Substituting the values given, the components of the velocity are

$$v_x(t) = v_{0x} + a_x t = (3.0\ \text{m/s}) - (1.0\ \text{m/s}^2)t$$
$$v_y(t) = v_{0y} + a_y t = -(0.50\ \text{m/s}^2)t$$

When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.0 - 1.0t_m = 0$ or $t_m = 3.0$ s. The y component of the velocity at this time is

$$v_y(t = 3.0\ \text{s}) = -(0.50\ \text{m/s}^2)(3.0) = -1.5\ \text{m/s}$$

Thus, $\vec{v}_m = (-1.5\ \text{m/s})\hat{\text{j}}$.

(b) At $t = 3.0$ s , the components of the position are

$$x(t=3.0\text{ s}) = v_{0x}t + \frac{1}{2}a_x t^2 = (3.0\text{ m/s})(3.0\text{ s}) + \frac{1}{2}(-1.0\text{ m/s}^2)(3.0\text{ s})^2 = 4.5\text{ m}$$
$$y(t=3.0\text{ s}) = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-0.5\text{ m/s}^2)(3.0\text{ s})^2 = -2.25\text{ m}$$

Using unit-vector notation, the results can be written as $\vec{r}_m = (4.50\text{ m})\,\hat{\text{i}} - (2.25\text{ m})\,\hat{\text{j}}$.

LEARN The motion of the particle in this problem is two-dimensional, and the kinematics in the *x*- and *y*-directions can be analyzed separately.

16. We make use of Eq. 4-16.

(a) The acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\left(6.0t - 4.0t^2\right)\hat{\text{i}} + 8.0\,\hat{\text{j}}\right) = \left(6.0 - 8.0t\right)\hat{\text{i}}$$

in SI units. Specifically, we find the acceleration vector at $t = 3.0\text{ s}$ to be $\left(6.0 - 8.0(3.0)\right)\hat{\text{i}} = (-18\text{ m/s}^2)\hat{\text{i}}$.

(b) The equation is $\vec{a} = \left(6.0 - 8.0t\right)\hat{\text{i}} = 0$; we find t = 0.75 s.

(c) Since the *y* component of the velocity, v_y = 8.0 m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have

$$v = |\vec{v}| = \sqrt{\left(6.0t - 4.0t^2\right)^2 + \left(8.0\right)^2} = 10$$

in SI units (m/s). To solve for *t*, we first square both sides of the above equation, followed by some rearrangement:

$$\left(6.0t - 4.0t^2\right)^2 + 64 = 100 \quad\Rightarrow\quad \left(6.0t - 4.0t^2\right)^2 = 36$$

Taking the square root of the new expression and making further simplification lead to

$$6.0t - 4.0t^2 = \pm 6.0 \quad\Rightarrow\quad 4.0t^2 - 6.0t \pm 6.0 = 0$$

Finally, using the quadratic formula, we obtain

$$t = \frac{6.0 \pm \sqrt{36 - 4(4.0)(\pm 6.0)}}{2(8.0)}$$

where the requirement of a real positive result leads to the unique answer: t = 2.2 s.

17. We find t by applying Eq. 2-11 to motion along the y axis (with v_y = 0 characterizing $y = y_{\max}$):

$$0 = (12 \text{ m/s}) + (-2.0 \text{ m/s}^2)t \quad \Rightarrow \quad t = 6.0 \text{ s}.$$

Then, Eq. 2-11 applies to motion along the x axis to determine the answer:

$$v_x = (8.0 \text{ m/s}) + (4.0 \text{ m/s}^2)(6.0 \text{ s}) = 32 \text{ m/s}.$$

Therefore, the velocity of the cart, when it reaches $y = y_{\max}$, is (32 m/s)$\hat{\text{i}}$.

18. We find t by solving $\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$:

$$12.0 \text{ m} = 0 + (4.00 \text{ m/s})t + \frac{1}{2}(5.00 \text{ m/s}^2)t^2$$

where we have used Δx = 12.0 m, v_x = 4.00 m/s, and a_x = 5.00 m/s^2 . We use the quadratic formula and find t = 1.53 s. Then, Eq. 2-11 (actually, its analog in two dimensions) applies with this value of t. Therefore, its velocity (when Δx = 12.00 m) is

$$\begin{aligned}\vec{v} &= \vec{v}_0 + \vec{a}t = (4.00 \text{ m/s})\hat{\text{i}} + (5.00 \text{ m/s}^2)(1.53 \text{ s})\hat{\text{i}} + (7.00 \text{ m/s}^2)(1.53 \text{ s})\hat{\text{j}} \\ &= (11.7 \text{ m/s})\hat{\text{i}} + (10.7 \text{ m/s})\hat{\text{j}}.\end{aligned}$$

Thus, the magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(11.7 \text{ m/s})^2 + (10.7 \text{ m/s})^2} = 15.8$ m/s.

(b) The angle of $\vec{v}$, measured from +x, is

$$\tan^{-1}\left(\frac{10.7 \text{ m/s}}{11.7 \text{ m/s}}\right) = 42.6°.$$

19. We make use of Eq. 4-16 and Eq. 4-10.

Using $\vec{a} = 3t\hat{\text{i}} + 4t\hat{\text{j}}$, we have (in m/s)

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}\; dt = (5.00\hat{\text{i}} + 2.00\hat{\text{j}}) + \int_0^t (3t\hat{\text{i}} + 4t\hat{\text{j}})\; dt = \left(5.00 + 3t^2/2\right)\hat{\text{i}} + \left(2.00 + 2t^2\right)\hat{\text{j}}$$

Integrating using Eq. 4-10 then yields (in meters)

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}\,dt = (20.0\hat{\text{i}} + 40.0\hat{\text{j}}) + \int_0^t [(5.00 + 3t^2/2)\hat{\text{i}} + (2.00 + 2t^2)\hat{\text{j}}]dt$$
$$= (20.0\hat{\text{i}} + 40.0\hat{\text{j}}) + (5.00t + t^3/2)\hat{\text{i}} + (2.00t + 2t^3/3)\hat{\text{j}}$$
$$= (20.0 + 5.00t + t^3/2)\hat{\text{i}} + (40.0 + 2.00t + 2t^3/3)\hat{\text{j}}$$

(a) At $t = 4.00$ s, we have $\vec{r}(t = 4.00\text{ s}) = (72.0\text{ m})\hat{\text{i}} + (90.7\text{ m})\hat{\text{j}}$.

(b) $\vec{v}(t = 4.00\text{ s}) = (29.0\text{ m/s})\hat{\text{i}} + (34.0\text{ m/s})\hat{\text{j}}$. Thus, the angle between the direction of travel and +x, measured counterclockwise, is $\theta = \tan^{-1}[(34.0\text{ m/s})/(29.0\text{ m/s})] = 49.5°$.

20. The acceleration is constant so that use of Table 2-1 (for both the x and y motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles A and B requires two things. First, the y motion of B must satisfy (using Eq. 2-15 and noting that θ is measured from the y axis)

$$y = \frac{1}{2}a_y t^2 \quad \Rightarrow \quad 30\text{ m} = \frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]t^2.$$

Second, the x motions of A and B must coincide:

$$vt = \frac{1}{2}a_x t^2 \quad \Rightarrow (3.0\text{ m/s})t = \frac{1}{2}\left[(0.40\text{ m/s}^2)\sin\theta\right]t^2.$$

We eliminate a factor of t in the last relationship and formally solve for time:

$$t = \frac{2v}{a_x} = \frac{2(3.0\text{ m/s})}{(0.40\text{ m/s}^2)\sin\theta}.$$

This is then plugged into the previous equation to produce

$$30\text{ m} = \frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]\left(\frac{2(3.0\text{ m/s})}{(0.40\text{ m/s}^2)\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1 - \cos^2\theta$, simplifies to

$$30 = \frac{9.0}{0.20}\,\frac{\cos\theta}{1 - \cos^2\theta} \quad \Rightarrow \; 1 - \cos^2\theta = \frac{9.0}{(0.20)(30)}\cos\theta.$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

which yields $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

21. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the *y* coordinate of the dart is given by $y = -\frac{1}{2}gt^2$, so that with *y* = –*PQ* we have $PQ = \frac{1}{2}\left(9.8 \text{ m/s}^2\right)\left(0.19 \text{ s}\right)^2 = 0.18 \text{ m}$.

(b) From $x = v_0 t$ we obtain $x = (10 \text{ m/s})(0.19 \text{ s}) = 1.9$ m.

22. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the *y* coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If *t* is the time of flight and $y = -1.20$ m indicates the level at which the ball hits the floor, then

$$t = \sqrt{\frac{2(-1.20 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.495 \text{s}.$$

(b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0 \hat{\text{i}}$. Since $x = 1.52$ m is the horizontal position of its impact point with the floor, we have $x = v_0 t$. Thus,

$$v_0 = \frac{x}{t} = \frac{1.52 \text{ m}}{0.495 \text{ s}} = 3.07 \text{ m/s}.$$

23. (a) From Eq. 4-22 (with $\theta_0 = 0$), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45.0 \text{ m})}{9.80 \text{ m/s}^2}} = 3.03 \text{ s}.$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0 t = (250 \text{ m/s})(3.03 \text{ s}) = 758 \text{ m}.$$

(c) And from Eq. 4-23, we find

$$\left|v_y\right| = gt = (9.80 \text{ m/s}^2)(3.03 \text{ s}) = 29.7 \text{ m/s}.$$

24. We use Eq. 4-26

$$R_{max} = \left(\frac{v_0^2}{g} \sin 2\theta_0 \right)_{max} = \frac{v_0^2}{g} = \frac{(9.50 \text{m/s})^2}{9.80 \text{m/s}^2} = 9.209 \text{ m} \approx 9.21 \text{m}$$

to compare with Powell's long jump; the difference from R_{max} is only ΔR =(9.21m – 8.95m) = 0.259 m.

25. Using Eq. (4-26), the take-off speed of the jumper is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.80 \text{ m/s}^2)(77.0 \text{ m})}{\sin 2(12.0°)}} = 43.1 \text{ m/s}$$

26. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is the throwing point (the stone's initial position). The x component of its initial velocity is given by $v_{0x} = v_0 \cos\theta_0$ and the y component is given by $v_{0y} = v_0 \sin\theta_0$, where v_0 = 20 m/s is the initial speed and θ_0 = 40.0° is the launch angle.

(a) At t = 1.10 s, its x coordinate is

$$x = v_0 t \cos\theta_0 = (20.0 \text{ m/s})(1.10 \text{ s}) \cos 40.0° = 16.9 \text{ m}$$

(b) Its y coordinate at that instant is

$$y = v_0 t \sin\theta_0 - \frac{1}{2} g t^2 = (20.0 \text{m/s})(1.10 \text{s}) \sin 40.0° - \frac{1}{2}(9.80 \text{m/s}^2)(1.10 \text{s})^2 = 8.21 \text{ m}.$$

(c) At t' = 1.80 s, its x coordinate is $x = (20.0 \text{ m/s})(1.80 \text{ s}) \cos 40.0° = 27.6 \text{ m}$.

(d) Its y coordinate at t' is

$$y = (20.0 \text{m/s})(1.80 \text{s}) \sin 40.0° - \frac{1}{2}(9.80 \text{m/s}^2)(1.80 \text{s}^2) = 7.26 \text{m}.$$

(e) The stone hits the ground earlier than t = 5.0 s. To find the time when it hits the ground solve $y = v_0 t \sin\theta_0 - \frac{1}{2} g t^2 = 0$ for t. We find

$$t = \frac{2v_0}{g} \sin\theta_0 = \frac{2(20.0 \text{ m/s})}{9.8 \text{ m/s}^2} \sin 40° = 2.62 \text{ s}.$$

Its x coordinate on landing is

$$x = v_0 t \cos\theta_0 = (20.0\text{ m/s})(2.62\text{ s})\cos 40° = 40.2\text{ m}.$$

(f) Assuming it stays where it lands, its vertical component at t = 5.00 s is y = 0.

27. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write θ_0 = –30.0° since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release: v_0 = 290 km/h, which we convert to SI units: (290)(1000/3600) = 80.6 m/s.

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0\cos\theta_0)\, t \quad \Rightarrow \quad t = \frac{700\text{ m}}{(80.6\text{ m/s})\cos(-30.0°)} = 10.0\text{ s}.$$

(b) And we use Eq. 4-22 to solve for the initial height y_0:

$$y - y_0 = (v_0\sin\theta_0)\, t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 - y_0 = (-40.3\text{ m/s})(10.0\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(10.0\text{ s})^2$$

which yields y_0 = 897 m.

28. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h = y_0 + v_0\sin\theta_0 t - \frac{1}{2}gt^2$$

which yields h = 51.8 m for y_0 = 0, v_0 = 42.0 m/s, θ_0 = 60.0°, and t = 5.50 s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0\cos\theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0\cos\theta_0)^2 + (v_0\sin\theta_0 - gt)^2} = 27.4\text{ m/s}.$$

(c) We use Eq. 4-24 with v_y = 0 and $y = H$:

$$H = \frac{(v_0\sin\theta_0)^2}{2g} = 67.5\text{ m}.$$

29. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to v_{0x}). In this notation, we have $v_0 = 5v$. Next, we observe $v_0 \cos\theta_0 = v_{0x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \;\Rightarrow\; \theta_0 = \cos^{-1}\left(\frac{1}{5}\right) = 78.5°.$$

30. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With $\Delta y = 0$, we have

$$\Delta y = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \;\Rightarrow\; t = \frac{(19.5 \text{ m/s}) \sin 45.0°}{(9.80 \text{ m/s}^2)/2} = 2.81 \text{ s}.$$

Then Eq. 4-21 yields $\Delta x = (v_0 \cos\theta_0)t = 38.7$ m. Thus, using Eq. 4-8, the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(38.7 \text{ m})\,\hat{\text{i}} - (55 \text{ m})\,\hat{\text{i}}}{2.81 \text{ s}} = (-5.8 \text{ m/s})\,\hat{\text{i}}$$

which means his average speed (assuming he ran in only one direction) is 5.8 m/s.

31. We first find the time it takes for the volleyball to hit the ground. Using Eq. 4-22, we have

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \;\Rightarrow\; 0 - 2.30 \text{ m} = (-20.0 \text{ m/s}) \sin(18.0°)t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

which gives $t = 0.30$ s. Thus, the range of the volleyball is

$$R = \left(v_0 \cos\theta_0\right)t = (20.0 \text{ m/s}) \cos 18.0°(0.30 \text{ s}) = 5.71 \text{ m}$$

On the other hand, when the angle is changed to $\theta_0' = 8.00°$, using the same procedure as shown above, we find

$$y - y_0 = (v_0 \sin\theta_0')\, t' - \frac{1}{2} g t'^2 \;\Rightarrow\; 0 - 2.30 \text{ m} = (-20.0 \text{ m/s}) \sin(8.00°)t' - \frac{1}{2}(9.80 \text{ m/s}^2)t'^2$$

which yields $t' = 0.46$ s, and the range is

$$R' = \left(v_0 \cos\theta_0\right)t' = (20.0 \text{ m/s}) \cos 18.0°(0.46 \text{ s}) = 9.06 \text{ m}$$

Thus, the ball travels an extra distance of

$$\Delta R = R' - R = 9.06 \text{ m} - 5.71 \text{ m} = 3.35 \text{ m}$$

32. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0 \cos 40.0°$, the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0 \text{ m}}{(25.0 \text{ m/s})\cos 40.0°} = 1.15 \text{ s}.$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = (25.0 \text{ m/s})\sin 40.0°(1.15 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.15 \text{ s})^2 = 12.0 \text{ m}.$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0° = 19.2$ m/s.

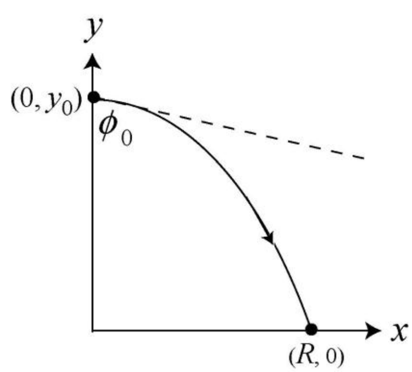
(c) The vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = (25.0 \text{ m/s}) \sin 40.0° - (9.80 \text{ m/s}^2)(1.15 \text{ s}) = 4.80 \text{ m/s}.$$

(d) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

33. **THINK** This problem deals with projectile motion. We're interested in the horizontal displacement and velocity of the projectile before it strikes the ground.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -37.0°$ for the angle measured from $+x$, since the angle $\phi_0 = 53.0°$ given in the problem is measured from the $-y$ direction. The initial setup of the problem is shown in the figure to the right (not to scale).



ANALYZE (a) The initial speed of the projectile is the plane's speed at the moment of release. Given that $y_0 = 730$ m and $y = 0$ at $t = 5.00$ s, we use Eq. 4-22 to find v_0:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \;\Rightarrow\; 0 - 730 \text{ m} = v_0 \sin(-37.0°)(5.00 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(5.00 \text{ s})^2$$

which yields v_0 = 202 m/s.

(b) The horizontal distance traveled is

$$R = v_x t = (v_0 \cos\theta_0)t = [(202 \text{ m/s})\cos(-37.0°)](5.00 \text{ s}) = 806 \text{ m}.$$

(c) The x component of the velocity (just before impact) is

$$v_x = v_0 \cos\theta_0 = (202 \text{ m/s})\cos(-37.0°) = 161 \text{ m/s}.$$

(d) The y component of the velocity (just before impact) is

$$v_y = v_0 \sin\theta_0 - gt = (202 \text{ m/s})\sin(-37.0°) - (9.80 \text{ m/s}^2)(5.00 \text{ s}) = -171 \text{ m/s}.$$

LEARN In this projectile problem we analyzed the kinematics in the vertical and horizontal directions separately since they do not affect each other. The x-component of the velocity, $v_x = v_0 \cos\theta_0$, remains unchanged throughout since there's no horizontal acceleration.

34. (a) Since the y-component of the velocity of the stone at the top of its path is zero, its speed is

$$v = \sqrt{v_x^2 + v_y^2} = v_x = v_0 \cos\theta_0 = (28.0 \text{ m/s})\cos 40.0° = 21.4 \text{ m/s}.$$

(b) Using the fact that $v_y = 0$ at the maximum height y_{max}, the amount of time it takes for the stone to reach y_{max} is given by Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \;\Rightarrow\; t = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{max} = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta_0}{g}\right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time the stone descends to $y = y_{max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2}y_{max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \;\Rightarrow\; t_\pm = \frac{(2 \pm \sqrt{2})v_0 \sin\theta_0}{2g}.$$

Choosing $t = t_+$ (for descending), we have

$$v_x = v_0 \cos\theta_0 = (28.0 \text{ m/s})\cos 40.0° = 21.4 \text{ m/s}$$
$$v_y = v_0 \sin\theta_0 - g\frac{(2+\sqrt{2})v_0 \sin\theta_0}{2g} = -\frac{\sqrt{2}}{2}v_0 \sin\theta_0 = -\frac{\sqrt{2}}{2}(28.0 \text{ m/s})\sin 40.0° = -12.7 \text{ m/s}$$

Thus, the speed of the stone when $y = y_{\max}/2$ is

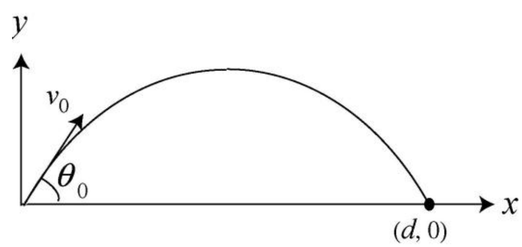
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21.4 \text{ m/s})^2 + (-12.7 \text{ m/s})^2} = 24.9 \text{ m/s}.$$

(c) The percentage difference is

$$\frac{24.9 \text{ m/s} - 21.4 \text{ m/s}}{21.4 \text{ m/s}} = 0.163 = 16.3\%.$$

35. **THINK** This problem deals with projectile motion of a bullet. We're interested in the firing angle that allows the bullet to strike a target at some distance away.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$.



The projectile motion equations lead to

$$d = (v_0 \cos\theta_0)t, \quad 0 = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$$

where θ_0 is the firing angle. The setup of the problem is shown in the figure above (scale exaggerated).

ANALYZE The time at which the bullet strikes the target is given by $t = d/(v_0 \cos\theta_0)$. Eliminating t leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. Using $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\ (2\theta_0) = gd \ \Rightarrow\ \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80\ \text{m/s}^2)(45.7\ \text{m})}{(460\ \text{m/s})^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$, or θ_0 = 0.0606°. If the gun is aimed at a point a distance ℓ above the target, then tan $\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7\ \text{m})\tan(0.0606°) = 0.0484\ \text{m} = 4.84\ \text{cm}.$$

LEARN Due to the downward gravitational acceleration, in order for the bullet to strike the target, the gun must be aimed at a point slightly above the target.

36. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

(a) We want to know how high the ball is above the court when it is at x = 12.0 m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{12.0\ \text{m}}{(23.6\ \text{m/s})\cos 0°} = 0.508\ \text{s}.$$

At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = 1.10\text{m}$$

which implies it does indeed clear the 0.90-m-high fence.

(b) At t = 0.508 s, the center of the ball is (1.10 m – 0.90 m) = 0.20 m above the net.

(c) Repeating the computation in part (a) with θ_0 = –5.0° results in t = 0.510 s and $y = 0.040$ m, which clearly indicates that it cannot clear the net.

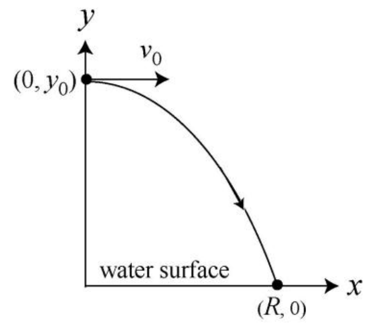
(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at t = 0.510 s is 0.90 m – 0.040 m = 0.86 m.

37. **THINK** The trajectory of the diver is a projectile motion. We are interested in the displacement of the diver at a later time.

EXPRESS The initial velocity has no vertical component ($\theta_0 = 0$), but only an x component. Eqs. 4-21 and 4-22 can be simplified to

$$x - x_0 = v_{0x}t$$
$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2.$$

where $x_0 = 0$, $v_{0x} = v_0 = +2.0\ \text{m/s}$ and y_0 = +10.0 m (taking the water surface to be at $y = 0$). The setup of the problem is shown in the figure below.



ANALYZE (a) At $t = 0.80\ \text{s}$, the horizontal distance of the diver from the edge is

$$x = x_0 + v_{0x}t = 0 + (2.0\ \text{m/s})(0.80\ \text{s}) = 1.60\ \text{m}.$$

(b) Similarly, using the second equation for the vertical motion, we obtain

$$y = y_0 - \frac{1}{2}gt^2 = 10.0\ \text{m} - \frac{1}{2}(9.80\ \text{m/s}^2)(0.80\ \text{s})^2 = 6.86\ \text{m}.$$

(c) At the instant the diver strikes the water surface, y = 0. Solving for t using the equation $y = y_0 - \frac{1}{2}gt^2 = 0$ leads to

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(10.0\ \text{m})}{9.80\ \text{m/s}^2}} = 1.43\ \text{s}.$$

During this time, the x-displacement of the diver is $R = x$ = (2.00 m/s)(1.43 s) = 2.86 m.

LEARN Using Eq. 4-25 with $\theta_0 = 0$, the trajectory of the diver can also be written as

$$y = y_0 - \frac{gx^2}{2v_0^2}\ .$$

Part (c) can also be solved by using this equation:

$$y = y_0 - \frac{gx^2}{2v_0^2} = 0 \ \Rightarrow\ x = R = \sqrt{\frac{2v_0^2 y_0}{g}} = \sqrt{\frac{2(2.0\ \text{m/s})^2(10.0\ \text{m})}{9.8\ \text{m/s}^2}} = 2.86\ \text{m}.$$

38. In this projectile motion problem, we have $v_0 = v_x$ = constant, and what is plotted is $v = \sqrt{v_x^2 + v_y^2}$. We infer from the plot that at t = 2.5 s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that v_x = 19 m/s.

(a) During t = 5 s, the horizontal motion is $x - x_0 = v_x t$ = 95 m.

(b) Since $\sqrt{(19\text{ m/s})^2 + v_{0y}^2} = 31\text{ m/s}$ (the first point on the graph), we find $v_{0y} = 24.5\text{ m/s}$. Thus, with t = 2.5 s, we can use $y_{max} - y_0 = v_{0y}t - \frac{1}{2}gt^2$ or $v_y^2 = 0 = v_{0y}^2 - 2g(y_{max} - y_0)$, or $y_{max} - y_0 = \frac{1}{2}(v_y + v_{0_y})t$ to solve. Here we will use the latter:

$$y_{max} - y_0 = \frac{1}{2}(v_y + v_{0y})\, t \Rightarrow y_{max} = \frac{1}{2}(0 + 24.5\text{m/s})(2.5\text{ s}) = 31\text{ m}$$

where we have taken y_0 = 0 as the ground level.

39. Following the hint, we have the time-reversed problem with the ball thrown from the ground, toward the right, at 60° measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with $+x$ as *rightward* and with positive angles measured counterclockwise.

(a) The x-equation (with x_0 = 0 and x = 25.0 m) leads to

$$25.0\text{ m} = (v_0 \cos 60.0°)(1.50\text{ s}),$$

so that v_0 = 33.3 m/s. And with y_0 = 0, and $y = h > 0$ at t = 1.50 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60.0°$. This leads to h = 32.3 m.

(b) We have

$$v_x = v_{0x} = (33.3\text{ m/s})\cos 60.0° = 16.7\text{ m/s}$$
$$v_y = v_{0y} - gt = (33.3\text{ m/s})\sin 60.0° - (9.80\text{ m/s}^2)(1.50\text{ s}) = 14.2\text{ m/s}.$$

The magnitude of $\vec{v}$ is given by

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.7\text{ m/s})^2 + (14.2\text{ m/s})^2} = 21.9\text{ m/s}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.2\text{ m/s}}{16.7\text{ m/s}}\right) = 40.4°.$$

(d) We interpret this result ("undoing" the time reversal) as an initial velocity (from the edge of the building) of magnitude 21.9 m/s with angle (down from leftward) of 40.4°.

40. (a) Solving the quadratic equation Eq. 4-22:

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \ \Rightarrow\ 0 - 2.160\text{ m} = (15.00\text{ m/s})\sin(45.00°)t - \frac{1}{2}(9.800\text{ m/s}^2)t^2$$

the total travel time of the shot in the air is found to be $t = 2.352\text{ s}$. Therefore, the horizontal distance traveled is

$$R = \left(v_0 \cos\theta_0\right)t = (15.00\text{ m/s})\cos 45.00°(2.352\text{ s}) = 24.95\text{ m}.$$

(b) Using the procedure outlined in (a) but for $\theta_0 = 42.00°$, we have

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \ \Rightarrow\ 0 - 2.160\text{ m} = (15.00\text{ m/s})\sin(42.00°)t - \frac{1}{2}(9.800\text{ m/s}^2)t^2$$

and the total travel time is $t = 2.245\text{ s}$. This gives

$$R = \left(v_0 \cos\theta_0\right)t = (15.00\text{ m/s})\cos 42.00°(2.245\text{ s}) = 25.02\text{ m}.$$

41. With the Archer fish set to be at the origin, the position of the insect is given by (x, y) where $x = R/2 = v_0^2 \sin 2\theta_0 / 2g$, and y corresponds to the maximum height of the parabolic trajectory: $y = y_{\max} = v_0^2 \sin^2\theta_0 / 2g$. From the figure, we have

$$\tan\phi = \frac{y}{x} = \frac{v_0^2 \sin^2\theta_0 / 2g}{v_0^2 \sin 2\theta_0 / 2g} = \frac{1}{2}\tan\theta_0$$

Given that $\phi = 36.0°$, we find the launch angle to be

$$\theta_0 = \tan^{-1}\left(2\tan\phi\right) = \tan^{-1}\left(2\tan 36.0°\right) = \tan^{-1}\left(1.453\right) = 55.46° \approx 55.5°.$$

Note that θ_0 depends only on ϕ and is independent of d.

42. (a) Using the fact that the person (as the projectile) reaches the maximum height over the middle wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5\text{ m}$, we can deduce the initial launch speed from Eq. 4-26:

$$x = \frac{R}{2} = \frac{v_0^2 \sin 2\theta_0}{2g} \quad \Rightarrow \quad v_0 = \sqrt{\frac{2gx}{\sin 2\theta_0}} = \sqrt{\frac{2(9.8\text{ m/s}^2)(34.5\text{ m})}{\sin(2\cdot 53°)}} = 26.5\text{ m/s}.$$

Upon substituting the value to Eq. 4-25, we obtain

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (23\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(23\text{ m})^2}{2(26.5\text{ m/s})^2(\cos 53°)^2} = 23.3\text{ m}.$$

Since the height of the wheel is $h_w = 18$ m, the clearance over the first wheel is $\Delta y = y - h_w = 23.3\text{ m} - 18\text{ m} = 5.3\text{ m}$.

(b) The height of the person when he is directly above the second wheel can be found by solving Eq. 4-24. With the second wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5$ m, we have

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (34.5\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(34.5\text{ m})^2}{2(26.52\text{ m/s})^2(\cos 53°)^2}$$
$$= 25.9\text{ m}.$$

Therefore, the clearance over the second wheel is $\Delta y = y - h_w = 25.9\text{ m} - 18\text{ m} = 7.9\text{ m}$.

(c) The location of the center of the net is given by

$$0 = y - y_0 = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} \;\Rightarrow\; x = \frac{v_0^2\sin 2\theta_0}{g} = \frac{(26.52\text{ m/s})^2\sin(2\cdot 53°)}{9.8\text{ m/s}^2} = 69\text{ m}.$$

43. We designate the given velocity $\vec{v} = (7.6\text{ m/s})\hat{\text{i}} + (6.1\text{ m/s})\hat{\text{j}}$ as $\vec{v}_1$, as opposed to the velocity when it reaches the max height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3$, and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial y velocity, that is how we will proceed. Using Eq. 2-16, we have

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y \;\Rightarrow\; (6.1\text{ m/s})^2 = v_{0y}^2 - 2(9.8\text{ m/s}^2)(9.1\text{ m})$$

which yields v_{0y} = 14.7 m/s. Knowing that v_{2y} must equal 0, we use Eq. 2-16 again but now with $\Delta y = h$ for the maximum height:

$$v_{2y}^2 = v_{0y}^2 - 2gh \;\Rightarrow\; 0 = (14.7\text{ m/s})^2 - 2(9.8\text{ m/s}^2)h$$

which yields h = 11 m.

(b) Recalling the derivation of Eq. 4-26, but using v_{0y} for $v_0\sin\theta_0$ and v_{0x} for $v_0\cos\theta_0$, we have

$$0 = v_{0y}t - \frac{1}{2}gt^2, \quad R = v_{0x}t$$

which leads to $R = 2v_{0x}v_{0y}/g$. Noting that $v_{0x} = v_{1x} = 7.6$ m/s, we plug in values and obtain

$$R = 2(7.6 \text{ m/s})(14.7 \text{ m/s})/(9.8 \text{ m/s}^2) = 23 \text{ m}.$$

(c) Since $v_{3x} = v_{1x} = 7.6$ m/s and $v_{3y} = -v_{0y} = -14.7$ m/s, we have

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{(7.6 \text{ m/s})^2 + (-14.7 \text{ m/s})^2} = 17 \text{ m/s}.$$

(d) The angle (measured from horizontal) for $\vec{v}_3$ is one of these possibilities:

$$\tan^{-1}\left(\frac{-14.7 \text{ m}}{7.6 \text{ m}}\right) = -63° \text{ or } 117°$$

where we settle on the first choice (–63°, which is equivalent to 297°) since the signs of its components imply that it is in the fourth quadrant.

44. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 161 \text{ km/h}$. Converting to SI units, this is $v_0 = 44.7$ m/s.

(a) With the origin at the initial point (where the ball leaves the pitcher's hand), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the x coordinate is given by $x = v_0t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if $x = 18.3/2$ m, then $t = (18.3/2 \text{ m})/(44.7 \text{ m/s}) = 0.205$ s.

(b) And the time to travel the next 18.3/2 m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_0\Delta t$ in order that this result can be seen more clearly.

(c) Using the equation $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $|-\frac{1}{2}(9.80 \text{ m/s}^2)(0.205 \text{ s})^2| = 0.205 \text{ m}$ at the moment the ball is halfway to the batter.

(d) The ball's height when it reaches the batter is $-\frac{1}{2}(9.80 \text{ m/s}^2)(0.409 \text{ s})^2 = -0.820 \text{m}$, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of y is not simply proportional to t, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial y-velocity for the first half of the motion is not the same as the "initial" y-velocity for the second half of the motion.

45. (a) Let $m = \frac{d_2}{d_1} = 0.600$ be the slope of the ramp, so $y = mx$ there. We choose our coordinate origin at the point of launch and use Eq. 4-25. Thus,

$$y = \tan(50.0°)x - \frac{(9.80 \text{ m/s}^2)x^2}{2(10.0 \text{ m/s})^2(\cos 50.0°)^2} = 0.600x$$

which yields $x = 4.99$ m. This is less than d_1 so the ball *does* land on the ramp.

(b) Using the value of x found in part (a), we obtain $y = mx = 2.99$ m. Thus, the Pythagorean theorem yields a displacement magnitude of $\sqrt{x^2 + y^2} = 5.82$ m.

(c) The angle is, of course, the angle of the ramp: $\tan^{-1}(m) = 31.0°$.

46. Using the fact that $v_y = 0$ when the player is at the maximum height $y_{\max}$, the amount of time it takes to reach $y_{\max}$ can be solved by using Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \quad t_{\max} = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin\theta_0)\, t_{\max} - \frac{1}{2} g t_{\max}^2 = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g}\right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time when the player is at $y = y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{\max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t_\pm = \frac{(2 \pm \sqrt{2}) v_0 \sin\theta_0}{2g}.$$

With $t = t_-$ (for ascending), the amount of time the player spends at a height $y \geq y_{\max}/2$ is

$$\Delta t = t_{\max} - t_- = \frac{v_0 \sin\theta_0}{g} - \frac{(2-\sqrt{2}) v_0 \sin\theta_0}{2g} = \frac{v_0 \sin\theta_0}{\sqrt{2} g} = \frac{t_{\max}}{\sqrt{2}} \quad \Rightarrow \quad \frac{\Delta t}{t_{\max}} = \frac{1}{\sqrt{2}} = 0.707.$$

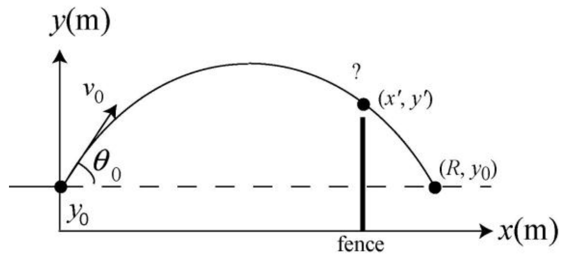
Therefore, the player spends about 70.7% of the time in the upper half of the jump. Note that the ratio $\Delta t / t_{\max}$ is independent of v_0 and θ_0, even though Δt and $t_{\max}$ depend on these quantities.

47. **THINK** The baseball undergoes projectile motion after being hit by the batter. We'd like to know if the ball clears a high fence at some distance away.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. In the absence of a fence, with $\theta_0 = 45°$, the horizontal range (same launch level) is $R = 107$ m. We want to know how high the ball is from the ground when it is at $x' = 97.5$ m, which requires knowing the initial velocity. The trajectory of the baseball can be described by Eq. 4-25:

$$y - y_0 = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}.$$

The setup of the problem is shown in the figure below (not to scale).



ANALYZE (a) We first solve for the initial speed v_0. Using the range information ($y = y_0$ when $x = R$) and $\theta_0 = 45°$, Eq. 4-25 gives

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8\text{ m/s}^2)(107\text{ m})}{\sin(2\cdot 45°)}} = 32.4\text{ m/s}.$$

Thus, the time at which the ball flies over the fence is:

$$x' = (v_0\cos\theta_0)t' \quad\Rightarrow\quad t' = \frac{x'}{v_0\cos\theta_0} = \frac{97.5\text{ m}}{(32.4\text{ m/s})\cos 45°} = 4.26\text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$\begin{aligned} y' &= y_0 + (v_0\sin\theta_0)t' - \frac{1}{2}gt'^2 \\ &= 1.22\text{ m} + [(32.4\text{ m/s})\sin 45°](4.26\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(4.26\text{ s})^2 \\ &= 9.88\text{ m} \end{aligned}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At $t' = 4.26$ s, the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

LEARN Using the trajectory equation above, one can show that the minimum initial velocity required to clear the fence is given by

$$y' - y_0 = (\tan\theta_0)x' - \frac{gx'^2}{2(v_0\cos\theta_0)^2},$$

or about 31.9 m/s.

48. Following the hint, we have the time-reversed problem with the ball thrown from the roof, toward the left, at 60° measured clockwise from a leftward axis. We see in this time-reversed situation that it is convenient to take $+x$ as *leftward* with positive angles measured clockwise. Lengths are in meters and time is in seconds.

(a) With y_0 = 20.0 m, and y = 0 at t = 4.00 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to v_0 = 16.9 m/s. This plugs into the x-equation $x - x_0 = v_{0x}t$ (with x_0 = 0 and $x = d$) to produce

$$d = (16.9 \text{ m/s})\cos 60°(4.00 \text{ s}) = 33.7 \text{ m}.$$

(b) We have

$$v_x = v_{0x} = (16.9 \text{ m/s})\cos 60.0° = 8.43 \text{ m/s}$$
$$v_y = v_{0y} - gt = (16.9 \text{ m/s})\sin 60.0° - (9.80\text{m/s}^2)(4.00 \text{ s}) = -24.6 \text{ m/s}.$$

The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(8.43 \text{ m/s})^2 + (-24.6 \text{ m/s})^2} = 26.0 \text{ m/s}.$

(c) The angle relative to horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-24.6 \text{ m/s}}{8.43 \text{ m/s}}\right) = -71.1°.$$

We may convert the result from rectangular components to magnitude-angle representation:

$$\vec{v} = (8.43, -24.6) \;\rightarrow\; (26.0 \angle -71.1°)$$

and we now interpret our result ("undoing" the time reversal) as an initial velocity of magnitude 26.0 m/s with angle (up from rightward) of 71.1°.

49. **THINK** In this problem a football is given an initial speed and it undergoes projectile motion. We'd like to know the smallest and greatest angles at which a field goal can be scored.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use x and y to denote the coordinates of the ball at the goalpost, and try to find the kicking angle(s) θ_0 so that y = 3.44 m when x = 50 m. Writing the kinematic equations for projectile motion:

$$x = v_0 \cos\theta_0, \quad y = v_0 t \sin\theta_0 - \tfrac{1}{2} g t^2,$$

we see the first equation gives $t = x/v_0 \cos\theta_0$, and when this is substituted into the second the result is

$$y = x \tan\theta_0 - \frac{gx^2}{2v_0^2 \cos^2\theta_0}.$$

ANALYZE One may solve the above equation by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity

$$1 / \cos^2\theta_0 = 1 + \tan^2\theta_0,$$

we obtain

$$\frac{1}{2}\frac{gx^2}{v_0^2}\tan^2\theta_0 - x\tan\theta_0 + y + \frac{1}{2}\frac{gx^2}{v_0^2} = 0$$

which is a second-order equation for $\tan\theta_0$. To simplify writing the solution, we denote

$$c = \frac{1}{2} gx^2 / v_0^2 = \frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(50\text{ m}\right)^2 / \left(25\text{ m/s}\right)^2 = 19.6\text{ m}.$$

Then the second-order equation becomes $c\tan^2\theta_0 - x\tan\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\tan\theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50\text{ m} \pm \sqrt{(50\text{ m})^2 - 4\left(3.44\text{ m} + 19.6\text{ m}\right)\left(19.6\text{ m}\right)}}{2\left(19.6\text{ m}\right)}.$$

The two solutions are given by $\tan\theta_0 = 1.95$ and $\tan\theta_0 = 0.605$. The corresponding (first-quadrant) angles are $\theta_0 = 63°$ and $\theta_0 = 31°$. Thus,

(a) The smallest elevation angle is $\theta_0 = 31°$, and

(b) The greatest elevation angle is $\theta_0 = 63°$.

LEARN If kicked at any angle between 31° and 63°, the ball will travel above the cross bar on the goalposts.

50. We apply Eq. 4-21, Eq. 4-22, and Eq. 4-23.

(a) From $\Delta x = v_{0x} t$, we find $v_{0x} = 40\text{ m}/2\text{ s} = 20\text{ m/s}$.

(b) From $\Delta y = v_{0y} t - \tfrac{1}{2} g t^2$, we find $v_{0y} = \left(53\text{ m} + \tfrac{1}{2}(9.8\text{ m/s}^2)(2\text{ s})^2\right)/2 = 36\text{ m/s}$.

(c) From $v_y = v_{0y} - gt'$ with $v_y = 0$ as the condition for maximum height, we obtain $t' = (36\ \text{m/s})\,/(9.8\ \text{m/s}^2) = 3.7\ \text{s}$. During that time the x-motion is constant, so $x' - x_0 = (20\ \text{m/s})(3.7\ \text{s}) = 74\ \text{m}$.

51. (a) The skier jumps up at an angle of $\theta_0 = 11.3°$ up from the horizontal and thus returns to the launch level with his velocity vector 11.3° below the horizontal. With the snow surface making an angle of $\alpha = 9.0°$ (downward) with the horizontal, the angle between the slope and the velocity vector is $\phi = \theta_0 - \alpha = 11.3° - 9.0° = 2.3°$.

(b) Suppose the skier lands at a distance d down the slope. Using Eq. 4-25 with $x = d\cos\alpha$ and $y = -d\sin\alpha$ (the edge of the track being the origin), we have

$$-d\sin\alpha = d\cos\alpha\,\tan\theta_0 - \frac{g(d\cos\alpha)^2}{2v_0^2\cos^2\theta_0}.$$

Solving for d, we obtain

$$d = \frac{2v_0^2\cos^2\theta_0}{g\cos^2\alpha}\left(\cos\alpha\,\tan\theta_0 + \sin\alpha\right) = \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\left(\cos\alpha\,\sin\theta_0 + \cos\theta_0\sin\alpha\right)$$
$$= \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\sin(\theta_0 + \alpha).$$

Substituting the values given, we find

$$d = \frac{2(10\ \text{m/s})^2\cos(11.3°)}{(9.8\ \text{m/s}^2)\cos^2(9.0°)}\sin(11.3° + 9.0°) = 7.117\ \text{m}.$$

which gives

$$y = -d\sin\alpha = -(7.117\ \text{m})\sin(9.0°) = -1.11\ \text{m}.$$

Therefore, at landing the skier is approximately 1.1 m below the launch level.

(c) The time it takes for the skier to land is

$$t = \frac{x}{v_x} = \frac{d\cos\alpha}{v_0\cos\theta_0} = \frac{(7.117\ \text{m})\cos(9.0°)}{(10\ \text{m/s})\cos(11.3°)} = 0.72\ \text{s}.$$

Using Eq. 4-23, the x-and y-components of the velocity at landing are

$$v_x = v_0\cos\theta_0 = (10\ \text{m/s})\cos(11.3°) = 9.81\ \text{m/s}$$
$$v_y = v_0\sin\theta_0 - gt = (10\ \text{m/s})\sin(11.3°) - (9.8\ \text{m/s}^2)(0.72\ \text{s}) = -5.07\ \text{m/s}$$

Thus, the direction of travel at landing is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.07\text{ m/s}}{9.81\text{ m/s}}\right) = -27.3°.$$

or 27.3° below the horizontal. The result implies that the angle between the skier's path and the slope is $\phi = 27.3° - 9.0° = 18.3°$, or approximately 18° to two significant figures.

52. From Eq. 4-21, we find $t = x / v_{0x}$. Then Eq. 4-23 leads to

$$v_y = v_{0y} - gt = v_{0y} - \frac{gx}{v_{0x}}.$$

Since the slope of the graph is –0.500, we conclude

$$\frac{g}{v_{0x}} = \frac{1}{2} \Rightarrow v_{ox} = 19.6\text{ m/s}.$$

And from the "y intercept" of the graph, we find v_{oy} = 5.00 m/s. Consequently,

$$\theta_o = \tan^{-1}(v_{oy} / v_{ox}) = 14.3° \approx 14°.$$

53. Let $y_0 = h_0$ = 1.00 m at x_0 = 0 when the ball is hit. Let $y_1 = h$ (the height of the wall) and x_1 describe the point where it first rises above the wall one second after being hit; similarly, $y_2 = h$ and x_2 describe the point where it passes back down behind the wall four seconds later. And y_f = 1.00 m at $x_f = R$ is where it is caught. Lengths are in meters and time is in seconds.

(a) Keeping in mind that v_x is constant, we have $x_2 - x_1$ = 50.0 m = v_{1x} (4.00 s), which leads to v_{1x} = 12.5 m/s. Thus, applied to the full six seconds of motion:

$$x_f - x_0 = R = v_x(6.00\text{ s}) = 75.0\text{ m}.$$

(b) We apply $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to the motion above the wall,

$$y_2 - y_1 = 0 = v_{1y}(4.00\text{ s}) - \frac{1}{2}g(4.00\text{ s})^2$$

and obtain v_{1y} = 19.6 m/s. One second earlier, using $v_{1y} = v_{0y} - g$(1.00 s), we find $v_{0y} = 29.4$ m/s. Therefore, the velocity of the ball just after being hit is

$$\vec{v} = v_{0x}\hat{\text{i}} + v_{0y}\hat{\text{j}} = (12.5\text{ m/s})\ \hat{\text{i}} + (29.4\text{ m/s})\hat{\text{j}}$$

Its magnitude is $|\vec{v}| = \sqrt{(12.5\text{ m/s})^2 + (29.4\text{ m/s})^2} = 31.9\text{ m/s}$.

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{29.4\text{ m/s}}{12.5\text{ m/s}}\right) = 67.0°.$$

We interpret this result as a velocity of magnitude 31.9 m/s, with angle (up from rightward) of 67.0°.

(d) During the first 1.00 s of motion, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ yields

$$h = 1.0\text{ m} + (29.4\text{ m/s})(1.00\text{ s}) - \tfrac{1}{2}(9.8\text{ m/s}^2)(1.00\text{ s})^2 = 25.5\text{ m}.$$

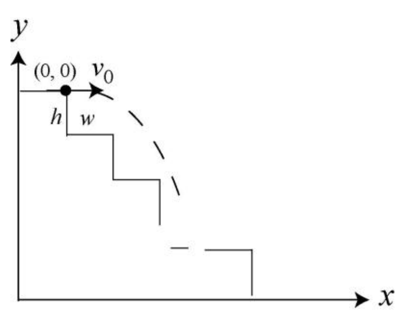
54. For $\Delta y = 0$, Eq. 4-22 leads to $t = 2v_o\sin\theta_o/g$, which immediately implies $t_{max} = 2v_o/g$ (which occurs for the "straight up" case: $\theta_o = 90°$). Thus,

$$\tfrac{1}{2}\,t_{max} = v_o/g \;\Rightarrow\; \tfrac{1}{2} = \sin\theta_o.$$

Therefore, the half-maximum-time flight is at angle $\theta_o = 30.0°$. Since the least speed occurs at the top of the trajectory, which is where the velocity is simply the x-component of the initial velocity ($v_o\cos\theta_o = v_o\cos30°$ for the half-maximum-time flight), then we need to refer to the graph in order to find v_o – in order that we may complete the solution. In the graph, we note that the range is 240 m when $\theta_o = 45.0°$. Equation 4-26 then leads to $v_o = 48.5$ m/s. The answer is thus (48.5 m/s)cos30.0° = 42.0 m/s.

55. **THINK** In this problem a ball rolls off the top of a stairway with an initial speed, and we'd like to know on which step it lands first.

EXPRESS We denote h as the height of a step and w as the width. To hit step n, the ball must fall a distance nh and travel horizontally a distance between $(n-1)w$ and nw. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the y axis to be positive in the upward direction, as shown in the figure.



The coordinates of the ball at time t are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$).

ANALYZE We equate y to $-nh$ and solve for the time to reach the level of step n:

$$t = \sqrt{\frac{2nh}{g}}.$$

The x coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.52\text{ m/s})\sqrt{\frac{2n(0.203\text{ m})}{9.8\text{ m/s}^2}} = (0.309\text{ m})\sqrt{n}.$$

The method is to try values of n until we find one for which x/w is less than n but greater than $n - 1$. For $n = 1$, $x = 0.309$ m and $x/w = 1.52$, which is greater than n. For $n = 2$, $x = 0.437$ m and $x/w = 2.15$, which is also greater than n. For $n = 3$, $x = 0.535$ m and $x/w = 2.64$. Now, this is less than n and greater than $n - 1$, so the ball hits the third step.

LEARN To check the consistency of our calculation, we can substitute $n = 3$ into the above equations. The results are $t = 0.353$ s, $y = 0.609$ m and $x = 0.535$ m. This indeed corresponds to the third step.

56. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find acceleration a.

(a) Since the radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is

$$r = (6.37 \times 10^6 + 640 \times 10^3)\text{ m} = 7.01 \times 10^6\text{ m}.$$

Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi\left(7.01 \times 10^6\text{ m}\right)}{(98.0\text{ min})(60\text{ s/min})} = 7.49 \times 10^3\text{ m/s}.$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{\left(7.49 \times 10^3\text{ m/s}\right)^2}{7.01 \times 10^6\text{ m}} = 8.00\text{ m/s}^2.$$

57. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (toward the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences $\vec{a} = 1.83\text{ m/s}^2$ east, then the center of the circle is *east* of this location. The distance is $r = v^2/a = (3.66\text{ m/s})^2/(1.83\text{ m/s}^2) = 7.32$ m.

(b) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

(c) If the direction of $\vec{a}$ experienced by the passenger is now *south*—indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the north.

58. (a) The circumference is $c = 2\pi r = 2\pi(0.15 \text{ m}) = 0.94$ m.

(b) With $T = (60 \text{ s})/1200 = 0.050$ s, the speed is $v = c/T = (0.94 \text{ m})/(0.050 \text{ s}) = 19$ m/s. This is equivalent to using Eq. 4-35.

(c) The magnitude of the acceleration is $a = v^2/r = (19 \text{ m/s})^2/(0.15 \text{ m}) = 2.4 \times 10^3 \text{ m/s}^2$.

(d) The period of revolution is $(1200 \text{ rev/min})^{-1} = 8.3 \times 10^{-4}$ min, which becomes, in SI units, $T = 0.050$ s = 50 ms.

59. (a) Since the wheel completes 5 turns each minute, its period is one-fifth of a minute, or 12 s.

(b) The magnitude of the centripetal acceleration is given by $a = v^2/R$, where R is the radius of the wheel, and v is the speed of the passenger. Since the passenger goes a distance $2\pi R$ for each revolution, his speed is

$$v = \frac{2\pi(15 \text{ m})}{12 \text{ s}} = 7.85 \text{ m/s}$$

and his centripetal acceleration is $a = \dfrac{(7.85 \text{ m/s})^2}{15 \text{ m}} = 4.1 \text{ m/s}^2$.

(c) When the passenger is at the highest point, his centripetal acceleration is downward, toward the center of the orbit.

(d) At the lowest point, the centripetal acceleration is $a = 4.1 \text{ m/s}^2$, same as part (b).

(e) The direction is up, toward the center of the orbit.

60. (a) During constant-speed circular motion, the velocity vector is perpendicular to the acceleration vector at every instant. Thus, $\vec{v} \cdot \vec{a} = 0$.

(b) The acceleration in this vector, at every instant, points toward the center of the circle, whereas the position vector points from the center of the circle to the object in motion. Thus, the angle between $\vec{r}$ and $\vec{a}$ is 180º at every instant, so $\vec{r} \times \vec{a} = 0$.

61. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find centripetal acceleration a.

(a) $v = 2\pi r/T = 2\pi(20 \text{ km})/1.0 \text{ s} = 126 \text{ km/s} = 1.3 \times 10^5$ m/s.

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(126 \text{ km/s})^2}{20 \text{ km}} = 7.9 \times 10^5 \text{ m/s}^2.$$

(c) Clearly, both v and a will increase if T is reduced.

62. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10 \text{ m/s})^2}{25 \text{ m}} = 4.0 \text{ m/s}^2.$$

63. We first note that $\vec{a}_1$ (the acceleration at t_1 = 2.00 s) is perpendicular to $\vec{a}_2$ (the acceleration at t_2=5.00 s), by taking their scalar (dot) product:

$$\vec{a}_1 \cdot \vec{a}_2 = [(6.00 \text{ m/s}^2)\hat{\text{i}} + (4.00 \text{ m/s}^2)\hat{\text{j}}] \cdot [(4.00 \text{ m/s}^2)\hat{\text{i}} + (-6.00 \text{ m/s}^2)\hat{\text{j}}] = 0.$$

Since the acceleration vectors are in the (negative) radial directions, then the two positions (at t_1 and t_2) are a quarter-circle apart (or three-quarters of a circle, depending on whether one measures clockwise or counterclockwise). A quick sketch leads to the conclusion that if the particle is moving counterclockwise (as the problem states) then it travels three-quarters of a circumference in moving from the position at time t_1 to the position at time t_2. Letting T stand for the period, then $t_2 - t_1$ = 3.00 s = $3T/4$. This gives T = 4.00 s. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(6.00 \text{ m/s}^2)^2 + (4.00 \text{ m/s})^2} = 7.21 \text{ m/s}^2.$$

Using Eqs. 4-34 and 4-35, we have $a = 4\pi^2 r / T^2$, which yields

$$r = \frac{aT^2}{4\pi^2} = \frac{(7.21 \text{ m/s}^2)(4.00 \text{ s})^2}{4\pi^2} = 2.92 \text{ m}.$$

64. When traveling in circular motion with constant speed, the instantaneous acceleration vector necessarily points toward the center. Thus, the center is "straight up" from the cited point.

(a) Since the center is "straight up" from (4.00 m, 4.00 m), the x coordinate of the center is 4.00 m.

(b) To find out "how far up" we need to know the radius. Using Eq. 4-34 we find

$$r = \frac{v^2}{a} = \frac{(5.00\text{ m/s})^2}{12.5\ \text{m/s}^2} = 2.00\text{ m}.$$

Thus, the y coordinate of the center is 2.00 m + 4.00 m = 6.00 m. Thus, the center may be written as (x, y) = (4.00 m, 6.00 m).

65. Since the period of a uniform circular motion is $T = 2\pi r / v$, where r is the radius and v is the speed, the centripetal acceleration can be written as

$$a = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2}.$$

Based on this expression, we compare the (magnitudes) of the wallet and purse accelerations, and find their ratio is the ratio of r values. Therefore, $a_{\text{wallet}} = 1.50\ a_{\text{purse}}$. Thus, the wallet acceleration vector is

$$a = 1.50[(2.00\text{ m/s}^2)\hat{\text{i}} + (4.00\text{ m/s}^2)\hat{\text{j}}] = (3.00\text{ m/s}^2)\hat{\text{i}} + (6.00\text{ m/s}^2)\hat{\text{j}}.$$

66. The fact that the velocity is in the $+y$ direction and the acceleration is in the $+x$ direction at t_1 = 4.00 s implies that the motion is clockwise. The position corresponds to the "9:00 position." On the other hand, the position at t_2 = 10.0 s is in the "6:00 position" since the velocity points in the $-x$ direction and the acceleration is in the $+y$ direction. The time interval $\Delta t = 10.0\text{ s} - 4.00\text{ s} = 6.00\text{ s}$ is equal to 3/4 of a period:

$$6.00\text{ s} = \frac{3}{4}T \quad \Rightarrow \quad T = 8.00\text{ s}.$$

Equation 4-35 then yields

$$r = \frac{vT}{2\pi} = \frac{(3.00\text{ m/s})(8.00\text{ s})}{2\pi} = 3.82\text{ m}.$$

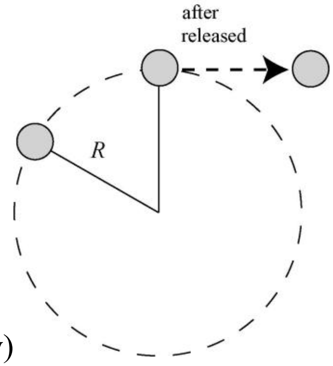
(a) The x coordinate of the center of the circular path is $x = 5.00\text{ m} + 3.82\text{ m} = 8.82\text{ m}.$

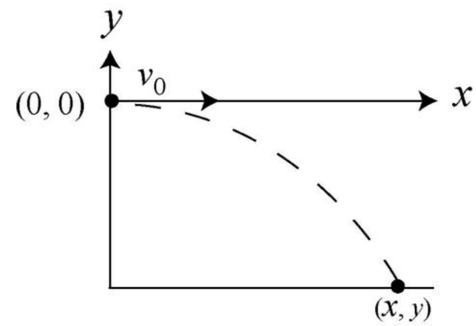
(b) The y coordinate of the center of the circular path is $y = 6.00\text{ m}.$

In other words, the center of the circle is at (x,y) = (8.82 m, 6.00 m).

67. **THINK** In this problem we have a stone whirled in a horizontal circle. After the string breaks, the stone undergoes projectile motion.

EXPRESS The stone moves in a circular path (top view shown below left) initially, but undergoes projectile motion after the string breaks (side view shown below right). Since $a = v^2 / R$, to calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed.





Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0t$ and $y = -\frac{1}{2}gt^2$ (since v_{0y} = 0). It hits the ground at x = 10 m and $y = -2.0$ m.

ANALYZE Formally solving the y-component equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\text{ m})\sqrt{-\frac{9.8\text{ m/s}^2}{2(-2.0\text{ m})}} = 15.7\text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v_0^2}{R} = \frac{(15.7\text{ m/s})^2}{1.5\text{ m}} = 160\text{ m/s}^2.$$

LEARN The above equations can be combined to give $a = \frac{gx^2}{-2yR}$. The equation implies that the greater the centripetal acceleration, the greater the initial speed of the projectile, and the greater the distance traveled by the stone. This is precisely what we expect.

68. We note that after three seconds have elapsed ($t_2 - t_1$ = 3.00 s) the velocity (for this object in circular motion of period T) is reversed; we infer that it takes three seconds to reach the opposite side of the circle. Thus, T = 2(3.00 s) = 6.00 s.

(a) Using Eq. 4-35, $r = vT/2\pi$, where $v = \sqrt{(3.00\text{ m/s})^2 + (4.00\text{ m/s})^2} = 5.00\text{ m/s}$, we obtain $r = 4.77$ m. The magnitude of the object's centripetal acceleration is therefore $a = v^2/r$ = 5.24 m/s^2.

(b) The average acceleration is given by Eq. 4-15:

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{(-3.00\hat{\text{i}} - 4.00\hat{\text{j}})\ \text{m/s} - (3.00\hat{\text{i}} + 4.00\hat{\text{j}})\ \text{m/s}}{5.00\ \text{s} - 2.00\ \text{s}} = (-2.00\ \text{m/s}^2)\hat{\text{i}} + (-2.67\ \text{m/s}^2)\hat{\text{j}}$$

which implies $|\vec{a}_{\text{avg}}| = \sqrt{(-2.00\ \text{m/s}^2)^2 + (-2.67\ \text{m/s}^2)^2} = 3.33\ \text{m/s}^2$.

69. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so $20\ \text{km/h} \to 5.6\ \text{m/s}$, $30\ \text{km/h} \to 8.3\ \text{m/s}$, and $45\ \text{km/h} \to 12.5\ \text{m/s}$. We choose east as the $+\hat{\text{i}}$ direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{\text{ct}} = \vec{v}_{\text{cg}} - \vec{v}_{\text{tg}} = (12.5\ \text{m/s})\ \hat{\text{i}} - (-5.6\ \text{m/s})\ \hat{\text{i}} = (18.1\ \text{m/s})\ \hat{\text{i}}$$

relative to the truck. Since the velocity of the cheetah relative to the truck at the beginning of the 2.0 s interval is $(-8.3\ \text{m/s})\hat{\text{i}}$, the (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{\text{avg}} = \frac{(18.1\ \text{m/s})\hat{\text{i}} - (-8.3\ \text{m/s})\hat{\text{i}}}{2.0\ \text{s}} = (13\ \text{m/s}^2)\hat{\text{i}},$$

or $|\vec{a}_{\text{avg}}| = 13\ \text{m/s}^2$.

(b) The direction of $\vec{a}_{\text{avg}}$ is $+\hat{\text{i}}$, or eastward.

(c) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{0\text{cg}} = \vec{v}_{0\text{ct}} + \vec{v}_{0\text{tg}} = (-8.3\ \text{m/s})\hat{\text{i}} + (-5.6\ \text{m/s})\hat{\text{i}} = (-13.9\ \text{m/s})\hat{\text{i}}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{\text{avg}} = \frac{(12.5\ \text{m/s})\hat{\text{i}} - (-13.9\ \text{m/s})\hat{\text{i}}}{2.0\ \text{s}} = (13\ \text{m/s}^2)\hat{\text{i}}, \quad |\vec{a}_{\text{avg}}| = 13\ \text{m/s}^2$$

identical to the result of part (a).

(d) The direction of $\vec{a}_{\text{avg}}$ is $+\hat{\text{i}}$, or eastward.

70. We use Eq. 4-44, noting that the upstream corresponds to the $+\hat{\text{i}}$ direction.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{\rm bg} = \vec{v}_{\rm bw} + \vec{v}_{\rm wg} = (14\ \text{km/h})\ \hat{\rm i} + (-9\ \text{km/h})\ \hat{\rm i} = (5\ \text{km/h})\ \hat{\rm i}.$$

Thus, the magnitude is $|\vec{v}_{\rm bg}| = 5\ \text{km/h}$.

(b) The direction of $\vec{v}_{\rm bg}$ is $+x$, or upstream.

(c) We use the subscript c for the child, and obtain

$$\vec{v}_{\rm c\,g} = \vec{v}_{\rm c\,b} + \vec{v}_{\rm b\,g} = (-6\ \text{km/h})\ \hat{\rm i} + (5\ \text{km/h})\ \hat{\rm i} = (-1\ \text{km/h})\ \hat{\rm i}\ .$$

The magnitude is $|\vec{v}_{\rm cg}| = 1\ \text{km/h}$.

(d) The direction of $\vec{v}_{\rm cg}$ is $-x$, or downstream.

71. While moving in the same direction as the sidewalk's motion (covering a distance d relative to the ground in time t_1 = 2.50 s), Eq. 4-44 leads to

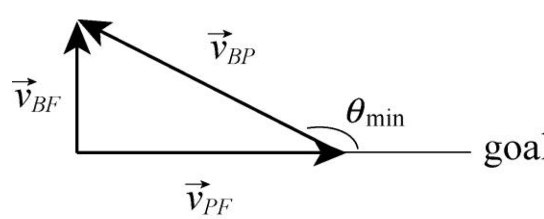
$$v_{\text{sidewalk}} + v_{\text{man running}} = \frac{d}{t_1}\ .$$

While he runs back (taking time t_2 = 10.0 s) we have

$$v_{\text{sidewalk}} - v_{\text{man running}} = -\frac{d}{t_2}\ .$$

Dividing these equations and solving for the desired ratio, we get $\frac{12.5}{7.5} = \frac{5}{3} = 1.67$.

72. We denote the velocity of the player with $\vec{v}_{PF}$ and the relative velocity between the player and the ball be $\vec{v}_{BP}$. Then the velocity $\vec{v}_{BF}$ of the ball relative to the field is given by $\vec{v}_{BF} = \vec{v}_{PF} + \vec{v}_{BP}$. The smallest angle $\theta_{\min}$ corresponds to the case when $\vec{v}_{BF} \perp \vec{v}_{PF}$. Hence,



$$\theta_{\min} = 180° - \cos^{-1}\left(\frac{|\vec{v}_{PF}|}{|\vec{v}_{BP}|}\right) = 180° - \cos^{-1}\left(\frac{4.0\ \text{m/s}}{6.0\ \text{m/s}}\right) = 130°.$$

73. We denote the police and the motorist with subscripts p and m, respectively. The coordinate system is indicated in Fig. 4-46.

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{m\,p} = \vec{v}_m - \vec{v}_p = (-60\ \text{km/h})\hat{\rm j} - (-80\ \text{km/h})\hat{\rm i} = (80\ \text{km/h})\hat{\rm i} - (60\ \text{km/h})\hat{\rm j}.$$

(b) $\vec{v}_{mp}$ does happen to be along the line of sight. Referring to Fig. 4-46, we find the vector pointing from one car to another is $\vec{r} = (800 \text{ m})\hat{\text{i}} - (600 \text{ m})\hat{\text{j}}$ (from M to P). Since the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{mp}$, they must point the same direction.

(c) No, they remain unchanged.

74. Velocities are taken to be constant; thus, the velocity of the plane relative to the ground is $\vec{v}_{PG} = (55 \text{ km})/(1/4 \text{ hour})\ \hat{\text{j}} = (220 \text{ km/h})\hat{\text{j}}$. In addition,

$$\vec{v}_{AG} = (42 \text{ km/h})(\cos 20°\hat{\text{i}} - \sin 20°\hat{\text{j}}) = (39 \text{ km/h})\hat{\text{i}} - (14 \text{ km/h})\hat{\text{j}}.$$

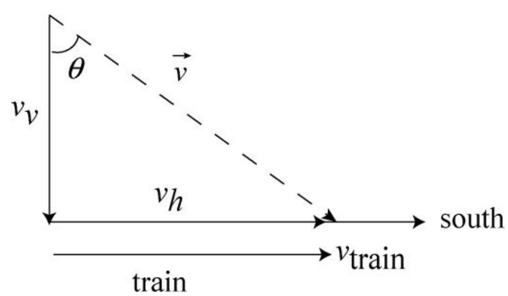
Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, we have

$$\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG} = -(39 \text{ km/h})\hat{\text{i}} + (234 \text{ km/h})\hat{\text{j}}.$$

which implies $|\vec{v}_{PA}| = 237 \text{ km/h}$, or 240 km/h (to two significant figures.)

75. **THINK** This problem deals with relative motion in two dimensions. Raindrops appear to fall vertically by an observer on a moving train.

EXPRESS Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop, v_h = 30 m/s, must be the same as the speed of the train, i.e., $v_h = v_{\text{train}}$ (see figure).



On the other hand, if v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then tan $\theta = v_h/v_v$. Knowing v_v and v_h allows us to determine the speed of the raindrops.

ANALYZE With $\theta = 70°$, we find the vertical component of the velocity to be

$$v_v = v_h/\tan\theta = (30 \text{ m/s})/\tan 70° = 10.9 \text{ m/s}.$$

Therefore, the speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30 \text{ m/s})^2 + (10.9 \text{ m/s})^2} = 32 \text{ m/s}.$$

LEARN As long as the horizontal component of the velocity of the raindrops coincides with the speed of the train, the passenger on board will see the rain falling perfectly vertically.

76. The destination is $\vec{D}$ = 800 km $\hat{\text{j}}$ where we orient axes so that +*y* points north and +*x* points east. This takes two hours, so the (constant) velocity of the plane (relative to the ground) is $\vec{v}_{\text{pg}}$ = (400 km/h) $\hat{\text{j}}$. This must be the vector sum of the plane's velocity with respect to the air which has (*x,y*) components (500cos70º, 500sin70º), and the velocity of the air (*wind*) relative to the ground $\vec{v}_{\text{ag}}$. Thus,

$$(400\text{ km/h})\,\hat{\text{j}} = (500\text{ km/h})\cos 70^\circ\,\hat{\text{i}} + (500\text{ km/h})\sin 70^\circ\,\hat{\text{j}} + \vec{v}_{\text{ag}}$$

which yields

$$\vec{v}_{\text{ag}} = (-171\text{ km/h})\hat{\text{i}} - (70.0\text{ km/h})\hat{\text{j}}\,.$$

(a) The magnitude of $\vec{v}_{\text{ag}}$ is $|\vec{v}_{\text{ag}}| = \sqrt{(-171\text{ km/h})^2 + (-70.0\text{ km/h})^2} = 185\text{ km/h}.$

(b) The direction of $\vec{v}_{\text{ag}}$ is

$$\theta = \tan^{-1}\left(\frac{-70.0\text{ km/h}}{-171\text{ km/h}}\right) = 22.3^\circ\ \text{(south of west)}.$$

77. **THINK** This problem deals with relative motion in two dimensions. Snowflakes falling vertically downward are seen to fall at an angle by a moving observer.

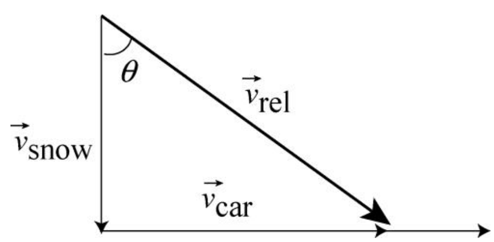
EXPRESS Relative to the car the velocity of the snowflakes has a vertical component of $v_v = 8.0\text{ m/s}$ and a horizontal component of $v_h = 50\text{ km/h} = 13.9\text{ m/s}$.

ANALYZE The angle θ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9\text{ m/s}}{8.0\text{ m/s}} = 1.74$$

which yields $\theta = 60°$.

LEARN The problem can also be solved by expressing the velocity relation in vector notation: $\vec{v}_{\text{rel}} = \vec{v}_{\text{car}} + \vec{v}_{\text{snow}}$, as shown in the figure.



78. We make use of Eq. 4-44 and Eq. 4-45.

The velocity of Jeep *P* relative to *A* at the instant is

$$\vec{v}_{PA} = (40.0 \text{ m/s})(\cos 60°\hat{\text{i}} + \sin 60°\hat{\text{j}}) = (20.0 \text{ m/s})\hat{\text{i}} + (34.6 \text{ m/s})\hat{\text{j}}.$$

Similarly, the velocity of Jeep *B* relative to *A* at the instant is

$$\vec{v}_{BA} = (20.0 \text{ m/s})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (17.3 \text{ m/s})\hat{\text{i}} + (10.0 \text{ m/s})\hat{\text{j}}.$$

Thus, the velocity of *P* relative to *B* is

$$\vec{v}_{PB} = \vec{v}_{PA} - \vec{v}_{BA} = (20.0\hat{\text{i}} + 34.6\hat{\text{j}}) \text{ m/s} - (17.3\hat{\text{i}} + 10.0\hat{\text{j}}) \text{ m/s} = (2.68 \text{ m/s})\hat{\text{i}} + (24.6 \text{ m/s})\hat{\text{j}}.$$

(a) The magnitude of $\vec{v}_{PB}$ is $|\vec{v}_{PB}| = \sqrt{(2.68 \text{ m/s})^2 + (24.6 \text{ m/s})^2} = 24.8 \text{ m/s}$.

(b) The direction of $\vec{v}_{PB}$ is $\theta = \tan^{-1}[(24.6 \text{ m/s})/(2.68 \text{ m/s})] = 83.8°$ north of east (or 6.2º east of north).

(c) The acceleration of *P* is

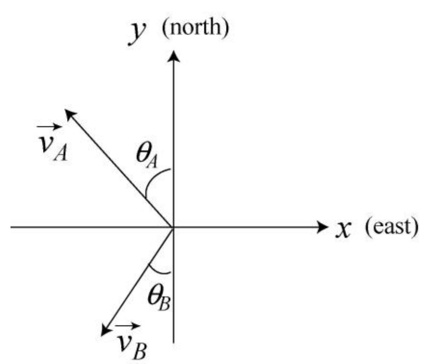
$$\vec{a}_{PA} = (0.400 \text{ m/s}^2)(\cos 60.0°\hat{\text{i}} + \sin 60.0°\hat{\text{j}}) = (0.200 \text{ m/s}^2)\hat{\text{i}} + (0.346 \text{ m/s}^2)\hat{\text{j}},$$

and $\vec{a}_{PA} = \vec{a}_{PB}$. Thus, we have $|\vec{a}_{PB}| = 0.400 \text{ m/s}^2$.

(d) The direction is 60.0° north of east (or 30.0° east of north).

79. **THINK** This problem involves analyzing the relative motion of two ships sailing in different directions.

EXPRESS Given that $\theta_A = 45°$, and $\theta_B = 40°$, as defined in the figure, the velocity vectors (relative to the shore) for ships *A* and *B* are given by



$$\vec{v}_A = -(v_A \cos 45°)\,\hat{\text{i}} + (v_A \sin 45°)\,\hat{\text{j}}$$
$$\vec{v}_B = -(v_B \sin 40°)\,\hat{\text{i}} - (v_B \cos 40°)\,\hat{\text{j}},$$

with v_A = 24 knots and v_B = 28 knots. We take east as $+\hat{\text{i}}$ and north as $\hat{\text{j}}$.

The velocity of ship *A* relative to ship *B* is simply given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.

ANALYZE (a) The relative velocity is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = (v_B \sin 40° - v_A \cos 45°)\hat{\mathrm{i}} + (v_B \cos 40° + v_A \sin 45°)\,\hat{\mathrm{j}}$$
$$= (1.03 \text{ knots})\hat{\mathrm{i}} + (38.4 \text{ knots})\hat{\mathrm{j}}$$

the magnitude of which is $|\vec{v}_{AB}| = \sqrt{(1.03 \text{ knots})^2 + (38.4 \text{ knots})^2} \approx 38.4$ knots.

(b) The angle θ_{AB} which $\vec{v}_{AB}$ makes with north is given by

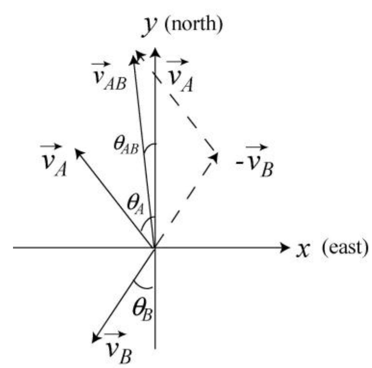
$$\theta_{AB} = \tan^{-1}\left(\frac{v_{AB,x}}{v_{AB,y}}\right) = \tan^{-1}\left(\frac{1.03 \text{ knots}}{38.4 \text{ knots}}\right) = 1.5°$$

which is to say that $\vec{v}_{AB}$ points 1.5° east of north.

(c) Since the two ships started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\Delta r_{AB}|}{|\vec{v}_{AB}|} = \frac{160 \text{ nautical miles}}{38.4 \text{ knots}} = 4.2 \text{ h.}$$

(d) The velocity $\vec{v}_{AB}$ does not change with time in this problem, and $\vec{r}_{AB}$ is in the same direction as $\vec{v}_{AB}$ since they started at the same time. Reversing the points of view, we have $\vec{v}_{AB} = -\vec{v}_{BA}$ so that $\vec{r}_{AB} = -\vec{r}_{BA}$ (i.e., they are 180° opposite to each other). Hence, we conclude that *B* stays at a bearing of 1.5° west of south relative to *A* during the journey (neglecting the curvature of Earth).



LEARN The relative velocity is depicted in the figure on the right. When analyzing relative motion in two dimensions, a vector diagram such as the one shown can be very helpful.

80. This is a classic problem involving two-dimensional relative motion. We align our coordinates so that *east* corresponds to +*x* and *north* corresponds to +*y*. We write the vector addition equation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. We have $\vec{v}_{WG} = (2.0\angle 0°)$ in the magnitude-angle notation (with the unit m/s understood), or $\vec{v}_{WG} = 2.0\hat{\mathrm{i}}$ in unit-vector notation. We also have $\vec{v}_{BW} = (8.0\angle 120°)$ where we have been careful to phrase the angle in the 'standard' way (measured counterclockwise from the +*x* axis), or $\vec{v}_{BW} = (-4.0\hat{\mathrm{i}}+6.9\hat{\mathrm{j}})$ m/s.

(a) We can solve the vector addition equation for $\vec{v}_{BG}$:

$$\vec{v}_{BG} = v_{BW} + \vec{v}_{WG} = (2.0\text{ m/s})\,\hat{\text{i}} + (-4.0\hat{\text{i}}+6.9\hat{\text{j}})\text{ m/s} = (-2.0\text{ m/s})\hat{\text{i}} + (6.9\text{ m/s})\,\hat{\text{j}}.$$

Thus, we find $|\vec{v}_{BG}| = 7.2$ m/s.

(b) The direction of $\vec{v}_{BG}$ is $\theta = \tan^{-1}[(6.9\text{ m/s})/(-2.0\text{ m/s})] = 106°$ (measured counterclockwise from the +*x* axis), or 16° west of north.

(c) The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have $(200\text{ m}) = (7.2\text{ m/s})\sin(106°)t \rightarrow t = 29$ s. Note: If a student obtains "28 s," then the student has probably neglected to take the *y* component properly (a common mistake).

81. Here, the subscript *W* refers to the water. Our coordinates are chosen with +*x* being *east* and +*y* being *north*. In these terms, the angle specifying *east* would be 0° and the angle specifying *south* would be –90° or 270°. Where the length unit is not displayed, km is to be understood.

(a) We have $\vec{v}_{A\,W} = \vec{v}_{A\,B} + \vec{v}_{B\,W}$, so that

$$\vec{v}_{A\,B} = (22\ \angle -90°) - (40\ \angle\ 37°) = (56\ \angle -125°)$$

in the magnitude-angle notation (conveniently done with a vector-capable calculator in polar mode). Converting to rectangular components, we obtain

$$\vec{v}_{A\,B} = (-32\text{km/h})\,\hat{\text{i}} - (46\text{ km/h})\,\hat{\text{j}}\,.$$

Of course, this could have been done in unit-vector notation from the outset.

(b) Since the velocity-components are constant, integrating them to obtain the position is straightforward ($\vec{r} - \vec{r}_0 = \int \vec{v}\,dt$)

$$\vec{r} = (2.5 - 32t)\,\hat{\text{i}} + (4.0 - 46t)\,\hat{\text{j}}$$

with lengths in kilometers and time in hours.

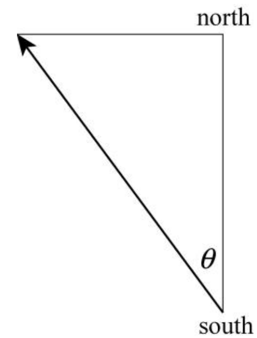
(c) The magnitude of this $\vec{r}$ is $r = \sqrt{(2.5-32t)^2 + (4.0-46t)^2}$. We minimize this by taking a derivative and requiring it to equal zero — which leaves us with an equation for *t*

$$\frac{dr}{dt} = \frac{1}{2}\frac{6286t - 528}{\sqrt{(2.5-32t)^2 + (4.0-46t)^2}} = 0$$

which yields $t = 0.084$ h.

(d) Plugging this value of t back into the expression for the distance between the ships (r), we obtain r = 0.2 km. Of course, the calculator offers more digits (r = 0.225…), but they are not significant; in fact, the uncertainties implicit in the given data, here, should make the ship captains worry.

82. We construct a right triangle starting from the clearing on the south bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance $(82\text{ m}) + (1.1\text{ m/s})t$, where the t-dependent contribution is the distance that the river will carry the boat downstream during time t.



The hypotenuse of this right triangle (the arrow in our sketch) also depends on t and on the boat's speed (relative to the water), and we set it equal to the Pythagorean "sum" of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

which leads to a quadratic equation for t

$$46724 + 180.4t - 14.8t^2 = 0.$$

(b) We solve for t first and find a positive value: t = 62.6 s.

(a) The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured "west of north") is then given by

$$\theta = \tan^{-1}\left(\frac{82 + 1.1t}{200}\right) = \tan^{-1}\left(\frac{151}{200}\right) = 37°.$$

83. We establish coordinates with $\hat{i}$ pointing to the far side of the river (perpendicular to the current) and $\hat{j}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}|$ = 6.4 km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = (3.2\text{ km/h})\hat{j}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} = v_{bg}\hat{i}$ where $v_{bg} > 0$ is unknown. Thus, all $\hat{j}$ components must cancel in the vector sum $\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$, which means the $\vec{v}_{bw}\sin\theta = (-3.2\text{ km/h})\ \hat{j}$, so

$$\theta = \sin^{-1}[(-3.2\text{ km/h})/(6.4\text{ km/h})] = -30°.$$

(b) Using the result from part (a), we find $v_{bg} = v_{bw} \cos\theta = 5.5$ km/h. Thus, traveling a distance of $\ell = 6.4$ km requires a time of (6.4 km)/(5.5 km/h) = 1.15 h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with $v_{wg} =$ 3.2 km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

where $D = 3.2$ km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{v_{bw}+v_{wg}} + \frac{D}{v_{bw}-v_{wg}} = \frac{D}{v_{bw}-v_{wg}} + \frac{D}{v_{bw}+v_{wg}}$$

the answer is the same as in the previous part, that is, $t_{\text{total}} = 80 \text{ min}$.

(e) The shortest-time path should have $\theta = 0°$. This can also be shown by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw}\cos\theta \,\hat{\text{i}} + (v_{bw}\sin\theta + v_{wg})\,\hat{\text{j}}$$

where the x component of $\vec{v}_{bg}$ must equal l/t. Thus,

$$t = \frac{l}{v_{bw}\cos\theta}$$

which can be minimized using $dt/d\theta = 0$.

(f) The above expression leads to $t = (6.4 \text{ km})/(6.4 \text{ km/h}) = 1.0$ h, or 60 min.

84. Relative to the sled, the launch velocity is $\vec{v}_{0\text{rel}} = v_{ox}\,\hat{\text{i}} + v_{oy}\,\hat{\text{j}}$. Since the sled's motion is in the negative direction with speed v_s (note that we are treating v_s as a positive number, so the sled's velocity is actually $-v_s\,\hat{\text{i}}$), then the launch velocity relative to the ground is $\vec{v}_0 = (v_{ox} - v_s)\,\hat{\text{i}} + v_{oy}\,\hat{\text{j}}$. The horizontal and vertical displacement (relative to the ground) are therefore

$$x_{\text{land}} - x_{\text{launch}} = \Delta x_{\text{bg}} = (v_{ox} - v_s)\, t_{\text{flight}}$$

$$y_{\text{land}} - y_{\text{launch}} = 0 = v_{oy}\, t_{\text{flight}} + \frac{1}{2}(-g)(t_{\text{flight}})^2 .$$

Combining these equations leads to

$$\Delta x_{\rm bg} = \frac{2v_{0x}v_{0y}}{g} - \left(\frac{2v_{0y}}{g}\right)v_s .$$

The first term corresponds to the "y intercept" on the graph, and the second term (in parentheses) corresponds to the magnitude of the "slope." From the figure, we have

$$\Delta x_{bg} = 40 - 4v_s .$$

This implies v_{oy} = (4.0 s)(9.8 m/s^2)/2 = 19.6 m/s, and that furnishes enough information to determine v_{ox}.

(a) $v_{ox} = 40g/2v_{oy}$ = (40 m)(9.8 m/s^2)/(39.2 m/s) = 10 m/s.

(b) As noted above, v_{oy} = 19.6 m/s.

(c) Relative to the sled, the displacement Δx_{bs} does not depend on the sled's speed, so $\Delta x_{bs} = v_{ox}\, t_{flight}$ = 40 m.

(d) As in (c), relative to the sled, the displacement Δx_{bs} does not depend on the sled's speed, and $\Delta x_{bs} = v_{ox}\, t_{flight}$ = 40 m.

85. Using displacement = velocity × time (for each constant-velocity part of the trip), along with the fact that 1 hour = 60 minutes, we have the following vector addition exercise (using notation appropriate to many vector-capable calculators):

(1667 m ∠ 0º) + (1333 m ∠ –90º) + (333 m ∠ 180º) + (833 m ∠ –90º) + (667 m ∠ 180º) + (417 m ∠ –90º) = (2668 m ∠ –76º).

(a) Thus, the magnitude of the net displacement is 2.7 km.

(b) Its direction is 76° clockwise (relative to the initial direction of motion).

86. We use a coordinate system with $+x$ eastward and $+y$ upward.

(a) We note that 123° is the angle between the initial position and later position vectors, so that the angle from $+x$ to the later position vector is 40° + 123° = 163°. In unit-vector notation, the position vectors are

$$\vec{r}_1 = (360\text{ m})\cos(40°)\,\hat{\mathrm{i}} + (360\text{ m})\sin(40°)\,\hat{\mathrm{j}} = (276\text{ m})\hat{\mathrm{i}} + (231\text{ m})\,\hat{\mathrm{j}}$$
$$\vec{r}_2 = (790\text{ m})\cos(163°)\,\hat{\mathrm{i}} + (790\text{ m})\sin(163°)\,\hat{\mathrm{j}} = (-755\text{ m})\hat{\mathrm{i}} + (231\text{ m})\hat{\mathrm{j}}$$

respectively. Consequently, we plug into Eq. 4-3

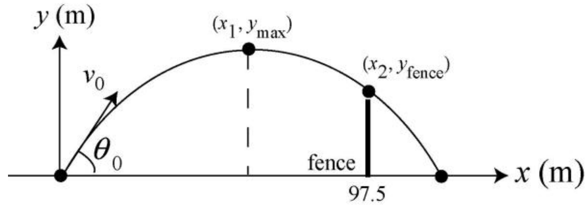
$$\Delta\vec{r} = [(-755 \text{ m}) - (276 \text{ m})]\hat{\text{i}} + (231 \text{ m} - 231 \text{ m})\hat{\text{j}} = -(1031 \text{ m})\,\hat{\text{i}}.$$

The magnitude of the displacement $\Delta\vec{r}$ is $|\Delta\vec{r}| = 1031$ m.

(b) The direction of $\Delta\vec{r}$ is $-\hat{\text{i}}$, or westward.

87. **THINK** This problem deals with the projectile motion of a baseball. Given the information on the position of the ball at two instants, we are asked to analyze its trajectory.

EXPRESS The trajectory of the baseball is shown in the figure on the right. According to the problem statement, at $t_1 = 3.0$ s, the ball reaches it maximum height y_{max}, and at $t_2 = t_1 + 2.5 \text{ s} = 5.5 \text{ s}$, it barely clears a fence at $x_2 = 97.5$ m.



Eq. 2-15 can be applied to the vertical (y axis) motion related to reaching the maximum height (when t_1 = 3.0 s and v_y = 0):

$$y_{max} - y_0 = v_y t - \frac{1}{2}gt^2 .$$

ANALYZE (a) With ground level chosen so y_0 = 0, this equation gives the result

$$y_{max} = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(3.0 \text{ s})^2 = 44.1 \text{ m}$$

(b) After the moment it reached maximum height, it is falling; at $t_2 = t_1 + 2.5 \text{ s} = 5.5 \text{ s}$, it will have fallen an amount given by Eq. 2-18:

$$y_{fence} - y_{max} = 0 - \frac{1}{2}g(t_2 - t_1)^2 .$$

Thus, the height of the fence is

$$y_{fence} = y_{max} - \frac{1}{2}g(t_2 - t_1)^2 = 44.1 \text{ m} - \frac{1}{2}(9.8 \text{ m/s}^2)(2.5 \text{ s})^2 = 13.48 \text{ m} .$$

(c) Since the horizontal component of velocity in a projectile-motion problem is constant (neglecting air friction), we find from 97.5 m = v_{0x}(5.5 s) that v_{0x} = 17.7 m/s. The total flight time of the ball is $T = 2t_1 = 2(3.0 \text{ s}) = 6.0 \text{ s}$. Thus, the range of the baseball is

$$R = v_{0x}T = (17.7 \text{ m/s})(6.0 \text{ s}) = 106.4 \text{ m}$$

which means that the ball travels an additional distance

$$\Delta x = R - x_2 = 106.4 \text{ m} - 97.5 \text{ m} = 8.86 \text{ m}$$

beyond the fence before striking the ground.

LEARN Part (c) can also be solved by noting that after passing the fence, the ball will strike the ground in 0.5 s (so that the total "fall-time" equals the "rise-time"). With v_{0x} = 17.7 m/s, we have Δx = (17.7 m/s)(0.5 s) = 8.86 m.

88. When moving in the same direction as the jet stream (of speed v_s), the time is

$$t_1 = \frac{d}{v_{ja} + v_s},$$

where d = 4000 km is the distance and v_{ja} is the speed of the jet relative to the air (1000 km/h). When moving against the jet stream, the time is

$$t_2 = \frac{d}{v_{ja} - v_s},$$

where $t_2 - t_1 = \frac{70}{60}$ h . Combining these equations and using the quadratic formula to solve gives v_s = 143 km/h.

89. **THINK** We have a particle moving in a two-dimensional plane with a constant acceleration. Since the x and y components of the acceleration are constants, we can use Table 2-1 for the motion along both axes.

EXPRESS Using vector notation with $\vec{r}_0 = 0$, the position and velocity of the particle as a function of time are given by $\vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ and $\vec{v}(t) = \vec{v}_0 + \vec{a}t$, respectively. Where units are not shown, SI units are to be understood.

ANALYZE (a) Given the initial velocity $\vec{v}_0 = (8.0\text{ m/s})\hat{\text{j}}$ and the acceleration $\vec{a} = (4.0\text{ m/s}^2)\hat{\text{i}} + (2.0\text{ m/s}^2)\hat{\text{j}}$, the position vector of the particle is

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \left(8.0\hat{\text{j}}\right)t + \frac{1}{2}\left(4.0\hat{\text{i}} + 2.0\hat{\text{j}}\right)t^2 = \left(2.0t^2\right)\hat{\text{i}} + \left(8.0t + 1.0t^2\right)\hat{\text{j}}.$$

Therefore, the time that corresponds to x = 29 m can be found by solving the equation $2.0t^2 = 29$, which leads to t = 3.8 s. The y coordinate at that time is

$$y = (8.0\text{ m/s})(3.8\text{ s}) + (1.0\text{ m/s}^2)(3.8\text{ s})^2 = 45\text{ m}.$$

(b) The velocity of the particle is given by $\vec{v} = \vec{v}_0 + \vec{a}t$. Thus, at t = 3.8 s, the velocity is

$$\vec{v} = (8.0\text{ m/s})\,\hat{\text{j}} + \left((4.0\text{ m/s}^2)\,\hat{\text{i}} + (2.0\text{ m/s}^2)\,\hat{\text{j}}\right)(3.8\text{ s}) = (15.2\text{ m/s})\,\hat{\text{i}} + (15.6\text{ m/s})\,\hat{\text{j}}$$

which has a magnitude of $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2\text{ m/s})^2 + (15.6\text{ m/s})^2} = 22\text{ m/s}.$

LEARN Instead of using the vector notation, we can also deal with the x- and the y-components individually.

90. Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2\,(x\tan\theta_0 - y)}}$$

which yields v_0 = 23 ft/s for g = 32 ft/s^2, x = 13 ft, y = 3 ft and θ_0 = 55°.

91. We make use of Eq. 4-25.

(a) By rearranging Eq. 4-25, we obtain the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2(x\tan\theta_0 - y)}}$$

which yields $v_0 = 255.5 \approx 2.6 \times 10^2$ m/s for x = 9400 m, y = –3300 m, and θ_0 = 35°.

(b) From Eq. 4-21, we obtain the time of flight:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{9400\text{ m}}{(255.5\text{ m/s})\cos 35°} = 45\text{ s}.$$

(c) We expect the air to provide resistance but no appreciable lift to the rock, so we would need a greater launching speed to reach the same target.

92. We apply Eq. 4-34 to solve for speed v and Eq. 4-35 to find the period T.

(a) We obtain

$$v = \sqrt{ra} = \sqrt{(5.0\text{ m})(7.0)(9.8\text{ m/s}^2)} = 19\text{ m/s}.$$

(b) The time to go around once (the period) is $T = 2\pi r/v$ = 1.7 s. Therefore, in one minute (t = 60 s), the astronaut executes

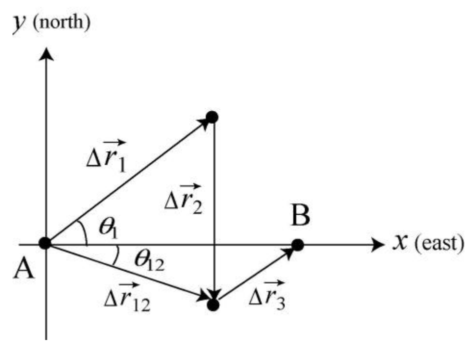
$$\frac{t}{T} = \frac{60\text{ s}}{1.7\text{ s}} = 35$$

revolutions. Thus, 35 rev/min is needed to produce a centripetal acceleration of 7g when the radius is 5.0 m.

(c) As noted above, T = 1.7 s.

93. **THINK** This problem deals with the two-dimensional kinematics of a desert camel moving from oasis A to oasis B.

EXPRESS The journey of the camel is illustrated in the figure on the right. We use a 'standard' coordinate system with +x East and +y North. Lengths are in kilometers and times are in hours. Using vector notation, we write the displacements for the first two segments of the trip as:



$$\Delta\vec{r}_1 = (75\text{ km})\cos(37°)\,\hat{\text{i}} + (75\text{ km})\sin(37°)\,\hat{\text{j}}$$

$$\Delta\vec{r}_2 = (-65\text{ km})\,\hat{\text{j}}$$

The net displacement is $\Delta\vec{r}_{12} = \Delta\vec{r}_1 + \Delta\vec{r}_2$. As can be seen from the figure, to reach oasis B requires an additional displacement $\Delta\vec{r}_3$.

ANALYZE (a) We perform the vector addition of individual displacements to find the net displacement of the camel: $\Delta\vec{r}_{12} = \Delta\vec{r}_1 + \Delta\vec{r}_2 = (60\text{ km})\,\hat{\text{i}} - (20\text{ km})\,\hat{\text{j}}$. Its corresponding magnitude is

$$|\Delta\vec{r}_{12}| = \sqrt{(60\text{ km})^2 + (-20\text{ km})^2} = 63\text{ km}.$$

(b) The direction of $\Delta\vec{r}_{12}$ is $\theta_{12} = \tan^{-1}[(-20\text{ km})/(60\text{ km})] = -18°$, or $18°$ south of east.

(c) To calculate the average velocity for the first two segments of the journey (including rest), we use the result from part (a) in Eq. 4-8 along with the fact that

$$\Delta t_{12} = \Delta t_1 + \Delta t_2 + \Delta t_{\text{rest}} = 50\text{ h} + 35\text{ h} + 5.0\text{ h} = 90\text{ h}.$$

In unit vector notation, we have $\vec{v}_{12,\text{avg}} = \dfrac{(60\,\hat{\text{i}} - 20\,\hat{\text{j}})\text{ km}}{90\text{ h}} = (0.67\,\hat{\text{i}} - 0.22\,\hat{\text{j}})\text{ km/h}.$

This leads to $|\vec{v}_{12,\text{avg}}| = 0.70\text{ km/h}$.

(d) The direction of $\vec{v}_{12,\text{avg}}$ is $\theta_{12} = \tan^{-1}[(-0.22\text{ km/h})/(0.67\text{ km/h})] = -18°$, or $18°$ south of east.

(e) The average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain (140 km)/(90 h) = 1.56 km/h $\approx$1.6 km/h.

(f) The net displacement is required to be the 90 km East from A to B. The displacement from the resting place to B is denoted $\Delta\vec{r}_3$. Thus, we must have

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (90\text{ km})\hat{\text{i}}$$

which produces $\Delta\vec{r}_3 = (30\text{ km})\hat{\text{i}} + (20\text{ km})\hat{\text{j}}$ in unit-vector notation, or $(36 \angle 33°)$ in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{3,\text{avg}}| = \frac{36\text{ km}}{(120-90)\text{ h}} = 1.2\text{ km/h}.$$

(g) The direction of $\vec{v}_{3,\text{avg}}$ is the same as $\Delta\vec{r}_3$ (that is, 33° north of east).

LEARN With a vector-capable calculator in polar mode, we could perform the vector addition of the displacements as $(75 \angle 37°) + (65 \angle -90°) = (63 \angle -18°)$. Note the distinction between average velocity and average speed.

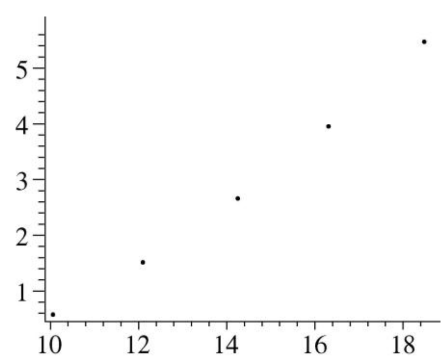
94. We compute the coordinate pairs (x, y) from $x = (v_0 \cos\theta)t$ and $y = v_0 \sin\theta t - \frac{1}{2}gt^2$ for t = 20 s and the speeds and angles given in the problem.

(a) We obtain

$$(x_A, y_A) = (10.1\text{ km}, 0.556\text{ km}) \qquad (x_B, y_B) = (12.1\text{ km}, 1.51\text{ km})$$
$$(x_C, y_C) = (14.3\text{ km}, 2.68\text{ km}) \qquad (x_D, y_D) = (16.4\text{ km}, 3.99\text{ km})$$

and (x_E, y_E) = (18.5 km, 5.53 km) which we plot in the next part.

(b) The vertical (y) and horizontal (x) axes are in kilometers. The graph does not start at the origin. The curve to "fit" the data is not shown, but is easily imagined (forming the "curtain of death").



95. (a) With Δx = 8.0 m, $t = \Delta t_1$, $a = a_x$, and v_{ox} = 0, Eq. 2-15 gives

$$8.0\text{ m} = \tfrac{1}{2} a_x(\Delta t_1)^2,$$

and the corresponding expression for motion along the y axis leads to

$$\Delta y = 12\text{ m} = \tfrac{1}{2} a_y(\Delta t_1)^2.$$

Dividing the second expression by the first leads to $a_y / a_x = 3/2 = 1.5$.

(b) Letting $t = 2\Delta t_1$, then Eq. 2-15 leads to $\Delta x = (8.0\text{ m})(2)^2 = 32$ m, which implies that its x coordinate is now (4.0 + 32) m = 36 m. Similarly, $\Delta y = (12\text{ m})(2)^2 = 48$ m, which means its y coordinate has become (6.0 + 48) m = 54 m.

96. We assume the ball's initial velocity is perpendicular to the plane of the net. We choose coordinates so that $(x_0, y_0) = (0, 3.0)$ m, and $v_x > 0$ (note that $v_{0y} = 0$).

(a) To (barely) clear the net, we have

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \;\Rightarrow\; 2.24\text{ m} - 3.0\text{ m} = 0 - \frac{1}{2}\left(9.8\text{ m/s}^2\right)t^2$$

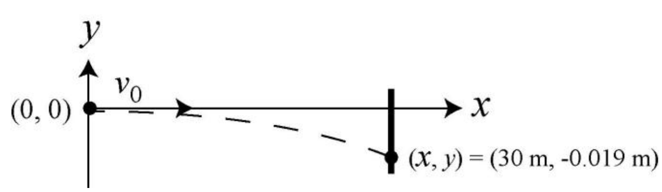
which gives $t = 0.39$ s for the time it is passing over the net. This is plugged into the x-equation to yield the (minimum) initial velocity $v_x = (8.0\text{ m})/(0.39\text{ s}) = 20.3$ m/s.

(b) We require $y = 0$ and find time t from the equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$. This value $\left(t = \sqrt{2(3.0\text{ m})/(9.8\text{ m/s}^2)} = 0.78\text{ s}\right)$ is plugged into the x-equation to yield the (maximum) initial velocity

$$v_x = (17.0\text{ m})/(0.78\text{ s}) = 21.7\text{ m/s}.$$

97. **THINK** A bullet fired horizontally from a rifle strikes the target at some distance below its aiming point. We're asked to find its total flight time and speed.

EXPRESS The trajectory of the bullet is shown in the figure on the right (not to scale). Note that the origin is chosen to be at the firing point. With this convention, the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. Knowing the coordinates (x, y) at the target allows us to calculate the total flight time and speed of the bullet.



ANALYZE (a) If t is the time of flight and $y = -0.019$ m indicates where the bullet hits the target, then

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-0.019\text{ m})}{9.8\text{ m/s}^2}} = 6.2\times10^{-2}\text{ s}.$$

(b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since x = 30 m is the horizontal position of the target, we have $x = v_0t$. Thus,

$$v_0 = \frac{x}{t} = \frac{30\text{ m}}{6.3\times10^{-2}\text{ s}} = 4.8\times10^2\text{ m/s}.$$

LEARN Alternatively, we may use Eq. 4-25 to solve for the initial velocity. With $\theta_0 = 0$ and $y_0 = 0$, the equation simplifies to $y = -\frac{gx^2}{2v_0^2}$, from which we find

$$v_0 = \sqrt{-\frac{gx^2}{2y}} = \sqrt{-\frac{(9.8\text{ m/s}^2)(30\text{ m})^2}{2(-0.019\text{ m})}} = 4.8\times10^2\text{ m/s},$$

in agreement with what we calculated in part (b).

98. For circular motion, we must have $\vec{v}$ with direction perpendicular to $\vec{r}$ and (since the speed is constant) magnitude $v = 2\pi r/T$ where $r = \sqrt{(2.00\text{ m})^2 + (-3.00\text{ m})^2}$ and $T = 7.00\text{ s}$. The $\vec{r}$ (given in the problem statement) specifies a point in the fourth quadrant, and since the motion is clockwise then the velocity must have both components negative. Our result, satisfying these three conditions, (using unit-vector notation which makes it easy to double-check that $\vec{r}\cdot\vec{v} = 0$) for $\vec{v} = (-2.69\text{ m/s})\hat{\text{i}} + (-1.80\text{ m/s})\hat{\text{j}}$.

99. Let $v_o = 2\pi(0.200\text{ m})/(0.00500\text{ s}) \approx 251$ m/s (using Eq. 4-35) be the speed it had in circular motion and $\theta_o = (1\text{ hr})(360°/12\text{ hr}$ [for full rotation]$) = 30.0°$. Then Eq. 4-25 leads to

$$y = (2.50\text{ m})\tan 30.0° - \frac{(9.8\text{ m/s}^2)(2.50\text{ m})^2}{2(251\text{ m/s})^2(\cos 30.0°)^2} \approx 1.44\text{ m}$$

which means its height above the floor is 1.44 m + 1.20 m = 2.64 m.

100. Noting that $\vec{v}_2 = 0$, then, using Eq. 4-15, the average acceleration is

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{0 - \left(6.30\,\hat{\text{i}} - 8.42\,\hat{\text{j}}\right)\text{m/s}}{3\text{ s}} = \left(-2.1\,\hat{\text{i}} + 2.8\,\hat{\text{j}}\right)\text{m/s}^2$$

101. Using Eq. 2-16, we obtain $v^2 = v_0^2 - 2gh$, or $h = (v_0^2 - v^2)/2g$.

(a) Since $v = 0$ at the maximum height of an upward motion, with $v_0 = 7.00$ m/s, we have

$$h = (7.00\text{ m/s})^2 / 2(9.80\text{ m/s}^2) = 2.50\text{ m}.$$

(b) The relative speed is $v_r = v_0 - v_c = 7.00\text{ m/s} - 3.00\text{ m/s} = 4.00\text{ m/s}$ with respect to the floor. Using the above equation we obtain $h = (4.00\text{ m/s})^2 / 2(9.80\text{ m/s}^2) = 0.82\text{ m}.$

(c) The acceleration, or the rate of change of speed of the ball with respect to the ground is 9.80 m/s^2 (downward).

(d) Since the elevator cab moves at constant velocity, the rate of change of speed of the ball with respect to the cab floor is also 9.80 m/s^2 (downward).

102. (a) With $r = 0.15$ m and $a = 3.0 \times 10^{14}$ m/s^2, Eq. 4-34 gives

$$v = \sqrt{ra} = 6.7 \times 10^6\text{ m/s}.$$

(b) The period is given by Eq. 4-35:

$$T = \frac{2\pi r}{v} = 1.4 \times 10^{-7}\text{ s}.$$

103. (a) The magnitude of the displacement vector $\Delta\vec{r}$ is given by

$$|\Delta\vec{r}| = \sqrt{(21.5\text{ km})^2 + (9.7\text{ km})^2 + (2.88\text{ km})^2} = 23.8\text{ km}.$$

Thus,

$$|\vec{v}_{\text{avg}}| = \frac{|\Delta\vec{r}|}{\Delta t} = \frac{23.8\text{ km}}{3.50\text{ h}} = 6.79\text{ km/h}.$$

(b) The angle θ in question is given by

$$\theta = \tan^{-1}\left(\frac{2.88\text{ km}}{\sqrt{(21.5\text{ km})^2 + (9.7\text{ km})^2}}\right) = 6.96°.$$

104. The initial velocity has magnitude v_0 and because it is horizontal, it is equal to v_x the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0$$

where $v_y = \sqrt{2gh}$ and we have used Eq. 2-16 with Δx replaced with $h = 20$ m. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = (3v_0)^2$$

which leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{(9.8\ \text{m/s}^2)(20\ \text{m})}/2 = 7.0$ m/s.

105. We choose horizontal x and vertical y axes such that both components of $\vec{v}_0$ are positive. Positive angles are counterclockwise from $+x$ and negative angles are clockwise from it. In unit-vector notation, the velocity at each instant during the projectile motion is

$$\vec{v} = v_0 \cos\theta_0\ \hat{\text{i}} + (v_0 \sin\theta_0 - gt)\ \hat{\text{j}}.$$

(a) With v_0 = 30 m/s and θ_0 = 60°, we obtain $\vec{v} = (15\hat{\text{i}} + 6.4\hat{\text{j}})$ m/s, for t = 2.0 s. The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(15\ \text{m/s})^2 + (6.4\ \text{m/s})^2} = 16$ m/s.

(b) The direction of $\vec{v}$ is

$$\theta = \tan^{-1}[(6.4\ \text{m/s})/(15\ \text{m/s})] = 23°,$$

measured counterclockwise from $+x$.

(c) Since the angle is positive, it is above the horizontal.

(d) With t = 5.0 s, we find $\vec{v} = (15\hat{\text{i}} - 23\hat{\text{j}})$ m/s, which yields

$$|\vec{v}| = \sqrt{(15\ \text{m/s})^2 + (-23\ \text{m/s})^2} = 27\ \text{m/s}.$$

(e) The direction of $\vec{v}$ is $\theta = \tan^{-1}[(-23\ \text{m/s})/(15\ \text{m/s})] = -57°$, or 57° measured *clockwise* from $+x$.

(f) Since the angle is negative, it is below the horizontal.

106. We use Eq. 4-2 and Eq. 4-3.

(a) With the initial position vector as $\vec{r}_1$ and the later vector as $\vec{r}_2$, Eq. 4-3 yields

$$\Delta r = [(-2.0\ \text{m}) - 5.0\ \text{m}]\hat{\text{i}} + [(6.0\ \text{m}) - (-6.0\ \text{m})]\hat{\text{j}} + (2.0\ \text{m} - 2.0\ \text{m})\hat{\text{k}} = (-7.0\ \text{m})\hat{\text{i}} + (12\ \text{m})\hat{\text{j}}$$

for the displacement vector in unit-vector notation.

(b) Since there is no z component (that is, the coefficient of $\hat{\text{k}}$ is zero), the displacement vector is in the xy plane.

107. We write our magnitude-angle results in the form $(R \angle \theta)$ with SI units for the magnitude understood (m for distances, m/s for speeds, m/s^2 for accelerations). All angles θ are measured counterclockwise from +x, but we will occasionally refer to angles ϕ, which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see r in the figure). We note that the speed of the particle is $v = 2\pi r/T$ where r = 3.00 m and T = 20.0 s; thus, v = 0.942 m/s. The particle is moving counterclockwise in Fig. 4-56.

(a) At t = 5.0 s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00\text{ s}}{20.0\text{ s}} = \frac{1}{4}$$

of a full revolution around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \frac{1}{4}(360°) = 90°$$

measured from vertical (as explained above). Referring to Fig. 4-56, we see that this position (which is the "3 o'clock" position on the circle) corresponds to x = 3.0 m and y = 3.0 m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R\angle\theta) = (4.2\angle 45°)$. Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of x and y relative to coordinate origin can be gotten from the angle ϕ from the relations

$$x = r\sin\phi, \quad y = r - r\cos\phi .$$

Of course, $R = \sqrt{x^2 + y^2}$ and θ comes from choosing the appropriate possibility from $\tan^{-1}(y/x)$ (or by using particular functions of vector-capable calculators).

(b) At t = 7.5 s, the particle has traveled a fraction of 7.5/20 = 3/8 of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at ϕ = 3/8 (360°) = 135° measured from vertical in the manner discussed above. Referring to Fig. 4-56, we compute that this position corresponds to

$$x = (3.00\text{ m})\sin 135° = 2.1\text{ m}$$
$$y = (3.0\text{ m}) - (3.0\text{ m})\cos 135° = 5.1\text{ m}$$

relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (5.5 \angle 68°)$.

(c) At t = 10.0 s, the particle has traveled a fraction of 10/20 = 1/2 of a revolution around the circle. Relative to the circle-center, the particle is at ϕ = 180° measured from vertical (see explanation above). Referring to Fig. 4-56, we see that this position corresponds to x

= 0 and y = 6.0 m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R\angle\theta) = (6.0\angle 90°)$.

(d) We subtract the position vector in part (a) from the position vector in part (c):

$$(6.0\angle 90°) - (4.2\angle 45°) = (4.2\angle 135°)$$

using magnitude-angle notation (convenient when using vector-capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta\vec{R} = (0 - 3.0 \text{ m})\ \hat{\text{i}} + (6.0 \text{ m} - 3.0 \text{ m})\ \hat{\text{j}} = (-3.0 \text{ m})\hat{\text{i}} + (3.0 \text{ m})\hat{\text{j}}$$

which leads to $|\Delta\vec{R}| = 4.2$ m and $\theta = 135°$.

(e) From Eq. 4-8, we have $\vec{v}_{\text{avg}} = \Delta\vec{R}/\Delta t$. With $\Delta t = 5.0$ s, we have

$$\vec{v}_{\text{avg}} = (-0.60 \text{ m/s})\ \hat{\text{i}} + (0.60 \text{ m/s})\ \hat{\text{j}}$$

in unit-vector notation or (0.85 $\angle$ 135°) in magnitude-angle notation.

(f) The speed has already been noted (v = 0.94 m/s), but its direction is best seen by referring again to Fig. 4-56. The velocity vector is tangent to the circle at its "3 o'clock position" (see part (a)), which means $\vec{v}$ is vertical. Thus, our result is $(0.94 \angle 90°)$.

(g) Again, the speed has been noted above (v = 0.94 m/s), but its direction is best seen by referring to Fig. 4-56. The velocity vector is tangent to the circle at its "12 o'clock position" (see part (c)), which means $\vec{v}$ is horizontal. Thus, our result is $(0.94 \angle 180°)$.

(h) The acceleration has magnitude $a = v^2/r = 0.30$ m/s^2, and at this instant (see part (a)) it is horizontal (toward the center of the circle). Thus, our result is $(0.30 \angle 180°)$.

(i) Again, $a = v^2/r = 0.30$ m/s^2, but at this instant (see part (c)) it is vertical (toward the center of the circle). Thus, our result is $(0.30 \angle 270°)$.

108. Equation 4-34 describes an inverse proportionality between r and a, so that a large acceleration results from a small radius. Thus, an upper limit for a corresponds to a lower limit for r.

(a) The minimum turning radius of the train is given by

$$r_{min} = \frac{v^2}{a_{max}} = \frac{(216\text{ km/h})^2}{(0.050)(9.8\text{ m/s}^2)} = 7.3 \times 10^3\text{ m}.$$

(b) The speed of the train must be reduced to no more than

$$v = \sqrt{a_{max} r} = \sqrt{0.050(9.8\text{ m/s}^2)(1.00 \times 10^3\text{ m})} = 22\text{ m/s}$$

which is roughly 80 km/h.

109. (a) Using the same coordinate system assumed in Eq. 4-25, we find

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0\cos\theta_0)^2} = -\frac{gx^2}{2v_0^2} \quad \text{if } \theta_0 = 0.$$

Thus, with $v_0 = 3.0 \times 10^6$ m/s and $x = 1.0$ m, we obtain $y = -5.4 \times 10^{-13}$ m, which is not practical to measure (and suggests why gravitational processes play such a small role in the fields of atomic and subatomic physics).

(b) It is clear from the above expression that $|y|$ decreases as v_0 is increased.

110. When the escalator is stalled the speed of the person is $v_p = \ell/t$, where ℓ is the length of the escalator and t is the time the person takes to walk up it. This is v_p = (15 m)/(90 s) = 0.167 m/s. The escalator moves at v_e = (15 m)/(60 s) = 0.250 m/s. The speed of the person walking up the moving escalator is

$$v = v_p + v_e = 0.167\text{ m/s} + 0.250\text{ m/s} = 0.417\text{ m/s}$$

and the time taken to move the length of the escalator is

$$t = \ell / v = (15\text{ m})/(0.417\text{ m/s}) = 36\text{ s}.$$

If the various times given are independent of the escalator length, then the answer does not depend on that length either. In terms of ℓ (in meters) the speed (in meters per second) of the person walking on the stalled escalator is $\ell/90$, the speed of the moving escalator is $\ell/60$, and the speed of the person walking on the moving escalator is $v = (\ell/90) + (\ell/60) = 0.0278\ell$. The time taken is $t = \ell/v = \ell/0.0278\ell = 36\text{s}$ and is independent of ℓ.

111. The radius of Earth may be found in Appendix C.

(a) The speed of an object at Earth's equator is $v = 2\pi R/T$, where R is the radius of Earth (6.37×10^6 m) and T is the length of a day (8.64×10^4 s):

$$v = 2\pi(6.37 \times 10^6 \text{ m})/(8.64 \times 10^4 \text{ s}) = 463 \text{ m/s}.$$

The magnitude of the acceleration is given by

$$a = \frac{v^2}{R} = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}} = 0.034 \text{ m/s}^2.$$

(b) If T is the period, then $v = 2\pi R/T$ is the speed and the magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}.$$

Thus,

$$T = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{6.37 \times 10^6 \text{ m}}{9.8 \text{ m/s}^2}} = 5.1 \times 10^3 \text{ s} = 84 \text{ min}.$$

112. With $g_B = 9.8128 \text{ m/s}^2$ and $g_M = 9.7999 \text{ m/s}^2$, we apply Eq. 4-26:

$$R_M - R_B = \frac{v_0^2 \sin 2\theta_0}{g_M} - \frac{v_0^2 \sin 2\theta_0}{g_B} = \frac{v_0^2 \sin 2\theta_0}{g_B}\left(\frac{g_B}{g_M} - 1\right)$$

which becomes

$$R_M - R_B = R_B\left(\frac{9.8128 \text{ m/s}^2}{9.7999 \text{ m/s}^2} - 1\right)$$

and yields (upon substituting $R_B = 8.09$ m) $R_M - R_B = 0.01$ m = 1 cm.

113. From the figure, the three displacements can be written as

$$\vec{d}_1 = d_1(\cos\theta_1\hat{\text{i}} + \sin\theta_1\hat{\text{j}}) = (5.00 \text{ m})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (4.33 \text{ m})\hat{\text{i}} + (2.50 \text{ m})\hat{\text{j}}$$

$$\vec{d}_2 = d_2[\cos(180° + \theta_1 - \theta_2)\hat{\text{i}} + \sin(180° + \theta_1 - \theta_2)\hat{\text{j}}] = (8.00 \text{ m})(\cos 160°\hat{\text{i}} + \sin 160°\hat{\text{j}})$$
$$= (-7.52 \text{ m})\hat{\text{i}} + (2.74 \text{ m})\hat{\text{j}}$$

$$\vec{d}_3 = d_3[\cos(360° - \theta_3 - \theta_2 + \theta_1)\hat{\text{i}} + \sin(360° - \theta_3 - \theta_2 + \theta_1)\hat{\text{j}}] = (12.0 \text{ m})(\cos 260°\hat{\text{i}} + \sin 260°\hat{\text{j}})$$
$$= (-2.08 \text{ m})\hat{\text{i}} - (11.8 \text{ m})\hat{\text{j}}$$

where the angles are measured from the $+x$ axis. The net displacement is

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (-5.27 \text{ m})\hat{\text{i}} - (6.58 \text{ m})\hat{\text{j}}.$$

(a) The magnitude of the net displacement is

$$|\vec{d}| = \sqrt{(-5.27\ \text{m})^2 + (-6.58\ \text{m})^2} = 8.43\ \text{m}.$$

(b) The direction of $\vec{d}$ is $\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-6.58\ \text{m}}{-5.27\ \text{m}}\right) = 51.3°$ or $231°$.

We choose $231°$ (measured counterclockwise from $+x$) since the desired angle is in the third quadrant. An equivalent answer is $-129°$ (measured clockwise from $+x$).

114. Taking derivatives of $\vec{r} = 2t\hat{\text{i}} + 2\sin(\pi t/4)\hat{\text{j}}$ (with lengths in meters, time in seconds, and angles in radians) provides expressions for velocity and acceleration:

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{\text{i}} + \frac{\pi}{2}\cos\left(\frac{\pi t}{4}\right)\hat{\text{j}}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{\pi^2}{8}\sin\left(\frac{\pi t}{4}\right)\hat{\text{j}}.$$

Thus, we obtain:

time t (s)			0.0	1.0	2.0	3.0	4.0
(a)	$\vec{r}$ position	x (m)	0.0	2.0	4.0	6.0	8.0
		y (m)	0.0	1.4	2.0	1.4	0.0
(b)	$\vec{v}$ velocity	v_x (m/s)		2.0	2.0	2.0	
		v_y (m/s)		1.1	0.0	−1.1	
(c)	$\vec{a}$ acceleration	a_x (m/s^2)		0.0	0.0	0.0	
		a_y (m/s^2)		−0.87	−1.2	−0.87	

115. Since this problem involves constant downward acceleration of magnitude a, similar to the projectile motion situation, we use the equations of §4-6 as long as we substitute a for g. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and

$$v_{0x} = v_0 = 1.00 \times 10^9\ \text{cm/s}.$$

(a) If ℓ is the length of a plate and t is the time an electron is between the plates, then $\ell = v_0 t$, where v_0 is the initial speed. Thus

$$t = \frac{\ell}{v_0} = \frac{2.00\,\text{cm}}{1.00 \times 10^9\,\text{cm/s}} = 2.00 \times 10^{-9}\ \text{s}.$$

(b) The vertical displacement of the electron is

$$y = -\frac{1}{2}at^2 = -\frac{1}{2}\left(1.00\times10^{17}\ \text{cm/s}^2\right)\left(2.00\times10^{-9}\ \text{s}\right)^2 = -0.20\,\text{cm} = -2.00\ \text{mm},$$

or $|y| = 2.00$ mm.

(c) The *x* component of velocity does not change:

$$v_x = v_0 = 1.00 \times 10^9\ \text{cm/s} = 1.00 \times 10^7\ \text{m/s}.$$

(d) The *y* component of the velocity is

$$v_y = a_y t = \left(1.00\times10^{17}\ \text{cm/s}^2\right)\left(2.00\times10^{-9}\ \text{s}\right) = 2.00\times10^8\ \text{cm/s}$$
$$= 2.00\times10^6\ \text{m/s}.$$

116. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion of the shot ball. We are allowed to use Table 2-1 (with Δy replacing Δx) because the ball has constant acceleration motion. We use primed variables (except *t*) with the constant-velocity elevator (so $v' = 10$ m/s), and unprimed variables with the ball (with initial velocity $v_0 = v' + 20 = 30$ m/s, relative to the ground). SI units are used throughout.

(a) Taking the time to be zero at the instant the ball is shot, we compute its maximum height *y* (relative to the ground) with $v^2 = v_0^2 - 2g(y - y_0)$, where the highest point is characterized by $v = 0$. Thus,

$$y = y_o + \frac{v_0^2}{2g} = 76\ \text{m}$$

where $y_o = y_o' + 2 = 30$ m (where $y_o' = 28$ m is given in the problem) and $v_0 = 30$ m/s relative to the ground as noted above.

(b) There are a variety of approaches to this question. One is to continue working in the frame of reference adopted in part (a) (which treats the ground as motionless and "fixes" the coordinate origin to it); in this case, one describes the elevator motion with $y' = y_o' + v't$ and the ball motion with Eq. 2-15, and solves them for the case where they reach the same point at the same time. Another is to work in the frame of reference of the elevator (the boy in the elevator might be oblivious to the fact the elevator is moving since it isn't accelerating), which is what we show here in detail:

$$\Delta y_e = v_{0_e} t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{v_{0_e} + \sqrt{v_{0e}^2 - 2g\Delta y_e}}{g}$$

where v_{0e} = 20 m/s is the initial velocity of the ball relative to the elevator and Δy_e = –2.0 m is the ball's displacement relative to the floor of the elevator. The positive root is chosen to yield a positive value for t; the result is t = 4.2 s.

117. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the initial position for the football as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of its initial velocity measured from the +x axis.

(a) x = 46 m and y = –1.5 m are the coordinates for the landing point; it lands at time t = 4.5 s. Since $x = v_{0x}t$,

$$v_{0x} = \frac{x}{t} = \frac{46 \text{ m}}{4.5 \text{ s}} = 10.2 \text{ m/s}.$$

Since $y = v_{0y}t - \frac{1}{2}gt^2$,

$$v_{0y} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{(-1.5 \text{ m}) + \frac{1}{2}(9.8 \text{ m/s}^2)(4.5 \text{ s})^2}{4.5 \text{ s}} = 21.7 \text{ m/s}.$$

The magnitude of the initial velocity is

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(10.2 \text{ m/s})^2 + (21.7 \text{ m/s})^2} = 24 \text{ m/s}.$$

(b) The initial angle satisfies tan $\theta_0 = v_{0y}/v_{0x}$. Thus,

$$\theta_0 = \tan^{-1} [(21.7 \text{ m/s})/(10.2 \text{ m/s})] = 65°.$$

118. The velocity of Larry is v_1 and that of Curly is v_2. Also, we denote the length of the corridor by L. Now, Larry's time of passage is t_1 = 150 s (which must equal L/v_1), and Curly's time of passage is t_2 = 70 s (which must equal L/v_2). The time Moe takes is therefore

$$t = \frac{L}{v_1 + v_2} = \frac{1}{v_1/L + v_2/L} = \frac{1}{\frac{1}{150\text{ s}} + \frac{1}{70\text{ s}}} = 48\text{s}.$$

119. The boxcar has velocity $\vec{v}_{cg} = v_1\,\hat{\text{i}}$ relative to the ground, and the bullet has velocity

$$\vec{v}_{0bg} = v_2 \cos\theta\,\hat{\text{i}} + v_2 \sin\theta\,\hat{\text{j}}$$

relative to the ground before entering the car (we are neglecting the effects of gravity on the bullet). While in the car, its velocity relative to the outside ground is

$$\vec{v}_{bg} = 0.8v_2 \cos\theta\,\hat{\text{i}} + 0.8v_2 \sin\theta\,\hat{\text{j}}$$

(due to the 20% reduction mentioned in the problem). The problem indicates that the velocity of the bullet in the car *relative to the car* is (with v_3 unspecified) $\vec{v}_{bc} = v_3\,\hat{\text{j}}$. Now, Eq. 4-44 provides the condition

$$\vec{v}_{bg} = \vec{v}_{bc} + \vec{v}_{cg}$$
$$0.8v_2 \cos\theta\ \hat{\text{i}} + 0.8v_2 \sin\theta\ \hat{\text{j}} = v_3\ \hat{\text{j}} + v_1\ \hat{\text{i}}$$

so that equating x components allows us to find θ. If one wished to find v_3 one could also equate the y components, and from this, if the car width were given, one could find the time spent by the bullet in the car, but this information is not asked for (which is why the width is irrelevant). Therefore, examining the x components in SI units leads to

$$\theta = \cos^{-1}\left(\frac{v_1}{0.8v_2}\right) = \cos^{-1}\left(\frac{85\text{ km/h}\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right)}{0.8\ (650\text{ m/s})}\right)$$

which yields 87° for the direction of $\vec{v}_{bg}$ (measured from $\hat{\text{i}}$, which is the direction of motion of the car). The problem asks, "from what direction was it fired?" — which means the answer is not 87° but rather its supplement 93° (measured from the direction of motion). Stating this more carefully, in the coordinate system we have adopted in our solution, the bullet velocity vector is in the first quadrant, at 87° measured counterclockwise from the $+x$ direction (the direction of train motion), which means that the direction from which the bullet came (where the sniper is) is in the third quadrant, at –93° (that is, 93° measured clockwise from $+x$).

120. (a) Using $a = v^2/R,$ the radius of the track is

$$R = \frac{v^2}{a} = \frac{(9.20\text{ m/s})^2}{3.80\text{ m/s}^2} = 22.3\text{ m}\,.$$

(b) Using $T = 2\pi R/v,$ the period of the circular motion is

$$T = \frac{2\pi R}{v} = \frac{2\pi(22.3\text{ m})}{9.20\text{ m/s}} = 15.2\text{ s}$$

121. (a) With $v = c/10 = 3\times10^7$ m/s and $a = 20g = 196\text{ m/s}^2$, Eq. 4-34 gives

$$r = v^2/a = 4.6\times10^{12}\text{ m}.$$

(b) The period is given by Eq. 4-35: $T = 2\pi r/v = 9.6\times10^5$ s. Thus, the time to make a quarter-turn is $T/4 = 2.4\times10^5$ s or about 2.8 days.

122. Since $v_y^2 = v_{0y}^2 - 2g\Delta y$, and v_y=0 at the target, we obtain

$$v_{0y} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(5.00\ \text{m}\right)} = 9.90\ \text{m/s}$$

(a) Since $v_0 \sin\theta_0 = v_{0y}$, with v_0 = 12.0 m/s, we find θ_0 = 55.6°.

(b) Now, $v_y = v_{0y} - gt$ gives t = (9.90 m/s)/(9.80 m/s^2) = 1.01 s. Thus,

$$\Delta x = (v_0 \cos\theta_0)t = 6.85\ \text{m}.$$

(c) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0 \cos\theta_0$ = 6.78 m/s.

123. With v_0 = 30.0 m/s and R = 20.0 m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.218.$$

Because $\sin\phi = \sin(180° - \phi)$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.218) = 12.58° \text{ and } 167.4°.$$

which correspond to the two possible launch angles that will hit the target (in the absence of air friction and related effects).

(a) The smallest angle is θ_0 = 6.29°.

(b) The greatest angle is and θ_0 = 83.7°.

An alternative approach to this problem in terms of Eq. 4-25 (with y = 0 and 1/cos^2 = 1 + tan^2) is possible — and leads to a quadratic equation for $\tan\theta_0$ with the roots providing these two possible θ_0 values.

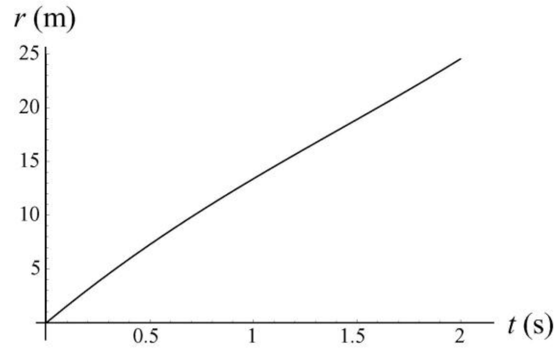
124. We make use of Eq. 4-21 and Eq.4-22.

(a) With v_o = 16 m/s, we square Eq. 4-21 and Eq. 4-22 and add them, then (using Pythagoras' theorem) take the square root to obtain r:

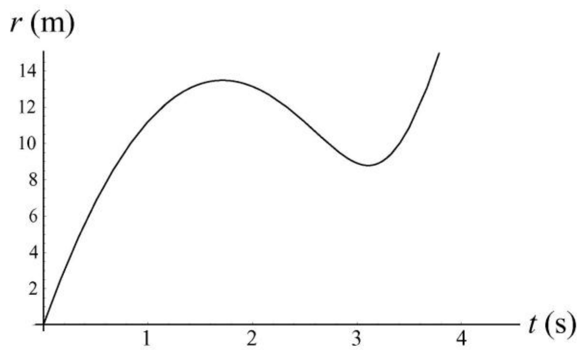
$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2} = \sqrt{(v_0 \cos\theta_0 t)^2 + (v_0 \sin\theta_0 t - gt^2/2)^2}$$

$$= t\sqrt{v_0^2 - v_0 g \sin\theta_0 t + g^2 t^2/4}$$

Below we plot r as a function of time for θ_o = 40.0°:



(b) For this next graph for r versus t we set $\theta_o = 80.0°$.



(c) Differentiating r with respect to t, we obtain

$$\frac{dr}{dt} = \frac{v_0^2 - 3v_0 g t \sin\theta_0 / 2 + g^2 t^2 / 2}{\sqrt{v_0^2 - v_0 g \sin\theta_0 t + g^2 t^2 / 4}}$$

Setting $dr/dt = 0$, with $v_0 = 16.0$ m/s and $\theta_0 = 40.0°$, we have $256 - 151t + 48t^2 = 0$. The equation has no real solution. This means that the maximum is reached at the end of the flight, with

$$t_{total} = 2v_0 \sin\theta_0 / g = 2(16.0\text{ m/s})\sin(40.0°)/(9.80\text{ m/s}^2) = 2.10\text{ s.}$$

(d) The value of r is given by

$$r = (2.10)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 40.0°(2.10) + (9.80)^2(2.10)^2/4} = 25.7\text{ m.}$$

(e) The horizontal distance is $r_x = v_0 \cos\theta_0 t = (16.0\text{ m/s})\cos 40.0°(2.10\text{ s}) = 25.7\text{ m.}$

(f) The vertical distance is $r_y = 0$.

(g) For the θ_0 = 80º launch, the condition for maximum r is $256 - 232t + 48t^2 = 0$, or $t = 1.71$ s (the other solution, t = 3.13 s, corresponds to a minimum.)

(h) The distance traveled is

$$r = (1.71)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 80.0°(1.71) + (9.80)^2(1.71)^2/4} = 13.5 \text{ m}.$$

(i) The horizontal distance is

$$r_x = v_0 \cos\theta_0 t = (16.0 \text{ m/s})\cos 80.0°(1.71 \text{ s}) = 4.75 \text{ m}.$$

(j) The vertical distance is

$$r_y = v_0 \sin\theta_0 t - \frac{gt^2}{2} = (16.0 \text{ m/s})\sin 80°(1.71 \text{ s}) - \frac{(9.80 \text{ m/s}^2)(1.71 \text{ s})^2}{2} = 12.6 \text{ m}.$$

125. Using the same coordinate system assumed in Eq. 4-25, we find x for the elevated cannon from

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0 \cos\theta_0)^2} \quad \text{where } y = -30 \text{ m}.$$

Using the quadratic formula (choosing the positive root), we find

$$x = v_0 \cos\theta_0 \left(\frac{v_0 \sin\theta_0 + \sqrt{(v_0 \sin\theta_0)^2 - 2gy}}{g} \right)$$

which yields x = 715 m for v_0 = 82 m/s and θ_0 = 45°. This is 29 m longer than the distance of 686 m.

126. At maximum height, the y-component of a projectile's velocity vanishes, so the given 10 m/s is the (constant) x-component of velocity.

(a) Using v_{0y} to denote the y-velocity 1.0 s before reaching the maximum height, then (with v_y = 0) the equation $v_y = v_{0y} - gt$ leads to v_{0y} = 9.8 m/s. The magnitude of the velocity vector (or *speed*) at that moment is therefore

$$\sqrt{v_x^2 + v_{0y}{}^2} = \sqrt{(10 \text{ m/s})^2 + (9.8 \text{ m/s})^2} = 14 \text{ m/s}.$$

(b) It is clear from the symmetry of the problem that the speed is the same 1.0 s after reaching the top, as it was 1.0 s before (14 m/s again). This may be verified by using $v_y = v_{0y} - gt$ again but now "starting the clock" at the highest point so that v_{0y} = 0 (and $t = 1.0$ s). This leads to v_y = –9.8 m/s and $\sqrt{(10 \text{ m/s})^2 + (-9.8 \text{ m/s})^2} = 14$ m/s.

(c) The x_0 value may be obtained from $x = 0 = x_0 + (10 \text{ m/s})(1.0\text{s})$, which yields $x_0 = -10\,\text{m}$.

(d) With v_{0y} = 9.8 m/s denoting the y-component of velocity one second before the top of the trajectory, then we have $y = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$ where t = 1.0 s. This yields $y_0 = -4.9$ m.

(e) By using $x - x_0 = (10 \text{ m/s})(1.0 \text{ s})$ where $x_0 = 0$, we obtain $x = 10$ m.

(f) Let $t = 0$ at the top with $y_0 = v_{0y} = 0$. From $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$, we have, for $t = 1.0$ s,

$$y = -(9.8 \text{ m/s}^2)(1.0 \text{ s})^2/2 = -4.9 \text{ m}.$$

127. With no acceleration in the x direction yet a constant acceleration of 1.40 m/s^2 in the y direction, the position (in meters) as a function of time (in seconds) must be

$$\vec{r} = (6.00t)\hat{\text{i}} + \left(\frac{1}{2}(1.40)t^2\right)\hat{\text{j}}$$

and $\vec{v}$ is its derivative with respect to t.

(a) At $t = 3.00$ s, therefore, $\vec{v} = (6.00\hat{\text{i}} + 4.20\hat{\text{j}})$ m/s.

(b) At $t = 3.00$ s, the position is $\vec{r} = (18.0\hat{\text{i}} + 6.30\hat{\text{j}})$ m.

128. We note that

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

describes a right triangle, with one leg being $\vec{v}_{PG}$ (east), another leg being $\vec{v}_{AG}$ (magnitude = 20, direction = south), and the hypotenuse being $\vec{v}_{PA}$ (magnitude = 70). Lengths are in kilometers and time is in hours. Using the Pythagorean theorem, we have

$$|\vec{v}_{PA}| = \sqrt{|\vec{v}_{PG}|^2 + |\vec{v}_{AG}|^2} \;\Rightarrow\; 70 \text{ km/h} = \sqrt{|\vec{v}_{PG}|^2 + (20 \text{ km/h})^2}$$

which can be solved to give the ground speed: $|\vec{v}_{PG}| = 67 \text{ km/h}$.

129. The figure offers many interesting points to analyze, and others are easily inferred (such as the point of maximum height). The focus here, to begin with, will be the final point shown (1.25 s after the ball is released) which is when the ball returns to its original height. In English units, g = 32 ft/s^2.

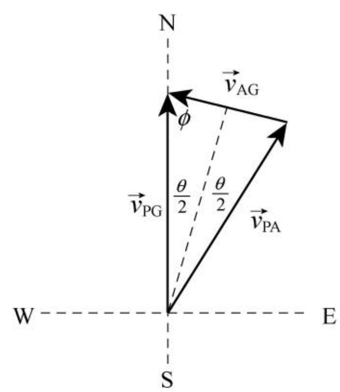
(a) Using $x - x_0 = v_x t$ we obtain v_x = (40 ft)/(1.25 s) = 32 ft/s. And $y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2$ yields $v_{0y} = \frac{1}{2}\left(32\ \text{ft/s}^2\right)\left(1.25\ \text{s}\right) = 20\ \text{ft/s}$. Thus, the initial speed is

$$v_0 = |\vec{v}_0| = \sqrt{(32\ \text{ft/s})^2 + (20\ \text{ft/s})^2} = 38\ \text{ft/s}.$$

(b) Since $v_y = 0$ at the maximum height and the horizontal velocity stays constant, then the speed at the top is the same as v_x = 32 ft/s.

(c) We can infer from the figure (or compute from $v_y = 0 = v_{0y} - gt$) that the time to reach the top is 0.625 s. With this, we can use $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to obtain 9.3 ft (where y_0 = 3 ft has been used). An alternative approach is to use $v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right)$.

130. We denote $\vec{v}_{\text{PG}}$ as the velocity of the plane relative to the ground, $\vec{v}_{\text{AG}}$ as the velocity of the air relative to the ground, and $\vec{v}_{\text{PA}}$ as the velocity of the plane relative to the air.



(a) The vector diagram is shown on the right: $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. Since the magnitudes v_{PG} and v_{PA} are equal the triangle is isosceles, with two sides of equal length.

Consider either of the right triangles formed when the bisector of θ is drawn (the dashed line). It bisects $\vec{v}_{\text{AG}}$, so

$$\sin\left(\theta/2\right) = \frac{v_{\text{AG}}}{2v_{\text{PG}}} = \frac{70.0\ \text{mi/h}}{2\left(135\ \text{mi/h}\right)}$$

which leads to θ = 30.1°. Now $\vec{v}_{\text{AG}}$ makes the same angle with the E-W line as the dashed line does with the N-S line. The wind is blowing in the direction 15.0° north of west. Thus, it is blowing *from* 75.0° east of south.

(b) The plane is headed along $\vec{v}_{\text{PA}}$, in the direction 30.0° east of north. There is another solution, with the plane headed 30.0° west of north and the wind blowing 15° north of east (that is, from 75° west of south).

131. We make use of Eq. 4-24 and Eq. 4-25.

(a) With x = 180 m, θ_o = 30º, and v_o = 43 m/s, we obtain

$$y = \tan(30°)(180\ \text{m}) - \frac{(9.8\ \text{m/s}^2)(180\ \text{m})^2}{2(43\ \text{m/s})^2(\cos 30°)^2} = -11\ \text{m}$$

or $|y| = 11$ m. This implies the rise is roughly eleven meters above the fairway.

(b) The horizontal component (in the absence of air friction) is unchanged, but the vertical component increases (see Eq. 4-24). The Pythagorean theorem then gives the magnitude of final velocity (right before striking the ground): 45 m/s.

132. We let g_p denote the magnitude of the gravitational acceleration on the planet. A number of the points on the graph (including some "inferred" points — such as the max height point at $x = 12.5$ m and $t = 1.25$ s) can be analyzed profitably; for future reference, we label (with subscripts) the first $((x_0, y_0) = (0, 2)$ at $t_0 = 0)$ and last ("final") points $((x_f, y_f) = (25, 2)$ at $t_f = 2.5)$, with lengths in meters and time in seconds.

(a) The x-component of the initial velocity is found from $x_f - x_0 = v_{0x} t_f$. Therefore, $v_{0x} = 25/2.5 = 10$ m/s. We try to obtain the y-component from

$$y_f - y_0 = 0 = v_{0y} t_f - \tfrac{1}{2} g_p t_f^2.$$

This gives us $v_{0y} = 1.25 g_p$, and we see we need another equation (by analyzing another point, say, the next-to-last one) $y - y_0 = v_{0y} t - \tfrac{1}{2} g_p t^2$ with $y = 6$ and $t = 2$; this produces our second equation $v_{0y} = 2 + g_p$. Simultaneous solution of these two equations produces results for v_{0y} and g_p (relevant to part (b)). Thus, our complete answer for the initial velocity is $\vec{v} = (10 \text{ m/s})\hat{\text{i}} + (10 \text{ m/s})\hat{\text{j}}$.

(b) As a by-product of the part (a) computations, we have $g_p = 8.0$ m/s^2.

(c) Solving for t_g (the time to reach the ground) in $y_g = 0 = y_0 + v_{0y} t_g - \tfrac{1}{2} g_p t_g^2$ leads to a positive answer: $t_g = 2.7$ s.

(d) With $g = 9.8$ m/s^2, the method employed in part (c) would produce the quadratic equation $-4.9 t_g^2 + 10 t_g + 2 = 0$ and then the positive result $t_g = 2.2$ s.

133. (a) The helicopter's speed is $v' = 6.2$ m/s, which implies that the speed of the package is $v_0 = 12 - v' = 5.8$ m/s, relative to the ground.

(b) Letting $+x$ be in the direction of $\vec{v}_0$ for the package and $+y$ be downward, we have (for the motion of the package)

$$\Delta x = v_0 t \quad \text{and} \quad \Delta y = \frac{1}{2} g t^2$$

where $\Delta y = 9.5$ m. From these, we find $t = 1.39$ s and $\Delta x = 8.08$ m for the package, while $\Delta x'$ (for the helicopter, which is moving in the opposite direction) is $-v' t = -8.63$ m. Thus, the horizontal separation between them is $8.08 - (-8.63) = 16.7 \text{ m} \approx 17$ m.

(c) The components of $\vec{v}$ at the moment of impact are $(v_x, v_y) = (5.8, 13.6)$ in SI units. The vertical component has been computed using Eq. 2-11. The angle (which is below horizontal) for this vector is $\tan^{-1}(13.6/5.8) = 67°$.

134. The type of acceleration involved in steady-speed circular motion is the centripetal acceleration $a = v^2/r$ which is at each moment directed towards the center of the circle. The radius of the circle is $r = (12)^2/3 = 48$ m.

(a) Thus, if at the instant the car is traveling *clockwise* around the circle, it is 48 m west of the center of its circular path.

(b) The same result holds here if at the instant the car is traveling *counterclockwise*. That is, it is 48 m west of the center of its circular path.

135. (a) Using the same coordinate system assumed in Eq. 4-21 and Eq. 4-22 (so that $\theta_0 = -20.0°$), we use $v_0 = 15.0$ m/s and find the horizontal displacement of the ball at $t = 2.30$ s:

$$\Delta x = (v_0 \cos\theta_0)t = 32.4 \text{ m}.$$

(b) The vertical displacement is $\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = -37.7$ m.

136. We take the initial (x, y) specification to be (0.000, 0.762) m, and the positive x direction to be towards the "green monster." The components of the initial velocity are $(33.53 \angle 55°) \rightarrow (19.23,\ 27.47)\ \text{m/s}$.

(a) With $t = 5.00$ s, we have $x = x_0 + v_x t = 96.2$ m.

(b) At that time, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 15.59\ \text{m}$, which is 4.31 m above the wall.

(c) The moment in question is specified by $t = 4.50$ s. At that time, $x - x_0 = (19.23)(4.50) = 86.5$ m.

(d) The vertical displacement is $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 25.1$ m.

137. When moving in the same direction as the jet stream (of speed v_s), the time is $t = d/(v_{ja} + v_s)$, where $d = 4350$ km is the distance and $v_{ja} = 966$ km/h is the speed of the jet relative to the air. When moving against the jet stream, the time is $t' = d/(v_{ja} - v_s)$, with $t' - t = 50 \text{ min} = (5/6)\text{h}$. Combining the expressions gives

$$t' - t = \frac{d}{v_{ja} - v_s} - \frac{d}{v_{ja} + v_s} = \frac{2dv_s}{v_{ja}^2 - v_s^2} = \frac{5}{6}\ \text{h}$$

Upon rearranging and using the quadratic formula to solve for v_s, we get $v_s = 88.63$ km/h.

138. We establish coordinates with $\hat{\text{i}}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\text{j}}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}| = 6.4$ km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = 3.2\hat{\text{j}}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} = v_{bg}\hat{\text{i}}$ where $v > 0$ is unknown. Thus, all $\hat{\text{j}}$ components must cancel in the vector sum

$$\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$$

which means the $u \sin\theta = -3.2$, so $\theta = \sin^{-1}(-3.2/6.4) = -30°$.

(b) Using the result from part (a), we find $v_{bg} = v_{bw}\cos\theta = 5.5$ km/h. Thus, traveling a distance of $\ell = 6.4$ km requires a time of 6.4/5.5 = 1.15 h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with v_{wg} = 3.2 km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

where $D = 3.2$ km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = \frac{D}{v_{bw} - v_{wg}} + \frac{D}{v_{bw} + v_{wg}}$$

the answer is the same as in the previous part, i.e., $t_{\text{total}} = 80$ min.

(e) The shortest-time path should have $\theta = 0$. This can also be shown by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw}\cos\theta\,\hat{\text{i}} + (v_{bw}\sin\theta + v_{wg})\,\hat{\text{j}}$$

where the x component of $\vec{v}_{bg}$ must equal l/t. Thus, $t = \dfrac{l}{v_{bw}\cos\theta}$, which can be minimized using the condition $dt/d\theta = 0$. The above expression leads to $t = 6.4/6.4 = 1.0$ h, or 60 min.

Chapter 5

1. We are only concerned with horizontal forces in this problem (gravity plays no direct role). We take East as the $+x$ direction and North as $+y$. This calculation is efficiently implemented on a vector-capable calculator, using magnitude-angle notation (with SI units understood).

$$\vec{a} = \frac{\vec{F}}{m} = \frac{(9.0 \angle 0°) + (8.0 \angle 118°)}{3.0} = (2.9 \angle 53°)$$

Therefore, the acceleration has a magnitude of 2.9 m/s^2.

2. We apply Newton's second law (Eq. 5-1 or, equivalently, Eq. 5-2). The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = (\vec{F}_1 + \vec{F}_2)/m$.

(a) In the first case

$$\vec{F}_1 + \vec{F}_2 = \left[(3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right] + \left[(-3.0\text{N})\hat{\text{i}} + (-4.0\text{N})\hat{\text{j}}\right] = 0$$

so $\vec{a} = 0$.

(b) In the second case, the acceleration $\vec{a}$ equals

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right) + \left((-3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right)}{2.0\,\text{kg}} = (4.0\,\text{m/s}^2)\hat{\text{j}}.$$

(c) In this final situation, $\vec{a}$ is

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right) + \left((3.0\text{N})\hat{\text{i}} + (-4.0\text{N})\hat{\text{j}}\right)}{2.0\,\text{kg}} = (3.0\,\text{m/s}^2)\hat{\text{i}}.$$

3. We apply Newton's second law (specifically, Eq. 5-2).

(a) We find the x component of the force is

$$F_x = ma_x = ma\cos 20.0° = (1.00\,\text{kg})\,(2.00\,\text{m/s}^2)\,\cos 20.0° = 1.88\text{N}.$$

(b) The y component of the force is

$$F_y = ma_y = ma \sin 20.0° = (1.0\,\text{kg})\left(2.00\,\text{m/s}^2\right) \sin 20.0° = 0.684\,\text{N}.$$

(c) In unit-vector notation, the force vector is

$$\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} = (1.88\text{ N})\hat{\text{i}} + (0.684\text{ N})\hat{\text{j}}\ .$$

4. Since $\vec{v}$ = constant, we have $\vec{a}$ = 0, which implies

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = m\vec{a} = 0\ .$$

Thus, the other force must be

$$\vec{F}_2 = -\vec{F}_1 = (-2\text{ N})\ \hat{\text{i}} + (6\text{ N})\ \hat{\text{j}}.$$

5. The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3)/m$.

(a) The forces exerted by the three astronauts can be expressed in unit-vector notation as follows:

$$\begin{aligned}
\vec{F}_1 &= (32\text{ N})\left(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}\right) = (27.7\text{ N})\,\hat{\text{i}} + (16\text{ N})\,\hat{\text{j}} \\
\vec{F}_2 &= (55\text{ N})\left(\cos 0°\hat{\text{i}} + \sin 0°\hat{\text{j}}\right) = (55\text{ N})\,\hat{\text{i}} \\
\vec{F}_3 &= (41\text{ N})\left(\cos(-60°)\,\hat{\text{i}} + \sin(-60°)\,\hat{\text{j}}\right) = (20.5\text{ N})\,\hat{\text{i}} - (35.5\text{ N})\,\hat{\text{j}}.
\end{aligned}$$

The resultant acceleration of the asteroid of mass m = 120 kg is therefore

$$\vec{a} = \frac{\left(27.7\hat{\text{i}} + 16\hat{\text{j}}\right)\text{N} + \left(55\hat{\text{i}}\right)\text{N} + \left(20.5\hat{\text{i}} - 35.5\hat{\text{j}}\right)\text{N}}{120\text{ kg}} = (0.86\text{ m/s}^2)\hat{\text{i}} - (0.16\text{ m/s}^2)\hat{\text{j}}\ .$$

(b) The magnitude of the acceleration vector is

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.86\text{ m/s}^2)^2 + \left(-0.16\text{ m/s}^2\right)^2} = 0.88\text{ m/s}^2\ .$$

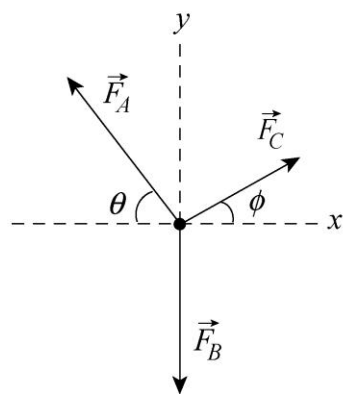
(c) The vector $\vec{a}$ makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.16\text{ m/s}^2}{0.86\text{ m/s}^2}\right) = -11°\ .$$

6. Since the tire remains stationary, by Newton's second law, the net force must be zero:

$$\vec{F}_{\text{net}} = \vec{F}_A + \vec{F}_B + \vec{F}_C = m\vec{a} = 0.$$

From the free-body diagram shown on the right, we have



$$0 = \sum F_{\text{net},x} = F_C \cos\phi - F_A \cos\theta$$
$$0 = \sum F_{\text{net},y} = F_A \sin\theta + F_C \sin\phi - F_B$$

To solve for F_B, we first compute ϕ. With $F_A = 220\text{ N}$, $F_C = 170\text{ N}$, and $\theta = 47°$, we get

$$\cos\phi = \frac{F_A\cos\theta}{F_C} = \frac{(220\text{ N})\cos 47.0°}{170\text{ N}} = 0.883 \quad \Rightarrow \phi = 28.0°$$

Substituting the value into the second force equation, we find

$$F_B = F_A\sin\theta + F_C\sin\phi = (220\text{ N})\sin 47.0° + (170\text{ N})\sin 28.0 = 241\text{ N}.$$

7. **THINK** A box is under acceleration by two applied forces. We use Newton's second law to solve for the unknown second force.

EXPRESS We denote the two forces as $\vec{F}_1$ and $\vec{F}_2$. According to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$, so the second force is $\vec{F}_2 = m\vec{a} - \vec{F}_1$. Note that since the acceleration is in the third quadrant, we expect $\vec{F}_2$ to be in the third quadrant as well.

ANALYZE (a) In unit vector notation $\vec{F}_1 = (20.0\text{ N})\hat{\text{i}}$ and

$$\vec{a} = -\left(12.0\sin 30.0°\text{m/s}^2\right)\hat{\text{i}} - \left(12.0\cos 30.0°\text{m/s}^2\right)\hat{\text{j}} = -\left(6.00\text{ m/s}^2\right)\hat{\text{i}} - \left(10.4\text{ m/s}^2\right)\hat{\text{j}}.$$

Therefore, we find the second force to be

$$\begin{aligned}\vec{F}_2 &= m\vec{a} - \vec{F}_1 \\ &= (2.00\text{ kg})\left(-6.00\text{ m/s}^2\right)\hat{\text{i}} + (2.00\text{ kg})\left(-10.4\text{ m/s}^2\right)\hat{\text{j}} - (20.0\text{ N})\hat{\text{i}} \\ &= (-32.0\text{ N})\hat{\text{i}} - (20.8\text{ N})\hat{\text{j}}.\end{aligned}$$

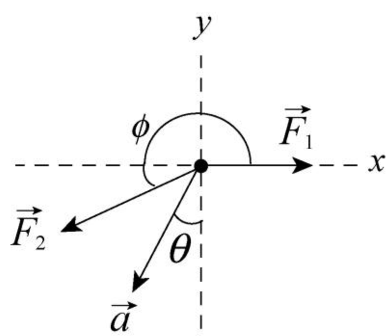
(b) The magnitude of $\vec{F}_2$ is $|\vec{F}_2| = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-32.0\text{ N})^2 + (-20.8\text{ N})^2} = 38.2\text{ N}.$

(c) The angle that $\vec{F}_2$ makes with the positive x-axis is found from

$$\tan\phi = \left(\frac{F_{2y}}{F_{2x}}\right) = \frac{-20.8\text{ N}}{-32.0\text{ N}} = 0.656\,.$$

Consequently, the angle is either 33.0° or 33.0° + 180° = 213°. Since both the x and y components are negative, the correct result is ϕ = 213° from the +x-axis. An alternative answer is $213° - 360° = -147°$.

LEARN The result is shown in the figure on the right. The calculation confirms our expectation that $\vec{F}_2$ lies in the third quadrant (same as $\vec{a}$). The net force is



$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = (20.0\text{ N})\hat{\mathrm{i}} + \left[(-32.0\text{ N})\hat{\mathrm{i}} - (20.8\text{ N})\hat{\mathrm{j}}\right]$$
$$= (-12.0\text{ N})\hat{\mathrm{i}} - (20.8\text{ N})\hat{\mathrm{j}}$$

which points in the same direction as $\vec{a}$.

8. We note that $m\vec{a}$ = (–16 N) $\hat{\mathrm{i}}$ + (12 N) $\hat{\mathrm{j}}$. With the other forces as specified in the problem, then Newton's second law gives the third force as

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2 = (-34\text{ N})\,\hat{\mathrm{i}} - (12\text{ N})\,\hat{\mathrm{j}}.$$

9. To solve the problem, we note that acceleration is the second time derivative of the position function; it is a vector and can be determined from its components. The net force is related to the acceleration via Newton's second law. Thus, differentiating $x(t) = -15.0 + 2.00t + 4.00t^3$ twice with respect to t, we get

$$\frac{dx}{dt} = 2.00 - 12.0t^2, \quad \frac{d^2x}{dt^2} = -24.0t$$

Similarly, differentiating $y(t) = 25.0 + 7.00t - 9.00t^2$ twice with respect to t yields

$$\frac{dy}{dt} = 7.00 - 18.0t, \quad \frac{d^2y}{dt^2} = -18.0$$

(a) The acceleration is

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} = \frac{d^2x}{dt^2}\hat{\mathrm{i}} + \frac{d^2y}{dt^2}\hat{\mathrm{j}} = (-24.0t)\hat{\mathrm{i}} + (-18.0)\hat{\mathrm{j}}.$$

At $t = 0.700$ s, we have $\vec{a} = (-16.8)\hat{\mathrm{i}} + (-18.0)\hat{\mathrm{j}}$ with a magnitude of

$$a = |\vec{a}| = \sqrt{(-16.8)^2 + (-18.0)^2} = 24.6\text{ m/s}^2.$$

Thus, the magnitude of the force is $F = ma = (0.34\text{ kg})(24.6\text{ m/s}^2) = 8.37\text{ N}.$

(b) The angle $\vec{F}$ or $\vec{a} = \vec{F}/m$ makes with $+x$ is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-18.0\text{ m/s}^2}{-16.8\text{ m/s}^2}\right) = 47.0° \text{ or } -133°.$$

We choose the latter ($-133°$) since $\vec{F}$ is in the third quadrant.

(c) The direction of travel is the direction of a tangent to the path, which is the direction of the velocity vector:

$$\vec{v}(t) = v_x\hat{\text{i}} + v_y\hat{\text{j}} = \frac{dx}{dt}\hat{\text{i}} + \frac{dy}{dt}\hat{\text{j}} = (2.00 - 12.0t^2)\hat{\text{i}} + (7.00 - 18.0t)\hat{\text{j}}.$$

At $t = 0.700\text{ s}$, we have $\vec{v}(t = 0.700\text{ s}) = (-3.88\text{ m/s})\hat{\text{i}} + (-5.60\text{ m/s})\hat{\text{j}}$. Therefore, the angle $\vec{v}$ makes with $+x$ is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.60\text{ m/s}}{-3.88\text{ m/s}}\right) = 55.3° \text{ or } -125°.$$

We choose the latter ($-125°$) since $\vec{v}$ is in the third quadrant.

10. To solve the problem, we note that acceleration is the second time derivative of the position function, and the net force is related to the acceleration via Newton's second law. Thus, differentiating

$$x(t) = -13.00 + 2.00t + 4.00t^2 - 3.00t^3$$

twice with respect to t, we get

$$\frac{dx}{dt} = 2.00 + 8.00t - 9.00t^2, \quad \frac{d^2x}{dt^2} = 8.00 - 18.0t$$

The net force acting on the particle at $t = 3.40\text{ s}$ is

$$\vec{F} = m\frac{d^2x}{dt^2}\hat{\text{i}} = (0.150)\left[8.00 - 18.0(3.40)\right]\hat{\text{i}} = (-7.98\text{ N})\hat{\text{i}}$$

11. The velocity is the derivative (with respect to time) of given function x, and the acceleration is the derivative of the velocity. Thus, $a = 2c - 3(2.0)(2.0)t$, which we use in Newton's second law: $F = (2.0\text{ kg})a = 4.0c - 24t$ (with SI units understood). At $t = 3.0$ s, we are told that $F = -36$ N. Thus, $-36 = 4.0c - 24(3.0)$ can be used to solve for c. The result is $c = +9.0\text{ m/s}^2$.

12. From the slope of the graph we find $a_x = 3.0$ m/s^2. Applying Newton's second law to the x axis (and taking θ to be the angle between F_1 and F_2), we have

$$F_1 + F_2 \cos\theta = m a_x \quad \Rightarrow \quad \theta = 56°.$$

13. (a) From the fact that $T_3 = 9.8$ N, we conclude the mass of disk D is 1.0 kg. Both this and that of disk C cause the tension $T_2 = 49$ N, which allows us to conclude that disk C has a mass of 4.0 kg. The weights of these two disks plus that of disk B determine the tension $T_1 = 58.8$ N, which leads to the conclusion that $m_B = 1.0$ kg. The weights of all the disks must add to the 98 N force described in the problem; therefore, disk A has mass 4.0 kg.

(b) $m_B = 1.0$ kg, as found in part (a).

(c) $m_C = 4.0$ kg, as found in part (a).

(d) $m_D = 1.0$ kg, as found in part (a).

14. Three vertical forces are acting on the block: the earth pulls down on the block with gravitational force 3.0 N; a spring pulls up on the block with elastic force 1.0 N; and, the surface pushes up on the block with normal force F_N. There is no acceleration, so

$$\sum F_y = 0 = F_N + (1.0\,\text{N}) + (-3.0\,\text{N})$$

yields $F_N = 2.0$ N.

(a) By Newton's third law, the force exerted by the block on the surface has that same magnitude but opposite direction: 2.0 N.

(b) The direction is down.

15. **THINK** We have a piece of salami hung to a spring scale in various ways. The problem is to explore the concept of weight.

EXPRESS We first note that the reading on the spring scale is proportional to the weight of the salami. In all three cases (a) – (c) depicted in Fig. 5-34, the scale is not accelerating, which means that the two cords exert forces of equal magnitude on it. The scale reads the magnitude of either of these forces. In each case the tension force of the cord attached to the salami must be the same in magnitude as the weight of the salami because the salami is not accelerating. Thus the scale reading is mg, where m is the mass of the salami.

ANALYZE In all three cases (a) – (c), the reading on the scale is

$$w = mg = (11.0\text{ kg})(9.8\text{ m/s}^2) = 108\text{ N}.$$

LEARN The weight of an object is measured when the object is not accelerating vertically relative to the ground. If it is, then the weight measured is called the apparent weight.

16. (a) There are six legs, and the vertical component of the tension force in each leg is $T\sin\theta$ where $\theta = 40°$. For vertical equilibrium (zero acceleration in the *y* direction) then Newton's second law leads to

$$6T\sin\theta = mg \Rightarrow T = \frac{mg}{6\sin\theta}$$

which (expressed as a multiple of the bug's weight *mg*) gives roughly $T/mg \approx 0.260$.

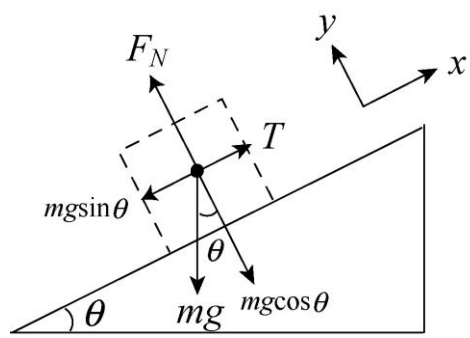
(b) The angle θ is measured from horizontal, so as the insect "straightens out the legs" θ will increase (getting closer to $90°$), which causes $\sin\theta$ to increase (getting closer to 1) and consequently (since $\sin\theta$ is in the denominator) causes *T* to decrease.

17. **THINK** A block attached to a cord is resting on an incline plane. We apply Newton's second law to solve for the tension in the cord and the normal force on the block.

EXPRESS The free-body diagram of the problem is shown to the right. Since the acceleration of the block is zero, the components of Newton's second law equation yield

$$T - mg\sin\theta = 0$$
$$F_N - mg\cos\theta = 0,$$

where *T* is the tension in the cord, and F_N is the normal force on the block.



ANALYZE (a) Solving the first equation for the tension in the string, we find

$$T = mg\sin\theta = (8.5\text{ kg})(9.8\text{ m/s}^2)\sin 30° = 42\text{ N}.$$

(b) We solve the second equation above for the normal force F_N:

$$F_N = mg\cos\theta = (8.5\text{ kg})(9.8\text{ m/s}^2)\cos 30° = 72\text{ N}.$$

(c) When the cord is cut, it no longer exerts a force on the block and the block accelerates. The *x* component of the second law becomes $-mg\sin\theta = ma$, so the acceleration becomes

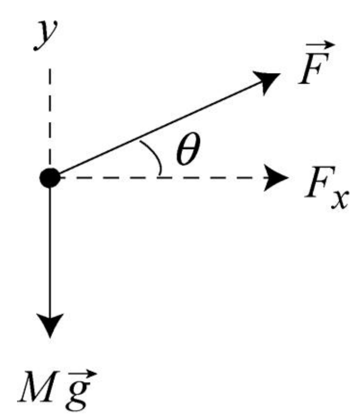
$$a = -g\sin\theta = -(9.8\text{ m/s}^2)\sin 30° = -4.9\text{ m/s}^2.$$

The negative sign indicates the acceleration is down the plane. The magnitude of the acceleration is 4.9 m/s^2.

LEARN The normal force F_N on the block must be equal to $mg\cos\theta$ so that the block is in contact with the surface of the incline at all time. When the cord is cut, the block has an acceleration $a=-g\sin\theta$, which in the limit $\theta\to 90°$ becomes $-g$, as in the case of a free fall.

18. The free-body diagram of the cars is shown on the right. The force exerted by John Massis is

$$F = 2.5mg = 2.5(80\text{ kg})(9.8\text{ m/s}^2) = 1960\text{ N}.$$



Since the motion is along the horizontal x-axis, using Newton's second law, we have $Fx = F\cos\theta = Ma_x$, where M is the total mass of the railroad cars. Thus, the acceleration of the cars is

$$a_x = \frac{F\cos\theta}{M} = \frac{(1960\text{ N})\cos 30°}{(7.0\times 10^5\text{ N}/9.8\text{ m/s}^2)} = 0.024\text{ m/s}^2.$$

Using Eq. 2-16, the speed of the car at the end of the pull is

$$v_x = \sqrt{2a_x\Delta x} = \sqrt{2(0.024\text{ m/s}^2)(1.0\text{ m})} = 0.22\text{ m/s}.$$

19. **THINK** In this problem we're interested in the force applied to a rocket sled to accelerate it from rest to a given speed in a given time interval.

EXPRESS In terms of magnitudes, Newton's second law is $F = ma$, where $F = \left|\vec{F}_{\text{net}}\right|$, $a = |\vec{a}\,|$, and m is the (always positive) mass. The magnitude of the acceleration can be found using constant acceleration kinematics (Table 2-1). Solving $v = v_0 + at$ for the case where it starts from rest, we have $a = v/t$ (which we interpret in terms of magnitudes, making specification of coordinate directions unnecessary). Thus, the required force is $F = ma = mv/t$.

ANALYZE Expressing the velocity in SI units as

$$v = (1600\text{ km/h})\,(1000\text{ m/km})/(3600\text{ s/h}) = 444\text{ m/s},$$

we find the force to be

$$F = m\frac{v}{t} = (500\,\text{kg})\,\frac{444\,\text{m/s}}{1.8\,\text{s}} = 1.2\times 10^5\text{ N}.$$

LEARN From the expression $F = mv/t$, we see that the shorter the time to attain a given speed, the greater the force required.

20. The stopping force $\vec{F}$ and the path of the passenger are horizontal. Our +*x* axis is in the direction of the passenger's motion, so that the passenger's acceleration (''deceleration'') is negative-valued and the stopping force is in the –*x* direction: $\vec{F} = -F\,\hat{\text{i}}$. Using Eq. 2-16 with

$$v_0 = (53 \text{ km/h})(1000 \text{ m/km})/(3600 \text{ s/h}) = 14.7 \text{ m/s}$$

and $v = 0$, the acceleration is found to be

$$v^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(14.7 \text{ m/s})^2}{2(0.65 \text{ m})} = -167 \text{ m/s}^2.$$

Assuming there are no significant horizontal forces other than the stopping force, Eq. 5-1 leads to

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad -F = (41 \text{ kg})(-167 \text{ m/s}^2)$$

which results in $F = 6.8 \times 10^3$ N.

21. (a) The slope of each graph gives the corresponding component of acceleration. Thus, we find $a_x = 3.00 \text{ m/s}^2$ and $a_y = -5.00 \text{ m/s}^2$. The magnitude of the acceleration vector is therefore

$$a = \sqrt{(3.00 \text{ m/s}^2)^2 + (-5.00 \text{ m/s}^2)^2} = 5.83 \text{ m/s}^2,$$

and the force is obtained from this by multiplying with the mass ($m = 2.00$ kg). The result is $F = ma = 11.7$ N.

(b) The direction of the force is the same as that of the acceleration:

$$\theta = \tan^{-1}[(-5.00 \text{ m/s}^2)/(3.00 \text{ m/s}^2)] = -59.0°.$$

22. (a) The coin undergoes free fall. Therefore, with respect to ground, its acceleration is

$$\vec{a}_{\text{coin}} = \vec{g} = (-9.8 \text{ m/s}^2)\hat{\text{j}}.$$

(b) Since the customer is being pulled down with an acceleration of $\vec{a}\,'_{\text{customer}} = 1.24\vec{g} = (-12.15 \text{ m/s}^2)\hat{\text{j}}$, the acceleration of the coin with respect to the customer is

$$\vec{a}_{\text{rel}} = \vec{a}_{\text{coin}} - \vec{a}\,'_{\text{customer}} = (-9.8 \text{ m/s}^2)\hat{\text{j}} - (-12.15 \text{ m/s}^2)\hat{\text{j}} = (+2.35 \text{ m/s}^2)\hat{\text{j}}.$$

(c) The time it takes for the coin to reach the ceiling is

$$t = \sqrt{\frac{2h}{a_{\rm rel}}} = \sqrt{\frac{2(2.20 \text{ m})}{2.35 \text{ m/s}^2}} = 1.37 \text{ s}.$$

(d) Since gravity is the only force acting on the coin, the actual force on the coin is

$$\vec{F}_{\rm coin} = m\vec{a}_{\rm coin} = m\vec{g} = (0.567\times10^{-3} \text{ kg})(-9.8 \text{ m/s}^2)\hat{\text{j}} = (-5.56\times10^{-3} \text{ N})\hat{\text{j}}.$$

(e) In the customer's frame, the coin travels upward at a constant acceleration. Therefore, the apparent force on the coin is

$$\vec{F}_{\rm app} = m\vec{a}_{\rm rel} = (0.567\times10^{-3} \text{ kg})(+2.35 \text{ m/s}^2)\hat{\text{j}} = (+1.33\times10^{-3} \text{ N})\hat{\text{j}}.$$

23. We note that the rope is 22.0° from vertical, and therefore 68.0° from horizontal.

(a) With T = 760 N, then its components are

$$\vec{T} = T\cos 68.0°\,\hat{\text{i}} + T\sin 68.0°\,\hat{\text{j}} = (285\text{N})\,\hat{\text{i}} + (705\text{N})\,\hat{\text{j}}.$$

(b) No longer in contact with the cliff, the only other force on Tarzan is due to earth's gravity (his weight). Thus,

$$\vec{F}_{\rm net} = \vec{T} + \vec{W} = (285 \text{ N})\,\hat{\text{i}} + (705 \text{ N})\,\hat{\text{j}} - (820 \text{ N})\,\hat{\text{j}} = (285\text{N})\,\hat{\text{i}} - (115 \text{ N})\,\hat{\text{j}}.$$

(c) In a manner that is efficiently implemented on a vector-capable calculator, we convert from rectangular (x, y) components to magnitude-angle notation:

$$\vec{F}_{\rm net} = (285, -115) \rightarrow (307\angle -22.0°)$$

so that the net force has a magnitude of 307 N.

(d) The angle (see part (c)) has been found to be −22.0°, or 22.0° below horizontal (away from the cliff).

(e) Since $\vec{a} = \vec{F}_{\rm net}/m$ where $m = W/g$ = 83.7 kg, we obtain $\vec{a} = 3.67 \text{ m/s}^2$.

(f) Eq. 5-1 requires that $\vec{a} \parallel \vec{F}_{\rm net}$ so that the angle is also −22.0°, or 22.0° below horizontal (away from the cliff).

24. We take rightward as the $+x$ direction. Thus, $\vec{F}_1 = (20 \text{ N})\hat{\text{i}}$. In each case, we use Newton's second law $\vec{F}_1 + \vec{F}_2 = m\vec{a}$ where m = 2.0 kg.

(a) If $\vec{a} = (+10 \text{ m/s}^2)\,\hat{\text{i}}$, then the equation above gives $\vec{F}_2 = 0$.

(b) If , $\vec{a} = (+20\text{m/s}^2)\,\hat{\text{i}}$, then that equation gives $\vec{F}_2 = (20\text{N})\hat{\text{i}}$.

(c) If $\vec{a} = 0$, then the equation gives $\vec{F}_2 = (-20\text{N})\hat{\text{i}}$.

(d) If $\vec{a} = (-10 \text{ m/s}^2)\,\hat{\text{i}}$, the equation gives $\vec{F}_2 = (-40\text{N})\hat{\text{i}}$.

(e) If $\vec{a} = (-20 \text{ m/s}^2)\,\hat{\text{i}}$, the equation gives $\vec{F}_2 = (-60\text{N})\hat{\text{i}}$.

25. (a) The acceleration is

$$a = \frac{F}{m} = \frac{20 \text{ N}}{900\text{kg}} = 0.022 \text{ m/s}^2 .$$

(b) The distance traveled in 1 day (= 86400 s) is

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(0.0222 \text{ m/s}^2\right)\left(86400 \text{ s}\right)^2 = 8.3 \times 10^7 \text{ m} .$$

(c) The speed it will be traveling is given by

$$v = at = \left(0.0222 \text{ m/s}^2\right)\left(86400 \text{ s}\right) = 1.9 \times 10^3 \text{ m/s} .$$

26. Some assumptions (not so much for realism but rather in the interest of using the given information efficiently) are needed in this calculation: we assume the fishing line and the path of the salmon are horizontal. Thus, the weight of the fish contributes only (via Eq. 5-12) to information about its mass ($m = W/g = 8.7$ kg). Our $+x$ axis is in the direction of the salmon's velocity (away from the fisherman), so that its acceleration (''deceleration'') is negative-valued and the force of tension is in the $-x$ direction: $\vec{T} = -T$. We use Eq. 2-16 and SI units (noting that $v = 0$).

$$v^2 = v_0^2 + 2a\Delta x \;\;\Rightarrow\;\; a = -\frac{v_0^2}{2\Delta x} = -\frac{(2.8 \text{ m/s})^2}{2\left(0.11 \text{ m}\right)} = -36 \text{ m/s}^2 .$$

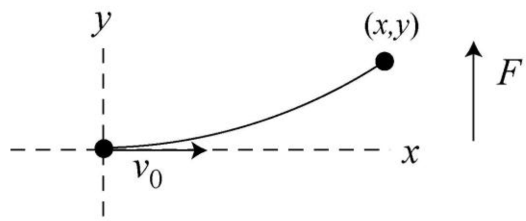
Assuming there are no significant horizontal forces other than the tension, Eq. 5-1 leads to

$$\vec{T} = m\vec{a} \;\;\Rightarrow\;\; -T = \left(8.7 \text{ kg}\right)\left(-36 \text{ m/s}^2\right)$$

which results in $T = 3.1 \times 10^2$ N.

27. **THINK** An electron moving horizontally is under the influence of a vertical force. Its path will be deflected toward the direction of the applied force.

EXPRESS The setup is shown in the figure below. The acceleration of the electron is vertical and for all practical purposes the only force acting on it is the electric force. The force of gravity is negligible. We take the $+x$ axis to be in the direction of the initial velocity v_0 and the $+y$ axis to be in the direction of the electrical force, and place the origin at the initial position of the electron.



Since the force and acceleration are constant, we use the equations from Table 2-1: $x = v_0 t$ and

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)t^2.$$

ANALYZE The time taken by the electron to travel a distance x (= 30 mm) horizontally is $t = x/v_0$ and its deflection in the direction of the force is

$$y = \frac{1}{2}\frac{F}{m}\left(\frac{x}{v_0}\right)^2 = \frac{1}{2}\left(\frac{4.5\times10^{-16}\text{ N}}{9.11\times10^{-31}\text{ kg}}\right)\left(\frac{30\times10^{-3}\text{ m}}{1.2\times10^{7}\text{ m/s}}\right)^2 = 1.5\times10^{-3}\text{ m}.$$

LEARN Since the applied force is constant, the acceleration in the y-direction is also constant and the path is parabolic with $y \propto x^2$.

28. The stopping force $\vec{F}$ and the path of the car are horizontal. Thus, the weight of the car contributes only (via Eq. 5-12) to information about its mass ($m = W/g = 1327$ kg). Our $+x$ axis is in the direction of the car's velocity, so that its acceleration (''deceleration'') is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F\,\hat{\text{i}}$.

(a) We use Eq. 2-16 and SI units (noting that $v = 0$ and $v_0 = 40(1000/3600) = 11.1$ m/s).

$$v^2 = v_0^2 + 2a\Delta x \quad\Rightarrow\quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(11.1\text{ m/s})^2}{2(15\text{ m})}$$

which yields $a = -4.12$ m/s^2. Assuming there are no significant horizontal forces other than the stopping force, Eq. 5-1 leads to

$$\vec{F} = m\vec{a} \;\Rightarrow\; -F = (1327 \text{ kg})(-4.12 \text{ m/s}^2)$$

which results in $F = 5.5 \times 10^3$ N.

(b) Equation 2-11 readily yields $t = -v_0/a = 2.7$ s.

(c) Keeping F the same means keeping a the same, in which case (since $v = 0$) Eq. 2-16 expresses a direct proportionality between Δx and v_0^2. Therefore, doubling v_0 means quadrupling Δx. That is, the new over the old stopping distances is a factor of 4.0.

(d) Equation 2-11 illustrates a direct proportionality between t and v_0 so that doubling one means doubling the other. That is, the new time of stopping is a factor of 2.0 greater than the one found in part (b).

29. We choose up as the $+y$ direction, so $\vec{a} = (-3.00 \text{ m/s}^2)\hat{j}$ (which, without the unit-vector, we denote as a since this is a 1-dimensional problem in which Table 2-1 applies). From Eq. 5-12, we obtain the firefighter's mass: $m = W/g = 72.7$ kg.

(a) We denote the force exerted by the pole on the firefighter $\vec{F}_{\text{fp}} = F_{\text{fp}}\,\hat{j}$ and apply Eq. 5-1. Since $\vec{F}_{\text{net}} = m\vec{a}$, we have

$$F_{\text{fp}} - F_g = ma \;\Rightarrow\; F_{\text{fp}} - 712 \text{ N} = (72.7 \text{ kg})(-3.00 \text{ m/s}^2)$$

which yields $F_{\text{fp}} = 494$ N.

(b) The fact that the result is positive means $\vec{F}_{\text{fp}}$ points up.

(c) Newton's third law indicates $\vec{F}_{\text{fp}} = -\vec{F}_{\text{pf}}$, which leads to the conclusion that $|\vec{F}_{\text{pf}}| = 494$ N.

(d) The direction of $\vec{F}_{\text{pf}}$ is down.

30. The stopping force $\vec{F}$ and the path of the toothpick are horizontal. Our $+x$ axis is in the direction of the toothpick's motion, so that the toothpick's acceleration (''deceleration'') is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F\,\hat{i}$. Using Eq. 2-16 with $v_0 = 220$ m/s and $v = 0$, the acceleration is found to be

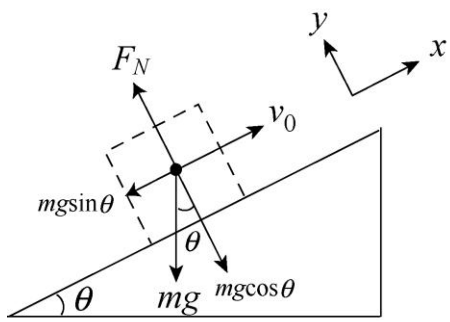
$$v^2 = v_0^2 + 2a\Delta x \;\Rightarrow\; a = -\frac{v_0^2}{2\Delta x} = -\frac{(220 \text{ m/s})^2}{2(0.015 \text{ m})} = -1.61\times10^6 \text{ m/s}^2.$$

Thus, the magnitude of the force exerted by the branch on the toothpick is

$$F = m|a| = (1.3\times10^{-4}\text{ kg})(1.61\times10^{6}\text{ m/s}^2) = 2.1\times10^{2}\text{ N}.$$

31. **THINK** In this problem we analyze the motion of a block sliding up an inclined plane and back down.

EXPRESS The free-body diagram is shown below. $\vec{F}_N$ is the normal force of the plane on the block and $m\vec{g}$ is the force of gravity on the block. We take the +x direction to be up the incline, and the +y direction to be in the direction of the normal force exerted by the incline on the block.



The x component of Newton's second law is then $mg\sin\theta = -ma$; thus, the acceleration is $a = -g\sin\theta$. Placing the origin at the bottom of the plane, the kinematic equations (Table 2-1) for motion along the x axis which we will use are $v^2 = v_0^2 + 2ax$ and $v = v_0 + at$. The block momentarily stops at its highest point, where $v = 0$; according to the second equation, this occurs at time $t = -v_0/a$.

ANALYZE (a) The position where the block stops is

$$x = v_0t + \frac{1}{2}at^2 = v_0\left(\frac{-v_0}{a}\right) + \frac{1}{2}a\left(\frac{-v_0}{a}\right)^2 = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left(\frac{(3.50\text{ m/s})^2}{-\left(9.8\text{ m/s}^2\right)\sin 32.0^\circ}\right) = 1.18\text{ m}.$$

(b) The time it takes for the block to get there is

$$t = \frac{v_0}{a} = -\frac{v_0}{-g\sin\theta} = -\frac{3.50\text{m/s}}{-(9.8\text{m/s}^2)\sin 32.0^\circ} = 0.674\text{ s}.$$

(c) That the return speed is identical to the initial speed is to be expected since there are no dissipative forces in this problem. In order to prove this, one approach is to set $x = 0$ and solve $x = v_0t + \frac{1}{2}at^2$ for the total time (up and back down) t. The result is

$$t = -\frac{2v_0}{a} = -\frac{2v_0}{-g\sin\theta} = -\frac{2(3.50\text{ m/s})}{-(9.8\text{ m/s}^2)\sin 32.0°} = 1.35\text{ s}.$$

The velocity when it returns is therefore

$$v = v_0 + at = v_0 - gt\sin\theta = 3.50\text{ m/s} - (9.8\text{ m/s}^2)(1.35\text{ s})\sin 32° = -3.50\text{ m/s}.$$

The negative sign indicates the direction is down the plane.

LEARN As expected, the speed of the block when it gets back to the bottom of the incline is the same as its initial speed. As we shall see in Chapter 8, this is a consequence of energy conservation. If friction is present, then the return speed will be smaller than the initial speed.

32. (a) Using notation suitable to a vector-capable calculator, the $\vec{F}_{\text{net}} = 0$ condition becomes

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (6.00 \angle 150°) + (7.00 \angle -60.0°) + \vec{F}_3 = 0 .$$

Thus, $\vec{F}_3 = (1.70\text{ N})\,\hat{\text{i}} + (3.06\text{ N})\hat{\text{j}}$.

(b) A constant velocity condition requires zero acceleration, so the answer is the same.

(c) Now, the acceleration is

$$\vec{a} = (13.0\text{ m/s}^2)\,\hat{\text{i}} - (14.0\text{ m/s}^2)\,\hat{\text{j}}.$$

Using $\vec{F}_{\text{net}} = m\,\vec{a}$ (with m = 0.025 kg) we now obtain

$$\vec{F}_3 = (2.02\text{ N})\,\hat{\text{i}} + (2.71\text{ N})\,\hat{\text{j}}.$$

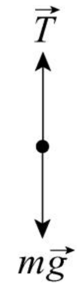
33. The free-body diagram is shown below. Let $\vec{T}$ be the tension of the cable and $m\vec{g}$ be the force of gravity. If the upward direction is positive, then Newton's second law is $T - mg = ma$, where a is the acceleration.

Thus, the tension is $T = m(g + a)$. We use constant acceleration kinematics (Table 2-1) to find the acceleration (where $v = 0$ is the final velocity, $v_0 = -12$ m/s is the initial velocity, and $y = -42$ m is the coordinate at the stopping point). Consequently, $v^2 = v_0^2 + 2ay$ leads to

$$a = -\frac{v_0^2}{2y} = -\frac{(-12\text{ m/s})^2}{2(-42\text{ m})} = 1.71\text{ m/s}^2.$$

We now return to calculate the tension:

$$T = m(g + a)$$
$$= (1600 \text{ kg})(9.8 \text{ m/s}^2 + 1.71 \text{ m/s}^2)$$
$$= 1.8 \times 10^4 \text{ N}.$$

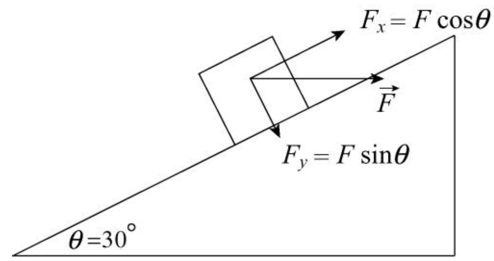


34. We resolve this horizontal force into appropriate components.

(a) Newton's second law applied to the *x*-axis produces

$$F\cos\theta - mg\sin\theta = ma.$$

For $a = 0$, this yields $F = 566$ N.



(b) Applying Newton's second law to the *y* axis (where there is no acceleration), we have

$$F_N - F\sin\theta - mg\cos\theta = 0$$

which yields the normal force $F_N = 1.13 \times 10^3$ N.

35. The acceleration vector as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(8.00t\,\hat{\text{i}} + 3.00t^2\,\hat{\text{j}}\right) \text{ m/s} = (8.00\,\hat{\text{i}} + 6.00t\,\hat{\text{j}}) \text{ m/s}^2.$$

(a) The magnitude of the force acting on the particle is

$$F = ma = m|\vec{a}| = (3.00)\sqrt{(8.00)^2 + (6.00t)^2} = (3.00)\sqrt{64.0 + 36.0\,t^2} \text{ N}.$$

Thus, $F = 35.0$ N corresponds to $t = 1.415$ s, and the acceleration vector at this instant is

$$\vec{a} = [8.00\,\hat{\text{i}} + 6.00(1.415)\,\hat{\text{j}}] \text{ m/s}^2 = (8.00 \text{ m/s}^2)\,\hat{\text{i}} + (8.49 \text{ m/s}^2)\,\hat{\text{j}}.$$

The angle $\vec{a}$ makes with $+x$ is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{8.49 \text{ m/s}^2}{8.00 \text{ m/s}^2}\right) = 46.7°.$$

(b) The velocity vector at $t = 1.415$ s is

$$\vec{v} = \left[8.00(1.415)\,\hat{\mathrm{i}} + 3.00(1.415)^2\,\hat{\mathrm{j}}\right]\mathrm{m/s} = (11.3\ \mathrm{m/s})\,\hat{\mathrm{i}} + (6.01\ \mathrm{m/s})\hat{\mathrm{j}}.$$

Therefore, the angle $\vec{v}$ makes with $+x$ is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{6.01\ \mathrm{m/s}}{11.3\ \mathrm{m/s}}\right) = 28.0°.$$

36. (a) Constant velocity implies zero acceleration, so the "uphill" force must equal (in magnitude) the "downhill" force: $T = mg \sin\theta$. Thus, with m = 50 kg and $\theta = 8.0°$, the tension in the rope equals 68 N.

(b) With an uphill acceleration of 0.10 m/s^2, Newton's second law (applied to the x axis) yields

$$T - mg\sin\theta = ma \quad\Rightarrow\quad T - (50\ \mathrm{kg})(9.8\ \mathrm{m/s^2})\sin 8.0° = (50\ \mathrm{kg})(0.10\ \mathrm{m/s^2})$$

which leads to T = 73 N.

37. (a) Since friction is negligible the force of the girl is the only horizontal force on the sled. The vertical forces (the force of gravity and the normal force of the ice) sum to zero. The acceleration of the sled is

$$a_s = \frac{F}{m_s} = \frac{5.2\ \mathrm{N}}{8.4\ \mathrm{kg}} = 0.62\ \mathrm{m/s^2}\ .$$

(b) According to Newton's third law, the force of the sled on the girl is also 5.2 N. Her acceleration is

$$a_g = \frac{F}{m_g} = \frac{5.2\ \mathrm{N}}{40\ \mathrm{kg}} = 0.13\ \mathrm{m/s^2}\ .$$

(c) The accelerations of the sled and girl are in opposite directions. Assuming the girl starts at the origin and moves in the $+x$ direction, her coordinate is given by $x_g = \frac{1}{2}a_g t^2$. The sled starts at x_0 = 15 m and moves in the $-x$ direction. Its coordinate is given by $x_s = x_0 - \frac{1}{2}a_s t^2$. They meet when $x_g = x_s$, or

$$\frac{1}{2}a_g t^2 = x_0 - \frac{1}{2}a_s t^2.$$

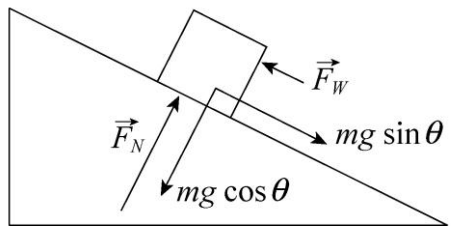
This occurs at time

$$t = \sqrt{\frac{2x_0}{a_g + a_s}}.$$

By then, the girl has gone the distance

$$x_g = \frac{1}{2}a_g t^2 = \frac{x_0 a_g}{a_g + a_s} = \frac{(15\text{ m})(0.13\text{ m/s}^2)}{0.13\text{ m/s}^2 + 0.62\text{ m/s}^2} = 2.6\text{ m}.$$

38. We label the 40 kg skier "m," which is represented as a block in the figure shown. The force of the wind is denoted $\vec{F}_w$ and might be either "uphill" or "downhill" (it is shown uphill in our sketch). The incline angle θ is 10°. The $-x$ direction is downhill.



(a) Constant velocity implies zero acceleration; thus, application of Newton's second law along the x axis leads to $mg\sin\theta - F_w = 0$. This yields $F_w = 68$ N (uphill).

(b) Given our coordinate choice, we have $a = |a| = 1.0\text{ m/s}^2$. Newton's second law

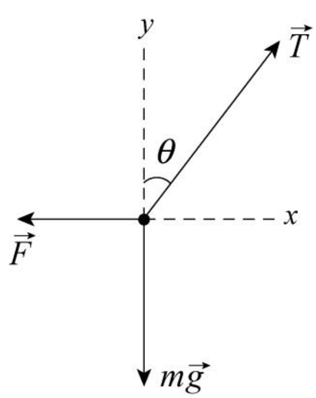
$$mg\sin\theta - F_w = ma$$

now leads to $F_w = 28$ N (uphill).

(c) Continuing with the forces as shown in our figure, the equation

$$mg\sin\theta - F_w = ma$$

will lead to $F_w = -12$ N when $|a| = 2.0\text{ m/s}^2$. This simply tells us that the wind is opposite to the direction shown in our sketch; in other words, $\vec{F}_w = 12\text{ N}$ *downhill*.



39. The solutions to parts (a) and (b) have been combined here. The free-body diagram is shown to the right, with the tension of the string $\vec{T}$, the force of gravity $m\vec{g}$, and the force of the air $\vec{F}$. Our coordinate system is shown. Since the sphere is motionless the net force on it is zero, and the x and the y components of the equations are:

$$T\sin\theta - F = 0$$
$$T\cos\theta - mg = 0,$$

where $\theta = 37°$. We answer the questions in the reverse order. Solving $T\cos\theta - mg = 0$ for the tension, we obtain

$$T = mg/\cos\theta = (3.0 \times 10^{-4}\text{ kg})(9.8\text{ m/s}^2)/\cos 37° = 3.7 \times 10^{-3}\text{ N}.$$

Solving $T\sin\theta - F = 0$ for the force of the air:

$$F = T\sin\theta = (3.7 \times 10^{-3}\text{ N})\sin 37° = 2.2 \times 10^{-3}\text{ N}.$$

40. The acceleration of an object (neither pushed nor pulled by any force other than gravity) on a smooth inclined plane of angle θ is $a = -g\sin\theta$. The slope of the graph shown with the problem statement indicates $a = -2.50$ m/s^2. Therefore, we find $\theta = 14.8°$. Examining the forces perpendicular to the incline (which must sum to zero since there is no component of acceleration in this direction) we find $F_N = mg\cos\theta$, where $m = 5.00$ kg. Thus, the normal (perpendicular) force exerted at the box/ramp interface is 47.4 N.

41. The mass of the bundle is $m = (449\text{ N})/(9.80\text{ m/s}^2) = 45.8$ kg and we choose $+y$ upward.

(a) Newton's second law, applied to the bundle, leads to

$$T - mg = ma \;\Rightarrow\; a = \frac{387\text{ N} - 449\text{ N}}{45.8\text{ kg}}$$

which yields $a = -1.4$ m/s^2 (or $|a| = 1.4$ m/s^2) for the acceleration. The minus sign in the result indicates the acceleration vector points down. Any downward acceleration of magnitude greater than this is also acceptable (since that would lead to even smaller values of tension).

(b) We use Eq. 2-16 (with Δx replaced by $\Delta y = -6.1$ m). We assume $v_0 = 0$.

$$|v| = \sqrt{2a\Delta y} = \sqrt{2\left(-1.35\text{ m/s}^2\right)\left(-6.1\text{ m}\right)} = 4.1\text{ m/s}.$$

For downward accelerations greater than 1.4 m/s^2, the speeds at impact will be larger than 4.1 m/s.

42. The direction of motion (the direction of the barge's acceleration) is $+\hat{\text{i}}$, and $+\vec{\text{j}}$ is chosen so that the pull $\vec{F}_\text{h}$ from the horse is in the first quadrant. The components of the unknown force of the water are denoted simply F_x and F_y.

(a) Newton's second law applied to the barge, in the x and y directions, leads to

$$\left(7900\text{N}\right)\cos 18° + F_x = ma$$
$$\left(7900\text{N}\right)\sin 18° + F_y = 0$$

respectively. Plugging in a = 0.12 m/s^2 and m = 9500 kg, we obtain $F_x = -6.4 \times 10^3$ N and $F_y = -2.4 \times 10^3$ N. The magnitude of the force of the water is therefore

$$F_{\text{water}} = \sqrt{F_x^2 + F_y^2} = 6.8 \times 10^3 \text{ N}.$$

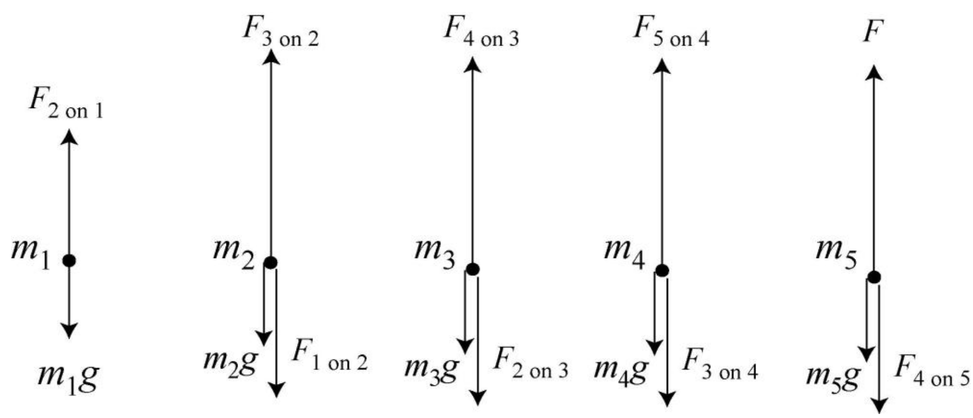
(b) Its angle measured from $+\hat{\text{i}}$ is either

$$\tan^{-1}\left(\frac{F_y}{F_x}\right) = +21° \text{ or } 201°.$$

The signs of the components indicate the latter is correct, so $\vec{F}_{\text{water}}$ is at 201° measured counterclockwise from the line of motion (+x axis).

43. **THINK** A chain of five links is accelerated vertically upward by an external force. We are interested in the forces exerted by one link on its adjacent one.

EXPRESS The links are numbered from bottom to top. The forces on the first link are the force of gravity $m\vec{g}$, downward, and the force $\vec{F}_{2\text{on}1}$ of link 2, upward, as shown in the free-body diagram below (not drawn to scale). Take the positive direction to be upward. Then Newton's second law for the first link is $F_{2\text{on}1} - m_1 g = m_1 a$. The equations for the other links can be written in a similar manner (see below).



ANALYZE (a) Given that $a = 2.50 \text{ m/s}^2$, from $F_{2\text{on}1} - m_1 g = m_1 a$, the force exerted by link 2 on link 1 is

$$F_{2\text{on}1} = m_1(a+g) = (0.100 \text{ kg})(2.5 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 1.23 \text{ N}.$$

(b) From the free-body diagram above, we see that the forces on the second link are the force of gravity $m_2\vec{g}$, downward, the force $\vec{F}_{1\text{on}2}$ of link 1, downward, and the force $\vec{F}_{3\text{on}2}$

of link 3, upward. According to Newton's third law $\vec{F}_{1on2}$ has the same magnitude as $\vec{F}_{2on1}$. Newton's second law for the second link is

$$F_{3on2} - F_{1on2} - m_2 g = m_2 a$$

so

$$F_{3on2} = m_2(a + g) + F_{1on2} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 1.23 \text{ N} = 2.46 \text{ N}.$$

(c) Newton's second law equation for link 3 is $F_{4on3} - F_{2on3} - m_3 g = m_3 a$, so

$$F_{4on3} = m_3(a + g) + F_{2on3} = (0.100 \text{ N})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 2.46 \text{ N} = 3.69 \text{ N},$$

where Newton's third law implies $F_{2on3} = F_{3on2}$ (since these are magnitudes of the force vectors).

(d) Newton's second law for link 4 is

$$F_{5on4} - F_{3on4} - m_4 g = m_4 a,$$

so

$$F_{5on4} = m_4(a + g) + F_{3on4} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 3.69 \text{ N} = 4.92 \text{ N},$$

where Newton's third law implies $F_{3on4} = F_{4on3}$.

(e) Newton's second law for the top link is $F - F_{4on5} - m_5 g = m_5 a$, so

$$F = m_5(a + g) + F_{4on5} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 4.92 \text{ N} = 6.15 \text{ N},$$

where $F_{4on5} = F_{5on4}$ by Newton's third law.

(f) Each link has the same mass ($m_1 = m_2 = m_3 = m_4 = m_5 = m$) and the same acceleration, so the same net force acts on each of them:

$$F_{net} = ma = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2) = 0.250 \text{ N}.$$

LEARN In solving this problem we have used both Newton's second and third laws. Each pair of links constitutes a third-law force pair, with $\vec{F}_{\text{i on j}} = -\vec{F}_{\text{j on i}}$.

44. (a) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is downward). Thus (with $+y$ upward) the acceleration is $a = +2.4 \text{ m/s}^2$. Newton's second law leads to

$$T - mg = ma \;\Rightarrow\; m = \frac{T}{g + a}$$

which yields $m = 7.3$ kg for the mass.

(b) Repeating the above computation (now to solve for the tension) with a = +2.4 m/s^2 will, of course, lead us right back to T = 89 N. Since the direction of the velocity did not enter our computation, this is to be expected.

45. (a) The mass of the elevator is m = (27800/9.80) = 2837 kg and (with +y upward) the acceleration is a = +1.22 m/s^2. Newton's second law leads to

$$T - mg = ma \;\Rightarrow\; T = m(g + a)$$

which yields $T = 3.13 \times 10^4$ N for the tension.

(b) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is upward). Thus (with +y upward) the acceleration is now a = –1.22 m/s^2, so that the tension is

$$T = m\,(g + a) = 2.43 \times 10^4 \text{ N}.$$

46. With a_{ce} meaning "the acceleration of the coin relative to the elevator" and a_{eg} meaning "the acceleration of the elevator relative to the ground," we have

$$a_{ce} + a_{eg} = a_{cg} \quad\Rightarrow\quad -8.00 \text{ m/s}^2 + a_{eg} = -9.80 \text{ m/s}^2$$

which leads to a_{eg} = –1.80 m/s^2. We have chosen upward as the positive y direction. Then Newton's second law (in the "ground" reference frame) yields $T - m\,g = m\,a_{eg}$, or

$$T = m\,g + m\,a_{eg} = m(g + a_{eg}) = (2000 \text{ kg})(8.00 \text{ m/s}^2) = 16.0 \text{ kN}.$$

47. Using Eq. 4-26, the launch speed of the projectile is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{(9.8 \text{ m/s}^2)(69 \text{ m})}{\sin 2(53°)}} = 26.52 \text{ m/s}.$$

The horizontal and vertical components of the speed are

$$v_x = v_0 \cos\theta = (26.52 \text{ m/s})\cos 53° = 15.96 \text{ m/s}$$
$$v_y = v_0 \sin\theta = (26.52 \text{ m/s})\sin 53° = 21.18 \text{ m/s}.$$

Since the acceleration is constant, we can use Eq. 2-16 to analyze the motion. The component of the acceleration in the horizontal direction is

$$a_x = \frac{v_x^2}{2x} = \frac{(15.96 \text{ m/s})^2}{2(5.2 \text{ m})\cos 53°} = 40.7 \text{ m/s}^2,$$

and the force component is

$$F_x = ma_x = (85\ \text{kg})(40.7\ \text{m/s}^2) = 3460\ \text{N}.$$

Similarly, in the vertical direction, we have

$$a_y = \frac{v_y^2}{2y} = \frac{(21.18\ \text{m/s})^2}{2(5.2\ \text{m})\sin 53°} = 54.0\ \text{m/s}^2.$$

and the force component is

$$F_y = ma_y + mg = (85\ \text{kg})(54.0\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = 5424\ \text{N}.$$

Thus, the magnitude of the force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(3460\ \text{N})^2 + (5424\ \text{N})^2} = 6434\ \text{N} \approx 6.4\times10^3\ \text{N},$$

to two significant figures.

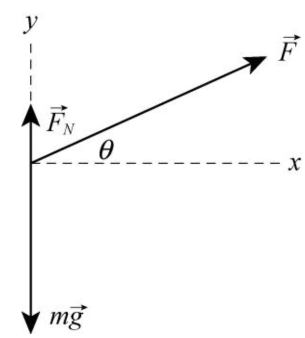
48. Applying Newton's second law to cab B (of mass m) we have

$$a = \frac{T}{m} - g = 4.89\ \text{m/s}^2.$$

Next, we apply it to the box (of mass m_b) to find the normal force:

$$F_N = m_b(g + a) = 176\ \text{N}.$$

49. The free-body diagram (not to scale) for the block is shown to the right. $\vec{F}_N$ is the normal force exerted by the floor and $m\vec{g}$ is the force of gravity.



(a) The x component of Newton's second law is $F\cos\theta = ma$, where m is the mass of the block and a is the x component of its acceleration. We obtain

$$a = \frac{F\cos\theta}{m} = \frac{(12.0\ \text{N})\cos 25.0°}{5.00\ \text{kg}} = 2.18\ \text{m/s}^2.$$

This is its acceleration provided it remains in contact with the floor. Assuming it does, we find the value of F_N (and if F_N is positive, then the assumption is true but if F_N is negative then the block leaves the floor). The y component of Newton's second law becomes

$$F_N + F\sin\theta - mg = 0,$$

so

$$F_N = mg - F\sin\theta = (5.00\ \text{kg})(9.80\ \text{m/s}^2) - (12.0\ \text{N})\sin 25.0° = 43.9\ \text{N}.$$

Hence the block remains on the floor and its acceleration is $a = 2.18\ \text{m/s}^2$.

(b) If F is the minimum force for which the block leaves the floor, then $F_N = 0$ and the y component of the acceleration vanishes. The y component of the second law becomes

$$F\sin\theta - mg = 0 \quad \Rightarrow \quad F = \frac{mg}{\sin\theta} = \frac{(5.00\ \text{kg})(9.80\ \text{m/s}^2)}{\sin 25.0°} = 116\ \text{N}.$$

(c) The acceleration is still in the x direction and is still given by the equation developed in part (a):

$$a = \frac{F\cos\theta}{m} = \frac{(116\ \text{N})\cos 25.0°}{5.00\ \text{kg}} = 21.0\,\text{m/s}^2.$$

50. (a) The net force on the *system* (of total mass $M = 80.0$ kg) is the force of gravity acting on the total overhanging mass ($m_{BC} = 50.0$ kg). The magnitude of the acceleration is therefore $a = (m_{BC}\,g)/M = 6.125\ \text{m/s}^2$. Next we apply Newton's second law to block C itself (choosing *down* as the $+y$ direction) and obtain

$$m_C g - T_{BC} = m_C a.$$

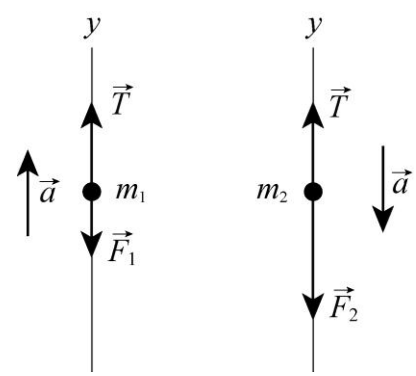
This leads to $T_{BC} = 36.8$ N.

(b) We use Eq. 2-15 (choosing *rightward* as the $+x$ direction): $\Delta x = 0 + \frac{1}{2}at^2 = 0.191$ m.

51. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$



which can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g$$

Substituting the result back, we have

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$$

(a) With $m_1 = 1.3$ kg and $m_2 = 2.8$ kg, the acceleration becomes

$$a = \left(\frac{2.80\text{ kg} - 1.30\text{ kg}}{2.80\text{ kg} + 1.30\text{ kg}}\right)(9.80\text{ m/s}^2) = 3.59\text{ m/s}^2 \approx 3.6\text{ m/s}^2.$$

(b) Similarly, the tension in the cord is

$$T = \frac{2(1.30\text{ kg})(2.80\text{ kg})}{1.30\text{ kg} + 2.80\text{ kg}}(9.80\text{ m/s}^2) = 17.4\text{ N} \approx 17\text{ N}.$$

52. Viewing the man-rope-sandbag as a system means that we should be careful to choose a consistent positive direction of motion (though there are other ways to proceed, say, starting with individual application of Newton's law to each mass). We take *down* as positive for the man's motion and *up* as positive for the sandbag's motion and, without ambiguity, denote their acceleration as a. The net force on the system is the different between the weight of the man and that of the sandbag. The system mass is m_{sys} = 85 kg + 65 kg = 150 kg. Thus, Eq. 5-1 leads to

$$(85\text{ kg})(9.8\text{ m/s}^2) - (65\text{ kg})(9.8\text{ m/s}^2) = m_{sys}\, a$$

which yields a = 1.3 m/s^2. Since the system starts from rest, Eq. 2-16 determines the speed (after traveling Δy = 10 m) as follows:

$$v = \sqrt{2a\Delta y} = \sqrt{2(1.3\text{ m/s}^2)(10\text{ m})} = 5.1\text{ m/s}.$$

53. We apply Newton's second law first to the three blocks as a single system and then to the individual blocks. The +x direction is to the right in Fig. 5-48.

(a) With $m_{sys} = m_1 + m_2 + m_3$ = 67.0 kg, we apply Eq. 5-2 to the x motion of the system, in which case, there is only one force $\vec{T}_3 = +\vec{T}_3\ \hat{\text{i}}$. Therefore,

$$T_3 = m_{sys} a \quad \Rightarrow \quad 65.0\text{ N} = (67.0\text{ kg})a$$

which yields a = 0.970 m/s^2 for the system (and for each of the blocks individually).

(b) Applying Eq. 5-2 to block 1, we find

$$T_1 = m_1 a = (12.0\text{kg})(0.970\text{m/s}^2) = 11.6\text{N}.$$

(c) In order to find T_2, we can either analyze the forces on block 3 or we can treat blocks 1 and 2 as a system and examine its forces. We choose the latter.

$$T_2 = (m_1 + m_2)a = (12.0\text{ kg} + 24.0\text{ kg})(0.970\text{ m/s}^2) = 34.9\text{ N}.$$

54. First, we consider all the penguins (1 through 4, counting left to right) as one system, to which we apply Newton's second law:

$$T_4 = \left(m_1 + m_2 + m_3 + m_4\right)a \quad \Rightarrow \quad 222\,\text{N} = \left(12\,\text{kg} + m_2 + 15\,\text{kg} + 20\,\text{kg}\right)a.$$

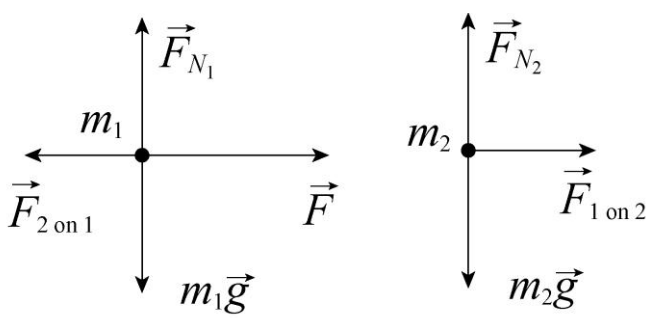
Second, we consider penguins 3 and 4 as one system, for which we have

$$T_4 - T_2 = \left(m_3 + m_4\right)a$$
$$111\,\text{N} = \left(15\text{ kg} + 20\,\text{kg}\right)a \quad \Rightarrow \quad a = 3.2\text{ m/s}^2.$$

Substituting the value, we obtain m_2 = 23 kg.

55. **THINK** In this problem a horizontal force is applied to block 1 which then pushes against block 2. Both blocks move together as a rigid connected system.

EXPRESS The free-body diagrams for the two blocks in (a) are shown below. $\vec{F}$ is the applied force and $\vec{F}_{1\text{on}2}$ is the force exerted by block 1 on block 2. We note that $\vec{F}$ is applied directly to block 1 and that block 2 exerts a force $\vec{F}_{2\text{on}1} = -\vec{F}_{1\text{on}2}$ on block 1 (taking Newton's third law into account).

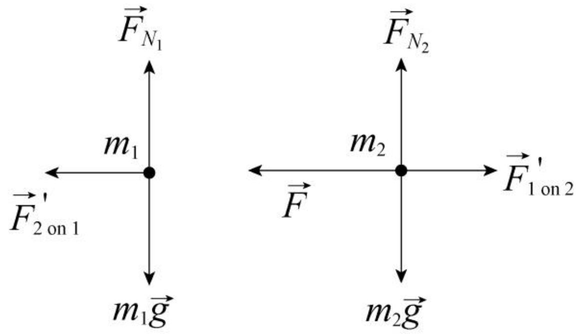


Newton's second law for block 1 is $F - F_{2\text{on}1} = m_1 a$, where a is the acceleration. The second law for block 2 is $F_{1\text{on}2} = m_2 a$. Since the blocks move together they have the same acceleration and the same symbol is used in both equations.

ANALYZE (a) From the second equation we obtain the expression $a = F_{1\text{on}2}/m_2$, which we substitute into the first equation to get $F - F_{2\text{on}1} = m_1 F_{1\text{on}2}/m_2$. Since $F_{2\text{on}1} = F_{1\text{on}2}$ (same magnitude for third-law force pair), we obtain

$$F_{2\text{on}1} = F_{1\text{on}2} = \frac{m_2}{m_1 + m_2}F = \frac{1.2\text{ kg}}{2.3\text{ kg} + 1.2\text{ kg}}\left(3.2\text{ N}\right) = 1.1\text{ N}.$$

(b) If $\vec{F}$ is applied to block 2 instead of block 1 (and in the opposite direction), the free-body diagrams would look like the following:



The corresponding force of contact between the blocks would be

$$F'_{2\text{on}1} = F'_{1\text{on}2} = \frac{m_1}{m_1 + m_2} F = \frac{2.3\text{ kg}}{2.3\text{ kg} + 1.2\text{ kg}}(3.2\text{ N}) = 2.1\text{ N}.$$

(c) We note that the acceleration of the blocks is the same in the two cases. In part (a), the force $F_{1\text{on}2}$ is the only horizontal force on the block of mass m_2 and in part (b) $F'_{2\text{on}1}$ is the only horizontal force on the block with $m_1 > m_2$. Since $F_{1\text{on}2} = m_2 a$ in part (a) and $F'_{2\text{on}1} = m_1 a$ in part (b), then for the accelerations to be the same, $F'_{2\text{on}1} > F_{1\text{on}2}$, i.e., force between blocks must be larger in part (b).

LEARN This problem demonstrates that when two blocks are being accelerated together under an external force, the contact force between the two blocks is greater if the smaller mass is pushing against the bigger one, as in part (b). In the special case where the two masses are equal, $m_1 = m_2 = m$, $F'_{2\text{on}1} = F_{2\text{on}1} = F/2$.

56. Both situations involve the same applied force and the same total mass, so the accelerations must be the same in both figures.

(a) The (direct) force causing *B* to have this acceleration in the first figure is twice as big as the (direct) force causing *A* to have that acceleration. Therefore, *B* has the twice the mass of *A*. Since their total is given as 12.0 kg then *B* has a mass of m_B = 8.00 kg and *A* has mass m_A = 4.00 kg. Considering the first figure, (20.0 N)/(8.00 kg) = 2.50 m/s^2. Of course, the same result comes from considering the second figure ((10.0 N)/(4.00 kg) = 2.50 m/s^2).

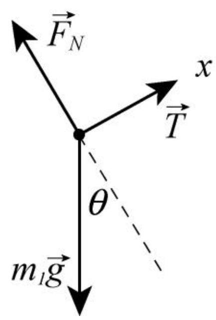
(b) F_a = (12.0 kg)(2.50 m/s^2) = 30.0 N

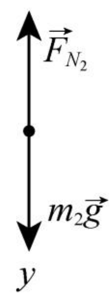
57. The free-body diagram for each block is shown below. *T* is the tension in the cord and θ = 30° is the angle of the incline. For block 1, we take the +*x* direction to be up the incline and the +*y* direction to be in the direction of the normal force $\vec{F}_N$ that the plane exerts on the block. For block 2, we take the +*y* direction to be down. In this way, the accelerations of the two blocks can be represented by the same symbol *a*, without

ambiguity. Applying Newton's second law to the *x* and *y* axes for block 1 and to the *y* axis of block 2, we obtain

$$T - m_1 g \sin\theta = m_1 a$$
$$F_N - m_1 g \cos\theta = 0$$
$$m_2 g - T = m_2 a$$

respectively. The first and third of these equations provide a simultaneous set for obtaining values of *a* and *T*. The second equation is not needed in this problem, since the normal force is neither asked for nor is it needed as part of some further computation (such as can occur in formulas for friction).





(a) We add the first and third equations above:

$$m_2 g - m_1 g \sin\theta = m_1 a + m_2 a.$$

Consequently, we find

$$a = \frac{(m_2 - m_1 \sin\theta) g}{m_1 + m_2} = \frac{[2.30\text{ kg} - (3.70\text{ kg})\sin 30.0°]\left(9.80\text{ m/s}^2\right)}{3.70\text{ kg} + 2.30\text{ kg}} = 0.735\,\text{m/s}^2.$$

(b) The result for *a* is positive, indicating that the acceleration of block 1 is indeed up the incline and that the acceleration of block 2 is vertically down.

(c) The tension in the cord is

$$T = m_1 a + m_1 g \sin\theta = (3.70\text{ kg})\left(0.735\text{ m/s}^2\right) + (3.70\text{ kg})\left(9.80\text{ m/s}^2\right)\sin 30.0° = 20.8\text{N}.$$

58. The motion of the man-and-chair is positive if upward.

(a) When the man is grasping the rope, pulling with a force equal to the tension *T* in the rope, the total upward force on the man-and-chair due its two contact points with the rope is 2*T*. Thus, Newton's second law leads to

$$2T - mg = ma$$

so that when $a = 0$, the tension is $T = 466$ N.

(b) When $a = +1.30$ m/s^2 the equation in part (a) predicts that the tension will be $T = 527$ N.

(c) When the man is not holding the rope (instead, the co-worker attached to the ground is pulling on the rope with a force equal to the tension T in it), there is only one contact point between the rope and the man-and-chair, and Newton's second law now leads to

$$T - mg = ma$$

so that when $a = 0$, the tension is $T = 931$ N.

(d) When $a = +1.30$ m/s^2, the equation in (c) yields $T = 1.05 \times 10^3$ N.

(e) The rope comes into contact (pulling down in each case) at the left edge and the right edge of the pulley, producing a total downward force of magnitude $2T$ on the ceiling. Thus, in part (a) this gives $2T = 931$ N.

(f) In part (b) the downward force on the ceiling has magnitude $2T = 1.05 \times 10^3$ N.

(g) In part (c) the downward force on the ceiling has magnitude $2T = 1.86 \times 10^3$ N.

(h) In part (d) the downward force on the ceiling has magnitude $2T = 2.11 \times 10^3$ N.

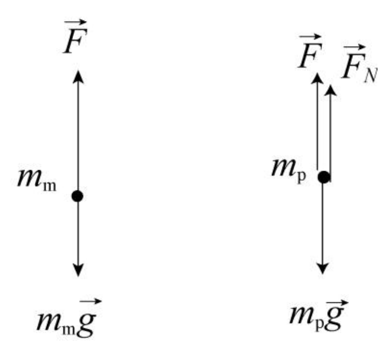
59. **THINK** This problem involves the application of Newton's third law. As the monkey climbs up a tree, it pulls downward on the rope, but the rope pulls upward on the monkey.

EXPRESS We take $+y$ to be up for both the monkey and the package. The force the monkey pulls downward on the rope has magnitude F.

The free-body diagrams for the monkey and the package are shown to the right (not to scale). According to Newton's third law, the rope pulls upward on the monkey with a force of the same magnitude, so Newton's second law for forces acting on the monkey leads to

$$F - m_m g = m_m a_m,$$

where m_m is the mass of the monkey and a_m is its acceleration.



Since the rope is massless, $F = T$ is the tension in the rope. The rope pulls upward on the package with a force of magnitude F, so Newton's second law for the package is

$$F + F_N - m_p g = m_p a_p,$$

where m_p is the mass of the package, a_p is its acceleration, and F_N is the normal force exerted by the ground on it. Now, if F is the minimum force required to lift the package, then $F_N = 0$ and $a_p = 0$. According to the second law equation for the package, this means $F = m_p g$.

ANALYZE (a) Substituting $m_p g$ for F in the equation for the monkey, we solve for a_m:

$$a_m = \frac{F - m_m g}{m_m} = \frac{(m_p - m_m)g}{m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{10\text{ kg}} = 4.9\text{ m/s}^2.$$

(b) As discussed, Newton's second law leads to $F - m_p g = m_p a'_p$ for the package and $F - m_m g = m_m a'_m$ for the monkey. If the acceleration of the package is downward, then the acceleration of the monkey is upward, so $a'_m = -a'_p$. Solving the first equation for F

$$F = m_p\left(g + a'_p\right) = m_p\left(g - a'_m\right)$$

and substituting this result into the second equation:

$$m_p(g - a'_m) - m_m g = m_m a'_m,$$

we solve for a'_m:

$$a'_m = \frac{(m_p - m_m)g}{m_p + m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{15\text{ kg} + 10\text{ kg}} = 2.0\text{ m/s}^2.$$

(c) The result is positive, indicating that the acceleration of the monkey is upward.

(d) Solving the second law equation for the package, the tension in the rope is

$$F = m_p\left(g - a'_m\right) = (15\text{ kg})\left(9.8\text{ m/s}^2 - 2.0\text{ m/s}^2\right) = 120\text{N}.$$

LEARN The situations described in (b)-(d) are similar to that of an Atwood machine. With $m_p > m_m$, the package accelerates downward while the monkey accelerates upward.

60. The horizontal component of the acceleration is determined by the net horizontal force.

(a) If the rate of change of the angle is

$$\frac{d\theta}{dt} = (2.00\times10^{-2})^\circ/\text{s} = (2.00\times10^{-2})^\circ/\text{s}\cdot\left(\frac{\pi\text{ rad}}{180^\circ}\right) = 3.49\times10^{-4}\text{rad/s},$$

then, using $F_x = F\cos\theta$, we find the rate of change of acceleration to be

$$\frac{da_x}{dt} = \frac{d}{dt}\left(\frac{F\cos\theta}{m}\right) = -\frac{F\sin\theta}{m}\frac{d\theta}{dt} = -\frac{(20.0\text{ N})\sin 25.0°}{5.00\text{ kg}}\left(3.49\times10^{-4}\text{ rad/s}\right)$$
$$= -5.90\times10^{-4}\text{ m/s}^3.$$

(b) If the rate of change of the angle is

$$\frac{d\theta}{dt} = -(2.00\times10^{-2})°/\text{s} = -(2.00\times10^{-2})°/\text{s}\cdot\left(\frac{\pi\text{ rad}}{180°}\right) = -3.49\times10^{-4}\text{ rad/s},$$

then the rate of change of acceleration would be

$$\frac{da_x}{dt} = \frac{d}{dt}\left(\frac{F\cos\theta}{m}\right) = -\frac{F\sin\theta}{m}\frac{d\theta}{dt} = -\frac{(20.0\text{ N})\sin 25.0°}{5.00\text{ kg}}\left(-3.49\times10^{-4}\text{ rad/s}\right)$$
$$= +5.90\times10^{-4}\text{ m/s}^3.$$

61. **THINK** As more mass is thrown out of the hot-air balloon, its upward acceleration increases.

EXPRESS The forces on the balloon are the force of gravity $m\vec{g}$ (down) and the force of the air $\vec{F}_a$ (up). We take the $+y$ to be up, and use a to mean the *magnitude* of the acceleration. When the mass is M (before the ballast is thrown out) the acceleration is downward and Newton's second law is

$$Mg - F_a = Ma$$

After the ballast is thrown out, the mass is $M - m$ (where m is the mass of the ballast) and the acceleration is now upward. Newton's second law leads to

$$F_a - (M - m)g = (M - m)a.$$

Combing the two equations allows us to solve for m.

ANALYZE The first equation gives $F_a = M(g - a)$, and this plugs into the new equation to give

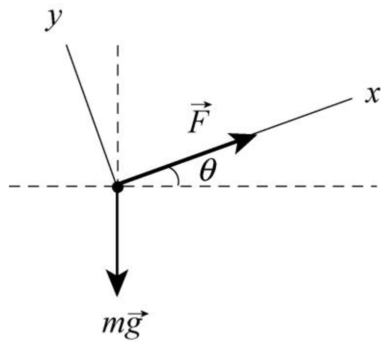
$$M(g - a) - (M - m)g = (M - m)a \;\Rightarrow\; m = \frac{2Ma}{g + a}.$$

LEARN More generally, if a ballast mass m' is tossed, the resulting acceleration is a' which is related to m' via:

$$m' = M\frac{a'+a}{g+a},$$

showing that the more mass thrown out, the greater is the upward acceleration. For $a' = a$, we get $m' = 2Ma/(g+a)$, which agrees with what was found above.

62. To solve the problem, we note that the acceleration along the slanted path depends on only the force components along the path, not the components perpendicular to the path.



(a) From the free-body diagram shown, we see that the net force on the putting shot along the +*x*-axis is

$$F_{\text{net},x} = F - mg\sin\theta = 380.0\text{ N} - (7.260\text{ kg})(9.80\text{ m/s}^2)\sin 30° = 344.4\text{ N},$$

which in turn gives

$$a_x = F_{\text{net},x}/m = (344.4\text{ N})/(7.260\text{ kg}) = 47.44\text{ m/s}^2.$$

Using Eq. 2-16 for constant-acceleration motion, the speed of the shot at the end of the acceleration phase is

$$v = \sqrt{v_0^2 + 2a_x\Delta x} = \sqrt{(2.500\text{ m/s})^2 + 2(47.44\text{ m/s}^2)(1.650\text{ m})} = 12.76\text{ m/s}.$$

(b) If $\theta = 42°$, then

$$a_x = \frac{F_{\text{net},x}}{m} = \frac{F - mg\sin\theta}{m} = \frac{380.0\text{ N} - (7.260\text{ kg})(9.80\text{ m/s}^2)\sin 42.00°}{7.260\text{ kg}} = 45.78\text{ m/s}^2,$$

and the final (launch) speed is

$$v = \sqrt{v_0^2 + 2a_x\Delta x} = \sqrt{(2.500\text{ m/s})^2 + 2(45.78\text{ m/s}^2)(1.650\text{ m})} = 12.54\text{ m/s}.$$

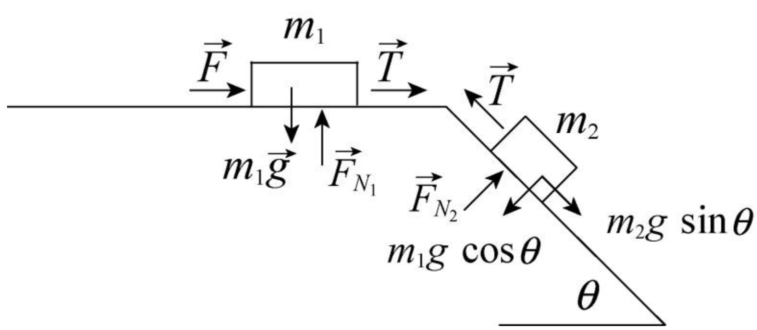
(c) The decrease in launch speed when changing the angle from $30.00°$ to $42.00°$ is

$$\frac{12.76 \text{ m/s} - 12.54 \text{ m/s}}{12.76 \text{ m/s}} = 0.0169 = 1.69\%.$$

63. (a) The acceleration (which equals *F/m* in this problem) is the derivative of the velocity. Thus, the velocity is the integral of *F/m*, so we find the "area" in the graph (15 units) and divide by the mass (3) to obtain $v - v_o = 15/3 = 5$. Since $v_o = 3.0$ m/s, then $v = 8.0\,\text{m/s}$.

(b) Our positive answer in part (a) implies $\vec{v}$ points in the $+x$ direction.

64. The $+x$ direction for $m_2 = 1.0$ kg is "downhill" and the $+x$ direction for $m_1 = 3.0$ kg is rightward; thus, they accelerate with the same sign.



(a) We apply Newton's second law to the *x* axis of each box:

$$m_2g\ \sin\theta - T = m_2a$$
$$F + T = m_1a$$

Adding the two equations allows us to solve for the acceleration:

$$a = \frac{m_2 g \sin\theta + F}{m_1 + m_2}$$

With $F = 2.3$ N and $\theta = 30°$, we have $a = 1.8$ m/s^2. We plug back in and find $T = 3.1$ N.

(b) We consider the "critical" case where the *F* has reached the *max* value, causing the tension to vanish. The first of the equations in part (a) shows that $a = g\sin 30°$ in this case; thus, $a = 4.9$ m/s^2. This implies (along with $T = 0$ in the second equation in part (a)) that

$$F = (3.0 \text{ kg})(4.9 \text{ m/s}^2) = 14.7 \text{ N} \approx 15 \text{ N}$$

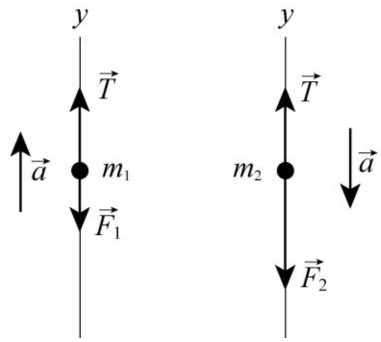
in the critical case.

65. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:

$$T - m_1 g = m_1 a$$
$$m_2 g - T = m_2 a$$

which can be solved to give

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$



(a) At $t = 0$, $m_{10} = 1.30$ kg. With $dm_1/dt = -0.200$ kg/s, we find the rate of change of acceleration to be

$$\frac{da}{dt} = \frac{da}{dm_1}\frac{dm_1}{dt} = -\frac{2m_2 g}{(m_2 + m_{10})^2}\frac{dm_1}{dt} = -\frac{2(2.80\text{ kg})(9.80\text{ m/s}^2)}{(2.80\text{ kg} + 1.30\text{ kg})^2}\left(-0.200\text{ kg/s}\right) = 0.653\text{ m/s}^3.$$

(b) At $t = 3.00$ s, $m_1 = m_{10} + (dm_1/dt)t = 1.30\text{ kg} + (-0.200\text{ kg/s})(3.00\text{ s}) = 0.700\text{ kg}$, and the rate of change of acceleration is

$$\frac{da}{dt} = \frac{da}{dm_1}\frac{dm_1}{dt} = -\frac{2m_2 g}{(m_2 + m_1)^2}\frac{dm_1}{dt} = -\frac{2(2.80\text{ kg})(9.80\text{ m/s}^2)}{(2.80\text{ kg} + 0.700\text{ kg})^2}\left(-0.200\text{ kg/s}\right) = 0.896\text{ m/s}^3.$$

(c) The acceleration reaches its maximum value when

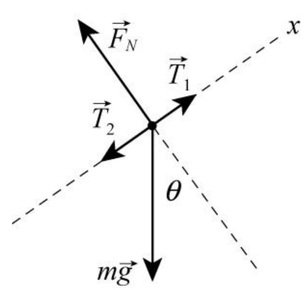
$$0 = m_1 = m_{10} + (dm_1/dt)t = 1.30\text{ kg} + (-0.200\text{ kg/s})t,$$

or $t = 6.50$ s.

66. The free-body diagram is shown to the right. Newton's second law for the mass m for the x direction leads to

$$T_1 - T_2 - mg\sin\theta = ma,$$

which gives the difference in the tension in the pull cable:



$$T_1 - T_2 = m\left(g\sin\theta + a\right) = \left(2800\text{ kg}\right)\left[(9.8\text{ m/s}^2)\sin 35° + 0.81\text{ m/s}^2\right] = 1.8 \times 10^4\text{ N}.$$

67. First we analyze the entire *system* with "clockwise" motion considered positive (that is, downward is positive for block *C*, rightward is positive for block *B*, and upward is positive for block *A*): $m_C g - m_A g = Ma$ (where M = mass of the *system* = 24.0 kg). This yields an acceleration of

$$a = g(m_C - m_A)/M = 1.63 \text{ m/s}^2.$$

Next we analyze the forces just on block C: $m_C g - T = m_C a$. Thus the tension is

$$T = m_C g(2m_A + m_B)/M = 81.7 \text{ N}.$$

68. We first use Eq. 4-26 to solve for the launch speed of the shot:

$$y - y_0 = (\tan\theta)x - \frac{gx^2}{2(v'\cos\theta)^2}.$$

With $\theta = 34.10°$, $y_0 = 2.11$ m, and $(x, y) = (15.90 \text{ m}, 0)$, we find the launch speed to be $v' = 11.85$ m/s. During this phase, the acceleration is

$$a = \frac{v'^2 - v_0^2}{2L} = \frac{(11.85 \text{ m/s})^2 - (2.50 \text{ m/s})^2}{2(1.65 \text{ m})} = 40.63 \text{ m/s}^2.$$

Since the acceleration along the slanted path depends on only the force components along the path, not the components perpendicular to the path, the average force on the shot during the acceleration phase is

$$F = m(a + g\sin\theta) = (7.260 \text{ kg})\left[40.63 \text{ m/s}^2 + (9.80 \text{ m/s}^2)\sin 34.10°\right] = 334.8 \text{ N}.$$

69. We begin by examining a slightly different problem: similar to this figure but without the string. The motivation is that if (without the string) block A is found to accelerate faster (or exactly as fast) as block B then (returning to the original problem) the tension in the string is trivially zero. In the absence of the string,

$$a_A = F_A / m_A = 3.0 \text{ m/s}^2$$

$$a_B = F_B / m_B = 4.0 \text{ m/s}^2$$

so the trivial case does not occur. We now (with the string) consider the net force on the *system*: $Ma = F_A + F_B = 36$ N. Since $M = 10$ kg (the total mass of the system) we obtain $a = 3.6$ m/s^2. The two forces on block A are F_A and T (in the same direction), so we have

$$m_A a = F_A + T \quad \Rightarrow \quad T = 2.4 \text{ N}.$$

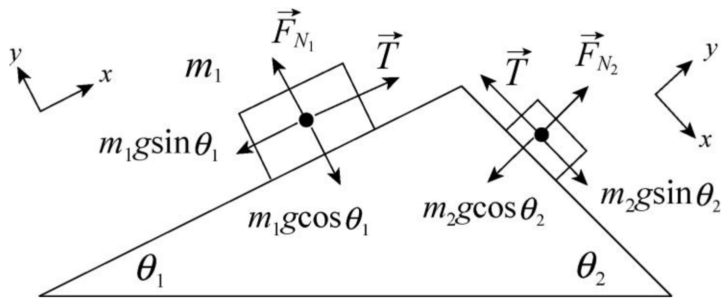
70. (a) For the 0.50 meter drop in "free fall," Eq. 2-16 yields a speed of 3.13 m/s. Using this as the "initial speed" for the final motion (over 0.02 meter) during which his motion slows at rate "a," we find the magnitude of his average acceleration from when his feet first touch the patio until the moment his body stops moving is $a = 245$ m/s^2.

(b) We apply Newton's second law: $F_{\text{stop}} - mg = ma \Rightarrow F_{\text{stop}} = 20.4$ kN.

71. **THINK** We have two boxes connected together by a cord and placed on a wedge. The system accelerates together and we'd like to know the tension in the cord.

EXPRESS The +*x* axis is "uphill" for m_1 = 3.0 kg and "downhill" for m_2 = 2.0 kg (so they both accelerate with the same sign). The *x* components of the two masses along the *x* axis are given by $m_1 g \sin\theta_1$ and $m_2 g \sin\theta_2$, respectively. The free-body diagram is shown below. Applying Newton's second law, we obtain

$$T - m_1 g \sin\theta_1 = m_1 a$$
$$m_2 g \sin\theta_2 - T = m_2 a$$



Adding the two equations allows us to solve for the acceleration:

$$a = \left(\frac{m_2 \sin\theta_2 - m_1 \sin\theta_1}{m_2 + m_1}\right) g$$

ANALYZE With $\theta_1 = 30°$ and $\theta_2 = 60°$, we have *a* = 0.45 m/s^2. This value is plugged back into either of the two equations to yield the tension

$$T = \frac{m_1 m_2 g}{m_2 + m_1}(\sin\theta_2 + \sin\theta_1) = 16.1\,\text{N}$$

LEARN In this problem we find $m_2 \sin\theta_2 > m_1 \sin\theta_1$, so that $a > 0$, indicating that m_2 slides down and m_1 slides up. The situation would reverse if $m_2 \sin\theta_2 < m_1 \sin\theta_1$. When $m_2 \sin\theta_2 = m_1 \sin\theta_1$, the acceleration is *a* = 0 and the two masses hang in balance. Notice also the symmetry between the two masses in the expression for *T*.

72. Since the velocity of the particle does not change, it undergoes no acceleration and must therefore be subject to zero net force. Therefore,

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0\,.$$

Thus, the third force $\vec{F}_3$ is given by

$$\vec{F}_3 = -\vec{F}_1 - \vec{F}_2 = -\left(2\hat{\text{i}} + 3\hat{\text{j}} - 2\hat{\text{k}}\right)\text{N} - \left(-5\hat{\text{i}} + 8\hat{\text{j}} - 2\hat{\text{k}}\right)\text{N} = \left(3\hat{\text{i}} - 11\hat{\text{j}} + 4\hat{\text{k}}\right)\text{N}.$$

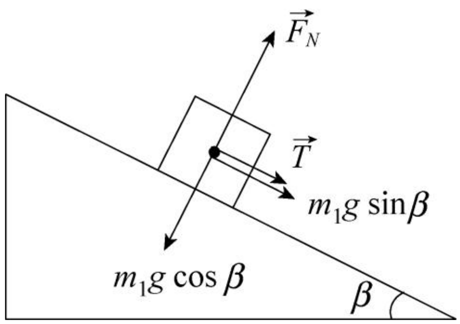
The specific value of the velocity is not used in the computation.

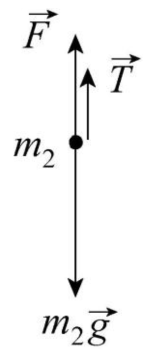
73. **THINK** We have two masses connected together by a cord. A force is applied to the second mass and the system accelerates together. We apply Newton's second law to solve the problem.

EXPRESS The free-body diagrams for the two masses are shown below (not to scale). We first analyze the forces on m_1=1.0 kg. The +x direction is "downhill" (parallel to $\vec{T}$). With an acceleration a = 5.5 m/s^2 in the positive x direction for m_1, Newton's second law applied to the x-axis gives

$$T + m_1 g \sin\beta = m_1 a .$$

On the other hand, for the second mass m_2=2.0 kg, we have $m_2 g - F - T = m_2 a$, where the tension comes in as an upward force (the cord can pull, not push). The two equations can be combined to solve for T and β.





ANALYZE We solve (b) first. By combining the two equations above, we obtain

$$\sin\beta = \frac{(m_1+m_2)a + F - m_2 g}{m_1 g} = \frac{(1.0\text{ kg}+2.0\text{ kg})(5.5\text{ m/s}^2)+6.0\text{ N}-(2.0\text{ kg})(9.8\text{ m/s}^2)}{(1.0\text{ kg})(9.8\text{ m/s}^2)}$$
$$= 0.296$$

which gives $\beta = 17.2°$.

(a) Substituting the value for β found in (a) into the first equation, we have

$$T = m_1(a - g\sin\beta) = (1.0\text{ kg})\left[5.5\text{ m/s}^2 - (9.8\text{ m/s}^2)\sin 17.2°\right] = 2.60\text{ N}.$$

LEARN For $\beta = 0$, the problem becomes the same as that discussed in Sample Problem "Block on table, block hanging." In this case, our results reduce to the familiar expressions: $a = m_2 g/(m_1+m_2)$, and $T = m_1 m_2 g/(m_1+m_2)$.

74. We are only concerned with horizontal forces in this problem (gravity plays no direct role). Without loss of generality, we take one of the forces along the $+x$ direction and the other at 80° (measured counterclockwise from the x axis). This calculation is efficiently implemented on a vector-capable calculator in polar mode, as follows (using magnitude-angle notation, with angles understood to be in degrees):

$$\vec{F}_{\text{net}} = (20 \angle 0) + (35 \angle 80) = (43 \angle 53) \Rightarrow |\vec{F}_{\text{net}}| = 43 \text{ N} .$$

Therefore, the mass is $m = (43 \text{ N})/(20 \text{ m/s}^2) = 2.2$ kg.

75. The goal is to arrive at the least magnitude of $\vec{F}_{\text{net}}$, and as long as the magnitudes of $\vec{F}_2$ and $\vec{F}_3$ are (in total) less than or equal to $|\vec{F}_1|$ then we should orient them opposite to the direction of $\vec{F}_1$ (which is the $+x$ direction).

(a) We orient both $\vec{F}_2$ and $\vec{F}_3$ in the $-x$ direction. Then, the magnitude of the net force is $50 - 30 - 20 = 0$, resulting in zero acceleration for the tire.

(b) We again orient $\vec{F}_2$ and $\vec{F}_3$ in the negative x direction. We obtain an acceleration along the $+x$ axis with magnitude

$$a = \frac{F_1 - F_2 - F_3}{m} = \frac{50\,\text{N} - 30\,\text{N} - 10\,\text{N}}{12\,\text{kg}} = 0.83 \text{ m/s}^2 .$$

(c) The least value is $a = 0$. In this case, the forces $\vec{F}_2$ and $\vec{F}_3$ are collectively strong enough to have y components (one positive and one negative) that cancel each other and still have enough x contributions (in the $-x$ direction) to cancel $\vec{F}_1$. Since $|\vec{F}_2| = |\vec{F}_3|$, we see that the angle above the $-x$ axis to one of them should equal the angle below the $-x$ axis to the other one (we denote this angle θ). We require

$$-50 \text{ N} = F_{2x} + F_{3x} = -(30\text{N})\cos\theta - (30\text{N})\cos\theta$$

which leads to

$$\theta = \cos^{-1}\left(\frac{50\,\text{N}}{60\,\text{N}}\right) = 34° .$$

76. (a) A small segment of the rope has mass and is pulled down by the gravitational force of the Earth. Equilibrium is reached because neighboring portions of the rope pull up sufficiently on it. Since tension is a force *along* the rope, at least one of the neighboring portions must slope up away from the segment we are considering. Then, the tension has an upward component, which means the rope sags.

(b) The only force acting with a horizontal component is the applied force $\vec{F}$. Treating the block and rope as a single object, we write Newton's second law for it: $F = (M + m)a$, where a is the acceleration and the positive direction is taken to be to the right. The acceleration is given by $a = F/(M + m)$.

(c) The force of the rope F_r is the only force with a horizontal component acting on the block. Then Newton's second law for the block gives

$$F_r = Ma = \frac{MF}{M+m}$$

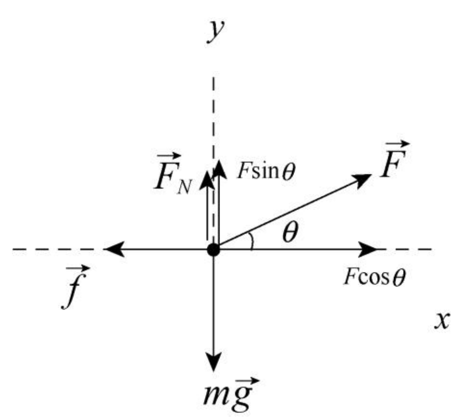
where the expression found above for a has been used.

(d) Treating the block and half the rope as a single object, with mass $M + \frac{1}{2}m$, where the horizontal force on it is the tension T_m at the midpoint of the rope, we use Newton's second law:

$$T_m = \left(M + \frac{1}{2}m\right)a = \frac{(M + m/2)F}{(M+m)} = \frac{(2M+m)F}{2(M+m)}.$$

77. **THINK** We have a crate that is being pulled at an angle. We apply Newton's second law to analyze the motion.

EXPRESS Although the full specification of $\vec{F}_{\text{net}} = m\vec{a}$ in this situation involves both x and y axes, only the x-application is needed to find what this particular problem asks for. We note that $a_y = 0$ so that there is no ambiguity denoting a_x simply as a. We choose $+x$ to the right and $+y$ up. The free-body diagram (not to scale) is shown to the right. The x component of the rope's tension (acting on the crate) is



$$F_x = F\cos\theta = (450\text{ N})\cos 38° = 355\text{ N},$$

and the resistive force (pointing in the $-x$ direction) has magnitude $f = 125$ N.

ANALYZE (a) Newton's second law leads to

$$F_x - f = ma \Rightarrow a = \frac{F\cos\theta - f}{m} = \frac{355\text{ N} - 125\text{ N}}{310\text{ kg}} = 0.74\text{m/s}^2.$$

(b) In this case, we use Eq. 5-12 to find the mass: $m' = W/g = 31.6\text{ kg}$. Newton's second law then leads to

$$F_x - f = m'a' \;\Rightarrow\; a' = \frac{F_x - f}{m'} = \frac{355\text{ N} - 125\text{ N}}{31.6\text{ kg}} = 7.3\text{ m/s}^2.$$

LEARN The resistive force opposing the motion is due to the friction between the crate and the floor. This topic is discussed in greater detail in Chapter 6.

78. We take +*x* uphill for the m_2 = 1.0 kg box and +*x* rightward for the m_1 = 3.0 kg box (so the accelerations of the two boxes have the same magnitude and the same sign). The uphill force on m_2 is F and the downhill forces on it are T and $m_2 g \sin\theta$, where $\theta = 37°$. The only horizontal force on m_1 is the rightward-pointed tension. Applying Newton's second law to each box, we find

$$F - T - m_2 g \sin\theta = m_2 a$$
$$T = m_1 a$$

which can be added to obtain

$$F - m_2 g \sin\theta = (m_1 + m_2)a.$$

This yields the acceleration

$$a = \frac{12\text{ N} - (1.0\text{ kg})(9.8\text{ m/s}^2)\sin 37°}{1.0\text{ kg} + 3.0\text{ kg}} = 1.53\text{ m/s}^2.$$

Thus, the tension is $T = m_1 a = (3.0\text{ kg})(1.53\text{ m/s}^2) = 4.6\text{ N}$.

79. We apply Eq. 5-12.

(a) The mass is

$$m = W/g = (22\text{ N})/(9.8\text{ m/s}^2) = 2.2\text{ kg}.$$

At a place where $g = 4.9\text{ m/s}^2$, the mass is still 2.2 kg but the gravitational force is

$$F_g = mg = (2.2\text{ kg})\,(4.0\text{ m/s}^2) = 11\text{ N}.$$

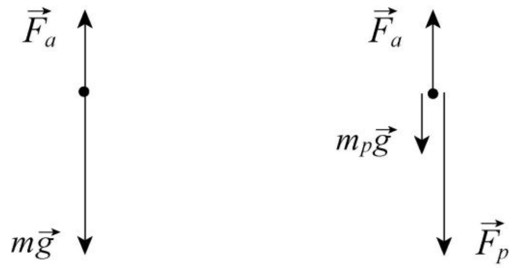
(b) As noted, m = 2.2 kg.

(c) At a place where g = 0 the gravitational force is zero.

(d) The mass is still 2.2 kg.

80. We take down to be the +*y* direction.

(a) The first diagram (shown below left) is the free-body diagram for the person and parachute, considered as a single object with a mass of 80 kg + 5.0 kg = 85 kg.



$\vec{F}_a$ is the force of the air on the parachute and $m\vec{g}$ is the force of gravity. Application of Newton's second law produces $mg - F_a = ma$, where a is the acceleration. Solving for F_a we find

$$F_a = m(g - a) = (85 \text{ kg})\left(9.8 \text{ m/s}^2 - 2.5 \text{ m/s}^2\right) = 620 \text{ N}.$$

(b) The second diagram (above right) is the free-body diagram for the parachute alone. $\vec{F}_a$ is the force of the air, $m_p\vec{g}$ is the force of gravity, and $\vec{F}_p$ is the force of the person. Now, Newton's second law leads to

$$m_p g + F_p - F_a = m_p a.$$

Solving for F_p, we obtain

$$F_p = m_p(a - g) + F_a = (5.0 \text{ kg})\left(2.5 \text{ m/s}^2 - 9.8 \text{ m/s}^2\right) + 620 \text{ N} = 580 \text{ N}.$$

81. The mass of the pilot is m = 735/9.8 = 75 kg. Denoting the upward force exerted by the spaceship (his seat, presumably) on the pilot as $\vec{F}$ and choosing upward as the $+y$ direction, then Newton's second law leads to

$$F - mg_{\text{moon}} = ma \quad \Rightarrow \quad F = (75 \text{ kg})\left(1.6 \text{ m/s}^2 + 1.0 \text{ m/s}^2\right) = 195 \text{ N}.$$

82. With SI units understood, the net force on the box is

$$\vec{F}_{\text{net}} = (3.0 + 14 \cos 30° - 11)\,\hat{\text{i}} + (14 \sin 30° + 5.0 - 17)\,\hat{\text{j}}$$

which yields $\vec{F}_{\text{net}} = (4.1 \text{ N})\,\hat{\text{i}} - (5.0 \text{ N})\,\hat{\text{j}}$.

(a) Newton's second law applied to the m = 4.0 kg box leads to

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = (1.0 \text{ m/s}^2)\hat{\text{i}} - (1.3 \text{ m/s}^2)\hat{\text{j}}.$$

(b) The magnitude of $\vec{a}$ is $a = \sqrt{(1.0 \text{ m/s}^2)^2 + \left(-1.3 \text{ m/s}^2\right)^2} = 1.6 \text{ m/s}^2$.

(c) Its angle is $\tan^{-1}$ [(–1.3 m/s^2)/(1.0 m/s^2)] = –50° (that is, 50° measured clockwise from the rightward axis).

83. **THINK** This problem deals with the relationship between the three quantities: force, mass and acceleration in Newton's second law $F = ma$.

EXPRESS The "certain force," denoted as F, is assumed to be the net force on the object when it gives m_1 an acceleration a_1 = 12 m/s^2 and when it gives m_2 an acceleration a_2 = 3.3 m/s^2, i.e., $F = m_1 a_1 = m_2 a_2$. The accelerations for $m_2 + m_1$ and $m_2 - m_1$ can be solved by substituting $m_1 = F/a_1$ and $m_2 = F/a_2$.

ANALYZE (a) Now we seek the acceleration a of an object of mass $m_2 - m_1$ when F is the net force on it. The result is

$$a = \frac{F}{m_2 - m_1} = \frac{F}{(F/a_2) - (F/a_1)} = \frac{a_1 a_2}{a_1 - a_2} = \frac{(12.0 \text{ m/s}^2)(3.30 \text{ m/s}^2)}{12.0 \text{ m/s}^2 - 3.30 \text{ m/s}^2} = 4.55 \text{ m/s}^2 .$$

(b) Similarly for an object of mass $m_2 + m_1$, we have:

$$a' = \frac{F}{m_2 + m_1} = \frac{F}{(F/a_2) + (F/a_1)} = \frac{a_1 a_2}{a_1 + a_2} = \frac{(12.0 \text{ m/s}^2)(3.30 \text{ m/s}^2)}{12.0 \text{ m/s}^2 + 3.30 \text{ m/s}^2} = 2.59 \text{ m/s}^2 .$$

LEARN With the same applied force, the greater the mass the smaller the acceleration. In this problem, we have $a_1 > a > a_2 > a'$. This implies $m_1 < m_2 - m_1 < m_2 < m_2 + m_1$.

84. We assume the direction of motion is $+x$ and assume the refrigerator starts from rest (so that the speed being discussed is the velocity $\vec{v}$ that results from the process). The only force along the x axis is the x component of the applied force $\vec{F}$.

(a) Since v_0 = 0, the combination of Eq. 2-11 and Eq. 5-2 leads simply to

$$F_x = m\left(\frac{v}{t}\right) \Rightarrow v_i = \left(\frac{F\cos\theta_i}{m}\right) t$$

for i = 1 or 2 (where we denote θ_1 = 0 and $\theta_2 = \theta$ for the two cases). Hence, we see that the ratio v_2 over v_1 is equal to cos θ.

(b) Since v_0 = 0, the combination of Eq. 2-16 and Eq. 5-2 leads to

$$F_x = m\left(\frac{v^2}{2\Delta x}\right) \Rightarrow v_i = \sqrt{2\left(\frac{F\cos\theta_i}{m}\right)\Delta x}$$

for i = 1 or 2 (again, $\theta_1 = 0$ and $\theta_2 = \theta$ is used for the two cases). In this scenario, we see that the ratio v_2 over v_1 is equal to $\sqrt{\cos\theta}$.

85. (a) Since the performer's weight is (52 kg)(9.8 m/s^2) = 510 N, the rope breaks.

(b) Setting T = 425 N in Newton's second law (with +y upward) leads to

$$T - mg = ma \;\Rightarrow\; a = \frac{T}{m} - g$$

which yields $|a|$ = 1.6 m/s^2.

86. We use $W_p = mg_p$, where W_p is the weight of an object of mass m on the surface of a certain planet p, and g_p is the acceleration of gravity on that planet.

(a) The weight of the space ranger on Earth is

$$W_e = mg_e = (75 \text{ kg})\,(9.8 \text{ m/s}^2) = 7.4 \times 10^2 \text{ N}.$$

(b) The weight of the space ranger on Mars is

$$W_m = mg_m = (75 \text{ kg})\,(3.7 \text{ m/s}^2) = 2.8 \times 10^2 \text{ N}.$$

(c) The weight of the space ranger in interplanetary space is zero, where the effects of gravity are negligible.

(d) The mass of the space ranger remains the same, m = 75 kg, at all the locations.

87. From the reading when the elevator was at rest, we know the mass of the object is m = (65 N)/(9.8 m/s^2) = 6.6 kg. We choose +y upward and note there are two forces on the object: mg downward and T upward (in the cord that connects it to the balance; T is the reading on the scale by Newton's third law).

(a) "Upward at constant speed" means constant velocity, which means no acceleration. Thus, the situation is just as it was at rest: T = 65 N.

(b) The term "deceleration" is used when the acceleration vector points in the direction opposite to the velocity vector. We're told the velocity is upward, so the acceleration vector points downward (a = –2.4 m/s^2). Newton's second law gives

$$T - mg = ma \;\Rightarrow\; T = (6.6 \text{ kg})(9.8 \text{ m/s}^2 - 2.4 \text{ m/s}^2) = 49 \text{ N}.$$

88. We use the notation g as the acceleration due to gravity near the surface of Callisto, m as the mass of the landing craft, a as the acceleration of the landing craft, and F as the rocket thrust. We take down to be the positive direction. Thus, Newton's second law takes the form $mg - F = ma$. If the thrust is F_1 (= 3260 N), then the acceleration is zero,

so $mg - F_1 = 0$. If the thrust is F_2 (= 2200 N), then the acceleration is a_2 (= 0.39 m/s^2), so $mg - F_2 = ma_2$.

(a) The first equation gives the weight of the landing craft: $mg = F_1 = 3260$ N.

(b) The second equation gives the mass:

$$m = \frac{mg - F_2}{a_2} = \frac{3260 \text{ N} - 2200 \text{ N}}{0.39 \text{ m/s}^2} = 2.7 \times 10^3 \text{ kg}.$$

(c) The weight divided by the mass gives the acceleration due to gravity:

$$g = (3260 \text{ N})/(2.7 \times 10^3 \text{ kg}) = 1.2 \text{ m/s}^2.$$

89. (a) When $\vec{F}_{\text{net}} = 3F - mg = 0$, we have

$$F = \frac{1}{3}mg = \frac{1}{3}(1400 \text{ kg})(9.8 \text{ m/s}^2) = 4.6 \times 10^3 \text{ N}$$

for the force exerted by each bolt on the engine.

(b) The force on each bolt now satisfies $3F - mg = ma$, which yields

$$F = \frac{1}{3}m(g + a) = \frac{1}{3}(1400 \text{ kg})(9.8 \text{ m/s}^2 + 2.6 \text{ m/s}^2) = 5.8 \times 10^3 \text{ N}.$$

90. We write the length unit light-month, the distance traveled by light in one month, as c·month in this solution.

(a) The magnitude of the required acceleration is given by

$$a = \frac{\Delta v}{\Delta t} = \frac{(0.10)(3.0 \times 10^8 \text{ m/s})}{(3.0 \text{ days})(86400 \text{ s/day})} = 1.2 \times 10^2 \text{ m/s}^2.$$

(b) The acceleration in terms of g is $a = \left(\frac{a}{g}\right) g = \left(\frac{1.2 \times 10^2 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) g = 12g$.

(c) The force needed is

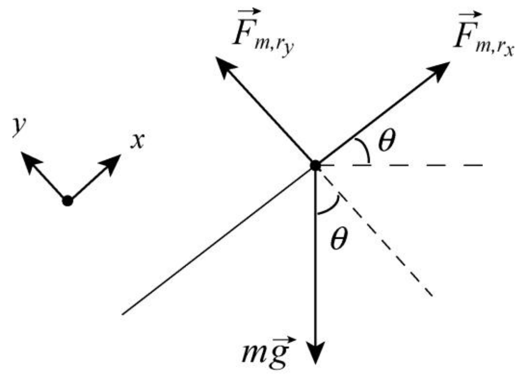
$$F = ma = (1.20 \times 10^6 \text{ kg})(1.2 \times 10^2 \text{ m/s}^2) = 1.4 \times 10^8 \text{ N}.$$

(d) The spaceship will travel a distance $d = 0.1$ c·month during one month. The time it takes for the spaceship to travel at constant speed for 5.0 light-months is

$$t = \frac{d}{v} = \frac{5.0 \text{ c}\cdot\text{months}}{0.1c} = 50 \text{ months} \approx 4.2 \text{ years.}$$

91. **THINK** We have a motorcycle going up a ramp at a constant acceleration. We apply Newton's second law to calculate the net force on the rider and the force on the rider from the motorcycle.

EXPRESS The free-body diagram is shown to the right (not to scale). Note that F_{m,r_y} and F_{m,r_x}, respectively, denote the *y* and *x* components of the force $\vec{F}_{m,r}$ exerted by the motorcycle on the rider. The net force on the rider is



$$F_{\text{net}} = ma.$$

ANALYZE (a) Since the net force equals *ma*, then the magnitude of the net force on the rider is

$$F_{\text{net}} = ma = (60.0 \text{ kg})\,(3.0 \text{ m/s}^2) = 1.8 \times 10^2 \text{ N.}$$

(b) To calculate the force by the motorcycle on the rider, we apply Newton's second law to the *x*- and the *y*-axes separately. For the *x*-axis, we have:

$$F_{m,r_x} - mg \sin\theta = ma$$

where m = 60.0 kg, a = 3.0 m/s^2, and θ = 10°. Thus, $F_{m,r_x} = 282$ N. Applying it to the *y*-axis (where there is no acceleration), we have

$$F_{m,r_y} - mg\cos\theta = 0$$

which gives $F_{m,r_y} = 579$ N. Using the Pythagorean theorem, we find

$$F_{m,r} = \sqrt{F_{m,r_x}^2 + F_{m,r_y}^2} = \sqrt{(282\text{ N})^2 + (579\text{ N})^2} = 644 \text{ N.}$$

Now, the magnitude of the force exerted on the rider by the motorcycle is the same magnitude of force exerted by the rider on the motorcycle, so the answer is 644 N.

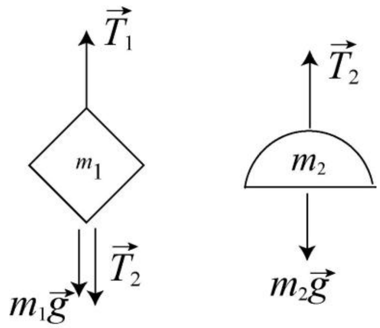
LEARN The force exerted by the motorcycle on the rider keeps the rider accelerating in the +*x*-direction, while maintaining contact with the inclines surface ($a_y = 0$).

92. We denote the thrust as *T* and choose +*y* upward. Newton's second law leads to

$$T - Mg = Ma \;\Rightarrow\; a = \frac{2.6\times10^5\ \text{N}}{1.3\times10^4\ \text{kg}} - 9.8\ \text{m/s}^2 = 10\ \text{m/s}^2.$$

93. **THINK** In this problem we have mobiles consisting of masses connected by cords. We apply Newton's second law to calculate the tensions in the cords.

EXPRESS The free-body diagrams for m_1 and m_2 for part (a) are shown to the right.



The bottom cord is only supporting m_2 = 4.5 kg against gravity, so its tension is $T_2 = m_2 g$. On the other hand, the top cord is supporting a total mass of $m_1 + m_2$ = (3.5 kg + 4.5 kg) = 8.0 kg against gravity. Applying Newton's second law gives

$$T_1 - T_2 - m_1 g = 0$$

so the tension is

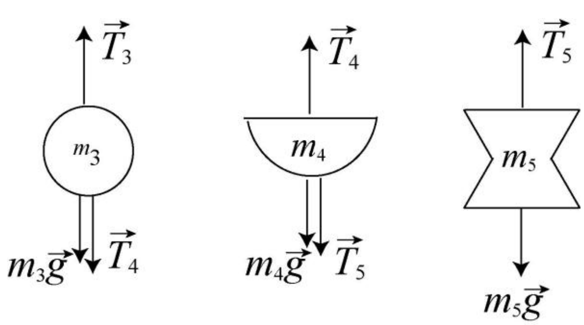
$$T_1 = m_1 g + T_2 = (m_1 + m_2)g.$$

ANALYZE (a) From the equations above, we find the tension in the bottom cord to be

$$T_2 = m_2 g = (4.5\ \text{kg})(9.8\ \text{m/s}^2) = 44\ \text{N}.$$

(b) Similarly, the tension in the top cord is $T_1 = (m_1 + m_2)g = (8.0\ \text{kg})(9.8\ \text{m/s}^2) = 78\ \text{N}$.

(c) The free-body diagrams for m_3, m_4 and m_5 for part (b) are shown below (not to scale).



From the diagram, we see that the lowest cord supports a mass of m_5 = 5.5 kg against gravity and consequently has a tension of

$$T_5 = m_5 g = (5.5 \text{ kg})(9.8 \text{ m/s}^2) = 54 \text{ N}.$$

(d) The top cord, as we are told, has a tension T_3 =199 N which supports a total of (199 N)/(9.80 m/s^2) = 20.3 kg, 10.3 kg of which is already accounted for in the figure. Thus, the unknown mass in the middle must be m_4 = 20.3 kg – 10.3 kg = 10.0 kg, and the tension in the cord above it must be enough to support

$$m_4 + m_5 = (10.0 \text{ kg} + 5.50 \text{ kg}) = 15.5 \text{ kg},$$

so T_4 = (15.5 kg)(9.80 m/s^2) = 152 N.

LEARN Another way to calculate T_4 is to examine the forces on m_3 − one of the downward forces on it is T_4. From Newton's second law, we have $T_3 - m_3 g - T_4 = 0$, which can be solved to give

$$T_4 = T_3 - m_3 g = 199 \text{ N} - (4.8 \text{ kg})(9.8 \text{ m/s}^2) = 152 \text{ N}.$$

94. The coordinate choices are made in the problem statement.

(a) We write the velocity of the armadillo as $\vec{v} = v_x \hat{\text{i}} + v_y \hat{\text{j}}$. Since there is no net force exerted on it in the x direction, the x component of the velocity of the armadillo is a constant: v_x = 5.0 m/s. In the y direction at t = 3.0 s, we have (using Eq. 2-11 with $v_{0y} = 0$)

$$v_y = v_{0y} + a_y t = v_{0y} + \left(\frac{F_y}{m}\right)t = \left(\frac{17 \text{ N}}{12 \text{ kg}}\right)(3.0 \text{ s}) = 4.3 \text{ m/s}.$$

Thus, $\vec{v} = (5.0 \text{ m/s})\hat{\text{i}} + (4.3 \text{ m/s})\hat{\text{j}}$.

(b) We write the position vector of the armadillo as $\vec{r} = r_x \hat{\text{i}} + r_y \hat{\text{j}}$. At t = 3.0 s we have r_x = (5.0 m/s) (3.0 s) = 15 m and (using Eq. 2-15 with v_{0y} = 0)

$$r_y = v_{0y} t + \frac{1}{2} a_y t^2 = \frac{1}{2}\left(\frac{F_y}{m}\right)t^2 = \frac{1}{2}\left(\frac{17 \text{ N}}{12 \text{ kg}}\right)(3.0 \text{ s})^2 = 6.4 \text{ m}.$$

The position vector at t = 3.0 s is therefore $\vec{r} = (15 \text{ m})\hat{\text{i}} + (6.4 \text{ m})\hat{\text{j}}$.

95. (a) Intuition readily leads to the conclusion that the heavier block should be the hanging one, for largest acceleration. The force that "drives" the system into motion is the weight of the hanging block (gravity acting on the block on the table has no effect on the dynamics, so long as we ignore friction). Thus, m = 4.0 kg.

The acceleration of the system and the tension in the cord can be readily obtained by solving

$$mg - T = ma, \quad T = Ma.$$

(b) The acceleration is given by $a = \left(\frac{m}{m+M}\right)g = 6.5 \text{ m/s}^2$.

(c) The tension is

$$T = Ma = \left(\frac{Mm}{m+M}\right)g = 13 \text{ N}.$$

96. According to Newton's second law, the magnitude of the force is given by $F = ma$, where a is the magnitude of the acceleration of the neutron. We use kinematics (Table 2-1) to find the acceleration that brings the neutron to rest in a distance d. Assuming the acceleration is constant, then $v^2 = v_0^2 + 2ad$ produces the value of a:

$$a = \frac{\left(v^2 - v_0^2\right)}{2d} = \frac{-\left(1.4 \times 10^7 \text{ m/s}\right)^2}{2\left(1.0 \times 10^{-14} \text{ m}\right)} = -9.8 \times 10^{27} \text{ m/s}^2.$$

The magnitude of the force is consequently

$$F = ma = \left(1.67 \times 10^{-27} \text{ kg}\right)\left(9.8 \times 10^{27} \text{ m/s}^2\right) = 16 \text{ N}.$$

97. (a) With SI units understood, the net force is

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(3.0 \text{ N} + (-2.0 \text{ N})\right)\hat{\text{i}} + \left(4.0 \text{ N} + (-6.0 \text{ N})\right)\hat{\text{j}}$$

which yields $\vec{F}_{\text{net}} = (1.0 \text{ N})\hat{\text{i}} - (2.0 \text{ N})\hat{\text{j}}$.

(b) The magnitude of $\vec{F}_{\text{net}}$ is $F_{\text{net}} = \sqrt{(1.0 \text{ N})^2 + (-2.0 \text{ N})^2} = 2.2 \text{ N}$.

(c) The angle of $\vec{F}_{\text{net}}$ is $\theta = \tan^{-1}\left(\frac{-2.0 \text{ N}}{1.0 \text{ N}}\right) = -63°$.

(d) The magnitude of $\vec{a}$ is $a = F_{\text{net}}/m = (2.2 \text{ N})/(1.0 \text{ kg}) = 2.2 \text{ m/s}^2$.

(e) Since $\vec{F}_{\text{net}}$ is equal to $\vec{a}$ multiplied by mass m, which is a positive scalar that cannot affect the direction of the vector it multiplies, $\vec{a}$ has the same angle as the net force, i.e, $\theta = -63°$. In magnitude-angle notation, we may write $\vec{a} = \left(2.2 \text{ m/s}^2 \angle -63°\right)$.

Chapter 6

1. The greatest deceleration (of magnitude a) is provided by the maximum friction force (Eq. 6-1, with $F_N = mg$ in this case). Using Newton's second law, we find

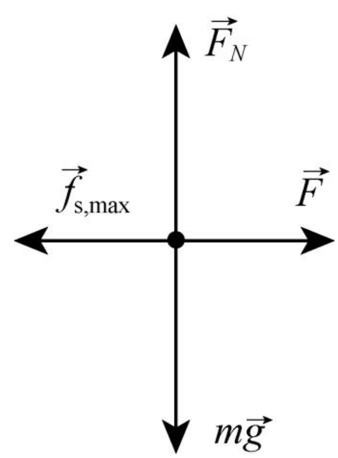
$$a = f_{s,\max}/m = \mu_s g.$$

Eq. 2-16 then gives the shortest distance to stop: $|\Delta x| = v^2/2a = 36$ m. In this calculation, it is important to first convert v to 13 m/s.

2. Applying Newton's second law to the horizontal motion, we have $F - \mu_k mg = ma$, where we have used Eq. 6-2, assuming that $F_N = mg$ (which is equivalent to assuming that the vertical force from the broom is negligible). Eq. 2-16 relates the distance traveled and the final speed to the acceleration: $v^2 = 2a\Delta x$. This gives $a = 1.4$ m/s^2. Returning to the force equation, we find (with $F = 25$ N and $m = 3.5$ kg) that $\mu_k = 0.58$.

3. **THINK** In the presence of friction between the floor and the bureau, a minimum horizontal force must be applied before the bureau would begin to move.

EXPRESS The free-body diagram for the bureau is shown to the right. We denote $\vec{F}$ as the horizontal force of the person, $\vec{f}_s$ is the force of static friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push $\vec{F}$ in the $+x$ direction). Applying Newton's second law to the x and y axes, we obtain



$$F - f_{s,\max} = ma$$
$$F_N - mg = 0$$

respectively.
The second equation yields the normal force $F_N = mg$, whereupon the maximum static friction is found to be (from Eq. 6-1) $f_{s,\max} = \mu_s mg$. Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set $a = 0$ to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.
ANALYZE (a) With $\mu_s = 0.45$ and $m = 45$ kg, the equation above leads to

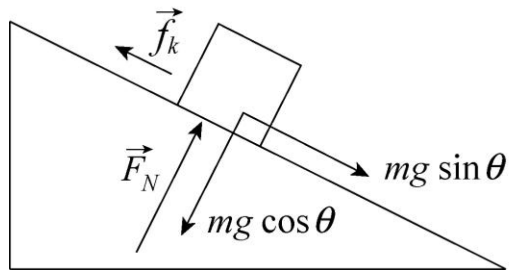
$$F = \mu_s mg = (0.45)(45\text{ kg})(9.8\text{ m/s}^2) = 198\text{ N}.$$

To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is $F = 2.0 \times 10^2$ N.

(b) Replacing m = 45 kg with m = 28 kg, the reasoning above leads to roughly $F = 1.2 \times 10^2$ N.

LEARN The values found above represent the minimum force required to overcome the friction. Applying a force greater than $\mu_s mg$ results in a net force in the +x-direction, and hence, non-zero acceleration.

4. We first analyze the forces on the pig of mass m. The incline angle is θ.



The +x direction is "downhill.'' Application of Newton's second law to the x- and y-axes leads to

$$mg \sin\theta - f_k = ma$$
$$F_N - mg \cos\theta = 0.$$

Solving these along with Eq. 6-2 ($f_k = \mu_k F_N$) produces the following result for the pig's downhill acceleration:

$$a = g(\sin\theta - \mu_k \cos\theta) .$$

To compute the time to slide from rest through a downhill distance ℓ, we use Eq. 2-15:

$$\ell = v_0 t + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\ell}{a}}.$$

We denote the frictionless ($\mu_k = 0$) case with a prime and set up a ratio:

$$\frac{t}{t'} = \frac{\sqrt{2\ell/a}}{\sqrt{2\ell/a'}} = \sqrt{\frac{a'}{a}}$$

which leads us to conclude that if $t/t' = 2$ then $a' = 4a$. Putting in what we found out above about the accelerations, we have

$$g\sin\theta = 4g\left(\sin\theta - \mu_k\cos\theta\right).$$

Using $\theta = 35°$, we obtain $\mu_k = 0.53$.

5. In addition to the forces already shown in Fig. 6-17, a free-body diagram would include an upward normal force $\vec{F}_N$ exerted by the floor on the block, a downward $m\vec{g}$ representing the gravitational pull exerted by Earth, and an assumed-leftward $\vec{f}$ for the kinetic or static friction. We choose $+x$ rightwards and $+y$ upwards. We apply Newton's second law to these axes:

$$F - f = ma$$
$$P + F_N - mg = 0$$

where $F = 6.0$ N and $m = 2.5$ kg is the mass of the block.

(a) In this case, $P = 8.0$ N leads to

$$F_N = (2.5 \text{ kg})(9.8 \text{ m/s}^2) - 8.0 \text{ N} = 16.5 \text{ N}.$$

Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 6.6$ N, which is larger than the 6.0 N rightward force – so the block (which was initially at rest) does not move. Putting $a = 0$ into the first of our equations above yields a static friction force of $f = P = 6.0$ N.

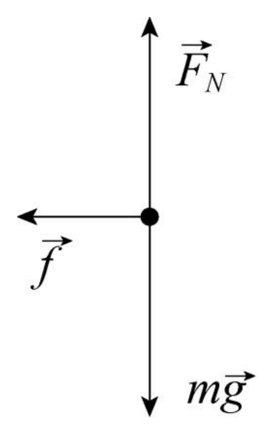
(b) In this case, $P = 10$ N, the normal force is

$$F_N = (2.5 \text{ kg})(9.8 \text{ m/s}^2) - 10 \text{ N} = 14.5 \text{ N}.$$

Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 5.8$ N, which is less than the 6.0 N rightward force – so the block does move. Hence, we are dealing not with static but with kinetic friction, which Eq. 6-2 reveals to be $f_k = \mu_k F_N = 3.6$ N.

(c) In this last case, $P = 12$ N leads to $F_N = 12.5$ N and thus to $f_{s,\max} = \mu_s F_N = 5.0$ N, which (as expected) is less than the 6.0 N rightward force – so the block moves. The kinetic friction force, then, is $f_k = \mu_k F_N = 3.1$ N.

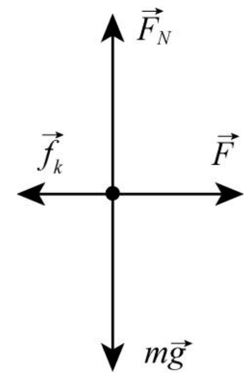
6. The free-body diagram for the player is shown to the right. $\vec{F}_N$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k F_N$. We use Newton's second law applied to the vertical axis to find the normal force. The vertical component of the acceleration is zero, so we obtain $F_N - mg = 0$; thus, $F_N = mg$. Consequently,



$$\mu_k = \frac{f}{F_N} = \frac{470\text{ N}}{(79\text{ kg})\,(9.8\text{ m/s}^2)} = 0.61.$$

7. **THINK** A force is being applied to accelerate a crate in the presence of friction. We apply Newton's second law to solve for the acceleration.

EXPRESS The free-body diagram for the crate is shown to the right. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the $+x$ direction), $\vec{f}_k$ is the force of kinetic friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by Eq. 6-2: $f_k = \mu_k F_N$. Applying Newton's second law to the x and y axes, we obtain



$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

ANALYZE (a) The second equation above yields the normal force $F_N = mg$, so that the friction is

$$f_k = \mu_k F_N = \mu_k mg = (0.35)(55\text{ kg})(9.8\text{ m/s}^2) = 1.9\times 10^2\text{ N}.$$

(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with $F = 220$ N) we solve to find

$$a = \frac{F}{m} - \mu_k g = \frac{220\text{ N}}{55\text{ kg}} - (0.35)(9.8\text{ m/s}^2) = 0.56\text{ m/s}^2.$$

LEARN For the crate to accelerate, the condition $F > f_k = \mu_k mg$ must be met. As can be seen from the equation above, the greater the value of μ_k, the smaller the acceleration under the same applied force.

8. To maintain the stone's motion, a horizontal force (in the $+x$ direction) is needed that cancels the retarding effect due to kinetic friction. Applying Newton's second to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively. The second equation yields the normal force $F_N = mg$, so that (using Eq. 6-2) the kinetic friction becomes $f_k = \mu_k\, mg$. Thus, the first equation becomes

$$F - \mu_k mg = ma = 0$$

where we have set $a = 0$ to be consistent with the idea that the horizontal velocity of the stone should remain constant. With $m = 20$ kg and $\mu_k = 0.80$, we find $F = 1.6 \times 10^2$ N.

9. We choose $+x$ horizontally rightwards and $+y$ upwards and observe that the 15 N force has components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$.

(a) We apply Newton's second law to the y axis:

$$F_N - F\sin\theta - mg = 0 \;\Rightarrow\; F_N = (15\text{ N})\sin 40° + (3.5\text{ kg})(9.8\text{ m/s}^2) = 44\text{ N}.$$

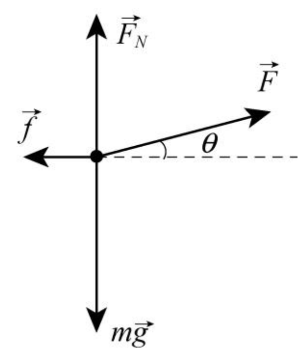
With $\mu_k = 0.25$, Eq. 6-2 leads to $f_k = 11$ N.

(b) We apply Newton's second law to the x axis:

$$F\cos\theta - f_k = ma \;\Rightarrow\; a = \frac{(15\text{ N})\cos 40° - 11\text{ N}}{3.5\text{ kg}} = 0.14\text{ m/s}^2.$$

Since the result is positive-valued, then the block is accelerating in the $+x$ (rightward) direction.

10. (a) The free-body diagram for the block is shown below, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction.



We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. The equations for the x and the y components of the force according to Newton's second law are:

$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta + F_N - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg - F\sin\theta$, which yields $f = \mu_k(mg - F\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$F\cos\theta - \mu_k(mg - F\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{F}{m}(\cos\theta + \mu_k\sin\theta) - \mu_k g.$$

(a) If $\mu_s = 0.600$ and $\mu_k = 0.500$, then the magnitude of $\vec{f}$ has a maximum value of

$$f_{s,\max} = \mu_s F_N = (0.600)(mg - 0.500mg\sin 20°) = 0.497mg.$$

On the other hand, $F\cos\theta = 0.500mg\cos 20° = 0.470mg$. Therefore, $F\cos\theta < f_{s,\max}$ and the block remains stationary with $a = 0$.

(b) If $\mu_s = 0.400$ and $\mu_k = 0.300$, then the magnitude of $\vec{f}$ has a maximum value of

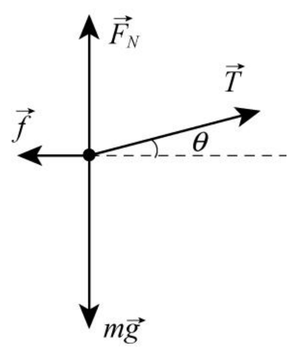
$$f_{s,\max} = \mu_s F_N = (0.400)(mg - 0.500mg\sin 20°) = 0.332mg.$$

In this case, $F\cos\theta = 0.500mg\cos 20° = 0.470mg > f_{s,\max}$. Therefore, the acceleration of the block is

$$\begin{aligned} a &= \frac{F}{m}(\cos\theta + \mu_k \sin\theta) - \mu_k g \\ &= (0.500)(9.80\ \text{m/s}^2)[\cos 20° + (0.300)\sin 20°] - (0.300)(9.80\ \text{m/s}^2) \\ &= 2.17\ \text{m/s}^2. \end{aligned}$$

11. **THINK** Since the crate is being pulled by a rope at an angle with the horizontal, we need to analyze the force components in both the x and y-directions.

EXPRESS The free-body diagram for the crate is shown to the right. Here $\vec{T}$ is the tension force of the rope on the crate, $\vec{F}_N$ is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. We assume the crate is motionless.



The equations for the x and the y components of the force according to Newton's second law are:

$$T\cos\theta - f = 0, \quad T\sin\theta + F_N - mg = 0$$

where $\theta = 15°$ is the angle between the rope and the horizontal. The first equation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s F_N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have $T\cos\theta = \mu_s(mg - T\sin\theta)$.

ANALYZE (a) We solve for the tension:

$$T=\frac{\mu_s mg}{\cos\theta+\mu_s\sin\theta}=\frac{(0.50)\,(68\text{ kg})\,(9.8\text{ m/s}^2)}{\cos 15°+0.50\sin 15°}=304\text{ N}\approx 3.0\times10^2\text{ N}.$$

(b) The second law equations for the moving crate are

$$T\cos\theta-f=ma,\qquad T\sin\theta+F_N-mg=0\,.$$

Now $f=\mu_k F_N$, and the second equation above gives $F_N=mg-T\sin\theta$, which then yields $f=\mu_k(mg-T\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$T\cos\theta-\mu_k(mg-T\sin\theta)=ma,$$

so the acceleration is

$$\begin{aligned}a&=\frac{T(\cos\theta+\mu_k\sin\theta)}{m}-\mu_k g\\&=\frac{(304\text{ N})(\cos 15°+0.35\sin 15°)}{68\text{ kg}}-(0.35)(9.8\text{ m/s}^2)=1.3\text{ m/s}^2.\end{aligned}$$

LEARN Let's check the limit where $\theta=0$. In this case, we recover the familiar expressions: $T=\mu_s mg$ and $a=(T-\mu_k mg)/m$.

12. There is no acceleration, so the (upward) static friction forces (there are four of them, one for each thumb and one for each set of opposing fingers) equals the magnitude of the (downward) pull of gravity. Using Eq. 6-1, we have

$$4\mu_s F_N=mg=(79\text{ kg})(9.8\text{ m/s}^2)$$

which, with μ_s = 0.70, yields $F_N = 2.8\times10^2$ N.

13. We denote the magnitude of 110 N force exerted by the worker on the crate as F. The magnitude of the static frictional force can vary between zero and $f_{s,\max}=\mu_s F_N$.

(a) In this case, application of Newton's second law in the vertical direction yields $F_N=mg$. Thus,

$$f_{s,\max}=\mu_s F_N=\mu_s mg=(0.37)(35\text{kg})(9.8\text{m}/\text{s}^2)=1.3\times10^2\text{ N}$$

which is greater than F.

(b) The block, which is initially at rest, stays at rest since $F < f_{s,\max}$. Thus, it does not move.

(c) By applying Newton's second law to the horizontal direction, that the magnitude of the frictional force exerted on the crate is $f_s=1.1\times10^2$ N.

(d) Denoting the upward force exerted by the second worker as F_2, then application of Newton's second law in the vertical direction yields $F_N = mg - F_2$, which leads to

$$f_{s,\max} = \mu_s F_N = \mu_s (mg - F_2) .$$

In order to move the crate, F must satisfy the condition $F > f_{s,\max} = \mu_s\ (mg - F_2)$,
or

$$110\text{N} > \left(0.37\right)\left[(35\text{kg})(9.8\text{m/s}^2) - F_2\right].$$

The minimum value of F_2 that satisfies this inequality is a value slightly bigger than 45.7 N, so we express our answer as $F_{2,\min} = 46$ N.

(e) In this final case, moving the crate requires a greater horizontal push from the worker than static friction (as computed in part (a)) can resist. Thus, Newton's law in the horizontal direction leads to

$$F + F_2 > f_{s,\max} \quad \Rightarrow \quad 110\text{ N} + F_2 > 126.9\text{ N}$$

which leads (after appropriate rounding) to $F_{2,\min} = 17$ N.

14. (a) Using the result obtained in Sample Problem – "Friction, applied force at an angle," the maximum angle for which static friction applies is

$$\theta_{\max} = \tan^{-1}\mu_s = \tan^{-1} 0.63 \approx 32° .$$

This is greater than the dip angle in the problem, so the block does not slide.

(b) Applying Newton's second law, we have

$$F + mg\sin\theta - f_{s,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0.$$

Along with Eq. 6-1 ($f_{s,\max} = \mu_s F_N$) we have enough information to solve for F. With $\theta = 24°$ and $m = 1.8 \times 10^7$ kg, we find

$$F = mg\left(\mu_s\cos\theta - \sin\theta\right) = 3.0\times10^7\text{ N}.$$

15. An excellent discussion and equation development related to this problem is given in Sample Problem – "Friction, applied force at an angle." We merely quote (and apply) their main result:

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.04 \approx 2° .$$

16. (a) In this situation, we take $\vec{f}_s$ to point uphill and to be equal to its maximum value, in which case $f_{s,\max} = \mu_s F_N$ applies, where μ_s = 0.25. Applying Newton's second law to the block of mass $m = W/g$ = 8.2 kg, in the x and y directions, produces

$$F_{\min 1} - mg\sin\theta + f_{s,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

which (with θ = 20°) leads to

$$F_{\min 1} - mg\left(\sin\theta + \mu_s \cos\theta\right) = 8.6 \text{ N}.$$

(b) Now we take $\vec{f}_s$ to point downhill and to be equal to its maximum value, in which case $f_{s,\max} = \mu_s F_N$ applies, where μ_s = 0.25. Applying Newton's second law to the block of mass $m = W/g$ = 8.2 kg, in the x and y directions, produces

$$F_{\min 2} = mg\sin\theta - f_{s,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

which (with θ = 20°) leads to

$$F_{\min 2} = mg\left(\sin\theta + \mu_s \cos\theta\right) = 46 \text{ N}.$$

A value slightly larger than the "exact" result of this calculation is required to make it accelerate uphill, but since we quote our results here to two significant figures, 46 N is a "good enough" answer.

(c) Finally, we are dealing with kinetic friction (pointing downhill), so that

$$0 = F - mg\sin\theta - f_k = ma$$
$$0 = F_N - mg\cos\theta$$

along with $f_k = \mu_k F_N$ (where μ_k = 0.15) brings us to

$$F = mg\left(\sin\theta + \mu_k \cos\theta\right) = 39 \text{ N}.$$

17. If the block is sliding then we compute the kinetic friction from Eq. 6-2; if it is not sliding, then we determine the extent of static friction from applying Newton's law, with zero acceleration, to the x axis (which is parallel to the incline surface). The question of whether or not it is sliding is therefore crucial, and depends on the maximum static friction force, as calculated from Eq. 6-1. The forces are resolved in the incline plane coordinate system in Figure 6-5 in the textbook. The acceleration, if there is any, is along the x axis, and we are taking uphill as $+x$. The net force along the y axis, then, is certainly zero, which provides the following relationship:

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

where $W = mg = 45$ N is the weight of the block, and $\theta = 15°$ is the incline angle. Thus, $F_N = 43.5$ N, which implies that the maximum static friction force should be

$$f_{s,\max} = (0.50)\,(43.5\text{ N}) = 21.7\text{ N}.$$

(a) For $\vec{P} = (-5.0\text{ N})\hat{\text{i}}$, Newton's second law, applied to the x axis becomes

$$f - |P| - mg\sin\theta = ma\,.$$

Here we are assuming $\vec{f}$ is pointing uphill, as shown in Figure 6-5, and if it turns out that it points downhill (which *is* a possibility), then the result for f_s will be negative. If $f = f_s$ then $a = 0$, we obtain

$$f_s = |P| + mg\sin\theta = 5.0\text{ N} + (43.5\text{ N})\sin 15° = 17\text{ N},$$

or $\vec{f}_s = (17\text{ N})\hat{\text{i}}$. This is clearly allowed since f_s is less than $f_{s,\max}$.

(b) For $\vec{P} = (-8.0\text{ N})\hat{\text{i}}$, we obtain (from the same equation) $\vec{f}_s = (20\text{ N})\hat{\text{i}}$, which is still allowed since it is less than $f_{s,\max}$.

(c) But for $\vec{P} = (-15\text{ N})\hat{\text{i}}$, we obtain (from the same equation) $f_s = 27$ N, which is not allowed since it is larger than $f_{s,\max}$. Thus, we conclude that it is the kinetic friction instead of the static friction that is relevant in this case. The result is

$$\vec{f}_k = \mu_k F_N \hat{\text{i}} = (0.34)(43.5\text{ N})\,\hat{\text{i}} = (15\text{ N})\,\hat{\text{i}}\,.$$

18. (a) We apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$

where, using Eq. 6-11,

$$f = f_k = \mu_k F_N = \mu_k\, mg\cos\theta\,.$$

Thus, with $\mu_k = 0.600$, we have

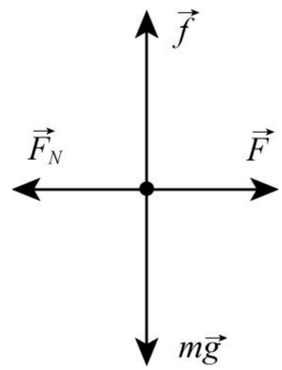
$$a = g\sin\theta - \mu_k\cos\theta = -3.72\text{ m/s}^2$$

which means, since we have chosen the positive direction in the direction of motion (down the slope) then the acceleration vector points "uphill"; it is decelerating. With $v_0 = 18.0\text{ m/s}$ and $\Delta x = d = 24.0$ m, Eq. 2-16 leads to

$$v = \sqrt{v_0^2 + 2ad} = 12.1 \text{ m/s}.$$

(b) In this case, we find $a = +1.1$ m/s^2, and the speed (when impact occurs) is 19.4 m/s.

19. (a) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{F}_N$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and $m\vec{g}$ is the force of gravity. To determine if the block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal force of the wall on the block. We compare f and $\mu_s F_N$. If $f < \mu_s F_N$, the block does not slide on the wall but if $f > \mu_s F_N$, the block does slide. The horizontal component of Newton's second law is $F - F_N = 0$, so $F_N = F = 12$ N and



$$\mu_s F_N = (0.60)(12 \text{ N}) = 7.2 \text{ N}.$$

The vertical component is $f - mg = 0$, so $f = mg = 5.0$ N. Since $f < \mu_s F_N$ the block does not slide.

(b) Since the block does not move $f = 5.0$ N and $F_N = 12$ N. The force of the wall on the block is

$$\vec{F}_w = -F_N\,\hat{\text{i}} + f\,\hat{\text{j}} = -(12\text{N})\,\hat{\text{i}} + (5.0\text{N})\,\hat{\text{j}}$$

where the axes are as shown on Fig. 6-26 of the text.

20. Treating the two boxes as a single system of total mass $m_C + m_W = 1.0 + 3.0 = 4.0$ kg, subject to a total (leftward) friction of magnitude 2.0 N + 4.0 N = 6.0 N, we apply Newton's second law (with $+x$ rightward):

$$F - f_{\text{total}} = m_{\text{total}}\, a \quad \Rightarrow \quad 12.0 \text{ N} - 6.0 \text{ N} = (4.0 \text{ kg})a$$

which yields the acceleration $a = 1.5$ m/s^2. We have treated F as if it were known to the nearest tenth of a Newton so that our acceleration is "good" to two significant figures. Turning our attention to the larger box (the Wheaties box of mass $m_W = 3.0$ kg) we apply Newton's second law to find the contact force F' exerted by the Cheerios box on it.

$$F' - f_W = m_W a \quad \Rightarrow \quad F' - 4.0 \text{ N} = (3.0 \text{ kg})(1.5 \text{ m/s}^2).$$

From the above equation, we find the contact force to be $F' = 8.5$ N.

21. Fig. 6-4 in the textbook shows a similar situation (using ϕ for the unknown angle) along with a free-body diagram. We use the same coordinate system as in that figure.

(a) Thus, Newton's second law leads to

$$x: \quad T\cos\phi - f = ma$$
$$y: \quad T\sin\phi + F_N - mg = 0$$

Setting a = 0 and $f = f_{s,\max} = \mu_s F_N$, we solve for the mass of the box-and-sand (as a function of angle):

$$m = \frac{T}{g}\left(\sin\phi + \frac{\cos\phi}{\mu_s}\right)$$

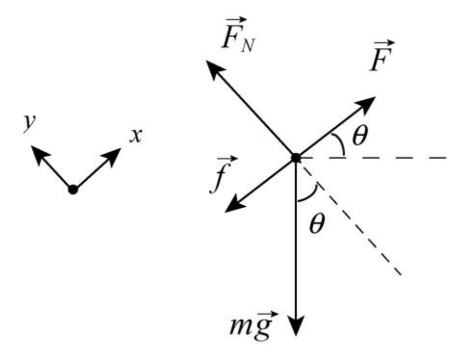
which we will solve with calculus techniques (to find the angle ϕ_m corresponding to the maximum mass that can be pulled).

$$\frac{dm}{dt} = \frac{T}{g}\left(\cos\phi_m - \frac{\sin\phi_m}{\mu_s}\right) = 0$$

This leads to tan $\phi_m = \mu_s$ which (for $\mu_s = 0.35$) yields $\phi_m = 19°$.

(b) Plugging our value for ϕ_m into the equation we found for the mass of the box-and-sand yields m = 340 kg. This corresponds to a weight of $mg = 3.3 \times 10^3$ N.

22. The free-body diagram for the sled is shown below, with $\vec{F}$ being the force applied to the sled, $\vec{F}_N$ the normal force of the inclined plane on the sled, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction.



We take the +x direction to be along the inclined plane and the +y direction to be in its normal direction. The equations for the x and the y components of the force according to Newton's second law are:

$$F_x = F - f - mg\sin\theta = ma = 0$$
$$F_y = F_N - mg\cos\theta = 0$$

Now $f = \mu F_N$, and the second equation gives $F_N = mg\cos\theta$, which yields $f = \mu mg\cos\theta$. This expression is substituted for f in the first equation to obtain

$$F = mg(\sin\theta + \mu\cos\theta)$$

From the figure, we see that $F = 2.0$ N when $\mu = 0$. This implies $mg\sin\theta = 2.0$ N. Similarly, we also find $F = 5.0$ N when $\mu = 0.5$:

$$5.0\text{ N} = mg(\sin\theta + 0.50\cos\theta) = 2.0\text{ N} + 0.50mg\cos\theta$$

which yields $mg\cos\theta = 6.0$ N. Combining the two results, we get

$$\tan\theta = \frac{2}{6} = \frac{1}{3} \quad \Rightarrow \quad \theta = 18°.$$

23. Let the tensions on the strings connecting m_2 and m_3 be T_{23}, and that connecting m_2 and m_1 be T_{12}, respectively. Applying Newton's second law (and Eq. 6-2, with $F_N = m_2g$ in this case) to the *system* we have

$$\begin{aligned} m_3g - T_{23} &= m_3a \\ T_{23} - \mu_k m_2 g - T_{12} &= m_2a \\ T_{12} - m_1g &= m_1a \end{aligned}$$

Adding up the three equations and using $m_1 = M, m_2 = m_3 = 2M$, we obtain

$$2Mg - 2\mu_k Mg - Mg = 5Ma .$$

With a = 0.500 m/s^2 this yields μ_k = 0.372. Thus, the coefficient of kinetic friction is roughly μ_k = 0.37.

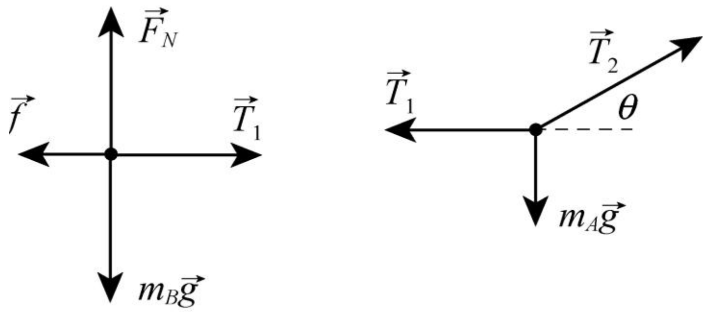
24. We find the acceleration from the slope of the graph (recall Eq. 2-11): a = 4.5 m/s^2. Thus, Newton's second law leads to

$$F - \mu_k mg = ma,$$

where F = 40.0 N is the constant horizontal force applied. With m = 4.1 kg, we arrive at μ_k =0.54.

25. **THINK** In order that the two blocks remain in equilibrium, friction must be present between block B and the surface.

EXPRESS The free-body diagrams for block B and for the knot just above block A are shown below. $\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle θ = 30°) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block B, $\vec{F}_N$ is normal force exerted by the surface on block B, W_A is the weight of block A (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block B (W_B = 711 N is the magnitude of $m_B\vec{g}$).



For each object we take +x horizontally rightward and +y upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{s,\max} &= 0 \\ F_N - W_B &= 0 \\ T_2 \cos\theta - T_1 &= 0 \\ T_2 \sin\theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). The above equations yield $T_1 = \mu_s F_N$, $F_N = W_B$ and $T_1 = T_2 \cos\theta$.

ANALYZE Solving these equations with $\mu_s = 0.25$, we obtain

$$\begin{aligned} W_A &= T_2 \sin\theta = T_1 \tan\theta = \mu_s F_N \tan\theta = \mu_s W_B \tan\theta \\ &= (0.25)(711\,\text{N})\tan 30° = 1.0\times10^2\ \text{N} \end{aligned}$$

LEARN As expected, the maximum weight of A is proportional to the weight of B, as well as the coefficient of static friction. In addition, we see that W_A is proportional to $\tan\theta$ (the larger the angle, the greater the vertical component of T_2 that supports its weight).

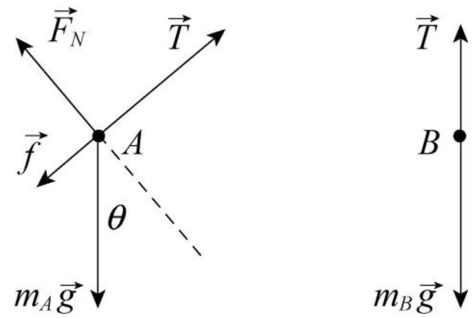
26. (a) Applying Newton's second law to the *system* (of total mass M = 60.0 kg) and using Eq. 6-2 (with $F_N = Mg$ in this case) we obtain

$$F - \mu_k Mg = Ma \ \Rightarrow\ a = 0.473\ \text{m/s}^2.$$

Next, we examine the forces just on m_3 and find $F_{32} = m_3(a + \mu_k g) = 147$ N. If the algebra steps are done more systematically, one ends up with the interesting relationship: $F_{32} = (m_3 / M)F$ (which is independent of the friction!).

(b) As remarked at the end of our solution to part (a), the result does not depend on the frictional parameters. The answer here is the same as in part (a).

27. First, we check to see if the bodies start to move. We assume they remain at rest and compute the force of (static) friction which holds them there, and compare its magnitude with the maximum value $\mu_s F_N$. The free-body diagrams are shown below.



T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on body A, F_N is the magnitude of the normal force of the plane on body A, $m_A\vec{g}$ is the force of gravity on body A (with magnitude W_A = 102 N), and $m_B\vec{g}$ is the force of gravity on body B (with magnitude W_B = 32 N). θ = 40° is the angle of incline. We are told the direction of $\vec{f}$ but we assume it is downhill. If we obtain a negative result for f, then we know the force is actually up the plane.

(a) For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force. The x and y components of Newton's second law become

$$T - f - W_A \sin\theta = 0$$
$$F_N - W_A \cos\theta = 0.$$

Taking the positive direction to be *downward* for body B, Newton's second law leads to $W_B - T = 0$. Solving these three equations leads to

$$f = W_B - W_A \sin\theta = 32 \text{ N} - (102 \text{ N}) \sin 40° = -34 \text{ N}$$

(indicating that the force of friction is *uphill*) and to

$$F_N = W_A \cos\theta = (102 \text{ N}) \cos 40° = 78\text{N}$$

which means that

$$f_{s,\max} = \mu_s F_N = (0.56)\,(78 \text{ N}) = 44 \text{ N}.$$

Since the magnitude f of the force of friction that holds the bodies motionless is less than $f_{s,\max}$ the bodies remain at rest. The acceleration is zero.

(b) Since A is moving up the incline, the force of friction is downhill with magnitude $f_k = \mu_k F_N$. Newton's second law, using the same coordinates as in part (a), leads to

$$T - f_k - W_A \sin\theta = m_A a$$
$$F_N - W_A \cos\theta = 0$$
$$W_B - T = m_B a$$

for the two bodies. We solve for the acceleration:

$$a = \frac{W_B - W_A \sin\theta - \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N})\sin 40° - (0.25)(102\text{N})\cos 40°}{(32\text{N}+102\text{N})\big/\left(9.8\ \text{m/s}^2\right)}$$
$$= -3.9\ \text{m/s}^2.$$

The acceleration is down the plane, i.e., $\vec{a} = (-3.9\ \text{m/s}^2)\hat{\text{i}}$, which is to say (since the initial velocity was uphill) that the objects are slowing down. We note that $m = W/g$ has been used to calculate the masses in the calculation above.

(c) Now body A is initially moving down the plane, so the force of friction is uphill with magnitude $f_k = \mu_k F_N$. The force equations become

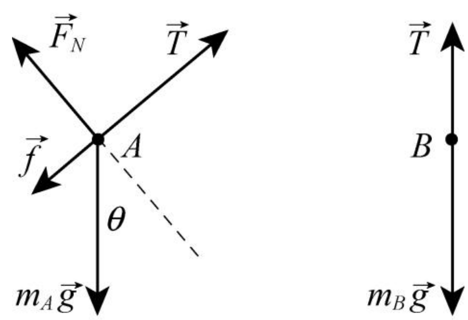
$$T + f_k - W_A \sin\theta = m_A a$$
$$F_N - W_A \cos\theta = 0$$
$$W_B - T = m_B a$$

which we solve to obtain

$$a = \frac{W_B - W_A \sin\theta + \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N})\sin 40° + (0.25)(102\text{N})\cos 40°}{(32\text{N}+102\text{N})\big/\left(9.8\ \text{m/s}^2\right)}$$
$$= -1.0\ \text{m/s}^2.$$

The acceleration is again downhill the plane, i.e., $\vec{a} = (-1.0\ \text{m/s}^2)\hat{\text{i}}$. In this case, the objects are speeding up.

28. The free-body diagrams are shown to the right, where T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on block A, F_N is the magnitude of the normal force of the plane on block A, $m_A\vec{g}$ is the force of gravity on body A (where m_A = 10 kg), and $m_B\vec{g}$ is the force of gravity on block B. θ = 30° is the angle of incline. For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force; the positive direction is chosen *downward* for block B.



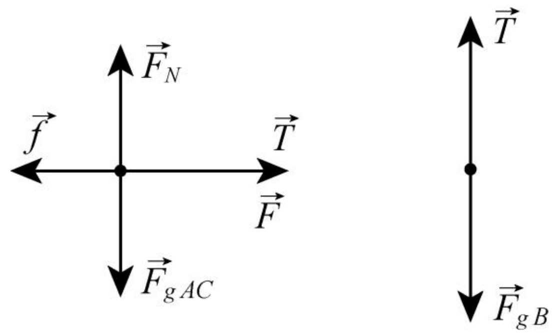
Since A is moving down the incline, the force of friction is uphill with magnitude $f_k = \mu_k F_N$ (where μ_k = 0.20). Newton's second law leads to

$$T - f_k + m_A g \sin\theta = m_A a = 0$$
$$F_N - m_A g \cos\theta = 0$$
$$m_B g - T = m_B a = 0$$

for the two bodies (where a = 0 is a consequence of the velocity being constant). We solve these for the mass of block B.

$$m_B = m_A(\sin\theta - \mu_k \cos\theta) = 3.3 \text{ kg.}$$

29. (a) Free-body diagrams for the blocks A and C, considered as a single object, and for the block B are shown below.



T is the magnitude of the tension force of the rope, F_N is the magnitude of the normal force of the table on block A, f is the magnitude of the force of friction, W_{AC} is the combined weight of blocks A and C (the magnitude of force $\vec{F}_{g\,AC}$ shown in the figure), and W_B is the weight of block B (the magnitude of force $\vec{F}_{g\,B}$ shown). Assume the blocks are not moving. For the blocks on the table we take the x axis to be to the right and the y axis to be upward. From Newton's second law, we have

$$x \text{ component:} \quad T - f = 0$$
$$y \text{ component:} \quad F_N - W_{AC} = 0.$$

For block B take the downward direction to be positive. Then Newton's second law for that block is $W_B - T = 0$. The third equation gives $T = W_B$ and the first gives $f = T = W_B$. The second equation gives $F_N = W_{AC}$. If sliding is not to occur, f must be less than $\mu_s F_N$, or $W_B < \mu_s W_{AC}$. The smallest that W_{AC} can be with the blocks still at rest is

$$W_{AC} = W_B/\mu_s = (22 \text{ N})/(0.20) = 110 \text{ N.}$$

Since the weight of block A is 44 N, the least weight for C is (110 – 44) N = 66 N.

(b) The second law equations become

$$T - f = (W_A/g)a$$

$$F_N - W_A = 0$$
$$W_B - T = (W_B/g)a.$$

In addition, $f = \mu_k F_N$. The second equation gives $F_N = W_A$, so $f = \mu_k W_A$. The third gives $T = W_B - (W_B/g)a$. Substituting these two expressions into the first equation, we obtain

$$W_B - (W_B/g)a - \mu_k W_A = (W_A/g)a.$$

Therefore,

$$a = \frac{g\left(W_B - \mu_k W_A\right)}{W_A + W_B} = \frac{(9.8 \text{ m/s}^2)\left(22 \text{ N} - (0.15)(44 \text{ N})\right)}{44 \text{ N} + 22 \text{ N}} = 2.3 \text{ m/s}^2.$$

30. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y-axis, where there is presumed to be no acceleration, leads to

$$F_N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\max}$ is assumed, then Newton's second law applied to the x axis (which also has $a = 0$ even though it is "verging" on moving) yields

$$F\cos\theta - f_s = ma \quad \Rightarrow \quad F\cos\theta - \mu_s(mg - F\sin\theta) = 0$$

which we solve, for $\theta = 42°$ and $\mu_s = 0.42$, to obtain $F = 74$ N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s \sin\theta} = \frac{(0.42)(180 \text{ N})}{\cos\theta + (0.42)\sin\theta} = \frac{76 \text{ N}}{\cos\theta + (0.42)\sin\theta}.$$

(c) We minimize the above expression for F by working through the condition:

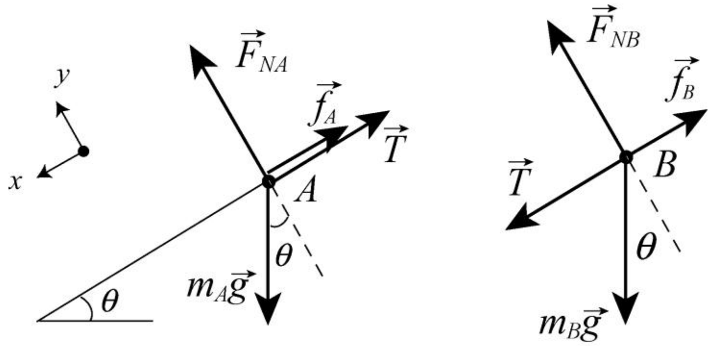
$$\frac{dF}{d\theta} = \frac{\mu_s W(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s \sin\theta)^2} = 0$$

which leads to the result $\theta = \tan^{-1}\mu_s = 23°$.

(d) Plugging $\theta = 23°$ into the above result for F, with $\mu_s = 0.42$ and $W = 180$ N, yields $F = 70$ N.

31. **THINK** In this problem we have two blocks connected by a string sliding down an inclined plane; the blocks have different coefficient of kinetic friction.

EXPRESS The free-body diagrams for the two blocks are shown below. T is the magnitude of the tension force of the string, $\vec{F}_{NA}$ is the normal force on block A (the leading block), $\vec{F}_{NB}$ is the normal force on block B, $\vec{f}_A$ is kinetic friction force on block A, $\vec{f}_B$ is kinetic friction force on block B. Also, m_A is the mass of block A (where $m_A = W_A/g$ and $W_A = 3.6$ N), and m_B is the mass of block B (where $m_B = W_B/g$ and $W_B = 7.2$ N). The angle of the incline is $\theta = 30°$.



For each block we take $+x$ downhill (which is toward the lower-left in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of both blocks A and B, we arrive at four equations:

$$W_A \sin\theta - f_A - T = m_A a$$
$$F_{NA} - W_A \cos\theta = 0$$
$$W_B \sin\theta - f_B + T = m_B a$$
$$F_{NB} - W_B \cos\theta = 0$$

which, when combined with Eq. 6-2 ($f_A = \mu_{kA} F_{NA}$ where $\mu_{k\,A}$ = 0.10 and $f_B = \mu_{kB} F_{NB} f_B$ where $\mu_{k\,B}$ = 0.20), fully describe the dynamics of the system so long as the blocks have the same acceleration and $T > 0$.

ANALYZE (a) From these equations, we find the acceleration to be

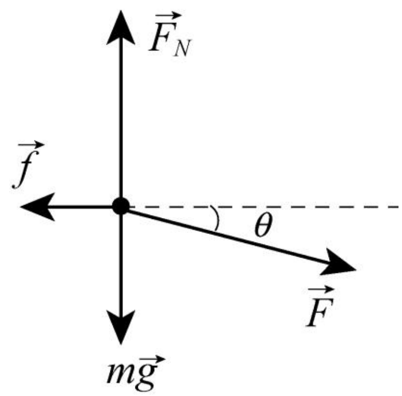
$$a = g\left(\sin\theta - \left(\frac{\mu_{k\,A}W_A + \mu_{k\,B}W_B}{W_A + W_B}\right)\cos\theta\right) = 3.5\ \text{m/s}^2.$$

(b) We solve the above equations for the tension and obtain

$$T = \left(\frac{W_A W_B}{W_A + W_B}\right)\ (\mu_{k\,B} - \mu_{k\,A})\cos\theta = \frac{(3.6\ \text{N})(7.2\ \text{N})}{3.6\ \text{N} + 7.2\ \text{N}}(0.20 - 0.10)\cos 30° = 0.21\ \text{N}.$$

LEARN The tension in the string is proportional to $\mu_{kB}-\mu_{kA}$, the difference in coefficients of kinetic friction for the two blocks. When the coefficients are equal ($\mu_{kB}=\mu_{kA}$), the two blocks can be viewed as moving independent of one another and the tension is zero. Similarly, when $\mu_{kB}<\mu_{kA}$ (the leading block *A* has larger coefficient than the *B*), the string is slack, so the tension is also zero.

32. The free-body diagram for the block is shown below, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the +*x* direction to be horizontal to the right and the +*y* direction to be up. The equations for the *x* and the *y* components of the force according to Newton's second law are:



$$F_x = F\cos\theta - f = ma$$
$$F_y = F_N - F\sin\theta - mg = 0$$

Now $f=\mu_k F_N$, and the second equation gives $F_N = mg + F\sin\theta$, which yields

$$f=\mu_k(mg+F\sin\theta).$$

This expression is substituted for *f* in the first equation to obtain

$$F\cos\theta - \mu_k(mg + F\sin\theta) = ma,$$

so the acceleration is

$$a=\frac{F}{m}\left(\cos\theta-\mu_k\sin\theta\right)-\mu_k g.$$

From the figure, we see that $a=3.0\text{ m/s}^2$ when $\mu_k=0$. This implies

$$3.0\text{ m/s}^2 = \frac{F}{m}\cos\theta.$$

We also find $a=0$ when $\mu_k=0.20$:

$$0=\frac{F}{m}\left(\cos\theta-(0.20)\sin\theta\right)-(0.20)(9.8\text{ m/s}^2)=3.00\text{ m/s}^2-0.20\frac{F}{m}\sin\theta-1.96\text{ m/s}^2$$
$$=1.04\text{ m/s}^2-0.20\frac{F}{m}\sin\theta$$

which yields $5.2\text{ m/s}^2=\dfrac{F}{m}\sin\theta.$ Combining the two results, we get

$$\tan\theta = \left(\frac{5.2\ \text{m/s}^2}{3.0\ \text{m/s}^2}\right) = 1.73 \quad \Rightarrow \quad \theta = 60°.$$

33. **THINK** In this problem, the frictional force is not a constant, but instead proportional to the speed of the boat. Integration is required to solve for the speed.

EXPRESS We denote the magnitude of the frictional force as αv, where $\alpha = 70\ \text{N}\cdot\text{s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{v} = -\frac{\alpha}{m}dt.$$

Integrating the equation gives

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

where v_0 is the velocity at time zero and v is the velocity at time t. Solving the integral allows us to calculate the time it takes for the boat to slow down to 45 km/h, or $v = v_0/2$, where $v_0 = 90\ \text{km/h}$.

ANALYZE The integrals are evaluated with the result

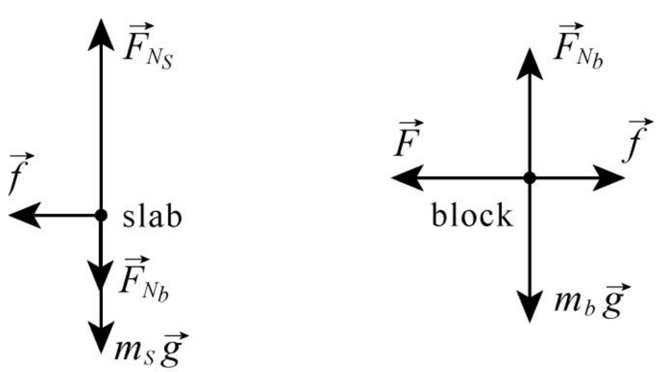
$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

With $v = v_0/2$ and $m = 1000$ kg, we find the time to be

$$t = -\frac{m}{\alpha}\ln\left(\frac{v}{v_0}\right) = -\frac{m}{\alpha}\ln\left(\frac{1}{2}\right) = -\frac{1000\ \text{kg}}{70\ \text{N}\cdot\text{s/m}}\ln\left(\frac{1}{2}\right) = 9.9\ \text{s}.$$

LEARN The speed of the boat is given by $v(t) = v_0 e^{-\alpha t/m}$, showing exponential decay with time. The greater the value of α, the more rapidly the boat slows down.

34. The free-body diagrams for the slab and block are shown below.



$\vec{F}$ is the 100 N force applied to the block, $\vec{F}_{Ns}$ is the normal force of the floor on the slab, F_{Nb} is the magnitude of the normal force between the slab and the block, $\vec{f}$ is the force of friction between the slab and the block, m_s is the mass of the slab, and m_b is the mass of the block. For both objects, we take the $+x$ direction to be to the right and the $+y$ direction to be up.

Applying Newton's second law for the x and y axes for (first) the slab and (second) the block results in four equations:

$$
\begin{aligned}
-f &= m_s a_s \\
F_{Ns} - F_{Ns} - m_s g &= 0 \\
f - F &= m_b a_b \\
F_{Nb} - m_b g &= 0
\end{aligned}
$$

from which we note that the maximum possible static friction magnitude would be

$$\mu_s F_{Nb} = \mu_s m_b g = (0.60)(10\ \text{kg})(9.8\ \text{m/s}^2) = 59\ \text{N}\ .$$

We check to see if the block slides on the slab. Assuming it does not, then $a_s = a_b$ (which we denote simply as a) and we solve for f:

$$f = \frac{m_s F}{m_s + m_b} = \frac{(40\ \text{kg})(100\ \text{N})}{40\ \text{kg} + 10\ \text{kg}} = 80\ \text{N}$$

which is greater than $f_{s,\max}$ so that we conclude the block is sliding across the slab (their accelerations are different).

(a) Using $f = \mu_k F_{Nb}$ the above equations yield

$$a_b = \frac{\mu_k m_b g - F}{m_b} = \frac{(0.40)(10\ \text{kg})(9.8\ \text{m/s}^2) - 100\ \text{N}}{10\ \text{kg}} = -6.1\ \text{m/s}^2.$$

The negative sign means that the acceleration is leftward. That is, $\vec{a}_b = (-6.1\ \text{m/s}^2)\hat{\text{i}}$

(b) We also obtain

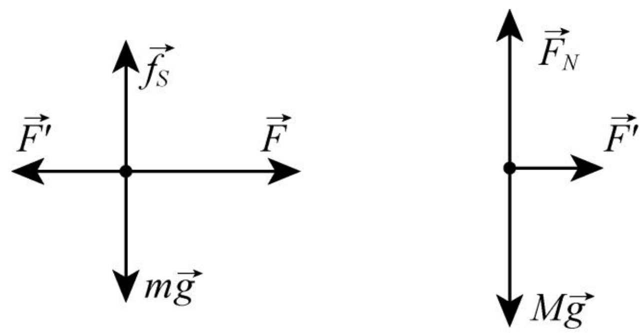
$$a_s = -\frac{\mu_k m_b g}{m_s} = -\frac{(0.40)(10\ \text{kg})(9.8\ \text{m/s}^2)}{40\ \text{kg}} = -0.98\ \text{m/s}^2.$$

As mentioned above, this means it accelerates to the left. That is, $\vec{a}_s = (-0.98\ \text{m/s}^2)\hat{\text{i}}$

35. The free-body diagrams for the two blocks, treated individually, are shown below (first m and then M). F' is the contact force between the two blocks, and the static friction force $\vec{f}_s$ is at its maximum value (so Eq. 6-1 leads to $f_s = f_{s,\max} = \mu_s F'$ where $\mu_s = 0.38$).

Treating the two blocks together as a single system (sliding across a frictionless floor), we apply Newton's second law (with $+x$ rightward) to find an expression for the acceleration:

$$F = m_{\text{total}}\, a \quad \Rightarrow a = \frac{F}{m+M}$$



This is equivalent to having analyzed the two blocks individually and then combined their equations. Now, when we analyze the small block individually, we apply Newton's second law to the x and y axes, substitute in the above expression for a, and use Eq. 6-1.

$$F - F' = ma \quad \Rightarrow \quad F' = F - m\left(\frac{F}{m+M}\right)$$
$$f_s - mg = 0 \quad \Rightarrow \quad \mu_s F' - mg = 0$$

These expressions are combined (to eliminate F') and we arrive at

$$F = \frac{mg}{\mu_s\left(1 - \dfrac{m}{m+M}\right)} = 4.9\times10^2\ \text{N}.$$

36. Using Eq. 6-16, we solve for the area $A \frac{2mg}{C\rho v_t^2}$ which illustrates the inverse proportionality between the area and the speed-squared. Thus, when we set up a ratio of areas – of the slower case to the faster case – we obtain

$$\frac{A_{\text{slow}}}{A_{\text{fast}}} = \left(\frac{310\ \text{km/h}}{160\ \text{km/h}}\right)^2 = 3.75.$$

37. In the solution to exercise 4, we found that the force provided by the wind needed to equal F = 157 N (where that last figure is not "significant'').

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v=\sqrt{\frac{2F}{C\rho A}}=\sqrt{\frac{2(157\text{ N})}{(0.80)(1.21\text{ kg/m}^3)(0.040\text{ m}^2)}}=90\text{ m/s }=3.2\times10^2\text{ km/h}.$$

(b) Doubling our previous result, we find the reported speed to be 6.5×10^2 km/h.

(c) The result is not reasonable for a terrestrial storm. A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.

38. (a) From Table 6-1 and Eq. 6-16, we have

$$v_t=\sqrt{\frac{2F_g}{C\rho A}}\;\Rightarrow C\rho A=2\frac{mg}{v_t^2}$$

where v_t = 60 m/s. We estimate the pilot's mass at about m = 70 kg. Now, we convert v = 1300(1000/3600) ≈ 360 m/s and plug into Eq. 6-14:

$$D=\frac{1}{2}C\rho A v^2=\frac{1}{2}\left(2\frac{mg}{v_t^2}\right)v^2=mg\left(\frac{v}{v_t}\right)^2$$

which yields $D = (70\text{ kg})(9.8\text{ m/s}^2)(360/60)^2 \approx 2 \times 10^4$ N.

(b) We assume the mass of the ejection seat is roughly equal to the mass of the pilot. Thus, Newton's second law (in the horizontal direction) applied to this system of mass $2m$ gives the magnitude of acceleration:

$$|a|=\frac{D}{2m}=\frac{g}{2}\left(\frac{v}{v_t}\right)^2=18g\;.$$

39. For the passenger jet $D_j=\frac{1}{2}C\rho_1 A v_j^2$, and for the prop-driven transport $D_t=\frac{1}{2}C\rho_2 A v_t^2$, where ρ_1 and ρ_2 represent the air density at 10 km and 5.0 km, respectively. Thus the ratio in question is

$$\frac{D_j}{D_t}=\frac{\rho_1 v_j^2}{\rho_2 v_t^2}=\frac{(0.38\text{ kg/m}^3)(1000\text{ km/h})^2}{(0.67\text{ kg/m}^3)(500\text{ km/h})^2}=2.3.$$

40. This problem involves Newton's second law for motion along the slope.

(a) The force along the slope is given by

$$\begin{aligned}F_g &= mg\sin\theta - \mu F_N = mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - \mu\cos\theta)\\ &= (85.0\text{ kg})(9.80\text{ m/s}^2)\left[\sin 40.0° - (0.04000)\cos 40.0°\right]\\ &= 510\text{ N}.\end{aligned}$$

Thus, the terminal speed of the skier is

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} = \sqrt{\frac{2(510\text{ N})}{(0.150)(1.20\text{ kg/m}^3)(1.30\text{ m}^2)}} = 66.0\text{ m/s}.$$

(b) Differentiating v_t with respect to C, we obtain

$$\begin{aligned}dv_t &= -\frac{1}{2}\sqrt{\frac{2F_g}{\rho A}}\,C^{-3/2}dC = -\frac{1}{2}\sqrt{\frac{2(510\text{ N})}{(1.20\text{ kg/m}^3)(1.30\text{ m}^2)}}(0.150)^{-3/2}dC\\ &= -(2.20\times10^2\text{ m/s})dC.\end{aligned}$$

41. Perhaps surprisingly, the equations pertaining to this situation are exactly those in Sample Problem – "Car in flat circular turn," although the logic is a little different. In the Sample Problem, the car moves along a (stationary) road, whereas in this problem the cat is stationary relative to the merry-go-around platform. But the static friction plays the same role in both cases since the bottom-most point of the car tire is instantaneously at rest with respect to the race track, just as static friction applies to the contact surface between cat and platform. Using Eq. 6-23 with Eq. 4-35, we find

$$\mu_s = (2\pi R/T)^2/gR = 4\pi^2 R/gT^2.$$

With $T = 6.0$ s and $R = 5.4$ m, we obtain $\mu_s = 0.60$.

42. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the car and m is the mass of the car, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad v \le \sqrt{\mu_s R g}.$$

Consequently, the maximum speed with which the car can round the curve without slipping is

$$v_{\max} = \sqrt{\mu_s R g} = \sqrt{(0.60)(30.5 \text{ m})(9.8 \text{ m/s}^2)} = 13 \text{ m/s} \approx 48 \text{ km/h}.$$

43. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the bicycle does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad R \ge \frac{v^2}{\mu_s g}.$$

Consequently, the minimum radius with which a cyclist moving at 29 km/h = 8.1 m/s can round the curve without slipping is

$$R_{\min} = \frac{v^2}{\mu_s g} = \frac{(8.1 \text{ m/s})^2}{(0.32)(9.8 \text{ m/s}^2)} = 21 \text{ m}.$$

44. With v = 96.6 km/h = 26.8 m/s, Eq. 6-17 readily yields

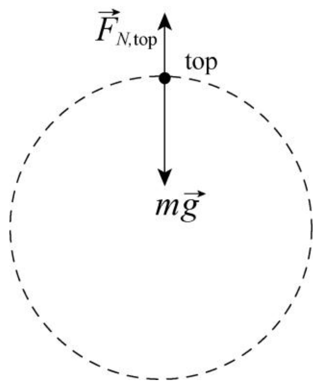
$$a = \frac{v^2}{R} = \frac{(26.8 \text{ m/s})^2}{7.6 \text{ m}} = 94.7 \text{ m/s}^2$$

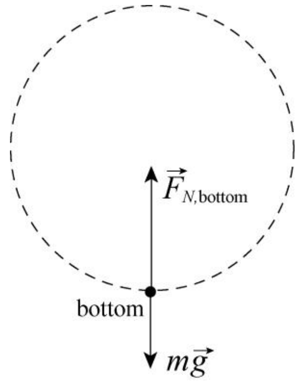
which we express as a multiple of g:

$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7 \text{ m/s}^2}{9.80 \text{ m/s}^2}\right) g = 9.7g.$$

45. **THINK** Ferris wheel ride is a vertical circular motion. The apparent weight of the rider varies with his position.

EXPRESS The free-body diagrams of the student at the top and bottom of the Ferris wheel are shown next:





At the top (the highest point in the circular motion) the seat pushes up on the student with a force of magnitude $F_{N,\text{top}}$, while the Earth pulls down with a force of magnitude mg. Newton's second law for the radial direction gives

$$mg - F_{N,\text{top}} = \frac{mv^2}{R}.$$

At the bottom of the ride, $F_{N,\text{bottom}}$ is the magnitude of the upward force exerted by the seat. The net force toward the center of the circle is (choosing upward as the positive direction):

$$F_{N,\text{bottom}} - mg = \frac{mv^2}{R}.$$

The Ferris wheel is "steadily rotating" so the value $F_c = mv^2/R$ is the same everywhere. The apparent weight of the student is given by F_N.

ANALYZE (a) At the top, we are told that $F_{N,\text{top}}$ = 556 N and mg = 667 N. This means that the seat is pushing up with a force that is smaller than the student's weight, and we say the student experiences a decrease in his "apparent weight" at the highest point. Thus, he feels "light."

(b) From (a), we find the centripetal force to be

$$F_c = \frac{mv^2}{R} = mg - F_{N,\text{top}} = 667\ \text{N} - 556\ \text{N} = 111\ \text{N}.$$

Thus, the normal force at the bottom is

$$F_{N,\text{bottom}} = \frac{mv^2}{R} + mg = F_c + mg = 111\ \text{N} + 667\ \text{N} = 778\ \text{N}.$$

(c) If the speed is doubled,

$$F_c' = \frac{m(2v)^2}{R} = 4(111\ \text{N}) = 444\ \text{N}.$$

Therefore, at the highest point we have

$$F'_{N,\text{top}} = mg - F'_c = 667\text{ N} - 444\text{ N} = 223\text{ N}.$$

(d) Similarly, the normal force at the lowest point is now found to be

$$F'_{N,\text{bottom}} = F'_c + mg = 444\text{ N} + 667\text{ N} = 1111\text{ N}.$$

LEARN The apparent weight of the student is the greatest at the bottom and smallest at the top of the ride. The speed $v = \sqrt{gR}$ would result in $F_{N,\text{top}} = 0$, giving the student a sudden sensation of "weightlessness" at the top of the ride.

46. (a) We note that the speed 80.0 km/h in SI units is roughly 22.2 m/s. The horizontal force that keeps her from sliding must equal the centripetal force (Eq. 6-18), and the upward force on her must equal *mg*. Thus,

$$F_{\text{net}} = \sqrt{(mg)^2 + (mv^2/R)^2} = 547\text{ N}.$$

(b) The angle is

$$\tan^{-1}[(mv^2/R)/(mg)] = \tan^{-1}(v^2/gR) = 9.53°$$

as measured from a vertical axis.

47. (a) Eq. 4-35 gives $T = 2\pi R/v = 2\pi(10\text{ m})/(6.1\text{ m/s}) = 10\text{ s}$.

(b) The situation is similar to that of Sample Problem – "Vertical circular loop, Diavolo," but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R) = 486\text{ N} \approx 4.9 \times 10^2\text{ N}.$$

(c) Now we reverse both the normal force direction and the acceleration direction (from what is shown in Sample Problem – "Vertical circular loop, Diavolo") and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 1081\text{ N} \approx 1.1\text{ kN}.$$

48. We will start by assuming that the normal force (on the car from the rail) points up. Note that gravity points down, and the y axis is chosen positive upwards. Also, the direction to the center of the circle (the direction of centripetal acceleration) is down. Thus, Newton's second law leads to

$$F_N - mg = m\left(-\frac{v^2}{r}\right).$$

(a) When $v = 11$ m/s, we obtain $F_N = 3.7 \times 10^3$ N.

(b) $\vec{F}_N$ points upward.

(c) When v = 14 m/s, we obtain $F_N = -1.3 \times 10^3$ N, or $|F_N| = 1.3 \times 10^3$ N.

(d) The fact that this answer is negative means that $\vec{F}_N$ points opposite to what we had assumed. Thus, the magnitude of $\vec{F}_N$ is $|\vec{F}_N| = 1.3$ kN and its direction is *down*.

49. At the top of the hill, the situation is similar to that of Sample Problem – "Vertical circular loop, Diavolo," but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R).$$

Since $F_N = 0$ there (as stated in the problem) then $v^2 = gR$. Later, at the bottom of the valley, we reverse both the normal force direction and the acceleration direction (from what is shown in the Sample Problem) and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 2mg = 1372 \text{ N} \approx 1.37 \times 10^3 \text{ N}.$$

50. The centripetal force on the passenger is $F = mv^2/r$.

(a) The slope of the plot at $v = 8.30$ m/s is

$$\left.\frac{dF}{dv}\right|_{v=8.30\text{ m/s}} = \left.\frac{2mv}{r}\right|_{v=8.30\text{ m/s}} = \frac{2(85.0\text{ kg})(8.30\text{ m/s})}{3.50\text{ m}} = 403\text{ N}\cdot\text{s/m}.$$

(b) The period of the circular ride is $T = 2\pi r/v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

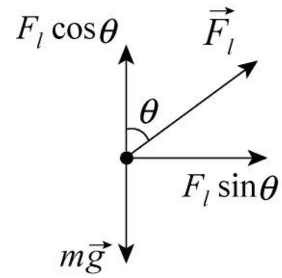
$$dF = -\frac{8\pi^2 mr}{T^3}dT.$$

The slope of the plot at $T = 2.50$ s is

$$\left.\frac{dF}{dT}\right|_{T=2.50\text{ s}} = -\left.\frac{8\pi^2 mr}{T^3}\right|_{T=2.50\text{ s}} = \frac{8\pi^2(85.0\text{ kg})(3.50\text{ m})}{(2.50\text{ s})^3} = -1.50\times 10^3\text{ N/s}.$$

51. **THINK** An airplane with its wings tilted at an angle is in a circular motion. Centripetal force is involved in this problem.

EXPRESS The free-body diagram for the airplane of mass m is shown to the right. We note that $\vec{F}_l$ is the force of aerodynamic lift and $\vec{a}$ points rightwards in the figure. We also note that $|\vec{a}| = v^2/R$. Applying Newton's law to the axes of the problem (+x rightward and +y upward) we obtain



$$F_l \sin\theta = m\frac{v^2}{R}$$
$$F_l \cos\theta = mg$$

Eliminating mass from these equations leads to $\tan\theta = \frac{v^2}{gR}$. The equation allows us to solve for the radius R.

ANALYZE With v = 480 km/h = 133 m/s and θ = 40°, we find

$$R = \frac{v^2}{g\tan\theta} = \frac{(133\text{ m/s})^2}{(9.8\text{ m/s}^2)\tan 40°} = 2151\text{ m} \approx 2.2\times10^3\text{ m}.$$

LEARN Our approach to solving this problem is identical to that discussed in the Sample Problem – "Car in banked circular turn." Do you see the similarities?

52. The situation is somewhat similar to that shown in the "loop-the-loop" example done in the textbook (see Figure 6-10) except that, instead of a downward normal force, we are dealing with the force of the boom $\vec{F}_B$ on the car – which is capable of pointing any direction. We will assume it to be upward as we apply Newton's second law to the car (of total weight 5000 N): $F_B - W = ma$ where $m = W/g$ and $a = -v^2/r$. Note that the centripetal acceleration is downward (our choice for negative direction) for a body at the top of its circular trajectory.

(a) If r = 10 m and v = 5.0 m/s, we obtain $F_B = 3.7 \times 10^3$ N = 3.7 kN.

(b) The direction of $\vec{F}_B$ is up.

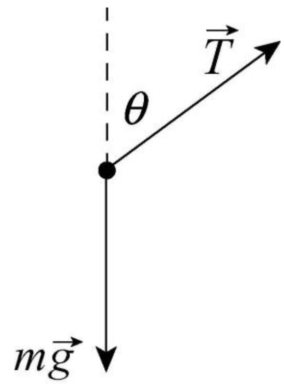
(c) If r = 10 m and v = 12 m/s, we obtain $F_B = -2.3 \times 10^3$ N = – 2.3 kN, or $|F_B|$ = 2.3 kN.

(d) The minus sign indicates that $\vec{F}_B$ points downward.

53. The free-body diagram (for the hand straps of mass m) is the view that a passenger might see if she was looking forward and the streetcar was curving towards the right (so $\vec{a}$ points rightwards in the figure). We note that $|\vec{a}| = v^2/R$ where v = 16 km/h = 4.4 m/s.

Applying Newton's law to the axes of the problem (+x is rightward and +y is upward) we obtain

$$T\sin\theta = m\frac{v^2}{R}$$
$$T\cos\theta = mg.$$



We solve these equations for the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$

which yields $\theta = 12°$.

54. The centripetal force on the passenger is $F = mv^2/r$.

(a) The variation of F with respect to r while holding v constant is $dF = -\frac{mv^2}{r^2}dr$.

(b) The variation of F with respect to v while holding r constant is $dF = \frac{2mv}{r}dv$.

(c) The period of the circular ride is $T = 2\pi r/v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

$$dF = -\frac{8\pi^2 mr}{T^3}dT = -8\pi^2 mr\left(\frac{v}{2\pi r}\right)^3 dT = -\left(\frac{mv^3}{\pi r^2}\right)dT.$$

55. We note that the period T is eight times the time between flashes ($\frac{1}{2000}$ s), so T = 0.0040 s. Combining Eq. 6-18 with Eq. 4-35 leads to

$$F = \frac{4m\pi^2 R}{T^2} = \frac{4(0.030\text{ kg})\pi^2(0.035\text{ m})}{(0.0040\text{ s})^2} = 2.6\times10^3\text{ N}.$$

56. We refer the reader to Sample Problem – "Car in banked circular turn," and use the result Eq. 6-26:

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

with v = 60(1000/3600) = 17 m/s and R = 200 m. The banking angle is therefore $\theta = 8.1°$. Now we consider a vehicle taking this banked curve at v' = 40(1000/3600) = 11 m/s. Its (horizontal) acceleration is $a' = v'^2/R$, which has components parallel the incline and perpendicular to it:

$$a_{||} = a'\cos\theta = \frac{v'^2\cos\theta}{R}$$
$$a_{\perp} = a'\sin\theta = \frac{v'^2\sin\theta}{R}.$$

These enter Newton's second law as follows (choosing downhill as the $+x$ direction and away-from-incline as $+y$):

$$mg\sin\theta - f_s = ma_{||}$$
$$F_N - mg\cos\theta = ma_{\perp}$$

and we are led to

$$\frac{f_s}{F_N} = \frac{mg\sin\theta - mv'^2\cos\theta/R}{mg\cos\theta + mv'^2\sin\theta/R}.$$

We cancel the mass and plug in, obtaining $f_s/F_N = 0.078$. The problem implies we should set $f_s = f_{s,\max}$ so that, by Eq. 6-1, we have $\mu_s = 0.078$.

57. For the puck to remain at rest the magnitude of the tension force T of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})}{1.50\text{ kg}}} = 1.81\text{ m/s}.$$

58. (a) Using the kinematic equation given in Table 2-1, the deceleration of the car is

$$v^2 = v_0^2 + 2ad \quad \Rightarrow \quad 0 = (35\text{ m/s})^2 + 2a(107\text{ m})$$

which gives $a = -5.72\text{ m/s}^2$. Thus, the force of friction required to stop by car is

$$f = m|a| = (1400\text{ kg})(5.72\text{ m/s}^2) \approx 8.0\times10^3\text{ N}.$$

(b) The maximum possible static friction is

$$f_{s,\max} = \mu_s mg = (0.50)(1400\text{ kg})(9.80\text{ m/s}^2) \approx 6.9\times10^3\text{ N}.$$

(c) If $\mu_k = 0.40$, then $f_k = \mu_k mg$ and the deceleration is $a = -\mu_k g$. Therefore, the speed of the car when it hits the wall is

$$v = \sqrt{v_0^2 + 2ad} = \sqrt{(35\text{ m/s})^2 - 2(0.40)(9.8\text{ m/s}^2)(107\text{ m})} \approx 20\text{ m/s}.$$

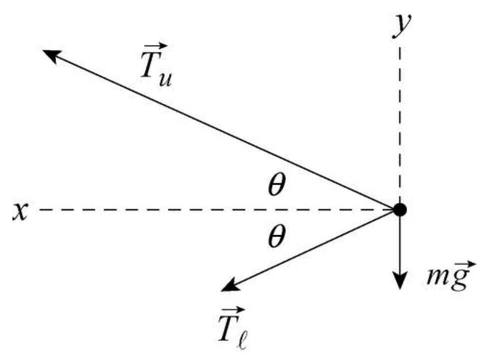
(d) The force required to keep the motion circular is

$$F_r = \frac{mv_0^2}{r} = \frac{(1400\text{ kg})(35.0\text{ m/s})^2}{107\text{ m}} = 1.6\times10^4\text{ N}.$$

(e) Since $F_r > f_{s,\max}$, no circular path is possible.

59. **THINK** As illustrated in Fig. 6-45, our system consists of a ball connected by two strings to a rotating rod. The tensions in the strings provide the source of centripetal force.

EXPRESS The free-body diagram for the ball is shown below. $\vec{T}_u$ is the tension exerted by the upper string on the ball, $\vec{T}_\ell$ is the tension in the lower string, and m is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



We take the $+x$ direction to be leftward (toward the center of the circular orbit) and $+y$ upward. Since the magnitude of the acceleration is $a = v^2/R$, the x component of Newton's second law is

$$T_u \cos\theta + T_\ell \cos\theta = \frac{mv^2}{R},$$

where v is the speed of the ball and R is the radius of its orbit. The y component is

$$T_u \sin\theta - T_\ell \sin\theta - mg = 0.$$

The second equation gives the tension in the lower string: $T_\ell = T_u - mg/\sin\theta$.

ANALYZE (a) Since the triangle is equilateral, the angle is $\theta = 30.0°$. Thus

$$T_\ell = T_u - \frac{mg}{\sin\theta} = 35.0\text{ N} - \frac{(1.34\text{ kg})(9.80\text{ m/s}^2)}{\sin 30.0°} = 8.74\text{ N}.$$

(b) The net force in the y-direction is zero. In the x-direction, the net force has magnitude

$$F_{\text{net,str}} = (T_u + T_\ell)\cos\theta = (35.0\text{ N} + 8.74\text{ N})\cos 30.0° = 37.9\text{ N}.$$

(c) The radius of the path is

$$R = L\cos\theta = (1.70\text{ m})\cos 30° = 1.47\text{ m}.$$

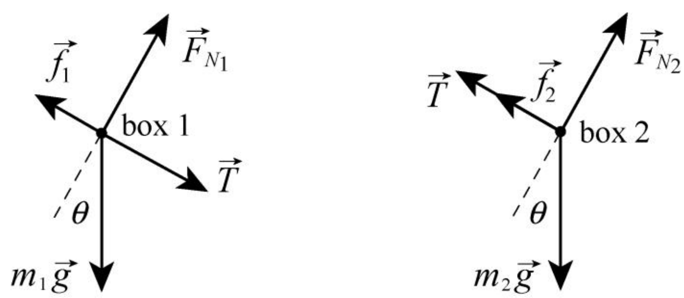
Using $F_{\text{net,str}} = mv^2/R$, we find the speed of the ball to be

$$v = \sqrt{\frac{RF_{\text{net,str}}}{m}} = \sqrt{\frac{(1.47\text{ m})(37.9\text{ N})}{1.34\text{ kg}}} = 6.45\text{ m/s}.$$

(d) The direction of $\vec{F}_{\text{net,str}}$ is leftward ("radially inward'').

LEARN The upper string, with a tension about 4 times that in the lower string ($T_u \approx 4T_\ell$), will break more easily than the lower one.

60. The free-body diagrams for the two boxes are shown below. T is the magnitude of the force in the rod (when $T > 0$ the rod is said to be in tension and when $T < 0$ the rod is under compression), $\vec{F}_{N2}$ is the normal force on box 2 (the uncle box), $\vec{F}_{N1}$ is the the normal force on the aunt box (box 1), $\vec{f}_1$ is kinetic friction force on the aunt box, and $\vec{f}_2$ is kinetic friction force on the uncle box. Also, m_1 = 1.65 kg is the mass of the aunt box and m_2 = 3.30 kg is the mass of the uncle box (which is a lot of ants!).



For each block we take $+x$ downhill (which is toward the lower-right in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of first box 2 and next box 1, we arrive at four equations:

$$m_2 g\sin\theta - f_2 - T = m_2\, a$$
$$F_{N2} - m_2 g\cos\theta = 0$$
$$m_1 g\sin\theta - f_1 + T = m_1 a$$
$$F_{N1} - m_1 g\cos\theta = 0$$

which, when combined with Eq. 6-2 ($f_1 = \mu_1 F_{N1}$ where $\mu_1 = 0.226$ and $f_2 = \mu_2 F_{N2}$ where $\mu_2 = 0.113$), fully describe the dynamics of the system.

(a) We solve the above equations for the tension and obtain

$$T = \left(\frac{m_2 m_1 g}{m_2 + m_1}\right)(\mu_1 - \mu_2)\cos\theta = 1.05 \text{ N}.$$

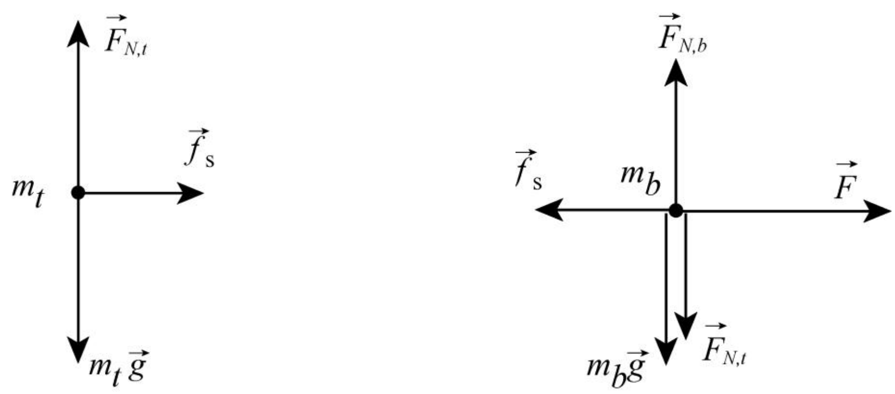
(b) These equations lead to an acceleration equal to

$$a = g\left(\sin\theta - \left(\frac{\mu_2 m_2 + \mu_1 m_1}{m_2 + m_1}\right)\cos\theta\right) = 3.62 \text{ m/s}^2.$$

(c) Reversing the blocks is equivalent to switching the labels. We see from our algebraic result in part (a) that this gives a negative value for T (equal in magnitude to the result we got before). Thus, the situation is as it was before except that the rod is now in a state of compression.

61. **THINK** Our system consists of two blocks, one on top of the other. If we pull the bottom block too hard, the top block will slip on the bottom one. We're interested in the maximum force that can be applied such that the two will move together.

EXPRESS The free-body diagrams for the two blocks are shown below.



We first calculate the coefficient of static friction for the surface between the two blocks. When the force applied is at a maximum, the frictional force between the two blocks must also be a maximum. Since $F_t = 12$ N of force has to be applied to the top block for slipping to take place, using $F_t = f_{s,\max} = \mu_s F_{N,t} = \mu_s m_t g$, we have

$$\mu_s = \frac{F_t}{m_t g} = \frac{12 \text{ N}}{(4.0 \text{ kg})(9.8 \text{ m/s}^2)} = 0.31.$$

Using the same reasoning, for the two masses to move together, the maximum applied force would be

$$F = \mu_s(m_t + m_b)g$$

ANALYZE (a) Substituting the value of μ_s found above, the maximum horizontal force has a magnitude

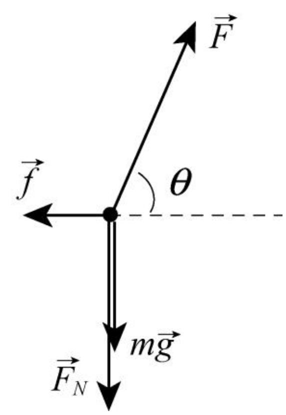
$$F = \mu_s(m_t + m_b)g = (0.31)(4.0\text{ kg} + 5.0\text{ kg})(9.8\text{ m/s}^2) = 27\text{ N}$$

(b) The maximum acceleration is

$$a_{\max} = \frac{F}{m_t + m_b} = \mu_s g = (0.31)(9.8\text{ m/s}^2) = 3.0\text{ m/s}^2 .$$

LEARN Slipping will occur if the applied force exceeds 27.3 N. In the absence of friction ($\mu_s = 0$) between the two blocks, any amount of force would cause the top block to slip.

62. The free-body diagram for the stone is shown to the right, with $\vec{F}$ being the force applied to the stone, $\vec{F}_N$ the *downward* normal force of the ceiling on the stone, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the +*x* direction to be horizontal to the right and the +*y* direction to be up. The equations for the *x* and the *y* components of the force according to Newton's second law are:



$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta - F_N - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = F\sin\theta - mg$, which yields $f = \mu_k(F\sin\theta - mg)$. This expression is substituted for *f* in the first equation to obtain

$$F\cos\theta - \mu_k(F\sin\theta - mg) = ma.$$

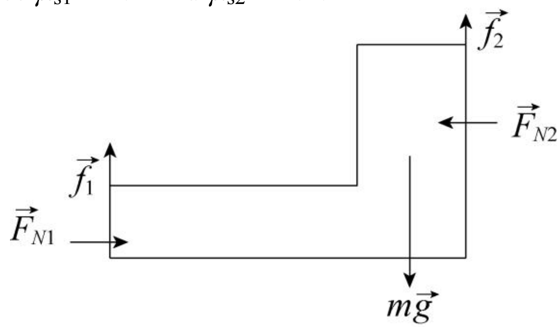
For $a = 0$, the force is

$$F = \frac{-\mu_k mg}{\cos\theta - \mu_k\sin\theta}.$$

With $\mu_k = 0.65$, m =5.0 kg, and $\theta = 70°$, we obtain F = 118 N.

63. (a) The free-body diagram for the person (shown as an L-shaped block) is shown below. The force that she exerts on the rock slabs is not directly shown (since the diagram should only show forces exerted on her), but it is related by Newton's third law) to the normal forces $\vec{F}_{N1}$ and $\vec{F}_{N2}$ exerted horizontally by the slabs onto her shoes and

back, respectively. We will show in part (b) that $F_{N1} = F_{N2}$ so that we there is no ambiguity in saying that the magnitude of her push is F_{N2}. The total upward force due to (maximum) static friction is $\vec{f} = \vec{f}_1 + \vec{f}_2$ where $f_1 = \mu_{s1} F_{N1}$ and $f_2 = \mu_{s2} F_{N2}$. The problem gives the values $\mu_{s1} = 1.2$ and $\mu_{s2} = 0.8$.



(b) We apply Newton's second law to the x and y axes (with $+x$ rightward and $+y$ upward and there is no acceleration in either direction).

$$F_{N1} - F_{N2} = 0$$
$$f_1 + f_2 - mg = 0$$

The first equation tells us that the normal forces are equal $F_{N1} = F_{N2} = F_N$. Consequently, from Eq. 6-1,

$$f_1 = \mu_{s1} F_N$$
$$f_2 = \mu_{s2} F_N$$

we conclude that

$$f_1 = \left(\frac{\mu_{s1}}{\mu_{s2}}\right) f_2 \, .$$

Therefore, $f_1 + f_2 - mg = 0$ leads to

$$\left(\frac{\mu_{s1}}{\mu_{s2}} + 1\right) f_2 = mg$$

which (with $m = 49$ kg) yields $f_2 = 192$ N. From this we find $F_N = f_2 / \mu_{s2} = 240$ N. This is equal to the magnitude of the push exerted by the rock climber.

(c) From the above calculation, we find $f_1 = \mu_{s1} F_N = 288$ N which amounts to a fraction

$$\frac{f_1}{W} = \frac{288}{(49)\,(9.8)} = 0.60$$

or 60% of her weight.

64. (a) The upward force exerted by the car on the passenger is equal to the downward force of gravity (W = 500 N) on the passenger. So the *net* force does not have a vertical contribution; it only has the contribution from the horizontal force (which is necessary for maintaining the circular motion). Thus $|\vec{F}_{\text{net}}| = F = 210$ N.

(b) Using Eq. 6-18, we have

$$v = \sqrt{\frac{FR}{m}} = \sqrt{\frac{(210\text{ N})(470\text{ m})}{51.0\text{ kg}}} = 44.0\text{ m/s}.$$

65. The layer of ice has a mass of

$$m_{\text{ice}} = \left(917\text{ kg/m}^3\right)\ (400\text{ m}\times 500\text{ m}\times 0.0040\text{ m}) = 7.34\times 10^5\text{ kg}.$$

This added to the mass of the hundred stones (at 20 kg each) comes to $m = 7.36 \times 10^5$ kg.

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v = \sqrt{\frac{\mu_k mg}{4C_{\text{ice}}\rho A_{\text{ice}}}} = \sqrt{\frac{(0.10)\left(7.36\times 10^5\text{ kg}\right)\left(9.8\text{ m/s}^2\right)}{4(0.002)\left(1.21\text{ kg/m}^3\right)\left(400\times 500\text{ m}^2\right)}} = 19\text{ m/s} \approx 69\text{ km/h}.$$

(b) Doubling our previous result, we find the reported speed to be 139 km/h.

(c) The result is reasonable for storm winds. (A category-5 hurricane has speeds on the order of 2.6×10^2 m/s.)

66. Note that since no static friction coefficient is mentioned, we assume f_s is not relevant to this computation. We apply Newton's second law to each block's x axis, which for m_1 is positive rightward and for m_2 is positive downhill:

$$\begin{aligned} T - f_k &= m_1 a \\ m_2 g\sin\theta - T &= m_2 a \end{aligned}$$

Adding the equations, we obtain the acceleration:

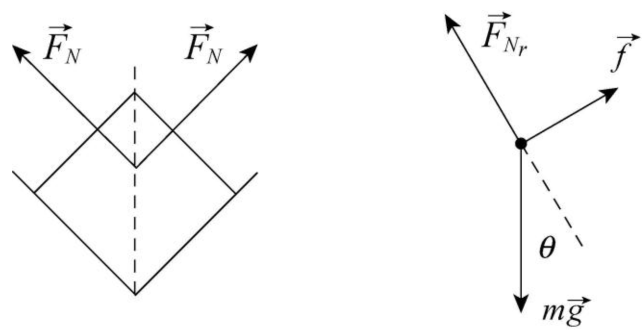
$$a = \frac{m_2 g\sin\theta - f_k}{m_1 + m_2}$$

For $f_k = \mu_k F_N = \mu_k m_1 g$, we obtain

$$a = \frac{(3.0\,\text{kg})(9.8\,\text{m/s}^2)\sin 30° - (0.25)(2.0\,\text{kg})(9.8\,\text{m/s}^2)}{3.0\,\text{kg} + 2.0\,\text{kg}} = 1.96\,\text{m/s}^2.$$

Returning this value to either of the above two equations, we find $T = 8.8$ N.

67. Each side of the trough exerts a normal force on the crate. The first diagram shows the view looking in toward a cross section.



The net force is along the dashed line. Since each of the normal forces makes an angle of 45° with the dashed line, the magnitude of the resultant normal force is given by

$$F_{Nr} = 2F_N \cos 45° = \sqrt{2}F_N.$$

The second diagram is the free-body diagram for the crate (from a "side" view, similar to that shown in the first picture in Fig. 6-51). The force of gravity has magnitude mg, where m is the mass of the crate, and the magnitude of the force of friction is denoted by f. We take the $+x$ direction to be down the incline and $+y$ to be in the direction of $\vec{F}_{Nr}$. Then the x and the y components of Newton's second law are

$$x: \quad mg \sin\theta - f = ma$$
$$y: \quad F_{Nr} - mg\cos\theta = 0.$$

Since the crate is moving, each side of the trough exerts a force of kinetic friction, so the total frictional force has magnitude

$$f = 2\mu_k F_N = 2\mu_k F_{Nr}/\sqrt{2} = \sqrt{2}\mu_k F_{Nr}$$

Combining this expression with $F_{Nr} = mg\cos\theta$ and substituting into the x component equation, we obtain

$$mg\sin\theta - \sqrt{2}mg\cos\theta = ma.$$

Therefore $a = g(\sin\theta - \sqrt{2}\mu_k\cos\theta)$.

68. (a) To be on the verge of sliding out means that the force of static friction is acting "down the bank" (in the sense explained in the problem statement) with maximum

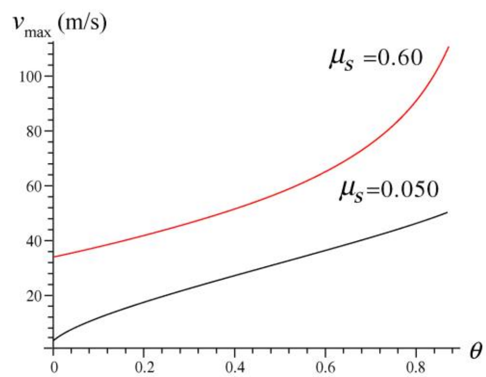
possible magnitude. We first consider the vector sum $\vec{F}$ of the (maximum) static friction force and the normal force. Due to the facts that they are perpendicular and their magnitudes are simply proportional (Eq. 6-1), we find $\vec{F}$ is at angle (measured from the *vertical* axis) $\phi = \theta + \theta_s$, where $\tan\theta_s = \mu_s$ (compare with Eq. 6-13), and θ is the bank angle (as stated in the problem). Now, the vector sum of $\vec{F}$ and the vertically downward pull (mg) of gravity must be equal to the (horizontal) centripetal force (mv^2/R), which leads to a surprisingly simple relationship:

$$\tan\phi = \frac{mv^2/R}{mg} = \frac{v^2}{Rg} \quad .$$

Writing this as an expression for the maximum speed, we have

$$v_{\max} = \sqrt{Rg\tan(\theta + \tan^{-1}\mu_s)} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta}}$$

(b) The graph is shown below (with θ in radians):



(c) Either estimating from the graph (μ_s = 0.60, upper curve) or calculated it more carefully leads to v = 41.3 m/s = 149 km/h when θ = 10º = 0.175 radian.

(d) Similarly (for μ_s = 0.050, the lower curve) we find v = 21.2 m/s = 76.2 km/h when θ = 10º = 0.175 radian.

69. For simplicity, we denote the 70° angle as θ and the magnitude of the push (80 N) as P. The vertical forces on the block are the downward normal force exerted on it by the ceiling, the downward pull of gravity (of magnitude mg) and the vertical component of $\vec{P}$ (which is upward with magnitude $P\sin\theta$). Since there is no acceleration in the vertical direction, we must have

$$F_N = P\sin\theta - mg$$

in which case the leftward-pointed kinetic friction has magnitude

$$f_k = \mu_k (P\sin\theta - mg).$$

Choosing +x rightward, Newton's second law leads to

$$P\cos\ \theta - f_k = ma \ \Rightarrow\ a = \frac{P\cos\theta - u_k (P\sin\theta - mg)}{m}$$

which yields a = 3.4 m/s^2 when μ_k = 0.40 and m = 5.0 kg.

70. (a) We note that R (the horizontal distance from the bob to the axis of rotation) is the circumference of the circular path divided by 2π; therefore, R = 0.94/2π = 0.15 m. The angle that the cord makes with the horizontal is now easily found:

$$\theta = \cos^{-1}(R/L) = \cos^{-1}(0.15\text{ m}/0.90\text{ m}) = 80^\circ.$$

The vertical component of the force of tension in the string is $T\sin\theta$ and must equal the downward pull of gravity (mg). Thus,

$$T = \frac{mg}{\sin\theta} = 0.40\text{ N}.$$

Note that we are using T for tension (not for the period).

(b) The horizontal component of that tension must supply the centripetal force (Eq. 6-18), so we have $T\cos\theta = mv^2/R$. This gives speed v = 0.49 m/s. This divided into the circumference gives the time for one revolution: 0.94/0.49 = 1.9 s.

71. (a) To be "on the verge of sliding" means the applied force is equal to the maximum possible force of static friction (Eq. 6-1, with $F_N = mg$ in this case):

$$f_{s,\max} = \mu_s mg = 35.3\text{ N}.$$

(b) In this case, the applied force $\vec{F}$ indirectly decreases the maximum possible value of friction (since its y component causes a reduction in the normal force) as well as directly opposing the friction force itself (because of its x component). The normal force turns out to be

$$F_N = mg - F\sin\theta$$

where θ = 60º, so that the horizontal equation (the x application of Newton's second law) becomes

$$F\cos\theta - f_{s,\max} = F\cos\theta - \mu_s(mg - F\sin\theta) = 0 \quad \Rightarrow F = 39.7\text{ N}.$$

(c) Now, the applied force $\vec{F}$ indirectly increases the maximum possible value of friction (since its y component causes a reduction in the normal force) as well as directly opposing the friction force itself (because of its x component). The normal force in this case turns out to be

$$F_N = mg + F\sin\theta,$$

where $\theta = 60°$, so that the horizontal equation becomes

$$F\cos\theta - f_{s,\max} = F\cos\theta - \mu_s(mg + F\sin\theta) = 0 \quad \Rightarrow F = 320 \text{ N}.$$

72. With $\theta = 40°$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

using Eq. 6-12. Thus,

$$a = 0.75 \text{ m/s}^2 = g(\sin\theta - \mu_k \cos\theta)$$

determines the coefficient of kinetic friction: $\mu_k = 0.74$.

73. (a) With $\theta = 60°$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma$$
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta.$$

Thus,

$$a = g(\sin\theta - \mu_k\cos\theta) = 7.5 \text{ m/s}^2.$$

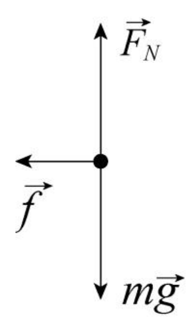
(b) The direction of the acceleration $\vec{a}$ is down the slope.

(c) Now the friction force is in the "downhill" direction (which is our positive direction) so that we obtain

$$a = g(\sin\theta + \mu_k\cos\theta) = 9.5 \text{ m/s}^2.$$

(d) The direction is down the slope.

74. The free-body diagram for the puck is shown on the right. $\vec{F}_N$ is the normal force of the ice on the puck, $\vec{f}$ is the force of friction (in the $-x$ direction), and $m\vec{g}$ is the force of gravity.



(a) The horizontal component of Newton's second law gives $-f = ma$, and constant acceleration kinematics (Table 2-1) can be used to find the acceleration.

Since the final velocity is zero, $v^2 = v_0^2 + 2ax$ leads to $a = -v_0^2/2x$. This is substituted into the Newton's law equation to obtain

$$f = \frac{mv_0^2}{2x} = \frac{(0.110 \text{ kg})(6.0 \text{ m/s})^2}{2(15 \text{ m})} = 0.13 \text{ N}.$$

(b) The vertical component of Newton's second law gives $F_N - mg = 0$, so $F_N = mg$ which implies (using Eq. 6-2) $f = \mu_k mg$. We solve for the coefficient:

$$\mu_k = \frac{f}{mg} = \frac{0.13 \text{ N}}{(0.110 \text{ kg})(9.8 \text{ m/s}^2)} = 0.12 .$$

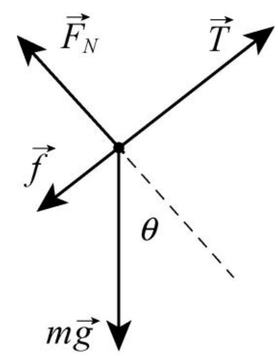
75. We may treat all 25 cars as a single object of mass $m = 25 \times 5.0 \times 10^4$ kg and (when the speed is 30 km/h = 8.3 m/s) subject to a friction force equal to

$$f = 25 \times 250 \times 8.3 = 5.2 \times 10^4 \text{ N}.$$

(a) Along the level track, this object experiences a "forward" force T exerted by the locomotive, so that Newton's second law leads to

$$T - f = ma \quad \Rightarrow \quad T = 5.2\times10^4 + (1.25\times10^6)(0.20) = 3.0\times10^5 \text{ N}.$$

(b) The free-body diagram is shown next, with θ as the angle of the incline. The $+x$ direction (which is the only direction to which we will be applying Newton's second law) is uphill (to the upper right in our sketch). Thus, we obtain



$$T - f - mg\sin\theta = ma$$

where we set $a = 0$ (implied by the problem statement) and solve for the angle. We obtain $\theta = 1.2°$.

76. An excellent discussion and equation development related to this problem is given in Sample Problem – "Friction, applied force at an angle." Using the result, we obtain

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.50 = 27°$$

which implies that the angle through which the slope should be *reduced* is

$$\phi = 45° - 27° \approx 20°.$$

77. We make use of Eq. 6-16 which yields

$$\sqrt{\frac{2mg}{C\rho\pi R^2}} = \sqrt{\frac{2(6)(9.8)}{(1.6)(1.2)\pi(0.03)^2}} = 147 \text{ m/s}.$$

78. (a) The coefficient of static friction is $\mu_s = \tan(\theta_{slip}) = 0.577 \approx 0.58$.

(b) Using

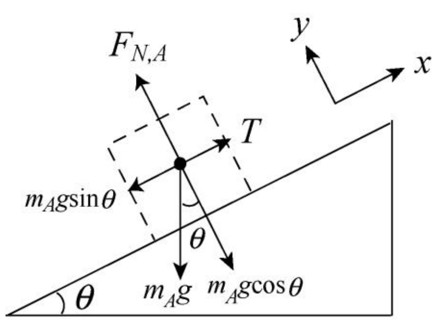
$$mg\sin\theta - f = ma$$

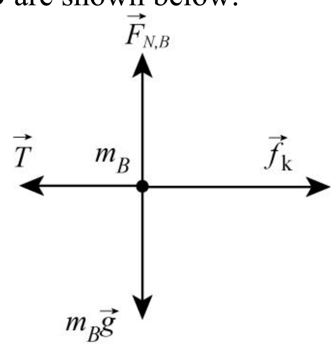
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

and $a = 2d/t^2$ (with d = 2.5 m and t = 4.0 s), we obtain μ_k = 0.54.

79. **THINK** We have two blocks connected by a cord, as shown in Fig. 6-56. As block *A* slides down the frictionless inclined plane, it pulls block *B*, so there's a tension in the cord.

EXPRESS The free-body diagrams for blocks *A* and *B* are shown below:





Newton's law gives

$$m_A g\sin\theta - T = m_A a$$

for block *A* (where θ = 30º). For block *B*, we have

$$T - f_k = m_B a$$

Now the frictional force is given by $f_k = \mu_k F_{N,B} = \mu_k m_B g$. The equations allow us to solve for the tension *T* and the acceleration *a*.

ANALYZE (a) Combining the above equations to solve for *T*, we obtain

$$T = \frac{m_A m_B}{m_A + m_B}(\sin\theta + \mu_k)g = \frac{(4.0\text{ kg})(2.0\text{ kg})}{4.0\text{ kg} + 2.0\text{ kg}}(\sin 30° + 0.50)(9.80\text{ m/s}^2) = 13\text{ N}.$$

(b) Similarly, the acceleration of the two-block system is

$$a = \left(\frac{m_A\sin\theta - \mu_k m_B}{m_A + m_B}\right)g = \frac{(4.0\text{ kg})\sin 30° - (0.50)(2.0\text{ kg})}{4.0\text{ kg} + 2.0\text{ kg}}(9.80\text{ m/s}^2) = 1.6\text{ m/s}^2.$$

LEARN In the case where $\theta = 90°$ and $\mu_k = 0$, we have

$$T = \frac{m_A m_B}{m_A + m_B} g, \quad a = \frac{m_A}{m_A + m_B} g$$

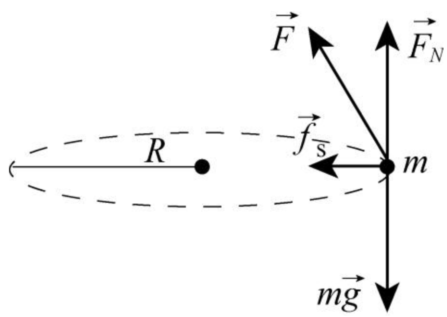
which correspond to the Sample Problem – "Block on table, block hanging," discussed in Chapter 5.

80. We use Eq. 6-14, $D = \frac{1}{2} C\rho A v^2$, where ρ is the air density, A is the cross-sectional area of the missile, v is the speed of the missile, and C is the drag coefficient. The area is given by $A = \pi R^2$, where R = 0.265 m is the radius of the missile. Thus

$$D = \frac{1}{2}(0.75)\left(1.2 \text{ kg/m}^3\right)\pi\left(0.265 \text{ m}\right)^2\left(250 \text{ m/s}\right)^2 = 6.2 \times 10^3 \text{ N}.$$

81. **THINK** How can a cyclist move in a circle? It is the force of friction that provides the centripetal force required for the circular motion.

EXPRESS The free-body diagram is shown below. The magnitude of the acceleration of the cyclist as it moves along the horizontal circular path is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve.



The horizontal component of Newton's second law is $f_s = mv^2/R$, where f_s is the static friction exerted horizontally by the ground on the tires. Similarly, if F_N is the vertical force of the ground on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg = 833 \text{ N}$.

ANALYZE (a) The frictional force is $f_s = \frac{mv^2}{R} = \frac{(85.0 \text{ kg})(9.00 \text{ m/s})^2}{25.0 \text{ m}} = 275 \text{ N}.$

(b) Since the frictional force $\vec{f}_s$ and $\vec{F}_N$, the normal force exerted by the road, are perpendicular to each other, the magnitude of the force exerted by the ground on the bicycle is

$$F = \sqrt{f_s^2 + F_N^2} = \sqrt{(275 \text{ N})^2 + (833 \text{ N})^2} = 877 \text{ N}.$$

LEARN The force exerted by the ground on the bicycle is at an angle $\theta = \tan^{-1}(275\text{ N}/833\text{ N}) = 18.3°$ with respect to the *vertical* axis.

82. At the top of the hill the vertical forces on the car are the upward normal force exerted by the ground and the downward pull of gravity. Designating $+y$ downward, we have

$$mg - F_N = \frac{mv^2}{R}$$

from Newton's second law. To find the greatest speed without leaving the hill, we set $F_N = 0$ and solve for v:

$$v = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(250\text{ m})} = 49.5\text{ m/s} = 49.5(3600/1000)\text{ km/h} = 178\text{ km/h}.$$

83. (a) The push (to get it moving) must be at least as big as $f_{s,max} = \mu_s F_N$ (Eq. 6-1, with $F_N = mg$ in this case), which equals (0.51)(165 N) = 84.2 N.

(b) While in motion, constant velocity (zero acceleration) is maintained if the push is equal to the kinetic friction force $f_k = \mu_k F_N = \mu_k mg = 52.8$ N.

(c) We note that the mass of the crate is 165/9.8 = 16.8 kg. The acceleration, using the push from part (a), is

$$a = (84.2\text{ N} - 52.8\text{ N})/(16.8\text{ kg}) \approx 1.87\text{ m/s}^2.$$

84. (a) The x component of $\vec{F}$ tries to move the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Newton's second law implies

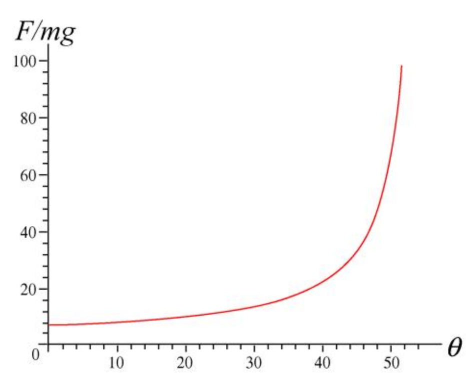
$$x\text{ direction: } F\cos\theta - f_s = 0$$

$$y\text{ direction: } F_N - F\sin\theta - mg = 0.$$

To be "on the verge of sliding" means $f_s = f_{s,max} = \mu_s F_N$ (Eq. 6-1). Solving these equations for F (actually, for the ratio of F to mg) yields

$$\frac{F}{mg} = \frac{\mu_s}{\cos\theta - \mu_s \sin\theta} .$$

This is plotted on the right (θ in degrees).



(b) The denominator of our expression (for F/mg) vanishes when

$$\cos\theta - \mu_s \sin\theta = 0 \quad \Rightarrow \quad \theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right)$$

For $\mu_s = 0.70$, we obtain $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 55°$.

(c) Reducing the coefficient means increasing the angle by the condition in part (b).

(d) For $\mu_s = 0.60$ we have $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 59°$.

85. The car is in "danger of sliding" down when

$$\mu_s = \tan\theta = \tan 35.0° = 0.700.$$

This value represents a 3.4% decrease from the given 0.725 value.

86. (a) The tension will be the greatest at the lowest point of the swing. Note that there is no substantive difference between the tension T in this problem and the normal force F_N in Sample Problem – "Vertical circular loop, Diavolo." Eq. 6-19 of that Sample Problem examines the situation at the top of the circular path (where F_N is the least), and rewriting that for the bottom of the path leads to

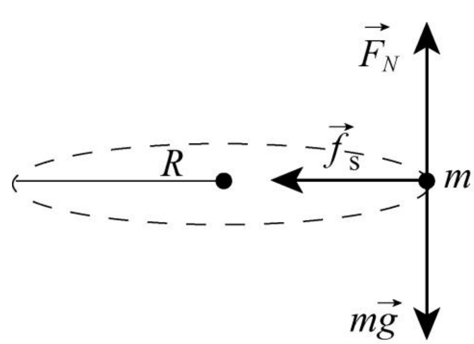
$$T = mg + mv^2/r$$

where F_N is at its greatest value.

(b) At the breaking point T = 33 N = $m(g + v^2/r)$ where m = 0.26 kg and r = 0.65 m. Solving for the speed, we find that the cord should break when the speed (at the lowest point) reaches 8.73 m/s.

87. **THINK** A car is making a turn on an unbanked curve. Friction is what provides the centripetal force needed for this circular motion.

EXPRESS The free-body diagram is shown to the right. The mass of the car is m = (10700/9.80) kg = 1.09 × 10^3 kg. We choose "inward" (horizontally toward the center of the circular path) as the positive direction. The normal force is $F_N = mg$ in this situation, and the required frictional force is $f_s = mv^2/R$.



ANALYZE (a) With a speed of v = 13.4 m/s and a radius R = 61 m, Newton's second law (using Eq. 6-18) leads to

$$f_s = \frac{mv^2}{R} = \frac{(1.09\times10^3\ \text{kg})(13.4\ \text{m/s})^2}{61.0\ \text{m}} = 3.21\times10^3\ \text{N}.$$

(b) The maximum possible static friction is found to be

$$f_{s,\max} = \mu_s mg = (0.35)(10700\ \text{N}) = 3.75\times10^3\ \text{N}$$

using Eq. 6-1. We see that the static friction found in part (a) is less than this, so the car rolls (no skidding) and successfully negotiates the curve.

LEARN From the above expressions, we see that with a coefficient of friction μ_s, the maximum speed of the car negotiating a curve of radius R is $v_{\max} = \sqrt{\mu_s gR}$. So in this case, the car can go up to a maximum speed of

$$v_{\max} = \sqrt{(0.35)(9.8\ \text{m/s}^2)(61\ \text{m})} = 14.5\ \text{m/s}$$

without skidding.

88. For the m_2 = 1.0 kg block, application of Newton's laws result in

$$F\cos\theta - T - f_k = m_2 a \quad x \text{ axis}$$
$$F_N - F\sin\theta - m_2 g = 0 \quad y \text{ axis}$$

Since $f_k = \mu_k F_N$, these equations can be combined into an equation to solve for a:

$$F(\cos\theta - \mu_k \sin\theta) - T - \mu_k m_2 g = m_2 a$$

Similarly (but without the applied push) we analyze the m_1= 2.0 kg block:

$$T - f'_k = m_1 a \quad x \text{ axis}$$
$$F'_N - m_1 g = 0 \quad y \text{ axis}$$

Using $f_k = \mu_k F'_N$, the equations can be combined:

$$T - \mu_k m_1 g = m_1 a$$

Subtracting the two equations for a and solving for the tension, we obtain

$$T = \frac{m_1(\cos\theta - \mu_k \sin\theta)}{m_1 + m_2} F = \frac{(2.0\ \text{kg})[\cos 35° - (0.20)\sin 35°]}{2.0\ \text{kg} + 1.0\ \text{kg}}(20\ \text{N}) = 9.4\ \text{N}.$$

89. **THINK** In order to move a filing cabinet, the force applied must be able to overcome the frictional force.

EXPRESS We apply Newton's second law (as $F_{\text{push}} - f = ma$). If we find the applied force F_{push} to be less than $f_{s,\max}$, the maximum static frictional force, our conclusion would then be "no, the cabinet does not move" (which means a is actually 0 and the frictional force is simply $f = F_{\text{push}}$). On the other hand, if we obtain $a > 0$ then the cabinet moves (so $f = f_k$). For $f_{s,\max}$ and f_k we use Eq. 6-1 and Eq. 6-2 (respectively), and in those formulas we set the magnitude of the normal force to the weight of the cabinet: $F_N = mg = 556\text{ N}$. Thus, the maximum static frictional force is

$$f_{s,\max} = \mu_s F_N = \mu_s mg = (0.68)(556\text{ N}) = 378\text{ N}.$$

and the kinetic frictional force is

$$f_k = \mu_k F_N = \mu_k mg = (0.56)(556\text{ N}) = 311\text{ N}.$$

ANALYZE (a) Here we find $F_{\text{push}} < f_{s,\max}$ which leads to $f = F_{\text{push}} = 222$ N. The cabinet does not move.

(b) Again we find $F_{\text{push}} < f_{s,\max}$ which leads to $f = F_{\text{push}} = 334$ N. The cabinet does not move.

(c) Now we have $F_{\text{push}} > f_{s,\max}$ which means the cabinet moves and $f = f_k = 311$ N.

(d) Again we have $F_{\text{push}} > f_{s,\max}$ which means the cabinet moves and $f = f_k = 311$ N.

(e) The cabinet moves in (c) and (d).

LEARN In summary, in order to make the cabinet move, the minimum applied force is equal to the maximum static frictional force $f_{s,\max}$.

90. Analysis of forces in the horizontal direction (where there can be no acceleration) leads to the conclusion that $F = F_N$; the magnitude of the normal force is 60 N. The maximum possible static friction force is therefore $\mu_s F_N = 33$ N, and the kinetic friction force (when applicable) is $\mu_k F_N = 23$ N.

(a) In this case, $\vec{P} = 34$ N upward. Assuming $\vec{f}$ points down, then Newton's second law for the y leads to

$$P - mg - f = ma.$$

if we assume $f = f_s$ and $a = 0$, we obtain $f = (34 - 22)$ N = 12 N. This is less than $f_{s,\max}$, which shows the consistency of our assumption. The answer is: $\vec{f_s} = 12$ N down.

(b) In this case, $\vec{P}$ = 12 N upward. The above equation, with the same assumptions as in part (a), leads to $f = (12 - 22)$ N = –10 N. Thus, $|f_s| < f_{s,\max}$, justifying our assumption that the block is stationary, but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. Thus, the answer is: $\vec{f_s}$ = 10 N up.

(c) In this case, $\vec{P}$ = 48 N upward. The above equation, with the same assumptions as in part (a), leads to $f = (48 - 22)$ N = 26 N. Thus, we again have $f_s < f_{s,\max}$, and our answer is: $\vec{f_s}$ = 26 N down.

(d) In this case, $\vec{P}$ = 62 N upward. The above equation, with the same assumptions as in part (a), leads to $f = (62 - 22)$ N = 40 N, which is larger than $f_{s,\max}$, -- invalidating our assumptions. Therefore, we take $f = f_k$ and $a \neq 0$ in the above equation; if we wished to find the value of a we would find it to be positive, as we should expect. The answer is: $\vec{f_k}$ = 23 N down.

(e) In this case, $\vec{P}$ = 10 N downward. The above equation (but with P replaced with $-P$) with the same assumptions as in part (a), leads to $f = (-10 - 22)$ N = –32 N. Thus, we have $|f_s| < f_{s,\max}$, justifying our assumption that the block is stationary, but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. Thus, the answer is: $\vec{f_s}$ = 32 N up.

(f) In this case, $\vec{P}$ = 18 N downward. The above equation (but with P replaced with $-P$) with the same assumptions as in part (a), leads to $f = (-18 - 22)$ N = –40 N, which is larger (in absolute value) than $f_{s,\max}$, -- invalidating our assumptions. Therefore, we take $f = f_k$ and $a \neq 0$ in the above equation; if we wished to find the value of a we would find it to be negative, as we should expect. The answer is: $\vec{f_k}$ = 23 N up.

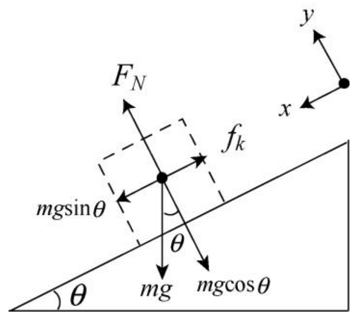
(g) The block moves up the wall in case (d) where $a > 0$.

(h) The block moves down the wall in case (f) where $a < 0$.

(i) The frictional force $\vec{f_s}$ is directed down in cases (a), (c) and (d).

91. **THINK** Whether the block is sliding down or up the incline, there is a frictional force in the opposite direction of the motion.

EXPRESS The free-body diagram for the first part of this problem (when the block is sliding downhill with zero acceleration) is shown next.

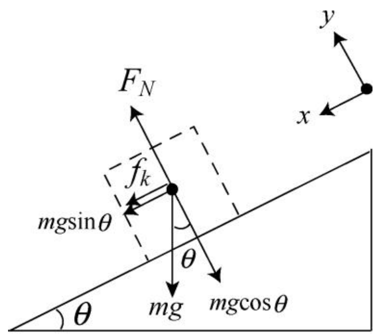


Newton's second law gives

$$mg\sin\theta - f_k = mg\sin\theta - \mu_k F_N = ma_x = 0$$
$$mg\cos\theta - F_N = ma_y = 0$$

The two equations can be combined to give

$$\mu_k = \tan\theta.$$



Now (for the second part of the problem, with the block projected uphill) the friction direction is reversed (see figure to the right). Newton's second law for the uphill motion (and Eq. 6-12) leads to

$$mg\sin\theta + f_k = mg\sin\theta + \mu_k F_N = ma_x$$
$$mg\cos\theta - F_N = ma_y = 0$$

Note that by our convention, $a_x > 0$ means that the acceleration is downhill, and therefore, the speed of the block will decrease as it moves up the incline.

ANALYZE (a) Using $\mu_k = \tan\theta$ and $F_N = mg\cos\theta$, we find the *x*-component of the acceleration to be

$$a_x = g\sin\theta + \frac{\mu_k F_N}{m} = g\sin\theta + \frac{(\tan\theta)(mg\cos\theta)}{m} = 2g\sin\theta.$$

The distance the block travels before coming to a stop can be found by using Eq. 2-16: $v_f^2 = v_0^2 - 2a_x\Delta x$, which yields

$$\Delta x = \frac{v_0^2}{2a_x} = \frac{v_0^2}{2(2g\sin\theta)} = \frac{v_0^2}{4g\sin\theta}.$$

(b) We usually expect $\mu_s > \mu_k$ (see the discussion in Section 6-1). The "angle of repose" (the minimum angle necessary for a stationary block to start sliding downhill) is $\mu_s = \tan(\theta_{\text{repose}})$. Therefore, we expect $\theta_{\text{repose}} > \theta$ found in part (a). Consequently, when the block comes to rest, the incline is not steep enough to cause it to start slipping down the incline again.

LEARN An alternative way to see that the block will not slide down again is to note that the downward force of gravitation is not large enough to overcome the force of friction, i.e., $mg\sin\theta = f_k < f_{s,\max}$.

92. Consider that the car is "on the verge of sliding out" – meaning that the force of static friction is acting "down the bank" (or "downhill" from the point of view of an ant on the banked curve) with maximum possible magnitude. We first consider the vector sum $\vec{F}$ of the (maximum) static friction force and the normal force. Due to the facts that they are perpendicular and their magnitudes are simply proportional (Eq. 6-1), we find $\vec{F}$ is at angle (measured from the <u>vertical</u> axis) $\phi = \theta + \theta_s$ where $\tan \theta_s = \mu_s$ (compare with Eq. 6-13), and θ is the bank angle. Now, the vector sum of $\vec{F}$ and the vertically downward pull (mg) of gravity must be equal to the (horizontal) centripetal force (mv^2/R), which leads to a surprisingly simple relationship:

$$\tan\phi = \frac{mv^2/R}{mg} = \frac{v^2}{Rg} .$$

Writing this as an expression for the maximum speed, we have

$$v_{\max} = \sqrt{Rg\tan(\theta + \tan^{-1}\mu_s)} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1-\mu_s\tan\theta}} .$$

(a) We note that the given speed is (in SI units) roughly 17 m/s. If we do not want the cars to "depend" on the static friction to keep from sliding out (that is, if we want the component "down the back" of gravity to be sufficient), then we can set $\mu_s = 0$ in the above expression and obtain $v = \sqrt{Rg\tan\theta}$. With R = 150 m, this leads to $\theta = 11°$.

(b) If, however, the curve is not banked (so $\theta = 0$) then the above expression becomes

$$v = \sqrt{Rg\tan(\tan^{-1}\mu_s)} = \sqrt{Rg\mu_s}$$

Solving this for the coefficient of static friction $\mu_s = 0.19$.

93. (a) The box doesn't move until t = 2.8 s, which is when the applied force $\vec{F}$ reaches a magnitude of F = (1.8)(2.8) = 5.0 N, implying therefore that $f_{s,\max}$ = 5.0 N. Analysis of the vertical forces on the block leads to the observation that the normal force magnitude equals the weight $F_N = mg$ = 15 N. Thus,

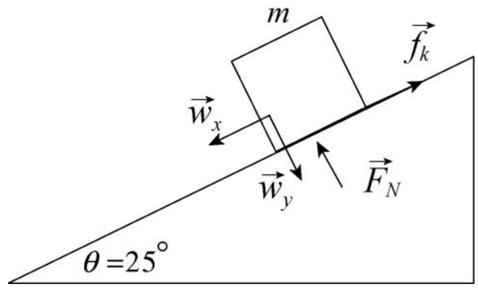
$$\mu_s = f_{s,\max}/F_N = 0.34.$$

(b) We apply Newton's second law to the horizontal x axis (positive in the direction of motion):

$$F - f_k = ma \Rightarrow 1.8t - f_k = (1.5)(1.2t - 2.4)$$

Thus, we find f_k = 3.6 N. Therefore, $\mu_k = f_k / F_N = 0.24$.

94. In the figure below, m = 140/9.8 = 14.3 kg is the mass of the child. We use $\vec{w}_x$ and $\vec{w}_y$ as the components of the gravitational pull of Earth on the block; their magnitudes are $w_x = mg \sin\theta$ and $w_y = mg \cos\theta$.



(a) With the x axis directed up along the incline (so that a = –0.86 m/s^2), Newton's second law leads to

$$f_k - 140\sin 25° = m(-0.86)$$

which yields f_k = 47 N. We also apply Newton's second law to the y axis (perpendicular to the incline surface), where the acceleration-component is zero:

$$F_N - 140\cos 25° = 0 \quad \Rightarrow \quad F_N = 127 \text{ N}.$$

Therefore, $\mu_k = f_k/F_N = 0.37$.

(b) Returning to our first equation in part (a), we see that if the downhill component of the weight force were insufficient to overcome static friction, the child would not slide at all. Therefore, we require 140 sin 25° > $f_{s,\max} = \mu_s F_N$, which leads to tan 25° = 0.47 > μ_s. The minimum value of μ_s equals μ_k and is more subtle; reference to §6-1 is recommended. If μ_k exceeded μ_s then when static friction were overcome (as the incline is raised) then it should start to move – which is impossible if f_k is large enough to cause deceleration! The bounds on μ_s are therefore given by 0.47 > μ_s > 0.37.

95. (a) The x component of $\vec{F}$ contributes to the motion of the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Along the y direction, we have $F_N - F\cos\theta - mg = 0$ and along the x direction we have $F\sin\theta - f_k = 0$ (since it is not accelerating, according to the problem). Also, Eq. 6-2 gives $f_k = \mu_k F_N$. Solving these equations for F yields

$$F = \frac{\mu_k mg}{\sin\theta - \mu_k \cos\theta} .$$

(b) When $\theta < \theta_0 = \tan^{-1}\mu_s$, F will not be able to move the mop head.

96. (a) The distance traveled in one revolution is $2\pi R = 2\pi(4.6 \text{ m}) = 29$ m. The (constant) speed is consequently $v = (29 \text{ m})/(30 \text{ s}) = 0.96$ m/s.

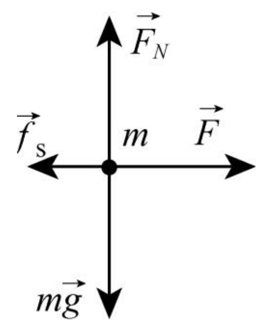
(b) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$f_s = m\left(\frac{v^2}{R}\right) = m(0.20)$$

in SI units. Noting that $F_N = mg$ in this situation, the maximum possible static friction is $f_{s,\max} = \mu_s mg$ using Eq. 6-1. Equating this with $f_s = m(0.20)$ we find the mass m cancels and we obtain $\mu_s = 0.20/9.8 = 0.021$.

97. **THINK** In this problem a force is applied to accelerate a box. From the distance traveled and the speed at that instant, we can calculate the coefficient of kinetic friction between the box and the floor.

EXPRESS The free-body diagram is shown to the right. We adopt the familiar axes with $+x$ rightward and $+y$ upward, and refer to the 85 N horizontal push of the worker as F (and assume it to be rightward). Applying Newton's second law to the x axis and y axis, respectively, produces



$$F - f_k = ma_x, \qquad F_N - mg = 0.$$

On the other hand, using Eq. 2-16 ($v^2 = v_0^2 + 2a_x\Delta x$), we find the acceleration to be

$$a_x = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(1.0 \text{ m/s})^2 - 0}{2(1.4 \text{ m})} = 0.357 \text{ m/s}^2.$$

The above equations can be combined to give μ_k.

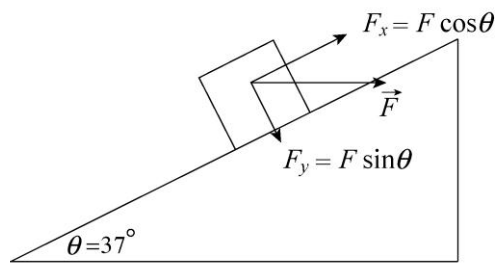
ANALYZE Using $f_k = \mu_k F_N$, we find the coefficient of kinetic friction between the box and the floor to be

$$\mu_k = \frac{f_k}{F_N} = \frac{F - ma_x}{mg} = \frac{85 \text{ N} - (40 \text{ kg})(0.357 \text{ m/s}^2)}{(40 \text{ kg})(9.8 \text{ m/s}^2)} = 0.18.$$

LEARN In general, the acceleration can be written as $a_x = (F/m) - \mu_k g$. We see that the smaller the value of μ_k, the greater the acceleration. In the limit $\mu_k = 0$, we simply have $a_x = F/m$.

98. We resolve this horizontal force into appropriate components.

(a) Applying Newton's second law to the x (directed uphill) and y (directed away from the incline surface) axes, we obtain



$$F\cos\theta - f_k - mg\sin\theta = ma$$
$$F_N - F\sin\theta - mg\cos\theta = 0.$$

Using $f_k = \mu_k F_N$, these equations lead to

$$a = \frac{F}{m}(\cos\theta - \mu_k \sin\theta) - g(\sin\theta + \mu_k \cos\theta)$$

which yields $a = -2.1\ \text{m/s}^2$, or $|a| = 2.1\ \text{m/s}^2$, for $\mu_k = 0.30$, $F = 50$ N and $m = 5.0$ kg.

(b) The direction of $\vec{a}$ is down the plane.

(c) With $v_0 = +4.0$ m/s and $v = 0$, Eq. 2-16 gives $\Delta x = -\dfrac{(4.0\ \text{m/s})^2}{2(-2.1\ \text{m/s}^2)} = 3.9$ m.

(d) We expect $\mu_s \geq \mu_k$; otherwise, an object started into motion would immediately start decelerating (before it gained any speed)! In the minimal expectation case, where $\mu_s = 0.30$, the maximum possible (downhill) static friction is, using Eq. 6-1,

$$f_{s,\max} = \mu_s F_N = \mu_s (F\sin\theta + mg\cos\theta)$$

which turns out to be 21 N. But in order to have no acceleration along the x axis, we must have

$$f_s = F\cos\theta - mg\sin\theta = 10\ \text{N}$$

(the fact that this is positive reinforces our suspicion that $\vec{f}_s$ points downhill). Since the f_s needed to remain at rest is less than $f_{s,\max}$ then it stays at that location.

99. (a) We note that $F_N = mg$ in this situation, so

$$f_{s,\max} = \mu_s mg = (0.52)(11\ \text{kg})(9.8\ \text{m/s}^2) = 56\ \text{N}.$$

Consequently, the horizontal force $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than 56 N.

(b) Analyzing vertical forces when $\vec{F}$ is at nonzero θ yields

$$F\sin\theta + F_N = mg \;\Rightarrow\; f_{s,\max} = \mu_s (mg - F\sin\theta).$$

Now, the horizontal component of $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than this, so

$$F\cos\theta = \mu_s(mg - F\sin\,\theta) \Rightarrow F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\,\theta}$$

which yields F = 59 N when θ = 60°.

(c) We now set θ = –60° and obtain

$$F = \frac{(0.52)(11\,\text{kg})(9.8\,\text{m/s}^2)}{\cos(-60°) + (0.52)\,\sin\,(-60°)} = 1.1\times10^3\ \text{N}.$$

100. (a) If the skier covers a distance L during time t with zero initial speed and a constant acceleration a, then $L = at^2/2$, which gives the acceleration a_1 for the first (old) pair of skis:

$$a_1 = \frac{2L}{t_1^2} = \frac{2(200\,\text{m})}{(61\ \text{s})^2} = 0.11\ \text{m/s}^2.$$

(b) The acceleration a_2 for the second (new) pair is

$$a_2 = \frac{2L}{t_2^2} = \frac{2(200\,\text{m})}{(42\,\text{s})^2} = 0.23\ \text{m/s}^2\ .$$

(c) The net force along the slope acting on the skier of mass m is

$$F_{\text{net}} = mg\sin\theta - f_k = mg(\sin\theta - \mu_k\cos\theta) = ma$$

which we solve for μ_{k1} for the first pair of skis:

$$\mu_{k1} = \tan\theta - \frac{a_1}{g\cos\theta} = \tan 3.0° - \frac{0.11\,\text{m/s}^2}{(9.8\,\text{m/s}^2)\cos 3.0°} = 0.041$$

(d) For the second pair, we have

$$\mu_{k2} = \tan\theta - \frac{a_2}{g\cos\theta} = \tan 3.0° - \frac{0.23\,\text{m/s}^2}{(9.8\,\text{m/s}^2)\cos 3.0°} = 0.029\ .$$

101. If we choose "downhill" positive, then Newton's law gives

$$m\,g\sin\theta - f_k = m\,a$$

for the sliding child. Now using Eq. 6-12

$$f_k = \mu_k F_N = \mu_k m g,$$

so we obtain $a = g(\sin\theta - \mu_k \cos\theta) = -0.5$ m/s^2 (note that the problem gives the direction of the acceleration vector as uphill, even though the child is sliding downhill, so it is a deceleration). With $\theta = 35°$, we solve for the coefficient and find $\mu_k = 0.76$.

102. (a) Our $+x$ direction is horizontal and is chosen (as we also do with $+y$) so that the components of the 100 N force $\vec{F}$ are non-negative. Thus, $F_x = F \cos\theta = 100$ N, which the textbook denotes F_h in this problem.

(b) Since there is no vertical acceleration, application of Newton's second law in the y direction gives

$$F_N + F_y = mg \Rightarrow F_N = mg - F\sin\theta$$

where $m = 25.0$ kg. This yields $F_N = 245$ N in this case ($\theta = 0°$).

(c) Now, $F_x = F_h = F\cos\theta = 86.6$ N for $\theta = 30.0°$.

(d) And $F_N = mg - F\sin\theta = 195$ N.

(e) We find $F_x = F_h = F\cos\theta = 50.0$ N for $\theta = 60.0°$.

(f) And $F_N = mg - F\sin\theta = 158$ N.

(g) The condition for the chair to slide is

$$F_x > f_{s,\max} = \mu_s F_N \quad \text{where} \quad \mu_s = 0.42.$$

For $\theta = 0°$, we have

$$F_x = 100 \text{ N} < f_{s,\max} = (0.42)(245 \text{ N}) = 103 \text{ N}$$

so the crate remains at rest.

(h) For $\theta = 30.0°$, we find $F_x = 86.6 \text{ N} > f_{s,\max} = (0.42)(195 \text{ N}) = 81.9 \text{ N}$, so the crate slides.

(i) For $\theta = 60°$, we get $F_x = 50.0 \text{ N} < f_{s,\max} = (0.42)(158 \text{ N}) = 66.4 \text{ N}$, which means the crate must remain at rest.

103. (a) The intuitive conclusion, that the tension is greatest at the bottom of the swing, is certainly supported by application of Newton's second law there:

$$T - mg = \frac{mv^2}{R} \Rightarrow T = m\left(g + \frac{v^2}{R}\right)$$

where Eq. 6-18 has been used. Increasing the speed eventually leads to the tension at the bottom of the circle reaching that breaking value of 40 N.

(b) Solving the above equation for the speed, we find

$$v = \sqrt{R\left(\frac{T}{m} - g\right)} = \sqrt{(0.91\text{ m})\left(\frac{40\text{ N}}{0.37\text{ kg}} - 9.8\text{ m/s}^2\right)}$$

which yields $v = 9.5$ m/s.

104. (a) The component of the weight along the incline (with downhill understood as the positive direction) is $mg\sin\theta$ where $m = 630$ kg and $\theta = 10.2°$. With $f = 62.0$ N, Newton's second law leads to $mg\sin\theta - f = ma$, which yields $a = 1.64$ m/s^2. Using Eq. 2-15, we have

$$80.0\text{ m} = \left(6.20\ \frac{\text{m}}{\text{s}}\right) t + \frac{1}{2}\left(1.64\ \frac{\text{m}}{\text{s}^2}\right) t^2 \ .$$

This is solved using the quadratic formula. The positive root is $t = 6.80$ s.

(b) Running through the calculation of part (a) with $f = 42.0$ N instead of $f = 62$ N results in $t = 6.76$ s.

105. Except for replacing f_s with f_k, Fig 6-5 in the textbook is appropriate. With that figure in mind, we choose uphill as the $+x$ direction. Applying Newton's second law to the x axis, we have

$$f_k - W\sin\theta = ma \quad \text{where} \quad m = \frac{W}{g},$$

and where $W = 40$ N, $a = +0.80$ m/s^2 and $\theta = 25°$. Thus, we find $f_k = 20$ N. Along the y-axis, we have

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

so that $\mu_k = f_k / F_N = 0.56$.

Chapter 7

1. **THINK** As the proton is being accelerated, its speed increases, and so does its kinetic energy.

EXPRESS To calculate the speed of the proton at a later time, we use the equation $v^2 = v_0^2 + 2a\Delta x$ from Table 2-1. The change in kinetic energy is then equal to

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)\,.$$

ANALYZE (a) With $\Delta x = 3.5\text{ cm} = 0.035\text{ m}$ and $a = 3.6\times 10^{15}\text{ m/s}^2$, we find the proton speed to be

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{\left(2.4\times 10^7\text{ m/s}\right)^2 + 2\,\left(3.6\times 10^{15}\text{ m/s}^2\right)\left(0.035\text{ m}\right)} = 2.9\times 10^7\text{ m/s.}$$

(b) The initial kinetic energy is

$$K_i = \frac{1}{2}mv_0^2 = \frac{1}{2}\,\left(1.67\times 10^{-27}\text{ kg}\right)\left(2.4\times 10^7\text{ m/s}\right)^2 = 4.8\times 10^{-13}\text{ J,}$$

and the final kinetic energy is

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}\,\left(1.67\times 10^{-27}\text{ kg}\right)\left(2.9\times 10^7\text{ m/s}\right)^2 = 6.9\times 10^{-13}\text{ J.}$$

Thus, the change in kinetic energy is

$$\Delta K = K_f - K_i = 6.9\times 10^{-13}\text{ J} - 4.8\times 10^{-13}\text{ J} = 2.1\times 10^{-13}\text{ J.}$$

LEARN The change in kinetic energy can be rewritten as

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m(2a\Delta x) = ma\Delta x = F\Delta x = W$$

which, according to the work-kinetic energy theorem, is simply the work done on the particle.

2. With speed $v = 11200$ m/s, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.9\times10^5 \text{ kg})\ (11200 \text{ m/s})^2 = 1.8\times10^{13} \text{ J}.$$

3. (a) The change in kinetic energy for the meteorite would be

$$\Delta K = K_f - K_i = -K_i = -\frac{1}{2}m_i v_i^2 = -\frac{1}{2}\left(4\times10^6 \text{ kg}\right)\left(15\times10^3 \text{ m/s}\right)^2 = -5\times10^{14} \text{ J},$$

or $|\Delta K| = 5\times10^{14}$ J. The negative sign indicates that kinetic energy is lost.

(b) The energy loss in units of megatons of TNT would be

$$-\Delta K = \left(5\times10^{14} \text{ J}\right)\left(\frac{1 \text{ megaton TNT}}{4.2\times10^{15} \text{ J}}\right) = 0.1 \text{megaton TNT}.$$

(c) The number of bombs N that the meteorite impact would correspond to is found by noting that megaton = 1000 kilotons and setting up the ratio:

$$N = \frac{0.1\times1000 \text{kiloton TNT}}{13 \text{kiloton TNT}} = 8.$$

4. (a) We set up the ratio

$$\frac{50 \text{ km}}{1 \text{ km}} = \left(\frac{E}{1 \text{ megaton}}\right)^{1/3}$$

and find $E = 50^3 \approx 1 \times 10^5$ megatons of TNT.

(b) We note that 15 kilotons is equivalent to 0.015 megatons. Dividing the result from part (a) by 0.013 yields about ten million (10^7) bombs.

5. We denote the mass of the father as m and his initial speed v_i. The initial kinetic energy of the father is

$$K_i = \frac{1}{2}K_{\text{son}}$$

and his final kinetic energy (when his speed is $v_f = v_i + 1.0$ m/s) is $K_f = K_{\text{son}}$. We use these relations along with Eq. 7-1 in our solution.

(a) We see from the above that $K_i = \frac{1}{2}K_f$, which (with SI units understood) leads to

$$\frac{1}{2}mv_i^2 = \frac{1}{2}\left[\frac{1}{2}m\ (v_i + 1.0\text{ m/s})^2\right].$$

The mass cancels and we find a second-degree equation for v_i : $\frac{1}{2}v_i^2 - v_i - \frac{1}{2} = 0$. The positive root (from the quadratic formula) yields v_i = 2.4 m/s.

(b) From the first relation above $\left(K_i = \frac{1}{2}K_{\text{son}}\right)$, we have

$$\frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{1}{2}\ (m/2)\ v_{\text{son}}^2\right)$$

and (after canceling m and one factor of 1/2) are led to $v_{\text{son}} = 2v_i = 4.8\text{ m/s}$.

6. We apply the equation $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$, found in Table 2-1. Since at t = 0 s, x_0 = 0, and $v_0 = 12\text{ m/s}$, the equation becomes (in unit of meters)

$$x(t) = 12t + \tfrac{1}{2}at^2.$$

With $x = 10\text{ m}$ when $t = 1.0\text{ s}$, the acceleration is found to be $a = -4.0\text{ m/s}^2$. The fact that $a < 0$ implies that the bead is decelerating. Thus, the position is described by $x(t) = 12t - 2.0t^2$. Differentiating x with respect to t then yields

$$v(t) = \frac{dx}{dt} = 12 - 4.0t.$$

Indeed at t =3.0 s, $v(t = 3.0) = 0$ and the bead stops momentarily. The speed at $t = 10\text{ s}$ is $v(t = 10) = -28\text{ m/s}$, and the corresponding kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.8\times 10^{-2}\text{kg})(-28\text{ m/s})^2 = 7.1\text{ J}.$$

7. Since this involves constant-acceleration motion, we can apply the equations of Table 2-1, such as $x = v_0 t + \frac{1}{2}at^2$ (where $x_0 = 0$). We choose to analyze the third and fifth points, obtaining

$$0.2\text{ m} = v_0(1.0\text{ s}) + \frac{1}{2}a\ (1.0\text{ s})^2$$
$$0.8\text{ m} = v_0(2.0\text{ s}) + \frac{1}{2}a\ (2.0\text{ s})^2.$$

Simultaneous solution of the equations leads to $v_0 = 0$ and $a = 0.40\,\text{m/s}^2$. We now have two ways to finish the problem. One is to compute force from $F = ma$ and then obtain the work from Eq. 7-7. The other is to find ΔK as a way of computing W (in accordance with Eq. 7-10). In this latter approach, we find the velocity at $t = 2.0\,\text{s}$ from $v = v_0 + at$ (so $v = 0.80\,\text{m/s}$). Thus,

$$W = \Delta K = \frac{1}{2}(3.0\,\text{kg})(0.80\,\text{m/s})^2 = 0.96\ \text{J}.$$

8. Using Eq. 7-8 (and Eq. 3-23), we find the work done by the water on the ice block:

$$W = \vec{F}\cdot\vec{d} = \left[(210\ \text{N})\,\hat{\text{i}} - (150\ \text{N})\,\hat{\text{j}}\right]\cdot\left[(15\ \text{m})\,\hat{\text{i}} - (12\ \text{m})\hat{\text{j}}\right] = (210\ \text{N})(15\ \text{m}) + (-150\ \text{N})(-12\ \text{m})$$
$$= 5.0\times10^3\ \text{J}.$$

9. By the work-kinetic energy theorem,

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\,\text{kg})\left((6.0\,\text{m/s})^2 - (4.0\,\text{m/s})^2\right) = 20\ \text{J}.$$

We note that the *directions* of $\vec{v}_f$ and $\vec{v}_i$ play no role in the calculation.

10. Equation 7-8 readily yields

$$W = F_x\Delta x + F_y\Delta y = (2.0\ \text{N})\cos(100^\circ)(3.0\ \text{m}) + (2.0\ \text{N})\sin(100^\circ)(4.0\ \text{m}) = 6.8\ \text{J}.$$

11. Using the work-kinetic energy theorem, we have

$$\Delta K = W = \vec{F}\cdot\vec{d} = Fd\cos\phi.$$

In addition, $F = 12\ \text{N}$ and $d = \sqrt{(2.00\ \text{m})^2 + (-4.00\ \text{m})^2 + (3.00\ \text{m})^2} = 5.39\ \text{m}$.

(a) If $\Delta K = +30.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{30.0\ \text{J}}{(12.0\ \text{N})(5.39\ \text{m})}\right) = 62.3^\circ.$$

(b) $\Delta K = -30.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{-30.0\ \text{J}}{(12.0\ \text{N})(5.39\ \text{m})}\right) = 118^\circ.$$

12. (a) From Eq. 7-6, $F = W/x = 3.00$ N (this is the slope of the graph).

(b) Equation 7-10 yields $K = K_i + W = 3.00$ J + 6.00 J = 9.00 J.

13. We choose $+x$ as the direction of motion (so $\vec{a}$ and $\vec{F}$ are negative-valued).

(a) Newton's second law readily yields $\vec{F} = (85\,\text{kg})(-2.0\,\text{m/s}^2)$ so that

$$F = |\vec{F}| = 1.7\times 10^2\ \text{N}.$$

(b) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \ \Rightarrow\ \Delta x = -\frac{(37\,\text{m/s})^2}{2(-2.0\,\text{m/s}^2)} = 3.4\times 10^2\ \text{m}.$$

Alternatively, this can be worked using the work-energy theorem.

(c) Since $\vec{F}$ is opposite to the direction of motion (so the angle ϕ between $\vec{F}$ and $\vec{d} = \Delta x$ is 180°) then Eq. 7-7 gives the work done as $W = -F\Delta x = -5.8\times 10^4$ J.

(d) In this case, Newton's second law yields $\vec{F} = (85\,\text{kg})(-4.0\,\text{m/s}^2)$ so that $F = |\vec{F}| = 3.4\times 10^2$ N.

(e) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37\,\text{m/s})^2}{2(-4.0\,\text{m/s}^2)} = 1.7\times 10^2\ \text{m}.$$

(f) The force $\vec{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to $W = -F\Delta x = -5.8\times 10^4$ J. The fact that this agrees with the result of part (c) provides insight into the concept of work.

14. The forces are all constant, so the total work done by them is given by $W = F_{\text{net}}\Delta x$, where F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. We add the three vectors, finding the x and y components of the net force:

$$\begin{aligned} F_{\text{net}\,x} &= -F_1 - F_2\sin 50.0° + F_3\cos 35.0° = -3.00\ \text{N} - (4.00\ \text{N})\sin 35.0° + (10.0\ \text{N})\cos 35.0° \\ &= 2.13\ \text{N} \end{aligned}$$

$$\begin{aligned} F_{\text{net}\,y} &= -F_2\cos 50.0° + F_3\sin 35.0° = -(4.00\ \text{N})\ \cos 50.0° + (10.0\ \text{N})\sin 35.0° \\ &= 3.17\ \text{N}. \end{aligned}$$

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{F_{\text{net}\,x}^2 + F_{\text{net}\,y}^2} = \sqrt{(2.13\text{ N})^2 + (3.17\text{ N})^2} = 3.82\text{ N}.$$

The work done by the net force is

$$W = F_{\text{net}}d = (3.82\text{ N})(4.00\text{ m}) = 15.3\text{ J}$$

where we have used the fact that $\vec{d} \,||\, \vec{F}_{\text{net}}$ (which follows from the fact that the canister started from rest and moved horizontally under the action of horizontal forces — the resultant effect of which is expressed by $\vec{F}_{\text{net}}$).

15. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F}\cdot\vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d\cos\phi_1 = (5.00\text{ N})(3.00\text{ m})\cos 0° = 15.0\text{ J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d\cos\phi_2 = (9.00\text{ N})(3.00\text{ m})\cos 120° = -13.5\text{ J}.$$

Force $\vec{F}_3$ is perpendicular to the displacement, so

$$W_3 = F_3 d\cos\phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\text{ J} - 13.5\text{ J} + 0 = +1.50\text{ J}.$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.

16. The change in kinetic energy can be written as

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m(2a\Delta x) = ma\Delta x$$

where we have used $v_f^2 = v_i^2 + 2a\Delta x$ from Table 2-1. From the figure, we see that $\Delta K = (0-30)\text{ J} = -30\text{ J}$ when $\Delta x = +5\text{ m}$. The acceleration can then be obtained as

$$a = \frac{\Delta K}{m\Delta x} = \frac{(-30\text{ J})}{(8.0\text{ kg})(5.0\text{ m})} = -0.75\text{ m/s}^2.$$

The negative sign indicates that the mass is decelerating. From the figure, we also see that when $x = 5$ m the kinetic energy becomes zero, implying that the mass comes to rest momentarily. Thus,

$$v_0^2 = v^2 - 2a\Delta x = 0 - 2(-0.75\text{ m/s}^2)(5.0\text{ m}) = 7.5\text{ m}^2/\text{s}^2,$$

or $v_0 = 2.7$ m/s. The speed of the object when $x = -3.0$ m is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{7.5\text{ m}^2/\text{s}^2 + 2(-0.75\text{ m/s}^2)(-3.0\text{ m})} = \sqrt{12}\text{ m/s} = 3.5\text{ m/s}.$$

17. **THINK** The helicopter does work to lift the astronaut upward against gravity. The work done on the astronaut is converted to the kinetic energy of the astronaut.

EXPRESS We use $\vec{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg downward. Furthermore, the acceleration of the astronaut is $a = g/10$ upward. According to Newton's second law, the force is given by

$$F - mg = ma \quad \Rightarrow \quad F = m(g + a) = \frac{11}{10}mg,$$

in the same direction as the displacement. On the other hand, the force of gravity has magnitude $F_g = mg$ and is opposite in direction to the displacement.

ANALYZE (a) Since the force of the cable $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11\,(72\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})}{10} = 1.164\times10^4\text{ J} \approx 1.2\times10^4\text{ J}.$$

(b) Using Eq. 7-7, the work done by gravity is

$$W_g = -F_g d = -mgd = -\,(72\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = -1.058\times10^4\text{ J} \approx -1.1\times10^4\text{ J}.$$

(c) The total work done is the sum of the two works:

$$W_{\text{net}} = W_F + W_g = 1.164\times10^4\text{ J} - 1.058\times10^4\text{ J} = 1.06\times10^3\text{ J} \approx 1.1\times10^3\text{ J}.$$

Since the astronaut started from rest, the work-kinetic energy theorem tells us that this is her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$, her final speed is $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(1.06\times10^3\text{ J})}{72\text{ kg}}} = 5.4\text{ m/s}.$

LEARN For a general upward acceleration *a*, the net work done is

$$W_{\text{net}} = W_F + W_g = Fd - F_g d = m(g+a)d - mgd = mad.$$

Since $W_{\text{net}} = \Delta K = mv^2/2$ by the work-kinetic energy theorem, the speed of the astronaut would be $v = \sqrt{2ad}$, which is independent of the mass of the astronaut. In our case, $v = \sqrt{2(9.8\text{ m/s}^2/10)(15\text{ m})} = 5.4\text{ m/s}$, which agrees with that calculated in (d).

18. In both cases, there is no acceleration, so the lifting force is equal to the weight of the object.

(a) Equation 7-8 leads to $W = \vec{F}\cdot\vec{d} = (360\,\text{kN})(0.10\,\text{m}) = 36\text{ kJ}$.

(b) In this case, we find $W = (4000\text{ N})(0.050\text{ m}) = 2.0\times10^2\text{ J}$.

19. Equation 7-15 applies, but the wording of the problem suggests that it is only necessary to examine the contribution from the rope (which would be the "W_a" term in Eq. 7-15):

$$W_a = -(50\text{ N})(0.50\text{ m}) = -25\text{ J}$$

(the minus sign arises from the fact that the pull from the rope is anti-parallel to the direction of motion of the block). Thus, the kinetic energy would have been 25 J greater if the rope had not been attached (given the same displacement).

20. From the figure, one may write the kinetic energy (in units of J) as a function of *x* as

$$K = K_s - 20x = 40 - 20x\,.$$

Since $W = \Delta K = \vec{F}_x \cdot \Delta x$, the component of the force along the force along $+x$ is $F_x = dK/dx = -20$ N. The normal force on the block is $F_N = F_y$, which is related to the gravitational force by

$$mg = \sqrt{F_x^2 + (-F_y)^2}\,.$$

(Note that F_N points in the opposite direction of the component of the gravitational force.) With an initial kinetic energy $K_s = 40.0\text{ J}$ and $v_0 = 4.00\text{ m/s}$, the mass of the block is

$$m = \frac{2K_s}{v_0^2} = \frac{2(40.0\text{ J})}{(4.00\text{ m/s})^2} = 5.00\text{ kg.}$$

Thus, the normal force is

$$F_y = \sqrt{(mg)^2 - F_x^2} = \sqrt{(5.0\text{ kg})^2(9.8\text{ m/s}^2)^2 - (20\text{ N})^2} = 44.7\text{ N} \approx 45\text{ N.}$$

21. **THINK** In this problem the cord is doing work on the block so that it does not undergo free fall.

EXPRESS We use F to denote the magnitude of the force of the cord on the block. This force is upward, opposite to the force of gravity (which has magnitude $F_g = Mg$), to prevent the block from undergoing free fall. The acceleration is $\vec{a} = g/4$ downward. Taking the downward direction to be positive, then Newton's second law yields

$$\vec{F}_{\text{net}} = m\vec{a} \Rightarrow Mg - F = M\left(\frac{g}{4}\right),$$

so $F = 3Mg/4$, in the opposite direction of the displacement. On the other hand, the force of gravity $F_g = mg$ is in the same direction to the displacement.

ANALYZE (a) Since the displacement is downward, the work done by the cord's force is, using Eq. 7-7,

$$W_F = -Fd = -\frac{3}{4}Mgd.$$

(b) Similarly, the work done by the force of gravity is $W_g = F_g d = Mgd$.

(c) The total work done on the block is simply the sum of the two works:

$$W_{\text{net}} = W_F + W_g = -\frac{3}{4}Mgd + Mgd = \frac{1}{4}Mgd .$$

Since the block starts from rest, we use Eq. 7-15 to conclude that this $(Mgd/4)$ is the block's kinetic energy K at the moment it has descended the distance d.

(d) With $K = \frac{1}{2}Mv^2$, the speed is

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2(Mgd/4)}{M}} = \sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

LEARN For a general downward acceleration a, the force exerted by the cord is $F = m(g-a)$, so that the net work done on the block is $W_{\text{net}} = F_{\text{net}}d = mad$. The speed of the block after falling a distance d is $v = \sqrt{2ad}$. In the special case where the block hangs still, $a = 0$, $F = mg$ and $v = 0$. In our case, $a = g/4$, and $v = \sqrt{2(g/4)d} = \sqrt{gd/2}$, which agrees with that calculated in (d).

22. We use d to denote the magnitude of the spelunker's displacement during each stage. The mass of the spelunker is m = 80.0 kg. The work done by the lifting force is denoted W_i where i = 1, 2, 3 for the three stages. We apply the work-energy theorem, Eq. 17-15.

(a) For stage 1, $W_1 - mgd = \Delta K_1 = \frac{1}{2}mv_1^2$, where $v_1 = 5.00$ m/s. This gives

$$W_1 = mgd + \frac{1}{2}mv_1^2 = (80.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) + \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2 = 8.84\times10^3\text{ J}.$$

(b) For stage 2, $W_2 - mgd = \Delta K_2 = 0$, which leads to

$$W_2 = mgd = (80.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) = 7.84\times10^3\text{ J}.$$

(c) For stage 3, $W_3 - mgd = \Delta K_3 = -\frac{1}{2}mv_1^2$. We obtain

$$W_3 = mgd - \frac{1}{2}mv_1^2 = (80.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) - \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2 = 6.84\times10^3\text{ J}.$$

23. The fact that the applied force $\vec{F}_a$ causes the box to move up a frictionless ramp at a constant speed implies that there is no net change in the kinetic energy: $\Delta K = 0$. Thus, the work done by $\vec{F}_a$ must be equal to the negative work done by gravity: $W_a = -W_g$. Since the box is displaced vertically upward by $h = 0.150$ m, we have

$$W_a = +mgh = (3.00\text{ kg})(9.80\text{ m/s}^2)(0.150\text{ m}) = 4.41\text{ J}$$

24. (a) Using notation common to many vector-capable calculators, we have (from Eq. 7-8) W = dot([20.0,0] + [0, –(3.00)(9.8)], [0.500 ∠ 30.0º]) = +1.31 J , where "dot" stands for dot product.

(b) Eq. 7-10 (along with Eq. 7-1) then leads to $v = \sqrt{2(1.31\text{ J})/(3.00\text{ kg})}$ = 0.935 m/s.

25. (a) The net upward force is given by

$$F + F_N - (m+M)g = (m+M)a$$

where m = 0.250 kg is the mass of the cheese, M = 900 kg is the mass of the elevator cab, F is the force from the cable, and $F_N = 3.00$ N is the normal force on the cheese. On the cheese alone, we have

$$F_N - mg = ma \;\Rightarrow\; a = \frac{3.00\text{ N} - (0.250\text{ kg})(9.80\text{ m/s}^2)}{0.250\text{ kg}} = 2.20\text{ m/s}^2.$$

Thus the force from the cable is $F=(m+M)(a+g)-F_N=1.08\times10^4$ N, and the work done by the cable on the cab is

$$W=Fd_1=(1.80\times10^4\text{ N})(2.40\text{ m})=2.59\times10^4\text{ J}.$$

(b) If $W=92.61$ kJ and $d_2=10.5$ m, the magnitude of the normal force is

$$F_N=(m+M)g-\frac{W}{d_2}=(0.250\text{ kg}+900\text{ kg})(9.80\text{ m/s}^2)-\frac{9.261\times10^4\text{ J}}{10.5\text{ m}}=2.45\text{ N}.$$

26. We make use of Eq. 7-25 and Eq. 7-28 since the block is stationary before and after the displacement. The work done by the applied force can be written as

$$W_a=-W_s=\frac{1}{2}k(x_f^2-x_i^2).$$

The spring constant is $k=(80\text{ N})/(2.0\text{ cm})=4.0\times10^3$N/m. With $W_a=4.0$ J, and $x_i=-2.0$ cm, we have

$$x_f=\pm\sqrt{\frac{2W_a}{k}+x_i^2}=\pm\sqrt{\frac{2(4.0\text{ J})}{(4.0\times10^3\text{ N/m})}+(-0.020\text{ m})^2}=\pm0.049\text{ m}=\pm4.9\text{ cm}.$$

27. From Eq. 7-25, we see that the work done by the spring force is given by

$$W_s=\frac{1}{2}k(x_i^2-x_f^2).$$

The fact that 360 N of force must be applied to pull the block to x = + 4.0 cm implies that the spring constant is

$$k=\frac{360\text{ N}}{4.0\text{ cm}}=90\text{ N/cm}=9.0\times10^3\text{ N/m}.$$

(a) When the block moves from $x_i=+5.0$ cm to $x=+3.0$ cm, we have

$$W_s=\frac{1}{2}(9.0\times10^3\text{ N/m})[(0.050\text{ m})^2-(0.030\text{ m})^2]=7.2\text{ J}.$$

(b) Moving from $x_i=+5.0$ cm to $x=-3.0$ cm, we have

$$W_s=\frac{1}{2}(9.0\times10^3\text{ N/m})[(0.050\text{ m})^2-(-0.030\text{ m})^2]=7.2\text{ J}.$$

(c) Moving from $x_i = +5.0$ cm to $x = -5.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.050 \text{ m})^2] = 0 \text{ J}.$$

(d) Moving from $x_i = +5.0$ cm to $x = -9.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.090 \text{ m})^2] = -25 \text{ J}.$$

28. The spring constant is k = 100 N/m and the maximum elongation is x_i = 5.00 m. Using Eq. 7-25 with $x_f = 0$, the work is found to be

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}(100 \text{ N/m})(5.00 \text{ m})^2 = 1.25\times10^3 \text{ J}.$$

29. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. The spring constant k can be deduced from the figure which shows the amount of work done to pull the block from 0 to x = 3.0 cm. The parabola $W_a = kx^2/2$ contains (0,0), (2.0 cm, 0.40 J) and (3.0 cm, 0.90 J). Thus, we may infer from the data that $k = 2.0\times10^3$ N/m.

(a) When the block moves from $x_i = +5.0$ cm to $x = +4.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (0.040 \text{ m})^2] = 0.90 \text{ J}.$$

(b) Moving from $x_i = +5.0$ cm to $x = -2.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.020 \text{ m})^2] = 2.1 \text{ J}.$$

(c) Moving from $x_i = +5.0$ cm to $x = -5.0$ cm, we have

$$W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.050 \text{ m})^2] = 0 \text{ J}.$$

30. Hooke's law and the work done by a spring is discussed in the chapter. We apply the work-kinetic energy theorem, in the form of $\Delta K = W_a + W_s$, to the points in the figure at x = 1.0 m and x = 2.0 m, respectively. The "applied" work W_a is that due to the constant force $\vec{P}$.

$$4\text{ J} = P(1.0\text{ m}) - \frac{1}{2}k(1.0\text{ m})^2$$
$$0 = P(2.0\text{ m}) - \frac{1}{2}k(2.0\text{ m})^2.$$

(a) Simultaneous solution leads to P = 8.0 N.

(b) Similarly, we find k = 8.0 N/m.

31. **THINK** The applied force varies with x, so an integration is required to calculate the work done on the body.

EXPRESS As the body moves along the x axis from x_i = 3.0 m to x_f = 4.0 m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x\ dx = \int_{x_i}^{x_f} -6x\ dx = -3(x_f^2 - x_i^2) = -3\ (4.0^2 - 3.0^2) = -21\text{ J}.$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). Given v_i, we can readily calculate v_f.

ANALYZE (a) The work-kinetic theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21\text{ J})}{2.0\text{ kg}} + (8.0\text{ m/s})^2} = 6.6\text{ m/s}.$$

(b) The velocity of the particle is v_f = 5.0 m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3\left(x_f^2 - x_i^2\right)$, so the theorem leads to

$$W = \Delta K \quad \Rightarrow \quad -3\left(x_f^2 - x_i^2\right) = \frac{1}{2}m\ \left(v_f^2 - v_i^2\right).$$

Thus,

$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.0\text{ kg}}{6\text{ N/m}}\left((5.0\text{ m/s})^2 - (8.0\text{ m/s})^2\right) + (3.0\text{ m})^2} = 4.7\text{ m}.$$

LEARN Since $x_f > x_i$, $W = -3(x_f^2 - x_i^2) < 0$, i.e., the work done by the force is negative. From the work-kinetic energy theorem, this implies $\Delta K < 0$. Hence, the speed of the particle will continue to decrease as it moves in the +x-direction.

32. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. Since $F_x = -kx$, the slope in Fig. 7-37 corresponds to the spring constant *k*. Its value is given by $k = 80 \text{ N/cm} = 8.0\times10^3 \text{ N/m}$.

(a) When the block moves from $x_i = +8.0 \text{ cm}$ to $x = +5.0 \text{ cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3 \text{ N/m})[(0.080 \text{ m})^2 - (0.050 \text{ m})^2] = 15.6 \text{ J} \approx 16 \text{ J}.$$

(b) Moving from $x_i = +8.0 \text{ cm}$ to $x = -5.0 \text{ cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3 \text{ N/m})[(0.080 \text{ m})^2 - (-0.050 \text{ m})^2] = 15.6 \text{ J} \approx 16 \text{ J}.$$

(c) Moving from $x_i = +8.0 \text{ cm}$ to $x = -8.0 \text{ cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3 \text{ N/m})[(0.080 \text{ m})^2 - (-0.080 \text{ m})^2] = 0 \text{ J}.$$

(d) Moving from $x_i = +8.0 \text{ cm}$ to $x = -10.0 \text{ cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3 \text{ N/m})[(0.080 \text{ m})^2 - (-0.10 \text{ m})^2] = -14.4 \text{ J} \approx -14 \text{ J}.$$

33. (a) This is a situation where Eq. 7-28 applies, so we have

$$Fx = \tfrac{1}{2}kx^2 \Rightarrow (3.0 \text{ N})\, x = \tfrac{1}{2}(50 \text{ N/m})x^2$$

which (other than the trivial root) gives $x = (3.0/25) \text{ m} = 0.12 \text{ m}$.

(b) The work done by the applied force is $W_a = Fx = (3.0 \text{ N})(0.12 \text{ m}) = 0.36 \text{ J}$.

(c) Eq. 7-28 immediately gives $W_s = -W_a = -0.36 \text{ J}$.

(d) With $K_f = K$ considered variable and $K_i = 0$, Eq. 7-27 gives $K = Fx - \tfrac{1}{2}kx^2$. We take the derivative of *K* with respect to *x* and set the resulting expression equal to zero, in order to find the position x_c taht corresponds to a maximum value of *K*:

$$x_c = \frac{F}{k} = (3.0/50) \text{ m} = 0.060 \text{ m}.$$

We note that x_c is also the point where the applied and spring forces "balance."

(e) At x_c we find $K = K_{max} = 0.090$ J.

34. According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = \frac{20 \text{ m/s}^2}{8.0 \text{ m}} = 2.5 \text{ s}^{-2}.$$

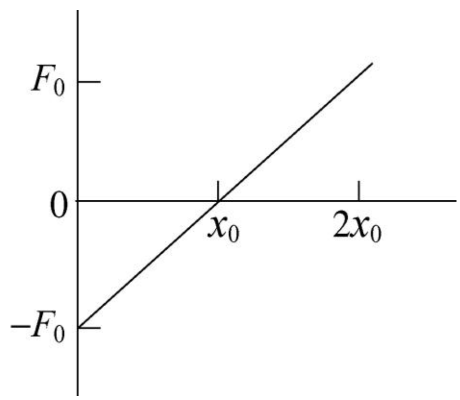
The force on the brick is in the positive x direction and, according to Newton's second law, its magnitude is given by $F = ma = m\alpha x$. If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F\ dx = m\alpha \int_0^{x_f} x\ dx = \frac{m\alpha}{2} x_f^2 = \frac{(10\text{ kg})(2.5\text{ s}^{-2})}{2}(8.0\text{ m})^2 = 8.0 \times 10^2\text{ J}.$$

35. **THINK** We have an applied force that varies with x. An integration is required to calculate the work done on the particle.

EXPRESS Given a one-dimensional force $F(x)$, the work done is simply equal to the area under the curve: $W = \int_{x_i}^{x_f} F(x)\ dx$.

ANALYZE (a) The plot of $F(x)$ is shown to the right. Here we take x_0 to be positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$.



Since the area of a triangle is (base)(altitude)/2, the work done from $x = 0$ to $x = x_0$ is $W_1 = -(x_0)(F_0)/2$ and the work done from $x = x_0$ to $x = 2x_0$ is

$$W_2 = (2x_0 - x_0)(F_0)/2 = (x_0)(F_0)/2$$

The total work is the sum of the two:

$$W = W_1 + W_2 = -\frac{1}{2}F_0x_0 + \frac{1}{2}F_0x_0 = 0.$$

(b) The integral for the work is

$$W = \int_0^{2x_0} F_0\left(\frac{x}{x_0} - 1\right)dx = F_0\left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{2x_0} = 0.$$

LEARN If the particle starts out at $x = 0$ with an initial speed v_i, with a negative work $W_1 = -F_0x_0/2 < 0$, its speed at $x = x_0$ will decrease to

$$v=\sqrt{v_i^2+\frac{2W_1}{m}}=\sqrt{v_i^2-\frac{F_0x_0}{m}}<v_i\,,$$

but return to v_i again at $x=2x_0$ with a positive work $W_2=F_0x_0/2>0$.

36. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. Finding that area (in terms of rectangular [length × width] and triangular $[\frac{1}{2}\text{ base}\times\text{height}]$ areas) we obtain

$$W=W_{0<x<2}+W_{2<x<4}+W_{4<x<6}+W_{6<x<8}=(20+10+0-5)\text{ J}=25\text{ J}.$$

37. (a) We first multiply the vertical axis by the mass, so that it becomes a graph of the applied force. Now, adding the triangular and rectangular "areas" in the graph (for $0 \le x \le 4$) gives 42 J for the work done.

(b) Counting the "areas" under the axis as negative contributions, we find (for $0 \le x \le 7$) the work to be 30 J at x = 7.0 m.

(c) And at x = 9.0 m, the work is 12 J.

(d) Equation 7-10 (along with Eq. 7-1) leads to speed v = 6.5 m/s at x = 4.0 m. Returning to the original graph (where a was plotted) we note that (since it started from rest) it has received acceleration(s) (up to this point) only in the $+x$ direction and consequently must have a velocity vector pointing in the $+x$ direction at x = 4.0 m.

(e) Now, using the result of part (b) and Eq. 7-10 (along with Eq. 7-1) we find the speed is 5.5 m/s at x = 7.0 m. Although it has experienced some deceleration during the $0 \le x \le 7$ interval, its velocity vector still points in the $+x$ direction.

(f) Finally, using the result of part (c) and Eq. 7-10 (along with Eq. 7-1) we find its speed v = 3.5 m/s at x = 9.0 m. It certainly has experienced a significant amount of deceleration during the $0 \le x \le 9$ interval; nonetheless, its velocity vector *still* points in the $+x$ direction.

38. (a) Using the work-kinetic energy theorem

$$K_f=K_i+\int_0^{2.0}(2.5-x^2)\,dx=0+(2.5)(2.0)-\frac{1}{3}(2.0)^3=2.3\text{ J}.$$

(b) For a variable end-point, we have K_f as a function of x, which could be differentiated to find the extremum value, but we recognize that this is equivalent to solving F = 0 for x:

$$F=0\;\Rightarrow\;2.5-x^2=0.$$

Thus, K is extremized at $x = \sqrt{2.5} \approx 1.6$ m and we obtain

$$K_f = K_i + \int_0^{\sqrt{2.5}} (2.5 - x^2)dx = 0 + (2.5)(\sqrt{2.5}) - \frac{1}{3}(\sqrt{2.5})^3 = 2.6 \text{ J}.$$

Recalling our answer for part (a), it is clear that this extreme value is a maximum.

39. As the body moves along the x axis from $x_i = 0$ m to $x_f = 3.00$ m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} (cx - 3.00x^2)dx = \left(\frac{c}{2}x^2 - x^3\right)_0^3 = \frac{c}{2}(3.00)^2 - (3.00)^3$$
$$= 4.50c - 27.0.$$

However, $W = \Delta K = (11.0 - 20.0) = -9.00$ J from the work-kinetic energy theorem. Thus,

$$4.50c - 27.0 = -9.00$$

or $c = 4.00$ N/m.

40. Using Eq. 7-32, we find

$$W = \int_{0.25}^{1.25} e^{-4x^2} \, dx = 0.21 \text{ J}$$

where the result has been obtained numerically. Many modern calculators have that capability, as well as most math software packages that a great many students have access to.

41. We choose to work this using Eq. 7-10 (the work-kinetic energy theorem). To find the initial and final kinetic energies, we need the speeds, so

$$v = \frac{dx}{dt} = 3.0 - 8.0t + 3.0t^2$$

in SI units. Thus, the initial speed is $v_i = 3.0$ m/s and the speed at $t = 4$ s is $v_f = 19$ m/s. The change in kinetic energy for the object of mass $m = 3.0$ kg is therefore

$$\Delta K = \frac{1}{2} m \left(v_f^2 - v_i^2\right) = 528 \text{ J}$$

which we round off to two figures and (using the work-kinetic energy theorem) conclude that the work done is $W = 5.3 \times 10^2$ J.

42. We solve the problem using the work-kinetic energy theorem, which states that the change in kinetic energy is equal to the work done by the applied force, $\Delta K = W$. In our

problem, the work done is $W = Fd$, where F is the tension in the cord and d is the length of the cord pulled as the cart slides from x_1 to x_2. From the figure, we have

$$d = \sqrt{x_1^2 + h^2} - \sqrt{x_2^2 + h^2} = \sqrt{(3.00 \text{ m})^2 + (1.20 \text{ m})^2} - \sqrt{(1.00 \text{ m})^2 + (1.20 \text{ m})^2}$$
$$= 3.23 \text{ m} - 1.56 \text{ m} = 1.67 \text{ m}$$

which yields $\Delta K = Fd = (25.0 \text{ N})(1.67 \text{ m}) = 41.7 \text{ J}$.

43. **THINK** This problem deals with the power and work done by a constant force.

EXPRESS The power done by a constant force F is given by $P = Fv$ and the work done by F from time t_1 to time t_2 is

$$W = \int_{t_1}^{t_2} P\, dt = \int_{t_1}^{t_2} Fv\, dt$$

Since F is the magnitude of the net force, the magnitude of the acceleration is $a = F/m$. Thus, if the initial velocity is $v_0 = 0$, then the velocity of the body as a function of time is given by $v = v_0 + at = (F/m)t$. Substituting the expression for v into the equation above, the work done during the time interval (t_1, t_2) becomes

$$W = \int_{t_1}^{t_2} (F^2/m)t\, dt = \frac{F^2}{2m}\left(t_2^2 - t_1^2\right).$$

ANALYZE (a) For $t_1 = 0$ and $t_2 = 1.0$ s, $W = \frac{1}{2}\left(\frac{(5.0 \text{ N})^2}{15 \text{ kg}}\right)[(1.0 \text{ s})^2 - 0] = 0.83 \text{ J}$.

(b) For $t_1 = 1.0$s, and $t_2 = 2.0$ s, $W = \frac{1}{2}\left(\frac{(5.0 \text{ N})^2}{15 \text{ kg}}\right)[(2.0 \text{ s})^2 - (1.0 \text{ s})^2] = 2.5 \text{ J}$.

(c) For $t_1 = 2.0$ s and $t_2 = 3.0$ s, $W = \frac{1}{2}\left(\frac{(5.0 \text{ N})^2}{15 \text{ kg}}\right)[(3.0 \text{ s})^2 - (2.0 \text{ s})^2] = 4.2 \text{ J}$.

(d) Substituting $v = (F/m)t$ into $P = Fv$ we obtain $P = F^2 t/m$ for the power at any time t. At the end of the third second, the instantaneous power is

$$P = \left(\frac{(5.0 \text{ N})^2 (3.0 \text{ s})}{15 \text{ kg}}\right) = 5.0 \text{ W}.$$

LEARN The work done here is quadratic in t. Therefore, from the definition $P = dW/dt$ for the instantaneous power, we see that P increases linearly with t.

44. (a) Since constant speed implies $\Delta K = 0$, we require $W_a = -W_g$, by Eq. 7-15. Since W_g is the same in both cases (same weight and same path), then $W_a = 9.0\times10^2$ J just as it was in the first case.

(b) Since the speed of 1.0 m/s is constant, then 8.0 meters is traveled in 8.0 seconds. Using Eq. 7-42, and noting that average power is *the* power when the work is being done at a steady rate, we have

$$P = \frac{W}{\Delta t} = \frac{900 \text{ J}}{8.0 \text{ s}} = 1.1\times10^2 \text{ W}.$$

(c) Since the speed of 2.0 m/s is constant, 8.0 meters is traveled in 4.0 seconds. Using Eq. 7-42, with *average power* replaced by *power*, we have

$$P = \frac{W}{\Delta t} = \frac{900 \text{ J}}{4.0 \text{ s}} = 225 \text{ W} \approx 2.3\times10^2 \text{ W}.$$

45. **THINK** A block is pulled at a constant speed by a force directed at some angle with respect to the direction of motion. The quantity we're interested in is the power, or the time rate at which work is done by the applied force.

EXPRESS The power associated with force $\vec{F}$ is given by $P = \vec{F}\cdot\vec{v} = Fv\cos\phi$, where $\vec{v}$ is the velocity of the object on which the force acts, and ϕ is the angle between $\vec{F}$ and $\vec{v}$.

ANALYZE With $F = 122$ N, $v = 5.0$ m/s and $\phi = 37.0°$, we find the power to be

$$P = Fv\cos\phi = (122 \text{ N})(5.0 \text{ m/s})\cos 37.0° = 4.9\times10^2 \text{ W}.$$

LEARN From the expression $P = Fv\cos\phi$, we see that the power is at a maximum when $\vec{F}$ and $\vec{v}$ are in the same direction ($\phi = 0$), and is zero when they are perpendicular of each other. In addition, we're told that the block moves at a constant speed, so $\Delta K = 0$, and the net work done on it must also be zero by the work-kinetic energy theorem. Thus, the applied force here must be compensating another force (e.g., friction) for the net rate to be zero.

46. Recognizing that the force in the cable must equal the total weight (since there is no acceleration), we employ Eq. 7-47:

$$P = Fv\cos\theta = mg\left(\frac{\Delta x}{\Delta t}\right)$$

where we have used the fact that $\theta = 0°$ (both the force of the cable and the elevator's motion are upward). Thus,

$$P = (3.0\times10^3 \text{ kg})(9.8 \text{ m/s}^2)\left(\frac{210 \text{ m}}{23 \text{ s}}\right) = 2.7\times10^5 \text{ W}.$$

47. (a) Equation 7-8 yields

$W = F_x\,\Delta x + F_y\,\Delta y + F_z\,\Delta z$
$= (2.00 \text{ N})(7.5 \text{ m} - 0.50 \text{ m}) + (4.00 \text{ N})(12.0 \text{ m} - 0.75 \text{ m}) + (6.00 \text{ N})(7.2\text{m} - 0.20 \text{ m})$
$= 101 \text{ J} \approx 1.0\times 10^2 \text{ J}.$

(b) Dividing this result by 12 s (see Eq. 7-42) yields $P = 8.4$ W.

48. (a) Since the force exerted by the spring on the mass is zero when the mass passes through the equilibrium position of the spring, the rate at which the spring is doing work on the mass at this instant is also zero.

(b) The rate is given by $P = \vec{F}\cdot\vec{v} = -Fv$, where the minus sign corresponds to the fact that $\vec{F}$ and $\vec{v}$ are anti-parallel to each other. The magnitude of the force is given by

$$F = kx = (500 \text{ N/m})(0.10 \text{ m}) = 50 \text{ N},$$

while v is obtained from conservation of energy for the spring-mass system:

$$E = K + U = 10 \text{ J} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.30 \text{ kg})v^2 + \frac{1}{2}(500 \text{ N/m})(0.10 \text{ m})^2$$

which gives $v = 7.1$ m/s. Thus,

$$P = -Fv = -(50 \text{ N})(7.1 \text{ m/s}) = -3.5\times10^2 \text{ W}.$$

49. **THINK** We have a loaded elevator moving upward at a constant speed. The forces involved are: gravitational force on the elevator, gravitational force on the counterweight, and the force by the motor via cable.

EXPRESS The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system:

$$W = W_e + W_c + W_m.$$

Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero, i.e., $W = \Delta K = 0$.

ANALYZE The elevator moves *upward* through 54 m, so the work done by gravity on it is

$$W_e = -m_e gd = -(1200 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = -6.35 \times 10^5 \text{ J}.$$

The counterweight moves *downward* the same distance, so the work done by gravity on it is

$$W_c = m_c gd = (950 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = 5.03\times10^5 \text{ J}.$$

Since $W = 0$, the work done by the motor on the system is

$$W_m = -W_e - W_c = 6.35\times10^5 \text{ J} - 5.03\times10^5 \text{ J} = 1.32\times10^5 \text{ J}.$$

This work is done in a time interval of $\Delta t = 3.0 \text{ min} = 180 \text{ s}$, so the power supplied by the motor to lift the elevator is

$$P = \frac{W_m}{\Delta t} = \frac{1.32 \times 10^5 \text{ J}}{180 \text{ s}} = 7.4 \times 10^2 \text{ W}.$$

LEARN In general, the work done by the motor is $W_m = (m_e - m_c)gd$. So when the counterweight mass balances the total mass, $m_c = m_e$, no work is required by the motor.

50. (a) Using Eq. 7-48 and Eq. 3-23, we obtain

$$P = \vec{F} \cdot \vec{v} = (4.0\,\text{N})(-2.0 \text{ m/s}) + (9.0 \text{ N})(4.0 \text{ m/s}) = 28 \text{ W}.$$

(b) We again use Eq. 7-48 and Eq. 3-23, but with a one-component velocity: $\vec{v} = v\hat{\text{j}}$.

$$P = \vec{F} \cdot \vec{v} \quad \Rightarrow -12 \text{ W} = (-2.0 \text{ N})v.$$

which yields $v = 6$ m/s.

51. (a) The object's displacement is

$$\vec{d} = \vec{d}_f - \vec{d}_i = (-8.00 \text{ m})\hat{\text{i}} + (6.00 \text{ m})\hat{\text{j}} + (2.00 \text{ m})\hat{\text{k}}\,.$$

Thus, Eq. 7-8 gives

$$W = \vec{F} \cdot \vec{d} = (3.00 \text{ N})(-8.00 \text{ m}) + (7.00 \text{ N})(6.00 \text{ m}) + (7.00 \text{ N})(2.00 \text{ m}) = 32.0 \text{ J}.$$

(b) The average power is given by Eq. 7-42:

$$P_{\text{avg}} = \frac{W}{t} = \frac{32.0}{4.00} = 8.00 \text{ W}.$$

(c) The distance from the coordinate origin to the initial position is

$$d_i = \sqrt{(3.00\text{ m})^2 + (-2.00\text{ m})^2 + (5.00\text{ m})^2} = 6.16\text{ m},$$

and the magnitude of the distance from the coordinate origin to the final position is

$$d_f = \sqrt{(-5.00\text{ m})^2 + (4.00\text{ m})^2 + (7.00\text{ m})^2} = 9.49\text{ m}.$$

Their scalar (dot) product is

$$\vec{d}_i \cdot \vec{d}_f = (3.00\text{ m})(-5.00\text{ m}) + (-2.00\text{ m})(4.00\text{ m}) + (5.00\text{ m})(7.00\text{ m}) = 12.0\text{ m}^2.$$

Thus, the angle between the two vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d}_i \cdot \vec{d}_f}{d_i d_f}\right) = \cos^{-1}\left(\frac{12.0}{(6.16)(9.49)}\right) = 78.2°.$$

52. According to the problem statement, the power of the car is

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = \text{constant}.$$

The condition implies $dt = mvdv/P$, which can be integrated to give

$$\int_0^T dt = \int_0^{v_T} \frac{mvdv}{P} \;\Rightarrow\; T = \frac{mv_T^2}{2P}$$

where v_T is the speed of the car at $t = T$. On the other hand, the total distance traveled can be written as

$$L = \int_0^T vdt = \int_0^{v_T} v\frac{mvdv}{P} = \frac{m}{P}\int_0^{v_T} v^2 dv = \frac{mv_T^3}{3P}.$$

By squaring the expression for L and substituting the expression for T, we obtain

$$L^2 = \left(\frac{mv_T^3}{3P}\right)^2 = \frac{8P}{9m}\left(\frac{mv_T^2}{2P}\right)^3 = \frac{8PT^3}{9m}$$

which implies that

$$PT^3 = \frac{9}{8}mL^2 = \text{constant}.$$

Differentiating the above equation gives $dPT^3 + 3PT^2dT = 0,$ or $dT = -\dfrac{T}{3P}dP.$

53. (a) Noting that the x component of the third force is F_{3x} = (4.00 N)cos(60º), we apply Eq. 7-8 to the problem:

$$W = [5.00 \text{ N} - 1.00 \text{ N} + (4.00 \text{ N})\cos 60º](0.20 \text{ m}) = 1.20 \text{ J}.$$

(b) Equation 7-10 (along with Eq. 7-1) then yields $v = \sqrt{2W/m}$ = 1.10 m/s.

54. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. We find the area in terms of rectangular [length × width] and triangular [$\frac{1}{2}$ base × height] areas and use the work-kinetic energy theorem appropriately. The initial point is taken to be x = 0, where v_0 = 4.0 m/s.

(a) With $K_i = \frac{1}{2}mv_0^2 = 16 \text{ J},$ we have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4.0 \text{ J}$$

so that K_3 (the kinetic energy when x = 3.0 m) is found to equal 12 J.

(b) With SI units understood, we write $W_{3<x<x_f}$ as $F_x \Delta x = (-4.0 \text{ N})(x_f - 3.0 \text{ m})$ and apply the work-kinetic energy theorem:

$$K_{x_f} - K_3 = W_{3<x<x_f}$$
$$K_{xf} - 12 = (-4)(x_f - 3.0)$$

so that the requirement $K_{xf} = 8.0 \text{ J}$ leads to $x_f = 4.0 \text{ m}$.

(c) As long as the work is positive, the kinetic energy grows. The graph shows this situation to hold until x = 1.0 m. At that location, the kinetic energy is

$$K_1 = K_0 + W_{0<x<1} = 16 \text{ J} + 2.0 \text{ J} = 18 \text{ J}.$$

55. **THINK** A horse is doing work to pull a cart at a constant speed. We'd like to know the work done during a time interval and the corresponding average power.

EXPRESS The horse pulls with a force $\vec{F}$. As the cart moves through a displacement $\vec{d}$, the work done by $\vec{F}$ is $W = \vec{F} \cdot \vec{d} = Fd\cos\phi,$ where ϕ is the angle between $\vec{F}$ and $\vec{d}$.

ANALYZE (a) In 10 min the cart moves a distance

$$d = v\Delta t = \left(6.0 \ \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft/mi}}{60 \text{ min/h}}\right)(10 \text{ min}) = 5280 \text{ ft}$$

so that Eq. 7-7 yields

$$W = Fd\cos\phi = (40 \text{ lb})(5280 \text{ ft}) \cos 30° = 1.8\times10^5 \text{ ft}\cdot\text{lb}.$$

(b) The average power is given by Eq. 7-42. With $\Delta t = 10 \text{ min} = 600 \text{ s}$, we obtain

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{1.8\times10^5 \text{ ft}\cdot\text{lb}}{600 \text{ s}} = 305 \text{ ft}\cdot\text{lb/s},$$

which (using the conversion factor $1 \text{ hp} = 550 \text{ ft}\cdot\text{lb/s}$ found on the inside back cover) converts to $P_{\text{avg}} = 0.55$ hp.

LEARN The average power can also be calculate by using Eq. 7-48: $P_{\text{avg}} = Fv\cos\phi$. Converting the speed to $v = (6.0 \text{ mi/h})\left(\dfrac{5280 \text{ ft/mi}}{3600 \text{ s/h}}\right) = 8.8 \text{ ft/s}$, we get

$$P_{\text{avg}} = Fv\cos\phi = (40 \text{ lb})(8.8 \text{ ft/s})\cos 30° = 305 \text{ ft}\cdot\text{lb} = 0.55 \text{ hp}$$

which agrees with that found in (b).

56. The acceleration is constant, so we may use the equations in Table 2-1. We choose the direction of motion as $+x$ and note that the displacement is the same as the distance traveled, in this problem. We designate the force (assumed singular) along the x direction acting on the m = 2.0 kg object as F.

(a) With $v_0 = 0$, Eq. 2-11 leads to $a = v/t$. And Eq. 2-17 gives $\Delta x = \frac{1}{2}vt$. Newton's second law yields the force $F = ma$. Equation 7-8, then, gives the work:

$$W = F\Delta x = m\left(\frac{v}{t}\right)\left(\frac{1}{2}vt\right) = \frac{1}{2}mv^2$$

as we expect from the work-kinetic energy theorem. With v = 10 m/s, this yields $W = 1.0\times10^2$ J.

(b) Instantaneous power is defined in Eq. 7-48. With t = 3.0 s, we find

$$P = Fv = m\left(\frac{v}{t}\right)v = 67 \text{ W}.$$

(c) The velocity at $t' = 1.5 \text{ s}$ is $v' = at' = 5.0 \text{ m/s}$. Thus, $P' = Fv' = 33$ W.

57. (a) To hold the crate at equilibrium in the final situation, $\vec{F}$ must have the same magnitude as the horizontal component of the rope's tension $T \sin\theta$, where θ is the angle between the rope (in the final position) and vertical:

$$\theta = \sin^{-1}\left(\frac{4.00}{12.0}\right) = 19.5°.$$

But the vertical component of the tension supports against the weight: $T\cos\theta = mg$. Thus, the tension is

$$T = (230\text{ kg})(9.80\text{ m/s}^2)/\cos 19.5° = 2391\text{ N}$$

and $F = (2391\text{ N})\sin 19.5° = 797$ N.

An alternative approach based on drawing a vector triangle (of forces) in the final situation provides a quick solution.

(b) Since there is no change in kinetic energy, the net work on it is zero.

(c) The work done by gravity is $W_g = \vec{F}_g \cdot \vec{d} = -mgh$, where $h = L(1 - \cos\theta)$ is the vertical component of the displacement. With L = 12.0 m, we obtain W_g = –1547 J, which should be rounded to three significant figures: –1.55 kJ.

(d) The tension vector is everywhere perpendicular to the direction of motion, so its work is zero (since cos 90° = 0).

(e) The implication of the previous three parts is that the work due to $\vec{F}$ is $-W_g$ (so the net work turns out to be zero). Thus, $W_F = -W_g$ = 1.55 kJ.

(f) Since $\vec{F}$ does not have constant magnitude, we cannot expect Eq. 7-8 to apply.

58. (a) The force of the worker on the crate is constant, so the work it does is given by $W_F = \vec{F}\cdot\vec{d} = Fd\cos\phi$, where $\vec{F}$ is the force, $\vec{d}$ is the displacement of the crate, and ϕ is the angle between the force and the displacement. Here F = 210 N, d = 3.0 m, and ϕ = 20°. Thus,

$$W_F = (210\text{ N})(3.0\text{ m})\cos 20° = 590\text{ J}.$$

(b) The force of gravity is downward, perpendicular to the displacement of the crate. The angle between this force and the displacement is 90° and cos 90° = 0, so the work done by the force of gravity is zero.

(c) The normal force of the floor on the crate is also perpendicular to the displacement, so the work done by this force is also zero.

(d) These are the only forces acting on the crate, so the total work done on it is 590 J.

59. The work done by the applied force $\vec{F}_a$ is given by $W = \vec{F}_a \cdot \vec{d} = F_a d\cos\phi$. From the figure, we see that $W = 25$ J when $\phi = 0$ and $d = 5.0$ cm. This yields the magnitude of $\vec{F}_a$:

$$F_a = \frac{W}{d} = \frac{25\ \text{J}}{0.050\ \text{m}} = 5.0\times10^2\ \text{N}.$$

(a) For $\phi = 64°$, we have $W = F_a d\cos\phi = (5.0\times10^2\,\text{N})(0.050\ \text{m})\cos 64° = 11$ J.

(b) For $\phi = 147°$, we have $W = F_a d\cos\phi = (5.0\times10^2\,\text{N})(0.050\ \text{m})\cos 147° = -21$ J.

60. (a) In the work-kinetic energy theorem, we include both the work due to an applied force W_a and work done by gravity W_g in order to find the latter quantity.

$$\Delta K = W_a + W_g \quad\Rightarrow\quad 30\ \text{J} = (100\ \text{N})(1.8\ \text{m})\cos 180° + W_g$$

leading to $W_g = 2.1\times10^2$ J.

(b) The value of W_g obtained in part (a) still applies since the weight and the path of the child remain the same, so $\Delta K = W_g = 2.1\times10^2$ J.

61. One approach is to assume a "path" from $\vec{r}_i$ to $\vec{r}_f$ and do the line-integral accordingly. Another approach is to simply use Eq. 7-36, which we demonstrate:

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy = \int_2^{-4} (2x)dx + \int_3^{-3} (3)\ dy$$

with SI units understood. Thus, we obtain W = 12 J – 18 J = – 6 J.

62. (a) The compression of the spring is d = 0.12 m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25\ \text{kg})\left(9.8\ \text{m/s}^2\right)(0.12\ \text{m}) = 0.29\ \text{J}.$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2}kd^2 = -\frac{1}{2}\ (250\ \text{N/m})\ (0.12\ \text{m})^2 = -1.8\ \text{J}.$$

(c) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15):

$$\Delta K = 0 - \frac{1}{2} m v_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29\text{ J} - 1.8\text{ J})}{0.25\text{ kg}}} = 3.5\text{ m/s}.$$

(d) If we instead had $v_i' = 7\,\text{m/s}$, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$0 - \frac{1}{2} m v_i'^2 = W_1' + W_2' = mgd' - \frac{1}{2} k d'^2$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2 g^2 + mk v_i'^2}}{k}$$

which yields $d' = 0.23$ m. In order to obtain this result, we have used more digits in our intermediate results than are shown above (so $v_i = \sqrt{12.048}\text{ m/s} = 3.471\text{ m/s}$ and v_i' = 6.942 m/s).

63. **THINK** A crate is being pushed up a frictionless inclined plane. The forces involved are: gravitational force on the crate, normal force on the crate, and the force applied by the worker.

EXPRESS The work done by a force $\vec{F}$ on an object as it moves through a displacement $\vec{d}$ is $W = \vec{F} \cdot \vec{d} = Fd\cos\phi$, where ϕ is the angle between $\vec{F}$ and $\vec{d}$.

ANALYZE (a) The applied force is parallel to the inclined plane. Thus, using Eq. 7-6, the work done on the crate by the worker's applied force is

$$W_a = Fd\cos 0° = (209\text{ N})(1.50\text{ m}) \approx 314\text{ J}.$$

(b) Using Eq. 7-12, we find the work done by the gravitational force to be

$$\begin{aligned} W_g &= F_g d\cos(90° + 25°) = mgd\cos 115° \\ &= (25.0\,\text{kg})(9.8\,\text{m/s}^2)(1.50\text{ m})\cos 115° \\ &\approx -155\text{ J}. \end{aligned}$$

(c) The angle between the normal force and the direction of motion remains 90º at all times, so the work it does is zero:

$$W_N = F_N d\cos 90° = 0.$$

(d) The total work done on the crate is the sum of all three works:

$$W = W_a + W_g + W_N = 314\ \text{J} + (-155\ \text{J}) + 0\ \text{J} = 158\ \text{J}.$$

LEARN By work-kinetic energy theorem, if the crate is initially at rest ($K_i = 0$), then its kinetic energy after having moved 1.50 m up the incline would be $K_f = W = 158\ \text{J}$, and the speed of the crate at that instant is

$$v = \sqrt{2K_f / m} = \sqrt{2(158\ \text{J})/25.0\ \text{kg}} = 3.56\ \text{m/s}.$$

64. (a) The force $\vec{F}$ of the incline is a combination of normal and friction force, which is serving to "cancel" the tendency of the box to fall downward (due to its 19.6 N weight). Thus, $\vec{F} = mg$ upward. In this part of the problem, the angle ϕ between the belt and $\vec{F}$ is 80°. From Eq. 7-47, we have

$$P = Fv\cos\phi = (19.6\ \text{N})(0.50\ \text{m/s})\cos 80° = 1.7\ \text{W}.$$

(b) Now the angle between the belt and $\vec{F}$ is 90°, so that $P = 0$.

(c) In this part, the angle between the belt and $\vec{F}$ is 100°, so that

$$P = (19.6\ \text{N})(0.50\ \text{m/s})\cos 100° = -1.7\ \text{W}.$$

65. There is no acceleration, so the lifting force is equal to the weight of the object. We note that the person's pull $\vec{F}$ is equal (in magnitude) to the tension in the cord.

(a) As indicated in the *hint*, tension contributes twice to the lifting of the canister: $2T = mg$. Since $|\vec{F}| = T$, we find $|\vec{F}| = 98\ \text{N}$.

(b) To rise 0.020 m, two segments of the cord (see Fig. 7-47) must shorten by that amount. Thus, the amount of string pulled down at the left end (this is the magnitude of $\vec{d}$, the downward displacement of the hand) is $d = 0.040$ m.

(c) Since (at the left end) both $\vec{F}$ and $\vec{d}$ are downward, then Eq. 7-7 leads to

$$W = \vec{F}\cdot\vec{d} = (98\ \text{N})(0.040\ \text{m}) = 3.9\ \text{J}.$$

(d) Since the force of gravity $\vec{F}_g$ (with magnitude mg) is opposite to the displacement $\vec{d}_c = 0.020$ m (up) of the canister, Eq. 7-7 leads to

$$W = \vec{F}_g \cdot \vec{d}_c = -\,(196\text{ N})(0.020\text{ m}) = -3.9\text{ J}.$$

This is consistent with Eq. 7-15 since there is no change in kinetic energy.

66. After converting the speed: $v = 120\text{ km/h} = 33.33\text{ m/s}$, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1200\,\text{kg})(33.33\,\text{m/s})^2 = 6.67\times10^5\text{ J}.$$

67. **THINK** In this problem we have packages hung from the spring. The extent of stretching can be determined from Hooke's law.

EXPRESS According to Hooke's law, the spring force is given by

$$F_x = -k(x - x_0) = -k\Delta x\,,$$

where Δx is the displacement from the equilibrium position. Thus, the first two situations in Fig. 7-48 can be written as

$$-110\text{ N} = -k(40\text{ mm} - x_0)$$
$$-240\text{ N} = -k(60\text{ mm} - x_0)$$

The two equations allow us to solve for *k*, the spring constant, as well as x_0, the relaxed position when no mass is hung.

ANALYZE (a) The two equations can be added to give

$$240\text{ N} - 110\text{ N} \;=\; k(60\text{ mm} - 40\text{ mm})$$

which yields $k = 6.5$ N/mm. Substituting the result into the first equation, we find

$$x_0 = 40\text{ mm} - \frac{110\text{ N}}{k} = 40\text{ mm} - \frac{110\text{ N}}{6.5\text{ N/mm}} = 23\text{ mm}.$$

(b) Using the results from part (a) to analyze that last picture, we find the weight to be

$$W = k(30\,\text{mm} - x_o) = (6.5\text{ N/mm})(30\text{ mm} - 23\text{ mm}) = 45\text{ N}.$$

LEARN An alternative method to calculate *W* in the third picture is to note that since the amount of stretching is proportional to the weight hung, we have $\dfrac{W}{W'} = \dfrac{\Delta x}{\Delta x'}$. Applying this relation to the second and the third pictures, the weight *W* is

$$W = \left(\frac{\Delta x_3}{\Delta x_2}\right) W_2 = \left(\frac{30\text{ mm} - 23\text{ mm}}{60\text{ mm} - 23\text{ mm}}\right)(240\text{ N}) = 45\text{ N},$$

in agreement with the result shown in (b).

68. Using Eq. 7-7, we have $W = Fd\cos\phi = 1504$ J. Then, by the work-kinetic energy theorem, we find the kinetic energy $K_f = K_i + W = 0 + 1504$ J. The answer is therefore 1.5 kJ.

69. The total weight is (100)(660 N) = 6.60×10^4 N, and the words "raises … at constant speed" imply zero acceleration, so the lift-force is equal to the total weight. Thus

$$P = Fv = (6.60 \times 10^4)(150\text{ m}/60.0\text{ s}) = 1.65 \times 10^5\text{ W}.$$

70. With SI units understood, Eq. 7-8 leads to $W = (4.0)(3.0) - c(2.0) = 12 - 2c$.

(a) If $W = 0$, then $c = 6.0$ N.

(b) If $W = 17$ J, then $c = -2.5$ N.

(c) If $W = -18$ J, then $c = 15$ N.

71. Using Eq. 7-8, we find

$$W = \vec{F}\cdot\vec{d} = (F\cos\theta\,\hat{\text{i}} + F\sin\theta\,\hat{\text{j}})\cdot(x\hat{\text{i}} + y\hat{\text{j}}) = Fx\cos\theta + Fy\sin\theta$$

where $x = 2.0$ m, $y = -4.0$ m, $F = 10$ N, and $\theta = 150°$. Thus, we obtain $W = -37$ J. Note that the given mass value (2.0 kg) is not used in the computation.

72. (a) Eq. 7-10 (along with Eq. 7-1 and Eq. 7-7) leads to

$$v_f = (2\,\frac{d}{m}F\cos\theta)^{1/2} = (\cos\theta)^{1/2},$$

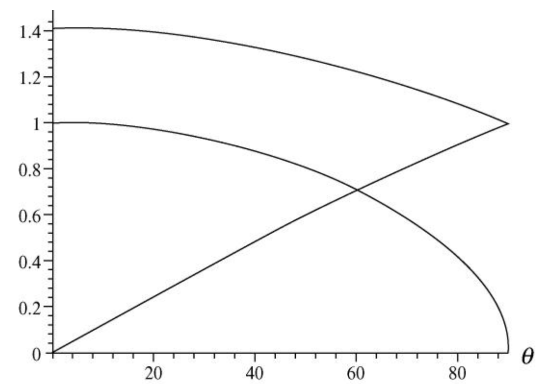
where we have substituted $F = 2.0$ N, $m = 4.0$ kg, and $d = 1.0$ m.

(b) With $v_i = 1$, those same steps lead to $v_f = (1 + \cos\theta)^{1/2}$.

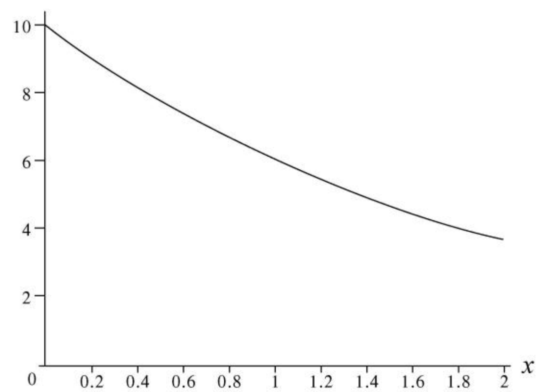
(c) Replacing θ with 180º – θ, and still using $v_i = 1$, we find

$$v_f = [1 + \cos(180° - \theta)]^{1/2} = (1 - \cos\theta)^{1/2}.$$

(d) The graphs are shown on the right. Note that as θ is increased in parts (a) and (b) the force provides less and less of a positive acceleration, whereas in part (c) the force provides less and less of a deceleration (as its θ value increases). The highest curve (which slowly decreases from 1.4 to 1) is the curve for part (b); the other decreasing curve (starting at 1 and ending at 0) is for part (a). The rising curve is for part (c); it is equal to 1 where $\theta = 90°$.



73. (a) The plot of the function (with SI units understood) is shown below.



Estimating the area under the curve allows for a range of answers. Estimates from 11 J to 14 J are typical.

(b) Evaluating the work analytically (using Eq. 7-32), we have

$$W = \int_0^2 10e^{-x/2}dx = -20e^{-x/2}\Big|_0^2 = 12.6\text{ J} \approx 13\text{ J}.$$

74. (a) Using Eq. 7-8 and SI units, we find

$$W = \vec{F}\cdot\vec{d} = (2\,\hat{\text{i}} - 4\,\hat{\text{j}})\cdot(8\,\hat{\text{i}} + c\,\hat{\text{j}}) = 16 - 4c$$

which, if equal zero, implies $c = 16/4 = 4$ m.

(b) If $W > 0$ then $16 > 4c$, which implies $c < 4$ m.

(c) If $W < 0$ then $16 < 4c$, which implies $c > 4$ m.

75. **THINK** Power must be supplied in order to lift the elevator with load upward at a constant speed.

EXPRESS For the elevator-load system to move upward at a constant speed (zero acceleration), the applied force F must exactly balance the gravitational force on the system, i.e., $F = F_g = (m_{\text{elev}} + m_{\text{load}})g$. The power required can then be calculated using Eq. 17-48: $P = Fv$.

ANALYZE With $m_{\text{elev}} = 4500\,\text{kg}$, $m_{\text{load}} = 1800\,\text{kg}$ and $v = 3.80\,\text{m/s}$, we find the power to be

$$P = Fv = (m_{\text{elev}} + m_{\text{load}})gv = (4500\,\text{kg} + 1800\,\text{kg})(9.8\,\text{m/s}^2)(3.80\,\text{m/s}) = 235\,\text{kW}.$$

LEARN The power required is proportional to the speed at which the system moves; the greater the speed, the greater the power that must be supplied.

76. (a) The component of the force of gravity exerted on the ice block (of mass m) along the incline is $mg \sin\theta$, where $\theta = \sin^{-1}(0.91/1.5)$ gives the angle of inclination for the inclined plane. Since the ice block slides down with uniform velocity, the worker must exert a force $\vec{F}$ "uphill" with a magnitude equal to $mg \sin\theta$. Consequently,

$$F = mg\sin\theta = (45\text{ kg})(9.8\text{ m/s}^2)\left(\frac{0.91\,\text{m}}{1.5\,\text{m}}\right) = 2.7\times10^2\text{ N}.$$

(b) Since the "downhill" displacement is opposite to $\vec{F}$, the work done by the worker is

$$W_1 = -\left(2.7\times10^2\,\text{N}\right)(1.5\text{ m}) = -4.0\times10^2\,\text{J}.$$

(c) Since the displacement has a vertically downward component of magnitude 0.91 m (in the same direction as the force of gravity), we find the work done by gravity to be

$$W_2 = (45\text{ kg})\left(9.8\text{ m/s}^2\right)(0.91\text{ m}) = 4.0\times10^2\,\text{J}.$$

(d) Since $\vec{F}_N$ is perpendicular to the direction of motion of the block, and $\cos 90° = 0$, work done by the normal force is $W_3 = 0$ by Eq. 7-7.

(e) The resultant force $\vec{F}_{\text{net}}$ is zero since there is no acceleration. Thus, its work is zero, as can be checked by adding the above results $W_1 + W_2 + W_3 = 0$.

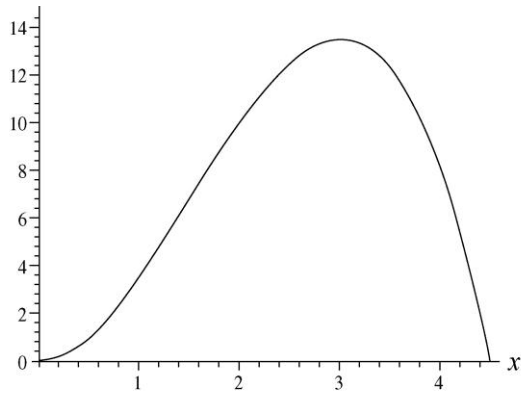
77. (a) To estimate the area under the curve between $x = 1$ m and $x = 3$ m (which should yield the value for the work done), one can try "counting squares" (or half-squares or thirds of squares) between the curve and the axis. Estimates between 5 J and 8 J are typical for this (crude) procedure.

(b) Equation 7-32 gives

$$\int_1^3 \frac{a}{x^2}\,dx = \frac{a}{3} - \frac{a}{1} = 6\text{ J}$$

where $a = -9$ N·m^2 is given in the problem statement.

78. (a) Using Eq. 7-32, the work becomes $W = \frac{9}{2}x^2 - x^3$ (SI units understood). The plot is shown below:



(b) We see from the graph that its peak value occurs at $x = 3.00$ m. This can be verified by taking the derivative of W and setting equal to zero, or simply by noting that this is where the force vanishes.

(c) The maximum value is $W = \frac{9}{2}(3.00)^2 - (3.00)^3 = 13.50$ J.

(d) We see from the graph (or from our analytic expression) that $W = 0$ at $x = 4.50$ m.

(e) The case is at rest when $v = 0$. Since $W = \Delta K = mv^2/2$, the condition implies $W = 0$. This happens at $x = 4.50$ m.

79. **THINK** A box sliding in the $+x$-direction is slowed down by a steady wind in the $-x$-direction. The problem involves graphical analysis.

EXPRESS Fig. 7-51 represents $x(t)$, the position of the lunch box as a function of time. It is convenient to fit the curve to a concave-downward parabola:

$$x(t) = \frac{1}{10}t(10-t) = t - \frac{1}{10}t^2.$$

By taking one and two derivatives, we find the velocity and acceleration to be

$$v(t) = \frac{dx}{dt} = 1 - \frac{t}{5} \text{ (in m/s)}, \quad a = \frac{d^2x}{dt^2} = -\frac{1}{5} = -0.2 \text{ (in m/s}^2\text{)}.$$

The equations imply that the initial speed of the box is $v_i = v(0) = 1.0\ \text{m/s}$, and the constant force by the wind is

$$F = ma = (2.0\ \text{kg})(-0.2\ \text{m/s}^2) = -0.40\ \text{N}.$$

The corresponding work is given by (SI units understood)

$$W(t) = F \cdot x(t) = -0.04t(10-t).$$

The initial kinetic energy of the lunch box is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\ \text{kg})(1.0\ \text{m/s})^2 = 1.0\ \text{J}.$$

With $\Delta K = K_f - K_i = W$, the kinetic energy at a later time is given by (in SI units)

$$K(t) = K_i + W = 1 - 0.04t(10-t)$$

ANALYZE (a) When t = 1.0 s, the above expression gives

$$K(1\ \text{s}) = 1 - 0.04(1)(10-1) = 1 - 0.36 = 0.64 \approx 0.6\ \text{J}$$

where the second significant figure is not to be taken too seriously.

(b) At t = 5.0 s, the above method gives $K(5.0\ \text{s}) = 1 - 0.04(5)(10-5) = 1 - 1 = 0.$

(c) The work done by the force from the wind from t = 1.0 s to t = 5.0 s is

$$W = K(5.0) - K(1.0\ \text{s}) = 0 - 0.6 \approx -0.6\ \text{J}.$$

LEARN The result in (c) can also be obtained by evaluating $W(t) = -0.04t(10-t)$ directly at t = 5.0 s and t = 1.0 s, and subtracting:

$$W(5) - W(1) = -0.04(5)(10-5) - \left[-0.04(1)(10-1)\right] = -1 - (-0.36) = -0.64 \approx -0.6\ \text{J}.$$

Note that at t = 5.0 s, K = 0, the box comes to a stop and then reverses its direction subsequently (with x decreasing).

80. The problem indicates that SI units are understood, so the result (of Eq. 7-23) is in joules. Done numerically, using features available on many modern calculators, the result is roughly 0.47 J. For the interested student it might be worthwhile to quote the "exact" answer (in terms of the "error function"):

$$\int_{.15}^{1.2} e^{-2x^2}\, dx = ¼\sqrt{2\pi}\,[\text{erf}(6\sqrt{2}/5) - \text{erf}(3\sqrt{2}/20)]\,.$$

81. (a) The work done by the spring force is $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. By energy conservation, when the block is at x_f= 0, the energy stored in the spring is completed converted to the kinetic energy of the block: $W_s = K = \frac{1}{2}mv^2$. Solving for v, we obtain

$$\frac{1}{2}k(x_i^2 - x_f^2) = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{500\text{ N/m}}{4.00\text{ kg}}}(0.300\text{ m}) = 3.35\text{ m/s}.$$

(b) The work done by the spring is

$$W_s = \frac{1}{2}kx_i^2 = \frac{1}{2}(500\text{ N/m})(0.300\text{ m})^2 = 22.5\text{ J}.$$

(c) The instantaneous power due to the spring can be written as

$$P = Fv = (kx)\sqrt{\frac{k}{m}\left(x_i^2 - x^2\right)}$$

At the release point x_i, the power is zero.

(d) Similarly, at x = 0, we also have P = 0.

(e) The position where the power is maximum can be found by differentiating P with respect to x, setting dP/dx = 0:

$$\frac{dP}{dx} = \frac{k^2(x_i^2 - 2x^2)}{\sqrt{\frac{k(x_i^2 - x^2)}{m}}} = 0$$

which gives $x = \dfrac{x_i}{\sqrt{2}} = \dfrac{(0.300\text{ m})}{\sqrt{2}} = 0.212\text{ m}$.

82. (a) Applying Newton's second law to the x (directed uphill) and y (normal to the inclined plane) axes, we obtain

$$F - mg\sin\theta = ma$$
$$F_N - mg\cos\theta = 0.$$

The second equation allows us to solve for the angle the inclined plane makes with the horizontal:

$$\theta = \cos^{-1}\left(\frac{F_N}{mg}\right) = \cos^{-1}\left(\frac{13.41\text{ N}}{(4.00\text{ kg})(9.8\text{ m/s}^2)}\right) = 70.0°$$

From the equation for the x-axis, we find the acceleration of the block to be

$$a=\frac{F}{m}-g\sin\theta=\frac{50\text{ N}}{4.00\text{ kg}}-(9.8\text{ m/s}^2)\sin 70.0°=3.29\text{ m/s}^2$$

Using the kinematic equation $v^2=v_0^2+2ad$, the speed of the block when d = 3.00 m is

$$v=\sqrt{2ad}=\sqrt{2(3.29\text{ m/s}^2)(3.00\text{ m})}=4.44\text{ m/s}$$

83. (a) The work done by the spring force (with spring constant $k=18\text{ N/cm}=1800\text{ N/m}$) is

$$W_s=\frac{1}{2}k(x_i^2-x_f^2)=-\frac{1}{2}kx_f^2=-\frac{1}{2}(1800\text{ N/m})(7.60\times10^{-3}\text{ m})^2=-5.20\times10^{-2}\text{ J}$$

(b) For $x'_f=2x_f$, the work done by the spring force is $W'_s=-\frac{1}{2}kx'^2_f=-\frac{1}{2}k(2x_f)^2$, so the additional work done is

$$\Delta W=W'_s-W_s=-\frac{1}{2}k(2x_f)^2-\left(-\frac{1}{2}kx_f^2\right)=-\frac{3}{2}kx_f^2=3W_s=3(-5.20\times10^{-2}\text{ J})=-0.156\text{ J}$$

84. (a) The displacement of the object is

$$\Delta\vec{r}=\vec{r}_2-\vec{r}_1=(-4.10\hat{\text{i}}+3.30\hat{\text{j}}+5.40\,\hat{\text{k}})-(2.70\hat{\text{i}}-2.90\hat{\text{j}}+5.50\,\hat{\text{k}})=(-6.80\hat{\text{i}}+6.20\hat{\text{j}}-0.10\,\hat{\text{k}})$$

The work done by $\vec{F}=(2.00\hat{\text{i}}+9.00\hat{\text{j}}+5.30\,\hat{\text{k}})\text{N}$ is (in SI units)

$$W=\vec{F}\cdot\Delta\vec{r}=(2.00\hat{\text{i}}+9.00\hat{\text{j}}+5.30\,\hat{\text{k}})\cdot(-6.80\hat{\text{i}}+6.20\hat{\text{j}}-0.10\,\hat{\text{k}})=41.7\text{ J}$$

(b) The average power due to the force during the time interval is

$$P=\frac{W}{\Delta t}=\frac{41.7\text{ J}}{2.10\text{ s}}=19.8\text{ W}$$

(c) The magnitudes of the position vectors are (in SI units)

$$r_1=|\vec{r}_1|=\sqrt{(2.70)^2+(-2.90)^2+(5.50)^2}=6.78$$
$$r_2=|\vec{r}_2|=\sqrt{(-4.10)^2+(3.30)^2+(5.40)^2}=7.54$$

and their dot product is

$$\vec{r}_1 \cdot \vec{r}_2 = (2.70\hat{\mathrm{i}} - 2.90\hat{\mathrm{j}} + 5.50\,\hat{\mathrm{k}}) \cdot (-4.10\hat{\mathrm{i}} + 3.30\hat{\mathrm{j}} + 5.40\,\hat{\mathrm{k}})$$
$$= (2.70)(-4.10) + (-2.90)(3.30) + (5.50)(5.40) = 9.06$$

Using $\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos\theta,$, the angle between $\vec{r}_1$ and $\vec{r}_2$ is

$$\theta = \cos^{-1}\left(\frac{\vec{r}_1 \cdot \vec{r}_2}{r_1 r_2}\right) = \cos^{-1}\left(\frac{9.06}{(6.78)(7.54)}\right) = 79.8°$$

85. The work done by the force is (in SI units)

$$W = \vec{F} \cdot \vec{d} = (-5.00\hat{\mathrm{i}} + 5.00\hat{\mathrm{j}} + 4.00\,\hat{\mathrm{k}}) \cdot (2.00\hat{\mathrm{i}} + 2.00\hat{\mathrm{j}} + 7.00\,\hat{\mathrm{k}}) = 28\ \mathrm{J}$$

By energy conservation, $W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2).$ Thus, the final speed of the particle is

$$v_f = \sqrt{v_i^2 + \frac{2W}{m}} = \sqrt{(4.00\ \mathrm{m/s})^2 + \frac{2(28\ \mathrm{J})}{2.00\ \mathrm{kg}}} = 6.63\ \mathrm{m/s}\ .$$

Chapter 8

1. **THINK** A compressed spring stores potential energy. This exercise explores the relationship between the energy stored and the spring constant.

EXPRESS The potential energy stored by the spring is given by $U = kx^2/2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is in equilibrium. Thus, the spring constant is $k = 2U/x^2$.

ANALYZE Substituting $U = 25\text{ J}$ and $x = 7.5\text{ m} = 0.075\text{ cm}$ into the above expression, we find the spring constant to be

$$k = \frac{2U}{x^2} = \frac{2(25\text{ J})}{(0.075\text{ m})^2} = 8.9\times10^3\text{ N/m}.$$

LEARN The spring constant k has units N/m. The quantity provides a measure of stiffness of the spring, for a given x, the greater the value of k, the greater the potential energy U.

2. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and A is horizontal, so $\phi = 90.0°$ and $W_g = 0$ (since cos 90.0° = 0).

(b) The displacement between the initial point and B has a vertical component of $h/2$ downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g\cdot\vec{d} = \frac{1}{2}mgh = \frac{1}{2}(825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 1.70\times10^5\text{ J}.$$

(c) The displacement between the initial point and C has a vertical component of h downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g\cdot\vec{d} = mgh = (825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 3.40\times10^5\text{ J}.$$

(d) With the reference position at C, we obtain

$$U_B = \frac{1}{2}mgh = \frac{1}{2}(825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 1.70\times10^5\text{ J}.$$

(e) Similarly, we find

$$U_A = mgh = (825\ \text{kg})(9.80\ \text{m/s}^2)(42.0\ \text{m}) = 3.40 \times 10^5\ \text{J}.$$

(f) All the answers are proportional to the mass of the object. If the mass is doubled, all answers are doubled.

3. (a) Noting that the vertical displacement is 10.0 m – 1.50 m = 8.50 m downward (same direction as $\vec{F}_g$), Eq. 7-12 yields

$$W_g = mgd\cos\phi = (2.00\ \text{kg})(9.80\ \text{m/s}^2)(8.50\ \text{m})\cos 0° = 167\ \text{J}.$$

(b) One approach (which is fairly trivial) is to use Eq. 8-1, but we feel it is instructive to instead calculate this as ΔU where $U = mgy$ (with upward understood to be the $+y$ direction). The result is

$$\Delta U = mg(y_f - y_i) = (2.00\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m} - 10.0\ \text{m}) = -167\ \text{J}.$$

(c) In part (b) we used the fact that $U_i = mgy_i$ =196 J.

(d) In part (b), we also used the fact $U_f = mgy_f$ = 29 J.

(e) The computation of W_g does not use the new information (that U = 100 J at the ground), so we again obtain W_g = 167 J.

(f) As a result of Eq. 8-1, we must again find $\Delta U = -W_g = -167$ J.

(g) With this new information (that U_0 = 100 J where y = 0) we have

$$U_i = mgy_i + U_0 = 296\ \text{J}.$$

(h) With this new information (that U_0 = 100 J where y = 0) we have

$$U_f = mgy_f + U_0 = 129\ \text{J}.$$

We can check part (f) by subtracting the new U_i from this result.

4. (a) The only force that does work on the ball is the force of gravity; the force of the rod is perpendicular to the path of the ball and so does no work. In going from its initial position to the lowest point on its path, the ball moves vertically through a distance equal to the length L of the rod, so the work done by the force of gravity is

$$W = mgL = (0.341\ \text{kg})(9.80\ \text{m/s}^2)(0.452\ \text{m}) = 1.51\ \text{J}.$$

(b) In going from its initial position to the highest point on its path, the ball moves vertically through a distance equal to L, but this time the displacement is upward, opposite the direction of the force of gravity. The work done by the force of gravity is

$$W = -mgL = -(0.341 \text{ kg})(9.80 \text{ m/s}^2)(0.452 \text{ m}) = -1.51 \text{ J}.$$

(c) The final position of the ball is at the same height as its initial position. The displacement is horizontal, perpendicular to the force of gravity. The force of gravity does no work during this displacement.

(d) The force of gravity is conservative. The change in the gravitational potential energy of the ball-Earth system is the negative of the work done by gravity:

$$\Delta U = -mgL = -(0.341 \text{ kg})(9.80 \text{ m/s}^2)(0.452 \text{ m}) = -1.51 \text{ J}$$

as the ball goes to the lowest point.

(e) Continuing this line of reasoning, we find

$$\Delta U = +mgL = (0.341 \text{ kg})(9.80 \text{ m/s}^2)(0.452 \text{ m}) = 1.51 \text{ J}$$

as it goes to the highest point.

(f) Continuing this line of reasoning, we have $\Delta U = 0$ as it goes to the point at the same height.

(g) The change in the gravitational potential energy depends only on the initial and final positions of the ball, not on its speed anywhere. The change in the potential energy is the *same* since the initial and final positions are the same.

5. **THINK** As the ice flake slides down the frictionless bowl, its potential energy changes due to the work done by the gravitational force.

EXPRESS The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the flake, so this reduces to $W = mgh$, where h is the height from which the flake falls. The force of gravity is conservative, so the change in gravitational potential energy of the flake-Earth system is the negative of the work done: $\Delta U = -W$.

ANALYZE (a) The ice flake falls a distance $h = r = 22.0 \text{ cm} = 0.22 \text{ m}$. Therefore, the work done by gravity is

$$W = mgr = (2.00 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)(22.0 \times 10^{-2} \text{ m}) = 4.31 \times 10^{-3} \text{J}.$$

(b) The change in gravitational potential energy is $\Delta U = -W = -4.31 \times 10^{-3}$ J.

(c) The potential energy when the flake is at the top is greater than when it is at the bottom by $|\Delta U|$. If $U = 0$ at the bottom, then $U = +4.31 \times 10^{-3}$ J at the top.

(d) If $U = 0$ at the top, then $U = -4.31 \times 10^{-3}$ J at the bottom.

(e) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

LEARN While the potential energy depends on the reference point (location where $U = 0$), the change in potential energy, ΔU, does not. In both (c) and (d), we find $\Delta U = -4.31 \times 10^{-3}$ J.

6. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and Q has a vertical component of $h - R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 4mgR = 4(3.20\times10^{-2}\text{ kg})(9.80\text{ m/s}^2)(0.12\text{ m}) = 0.15\text{ J}.$$

(b) The displacement between the initial point and the top of the loop has a vertical component of $h - 2R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 3mgR = 3(3.20\times10^{-2}\text{ kg})(9.80\text{ m/s}^2)(0.12\text{ m}) = 0.11\text{ J}.$$

(c) With $y = h = 5R$, at P we find

$$U = 5mgR = 5(3.20\times10^{-2}\text{ kg})(9.80\text{ m/s}^2)(0.12\text{ m}) = 0.19\text{ J}.$$

(d) With $y = R$, at Q we have

$$U = mgR = (3.20\times10^{-2}\text{ kg})(9.80\text{ m/s}^2)(0.12\text{ m}) = 0.038\text{ J}.$$

(e) With $y = 2R$, at the top of the loop, we find

$$U = 2mgR = 2(3.20\times10^{-2}\text{ kg})(9.80\text{ m/s}^2)(0.12\text{ m}) = 0.075\text{ J}.$$

(f) The new information $(v_i \neq 0)$ is not involved in any of the preceding computations; the above results are unchanged.

7. The main challenge for students in this type of problem seems to be working out the trigonometry in order to obtain the height of the ball (relative to the low point of the

swing) $h = L - L\cos\theta$ (for angle θ measured from vertical as shown in Fig. 8-34). Once this relation (which we will not derive here since we have found this to be most easily illustrated at the blackboard) is established, then the principal results of this problem follow from Eq. 7-12 (for W_g) and Eq. 8-9 (for U).

(a) The vertical component of the displacement vector is downward with magnitude h, so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL(1-\cos\theta)$$
$$= (5.00 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m})(1-\cos 30°) = 13.1 \text{ J}.$$

(b) From Eq. 8-1, we have $\Delta U = -W_g = -mgL(1 - \cos\theta) = -13.1$ J.

(c) With $y = h$, Eq. 8-9 yields $U = mgL(1 - \cos\theta) = 13.1$ J.

(d) As the angle increases, we intuitively see that the height h increases (and, less obviously, from the mathematics, we see that $\cos\theta$ decreases so that $1 - \cos\theta$ increases), so the answers to parts (a) and (c) increase, and the absolute value of the answer to part (b) also increases.

8. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F}\cdot\vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the snowball. The expression for the work reduces to $W = mgh$, where h is the height through which the snowball drops. Thus

$$W = mgh = (1.50 \text{ kg})(9.80 \text{ m/s}^2)(12.5 \text{ m}) = 184 \text{ J}.$$

(b) The force of gravity is conservative, so the change in the potential energy of the snowball-Earth system is the negative of the work it does: $\Delta U = -W = -184$ J.

(c) The potential energy when it reaches the ground is less than the potential energy when it is fired by $|\Delta U|$, so $U = -184$ J when the snowball hits the ground.

9. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In Problem 9-2, we found $U_A = mgh$ (with the reference position at C). Referring again to Fig. 8-29, we see that this is the same as U_0, which implies that $K_A = K_0$ and thus that

$$v_A = v_0 = 17.0 \text{ m/s}.$$

(b) In the solution to Problem 9-2, we also found $U_B = mgh/2$. In this case, we have

$$K_0 + U_0 = K_B + U_B$$

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_B^2 + mg\left(\frac{h}{2}\right)$$

which leads to

$$v_B = \sqrt{v_0^2 + gh} = \sqrt{(17.0\text{ m/s})^2 + (9.80\text{ m/s}^2)(42.0\text{ m})} = 26.5\text{ m/s}.$$

(c) Similarly, $v_C = \sqrt{v_0^2 + 2gh} = \sqrt{(17.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(42.0\text{ m})} = 33.4\text{ m/s}.$

(d) To find the "final" height, we set $K_f = 0$. In this case, we have

$$K_0 + U_0 = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + mgh = 0 + mgh_f$$

which yields $h_f = h + \frac{v_0^2}{2g} = 42.0\text{ m} + \frac{(17.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 56.7\text{ m}.$

(e) It is evident that the above results do not depend on mass. Thus, a different mass for the coaster must lead to the same results.

10. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In the solution to Problem 9-3 (to which this problem refers), we found $U_i = mgy_i$ = 196 J and $U_f = mgy_f$ = 29.0 J (assuming the reference position is at the ground). Since $K_i = 0$ in this case, we have

$$0 + 196\text{ J} = K_f + 29.0\text{ J}$$

which gives $K_f = 167\text{ J}$ and thus leads to $v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(167\text{ J})}{2.00\text{ kg}}} = 12.9\text{ m/s}.$

(b) If we proceed algebraically through the calculation in part (a), we find $K_f = -\Delta U = mgh$ where $h = y_i - y_f$ and is positive-valued. Thus,

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gh}$$

as we might also have derived from the equations of Table 2-1 (particularly Eq. 2-16). The fact that the answer is independent of mass means that the answer to part (b) is identical to that of part (a), that is, $v = 12.9\text{ m/s}$.

(c) If $K_i \neq 0$, then we find $K_f = mgh + K_i$ (where K_i is necessarily positive-valued). This represents a larger value for K_f than in the previous parts, and thus leads to a larger value for v.

11. **THINK** As the ice flake slides down the frictionless bowl, its potential energy decreases (discussed in Problem 8-5). By conservation of mechanical energy, its kinetic energy must increase.

EXPRESS If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then

$$K_f + U_f = K_i + U_i .$$

Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where $r = 0.220$ m is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $K_f = mv^2/2$.

ANALYZE (a) Energy conservation leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow \quad \frac{1}{2}mv^2 + 0 = 0 + mgr .$$

The speed is $v = \sqrt{2gr} = 2.08 \text{ m/s}$.

(b) Since the expression for speed is $v = \sqrt{2gr}$, which does not contain the mass of the flake, the speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. If K_i is greater than before, then K_f will also be greater. This means the final speed of the flake is greater.

LEARN The mechanical energy conservation principle can also be expressed as $\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$, which implies $\Delta K = -\Delta U$, i.e., the increase in kinetic energy is equal to the negative of the change in potential energy.

12. We use Eq. 8-18, representing the conservation of mechanical energy. We choose the reference position for computing U to be at the ground below the cliff; it is also regarded as the "final" position in our calculations.

(a) Using Eq. 8-9, the initial potential energy is given by $U_i = mgh$ where $h = 12.5$ m and $m = 1.50 \text{ kg}$. Thus, we have

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv^2 + 0$$

which leads to the speed of the snowball at the instant before striking the ground:

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_i^2 + mgh\right)} = \sqrt{v_i^2 + 2gh}$$

where v_i = 14.0 m/s is the magnitude of its initial velocity (not just one component of it). Thus we find v = 21.0 m/s.

(b) As noted above, v_i is the magnitude of its initial velocity and not just one component of it; therefore, there is no dependence on launch angle. The answer is again 21.0 m/s.

(c) It is evident that the result for v in part (a) does not depend on mass. Thus, changing the mass of the snowball does not change the result for v.

13. **THINK** As the marble moves vertically upward, its gravitational potential energy increases. This energy comes from the release of elastic potential energy stored in the spring.

EXPRESS We take the reference point for gravitational potential energy to be at the position of the marble when the spring is compressed. The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$. On the other had, the energy stored in the spring is $U_s = kx^2/2$. Applying mechanical energy conservation principle allows us to solve the problem.

ANALYZE (a) The height of the highest point is h = 20 m. With initial gravitational potential energy set to zero, we find

$$\Delta U_g = mgh = (5.0\times10^{-3}\text{ kg})(9.8\text{ m/s}^2)(20\text{ m}) = 0.98\text{ J}.$$

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, $\Delta U_s = -\Delta U_g = -0.98$ J.

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is U_s = 0.98 J. This must be $\frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{0.98\text{ J}}{(0.080\text{ m})^2} = 3.1\times10^2\text{ N/m} = 3.1\text{ N/cm}.$$

LEARN In general, the marble has both kinetic and potential energies:

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + mgy$$

At the maximum height $y_{\max} = h,\ \ v = 0$ and $mgh = kx^2/2$, or $h = \dfrac{kx^2}{2mg}$.

14. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) The change in potential energy is $\Delta U = mgL$ as it goes to the highest point. Thus, we have

$$\Delta K + \Delta U = 0$$
$$K_{\text{top}} - K_0 + mgL = 0$$

which, upon requiring $K_{\text{top}} = 0$, gives $K_0 = mgL$ and thus leads to

$$v_0 = \sqrt{\frac{2K_0}{m}} = \sqrt{2gL} = \sqrt{2(9.80\ \text{m/s}^2)(0.452\ \text{m})} = 2.98\ \text{m/s}\,.$$

(b) We also found in Problem 9-4 that the potential energy change is $\Delta U = -mgL$ in going from the initial point to the lowest point (the bottom). Thus,

$$\Delta K + \Delta U = 0$$
$$K_{\text{bottom}} - K_0 - mgL = 0$$

which, with $K_0 = mgL$, leads to $K_{\text{bottom}} = 2mgL$. Therefore,

$$v_{\text{bottom}} = \sqrt{\frac{2K_{\text{bottom}}}{m}} = \sqrt{4gL} = \sqrt{4(9.80\ \text{m/s}^2)(0.452\ \text{m})} = 4.21\ \text{m/s}\,.$$

(c) Since there is no change in height (going from initial point to the rightmost point), then $\Delta U = 0$, which implies $\Delta K = 0$. Consequently, the speed is the same as what it was initially,

$$v_{\text{right}} = v_0 = 2.98\ \text{m/s}\,.$$

(d) It is evident from the above manipulations that the results do not depend on mass. Thus, a different mass for the ball must lead to the same results.

15. **THINK** The truck with failed brakes is moving up an escape ramp. In order for it to come to a complete stop, all of its kinetic energy must be converted into gravitational potential energy.

EXPRESS We ignore any work done by friction. In SI units, the initial speed of the truck just before entering the escape ramp is v_i = 130(1000/3600) = 36.1 m/s. When the truck comes to a stop, its kinetic and potential energies are $K_f = 0$ and $U_f = mgh$. We apply mechanical energy conservation to solve the problem.

ANALYZE (a) Energy conservation implies $K_f + U_f = K_i + U_i$. With U_i = 0, and $K_i = \frac{1}{2}mv_i^2$, we obtain

$$\frac{1}{2}mv_i^2 + 0 = 0 + mgh \quad \Rightarrow \quad h = \frac{v_i^2}{2g} = \frac{(36.1\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 66.5\ \text{m}.$$

If L is the minimum length of the ramp, then $L\sin\theta = h$, or L sin 15° = 66.5 m so that $L = (66.5\ \text{m})/\sin 15° = 257$ m. That is, the ramp must be about 2.6×10^2 m long if friction is negligible.

(b) The minimum length is $L = \dfrac{h}{\sin\theta} = \dfrac{v_i^2}{2g\sin\theta}$ which does not depend on the mass of the truck. Thus, the answer remains the same if the mass is reduced.

(c) If the speed is decreased, then h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

LEARN The greater the speed of the truck, the longer the ramp required. This length can be shortened considerably if the friction between the tires and the ramp surface is factored in.

16. We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use x for the spring's compression, measured positively downward (so $x > 0$ means it is compressed).

(a) With x = 0.190 m, Eq. 7-26 gives

$$W_s = -\frac{1}{2}kx^2 = -7.22\ \text{J} \approx -7.2\ \text{J}$$

for the work done by the spring force. Using Newton's third law, we see that the work done on the spring is 7.2 J.

(b) As noted above, $W_s = -7.2$ J.

(c) Energy conservation leads to

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad 0 + mgh_0 = \frac{1}{2}kx^2 - mgx$$

which (with $m = 0.70$ kg) yields $h_0 = 0.86$ m.

(d) With a new value for the height $h_0' = 2h_0 = 1.72$ m, we solve for a new value of x using the quadratic formula (taking its positive root so that $x > 0$).

$$mgh_0' = -mgx + \frac{1}{2}kx^2 \;\Rightarrow\; x = \frac{mg + \sqrt{(mg)^2 + 2mgkh_0'}}{k}$$

which yields $x = 0.26$ m.

17. (a) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$K_P + U_P = K_Q + U_Q$$
$$0 + mgh = \frac{1}{2}mv^2 + mgR$$

Using the fact that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is

$$F = mv^2/R = 8mg = 8(0.032\text{ kg})(9.8\text{ m/s}^2) = 2.5\text{ N}.$$

The direction is to the left (in the same direction as $\vec{a}$).

(b) The downward component of the net force on the block at Q is the downward force of gravity

$$F = mg = (0.032\text{ kg})(9.8\text{ m/s}^2) = 0.31\text{ N}.$$

(c) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$\frac{mv_t^2}{R} = mg \;\Rightarrow\; mv_t^2 = mgR.$$

This requires a different value of h than what was used above.

$$K_P + U_P = K_t + U_t$$
$$0 + mgh = \frac{1}{2}mv_t^2 + mgh_t$$
$$mgh = \frac{1}{2}(mgR) + mg(2R)$$

Consequently, $h = 2.5R = (2.5)(0.12\text{ m}) = 0.30$ m.

(d) The normal force F_N, for speeds v_t greater than $\sqrt{gR}$ (which are the only possibilities for nonzero F_N — see the solution in the previous part), obeys

$$F_N = \frac{mv_t^2}{R} - mg$$

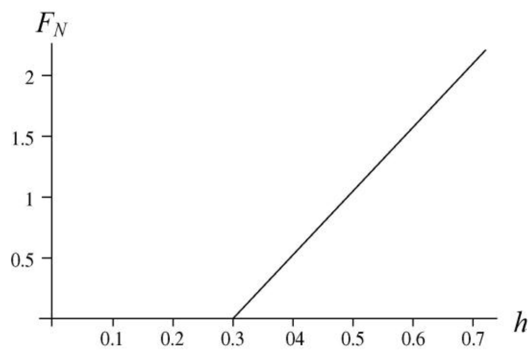
from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \Rightarrow gh = \frac{1}{2}v_t^2 + 2gR$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ — see the solution in the previous part), becomes

$$F_N = \frac{2mgh}{R} - 5mg .$$

Thus, the graph for $h \geq 2.5R = 0.30$ m consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes). Note that for $h \leq 2.5R$, the normal force is zero.



18. We use Eq. 8-18, representing the conservation of mechanical energy. The reference position for computing U is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) The potential energy is $U = mgL(1 - \cos\theta)$ at the position shown in Fig. 8-34 (which we consider to be the initial position). Thus, we have

$$K_i + U_i = K_f + U_f$$

$$0 + mgL(1-\cos\theta) = \frac{1}{2}mv^2 + 0$$

which leads to

$$v = \sqrt{\frac{2mgL(1-\cos\theta)}{m}} = \sqrt{2gL(1-\cos\theta)}.$$

Plugging in $L = 2.00$ m and $\theta = 30.0°$ we find $v = 2.29$ m/s.

(b) It is evident that the result for v does not depend on mass. Thus, a different mass for the ball must not change the result.

19. We convert to SI units and choose upward as the $+y$ direction. Also, the relaxed position of the top end of the spring is the origin, so the initial compression of the spring (defining an equilibrium situation between the spring force and the force of gravity) is $y_0 = -0.100$ m and the additional compression brings it to the position $y_1 = -0.400$ m.

(a) When the stone is in the equilibrium ($a = 0$) position, Newton's second law becomes

$$\vec{F}_{\text{net}} = ma$$
$$F_{\text{spring}} - mg = 0$$
$$-k(-0.100) - (8.00)(9.8) = 0$$

where Hooke's law (Eq. 7-21) has been used. This leads to a spring constant equal to $k = 784 \text{ N/m}$.

(b) With the additional compression (and release) the acceleration is no longer zero, and the stone will start moving upward, turning some of its elastic potential energy (stored in the spring) into kinetic energy. The amount of elastic potential energy at the moment of release is, using Eq. 8-11,

$$U = \frac{1}{2}ky_1^2 = \frac{1}{2}(784 \text{ N/m})(-0.400)^2 = 62.7 \text{ J}.$$

(c) Its maximum height y_2 is beyond the point that the stone separates from the spring (entering free-fall motion). As usual, it is characterized by having (momentarily) zero speed. If we choose the y_1 position as the reference position in computing the gravitational potential energy, then

$$K_1 + U_1 = K_2 + U_2$$
$$0 + \frac{1}{2}ky_1^2 = 0 + mgh$$

where $h = y_2 - y_1$ is the height above the release point. Thus, mgh (the gravitational potential energy) is seen to be equal to the previous answer, 62.7 J, and we proceed with the solution in the next part.

(d) We find $h = ky_1^2/2mg = 0.800 \text{ m}$, or 80.0 cm.

20. (a) We take the reference point for gravitational energy to be at the lowest point of the swing. Let θ be the angle measured from vertical. Then the height y of the pendulum "bob" (the object at the end of the pendulum, which in this problem is the stone) is given by $L(1 - \cos\theta) = y$. Hence, the gravitational potential energy is

$$mg\,y = mgL(1 - \cos\theta).$$

When θ = 0º (the string at its lowest point) we are told that its speed is 8.0 m/s; its kinetic energy there is therefore 64 J (using Eq. 7-1). At θ = 60º its mechanical energy is

$$E_{mech} = \frac{1}{2}\,mv^2 + mgL(1 - \cos\theta)\,.$$

Energy conservation (since there is no friction) requires that this be equal to 64 J. Solving for the speed, we find v = 5.0 m/s.

(b) We now set the above expression again equal to 64 J (with θ being the unknown) but with zero speed (which gives the condition for the maximum point, or "turning point" that it reaches). This leads to θ_{max} = 79°.

(c) As observed in our solution to part (a), the total mechanical energy is 64 J.

21. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects). The reference position for computing U (and height h) is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) Careful examination of the figure leads to the trigonometric relation $h = L - L\cos\theta$ when the angle is measured from vertical as shown. Thus, the gravitational potential energy is $U = mgL(1 - \cos\theta_0)$ at the position shown in Fig. 8-34 (the initial position). Thus, we have

$$K_0 + U_0 = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = \frac{1}{2}mv^2 + 0$$

which leads to

$$v = \sqrt{\frac{2}{m}\left[\frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0)\right]} = \sqrt{v_0^2 + 2gL(1-\cos\theta_0)}$$

$$= \sqrt{(8.00\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(1.25\text{ m})(1-\cos 40°)} = 8.35\text{ m/s}.$$

(b) We look for the initial speed required to barely reach the horizontal position — described by v_h = 0 and θ = 90° (or θ = –90°, if one prefers, but since cos(–ϕ) = cos ϕ, the sign of the angle is not a concern).

$$K_0 + U_0 = K_h + U_h$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = 0 + mgL$$

which yields

$$v_0 = \sqrt{2gL\cos\theta_0} = \sqrt{2(9.80\text{ m/s}^2)(1.25\text{ m})\cos 40°} = 4.33\text{ m/s}.$$

(c) For the cord to remain straight, then the centripetal force (at the top) must be (at least) equal to gravitational force:

$$\frac{mv_t^2}{r} = mg \Rightarrow mv_t^2 = mgL$$

where we recognize that $r = L$. We plug this into the expression for the kinetic energy (at the top, where $\theta = 180°$).

$$K_0 + U_0 = K_t + U_t$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = \frac{1}{2}mv_t^2 + mg\left(1 - \cos 180°\right)$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = \frac{1}{2}(mgL) + mg(2L)$$

which leads to

$$v_0 = \sqrt{gL(3 + 2\cos\theta_0)} = \sqrt{(9.80 \text{ m/s}^2)(1.25 \text{ m})(3 + 2\cos 40°)} = 7.45 \text{ m/s}.$$

(d) The more initial potential energy there is, the less initial kinetic energy there needs to be, in order to reach the positions described in parts (b) and (c). Increasing θ_0 amounts to increasing U_0, so we see that a greater value of θ_0 leads to smaller results for v_0 in parts (b) and (c).

22. From Chapter 4, we know the height h of the skier's jump can be found from $v_y^2 = 0 = v_{0y}^2 - 2gh$ where $v_{0y} = v_0 \sin 28°$ is the upward component of the skier's "launch velocity." To find v_0 we use energy conservation.

(a) The skier starts at rest $y = 20$ m above the point of "launch" so energy conservation leads to

$$mgy = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gy} = 20 \text{ m/s}$$

which becomes the initial speed v_0 for the launch. Hence, the above equation relating h to v_0 yields

$$h = \frac{\left(v_0 \sin 28°\right)^2}{2g} = 4.4 \text{ m}.$$

(b) We see that all reference to mass cancels from the above computations, so a new value for the mass will yield the same result as before.

23. (a) As the string reaches its lowest point, its original potential energy $U = mgL$ (measured relative to the lowest point) is converted into kinetic energy. Thus,

$$mgL = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gL} \ .$$

With L = 1.20 m we obtain $v = \sqrt{2gL} = \sqrt{2(9.80\ \text{m/s}^2)(1.20\ \text{m})} = 4.85\ \text{m/s}$.

(b) In this case, the total mechanical energy is shared between kinetic $\frac{1}{2}mv_b^2$ and potential mgy_b. We note that $y_b = 2r$ where $r = L - d$ = 0.450 m. Energy conservation leads to

$$mgL = \frac{1}{2}mv_b^2 + mgy_b$$

which yields $v_b = \sqrt{2gL - 2g(2r)} = 2.42\ \text{m/s}$.

24. We denote m as the mass of the block, h = 0.40 m as the height from which it dropped (measured from the relaxed position of the spring), and x as the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is $-mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$K_i + U_i = K_f + U_f$$
$$0 = -mg(h+x) + \frac{1}{2}kx^2$$

which is a second degree equation in x. Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k} \ .$$

Now mg = 19.6 N, h = 0.40 m, and $k = 1960\ \text{N/m}$, and we choose the positive root so that $x > 0$.

$$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10\ \text{m} \ .$$

25. Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this t = 6.0 s flight.

$$\Delta y = v_{0_y}t - \frac{1}{2}gt^2$$

This leads to $\Delta y = -32\ \text{m}$. Therefore $\Delta U = mg\Delta y = -318\ \text{J} \approx -3.2\times 10^{-2}\ \text{J}$.

26. (a) With energy in joules and length in meters, we have

$$\Delta U = U(x) - U(0) = -\int_0^x (6x' - 12)\,dx' \ .$$

Therefore, with U (0) = 27 J, we obtain $U(x)$ (written simply as U) by integrating and rearranging:

$$U = 27 + 12x - 3x^2 \ .$$

(b) We can maximize the above function by working through the $dU/dx = 0$ condition, or we can treat this as a force equilibrium situation — which is the approach we show.

$$F = 0 \Rightarrow 6x_{eq} - 12 = 0$$

Thus, x_{eq} = 2.0 m, and the above expression for the potential energy becomes U = 39 J.

(c) Using the quadratic formula or using the polynomial solver on an appropriate calculator, we find the negative value of x for which U = 0 to be x = –1.6 m.

(d) Similarly, we find the positive value of x for which U = 0 to be x = 5.6 m.

27. (a) To find out whether or not the vine breaks, it is sufficient to examine it at the moment Tarzan swings through the lowest point, which is when the vine — if it didn't break — would have the greatest tension. Choosing upward positive, Newton's second law leads to

$$T - mg = m\frac{v^2}{r}$$

where r = 18.0 m and $m = W/g = 688/9.8 = 70.2$ kg. We find the v^2 from energy conservation (where the reference position for the potential energy is at the lowest point).

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = 2gh$$

where h = 3.20 m. Combining these results, we have

$$T = mg + m\frac{2gh}{r} = mg\left(1 + \frac{2h}{r}\right)$$

which yields 933 N. Thus, the vine does not break.

(b) Rounding to an appropriate number of significant figures, we see the maximum tension is roughly 9.3×10^2 N.

28. From the slope of the graph, we find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10\,\text{N/cm} = 10\,\text{N/m}.$$

(a) Equating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$$

which yields v = 2.8 m/s for m = 0.0038 kg and x = 0.055 m.

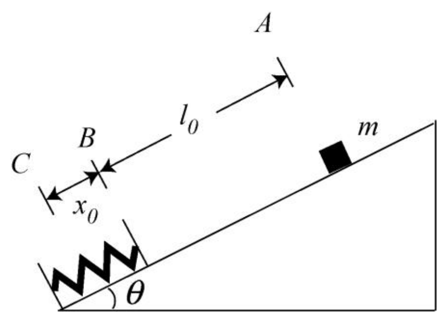
(b) The new scenario involves some potential energy at the moment of release. With d = 0.015 m, energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \Rightarrow v = \sqrt{\frac{k}{m}\left(x^2 - d^2\right)}$$

which yields v = 2.7 m/s.

29. **THINK** As the block slides down the inclined plane, it compresses the spring, then stops momentarily before sliding back up again.

EXPRESS We refer to its starting point as A, the point where it first comes into contact with the spring as B, and the point where the spring is compressed by $x_0 = 0.055$ m as C (see the figure below). Point C is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed.



Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270\text{ N}}{0.02\text{ m}} = 1.35\times10^4\text{ N/m}.$$

The distance between points *A* and *B* is l_0 and we note that the total sliding distance $l_0 + x_0$ is related to the initial height h_A of the block (measured relative to *C*) by $\sin\theta = \dfrac{h_A}{l_0 + x_0}$, where the incline angle θ is 30°.

ANALYZE (a) Mechanical energy conservation leads to

$$K_A + U_A = K_C + U_C \quad \Rightarrow \quad 0 + mgh_A = \frac{1}{2}kx_0^2$$

which yields

$$h_A = \frac{kx_0^2}{2mg} = \frac{(1.35\times10^4 \text{ N/m})(0.055 \text{ m})^2}{2(12 \text{ kg})(9.8 \text{ m/s}^2)} = 0.174 \text{ m}.$$

Therefore, the total distance traveled by the block before coming to a stop is

$$l_0 + x_0 = \frac{h_A}{\sin 30°} = \frac{0.174 \text{ m}}{\sin 30°} = 0.347 \text{ m} \approx 0.35 \text{ m}.$$

(b) From this result, we find $l_0 = x_0 = 0.347 \text{ m} - 0.055 \text{ m} = 0.292 \text{ m}$, which means that the block has descended a vertical distance

$$|\Delta y| = h_A - h_B = l_0 \sin\theta = (0.292 \text{ m})\sin 30° = 0.146 \text{ m}$$

in sliding from point *A* to point *B*. Thus, using Eq. 8-18, we have

$$0 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B \quad \Rightarrow \quad \frac{1}{2}mv_B^2 = mg|\Delta y|$$

which yields $v_B = \sqrt{2g|\Delta y|} = \sqrt{2(9.8 \text{ m/s}^2)(0.146 \text{ m})} = 1.69 \text{ m/s} \approx 1.7 \text{ m/s}$.

LEARN Energy is conserved in the process. The total energy of the block at position *B* is

$$E_B = \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}(12 \text{ kg})(1.69 \text{ m/s})^2 + (12 \text{ kg})(9.8 \text{ m/s}^2)(0.028 \text{ m}) = 20.4 \text{ J},$$

which is equal to the elastic potential energy in the spring:

$$\frac{1}{2}kx_0^2 = \frac{1}{2}(1.35\times10^4 \text{ N/m})(0.055 \text{ m})^2 = 20.4 \text{ J}.$$

30. We take the original height of the box to be the $y = 0$ reference level and observe that, in general, the height of the box (when the box has moved a distance d downhill) is $y = -d\sin 40°$.

(a) Using the conservation of energy, we have

$$K_i + U_i = K + U \Rightarrow 0 + 0 = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kd^2.$$

Therefore, with $d = 0.10$ m, we obtain $v = 0.81$ m/s.

(b) We look for a value of $d \neq 0$ such that $K = 0$.

$$K_i + U_i = K + U \Rightarrow 0 + 0 = 0 + mgy + \frac{1}{2}kd^2.$$

Thus, we obtain $mgd\sin 40° = \frac{1}{2}kd^2$ and find $d = 0.21$ m.

(c) The uphill force is caused by the spring (Hooke's law) and has magnitude $kd = 25.2$ N. The downhill force is the component of gravity $mg\sin 40° = 12.6$ N. Thus, the net force on the box is (25.2 – 12.6) N = 12.6 N uphill, with

$$a = F/m = (12.6 \text{ N})/(2.0 \text{ kg}) = 6.3 \text{ m/s}^2.$$

(d) The acceleration is up the incline.

31. The reference point for the gravitational potential energy U_g (and height h) is at the block when the spring is maximally compressed. When the block is moving to its highest point, it is first accelerated by the spring; later, it separates from the spring and finally reaches a point where its speed v_f is (momentarily) zero. The x axis is along the incline, pointing uphill (so x_0 for the initial compression is negative-valued); its origin is at the relaxed position of the spring. We use SI units, so $k = 1960$ N/m and $x_0 = -0.200$ m.

(a) The elastic potential energy is $\frac{1}{2}kx_0^2 = 39.2$ J.

(b) Since initially $U_g = 0$, the change in U_g is the same as its final value mgh where $m = 2.00$ kg. That this must equal the result in part (a) is made clear in the steps shown in the next part. Thus, $\Delta U_g = U_g = 39.2$ J.

(c) The principle of mechanical energy conservation leads to

$$K_0 + U_0 = K_f + U_f$$

$$0 + \frac{1}{2}kx_0^2 = 0 + mgh$$

which yields h = 2.00 m. The problem asks for the distance *along the incline*, so we have $d = h/\sin 30° = 4.00$ m.

32. The work required is the change in the gravitational potential energy as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length dy, we note that the mass of a segment is $(m/L)\,dy$ and the change in potential energy of a segment when it is a distance $|y|$ below the table top is

$$dU = (m/L)g|y|\,dy = -(m/L)gy\,dy$$

since y is negative-valued (we have $+y$ upward and the origin is at the tabletop). The total potential energy change is

$$\Delta U = -\frac{mg}{L}\int_{-L/4}^{0} y\,dy = \frac{1}{2}\frac{mg}{L}(L/4)^2 = mgL/32.$$

The work required to pull the chain onto the table is therefore

$$W = \Delta U = mgL/32 = (0.012\text{ kg})(9.8\text{ m/s}^2)(0.28\text{ m})/32 = 0.0010\text{ J}.$$

33. All heights h are measured from the lower end of the incline (which is our reference position for computing gravitational potential energy mgh). Our x axis is along the incline, with $+x$ being uphill (so spring compression corresponds to $x > 0$) and its origin being at the relaxed end of the spring. The height that corresponds to the canister's initial position (with spring compressed amount x = 0.200 m) is given by $h_1 = (D+x)\sin\theta$, where $\theta = 37°$.

(a) Energy conservation leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + mg(D+x)\sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_2^2 + mgD\sin\theta$$

which yields, using the data m = 2.00 kg and k = 170 N/m,

$$v_2 = \sqrt{2gx\sin\theta + kx^2/m} = 2.40\text{ m/s}.$$

(b) In this case, energy conservation leads to

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mg(D+x)\sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_3^2 + 0$$

which yields $v_3 = \sqrt{2g(D+x)\sin\theta + kx^2/m} = 4.19$ m/s.

34. Let $\vec{F}_N$ be the normal force of the ice on him and m is his mass. The net inward force is $mg\cos\theta - F_N$ and, according to Newton's second law, this must be equal to mv^2/R, where v is the speed of the boy. At the point where the boy leaves the ice $F_N = 0$, so $g\cos\theta = v^2/R$. We wish to find his speed. If the gravitational potential energy is taken to be zero when he is at the top of the ice mound, then his potential energy at the time shown is

$$U = -mgR(1 - \cos\theta).$$

He starts from rest and his kinetic energy at the time shown is $\frac{1}{2}mv^2$. Thus conservation of energy gives

$$0 = \tfrac{1}{2}mv^2 - mgR(1 - \cos\theta),$$

or $v^2 = 2gR(1 - \cos\theta)$. We substitute this expression into the equation developed from the second law to obtain $g\cos\theta = 2g(1 - \cos\theta)$. This gives $\cos\theta = 2/3$. The height of the boy above the bottom of the mound is

$$h = R\cos\theta = \frac{2}{3}R = \frac{2}{3}(13.8\text{ m}) = 9.20\text{ m}.$$

35. (a) The (final) elastic potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(431\text{ N/m})(0.210\text{ m})^2 = 9.50\text{ J}.$$

Ultimately this must come from the original (gravitational) energy in the system mgy (where we are measuring y from the lowest "elevation" reached by the block, so

$$y = (d + x)\sin(30^\circ).$$

Thus,

$$mg(d + x)\sin(30^\circ) = 9.50\text{ J} \quad \Rightarrow \quad d = 0.396\text{ m}.$$

(b) The block is still accelerating (due to the component of gravity along the incline, $mg\sin(30^\circ)$) for a few moments after coming into contact with the spring (which exerts the Hooke's law force kx), until the Hooke's law force is strong enough to cause the block to begin decelerating. This point is reached when

$$kx = mg\sin 30^\circ$$

which leads to $x = 0.0364$ m = 3.64 cm; this is long before the block finally stops (36.0 cm before it stops).

36. The distance the marble travels is determined by its initial speed (and the methods of Chapter 4), and the initial speed is determined (using energy conservation) by the original compression of the spring. We denote h as the height of the table, and x as the horizontal

distance to the point where the marble lands. Then $x = v_0\, t$ and $h = \frac{1}{2} g t^2$ (since the vertical component of the marble's "launch velocity" is zero). From these we find $x = v_0 \sqrt{2h/g}$. We note from this that the distance to the landing point is directly proportional to the initial speed. We denote v_{01} be the initial speed of the first shot and D_1 = (2.20 – 0.27) m = 1.93 m be the horizontal distance to its landing point; similarly, v_{02} is the initial speed of the second shot and D = 2.20 m is the horizontal distance to its landing spot. Then

$$\frac{v_{02}}{v_{01}} = \frac{D}{D_1} \quad \Rightarrow \quad v_{02} = \frac{D}{D_1} v_{01}$$

When the spring is compressed an amount ℓ, the elastic potential energy is $\frac{1}{2}k\ell^2$. When the marble leaves the spring its kinetic energy is $\frac{1}{2}mv_0^2$. Mechanical energy is conserved: $\frac{1}{2}mv_0^2 = \frac{1}{2}k\ell^2$, and we see that the initial speed of the marble is directly proportional to the original compression of the spring. If ℓ_1 is the compression for the first shot and ℓ_2 is the compression for the second, then $v_{02} = (\ell_2/\ell_1)v_{01}$. Relating this to the previous result, we obtain

$$\ell_2 = \frac{D}{D_1}\ell_1 = \left(\frac{2.20 \text{ m}}{1.93 \text{ m}}\right)(1.10 \text{ cm}) = 1.25 \text{ cm}.$$

37. Consider a differential element of length dx at a distance x from one end (the end that remains stuck) of the cord. As the cord turns vertical, its change in potential energy is given by

$$dU = -(\lambda dx)gx$$

where $\lambda = m/h$ is the mass/unit length and the negative sign indicates that the potential energy decreases. Integrating over the entire length, we obtain the total change in the potential energy:

$$\Delta U = \int dU = -\int_0^h \lambda g x dx = -\frac{1}{2}\lambda g h^2 = -\frac{1}{2}mgh.$$

With m = 15 g and h = 25 cm, we have $\Delta U = -0.018$ J.

38. In this problem, the mechanical energy (the sum of K and U) remains constant as the particle moves.

(a) Since mechanical energy is conserved, $U_B + K_B = U_A + K_A$, the kinetic energy of the particle in region A ($3.00 \text{ m} \le x \le 4.00 \text{ m}$) is

$$K_A = U_B - U_A + K_B = 12.0 \text{ J} - 9.00 \text{ J} + 4.00 \text{ J} = 7.00 \text{ J}.$$

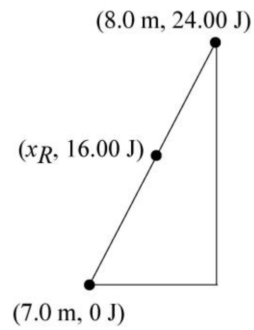
With $K_A = mv_A^2/2$, the speed of the particle at $x = 3.5$ m (within region A) is

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2(7.00\text{ J})}{0.200\text{ kg}}} = 8.37\text{ m/s}.$$

(b) At $x = 6.5$ m, $U = 0$ and $K = U_B + K_B = 12.0\text{ J} + 4.00\text{ J} = 16.0\text{ J}$ by mechanical energy conservation. Therefore, the speed at this point is

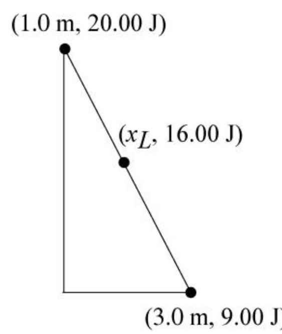
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(16.0\text{ J})}{0.200\text{ kg}}} = 12.6\text{ m/s}.$$

(c) At the turning point, the speed of the particle is zero. Let the position of the right turning point be x_R. From the figure shown on the right, we find x_R to be



$$\frac{16.00\text{ J} - 0}{x_R - 7.00\text{ m}} = \frac{24.00\text{ J} - 16.00\text{ J}}{8.00\text{ m} - x_R} \quad \Rightarrow \quad x_R = 7.67\text{ m}.$$

(d) Let the position of the left turning point be x_L. From the figure shown, we find x_L to be



$$\frac{16.00\text{ J} - 20.00\text{ J}}{x_L - 1.00\text{ m}} = \frac{9.00\text{ J} - 16.00\text{ J}}{3.00\text{ m} - x_L} \quad \Rightarrow \quad x_L = 1.73\text{ m}.$$

39. From the figure, we see that at x = 4.5 m, the potential energy is U_1 = 15 J. If the speed is v = 7.0 m/s, then the kinetic energy is

$$K_1 = mv^2/2 = (0.90\text{ kg})(7.0\text{ m/s})^2/2 = 22\text{ J}.$$

The total energy is $E_1 = U_1 + K_1$ = (15 + 22) J = 37 J.

(a) At x = 1.0 m, the potential energy is U_2 = 35 J. By energy conservation, we have K_2 = 2.0 J > 0. This means that the particle can reach there with a corresponding speed

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(2.0\text{ J})}{0.90\text{ kg}}} = 2.1\text{ m/s}.$$

(b) The force acting on the particle is related to the potential energy by the negative of the slope:

$$F_x = -\frac{\Delta U}{\Delta x}$$

From the figure we have $F_x = -\frac{35\text{ J} - 15\text{ J}}{2\text{ m} - 4\text{ m}} = +10\text{ N}$.

(c) Since the magnitude $F_x > 0$, the force points in the $+x$ direction.

(d) At x = 7.0 m, the potential energy is U_3 = 45 J, which exceeds the initial total energy E_1. Thus, the particle can never reach there. At the turning point, the kinetic energy is zero. Between x = 5 and 6 m, the potential energy is given by

$$U(x) = 15 + 30(x-5), \qquad 5 \le x \le 6.$$

Thus, the turning point is found by solving $37 = 15 + 30(x-5)$, which yields x = 5.7 m.

(e) At x = 5.0 m, the force acting on the particle is

$$F_x = -\frac{\Delta U}{\Delta x} = -\frac{(45-15)\text{ J}}{(6-5)\text{ m}} = -30\text{ N}.$$

The magnitude is $|F_x| = 30\text{ N}$.

(f) The fact that $F_x < 0$ indicated that the force points in the $-x$ direction.

40. (a) The force at the equilibrium position $r = r_{eq}$ is

$$F = -\left.\frac{dU}{dr}\right|_{r=r_{eq}} = 0 \quad \Rightarrow \quad -\frac{12A}{r_{eq}^{13}} + \frac{6B}{r_{eq}^{7}} = 0$$

which leads to the result

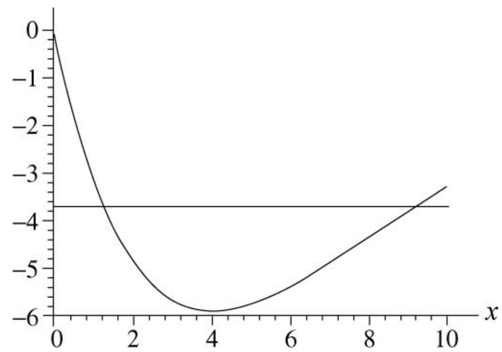
$$r_{eq} = \left(\frac{2A}{B}\right)^{\frac{1}{6}} = 1.12\left(\frac{A}{B}\right)^{\frac{1}{6}}.$$

(b) This defines a minimum in the potential energy curve (as can be verified either by a graph or by taking another derivative and verifying that it is concave upward at this point), which means that for values of r slightly smaller than r_{eq} the slope of the curve is negative (so the force is positive, repulsive).

(c) And for values of r slightly larger than r_{eq} the slope of the curve must be positive (so the force is negative, attractive).

41. (a) The energy at x = 5.0 m is $E = K + U$ = 2.0 J – 5.7 J = –3.7 J.

(b) A plot of the potential energy curve (SI units understood) and the energy E (the horizontal line) is shown for $0 \le x \le 10$ m.



(c) The problem asks for a graphical determination of the turning points, which are the points on the curve corresponding to the total energy computed in part (a). The result for the smallest turning point (determined, to be honest, by more careful means) is x = 1.3 m.

(d) And the result for the largest turning point is x = 9.1 m.

(e) Since $K = E - U$, then maximizing K involves finding the minimum of U. A graphical determination suggests that this occurs at x = 4.0 m, which plugs into the expression $E - U = -3.7 - (-4xe^{-x/4})$ to give $K = 2.16 \text{ J} \approx 2.2 \text{ J}$. Alternatively, one can measure from the graph from the minimum of the U curve up to the level representing the total energy E and thereby obtain an estimate of K at that point.

(f) As mentioned in the previous part, the minimum of the U curve occurs at x = 4.0 m.

(g) The force (understood to be in newtons) follows from the potential energy, using Eq. 8-20 (and Appendix E if students are unfamiliar with such derivatives).

$$F = \frac{dU}{dx} = (4 - x)e^{-x/4}$$

(h) This revisits the considerations of parts (d) and (e) (since we are returning to the minimum of $U(x)$) — but now with the advantage of having the analytic result of part (g). We see that the location that produces F = 0 is exactly x = 4.0 m.

42. Since the velocity is constant, $\vec{a} = 0$ and the horizontal component of the worker's push $F \cos\theta$ (where θ = 32°) must equal the friction force magnitude $f_k = \mu_k F_N$. Also, the vertical forces must cancel, implying

$$W_{\text{applied}} = (8.0\text{N})(0.70\text{m}) = 5.6 \text{ J}$$

which is solved to find F = 71 N.

(a) The work done on the block by the worker is, using Eq. 7-7,

$$W = Fd\cos\theta = (71\text{ N})(9.2\text{ m})\cos 32° = 5.6\times 10^2\text{ J}.$$

(b) Since $f_k = \mu_k (mg + F\sin\theta)$, we find $\Delta E_{\text{th}} = f_k d = (60\text{ N})(9.2\text{ m}) = 5.6\times 10^2$ J.

43. (a) Using Eq. 7-8, we have $W_{\text{applied}} = (8.0\text{ N})(0.70\text{ m}) = 5.6$ J.

(b) Using Eq. 8-31, the thermal energy generated is $\Delta E_{\text{th}} = f_k d = (5.0\text{ N})(0.70\text{ m}) = 3.5$ J.

44. (a) The work is $W = Fd = (35.0\text{ N})(3.00\text{ m}) = 105$ J.

(b) The total amount of energy that has gone to thermal forms is (see Eq. 8-31 and Eq. 6-2)

$$\Delta E_{\text{th}} = \mu_k mgd = (0.600)(4.00\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 70.6\text{ J}.$$

If 40.0 J has gone to the block then (70.6 – 40.0) J = 30.6 J has gone to the floor.

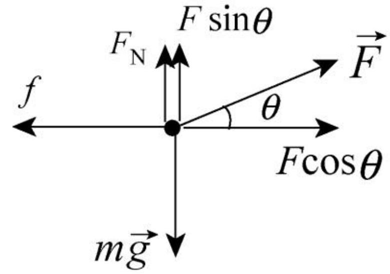
(c) Much of the work (105 J) has been "wasted" due to the 70.6 J of thermal energy generated, but there still remains (105 – 70.6) J = 34.4 J that has gone into increasing the kinetic energy of the block. (It has not gone into increasing the potential energy of the block because the floor is presumed to be horizontal.)

45. **THINK** Work is done against friction while pulling a block along the floor at a constant speed.

EXPRESS Place the *x*-axis along the path of the block and the *y*-axis normal to the floor. The free-body diagram is shown below. The *x* and the *y* component of Newton's second law are

$$x:\quad F\cos\theta - f = 0$$
$$y:\quad F_N + F\sin\theta - mg = 0,$$

where *m* is the mass of the block, *F* is the force exerted by the rope, *f* is the magnitude of the kinetic friction force, and θ is the angle between that force and the horizontal.



The work done on the block by the force in the rope is $W = Fd\cos\theta$. Similarly, the increase in thermal energy of the block-floor system due to the frictional force is given by Eq. 8-29, $\Delta E_{\text{th}} = fd$.

ANALYZE (a) Substituting the values given, we find the work done on the block by the rope's force to be

$$W = Fd\cos\theta = (7.68\,\text{N})(4.06\,\text{m})\cos 15.0° = 30.1\ \text{J}.$$

(b) The increase in thermal energy is $\Delta E_{\text{th}} = fd = (7.42\,\text{N})(4.06\,\text{m}) = 30.1\ \text{J}$.

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/F_N$ to obtain the coefficient of friction. The x-component of Newton's law gives

$$f = F\cos\theta = (7.68\ \text{N})\cos 15.0° = 7.42\ \text{N}.$$

Similarly, the y-component yields

$$F_N = mg - F\sin\theta = (3.57\ \text{kg})(9.8\ \text{m/s}^2) - (7.68\ \text{N})\sin 15.0° = 33.0\ \text{N}.$$

Thus, the coefficient of kinetic friction is

$$\mu_k = \frac{f}{F_N} = \frac{7.42\ \text{N}}{33.0\ \text{N}} = 0.225.$$

LEARN In this problem, the block moves at a constant speed so that $\Delta K = 0$, i.e., no change in kinetic energy. The work done by the external force is converted into thermal energy of the system, $W = \Delta E_{\text{th}}$.

46. We work this using English units (with g = 32 ft/s), but for consistency we convert the weight to pounds

$$mg = (9.0)\,\text{oz}\left(\frac{1\ \text{1b}}{16\,\text{oz}}\right) = 0.56\,\text{lb}$$

which implies $m = 0.018\ \text{lb}\cdot\text{s}^2/\text{ft}$ (which can be phrased as 0.018 slug as explained in Appendix D). And we convert the initial speed to feet-per-second

$$v_i = (81.8\,\text{mi/h})\left(\frac{5280\ \text{ft/mi}}{3600\ \text{s/h}}\right) = 120\ \text{ft/s}$$

or a more "direct" conversion from Appendix D can be used. Equation 8-30 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\Delta E_{\text{th}} = \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) = \frac{1}{2}(0.018)(120^2 - 110^2) + 0 = 20 \text{ ft}\cdot\text{lb}.$$

47. We use SI units so m = 0.075 kg. Equation 8-33 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) \\ &= \frac{1}{2}(0.075 \text{ kg})[(12 \text{ m/s})^2 - (10.5 \text{ m/s})^2] + (0.075 \text{ kg})(9.8 \text{ m/s}^2)(1.1 \text{ m} - 2.1 \text{ m}) \\ &= 0.53 \text{ J}.\end{aligned}$$

48. We use Eq. 8-31 to obtain $\Delta E_{\text{th}} = f_k d = (10\,\text{N})(5.0\,\text{m}) = 50 \text{ J}$, and Eq. 7-8 to get

$$W = Fd = (2.0\,\text{N})(5.0\,\text{m}) = 10 \text{ J}.$$

Similarly, Eq. 8-31 gives

$$\begin{aligned}W &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ 10 &= 35 + \Delta U + 50\end{aligned}$$

which yields $\Delta U = -75$ J. By Eq. 8-1, then, the work done by gravity is $W = -\Delta U = 75$ J.

49. **THINK** As the bear slides down the tree, its gravitational potential energy is converted into both kinetic energy and thermal energy.

EXPRESS We take the initial gravitational potential energy to be $U_i = mgL$, where L is the length of the tree, and final gravitational potential energy at the bottom to be $U_f = 0$. To solve this problem, we note that the changes in the mechanical and thermal energies must sum to zero.

ANALYZE (a) Substituting the values given, the change in gravitational potential energy is

$$\Delta U = U_f - U_i = -mgL = -(25\,\text{kg})(9.8\,\text{m/s}^2)(12 \text{ m}) = -2.9\times10^3 \text{ J}.$$

(b) The final speed is $v_f = 5.6\,\text{m/s}$. Therefore, the kinetic energy is

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(25 \text{ kg})(5.6 \text{ m/s})^2 = 3.9\times10^2 \text{ J}.$$

(c) The change in thermal energy is $\Delta E_{\text{th}} = fL$, where f is the magnitude of the average frictional force; therefore, from $\Delta E_{\text{th}} + \Delta K + \Delta U = 0$, we find f to be

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.9\times10^2 \text{ J} - 2.9\times10^3 \text{ J}}{12 \text{ m}} = 2.1\times10^2 \text{ N}.$$

LEARN In this problem, no external work is done to the bear. Therefore,

$$W = \Delta E_{\text{th}} + \Delta E_{\text{mech}} = \Delta E_{\text{th}} + \Delta K + \Delta U = 0,$$

which implies $\Delta K = -\Delta U - \Delta E_{\text{th}} = -\Delta U - fL$. Thus, $\Delta E_{\text{th}} = fL$ can be interpreted as the additional change (decrease) in kinetic energy due to frictional force.

50. Equation 8-33 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f)\\ &= \frac{1}{2}(60\ \text{kg})[(24\ \text{m/s})^2 - (22\ \text{m/s})^2] + (60\ \text{kg})(9.8\ \text{m/s}^2)(14\ \text{m})\\ &= 1.1\times10^4\ \text{J}.\end{aligned}$$

That the angle of 25° is nowhere used in this calculation is indicative of the fact that energy is a scalar quantity.

51. (a) The initial potential energy is

$$U_i = mgy_i = (520\ \text{kg})\left(9.8\ \text{m/s}^2\right)(300\ \text{m}) = 1.53\times10^6\ \text{J}$$

where $+y$ is upward and $y = 0$ at the bottom (so that $U_f = 0$).

(b) Since $f_k = \mu_k F_N = \mu_k mg\cos\theta$ we have $\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta$ from Eq. 8-31. Now, the hillside surface (of length $d = 500$ m) is treated as an hypotenuse of a 3-4-5 triangle, so $\cos\theta = x/d$ where $x = 400$ m. Therefore,

$$\Delta E_{\text{th}} = \mu_k mgd\frac{x}{d} = \mu_k mgx = (0.25)(520)(9.8)(400) = 5.1\times10^5\ \text{J}.$$

(c) Using Eq. 8-31 (with $W = 0$) we find

$$K_f = K_i + U_i - U_f - \Delta E_{\text{th}} = 0 + (1.53\times10^6\ \text{J}) - 0 - (5.1\times10^6\ \text{J}) = 1.02\times10^6\ \text{J}.$$

(d) From $K_f = mv^2/2$, we obtain $v = 63$ m/s.

52. (a) An appropriate picture (once friction is included) for this problem is Figure 8-3 in the textbook. We apply Eq. 8-31, $\Delta E_{\text{th}} = f_k d$, and relate initial kinetic energy K_i to the "resting" potential energy U_r:

$$K_i + U_i = f_k d + K_r + U_r \quad\Rightarrow\quad 20.0\ \text{J} + 0 = f_k d + 0 + \frac{1}{2}kd^2$$

where f_k = 10.0 N and k = 400 N/m. We solve the equation for d using the quadratic formula or by using the polynomial solver on an appropriate calculator, with d = 0.292 m being the only positive root.

(b) We apply Eq. 8-31 again and relate U_r to the "second" kinetic energy K_s it has at the unstretched position.

$$K_r + U_r = f_k d + K_s + U_s \quad \Rightarrow \quad \frac{1}{2}kd^2 = f_k d + K_s + 0$$

Using the result from part (a), this yields K_s = 14.2 J.

53. (a) The vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law requires $F_N = mg$, where m is the mass of the block. Thus $f = \mu_k F_N = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{th} = fd = \mu_k mgD$, where D is the distance the block moves before coming to rest. Using Eq. 8-29, we have

$$\Delta E_{\text{th}} = (0.25)(3.5\,\text{kg})(9.8\,\text{m/s}^2)(7.8\,\text{m}) = 67\,\text{J}.$$

(b) The block has its maximum kinetic energy K_{max} just as it leaves the spring and enters the region where friction acts. Therefore, the maximum kinetic energy equals the thermal energy generated in bringing the block back to rest, 67 J.

(c) The energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. Thus, $K_{\max} = U_i = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Thus,

$$x = \sqrt{\frac{2K_{\max}}{k}} = \sqrt{\frac{2(67\,\text{J})}{640\,\text{N/m}}} = 0.46\,\text{m}.$$

54. (a) Using the force analysis shown in Chapter 6, we find the normal force $F_N = mg\cos\theta$ (where mg = 267 N) which means

$$f_k = \mu_k F_N = \mu_k mg \cos\theta.$$

Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta = (0.10)(267)(6.1)\cos 20° = 1.5\times 10^2\,\text{J}.$$

(b) The potential energy change is

$$\Delta U = mg(-d\sin\theta) = (267\text{ N})(-6.1\text{ m})\sin 20° = -5.6\times 10^2\text{ J}.$$

The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{267\,\text{N}}{9.8\,\text{m/s}^2}\right)(0.457\,\text{m/s}^2) = 2.8\ \text{J}.$$

Therefore, using Eq. 8-33 (with $W = 0$), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - \left(-5.6\times10^2\right) - 1.5\times10^2 = 4.1\times10^2\,\text{J}.$$

Consequently, the final speed is $v_f = \sqrt{2K_f/m} = 5.5\ \text{m/s}$.

55. (a) With $x = 0.075$ m and $k = 320\,\text{N/m}$, Eq. 7-26 yields $W_s = -\frac{1}{2}kx^2 = -0.90$ J. For later reference, this is equal to the negative of ΔU.

(b) Analyzing forces, we find $F_N = mg$, which means $f_k = \mu_k F_N = \mu_k mg$. With $d = x$, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgx = (0.25)(2.5)(9.8)(0.075) = 0.46\ \text{J}.$$

(c) Equation 8-33 (with $W = 0$) indicates that the initial kinetic energy is

$$K_i = \Delta U + \Delta E_{\text{th}} = 0.90 + 0.46 = 1.36\ \text{J}$$

which leads to $v_i = \sqrt{2K_i/m} = 1.0\ \text{m/s}$.

56. Energy conservation, as expressed by Eq. 8-33 (with $W = 0$) leads to

$$\Delta E_{\text{th}} = K_i - K_f + U_i - U_f \;\Rightarrow\; f_k d = 0 - 0 + \frac{1}{2}kx^2 - 0$$

$$\Rightarrow\; \mu_k mgd = \frac{1}{2}(200\,\text{N/m})(0.15\,\text{m})^2 \;\Rightarrow\; \mu_k(2.0\,\text{kg})(9.8\,\text{m/s}^2)(0.75\,\text{m}) = 2.25\ \text{J}$$

which yields $\mu_k = 0.15$ as the coefficient of kinetic friction.

57. Since the valley is frictionless, the only reason for the speed being less when it reaches the higher level is the gain in potential energy $\Delta U = mgh$ where $h = 1.1$ m. Sliding along the rough surface of the higher level, the block finally stops since its remaining kinetic energy has turned to thermal energy $\Delta E_{\text{th}} = f_k d = \mu mgd$, where $\mu = 0.60$. Thus, Eq. 8-33 (with $W = 0$) provides us with an equation to solve for the distance d:

$$K_i = \Delta U + \Delta E_{\text{th}} = mg\left(h + \mu d\right)$$

where $K_i = mv_i^2/2$ and $v_i = 6.0$ m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2\mu g} - \frac{h}{\mu} = 1.2\,\text{m}.$$

58. This can be worked entirely by the methods of Chapters 2–6, but we will use energy methods in as many steps as possible.

(a) By a force analysis of the style done in Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$ (where $\theta = 40°$), which means $f_k = \mu_k F_N = \mu_k mg\cos\theta$ where $\mu_k = 0.15$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta.$$

Also, elementary trigonometry leads us to conclude that $\Delta U = mgd\sin\theta$. Eq. 8-33 (with $W = 0$ and $K_f = 0$) provides an equation for determining d:

$$K_i = \Delta U + \Delta E_{\text{th}}$$
$$\frac{1}{2}mv_i^2 = mgd\left(\sin\theta + \mu_k\cos\theta\right)$$

where $v_i = 1.4\,\text{m/s}$. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2g(\sin\theta + \mu_k\cos\theta)} = 0.13\,\text{m}.$$

(b) Now that we know where on the incline it stops ($d' = 0.13 + 0.55 = 0.68$ m from the bottom), we can use Eq. 8-33 again (with $W = 0$ and now with $K_i = 0$) to describe the final kinetic energy (at the bottom):

$$K_f = -\Delta U - \Delta E_{\text{th}}$$
$$\frac{1}{2}mv^2 = mgd'\left(\sin\theta - \mu_k\cos\theta\right)$$

which — after dividing by the mass and rearranging — yields

$$v = \sqrt{2gd'\left(\sin\theta - \mu_k\cos\theta\right)} = 2.7\,\text{m/s}.$$

(c) In part (a) it is clear that d increases if μ_k decreases — both mathematically (since it is a positive term in the denominator) and intuitively (less friction — less energy "lost"). In part (b), there are two terms in the expression for v that imply that it should increase if μ_k were smaller: the increased value of $d' = d_0 + d$ and that last factor $\sin\theta - \mu_k\cos\theta$, which indicates that less is being subtracted from $\sin\theta$ when μ_k is less (so the factor itself increases in value).

59. (a) The maximum height reached is *h*. The thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{th} = fh$ by Eq. 8-31. We use energy conservation in the form of Eq. 8-33 (with $W = 0$):

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i$$

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is $K_i = \frac{1}{2} m v_0^2$, the initial potential energy is $U_i = 0$, the final kinetic energy is $K_f = 0$, and the final potential energy is $U_f = wh$, where $w = mg$ is the weight of the stone. Thus, $wh + fh = \frac{1}{2} m v_0^2$, and we solve for the height:

$$h = \frac{m v_0^2}{2(w+f)} = \frac{v_0^2}{2g(1+f/w)}.$$

Numerically, we have, with $m = (5.29 \text{ N})/(9.80 \text{ m/s}^2) = 0.54$ kg,

$$h = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(1+0.265/5.29)} = 19.4 \text{ m}.$$

(b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{th} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2} m v^2$, where v is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2} m v^2 + 2fh = \frac{1}{2} m v_0^2.$$

We substitute the expression found for *h* to obtain

$$\frac{2 f v_0^2}{2g(1+f/w)} = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2 f v_0^2}{mg(1+f/w)} = v_0^2 - \frac{2 f v_0^2}{w(1+f/w)} = v_0^2 \left(1 - \frac{2f}{w+f}\right) = v_0^2 \frac{w-f}{w+f}$$

where w was substituted for mg and some algebraic manipulations were carried out. Therefore,

$$v = v_0\sqrt{\frac{w-f}{w+f}} = (20.0 \text{ m/s})\sqrt{\frac{5.29 \text{ N} - 0.265 \text{ N}}{5.29 \text{ N} + 0.265 \text{ N}}} = 19.0 \text{ m/s} .$$

60. We look for the distance along the incline d, which is related to the height ascended by $\Delta h = d \sin\theta$. By a force analysis of the style done in Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$, which means $f_k = \mu_k mg\cos\theta$. Thus, Eq. 8-33 (with W = 0) leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{\text{th}} \\ &= 0 - K_i + mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

which leads to

$$d = \frac{K_i}{mg\left(\sin\theta + \mu_k\cos\theta\right)} = \frac{128}{\left(4.0\right)\left(9.8\right)\left(\sin 30° + 0.30\cos 30°\right)} = 4.3 \text{ m}.$$

61. Before the launch, the mechanical energy is $\Delta E_{\text{mech},0} = 0$. At the maximum height h where the speed of the beetle vanishes, the mechanical energy is $\Delta E_{\text{mech},1} = mgh$. The change of the mechanical energy is related to the external force by

$$\Delta E_{\text{mech}} = \Delta E_{\text{mech},1} - \Delta E_{\text{mech},0} = mgh = F_{avg} d\cos\phi ,$$

where F_{avg} is the average magnitude of the external force on the beetle.

(a) From the above equation, we have

$$F_{avg} = \frac{mgh}{d\cos\phi} = \frac{(4.0\times10^{-6} \text{ kg})(9.80 \text{ m/s}^2)(0.30 \text{ m})}{(7.7\times10^{-4} \text{ m})(\cos 0°)} = 1.5\times10^{-2} \text{ N}.$$

(b) Dividing the above result by the mass of the beetle, we obtain

$$a = \frac{F_{avg}}{m} = \frac{h}{d\cos\phi}\, g = \frac{(0.30 \text{ m})}{(7.7\times10^{-4} \text{ m})(\cos 0°)}\, g = 3.8\times10^2 g.$$

62. We will refer to the point where it first encounters the "rough region" as point C (this is the point at a height h above the reference level). From Eq. 8-17, we find the speed it has at point C to be

$$v_C = \sqrt{v_A^2 - 2gh} = \sqrt{(8.0)^2 - 2(9.8)(2.0)} = 4.980 \approx 5.0 \text{ m/s}.$$

Thus, we see that its kinetic energy right at the beginning of its "rough slide" (heading uphill towards B) is

$$K_C = \frac{1}{2}\, m(4.980 \text{ m/s})^2 = 12.4m$$

(with SI units understood). Note that we "carry along" the mass (as if it were a known quantity); as we will see, it will cancel out, shortly. Using Eq. 8-37 (and Eq. 6-2 with $F_N = mg\cos\theta$) and $y = d\sin\theta$, we note that if $d < L$ (the block does not reach point B), this kinetic energy will turn entirely into thermal (and potential) energy

$$K_C = mgy + f_k d \quad \Rightarrow \quad 12.4m = mgd\sin\theta + \mu_k mgd\cos\theta.$$

With $\mu_k = 0.40$ and $\theta = 30°$, we find $d = 1.49$ m, which is greater than L (given in the problem as 0.75 m), so our assumption that $d < L$ is incorrect. What is its kinetic energy as it reaches point B? The calculation is similar to the above, but with d replaced by L and the final v^2 term being the unknown (instead of assumed zero):

$$\frac{1}{2}\, m v^2 = K_C - (mgL\sin\theta + \mu_k mgL\cos\theta)\ .$$

This determines the speed with which it arrives at point B:

$$\begin{aligned} v_B &= \sqrt{v_C^2 - 2gL(\sin\theta + \mu_k\cos\theta)} \\ &= \sqrt{(4.98\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.75\text{ m})(\sin 30° + 0.4\cos 30°)} = 3.5\text{ m/s}. \end{aligned}$$

63. We observe that the last line of the problem indicates that static friction is not to be considered a factor in this problem. The friction force of magnitude $f = 4400$ N mentioned in the problem is kinetic friction and (as mentioned) is constant (and directed upward), and the thermal energy change associated with it is $\Delta E_{th} = fd$ (Eq. 8-31) where $d = 3.7$ m in part (a) (but will be replaced by x, the spring compression, in part (b)).

(a) With $W = 0$ and the reference level for computing $U = mgy$ set at the top of the (relaxed) spring, Eq. 8-33 leads to

$$U_i = K + \Delta E_{\text{th}} \Rightarrow v = \sqrt{2d\left(g - \frac{f}{m}\right)}$$

which yields $v = 7.4\text{ m/s}$ for $m = 1800$ kg.

(b) We again utilize Eq. 8-33 (with $W = 0$), now relating its kinetic energy at the moment it makes contact with the spring to the system energy at the bottom-most point. Using the same reference level for computing $U = mgy$ as we did in part (a), we end up with gravitational potential energy equal to $mg(-x)$ at that bottom-most point, where the spring (with spring constant $k = 1.5\times10^5$ N/m) is fully compressed.

$$K = mg(-x) + \frac{1}{2}kx^2 + fx$$

where $K = \frac{1}{2}mv^2 = 4.9 \times 10^4$ J using the speed found in part (a). Using the abbreviation $\xi = mg - f = 1.3 \times 10^4$ N, the quadratic formula yields

$$x = \frac{\xi \pm \sqrt{\xi^2 + 2kK}}{k} = 0.90\,\text{m}$$

where we have taken the positive root.

(c) We relate the energy at the bottom-most point to that of the highest point of rebound (a distance d' above the relaxed position of the spring). We assume $d' > x$. We now use the bottom-most point as the reference level for computing gravitational potential energy.

$$\frac{1}{2}kx^2 = mgd' + fd' \Rightarrow d' = \frac{kx^2}{2(mg + d)} = 2.8\,\text{m}.$$

(d) The non-conservative force (§8-1) is friction, and the energy term associated with it is the one that keeps track of the total distance traveled (whereas the potential energy terms, coming as they do from conservative forces, depend on positions — but not on the paths that led to them). We assume the elevator comes to final rest at the equilibrium position of the spring, with the spring compressed an amount d_{eq} given by

$$mg = kd_{\text{eq}} \Rightarrow d_{\text{eq}} = \frac{mg}{k} = 0.12\,\text{m}.$$

In this part, we use that final-rest point as the reference level for computing gravitational potential energy, so the original $U = mgy$ becomes $mg(d_{\text{eq}} + d)$. In that final position, then, the gravitational energy is zero and the spring energy is $kd_{\text{eq}}^2/2$. Thus, Eq. 8-33 becomes

$$mg(d_{\text{eq}} + d) = \frac{1}{2}kd_{\text{eq}}^2 + fd_{\text{total}}$$

$$(1800)(9.8)(0.12 + 3.7) = \frac{1}{2}(1.5 \times 10^5)(0.12)^2 + (4400)d_{\text{total}}$$

which yields $d_{\text{total}} = 15$ m.

64. In the absence of friction, we have a simple conversion (as it moves along the inclined ramps) of energy between the kinetic form (Eq. 7-1) and the potential form (Eq. 8-9). Along the horizontal plateaus, however, there is friction that causes some of the kinetic energy to dissipate in accordance with Eq. 8-31 (along with Eq. 6-2 where $\mu_k = 0.50$ and $F_N = mg$ in this situation). Thus, after it slides down a (vertical) distance d it has gained $K = \frac{1}{2}mv^2 = mgd$, some of which ($\Delta E_{\text{th}} = \mu_k\, mgd$) is dissipated, so that the value of kinetic energy at the end of the first plateau (just before it starts descending towards the lowest plateau) is

$$K = mgd - \mu_k mgd = \frac{1}{2} mgd \ .$$

In its descent to the lowest plateau, it gains *mgd/*2 more kinetic energy, but as it slides across it "loses" $\mu_k\,mgd/2$ of it. Therefore, as it starts its climb up the right ramp, it has kinetic energy equal to

$$K = \frac{1}{2} mgd + \frac{1}{2} mgd - \frac{1}{2} \mu_k mgd = \frac{3}{4} mgd \ .$$

Setting this equal to Eq. 8-9 (to find the height to which it climbs) we get $H = ¾d$. Thus, the block (momentarily) stops on the inclined ramp at the right, at a height of

$$H = 0.75d = 0.75\ (40\ \text{cm}) = 30\ \text{cm}$$

measured from the lowest plateau.

65. The initial and final kinetic energies are zero, and we set up energy conservation in the form of Eq. 8-33 (with $W = 0$) according to our assumptions. Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on. If it occurs during its first pass through, then the thermal energy generated is $\Delta E_{th} = f_k d$ where $d \le L$ and $f_k = \mu_k mg$. If it occurs during its second pass through, then the total thermal energy is $\Delta E_{th} = \mu_k\, mg(L + d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \le d \le L$). Generalizing to the n^{th} pass through, we see that

$$\Delta E_{th} = \mu_k\, mg[(n - 1)L + d].$$

In this way, we have

$$mgh = \mu_k mg\left((n-1)L + d\right)$$

which simplifies (when $h = L/2$ is inserted) to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n.$$

The first two terms give $1 + 1/2\mu_k = 3.5,$ so that the requirement $0 \le d/L \le 1$ demands that $n = 3$. We arrive at the conclusion that $d/L = \frac{1}{2}$, or

$$d = \frac{1}{2} L = \frac{1}{2}(40\ \text{cm}) = 20\ \text{cm}$$

and that this occurs on its third pass through the flat region.

66. (a) Equation 8-9 gives $U = mgh$ = (3.2 kg)(9.8 m/s^2)(3.0 m) = 94 J.

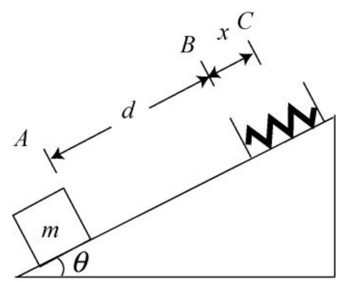
(b) The mechanical energy is conserved, so K = 94 J.

(c) The speed (from solving Eq. 7-1) is

$$v = \sqrt{2K/m} = \sqrt{2(94\text{ J})/(32\text{ kg})} = 7.7\text{ m/s}.$$

67. **THINK** As the block is projected up the inclined plane, its kinetic energy is converted into gravitational potential energy and elastic potential energy of the spring. The block compresses the spring, stopping momentarily before sliding back down again.

EXPRESS Let A be the starting point and the reference point for computing gravitational potential energy ($U_A = 0$). The block first comes into contact with the spring at B. The spring is compressed by an additional amount x at C, as shown in the figure below.



By energy conservation, $K_A + U_A = K_B + U_B = K_C + U_C$. Note that

$$U = U_g + U_s = mgy + \frac{1}{2}kx^2,$$

i.e., the total potential energy is the sum of gravitational potential energy and elastic potential energy of the spring.

ANALYZE (a) At the instant when $x_C = 0.20$ m, the vertical height is

$$y_C = (d + x_C)\sin\theta = (0.60\text{ m} + 0.20\text{ m})\sin 40° = 0.514\text{ m}.$$

Applying energy conservation principle gives

$$K_A + U_A = K_C + U_C \;\Rightarrow\; 16\text{ J} + 0 = K_C + mgy_C + \frac{1}{2}kx_C^2$$

from which we obtain

$$K_C = K_A - mgy_C - \frac{1}{2}kx_C^2$$
$$= 16\text{ J} - (1.0\text{ kg})(9.8\text{ m/s}^2)(0.514\text{ m}) - \frac{1}{2}(200\text{ N/m})(0.20\text{ m})^2 = 6.96\text{ J} \approx 7.0\text{ J}.$$

(b) At the instant when $x'_C = 0.40$ m, the vertical height is

$$y'_C = (d + x'_C)\sin\theta = (0.60\text{ m} + 0.40\text{ m})\sin 40° = 0.64\text{ m}.$$

Applying energy conservation principle, we have $K'_A + U'_A = K'_C + U'_C$. Since $U'_A = 0$, the initial kinetic energy that gives $K'_C = 0$ is

$$K'_A = U'_C = mgy'_C + \frac{1}{2}kx'^2_C$$
$$= (1.0\text{ kg})(9.8\text{ m/s}^2)(0.64\text{ m}) + \frac{1}{2}(200\text{ N/m})(0.40\text{ m})^2$$
$$= 22\text{ J}.$$

LEARN Comparing the results found in (a) and (b), we see that more kinetic energy is required to move the block higher in the inclined plane to achieve a greater spring compression.

68. (a) At the point of maximum height, where *y* = 140 m, the vertical component of velocity vanishes but the horizontal component remains what it was when it was launched (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(0.55\text{ kg})v_x^2.$$

Also, its potential energy (with the reference level chosen at the level of the cliff edge) at that moment is $U = mgy = 755$ J. Thus, by mechanical energy conservation,

$$K = K_i - U = 1550 - 755 \Rightarrow v_x = \sqrt{\frac{2(1550 - 755)}{0.55}} = 54\text{ m/s}.$$

(b) As mentioned, $v_x = v_{ix}$ so that the initial kinetic energy

$$K_i = \frac{1}{2}m\left(v_{ix}^2 + v_{iy}^2\right)$$

can be used to find v_{iy}. We obtain $v_{iy} = 52$ m/s.

(c) Applying Eq. 2-16 to the vertical direction (with +*y* upward), we have

$$v_y^2 = v_{iy}^2 - 2g\Delta y \quad \Rightarrow \quad (65\text{ m/s})^2 = (52\text{ m/s})^2 - 2(9.8\text{ m/s}^2)\Delta y$$

which yields $\Delta y = -76$ m. The minus sign tells us it is below its launch point.

69. **THINK** The two blocks are connected by a cord. As block *B* falls, block *A* moves up the incline.

EXPRESS If the larger mass (block *B*, m_B = 2.0 kg) falls a vertical distance $d = 0.25$ m, then the smaller mass (blocks *A*, m_A = 1.0 kg) must increase its height by $h = d\sin 30°$. The change in gravitational potential energy is

$$\Delta U = -m_B gd + m_A gh.$$

By mechanical energy conservation, $\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$, the change in kinetic energy of the system is $\Delta K = -\Delta U$.

ANALYZE Since the initial kinetic energy is zero, the final kinetic energy is

$$\begin{aligned} K_f &= \Delta K = m_B gd - m_A gh = m_B gd - m_A gd\sin\theta \\ &= (m_B - m_A\sin\theta)gd = [2.0\text{ kg} - (1.0\text{ kg})\sin 30°](9.8\text{ m/s}^2)(0.25\text{ m}) \\ &= 3.7\text{ J}. \end{aligned}$$

LEARN From the above expression, we see that in the special case where $m_B = m_A\sin\theta$, the two-block system would remain stationary. On the other hand, if $m_A\sin\theta > m_B$, block *A* will slide down the incline, with block *B* moving vertically upward.

70. We use conservation of mechanical energy: the mechanical energy must be the same at the top of the swing as it is initially. Newton's second law is used to find the speed, and hence the kinetic energy, at the top. There the tension force *T* of the string and the force of gravity are both downward, toward the center of the circle. We notice that the radius of the circle is $r = L - d$, so the law can be written

$$T + mg = mv^2/(L-d),$$

where *v* is the speed and *m* is the mass of the ball. When the ball passes the highest point with the least possible speed, the tension is zero. Then

$$mg = m\frac{v^2}{L-d} \Rightarrow v = \sqrt{g(L-d)}.$$

We take the gravitational potential energy of the ball-Earth system to be zero when the ball is at the bottom of its swing. Then the initial potential energy is *mgL*. The initial kinetic energy is zero since the ball starts from rest. The final potential energy, at the top of the swing, is $2mg(L - d)$ and the final kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}mg(L-d)$ using the above result for *v*. Conservation of energy yields

$$mgL = 2mg(L-d) + \frac{1}{2}mg(L-d) \Rightarrow d = 3L/5 .$$

With L = 1.20 m, we have d = 0.60(1.20 m) = 0.72 m.

Notice that if *d* is greater than this value, so the highest point is lower, then the speed of the ball is greater as it reaches that point and the ball passes the point. If *d* is less, the ball cannot go around. Thus the value we found for *d* is a lower limit.

71. **THINK** As the block slides down the frictionless incline, its gravitational potential energy is converted to kinetic energy, so the speed of the block increases.

EXPRESS By energy conservation, $K_A + U_A = K_B + U_B$. Thus, the change in kinetic energy as the block moves from points *A* to *B* is

$$\Delta K = K_B - K_A = -\Delta U = -(U_B - U_A).$$

In both circumstances, we have the same potential energy change. Thus, $\Delta K_1 = \Delta K_2$.

ANALYZE With $\Delta K_1 = \Delta K_2$, the speed of the block at B the second time is given by

$$\frac{1}{2}mv_{B,1}^2 - \frac{1}{2}mv_{A,1}^2 = \frac{1}{2}mv_{B,2}^2 \; \frac{1}{2}mv_{A,2}^2$$

or

$$v_{B,2} = \sqrt{v_{B,1}^2 - v_{A,1}^2 + v_{A,2}^2} = \sqrt{(2.60\text{ m/s})^2 - (2.00\text{ m/s})^2 + (4.00\text{ m/s})^2} = 4.33\text{ m/s} .$$

LEARN The speed of the block at *A* is greater the second time, $v_{A,2} > v_{A,1}$. This can happen if the block slides down from a higher position with greater initial gravitational potential energy.

72. (a) We take the gravitational potential energy of the skier-Earth system to be zero when the skier is at the bottom of the peaks. The initial potential energy is $U_i = mgH$, where *m* is the mass of the skier, and *H* is the height of the higher peak. The final potential energy is $U_f = mgh$, where *h* is the height of the lower peak. The skier initially has a kinetic energy of $K_i = 0$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where *v* is the speed of the skier at the top of the lower peak. The normal force of the slope on the skier does no work and friction is negligible, so mechanical energy is conserved:

$$U_i + K_i = U_f + K_f \quad \Rightarrow \quad mgH = mgh + \frac{1}{2}mv^2 .$$

Thus,

$$v = \sqrt{2g(H-h)} = \sqrt{2(9.8\text{ m/s}^2)(850\text{ m} - 750\text{ m})} = 44\text{ m/s} .$$

(b) We recall from analyzing objects sliding down inclined planes that the normal force of the slope on the skier is given by $F_N = mg\cos\theta$, where θ is the angle of the slope from the horizontal, 30° for each of the slopes shown. The magnitude of the force of friction is given by $f = \mu_k F_N = \mu_k mg\cos\theta$. The thermal energy generated by the force of friction is $fd = \mu_k mgd\cos\theta$, where d is the total distance along the path. Since the skier gets to the top of the lower peak with no kinetic energy, the increase in thermal energy is equal to the decrease in potential energy. That is, $\mu_k mgd\cos\theta = mg(H-h)$. Consequently,

$$\mu_k = \frac{H-h}{d\cos\theta} = \frac{(850\text{ m} - 750\text{ m})}{(3.2\times10^3\text{ m})\cos 30°} = 0.036 .$$

73. **THINK** As the cube is pushed across the floor, both the thermal energies of floor and the cube increase because of friction.

EXPRESS By law of conservation of energy, we have $W = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$ for the floor-cube system. Since the speed is constant, $\Delta K = 0$, Eq. 8-33 (an application of the energy conservation concept) implies

$$W = \Delta E_{\text{mech}} + \Delta E_{\text{th}} = \Delta E_{\text{th}} = \Delta E_{\text{th (cube)}} + \Delta E_{\text{th (floor)}} .$$

ANALYZE With $W = (15\text{ N})(3.0\text{ m}) = 45\text{ J}$, and we are told that $\Delta E_{\text{th (cube)}} = 20$ J, then we conclude that $\Delta E_{\text{th (floor)}} = 25$ J.

LEARN The applied work here has all been converted into thermal energies of the floor and the cube. The amount of thermal energy transferred to a material depends on its thermal properties, as we shall discuss in Chapter 18.

74. We take her original elevation to be the $y = 0$ reference level and observe that the top of the hill must consequently have $y_A = R(1 - \cos 20°) = 1.2$ m, where R is the radius of the hill. The mass of the skier is $m = (600\text{ N})/(9.8\text{ m/s}^2) = 61\text{ kg}$.

(a) Applying energy conservation, Eq. 8-17, we have

$$K_B + U_B = K_A + U_A \Rightarrow K_B + 0 = K_A + mgy_A .$$

Using $K_B = \frac{1}{2}(61\text{ kg})(8.0\text{ m/s})^2$, we obtain $K_A = 1.2\times10^3$ J. Thus, we find the speed at the hilltop is

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2(1.2\times10^3\ \text{J})}{61\,\text{kg}}} = 6.4\ \text{m/s}.$$

Note: One might wish to check that the skier stays in contact with the hill — which is indeed the case here. For instance, at A we find $v^2/r \approx 2\ \text{m/s}^2$, which is considerably less than g.

(b) With $K_A = 0$, we have

$$K_B + U_B = K_A + U_A \Rightarrow K_B + 0 = 0 + mgy_A$$

which yields $K_B = 724$ J, and the corresponding speed is

$$v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(724\ \text{J})}{61\,\text{kg}}} = 4.9\ \text{m/s}.$$

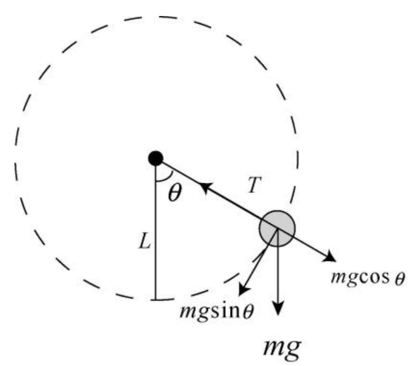
(c) Expressed in terms of mass, we have

$$K_B + U_B = K_A + U_A \Rightarrow$$
$$\frac{1}{2}mv_B^2 + mgy_B = \frac{1}{2}mv_A^2 + mgy_A\ .$$

Thus, the mass m cancels, and we observe that solving for speed does not depend on the value of mass (or weight).

75. **THINK** This problem deals with pendulum motion. The kinetic and potential energies of the ball attached to the rod change with position, but the mechanical energy remains conserved throughout the process.

EXPRESS Let L be the length of the pendulum. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy mgh) is given by $h = L(1 - \cos\theta)$.



The free-body diagram is shown above. The initial height is at $h_1 = 2L$, and at the lowest point, we have $h_2 = 0$. The total mechanical energy is conserved throughout.

ANALYZE (a) Initially the ball is at $h_1 = 2L$ with $K_1 = 0$ and $U_1 = mgh_1 = mg(2L)$. At the lowest point $h_2 = 0$, we have $K_2 = \frac{1}{2}mv_2^2$ and $U_2 = 0$. Using energy conservation in the form of Eq. 8-17 leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + 2mgL = \frac{1}{2}mv_2^2 + 0$$

This leads to $v_2 = 2\sqrt{gL}$. With $L = 0.62$ m, we have

$$v_2 = 2\sqrt{(9.8 \text{ m/s}^2)(0.62 \text{ m})} = 4.9 \text{ m/s}.$$

(b) At the lowest point, the ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2 / r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m\frac{v^2}{r} \Rightarrow T = m\left(g + \frac{4gL}{L}\right) = 5\ mg.$$

With $m = 0.092$ kg, the tension is $T = 4.5$ N.

(c) The pendulum is now started (with zero speed) at $\theta_i = 90°$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ, as can be seen from the free-body diagram shown above, Newton's second law applied to the axis along the rod yields

$$\frac{mv^2}{r} = T - mg\cos\theta = mg(1 - \cos\theta)$$

which (since $r = L$) implies $v^2 = gL(1 - \cos\theta)$ at the position we are looking for. Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 + mgL &= \frac{1}{2}mv^2 + mgL\ (1 - \cos\theta) \\ gL &= \frac{1}{2}(gL(1 - \cos\theta)) + gL\ (1 - \cos\theta) \end{aligned}$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 71°.$$

(d) Since the angle found in (c) is independent of the mass, the result remains the same if the mass of the ball is changed.

LEARN At a given angle θ with respect to the vertical, the tension in the rod is

$$T = m\left(\frac{v^2}{r} + g\cos\theta\right)$$

The tangential acceleration, $a_t = g\sin\theta$, is what causes the speed and, therefore, the kinetic energy to change with time. Nonetheless, mechanical energy is conserved.

76. (a) The table shows that the force is $+(3.0\text{ N})\hat{\mathrm{i}}$ while the displacement is in the $+x$ direction ($\vec{d} = +(3.0\text{ m})\hat{\mathrm{i}}$), and it is $-(3.0\text{ N})\hat{\mathrm{i}}$ while the displacement is in the $-x$ direction. Using Eq. 7-8 for each part of the trip, and adding the results, we find the work done is 18 J. This is not a conservative force field; if it had been, then the net work done would have been zero (since it returned to where it started).

(b) This, however, is a conservative force field, as can be easily verified by calculating that the net work done here is zero.

(c) The two integrations that need to be performed are each of the form $\int 2x\,dx$ so that we are adding two equivalent terms, where each equals x^2 (evaluated at $x = 4$, minus its value at $x = 1$). Thus, the work done is $2(4^2 - 1^2) = 30$ J.

(d) This is another conservative force field, as can be easily verified by calculating that the net work done here is zero.

(e) The forces in (b) and (d) are conservative.

77. **THINK** This problem involves graphical analyses. From the graph of potential energy as a function of position, the conservative force can de deduced.

EXPRESS The connection between the potential energy function $U(x)$ and the conservative force $F(x)$ is given by Eq. 8-22: $F(x) = -dU/dx$. A positive slope of $U(x)$ at a point means that $F(x)$ is negative, and vice versa.

ANALYZE (a) The force at $x = 2.0$ m is

$$F = -\frac{dU}{dx} \approx -\frac{\Delta U}{\Delta x} = -\frac{U(x=4\text{ m}) - U(x=1\text{ m})}{4.0\text{ m} - 1.0\text{ m}} = -\frac{-(17.5\text{ J}) - (-2.8\text{ J})}{4.0\text{ m} - 1.0\text{ m}} = 4.9\text{ N}.$$

(b) Since the slope of $U(x)$ at $x = 2.0$ m is negative, the force points in the $+x$ direction (but there is some uncertainty in reading the graph which makes the last digit not very significant).

(c) At x = 2.0 m, we estimate the potential energy to be

$$U(x=2.0\text{ m}) \approx U(x=1.0\text{ m}) + (-4.9\text{ J/m})(1.0\text{ m}) = -7.7\text{ J}$$

Thus, the total mechanical energy is

$$E = K + U = \frac{1}{2}mv^2 + U = \frac{1}{2}(2.0\text{ kg})(-1.5\text{ m/s})^2 + (-7.7\text{ J}) = -5.5\text{ J}.$$

Again, there is some uncertainty in reading the graph which makes the last digit not very significant. At that level (–5.5 J) on the graph, we find two points where the potential energy curve has that value — at $x \approx 1.5$ m and $x \approx 13.5$ m. Therefore, the particle remains in the region $1.5 < x < 13.5$ m. The left boundary is at x = 1.5 m.

(d) From the above results, the right boundary is at x = 13.5 m.

(e) At x = 7.0 m, we read $U \approx -17.5$ J. Thus, if its total energy (calculated in the previous part) is $E \approx -5.5$ J, then we find

$$\frac{1}{2}mv^2 = E - U \approx 12\text{ J} \Rightarrow v = \sqrt{\frac{2}{m}(E-U)} \approx 3.5\text{ m/s}$$

where there is certainly room for disagreement on that last digit for the reasons cited above.

LEARN Since the total mechanical energy is negative, the particle is bounded by the potential, with its motion confined to the region 1.5 m < x < 13.5 m. At the turning points (1.5 m and 13.5 m), kinetic energy is zero and the particle is momentarily at rest.

78. (a) Since the speed of the crate of mass m increases from 0 to 1.20 m/s relative to the factory ground, the kinetic energy supplied to it is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(300\text{ kg})(120\text{ m/s})^2 = 216\text{ J}.$$

(b) The magnitude of the kinetic frictional force is

$$f = \mu F_N = \mu mg = (0.400)(300\text{ kg})(9.8\text{ m/s}^2) = 1.18\times10^3\text{ N}.$$

(c) Let the distance the crate moved relative to the conveyor belt before it stops slipping be d. Then from Eq. 2-16 ($v^2 = 2ad = 2(f/m)d$) we find

$$\Delta E_{\text{th}} = fd = \frac{1}{2}mv^2 = K.$$

Thus, the total energy that must be supplied by the motor is

$$W = K + \Delta E_{\text{th}} = 2K = (2)(216\,\text{J}) = 432\ \text{J}.$$

(d) The energy supplied by the motor is the work W it does on the system, and must be greater than the kinetic energy gained by the crate computed in part (b). This is due to the fact that part of the energy supplied by the motor is being used to compensate for the energy dissipated ΔE_{th} while it was slipping.

79. **THINK** As the car slides down the incline, due to the presence of frictional force, some of its mechanical energy is converted into thermal energy.

EXPRESS The incline angle is $\theta = 5.0°$. Thus, the change in height between the car's highest and lowest points is $\Delta y = -(50\text{ m})\sin\theta = -4.4$ m. We take the lowest point (the car's final reported location) to correspond to the $y = 0$ reference level. The change in potential energy is given by $\Delta U = mg\Delta y$.

As for the kinetic energy, we first convert the speeds to SI units, $v_0 = 8.3\ \text{m/s}$ and $v = 11.1\ \text{m/s}$. The change in kinetic energy is $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$. The total change in mechanical energy is $\Delta E_{\text{mech}} = \Delta K + \Delta U$.

ANALYZE (a) Substituting the values given, we find ΔE_{mech} to be

$$\begin{aligned}\Delta E_{\text{mech}} &= \Delta K + \Delta U = \frac{1}{2}m(v_f^2 - v_i^2) + mg\Delta y \\ &= \frac{1}{2}(1500\,\text{kg})\left[(11.1\,\text{m/s})^2 - (8.3\,\text{m/s})^2\right] + (1500\,\text{kg})(9.8\,\text{m/s}^2)(-4.4\ \text{m}) \\ &= -23940\ \text{J} \approx -2.4\times10^4\ \text{J}\end{aligned}$$

That is, the mechanical energy decreases (due to friction) by 2.4×10^4 J.

(b) Using Eq. 8-31 and Eq. 8-33, we find $\Delta E_{\text{th}} = f_k d = -\Delta E_{\text{mech}}$. With $d = 50$ m, we solve for f_k and obtain

$$f_k = \frac{-\Delta E_{\text{mech}}}{d} = \frac{-(-2.4\times10^4\ \text{J})}{50\ \text{m}} = 4.8\times10^2\ \text{N}.$$

LEARN The amount of mechanical energy lost is proportional to the frictional force; in the absence of friction, mechanical energy would have been conserved.

80. We note that in one second, the block slides $d = 1.34$ m up the incline, which means its height increase is $h = d\sin\theta$ where

$$\theta = \tan^{-1}\left(\frac{30}{40}\right) = 37°.$$

We also note that the force of kinetic friction in this inclined plane problem is $f_k = \mu_k mg\cos\theta$, where μ_k = 0.40 and m = 1400 kg. Thus, using Eq. 8-31 and Eq. 8-33, we find

$$W = mgh + f_k d = mgd\left(\sin\theta + \mu_k\cos\theta\right)$$

or $W = 1.69 \times 10^4$ J for this one-second interval. Thus, the power associated with this is

$$P = \frac{1.69\times10^4\ \text{J}}{1\ \text{s}} = 1.69\times10^4\ \text{W} \approx 1.7\times10^4\ \text{W}.$$

81. (a) The remark in the problem statement that the forces can be associated with potential energies is illustrated as follows: the work from x = 3.00 m to x = 2.00 m is

$$W = F_2\,\Delta x = (5.00\ \text{N})(-1.00\ \text{m}) = -5.00\ \text{J},$$

so the potential energy at x = 2.00 m is U_2 = +5.00 J.

(b) Now, it is evident from the problem statement that E_{max} = 14.0 J, so the kinetic energy at x = 2.00 m is

$$K_2 = E_{max} - U_2 = 14.0 - 5.00 = 9.00\ \text{J}.$$

(c) The work from x = 2.00 m to x = 0 is $W = F_1\,\Delta x$ =(3.00 N)(–2.00 m) = – 6.00 J, so the potential energy at x = 0 is

$$U_0 = 6.00\ \text{J} + U_2 = (6.00 + 5.00)\ \text{J} = 11.0\ \text{J}.$$

(d) Similar reasoning to that presented in part (a) then gives

$$K_0 = E_{max} - U_0 = (14.0 - 11.0)\ \text{J} = 3.00\ \text{J}.$$

(e) The work from x = 8.00 m to x = 11.0 m is $W = F_3\,\Delta x$ =(– 4.00 N)(3.00 m) = –12.0 J, so the potential energy at x = 11.0 m is U_{11} = 12.0 J.

(f) The kinetic energy at x = 11.0 m is therefore

$$K_{11} = E_{max} - U_{11} = (14.0 - 12.0)\ \text{J} = 2.00\ \text{J}.$$

(g) Now we have $W = F_4\,\Delta x$ =(–1.00 N)(1.00 m) = –1.00 J, so the potential energy at $x = 12.0$ m is

$$U_{12} = 1.00\ \text{J} + U_{11} = (1.00 + 12.0)\ \text{J} = 13.0\ \text{J}.$$

(h) Thus, the kinetic energy at x = 12.0 m is

$$K_{12} = E_{max} - U_{12} = (14.0 - 13.0) = 1.00 \text{ J}.$$

(i) There is no work done in this interval (from x = 12.0 m to x = 13.0 m) so the answers are the same as in part (g): U_{12} = 13.0 J.

(j) There is no work done in this interval (from x = 12.0 m to x = 13.0 m) so the answers are the same as in part (h): K_{12} = 1.00 J.

(k) Although the plot is not shown here, it would look like a "potential well" with piecewise-sloping sides: from x = 0 to x = 2 (SI units understood) the graph of U is a decreasing line segment from 11 to 5, and from x = 2 to x = 3, it then heads down to zero, where it stays until x = 8, where it starts increasing to a value of 12 (at x = 11), and then in another positive-slope line segment it increases to a value of 13 (at x = 12). For $x > 12$ its value does not change (this is the "top of the well").

(l) The particle can be thought of as "falling" down the $0 < x < 3$ slopes of the well, gaining kinetic energy as it does so, and certainly is able to reach x = 5. Since U = 0 at x = 5, then its initial potential energy (11 J) has completely converted to kinetic: now $K = 11.0 \text{ J}$.

(m) This is not sufficient to climb up and out of the well on the large x side (x > 8), but does allow it to reach a "height" of 11 at x = 10.8 m. As discussed in section 8-5, this is a "turning point" of the motion.

(n) Next it "falls" back down and rises back up the small x slope until it comes back to its original position. Stating this more carefully, when it is (momentarily) stopped at x = 10.8 m it is accelerated to the left by the force $\vec{F}_3$; it gains enough speed as a result that it eventually is able to return to x = 0, where it stops again.

82. (a) At x = 5.00 m the potential energy is zero, and the kinetic energy is

$$K = \frac{1}{2} mv^2 = \frac{1}{2} (2.00 \text{ kg})(3.45 \text{ m/s})^2 = 11.9 \text{ J}.$$

The total energy, therefore, is great enough to reach the point x = 0 where U = 11.0 J, with a little "left over" (11.9 J – 11.0 J = 0.9025 J). This is the kinetic energy at x = 0, which means the speed there is

$$v = \sqrt{2(0.9025 \text{ J})/(2 \text{ kg})} = 0.950 \text{ m/s}.$$

It has now come to a stop, therefore, so it has not encountered a turning point.

(b) The total energy (11.9 J) is equal to the potential energy (in the scenario where it is initially moving rightward) at x = 10.9756 ≈ 11.0 m. This point may be found by interpolation or simply by using the work-kinetic energy theorem:

$$K_f = K_i + W = 0 \quad \Rightarrow \quad 11.9025 + (-4)d = 0 \quad \Rightarrow \quad d = 2.9756 \approx 2.98$$

(which when added to x = 8.00 [the point where F_3 begins to act] gives the correct result). This provides a turning point for the particle's motion.

83. **THINK** Energy is transferred from an external agent to the block so that its speed continues to increase.

EXPRESS According to Eq. 8-25, the work done by the external force is $W = \Delta E_{\text{mech}} = \Delta K + \Delta U$. When there is no change in potential energy, $\Delta U = 0$, the expression simplifies to

$$W = \Delta E_{\text{mech}} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2).$$

The average power, or average rate of work done, is given by $P_{\text{avg}} = W/\Delta t$.

ANALYZE (a) Substituting the values given, the change in mechanical energy is

$$\Delta E_{\text{mech}} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(15\text{ kg})[(30\text{ m/s})^2 - (10\text{ m/s})^2] = 6000\text{ J} = 6.0\times10^3\text{ J}$$

(b) From the above, we have $W = 6.0 \times 10^3$ J. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10$ s. Thus, using Eq. 7-42, the average rate at which energy is transferred to the block is

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0\times10^3\text{ J}}{10.0\text{ s}} = 600\text{ W}.$$

(c) and (d) The constant applied force is $F = ma$ = 30 N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power:

$$P = \vec{F}\cdot\vec{v} = \begin{cases} 300\text{ W} & \text{for } v = 10\text{ m/s} \\ 900\text{ W} & \text{for } v = 30\text{ m/s} \end{cases}$$

LEARN The average of these two values found in (c) and (d) agrees with the result in part (b). Note that the expression for the instantaneous rate used above can be derived from:

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = m\vec{v}\cdot\frac{d\vec{v}}{dt} = m\vec{v}\cdot\vec{a} = \vec{F}\cdot\vec{v}$$

84. (a) To stretch the spring an external force, equal in magnitude to the force of the spring but opposite to its direction, is applied. Since a spring stretched in the positive x direction exerts a force in the negative x direction, the applied force must be $F = 52.8x + 38.4x^2$, in the $+x$ direction. The work it does is

$$W=\int_{0.50}^{1.00}(52.8x+38.4x^2)dx=\left(\frac{52.8}{2}x^2+\frac{38.4}{3}x^3\right)\Bigg|_{0.50}^{1.00}=31.0\ \text{J}.$$

(b) The spring does 31.0 J of work and this must be the increase in the kinetic energy of the particle. Its speed is then

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(31.0\,\text{J})}{2.17\,\text{kg}}}=5.35\,\text{m/s}.$$

(c) The force is conservative since the work it does as the particle goes from any point x_1 to any other point x_2 depends only on x_1 and x_2, not on details of the motion between x_1 and x_2.

85. **THINK** This problem deals with the concept of hydroelectric generator – kinetic energy of water can be converted into electrical energy.

EXPRESS By energy conservation, the change in kinetic energy of water in one second is

$$\Delta K=-\Delta U=mgh=\rho Vgh=(10^3\,\text{kg/m}^3)(1200\,\text{m}^3)(9.8\,\text{m/s}^2)(100\,\text{m})=1.176\times10^9\ \text{J}$$

Only 3/4 of this amount is transferred to electrical energy.

ANALYZE The power generation (assumed constant, so average power is the same as instantaneous power) is

$$P_{\text{avg}}=\frac{(3/4)\Delta K}{t}=\frac{(3/4)(1.176\times10^9\ \text{J})}{1.0\,\text{s}}=8.82\times10^8\ \text{W}.$$

LEARN Hydroelectricity is the most widely used renewable energy; it accounts for almost 20% of the world's electricity supply.

86. (a) At *B* the speed is (from Eq. 8-17)

$$v=\sqrt{v_0^2+2gh_1}=\sqrt{(7.0\text{ m/s})^2+2(9.8\text{ m/s}^2)(6.0\text{ m})}=13\text{ m/s}.$$

(a) Here what matters is the difference in heights (between *A* and *C*):

$$v=\sqrt{v_0^2+2g(h_1-h_2)}=\sqrt{(7.0\text{ m/s})^2+2(9.8\text{ m/s}^2)(4.0\text{ m})}=11.29\text{ m/s}\approx11\text{ m/s}.$$

(c) Using the result from part (b), we see that its kinetic energy right at the beginning of its "rough slide" (heading horizontally toward *D*) is $\frac{1}{2}$ $m(11.29\text{ m/s})^2$ = 63.7m (with SI units understood). Note that we "carry along" the mass (as if it were a known quantity);

as we will see, it will cancel out, shortly. Using Eq. 8-31 (and Eq. 6-2 with $F_N = mg$) we note that this kinetic energy will turn entirely into thermal energy

$$63.7m = \mu_k mgd$$

if $d < L$. With $\mu_k = 0.70$, we find $d = 9.3$ m, which is indeed less than L (given in the problem as 12 m). We conclude that the block stops before passing out of the "rough" region (and thus does not arrive at point D).

87. **THINK** We have a ball attached to a rod that moves in a vertical circle. The total mechanical energy of the system is conserved.

EXPRESS Let position A be the reference point for potential energy, $U_A = 0$. The total mechanical energies at A, B and C are:

$$E_A = \frac{1}{2}mv_A^2 + U_A = \frac{1}{2}mv_0^2$$
$$E_B = \frac{1}{2}mv_B^2 + U_B = \frac{1}{2}mv_B^2 - mgL$$
$$E_D = \frac{1}{2}mv_D^2 + U_D = mgL$$

where $v_D = 0$. The problem can be analyzed by applying energy conservation: $E_A = E_B = E_D$.

ANALYZE (a) The condition $E_A = E_D$ gives

$$\frac{1}{2}mv_0^2 = mgL \quad \Rightarrow \quad v_0 = \sqrt{2gL}$$

(b) To find the tension in the rod when the ball passes through B, we first calculate the speed at B. Using $E_B = E_D$, we find

$$\frac{1}{2}mv_B^2 - mgL = mgL$$

or $v_B = \sqrt{4gL}$. The direction of the centripetal acceleration is upward (at that moment), as is the tension force. Thus, Newton's second law gives

$$T - mg = \frac{mv_B^2}{r} = \frac{m(4gL)}{L} = 4mg$$

or $T = 5mg$.

(c) The difference in height between C and D is L, so the "loss" of mechanical energy (which goes into thermal energy) is $-mgL$.

(d) The difference in height between *B* and *D* is 2*L*, so the total "loss" of mechanical energy (which all goes into thermal energy) is –2*mgL*.

LEARN An alternative way to calculate the energy loss in (d) is to note that

$$E'_B = \frac{1}{2}mv_B'^2 + U_B = 0 - mgL = -mgL$$

which gives

$$\Delta E = E'_B - E_A = -mgL - mgL = -2mgL.$$

88. (a) The initial kinetic energy is $K_i = \frac{1}{2}(1.5)(3)^2 = 6.75\text{ J}$.

(b) The work of gravity is the negative of its change in potential energy. At the highest point, all of K_i has converted into *U* (if we neglect air friction) so we conclude the work of gravity is –6.75 J.

(c) And we conclude that $\Delta U = 6.75\text{ J}$.

(d) The potential energy there is $U_f = U_i + \Delta U = 6.75\text{ J}$.

(e) If $U_f = 0$, then $U_i = U_f - \Delta U = -6.75\text{ J}$.

(f) Since $mg\Delta y = \Delta U$, we obtain $\Delta y = 0.459\text{ m}$.

89. (a) By mechanical energy conversation, the kinetic energy as it reaches the floor (which we choose to be the *U* = 0 level) is the sum of the initial kinetic and potential energies:

$$K = K_i + U_i = \frac{1}{2}(2.50\text{ kg})(3.00\text{ m/s})^2 + (2.50\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m}) = 109\text{ J}.$$

For later use, we note that the speed with which it reaches the ground is

$$v = \sqrt{2K/m} = 9.35\text{ m/s}.$$

(b) When the drop in height is 2.00 m instead of 4.00 m, the kinetic energy is

$$K = \frac{1}{2}(2.50\text{ kg})(3.00\text{ m/s})^2 + (2.50\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 60.3\text{ J}.$$

(c) A simple way to approach this is to imagine the can being *launched* from the ground at $t = 0$ with a speed 9.35 m/s (see above) and calculate the height and speed at t = 0.200 s, using Eq. 2-15 and Eq. 2-11:

$$y = (9.35 \text{ m/s})(0.200 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(0.200 \text{ s})^2 = 1.67 \text{ m},$$

$$v = 9.35 \text{ m/s} - (9.80 \text{ m/s}^2)(0.200 \text{ s}) = 7.39 \text{ m/s}.$$

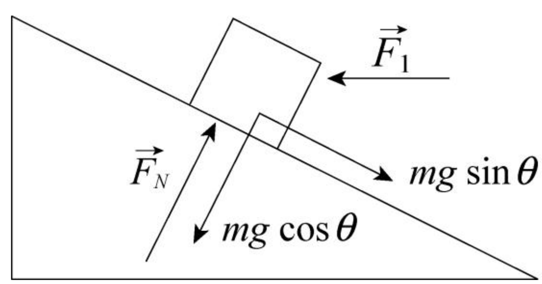
The kinetic energy is $K = \frac{1}{2}(2.50 \text{ kg})(7.39 \text{ m/s})^2 = 68.2 \text{ J}$.

(d) The gravitational potential energy is

$$U = mgy = (2.5 \text{ kg})(9.8 \text{ m/s}^2)(1.67 \text{ m}) = 41.0 \text{ J}.$$

90. The free-body diagram for the trunk is shown below. The x and y applications of Newton's second law provide two equations:

$$F_1 \cos\theta - f_k - mg \sin\theta = ma$$
$$F_N - F_1 \sin\theta - mg\cos\theta = 0.$$



(a) The trunk is moving up the incline at constant velocity, so $a = 0$. Using $f_k = \mu_k F_N$, we solve for the push-force F_1 and obtain

$$F_1 = \frac{mg(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta}.$$

The work done by the push-force $\vec{F}_1$ as the trunk is pushed through a distance ℓ up the inclined plane is therefore

$$\begin{aligned} W_1 = F_1 \ell \cos\theta &= \frac{(mg\ell\cos\theta)(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta} \\ &= \frac{(50 \text{ kg})(9.8 \text{ m/s}^2)(6.0 \text{ m})(\cos 30°)(\sin 30° + (0.20)\cos 30°)}{\cos 30° - (0.20)\sin 30°} \\ &= 2.2\times10^3 \text{ J}. \end{aligned}$$

(b) The increase in the gravitational potential energy of the trunk is

$$\Delta U = mg\ell \sin\theta = (50\,\text{kg})(9.8\,\text{m/s}^2)(6.0\,\text{m})\sin 30° = 1.5\times10^3\ \text{J}.$$

Since the speed (and, therefore, the kinetic energy) of the trunk is unchanged, Eq. 8-33 leads to

$$W_1 = \Delta U + \Delta E_{\text{th}}.$$

Thus, using more precise numbers than are shown above, the increase in thermal energy (generated by the kinetic friction) is 2.24×10^3 J – 1.47×10^3 J = 7.7×10^2 J. An alternate way to this result is to use $\Delta E_{\text{th}} = f_k \ell$ (Eq. 8-31).

91. The initial height of the $2M$ block, shown in Fig. 8-69, is the $y = 0$ level in our computations of its value of U_g. As that block drops, the spring stretches accordingly. Also, the kinetic energy K_{sys} is evaluated for the *system*, that is, for a total moving mass of $3M$.

(a) The conservation of energy, Eq. 8-17, leads to

$$K_i + U_i = K_{sys} + U_{sys} \quad\Rightarrow\quad 0 + 0 = K_{sys} + (2M)g(-0.090) + \frac{1}{2}k(0.090)^2 .$$

Thus, with $M = 2.0$ kg, we obtain $K_{sys} = 2.7$ J.

(b) The kinetic energy of the $2M$ block represents a fraction of the total kinetic energy:

$$\frac{K_{2M}}{K_{sys}} = \frac{(2M)v^2/2}{(3M)v^2/2} = \frac{2}{3}.$$

Therefore, $K_{2M} = \frac{2}{3}(2.7\ \text{J}) = 1.8$ J.

(c) Here we let $y = -d$ and solve for d.

$$K_i + U_i = K_{sys} + U_{sys} \quad\Rightarrow\quad 0 + 0 = 0 + (2M)g(-d) + \frac{1}{2}kd^2 .$$

Thus, with $M = 2.0$ kg, we obtain $d = 0.39$ m.

92. By energy conservation, $mgh = mv^2/2$, the speed of the volcanic ash is given by $v = \sqrt{2gh}$. In our present problem, the height is related to the distance (on the $\theta = 10°$ slope) $d = 920$ m by the trigonometric relation $h = d\sin\theta$. Thus,

$$v = \sqrt{2(9.8\ \text{m/s}^2)(920\ \text{m})\sin 10°} = 56\ \text{m/s}.$$

93. (a) The assumption is that the slope of the bottom of the slide is horizontal, like the ground. A useful analogy is that of the pendulum of length $R = 12$ m that is pulled

leftward to an angle θ (corresponding to being at the top of the slide at height h = 4.0 m) and released so that the pendulum swings to the lowest point (zero height) gaining speed $v = 6.2\text{ m/s}$. Exactly as we would analyze the trigonometric relations in the pendulum problem, we find

$$h = R(1-\cos\theta) \Rightarrow \theta = \cos^{-1}\left(1-\frac{h}{R}\right) = 48°$$

or 0.84 radians. The slide, representing a circular arc of length $s = R\theta$, is therefore (12 m)(0.84) = 10 m long.

(b) To find the magnitude f of the frictional force, we use Eq. 8-31 (with W = 0):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + fs \end{aligned}$$

so that (with m = 25 kg) we obtain f = 49 N.

(c) The assumption is no longer that the slope of the bottom of the slide is horizontal, but rather that the slope of the top of the slide is vertical (and 12 m to the left of the center of curvature). Returning to the pendulum analogy, this corresponds to releasing the pendulum from horizontal (at θ_1 = 90° measured from vertical) and taking a snapshot of its motion a few moments later when it is at angle θ_2 with speed v = 6.2 m/s. The difference in height between these two positions is (just as we would figure for the pendulum of length R)

$$\Delta h = R(1-\cos\theta_2) - R(1-\cos\theta_1) = -R\cos\theta_2$$

where we have used the fact that cos θ_1 = 0. Thus, with Δh = –4.0 m, we obtain θ_2 = 70.5° which means the arc subtends an angle of $|\Delta\theta|$ = 19.5° or 0.34 radians. Multiplying this by the radius gives a slide length of s' = 4.1 m.

(d) We again find the magnitude f' of the frictional force by using Eq. 8-31 (with W = 0):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + f's' \end{aligned}$$

so that we obtain $f' = 1.2 \times 10^2$ N.

94. We use $P = Fv$ to compute the force:

$$F = \frac{P}{v} = \frac{92\times10^6\text{ W}}{(32.5\text{ knot})\left(1.852\,\frac{\text{km/h}}{\text{knot}}\right)\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right)} = 5.5\times10^6\text{ N}.$$

95. This can be worked entirely by the methods of Chapters 2–6, but we will use energy methods in as many steps as possible.

(a) By a force analysis in the style of Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$ (where $\theta = 39°$), which means $f_k = \mu_k mg\cos\theta$ where $\mu_k = 0.28$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta.$$

Also, elementary trigonometry leads us to conclude that $\Delta U = -mgd\sin\theta$ where $d = 3.7$ m. Since $K_i = 0$, Eq. 8-33 (with $W = 0$) indicates that the final kinetic energy is

$$K_f = -\Delta U - \Delta E_{\text{th}} = mgd\,(\sin\theta - \mu_k\cos\theta)$$

which leads to the speed at the bottom of the ramp

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gd\,(\sin\theta - \mu_k\cos\theta)} = 5.5\ \text{m/s}.$$

(b) This speed begins its horizontal motion, where $f_k = \mu_k mg$ and $\Delta U = 0$. It slides a distance d' before it stops. According to Eq. 8-31 (with $W = 0$),

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= 0 - \frac{1}{2}mv^2 + 0 + \mu_k mgd' \\ &= -\frac{1}{2}\left(2gd\,(\sin\theta - \mu_k\cos\theta)\right) + \mu_k gd' \end{aligned}$$

where we have divided by mass and substituted from part (a) in the last step. Therefore,

$$d' = \frac{d\,(\sin\theta - \mu_k\cos\theta)}{\mu_k} = 5.4\ \text{m}.$$

(c) We see from the algebraic form of the results, above, that the answers do not depend on mass. A 90 kg crate should have the same speed at the bottom and sliding distance across the floor, to the extent that the friction relations in Chapter 6 are accurate. Interestingly, since g does not appear in the relation for d', the sliding distance would seem to be the same if the experiment were performed on Mars!

96. (a) The loss of the initial $K = \frac{1}{2}mv^2 = \frac{1}{2}(70\text{ kg})(10\text{ m/s})^2$ is 3500 J, or 3.5 kJ.

(b) This is dissipated as thermal energy; $\Delta E_{\text{th}} = 3500$ J = 3.5 kJ.

97. Eq. 8-33 gives $mgy_f = K_i + mgy_i - \Delta E_{\text{th}}$, or

$$(0.50\text{ kg})(9.8\text{ m/s}^2)(0.80\text{ m}) = \frac{1}{2}(0.50\text{ kg})(4.00\text{ /s})^2 + (0.50\text{ kg})(9.8\text{ m/s}^2)(0) - \Delta E_{\text{th}}$$

which yields ΔE_{th} = 4.00 J – 3.92 J = 0.080 J.

98. Since the period T is (2.5 rev/s)$^{-1}$ = 0.40 s, then Eq. 4-33 leads to v = 3.14 m/s. The frictional force has magnitude (using Eq. 6-2)

$$f = \mu_k F_N = (0.320)(180\text{ N}) = 57.6\text{ N}.$$

The power dissipated by the friction must equal that supplied by the motor, so Eq. 7-48 gives P = (57.6 N)(3.14 m/s) = 181 W.

99. To swim at constant velocity the swimmer must push back against the water with a force of 110 N. Relative to him the water is going at 0.22 m/s toward his rear, in the same direction as his force. Using Eq. 7-48, his power output is obtained:

$$P = \vec{F}\cdot\vec{v} = Fv = (110\text{ N})(0.22\text{ m/s}) = 24\text{ W}.$$

100. The initial kinetic energy of the automobile of mass m moving at speed v_i is $K_i = \frac{1}{2}mv_i^2$, where m = 16400/9.8 = 1673 kg. Using Eq. 8-31 and Eq. 8-33, this relates to the effect of friction force f in stopping the auto over a distance d by $K_i = fd$, where the road is assumed level (so ΔU = 0). With

$$v_i = (113\text{ km/h}) = (113\text{ km/h})(1000\text{ m/km})(1\text{ h}/3600\text{ s}) = 31.4\text{ m/s},$$

we obtain

$$d = \frac{K_i}{f} = \frac{mv_i^2}{2f} = \frac{(1673\text{ kg})(31.4\text{ m/s})^2}{2(8230\text{ N})} = 100\text{ m}.$$

101. With the potential energy reference level set at the point of throwing, we have (with SI units understood)

$$\Delta E = mgh - \frac{1}{2}mv_0^2 = m\left[(9.8)(8.1) - \frac{1}{2}(14)^2\right]$$

which yields ΔE = –12 J for m = 0.63 kg. This "loss" of mechanical energy is presumably due to air friction.

102. (a) The (internal) energy the climber must convert to gravitational potential energy is

$$\Delta U = mgh = (90\text{ kg})(9.80\text{ m/s}^2)(8850\text{ m}) = 7.8\times10^6\text{ J}.$$

(b) The number of candy bars this corresponds to is

$$N = \frac{7.8\times10^6\text{ J}}{1.25\times10^6\text{ J/bar}} \approx 6.2\,\text{bars}.$$

103. (a) The acceleration of the sprinter is (using Eq. 2-15)

$$a = \frac{2\Delta x}{t^2} = \frac{(2)(7.0\text{ m})}{(1.6\text{ s})^2} = 5.47\text{ m/s}^2.$$

Consequently, the speed at t = 1.6s is $v = at = (5.47\text{ m/s}^2)(1.6\text{ s}) = 8.8\text{ m/s}$. Alternatively, Eq. 2-17 could be used.

(b) The kinetic energy of the sprinter (of weight w and mass $m = w/g$) is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{1}{2}\left(670\text{ N/(9.8 m/s}^2)\right)(8.8\text{ m/s})^2 = 2.6\times10^3\text{ J}.$$

(c) The average power is

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{2.6\times10^3\text{ J}}{1.6\text{ s}} = 1.6\times10^3\text{ W}.$$

104. From Eq. 8-6, we find (with SI units understood)

$$U(\xi) = -\int_0^{\xi}\left(-3x - 5x^2\right)dx = \frac{3}{2}\xi^2 + \frac{5}{3}\xi^3.$$

(a) Using the above formula, we obtain $U(2) \approx 19$ J.

(b) When its speed is v = 4 m/s, its mechanical energy is $\frac{1}{2}mv^2 + U(5)$. This must equal the energy at the origin:

$$\frac{1}{2}mv^2 + U(5) = \frac{1}{2}mv_o^2 + U(0)$$

so that the speed at the origin is

$$v_o = \sqrt{v^2 + \frac{2}{m}\left(U(5) - U(0)\right)}.$$

Thus, with $U(5)$ = 246 J, $U(0)$ = 0 and m = 20 kg, we obtain v_o = 6.4 m/s.

(c) Our original formula for U is changed to

$$U(x) = -8 + \frac{3}{2}x^2 + \frac{5}{3}x^3$$

in this case. Therefore, $U(2) = 11$ J. But we still have $v_o = 6.4$ m/s since that calculation only depended on the difference of potential energy values (specifically, $U(5) - U(0)$).

105. (a) Resolving the gravitational force into components and applying Newton's second law (as well as Eq. 6-2), we find

$$F_{\text{machine}} - mg\sin\theta - \mu_k mg\cos\theta = ma.$$

In the situation described in the problem, we have $a = 0$, so

$$F_{\text{machine}} = mg\sin\theta + \mu_k mg\cos\theta = 372 \text{ N}.$$

Thus, the work done by the machine is $F_{\text{machine}}d = 744 \text{ J} = 7.4 \times 10^2$ J.

(b) The thermal energy generated is $(\mu_k mg\cos\theta)\,d = 240 \text{ J} = 2.4 \times 10^2$ J.

106. (a) At the highest point, the velocity $v = v_x$ is purely horizontal and is equal to the horizontal component of the launch velocity (see section 4-6): $v_{ox} = v_o\cos\theta$, where $\theta = 30°$ in this problem. Equation 8-17 relates the kinetic energy at the highest point to the launch kinetic energy:

$$K_o = mgy + \frac{1}{2}mv^2 = \frac{1}{2}mv_{ox}^2 + \frac{1}{2}mv_{oy}^2,$$

with $y = 1.83$ m. Since the $mv_{ox}^2/2$ term on the left-hand side cancels the $mv^2/2$ term on the right-hand side, this yields $v_{oy} = \sqrt{2gy} \approx 6$ m/s. With $v_{oy} = v_o\sin\theta$, we obtain

$$v_o = 11.98 \text{ m/s} \approx 12 \text{ m/s}.$$

(b) Energy conservation (including now the energy stored elastically in the spring, Eq. 8-11) also applies to the motion along the muzzle (through a distance d that corresponds to a vertical height increase of $d\sin\theta$):

$$\frac{1}{2}kd^2 = K_o + mgd\sin\theta \quad \Rightarrow \quad d = 0.11 \text{ m}.$$

107. The work done by $\vec{F}$ is the negative of its potential energy change (see Eq. 8-6), so $U_B = U_A - 25 = 15$ J.

108. (a) We assume his mass is between m_1 = 50 kg and m_2 = 70 kg (corresponding to a weight between 110 lb and 154 lb). His increase in gravitational potential energy is therefore in the range

$$m_1gh \le \Delta U \le m_2gh \quad \Rightarrow \quad 2\times10^5 \le \Delta U \le 3\times10^5$$

in SI units (J), where h = 443 m.

(b) The problem only asks for the amount of internal energy that converts into gravitational potential energy, so this result is the same as in part (a). But if we were to consider his *total* internal energy "output" (much of which converts to heat) we can expect that external climb is quite different from taking the stairs.

109. (a) We implement Eq. 8-37 as

$$K_f = K_i + mgy_i - f_k d = 0 + (60\text{ kg})(9.8\text{ m/s}^2)(4.0\text{ m}) - 0 = 2.35 \times 10^3\text{ J}.$$

(b) Now it applies with a nonzero thermal term:

$$K_f = K_i + mgy_i - f_k d = 0 + (60\text{ kg})(9.8\text{ m/s}^2)(4.0\text{ m}) - (500\text{ N})(4.0\text{ m}) = 352\text{ J}.$$

110. We take the bottom of the incline to be the y = 0 reference level. The incline angle is $\theta = 30°$. The distance along the incline d (measured from the bottom) is related to height y by the relation $y = d \sin\theta$.

(a) Using the conservation of energy, we have

$$K_0 + U_0 = K_{\text{top}} + U_{\text{top}} \Rightarrow \frac{1}{2}mv_0^2 + 0 = 0 + mgy$$

with $v_0 = 5.0\text{ m/s}$. This yields y = 1.3 m, from which we obtain d = 2.6 m.

(b) An analysis of forces in the manner of Chapter 6 reveals that the magnitude of the friction force is $f_k = \mu_k mg \cos\theta$. Now, we write Eq. 8-33 as

$$K_0 + U_0 = K_{\text{top}} + U_{\text{top}} + f_k d$$
$$\frac{1}{2}mv_0^2 + 0 = 0 + mgy + f_k d$$
$$\frac{1}{2}mv_0^2 = mgd\sin\theta + \mu_k mgd\cos\theta$$

which — upon canceling the mass and rearranging — provides the result for d:

$$d = \frac{v_0^2}{2g(\mu_k \cos\theta + \sin\theta)} = 1.5 \text{ m} .$$

(c) The thermal energy generated by friction is $f_k d = \mu_k mgd \cos\theta = 26$ J.

(d) The slide back down, from the height y = 1.5 sin 30º, is also described by Eq. 8-33. With ΔE_{th} again equal to 26 J, we have

$$K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}} + f_k d \Rightarrow 0 + mgy = \frac{1}{2}mv_{\text{bot}}^2 + 0 + 26$$

from which we find $v_{\text{bot}} = 2.1 \text{ m/s}$.

111. Equation 8-8 leads directly to $\Delta y = \dfrac{68000 \text{ J}}{(9.4 \text{ kg})(9.8 \text{ m/s}^2)} = 738$ m.

112. We assume his initial kinetic energy (when he jumps) is negligible. Then, his initial gravitational potential energy measured relative to where he momentarily stops is what becomes the elastic potential energy of the stretched net (neglecting air friction). Thus,

$$U_{\text{net}} = U_{\text{grav}} = mgh$$

where h = 11.0 m + 1.5 m = 12.5 m. With m = 70 kg, we obtain U_{net} = 8580 J.

113. We use SI units so m = 0.030 kg and d = 0.12 m.

(a) Since there is no change in height (and we assume no changes in elastic potential energy), then $\Delta U = 0$ and we have

$$\Delta E_{\text{mech}} = \Delta K = -\frac{1}{2} mv_0^2 = -3.8 \times 10^3 \text{ J}$$

where v_0 = 500 m/s and the final speed is zero.

(b) By Eq. 8-33 (with $W = 0$) we have $\Delta E_{\text{th}} = 3.8 \times 10^3$ J, which implies

$$f = \frac{\Delta E_{\text{th}}}{d} = 3.1 \times 10^4 \text{ N}$$

using Eq. 8-31 with f_k replaced by f (effectively generalizing that equation to include a greater variety of dissipative forces than just those obeying Eq. 6-2).

114. (a) The kinetic energy K of the automobile of mass m at t = 30 s is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1500\,\text{kg})\left[(72\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)\right]^2 = 3.0\times 10^5\,\text{J}.$$

(b) The average power required is

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{3.0\times 10^5\,\text{J}}{30\,\text{s}} = 1.0\times 10^4\,\text{W}.$$

(c) Since the acceleration a is constant, the power is $P = Fv = mav = ma(at) = ma^2t$ using Eq. 2-11. By contrast, from part (b), the average power is $P_{\text{avg}} = \frac{mv^2}{2t}$, which becomes $\frac{1}{2}ma^2t$ when $v = at$ is again utilized. Thus, the instantaneous power at the end of the interval is twice the average power during it:

$$P = 2P_{\text{avg}} = (2)(1.0\times 10^4\,\text{W}) = 2.0\times 10^4\,\text{W}.$$

115. (a) The initial kinetic energy is $K_i = (1.5\,\text{kg})(20\,\text{m/s})^2/2 = 300\,\text{J}$.

(b) At the point of maximum height, the vertical component of velocity vanishes but the horizontal component remains what it was when it was "shot" (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(1.5\,\text{kg})\left[(20\,\text{m/s})\cos 34°\right]^2 = 206\,\text{J}.$$

Thus, $\Delta U = K_i - K = 300\,\text{J} - 206\,\text{J} = 93.8\,\text{J}$.

(c) Since $\Delta U = mg\,\Delta y$, we obtain $\Delta y = \frac{94\,\text{J}}{(1.5\,\text{kg})(9.8\,\text{m/s}^2)} = 6.38\,\text{m}$.

116. (a) The rate of change of the gravitational potential energy is

$$\frac{dU}{dt} = mg\frac{dy}{dt} = -mg|v| = -(68)(9.8)(59) = -3.9\times 10^4\,\text{J/s}.$$

Thus, the gravitational energy is being reduced at the rate of 3.9×10^4 W.

(b) Since the velocity is constant, the rate of change of the kinetic energy is zero. Thus the rate at which the mechanical energy is being dissipated is the same as that of the gravitational potential energy (3.9×10^4 W).

117. (a) The effect of (sliding) friction is described in terms of energy dissipated as shown in Eq. 8-31. We have

$$\Delta E = K + \frac{1}{2}k(0.08)^2 - \frac{1}{2}k(0.10)^2 = -f_k(0.02)$$

where distances are in meters and energies are in joules. With k = 4000 N/m and $f_k = 80$ N, we obtain K = 5.6 J.

(b) In this case, we have d = 0.10 m. Thus,

$$\Delta E = K + 0 - \frac{1}{2}k(0.10)^2 = -f_k(0.10)$$

which leads to K = 12 J.

(c) We can approach this two ways. One way is to examine the dependence of energy on the variable d:

$$\Delta E = K + \frac{1}{2}k(d_0 - d)^2 - \frac{1}{2}kd_0^2 = -f_k d$$

where d_0 = 0.10 m, and solving for K as a function of d:

$$K = -\frac{1}{2}kd^2 + (kd_0)d - f_k d .$$

In this first approach, we could work through the $dK/d(d) = 0$ condition (or with the special capabilities of a graphing calculator) to obtain the answer $K_{\max} = \frac{1}{2k}(kd_0 - f_k)^2$. In the second (and perhaps easier) approach, we note that K is maximum where v is maximum — which is where $a = 0 \Rightarrow$ equilibrium of forces. Thus, the second approach simply solves for the equilibrium position

$$|F_{\text{spring}}| = f_k \Rightarrow kx = 80.$$

Thus, with k = 4000 N/m we obtain x = 0.02 m. But $x = d_0 - d$ so this corresponds to d = 0.08 m. Then the methods of part (a) lead to the answer $K_{\max}$ = 12.8 J ≈ 13 J.

118. We work this in SI units and convert to horsepower in the last step. Thus,

$$v = (80\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 22.2\,\text{m/s}.$$

The force F_P needed to propel the car (of weight w and mass $m = w/g$) is found from Newton's second law:

$$F_{\text{net}} = F_P - F = ma = \frac{wa}{g}$$

where $F = 300 + 1.8v^2$ in SI units. Therefore, the power required is

$$P = \vec{F}_P \cdot \vec{v} = \left(F + \frac{wa}{g}\right)v = \left(300 + 1.8(22.2)^2 + \frac{(12000)(0.92)}{9.8}\right)(22.2) = 5.14\times10^4 \text{ W}$$
$$= \left(5.14\times10^4 \text{ W}\right)\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 69 \text{ hp}.$$

119. **THINK** We apply energy method to analyze the projectile motion of a ball.

EXPRESS We choose the initial position at the window to be our reference point for calculating the potential energy. The initial energy of the ball is $E_0 = \frac{1}{2}mv_0^2$. At the top of its flight, the vertical component of the velocity is zero, and the horizontal component (neglecting air friction) is the same as it was when it was thrown: $v_x = v_0\cos\theta$. At a position h below the window, the energy of the ball is

$$E = K + U = \frac{1}{2}mv^2 - mgh$$

where v is the speed of the ball.

ANALYZE (a) The kinetic energy of the ball at the top of the flight is

$$K_{\text{top}} = \frac{1}{2}mv_x^2 = \frac{1}{2}m(v_0\cos\theta)^2 = \frac{1}{2}(0.050\text{ kg})[(8.0\text{ m/s})\cos30°]^2 = 1.2\text{ J}.$$

(b) When the ball is h = 3.0 m below the window, by energy conservation, we have

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 - mgh$$

or

$$v = \sqrt{v_0^2 + 2gh} = \sqrt{(8.0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(3.0\text{ m})} = 11.1\text{ m/s}.$$

(c) As can be seen from our expression above, $v = \sqrt{v_0^2 + 2gh}$, which is independent of the mass m.

(d) Similarly, the speed v is independent of the initial angle θ.

LEARN Our results demonstrate that the quantity v in the kinetic energy formula is the magnitude of the velocity vector; it does not depend on direction. In addition, mass cancels out in the energy conservation equation, so that v is independent of m.

120. (a) In the initial situation, the elongation was (using Eq. 8-11)

$$x_i = \sqrt{2(1.44)/3200} = 0.030 \text{ m (or 3.0 cm)}.$$

In the next situation, the elongation is only 2.0 cm (or 0.020 m), so we now have less stored energy (relative to what we had initially). Specifically,

$$\Delta U = \frac{1}{2}(3200 \text{ N/m})(0.020 \text{ m})^2 - 1.44 \text{ J} = -0.80 \text{ J}.$$

(b) The elastic stored energy for $|x| = 0.020$ m does not depend on whether this represents a stretch or a compression. The answer is the same as in part (a), $\Delta U = -0.80$ J.

(c) Now we have $|x| = 0.040$ m, which is greater than x_i, so this represents an increase in the potential energy (relative to what we had initially). Specifically,

$$\Delta U = \frac{1}{2}(3200 \text{ N/m})(0.040 \text{ m})^2 - 1.44 \text{ J} = +1.12 \text{ J} \approx 1.1 \text{ J}.$$

121. (a) With $P = 1.5$ MW $= 1.5 \times 10^6$ W (assumed constant) and $t = 6.0$ min $= 360$ s, the work-kinetic energy theorem becomes

$$W = Pt = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right).$$

The mass of the locomotive is then

$$m = \frac{2Pt}{v_f^2 - v_i^2} = \frac{(2)(1.5\times10^6 \text{ W})(360 \text{ s})}{(25 \text{ m/s})^2 - (10 \text{ m/s})^2} = 2.1\times10^6 \text{ kg}.$$

(b) With t arbitrary, we use $Pt = \frac{1}{2}m\left(v^2 - v_i^2\right)$ to solve for the speed $v = v(t)$ as a function of time and obtain

$$v(t) = \sqrt{v_i^2 + \frac{2Pt}{m}} = \sqrt{(10)^2 + \frac{(2)(1.5\times10^6)t}{2.1\times10^6}} = \sqrt{100 + 1.5t}$$

in SI units (v in m/s and t in s).

(c) The force $F(t)$ as a function of time is

$$F(t) = \frac{P}{v(t)} = \frac{1.5\times 10^6}{\sqrt{100+1.5t}}$$

in SI units (F in N and t in s).

(d) The distance d the train moved is given by

$$d = \int_0^t v(t')dt' = \int_0^{360}\left(100+\frac{3}{2}t\right)^{1/2} dt = \frac{4}{9}\left(100+\frac{3}{2}t\right)^{3/2}\Bigg|_0^{360} = 6.7\times 10^3 \text{ m.}$$

122. **THINK** A shuffleboard disk is accelerated over some distance by an external force, but it eventually comes to rest due to the frictional force.

EXPRESS In the presence of frictional force, the work done on a system is $W = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$, where $\Delta E_{\text{mech}} = \Delta K + \Delta U$ and $\Delta E_{\text{th}} = f_k d$. In our situation, work has been done by the cue only to the first 2.0 m, and not to the subsequent 12 m of distance traveled.

ANALYZE (a) During the final d = 12 m of motion, $W = 0$ and we use

$$K_1 + U_1 = K_2 + U_2 + f_k d$$
$$\frac{1}{2}mv^2 + 0 = 0 + 0 + f_k d$$

where $m = 0.42$ kg and v = 4.2 m/s. This gives f_k = 0.31 N. Therefore, the thermal energy change is $\Delta E_{\text{th}} = f_k d = 3.7$ J.

(b) Using f_k = 0.31 N for the entire distance d_{total} = 14 m, we obtain

$$\Delta E_{\text{th,total}} = f_k d_{\text{total}} = (0.31 \text{ N})(14 \text{ m}) = 4.3 \text{ J}$$

for the thermal energy generated by friction.

(c) During the initial d' = 2 m of motion, we have

$$W = \Delta E_{\text{mech}} + \Delta E'_{\text{th}} = \Delta K + \Delta U + f_k d' = \frac{1}{2}mv^2 + 0 + f_k d'$$

which essentially combines Eq. 8-31 and Eq. 8-33. Thus, the work done on the disk by the cue is

$$W = \frac{1}{2}mv^2 + f_k d' = \frac{1}{2}(0.42 \text{ kg})(4.2 \text{ m/s})^2 + (0.31 \text{ N})(2.0 \text{ m}) = 4.3 \text{ J}.$$

LEARN Our answer in (c) is the same as that in (b). This is expected because all the work done becomes thermal energy at the end.

123. The water has gained

$$\Delta K = \frac{1}{2}\,(10\text{ kg})(13\text{ m/s})^2 - \frac{1}{2}\,(10\text{ kg})(3.2\text{ m/s})^2 = 794\text{ J}$$

of kinetic energy, and it has lost $\Delta U = (10\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = 1470\text{ J}$.

of potential energy (the lack of agreement between these two values is presumably due to transfer of energy into thermal forms). The ratio of these values is 0.54 = 54%. The mass of the water cancels when we take the ratio, so that the assumption (stated at the end of the problem: $m = 10$ kg) is not needed for the final result.

124. (a) The integral (see Eq. 8-6, where the value of U at $x = \infty$ is required to vanish) is straightforward. The result is $U(x) = -Gm_1m_2/x$.

(b) One approach is to use Eq. 8-5, which means that we are effectively doing the integral of part (a) all over again. Another approach is to use our result from part (a) (and thus use Eq. 8-1). Either way, we arrive at

$$W = \frac{G\,m_1\,m_2}{x_1} - \frac{G\,m_1\,m_2}{x_1 + d} = \frac{G\,m_1\,m_2\,d}{x_1(x_1 + d)}\ .$$

125. (a) During one second, the decrease in potential energy is

$$-\Delta U = mg(-\Delta y) = (5.5\times10^6\text{ kg})\left(9.8\text{ m/s}^2\right)(50\text{ m}) = 2.7\times10^9\text{ J}$$

where $+y$ is upward and $\Delta y = y_f - y_i$.

(b) The information relating mass to volume is not needed in the computation. By Eq. 8-40 (and the SI relation W = J/s), the result follows:

$$P = (2.7\times10^9\text{ J})/(1\text{ s}) = 2.7\times10^9\text{ W}.$$

(c) One year is equivalent to 24 × 365.25 = 8766 h which we write as 8.77 kh. Thus, the energy supply rate multiplied by the cost and by the time is

$$(2.7\times10^9\text{ W})(8.77\text{ kh})\left(\frac{1\text{ cent}}{1\text{ kWh}}\right) = 2.4\times10^{10}\text{ cents} = \$2.4\times10^8.$$

126. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the

gravitational potential energy) is given by $h = L(1 - \cos\,\theta)$ where L is the length of the pendulum.

(a) We use energy conservation in the form of Eq. 8-17.

$$K_1 + U_1 = K_2 + U_2$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_2^2 + mgL(1 - \cos\theta_2)$$

With L = 1.4 m, θ_1 = 30°, and θ_2 = 20°, we have

$$v_2 = \sqrt{2gL(\cos\theta_2 - \cos\theta_1)} = 1.4 \text{ m/s}.$$

(b) The maximum speed v_3 is at the lowest point. Our formula for h gives $h_3 = 0$ when θ_3 = 0°, as expected. From

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_3^2 + 0$$

we obtain $v_3 = 1.9 \text{ m/s}$.

(c) We look for an angle θ_4 such that the speed there is $v_4 = v_3/3$. To be as accurate as possible, we proceed algebraically (substituting $v_3^2 = 2gL(1 - \cos\theta_1)$ at the appropriate place) and plug numbers in at the end. Energy conservation leads to

$$K_1 + U_1 = K_4 + U_4$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_4^2 + mgL(1 - \cos\theta_4)$$
$$mgL(1 - \cos\theta_1) = \frac{1}{2}m\frac{v_3^2}{9} + mgL(1 - \cos\theta_4)$$
$$-gL\cos\theta_1 = \frac{1}{2}\frac{2gL(1 - \cos\theta_1)}{9} - gL\cos\theta_4$$

where in the last step we have subtracted out mgL and then divided by m. Thus, we obtain

$$\theta_4 = \cos^{-1}\left(\frac{1}{9} + \frac{8}{9}\cos\theta_1\right) = 28.2° \approx 28°.$$

127. Equating the mechanical energy at his initial position (as he emerges from the canon, where we set the reference level for computing potential energy) to his energy as he lands, we obtain

$$K_i = K_f + U_f$$

$$\frac{1}{2}(60\,\text{kg})(16\,\text{m/s})^2 = K_f + (60\,\text{kg})(9.8\,\text{m/s}^2)(3.9\,\text{m})$$

which leads to $K_f = 5.4 \times 10^3$ J.

128. (a) This part is essentially a free-fall problem, which can be easily done with Chapter 2 methods. Instead, choosing energy methods, we take $y = 0$ to be the ground level.

$$K_i + U_i = K + U \Rightarrow 0 + mgy_i = \frac{1}{2}mv^2 + 0$$

Therefore $v = \sqrt{2gy_i} = 9.2\,\text{m/s}$, where $y_i = 4.3$ m.

(b) Eq. 8-29 provides $\Delta E_{\text{th}} = f_k d$ for thermal energy generated by the kinetic friction force. We apply Eq. 8-31:

$$K_i + U_i = K + U \Rightarrow 0 + mgy_i = \frac{1}{2}mv^2 + 0 + f_k d\,.$$

With $d = y_i$, $m = 70$ kg and $f_k = 500$ N, this yields $v = 4.8$ m/s.

129. We want to convert (at least in theory) the water that falls through $h = 500$ m into electrical energy. The problem indicates that in one year, a volume of water equal to $A\Delta z$ lands in the form of rain on the country, where $A = 8 \times 10^{12}$ m^2 and $\Delta z = 0.75$ m. Multiplying this volume by the density $\rho = 1000$ kg/m^3 leads to

$$m_{\text{total}} = \rho A\Delta z = (1000)(8 \times 10^{12})(0.75) = 6 \times 10^{15}\,\text{kg}$$

for the mass of rainwater. One-third of this "falls" to the ocean, so it is $m = 2 \times 10^{15}$ kg that we want to use in computing the gravitational potential energy mgh (which will turn into electrical energy during the year). Since a year is equivalent to 3.2×10^7 s, we obtain

$$P_{\text{avg}} = \frac{(2 \times 10^{15})(9.8)(500)}{3.2 \times 10^7} = 3.1 \times 10^{11}\,\text{W}.$$

130. The spring is relaxed at $y = 0$, so the elastic potential energy (Eq. 8-11) is $U_{\text{el}} = \frac{1}{2}ky^2$. The total energy is conserved, and is zero (determined by evaluating it at its initial position). We note that U is the same as ΔU in these manipulations. Thus, we have

$$0 = K + U_g + U_e \quad \Rightarrow \quad K = -U_g - U_e$$

where U_g = mgy = (20 N)y with y in meters (so that the energies are in Joules). We arrange the results in a table:

position y	–0.05	–0.10	–0.15	–0.20
K	(a) 0.75	(d) 1.0	(g) 0.75	(j) 0
U_g	(b) –1.0	(e) –2.0	(h) –3.0	(k) –4.0
U_e	(c) 0.25	(f) 1.0	(i) 2.25	(l) 4.0

131. Let the amount of stretch of the spring be x. For the object to be in equilibrium

$$kx - mg = 0 \Rightarrow x = mg/k .$$

Thus the gain in elastic potential energy for the spring is

$$\Delta U_e = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

while the loss in the gravitational potential energy of the system is

$$-\Delta U_g = mgx = mg\left(\frac{mg}{k}\right) = \frac{m^2g^2}{k}$$

which we see (by comparing with the previous expression) is equal to $2\Delta U_e$. The reason why $|\Delta U_g| \neq \Delta U_e$ is that, since the object is slowly lowered, an upward external force (e.g., due to the hand) must have been exerted on the object during the lowering process, preventing it from accelerating downward. This force does *negative* work on the object, reducing the total mechanical energy of the system.

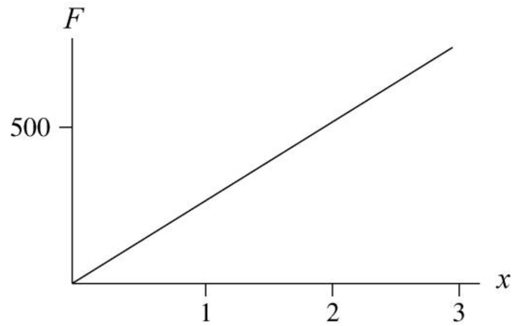
132. (a) The compression is "spring-like" so the maximum force relates to the distance x by Hooke's law:

$$F_x = kx \Rightarrow x = \frac{750}{2.5\times10^5} = 0.0030\,\text{m}.$$

(b) The work is what produces the "spring-like" potential energy associated with the compression. Thus, using Eq. 8-11,

$$W = \frac{1}{2}kx^2 = \frac{1}{2}(2.5\times10^5)(0.0030)^2 = 1.1\,\text{J}.$$

(c) By Newton's third law, the force F exerted by the tooth is equal and opposite to the "spring-like" force exerted by the licorice, so the graph of F is a straight line of slope k. We plot F (in newtons) versus x (in millimeters); both are taken as positive.

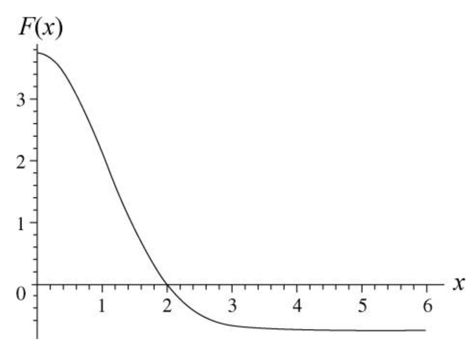


(d) As mentioned in part (b), the spring potential energy expression is relevant. Now, whether or not we can ignore dissipative processes is a deeper question. In other words, it seems unlikely that — if the tooth at any moment were to reverse its motion — that the licorice could "spring back" to its original shape. Still, to the extent that $U = \frac{1}{2}kx^2$ applies, the graph is a parabola (not shown here) which has its vertex at the origin and is either concave upward or concave downward depending on how one wishes to define the sign of F (the connection being $F = -dU/dx$).

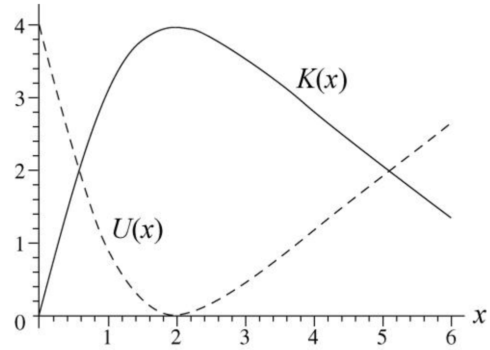
(e) As a crude estimate, the area under the curve is roughly half the area of the entire plotting-area (8000 N by 12 mm). This leads to an approximate work of $\frac{1}{2}$ (8000 N) (0.012 m) ≈ 50 J. Estimates in the range $40 \le W \le 50$ J are acceptable.

(f) Certainly dissipative effects dominate this process, and we cannot assign it a meaningful potential energy.

133. (a) The force (SI units understood) from Eq. 8-20 is plotted in the graph below.



(b) The potential energy U(x) and the kinetic energy K(x) are shown in the next. The potential energy curve begins at 4 and drops (until about x = 2); the kinetic energy curve is the one that starts at zero and rises (until about x = 2).



134. The style of reasoning used here is presented in Section 8-5.

(a) The horizontal line representing E_1 intersects the potential energy curve at a value of $r \approx 0.07$ nm and seems not to intersect the curve at larger r (though this is somewhat unclear since $U(r)$ is graphed only up to $r = 0.4$ nm). Thus, if m were propelled towards M from large r with energy E_1 it would "turn around" at 0.07 nm and head back in the direction from which it came.

(b) The line representing E_2 has two intersection points $r_1 \approx 0.16$ nm and $r_2 \approx 0.28$ nm with the $U(r)$ plot. Thus, if m starts in the region $r_1 < r < r_2$ with energy E_2 it will bounce back and forth between these two points, presumably forever.

(c) At $r = 0.3$ nm, the potential energy is roughly $U = -1.1 \times 10^{-19}$ J.

(d) With $M >> m$, the kinetic energy is essentially just that of m. Since $E = 1 \times 10^{-19}$ J, its kinetic energy is $K = E - U \approx 2.1 \times 10^{-19}$ J.

(e) Since force is related to the slope of the curve, we must (crudely) estimate $|F| \approx 1\times 10^{-9}\,\text{N}$ at this point. The sign of the slope is positive, so by Eq. 8-20, the force is negative-valued. This is interpreted to mean that the atoms are attracted to each other.

(f) Recalling our remarks in the previous part, we see that the sign of F is positive (meaning it's repulsive) for $r < 0.2$ nm.

(g) And the sign of F is negative (attractive) for $r > 0.2$ nm.

(h) At $r = 0.2$ nm, the slope (hence, F) vanishes.

135. The distance traveled up the incline can be calculated using the kinematic equations discussed in Chapter 2:

$$v^2 = v_0^2 + 2a\Delta x \;\; \to \Delta x = 200\text{ m}.$$

This corresponds to an increase in height equal to $y = (200\text{ m})\sin\theta = 17$ m, where $\theta = 5.0°$. We take its initial height to be $y = 0$.

(a) Eq. 8-24 leads to

$$W_{\text{app}} = \Delta E = \frac{1}{2}m\left(v^2 - v_0^2\right) + mgy \; .$$

Therefore, $\Delta E = 8.6 \times 10^3$ J .

(b) From the above manipulation, we see W_{app} = 8.6 × 10[3] J. Also, from Chapter 2, we know that $\Delta t = \Delta v / a = 10$ s. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{8.6 \times 10^3}{10} = 860 \text{ W}$$

where the answer has been rounded off (from the 856 value that is provided by the calculator).

(c) and (d) Taking into account the component of gravity along the incline surface, the applied force is $ma + mg \sin\theta = 43$ N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 430 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 1300 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

where these answers have been rounded off (from 428 and 1284, respectively). We note that the average of these two values agrees with the result in part (b).

136. (a) Conservation of mechanical energy leads to

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad 0 + \frac{1}{2}ky_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}k(y_f - y_i)^2 + mgy_f$$

where $y_i = 0.25$ m is the initial depression of the spring, and $y_f - y_i$ is the displacement of the spring from its equilibrium position when the block is at y_f. Thus, the kinetic energy of the block can be written as

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f \, .$$

For $y_f = 0$, the kinetic energy is $K_f = 0$, as expected, since this corresponds to the initial release point.

(b) At $y_f = 0.050$ m, we have

$$K_f = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f$$
$$= \frac{1}{2}(620\text{ N/m})\left[(0.250\text{ m})^2 - (0.050\text{ m} - 0.250\text{ m})^2\right] - (50\text{ N})(0.050\text{ m}) = 4.48\text{ J}$$

(c) At $y_f = 0.100$ m, we have

$$K_f = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f$$
$$= \frac{1}{2}(620\text{ N/m})\left[(0.250\text{ m})^2 - (0.100\text{ m} - 0.250\text{ m})^2\right] - (50\text{ N})(0.100\text{ m}) = 7.40\text{ J}$$

(d) Similarly, the kinetic energy at $y_f = 0.150$ m is

$$K_f = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f$$
$$= \frac{1}{2}(620\text{ N/m})\left[(0.250\text{ m})^2 - (0.150\text{ m} - 0.250\text{ m})^2\right] - (50\text{ N})(0.150\text{ m}) = 8.78\text{ J}$$

(e) At $y_f = 0.200$ m, the kinetic energy of the block is

$$K_f = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f$$
$$= \frac{1}{2}(620\text{ N/m})\left[(0.250\text{ m})^2 - (0.200\text{ m} - 0.250\text{ m})^2\right] - (50\text{ N})(0.200\text{ m}) = 8.60\text{ J}$$

(f) The spring returns to its uncompressed state once $y_f \ge y_i$. Since the block becomes detached from the spring beyond that point, at its maximum height, $K = 0$, and we have

$$\frac{1}{2}ky_i^2 = mgy_{\text{max}} \quad \Rightarrow \quad y_{\text{max}} = \frac{ky_i^2}{2mg} = \frac{(620\text{ N/m})(0.250\text{ m})^2}{2(50\text{ N})} = 0.388\text{ m}.$$

Chapter 9

1. We use Eq. 9-5 to solve for (x_3, y_3).

(a) The *x* coordinate of the system's center of mass is:

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(-1.20\text{ m}) + (4.00\text{ kg})(0.600\text{ m}) + (3.00\text{ kg})x_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.500\text{ m}.$$

Solving the equation yields $x_3 = -1.50$ m.

(b) The *y* coordinate of the system's center of mass is:

$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(0.500\text{ m}) + (4.00\text{ kg})(-0.750\text{ m}) + (3.00\text{ kg})y_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.700\text{ m}.$$

Solving the equation yields $y_3 = -1.43$ m.

2. Our notation is as follows: $x_1 = 0$ and $y_1 = 0$ are the coordinates of the $m_1 = 3.0$ kg particle; $x_2 = 2.0$ m and $y_2 = 1.0$ m are the coordinates of the $m_2 = 4.0$ kg particle; and $x_3 = 1.0$ m and $y_3 = 2.0$ m are the coordinates of the $m_3 = 8.0$ kg particle.

(a) The *x* coordinate of the center of mass is

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(2.0\text{ m}) + (8.0\text{ kg})(1.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.1\text{ m}.$$

(b) The *y* coordinate of the center of mass is

$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(1.0\text{ m}) + (8.0\text{ kg})(2.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.3\text{ m}.$$

(c) As the mass of m_3, the topmost particle, is increased, the center of mass shifts toward that particle. As we approach the limit where m_3 is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of m_3.

3. We use Eq. 9-5 to locate the coordinates.

(a) By symmetry $x_{com} = -d_1/2 = -(13\text{ cm})/2 = -6.5$ cm. The negative value is due to our choice of the origin.

(b) We find y_{com} as

$$y_{com} = \frac{m_i y_{com,i} + m_a y_{com,a}}{m_i + m_a} = \frac{\rho_i V_i y_{com,i} + \rho_a V_a y_{cm,a}}{\rho_i V_i + \rho_a V_a}$$

$$= \frac{(11\text{ cm}/2)(7.85\text{ g/cm}^3) + 3(11\text{ cm}/2)(2.7\text{ g/cm}^3)}{7.85\text{ g/cm}^3 + 2.7\text{ g/cm}^3} = 8.3\text{ cm.}$$

(c) Again by symmetry, we have $z_{com} = (2.8\text{ cm})/2 = 1.4$ cm.

4. We will refer to the arrangement as a "table." We locate the coordinate origin at the left end of the tabletop (as shown in Fig. 9-37). With $+x$ rightward and $+y$ upward, then the center of mass of the right leg is at $(x,y) = (+L, -L/2)$, the center of mass of the left leg is at $(x,y) = (0, -L/2)$, and the center of mass of the tabletop is at $(x,y) = (L/2, 0)$.

(a) The x coordinate of the (whole table) center of mass is

$$x_{com} = \frac{M(+L) + M(0) + 3M(+L/2)}{M + M + 3M} = \frac{L}{2}.$$

With $L = 22$ cm, we have $x_{com} = (22\text{ cm})/2 = 11$ cm.

(b) The y coordinate of the (whole table) center of mass is

$$y_{com} = \frac{M(-L/2) + M(-L/2) + 3M(0)}{M + M + 3M} = -\frac{L}{5},$$

or $y_{com} = -(22\text{ cm})/5 = -4.4$ cm.

From the coordinates, we see that the whole table center of mass is a small distance 4.4 cm directly below the middle of the tabletop.

5. Since the plate is uniform, we can split it up into three rectangular pieces, with the mass of each piece being proportional to its area and its center of mass being at its geometric center. We'll refer to the large 35 cm × 10 cm piece (shown to the left of the y axis in Fig. 9-38) as section 1; it has 63.6% of the total area and its center of mass is at $(x_1, y_1) = (-5.0\text{ cm}, -2.5\text{ cm})$. The top 20 cm × 5 cm piece (section 2, in the first quadrant) has 18.2% of the total area; its center of mass is at $(x_2, y_2) = (10\text{ cm}, 12.5\text{ cm})$. The bottom 10 cm x 10 cm piece (section 3) also has 18.2% of the total area; its center of mass is at $(x_3, y_3) = (5\text{ cm}, -15\text{ cm})$.

(a) The x coordinate of the center of mass for the plate is

$$x_{com} = (0.636)x_1 + (0.182)x_2 + (0.182)x_3 = -0.45 \text{ cm} .$$

(b) The y coordinate of the center of mass for the plate is

$$y_{com} = (0.636)y_1 + (0.182)y_2 + (0.182)y_3 = -2.0 \text{ cm} .$$

6. The centers of mass (with centimeters understood) for each of the five sides are as follows:

$(x_1, y_1, z_1) = (0, 20, 20)$	for the side in the yz plane
$(x_2, y_2, z_2) = (20, 0, 20)$	for the side in the xz plane
$(x_3, y_3, z_3) = (20, 20, 0)$	for the side in the xy plane
$(x_4, y_4, z_4) = (40, 20, 20)$	for the remaining side parallel to side 1
$(x_5, y_5, z_5) = (20, 40, 20)$	for the remaining side parallel to side 2

Recognizing that all sides have the same mass m, we plug these into Eq. 9-5 to obtain the results (the first two being expected based on the symmetry of the problem).

(a) The x coordinate of the center of mass is

$$x_{com} = \frac{mx_1 + mx_2 + mx_3 + mx_4 + mx_5}{5m} = \frac{0 + 20 + 20 + 40 + 20}{5} = 20 \text{ cm}$$

(b) The y coordinate of the center of mass is

$$y_{com} = \frac{my_1 + my_2 + my_3 + my_4 + my_5}{5m} = \frac{20 + 0 + 20 + 20 + 40}{5} = 20 \text{ cm}$$

(c) The z coordinate of the center of mass is

$$z_{com} = \frac{mz_1 + mz_2 + mz_3 + mz_4 + mz_5}{5m} = \frac{20 + 20 + 0 + 20 + 20}{5} = 16 \text{ cm}$$

7. (a) By symmetry the center of mass is located on the axis of symmetry of the molecule – the y axis. Therefore $x_{com} = 0$.

(b) To find y_{com}, we note that $3m_H y_{com} = m_N(y_N - y_{com})$, where y_N is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:

$$y_N = \sqrt{(10.14\times10^{-11} \text{ m})^2 - (9.4\times10^{-11} \text{ m})^2} = 3.803\times10^{-11} \text{ m}.$$

Thus,

$$y_{\text{com}} = \frac{m_N y_N}{m_N + 3m_H} = \frac{(14.0067)(3.803\times10^{-11}\text{ m})}{14.0067+3(1.00797)} = 3.13\times10^{-11}\text{ m}$$

where Appendix F has been used to find the masses.

8. (a) Since the can is uniform, its center of mass is at its geometrical center, a distance $H/2$ above its base. The center of mass of the soda alone is at its geometrical center, a distance $x/2$ above the base of the can. When the can is full this is $H/2$. Thus the center of mass of the can and the soda it contains is a distance

$$h = \frac{M(H/2) + m(H/2)}{M+m} = \frac{H}{2}$$

above the base, on the cylinder axis. With H = 12 cm, we obtain h = 6.0 cm.

(b) We now consider the can alone. The center of mass is $H/2$ = 6.0 cm above the base, on the cylinder axis.

(c) As x decreases the center of mass of the soda in the can at first drops, then rises to $H/2$ = 6.0 cm again.

(d) When the top surface of the soda is a distance x above the base of the can, the mass of the soda in the can is $m_p = m(x/H)$, where m is the mass when the can is full ($x = H$). The center of mass of the soda alone is a distance $x/2$ above the base of the can. Hence

$$h = \frac{M(H/2) + m_p(x/2)}{M+m_p} = \frac{M(H/2) + m(x/H)(x/2)}{M+(mx/H)} = \frac{MH^2 + mx^2}{2(MH+mx)}.$$

We find the lowest position of the center of mass of the can and soda by setting the derivative of h with respect to x equal to 0 and solving for x. The derivative is

$$\frac{dh}{dx} = \frac{2mx}{2(MH+mx)} - \frac{(MH^2+mx^2)m}{2(MH+mx)^2} = \frac{m^2x^2 + 2MmHx - MmH^2}{2(MH+mx)^2}.$$

The solution to $m^2x^2 + 2MmHx - MmH^2 = 0$ is

$$x = \frac{MH}{m}\left(-1+\sqrt{1+\frac{m}{M}}\right).$$

The positive root is used since x must be positive. Next, we substitute the expression found for x into $h = (MH^2 + mx^2)/2(MH + mx)$. After some algebraic manipulation we obtain

$$h = \frac{HM}{m}\left(\sqrt{1+\frac{m}{M}}-1\right) = \frac{(12\text{ cm})(0.14\text{ kg})}{0.354\text{ kg}}\left(\sqrt{1+\frac{0.354\text{ kg}}{0.14\text{ kg}}}-1\right) = 4.2\text{ cm}.$$

9. We use the constant-acceleration equations of Table 2-1 (with $+y$ downward and the origin at the release point), Eq. 9-5 for y_{com} and Eq. 9-17 for $\vec{v}_{com}$.

(a) The location of the first stone (of mass m_1) at $t = 300 \times 10^{-3}$ s is

$$y_1 = (1/2)gt^2 = (1/2)(9.8\text{ m/s}^2)\,(300 \times 10^{-3}\text{ s})^2 = 0.44\text{ m},$$

and the location of the second stone (of mass $m_2 = 2m_1$) at $t = 300 \times 10^{-3}$ s is

$$y_2 = (1/2)gt^2 = (1/2)(9.8\text{ m/s}^2)(300 \times 10^{-3}\text{ s} - 100 \times 10^{-3}\text{ s})^2 = 0.20\text{ m}.$$

Thus, the center of mass is at

$$y_{com} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1(0.44\text{ m}) + 2m_1(0.20\text{ m})}{m_1 + 2m_2} = 0.28\text{ m}.$$

(b) The speed of the first stone at time t is $v_1 = gt$, while that of the second stone is

$$v_2 = g(t - 100 \times 10^{-3}\text{ s}).$$

Thus, the center-of-mass speed at $t = 300 \times 10^{-3}$ s is

$$\begin{aligned} v_{com} &= \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m_1\left(9.8\text{ m/s}^2\right)\left(300\times10^{-3}\text{ s}\right) + 2m_1\left(9.8\text{ m/s}^2\right)\left(300\times10^{-3}\text{ s} - 100\times10^{-3}\text{ s}\right)}{m_1 + 2m_1} \\ &= 2.3\text{ m/s}. \end{aligned}$$

10. We use the constant-acceleration equations of Table 2-1 (with the origin at the traffic light), Eq. 9-5 for x_{com} and Eq. 9-17 for $\vec{v}_{com}$. At $t = 3.0$ s, the location of the automobile (of mass m_1) is

$$x_1 = \tfrac{1}{2}at^2 = \tfrac{1}{2}\left(4.0\text{ m/s}^2\right)\left(3.0\text{ s}\right)^2 = 18\text{ m},$$

while that of the truck (of mass m_2) is $x_2 = vt = (8.0\text{ m/s})(3.0\text{s}) = 24$ m. The speed of the automobile then is $v_1 = at = \left(4.0\text{ m/s}^2\right)\left(3.0\text{ s}\right) = 12$ m/s, while the speed of the truck remains $v_2 = 8.0$ m/s.

(a) The location of their center of mass is

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(1000\text{ kg})(18\text{ m}) + (2000\text{ kg})(24\text{ m})}{1000\text{ kg} + 2000\text{ kg}} = 22\text{ m}.$$

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1000\text{ kg})(12\text{ m/s}) + (2000\text{ kg})(8.0\text{ m/s})}{1000\text{ kg} + 2000\text{ kg}} = 9.3\text{ m/s}.$$

11. The implication in the problem regarding $\vec{v}_0$ is that the olive and the nut start at rest. Although we could proceed by analyzing the forces on each object, we prefer to approach this using Eq. 9-14. The total force on the nut-olive system is $\vec{F}_o + \vec{F}_n = (-\hat{\text{i}} + \hat{\text{j}})\text{ N}$. Thus, Eq. 9-14 becomes

$$(-\hat{\text{i}} + \hat{\text{j}})\text{ N} = M\vec{a}_{\text{com}}$$

where M = 2.0 kg. Thus, $\vec{a}_{\text{com}} = (-\frac{1}{2}\hat{\text{i}} + \frac{1}{2}\hat{\text{j}})\text{ m/s}^2$. Each component is constant, so we apply the equations discussed in Chapters 2 and 4 and obtain

$$\Delta\vec{r}_{\text{com}} = \frac{1}{2}\vec{a}_{\text{com}}t^2 = (-4.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}$$

when t = 4.0 s. It is perhaps instructive to work through this problem the *long way* (separate analysis for the olive and the nut and then application of Eq. 9-5) since it helps to point out the computational advantage of Eq. 9-14.

12. Since the center of mass of the two-skater system does not move, both skaters will end up at the center of mass of the system. Let the center of mass be a distance x from the 40-kg skater, then

$$(65\text{ kg})(10\text{ m} - x) = (40\text{ kg})x \Rightarrow x = 6.2\text{ m}.$$

Thus the 40-kg skater will move by 6.2 m.

13. **THINK** A shell explodes into two segments at the top of its trajectory. Knowing the motion of one segment allows us to analyze the motion of the other using the momentum conservation principle.

EXPRESS We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0y} - gt$ and this is zero at time $t = v_{0y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin 60°\cos 60° = 17.7\text{ m}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin^2 60° = 15.3\text{ m}.$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. In addition, since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then

$$Mv_0\cos\theta_0 = MV_0/2,$$

since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\text{ m/s})\cos 60° = 20\text{ m/s}.$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands.

ANALYZE Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7$ m, $y_0 = 15.3$ m. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\text{ m} + (20\text{ m/s})\sqrt{\frac{2(15.3\text{ m})}{9.8\text{ m/s}^2}} = 53\text{ m}.$$

LEARN In the absence of explosion, the shell with a mass M would have landed at

$$R = 2x_0 = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin[2(60°)] = 35.3\text{ m}$$

which is shorter than $x = 53$ m found above. This makes sense because the broken fragment, having a smaller mass but greater horizontal speed, can travel much farther than the original shell.

14. (a) The phrase (in the problem statement) "such that it [particle 2] always stays directly above particle 1 during the flight" means that the shadow (as if a light were directly above the particles shining down on them) of particle 2 coincides with the position of particle 1, at each moment. We say, in this case, that they are vertically

aligned. Because of that alignment, $v_{2x} = v_1 = 10.0$ m/s. Because the initial value of v_2 is given as 20.0 m/s, then (using the Pythagorean theorem) we must have

$$v_{2y} = \sqrt{v_2^2 - v_{2x}^2} = \sqrt{300} \text{ m/s}$$

for the initial value of the y component of particle 2's velocity. Equation 2-16 (or conservation of energy) readily yields $y_{max} = 300/19.6 = 15.3$ m. Thus, we obtain

$$H_{max} = m_2 y_{max} / m_{total} = (3.00 \text{ g})(15.3 \text{ m})/(8.00 \text{ g}) = 5.74 \text{ m}.$$

(b) Since both particles have the same horizontal velocity, and particle 2's vertical component of velocity vanishes at that highest point, then the center of mass velocity then is simply $(10.0 \text{ m/s})\hat{\text{i}}$ (as one can verify using Eq. 9-17).

(c) Only particle 2 experiences any acceleration (the free fall acceleration downward), so Eq. 9-18 (or Eq. 9-19) leads to

$$a_{com} = m_2 g / m_{total} = (3.00 \text{ g})(9.8 \text{ m/s}^2)/(8.00 \text{ g}) = 3.68 \text{ m/s}^2$$

for the magnitude of the downward acceleration of the center of mass of this system. Thus, $\vec{a}_{com} = (-3.68 \text{ m/s}^2)\hat{\text{j}}$.

15. (a) The net force on the *system* (of total mass $m_1 + m_2$) is $m_2 g$. Thus, Newton's second law leads to $a = g(m_2/(m_1 + m_2)) = 0.4g$. For block 1, this acceleration is to the right (the $\hat{\text{i}}$ direction), and for block 2 this is an acceleration downward (the $-\hat{\text{j}}$ direction). Therefore, Eq. 9-18 gives

$$\vec{a}_{com} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2} = \frac{(0.6)(0.4g\hat{\text{i}}) + (0.4)(-0.4g\hat{\text{j}})}{0.6 + 0.4} = (2.35\,\hat{\text{i}} - 1.57\,\hat{\text{j}}) \text{ m/s}^2 .$$

(b) Integrating Eq. 4-16, we obtain

$$\vec{v}_{com} = (2.35\,\hat{\text{i}} - 1.57\hat{\text{j}})\, t$$

(with SI units understood), since it started at rest. We note that the *ratio* of the y-component to the x-component (for the velocity vector) does not change with time, and it is that ratio which determines the angle of the velocity vector (by Eq. 3-6), and thus the direction of motion for the center of mass of the system.

(c) The last sentence of our answer for part (b) implies that the path of the center-of-mass is a straight line.

(d) Equation 3-6 leads to $\theta = -34^\circ$. The path of the center of mass is therefore straight, at downward angle 34°.

16. We denote the mass of Ricardo as M_R and that of Carmelita as M_C. Let the center of mass of the two-person system (assumed to be closer to Ricardo) be a distance x from the middle of the canoe of length L and mass m. Then

$$M_R(L/2 - x) = mx + M_C(L/2 + x).$$

Now, after they switch positions, the center of the canoe has moved a distance $2x$ from its initial position. Therefore, x = 40 cm/2 = 0.20 m, which we substitute into the above equation to solve for M_C:

$$M_C = \frac{M_R\left(L/2 - x\right) - mx}{L/2 + x} = \frac{(80)\left(\frac{3.0}{2} - 0.20\right) - (30)(0.20)}{(3.0/2) + 0.20} = 58 \text{ kg.}$$

17. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16,

$$M\Delta x_{\text{com}} = 0 = m_b \Delta x_b + m_d \Delta x_d \,,$$

which implies

$$\left|\Delta x_b\right| = \frac{m_d}{m_b}\left|\Delta x_d\right|.$$

Now we express the geometrical condition that *relative to the boat* the dog has moved a distance d = 2.4 m:

$$\left|\Delta x_b\right| + \left|\Delta x_d\right| = d$$

which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}\left|\left(\Delta x_d\right)\right| + \left|\Delta x_d\right| = d$$

which leads to $\left|\Delta x_d\right| = \dfrac{d}{1 + m_d / m_b} = \dfrac{2.4 \text{ m}}{1 + (4.5/18)} = 1.92 \text{ m.}$

The dog is therefore 1.9 m closer to the shore than initially (where it was D = 6.1 m from it). Thus, it is now $D - |\Delta x_d|$ = 4.2 m from the shore.

18. The magnitude of the ball's momentum change is

$$\Delta p = m\left|v_i - v_f\right| = (0.70 \text{ kg})\left|(5.0 \text{ m/s}) - (-2.0 \text{ m/s})\right| = 4.9 \text{ kg}\cdot\text{m/s}.$$

19. (a) The change in kinetic energy is

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2100 \text{ kg})\left((51 \text{ km/h})^2 - (41 \text{ km/h})^2\right)$$
$$= 9.66\times10^4 \text{ kg}\cdot(\text{km/h})^2\left((10^3 \text{ m/km})(1 \text{ h/3600 s})\right)^2$$
$$= 7.5\times10^4 \text{ J}.$$

(b) The magnitude of the change in velocity is

$$|\Delta\vec{v}| = \sqrt{(-v_i)^2 + (v_f)^2} = \sqrt{(-41 \text{ km/h})^2 + (51 \text{ km/h})^2} = 65.4 \text{ km/h}$$

so the magnitude of the change in momentum is

$$|\Delta\vec{p}| = m|\Delta\vec{v}| = (2100 \text{ kg})(65.4 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right) = 3.8\times10^4 \text{ kg}\cdot\text{m/s}.$$

(c) The vector $\Delta\vec{p}$ points at an angle θ south of east, where

$$\theta = \tan^{-1}\left(\frac{v_i}{v_f}\right) = \tan^{-1}\left(\frac{41 \text{ km/h}}{51 \text{ km/h}}\right) = 39°.$$

20. We infer from the graph that the horizontal component of momentum p_x is 4.0 $\text{kg}\cdot\text{m/s}$. Also, its initial magnitude of momentum p_o is 6.0 $\text{kg}\cdot\text{m/s}$. Thus,

$$\cos\theta_o = \frac{p_x}{p_o} \quad\Rightarrow\quad \theta_o = 48°.$$

21. We use coordinates with +x horizontally toward the pitcher and +y upward. Angles are measured counterclockwise from the +x axis. Mass, velocity, and momentum units are SI. Thus, the initial momentum can be written $\vec{p}_0 = (4.5\angle 215°)$ in magnitude-angle notation.

(a) In magnitude-angle notation, the momentum change is

$$(6.0 \angle -90°) - (4.5 \angle 215°) = (5.0 \angle -43°)$$

(efficiently done with a vector-capable calculator in polar mode). The magnitude of the momentum change is therefore 5.0 $\text{kg}\cdot\text{m/s}$.

(b) The momentum change is (6.0 ∠ 0°) – (4.5 ∠ 215°) = (10 ∠ 15°). Thus, the magnitude of the momentum change is 10 $\text{kg}\cdot\text{m/s}$.

22. (a) Since the force of impact on the ball is in the y direction, p_x is conserved:

$$p_{xi} = p_{xf} \quad\Rightarrow\quad mv_i\sin\theta_1 = mv_i\sin\theta_2.$$

With θ_1 = 30.0°, we find θ_2 = 30.0°.

(b) The momentum change is

$$\Delta\vec{p} = mv_i \cos\theta_2 \left(-\hat{\mathrm{j}}\right) - mv_i \cos\theta_2 \left(+\hat{\mathrm{j}}\right) = -2\left(0.165 \text{ kg}\right)\left(2.00 \text{ m/s}\right)\left(\cos 30°\right)\hat{\mathrm{j}}$$
$$= (-0.572 \text{ kg}\cdot\text{m/s})\hat{\mathrm{j}}.$$

23. We estimate his mass in the neighborhood of 70 kg and compute the upward force F of the water from Newton's second law: $F - mg = ma$, where we have chosen $+y$ upward, so that $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the water). His speed when he arrives at the surface of the water is found either from Eq. 2-16 or from energy conservation: $v = \sqrt{2gh}$, where $h = 12$ m, and since the deceleration a reduces the speed to zero over a distance d = 0.30 m we also obtain $v = \sqrt{2ad}$. We use these observations in the following.

Equating our two expressions for v leads to $a = gh/d$. Our force equation, then, leads to

$$F = mg + m\left(g\frac{h}{d}\right) = mg\left(1 + \frac{h}{d}\right)$$

which yields $F \approx 2.8 \times 10^4$ kg. Since we are not at all certain of his mass, we express this as a guessed-at range (in kN) $25 < F < 30$.

Since $F \gg mg$, the impulse $\vec{J}$ due to the net force (while he is in contact with the water) is overwhelmingly caused by the upward force of the water: $\int F\,dt = \vec{J}$ to a good approximation. Thus, by Eq. 9-29,

$$\int F dt = \vec{p}_f - \vec{p}_i = 0 - m\left(-\sqrt{2gh}\right)$$

(the minus sign with the initial velocity is due to the fact that downward is the negative direction), which yields $(70\text{ kg})\sqrt{2\left(9.8\text{ m/s}^2\right)\left(12\text{ m}\right)} = 1.1\times10^3 \text{kg}\cdot\text{m/s}$. Expressing this as a range we estimate

$$1.0\times10^3 \text{kg}\cdot\text{m/s} < \int F\,dt < 1.2\times10^3 \text{kg}\cdot\text{m/s}.$$

24. We choose $+y$ upward, which implies $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the snow).

(a) The maximum deceleration a_{max} of the paratrooper (of mass m and initial speed v = 56 m/s) is found from Newton's second law

$$F_{\text{snow}} - mg = ma_{\text{max}}$$

where we require $F_{\text{snow}} = 1.2 \times 10^5$ N. Using Eq. 2-15 $v^2 = 2a_{\text{max}}d$, we find the minimum depth of snow for the man to survive:

$$d = \frac{v^2}{2a_{\text{max}}} = \frac{mv^2}{2(F_{\text{snow}} - mg)} \approx \frac{(85\text{kg})(56\,\text{m/s})^2}{2(1.2\times 10^5\,\text{N})} = 1.1\ \text{m}.$$

(b) His short trip through the snow involves a change in momentum

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - (85\,\text{kg})(-56\,\text{m/s}) = -4.8\times 10^3\ \text{kg}\cdot\text{m/s},$$

or $|\Delta\vec{p}| = 4.8\times 10^3\ \text{kg}\cdot\text{m/s}$. The negative value of the initial velocity is due to the fact that downward is the negative direction. By the impulse-momentum theorem, this equals the impulse due to the net force $F_{\text{snow}} - mg$, but since $F_{\text{snow}} \gg mg$ we can approximate this as the impulse on him just from the snow.

25. We choose $+y$ upward, which means $\vec{v}_i = -25\,\text{m/s}$ and $\vec{v}_f = +10\,\text{m/s}$. During the collision, we make the reasonable approximation that the net force on the ball is equal to F_{avg}, the average force exerted by the floor up on the ball.

(a) Using the impulse momentum theorem (Eq. 9-31) we find

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (1.2)(10) - (1.2)(-25) = 42\,\text{kg}\cdot\text{m/s}.$$

(b) From Eq. 9-35, we obtain

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{42}{0.020} = 2.1\times 10^3\,\text{N}.$$

26. (a) By energy conservation, the speed of the victim when he falls to the floor is

$$\frac{1}{2}mv^2 = mgh \;\Rightarrow\; v = \sqrt{2gh} = \sqrt{2(9.8\ \text{m/s}^2)(0.50\ \text{m})} = 3.1\,\text{m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (70\,\text{kg})(3.1\,\text{m/s}) \approx 2.2\times 10^2\ \text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 0.082\ \text{s}$ for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.2\times 10^2\ \text{N}\cdot\text{s}}{0.082\ \text{s}} \approx 2.7\times 10^3\ \text{N}.$$

27. **THINK** The velocity of the ball is changing because of the external force applied. Impulse-linear momentum theorem is involved.

EXPRESS The initial direction of motion is in the +x direction. The magnitude of the average force F_{avg} is given by

$$F_{avg} = \frac{J}{\Delta t} = \frac{32.4 \text{ N}\cdot\text{s}}{2.70\times10^{-2} \text{ s}} = 1.20\times10^{3} \text{ N}.$$

The force is in the negative direction. Using the linear momentum-impulse theorem stated in Eq. 9-31, we have

$$-F_{avg}\Delta t = J = \Delta p = m(v_f - v_i).$$

where m is the mass, v_i the initial velocity, and v_f the final velocity of the ball. The equation can be used to solve for v_f.

ANALYZE (a) Using the above expression, we find

$$v_f = \frac{mv_i - F_{avg}\Delta t}{m} = \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})(27\times10^{-3}\text{s})}{0.40\,\text{kg}} = -67\,\text{m/s}.$$

The final speed of the ball is $|v_f| = 67$ m/s.

(b) The negative sign in v_f indicates that the velocity is in the $-x$ direction, which is opposite to the initial direction of travel.

(c) From the above, the average magnitude of the force is $F_{avg} = 1.20\times10^{3}$ N.

(d) The direction of the impulse on the ball is $-x$, same as the applied force.

LEARN In vector notation, $\vec{F}_{avg}\Delta t = \vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$, which gives

$$\vec{v}_f = \vec{v}_i + \frac{\vec{J}}{m} = \vec{v}_i + \frac{\vec{F}_{avg}\Delta t}{m}$$

Since $\vec{J}$ or $\vec{F}_{avg}$ is in the opposite direction of $\vec{v}_i$, the velocity of the ball will decrease under the applied force. The ball first moves in the $+x$-direction, but then slows down and comes to a stop, and then reverses its direction of travel.

28. (a) The magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (0.70\text{ kg})(13\text{ m/s}) \approx 9.1\text{ kg}\cdot\text{m/s} = 9.1\text{ N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.1\ \text{N}\cdot\text{s}}{5.0\times10^{-3}\ \text{s}} \approx 1.8\times10^{3}\ \text{N}.$$

29. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed v, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass m is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(200)(3\times10^{-3}\,\text{kg})(500\,\text{m/s})}{(1\,\text{min})(60\,\text{s/min})} \approx 5\,\text{N}.$$

30. (a) By Eq. 9-30, impulse can be determined from the "area" under the $F(t)$ curve. Keeping in mind that the area of a triangle is $\frac{1}{2}$(base)(height), we find the impulse in this case is $1.00\ \text{N}\cdot\text{s}$.

(b) By definition (of the average of function, in the calculus sense) the average force must be the result of part (a) divided by the time (0.010 s). Thus, the average force is found to be 100 N.

(c) Consider ten hits. Thinking of ten hits as 10 $F(t)$ triangles, our total time interval is 10(0.050 s) = 0.50 s, and the total area is $10(1.0\ \text{N}\cdot\text{s})$. We thus obtain an average force of 10/0.50 = 20.0 N. One could consider 15 hits, 17 hits, and so on, and still arrive at this same answer.

31. (a) By energy conservation, the speed of the passenger when the elevator hits the floor is

$$\frac{1}{2}mv^2 = mgh \;\Rightarrow\; v = \sqrt{2gh} = \sqrt{2(9.8\ \text{m/s}^2)(36\ \text{m})} = 26.6\ \text{m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (90\ \text{kg})(26.6\ \text{m/s}) \approx 2.39\times10^{3}\ \text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.39\times10^{3}\ \text{N}\cdot\text{s}}{5.0\times10^{-3}\ \text{s}} \approx 4.78\times10^{5}\ \text{N}.$$

(c) If the passenger were to jump upward with a speed of $v' = 7.0\text{ m/s}$, then the resulting downward velocity would be

$$v'' = v - v' = 26.6\text{ m/s} - 7.0\text{ m/s} = 19.6\text{ m/s},$$

and the magnitude of the impulse becomes

$$J'' = |\Delta p''| = m\,|\Delta v''| = mv'' = (90\text{ kg})(19.6\text{ m/s}) \approx 1.76\times10^{3}\text{ N}\cdot\text{s}.$$

(d) The corresponding average force would be

$$F''_{\text{avg}} = \frac{J''}{\Delta t} = \frac{1.76\times10^{3}\text{ N}\cdot\text{s}}{5.0\times10^{-3}\text{ s}} \approx 3.52\times10^{5}\text{ N}.$$

32. (a) By the impulse-momentum theorem (Eq. 9-31) the change in momentum must equal the "area" under the $F(t)$ curve. Using the facts that the area of a triangle is $\frac{1}{2}$ (base)(height), and that of a rectangle is (height)(width), we find the momentum at t = 4 s to be (30 kg·m/s)$\hat{\text{i}}$.

(b) Similarly (but keeping in mind that areas beneath the axis are counted negatively) we find the momentum at t = 7 s is (38 kg·m/s)$\hat{\text{i}}$.

(c) At t = 9 s, we obtain $\vec{v}$ = (6.0 m/s)$\hat{\text{i}}$.

33. We use coordinates with +x rightward and +y upward, with the usual conventions for measuring the angles (so that the initial angle becomes 180 + 35 = 215°). Using SI units and magnitude-angle notation (efficient to work with when using a vector-capable calculator), the change in momentum is

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = \left(3.00\angle 90^\circ\right) - \left(3.60\angle 215^\circ\right) = \left(5.86\angle 59.8^\circ\right).$$

(a) The magnitude of the impulse is $J = \Delta p = 5.86\text{ kg}\cdot\text{m/s} = 5.86\text{ N}\cdot\text{s}$.

(b) The direction of $\vec{J}$ is 59.8° measured counterclockwise from the +x axis.

(c) Equation 9-35 leads to

$$J = F_{\text{avg}}\Delta t = 5.86\text{ N}\cdot\text{s} \;\Rightarrow\; F_{\text{avg}} = \frac{5.86\text{ N}\cdot\text{s}}{2.00\times10^{-3}\text{ s}} \approx 2.93\times10^{3}\text{N}.$$

We note that this force is very much larger than the weight of the ball, which justifies our (implicit) assumption that gravity played no significant role in the collision.

(d) The direction of $\vec{F}_{\text{avg}}$ is the same as $\vec{J}$, 59.8° measured counterclockwise from the +x axis.

34. (a) Choosing upward as the positive direction, the momentum change of the foot is

$$\Delta\vec{p} = 0 - m_{\text{foot}}\vec{v}_i = -(0.003 \text{ kg})\,(-1.50 \text{ m/s}) = 4.50\times10^{-3} \text{ N}\cdot\text{s}.$$

(b) Using Eq. 9-35 and now treating *downward* as the positive direction, we have

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = m_{\text{lizard}}g\ \Delta t = (0.090 \text{ kg})(9.80 \text{ m/s}^2)(0.60 \text{ s}) = 0.529 \text{ N}\cdot\text{s}.$$

(c) Push is what provides the primary support.

35. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued). We evaluate the integral $J = \int F\,dt$ by adding the appropriate areas (of a triangle, a rectangle, and another triangle) shown in the graph (but with the t converted to seconds). With m = 0.058 kg and v = 34 m/s, we apply the impulse-momentum theorem:

$$\int F_{\text{wall}}\,dt = m\vec{v}_f - m\vec{v}_i \quad\Rightarrow\quad \int_0^{0.002} F\,dt + \int_{0.002}^{0.004} F\,dt + \int_{0.004}^{0.006} F\,dt = m(+v) - m(-v)$$
$$\Rightarrow\quad \frac{1}{2}F_{\max}(0.002\,\text{s}) + F_{\max}(0.002\,\text{s}) + \frac{1}{2}F_{\max}(0.002\,\text{s}) = 2mv$$

which yields $F_{\max}(0.004\,\text{s}) = 2(0.058\,\text{kg})(34\,\text{m/s}) = 9.9\times10^2$ N.

36. (a) Performing the integral (from time a to time b) indicated in Eq. 9-30, we obtain

$$\int_a^b (12 - 3t^2)\,dt = 12(b-a) - (b^3 - a^3)$$

in SI units. If b = 1.25 s and a = 0.50 s, this gives $7.17\ \text{N}\cdot\text{s}$.

(b) This integral (the impulse) relates to the change of momentum in Eq. 9-31. We note that the force is zero at t = 2.00 s. Evaluating the above expression for a = 0 and b = 2.00 gives an answer of $16.0\ \text{kg}\cdot\text{m/s}$.

37. **THINK** We're given the force as a function of time, and asked to calculate the corresponding impulse, the average force and the maximum force.

EXPRESS Since the motion is one-dimensional, we work with the magnitudes of the vector quantities. The impulse J due to a force $F(t)$ exerted on a body is

$$J = \int_{t_i}^{t_f} F(t)dt = F_{\text{avg}}\Delta t\,,$$

where F_{avg} is the average force and $\Delta t = t_f - t_i$. To find the time at which the maximum force occurs, we set the derivative of *F* with respect to time equal to zero, and solve for *t*.

ANALYZE (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$J = \int_0^{3.0\times10^{-3}} F dt = \int_0^{3.0\times10^{-3}} \left[(6.0\times10^6)\,t - (2.0\times10^9)t^2\right]dt$$
$$= \left[\frac{1}{2}(6.0\times10^6)t^2 - \frac{1}{3}(2.0\times10^9)t^3\right]\Bigg|_0^{3.0\times10^{-3}} = 9.0\,\text{N}\cdot\text{s}.$$

(b) Using $J = F_{\text{avg}}\,\Delta t$, we find the average force to be

$$F_{\text{avg}}\,\frac{J}{\Delta t} = \frac{9.0\,\text{N}\cdot\text{s}}{3.0\times10^{-3}\ \text{s}} = 3.0\times10^3\ \text{N}.$$

(c) Differentiating $F(t)$ with respect to *t* and setting it to zero, we have

$$\frac{dF}{dt} = \frac{d}{dt}\left[(6.0\times10^6)\,t - (2.0\times10^9)t^2\right] = (6.0\times10^6) - (4.0\times10^9)t = 0\,,$$

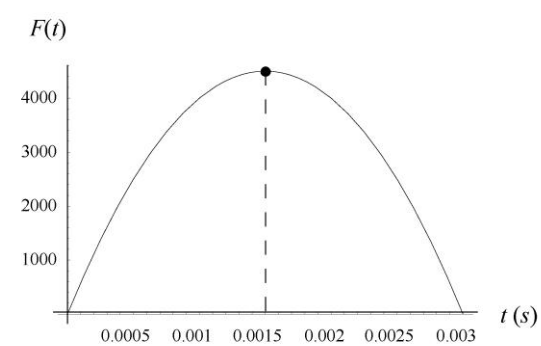
which can be solved to give $t = 1.5\times10^{-3}$ s. At that time the force is

$$F_{\text{max}} = (6.0\times10^6)(1.5\times10^{-3}) - (2.0\times10^9)(1.5\times10^{-3})^2 = 4.5\times10^3\ \text{N}.$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let *m* be the mass of the ball and *v* its speed as it leaves the foot. The speed of the ball immediately after it loses contact with the player's foot is

$$v = \frac{p}{m} = \frac{J}{m} = \frac{9.0\,\text{N}\cdot\text{s}}{0.45\ \text{kg}} = 20\ \text{m/s}.$$

LEARN The force as function of time is shown below. The area under the curve is the impulse *J*. From the plot, we readily see that $F(t)$ is a maximum at $t = 0.0015\,\text{s}$, with $F_{\text{max}} = 4500\,\text{N}$.



38. From Fig. 9-54, +*y* corresponds to the direction of the rebound (directly away from the wall) and +*x* toward the right. Using unit-vector notation, the ball's initial and final velocities are

$$\vec{v}_i = v\cos\theta\,\hat{\text{i}} - v\sin\theta\,\hat{\text{j}} = 5.2\,\hat{\text{i}} - 3.0\,\hat{\text{j}}$$
$$\vec{v}_f = v\cos\theta\,\hat{\text{i}} + v\sin\theta\,\hat{\text{j}} = 5.2\,\hat{\text{i}} + 3.0\,\hat{\text{j}}$$

respectively (with SI units understood).

(a) With m = 0.30 kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = 2(0.30\text{ kg})(3.0\text{ m/s }\hat{\text{j}}) = (1.8\text{ N}\cdot\text{s})\hat{\text{j}}.$$

(b) Using Eq. 9-35, the force on the ball by the wall is $\vec{J}/\Delta t = (1.8/0.010)\hat{\text{j}} = (180\,\text{N})\,\hat{\text{j}}$. By Newton's third law, the force on the wall by the ball is $(-180\text{ N})\hat{\text{j}}$ (that is, its magnitude is 180 N and its direction is directly into the wall, or "down" in the view provided by Fig. 9-54).

39. **THINK** This problem deals with momentum conservation. Since no external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, the total momentum of the system is conserved.

EXPRESS Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked. Also, let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then, by momentum conservation,

$$m_s v_s + m_m v_m = 0 \;\Rightarrow\; v_m = -\frac{m_s}{m_m}v_s.$$

ANALYZE We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{m_s}{m_m}v_s = -\frac{0.068\text{ kg}}{91\text{ kg}}(4.0\text{ m/s}) = -3.0\times10^{-3}\text{ m/s}$$

or $|v_m| = 3.0\times10^{-3}\text{ m/s}$.

LEARN The negative sign in v_m indicates that the man moves in the direction opposite to the motion of the stone. Note that his speed is much smaller (by a factor of m_s/m_m) compared to the speed of the stone.

40. Our notation is as follows: the mass of the motor is M; the mass of the module is m; the initial speed of the system is v_0; the relative speed between the motor and the module

is v_r; and, the speed of the module relative to the Earth is v after the separation. Conservation of linear momentum requires

$$(M + m)v_0 = mv + M(v - v_r).$$

Therefore,

$$v = v_0 + \frac{Mv_r}{M+m} = 4300 \text{ km/h} + \frac{(4m)(82 \text{ km/h})}{4m+m} = 4.4 \times 10^3 \text{ km/h}.$$

41. (a) With SI units understood, the velocity of block L (in the frame of reference indicated in the figure that goes with the problem) is $(v_1 - 3)\hat{\text{i}}$. Thus, momentum conservation (for the explosion at $t = 0$) gives

$$m_L(v_1 - 3) + (m_C + m_R)v_1 = 0$$

which leads to

$$v_1 = \frac{3\, m_L}{m_L + m_C + m_R} = \frac{3(2 \text{ kg})}{10 \text{ kg}} = 0.60 \text{ m/s}.$$

Next, at $t = 0.80$ s, momentum conservation (for the second explosion) gives

$$m_C\, v_2 + m_R(v_2 + 3) = (m_C + m_R)v_1 = (8 \text{ kg})(0.60 \text{ m/s}) = 4.8 \text{ kg} \cdot \text{m/s}.$$

This yields $v_2 = -0.15$. Thus, the velocity of block C after the second explosion is

$$v_2 = -(0.15 \text{ m/s})\hat{\text{i}}.$$

(b) Between $t = 0$ and $t = 0.80$ s, the block moves $v_1\Delta t = (0.60 \text{ m/s})(0.80 \text{ s}) = 0.48$ m. Between $t = 0.80$ s and $t = 2.80$ s, it moves an additional

$$v_2\Delta t = (-0.15 \text{ m/s})(2.00 \text{ s}) = -0.30 \text{ m}.$$

Its net displacement since $t = 0$ is therefore 0.48 m – 0.30 m = 0.18 m.

42. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of the original body is m; its initial velocity is $\vec{v}_0 = v\,\hat{\text{i}}$; the mass of the less massive piece is m_1; its velocity is $\vec{v}_1 = 0$; and, the mass of the more massive piece is m_2. We note that the conditions $m_2 = 3m_1$ (specified in the problem) and $m_1 + m_2 = m$ generally assumed in classical physics (before Einstein) lead us to conclude

$$m_1 = \frac{1}{4}m \text{ and } m_2 = \frac{3}{4}m.$$

Conservation of linear momentum requires

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 \;\Rightarrow\; mv\,\hat{\text{i}} = 0 + \frac{3}{4}m\vec{v}_2$$

which leads to $\vec{v}_2 = \frac{4}{3}v\,\hat{\text{i}}$. The increase in the system's kinetic energy is therefore

$$\Delta K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}mv_0^2 = 0 + \frac{1}{2}\left(\frac{3}{4}m\right)\left(\frac{4}{3}v\right)^2 - \frac{1}{2}mv^2 = \frac{1}{6}mv^2.$$

43. With $\vec{v}_0 = (9.5\,\hat{\text{i}} + 4.0\,\hat{\text{j}})$ m/s, the initial speed is

$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = \sqrt{(9.5\text{ m/s})^2 + (4.0\text{ m/s})^2} = 10.31\text{ m/s}$$

and the takeoff angle of the athlete is

$$\theta_0 = \tan^{-1}\left(\frac{v_{y0}}{v_{x0}}\right) = \tan^{-1}\left(\frac{4.0}{9.5}\right) = 22.8°.$$

Using Equation 4-26, the range of the athlete without using halteres is

$$R_0 = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(10.31\text{ m/s})^2 \sin 2(22.8°)}{9.8\text{ m/s}^2} = 7.75\text{ m}.$$

On the other hand, if two halteres of mass m = 5.50 kg were thrown at the maximum height, then, by momentum conservation, the subsequent speed of the athlete would be

$$(M + 2m)v_{x0} = Mv'_x \quad \Rightarrow v'_x = \frac{M + 2m}{M}v_{x0}$$

Thus, the change in the x-component of the velocity is

$$\Delta v_x = v'_x - v_{x0} = \frac{M + 2m}{M}v_{x0} - v_{x0} = \frac{2m}{M}v_{x0} = \frac{2(5.5\text{ kg})}{78\text{ kg}}(9.5\text{ m/s}) = 1.34\text{ m/s}.$$

The maximum height is attained when $v_y = v_{y0} - gt = 0$, or

$$t = \frac{v_{y0}}{g} = \frac{4.0\text{ m/s}}{9.8\text{ m/s}^2} = 0.41\text{ s}.$$

Therefore, the increase in range with use of halteres is

$$\Delta R = (\Delta v'_x)t = (1.34\text{ m/s})(0.41\text{ s}) = 0.55\text{ m}.$$

44. We can think of the sliding-until-stopping as an example of kinetic energy converting into thermal energy (see Eq. 8-29 and Eq. 6-2, with $F_N = mg$). This leads to $v^2 = 2\mu gd$ being true separately for each piece. Thus we can set up a ratio:

$$\left(\frac{v_L}{v_R}\right)^2 = \frac{2\mu_L g d_L}{2\mu_R g d_R} = \frac{12}{25} .$$

But (by the conservation of momentum) the ratio of speeds must be inversely proportional to the ratio of masses (since the initial momentum before the explosion was zero). Consequently,

$$\left(\frac{m_R}{m_L}\right)^2 = \frac{12}{25} \quad \Rightarrow \quad m_R = \tfrac{2}{5}\sqrt{3}\, m_L = 1.39 \text{ kg}.$$

Therefore, the total mass is $m_R + m_L \approx 3.4$ kg.

45. **THINK** The moving body is an isolated system with no external force acting on it. When it breaks up into three pieces, momentum remains conserved, both in the *x*- and the *y*-directions.

EXPRESS Our notation is as follows: the mass of the original body is $M = 20.0$ kg; its initial velocity is $\vec{v}_0 = (200 \text{ m/s})\hat{\text{i}}$; the mass of one fragment is $m_1 = 10.0$ kg; its velocity is $\vec{v}_1 = (100 \text{ m/s})\hat{\text{j}}$; the mass of the second fragment is $m_2 = 4.0$ kg; its velocity is $\vec{v}_2 = (-500 \text{ m/s})\hat{\text{i}}$; and, the mass of the third fragment is $m_3 = 6.00$ kg. Conservation of linear momentum requires

$$M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 .$$

The energy released in the explosion is equal to ΔK, the change in kinetic energy.

ANALYZE (a) The above momentum-conservation equation leads to

$$\begin{aligned}\vec{v}_3 &= \frac{M\vec{v}_0 - m_1\vec{v}_1 - m_2\vec{v}_2}{m_3} \\ &= \frac{(20.0\text{ kg})(200\text{ m/s})\hat{\text{i}} - (10.0\text{ kg})(100\text{ m/s})\hat{\text{j}} - (4.0\text{ kg})(-500\text{ m/s})\hat{\text{i}}}{6.00\text{ kg}} . \\ &= (1.00\times10^3 \text{ m/s})\,\hat{\text{i}} - (0.167\times10^3 \text{ m/s})\hat{\text{j}}\end{aligned}$$

The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{(1000 \text{ m/s})^2 + (-167 \text{ m/s})^2} = 1.01\times10^3 \text{ m/s}$. It points at $\theta = \tan^{-1}(-167/1000) = -9.48°$ (that is, at 9.5° measured clockwise from the +*x* axis).

(b) The energy released is ΔK:

$$\Delta K = K_f - K_i = \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2\right) - \frac{1}{2}Mv_0^2 = 3.23\times10^6\ \text{J}.$$

LEARN The energy released in the explosion, of chemical nature, is converted into the kinetic energy of the fragments.

46. Our $+x$ direction is east and $+y$ direction is north. The linear momenta for the two m = 2.0 kg parts are then

$$\vec{p}_1 = m\vec{v}_1 = mv_1\,\hat{\text{j}}$$

where v_1 = 3.0 m/s, and

$$\vec{p}_2 = m\vec{v}_2 = m\left(v_{2x}\,\hat{\text{i}} + v_{2y}\,\hat{\text{j}}\right) = mv_2\left(\cos\theta\,\hat{\text{i}} + \sin\theta\,\hat{\text{j}}\right)$$

where v_2 = 5.0 m/s and θ = 30°. The combined linear momentum of both parts is then

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 = mv_1\,\hat{\text{j}} + mv_2\left(\cos\theta\,\hat{\text{i}} + \sin\theta\,\hat{\text{j}}\right) = \left(mv_2\cos\theta\right)\hat{\text{i}} + \left(mv_1 + mv_2\sin\theta\right)\hat{\text{j}}\\ &= (2.0\text{ kg})(5.0\text{ m/s})(\cos 30°)\,\hat{\text{i}} + (2.0\text{ kg})\left(3.0\text{ m/s} + (5.0\text{ m/s})(\sin 30°)\right)\hat{\text{j}}\\ &= \left(8.66\,\hat{\text{i}} + 11\,\hat{\text{j}}\right)\text{ kg}\cdot\text{m/s}.\end{aligned}$$

From conservation of linear momentum we know that this is also the linear momentum of the whole kit before it splits. Thus the speed of the 4.0-kg kit is

$$v = \frac{P}{M} = \frac{\sqrt{P_x^2 + P_y^2}}{M} = \frac{\sqrt{(8.66\text{ kg}\cdot\text{m/s})^2 + (11\text{ kg}\cdot\text{m/s})^2}}{4.0\text{ kg}} = 3.5\text{ m/s}.$$

47. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of one piece is $m_1 = m$; its velocity is $\vec{v}_1 = (-30\text{ m/s})\hat{\text{i}}$; the mass of the second piece is $m_2 = m$; its velocity is $\vec{v}_2 = (-30\text{ m/s})\hat{\text{j}}$; and, the mass of the third piece is $m_3 = 3m$.

(a) Conservation of linear momentum requires

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \quad\Rightarrow\quad 0 = m\left(-30\hat{\text{i}}\right) + m\left(-30\hat{\text{j}}\right) + 3m\vec{v}_3$$

which leads to $\vec{v}_3 = (10\hat{\text{i}} + 10\hat{\text{j}})$ m/s. Its magnitude is $v_3 = 10\sqrt{2} \approx 14$ m/s.

(b) The direction is 45° *counterclockwise* from $+x$ (in this system where we have m_1 flying off in the $-x$ direction and m_2 flying off in the $-y$ direction).

48. This problem involves both mechanical energy conservation $U_i = K_1 + K_2$, where U_i = 60 J, and momentum conservation

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $m_2 = 2m_1$. From the second equation, we find $|\vec{v}_1| = 2|\vec{v}_2|$, which in turn implies (since $v_1 = |\vec{v}_1|$ and likewise for v_2)

$$K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\left(\frac{1}{2}m_2\right)(2v_2)^2 = 2\left(\frac{1}{2}m_2v_2^2\right) = 2K_2.$$

(a) We substitute $K_1 = 2K_2$ into the energy conservation relation and find

$$U_i = 2K_2 + K_2 \Rightarrow K_2 = \frac{1}{3}U_i = 20 \text{ J}.$$

(b) And we obtain $K_1 = 2(20) = 40$ J.

49. We refer to the discussion in the textbook (see Sample Problem – "Conservation of momentum, ballistic pendulum," which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units):

$$v = \frac{m+M}{m}\sqrt{2gh} = \frac{2.010}{0.010}\sqrt{2(9.8)(0.12)} = 3.1\times 10^2 \text{ m/s}.$$

50. (a) We choose $+x$ along the initial direction of motion and apply momentum conservation:

$$m_{\text{bullet}}\vec{v}_i = m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2$$
$$(5.2 \text{ g})(672 \text{ m/s}) = (5.2 \text{ g})(428 \text{ m/s}) + (700 \text{ g})\vec{v}_2$$

which yields $v_2 = 1.81$ m/s.

(b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}}\vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2 \text{ g})(672 \text{ m/s})}{5.2 \text{ g} + 700 \text{ g}} = 4.96 \text{ m/s}.$$

51. In solving this problem, our $+x$ direction is to the right (so all velocities are positive-valued).

(a) We apply momentum conservation to relate the situation just before the bullet strikes the second block to the situation where the bullet is embedded within the block.

$$(0.0035 \text{ kg})v = (1.8035 \text{ kg})(1.4 \text{ m/s}) \;\Rightarrow\; v = 721 \text{ m/s}.$$

(b) We apply momentum conservation to relate the situation just before the bullet strikes the first block to the instant it has passed through it (having speed v found in part (a)).

$$(0.0035 \text{ kg})v_0 = (1.20 \text{ kg})(0.630 \text{ m/s}) + (0.00350 \text{ kg})(721 \text{ m/s})$$

which yields v_0 = 937 m/s.

52. We think of this as having two parts: the first is the collision itself – where the bullet passes through the block so quickly that the block has not had time to move through any distance yet – and then the subsequent "leap" of the block into the air (up to height h measured from its initial position). The first part involves momentum conservation (with $+y$ upward):

$$(0.01 \text{ kg})(1000 \text{ m/s}) = (5.0 \text{ kg})\vec{v} + (0.01 \text{ kg})(400 \text{ m/s})$$

which yields $\vec{v} = 1.2 \text{ m/s}$. The second part involves either the free-fall equations from Ch. 2 (since we are ignoring air friction) or simple energy conservation from Ch. 8. Choosing the latter approach, we have

$$\frac{1}{2}(5.0 \text{ kg})(1.2 \text{ m/s})^2 = (5.0 \text{ kg})(9.8 \text{ m/s}^2)h$$

which gives the result h = 0.073 m.

53. With an initial speed of v_i, the initial kinetic energy of the car is $K_i = m_c v_i^2/2$. After a totally inelastic collision with a moose of mass m_m, by momentum conservation, the speed of the combined system is

$$m_c v_i = (m_c + m_m)v_f \;\Rightarrow\; v_f = \frac{m_c v_i}{m_c + m_m},$$

with final kinetic energy

$$K_f = \frac{1}{2}(m_c + m_m)v_f^2 = \frac{1}{2}(m_c + m_m)\left(\frac{m_c v_i}{m_c + m_m}\right)^2 = \frac{1}{2}\frac{m_c^2}{m_c + m_m}v_i^2.$$

(a) The percentage loss of kinetic energy due to collision is

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{m_c}{m_c + m_m} = \frac{m_m}{m_c + m_m} = \frac{500 \text{ kg}}{1000 \text{ kg} + 500 \text{ kg}} = \frac{1}{3} = 33.3\%.$$

(b) If the collision were with a camel of mass $m_{\text{camel}} = 300 \text{ kg}$, then the percentage loss of kinetic energy would be

$$\frac{\Delta K}{K_i} = \frac{m_{\text{camel}}}{m_c + m_{\text{camel}}} = \frac{300\text{ kg}}{1000\text{ kg} + 300\text{ kg}} = \frac{3}{13} = 23\%.$$

(c) As the animal mass decreases, the percentage loss of kinetic energy also decreases.

54. The total momentum immediately before the collision (with +x upward) is

$$p_i = (3.0\text{ kg})(20\text{ m/s}) + (2.0\text{ kg})(-12\text{ m/s}) = 36\ \text{kg}\cdot\text{m/s}.$$

Their momentum immediately after, when they constitute a combined mass of M = 5.0 kg, is $p_f = (5.0\text{ kg})\vec{v}$. By conservation of momentum, then, we obtain $\vec{v}$ = 7.2 m/s, which becomes their "initial" velocity for their subsequent free-fall motion. We can use Ch. 2 methods or energy methods to analyze this subsequent motion; we choose the latter. The level of their collision provides the reference (y = 0) position for the gravitational potential energy, and we obtain

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}Mv_0^2 + 0 = 0 + Mgy_{\max}.$$

Thus, with v_0 = 7.2 m/s, we find $y_{\max}$ = 2.6 m.

55. We choose +x in the direction of (initial) motion of the blocks, which have masses m_1 = 5 kg and m_2 = 10 kg. Where units are not shown in the following, SI units are to be understood.

(a) Momentum conservation leads to

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$
$$(5\text{ kg})(3.0\text{ m/s}) + (10\text{ kg})(2.0\text{ m/s}) = (5\text{ kg})\vec{v}_{1f} + (10\text{ kg})(2.5\text{ m/s})$$

which yields $\vec{v}_{1f} = 2.0\text{ m/s}$. Thus, the speed of the 5.0 kg block immediately after the collision is 2.0 m/s.

(b) We find the reduction in total kinetic energy:

$$K_i - K_f = \frac{1}{2}(5\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(10\text{ kg})(2\text{ m/s})^2 - \frac{1}{2}(5\text{ kg})(2\text{ m/s})^2 - \frac{1}{2}(10\text{ kg})(2.5\text{ m/s})^2$$
$$= -1.25\text{ J} \approx -1.3\text{ J}.$$

(c) In this new scenario where $\vec{v}_{2f} = 4.0\text{ m/s}$, momentum conservation leads to $\vec{v}_{1f} = -1.0\text{ m/s}$ and we obtain $\Delta K = +40\text{ J}$.

(d) The creation of additional kinetic energy is possible if, say, some gunpowder were on the surface where the impact occurred (initially stored chemical energy would then be contributing to the result).

56. (a) The magnitude of the deceleration of each of the cars is $a = f/m = \mu_k mg/m = \mu_k g$. If a car stops in distance d, then its speed v just after impact is obtained from Eq. 2-16:

$$v^2 = v_0^2 + 2ad \Rightarrow v = \sqrt{2ad} = \sqrt{2\mu_k gd}$$

since $v_0 = 0$ (this could alternatively have been derived using Eq. 8-31). Thus,

$$v_A = \sqrt{2\mu_k gd_A} = \sqrt{2(0.13)(9.8\text{ m/s}^2)(8.2\text{ m})} = 4.6\text{ m/s}.$$

(b) Similarly, $v_B = \sqrt{2\mu_k gd_B} = \sqrt{2(0.13)(9.8\text{ m/s}^2)(6.1\text{ m})} = 3.9\text{ m/s}.$

(c) Let the speed of car B be v just before the impact. Conservation of linear momentum gives $m_B v = m_A v_A + m_B v_B$, or

$$v = \frac{(m_A v_A + m_B v_B)}{m_B} = \frac{(1100)(4.6) + (1400)(3.9)}{1400} = 7.5\text{ m/s}.$$

(d) The conservation of linear momentum during the impact depends on the fact that the only significant force (during impact of duration Δt) is the force of contact between the bodies. In this case, that implies that the force of friction exerted by the road on the cars is neglected during the brief Δt. This neglect would introduce some error in the analysis. Related to this is the assumption we are making that the transfer of momentum occurs at one location, that the cars do not slide appreciably during Δt, which is certainly an approximation (though probably a good one). Another source of error is the application of the friction relation Eq. 6-2 for the sliding portion of the problem (after the impact); friction is a complex force that Eq. 6-2 only partially describes.

57. (a) Let v be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_i = (m + M)v$. Therefore,

$$v = \frac{mv_i}{m+M} = \frac{(60\text{ g})(22\text{ m/s})}{60\text{ g} + 240\text{ g}} = 4.4\text{ m/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$ and the final kinetic energy is

$$K_f = \tfrac{1}{2}(m+M)v^2 = \tfrac{1}{2}m^2v_i^2/(m+M).$$

The problem indicates $\Delta E_{\text{th}} = 0$, so the difference $K_i - K_f$ must equal the energy U_s stored in the spring:

$$U_s = \frac{1}{2}mv_i^2 - \frac{1}{2}\left(\frac{m^2v_i^2}{m+M}\right) = \frac{1}{2}mv_i^2\left[1 - \frac{m}{m+M}\right] = \frac{1}{2}mv_i^2\,\frac{M}{m+M}.$$

Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is

$$\frac{U_s}{K_i} = \frac{M}{m+M} = \frac{240}{60+240} = 0.80.$$

58. We think of this as having two parts: the first is the collision itself, where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet, and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount x_m. The first part involves momentum conservation (with $+x$ rightward):

$$m_1v_1 = (m_1+m_2)v \quad \Rightarrow \quad (2.0\text{ kg})(4.0\text{ m/s}) = (3.0\text{ kg})\vec{v}$$

which yields $\vec{v} = 2.7\text{ m/s}$. The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0\text{ kg})\,(2.7\text{ m/s})^2 = \frac{1}{2}\,(200\text{ N/m})x_m^2$$

which gives the result $x_m = 0.33$ m.

59. As hinted in the problem statement, the velocity v of the system as a whole, when the spring reaches the maximum compression x_m, satisfies

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v.$$

The change in kinetic energy of the system is therefore

$$\Delta K = \frac{1}{2}(m_1+m_2)v^2 - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2 = \frac{(m_1v_{1i}+m_2v_{2i})^2}{2(m_1+m_2)} - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2$$

which yields $\Delta K = -35$ J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $|\Delta K| = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_{rel}^2$ where $v_{rel} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2}kx_m^2 = -\Delta K \Rightarrow x_m = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35\text{ J})}{1120\text{ N/m}}} = 0.25\text{ m}.$$

60. (a) Let m_A be the mass of the block on the left, v_{Ai} be its initial velocity, and v_{Af} be its final velocity. Let m_B be the mass of the block on the right, v_{Bi} be its initial velocity, and v_{Bf} be its final velocity. The momentum of the two-block system is conserved, so

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

and

$$v_{Af} = \frac{m_A v_{Ai} + m_B v_{Bi} - m_B v_{Bf}}{m_A} = \frac{(1.6\text{ kg})(5.5\text{ m/s}) + (2.4\text{ kg})(2.5\text{ m/s}) - (2.4\text{ kg})(4.9\text{ m/s})}{1.6\text{ kg}}$$
$$= 1.9\text{ m/s}.$$

(b) The block continues going to the right after the collision.

(c) To see whether the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

$$K_i = \frac{1}{2}m_A v_{Ai}^2 + \frac{1}{2}m_B v_{Bi}^2 = \frac{1}{2}(1.6\text{ kg})(5.5\text{ m/s})^2 + \frac{1}{2}(2.4\text{ kg})(2.5\text{ m/s})^2 = 31.7\text{ J}.$$

The total kinetic energy after is

$$K_f = \frac{1}{2}m_A v_{Af}^2 + \frac{1}{2}m_B v_{Bf}^2 = \frac{1}{2}(1.6\text{ kg})(1.9\text{ m/s})^2 + \frac{1}{2}(2.4\text{ kg})(4.9\text{ m/s})^2 = 31.7\text{ J}.$$

Since $K_i = K_f$ the collision is found to be elastic.

61. **THINK** We have a moving cart colliding with a stationary cart. Since the collision is elastic, the total kinetic energy of the system remains unchanged.

EXPRESS Let m_1 be the mass of the cart that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the cart that is originally at rest and v_{2f} be its velocity after the collision. Conservation of linear momentum gives $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$. Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2.$$

Solving for v_{1f} and v_{2f}, we obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

The speed of the center of mass is $v_{\text{com}} = \dfrac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$.

ANALYZE (a) With $m_1 = 0.34$ kg, $v_{1i} = 1.2$ m/s and $v_{1f} = 0.66$ m/s, we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{1.2 \text{ m/s} - 0.66 \text{ m/s}}{1.2 \text{ m/s} + 0.66 \text{ m/s}} \right) (0.34 \text{ kg}) = 0.0987 \text{ kg} \approx 0.099 \text{ kg}.$$

(b) The velocity of the second cart is:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(0.34 \text{ kg})}{0.34 \text{ kg} + 0.099 \text{ kg}} \right) (1.2 \text{ m/s}) = 1.9 \text{ m/s}.$$

(c) From the above, we find the speed of the center of mass to be

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34 \text{ kg})(1.2 \text{ m/s}) + 0}{0.34 \text{ kg} + 0.099 \text{ kg}} = 0.93 \text{ m/s}.$$

LEARN In solving for v_{com}, values for the initial velocities were used. Since the system is isolated with no external force acting on it, v_{com} remains the same after the collision, so the same result is obtained if values for the final velocities are used. That is,

$$v_{\text{com}} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2} = \frac{(0.34 \text{ kg})(0.66 \text{ m/s}) + (0.099 \text{ kg})(1.9 \text{ m/s})}{0.34 \text{ kg} + 0.099 \text{ kg}} = 0.93 \text{ m/s}.$$

62. (a) Let m_1 be the mass of one sphere, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the other sphere, v_{2i} be its velocity before the collision, and v_{2f} be its velocity after the collision. Then, according to Eq. 9-75,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}.$$

Suppose sphere 1 is originally traveling in the positive direction and is at rest after the collision. Sphere 2 is originally traveling in the negative direction. Replace v_{1i} with v, v_{2i} with $-v$, and v_{1f} with zero to obtain $0 = m_1 - 3m_2$. Thus,

$$m_2 = m_1 / 3 = (300 \text{ g}) / 3 = 100 \text{ g}.$$

(b) We use the velocities before the collision to compute the velocity of the center of mass:

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(300 \text{ g})(2.00 \text{ m/s}) + (100 \text{ g})(-2.00 \text{ m/s})}{300 \text{ g} + 100 \text{ g}} = 1.00 \text{ m/s}.$$

63. (a) The center of mass velocity does not change in the absence of external forces. In this collision, only forces of one block on the other (both being part of the same system) are exerted, so the center of mass velocity is 3.00 m/s before and after the collision.

(b) We can find the velocity v_{1i} of block 1 before the collision (when the velocity of block 2 is known to be zero) using Eq. 9-17:

$$(m_1 + m_2)v_{com} = m_1 v_{1i} + 0 \quad \Rightarrow \quad v_{1i} = 12.0 \text{ m/s} .$$

Now we use Eq. 9-68 to find v_{2f}:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = 6.00 \text{ m/s} .$$

64. First, we find the speed v of the ball of mass m_1 right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with h = 0.700 m) leads to

$$m_1 gh = \frac{1}{2} m_1 v^2 \Rightarrow v = \sqrt{2gh} = 3.7 \text{ m/s}.$$

(a) We now treat the elastic collision using Eq. 9-67:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 \text{ kg} - 2.5 \text{ kg}}{0.5 \text{ kg} + 2.5 \text{ kg}} (3.7 \text{ m/s}) = -2.47 \text{ m/s}$$

which means the final speed of the ball is 2.47 m/s.

(b) Finally, we use Eq. 9-68 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5 \text{ kg})}{0.5 \text{ kg} + 2.5 \text{ kg}} (3.7 \text{ m/s}) = 1.23 \text{ m/s}.$$

65. **THINK** We have a mass colliding with another stationary mass. Since the collision is elastic, the total kinetic energy of the system remains unchanged.

EXPRESS Let m_1 be the mass of the body that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the body that is originally at rest and v_{2f} be its velocity after the collision. Conservation of linear momentum gives

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} .$$

Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 .$$

The solution to v_{1f} is given by Eq. 9-67: $v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1i}$. We solve for m_2 to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 .$$

The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} .$$

ANALYZE (a) given that $v_{1f} = v_{1i}/4$, we find the second mass to be

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{v_{1i} - v_{1i}/4}{v_{1i} + v_{1i}/4}\right) m_1 = \frac{3}{5} m_1 = \frac{3}{5}(2.0 \text{ kg}) = 1.2 \text{ kg} .$$

(b) The speed of the center of mass is $v_{\text{com}} = \dfrac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \dfrac{(2.0 \text{ kg})(4.0 \text{ m/s})}{2.0 \text{ kg} + 1.2 \text{ kg}} = 2.5 \text{ m/s}$.

LEARN The final speed of the second mass is

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(2.0 \text{ kg})}{2.0 \text{ kg} + 1.2 \text{ kg}}\right)(4.0 \text{ m/s}) = 5.0 \text{ m/s}.$$

Since the system is isolated with no external force acting on it, v_{com} remains the same after the collision, so the same result is obtained if values for the final velocities are used:

$$v_{\text{com}} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2} = \frac{(2.0 \text{ kg})(1.0 \text{ m/s}) + (1.2 \text{ kg})(5.0 \text{ kg})}{2.0 \text{ kg} + 1.2 \text{ kg}} = 2.5 \text{ m/s} .$$

66. Using Eq. 9-67 and Eq. 9-68, we have after the collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - 0.40 m_1}{m_1 + 0.40 m_1}(4.0 \text{ m/s}) = 1.71 \text{ m/s}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + 0.40 m_1}(4.0 \text{ m/s}) = 5.71 \text{ m/s}.$$

(a) During the (subsequent) sliding, the kinetic energy of block 1 $K_{1f} = \frac{1}{2} m_1 v_{1f}^2$ is converted into thermal form ($\Delta E_{\text{th}} = \mu_k m_1 g d_1$). Solving for the sliding distance d_1 we obtain $d_1 = 0.2999$ m ≈ 30 cm.

(b) A very similar computation (but with subscript 2 replacing subscript 1) leads to block 2's sliding distance $d_2 = 3.332$ m ≈ 3.3 m.

67. We use Eq 9-67 and 9-68 to find the velocities of the particles after their first collision (at x = 0 and t = 0):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{0.30\text{ kg} - 0.40\text{ kg}}{0.30\text{ kg} + 0.40\text{ kg}} (2.0\text{ m/s}) = -0.29\text{ m/s}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2(0.30\text{ kg})}{0.30\text{ kg} + 0.40\text{ kg}} (2.0\text{ m/s}) = 1.7\text{ m/s}.$$

At a rate of motion of 1.7 m/s, $2x_w$ = 140 cm (the distance to the wall and back to x = 0) will be traversed by particle 2 in 0.82 s. At t = 0.82 s, particle 1 is located at

$$x = (-2/7)(0.82) = -23\text{ cm},$$

and particle 2 is "gaining" at a rate of (10/7) m/s leftward; this is their relative velocity at that time. Thus, this "gap" of 23 cm between them will be closed after an additional time of (0.23 m)/(10/7 m/s) = 0.16 s has passed. At this time (t = 0.82 + 0.16 = 0.98 s) the two particles are at $x = (-2/7)(0.98) = -28$ cm.

68. (a) If the collision is perfectly elastic, then Eq. 9-68 applies

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + (2.00)m_1} \sqrt{2gh} = \frac{2}{3}\sqrt{2gh}$$

where we have used the fact (found most easily from energy conservation) that the speed of block 1 at the bottom of the frictionless ramp is $\sqrt{2gh}$ (where h = 2.50 m). Next, for block 2's "rough slide" we use Eq. 8-37:

$$\tfrac{1}{2} m_2 v_2^2 = \Delta E_{th} = f_k d = \mu_k m_2 g d$$

where μ_k = 0.500. Solving for the sliding distance d, we find that m_2 cancels out and we obtain d = 2.22 m.

(b) In a completely inelastic collision, we apply Eq. 9-53: $v_2 = \dfrac{m_1}{m_1 + m_2} v_{1i}$ (where, as above, $v_{1i} = \sqrt{2gh}$). Thus, in this case we have $v_2 = \sqrt{2gh}/3$. Now, Eq. 8-37 (using the total mass since the blocks are now joined together) leads to a sliding distance of $d = 0.556$ m (one-fourth of the part (a) answer).

69. (a) We use conservation of mechanical energy to find the speed of either ball after it has fallen a distance h. The initial kinetic energy is zero, the initial gravitational potential energy is $M gh$, the final kinetic energy is $\tfrac{1}{2} Mv^2$, and the final potential energy is zero. Thus $Mgh = \tfrac{1}{2} Mv^2$ and $v = \sqrt{2gh}$. The collision of the ball of M with the floor is an elastic collision of a light object with a stationary massive object. The velocity of the light object reverses direction without change in magnitude. After the collision, the ball is

traveling upward with a speed of $\sqrt{2gh}$. The ball of mass m is traveling downward with the same speed. We use Eq. 9-75 to find an expression for the velocity of the ball of mass M after the collision:

$$v_{Mf} = \frac{M-m}{M+m} v_{Mi} + \frac{2m}{M+m} v_{mi} = \frac{M-m}{M+m}\sqrt{2gh} - \frac{2m}{M+m}\sqrt{2gh} = \frac{M-3m}{M+m}\sqrt{2gh} \ .$$

For this to be zero, $m = M/3$. With $M = 0.63$ kg, we have $m = 0.21$ kg.

(b) We use the same equation to find the velocity of the ball of mass m after the collision:

$$v_{mf} = -\frac{m-M}{M+m}\sqrt{2gh} + \frac{2M}{M+m}\sqrt{2gh} = \frac{3M-m}{M+m}\sqrt{2gh}$$

which becomes (upon substituting $M = 3m$) $v_{mf} = 2\sqrt{2gh}$. We next use conservation of mechanical energy to find the height h' to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is mgh'. Thus,

$$\frac{1}{2}mv_{mf}^2 = mgh' \Rightarrow h' = \frac{v_{mf}^2}{2g} = 4h \ .$$

With $h = 1.8$ m, we have $h' = 7.2 \text{ m}$.

70. We use Eqs. 9-67, 9-68, and 4-21 for the elastic collision and the subsequent projectile motion. We note that both pucks have the same time-of-fall t (during their projectile motions). Thus, we have

$$\Delta x_2 = v_2\, t \quad \text{where } \Delta x_2 = d \text{ and } v_2 = \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$\Delta x_1 = v_1\, t \quad \text{where } \Delta x_1 = -2d \text{ and } v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \ .$$

Dividing the first equation by the second, we arrive at

$$\frac{d}{-2d} = \frac{\dfrac{2m_1}{m_1+m_2} v_{1i}\, t}{\dfrac{m_1-m_2}{m_1+m_2} v_{1i}\, t} \ .$$

After canceling v_{1i}, t, and d, and solving, we obtain $m_2 = 1.0$ kg.

71. We apply the conservation of linear momentum to the x and y axes respectively.

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$
$$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2.$$

We are given $v_{2f} = 1.20\times10^5$ m/s, $\theta_1 = 64.0°$ and $\theta_2 = 51.0°$. Thus, we are left with two unknowns and two equations, which can be readily solved.

(a) We solve for the final alpha particle speed using the *y*-momentum equation:

$$v_{1f} = \frac{m_2 v_{2f} \sin\theta_2}{m_1 \sin\theta_1} = \frac{(16.0)\left(1.20\times10^5\right)\sin(51.0°)}{(4.00)\sin(64.0°)} = 4.15\times10^5 \text{ m/s}.$$

(b) Plugging our result from part (a) into the *x*-momentum equation produces the initial alpha particle speed:

$$\begin{aligned} v_{1i} &= \frac{m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2}{m_{1i}} \\ &= \frac{(4.00)\left(4.15\times10^5\right)\cos(64.0°) + (16.0)\left(1.2\times10^5\right)\cos(51.0°)}{4.00} \\ &= 4.84\times10^5 \text{ m/s}. \end{aligned}$$

72. We orient our +*x* axis along the initial direction of motion, and specify angles in the "standard" way — so θ = –90° for the particle *B*, which is assumed to scatter "downward" and ϕ > 0 for particle *A*, which presumably goes into the first quadrant. We apply the conservation of linear momentum to the *x* and *y* axes, respectively.

$$m_B v_B = m_B v'_B \cos\theta + m_A v'_A \cos\phi$$
$$0 = m_B v'_B \sin\theta + m_A v'_A \sin\phi$$

(a) Setting $v_B = v$ and $v'_B = v/2$, the *y*-momentum equation yields

$$m_A v'_A \sin\phi = m_B \frac{v}{2}$$

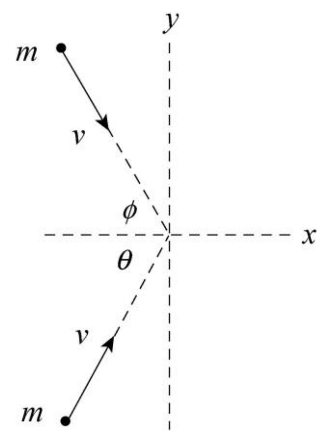
and the *x*-momentum equation yields $m_A v'_A \cos\phi = m_B v$. Dividing these two equations, we find $\tan\phi = \frac{1}{2}$, which yields $\phi = 27°$.

(b) We can *formally* solve for v'_A (using the *y*-momentum equation and the fact that $\phi = 1/\sqrt{5}$)

$$v'_A = \frac{\sqrt{5}}{2}\frac{m_B}{m_A} v$$

but lacking numerical values for v and the mass ratio, we cannot fully determine the final speed of A. Note: substituting $\cos\phi = 2/\sqrt{5}$, into the x-momentum equation leads to exactly this same relation (that is, no new information is obtained that might help us determine an answer).

73. Suppose the objects enter the collision along lines that make the angles $\theta > 0$ and $\phi > 0$ with the x axis, as shown in the diagram that follows. Both have the same mass m and the same initial speed v. We suppose that after the collision the combined object moves in the positive x direction with speed V.



Since the y component of the total momentum of the two-object system is conserved,

$$mv \sin\theta - mv \sin\phi = 0.$$

This means $\phi = \theta$. Since the x component is conserved,

$$2mv \cos\theta = 2mV.$$

We now use $V = v/2$ to find that $\cos\theta = 1/2$. This means $\theta = 60°$. The angle between the initial velocities is 120°.

74. (a) Conservation of linear momentum implies

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B .$$

Since $m_A = m_B = m = 2.0$ kg, the masses divide out and we obtain

$$\begin{aligned}\vec{v}\,'_B &= \vec{v}_A + \vec{v}_B - \vec{v}\,'_A = (15\hat{\mathrm{i}} + 30\hat{\mathrm{j}})\text{ m/s} + (-10\hat{\mathrm{i}} + 5\hat{\mathrm{j}})\text{ m/s} - (-5\hat{\mathrm{i}} + 20\hat{\mathrm{j}})\text{ m/s} \\ &= (10\hat{\mathrm{i}} + 15\hat{\mathrm{j}})\text{ m/s} .\end{aligned}$$

(b) The final and initial kinetic energies are

$$K_f = \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B = \frac{1}{2}(2.0)\left((-5)^2 + 20^2 + 10^2 + 15^2\right) = 8.0\times 10^2 \text{ J}$$

$$K_i = \frac{1}{2}mv^2_A + \frac{1}{2}mv^2_B = \frac{1}{2}(2.0)\left(15^2 + 30^2 + (-10)^2 + 5^2\right) = 1.3\times 10^3 \text{ J} .$$

The change kinetic energy is then $\Delta K = -5.0 \times 10^2$ J (that is, 500 J of the initial kinetic energy is lost).

75. We orient our +x axis along the initial direction of motion, and specify angles in the "standard" way — so θ = +60° for the proton (1), which is assumed to scatter into the first quadrant and ϕ = –30° for the target proton (2), which scatters into the fourth quadrant (recall that the problem has told us that this is perpendicular to θ). We apply the conservation of linear momentum to the x and y axes, respectively.

$$\begin{aligned} m_1 v_1 &= m_1 v_1' \cos\theta + m_2 v_2' \cos\phi \\ 0 &= m_1 v_1' \sin\theta + m_2 v_2' \sin\phi. \end{aligned}$$

We are given v_1 = 500 m/s, which provides us with two unknowns and two equations, which is sufficient for solving. Since $m_1 = m_2$ we can cancel the mass out of the equations entirely.

(a) Combining the above equations and solving for v_2' we obtain

$$v_2' = \frac{v_1 \sin\theta}{\sin(\theta - \phi)} = \frac{(500 \text{ m/s}) \sin(60°)}{\sin(90°)} = 433 \text{ m/s}.$$

We used the identity $\sin\theta\cos\phi - \cos\theta\sin\phi = \sin(\theta - \phi)$ in simplifying our final expression.

(b) In a similar manner, we find

$$v_1' = \frac{v_1 \sin\theta}{\sin(\phi - \theta)} = \frac{(500 \text{ m/s}) \sin(-30°)}{\sin(-90°)} = 250 \text{ m/s}.$$

76. We use Eq. 9-88. Then

$$v_f = v_i + v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) = 105 \text{ m/s} + (253 \text{ m/s}) \ln\left(\frac{6090 \text{ kg}}{6010 \text{ kg}}\right) = 108 \text{ m/s}.$$

77. **THINK** The mass of the faster barge is increasing at a constant rate. Additional force must be provided in order to maintain a constant speed.

EXPRESS We consider what must happen to the coal that lands on the faster barge during a time interval Δt. In that time, a total of Δm of coal must experience a change of velocity (from slow to fast) $\Delta v = v_{\text{fast}} - v_{\text{slow}}$, where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta p}{\Delta t} = \frac{(\Delta m)}{\Delta t}\Delta v = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}})$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$. Note that we ignore the transverse speed of the coal as it is shoveled from the slower barge to the faster one.

ANALYZE (a) With $v_{\text{fast}} = 20\,\text{km/h} = 5.56\,\text{m/s}$, $v_{\text{slow}} = 10\,\text{km/h} = 2.78\,\text{m/s}$ and the rate of mass change $(\Delta m/\Delta t) = 1000\,\text{kg/min} = (16.67\,\text{kg/s})$, the force that must be applied to the faster barge is

$$F_{\text{fast}} = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}}) = (16.67\text{ kg/s})(5.56\text{ m/s} - 2.78\text{ m/s}) = 46.3\,\text{N}$$

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

LEARN The force that must be applied to the faster barge in order to maintain a constant speed is equal to the rate of change of momentum of the coal.

78. We use Eq. 9-88 and simplify with $v_i = 0$, $v_f = v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \Rightarrow \frac{M_i}{M_f} = e^{v/u}$$

(a) If $v = u$ we obtain $\frac{M_i}{M_f} = e^1 \approx 2.7$.

(b) If $v = 2u$ we obtain $\frac{M_i}{M_f} = e^2 \approx 7.4$.

79. **THINK** As fuel is consumed, both the mass and the speed of the rocket will change.

EXPRESS The thrust of the rocket is given by $T = Rv_{\text{rel}}$ where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket. On the other hand, the mass of fuel ejected is given by $M_{\text{fuel}} = R\Delta t$, where Δt is the time interval of the burn. Thus, the mass of the rocket after the burn is

$$M_f = M_i - M_{\text{fuel}} .$$

ANALYZE (a) Given that $R = 480$ kg/s and $v_{\text{rel}} = 3.27 \times 10^3$ m/s, we find the thrust to be

$$T = Rv_{\text{rel}} = (480\,\text{kg/s})(3.27\times10^3\,\text{m/s}) = 1.57\times10^6\,\text{N}.$$

(b) With the mass of fuel ejected given by $M_{\text{fuel}} = R\Delta t =$ (480 kg/s)(250 s) = 1.20×10^5 kg, the final mass of the rocket is

$$M_f = M_i - M_{\text{fuel}} = (2.55 \times 10^5 \text{ kg}) - (1.20 \times 10^5 \text{ kg}) = 1.35 \times 10^5 \text{ kg}.$$

(c) Since the initial speed is zero, the final speed of the rocket is

$$v_f = v_{\text{rel}} \ln\frac{M_i}{M_f} = \left(3.27\times10^3 \text{ m/s}\right)\ln\left(\frac{2.55\times10^5 \text{ kg}}{1.35\times10^5 \text{ kg}}\right) = 2.08\times10^3 \text{ m/s}.$$

LEARN The speed of the rocket continues to rise as the fuel is consumed. From the first rocket equation given in Eq. 9-87, the thrust of the rocket is related to the acceleration by $T = Ma$. Using this equation, we find the initial acceleration to be

$$a_i = \frac{T}{M_i} = \frac{1.57\times10^6 \text{ N}}{2.55\times10^5 \text{ kg}} = 6.16 \text{ m/s}^2.$$

80. The velocity of the object is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left((3500-160t)\,\hat{\text{i}} + 2700\,\hat{\text{j}} + 300\,\hat{\text{k}}\right) = -(160 \text{ m/s})\hat{\text{i}}.$$

(a) The linear momentum is $\vec{p} = m\vec{v} = \left(250 \text{ kg}\right)\left(-160 \text{ m/s}\,\hat{\text{i}}\right) = (-4.0\times10^4 \text{ kg}\cdot\text{m/s})\,\hat{\text{i}}.$

(b) The object is moving west (our $-\hat{\text{i}}$ direction).

(c) Since the value of $\vec{p}$ does not change with time, the net force exerted on the object is zero, by Eq. 9-23.

81. We assume no external forces act on the system composed of the two parts of the last stage. Hence, the total momentum of the system is conserved. Let m_c be the mass of the rocket case and m_p the mass of the payload. At first they are traveling together with velocity v. After the clamp is released m_c has velocity v_c and m_p has velocity v_p. Conservation of momentum yields

$$(m_c + m_p)v = m_c v_c + m_p v_p.$$

(a) After the clamp is released the payload, having the lesser mass, will be traveling at the greater speed. We write $v_p = v_c + v_{\text{rel}}$, where v_{rel} is the relative velocity. When this expression is substituted into the conservation of momentum condition, the result is

$$\left(m_c + m_p\right)v = m_c v_c + m_p v_c + m_p v_{\text{rel}}.$$

Therefore,

$$v_c = \frac{(m_c + m_p)v - m_p v_{\text{rel}}}{m_c + m_p} = \frac{(290.0 \text{ kg} + 150.0 \text{ kg})(7600 \text{ m/s}) - (150.0 \text{ kg})(910.0 \text{ m/s})}{290.0 \text{ kg} + 150.0 \text{ kg}}$$
$$= 7290 \text{ m/s}.$$

(b) The final speed of the payload is $v_p = v_c + v_{\text{rel}}$ = 7290 m/s + 910.0 m/s = 8200 m/s.

(c) The total kinetic energy before the clamp is released is

$$K_i = \frac{1}{2}(m_c + m_p)v^2 = \frac{1}{2}(290.0 \text{ kg} + 150.0 \text{ kg})(7600 \text{ m/s})^2 = 1.271 \times 10^{10} \text{ J}.$$

(d) The total kinetic energy after the clamp is released is

$$K_f = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_p v_p^2 = \frac{1}{2}(290.0 \text{ kg})(7290 \text{ m/s})^2 + \frac{1}{2}(150.0 \text{ kg})(8200 \text{ m/s})^2$$
$$= 1.275 \times 10^{10} \text{ J}.$$

The total kinetic energy increased slightly. Energy originally stored in the spring is converted to kinetic energy of the rocket parts.

82. Let m be the mass of the higher floors. By energy conservation, the speed of the higher floors just before impact is

$$mgd = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gd}.$$

The magnitude of the impulse during the impact is

$$J = |\Delta p| = m|\Delta v| = mv = m\sqrt{2gd} = mg\sqrt{\frac{2d}{g}} = W\sqrt{\frac{2d}{g}}$$

where $W = mg$ represents the weight of the higher floors. Thus, the average force exerted on the lower floor is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{W}{\Delta t}\sqrt{\frac{2d}{g}}$$

With $F_{\text{avg}} = sW$, where s is the safety factor, we have

$$s = \frac{1}{\Delta t}\sqrt{\frac{2d}{g}} = \frac{1}{1.5 \times 10^{-3} \text{ s}}\sqrt{\frac{2(4.0 \text{ m})}{9.8 \text{ m/s}^2}} = 6.0 \times 10^2.$$

83. (a) Momentum conservation gives

$$m_R v_R + m_L v_L = 0 \Rightarrow (0.500 \text{ kg})\, v_R + (1.00 \text{ kg})(-1.20 \text{ m/s}) = 0$$

which yields v_R = 2.40 m/s. Thus, $\Delta x = v_R\, t$ = (2.40 m/s)(0.800 s) = 1.92 m.

(b) Now we have $m_R v_R + m_L (v_R - 1.20 \text{ m/s}) = 0$, which yields

$$v_R = \frac{(1.2 \text{ m/s})m_L}{m_L + m_R} = \frac{(1.20 \text{ m/s})(1.00 \text{ kg})}{1.00 \text{ kg} + 0.500 \text{ kg}} = 0.800 \text{ m/s}.$$

Consequently, $\Delta x = v_R\, t$ = 0.640 m.

84. (a) This is a highly symmetric collision, and when we analyze the y-components of momentum we find their net value is zero. Thus, the stuck-together particles travel along the x axis.

(b) Since it is an elastic collision with identical particles, the final speeds are the same as the initial values. Conservation of momentum along each axis then assures that the angles of approach are the same as the angles of scattering. Therefore, one particle travels along line 2, the other along line 3.

(c) Here the final speeds are less than they were initially. The total x-component cannot be less, however, by momentum conservation, so the loss of speed shows up as a decrease in their y-velocity-components. This leads to smaller angles of scattering. Consequently, one particle travels through region B, the other through region C; the paths are symmetric about the x-axis. We note that this is intermediate between the final states described in parts (b) and (a).

(d) Conservation of momentum along the x-axis leads (because these are identical particles) to the simple observation that the x-component of each particle remains constant:

$$v_{fx} = v\cos\theta = 3.06 \text{ m/s}.$$

(e) As noted above, in this case the speeds are unchanged; both particles are moving at 4.00 m/s in the final state.

85. Using Eq. 9-67 and Eq. 9-68, we have after the first collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - 2m_1}{m_1 + 2m_1} v_{1i} = -\frac{1}{3} v_{1i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + 2m_1} v_{1i} = \frac{2}{3} v_{1i}.$$

After the second collision, the velocities are

$$v_{2ff} = \frac{m_2 - m_3}{m_2 + m_3} v_{2f} = \frac{-m_2}{3m_2} \frac{2}{3} v_{1i} = -\frac{2}{9} v_{1i}$$

$$v_{3ff} = \frac{2m_2}{m_2 + m_3} v_{2f} = \frac{2m_2}{3m_2} \frac{2}{3} v_{1i} = \frac{4}{9} v_{1i} \ .$$

(a) Setting $v_{1i} = 4$ m/s, we find $v_{3ff} \approx 1.78$ m/s.

(b) We see that v_{3ff} is less than v_{1i} .

(c) The final kinetic energy of block 3 (expressed in terms of the initial kinetic energy of block 1) is

$$K_{3ff} = \frac{1}{2} m_3 v_3^2 = \frac{1}{2} (4m_1) \left(\frac{4}{9}\right)^2 v_{1i}^2 = \frac{64}{81} K_{1i} \, .$$

We see that this is less than K_{1i} .

(d) The final momentum of block 3 is $p_{3ff} = m_3 v_{3ff} = (4m_1)\left(\frac{16}{9}\right) v_1 > m_1 v_1$.

86. (a) We use Eq. 9-68 twice:

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{1.5m_1} (4.00 \text{ m/s}) = \frac{16}{3} \text{ m/s}$$

$$v_3 = \frac{2m_2}{m_2 + m_3} v_2 = \frac{2m_2}{1.5m_2} (16/3 \text{ m/s}) = \frac{64}{9} \text{ m/s} = 7.11 \text{ m/s} \ .$$

(b) Clearly, the speed of block 3 is greater than the (initial) speed of block 1.

(c) The kinetic energy of block 3 is

$$K_{3f} = \frac{1}{2} m_3 v_3^2 = \left(\frac{1}{2}\right)^3 m_1 \left(\frac{16}{9}\right)^2 v_{1i}^2 = \frac{64}{81} K_{1i} \, .$$

We see the kinetic energy of block 3 is less than the (initial) K of block 1. In the final situation, the initial K is being shared among the three blocks (which are all in motion), so this is not a surprising conclusion.

(d) The momentum of block 3 is

$$p_{3f} = m_3 v_3 = \left(\frac{1}{2}\right)^2 m_1 \left(\frac{16}{9}\right) v_{1i} = \frac{4}{9} p_{1i}$$

and is therefore less than the initial momentum (both of these being considered in magnitude, so questions about ± sign do not enter the discussion).

87. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued $\vec{v}_i = -5.2$ m/s).

(a) The speed of the ball right after the collision is

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(K_i/2)}{m}} = \sqrt{\frac{mv_i^2/2}{m}} = \frac{v_i}{\sqrt{2}} \approx 3.7 \text{ m/s}.$$

(b) With m = 0.15 kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (0.15 \text{ kg})(3.7 \text{ m/s}) - (0.15 \text{ kg})(-5.2 \text{ m/s}) = 1.3 \text{ N}\cdot\text{s}.$$

(c) Equation 9-35 leads to $F_{avg} = J/\Delta t$ = 1.3/0.0076 = 1.8 × 10^2 N.

88. We first consider the 1200 kg part. The impulse has magnitude J and is (by our choice of coordinates) in the positive direction. Let m_1 be the mass of the part and v_1 be its velocity after the bolts are exploded. We assume both parts are at rest before the explosion. Then $J = m_1v_1$, so

$$v_1 = \frac{J}{m_1} = \frac{300 \text{ N}\cdot\text{s}}{1200 \text{ kg}} = 0.25 \text{ m/s}.$$

The impulse on the 1800 kg part has the same magnitude but is in the opposite direction, so $-J = m_2v_2$, where m_2 is the mass and v_2 is the velocity of the part. Therefore,

$$v_2 = -\frac{J}{m_2} = -\frac{300 \text{N}\cdot\text{s}}{1800 \text{kg}} = -0.167 \text{ m/s}.$$

Consequently, the relative speed of the parts after the explosion is

$$u = 0.25 \text{ m/s} - (-0.167 \text{ m/s}) = 0.417 \text{ m/s}.$$

89. **THINK** The momentum of the car changes as it turns and collides with a tree.

EXPRESS Let the initial and final momenta of the car be $\vec{p}_i = m\vec{v}_i$ and $\vec{p}_f = m\vec{v}_f$, respectively. The impulse on it equals the change in its momentum:

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i).$$

The average force over the duration Δt is given by $\vec{F}_{avg} = \vec{J}/\Delta t$.

ANALYZE (a) The initial momentum of the car is

$$\vec{p}_i = m\vec{v}_i = (1400\,\text{kg})(5.3\,\text{m/s})\hat{\text{j}} = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{j}}$$

and the final momentum after making the turn is $\vec{p}_f = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$ (note that the magnitude remains the same, only the direction is changed). Thus, the impulse is

$$\vec{J} = \vec{p}_f - \vec{p}_i = (7.4\times10^3\,\text{N}\cdot\text{s})(\hat{\text{i}} - \hat{\text{j}}).$$

(b) The initial momentum of the car after the turn is $\vec{p}_i' = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$ and the final momentum after colliding with a tree is $\vec{p}_f' = 0$. The impulse acting on it is

$$\vec{J}' = \vec{p}_f' - \vec{p}_i' = (-7.4\times10^3\,\text{N}\cdot\text{s})\hat{\text{i}}.$$

(c) The average force on the car during the turn is

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t} = \frac{(7400\,\text{kg}\cdot\text{m/s})(\hat{\text{i}} - \hat{\text{j}})}{4.6\ \text{s}} = (1600\,\text{N})(\hat{\text{i}} - \hat{\text{j}})$$

and its magnitude is

$$F_{\text{avg}} = (1600\,\text{N})\sqrt{2} = 2.3\times10^3\,\text{N}.$$

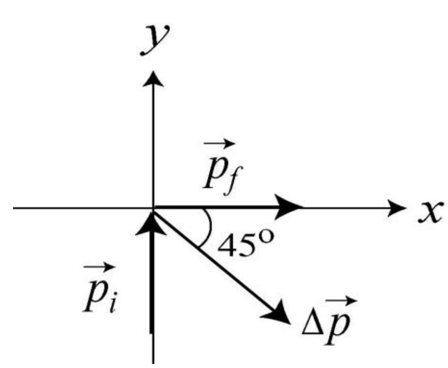
(d) The average force during the collision with the tree is

$$\vec{F}_{\text{avg}}' = \frac{\vec{J}'}{\Delta t} = \frac{(-7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}}{350\times10^{-3}\text{s}} = (-2.1\times10^4\,\text{N})\hat{\text{i}}$$

and its magnitude is $F_{\text{avg}}' = 2.1\times10^4\,\text{N}$.

(e) As shown in (c), the average force during the turn, in unit vector notation, is $\vec{F}_{\text{avg}} = (1600\,\text{N})(\hat{\text{i}} - \hat{\text{j}})$. The force is 45° below the positive x axis.

LEARN During the turn, the average force $\vec{F}_{\text{avg}}$ is in the same direction as $\vec{J}$, or $\Delta\vec{p}$. Its x and y components have equal magnitudes. The x component is positive and the y component is negative, so the force is 45° below the positive x axis.



90. (a) We find the momentum $\vec{p}_{n\,r}$ of the residual nucleus from momentum conservation.

$$\vec{p}_{n\,i} = \vec{p}_e + \vec{p}_\nu + \vec{p}_{n\,r} \quad \Rightarrow \quad 0 = (-1.2\times10^{-22}\ \text{kg}\cdot\text{m/s})\,\hat{\text{i}} + (-6.4\times10^{-23}\ \text{kg}\cdot\text{m/s})\,\hat{\text{j}} + \vec{p}_{n\,r}$$

Thus, $\vec{p}_{n\,r} = (1.2\times10^{-22}\,\text{kg}\cdot\text{m/s})\,\hat{\text{i}} + (6.4\times10^{-23}\,\text{kg}\cdot\text{m/s})\,\hat{\text{j}}$. Its magnitude is

$$|\,\vec{p}_{n\,r}\,| = \sqrt{\left(1.2\times10^{-22}\ \text{kg}\cdot\text{m/s}\right)^2 + \left(6.4\times10^{-23}\ \text{kg}\cdot\text{m/s}\right)^2} = 1.4\times10^{-22}\ \text{kg}\cdot\text{m/s}.$$

(b) The angle measured from the +x axis to $\vec{p}_{n\,r}$ is

$$\theta = \tan^{-1}\left(\frac{6.4\times10^{-23}\ \text{kg}\cdot\text{m/s}}{1.2\times10^{-22}\ \text{kg}\cdot\text{m/s}}\right) = 28°.$$

(c) Combining the two equations $p = mv$ and $K = \frac{1}{2}mv^2$, we obtain (with $p = p_{n\,r}$ and $m = m_{n\,r}$)

$$K = \frac{p^2}{2m} = \frac{\left(1.4\times10^{-22}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(5.8\times10^{-26}\ \text{kg}\right)} = 1.6\times10^{-19}\ \text{J}.$$

91. No external forces with horizontal components act on the cart-man system and the vertical forces sum to zero, so the total momentum of the system is conserved. Let m_c be the mass of the cart, v be its initial velocity, and v_c be its final velocity (after the man jumps off). Let m_m be the mass of the man. His initial velocity is the same as that of the cart and his final velocity is zero. Conservation of momentum yields $(m_m + m_c)v = m_c v_c$. Consequently, the final speed of the cart is

$$v_c = \frac{v(m_m + m_c)}{m_c} = \frac{(2.3\ \text{m/s})(75\ \text{kg} + 39\ \text{kg})}{39\ \text{kg}} = 6.7\ \text{m/s}.$$

The cart speeds up by 6.7 m/s – 2.3 m/s = + 4.4 m/s. In order to slow himself, the man gets the cart to push backward on him by pushing forward on it, so the cart speeds up.

92. The fact that they are connected by a spring is not used in the solution. We use Eq. 9-17 for $\vec{v}_{\text{com}}$:

$$M\vec{v}_{\text{com}} = m_1\vec{v}_1 + m_2\vec{v}_2 = \left(1.0\ \text{kg}\right)\left(1.7\ \text{m/s}\right) + \left(3.0\ \text{kg}\right)\vec{v}_2$$

which yields $|\vec{v}_2| = 0.57\ \text{m/s}$. The direction of $\vec{v}_2$ is opposite that of $\vec{v}_1$ (that is, they are both headed toward the center of mass, but from opposite directions).

93. **THINK** A completely inelastic collision means that the railroad freight car and the caboose car move together after the collision. The motion is one-dimensional.

EXPRESS Let m_F be the mass of the freight car and v_F be its initial velocity. Let m_C be the mass of the caboose and v be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to

$$m_F v_F = (m_F + m_C)v \;\Rightarrow\; v = \frac{m_F v_F}{m_F + m_C}.$$

The initial kinetic energy of the system is $K_i = \frac{1}{2} m_F v_F^2$ and the final kinetic energy is

$$K_f = \frac{1}{2}(m_F + m_C)v^2 = \frac{1}{2}(m_F + m_C)\frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)}.$$

Since 27% of the original kinetic energy is lost, we have $K_f = 0.73K_i$. Combining with the two equations above allows us to solve for m_C, the mass of the caboose.

ANALYZE With $K_f = 0.73K_i$, or

$$\frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73)\left(\frac{1}{2} m_F v_F^2\right)$$

we obtain $m_F/(m_F + m_C) = 0.73,$ which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73} m_F = 0.37 m_F = (0.37)(3.18 \times 10^4 \text{ kg}) = 1.18 \times 10^4 \text{ kg}.$$

LEARN Energy is lost during an inelastic collision, but momentum is still conserved because there's no external force acting on the two-car system.

94. Let m_c be the mass of the Chrysler and v_c be its velocity. Let m_f be the mass of the Ford and v_f be its velocity. Then the velocity of the center of mass is

$$v_{\text{com}} = \frac{m_c v_c + m_f v_f}{m_c + m_f} = \frac{(2400 \text{ kg})(80 \text{ km/h}) + (1600 \text{ kg})(60 \text{ km/h})}{2400 \text{ kg} + 1600 \text{ kg}} = 72 \text{ km/h}.$$

We note that the two velocities are in the same direction, so the two terms in the numerator have the same sign.

95. **THINK** A billiard ball undergoes glancing collision with another identical billiard ball. The collision is two-dimensional.

EXPRESS The mass of each ball is *m*, and the initial speed of one of the balls is $v_{1i} = 2.2\,\text{m/s}$. We apply the conservation of linear momentum to the *x* and *y* axes respectively:

$$mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$
$$0 = mv_{1f}\sin\theta_1 - mv_{2f}\sin\theta_2$$

The mass *m* cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

ANALYZE (a) Solving the simultaneous equations leads to

$$v_{1f} = \frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}v_{1i}, \quad v_{2f} = \frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}v_{1i}$$

Since $v_{2f} = v_{1i}/2 = 1.1\,\text{m/s}$ and $\theta_2 = 60°$, we have

$$\frac{\sin\theta_1}{\sin(\theta_1+60°)} = \frac{1}{2} \quad \Rightarrow \quad \tan\theta_1 = \frac{1}{\sqrt{3}}$$

or $\theta_1 = 30°$. Thus, the speed of ball 1 after collision is

$$v_{1f} = \frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}v_{1i} = \frac{\sin 60°}{\sin(30°+60°)}v_{1i} = \frac{\sqrt{3}}{2}v_{1i} = \frac{\sqrt{3}}{2}(2.2\,\text{m/s}) = 1.9\,\text{m/s}\,.$$

(b) From the above, we have θ_1 = 30°, measured *clockwise* from the +*x*-axis, or equivalently, –30°, measured *counterclockwise* from the +*x*-axis.

(c) The kinetic energy before collision is $K_i = \frac{1}{2}mv_{1i}^2$. After the collision, we have

$$K_f = \frac{1}{2}m\left(v_{1f}^2 + v_{2f}^2\right)$$

Substituting the expressions for v_{1f} and v_{2f} found above gives

$$K_f = \frac{1}{2}m\left[\frac{\sin^2\theta_2}{\sin^2(\theta_1+\theta_2)} + \frac{\sin^2\theta_1}{\sin^2(\theta_1+\theta_2)}\right]v_{1i}^2$$

Since $\theta_1 = 30°$ and $\theta_2 = 60°$, $\sin(\theta_1+\theta_2) = 1$ and $\sin^2\theta_1 + \sin^2\theta_2 = \sin^2\theta_1 + \cos^2\theta_1 = 1$, and indeed, we have $K_f = \frac{1}{2}mv_{1i}^2 = K_i$, which means that energy is conserved.

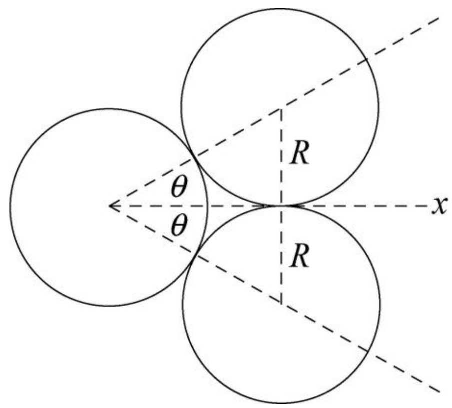
LEARN One may verify that when two identical masses collide elastically, they will move off perpendicularly to each other with $\theta_1 + \theta_2 = 90°$.

96. (a) We use Eq. 9-87. The thrust is

$$Rv_{\text{rel}} = Ma = \left(4.0\times10^4\ \text{kg}\right)\left(2.0\ \text{m/s}^2\right) = 8.0\times10^4\ \text{N}.$$

(b) Since v_{rel} = 3000 m/s, we see from part (a) that $R \approx 27$ kg/s.

97. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two.



It exerts an impulse of the same magnitude on each ball, along the line that joins the centers of the incident ball and the target ball. The target balls leave the collision along those lines, while the incident ball leaves the collision along the x axis. The three dashed lines that join the centers of the balls in contact form an equilateral triangle, so both of the angles marked θ are 30°. Let v_0 be the velocity of the incident ball before the collision and V be its velocity afterward. The two target balls leave the collision with the same speed. Let v represent that speed. Each ball has mass m. Since the x component of the total momentum of the three-ball system is conserved,

$$mv_0 = mV + 2mv\cos\theta$$

and since the total kinetic energy is conserved,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mV^2 + 2\left(\frac{1}{2}mv^2\right).$$

We know the directions in which the target balls leave the collision so we first eliminate V and solve for v. The momentum equation gives $V = v_0 - 2v\cos\theta$, so

$$V^2 = v_0^2 - 4v_0 v\cos\theta + 4v^2\cos^2\theta$$

and the energy equation becomes $v_0^2 = v_0^2 - 4v_0 v\cos\theta + 4v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2v_0\cos\theta}{1+2\cos^2\theta} = \frac{2(10\ \text{m/s})\cos 30°}{1+2\cos^2 30°} = 6.93\ \text{m/s}.$$

(a) The discussion and computation above determines the final speed of ball 2 (as labeled in Fig. 9-76) to be 6.9 m/s.

(b) The direction of ball 2 is at 30° counterclockwise from the $+x$ axis.

(c) Similarly, the final speed of ball 3 is 6.9 m/s.

(d) The direction of ball 3 is at −30° counterclockwise from the $+x$ axis.

(e) Now we use the momentum equation to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10\ \text{m/s} - 2(6.93\ \text{m/s})\ \cos 30° = -2.0\,\text{m/s}.$$

So the speed of ball 1 is $|V| = 2.0$ m/s.

(f) The minus sign indicates that it bounces back in the $-x$ direction. The angle is −180°.

98. (a) The momentum change for the 0.15 kg object is

$$\Delta\vec{p} = (0.15)[2\,\hat{i} + 3.5\,\hat{j} - 3.2\,\hat{k} - (5\,\hat{i} + 6.5\,\hat{j} + 4\,\hat{k})] = (-0.450\hat{i} - 0.450\hat{j} - 1.08\hat{k})\ \text{kg}\cdot\text{m/s}.$$

(b) By the impulse-momentum theorem (Eq. 9-31), $\vec{J} = \Delta\vec{p}$, we have

$$\vec{J} = (-0.450\hat{i} - 0.450\hat{j} - 1.08\hat{k})\ \text{N}\cdot\text{s}.$$

(c) Newton's third law implies $\vec{J}_{\text{wall}} = -\vec{J}_{\text{ball}}$ (where $\vec{J}_{\text{ball}}$ is the result of part (b)), so

$$\vec{J}_{\text{wall}} = (0.450\hat{i} + 0.450\hat{j} + 1.08\hat{k})\ \text{N}\cdot\text{s}.$$

99. (a) We place the origin of a coordinate system at the center of the pulley, with the x axis horizontal and to the right and with the y axis downward. The center of mass is halfway between the containers, at $x = 0$ and $y = \ell$, where ℓ is the vertical distance from the pulley center to either of the containers. Since the diameter of the pulley is 50 mm, the center of mass is at a horizontal distance of 25 mm from each container.

(b) Suppose 20 g is transferred from the container on the left to the container on the right. The container on the left has mass $m_1 = 480$ g and is at $x_1 = -25$ mm. The container on the right has mass m_2 = 520 g and is at x_2 = +25 mm. The x coordinate of the center of mass is then

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(480 \text{ g})(-25 \text{ mm}) + (520 \text{ g})(25 \text{ mm})}{480 \text{ g} + 520 \text{ g}} = 1.0 \text{ mm}.$$

The y coordinate is still ℓ. The center of mass is 26 mm from the lighter container, along the line that joins the bodies.

(c) When they are released the heavier container moves downward and the lighter container moves upward, so the center of mass, which must remain closer to the heavier container, moves downward.

(d) Because the containers are connected by the string, which runs over the pulley, their accelerations have the same magnitude but are in opposite directions. If a is the acceleration of m_2, then $-a$ is the acceleration of m_1. The acceleration of the center of mass is

$$a_{com} = \frac{m_1(-a) + m_2 a}{m_1 + m_2} = a\frac{m_2 - m_1}{m_1 + m_2}.$$

We must resort to Newton's second law to find the acceleration of each container. The force of gravity $m_1 g$, down, and the tension force of the string T, up, act on the lighter container. The second law for it is $m_1 g - T = -m_1 a$. The negative sign appears because a is the acceleration of the heavier container. The same forces act on the heavier container and for it the second law is $m_2 g - T = m_2 a$. The first equation gives $T = m_1 g + m_1 a$. This is substituted into the second equation to obtain $m_2 g - m_1 g - m_1 a = m_2 a$, so

$$a = (m_2 - m_1)g/(m_1 + m_2).$$

Thus,

$$a_{com} = \frac{g(m_2 - m_1)^2}{(m_1 + m_2)^2} = \frac{(9.8 \text{ m/s}^2)(520 \text{ g} - 480 \text{ g})^2}{(480 \text{ g} + 520 \text{ g})^2} = 1.6 \times 10^{-2} \text{ m/s}^2.$$

The acceleration is downward.

100. (a) We use Fig. 9-21 of the text (which treats both angles as positive-valued, even though one of them is in the fourth quadrant; this is why there is an explicit minus sign in Eq. 9-80 as opposed to it being implicitly in the angle). We take the cue ball to be body 1 and the other ball to be body 2. Conservation of the x and the components of the total momentum of the two-ball system leads to:

$$mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$

$$0 = -mv_{1f}\sin\theta_1 + mv_{2f}\sin\theta_2.$$

The masses are the same and cancel from the equations. We solve the second equation for $\sin\theta_2$:

$$\sin\theta_2 = \frac{v_{1f}}{v_{2f}}\sin\theta_1 = \left(\frac{3.50\text{ m/s}}{2.00\text{ m/s}}\right)\sin 22.0° = 0.656\ .$$

Consequently, the angle between the second ball and the initial direction of the first is θ_2 = 41.0°.

(b) We solve the first momentum conservation equation for the initial speed of the cue ball.

$$v_{1i} = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 = (3.50\text{ m/s})\cos 22.0° + (2.00\text{ m/s})\cos 41.0° = 4.75\text{ m/s}\ .$$

(c) With SI units understood, the initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}m(4.75)^2 = 11.3m$$

and the final kinetic energy is

$$K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{1}{2}m\left((3.50)^2 + (2.00)^2\right) = 8.1m.$$

Kinetic energy is not conserved.

101. This is a completely inelastic collision, followed by projectile motion. In the collision, we use momentum conservation.

$$\vec{p}_{\text{shoes}} = \vec{p}_{\text{together}} \quad \Rightarrow \quad (3.2\text{ kg})(3.0\text{ m/s}) = (5.2\text{ kg})\vec{v}$$

Therefore, $\vec{v} = 1.8\text{ m/s}$ toward the right as the combined system is projected from the edge of the table. Next, we can use the projectile motion material from Ch. 4 or the energy techniques of Ch. 8; we choose the latter.

$$K_{\text{edge}} + U_{\text{edge}} = K_{\text{floor}} + U_{\text{floor}}$$

$$\frac{1}{2}(5.2\text{ kg})(1.8\text{ m/s})^2 + (5.2\text{ kg})(9.8\text{ m/s}^2)(0.40\text{ m}) = K_{\text{floor}} + 0$$

Therefore, the kinetic energy of the system right before hitting the floor is K_{floor} = 29 J.

102. (a) Since the center of mass of the man-balloon system does not move, the balloon will move downward with a certain speed u relative to the ground as the man climbs up the ladder.

(b) The speed of the man relative to the ground is $v_g = v - u$. Thus, the speed of the center of mass of the system is

$$v_{\text{com}} = \frac{mv_g - Mu}{M+m} = \frac{m(v-u) - Mu}{M+m} = 0.$$

This yields

$$u = \frac{mv}{M+m} = \frac{(80\text{ kg})(2.5\text{ m/s})}{320\text{ kg} + 80\text{ kg}} = 0.50\text{ m/s}.$$

(c) Now that there is no relative motion within the system, the speed of both the balloon and the man is equal to v_{com}, which is zero. So the balloon will again be stationary.

103. The velocities of m_1 and m_2 just after the collision with each other are given by Eq. 9-75 and Eq. 9-76 (setting $v_{1i} = 0$):

$$v_{1f} = \frac{2m_2}{m_1+m_2}v_{2i}, \quad v_{2f} = \frac{m_2-m_1}{m_1+m_2}v_{2i}$$

After bouncing off the wall, the velocity of m_2 becomes $-v_{2f}$. In these terms, the problem requires $v_{1f} = -v_{2f}$, or

$$\frac{2m_2}{m_1+m_2}v_{2i} = -\frac{m_2-m_1}{m_1+m_2}v_{2i}$$

which simplifies to

$$2m_2 = -(m_2 - m_1) \Rightarrow m_2 = \frac{m_1}{3} \ .$$

With $m_1 = 6.6$ kg, we have $m_2 = 2.2$ kg.

104. We treat the car (of mass m_1) as a "point-mass" (which is initially 1.5 m from the right end of the boat). The left end of the boat (of mass m_2) is initially at $x = 0$ (where the dock is), and its left end is at $x = 14$ m. The boat's center of mass (in the absence of the car) is initially at $x = 7.0$ m. We use Eq. 9-5 to calculate the center of mass of the system:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1500\text{ kg})(14\text{ m} - 1.5\text{ m}) + (4000\text{ kg})(7\text{ m})}{1500\text{ kg} + 4000\text{ kg}} = 8.5\text{ m}.$$

In the absence of *external* forces, the center of mass of the system does not change. Later, when the car (about to make the jump) is near the left end of the boat (which has moved from the shore an amount δx), the value of the system center of mass is still 8.5 m. The car (at this moment) is thought of as a "point-mass" 1.5 m from the left end, so we must have

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1500\text{ kg})(\delta x + 1.5\text{ m}) + (4000\text{ kg})(7\text{ m} + \delta x)}{1500\text{ kg} + 4000\text{ kg}} = 8.5\text{ m}.$$

Solving this for δx, we find $\delta x = 3.0$ m.

105. **THINK** Both momentum and energy are conserved during an elastic collision.

EXPRESS Let m_1 be the mass of the object that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let $m_2 = M$ be the mass of the object that is originally at rest and v_{2f} be its velocity after the collision. Conservation of linear momentum gives $m_1v_{1i} = m_1v_{1f} + m_2v_{2f}$. Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2.$$

Solving for v_{1f} and v_{2f}, we obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}$$

The second equation can be inverted to give $m_2 = m_1\left(\frac{2v_{1i}}{v_{2f}} - 1\right)$.

ANALYZE With m_1 = 3.0 kg, v_{1i} = 8.0 m/s and v_{2f} = 6.0 m/s, the above expression leads to

$$m_2 = M = m_1\left(\frac{2v_{1i}}{v_{2f}} - 1\right) = (3.0\text{ kg})\left(\frac{2(8.0\text{ m/s})}{6.0\text{ m/s}} - 1\right) = 5.0\text{ kg}$$

LEARN Our analytic expression for m_2 shows that if the two masses are equal, then $v_{2f} = v_{1i}$, and the pool player's result is recovered.

106. We denote the mass of the car as M and that of the sumo wrestler as m. Let the initial velocity of the sumo wrestler be $v_0 > 0$ and the final velocity of the car be v. We apply the momentum conservation law.

(a) From $mv_0 = (M + m)v$ we get

$$v = \frac{mv_0}{M+m} = \frac{(242\text{ kg})(5.3\text{ m/s})}{2140\text{ kg} + 242\text{ kg}} = 0.54\text{ m/s}.$$

(b) Since $v_{\text{rel}} = v_0$, we have

$$mv_0 = Mv + m(v + v_{\text{rel}}) = mv_0 + (M + m)v,$$

and obtain $v = 0$ for the final speed of the flatcar.

(c) Now $mv_0 = Mv + m(v - v_{\text{rel}})$, which leads to

$$v = \frac{m(v_0 + v_{\text{rel}})}{m + M} = \frac{(242\text{ kg})(5.3\text{ m/s} + 5.3\text{ m/s})}{242\text{ kg} + 2140\text{ kg}} = 1.1\text{ m/s}.$$

107. **THINK** To successfully launch a rocket from the ground, fuel is consumed at a rate that results in a thrust big enough to overcome the gravitational force.

EXPRESS The thrust of the rocket is given by $T = Rv_{rel}$ where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket.

ANALYZE (a) The exhaust speed is v_{rel} = 1200 m/s. For the thrust to equal the weight Mg where M = 6100 kg, we must have

$$T = Rv_{rel} = Mg \quad \Rightarrow \quad R = \frac{Mg}{v_{rel}} = \frac{(6100\,\text{kg})(9.8\,\text{m/s}^2)}{1200\,\text{m/s}} = 49.8\,\text{kg/s} \approx 50\,\text{kg/s}\,.$$

(b) Using Eq. 9-42 with the additional effect due to gravity, we have

$$Rv_{rel} - Mg = Ma$$

so that requiring a = 21 m/s^2 leads to

$$R = \frac{M(g+a)}{v_{rel}} = \frac{(6100\,\text{kg})(9.8\,\text{m/s}^2 + 21\,\text{m/s}^2)}{1200\,\text{m/s}} = 156.6\,\text{kg/s} \approx 1.6\times10^2\,\text{kg/s}\,.$$

LEARN A greater upward acceleration requires a greater fuel consumption rate. To be launched from Earth's surface, the initial acceleration of the rocket must exceed $g = 9.8\,\text{m/s}^2$. This means that the rate R must be greater than 50 kg/s.

108. Conservation of momentum leads to

$$(900\text{ kg})(1000\text{ m/s}) = (500\text{ kg})(v_{shuttle} - 100\text{ m/s}) + (400\text{ kg})(v_{shuttle})$$

which yields $v_{shuttle}$ = 1055.6 m/s for the shuttle speed and $v_{shuttle}$ – 100 m/s = 955.6 m/s for the module speed (all measured in the frame of reference of the stationary main spaceship). The fractional increase in the kinetic energy is

$$\frac{\Delta K}{K_i} = \frac{K_f}{K_i} - 1 = \frac{(500\text{ kg})(955.6\text{ m/s})^2/2 + (400\text{ kg})(1055.6\text{ m/s})^2/2}{(900\text{ kg})(1000\text{ m/s})^2/2} = 2.5\times10^{-3}.$$

109. **THINK** In this problem, we are asked to locate the center of mass of the Earth-Moon system.

EXPRESS We locate the coordinate origin at the center of Earth. Then the distance r_{com} of the center of mass of the Earth-Moon system is given by

$$r_{com} = \frac{m_M r_{ME}}{m_M + m_E}$$

where m_M is the mass of the Moon, m_E is the mass of Earth, and r_{ME} is their separation.

ANALYZE (a) With $m_E = 5.98\times10^{24}$ kg , $m_M = 7.36\times10^{22}$ kg and $r_{ME} = 3.82\times10^{8}$ m (these values are given in Appendix C), we find the center of mass to be at

$$r_{\text{com}} = \frac{\left(7.36\times10^{22}\ \text{kg}\right)\left(3.82\times10^{8}\ \text{m}\right)}{7.36\times10^{22}\ \text{kg} + 5.98\times10^{24}\ \text{kg}} = 4.64\times10^{6}\ \text{m} \approx 4.6\times10^{3}\ \text{km}.$$

(b) The radius of Earth is $R_E = 6.37 \times 10^6$ m, so $r_{\text{com}} / R_E = 0.73 = 73\%$.

LEARN The center of mass of the Earth-Moon system is located inside the Earth!

110. (a) The magnitude of the impulse is equal to the change in momentum:

$$J = mv - m(-v) = 2mv = 2(0.140\ \text{kg})(7.80\ \text{m/s}) = 2.18\ \text{kg}\cdot\text{m/s}$$

(b) Since in the calculus sense the average of a function is the integral of it divided by the corresponding interval, then the average force is the impulse divided by the time Δt. Thus, our result for the magnitude of the average force is $2mv/\Delta t$. With the given values, we obtain

$$F_{\text{avg}} = \frac{2(0.140\ \text{kg})(7.80\ \text{m/s})}{0.00380\ \text{s}} = 575\ \text{N}.$$

111. **THINK** The water added to the sled will move at the same speed as the sled.

EXPRESS Let the mass of the sled be m_s and its initial speed be v_i. If the total mass of water being scooped up is m_w, then by momentum conservation, $m_s v_i = (m_s + m_w)v_f$, where v_f is the final speed of the sled-water system.

ANALYZE With $m_s = 2900$ kg, $m_w = 920$ kg and $v_i = 250$ m/s, we obtain

$$v_f = \frac{m_s v_i}{m_s + m_w} = \frac{\left(2900\ \text{kg}\right)\left(250\ \text{m/s}\right)}{2900\ \text{kg} + 920\ \text{kg}} = 189.8\ \text{m/s} \approx 190\ \text{m/s}.$$

LEARN The water added to the sled can be regarded as undergoing completely inelastic collision with the sled. Some kinetic energy is converted into other forms of energy (thermal, sound, etc.) and the final speed of the sled-water system is smaller than the initial speed of the sled alone.

112. **THINK** The pellets that were fired carry both kinetic energy and momentum. Force is exerted by the rigid wall in stopping the pellets.

EXPRESS Let m be the mass of a pellet and v be its velocity as it hits the wall, then its momentum is $p = mv$, toward the wall. The kinetic energy of a pellet is $K = mv^2/2$. The force on the wall is given by the rate at which momentum is transferred from the pellets to the wall. Since the pellets do not rebound, each pellet that hits transfers p. If ΔN pellets hit in time Δt, then the average rate at which momentum is transferred would be $F_{avg} = p(\Delta N/\Delta t)$.

ANALYZE (a) With $m = 2.0 \times 10^{-3}$ kg and $v = 500$ m/s, the momentum of a pellet is

$$p = mv = (2.0 \times 10^{-3}\text{ kg})(500\text{ m/s}) = 1.0\text{ kg}\cdot\text{m/s}.$$

(b) The kinetic energy of a pellet is $K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2.0\times10^{-3}\text{ kg}\right)\left(500\text{ m/s}\right)^2 = 2.5\times10^2\text{ J}$.

(c) With $(\Delta N/\Delta t) = 10/\text{s}$, the average force on the wall from the stream of pellets is

$$F_{avg} = p\left(\frac{\Delta N}{\Delta t}\right) = (1.0\text{ kg}\cdot\text{m/s})(10\text{ s}^{-1}) = 10\text{ N}.$$

The force on the wall is in the direction of the initial velocity of the pellets.

(d) If $\Delta t'$ is the time interval for a pellet to be brought to rest by the wall, then the average force exerted on the wall by a pellet is

$$F'_{avg} = \frac{p}{\Delta t'} = \frac{1.0\text{ kg}\cdot\text{m/s}}{0.6\times10^{-3}\text{ s}} = 1.7\times10^3\text{ N}.$$

The force is in the direction of the initial velocity of the pellet.

(e) In part (d) the force is averaged over the time a pellet is in contact with the wall, while in part (c) it is averaged over the time for many pellets to hit the wall. Hence, $F'_{avg} \neq F_{avg}$.

LEARN During the majority of this time, no pellet is in contact with the wall, so the average force in part (c) is much less than the average force in part (d).

113. We convert mass rate to SI units: R = (540 kg/min)/(60 s/min) = 9.00 kg/s. In the absence of the asked-for additional force, the car would decelerate with a magnitude given by Eq. 9-87: $R\,v_{rel} = M|a|$, so that if $a = 0$ is desired then the additional force must have a magnitude equal to $R\,v_{rel}$ (so as to cancel that effect):

$$F = Rv_{rel} = (9.00\text{ kg/s})(3.20\text{ m/s}) = 28.8\text{ N}.$$

114. First, we imagine that the small square piece (of mass m) that was cut from the large plate is returned to it so that the large plate is again a complete 6 m × 6 m (d =1.0 m) square plate (which has its center of mass at the origin). Then we "add" a square piece of

"negative mass" ($-m$) at the appropriate location to obtain what is shown in the figure. If the mass of the whole plate is M, then the mass of the small square piece cut from it is obtained from a simple ratio of areas:

$$m = \left(\frac{2.0\,\text{m}}{6.0\,\text{m}}\right)^2 M \Rightarrow M = 9m.$$

(a) The x coordinate of the small square piece is x = 2.0 m (the middle of that square "gap" in the figure). Thus the x coordinate of the center of mass of the remaining piece is

$$x_{\text{com}} = \frac{(-m)x}{M + (-m)} = \frac{-m(2.0\text{ m})}{9m - m} = -0.25\text{ m}.$$

(b) Since the y coordinate of the small square piece is zero, we have $y_{\text{com}} = 0$.

115. **THINK** We have two forces acting on two masses separately. The masses will move according to Newton's second law.

EXPRESS Let $\vec{F}_1$ be the force acting on m_1, and $\vec{F}_2$ the force acting on m_2. According to Newton's second law, their displacements are

$$\vec{d}_1 = \frac{1}{2}\vec{a}_1 t^2 = \frac{1}{2}\left(\frac{\vec{F}_1}{m_1}\right)t^2, \quad \vec{d}_2 = \frac{1}{2}\vec{a}_2 t^2 = \frac{1}{2}\left(\frac{\vec{F}_2}{m_2}\right)t^2$$

The corresponding displacement of the center of mass is

$$\vec{d}_{\text{cm}} = \frac{m_1\vec{d}_1 + m_2\vec{d}_2}{m_1 + m_2} = \frac{1}{2}\frac{m_1}{m_1+m_2}\left(\frac{\vec{F}_1}{m_1}\right)t^2 + \frac{1}{2}\frac{m_2}{m_1+m_2}\left(\frac{\vec{F}_2}{m_2}\right)t^2 = \frac{1}{2}\left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t^2.$$

ANALYZE (a) The two masses are $m_1 = 2.00\times10^{-3}$ kg and $m_2 = 4.00\times10^{-3}$ kg. With the forces given by $\vec{F}_1 = (-4.00\text{ N})\hat{\text{i}} + (5.00\text{ N})\hat{\text{j}}$ and $\vec{F}_2 = (2.00\text{ N})\hat{\text{i}} - (4.00\text{ N})\hat{\text{j}}$, and $t = 2.00\times10^{-3}$ s, we obtain

$$\begin{aligned}\vec{d}_{\text{cm}} &= \frac{1}{2}\left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t^2 = \frac{1}{2}\frac{(-4.00\text{ N}+2.00\text{ N})\hat{\text{i}} + (5.00\text{ N}-4.00\text{ N})\hat{\text{j}}}{2.00\times10^{-3}\text{ kg}+4.00\times10^{-3}\text{ kg}}(2.00\times10^{-3}\text{ s})^2 \\ &= (-6.67\times10^{-4}\text{ m})\hat{\text{i}} + (3.33\times10^{-4}\text{ m})\hat{\text{j}}.\end{aligned}$$

The magnitude of $\vec{d}_{\text{cm}}$ is

$$d_{\text{cm}} = \sqrt{(-6.67\times10^{-4}\text{ m})^2 + (3.33\times10^{-4}\text{ m})^2} = 7.45\times10^{-4}\text{ m}$$

or 0.745 mm.

(b) The angle of $\vec{d}_{\text{cm}}$ is given by

$$\theta = \tan^{-1}\left(\frac{3.33\times10^{-4}\text{ m}}{-6.67\times10^{-4}\text{ m}}\right) = \tan^{-1}\left(-\frac{1}{2}\right) = 153°,$$

measured counterclockwise from +x-axis.

(c) The velocities of the two masses are

$$\vec{v}_1 = \vec{a}_1 t = \frac{\vec{F}_1 t}{m_1}, \quad \vec{v}_2 = \vec{a}_2 t = \frac{\vec{F}_2 t}{m_2},$$

and the velocity of the center of mass is

$$\vec{v}_{\text{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{m_1}{m_1+m_2}\left(\frac{\vec{F}_1 t}{m_1}\right) + \frac{m_2}{m_1+m_2}\left(\frac{\vec{F}_2 t}{m_2}\right) = \left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t.$$

The corresponding kinetic energy of the center of mass is

$$K_{\text{cm}} = \frac{1}{2}(m_1+m_2)v_{\text{cm}}^2 = \frac{1}{2}\frac{|\vec{F}_1+\vec{F}_2|^2}{m_1+m_2}t^2$$

With $|\vec{F}_1+\vec{F}_2| = |(-2.00\text{ N})\hat{\text{i}} + (1.00\text{ N})\hat{\text{j}}| = \sqrt{5}\text{ N}$, we get

$$K_{\text{cm}} = \frac{1}{2}\frac{|\vec{F}_1+\vec{F}_2|^2}{m_1+m_2}t^2 = \frac{1}{2}\frac{(\sqrt{5}\text{ N})^2}{2.00\times10^{-3}\text{ kg}+4.00\times10^{-3}\text{ kg}}(2.00\times10^{-3}\text{ s})^2 = 1.67\times10^{-3}\text{ J}.$$

LEARN The motion of the center of the mass could be analyzed as though a force $\vec{F} = \vec{F}_1 + \vec{F}_2$ is acting on a mass $M = m_1 + m_2$. Thus, the acceleration of the center of the mass is $\vec{a}_{\text{cm}} = \dfrac{\vec{F}_1+\vec{F}_2}{m_1+m_2}$.

116. (a) The center of mass does not move in the absence of external forces (since it was initially at rest).

(b) They collide at their center of mass. If the initial coordinate of P is $x = 0$ and the initial coordinate of Q is $x = 1.0$ m, then Eq. 9-5 gives

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{0 + (0.30 \text{ kg})(1.0 \text{ m})}{0.1 \text{ kg} + 0.3 \text{ kg}} = 0.75 \text{ m}.$$

Thus, they collide at a point 0.75 m from *P*'s original position.

117. This is a completely inelastic collision, but Eq. 9-53 ($V = \frac{m_1}{m_1 + m_2} v_{1i}$) is not easily applied since that equation is designed for use when the struck particle is initially stationary. To deal with this case (where particle 2 is already in motion), we return to the principle of momentum conservation:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{V} \quad \Rightarrow \quad \vec{V} = \frac{2(4\hat{\text{i}} - 5\hat{\text{j}}) + 4(6\hat{\text{i}} - 2\hat{\text{j}})}{2+4}.$$

(a) In unit-vector notation, then, $\vec{V} = (2.67 \text{ m/s})\hat{\text{i}} + (-3.00 \text{ m/s})\hat{\text{j}}$.

(b) The magnitude of $\vec{V}$ is $|\vec{V}| = 4.01$ m/s.

(c) The direction of $\vec{V}$ is 48.4° (measured *clockwise* from the +*x* axis).

118. We refer to the discussion in the textbook (Sample Problem – "Elastic collision, two pendulums," which uses the same notation that we use here) for some important details in the reasoning. We choose rightward in Fig. 9-20 as our +*x* direction. We use the notation $\vec{v}$ when we refer to velocities and v when we refer to speeds (which are necessarily positive). Since the algebra is fairly involved, we find it convenient to introduce the notation $\Delta m = m_2 - m_1$ (which, we note for later reference, is a positive-valued quantity).

(a) Since $\vec{v}_{1i} = +\sqrt{2gh_1}$ where h_1 = 9.0 cm, we have

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = -\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}$$

which is to say that the *speed* of sphere 1 immediately after the collision is

$$v_{1f} = \left(\Delta m / \left(m_1 + m_2\right)\right)\sqrt{2gh_1}$$

and that $\vec{v}_{1f}$ points in the $-x$ direction. This leads (by energy conservation $m_1gh_{1f} = \frac{1}{2}m_1v_{1f}^2$) to

$$h_{1f} = \frac{v_{1f}^2}{2g} = \left(\frac{\Delta m}{m_1 + m_2}\right)^2 h_1 .$$

With m_1 = 50 g and m_2 = 85 g, this becomes $h_{1f} \approx 0.60$ cm.

(b) Equation 9-68 gives

$$v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i} = \frac{2m_1}{m_1+m_2}\sqrt{2gh_1}$$

which leads (by energy conservation $m_2 g h_{2f} = \frac{1}{2} m_2 v_{2f}^2$) to

$$h_{2f} = \frac{v_{2f}^2}{2g} = \left(\frac{2m_1}{m_1+m_2}\right)^2 h_1 \ .$$

With m_1 = 50 g and m_2 = 85 g, this becomes $h_{2f} \approx 4.9 \text{ cm}$.

(c) Fortunately, they hit again at the lowest point (as long as their amplitude of swing was "small," this is further discussed in Chapter 16). At the risk of using cumbersome notation, we refer to the *next* set of heights as h_{1ff} and h_{2ff}. At the lowest point (before this second collision) sphere 1 has velocity $+\sqrt{2gh_{1f}}$ (rightward in Fig. 9-20) and sphere 2 has velocity $-\sqrt{2gh_{1f}}$ (that is, it points in the $-x$ direction). Thus, the velocity of sphere 1 immediately after the second collision is, using Eq. 9-75,

$$\begin{aligned}\vec{v}_{1ff} &= \frac{m_1 - m_2}{m_1+m_2}\sqrt{2gh_{1f}} + \frac{2m_2}{m_1+m_2}\left(-\sqrt{2gh_{2f}}\right) \\ &= \frac{-\Delta m}{m_1+m_2}\left(\frac{\Delta m}{m_1+m_2}\sqrt{2gh_1}\right) - \frac{2m_2}{m_1+m_2}\left(\frac{2m_1}{m_1+m_2}\sqrt{2gh_1}\right) \\ &= -\frac{(\Delta m)^2 + 4m_1 m_2}{(m_1+m_2)^2}\sqrt{2gh_1} \ .\end{aligned}$$

This can be greatly simplified (by expanding $(\Delta m)^2$ and $(m_1 + m_2)^2$) to arrive at the conclusion that the speed of sphere 1 immediately after the second collision is simply $v_{1ff} = \sqrt{2gh_1}$ and that $\vec{v}_{1ff}$ points in the $-x$ direction. Energy conservation $\left(m_1 g h_{1ff} = \frac{1}{2} m_1 v_{1ff}^2\right)$ leads to

$$h_{1ff} = \frac{v_{1ff}^2}{2g} = h_1 = 9.0 \text{ cm} \ .$$

(d) One can reason (energy-wise) that $h_{1ff} = 0$ simply based on what we found in part (c). Still, it might be useful to see how this shakes out of the algebra. Equation 9-76 gives the velocity of sphere 2 immediately after the second collision:

$$
\begin{aligned}
v_{2ff} &= \frac{2m_1}{m_1+m_2}\sqrt{2gh_{1f}} + \frac{m_2-m_1}{m_1+m_2}\left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{2m_1}{m_1+m_2}\left(\frac{\Delta m}{m_1+m_2}\sqrt{2gh_1}\right) + \frac{\Delta m}{m_1+m_2}\left(\frac{-2m_1}{m_1+m_2}\sqrt{2gh_1}\right)
\end{aligned}
$$

which vanishes since $(2m_1)(\Delta m)-(\Delta m)(2m_1)=0$. Thus, the second sphere (after the second collision) stays at the lowest point, which basically recreates the conditions at the start of the problem (so all subsequent swings-and-impacts, neglecting friction, can be easily predicted, as they are just replays of the first two collisions).

119. (a) Each block is assumed to have uniform density, so that the center of mass of each block is at its geometric center (the positions of which are given in the table [see problem statement] at $t = 0$). Plugging these positions (and the block masses) into Eq. 9-29 readily gives $x_{com} = -0.50$ m (at $t = 0$).

(b) Note that the left edge of block 2 (the middle of which is still at $x = 0$) is at $x = -2.5$ cm, so that at the moment they touch the right edge of block 1 is at $x = -2.5$ cm and thus the middle of block 1 is at $x = -5.5$ cm. Putting these positions (for the middles) and the block masses into Eq. 9-29 leads to $x_{com} = -1.83$ cm or -0.018 m (at $t = (1.445 \text{ m})/(0.75 \text{ m/s}) = 1.93$ s).

(c) We could figure where the blocks are at $t = 4.0$ s and use Eq. 9-29 again, but it is easier (and provides more insight) to note that in the absence of *external* forces on the system the center of mass should move at constant velocity:

$$\vec{v}_{com} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = 0.25 \text{ m/s } \hat{\text{i}}$$

as can be easily verified by putting in the values at $t = 0$. Thus,

$$x_{com} = x_{com\ initial} + \vec{v}_{com}\, t = (-0.50 \text{ m}) + (0.25 \text{ m/s})(4.0 \text{ s}) = +0.50 \text{ m} .$$

120. One approach is to choose a *moving* coordinate system that travels the center of mass of the body, and another is to do a little extra algebra analyzing it in the original coordinate system (in which the speed of the $m = 8.0$ kg mass is $v_0 = 2$ m/s, as given). Our solution is in terms of the latter approach since we are assuming that this is the approach most students would take. Conservation of linear momentum (along the direction of motion) requires

$$mv_0 = m_1v_1 + m_2v_2 \quad \Rightarrow \quad (8.0)(2.0) = (4.0)v_1 + (4.0)v_2$$

which leads to $v_2 = 4 - v_1$ in SI units (m/s). We require

$$\Delta K = \left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2\right) - \frac{1}{2}mv_0^2 \quad \Rightarrow \quad 16 = \left(\frac{1}{2}(4.0)v_1^2 + \frac{1}{2}(4.0)v_2^2\right) - \frac{1}{2}(8.0)(2.0)^2$$

which simplifies to $v_2^2 = 16 - v_1^2$ in SI units. If we substitute for v_2 from above, we find

$$(4 - v_1)^2 = 16 - v_1^2$$

which simplifies to $2v_1^2 - 8v_1 = 0$, and yields either $v_1 = 0$ or $v_1 = 4$ m/s. If $v_1 = 0$ then $v_2 = 4 - v_1 = 4$ m/s, and if $v_1 = 4$ m/s then $v_2 = 0$.

(a) Since the forward part continues to move in the original direction of motion, the speed of the rear part must be zero.

(b) The forward part has a velocity of 4.0 m/s along the original direction of motion.

121. We use m_1 for the mass of the electron and $m_2 = 1840m_1$ for the mass of the hydrogen atom. Using Eq. 9-68,

$$v_{2f} = \frac{2m_1}{m_1 + 1840m_1} v_{1i} = \frac{2}{1841} v_{1i}$$

we compute the final kinetic energy of the hydrogen atom:

$$K_{2f} = \frac{1}{2}(1840m_1)\left(\frac{2v_{1i}}{1841}\right)^2 = \frac{(1840)(4)}{1841^2}\left(\frac{1}{2}(1840m_1)v_{1i}^2\right)$$

so we find the fraction to be $(1840)(4)/1841^2 \approx 2.2 \times 10^{-3}$, or 0.22%.

122. Denoting the new speed of the car as v, then the new speed of the man relative to the ground is $v - v_{rel}$. Conservation of momentum requires

$$\left(\frac{W}{g} + \frac{w}{g}\right) v_0 = \left(\frac{W}{g}\right) v + \left(\frac{w}{g}\right)(v - v_{rel}).$$

Consequently, the change of velocity is

$$\Delta\vec{v} = v - v_0 = \frac{w\, v_{rel}}{W + w} = \frac{(915 \text{ N})(4.00 \text{ m/s})}{(2415 \text{ N}) + (915 \text{ N})} = 1.10 \text{ m/s}.$$

123. Conservation of linear momentum gives $mv + MV_J = mv_f + MV_{Jf}$. Similarly, the total kinetic energy is conserved:

$$\frac{1}{2}mv^2 + \frac{1}{2}MV_J^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}MV_{Jf}^2 .$$

Solving for v_f and V_{Jf}, we obtain:

$$v_{1f} = \frac{m-M}{m+M}v + \frac{2M}{m+M}V_J, \quad V_{Jf} = \frac{2m}{m+M}v + \frac{M-m}{m+M}V_J$$

Since $m \ll M$, the above expressions can be simplified to

$$v_{1f} \approx -v + 2V_J, \quad V_{Jf} \approx V_J$$

The velocity of the probe relative to the Sun is

$$v_{1f} \approx -v + 2V_J = -(10.5\text{ km/s}) + 2(-13.0\text{ km/s}) = -36.5\text{ km/s}.$$

The speed is $|v_{1f}| = 36.5\text{ km/s}$.

124. (a) The change in momentum (taking upwards to be the positive direction) is

$$\Delta\vec{p} = (0.550\text{ kg})[(3\text{ m/s})\hat{\text{j}} - (-12\text{ m/s})\hat{\text{j}}] = (+8.25\text{ kg}\cdot\text{m/s})\,\hat{\text{j}}.$$

(b) By the impulse-momentum theorem (Eq. 9-31) $\vec{J} = \Delta\vec{p} = (+8.25\text{ N}\cdot\text{s})\,\hat{\text{j}}$.

(c) By Newton's third law, $\vec{J}_\text{c} = -\vec{J}_\text{b} = (-8.25\text{ N}\cdot\text{s})\,\hat{\text{j}}$.

125. (a) Since the initial momentum is zero, then the final momenta must add (in the vector sense) to 0. Therefore, with SI units understood, we have

$$\begin{aligned}\vec{p}_3 &= -\vec{p}_1 - \vec{p}_2 = -m_1\vec{v}_1 - m_2\vec{v}_2 \\ &= -\left(16.7\times10^{-27}\right)\left(6.00\times10^6\,\hat{\text{i}}\right) - \left(8.35\times10^{-27}\right)\left(-8.00\times10^6\,\hat{\text{j}}\right) \\ &= \left(-1.00\times10^{-19}\,\hat{\text{i}} + 0.67\times10^{-19}\,\hat{\text{j}}\right)\text{kg}\cdot\text{m/s}.\end{aligned}$$

(b) Dividing by $m_3 = 11.7 \times 10^{-27}$ kg and using the Pythagorean theorem we find the speed of the third particle to be $v_3 = 1.03 \times 10^7$ m/s. The total amount of kinetic energy is

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 = 1.19\times10^{-12}\text{ J}.$$

126. Using Eq. 9-67, we have after the elastic collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{-200 \text{ g}}{600 \text{ g}} v_{1i} = -\tfrac{1}{3}(3.00 \text{ m/s}) = -1.00 \text{ m/s} .$$

(a) The impulse is therefore

$$J = m_1 v_{1f} - m_1 v_{1i} = (0.200 \text{ kg})(-1.00 \text{ m/s}) - (0.200 \text{ kg})(3.00 \text{ m/s}) = -0.800 \text{ N·s}$$
$$= -0.800 \text{ kg·m/s},$$

or $|J| = -0.800$ kg·m/s.

(b) For the completely inelastic collision Eq. 9-75 applies

$$v_{1f} = V = \frac{m_1}{m_1 + m_2} v_{1i} = +1.00 \text{ m/s} .$$

Now the impulse is

$$J = m_1 v_{1f} - m_1 v_{1i} = (0.200 \text{ kg})(1.00 \text{ m/s}) - (0.200 \text{ kg})(3.00 \text{ m/s}) = 0.400 \text{ N·s}$$
$$= 0.400 \text{ kg·m/s}.$$

127. We use Eq. 9-88 and simplify with $v_f - v_i = \Delta v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) \quad \Rightarrow \quad \frac{M_f}{M_i} = e^{-\Delta v/u}$$

If $\Delta v = 2.2$ m/s and $u = 1000$ m/s, we obtain $\dfrac{M_i - M_f}{M_i} = 1 - e^{-0.0022} \approx 0.0022.$

128. Using the linear momentum-impulse theorem, we have

$$J = F_{\text{avg}} \Delta t = \Delta p = m(v_f - v_i) .$$

where m is the mass, v_i the initial velocity, and v_f the final velocity of the ball. With $v_i = 0$, we obtain

$$v_f = \frac{F_{\text{avg}} \Delta t}{m} = \frac{(32 \text{ N})(14 \times 10^{-3} \text{ s})}{0.20 \text{ kg}} = 2.24 \text{ m/s}.$$

Chapter 10

1. The problem asks us to assume v_{com} and ω are constant. For consistency of units, we write

$$v_{com} = (85\,\text{mi/h})\left(\frac{5280\,\text{ft/mi}}{60\,\text{min/h}}\right) = 7480\,\text{ft/min}\ .$$

Thus, with $\Delta x = 60\,\text{ft}$, the time of flight is

$$t = \Delta x/v_{com} = (60\ \text{ft})/(7480\ \text{ft/min}) = 0.00802\,\text{min}\ .$$

During that time, the angular displacement of a point on the ball's surface is

$$\theta = \omega t = (1800\,\text{rev/min})(0.00802\,\text{min}) \approx 14\,\text{rev}\ .$$

2. (a) The second hand of the smoothly running watch turns through 2π radians during $60\ \text{s}$. Thus,

$$\omega = \frac{2\pi}{60} = 0.105\ \text{rad/s}.$$

(b) The minute hand of the smoothly running watch turns through 2π radians during $3600\ \text{s}$. Thus,

$$\omega = \frac{2\pi}{3600} = 1.75\times10^{-3}\ \text{rad / s}.$$

(c) The hour hand of the smoothly running 12-hour watch turns through 2π radians during 43200 s. Thus,

$$\omega = \frac{2\pi}{43200} = 1.45\times10^{-4}\ \text{rad / s}.$$

3. The falling is the type of constant-acceleration motion you had in Chapter 2. The time it takes for the buttered toast to hit the floor is

$$\Delta t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.76\ \text{m})}{9.8\ \text{m/s}^2}} = 0.394\ \text{s}.$$

(a) The smallest angle turned for the toast to land butter-side down is $\Delta\theta_{min} = 0.25\ \text{rev} = \pi/2\ \text{rad}.$ This corresponds to an angular speed of

$$\omega_{\min} = \frac{\Delta\theta_{\min}}{\Delta t} = \frac{\pi/2 \text{ rad}}{0.394 \text{ s}} = 4.0 \text{ rad/s}.$$

(b) The largest angle (less than 1 revolution) turned for the toast to land butter-side down is $\Delta\theta_{\max} = 0.75 \text{ rev} = 3\pi/2$ rad. This corresponds to an angular speed of

$$\omega_{\max} = \frac{\Delta\theta_{\max}}{\Delta t} = \frac{3\pi/2 \text{ rad}}{0.394 \text{ s}} = 12.0 \text{ rad/s}.$$

4. If we make the units explicit, the function is

$$\theta = 2.0 \text{ rad} + \left(4.0 \text{ rad/s}^2\right)t^2 + \left(2.0 \text{ rad/s}^3\right)t^3$$

but in some places we will proceed as indicated in the problem—by letting these units be understood.

(a) We evaluate the function θ at $t = 0$ to obtain $\theta_0 = 2.0$ rad.

(b) The angular velocity as a function of time is given by Eq. 10-6:

$$\omega = \frac{d\theta}{dt} = \left(8.0 \text{ rad/s}^2\right)t + \left(6.0 \text{ rad/s}^3\right)t^2$$

which we evaluate at $t = 0$ to obtain $\omega_0 = 0$.

(c) For $t = 4.0$ s, the function found in the previous part is

$$\omega_4 = (8.0)(4.0) + (6.0)(4.0)^2 = 128 \text{ rad/s}.$$

If we round this to two figures, we obtain $\omega_4 \approx 1.3 \times 10^2$ rad/s.

(d) The angular acceleration as a function of time is given by Eq. 10-8:

$$\alpha = \frac{d\omega}{dt} = 8.0 \text{ rad/s}^2 + \left(12 \text{ rad/s}^3\right)t$$

which yields $\alpha_2 = 8.0 + (12)(2.0) = 32 \text{ rad/s}^2$ at $t = 2.0$ s.

(e) The angular acceleration, given by the function obtained in the previous part, depends on time; it is not constant.

5. Applying Eq. 2-15 to the vertical axis (with $+y$ downward) we obtain the free-fall time:

$$\Delta y = v_{0y}t + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(10\text{ m})}{9.8\text{ m/s}^2}} = 1.4\text{ s}.$$

Thus, by Eq. 10-5, the magnitude of the average angular velocity is

$$\omega_{\text{avg}} = \frac{(2.5\text{ rev})(2\pi\text{ rad/rev})}{1.4\text{ s}} = 11\text{ rad/s}.$$

6. If we make the units explicit, the function is

$$\theta = (4.0\text{ rad/s})t - (3.0\text{ rad/s}^2)t^2 + (1.0\text{ rad/s}^3)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Equation 10-6 leads to

$$\omega = \frac{d}{dt}(4t - 3t^2 + t^3) = 4 - 6t + 3t^2.$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4-2} = 12\text{ rad/s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(4 - 6t + 3t^2) = -6 + 6t.$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s^2.

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

7. (a) To avoid touching the spokes, the arrow must go through the wheel in not more than

$$\Delta t = \frac{1/8\text{ rev}}{2.5\text{ rev/s}} = 0.050\text{ s}.$$

The minimum speed of the arrow is then $v_{\min} = \dfrac{20\text{ cm}}{0.050\text{ s}} = 400\text{ cm/s} = 4.0\text{ m/s}.$

(b) No—there is no dependence on radial position in the above computation.

8. (a) We integrate (with respect to time) the $\alpha = 6.0t^4 - 4.0t^2$ expression, taking into account that the initial angular velocity is 2.0 rad/s. The result is

$$\omega = 1.2\,t^5 - 1.33\,t^3 + 2.0.$$

(b) Integrating again (and keeping in mind that $\theta_o = 1$) we get

$$\theta = 0.20t^6 - 0.33\,t^4 + 2.0\,t + 1.0\,.$$

9. (a) With $\omega = 0$ and $\alpha = -4.2$ rad/s^2, Eq. 10-12 yields $t = -\omega_o/\alpha = 3.00$ s.

(b) Eq. 10-4 gives $\theta - \theta_o = -\,\omega_o^2 / 2\alpha = 18.9$ rad.

10. We assume the sense of rotation is positive, which (since it starts from rest) means all quantities (angular displacements, accelerations, etc.) are positive-valued.

(a) The angular acceleration satisfies Eq. 10-13:

$$25\text{ rad} = \frac{1}{2}\alpha(5.0\text{ s})^2 \;\;\Rightarrow\;\; \alpha = 2.0\text{ rad/s}^2.$$

(b) The average angular velocity is given by Eq. 10-5:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{25\text{ rad}}{5.0\text{ s}} = 5.0\text{ rad/s}.$$

(c) Using Eq. 10-12, the instantaneous angular velocity at $t = 5.0$ s is

$$\omega = \left(2.0\text{ rad/s}^2\right)(5.0\text{ s}) = 10\text{ rad/s}\,.$$

(d) According to Eq. 10-13, the angular displacement at $t = 10$ s is

$$\theta = \omega_0 + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(2.0\text{ rad/s}^2)(10\text{ s})^2 = 100\text{ rad}.$$

Thus, the displacement between $t = 5$ s and $t = 10$ s is $\Delta\theta = 100$ rad – 25 rad = 75 rad.

11. We assume the sense of initial rotation is positive. Then, with ω_0 = +120 rad/s and ω = 0 (since it stops at time t), our angular acceleration (''deceleration'') will be negative-valued: $\alpha = -4.0$ rad/s^2.

(a) We apply Eq. 10-12 to obtain t.

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad t = \frac{0 - 120 \text{ rad/s}}{-4.0 \text{ rad/s}^2} = 30 \text{ s}.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(120 \text{ rad/s} + 0)(30 \text{ s}) = 1.8 \times 10^3 \text{ rad}.$$

Alternatively, Eq. 10-14 could be used if it is desired to only use the given information (as opposed to using the result from part (a)) in obtaining θ. If using the result of part (a) is acceptable, then any angular equation in Table 10-1 (except Eq. 10-12) can be used to find θ.

12. (a) We assume the sense of rotation is positive. Applying Eq. 10-12, we obtain

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{(3000 - 1200) \text{ rev/min}}{(12/60) \text{ min}} = 9.0 \times 10^3 \text{ rev/min}^2.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(1200 \text{ rev/min} + 3000 \text{ rev/min})\left(\frac{12}{60} \text{ min}\right) = 4.2 \times 10^2 \text{ rev}.$$

13. The wheel has angular velocity ω_0 = +1.5 rad/s = +0.239 rev/s at t = 0, and has constant value of angular acceleration $\alpha < 0$, which indicates our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at t = 0) is θ_1 = +20 rev, and at t_2 its angular displacement is θ_2 = +40 rev and its angular velocity is $\omega_2 = 0$.

(a) We obtain t_2 using Eq. 10-15:

$$\theta_2 = \frac{1}{2}(\omega_0 + \omega_2)t_2 \quad \Rightarrow \quad t_2 = \frac{2(40 \text{ rev})}{0.239 \text{ rev/s}} = 335 \text{ s}$$

which we round off to $t_2 \approx 3.4 \times 10^2$ s.

(b) Any equation in Table 10-1 involving α can be used to find the angular acceleration; we select Eq. 10-16.

$$\theta_2 = \omega_2 t_2 - \frac{1}{2}\alpha t_2^2 \quad \Rightarrow \quad \alpha = -\frac{2(40 \text{ rev})}{(335 \text{ s})^2} = -7.12\times10^{-4} \text{ rev/s}^2$$

which we convert to $\alpha = -4.5 \times 10^{-3}$ rad/s^2.

(c) Using $\theta_1 = \omega_0 t_1 + \frac{1}{2}\alpha t_1^2$ (Eq. 10-13) and the quadratic formula, we have

$$t_1 = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\theta_1\alpha}}{\alpha} = \frac{-(0.239 \text{ rev/s}) \pm \sqrt{(0.239 \text{ rev/s})^2 + 2(20 \text{ rev})(-7.12\times10^{-4} \text{ rev/s}^2)}}{-7.12\times10^{-4} \text{ rev/s}^2}$$

which yields two positive roots: 98 s and 572 s. Since the question makes sense only if $t_1 < t_2$ we conclude the correct result is $t_1 = 98$ s.

14. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha > 0$, which makes our choice for positive sense of rotation. At t_1 its angular velocity is $\omega_1 = +10$ rev/s, and at t_2 its angular velocity is $\omega_2 = +15$ rev/s. Between t_1 and t_2 it turns through $\Delta\theta = 60$ rev, where $t_2 - t_1 = \Delta t$.

(a) We find α using Eq. 10-14:

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta \quad \Rightarrow \quad \alpha = \frac{(15 \text{ rev/s})^2 - (10 \text{ rev/s})^2}{2(60 \text{ rev})} = 1.04 \text{ rev/s}^2$$

which we round off to 1.0 rev/s^2.

(b) We find Δt using Eq. 10-15: $\Delta\theta = \frac{1}{2}(\omega_1 + \omega_2)\Delta t \Rightarrow \Delta t = \frac{2(60 \text{ rev})}{10 \text{ rev/s} + 15 \text{ rev/s}} = 4.8$ s.

(c) We obtain t_1 using Eq. 10-12: $\omega_1 = \omega_0 + \alpha t_1 \Rightarrow t_1 = \frac{10 \text{ rev/s}}{1.04 \text{ rev/s}^2} = 9.6$ s.

(d) Any equation in Table 10-1 involving θ can be used to find θ_1 (the angular displacement during $0 \le t \le t_1$); we select Eq. 10-14.

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta_1 \Rightarrow \theta_1 = \frac{(10 \text{ rev/s})^2}{2(1.04 \text{ rev/s}^2)} = 48 \text{ rev}.$$

15. **THINK** We have a wheel rotating with constant angular acceleration. We can apply the equations given in Table 10-1 to analyze the motion.

EXPRESS Since the wheel starts from rest, its angular displacement as a function of time is given by $\theta = \frac{1}{2}\alpha t^2$. We take t_1 to be the start time of the interval so that $t_2 = t_1 + 4.0$ s. The corresponding angular displacements at these times are

$$\theta_1 = \frac{1}{2}\alpha t_1^2, \quad \theta_2 = \frac{1}{2}\alpha t_2^2$$

Given $\Delta\theta = \theta_2 - \theta_1$, we can solve for t_1, which tells us how long the wheel has been in motion up to the beginning of the 4.0 s-interval.

ANALYZE The above expressions can be combined to give

$$\Delta\theta = \theta_2 - \theta_1 = \frac{1}{2}\alpha\left(t_2^2 - t_1^2\right) = \frac{1}{2}\alpha(t_2 + t_1)(t_2 - t_1)$$

With $\Delta\theta = 120\,\text{rad}$, $\alpha = 3.0\,\text{rad/s}^2$, and $t_2 - t_1 = 4.0\,\text{s}$, we obtain

$$t_2 + t_1 = \frac{2(\Delta\theta)}{\alpha(t_2 - t_1)} = \frac{2(120\,\text{rad})}{(3.0\,\text{rad/s}^2)(4.0\,\text{s})} = 20\,\text{s},$$

which can be further solved to give $t_2 = 12.0\,\text{s}$ and $t_1 = 8.0\,\text{s}$. So, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

LEARN We can readily verify the results by calculating θ_1 and θ_2 explicitly:

$$\theta_1 = \frac{1}{2}\alpha t_1^2 = \frac{1}{2}(3.0\,\text{rad/s}^2)(8.0\,\text{s})^2 = 96\,\text{rad}$$
$$\theta_2 = \frac{1}{2}\alpha t_2^2 = \frac{1}{2}(3.0\,\text{rad/s}^2)(12.0\,\text{s})^2 = 216\,\text{rad}.$$

Indeed the difference is $\Delta\theta = \theta_2 - \theta_1 = 120\,\text{rad}$.

16. (a) Eq. 10-13 gives

$$\theta - \theta_o = \omega_o t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}(1.5\text{ rad/s}^2)\,t_1^2$$

where $\theta - \theta_o$ = (2 rev)(2π rad/rev). Therefore, t_1 = 4.09 s.

(b) We can find the time to go through a full 4 rev (using the same equation to solve for a new time t_2) and then subtract the result of part (a) for t_1 in order to find this answer.

$$(4\text{ rev})(2\pi\text{ rad/rev}) = 0 + \tfrac{1}{2}(1.5\text{ rad/s}^2)\,t_2^2 \quad\Rightarrow\quad t_2 = 5.789\text{ s}.$$

Thus, the answer is 5.789 s – 4.093 s ≈ 1.70 s.

17. The problem has (implicitly) specified the positive sense of rotation. The angular acceleration of magnitude 0.25 rad/s^2 in the negative direction is assumed to be constant over a large time interval, including negative values (for t).

(a) We specify θ_{max} with the condition $\omega = 0$ (this is when the wheel reverses from positive rotation to rotation in the negative direction). We obtain θ_{max} using Eq. 10-14:

$$\theta_{max} = -\frac{\omega_o^2}{2\alpha} = -\frac{(4.7 \text{ rad/s})^2}{2(-0.25 \text{ rad/s}^2)} = 44 \text{ rad}.$$

(b) We find values for t_1 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_1 = 22$ rad (or 22.09 rad if we wish to keep track of accurate values in all intermediate steps and only round off on the final answers). Using Eq. 10-13 and the quadratic formula, we have

$$\theta_1 = \omega_o t_1 + \frac{1}{2}\alpha t_1^2 \Rightarrow t_1 = \frac{-\omega_o \pm \sqrt{\omega_o^2 + 2\theta_1\alpha}}{\alpha}$$

which yields the two roots 5.5 s and 32 s. Thus, the first time the reference line will be at $\theta_1 = 22$ rad is $t = 5.5$ s.

(c) The second time the reference line will be at $\theta_1 = 22$ rad is $t = 32$ s.

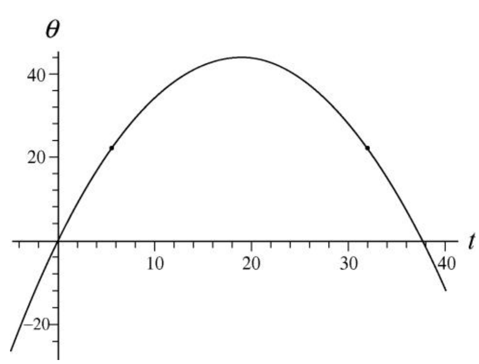
(d) We find values for t_2 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_2 = -10.5$ rad. Using Eq. 10-13 and the quadratic formula, we have

$$\theta_2 = \omega_o t_2 + \frac{1}{2}\alpha t_2^2 \Rightarrow t_2 = \frac{-\omega_o \pm \sqrt{\omega_o^2 + 2\theta_2\alpha}}{\alpha}$$

which yields the two roots –2.1 s and 40 s. Thus, at $t = -2.1$ s the reference line will be at $\theta_2 = -10.5$ rad.

(e) At $t = 40$ s the reference line will be at $\theta_2 = -10.5$ rad.

(f) With radians and seconds understood, the graph of θ versus t is shown below (with the points found in the previous parts indicated as small dots).



18. (a) A complete revolution is an angular displacement of $\Delta\theta = 2\pi$ rad, so the angular velocity in rad/s is given by $\omega = \Delta\theta/T = 2\pi/T$. The angular acceleration is given by

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt}.$$

For the pulsar described in the problem, we have

$$\frac{dT}{dt} = \frac{1.26\times10^{-5}\ \text{s/y}}{3.16\times10^{7}\ \text{s/y}} = 4.00\times10^{-13}.$$

Therefore,

$$\alpha = -\left(\frac{2\pi}{(0.033\ \text{s})^2}\right)(4.00\times10^{-13}) = -2.3\times10^{-9}\ \text{rad/s}^2.$$

The negative sign indicates that the angular acceleration is opposite the angular velocity and the pulsar is slowing down.

(b) We solve $\omega = \omega_0 + \alpha t$ for the time t when $\omega = 0$:

$$t = -\frac{\omega_0}{\alpha} = -\frac{2\pi}{\alpha T} = -\frac{2\pi}{(-2.3\times10^{-9}\ \text{rad/s}^2)(0.033\ \text{s})} = 8.3\times10^{10}\ \text{s} \approx 2.6\times10^{3}\ \text{years}$$

(c) The pulsar was born 1992–1054 = 938 years ago. This is equivalent to (938 y)(3.16 × 10^7 s/y) = 2.96 × 10^{10} s. Its angular velocity at that time was

$$\omega = \omega_0 + \alpha t + \frac{2\pi}{T} + \alpha t = \frac{2\pi}{0.033\ \text{s}} + (-2.3\times10^{-9}\ \text{rad/s}^2)(-2.96\times10^{10}\ \text{s}) = 258\ \text{rad/s}.$$

Its period was

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{258\ \text{rad/s}} = 2.4\times10^{-2}\ \text{s}.$$

19. (a) Converting from hours to seconds, we find the angular velocity (assuming it is positive) from Eq. 10-18:

$$\omega = \frac{v}{r} = \frac{\left(2.90\times10^{4}\ \text{km/h}\right)\left(1.000\ \text{h}/3600\ \text{s}\right)}{3.22\times10^{3}\ \text{km}} = 2.50\times10^{-3}\ \text{rad/s}.$$

(b) The radial (or centripetal) acceleration is computed according to Eq. 10-23:

$$a_r = \omega^2 r = \left(2.50\times10^{-3}\ \text{rad/s}\right)^2\left(3.22\times10^{6}\ \text{m}\right) = 20.2\ \text{m/s}^2.$$

(c) Assuming the angular velocity is constant, then the angular acceleration and the tangential acceleration vanish, since

$$\alpha = \frac{d\omega}{dt} = 0 \text{ and } a_t = r\alpha = 0.$$

20. The function $\theta = \xi e^{\beta t}$ where ξ = 0.40 rad and β = 2 s^{-1} is describing the angular coordinate of a line (which is marked in such a way that all points on it have the same value of angle at a given time) on the object. Taking derivatives with respect to time leads to $\frac{d\theta}{dt} = \xi\beta e^{\beta t}$ and $\frac{d^2\theta}{dt^2} = \xi\beta^2 e^{\beta t}$.

(a) Using Eq. 10-22, we have $a_t = \alpha r = \frac{d^2\theta}{dt^2} r = 6.4 \text{ cm/s}^2$.

(b) Using Eq. 10-23, we get $a_r = \omega^2 r = \left(\frac{d\theta}{dt}\right)^2 r = 2.6 \text{ cm/s}^2$.

21. We assume the given rate of 1.2 × 10^{-3} m/y is the linear speed of the top; it is also possible to interpret it as just the horizontal component of the linear speed but the difference between these interpretations is arguably negligible. Thus, Eq. 10-18 leads to

$$\omega = \frac{1.2\times10^{-3} \text{ m/y}}{55 \text{ m}} = 2.18\times10^{-5} \text{ rad/y}$$

which we convert (since there are about 3.16 × 10^7 s in a year) to ω = 6.9 × 10^{-13} rad/s.

22. (a) Using Eq. 10-6, the angular velocity at t = 5.0s is

$$\omega = \left.\frac{d\theta}{dt}\right|_{t=5.0} = \left.\frac{d}{dt}\left(0.30t^2\right)\right|_{t=5.0} = 2(0.30)(5.0) = 3.0 \text{ rad/s}.$$

(b) Equation 10-18 gives the linear speed at t = 5.0s: $v = \omega r = (3.0 \text{ rad/s})(10 \text{ m}) = 30 \text{ m/s}$.

(c) The angular acceleration is, from Eq. 10-8,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.60t) = 0.60 \text{ rad/s}^2.$$

Then, the tangential acceleration at t = 5.0s is, using Eq. 10-22,

$$a_t = r\alpha = (10 \text{ m})\left(0.60 \text{ rad/s}^2\right) = 6.0 \text{ m/s}^2.$$

(d) The radial (centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = (3.0\text{ rad/s})^2(10\text{ m}) = 90\text{ m/s}^2.$$

23. **THINK** A positive angular acceleration is required in order to increase the angular speed of the flywheel.

EXPRESS The linear speed of the flywheel is related to its angular speed by $v = \omega r$, where r is the radius of the wheel. As the wheel is accelerated, its angular speed at a later time is $\omega = \omega_0 + \alpha t$.

ANALYZE (a) The angular speed of the wheel, expressed in rad/s, is

$$\omega_0 = \frac{(200\text{ rev/min})(2\pi\text{ rad/rev})}{60\text{ s/min}} = 20.9\text{ rad/s}.$$

(b) With r = (1.20 m)/2 = 0.60 m, using Eq. 10-18, we find the linear speed to be

$$v = r\omega_0 = (0.60\text{ m})(20.9\text{ rad/s}) = 12.5\text{ m/s}.$$

(c) With t = 1 min, ω = 1000 rev/min and ω_0 = 200 rev/min, Eq. 10-12 gives the required acceleration:

$$\alpha = \frac{\omega - \omega_0}{t} = 800\text{ rev/min}^2.$$

(d) With the same values used in part (c), Eq. 10-15 becomes

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(200\text{ rev/min} + 1000\text{ rev/min})(1.0\text{ min}) = 600\text{ rev}.$$

LEARN An alternative way to solve for (d) is to use Eq. 10-13:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + (200\text{ rev/min})(1.0\text{ min}) + \frac{1}{2}(800\text{ rev/min}^2)(1.0\text{ min})^2 = 600\text{ rev}.$$

24. Converting $33\frac{1}{3}$ rev/min to radians-per-second, we get ω = 3.49 rad/s. Combining $v = \omega r$ (Eq. 10-18) with $\Delta t = d/v$ where Δt is the time between bumps (a distance d apart), we arrive at the rate of striking bumps:

$$\frac{1}{\Delta t} = \frac{\omega r}{d} \approx 199/\text{s}.$$

25. **THINK** The linear speed of a point on Earth's surface depends on its distance from the Earth's axis of rotation.

EXPRESS To solve for the linear speed, we use $v = \omega r$, where r is the radius of its orbit. A point on Earth at a latitude of 40° moves along a circular path of radius $r = R \cos 40°$, where R is the radius of Earth (6.4×10^6 m). On the other hand, $r = R$ at the equator.

ANALYZE (a) Earth makes one rotation per day and 1 d is (24 h) (3600 s/h) = 8.64×10^4 s, so the angular speed of Earth is

$$\omega = \frac{2\pi \text{ rad}}{8.64\times10^4 \text{ s}} = 7.3\times10^{-5} \text{ rad/s}.$$

(b) At latitude of 40°, the linear speed is

$$v = \omega(R\cos 40°) = (7.3\times10^{-5} \text{ rad/s})(6.4\times10^6 \text{ m})\cos 40° = 3.5\times10^2 \text{ m/s}.$$

(c) At the equator (and all other points on Earth) the value of ω is the same (7.3×10^{-5} rad/s).

(d) The latitude at the equator is 0° and the speed is

$$v = \omega R = (7.3\times10^{-5} \text{ rad/s})(6.4\times10^6 \text{ m}) = 4.6\times10^2 \text{ m/s}.$$

LEARN The linear speed at the poles is zero since $r = R\cos 90° = 0$.

26. (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 150 \text{ rev/min}}{(2.2 \text{ h})(60 \text{ min/1h})} = -1.14 \text{ rev/min}^2.$$

(b) Using Eq. 10-13 with t = (2.2) (60) = 132 min, the number of revolutions is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (150 \text{ rev/min})(132 \text{ min}) + \frac{1}{2}\left(-1.14 \text{ rev/min}^2\right)\left(132 \text{ min}\right)^2 = 9.9\times10^3 \text{ rev}.$$

(c) With r = 500 mm, the tangential acceleration is

$$a_t = \alpha r = \left(-1.14 \text{ rev/min}^2\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2 (500 \text{ mm})$$

which yields $a_t = -0.99$ mm/s^2.

(d) The angular speed of the flywheel is

$$\omega = (75 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min/ } 60 \text{ s}) = 7.85 \text{ rad/s}.$$

With $r = 0.50$ m, the radial (or centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = (7.85 \text{ rad/s})^2 (0.50 \text{ m}) \approx 31 \text{ m/s}^2$$

which is much bigger than a_t. Consequently, the magnitude of the acceleration is

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} \approx a_r = 31 \text{ m/s}^2.$$

27. (a) The angular speed in rad/s is

$$\omega = \left(33\frac{1}{3} \text{ rev/min}\right)\left(\frac{2\pi \text{ rad/rev}}{60 \text{ s/min}}\right) = 3.49 \text{ rad/s}.$$

Consequently, the radial (centripetal) acceleration is (using Eq. 10-23)

$$a = \omega^2 r = \left(3.49 \text{ rad/s}\right)^2 (6.0 \times 10^{-2} \text{ m}) = 0.73 \text{ m/s}^2.$$

(b) Using Ch. 6 methods, we have $ma = f_s \le f_{s,\max} = \mu_s mg$, which is used to obtain the (minimum allowable) coefficient of friction:

$$\mu_{s,\min} = \frac{a}{g} = \frac{0.73}{9.8} = 0.075.$$

(c) The radial acceleration of the object is $a_r = \omega^2 r$, while the tangential acceleration is $a_t = \alpha r$. Thus,

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2} = r\sqrt{\omega^4 + \alpha^2}.$$

If the object is not to slip at any time, we require

$$f_{s,\max} = \mu_s mg = ma_{\max} = mr\sqrt{\omega_{\max}^4 + \alpha^2}.$$

Thus, since $\alpha = \omega/t$ (from Eq. 10-12), we find

$$\mu_{s,\min} = \frac{r\sqrt{\omega_{\max}^4 + \alpha^2}}{g} = \frac{r\sqrt{\omega_{\max}^4 + (\omega_{\max}/t)^2}}{g} = \frac{(0.060)\sqrt{3.49^4 + (3.4/0.25)^2}}{9.8} = 0.11.$$

28. Since the belt does not slip, a point on the rim of wheel C has the same tangential acceleration as a point on the rim of wheel A. This means that $\alpha_A r_A = \alpha_C r_C$, where α_A is the angular acceleration of wheel A and α_C is the angular acceleration of wheel C. Thus,

$$\alpha_C = \left(\frac{r_A}{r_C}\right)\alpha_C = \left(\frac{10\text{ cm}}{25\text{ cm}}\right)(1.6\text{ rad/s}^2) = 0.64\text{ rad/s}^2.$$

With the angular speed of wheel C given by $\omega_C = \alpha_C t$, the time for it to reach an angular speed of ω = 100 rev/min = 10.5 rad/s starting from rest is

$$t = \frac{\omega_C}{\alpha_C} = \frac{10.5\text{ rad/s}}{0.64\text{ rad/s}^2} = 16\text{ s}.$$

29. (a) In the time light takes to go from the wheel to the mirror and back again, the wheel turns through an angle of $\theta = 2\pi/500 = 1.26 \times 10^{-2}$ rad. That time is

$$t = \frac{2\ell}{c} = \frac{2(500\text{ m})}{2.998\times10^8\text{ m/s}} = 3.34\times10^{-6}\text{ s}$$

so the angular velocity of the wheel is

$$\omega = \frac{\theta}{t} = \frac{1.26\times10^{-2}\text{ rad}}{3.34\times10^{-6}\text{ s}} = 3.8\times10^3\text{ rad/s}.$$

(b) If r is the radius of the wheel, the linear speed of a point on its rim is

$$v = \omega r = \left(3.8\times10^3\text{ rad/s}\right)\left(0.050\text{ m}\right) = 1.9\times10^2\text{ m/s}.$$

30. (a) The tangential acceleration, using Eq. 10-22, is

$$a_t = \alpha r = \left(14.2\text{ rad/s}^2\right)(2.83\text{ cm}) = 40.2\text{ cm/s}^2.$$

(b) In rad/s, the angular velocity is ω = (2760)(2π/60) = 289 rad/s, so

$$a_r = \omega^2 r = (289\text{ rad/s})^2(0.0283\text{ m}) = 2.36\times10^3\text{ m/s}^2.$$

(c) The angular displacement is, using Eq. 10-14,

$$\theta = \frac{\omega^2}{2\alpha} = \frac{(289\text{ rad/s})^2}{2(14.2\text{ rad/s}^2)} = 2.94\times10^3\text{ rad}.$$

Then, using Eq. 10-1, the distance traveled is

$$s = r\theta = (0.0283\text{ m})(2.94\times10^3\text{ rad}) = 83.2\text{ m}.$$

31. (a) The upper limit for centripetal acceleration (same as the radial acceleration – see Eq. 10-23) places an upper limit of the rate of spin (the angular velocity ω) by considering a point at the rim (r = 0.25 m). Thus, $\omega_{max} = \sqrt{a/r}$ = 40 rad/s. Now we apply Eq. 10-15 to first half of the motion (where $\omega_o = 0$):

$$\theta - \theta_o = \tfrac{1}{2}(\omega_o + \omega)t \;\Rightarrow\; 400 \text{ rad} = \tfrac{1}{2}(0 + 40 \text{ rad/s})t$$

which leads to t = 20 s. The second half of the motion takes the same amount of time (the process is essentially the reverse of the first); the total time is therefore 40 s.

(b) Considering the first half of the motion again, Eq. 10-11 leads to

$$\omega = \omega_o + \alpha t \;\Rightarrow\; \alpha = \frac{40 \text{ rad/s}}{20 \text{ s}} = 2.0 \text{ rad/s}^2 .$$

32. (a) The linear speed at t = 15.0 s is

$$v = a_t t = \left(0.500\,\text{m/s}^2\right)\left(15.0\,\text{s}\right) = 7.50\,\text{m/s} .$$

The radial (centripetal) acceleration at that moment is

$$a_r = \frac{v^2}{r} = \frac{\left(7.50\,\text{m/s}\right)^2}{30.0\,\text{m}} = 1.875\,\text{m/s}^2 .$$

Thus, the net acceleration has magnitude:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(0.500\,\text{m/s}^2\right)^2 + \left(1.875\,\text{m/s}^2\right)^2} = 1.94\,\text{m/s}^2 .$$

(b) We note that $\vec{a}_t \parallel \vec{v}$. Therefore, the angle between $\vec{v}$ and $\vec{a}$ is

$$\tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{1.875}{0.5}\right) = 75.1°$$

so that the vector is pointing more toward the center of the track than in the direction of motion.

33. **THINK** We want to calculate the rotational inertia of a wheel, given its rotational energy and rotational speed.

EXPRESS The kinetic energy (in J) is given by $K = \tfrac{1}{2} I\omega^2$, where I is the rotational inertia (in $\text{kg}\cdot\text{m}^2$) and ω is the angular velocity (in rad/s).

ANALYZE Expressing the angular speed as

$$\omega = \frac{(602 \text{ rev/min})(2\pi \text{rad/rev})}{60 \text{ s/min}} = 63.0 \text{ rad/s},$$

we find the rotational inertia to be $I = \frac{2K}{\omega^2} = \frac{2(24400 \text{ J})}{(63.0 \text{ rad/s})^2} = 12.3 \text{ kg}\cdot\text{m}^2$.

LEARN Note the analogy between rotational kinetic energy $\frac{1}{2}I\omega^2$ and $\frac{1}{2}mv^2$, the kinetic energy associated with linear motion.

34. (a) Equation 10-12 implies that the angular acceleration α should be the slope of the ω vs t graph. Thus, α = 9/6 = 1.5 rad/s^2.

(b) By Eq. 10-34, K is proportional to ω^2. Since the angular velocity at t = 0 is –2 rad/s (and this value squared is 4) and the angular velocity at t = 4 s is 4 rad/s (and this value squared is 16), then the ratio of the corresponding kinetic energies must be

$$\frac{K_o}{K_4} = \frac{4}{16} \Rightarrow K_o = K_4/4 = 0.40 \text{ J}.$$

35. **THINK** The rotational inertia of a rigid body depends on how its mass is distributed.

EXPRESS Since the rotational inertia of a cylinder is $I = \frac{1}{2}MR^2$ (Table 10-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{4}MR^2\omega^2.$$

ANALYZE (a) For the smaller cylinder, we have

$$K_1 = \frac{1}{4}(1.25 \text{ kg})(0.25 \text{ m})^2(235 \text{ rad/s})^2 = 1.08\times10^3 \text{ J}.$$

(b) For the larger cylinder, we obtain

$$K_2 = \frac{1}{4}(1.25 \text{ kg})(0.75 \text{ m})^2(235 \text{ rad/s})^2 = 9.71\times10^3 \text{ J}.$$

LEARN The ratio of the rotational kinetic energies of the two cylinders having the same mass and angular speed is

$$\frac{K_2}{K_1} = \left(\frac{R_2}{R_1}\right)^2 = \left(\frac{0.75 \text{ m}}{0.25 \text{ m}}\right)^2 = (3)^2 = 9.$$

36. The parallel axis theorem (Eq. 10-36) shows that I increases with h. The phrase "out to the edge of the disk" (in the problem statement) implies that the maximum h in the graph is, in fact, the radius R of the disk. Thus, R = 0.20 m. Now we can examine, say, the h = 0 datum and use the formula for I_{com} (see Table 10-2(c)) for a solid disk, or (which might be a little better, since this is independent of whether it is really a solid disk) we can the difference between the h = 0 datum and the $h = h_{max} = R$ datum and relate that difference to the parallel axis theorem (thus the difference is $M(h_{max})^2 = 0.10\ \text{kg}\cdot\text{m}^2$). In either case, we arrive at M = 2.5 kg.

37. **THINK** We want to calculate the rotational inertia of a meter stick about an axis perpendicular to the stick but not through its center.

EXPRESS We use the parallel-axis theorem: $I = I_{com} + Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 10-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies h = 0.50 m – 0.20 m = 0.30 m.

ANALYZE With $M = 0.56\,\text{kg}$ and $L = 1.0\ \text{m},$ we have

$$I_{com} = \frac{1}{12}ML^2 = \frac{1}{12}(0.56\ \text{kg})(1.0\ \text{m})^2 = 4.67\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

Consequently, the parallel-axis theorem yields

$$I = 4.67\times10^{-2}\ \text{kg}\cdot\text{m}^2 + (0.56\ \text{kg})(0.30\ \text{m})^2 = 9.7\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

LEARN A greater moment of inertia $I > I_{com}$ means that it is more difficult to rotate the meter stick about this axis than the case where the axis passes through the center.

38. (a) Equation 10-33 gives

$$I_{total} = md^2 + m(2d)^2 + m(3d)^2 = 14\ md^2.$$

If the innermost one is removed then we would only obtain $m(2d)^2 + m(3d)^2 = 13\ md^2$. The percentage difference between these is (13 – 14)/14 = 0.0714 ≈ 7.1%.

(b) If, instead, the outermost particle is removed, we would have $md^2 + m(2d)^2 = 5\ md^2$. The percentage difference in this case is 0.643 ≈ 64%.

39. (a) Using Table 10-2(c) and Eq. 10-34, the rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}(500\,\text{kg})(200\pi\ \text{rad/s})^2(1.0\,\text{m})^2 = 4.9\times10^7\ \text{J}.$$

(b) We solve $P = K/t$ (where P is the average power) for the operating time t.

$$t = \frac{K}{P} = \frac{4.9\times10^7\,\text{J}}{8.0\times10^3\ \text{W}} = 6.2\times10^3\ \text{s}$$

which we rewrite as $t \approx 1.0 \times 10^2$ min.

40. (a) Consider three of the disks (starting with the one at point O): ⊕OO . The first one (the one at point O, shown here with the plus sign inside) has rotational inertial (see item (c) in Table 10-2) $I = \frac{1}{2}mR^2$. The next one (using the parallel-axis theorem) has

$$I = \tfrac{1}{2}mR^2 + mh^2$$

where $h = 2R$. The third one has $I = \frac{1}{2}mR^2 + m(4R)^2$. If we had considered five of the disks OO⊕OO with the one at O in the middle, then the total rotational inertia is

$$I = 5(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2).$$

The pattern is now clear and we can write down the total I for the collection of fifteen disks:

$$I = 15(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2 + m(6R)^2 + \ldots + m(14R)^2) = \tfrac{2255}{2}mR^2.$$

The generalization to N disks (where N is assumed to be an odd number) is

$$I = \tfrac{1}{6}(2N^2 + 1)NmR^2.$$

In terms of the total mass ($m = M/15$) and the total length ($R = L/30$), we obtain

$$I = 0.083519ML^2 \approx (0.08352)(0.1000\ \text{kg})(1.0000\ \text{m})^2 = 8.352\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

(b) Comparing to the formula (e) in Table 10-2 (which gives roughly $I = 0.08333\ ML^2$), we find our answer to part (a) is 0.22% lower.

41. The particles are treated "point-like" in the sense that Eq. 10-33 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 10-2(e) and the parallel-axis theorem (Eq. 10-36).

(a) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$I = I_1 + I_2 + I_3 + I_4 = \left(\frac{1}{12}Md^2 + M\left(\frac{1}{2}d\right)^2\right) + md^2 + \left(\frac{1}{12}Md^2 + M\left(\frac{3}{2}d\right)^2\right) + m(2d)^2$$
$$= \frac{8}{3}Md^2 + 5md^2 = \frac{8}{3}(1.2 \text{ kg})(0.056 \text{ m})^2 + 5(0.85 \text{ kg})(0.056 \text{ m})^2$$
$$= 0.023 \text{ kg}\cdot\text{m}^2.$$

(b) Using Eq. 10-34, we have

$$K = \frac{1}{2}I\omega^2 = \left(\frac{4}{3}M + \frac{5}{2}m\right)d^2\omega^2 = \left[\frac{4}{3}(1.2 \text{ kg}) + \frac{5}{2}(0.85 \text{ kg})\right](0.056 \text{ m})^2(0.30 \text{ rad/s})^2$$
$$= 1.1\times10^{-3} \text{ J}.$$

42. (a) We apply Eq. 10-33:

$$I_x = \sum_{i=1}^{4} m_i y_i^2 = \left[50(2.0)^2 + (25)(4.0)^2 + 25(-3.0)^2 + 30(4.0)^2\right]\text{g}\cdot\text{cm}^2 = 1.3\times10^3 \text{ g}\cdot\text{cm}^2.$$

(b) For rotation about the y axis we obtain

$$I_y = \sum_{i=1}^{4} m_i x_i^2 = 50(2.0)^2 + (25)(0)^2 + 25(3.0)^2 + 30(2.0)^2 = 5.5\times10^2 \text{ g}\cdot\text{cm}^2.$$

(c) And about the z axis, we find (using the fact that the distance from the z axis is $\sqrt{x^2+y^2}$)

$$I_z = \sum_{i=1}^{4} m_i\left(x_i^2 + y_i^2\right) = I_x + I_y = 1.3\times10^3 + 5.5\times10^2 = 1.9\times10^2 \text{ g}\cdot\text{cm}^2.$$

(d) Clearly, the answer to part (c) is $A + B$.

43. **THINK** Since the rotation axis does not pass through the center of the block, we use the parallel-axis theorem to calculate the rotational inertia.

EXPRESS According to Table 10-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by $I_{\text{com}} = \frac{M}{12}\left(a^2 + b^2\right)$. A parallel axis through the corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center. Therefore,

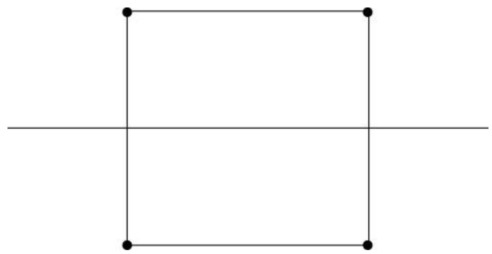
$$I = I_{\text{com}} + Mh^2 = \frac{M}{12}\left(a^2+b^2\right) + \frac{M}{4}\left(a^2+b^2\right) = \frac{M}{3}\left(a^2+b^2\right).$$

ANALYZE With $M = 0.172$ kg, $a = 3.5$ cm and $b = 8.4$ cm, we have

$$I = \frac{M}{3}\left(a^2 + b^2\right) = \frac{0.172 \text{ kg}}{3}[(0.035 \text{ m})^2 + (0.084 \text{ m})^2] = 4.7\times 10^{-4} \text{ kg}\cdot\text{m}^2.$$

LEARN A greater moment of inertia $I > I_{\text{com}}$ means that it is more difficult to rotate the block about the axis through the corner than the case where the axis passes through the center.

44. (a) We show the figure with its axis of rotation (the thin horizontal line).



We note that each mass is $r = 1.0$ m from the axis. Therefore, using Eq. 10-26, we obtain

$$I = \sum m_i r_i^2 = 4\ (0.50 \text{ kg})\ (1.0 \text{ m})^2 = 2.0 \text{ kg}\cdot\text{m}^2.$$

(b) In this case, the two masses nearest the axis are r = 1.0 m away from it, but the two furthest from the axis are $r = \sqrt{(1.0 \text{ m})^2 + (2.0 \text{ m})^2}$ from it. Here, then, Eq. 10-33 leads to

$$I = \sum m_i r_i^2 = 2(0.50 \text{ kg})(1.0 \text{ m}^2) + 2(0.50 \text{ kg})(5.0 \text{ m}^2) = 6.0 \text{ kg}\cdot\text{m}^2.$$

(c) Now, two masses are on the axis (with r = 0) and the other two are a distance $r = \sqrt{(1.0 \text{ m})^2 + (1.0 \text{ m})^2}$ away. Now we obtain $I = 2.0 \text{ kg}\cdot\text{m}^2$.

45. **THINK** Torque is the product of the force applied and the moment arm. When two torques act on a body, the net torque is their vector sum.

EXPRESS We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1 F_1 \sin\theta_1$ is associated with $\vec{F}_1$ and a negative torque of magnitude $r_2F_2 \sin\theta_2$ is associated with $\vec{F}_2$. The net torque is consequently

$$\tau = r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2.$$

ANALYZE Substituting the given values, we obtain

$$\tau = r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2 = (1.30 \text{ m})(4.20 \text{ N})\sin 75° - (2.15 \text{ m})(4.90 \text{ N})\sin 60°$$
$$= -3.85 \text{ N}\cdot\text{m}.$$

LEARN Since $\tau < 0$, the body will rotate clockwise about the pivot point.

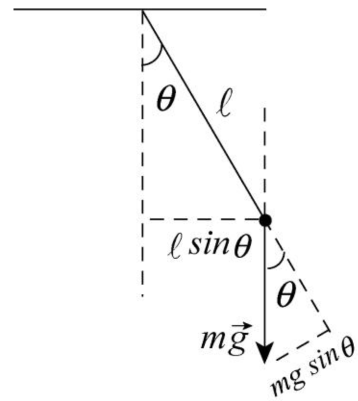
46. The net torque is

$$\begin{aligned}\tau &= \tau_A + \tau_B + \tau_C = F_A r_A \sin\phi_A - F_B r_B \sin\phi_B + F_C r_C \sin\phi_C \\ &= (10)(8.0)\sin 135° - (16)(4.0)\sin 90° + (19)(3.0)\sin 160° \\ &= 12\ \text{N}\cdot\text{m}.\end{aligned}$$

47. **THINK** In this problem we have a pendulum made up of a ball attached to a massless rod. There are two forces acting on the ball, the force of the rod and the force of gravity.

EXPRESS No torque about the pivot point is associated with the force of the rod since that force is along the line from the pivot point to the ball. As can be seen from the diagram, the component of the force of gravity that is perpendicular to the rod is *mg* sin *θ*. If ℓ is the length of the rod, then the torque associated with this force has magnitude

$$\tau = mg\ell \sin\theta .$$



ANALYZE With $m = 0.75$ kg, $\ell = 1.25$ m and $\theta = 30°$, we find the torque to be

$$\tau = mg\ell \sin\theta = (0.75)(9.8)(1.25)\sin 30° = 4.6\ \text{N}\cdot\text{m}.$$

LEARN The moment arm of the gravitational force mg is $\ell \sin\theta$. Alternatively, we may say that ℓ is the moment arm of $mg\sin\theta$, the tangential component of the gravitational force. Both interpretations lead to the same result: $\tau = (mg)(\ell \sin\theta) = (mg\sin\theta)(\ell)$.

48. We compute the torques using τ = *rF* sin *ϕ*.

(a) For $\phi = 30°$, $\tau_a = (0.152\ \text{m})(111\ \text{N})\sin 30° = 8.4\ \text{N}\cdot\text{m}$.

(b) For $\phi = 90°$, $\tau_b = (0.152\ \text{m})(111\ \text{N})\sin 90° = 17\ \text{N}\cdot\text{m}$.

(c) For $\phi = 180°$, $\tau_c = (0.152\ \text{m})(111\text{N})\sin 180° = 0$.

49. **THINK** Since the angular velocity of the diver changes with time, there must be a non-vanishing angular acceleration.

EXPRESS To calculate the angular acceleration α, we use the kinematic equation $\omega = \omega_0 + \alpha t$, where ω_0 is the initial angular velocity, ω is the final angular velocity and t is the time. If I is the rotational inertia of the diver, then the magnitude of the torque acting on her is $\tau = I\alpha$.

ANALYZE (a) Using the values given, the angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.20 \text{ rad/s}}{220 \times 10^{-3} \text{ s}} = 28.2 \text{ rad/s}^2.$$

(b) Similarly, we find the magnitude of the torque on the diver to be

$$\tau = I\alpha = (12.0 \text{ kg}\cdot\text{m}^2)(28.2 \text{ rad/s}^2) = 3.38 \times 10^2 \text{ N}\cdot\text{m}.$$

LEARN A net toque results in an angular acceleration that changes angular velocity. The equation $\tau = I\alpha$ implies that the greater the rotational inertia I, the greater the torque required for a given angular acceleration α.

50. The rotational inertia is found from Eq. 10-45.

$$I = \frac{\tau}{\alpha} = \frac{32.0}{25.0} = 1.28 \text{ kg}\cdot\text{m}^2$$

51. (a) We use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block m_2, then its coordinate is given by $y = \frac{1}{2}at^2$, so

$$a = \frac{2y}{t^2} = \frac{2(0.750 \text{ m})}{(5.00 \text{ s})^2} = 6.00 \times 10^{-2} \text{ m/s}^2.$$

Block 1 has an acceleration of 6.00×10^{-2} m/s^2 upward.

(b) Newton's second law for block 2 is $m_2 g - T_2 = m_2 a$, where m_2 is its mass and T_2 is the tension force on the block. Thus,

$$T_2 = m_2(g - a) = (0.500 \text{ kg})\left(9.8 \text{ m/s}^2 - 6.00\times10^{-2} \text{ m/s}^2\right) = 4.87 \text{ N}.$$

(c) Newton's second law for block 1 is $m_1 g - T_1 = -m_1 a$, where T_1 is the tension force on the block. Thus,

$$T_1 = m_1(g + a) = (0.460 \text{ kg})\left(9.8 \text{ m/s}^2 + 6.00\times10^{-2} \text{ m/s}^2\right) = 4.54 \text{ N}.$$

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so

$$\alpha = \frac{a}{R} = \frac{6.00\times10^{-2}\ \text{m/s}^2}{5.00\times10^{-2}\ \text{m}} = 1.20\ \text{rad/s}^2.$$

(e) The net torque acting on the pulley is $\tau = (T_2 - T_1)R$. Equating this to $I\alpha$ we solve for the rotational inertia:

$$I = \frac{(T_2 - T_1)R}{\alpha} = \frac{(4.87\ \text{N} - 4.54\ \text{N})(5.00\times10^{-2}\ \text{m})}{1.20\ \text{rad/s}^2} = 1.38\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

52. According to the sign conventions used in the book, the magnitude of the net torque exerted on the cylinder of mass m and radius R is

$$\tau_{\text{net}} = F_1R - F_2R - F_3r = (6.0\ \text{N})(0.12\ \text{m}) - (4.0\ \text{N})(0.12\ \text{m}) - (2.0\ \text{N})(0.050\ \text{m}) = 71\,\text{N}\cdot\text{m}.$$

(a) The resulting angular acceleration of the cylinder (with $I = \frac{1}{2}MR^2$ according to Table 10-2(c)) is

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{71\,\text{N}\cdot\text{m}}{\frac{1}{2}(2.0\,\text{kg})(0.12\ \text{m})^2} = 9.7\,\text{rad/s}^2.$$

(b) The direction is counterclockwise (which is the positive sense of rotation).

53. Combining Eq. 10-45 ($\tau_{\text{net}} = I\ \alpha$) with Eq. 10-38 gives $RF_2 - RF_1 = I\alpha$, where $\alpha = \omega/t$ by Eq. 10-12 (with $\omega_o = 0$). Using item (c) in Table 10-2 and solving for F_2 we find

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(0.02)(0.02)(250)}{2(1.25)} + 0.1 = 0.140\ \text{N}.$$

54. (a) In this case, the force is mg = (70 kg)(9.8 m/s^2), and the "lever arm" (the perpendicular distance from point O to the line of action of the force) is 0.28 m. Thus, the torque (in absolute value) is (70 kg)(9.8 m/s^2)(0.28 m). Since the moment-of-inertia is $I = 65\ \text{kg}\cdot\text{m}^2$, then Eq. 10-45 gives $|\alpha| = 2.955 \approx 3.0$ rad/s^2.

(b) Now we have another contribution (1.4 m × 300 N) to the net torque, so

$$|\tau_{\text{net}}| = (70\ \text{kg})(9.8\ \text{m/s}^2)(0.28\ \text{m}) + (1.4\ \text{m})(300\ \text{N}) = (65\ \text{kg}\cdot\text{m}^2)\,|\alpha|$$

which leads to $|\alpha| = 9.4$ rad/s^2.

55. Combining Eq. 10-34 and Eq. 10-45, we have $RF = I\alpha$, where α is given by ω/t (according to Eq. 10-12, since $\omega_o = 0$ in this case). We also use the fact that

$$I = I_{\text{plate}} + I_{\text{disk}}$$

where $I_{\text{disk}} = \frac{1}{2}MR^2$ (item (c) in Table 10-2). Therefore,

$$I_{\text{plate}} = \frac{RFt}{\omega} - \frac{1}{2}MR^2 = 2.51 \times 10^{-4}\ \text{kg}\cdot\text{m}^2.$$

56. With counterclockwise positive, the angular acceleration α for both masses satisfies

$$\tau = mgL_1 - mgL_2 = I\alpha = \left(mL_1^2 + mL_2^2\right)\alpha,$$

by combining Eq. 10-45 with Eq. 10-39 and Eq. 10-33. Therefore, using SI units,

$$\alpha = \frac{g\left(L_1 - L_2\right)}{L_1^2 + L_2^2} = \frac{\left(9.8\ \text{m/s}^2\right)\left(0.20\ \text{m} - 0.80\ \text{m}\right)}{(0.20\ \text{m})^2 + (0.80\ \text{m})^2} = -\,8.65\ \text{rad/s}^2$$

where the negative sign indicates the system starts turning in the clockwise sense. The magnitude of the acceleration vector involves no radial component (yet) since it is evaluated at $t = 0$ when the instantaneous velocity is zero. Thus, for the two masses, we apply Eq. 10-22:

(a) $\left|\vec{a}_1\right| = |\alpha| L_1 = \left(8.65\ \text{rad/s}^2\right)\left(0.20\ \text{m}\right) = 1.7\ \text{m/s}.$

(b) $\left|\vec{a}_2\right| = |\alpha| L_2 = \left(8.65\ \text{rad/s}^2\right)\left(0.80\ \text{m}\right) = 6.9\ \text{m/s}^2.$

57. Since the force acts tangentially at $r = 0.10$ m, the angular acceleration (presumed positive) is

$$\alpha = \frac{\tau}{I} = \frac{Fr}{I} = \frac{\left(0.5t + 0.3t^2\right)\left(0.10\right)}{1.0 \times 10^{-3}} = 50t + 30t^2$$

in SI units (rad/s^2).

(a) At $t = 3$ s, the above expression becomes $\alpha = 4.2 \times 10^2$ rad/s^2.

(b) We integrate the above expression, noting that $\omega_0 = 0$, to obtain the angular speed at t = 3 s:

$$\omega = \int_0^3 \alpha\, dt = \left(25t^2 + 10t^3\right)\Big|_0^3 = 5.0 \times 10^2\ \text{rad/s}.$$

58. (a) The speed of v of the mass m after it has descended $d = 50$ cm is given by $v^2 = 2ad$ (Eq. 2-16). Thus, using $g = 980$ cm/s^2, we have

$$v = \sqrt{2ad} = \sqrt{\frac{2(2mg)d}{M+2m}} = \sqrt{\frac{4(50)(980)(50)}{400+2(50)}} = 1.4\times10^2 \text{ cm/s}.$$

(b) The answer is still 1.4×10^2 cm/s = 1.4 m/s, since it is independent of R.

59. With ω = (1800)(2π/60) = 188.5 rad/s, we apply Eq. 10-55:

$$P = \tau\omega \quad \Rightarrow \quad \tau = \frac{74600 \text{ W}}{188.5 \text{ rad/s}} = 396 \text{ N}\cdot\text{m}.$$

60. (a) We apply Eq. 10-34:

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 = \frac{1}{6}mL^2\omega^2$$
$$= \frac{1}{6}(0.42 \text{ kg})(0.75 \text{ m})^2(4.0 \text{ rad/s})^2 = 0.63 \text{ J}.$$

(b) Simple conservation of mechanical energy leads to $K = mgh$. Consequently, the center of mass rises by

$$h = \frac{K}{mg} = \frac{mL^2\omega^2}{6mg} = \frac{L^2\omega^2}{6g} = \frac{(0.75 \text{ m})^2(4.0 \text{ rad/s})^2}{6(9.8 \text{ m/s}^2)} = 0.153 \text{ m} \approx 0.15 \text{ m}.$$

61. The initial angular speed is ω = (280 rev/min)(2π/60) = 29.3 rad/s.

(a) Since the rotational inertia is (Table 10-2(a)) $I = (32 \text{ kg})(1.2 \text{ m})^2 = 46.1 \text{ kg}\cdot\text{m}^2$, the work done is

$$W = \Delta K = 0 - \frac{1}{2}I\omega^2 = -\frac{1}{2}(46.1 \text{ kg}\cdot\text{m}^2)(29.3 \text{ rad/s})^2 = -1.98\times10^4 \text{ J}.$$

(b) The average power (in absolute value) is therefore

$$|P| = \frac{|W|}{\Delta t} = \frac{19.8\times10^3}{15} = 1.32\times10^3 \text{ W}.$$

62. (a) Eq. 10-33 gives

$$I_{\text{total}} = md^2 + m(2d)^2 + m(3d)^2 = 14\, md^2,$$

where d = 0.020 m and m = 0.010 kg. The work done is

$$W = \Delta K = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2,$$

where ω_f = 20 rad/s and ω_i = 0. This gives W = 11.2 mJ.

(b) Now, ω_f = 40 rad/s and ω_i = 20 rad/s, and we get W = 33.6 mJ.

(c) In this case, ω_f = 60 rad/s and ω_i = 40 rad/s. This gives W = 56.0 mJ.

(d) Equation 10-34 indicates that the slope should be $\frac{1}{2}I$. Therefore, it should be

$$7md^2 = 2.80 \times 10^{-5}\ \text{J}\cdot\text{s}^2/\ \text{rad}^2.$$

63. **THINK** As the meter stick falls by rotating about the axis passing through one end of the stick, its potential energy is converted into rotational kinetic energy.

EXPRESS We use ℓ to denote the length of the stick. The meter stick is initially at rest so its initial kinetic energy is zero. Since its center of mass is $\ell/2$ from either end, its initial potential energy is $U_g = \frac{1}{2}mg\ell$, where m is its mass. Just before the stick hits the floor, its final potential energy is zero, and its final kinetic energy is $\frac{1}{2}I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity. Conservation of energy yields

$$\frac{1}{2}mg\ell = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{mg\ell}{I}}.$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 10-18)

$$v = \omega\ell = \sqrt{\frac{mg\ell^3}{I}}.$$

ANALYZE Using Table 10-2 and the parallel-axis theorem, the rotational inertial is $I = \frac{1}{3}m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3\left(9.8\ \text{m/s}^2\right)\left(1.00\ \text{m}\right)} = 5.42\ \text{m/s}.$$

LEARN The linear speed of a point on the meter stick depends on its distance from the axis of rotation. One may show that the speed of the center of mass is

$$v_{\text{cm}} = \omega(\ell/2) = \frac{1}{2}\sqrt{3g\ell}.$$

64. (a) We use the parallel-axis theorem to find the rotational inertia:

$$I = I_{\text{com}} + Mh^2 = \frac{1}{2}MR^2 + Mh^2 = \frac{1}{2}(20\ \text{kg})(0.10\ \text{m})^2 + (20\ \text{kg})(0.50\ \text{m})^2 = 0.15\ \text{kg}\cdot\text{m}^2.$$

(b) Conservation of energy requires that $Mgh = \frac{1}{2} I\omega^2$, where ω is the angular speed of the cylinder as it passes through the lowest position. Therefore,

$$\omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2(20\ \text{kg})(9.8\ \text{m/s}^2)(0.050\ \text{m})}{0.15\ \text{kg}\cdot\text{m}^2}} = 11\ \text{rad/s}.$$

65. (a) We use conservation of mechanical energy to find an expression for ω^2 as a function of the angle θ that the chimney makes with the vertical. The potential energy of the chimney is given by $U = Mgh$, where M is its mass and h is the altitude of its center of mass above the ground. When the chimney makes the angle θ with the vertical, $h = (H/2)\cos\theta$. Initially the potential energy is $U_i = Mg(H/2)$ and the kinetic energy is zero. The kinetic energy is $\frac{1}{2} I\omega^2$ when the chimney makes the angle θ with the vertical, where I is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2)\cos\theta + \frac{1}{2} I\omega^2 \Rightarrow \omega^2 = (MgH/I)(1-\cos\theta).$$

The rotational inertia of the chimney about its base is $I = MH^2/3$ (found using Table 10-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1-\cos\theta)} = \sqrt{\frac{3(9.80\ \text{m/s}^2)}{55.0\ \text{m}}(1-\cos 35.0°)} = 0.311\ \text{rad/s}.$$

(b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so

$$a_r = 3g\,(1 - \cos\theta) = 3\,(9.80\ \text{m/s}^2)(1 - \cos 35.0°) = 5.32\ \text{m/s}^2.$$

(c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 10-1 since the acceleration is not uniform. Hence, we differentiate

$$\omega^2 = (3g/H)(1 - \cos\theta)$$

with respect to time, replacing $d\omega/dt$ with α, and $d\theta/dt$ with ω, and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega\sin\theta \Rightarrow \alpha = (3g/2H)\sin\theta.$$

Consequently,

$$a_t = H\alpha = \frac{3g}{2}\sin\theta = \frac{3(9.80\ \text{m/s}^2)}{2}\sin 35.0° = 8.43\ \text{m/s}^2.$$

(d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2}\sin\theta = g$. Thus, sin θ = 2/3 and we obtain θ = 41.8°.

66. From Table 10-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega^2_{\text{sphere}} + \frac{1}{2}I\omega^2_{\text{pulley}} + \frac{1}{2}mv^2.$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$
$$= \sqrt{\frac{2(9.8)(0.82)}{1 + 3.0\times10^{-3}/((0.60)(0.050)^2) + 2(4.5)/3(0.60)}} = 1.4 \text{ m/s}.$$

67. Using the parallel axis theorem and items (e) and (h) in Table 10-2, the rotational inertia is

$$I = \frac{1}{12}mL^2 + m(L/2)^2 + \frac{1}{2}mR^2 + m(R+L)^2 = 10.83mR^2,$$

where $L = 2R$ has been used. If we take the base of the rod to be at the coordinate origin (x = 0, y = 0) then the center of mass is at

$$y = \frac{mL/2 + m(L+R)}{m+m} = 2R.$$

Comparing the position shown in the textbook figure to its upside down (inverted) position shows that the change in center of mass position (in absolute value) is $|\Delta y| = 4R$. The corresponding loss in gravitational potential energy is converted into kinetic energy. Thus,

$$K = (2m)g(4R) \quad \Rightarrow \quad \omega = 9.82 \text{ rad/s}$$

where Eq. 10-34 has been used.

68. We choose ± directions such that the initial angular velocity is $\omega_0 = -317$ rad/s and the values for α, τ, and F are positive.

(a) Combining Eq. 10-12 with Eq. 10-45 and Table 10-2(f) (and using the fact that $\omega = 0$) we arrive at the expression

$$\tau = \left(\frac{2}{5}MR^2\right)\left(-\frac{\omega_0}{t}\right) = -\frac{2}{5}\frac{MR^2\omega_0}{t}.$$

With t = 15.5 s, R = 0.226 m, and M = 1.65 kg, we obtain τ = 0.689 N · m.

(b) From Eq. 10-40, we find $F = \tau/R$ = 3.05 N.

(c) Using again the expression found in part (a), but this time with R = 0.854 m, we get $\tau = 9.84\ \text{N}\cdot\text{m}$.

(d) Now, $F = \tau / R$ = 11.5 N.

69. The volume of each disk is $\pi r^2 h$ where we are using h to denote the thickness (which equals 0.00500 m). If we use R (which equals 0.0400 m) for the radius of the larger disk and r (which equals 0.0200 m) for the radius of the smaller one, then the mass of each is $m = \rho\pi r^2 h$ and $M = \rho\pi R^2 h$ where ρ = 1400 kg/m^3 is the given density. We now use the parallel axis theorem as well as item (c) in Table 10-2 to obtain the rotation inertia of the two-disk assembly:

$$I = \tfrac{1}{2}MR^2 + \tfrac{1}{2}mr^2 + m(r+R)^2 = \rho\pi h[\tfrac{1}{2}R^4 + \tfrac{1}{2}r^4 + r^2(r+R)^2] = 6.16 \times 10^{-5}\ \text{kg}\cdot\text{m}^2.$$

70. The wheel starts turning from rest (ω_0 = 0) at t = 0, and accelerates uniformly at $\alpha = 2.00\ \text{rad/s}^2$. Between t_1 and t_2 the wheel turns through $\Delta\theta$ = 90.0 rad, where $t_2 - t_1 = \Delta t$ = 3.00 s. We solve (b) first.

(b) We use Eq. 10-13 (with a slight change in notation) to describe the motion for $t_1 \le t \le t_2$:

$$\Delta\theta = \omega_1\Delta t + \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow \omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2}$$

which we plug into Eq. 10-12, set up to describe the motion during $0 \le t \le t_1$:

$$\omega_1 = \omega_0 + \alpha t_1 \;\Rightarrow\; \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \alpha t_1 \;\Rightarrow\; \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = (2.00)\,t_1$$

yielding t_1 = 13.5 s.

(a) Plugging into our expression for ω_1 (in previous part) we obtain

$$\omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = 27.0\ \text{rad/s}.$$

71. **THINK** Since the string that connects the two blocks does not slip, the pulley rotates about its axel as the blocks move.

EXPRESS We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_2 = a_1 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose rightward positive for $m_2 = M$ (the block on the table), downward positive for $m_1 = M$ (the block at the end of the string) and (somewhat unconventionally) clockwise for positive sense of disk rotation. This means that we interpret θ given in the problem as a positive-valued quantity. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations (where we allow for the possibility of friction f_2 acting on m_2):

$$m_1 g - T_1 = m_1 a_1$$
$$T_2 - f_2 = m_2 a_2$$
$$T_1 R - T_2 R = I\alpha$$

ANALYZE (a) From Eq. 10-13 (with $\omega_0 = 0$) we find the magnitude of the pulley's angular acceleration to be

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \;\Rightarrow\; \alpha = \frac{2\theta}{t^2} = \frac{2(0.130 \text{ rad})}{(0.0910 \text{ s})^2} = 31.4 \text{ rad/s}^2.$$

(b) From the fact that $a = R\alpha$ (noted above), the acceleration of the blocks is

$$a = \frac{2R\theta}{t^2} = \frac{2(0.024 \text{ m})(0.130 \text{ rad})}{(0.0910 \text{ s})^2} = 0.754 \text{ m/s}^2.$$

(c) From the first of the above equations, we find the string tension T_1 to be

$$T_1 = m_1(g - a_1) = M\left(g - \frac{2R\theta}{t^2}\right) = (6.20 \text{ kg})\left(9.80 \text{ m/s}^2 - \frac{2(0.024 \text{ m})(0.130 \text{ rad})}{(0.0910 \text{ s})^2}\right) = 56.1 \text{ N}.$$

(d) From the last of the above equations, we obtain the second tension:

$$T_2 = T_1 - \frac{I\alpha}{R} = 56.1 \text{ N} - \frac{(7.40\times10^{-4} \text{kg}\cdot\text{m}^2)(31.4 \text{ rad/s}^2)}{0.024 \text{ m}} = 55.1 \text{ N}.$$

LEARN The torque acting on the pulley is $\tau = I\alpha = (T_1 - T_2)R$. If the pulley becomes massless, then $I = 0$ and we recover the expected result: $T_1 = T_2$.

72. (a) Constant angular acceleration kinematics can be used to compute the angular acceleration α. If ω_0 is the initial angular velocity and t is the time to come to rest, then $0 = \omega_0 + \alpha t$, which gives

$$\alpha=-\frac{\omega_0}{t}=-\frac{39.0\,\text{rev/s}}{32.0\,\text{s}}=-1.22\,\text{rev/s}^2=-7.66\,\text{rad/s}^2\ .$$

(b) We use τ = $I\alpha$, where τ is the torque and I is the rotational inertia. The contribution of the rod to I is $M\ell^2/12$ (Table 10-2(e)), where M is its mass and ℓ is its length. The contribution of each ball is $m(\ell/2)^2$, where m is the mass of a ball. The total rotational inertia is

$$I=\frac{M\ell^2}{12}+2\frac{m\ell^2}{4}=\frac{(6.40\text{ kg})(1.20\text{ m})^2}{12}+\frac{(1.06\text{ kg})(1.20\text{ m})^2}{2}$$

which yields $I = 1.53\ \text{kg}\cdot\text{m}^2$. The torque, therefore, is

$$\tau=\left(1.53\text{ kg}\cdot\text{m}^2\right)\left(-7.66\text{ rad/s}^2\right)=-11.7\text{ N}\cdot\text{m}.$$

(c) Since the system comes to rest the mechanical energy that is converted to thermal energy is simply the initial kinetic energy

$$K_i=\frac{1}{2}I\omega_0^2=\frac{1}{2}\left(1.53\text{ kg}\cdot\text{m}^2\right)\left((2\pi)(39)\text{ rad/s}\right)^2=4.59\times10^4\text{ J}.$$

(d) We apply Eq. 10-13:

$$\theta=\omega_0 t+\frac{1}{2}\alpha t^2=\left((2\pi)(39)\text{ rad/s}\right)(32.0\text{ s})+\frac{1}{2}\left(-7.66\text{ rad/s}^2\right)(32.0\text{ s})^2$$

which yields 3920 rad or (dividing by 2π) 624 rev for the value of angular displacement θ.

(e) Only the mechanical energy that is converted to thermal energy can still be computed without additional information. It is 4.59×10^4 J no matter how τ varies with time, as long as the system comes to rest.

73. The *Hint* given in the problem would make the computation in part (a) very straightforward (without doing the integration as we show here), but we present this further level of detail in case that hint is not obvious or — simply — in case one wishes to see how the calculus supports our intuition.

(a) The (centripetal) force exerted on an infinitesimal portion of the blade with mass dm located a distance r from the rotational axis is (Newton's second law) $dF = (dm)\omega^2 r$, where dm can be written as $(M/L)dr$ and the angular speed is

$$\omega=(320)(2\pi/60)=33.5\text{ rad/s}\ .$$

Thus for the entire blade of mass M and length L the total force is given by

$$F = \int dF = \int \omega^2 r dm = \frac{M}{L}\int_0^L \omega^2 r dr = \frac{M\omega^2 L}{2} = \frac{(110\,\text{kg})\,(33.5\ \text{rad/s})^2\,(7.80\,\text{m})}{2}$$
$$= 4.81\times10^5\,\text{N}.$$

(b) About its center of mass, the blade has $I = ML^2/12$ according to Table 10-2(e), and using the parallel-axis theorem to "move" the axis of rotation to its end-point, we find the rotational inertia becomes $I = ML^2/3$. Using Eq. 10-45, the torque (assumed constant) is

$$\tau = I\alpha = \left(\frac{1}{3}ML^2\right)\left(\frac{\Delta\omega}{\Delta t}\right) = \frac{1}{3}(110\,\text{kg})(7.8\ \text{m})^2\left(\frac{33.5\,\text{rad/s}}{6.7\,\text{s}}\right) = 1.12\times10^4\,\text{N}\cdot\text{m}.$$

(c) Using Eq. 10-52, the work done is

$$W = \Delta K = \frac{1}{2}I\omega^2 - 0 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = \frac{1}{6}(110\,\text{kg})(7.80\,\text{m})^2(33.5\,\text{rad/s})^2 = 1.25\times10^6\ \text{J}.$$

74. The angular displacements of disks A and B can be written as:

$$\theta_A = \omega_A t, \quad \theta_B = \frac{1}{2}\alpha_B t^2.$$

(a) The time when $\theta_A = \theta_B$ is given by

$$\omega_A t = \frac{1}{2}\alpha_B t^2 \quad \Rightarrow \quad t = \frac{2\omega_A}{\alpha_B} = \frac{2(9.5\ \text{rad/s})}{(2.2\ \text{rad/s}^2)} = 8.6\ \text{s}.$$

(b) The difference in the angular displacement is

$$\Delta\theta = \theta_A - \theta_B = \omega_A t - \frac{1}{2}\alpha_B t^2 = 9.5t - 1.1t^2.$$

For their reference lines to align momentarily, we only require $\Delta\theta = 2\pi N$, where N is an integer. The quadratic equation can be readily solve to yield

$$t_N = \frac{9.5 \pm \sqrt{(9.5)^2 - 4(1.1)(2\pi N)}}{2(1.1)} = \frac{9.5 \pm \sqrt{90.25 - 27.6N}}{2.2}.$$

The solution $t_0 = 8.63$ s (taking the positive root) coincides with the result obtained in (a), while $t_0 = 0$ (taking the negative root) is the moment when both disks begin to rotate. In fact, two solutions exist for N = 0, 1, 2, and 3.

75. The magnitude of torque is the product of the force magnitude and the distance from the pivot to the line of action of the force. In our case, it is the gravitational force that passes through the walker's center of mass. Thus,

$$\tau = I\alpha = rF = rmg.$$

(a) Without the pole, with $I = 15\ \text{kg}\cdot\text{m}^2$, the angular acceleration is

$$\alpha = \frac{rF}{I} = \frac{rmg}{I} = \frac{(0.050\ \text{m})(70\ \text{kg})(9.8\ \text{m/s}^2)}{15\ \text{kg}\cdot\text{m}^2} = 2.3\ \text{rad/s}^2.$$

(b) When the walker carries a pole, the torque due to the gravitational force through the pole's center of mass opposes the torque due to the gravitational force that passes through the walker's center of mass. Therefore,

$$\tau_{\text{net}} = \sum_i r_i F_i = (0.050\ \text{m})(70\ \text{kg})(9.8\ \text{m/s}^2) - (0.10\ \text{m})(14\ \text{kg})(9.8\ \text{m/s}^2) = 20.58\ \text{N}\cdot\text{m},$$

and the resulting angular acceleration is

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{20.58\ \text{N}\cdot\text{m}}{15\ \text{kg}\cdot\text{m}^2} \approx 1.4\ \text{rad/s}^2.$$

76. The motion consists of two stages. The first, the interval $0 \le t \le 20$ s, consists of constant angular acceleration given by

$$\alpha = \frac{5.0\ \text{rad/s}}{2.0\ \text{s}} = 2.5\ \text{rad/s}^2.$$

The second stage, $20 < t \le 40$ s, consists of constant angular velocity $\omega = \Delta\theta / \Delta t$. Analyzing the first stage, we find

$$\theta_1 = \frac{1}{2}\alpha t^2\Big|_{t=20} = 500\ \text{rad}, \quad \omega = \alpha t\big|_{t=20} = 50\ \text{rad/s}.$$

Analyzing the second stage, we obtain

$$\theta_2 = \theta_1 + \omega\Delta t = 500\ \text{rad} + \left(50\ \text{rad/s}\right)\left(20\ \text{s}\right) = 1.5\times10^3\ \text{rad}.$$

77. **THINK** The record turntable comes to a stop due to a constant angular acceleration. We apply equations given in Table 10-1 to analyze the rotational motion.

EXPRESS We take the sense of initial rotation to be positive. Then, with $\omega_0 > 0$ and $\omega = 0$ (since it stops at time t), our angular acceleration is negative-valued. The angular acceleration is constant, so we can apply Eq. 10-12 ($\omega = \omega_0 + \alpha t$), which gives $\alpha = (\omega - \omega_0)/t$. Similarly, the angular displacement can be found by using Eq. 10-13: $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$.

ANALYZE (a) To obtain the requested units, we use t = 30 s = 0.50 min. With $\omega_0 = 33.33$ rev/min, we find the angular acceleration to be

$$\alpha = -\frac{33.33\ \text{rev/min}}{0.50\,\text{min}} = -66.7\ \text{rev/min}^2 \approx -67\ \text{rev/min}^2.$$

(b) Substituting the value of α obtained above into Eq. 10-13, we get

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (33.33\ \text{rev/min})(0.50\ \text{min}) + \frac{1}{2}(-66.7\text{rev/min}^2)(0.50\ \text{min})^2 = 8.33\ \text{rev}.$$

LEARN To solve for the angular displacement in (b), we may also use Eq. 10-15:

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(33.33\ \text{rev/min} + 0)(0.50\ \text{min}) = 8.33\ \text{rev}.$$

78. We use conservation of mechanical energy. The center of mass is at the midpoint of the cross bar of the **H** and it drops by $L/2$, where L is the length of any one of the rods. The gravitational potential energy decreases by $MgL/2$, where M is the mass of the body. The initial kinetic energy is zero and the final kinetic energy may be written $\frac{1}{2}I\omega^2$, where I is the rotational inertia of the body and ω is its angular velocity when it is vertical. Thus,

$$0 = -MgL/2 + \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{MgL/I}.$$

Since the rods are thin the one along the axis of rotation does not contribute to the rotational inertia. All points on the other leg are the same distance from the axis of rotation, so that leg contributes $(M/3)L^2$, where $M/3$ is its mass. The cross bar is a rod that rotates around one end, so its contribution is $(M/3)L^2/3 = ML^2/9$. The total rotational inertia is

$$I = (ML^2/3) + (ML^2/9) = 4ML^2/9.$$

Consequently, the angular velocity is

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{4ML^2/9}} = \sqrt{\frac{9g}{4L}} = \sqrt{\frac{9(9.800\ \text{m/s}^2)}{4(0.600\ \text{m})}} = 6.06\ \text{rad/s}.$$

79. **THINK** In this problem we compare the rotational inertia between a solid cylinder and a hoop.

EXPRESS According to Table 10-2, the rotational inertia formulas for a cylinder of radius *R* and mass *M*, and a hoop of radius *r* and mass *M* are

$$I_C = \frac{1}{2}MR^2, \quad I_H = Mr^2.$$

Equating $I_C = I_H$ allows us to deduce the relationship between *r* and *R*.

ANALYZE (a) Since both the cylinder and the hoop have the same mass, then they will have the same rotational inertia ($I_C = I_H$) if $R^2/2 = r^2 \rightarrow r = R/\sqrt{2}$.

(b) We require the rotational inertia of any given body to be written as $I = Mk^2$, where *M* is the mass of the given body and *k* is the radius of the "equivalent hoop." It follows directly that $k = \sqrt{I/M}$.

LEARN Listed below are some examples of equivalent hoop and their radii:

$$I_C = \frac{1}{2}MR^2 = M(R/\sqrt{2})^2 \quad \Rightarrow \quad k_C = R/\sqrt{2}$$
$$I_S = \frac{2}{5}MR^2 = M\left(\sqrt{\frac{2}{5}}R\right)^2 \quad \Rightarrow \quad k_S = \sqrt{\frac{2}{5}}R$$

80. (a) Using Eq. 10-15, we have 60.0 rad = $\frac{1}{2}(\omega_1 + \omega_2)(6.00 \text{ s})$. With ω_2 = 15.0 rad/s, then ω_1 = 5.00 rad/s.

(b) Eq. 10-12 gives α = (15.0 rad/s – 5.0 rad/s)/(6.00 s) = 1.67 rad/s^2.

(c) Interpreting ω now as ω_1 and θ as θ_1 = 10.0 rad (and ω_o = 0) Eq. 10-14 leads to

$$\theta_o = -\frac{\omega_1^2}{2\alpha} + \theta_1 = 2.50 \text{ rad}.$$

81. The center of mass is initially at height $h = \frac{L}{2}\sin 40°$ when the system is released (where *L* = 2.0 m). The corresponding potential energy *Mgh* (where *M* = 1.5 kg) becomes rotational kinetic energy $\frac{1}{2}I\omega^2$ as it passes the horizontal position (where *I* is the rotational inertia about the pin). Using Table 10-2 (e) and the parallel axis theorem, we find

$$I = \tfrac{1}{12}ML^2 + M(L/2)^2 = \tfrac{1}{3}ML^2.$$

Therefore,

$$Mg\frac{L}{2}\sin 40° = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{3g\sin 40°}{L}} = 3.1 \text{ rad/s.}$$

82. The rotational inertia of the passengers is (to a good approximation) given by Eq. 10-53: $I = \sum mR^2 = NmR^2$ where N is the number of people and m is the (estimated) mass per person. We apply Eq. 10-52:

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}NmR^2\omega^2$$

where R = 38 m and N = 36 × 60 = 2160 persons. The rotation rate is constant so that $\omega = \theta/t$ which leads to $\omega = 2\pi/120 = 0.052$ rad/s. The mass (in kg) of the average person is probably in the range $50 \le m \le 100$, so the work should be in the range

$$\frac{1}{2}(2160)(50)(38)^2(0.052)^2 \le W \le \frac{1}{2}(2160)(100)(38)^2(0.052)^2$$
$$2\times 10^5 \text{ J} \le W \le 4\times 10^5 \text{ J.}$$

83. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_1 = a_2 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose upward positive for m_1, downward positive for m_2, and (somewhat unconventionally) clockwise for positive sense of disk rotation. Applying Newton's second law to m_1m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations.

$$T_1 - m_1g = m_1a_1$$
$$m_2g - T_2 = m_2a_2$$
$$T_2R - T_1R = I\alpha$$

(a) The rotational inertia of the disk is $I = \frac{1}{2}MR^2$ (Table 10-2(c)), so we divide the third equation (above) by R, add them all, and use the earlier equality among accelerations — to obtain:

$$m_2g - m_1g = \left(m_1 + m_2 + \frac{1}{2}M\right)a$$

which yields $a = \frac{4}{25}g = 1.57$ m/s^2.

(b) Plugging back in to the first equation, we find

$$T_1 = \frac{29}{25}m_1g = 4.55 \text{ N}$$

where it is important in this step to have the mass in SI units: m_1 = 0.40 kg.

(c) Similarly, with m_2 = 0.60 kg, we find $T_2 = \frac{5}{6}m_2 g = 4.94$ N.

84. (a) The longitudinal separation between Helsinki and the explosion site is $\Delta\theta = 102° - 25° = 77°$. The spin of the Earth is constant at

$$\omega = \frac{1 \text{ rev}}{1 \text{ day}} = \frac{360°}{24 \text{ h}}$$

so that an angular displacement of $\Delta\theta$ corresponds to a time interval of

$$\Delta t = (77°)\left(\frac{24 \text{ h}}{360°}\right) = 5.1 \text{ h}.$$

(b) Now $\Delta\theta = 102° - (-20°) = 122°$ so the required time shift would be

$$\Delta t = (122°)\left(\frac{24 \text{ h}}{360°}\right) = 8.1 \text{ h}.$$

85. To get the time to reach the maximum height, we use Eq. 4-23, setting the left-hand side to zero. Thus, we find

$$t = \frac{(60 \text{ m/s})\sin(20°)}{9.8 \text{ m/s}^2} = 2.094 \text{ s}.$$

Then (assuming α = 0) Eq. 10-13 gives

$$\theta - \theta_0 = \omega_0 t = (90 \text{ rad/s})(2.094 \text{ s}) = 188 \text{ rad},$$

which is equivalent to roughly 30 rev.

86. In the calculation below, M_1 and M_2 are the ring masses, R_{1i} and R_{2i} are their inner radii, and R_{1o} and R_{2o} are their outer radii. Referring to item (b) in Table 10-2, we compute

$$I = \tfrac{1}{2}M_1(R_{1i}^2 + R_{1o}^2) + \tfrac{1}{2}M_2(R_{2i}^2 + R_{2o}^2) = 0.00346 \text{ kg}\cdot\text{m}^2 .$$

Thus, with Eq. 10-38 ($\tau = rF$ where $r = R_{2o}$) and $\tau = I\alpha$ (Eq. 10-45), we find

$$\alpha = \frac{(0.140)(12.0)}{0.00346} = 485 \text{ rad/s}^2 .$$

Then Eq. 10-12 gives $\omega = \alpha t$ = 146 rad/s.

87. We choose positive coordinate directions so that each is accelerating positively, which will allow us to set $a_{box} = R\alpha$ (for simplicity, we denote this as a). Thus, we choose downhill positive for the m = 2.0 kg box and (as is conventional) counterclockwise for positive sense of wheel rotation. Applying Newton's second law to the box and (in the form of Eq. 10-45) to the wheel, respectively, we arrive at the following two equations (using θ as the incline angle 20°, not as the angular displacement of the wheel).

$$mg\sin\theta - T = ma$$
$$TR = I\alpha$$

Since the problem gives a = 2.0 m/s^2, the first equation gives the tension $T = m(g \sin\theta - a)$ = 2.7 N. Plugging this and R = 0.20 m into the second equation (along with the fact that $\alpha = a/R$) we find the rotational inertia

$$I = TR^2/a = 0.054 \text{ kg}\cdot\text{m}^2.$$

88. (a) We use $\tau = I\alpha$, where τ is the net torque acting on the shell, I is the rotational inertia of the shell, and α is its angular acceleration. Therefore,

$$I = \frac{\tau}{\alpha} = \frac{960 \text{ N}\cdot\text{m}}{6.20 \text{ rad/s}^2} = 155 \text{ kg}\cdot\text{m}^2.$$

(b) The rotational inertia of the shell is given by $I = (2/3) MR^2$ (see Table 10-2 of the text). This implies

$$M = \frac{3I}{2R^2} = \frac{3\left(155 \text{ kg}\cdot\text{m}^2\right)}{2\left(1.90 \text{ m}\right)^2} = 64.4 \text{ kg}.$$

89. Equation 10-40 leads to $\tau = mgr$ = (70 kg) (9.8 m/s^2) (0.20 m) = 1.4×10^2 N·m.

90. (a) Equation 10-12 leads to $\alpha = -\omega_o / t = -(25.0 \text{ rad/s})/(20.0 \text{ s}) = -1.25 \text{ rad/s}^2$.

(b) Equation 10-15 leads to $\theta = \frac{1}{2}\omega_o t = \frac{1}{2}(25.0 \text{ rad/s})(20.0 \text{ s}) = 250 \text{ rad}.$

(c) Dividing the previous result by 2π we obtain θ = 39.8 rev.

91. **THINK** As the box falls, gravitational force gives rise to a torque that causes the wheel to rotate.

EXPRESS We employ energy methods to solve this problem; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, where $K_{\text{box}} = m_{\text{box}}v^2/2$. The rotational kinetic energy of the wheel is $K_{\text{rot}} = I\omega^2/2$.

ANALYZE (a) With $K_{\text{box}} = 0.60\ \text{J}$, we find the speed of the box to be

$$K_{\text{box}} = \frac{1}{2}m_{\text{box}}v^2 \Rightarrow v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = \sqrt{\frac{2(6.0\ \text{J})}{6.0\ \text{kg}}} = 1.41\ \text{m/s},$$

implying that the angular speed is ω = (1.41 m/s)/(0.20 m) = 7.07 rad/s. Thus, the kinetic energy of rotation is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.40\ \text{kg}\cdot\text{m}^2)(7.07\ \text{rad/s})^2 = 10.0\ \text{J}.$$

(b) Since it was released from rest, we will take the initial position to be our reference point for gravitational potential. Energy conservation requires

$$K_0 + U_0 = K + U \quad \Rightarrow \quad 0 + 0 = \left(6.0\ \text{J} + 10.0\ \text{J}\right) + m_{\text{box}}g\left(-h\right).$$

Therefore,

$$h = \frac{K}{m_{\text{box}}g} = \frac{6.0\ \text{J} + 10.0\ \text{J}}{(6.0\ \text{kg})(9.8\ \text{m/s}^2)} = 0.27\ \text{m}.$$

LEARN As the box falls, its gravitational potential energy gets converted into kinetic energy of the box as well as rotational kinetic energy of the wheel; the total energy remains conserved.

92. (a) The time for one revolution is the circumference of the orbit divided by the speed v of the Sun: $T = 2\pi R/v$, where R is the radius of the orbit. We convert the radius:

$$R = \left(2.3\times10^4\ \text{ly}\right)\left(9.46\times10^{12}\ \text{km/ly}\right) = 2.18\times10^{17}\ \text{km}$$

where the ly ↔ km conversion can be found in Appendix D or figured “from basics” (knowing the speed of light). Therefore, we obtain

$$T = \frac{2\pi\left(2.18\times10^{17}\ \text{km}\right)}{250\ \text{km/s}} = 5.5\times10^{15}\ \text{s}.$$

(b) The number of revolutions N is the total time t divided by the time T for one revolution; that is, $N = t/T$. We convert the total time from years to seconds and obtain

$$N = \frac{\left(4.5\times10^9\ \text{y}\right)\left(3.16\times10^7\ \text{s/y}\right)}{5.5\times10^{15}\ \text{s}} = 26.$$

93. **THINK** The applied force P accelerates the block. In addition, it gives rise to a torque that causes the wheel to undergo angular acceleration.

EXPRESS We take rightward to be positive for the block and clockwise negative for the wheel (as is conventional). With this convention, we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to $P - T = ma$, where T is the tension in the cord. Similarly, applying Newton's second law (for rotation) to the wheel leads to $-TR = I\alpha$. Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \Rightarrow T = a\frac{I}{R^2}.$$

Adding this to the above equation (for the block) leads to $P = (m + I/R^2)a$. Thus, the angular acceleration is

$$\alpha = -\frac{a}{R} = -\frac{P}{(m + I/R^2)R}$$

ANALYZE With $m = 2.0$ kg, $I = 0.050\ \text{kg}\cdot\text{m}^2$, $P = 3.0$ N and $R = 0.20$ m, we find

$$\alpha = -\frac{P}{(m + I/R^2)R} = -\frac{3.0\ \text{N}}{[2.0\ \text{kg} + (0.050\ \text{kg}\cdot\text{m}^2)/(0.20\ \text{m})^2](0.20\ \text{m})} = -4.62\ \text{rad/s}^2\ ,$$

or $|\alpha| = 4.62\ \text{rad/s}^2$.

LEARN The greater the applied force P, the greater the (magnitude of) angular acceleration. Note that the negative sign in α should not be mistaken for a deceleration; it simply indicates the clockwise sense to the motion.

94. First, we convert the angular velocity: ω = (2000 rev/min)(2π/60) = 209 rad/s. Also, we convert the plane's speed to SI units: (480)(1000/3600) = 133 m/s. We use Eq. 10-18 in part (a) and (implicitly) Eq. 4-39 in part (b).

(a) The speed of the tip as seen by the pilot is $v_t = \omega r = (209\ \text{rad/s})(1.5\ \text{m}) = 314\ \text{m/s}$, which (since the radius is given to only two significant figures) we write as $v_t = 3.1\times 10^2\ \text{m/s}$.

(b) The plane's velocity $\vec{v}_p$ and the velocity of the tip $\vec{v}_t$ (found in the plane's frame of reference), in any of the tip's positions, must be perpendicular to each other. Thus, the speed as seen by an observer on the ground is

$$v = \sqrt{v_p^2 + v_t^2} = \sqrt{(133\ \text{m/s})^2 + (314\ \text{m/s})^2} = 3.4\times 10^2\ \text{m/s}\ .$$

95. The distances from P to the particles are as follows:

$$r_1 = a \text{ for } m_1 = 2M \text{ (lower left)}$$
$$r_2 = \sqrt{b^2 - a^2} \text{ for } m_2 = M \text{ (top)}$$
$$r_3 = a \text{ for } m_1 = 2M \text{ (lower right)}$$

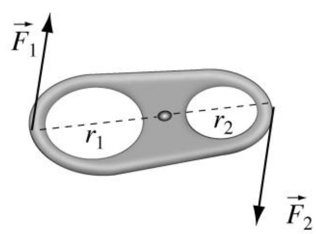
The rotational inertia of the system about P is

$$I = \sum_{i=1}^{3} m_i r_i^2 = \left(3a^2 + b^2\right)M,$$

which yields $I = 0.208 \text{ kg} \cdot \text{m}^2$ for M = 0.40 kg, a = 0.30 m, and b = 0.50 m. Applying Eq. 10-52, we find

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(0.208 \text{ kg} \cdot \text{m}^2\right)\left(5.0 \text{ rad/s}\right)^2 = 2.6 \text{ J}.$$

96. In the figure below, we show a pull tab of a beverage can. Since the tab is pivoted, when pulling on one end upward with a force $\vec{F}_1$, a force $\vec{F}_2$ will be exerted on the other end. The torque produced by $\vec{F}_1$ must be balanced by the torque produced by $\vec{F}_2$ so that the tab does not rotate.



The two forces are related by

$$r_1 F_1 = r_2 F_2$$

where $r_1 \approx 1.8 \text{ cm}$ and $r_2 \approx 0.73 \text{ cm}$. Thus, if F_1 = 10 N,

$$F_2 = \left(\frac{r_1}{r_2}\right)F_1 \approx \left(\frac{1.8 \text{ cm}}{0.73 \text{ cm}}\right)(10 \text{ N}) \approx 25 \text{ N}.$$

97. The centripetal acceleration at a point P that is r away from the axis of rotation is given by Eq. 10-23: $a = v^2 / r = \omega^2 r$, where $v = \omega r$, with $\omega = 2000$ rev/min $\approx$ 209.4 rad/s.

(a) If points A and P are at a radial distance r_A = 1.50 m and r = 0.150 m from the axis, the difference in their acceleration is

$$\Delta a = a_A - a = \omega^2 (r_A - r) = (209.4 \text{ rad/s})^2 (1.50 \text{ m} - 0.150 \text{ m}) \approx 5.92 \times 10^4 \text{ m/s}^2.$$

(b) The slope is given by $a/r = \omega^2 = 4.39\times10^4/\text{s}^2$.

98. Let T be the tension on the rope. From Newton's second law, we have

$$T - mg = ma \;\Rightarrow\; T = m(g+a).$$

Since the box has an upward acceleration a = 0.80 m/s^2, the tension is given by

$$T = (30 \text{ kg})(9.8 \text{ m/s}^2 + 0.8 \text{ m/s}^2) = 318 \text{ N}.$$

The rotation of the device is described by $F_{\text{app}}R - Tr = I\alpha = Ia/r$. The moment of inertia can then be obtained as

$$I = \frac{r(F_{\text{app}}R - Tr)}{a} = \frac{(0.20 \text{ m})[(140 \text{ N})(0.50 \text{ m}) - (318 \text{ N})(0.20 \text{ m})]}{0.80 \text{ m/s}^2} = 1.6 \text{ kg}\cdot\text{m}^2$$

99. (a) With r = 0.780 m, the rotational inertia is

$$I = Mr^2 = (1.30 \text{ kg})(0.780 \text{ m})^2 = 0.791 \text{ kg}\cdot\text{m}^2.$$

(b) The torque that must be applied to counteract the effect of the drag is

$$\tau = rf = (0.780 \text{ m})(2.30\times10^{-2} \text{ N}) = 1.79\times10^{-2} \text{ N}\cdot\text{m}.$$

100. We make use of Table 10-2(e) as well as the parallel-axis theorem, Eq. 10-34, where needed. We use ℓ (as a subscript) to refer to the long rod and s to refer to the short rod.

(a) The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12}m_s L_s^2 + \frac{1}{3}m_\ell L_\ell^2 = 0.019 \text{ kg}\cdot\text{m}^2.$$

(b) We note that the center of the short rod is a distance of h = 0.25 m from the axis. The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12}m_s L_s^2 + m_s h^2 + \frac{1}{12}m_\ell L_\ell^2$$

which again yields $I = 0.019 \text{ kg}\cdot\text{m}^2$.

101. (a) The linear speed of a point on belt 1 is

$$v_1 = r_A\omega_A = (15 \text{ cm})(10 \text{ rad/s}) = 1.5\times10^2 \text{ cm/s}.$$

(b) The angular speed of pulley *B* is

$$r_B\omega_B = r_A\omega_A \quad \Rightarrow \quad \omega_B = \frac{r_A\omega_A}{r_B} = \left(\frac{15\text{ cm}}{10\text{ cm}}\right)(10\text{ rad/s}) = 15\text{ rad/s}.$$

(c) Since the two pulleys are rigidly attached to each other, the angular speed of pulley B' is the same as that of pulley *B*, that is, $\omega'_B = 15\text{ rad/s}$.

(d) The linear speed of a point on belt 2 is

$$v_2 = r_{B'}\omega'_B = (5\text{ cm})(15\text{ rad/s}) = 75\text{ cm/s}.$$

(e) The angular speed of pulley *C* is

$$r_C\omega_C = r_{B'}\omega'_B \quad \Rightarrow \quad \omega_C = \frac{r_{B'}\omega'_B}{r_C} = \left(\frac{5\text{ cm}}{25\text{ cm}}\right)(15\text{ rad/s}) = 3.0\text{ rad/s}$$

102. (a) The rotational inertia relative to the specified axis is

$$I = \sum m_i r_i^2 = (2M)L^2 + (2M)L^2 + M(2L)^2$$

which is found to be I = 4.6 kg·m^2. Then, with ω = 1.2 rad/s, we obtain the kinetic energy from Eq. 10-34:

$$K = \frac{1}{2}I\omega^2 = 3.3\text{ J}.$$

(b) In this case the axis of rotation would appear as a standard *y* axis with origin at *P*. Each of the 2*M* balls are a distance of $r = L\cos 30°$ from that axis. Thus, the rotational inertia in this case is

$$I = \sum m_i r_i^2 = (2M)r^2 + (2M)r^2 + M(2L)^2$$

which is found to be I = 4.0 kg·m^2. Again, from Eq. 10-34 we obtain the kinetic energy

$$K = \frac{1}{2}I\omega^2 = 2.9\text{ J}.$$

103. We make use of Table 10-2(e) and the parallel-axis theorem in Eq. 10-36.

(a) The moment of inertia is

$$I = \frac{1}{12}ML^2 + Mh^2 = \frac{1}{12}(3.0\text{ kg})(4.0\text{ m})^2 + (3.0\text{ kg})(1.0\text{ m})^2 = 7.0\text{ kg}\cdot\text{m}^2.$$

(b) The rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2} I\omega^2 \;\Rightarrow\; \omega = \sqrt{\frac{2K_{\text{rot}}}{I}} = \sqrt{\frac{2(20\text{ J})}{7\text{ kg}\cdot\text{m}^2}} = 2.4\text{ rad/s}.$$

The linear speed of the end B is given by $v_B = \omega r_{AB} = (2.4\text{ rad/s})(3.00\text{ m}) = 7.2\text{ m/s}$, where r_{AB} is the distance between A and B.

(c) The maximum angle θ is attained when all the rotational kinetic energy is transformed into potential energy. Moving from the vertical position ($\theta = 0$) to the maximum angle θ, the center of mass is elevated by $\Delta y = d_{AC}(1-\cos\theta)$, where d_{AC} = 1.00 m is the distance between A and the center of mass of the rod. Thus, the change in potential energy is

$$\Delta U = mg\Delta y = mgd_{AC}(1-\cos\theta) \quad\Rightarrow\quad 20\text{ J} = (3.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m})(1-\cos\theta)$$

which yields $\cos\theta = 0.32$, or $\theta \approx 71°$.

104. (a) The particle at A has $r = 0$ with respect to the axis of rotation. The particle at B is $r = L = 0.50$ m from the axis; similarly for the particle directly above A in the figure. The particle diagonally opposite A is a distance $r = \sqrt{2}L = 0.71\text{ m}$ from the axis. Therefore,

$$I = \sum m_i r_i^2 = 2mL^2 + m\left(\sqrt{2}L\right)^2 = 0.20\text{ kg}\cdot\text{m}^2.$$

(b) One imagines rotating the figure (about point A) clockwise by 90° and noting that the center of mass has fallen a distance equal to L as a result. If we let our reference position for gravitational potential be the height of the center of mass at the instant AB swings through vertical orientation, then

$$K_0 + U_0 = K + U \;\Rightarrow\; 0 + (4m)gh_0 = K + 0.$$

Since $h_0 = L = 0.50$ m, we find $K = 3.9$ J. Then, using Eq. 10-34, we obtain

$$K = \frac{1}{2} I_A\omega^2 \Rightarrow \omega = 6.3\text{ rad/s}.$$

105. (a) We apply Eq. 10-18, using the subscript J for the Jeep.

$$\omega = \frac{v_J}{r_J} = \frac{114\text{ km/h}}{0.100\text{ km}}$$

which yields 1140 rad/h or (dividing by 3600) 0.32 rad/s for the value of the angular speed ω.

(b) Since the cheetah has the same angular speed, we again apply Eq. 10-18, using the subscript c for the cheetah.

$$v_c = r_c\omega = (92\,\text{m})\,(1140\ \text{rad/h}) = 1.048 \times 10^5\ \text{m/h} \approx 1.0\times10^2\ \text{km/h}$$

for the cheetah's speed.

106. Using Eq. 10-7 and Eq. 10-18, the average angular acceleration is

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{r\Delta t} = \frac{25-12}{(0.75/2)(6.2)} = 5.6\,\text{rad/s}^2\ .$$

107. (a) Using Eq. 10-1, the angular displacement is

$$\theta = \frac{5.6\,\text{m}}{8.0\times10^{-2}\ \text{m}} = 1.4\times10^2\ \text{rad}\ .$$

(b) We use $\theta = \frac{1}{2}\alpha t^2$ (Eq. 10-13) to obtain *t*:

$$t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2(1.4\times10^2\ \text{rad})}{1.5\,\text{rad/s}^2}} = 14\,\text{s}\ .$$

108. (a) We obtain

$$\omega = \frac{(33.33\ \text{rev/min})\ (2\pi\ \text{rad/rev})}{60\ \text{s/min}} = 3.5\ \text{rad/s}.$$

(b) Using Eq. 10-18, we have $v = r\omega = (15)(3.49) = 52$ cm/s.

(c) Similarly, when r = 7.4 cm we find $v = r\omega$ = 26 cm/s. The goal of this exercise is to observe what is and is not the same at different locations on a body in rotational motion (ω is the same, v is not), as well as to emphasize the importance of radians when working with equations such as Eq. 10-18.

Chapter 11

1. The velocity of the car is a constant

$$\vec{v} = +\left(80 \text{ km/h}\right)(1000 \text{ m/km})(1 \text{ h/3600 s})\,\hat{\text{i}} = (+22\,\text{m/s})\hat{\text{i}},$$

and the radius of the wheel is $r = 0.66/2 = 0.33$ m.

(a) In the car's reference frame (where the lady perceives herself to be at rest) the road is moving toward the rear at $\vec{v}_{\text{road}} = -v = -22\,\text{m/s}$, and the motion of the tire is purely rotational. In this frame, the center of the tire is "fixed" so $v_{\text{center}} = 0$.

(b) Since the tire's motion is only rotational (not translational) in this frame, Eq. 10-18 gives $\vec{v}_{\text{top}} = (+22\,\text{m/s})\hat{\text{i}}$.

(c) The bottom-most point of the tire is (momentarily) in firm contact with the road (not skidding) and has the same velocity as the road: $\vec{v}_{\text{bottom}} = (-22\,\text{m/s})\hat{\text{i}}$. This also follows from Eq. 10-18.

(d) This frame of reference is not accelerating, so "fixed" points within it have zero acceleration; thus, $a_{\text{center}} = 0$.

(e) Not only is the motion purely rotational in this frame, but we also have ω = constant, which means the only acceleration for points on the rim is radial (centripetal). Therefore, the magnitude of the acceleration is

$$a_{\text{top}} = \frac{v^2}{r} = \frac{(22 \text{ m/s})^2}{0.33 \text{ m}} = 1.5\times10^3 \text{ m/s}^2 .$$

(f) The magnitude of the acceleration is the same as in part (d): $a_{\text{bottom}} = 1.5 \times 10^3$ m/s^2.

(g) Now we examine the situation in the road's frame of reference (where the road is "fixed" and it is the car that appears to be moving). The center of the tire undergoes purely translational motion while points at the rim undergo a combination of translational and rotational motions. The velocity of the center of the tire is $\vec{v} = (+22\,\text{m/s})\hat{\text{i}}$.

(h) In part (b), we found $\vec{v}_{\text{top,car}} = +v$ and we use Eq. 4-39:

$$\vec{v}_{\text{top, ground}} = \vec{v}_{\text{top, car}} + \vec{v}_{\text{car, ground}} = v\,\hat{\text{i}} + v\,\hat{\text{i}} = 2v\,\hat{\text{i}}$$

which yields $2v = +44$ m/s.

(i) We can proceed as in part (h) or simply recall that the bottom-most point is in firm contact with the (zero-velocity) road. Either way, the answer is zero.

(j) The translational motion of the center is constant; it does not accelerate.

(k) Since we are transforming between constant-velocity frames of reference, the accelerations are unaffected. The answer is as it was in part (e): 1.5×10^3 m/s^2.

(1) As explained in part (k), $a = 1.5 \times 10^3$ m/s^2.

2. The initial speed of the car is

$$v = \left(80 \text{ km/h}\right)(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 22.2 \text{ m/s}.$$

The tire radius is $R = 0.750/2 = 0.375$ m.

(a) The initial speed of the car is the initial speed of the center of mass of the tire, so Eq. 11-2 leads to

$$\omega_0 = \frac{v_{\text{com}0}}{R} = \frac{22.2 \text{ m/s}}{0.375 \text{ m}} = 59.3 \text{ rad/s}.$$

(b) With $\theta = (30.0)(2\pi) = 188$ rad and $\omega = 0$, Eq. 10-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \;\Rightarrow\; |\alpha| = \frac{(59.3 \text{ rad/s})^2}{2\left(188 \text{ rad}\right)} = 9.31 \text{ rad/s}^2.$$

(c) Equation 11-1 gives $R\theta = 70.7$ m for the distance traveled.

3. **THINK** The work required to stop the hoop is the negative of the initial kinetic energy of the hoop.

EXPRESS From Eq. 11-5, the initial kinetic energy of the hoop is $K_i = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, where $I = mR^2$ is its rotational inertia about the center of mass. Eq. 11-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K_i = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}(mR^2)\left(\frac{v}{R}\right)^2 + \frac{1}{2}mv^2 = mv^2$$

ANALYZE With $m = 140$ kg, and the speed of its center of mass $v = 0.150$ m/s, we find the initial kinetic energy to be

$$K_i = mv^2 = (140\text{ kg})(0.150\text{ m/s})^2 = 3.15\text{ J}$$

which implies that the work required is $W = \Delta K = K_f - K_i = -K_i = -3.15\text{ J}$.

LEARN By the work-kinetic energy theorem, the work done is negative since it decreases the kinetic energy. A rolling body has two types of kinetic energy: rotational and translational.

4. We use the results from section 11.3.

(a) We substitute $I = \frac{2}{5}MR^2$ (Table 10-2(f)) and $a = -0.10g$ into Eq. 11-10:

$$-0.10g = -\frac{g\sin\theta}{1 + \left(\frac{2}{5}MR^2\right)/MR^2} = -\frac{g\sin\theta}{7/5}$$

which yields $\theta = \sin^{-1}(0.14) = 8.0°$.

(b) The acceleration would be more. We can look at this in terms of forces or in terms of energy. In terms of forces, the uphill static friction would then be absent so the downhill acceleration would be due only to the downhill gravitational pull. In terms of energy, the rotational term in Eq. 11-5 would be absent so that the potential energy it started with would simply become $\frac{1}{2}mv^2$ (without it being "shared" with another term) resulting in a greater speed (and, because of Eq. 2-16, greater acceleration).

5. Let M be the mass of the car (presumably including the mass of the wheels) and v be its speed. Let I be the rotational inertia of one wheel and ω be the angular speed of each wheel. The kinetic energy of rotation is

$$K_{\text{rot}} = 4\left(\frac{1}{2}I\omega^2\right),$$

where the factor 4 appears because there are four wheels. The total kinetic energy is given by

$$K = \frac{1}{2}Mv^2 + 4\left(\frac{1}{2}I\omega^2\right).$$

The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{\text{rot}}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2}.$$

For a uniform disk (relative to its center of mass) $I = \frac{1}{2}mR^2$ (Table 10-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 11-2). Thus the numerator of our fraction is

$$4I\omega^2 = 4\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = 2mv^2$$

and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50} = 0.020.$$

The wheel radius cancels from the equations and is not needed in the computation.

6. We plug $a = -3.5$ m/s^2 (where the magnitude of this number was estimated from the "rise over run" in the graph), $\theta = 30°$, $M = 0.50$ kg, and $R = 0.060$ m into Eq. 11-10 and solve for the rotational inertia. We find $I = 7.2 \times 10^{-4}$ kg·m^2.

7. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0 \sin 30° = 3.0$ m (we are using the edge of the roof as our reference level for computing U). Its final kinetic energy (as it leaves the roof) is (Eq. 11-5)

$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 .$$

Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where $R = 0.10$ m. Using $I = \tfrac{1}{2}MR^2$ (Table 10-2(c)), conservation of energy leads to

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2.$$

The mass M cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10\text{ m}}\sqrt{\frac{4}{3}\left(9.8\text{ m/s}^2\right)\left(3.0\text{ m}\right)} = 63\text{ rad/s}.$$

(b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ leftward and $+y$ downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$v_{0x} = v_0 \cos 30° = 5.4\text{ m/s}$$
$$v_{0y} = v_0 \sin 30° = 3.1\text{ m/s}.$$

The projectile motion equations become

$$x = v_{0x}t \quad \text{and} \quad y = v_{0y}t + \frac{1}{2}gt^2 .$$

We first find the time when $y = H = 5.0$ m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gH}}{g} = 0.74\,\text{s}.$$

Then we substitute this into the x equation and obtain $x = (5.4\ \text{m/s})(0.74\ \text{s}) = 4.0\ \text{m}.$

8. (a) Let the turning point be designated P. By energy conservation, the mechanical energy at x = 7.0 m is equal to the mechanical energy at P. Thus, with Eq. 11-5, we have

$$75\ \text{J} = \tfrac{1}{2}mv_p^2 + \tfrac{1}{2}I_{\text{com}}\,\omega_p^2 + U_p .$$

Using item (*f*) of Table 10-2 and Eq. 11-2 (which means, if this is to be a turning point, that $\omega_p = v_p = 0$), we find U_p = 75 J. On the graph, this seems to correspond to x = 2.0 m, and we conclude that there is a turning point (and this is it). The ball, therefore, does not reach the origin.

(b) We note that there is no point (on the graph, to the right of x = 7.0 m) taht is shown "higher" than 75 J, so we suspect that there is no turning point in this direction, and we seek the velocity v_p at x = 13 m. If we obtain a real, nonzero answer, then our suspicion is correct (that it does reach this point P at x = 13 m). By energy conservation, the mechanical energy at x = 7.0 m is equal to the mechanical energy at P. Therefore,

$$75\ \text{J} = \tfrac{1}{2}mv_p^2 + \tfrac{1}{2}I_{\text{com}}\,\omega_p^2 + U_p .$$

Again, using item (*f*) of Table 11-2, Eq. 11-2 (less trivially this time) and U_p = 60 J (from the graph), as well as the numerical data given in the problem, we find v_p = 7.3 m/s.

9. To find where the ball lands, we need to know its speed as it leaves the track (using conservation of energy). Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = M\,gH$. Its final kinetic energy (as it leaves the track) is given by Eq. 11-5:

$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2$$

and its final potential energy is $M\,gh$. Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the track. Since (up to that moment) the ball rolls without sliding we can set $\omega = v/R$. Using $I = \tfrac{2}{5}MR^2$ (Table 10-2(f)), conservation of energy leads to

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 + Mgh$$
$$= \frac{7}{10}Mv^2 + Mgh.$$

The mass M cancels from the equation, and we obtain

$$v = \sqrt{\frac{10}{7}g(H-h)} = \sqrt{\frac{10}{7}(9.8\ \text{m/s}^2)(6.0\ \text{m} - 2.0\ \text{m})} = 7.48\ \text{m/s}.$$

Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ rightward and $+y$ downward. Then (since the initial velocity is purely horizontal) the projectile motion equations become

$$x = vt, \quad y = -\frac{1}{2}gt^2.$$

Solving for x at the time when $y = h$, the second equation gives $t = \sqrt{2h/g}$. Then, substituting this into the first equation, we find

$$x = v\sqrt{\frac{2h}{g}} = (7.48\ \text{m/s})\sqrt{\frac{2(2.0\ \text{m})}{9.8\ \text{m/s}^2}} = 4.8\ \text{m}.$$

10. From $I = \frac{2}{3}MR^2$ (Table 10-2(g)) we find

$$M = \frac{3I}{2R^2} = \frac{3(0.040\ \text{kg}\cdot\text{m}^2)}{2(0.15\ \text{m})^2} = 2.7\ \text{kg}.$$

It also follows from the rotational inertia expression that $\frac{1}{2}I\omega^2 = \frac{1}{3}MR^2\omega^2$. Furthermore, it rolls without slipping, $v_{com} = R\omega$, and we find

$$\frac{K_{rot}}{K_{com} + K_{rot}} = \frac{\frac{1}{3}MR^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{3}MR^2\omega^2}.$$

(a) Simplifying the above ratio, we find $K_{rot}/K = 0.4$. Thus, 40% of the kinetic energy is rotational, or

$$K_{rot} = (0.4)(20\ \text{J}) = 8.0\ \text{J}.$$

(b) From $K_{rot} = \frac{1}{3}M R^2\omega^2 = 8.0\,\text{J}$ (and using the above result for M) we find

$$\omega = \frac{1}{0.15\text{ m}}\sqrt{\frac{3(8.0\text{ J})}{2.7\text{ kg}}} = 20\text{ rad/s}$$

which leads to v_{com} = (0.15 m)(20 rad/s) = 3.0 m/s.

(c) We note that the inclined distance of 1.0 m corresponds to a height h = 1.0 sin 30° = 0.50 m. Mechanical energy conservation leads to

$$K_i = K_f + U_f \quad \Rightarrow \quad 20\,\text{J} = K_f + Mgh$$

which yields (using the values of M and h found above) K_f = 6.9 J.

(d) We found in part (a) that 40% of this must be rotational, so

$$\frac{1}{3}MR^2\omega_f^2 = (0.40)K_f \quad \Rightarrow \quad \omega_f = \frac{1}{0.15\text{ m}}\sqrt{\frac{3(0.40)(6.9\text{ J})}{2.7\text{ kg}}}$$

which yields ω_f = 12 rad/s and leads to

$$v_{com\,f} = R\omega_f = (0.15\text{ m})(12\text{ rad/s}) = 1.8\text{ m/s}.$$

11. With $\vec{F}_{app} = (10\text{ N})\hat{\text{i}}$, we solve the problem by applying Eq. 9-14 and Eq. 11-37.

(a) Newton's second law in the x direction leads to

$$F_{app} - f_s = ma \quad \Rightarrow f_s = 10\,\text{N} - (10\,\text{kg})(0.60\text{ m/s}^2) = 4.0\text{ N}.$$

In unit vector notation, we have $\vec{f}_s = (-4.0\text{ N})\hat{\text{i}}$, which points leftward.

(b) With R = 0.30 m, we find the magnitude of the angular acceleration to be

$$|\alpha| = |a_{com}| / R = 2.0\text{ rad/s}^2,$$

from Eq. 11-6. The only force not directed toward (or away from) the center of mass is $\vec{f}_s$, and the torque it produces is clockwise:

$$|\tau| = I|\alpha| \quad \Rightarrow \quad (0.30\,\text{m})(4.0\,\text{N}) = I(2.0\,\text{rad/s}^2)$$

which yields the wheel's rotational inertia about its center of mass: $I = 0.60\,\text{kg}\cdot\text{m}^2$.

12. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$U_{\text{release}} = K_{\text{top}} + U_{\text{top}} \quad \Rightarrow \quad mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg(2R).$$

Substituting $I = \frac{2}{5}mr^2$ (Table 10-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 11-2), we obtain

$$mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \quad \Rightarrow \quad gh = \frac{7}{10}v_{\text{com}}^2 + 2gR$$

where we have canceled out mass *m* in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is nearly zero. In this case, Newton's second law along the vertical direction (+*y* downward) leads to

$$mg = ma_r \Rightarrow g = \frac{v_{\text{com}}^2}{R - r}$$

where we have used Eq. 10-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R - r$ from the center of the loop). Plugging the result $v_{\text{com}}^2 = g(R - r)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}(g)(R - r) + 2gR$$

which leads to $h = 2.7R - 0.7r \approx 2.7R.$ With R = 14.0 cm , we have

$$h = (2.7)(14.0 \text{ cm}) = 37.8 \text{ cm}.$$

(b) The energy considerations shown above (now with $h = 6R$) can be applied to point *Q* (which, however, is only at a height of *R*) yielding the condition

$$g(6R) = \frac{7}{10}v_{\text{com}}^2 + gR$$

which gives us $v_{\text{com}}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at *Q* leads to

$$N = m\frac{v_{\text{com}}^2}{R - r} = m\frac{50gR}{7(R - r)}$$

which (for $R \gg r$) gives

$$N \approx \frac{50mg}{7} = \frac{50(2.80\times10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{7} = 1.96\times10^{-2}\ \text{N}.$$

(b) The direction is toward the center of the loop.

13. The physics of a rolling object usually requires a separate and very careful discussion (above and beyond the basics of rotation discussed in Chapter 10); this is done in the first three sections of Chapter 11. Also, the normal force on something (which is here the center of mass of the ball) following a circular trajectory is discussed in Section 6-6. Adapting Eq. 6-19 to the consideration of forces at the *bottom* of an arc, we have

$$F_N - Mg = Mv^2/r$$

which tells us (since we are given $F_N = 2Mg$) that the center of mass speed (squared) is $v^2 = gr$, where r is the arc radius (0.48 m) Thus, the ball's angular speed (squared) is

$$\omega^2 = v^2/R^2 = gr/R^2,$$

where R is the ball's radius. Plugging this into Eq. 10-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{\text{com}} = 2MhR^2/r - MR^2 = MR^2[2(0.36/0.48) - 1] .$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2(0.36/0.48) - 1 = 0.50.$$

14. To find the center of mass speed v on the plateau, we use the projectile motion equations of Chapter 4. With $v_{oy} = 0$ (and using "h" for h_2) Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Now, to find the speed v_p at point P, we apply energy conservation, that is, mechanical energy on the plateau is equal to the mechanical energy at P. With Eq. 11-5, we obtain

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{\text{com}}\,\omega^2 + mgh_1 = \tfrac{1}{2}mv_p{}^2 + \tfrac{1}{2}I_{\text{com}}\,\omega_p{}^2 .$$

Using item (*f*) of Table 10-2, Eq. 11-2, and our expression (above) $v^2 = gd^2/2h$, we obtain

$$gd^2/2h + 10gh_1/7 = v_p{}^2$$

which yields (using the values stated in the problem) $v_p = 1.34$ m/s.

15. (a) We choose clockwise as the negative rotational sense and rightward as the positive translational direction. Thus, since this is the moment when it begins to roll smoothly, Eq. 11-2 becomes

$$v_{\text{com}} = -R\omega = (-0.11\ \text{m})\omega.$$

This velocity is positive-valued (rightward) since ω is negative-valued (clockwise) as shown in the figure.

(b) The force of friction exerted on the ball of mass m is $-\mu_k mg$ (negative since it points left), and setting this equal to ma_{com} leads to

$$a_{\text{com}} = -\mu g = -(0.21)(9.8 \text{ m/s}^2) = -2.1 \text{ m/s}^2$$

where the minus sign indicates that the center of mass acceleration points left, opposite to its velocity, so that the ball is decelerating.

(c) Measured about the center of mass, the torque exerted on the ball due to the frictional force is given by $\tau = -\mu mgR$. Using Table 10-2(f) for the rotational inertia, the angular acceleration becomes (using Eq. 10-45)

$$\alpha = \frac{\tau}{I} = \frac{-\mu mgR}{2mR^2/5} = \frac{-5\mu g}{2R} = \frac{-5(0.21)(9.8 \text{ m/s}^2)}{2(0.11 \text{ m})} = -47 \text{ rad/s}^2$$

where the minus sign indicates that the angular acceleration is clockwise, the same direction as ω (so its angular motion is "speeding up'').

(d) The center of mass of the sliding ball decelerates from $v_{\text{com},0}$ to v_{com} during time t according to Eq. 2-11: $v_{\text{com}} = v_{\text{com},0} - \mu gt$. During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 10-12:

$$|\omega| = |\alpha|t = \frac{5\mu gt}{2R} = \frac{v_{\text{com}}}{R}$$

where we have made use of our part (a) result in the last equality. We have two equations involving v_{com}, so we eliminate that variable and find

$$t = \frac{2v_{\text{com},0}}{7\mu g} = \frac{2(8.5 \text{ m/s})}{7(0.21)(9.8 \text{ m/s}^2)} = 1.2 \text{ s}.$$

(e) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{\text{com},0}t - \frac{1}{2}(\mu g)t^2 = (8.5 \text{ m/s})(1.2 \text{ s}) - \frac{1}{2}(0.21)(9.8 \text{ m/s}^2)(1.2 \text{ s})^2 = 8.6 \text{ m}.$$

(f) The center of mass velocity at the time found in part (d) is

$$v_{com} = v_{com,0} - \mu g t = 8.5\text{ m/s} - (0.21)(9.8\text{ m/s}^2)(1.2\text{ s}) = 6.1\text{ m/s}.$$

16. Using energy conservation with Eq. 11-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{com} = 2MhR^2/r - MR^2 = MR^2[2g(H-h)/v^2 - 1] .$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2g(H-h)/v^2 - 1.$$

To proceed further, we need to find the center of mass speed v, which we do using the projectile motion equations of Chapter 4. With $v_{oy} = 0$, Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Plugging this into our expression for β gives

$$2g(H-h)/v^2 - 1 = 4h(H-h)/d^2 - 1.$$

Therefore, with the values given in the problem, we find β = 0.25.

17. **THINK** The yo-yo has both translational and rotational types of motion.

EXPRESS The derivation of the acceleration is given by Eq. 11-13:

$$a_{com} = -\frac{g}{1 + I_{com}/MR_0^2}$$

where M is the mass of the yo-yo, I_{cm} is the rotational inertia and R_0 is the radius of the axel. The positive direction is upward. The time it takes for the yo-yo to reach the end of the string can be found by solving the kinematic equation $y_{com} = \frac{1}{2}a_{com}t^2$.

ANALYZE (a) With $I_{com} = 950\ \text{g}\cdot\text{cm}^2$, M =120 g, R_0 = 0.320 cm and g = 980 cm/s^2, we obtain

$$|a_{com}| = \frac{980\text{ cm/s}^2}{1 + (950\text{ g}\cdot\text{cm}^2)/(120\text{ g})(0.32\text{ cm})^2} = 12.5\text{ cm/s}^2 \approx 13\text{ cm/s}^2.$$

(b) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{com} = \frac{1}{2}a_{com}t^2$. Thus, we set y_{com} = – 120 cm and find

$$t = \sqrt{\frac{2y_{com}}{a_{com}}} = \sqrt{\frac{2(-120\text{cm})}{-12.5\text{ cm/s}^2}} = 4.38\text{ s} \approx 4.4\text{ s}.$$

(c) As the yo-yo reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{com} = a_{com}t = \left(-12.5 \text{ cm/s}^2\right)\left(4.38 \text{s}\right) = -54.8 \text{ cm/s},$$

so its linear speed then is approximately $|v_{com}| = 55$ cm/s.

(d) The translational kinetic energy of the yo-yo is

$$K_{trans} = \frac{1}{2}mv_{com}^2 = \frac{1}{2}(0.120 \text{ kg})(0.548 \text{ m/s})^2 = 1.8\times10^{-2} \text{ J}.$$

(e) The angular velocity is $\omega = -v_{com}/R_0$, so the rotational kinetic energy is

$$K_{rot} = \frac{1}{2}I_{com}\omega^2 = \frac{1}{2}I_{com}\left(\frac{v_{com}}{R_0}\right)^2 = \frac{1}{2}(9.50\times10^{-5} \text{ kg}\cdot\text{m}^2)\left(\frac{0.548 \text{ m/s}}{3.2\times10^{-3} \text{ m}}\right)^2$$
$$= 1.393 \text{ J} \approx 1.4 \text{ J}$$

(f) The angular speed is

$$\omega = \frac{|v_{com}|}{R_0} = \frac{0.548 \text{ m/s}}{3.2\times10^{-3} \text{ m}} = 1.7\times10^2 \text{ rad/s} = 27 \text{ rev/s}.$$

LEARN As the yo-yo rolls down, its gravitational potential energy gets converted into both translational kinetic energy as well as rotational kinetic energy of the wheel. To show that the total energy remains conserved, we note that the initial energy is

$$U_i = Mgy_i = (0.120 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m}) = 1.411 \text{ J}$$

which is equal to the sum of K_{trans} (= 0.018 J) and K_{rot} (= 1.393 J).

18. (a) The derivation of the acceleration is found in § 11-4; Eq. 11-13 gives

$$a_{com} = -\frac{g}{1 + I_{com}/MR_0^2}$$

where the positive direction is upward. We use $I_{com} = MR^2/2$ where the radius is R = 0.32 m and M = 116 kg is the *total* mass (thus including the fact that there are two disks) and obtain

$$a = -\frac{g}{1 + (MR^2/2)/MR_0^2} = \frac{g}{1 + \left(R/R_0\right)^2/2}$$

which yields $a = -g/51$ upon plugging in $R_0 = R/10 = 0.032$ m. Thus, the magnitude of the center of mass acceleration is 0.19 m/s^2.

(b) As observed in §11-4, our result in part (a) applies to both the descending and the rising yo-yo motions.

(c) The external forces on the center of mass consist of the cord tension (upward) and the pull of gravity (downward). Newton's second law leads to

$$T - Mg = ma \Rightarrow T = M\left(g - \frac{g}{51}\right) = 1.1 \times 10^3 \text{ N}.$$

(d) Our result in part (c) indicates that the tension is well below the ultimate limit for the cord.

(e) As we saw in our acceleration computation, all that mattered was the ratio R/R_0 (and, of course, g). So if it's a scaled-up version, then such ratios are unchanged and we obtain the same result.

(f) Since the tension also depends on mass, then the larger yo-yo will involve a larger cord tension.

19. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

With (using SI units) $x = 0$, $y = -4.0$, $z = 5.0$, $F_x = 0$, $F_y = -2.0$, and $F_z = 3.0$ (these latter terms being the individual forces that contribute to the net force), the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = (-2.0\,\text{N}\cdot\text{m})\hat{\text{i}}.$$

20. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

(a) In the above expression, we set (with SI units understood) $x = -2.0$, $y = 0$, $z = 4.0$, $F_x = 6.0$, $F_y = 0$, and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (24\,\text{N}\cdot\text{m})\hat{\text{j}}$.

(b) The values are just as in part (a) with the exception that now $F_x = -6.0$. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-24\,\text{N}\cdot\text{m})\hat{\text{j}}$.

(c) In the above expression, we set $x = -2.0$, $y = 0$, $z = 4.0$, $F_x = 0$, $F_y = 0$, and $F_z = 6.0$. We get $\vec{\tau} = \vec{r} \times \vec{F} = (12\,\text{N}\cdot\text{m})\hat{\text{j}}$.

(d) The values are just as in part (c) with the exception that now F_z = –6.0. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-12\,\text{N}\cdot\text{m})\hat{\text{j}}$.

21. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

(a) In the above expression, we set (with SI units understood) $x = 0$, $y = -4.0$, $z = 3.0$, $F_x = 2.0$, $F_y = 0$, and $F_z = 0$. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(6.0\hat{\text{j}} + 8.0\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

This has magnitude $\sqrt{(6.0\ \text{N}\cdot\text{m})^2 + (8.0\ \text{N}\cdot\text{m})^2} = 10\ \text{N}\cdot\text{m}$ and is seen to be parallel to the yz plane. Its angle (measured counterclockwise from the $+y$ direction) is $\tan^{-1}\left(8/6\right) = 53°$.

(b) In the above expression, we set $x = 0$, $y = -4.0$, $z = 3.0$, $F_x = 0$, $F_y = 2.0$, and $F_z = 4.0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (-22\,\text{N}\cdot\text{m})\hat{\text{i}}$. The torque has magnitude $22\ \text{N}\cdot\text{m}$ and points in the $-x$ direction.

22. Equation 11-14 (along with Eq. 3-30) gives

$$\vec{\tau} = \vec{r} \times \vec{F} = 4.00\hat{\text{i}} + (12.0 + 2.00F_x)\hat{\text{j}} + (14.0 + 3.00F_x)\hat{\text{k}}$$

with SI units understood. Comparing this with the known expression for the torque (given in the problem statement), we see that F_x must satisfy two conditions:

$$12.0 + 2.00F_x = 2.00 \quad \text{and} \quad 14.0 + 3.00F_x = -1.00.$$

The answer (F_x = –5.00 N) satisfies both conditions.

23. We use the notation $\vec{r}\,'$ to indicate the vector pointing from the axis of rotation directly to the position of the particle. If we write $\vec{r}\,' = x'\,\hat{\text{i}} + y'\,\hat{\text{j}} + z'\,\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\text{i}} + \left(z'F_x - x'F_z\right)\hat{\text{j}} + \left(x'F_y - y'F_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$. Dropping the primes in the above expression, we set (with SI units understood) $x = 0$, $y = 0.5$, $z = -2.0$, $F_x = 2.0$, $F_y = 0$, and $F_z = -3.0$. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(-1.5\hat{\text{i}} - 4.0\hat{\text{j}} - 1.0\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\hat{\text{i}} - 3.0\hat{\text{k}}$. Therefore, in the above expression, we set $x' = -2.0, y' = 0.5, z' = 1.0, F_x = 2.0, F_y = 0$, and $F_z = -3.0$. Thus, we obtain

$$\vec{\tau} = \vec{r}\,' \times \vec{F} = \left(-1.5\hat{\text{i}} - 4.0\hat{\text{j}} - 1.0\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

24. If we write $\vec{r}\,' = x'\hat{\text{i}} + y'\hat{\text{j}} + z'\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\text{i}} + \left(z'F_x - x'F_z\right)\hat{\text{j}} + \left(x'F_y - y'F_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3.0\hat{\text{i}} - 2.0\hat{\text{j}} + 4.0\hat{\text{k}}$, and $\vec{F} = \vec{F}_1$. Thus, dropping the prime in the above expression, we set (with SI units understood) $x = 3.0$, $y = -2.0$, $z = 4.0$, $F_x = 3.0$, $F_y = -4.0$, and $F_z = 5.0$. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F}_1 = \left(6.0\hat{\text{i}} - 3.0\hat{\text{j}} - 6.0\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

(b) This is like part (a) but with $\vec{F} = \vec{F}_2$. We plug in $F_x = -3.0$, $F_y = -4.0$, and $F_z = -5.0$ and obtain

$$\vec{\tau} = \vec{r} \times \vec{F}_2 = \left(26\hat{\text{i}} + 3.0\hat{\text{j}} - 18\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

(c) We can proceed in either of two ways. We can add (vectorially) the answers from parts (a) and (b), or we can first add the two force vectors and then compute $\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$ (these total force components are computed in the next part). The result is

$$\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right) = \left(32\hat{\text{i}} - 24\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

(d) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 3.0\hat{\text{i}} + 2.0\hat{\text{j}} + 4.0\hat{\text{k}}$. Therefore, in the above expression, we set $x' = 0, y' = -4.0, z' = 0$, and

$$F_x = 3.0 - 3.0 = 0$$
$$F_y = -4.0 - 4.0 = -8.0$$
$$F_z = 5.0 - 5.0 = 0.$$

We get $\vec{\tau} = \vec{r}\,' \times \left(\vec{F}_1 + \vec{F}_2\right) = 0$.

25. **THINK** We take the cross product of $\vec{r}$ and $\vec{F}$ to find the torque $\vec{\tau}$ on a particle.

EXPRESS If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$ and $\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} + F_z\hat{\text{k}}$, then (using Eq. 3-30) the general expression for torque can be written as

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

ANALYZE (a) With $\vec{r} = (3.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}$ and $\vec{F} = (-8.0\text{ N})\hat{\text{i}} + (6.0\text{ N})\hat{\text{j}}$, we have

$$\vec{\tau} = \left[(3.0\text{m})(6.0\text{N}) - (4.0\text{m})(-8.0\text{N})\right]\hat{\text{k}} = (50\,\text{N}\cdot\text{m})\,\hat{\text{k}}.$$

(b) To find the angle ϕ between $\vec{r}$ and $\vec{F}$, we use Eq. 3-27: $|\vec{r} \times \vec{F}| = rF\sin\phi$. Now $r = \sqrt{x^2 + y^2} = 5.0$ m and $F = \sqrt{F_x^2 + F_y^2} = 10$ N. Thus,

$$rF = (5.0\text{ m})(10\text{ N}) = 50\text{ N}\cdot\text{m},$$

the same as the magnitude of the vector product calculated in part (a). This implies sin ϕ = 1 and ϕ = 90°.

LEARN Our result (ϕ = 90°) implies that $\vec{r}$ and $\vec{F}$ are perpendicular to each other. A useful check is to show that their dot product is zero. This is indeed the case:

$$\begin{aligned}\vec{r}\cdot\vec{F} &= [(3.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}]\cdot[(-8.0\text{ N})\hat{\text{i}} + (6.0\text{ N})\hat{\text{j}}] \\ &= (3.0\text{ m})(-8.0\text{ N}) + (4.0\text{ m})(6.0\text{ N}) = 0.\end{aligned}$$

26. We note that the component of $\vec{v}$ perpendicular to $\vec{r}$ has magnitude v sin θ_2 where θ_2= 30°. A similar observation applies to $\vec{F}$.

(a) Eq. 11-20 leads to

$$\ell = rmv_\perp = (3.0\text{ m})(2.0\text{ kg})(4.0\text{ m/s})\sin 30° = 12\text{ kg}\cdot\text{m}^2/\text{s}.$$

(b) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{p}$ points out of the page, or along the +z axis, perpendicular to the plane of the figure.

(c) Similarly, Eq. 10-38 leads to

$$\tau = rF\sin\theta_2 = (3.0\text{ m})(2.0\text{ N})\sin 30° = 3.0\text{N}\cdot\text{m}.$$

(d) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{F}$ is also out of the page, or along the +z axis, perpendicular to the plane of the figure.

27. **THINK** We evaluate the cross product $\vec{\ell} = m\vec{r}\times\vec{v}$ to find the angular momentum $\vec{\ell}$ on the object, and the cross product of $\vec{r}\times\vec{F}$ for the torque $\vec{\tau}$.

EXPRESS Let $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$ be the position vector of the object, $\vec{v} = v_x\hat{\text{i}} + v_y\hat{\text{j}} + v_z\hat{\text{k}}$ its velocity vector, and m its mass. The cross product of $\vec{r}$ and $\vec{v}$ is (using Eq. 3-30)

$$\vec{r}\times\vec{v} = \left(yv_z - zv_y\right)\hat{\text{i}} + \left(zv_x - xv_z\right)\hat{\text{j}} + \left(xv_y - yv_x\right)\hat{\text{k}}.$$

Since only the x and z components of the position and velocity vectors are nonzero (i.e., $y = 0$ and $v_y = 0$), the above expression becomes $\vec{r}\times\vec{v} = \left(-xv_z + zv_z\right)\hat{\text{j}}$. As for the torque, writing $\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} + F_z\hat{\text{k}}$, we find $\vec{r}\times\vec{F}$ to be

$$\vec{\tau} = \vec{r}\times\vec{F} = \left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

ANALYZE (a) With $\vec{r} = (2.0\text{ m})\hat{\text{i}} - (2.0\text{ m})\hat{\text{k}}$ and $\vec{v} = (-5.0\text{ m/s})\hat{\text{i}} + (5.0\text{ m/s})\hat{\text{k}}$, in unit-vector notation, the angular momentum of the object is

$$\vec{\ell} = m\left(-xv_z + zv_x\right)\hat{\text{j}} = \left(0.25\text{ kg}\right)\left(-\left(2.0\text{ m}\right)\left(5.0\text{ m/s}\right) + \left(-2.0\text{ m}\right)\left(-5.0\text{ m/s}\right)\right)\hat{\text{j}} = 0.$$

(b) With x = 2.0 m, z = –2.0 m, F_y = 4.0 N and all other components zero, the expression above yields

$$\vec{\tau} = \vec{r}\times\vec{F} = (8.0\text{ N}\cdot\text{m})\hat{\text{i}} + (8.0\text{ N}\cdot\text{m})\hat{\text{k}}\,.$$

LEARN The fact that $\vec{\ell} = 0$ implies that $\vec{r}$ and $\vec{v}$ are parallel to each other ($\vec{r}\times\vec{v} = 0$). Using $\tau = |\vec{r}\times\vec{F}| = rF\sin\phi$, we find the angle between $\vec{r}$ and $\vec{F}$ to be

$$\sin\phi = \frac{\tau}{rF} = \frac{8\sqrt{2}\text{ N}\cdot\text{m}}{(2\sqrt{2}\text{ m})(4.0\text{ N})} = 1 \;\Rightarrow\; \phi = 90°$$

That is, $\vec{r}$ and $\vec{F}$ are perpendicular to each other.

28. If we write $\vec{r}\,' = x'\,\hat{\text{i}} + y'\,\hat{\text{j}} + z'\,\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' = \vec{v}$ is equal to

$$\left(y'v_z - z'v_y\right)\hat{\text{i}} + \left(z'v_x - x'v_z\right)\hat{\text{j}} + \left(x'v_y - y'v_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3.0\hat{\text{i}} - 4.0\hat{\text{j}}$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3.0, y = -4.0, z = 0, v_x = 30, v_y = 60$, and v_z = 0. Then (with m = 2.0 kg) we obtain

$$\vec{\ell} = m(\vec{r} \times \vec{v}) = (6.0\times10^2\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_\text{o}$ where $\vec{r}_\text{o} = -2.0\hat{\text{i}} - 2.0\hat{\text{j}}$. Therefore, in the above expression, we set $x' = 5.0, y' = -2.0, z' = 0, v_x = 30, v_y = 60$, and $v_z = 0$. We get

$$\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = (7.2\times10^2\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

29. For the 3.1 kg particle, Eq. 11-21 yields

$$\ell_1 = r_{\perp 1} m v_1 = (2.8\ \text{m})(3.1\ \text{kg})(3.6\ \text{m/s}) = 31.2\ \text{kg}\cdot\text{m}^2/\text{s}.$$

Using the right-hand rule for vector products, we find this $(\vec{r}_1 \times \vec{p}_1)$ is out of the page, or along the +*z* axis, perpendicular to the plane of Fig. 11-41. And for the 6.5 kg particle, we find

$$\ell_2 = r_{\perp 2} m v_2 = (1.5\ \text{m})(6.5\ \text{kg})(2.2\ \text{m/s}) = 21.4\ \text{kg}\cdot\text{m}^2/\text{s}.$$

And we use the right-hand rule again, finding that this $(\vec{r}_2 \times \vec{p}_2)$ is into the page, or in the –*z* direction.

(a) The two angular momentum vectors are in opposite directions, so their vector sum is the *difference* of their magnitudes: $L = \ell_1 - \ell_2 = 9.8\ \text{kg}\cdot\text{m}^2/\text{s}$.

(b) The direction of the net angular momentum is along the +*z* axis.

30. (a) The acceleration vector is obtained by dividing the force vector by the (scalar) mass:

$$\vec{a} = \vec{F}/m = (3.00\ \text{m/s}^2)\hat{\text{i}} - (4.00\ \text{m/s}^2)\hat{\text{j}} + (2.00\ \text{m/s}^2)\hat{\text{k}}.$$

(b) Use of Eq. 11-18 leads directly to

$$\vec{L} = (42.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{i}} + (24.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{j}} + (60.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(c) Similarly, the torque is

$$\vec{\tau} = \vec{r} \times \vec{F} = (-8.00\ \text{N}\cdot\text{m})\hat{\text{i}} - (26.0\ \text{N}\cdot\text{m})\hat{\text{j}} - (40.0\ \text{N}\cdot\text{m})\hat{\text{k}}.$$

(d) We note (using the Pythagorean theorem) that the magnitude of the velocity vector is 7.35 m/s and that of the force is 10.8 N. The dot product of these two vectors is $\vec{v}\cdot\vec{F} = -48$ (in SI units). Thus, Eq. 3-20 yields

$$\theta = \cos^{-1}[-48.0/(7.35 \times 10.8)] = 127°.$$

31. (a) Since the speed is (momentarily) zero when it reaches maximum height, the angular momentum is zero then.

(b) With the convention (used in several places in the book) that clockwise sense is to be associated with the negative sign, we have $L = -r_\perp m v$ where $r_\perp = 2.00$ m, $m = 0.400$ kg, and v is given by free-fall considerations (as in Chapter 2). Specifically, y_{max} is determined by Eq. 2-16 with the speed at max height set to zero; we find $y_{max} = v_o^2/2g$ where $v_o = 40.0$ m/s. Then with $y = \frac{1}{2} y_{max}$, Eq. 2-16 can be used to give $v = v_o/\sqrt{2}$. In this way we arrive at $L = -22.6\ \text{kg}\cdot\text{m}^2/\text{s}$.

(c) As mentioned in the previous part, we use the minus sign in writing $\tau = -r_\perp F$ with the force F being equal (in magnitude) to mg. Thus, $\tau = -7.84\ \text{N}\cdot\text{m}$.

(d) Due to the way $r_\perp$ is defined it does not matter how far up the ball is. The answer is the same as in part (c), $\tau = -7.84\ \text{N}\cdot\text{m}$.

32. The rate of change of the angular momentum is

$$\frac{d\vec{\ell}}{dt} = \vec{\tau}_1 + \vec{\tau}_2 = (2.0\ \text{N}\cdot\text{m})\hat{\text{i}} - (4.0\ \text{N}\cdot\text{m})\hat{\text{j}}.$$

Consequently, the vector $d\vec{\ell}/dt$ has a magnitude $\sqrt{(2.0\ \text{N}\cdot\text{m})^2 + (-4.0\ \text{N}\cdot\text{m})^2} = 4.5\ \text{N}\cdot\text{m}$ and is at an angle θ (in the xy plane, or a plane parallel to it) measured from the positive x axis, where

$$\theta = \tan^{-1}\left(\frac{-4.0\ \text{N}\cdot\text{m}}{2.0\ \text{N}\cdot\text{m}}\right) = -63°,$$

the negative sign indicating that the angle is measured clockwise as viewed "from above" (by a person on the $+z$ axis).

33. **THINK** We evaluate the cross product $\vec{\ell} = m\vec{r}\times\vec{v}$ to find the angular momentum $\vec{\ell}$ on the particle, and the cross product of $\vec{r}\times\vec{F}$ for the torque $\vec{\tau}$.

EXPRESS Let $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$ be the position vector of the object, $\vec{v} = v_x\hat{\text{i}} + v_y\hat{\text{j}} + v_z\hat{\text{k}}$ its velocity vector, and m its mass. The cross product of $\vec{r}$ and $\vec{v}$ is

$$\vec{r}\times\vec{v} = (yv_z - zv_y)\hat{\text{i}} + (zv_x - xv_z)\hat{\text{j}} + (xv_y - yv_x)\hat{\text{k}}.$$

The angular momentum is given by the vector product $\vec{\ell} = m\vec{r}\times\vec{v}$. As for the torque, writing $\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} + F_z\hat{\text{k}}$, then we find $\vec{r}\times\vec{F}$ to be

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

ANALYZE (a) Substituting m = 3.0 kg, x = 3.0 m, y = 8.0 m, z = 0, v_x = 5.0 m/s, $v_y = -6.0$ m/s and v_z = 0 into the above expression, we obtain

$$\vec{\ell} = (3.0\text{ kg})\,[(3.0\text{ m})(-6.0\text{ m/s}) - (8.0\text{ m})(5.0\text{ m/s})]\hat{\mathrm{k}} = (-174\text{ kg}\cdot\text{m}^2/\text{s})\hat{\mathrm{k}}.$$

(b) Given that $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}}$ and $\vec{F} = F_x\hat{\mathrm{i}}$, the corresponding torque is

$$\vec{\tau} = \left(x\hat{\mathrm{i}} + y\hat{\mathrm{j}}\right) \times \left(F_x\hat{\mathrm{i}}\right) = -yF_x\hat{\mathrm{k}}.$$

Substituting the values given, we find

$$\vec{\tau} = -(8.0\text{m})(-7.0\text{N})\,\hat{\mathrm{k}} = (56\text{ N}\cdot\text{m})\hat{\mathrm{k}}.$$

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the angular momentum is 56 $\text{kg}\cdot\text{m}^2/\text{s}^2$, in the positive z direction.

LEARN The direction of $\vec{\ell}$ is in the −z-direction, which is perpendicular to both $\vec{r}$ and $\vec{v}$. Similarly, the torque $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$ (i.e, $\vec{\tau}$ is in the direction normal to the plane formed by $\vec{r}$ and $\vec{F}$).

34. We use a right-handed coordinate system with $\hat{\mathrm{k}}$ directed out of the xy plane so as to be consistent with counterclockwise rotation (and the right-hand rule). Thus, all the angular momenta being considered are along the $-\hat{\mathrm{k}}$ direction; for example, in part (b) $\vec{\ell} = -4.0t^2\,\hat{\mathrm{k}}$ in SI units. We use Eq. 11-23.

(a) The angular momentum is constant so its derivative is zero. There is no torque in this instance.

(b) Taking the derivative with respect to time, we obtain the torque:

$$\vec{\tau} = \frac{d\vec{\ell}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\frac{dt^2}{dt} = (-8.0t\text{ N}\cdot\text{m})\hat{\mathrm{k}}\,.$$

This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$.

(c) With $\vec{\ell} = (-4.0\sqrt{t})\hat{\mathrm{k}}$ in SI units, the torque is

$$\vec{\tau} = \left(-4.0\hat{k}\right)\frac{d\sqrt{t}}{dt} = \left(-4.0\hat{k}\right)\left(\frac{1}{2\sqrt{t}}\right) = \left(-\frac{2.0}{\sqrt{t}}\hat{k}\right)\text{N}\cdot\text{m}.$$

This vector points in the $-\hat{k}$ direction (causing the clockwise motion to speed up) for all $t > 0$ (and it is undefined for $t < 0$).

(d) Finally, we have

$$\vec{\tau} = \left(-4.0\hat{k}\right)\frac{dt^{-2}}{dt} = \left(-4.0\hat{k}\right)\left(\frac{-2}{t^3}\right) = \left(\frac{8.0}{t^3}\hat{k}\right)\text{N}\cdot\text{m}.$$

This vector points in the $+\hat{k}$ direction (causing the initially clockwise motion to slow down) for all $t > 0$.

35. (a) We note that

$$\vec{v} = \frac{d\vec{r}}{dt} = 8.0t\,\hat{i} - (2.0 + 12t)\hat{j}$$

with SI units understood. From Eq. 11-18 (for the angular momentum) and Eq. 3-30, we find the particle's angular momentum is $8t^2\hat{k}$. Using Eq. 11-23 (relating its time-derivative to the (single) torque) then yields $\vec{\tau} = (48t\,\hat{k})$ N·m.

(b) From our (intermediate) result in part (a), we see the angular momentum increases in proportion to t^2.

36. We relate the motions of the various disks by examining their linear speeds (using Eq. 10-18). The fact that the linear speed at the rim of disk A must equal the linear speed at the rim of disk C leads to $\omega_A = 2\omega_C$. The fact that the linear speed at the hub of disk A must equal the linear speed at the rim of disk B leads to $\omega_A = \frac{1}{2}\omega_B$. Thus, $\omega_B = 4\omega_C$. The ratio of their angular momenta depend on these angular velocities as well as their rotational inertias (see item (c) in Table 11-2), which themselves depend on their masses. If h is the thickness and ρ is the density of each disk, then each mass is $\rho\pi R^2 h$. Therefore,

$$\frac{L_C}{L_B} = \frac{(½)\rho\pi R_C^2\, h\, R_C^2\, \omega_C}{(½)\rho\pi R_B^2\, h\, R_B^2\, \omega_B} = 1024.$$

37. (a) A particle contributes mr_2 to the rotational inertia. Here r is the distance from the origin O to the particle. The total rotational inertia is

$$I = m(3d)^2 + m(2d)^2 + m(d)^2 = 14md^2 = 14(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2$$
$$= 4.6\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

(b) The angular momentum of the middle particle is given by $L_m = I_m\omega$, where $I_m = 4md^2$ is its rotational inertia. Thus

$$L_m = 4md^2\omega = 4(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2(0.85\text{ rad/s}) = 1.1\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}.$$

(c) The total angular momentum is

$$I\omega = 14md^2\omega = 14(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2(0.85\text{ rad/s}) = 3.9\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}.$$

38. (a) Equation 10-34 gives $\alpha = \tau/I$ and Eq. 10-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at $t = 0.033$ s is

$$I\omega = \tau\, t = (16\,\text{N}\cdot\text{m})(0.033\,\text{s}) = 0.53\,\text{kg}\cdot\text{m}^2/\text{s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau\, t}{I} = \frac{(16\text{ N}\cdot\text{m})(0.033\text{ s})}{1.2\times10^{-3}\text{ kg}\cdot\text{m}^2} = 440\,\text{rad/s}$$

which we convert as follows:

$$\omega = (440\text{ rad/s})(60\text{ s/min})(1\text{ rev}/2\pi\text{ rad}) \approx 4.2\times10^3\text{ rev/min}.$$

39. **THINK** A non-zero torque is required to change the angular momentum of the flywheel. We analyze the rotational motion of the wheel using the equations given in Table 10-1.

EXPRESS Since the torque is equal to the rate of change of angular momentum, $\tau = dL/dt$, the average torque acting during any interval Δt is simply given by $\tau_{\text{avg}} = (L_f - L_i)/\Delta t$, where L_i is the initial angular momentum and L_f is the final angular momentum. For uniform angular acceleration, the angle turned is $\theta = \omega_0 t + \alpha t^2/2$, and the work done on the wheel is $W = \tau\theta$.

ANALYZE (a) Substituting the values given, the average torque is

$$\tau_{\text{avg}} = \frac{L_f - L_i}{\Delta t} = \frac{(0.800\text{ kg}\cdot\text{m}^2/\text{s}) - (3.00\text{ kg}\cdot\text{m}^2/\text{s})}{1.50\text{ s}} = -1.47\text{ N}\cdot\text{m},$$

or $|\tau_{\text{avg}}| = 1.47\text{ N}\cdot\text{m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) If the angular acceleration α is uniform, so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\theta = \frac{L_i t + \tau\, t^2/2}{I} = \frac{(3.00\,\text{kg}\cdot\text{m}^2/\text{s})(1.50\ \text{s}) + (-1.467\,\text{N}\cdot\text{m})(1.50\ \text{s})^2/2}{0.140\,\text{kg}\cdot\text{m}^2} = 20.4\,\text{rad}.$$

(c) Using the values of τ and θ found above, we find the work done on the wheel to be

$$W = \tau\theta = (-1.47\,\text{N}\cdot\text{m})(20.4\,\text{rad}) = -29.9\ \text{J}.$$

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

$$P_{\text{avg}} = -\frac{W}{\Delta t} = -\frac{-29.9\ \text{J}}{1.50\,\text{s}} = 19.9\,\text{W}.$$

LEARN An alternative way to calculate the work done on the wheel is to apply the work-kinetic energy theorem:

$$W = \Delta K = K_f - K_i = \frac{1}{2}I(\omega_f^2 - \omega_i^2) = \frac{1}{2}I\left[\left(\frac{L_f}{I}\right)^2 - \left(\frac{L_i}{I}\right)^2\right] = \frac{L_f^2 - L_i^2}{2I}$$

Substituting the values given, we have

$$W = \frac{L_f^2 - L_i^2}{2I} = \frac{(0.800\ \text{kg}\cdot\text{m}^2/\text{s})^2 - (3.00\ \text{kg}\cdot\text{m}^2/\text{s})^2}{2(0.140\,\text{kg}\cdot\text{m}^2)} = -29.9\ \text{J}$$

which agrees with that calculated in part (c).

40. Torque is the time derivative of the angular momentum. Thus, the change in the angular momentum is equal to the time integral of the torque. With $\tau = (5.00 + 2.00t)\ \text{N}\cdot\text{m}$, the angular momentum (in units $\text{kg}\cdot\text{m}^2/\text{s}$) as a function of time is

$$L(t) = \int \tau\, dt = \int (5.00 + 2.00t)\, dt = L_0 + 5.00t + 1.00t^2.$$

Since $L = 5.00\ \text{kg}\cdot\text{m}^2/\text{s}$ when $t = 1.00\ \text{s}$, the integration constant is $L_0 = -1$. Thus, the complete expression of the angular momentum is

$$L(t) = -1 + 5.00t + 1.00t^2.$$

At $t = 3.00\ \text{s}$, we have $L(t = 3.00) = -1 + 5.00(3.00) + 1.00(3.00)^2 = 23.0\ \text{kg}\cdot\text{m}^2/\text{s}$.

41. (a) For the hoop, we use Table 10-2(h) and the parallel-axis theorem to obtain

$$I_1 = I_{\text{com}} + mh^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 .$$

Of the thin bars (in the form of a square), the member along the rotation axis has (approximately) no rotational inertia about that axis (since it is thin), and the member farthest from it is very much like it (by being parallel to it) except that it is displaced by a distance h; it has rotational inertia given by the parallel axis theorem:

$$I_2 = I_{\text{com}} + mh^2 = 0 + mR^2 = mR^2 .$$

Now the two members of the square perpendicular to the axis have the same rotational inertia (that is $I_3 = I_4$). We find I_3 using Table 10-2(e) and the parallel-axis theorem:

$$I_3 = I_{\text{com}} + mh^2 = \frac{1}{12}mR^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{3}mR^2 .$$

Therefore, the total rotational inertia is

$$I_1 + I_2 + I_3 + I_4 = \frac{19}{6}mR^2 = 1.6\,\text{kg}\cdot\text{m}^2 .$$

(b) The angular speed is constant:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.5} = 2.5\,\text{rad/s}.$$

Thus, $L = I_{\text{total}}\omega = 4.0\,\text{kg}\cdot\text{m}^2/\text{s}$.

42. The results may be found by integrating Eq. 11-29 with respect to time, keeping in mind that $\vec{L}_i = 0$ and that the integration may be thought of as "adding the areas" under the line-segments (in the plot of the torque versus time, with "areas" under the time axis contributing negatively). It is helpful to keep in mind, also, that the area of a triangle is $\frac{1}{2}$ (base)(height).

(a) We find that $\vec{L} = 24\ \text{kg}\cdot\text{m}^2/\text{s}$ at $t = 7.0$ s.

(b) Similarly, $\vec{L} = 1.5\ \text{kg}\cdot\text{m}^2/\text{s}$ at $t = 20$ s.

43. We assume that from the moment of grabbing the stick onward, they maintain rigid postures so that the system can be analyzed as a symmetrical rigid body with center of mass midway between the skaters.

(a) The total linear momentum is zero (the skaters have the same mass and equal and opposite velocities). Thus, their center of mass (the middle of the 3.0 m long stick) remains fixed and they execute circular motion (of radius $r = 1.5$ m) about it.

(b) Using Eq. 10-18, their angular velocity (counterclockwise as seen in Fig. 11-47) is

$$\omega = \frac{v}{r} = \frac{1.4 \text{ m/s}}{1.5 \text{ m}} = 0.93 \text{ rad/s}.$$

(c) Their rotational inertia is that of two particles in circular motion at r = 1.5 m, so Eq. 10-33 yields

$$I = \sum mr^2 = 2(50 \text{ kg})(1.5 \text{ m})^2 = 225 \text{ kg}\cdot\text{m}^2.$$

Therefore, Eq. 10-34 leads to

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(225 \text{ kg}\cdot\text{m}^2)(0.93 \text{ rad/s})^2 = 98 \text{ J}.$$

(d) Angular momentum is conserved in this process. If we label the angular velocity found in part (a) ω_i and the rotational inertia of part (b) as I_i, we have

$$I_i\omega_i = (225 \text{ kg}\cdot\text{m}^2)(0.93 \text{ rad/s}) = I_f\omega_f.$$

The final rotational inertia is $\sum mr_f^2$ where r_f = 0.5 m so $I_f = 25 \text{ kg}\cdot\text{m}^2$. Using this value, the above expression gives ω_f = 8.4 rad/s.

(e) We find

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(25 \text{ kg}\cdot\text{m}^2)(8.4 \text{ rad/s})^2 = 8.8\times10^2 \text{ J}.$$

(f) We account for the large increase in kinetic energy (part (e) minus part (c)) by noting that the skaters do a great deal of work (converting their internal energy into mechanical energy) as they pull themselves closer — "fighting" what appears to them to be large "centrifugal forces" trying to keep them apart.

44. So that we don't get confused about ± signs, we write the angular *speed* to the lazy Susan as $|\omega|$ and reserve the ω symbol for the angular velocity (which, using a common convention, is negative-valued when the rotation is clockwise). When the roach "stops" we recognize that it comes to rest relative to the lazy Susan (not relative to the ground).

(a) Angular momentum conservation leads to

$$mvR + I\omega_0 = (mR^2 + I)\omega_f$$

which we can write (recalling our discussion about angular speed versus angular velocity) as

$$mvR - I|\omega_0| = -(mR^2 + I)|\omega_f|.$$

We solve for the final angular speed of the system:

$$|\omega_f| = \frac{mvR - I|\omega_0|}{mR^2 + I} = \frac{(0.17\text{ kg})(2.0\text{ m/s})(0.15\text{ m}) - (5.0\times10^{-3}\text{ kg}\cdot\text{m}^2)(2.8\text{ rad/s})}{(5.0\times10^{-3}\text{ kg}\cdot\text{m}^2) + (0.17\text{ kg})(0.15\text{ m})^2}$$
$$= 4.2\text{ rad/s}.$$

(b) No, $K_f \neq K_i$ and — if desired — we can solve for the difference:

$$K_i - K_f = \frac{mI}{2}\,\frac{v^2 + \omega_0^2 R^2 + 2Rv|\omega_0|}{mR^2 + I}$$

which is clearly positive. Thus, some of the initial kinetic energy is "lost" — that is, transferred to another form. And the culprit is the roach, who must find it difficult to stop (and "internalize" that energy).

45. **THINK** No external torque acts on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved.

EXPRESS Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. Then $I_i\omega_i = I_f\omega_f$ by angular momentum conservation. The kinetic energy (of rotational nature) is given by $K = I\omega^2/2$.

ANALYZE (a) The final angular momentum of the system is

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{6.0\text{ kg}\cdot\text{m}^2}{2.0\text{ kg}\cdot\text{m}^2}\right)(1.2\,\text{rev/s}) = 3.6\,\text{rev/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_i\omega_i^2$, and the final kinetic energy is $K_f = \frac{1}{2}I_f\omega_f^2$, so that their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{(2.0\,\text{kg}\cdot\text{m}^2)(3.6\,\text{rev/s})^2/2}{(6.0\,\text{kg}\cdot\text{m}^2)(1.2\,\text{rev/s})^2/2} = 3.0.$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's internal energy.

LEARN The work done by the person is equal to the change in kinetic energy:

$$W = K_f - K_i = 3K_i - K_i = 2K_i = I_i\omega_i^2 = (6.0\,\text{kg}\cdot\text{m}^2)(2\pi\cdot 1.2\,\text{rad/s})^2 = 341\text{ J}.$$

46. Angular momentum conservation $I_i\omega_i = I_f\omega_f$ leads to

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}\omega_i = 3$$

which implies

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{I_f}{I_i}\left(\frac{\omega_f}{\omega_i}\right)^2 = 3.$$

47. **THINK** No external torque acts on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero.

EXPRESS Let $I = MR^2$ be the rotational inertia of the wheel (which we treat as a hoop). Its angular momentum is

$$\vec{L}_{\text{wheel}} = (I\omega)\hat{\text{k}} = -M\,R^2\,|\omega|\,\hat{\text{k}},$$

where $\hat{\text{k}}$ is *up* in Fig. 11-48 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is $-|\omega|R$ and the speed of the train (going counterclockwise in Fig. 11-48 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $\vec{L}_{\text{train}} = m\left(v - |\omega|R\right)R\,\hat{\text{k}}$. Conservation of angular momentum yields

$$0 = \vec{L}_{\text{wheel}} + \vec{L}_{\text{train}} = -MR^2\,|\omega|\,\hat{\text{k}} + m\left(v - |\omega|R\right)R\hat{\text{k}}$$

which we can use to solve for $|\omega|$.

ANALYZE Solving for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{v}{(M/m+1)R} = \frac{0.15\text{ m/s}}{(1.1+1)(0.43\text{ m})} = 0.17\text{ rad/s}.$$

LEARN By angular momentum conservation, we must have $\vec{L}_{\text{wheel}} = -\vec{L}_{\text{train}}$, which means that train and the wheel must have opposite senses of rotation.

48. Using Eq. 11-31 with angular momentum conservation, $\vec{L}_i = \vec{L}_f$ (Eq. 11-33) leads to the ratio of rotational inertias being inversely proportional to the ratio of angular velocities. Thus, I_f/I_i = 6/5 = 1.0 + 0.2. We interpret the "1.0" as the ratio of disk rotational inertias (which does not change in this problem) and the "0.2" as the ratio of the roach rotational inertial to that of the disk. Thus, the answer is 0.20.

49. (a) We apply conservation of angular momentum:

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega.$$

The angular speed after coupling is therefore

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{\left(3.3\,\text{kg}\cdot\text{m}^2\right)\left(450\text{ rev/min}\right) + \left(6.6\,\text{kg}\cdot\text{m}^2\right)\left(900\text{ rev/min}\right)}{3.3\,\text{kg}\cdot\text{m}^2 + 6.6\,\text{kg}\cdot\text{m}^2}$$
$$= 750\text{ rev/min}.$$

(b) In this case, we obtain

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{\left(3.3\text{ kg}\cdot\text{m}^2\right)\left(450\text{ rev/min}\right) + \left(6.6\text{ kg}\cdot\text{m}^2\right)\left(-900\text{ rev/min}\right)}{3.3\text{ kg}\cdot\text{m}^2 + 6.6\text{ kg}\cdot\text{m}^2}$$
$$= -450\text{ rev/min}$$

or $|\omega| = 450\text{ rev/min}$.

(c) The minus sign indicates that $\vec{\omega}$ is clockwise, that is, in the direction of the second disk's initial angular velocity.

50. We use conservation of angular momentum:

$$I_m\omega_m = I_p\omega_p.$$

The respective angles θ_m and θ_p by which the motor and probe rotate are therefore related by

$$\int I_m\omega_m dt = I_m\theta_m = \int I_p\omega_p dt = I_p\theta_p$$

which gives

$$\theta_m = \frac{I_p\theta_p}{I_m} = \frac{\left(12\text{ kg}\cdot\text{m}^2\right)\left(30°\right)}{2.0\times10^{-3}\text{ kg}\cdot\text{m}^2} = 180000°.$$

The number of revolutions for the rotor is then

$$N = (1.8\times10^5)°/(360°/\text{rev}) = 5.0\times10^2\text{ rev}.$$

51. **THINK** No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved.

EXPRESS Let I_1 be the rotational inertia of the wheel that is originally spinning $(\text{at } \omega_i)$ and I_2 be the rotational inertia of the wheel that is initially at rest. Then by angular momentum conservation, $L_i = L_f$, or $I_1\omega_i = (I_1 + I_2)\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i$$

where ω_f is the common final angular velocity of the wheels.

ANALYZE (a) Substituting $I_2 = 2I_1$ and $\omega_i = 800$ rev/min, we obtain

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i = \frac{I_1}{I_1 + 2(I_1)}(800\ \text{rev/min}) = \frac{1}{3}(800\ \text{rev/min}) = 267\ \text{rev/min}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_1\omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6}I\omega_i^2.$$

Therefore, the fraction lost is

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{I\omega_i^2/6}{I\omega_i^2/2} = \frac{2}{3} = 0.667.$$

LEARN The situation here is analogous to the case of completely inelastic collision, in which some energy is lost but momentum remains conserved.

52. We denote the cockroach with subscript 1 and the disk with subscript 2. The cockroach has a mass $m_1 = m$, while the mass of the disk is $m_2 = 4.00\ m$.

(a) Initially the angular momentum of the system consisting of the cockroach and the disk is

$$L_i = m_1 v_{1i} r_{1i} + I_2\omega_{2i} = m_1\omega_0 R^2 + \frac{1}{2}m_2\omega_0 R^2.$$

After the cockroach has completed its walk, its position (relative to the axis) is $r_{1f} = R/2$ so the final angular momentum of the system is

$$L_f = m_1\omega_f\left(\frac{R}{2}\right)^2 + \frac{1}{2}m_2\omega_f R^2.$$

Then from $L_f = L_i$ we obtain

$$\omega_f\left[\frac{1}{4}m_1R^2+\frac{1}{2}m_2R^2\right]=\omega_0\left[m_1R^2+\frac{1}{2}m_2R^2\right].$$

Thus,

$$\omega_f=\left(\frac{m_1R^2+m_2R^2/2}{m_1R^2/4+m_2R^2/2}\right)\omega_0=\left(\frac{1+(m_2/m_1)/2}{1/4+(m_2/m_1)/2}\right)\omega_0=\left(\frac{1+2}{1/4\ +\ 2}\right)\omega_0=1.33\omega_0.$$

With ω_0 = 0.260 rad/s, we have ω_f =0.347 rad/s.

(b) We substitute $I = L/\omega$ into $K=\frac{1}{2}I\omega^2$ and obtain $K=\frac{1}{2}L\omega$. Since we have $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K}{K_0}=\frac{L_f\omega_f/2}{L_i\omega_i/2}=\frac{\omega_f}{\omega_0}=1.33.$$

(c) The cockroach does positive work while walking toward the center of the disk, increasing the total kinetic energy of the system.

53. The axis of rotation is in the middle of the rod, with r = 0.25 m from either end. By Eq. 11-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv\sin\theta$ where m = 0.003 kg and θ = 60°. Relative to the axis, this is counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is

$$I = I_{\text{rod}} + mr^2$$

where $I_{\text{rod}} = ML^2/12$ by Table 10-2(e), with M = 4.0 kg and L = 0.5 m. Angular momentum conservation leads to

$$rmv\sin\theta=\left(\frac{1}{12}ML^2+mr^2\right)\omega.$$

Thus, with ω = 10 rad/s, we obtain

$$v=\frac{\left(\frac{1}{12}(4.0\text{ kg})(0.5\text{ m})^2+(0.003\text{ kg})(0.25\text{ m})^2\right)(10\text{ rad/s})}{(0.25\text{ m})(0.003\text{ kg})\sin 60°}=1.3\times10^3\text{ m/s}.$$

54. We denote the cat with subscript 1 and the ring with subscript 2. The cat has a mass $m_1 = M/4$, while the mass of the ring is $m_2 = M$ = 8.00 kg. The moment of inertia of the ring is $I_2=m_2(R_1^2+R_2^2)/2$ (Table 10-2), and $I_1 = m_1r^2$ for the cat, where r is the perpendicular distance from the axis of rotation.

Initially the angular momentum of the system consisting of the cat (at $r = R_2$) and the ring is

$$L_i = m_1 v_{1i} r_{1i} + I_2\omega_{2i} = m_1\omega_0 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_0 = m_1 R_2^2\,\omega_0 \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right].$$

After the cat has crawled to the inner edge at $r = R_1$ the final angular momentum of the system is

$$L_f = m_1\omega_f R_1^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_f = m_1 R_1^2\,\omega_f \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)\right].$$

Then from $L_f = L_i$ we obtain

$$\frac{\omega_f}{\omega_0} = \left(\frac{R_2}{R_1}\right)^2 \frac{1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)}{1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)} = (2.0)^2 \frac{1 + 2(0.25 + 1)}{1 + 2(1 + 4)} = 1.273\,.$$

Thus, $\omega_f = 1.273\omega_0$. Using ω_0 =8.00 rad/s, we have ω_f =10.2 rad/s. By substituting $I = L/\omega$ into $K = I\omega^2/2$, we obtain $K = L\omega/2$. Since $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K_f}{K_i} = \frac{L_f\omega_f/2}{L_i\omega_i/2} = \frac{\omega_f}{\omega_0} = 1.273.$$

which implies $\Delta K = K_f - K_i = 0.273 K_i$. The cat does positive work while walking toward the center of the ring, increasing the total kinetic energy of the system.

Since the initial kinetic energy is given by

$$K_i = \frac{1}{2}\left[m_1 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\right]\omega_0^2 = \frac{1}{2} m_1 R_2^2 \omega_0^2 \left[1 + \frac{1}{2}\,\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right]$$
$$= \frac{1}{2}(2.00\text{ kg})(0.800\text{ m})^2 (8.00\text{ rad/s})^2 [1+(1/2)(4)(0.5^2+1)]$$
$$= 143.36\text{ J},$$

the increase in kinetic energy is

$$\Delta K = (0.273)(143.36\text{ J}) = 39.1\text{ J}.$$

55. For simplicity, we assume the record is turning freely, without any work being done by its motor (and without any friction at the bearings or at the stylus trying to slow it down). Before the collision, the angular momentum of the system (presumed positive) is $I_i\omega_i$ where $I_i = 5.0\times10^{-4}\ \text{kg}\cdot\text{m}^2$ and $\omega_i = 4.7\ \text{rad/s}$. The rotational inertia afterward is

$$I_f = I_i + mR^2$$

where m = 0.020 kg and R = 0.10 m. The mass of the record (0.10 kg), although given in the problem, is not used in the solution. Angular momentum conservation leads to

$$I_i \omega_i = I_f \omega_f \Rightarrow \omega_f = \frac{I_i \omega_i}{I_i + mR^2} = 3.4 \text{ rad / s.}$$

56. Table 10-2 gives the rotational inertia of a thin rod rotating about a perpendicular axis through its center. The angular speeds of the two arms are, respectively,

$$\omega_1 = \frac{(0.500 \text{ rev})(2\pi \text{ rad/rev})}{0.700 \text{ s}} = 4.49 \text{ rad/s}$$
$$\omega_2 = \frac{(1.00 \text{ rev})(2\pi \text{ rad/rev})}{0.700 \text{ s}} = 8.98 \text{ rad/s.}$$

Treating each arm as a thin rod of mass 4.0 kg and length 0.60 m, the angular momenta of the two arms are

$$L_1 = I\omega_1 = mr^2\omega_1 = (4.0 \text{ kg})(0.60 \text{ m})^2(4.49 \text{ rad/s}) = 6.46 \text{ kg}\cdot\text{m}^2\text{/s}$$
$$L_2 = I\omega_2 = mr^2\omega_2 = (4.0 \text{ kg})(0.60 \text{ m})^2(8.98 \text{ rad/s}) = 12.92 \text{ kg}\cdot\text{m}^2\text{/s.}$$

From the athlete's reference frame, one arm rotates clockwise, while the other rotates counterclockwise. Thus, the total angular momentum about the common rotation axis though the shoulders is

$$L = L_2 - L_1 = 12.92 \text{ kg}\cdot\text{m}^2\text{/s} - 6.46 \text{ kg}\cdot\text{m}^2\text{/s} = 6.46 \text{ kg}\cdot\text{m}^2\text{/s.}$$

57. Their angular velocities, when they are stuck to each other, are equal, regardless of whether they share the same central axis. The initial rotational inertia of the system is, using Table 10-2(c),

$$I_0 = I_{\text{bigdisk}} + I_{\text{smalldisk}}$$

where $I_{\text{bigdisk}} = MR^2/2$. Similarly, since the small disk is initially concentric with the big one, $I_{\text{smalldisk}} = \frac{1}{2}mr^2$. After it slides, the rotational inertia of the small disk is found from the parallel axis theorem (using $h = R - r$). Thus, the new rotational inertia of the system is

$$I = \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + m(R-r)^2 .$$

(a) Angular momentum conservation, $I_0\omega_0 = I\omega$, leads to the new angular velocity:

$$\omega = \omega_0 \frac{(MR^2/2) + (mr^2/2)}{(MR^2/2) + (mr^2/2) + m(R-r)^2}.$$

Substituting $M = 10m$ and $R = 3r$, this becomes $\omega = \omega_0(91/99)$. Thus, with $\omega_0 = 20$ rad/s, we find $\omega = 18$ rad/s.

(b) From the previous part, we know that

$$\frac{I_0}{I} = \frac{91}{99}, \quad \frac{\omega}{\omega_0} = \frac{91}{99}.$$

Plugging these into the ratio of kinetic energies, we have

$$\frac{K}{K_0} = \frac{I\omega^2/2}{I_0\omega_0^2/2} = \frac{I}{I_0}\left(\frac{\omega}{\omega_0}\right)^2 = \frac{99}{91}\left(\frac{91}{99}\right)^2 = 0.92.$$

58. The initial rotational inertia of the system is $I_i = I_{\text{disk}} + I_{\text{student}}$, where $I_{\text{disk}} = 300$ $\text{kg}\cdot\text{m}^2$ (which, incidentally, does agree with Table 10-2(c)) and $I_{\text{student}} = mR^2$ where $m = 60$ kg and $R = 2.0$ m.

The rotational inertia when the student reaches $r = 0.5$ m is $I_f = I_{\text{disk}} + mr^2$. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \omega_i \frac{I_{\text{disk}} + mR^2}{I_{\text{disk}} + mr^2}$$

which yields, for $\omega_i = 1.5$ rad/s, a final angular velocity of $\omega_f = 2.6$ rad/s.

59. By angular momentum conservation (Eq. 11-33), the total angular momentum after the explosion must be equal to that before the explosion:

$$L'_p + L'_r = L_p + L_r$$

$$\left(\tfrac{L}{2}\right)mv_{\text{p}} + \tfrac{1}{12}ML^2\,\omega' = I_{\text{p}}\,\omega + \tfrac{1}{12}ML^2\,\omega$$

where one must be careful to avoid confusing the length of the rod ($L = 0.800$ m) with the angular momentum symbol. Note that $I_{\text{p}} = m(L/2)^2$ by Eq.10-33, and

$$\omega' = v_{\text{end}}/r = (v_{\text{p}} - 6)/(L/2),$$

where the latter relation follows from the penultimate sentence in the problem (and "6" stands for "6.00 m/s" here). Since $M = 3m$ and $\omega = 20$ rad/s, we end up with enough information to solve for the particle speed: $v_{\text{p}} = 11.0$ m/s.

60. (a) With r = 0.60 m, we obtain $I = 0.060 + (0.501)r^2 = 0.24$ kg · m^2.

(b) Invoking angular momentum conservation, with SI units understood,

$$\ell_0 = L_f \;\Rightarrow\; mv_0 r = I\omega \;\;\Rightarrow\;\; (0.001)v_0(0.60) = (0.24)(4.5)$$

which leads to $v_0 = 1.8 \times 10^3$ m/s.

61. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities in this problem are positive. With r = 0.60 m and I_0 = 0.12 kg · m^2, the rotational inertia of the putty-rod system (after the collision) is

$$I = I_0 + (0.20)r^2 = 0.19 \text{ kg} \cdot \text{m}^2.$$

Invoking angular momentum conservation $L_0 = L_f$ or $I_0\omega_0 = I\omega$, we have

$$\omega = \frac{I_0}{I}\omega_0 = \frac{0.12 \text{ kg} \cdot \text{m}^2}{0.19 \text{ kg} \cdot \text{m}^2}(2.4 \text{ rad/s}) = 1.5 \text{ rad/s}.$$

62. The aerialist is in extended position with $I_1 = 19.9 \text{ kg} \cdot \text{m}^2$ during the first and last quarter of the turn, so the total angle rotated in t_1 is $\theta_1 = 0.500$ rev. In t_2 he is in a tuck position with $I_2 = 3.93 \text{ kg} \cdot \text{m}^2$, and the total angle rotated is $\theta_2 = 3.500$ rev. Since there is no external torque about his center of mass, angular momentum is conserved, $I_1\omega_1 = I_2\omega_2$. Therefore, the total flight time can be written as

$$t = t_1 + t_2 = \frac{\theta_1}{\omega_1} + \frac{\theta_2}{\omega_2} = \frac{\theta_1}{I_2\omega_2 / I_1} + \frac{\theta_2}{\omega_2} = \frac{1}{\omega_2}\left(\frac{I_1}{I_2}\theta_1 + \theta_2\right).$$

Substituting the values given, we find ω_2 to be

$$\omega_2 = \frac{1}{t}\left(\frac{I_1}{I_2}\theta_1 + \theta_2\right) = \frac{1}{1.87 \text{ s}}\left(\frac{19.9 \text{ kg} \cdot \text{m}^2}{3.93 \text{ kg} \cdot \text{m}^2}(0.500 \text{ rev}) + 3.50 \text{ rev}\right) = 3.23 \text{ rev/s}.$$

63. This is a completely inelastic collision, which we analyze using angular momentum conservation. Let m and v_0 be the mass and initial speed of the ball and R the radius of the merry-go-round. The initial angular momentum is

$$\vec{\ell}_0 = \vec{r}_0 \times \vec{p}_0 \Rightarrow \ell_0 = R(mv_0)\cos 37°$$

where ϕ = 37° is the angle between $\vec{v}_0$ and the line tangent to the outer edge of the merry-go-around. Thus, $\ell_0 = 19 \text{ kg} \cdot \text{m}^2/\text{s}$. Now, with SI units understood,

$$\ell_0 = L_f \quad \Rightarrow \quad 19\ \text{kg}\cdot\text{m}^2 = I\omega = \left(150 + (30)R^2 + (1.0)R^2\right)\omega$$

so that $\omega = 0.070$ rad/s.

64. We treat the ballerina as a rigid object rotating around a fixed axis, initially and then again near maximum height. Her initial rotational inertia (trunk and one leg extending outward at a $90°$ angle) is

$$I_i = I_{\text{trunk}} + I_{\text{leg}} = 0.660\ \text{kg}\cdot\text{m}^2 + 1.44\ \text{kg}\cdot\text{m}^2 = 2.10\ \text{kg}\cdot\text{m}^2.$$

Similarly, her final rotational inertia (trunk and *both* legs extending outward at a $\theta = 30°$ angle) is

$$I_f = I_{\text{trunk}} + 2I_{\text{leg}}\sin^2\theta = 0.660\ \text{kg}\cdot\text{m}^2 + 2(1.44\ \text{kg}\cdot\text{m}^2)\sin^2 30° = 1.38\ \text{kg}\cdot\text{m}^2,$$

where we have used the fact that the effective length of the extended leg at an angle θ is $L_\perp = L\sin\theta$ and $I \sim L_\perp^2$. Once airborne, there is no external torque about the ballerina's center of mass and her angular momentum cannot change. Therefore, $L_i = L_f$ or $I_i\omega_i = I_f\omega_f$, and the ratio of the angular speeds is

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{2.10\ \text{kg}\cdot\text{m}^2}{1.38\ \text{kg}\cdot\text{m}^2} = 1.52.$$

65. **THINK** If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad.

EXPRESS The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where d is the length of the rod. The angular momentum of the wad is $mvd/2$ where m and v are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls (each having a mass M) and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$ where $I = (2M + m)(d/2)^2$. The equation allows us to solve for ω.

ANALYZE (a) With $M = 2.00$ kg, $d = 0.500$ m, $m = 0.0500$ kg, and $v = 3.00$ m/s, we find the angular speed to be

$$\omega = \frac{mvd}{2I} = \frac{2mv}{(2M + m)d} = \frac{2(0.0500\ \text{kg})(3.00\ \text{m/s})}{\left(2(2.00\ \text{kg}) + 0.0500\ \text{kg}\right)(0.500\ \text{m})}$$
$$= 0.148\ \text{rad/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is

$$K_f/K_i = I\omega^2/mv^2.$$

When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, the ratio becomes

$$\frac{K_f}{K_i} = \frac{m}{2M+m} = \frac{0.0500\text{ kg}}{2(2.00\text{ kg}) + 0.0500\text{ kg}} = 0.0123.$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance h, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a 90° arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mg(d/2)$. If it swings up to the angular position θ, as measured from its lowest point, then its final height is $(d/2)(1 - \cos\theta)$ above the lowest point and its final potential energy is

$$U_f = mg(d/2)(1 - \cos\theta).$$

The initial kinetic energy is the sum of that of the balls and wad:

$$K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M+m)(d/2)^2\,\omega^2.$$

At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$U_i + K_i = U_f + K_f \quad \Rightarrow \quad mg\frac{d}{2} + \frac{1}{2}(2M+m)\left(\frac{d}{2}\right)^2\omega^2 = mg\frac{d}{2}(1-\cos\theta).$$

When this equation is solved for $\cos\theta$, the result is

$$\begin{aligned}\cos\theta &= -\frac{1}{2}\left(\frac{2M+m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 \\ &= -\frac{1}{2}\left(\frac{2(2.00\text{ kg}) + 0.0500\text{ kg}}{(0.0500\text{ kg})(9.8\text{ m/s}^2)}\right)\left(\frac{0.500\text{ m}}{2}\right)(0.148\text{ rad/s})^2 \\ &= -0.0226.\end{aligned}$$

Consequently, the result for θ is 91.3°. The total angle through which it has swung is 90° + 91.3° = 181°.

LEARN This problem is rather involved. To summarize, we calculated ω using angular momentum conservation. Some energy is lost due to the inelastic collision between the putty wad and one of the balls. However, in the subsequent motion, energy is conserved, and we apply energy conservation to find the angle at which the system comes to rest momentarily.

66. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = \left(I_{\text{rod}} + md^2\right)\omega$$

where I_{rod} is found using Table 10-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2.$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{(Md^2/3) + md^2}$$

which means the system has kinetic energy $\left(I_{\text{rod}} + md^2\right)\omega^2/2$, which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 = mgH + Mg\frac{H}{2} \;\Rightarrow\; \frac{1}{2}\frac{m^2d^2\left(2gh\right)}{(Md^2/3) + md^2} = \left(m + \frac{M}{2}\right)gd\left(1 - \cos\theta\right)$$

from which we obtain

$$\theta = \cos^{-1}\left(1 - \frac{m^2 h}{(m + M/2)(m + M/3)}\right) = \cos^{-1}\left(1 - \frac{h/d}{(1 + M/2m)(1 + M/3m)}\right)$$

$$= \cos^{-1}\left(1 - \frac{(20\text{ cm}/40\text{ cm})}{(1+1)(1+2/3)}\right) = \cos^{-1}(0.85)$$

$$= 32°.$$

67. (a) We consider conservation of angular momentum (Eq. 11-33) about the center of the rod:

$$L_i = L_f \;\Rightarrow\; -dmv + \frac{1}{12}ML^2\omega = 0$$

where negative is used for "clockwise." Item (e) in Table 11-2 and Eq. 11-21 (with $r_\perp = d$) have also been used. This leads to

$$d = \frac{ML^2\,\omega}{12\,m\,\text{v}} = \frac{M(0.60\text{ m})^2\,(80\text{ rad/s})}{12(M/3)(40\text{ m/s})} = 0.180\text{ m}\,.$$

(b) Increasing d causes the magnitude of the negative (clockwise) term in the above equation to increase. This would make the total angular momentum negative before the collision, and (by Eq. 11-33) also negative afterward. Thus, the system would rotate clockwise if d were greater.

68. (a) The angular speed of the top is $\omega = 30\text{ rev/s} = 30(2\pi)\text{ rad/s}$. The precession rate of the top can be obtained by using Eq. 11-46:

$$\Omega = \frac{Mgr}{I\omega} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)(0.040\text{ m})}{(5.0\times10^{-4}\text{ kg}\cdot\text{m}^2)(60\pi\text{ rad/s})} = 2.08\text{ rad/s} \approx 0.33\text{ rev/s}.$$

(b) The direction of the precession is clockwise as viewed from overhead.

69. The precession rate can be obtained by using Eq. 11-46 with r = (11/2) cm = 0.055 m. Noting that $I_{\text{disk}} = MR^2/2$ and its angular speed is

$$\omega = 1000\text{ rev/min} = \frac{2\pi(1000)}{60}\text{ rad/s} \approx 1.0\times10^2\text{ rad/s},$$

we have

$$\Omega = \frac{Mgr}{(MR^2/2)\omega} = \frac{2gr}{R^2\omega} = \frac{2(9.8\text{ m/s}^2)(0.055\text{ m})}{(0.50\text{ m})^2(1.0\times10^2\text{ rad/s})} \approx 0.041\text{ rad/s}.$$

70. Conservation of energy implies that mechanical energy at maximum height up the ramp is equal to the mechanical energy on the floor. Thus, using Eq. 11-5, we have

$$\frac{1}{2}mv_f^2 + \frac{1}{2}I_{\text{com}}\omega_f^2 + mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{com}}\omega^2$$

where $v_f = \omega_f = 0$ at the point on the ramp where it (momentarily) stops. We note that the height h relates to the distance traveled along the ramp d by $h = d\sin(15°)$. Using item (*f*) in Table 10-2 and Eq. 11-2, we obtain

$$mgd\sin 15° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2.$$

After canceling m and plugging in $d = 1.5$ m, we find $v = 2.33$ m/s.

71. **THINK** The applied force gives rise to a torque that causes the cylinder to rotate to the right at a constant angular acceleration.

EXPRESS We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration is positive (as is the linear acceleration of the center of mass, since we take rightwards as positive). We approach this in the manner of Eq. 11-3 (*pure rotation* about point P) but use torques instead of energy. The torque (relative to point P) is $\tau = I_P\alpha$, where

$$I_P = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

with the use of the parallel-axis theorem and Table 10-2(c). The torque is due to the F_{app} force and can be written as $\tau = F_{\text{app}}(2R)$. In this way, we find

$$\tau = I_P\alpha = \left(\frac{3}{2}MR^2\right)\alpha = 2RF_{\text{app}}.$$

The equation allows us to solve for the angular acceleration α, which is related to the acceleration of the center of mass as $\alpha = a_{\text{com}}/R$.

ANALYZE (a) With $M = 10\text{ kg}$, $R = 0.10$ m and $F_{\text{app}} = 12\text{ N}$, we obtain

$$a_{\text{com}} = \alpha R = \frac{2R^2F_{\text{app}}}{3MR^2/2} = \frac{4F_{\text{app}}}{3M} = \frac{4(12\text{ N})}{3(10\text{ kg})} = 1.6\text{ m/s}^2.$$

(b) The magnitude of the angular acceleration is

$$\alpha = a_{\text{com}}/R = (1.6\text{ m/s}^2)/(0.10\text{ m}) = 16\text{ rad/s}^2.$$

(c) Applying Newton's second law in its linear form yields $(12\,\text{N}) - f = Ma_{\text{com}}$. Therefore, $f = -4.0$ N. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* with magnitude 4.0 N, i.e., $\vec{f} = (4.0\ \text{N})\hat{\text{i}}$.

LEARN As the cylinder rolls to the right, the frictional force also points to the right to oppose the tendency to slip.

72. The rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, where $I = mR^2$ is its rotational inertia about the center of mass (Table 10-2(a)), $m = 140$ kg, and $\omega = v_{\text{com}}/R$ (Eq. 11-2). The ratio is

$$\frac{K_{\text{transl}}}{K_{\text{rot}}} = \frac{\frac{1}{2}mv_{\text{com}}^2}{\frac{1}{2}\left(mR^2\right)\left(v_{\text{com}}/R\right)^2} = 1.00.$$

73. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r}\,' = x'\hat{\text{i}} + y'\hat{\text{j}}$, then (using Eq. 3-30) we find

$$\vec{r}\,' \times \vec{v} = \left(x'v_y - y'v_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,'$ points in either the $+\hat{\text{i}}$ or the $-\hat{\text{i}}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{\text{i}} \times \hat{\text{i}} = 0$) that $\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = 0$ in this case.

(b) The net force is in the $-\hat{\text{i}}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\tau = \vec{r}\,' \times \vec{F} = 0$.

(c) Now, $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\hat{\text{i}} + 5.0\hat{\text{j}}$ (with SI units understood) and points from (2.0, 5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$, which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = -m\left(\vec{r}_o \times \vec{v}\right) = -\left(3.0\right)\left(\left(2.0\right)\left(0\right) - \left(5.0\right)\left(-2.0t^3\right)\right)\hat{\text{k}}$$

which yields $\vec{\ell} = (-30t^3\hat{\text{k}})\ \text{kg}\cdot\text{m/s}^2$.

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\hat{\text{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\left(\vec{r}\,' \times \vec{a}\right) = -m\left(\vec{r}_o \times \vec{a}\right) = -\left(3.0\right)\left(\left(2.0\right)\left(0\right) - \left(5.0\right)\left(-6.0t^2\right)\right)\hat{\text{k}}$$

which yields $\vec{\tau} = (-90t^2\hat{\text{k}})\ \text{N}\cdot\text{m}$.

(e) In this situation, $\vec{r}\,' = \vec{r} - \vec{r}_\text{o}$ where $\vec{r}_\text{o} = 2.0\hat{\text{i}} - 5.0\hat{\text{j}}$ (with SI units understood) and points from (2.0, –5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$, which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r}\times\vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = -m(\vec{r}_\text{o} \times \vec{v}) = -(3.0)\left[(2.0)(0) - (-5.0)(-2.0t^3)\right]\hat{\text{k}}$$

which yields $\vec{\ell} = (30t^3\hat{\text{k}})\ \text{kg}\cdot\text{m}^2/\text{s}$.

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\hat{\text{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m(\vec{r}\,' \times \vec{a}) = -m(\vec{r}_\text{o} \times \vec{a}) = -(3.0)\left[(2.0)(0) - (-5.0)(-6.0t^2)\right]\hat{\text{k}}$$

which yields $\vec{\tau} = (90t^2\hat{\text{k}})\ \text{N}\cdot\text{m}$.

74. For a constant (single) torque, Eq. 11-29 becomes

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{\Delta\vec{L}}{\Delta t}.$$

Thus, we obtain

$$\Delta t = \frac{\Delta L}{\tau} = \frac{600\ \text{kg}\cdot\text{m}^2/\text{s}}{50\ \text{N}\cdot\text{m}} = 12\ \text{s}.$$

75. **THINK** No external torque acts on the system consisting of the child and the merry-go-round, so the total angular momentum of the system is conserved.

EXPRESS An object moving along a straight line has angular momentum about any point that is not on the line. The magnitude of the angular momentum of the child about the center of the merry-go-round is given by Eq. 11-21, mvR, where R is the radius of the merry-go-round.

ANALYZE (a) In terms of the radius of gyration k, the rotational inertia of the merry-go-round is $I = Mk^2$. With M = 180 kg and k = 0.91 m, we obtain

$$I = (180\ \text{kg})\ (0.910\ \text{m})^2 = 149\ \text{kg}\cdot\text{m}^2.$$

(b) The magnitude of angular momentum of the running child about the axis of rotation of the merry-go-round is

$$L_{\text{child}} = mvR = (44.0 \text{ kg})(3.00 \text{ m/s})(1.20 \text{ m}) = 158 \text{ kg}\cdot\text{m}^2/\text{s}.$$

(c) The initial angular momentum is given by $L_i = L_{\text{child}} = mvR$; the final angular momentum is given by $L_f = (I + mR^2)\,\omega$, where ω is the final common angular velocity of the merry-go-round and child. Thus $mvR = (I + mR^2)\omega$ and

$$\omega = \frac{mvR}{I + mR^2} = \frac{158 \text{ kg}\cdot\text{m}^2/\text{s}}{149 \text{ kg}\cdot\text{m}^2 + (44.0 \text{ kg})(1.20 \text{ m})^2} = 0.744 \text{ rad/s}.$$

LEARN The child initially had an angular velocity of

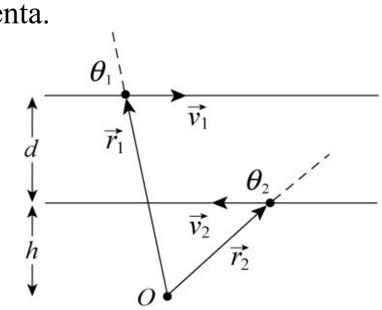
$$\omega_0 = \frac{v}{R} = \frac{3.00 \text{ m/s}}{1.20 \text{ m}} = 2.5 \text{ rad/s}.$$

After he jumped onto the merry-go-round, the rotational inertia of the system (merry-go-round + child) increases, so the angular velocity decreases by angular momentum conservation.

76. Item (*i*) in Table 10-2 gives the moment of inertia about the center of mass in terms of width *a* (0.15 m) and length *b* (0.20 m). In using the parallel axis theorem, the distance from the center to the point about which it spins (as described in the problem) is $\sqrt{(a/4)^2 + (b/4)^2}$. If we denote the thickness as *h* (0.012 m) then the volume is *abh*, which means the mass is ρabh (where ρ = 2640 kg/m^3 is the density). We can write the kinetic energy in terms of the angular momentum by substituting $\omega = L/I$ into Eq. 10-34:

$$K = \frac{1}{2}\frac{L^2}{I} = \frac{1}{2}\frac{(0.104)^2}{\rho abh((a^2 + b^2)/12 + (a/4)^2 + (b/4)^2)} = 0.62 \text{ J}.$$

77. **THINK** Our system consists of two particles moving in opposite directions along parallel lines. The angular momentum of the system about a point is the vector sum of the two individual angular momenta.



EXPRESS The diagram above shows the particles and their lines of motion. The origin is marked *O* and may be anywhere. We set up our coordinate system in such a way that

+x is to the right, +y up and +z out of the page. The angular momentum of the system about O is

$$\vec{\ell} = \vec{\ell}_1 + \vec{\ell}_2 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 = m(\vec{r}_1 \times \vec{v}_1 + \vec{r}_2 \times \vec{v}_2)$$

since $m_1 = m_2 = m$.

ANALYZE (a) With $\vec{v}_1 = v_1\hat{\mathrm{i}}$, the angular momentum of particle 1 has magnitude

$$\ell_1 = mvr_1 \sin\theta_1 = mv\left(d+h\right)$$

and is in the –z-direction, or into the page. On the other hand, with $\vec{v}_2 = -v_2\hat{\mathrm{i}}$, the angular momentum of particle 2 has magnitude $\ell_2 = mvr_2 \sin\theta_2 = mvh$, and is in the +$z$-direction, or out of the page. The net angular momentum has magnitude

$$\ell = mv(d+h) - mvh = mvd$$

which depends only on the separation between the two lines and not on the location of the origin. Thus, if O is midway between the two lines, the total angular momentum is

$$\ell = mvd = (2.90\times10^{-4}\ \text{kg})(5.46\ \text{m/s})(0.042\ \text{m}) = 6.65\times10^{-5}\ \text{kg}\cdot\text{m}^2/\text{s}$$

and is into the page.

(b) As indicated above, the expression does not change.

(c) Suppose particle 2 is traveling to the right. Then

$$\ell = mv(d+h) + mvh = mv(d+2h).$$

This result now depends on h, the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $\ell = 0$.

(d) As we have seen in part (c), the result depends on the choice of origin.

LEARN Angular momentum is a vector quantity. For a system of many particles, the total angular momentum about a point is

$$\vec{\ell} = \vec{\ell}_1 + \vec{\ell}_2 + \cdots = \sum_i \vec{\ell}_i = \sum_i m_i \vec{r}_i \times \vec{v}_i .$$

78. (a) Using Eq. 2-16 for the translational (center-of-mass) motion, we find

$$v^2 = v_0^2 + 2a\Delta x \Rightarrow a = -\frac{v_0^2}{2\Delta x}$$

which yields $a = –4.11$ for $v_0 = 43$ and $\Delta x = 225$ (SI units understood). The magnitude of the linear acceleration of the center of mass is therefore 4.11 m/s^2.

(b) With R = 0.250 m, Eq. 11-6 gives

$$|\alpha| = |a|/R = 16.4\,\text{rad/s}^2.$$

If the wheel is going rightward, it is rotating in a clockwise sense. Since it is slowing down, this angular acceleration is counterclockwise (opposite to ω) so (with the usual convention that counterclockwise is positive) there is no need for the absolute value signs for α.

(c) Equation 11-8 applies with Rf_s representing the magnitude of the frictional torque. Thus,

$$Rf_s = I\alpha = (0.155\ \text{kg·m}^2)\,(16.4\ \text{rad/s}^2) = 2.55\ \text{N}\cdot\text{m}.$$

79. We use $L = I\omega$ and $K = \frac{1}{2}I\omega^2$ and observe that the speed of points on the rim (corresponding to the speed of points on the belt) of wheels A and B must be the same (so $\omega_A R_A = \omega_B r_B$).

(a) If $L_A = L_B$ (call it L) then the ratio of rotational inertias is

$$\frac{I_A}{I_B} = \frac{L/\omega_A}{L/\omega_B} = \frac{\omega_A}{\omega_B} = \frac{R_A}{R_B} = \frac{1}{3} = 0.333.$$

(b) If we have $K_A = K_B$ (call it K) then the ratio of rotational inertias becomes

$$\frac{I_A}{I_B} = \frac{2K/\omega_A^2}{2K/\omega_B^2} = \left(\frac{\omega_B}{\omega_A}\right)^2 = \left(\frac{R_A}{R_B}\right)^2 = \frac{1}{9} = 0.111.$$

80. The total angular momentum (about the origin) before the collision (using Eq. 11-18 and Eq. 3-30 for each particle and then adding the terms) is

$$\vec{L}_i = [(0.5\ \text{m})(2.5\ \text{kg})(3.0\ \text{m/s}) + (0.1\ \text{m})(4.0\ \text{kg})(4.5\ \text{m/s})]\hat{\text{k}}.$$

The final angular momentum of the stuck-together particles (after the collision) measured relative to the origin is (using Eq. 11-33)

$$\vec{L}_f = \vec{L}_i = (5.55\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

81. **THINK** As the wheel rolls without slipping down an inclined plane, its gravitational potential energy is converted into translational and rotational kinetic energies.

EXPRESS As the wheel-axel system rolls down the inclined plane by a distance d, the change in potential energy is $\Delta U = -mgd\sin\theta$. By energy conservation, the total kinetic energy gained is

$$-\Delta U = \Delta K = \Delta K_{\text{trans}} + \Delta K_{\text{rot}} \quad \Rightarrow \quad mgd\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 .$$

Since the axel rolls without slipping, the angular speed is given by $\omega = v/r$, where r is the radius of the axel. The above equation then becomes

$$mgd\sin\theta = \frac{1}{2}I\omega^2\left(\frac{mr^2}{I}+1\right) = \Delta K_{\text{rot}}\left(\frac{mr^2}{I}+1\right).$$

ANALYZE (a) With m=10.0 kg, d = 2.00 m, r = 0.200 m, and $I = 0.600\ \text{kg}\cdot\text{m}^2$, the rotational kinetic energy may be obtained as

$$\Delta K_{\text{rot}} = \frac{mgd\sin\theta}{\dfrac{mr^2}{I}+1} = \frac{(10.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})\sin 30.0^\circ}{\dfrac{(10.0\ \text{kg})(0.200\ \text{m})^2}{0.600\ \text{kg}\cdot\text{m}^2}+1} = 58.8\ \text{J} .$$

(b) The translational kinetic energy is $\Delta K_{\text{trans}} = \Delta K - \Delta K_{\text{rot}} = 98\ \text{J} - 58.8\ \text{J} = 39.2\ \text{J}$.

LEARN One may show that $mr^2/I = 2/3$, which implies that $\Delta K_{\text{trans}}/\Delta K_{\text{rot}} = 2/3$. Equivalently, we may write $\Delta K_{\text{trans}}/\Delta K = 2/5$ and $\Delta K_{\text{rot}}/\Delta K = 3/5$. So as the wheel rolls down, 40% of the kinetic energy is translational while the other 60% is rotational.

82. (a) We use Table 10-2(e) and the parallel-axis theorem to obtain the rod's rotational inertia about an axis through one end:

$$I = I_{\text{com}} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

where L = 6.00 m and M = 10.0/9.8 = 1.02 kg. Thus, the inertia is $I = 12.2\ \text{kg}\cdot\text{m}^2$.

(b) Using ω = (240)(2π/60) = 25.1 rad/s, Eq. 11-31 gives the magnitude of the angular momentum as

$$I\omega = \left(12.2\ \text{kg}\cdot\text{m}^2\right)\left(25.1\,\text{rad/s}\right) = 308\ \ \text{kg}\cdot\text{m}^2/\text{s} .$$

Since it is rotating clockwise as viewed from above, then the right-hand rule indicates that its direction is down.

83. We note that its mass is M = 36/9.8 = 3.67 kg and its rotational inertia is $I_{\text{com}} = \frac{2}{5}MR^2$ (Table 10-2(f)).

(a) Using Eq. 11-2, Eq. 11-5 becomes

$$K = \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{com}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{com}}^2 = \frac{7}{10}Mv_{\text{com}}^2$$

which yields K = 61.7 J for v_{com} = 4.9 m/s.

(b) This kinetic energy turns into potential energy Mgh at some height $h = d\sin\theta$ where the sphere comes to rest. Therefore, we find the distance traveled up the θ = 30° incline from energy conservation:

$$\frac{7}{10}Mv_{\text{com}}^2 = Mgd\sin\theta \quad\Rightarrow\quad d = \frac{7v_{\text{com}}^2}{10g\sin\theta} = 3.43\,\text{m}.$$

(c) As shown in the previous part, M cancels in the calculation for d. Since the answer is independent of mass, then it is also independent of the sphere's weight.

84. (a) The acceleration is given by Eq. 11-13:

$$a_{\text{com}} = \frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where upward is the positive translational direction. Taking the coordinate origin at the initial position, Eq. 2-15 leads to

$$y_{\text{com}} = v_{\text{com},0}t + \frac{1}{2}a_{\text{com}}t^2 = v_{\text{com},0}t - \frac{\frac{1}{2}gt^2}{1 + I_{\text{com}}/MR_0^2}$$

where y_{com} = – 1.2 m and $v_{\text{com},0}$ = – 1.3 m/s. Substituting $I_{\text{com}} = 0.000095\ \text{kg}\cdot\text{m}^2$, M = 0.12 kg, R_0 = 0.0032 m, and g = 9.8 m/s^2, we use the quadratic formula and find

$$\begin{aligned} t &= \frac{\left(1 + \frac{I_{\text{com}}}{MR_0^2}\right)\left(v_{\text{com},0} \mp \sqrt{v_{\text{com},0}^2 - \frac{2gy_{\text{com}}}{1+I_{\text{com}}/MR_0^2}}\right)}{g} \\ &= \frac{\left(1 + \frac{0.000095}{(0.12)(0.0032)^2}\right)\left(-1.3 \mp \sqrt{(1.3)^2 - \frac{2(9.8)(-1.2)}{1+0.000095/(0.12)(0.0032)^2}}\right)}{9.8} \\ &= -21.7 \text{ or } 0.885 \end{aligned}$$

where we choose $t = 0.89$ s as the answer.

(b) We note that the initial potential energy is $U_i = Mgh$ and $h = 1.2$ m (using the bottom as the reference level for computing U). The initial kinetic energy is as shown in Eq. 11-5, where the initial angular and linear speeds are related by Eq. 11-2. Energy conservation leads to

$$\begin{aligned}K_f &= K_i + U_i = \frac{1}{2}mv_{\text{com},0}^2 + \frac{1}{2}I\left(\frac{v_{\text{com},0}}{R_0}\right)^2 + Mgh \\ &= \frac{1}{2}(0.12\text{ kg})(1.3\text{ m/s})^2 + \frac{1}{2}\left(9.5\times10^{-5}\text{ kg}\cdot\text{m}^2\right)\left(\frac{1.3\text{ m/s}}{0.0032\text{ m}}\right)^2 + (0.12\text{ kg})\left(9.8\text{ m/s}^2\right)(1.2\text{ m}) \\ &= 9.4\text{ J}.\end{aligned}$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} + a_{\text{com}}t = v_{\text{com},0} - \frac{gt}{1 + I_{\text{com}}/MR_0^2}.$$

Thus, we obtain

$$v_{\text{com}} = -1.3\text{ m/s} - \frac{\left(9.8\text{ m/s}^2\right)(0.885\text{ s})}{1 + \dfrac{0.000095\text{ kg}\cdot\text{m}^2}{(0.12\text{ kg})(0.0032\text{ m})^2}} = -1.41\text{ m/s}$$

so its linear speed at that moment is approximately 1.4 m/s.

(d) The translational kinetic energy is

$$\tfrac{1}{2}mv_{\text{com}}^2 = \tfrac{1}{2}(0.12\text{ kg})(-1.41\text{ m/s})^2 = 0.12\text{ J}.$$

(e) The angular velocity at that moment is given by

$$\omega = -\frac{v_{\text{com}}}{R_0} = -\frac{-1.41\text{ m/s}}{0.0032\text{ m}} = 441\text{ rad/s} \approx 4.4\times10^2\text{ rad/s}.$$

(f) And the rotational kinetic energy is

$$\frac{1}{2}I_{\text{com}}\omega^2 = \frac{1}{2}\left(9.50\times10^{-5}\text{ kg}\cdot\text{m}^2\right)(441\text{ rad/s})^2 = 9.2\text{ J}.$$

85. The initial angular momentum of the system is zero. The final angular momentum of the girl-plus-merry-go-round is $(I + MR^2)\,\omega$, which we will take to be positive. The final angular momentum we associate with the thrown rock is negative: $-mRv$, where v is the speed (positive, by definition) of the rock relative to the ground.

(a) Angular momentum conservation leads to

$$0=\left(I+MR^2\right)\omega-mRv \quad \Rightarrow \quad \omega=\frac{mRv}{I+MR^2}.$$

(b) The girl's linear speed is given by Eq. 10-18:

$$R\omega=\frac{mvR^2}{I+MR^2}.$$

86. (a) Interpreting *h* as the height increase for the center of mass of the body, then (using Eq. 11-5) mechanical energy conservation, $K_i=U_f$, leads to

$$\frac{1}{2}mv_{\text{com}}^2+\frac{1}{2}I\omega^2=mgh \quad \Rightarrow \quad \frac{1}{2}mv^2+\frac{1}{2}I\left(\frac{v}{R}\right)^2=mg\left(\frac{3v^2}{4g}\right)$$

from which *v* cancels and we obtain $I=\frac{1}{2}mR^2$.

(b) From Table 10-2(c), we see that the body could be a solid cylinder.

Chapter 12

1. (a) The center of mass is given by

$$x_{\text{com}} = \frac{0+0+0+(m)(2.00\text{ m})+(m)(2.00\text{ m})+(m)(2.00\text{ m})}{6m} = 1.00\text{ m}.$$

(b) Similarly, we have

$$y_{\text{com}} = \frac{0+(m)(2.00\text{ m})+(m)(4.00\text{ m})+(m)(4.00\text{ m})+(m)(2.00\text{ m})+0}{6m} = 2.00\text{ m}.$$

(c) Using Eq. 12-14 and noting that the gravitational effects are different at the different locations in this problem, we have

$$x_{\text{cog}} = \frac{\sum_{i=1}^{6} x_i m_i g_i}{\sum_{i=1}^{6} m_i g_i} = \frac{x_1m_1g_1 + x_2m_2g_2 + x_3m_3g_3 + x_4m_4g_4 + x_5m_5g_5 + x_6m_6g_6}{m_1g_1 + m_2g_2 + m_3g_3 + m_4g_4 + m_5g_5 + m_6g_6} = 0.987\text{ m}.$$

(d) Similarly, we have

$$\begin{aligned} y_{\text{cog}} &= \frac{\sum_{i=1}^{6} y_i m_i g_i}{\sum_{i=1}^{6} m_i g_i} = \frac{y_1m_1g_1 + y_2m_2g_2 + y_3m_3g_3 + y_4m_4g_4 + y_5m_5g_5 + y_6m_6g_6}{m_1g_1 + m_2g_2 + m_3g_3 + m_4g_4 + m_5g_5 + m_6g_6} \\ &= \frac{0+(2.00)(7.80m)+(4.00)(7.60m)+(4.00)(7.40m)+(2.00)(7.60m)+0}{8.0m+7.80m+7.60m+7.40m+7.60m+7.80m} \\ &= 1.97\text{ m}. \end{aligned}$$

2. Our notation is as follows: M = 1360 kg is the mass of the automobile; L = 3.05 m is the horizontal distance between the axles; $\ell = (3.05 - 1.78)\text{ m} = 1.27\text{ m}$ is the horizontal distance from the rear axle to the center of mass; F_1 is the force exerted on each front wheel; and F_2 is the force exerted on each back wheel.

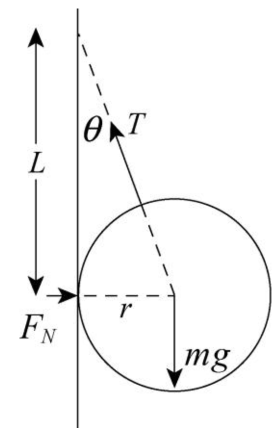
(a) Taking torques about the rear axle, we find

$$F_1 = \frac{Mg\ell}{2L} = \frac{(1360\text{ kg})(9.80\text{ m/s}^2)(1.27\text{ m})}{2(3.05\text{ m})} = 2.77\times10^3\text{ N}.$$

(b) Equilibrium of forces leads to $2F_1 + 2F_2 = Mg$, from which we obtain $F_2 = 3.89 \times 10^3\,\text{N}$.

3. **THINK** Three forces act on the sphere: the tension force $\vec{T}$ of the rope, the force of the wall $\vec{F}_N$, and the force of gravity $m\vec{g}$.

EXPRESS The free-body diagram is shown to the right. The tension force $\vec{T}$ acts along the rope, the force of the wall $\vec{F}_N$ acts horizontally away from the wall, and the force of gravity $m\vec{g}$ acts downward. Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then Newton's second law gives



vertical component : $T\cos\theta - mg = 0$

horizontal component : $F_N - T\sin\theta = 0$.

ANALYZE (a) We solve the first equation for the tension: $T = mg/\cos\theta$. We substitute $\cos\theta = L/\sqrt{L^2+r^2}$ to obtain

$$T = \frac{mg\sqrt{L^2+r^2}}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)\sqrt{(0.080\text{ m})^2+(0.042\text{ m})^2}}{0.080\text{ m}} = 9.4\text{ N}.$$

(b) We solve the second equation for the normal force: $F_N = T\sin\theta$. Using $\sin\theta = r/\sqrt{L^2+r^2}$, we obtain

$$F_N = \frac{Tr}{\sqrt{L^2+r^2}} = \frac{mg\sqrt{L^2+r^2}}{L}\frac{r}{\sqrt{L^2+r^2}} = \frac{mgr}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)(0.042\text{ m})}{(0.080\text{ m})} = 4.4\text{ N}.$$

LEARN Since the sphere is in static equilibrium, the vector sum of all external forces acting on it must be zero.

4. The situation is somewhat similar to that depicted for problem 10 (see the figure that accompanies that problem in the text). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T\sin\theta = F$, where θ is the angle (taken positive) between each segment of the string and its "relaxed" position (when the two segments are collinear). Setting $T = F$ therefore yields $\theta = 30°$. Since $\alpha = 180° - 2\theta$ is the angle between the two segments, then we find $\alpha = 120°$.

5. The object exerts a downward force of magnitude $F = 3160$ N at the midpoint of the rope, causing a "kink" similar to that shown for problem 10 (see the figure that accompanies that problem in the text). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T\sin\theta = F$, where θ is the angle (taken

positive) between each segment of the string and its "relaxed" position (when the two segments are collinear). In this problem, we have

$$\theta = \tan^{-1}\left(\frac{0.35\,\text{m}}{1.72\,\text{m}}\right) = 11.5°.$$

Therefore, $T = F/(2\sin\theta) = 7.92 \times 10^3$ N.

6. Let $\ell_1 = 1.5\,\text{m}$ and $\ell_2 = (5.0 - 1.5)\text{ m} = 3.5\text{ m}$. We denote tension in the cable closer to the window as F_1 and that in the other cable as F_2. The force of gravity on the scaffold itself (of magnitude $m_s g$) is at its midpoint, $\ell_3 = 2.5\,\text{m}$ from either end.

(a) Taking torques about the end of the plank farthest from the window washer, we find

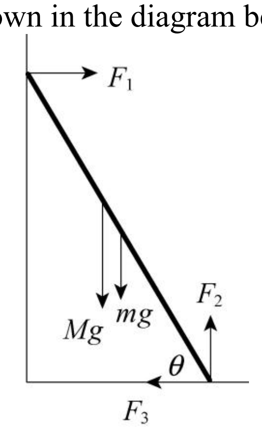
$$F_1 = \frac{m_w g \ell_2 + m_s g \ell_3}{\ell_1 + \ell_2} = \frac{(80\,\text{kg})(9.8\,\text{m/s}^2)(3.5\,\text{m}) + (60\,\text{kg})(9.8\,\text{m/s}^2)(2.5\,\text{m})}{5.0\,\text{m}}$$
$$= 8.4 \times 10^2\text{ N}.$$

(b) Equilibrium of forces leads to

$$F_1 + F_2 = m_s g + m_w g = (60\,\text{kg} + 80\,\text{kg})(9.8\,\text{m/s}^2) = 1.4 \times 10^3\text{ N}$$

which (using our result from part (a)) yields $F_2 = 5.3 \times 10^2\,\text{N}$.

7. The forces on the ladder are shown in the diagram below.



F_1 is the force of the window, horizontal because the window is frictionless. F_2 and F_3 are components of the force of the ground on the ladder. M is the mass of the window cleaner and m is the mass of the ladder.

The force of gravity on the man acts at a point 3.0 m up the ladder and the force of gravity on the ladder acts at the center of the ladder. Let θ be the angle between the ladder and the ground. We use $\cos\theta = d/L$ or $\sin\theta = \sqrt{L^2 - d^2}/L$ to find $\theta = 60°$. Here L

is the length of the ladder (5.0 m) and d is the distance from the wall to the foot of the ladder (2.5 m).

(a) Since the ladder is in equilibrium the sum of the torques about its foot (or any other point) vanishes. Let ℓ be the distance from the foot of the ladder to the position of the window cleaner. Then,

$$Mg\ell\cos\theta + mg\left(L/2\right)\cos\theta - F_1 L\sin\theta = 0\,,$$

and

$$F_1 = \frac{(M\ell + mL/2)g\cos\theta}{L\sin\theta} = \frac{[(75\,\text{kg})(3.0\,\text{m})+(10\,\text{kg})(2.5\,\text{m})](9.8\,\text{m/s}^2)\cos 60°}{(5.0\,\text{m})\sin 60°}$$
$$= 2.8\times10^2\ \text{N}.$$

This force is outward, away from the wall. The force of the ladder on the window has the same magnitude but is in the opposite direction: it is approximately 280 N, inward.

(b) The sum of the horizontal forces and the sum of the vertical forces also vanish:

$$F_1 - F_3 = 0$$
$$F_2 - Mg - mg = 0$$

The first of these equations gives $F_3 = F_1 = 2.8\times10^2\,\text{N}$ and the second gives

$$F_2 = (M+m)g = (75\,\text{kg} + 10\,\text{kg})(9.8\,\text{m/s}^2) = 8.3\times10^2\,\text{N}\,.$$

The magnitude of the force of the ground on the ladder is given by the square root of the sum of the squares of its components:

$$F = \sqrt{F_2^2 + F_3^2} = \sqrt{(2.8\times10^2\,\text{N})^2 + (8.3\times10^2\,\text{N})^2} = 8.8\times10^2\,\text{N}.$$

(c) The angle ϕ between the force and the horizontal is given by

$$\tan\phi = F_3/F_2 = (830\ \text{N})/(280\ \text{N}) = 2.94,$$

so ϕ = 71º. The force points to the left and upward, 71º above the horizontal. We note that this force is not directed along the ladder.

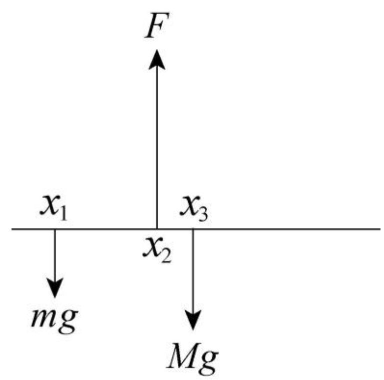
8. From $\vec{\tau} = \vec{r}\times\vec{F}$, we note that persons 1 through 4 exert torques pointing out of the page (relative to the fulcrum), and persons 5 through 8 exert torques pointing into the page.

(a) Among persons 1 through 4, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 2.

(b) Among persons 5 through 8, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 7.

9. **THINK** In order for the meter stick to remain in equilibrium, the net force acting on it must be zero. In addition, the net torque about any point must also be zero.

EXPRESS Let the x axis be along the meter stick, with the origin at the zero position on the scale. The forces acting on it are shown to the right. The coins are at $x = x_1$ = 0.120 m, and $m = 10.0$ g is their total mass. The knife edge is at $x = x_2$ = 0.455 m and exerts force $\vec{F}$. The mass of the meter stick is M, and the force of gravity acts at the center of the stick, $x = x_3$ = 0.500 m.



Since the meter stick is in equilibrium, the sum of the torques about x_2 must vanish:

$$Mg(x_3 - x_2) - mg(x_2 - x_1) = 0.$$

ANALYZE Solving the equation above for M, we find the mass of the meter stick to be

$$M = \left(\frac{x_2 - x_1}{x_3 - x_2}\right)m = \left(\frac{0.455\,\text{m} - 0.120\,\text{m}}{0.500\,\text{m} - 0.455\,\text{m}}\right)(10.0\text{g}) = 74.4 \text{ g}.$$

LEARN Since the torque about any point is zero, we could have chosen x_1. In this case, balance of torques requires that

$$F(x_2 - x_1) - Mg(x_3 - x_1) = 0$$

The fact that the net force is zero implies $F = (M + m)g$. Substituting this into the above equation gives the same result as before:

$$M = \left(\frac{x_2 - x_1}{x_3 - x_2}\right)m.$$

10. (a) Analyzing vertical forces where string 1 and string 2 meet, we find

$$T_1 = \frac{w_A}{\cos\phi} = \frac{40\text{N}}{\cos 35°} = 49\text{N}.$$

(b) Looking at the horizontal forces at that point leads to

$$T_2 = T_1 \sin 35° = (49\text{N})\sin 35° = 28\,\text{N}.$$

(c) We denote the components of T_3 as T_x (rightward) and T_y (upward). Analyzing horizontal forces where string 2 and string 3 meet, we find $T_x = T_2 = 28$ N. From the vertical forces there, we conclude $T_y = w_B = 50$ N. Therefore,

$$T_3 = \sqrt{T_x^2 + T_y^2} = 57\,\text{N}.$$

(d) The angle of string 3 (measured from vertical) is

$$\theta = \tan^{-1}\left(\frac{T_x}{T_y}\right) = \tan^{-1}\left(\frac{28}{50}\right) = 29°.$$

11. **THINK** The diving board is in equilibrium, so the net force and net torque must be zero.

EXPRESS We take the force of the left pedestal to be F_1 at $x = 0$, where the x axis is along the diving board. We take the force of the right pedestal to be F_2 and denote its position as $x = d$. Upward direction is taken to be positive and W is the weight of the diver, located at $x = L$. The following two equations result from setting the sum of forces equal to zero (with upwards positive), and the sum of torques (about x_2) equal to zero:

$$F_1 + F_2 - W = 0$$
$$F_1 d + W(L - d) = 0$$

ANALYZE (a) The second equation gives

$$F_1 = -\left(\frac{L-d}{d}\right)W = -\left(\frac{3.0\,\text{m}}{1.5\,\text{m}}\right)(580\,\text{N}) = -1160\,\text{N}$$

which should be rounded off to $F_1 = -1.2\times10^3$ N. Thus, $|F_1| = 1.2\times10^3$ N.

(b) Since F_1 is negative, this force is downward.

(c) The first equation gives $F_2 = W - F_1 = 580\,\text{N} + 1160\,\text{N} = 1740\,\text{N}$.

which should be rounded off to $F_2 = 1.7\times10^3$ N. Thus, $|F_2| = 1.7\times10^3$ N.

(d) The result is positive, indicating that this force is upward.

(e) The force of the diving board on the left pedestal is upward (opposite to the force of the pedestal on the diving board), so this pedestal is being stretched.

(f) The force of the diving board on the right pedestal is downward, so this pedestal is being compressed.

LEARN We can relate F_1 and F_2 via $F_1 = -\left(\frac{L-d}{L}\right)F_2$. The expression makes it clear that the two forces must be of opposite signs, i.e., one acting downward and the other upward.

12. The angle of each half of the rope, measured from the dashed line, is

$$\theta = \tan^{-1}\left(\frac{0.30\,\text{m}}{9.0\,\text{m}}\right) = 1.9°.$$

Analyzing forces at the "kink" (where $\vec{F}$ is exerted) we find

$$T = \frac{F}{2\sin\theta} = \frac{550\,\text{N}}{2\sin 1.9°} = 8.3\times10^3\ \text{N}.$$

13. The (vertical) forces at points A, B, and P are F_A, F_B, and F_P, respectively. We note that $F_P = W$ and is upward. Equilibrium of forces and torques (about point B) lead to

$$F_A + F_B + W = 0$$
$$bW - aF_A = 0.$$

(a) From the second equation, we find

$$F_A = bW/a = (15/5)W = 3W = 3(900\ \text{N}) = 2.7\times10^3\ \text{N}.$$

(b) The direction is upward since $F_A > 0$.

(c) Using this result in the first equation above, we obtain

$$F_B = W - F_A = -4W = -4(900\ \text{N}) = -3.6\times10^3\,\text{N},$$

or $|F_B| = 3.6\times10^3\,\text{N}$.

(d) F_B points downward, as indicated by the negative sign.

14. With pivot at the left end, Eq. 12-9 leads to

$$-m_s g\frac{L}{2} - Mgx + T_R L = 0$$

where m_s is the scaffold's mass (50 kg) and M is the total mass of the paint cans (75 kg). The variable x indicates the center of mass of the paint can collection (as measured from the left end), and T_R is the tension in the right cable (722 N). Thus we obtain $x = 0.702$ m.

15. (a) Analyzing the horizontal forces (which add to zero) we find $F_h = F_3 = 5.0$ N.

(b) Equilibrium of vertical forces leads to $F_v = F_1 + F_2 = 30$ N.

(c) Computing torques about point O, we obtain

$$F_v d = F_2 b + F_3 a \;\Rightarrow\; d = \frac{(10\text{ N})(3.0\text{ m}) + (5.0\text{ N})(2.0\text{ m})}{30\text{ N}} = 1.3\text{ m}.$$

16. The forces exerted horizontally by the obstruction and vertically (upward) by the floor are applied at the bottom front corner C of the crate, as it verges on tipping. The center of the crate, which is where we locate the gravity force of magnitude $mg = 500$ N, is a horizontal distance $\ell = 0.375\text{ m}$ from C. The applied force of magnitude $F = 350$ N is a vertical distance h from C. Taking torques about C, we obtain

$$h = \frac{mg\ell}{F} = \frac{(500\text{ N})(0.375\text{ m})}{350\text{ N}} = 0.536\text{ m}.$$

17. (a) With the pivot at the hinge, Eq. 12-9 gives

$$TL\cos\theta - mg\frac{L}{2} = 0.$$

This leads to $\theta = 78°$. Then the geometric relation $\tan\theta = L/D$ gives $D = 0.64$ m.

(b) A higher (steeper) slope for the cable results in a smaller tension. Thus, making D greater than the value of part (a) should prevent rupture.

18. With pivot at the left end of the lower scaffold, Eq. 12-9 leads to

$$-m_2 g\frac{L_2}{2} - mgd + T_R L_2 = 0$$

where m_2 is the lower scaffold's mass (30 kg) and L_2 is the lower scaffold's length (2.00 m). The mass of the package ($m = 20$ kg) is a distance $d = 0.50$ m from the pivot, and T_R is the tension in the rope connecting the right end of the lower scaffold to the larger scaffold above it. This equation yields $T_R = 196$ N. Then Eq. 12-8 determines T_L (the tension in the cable connecting the right end of the lower scaffold to the larger scaffold above it): $T_L = 294$ N. Next, we analyze the larger scaffold (of length $L_1 = L_2 + 2d$ and mass m_1, given in the problem statement) placing our pivot at its left end and using Eq. 12-9:

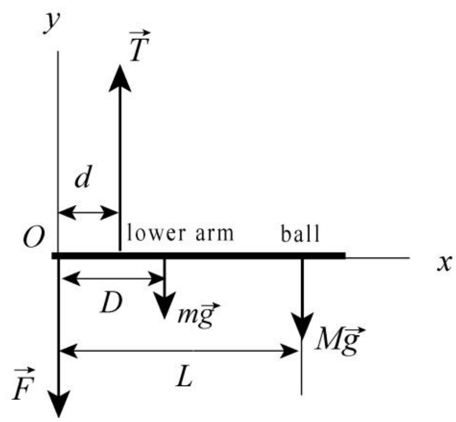
$$-m_1 g\frac{L_1}{2} - T_L d - T_R(L_1 - d) + TL_1 = 0.$$

This yields $T = 457$ N.

19. Setting up equilibrium of torques leads to a simple "level principle" ratio:

$$F_{\perp} = (40\,\text{N})\frac{d}{L} = (40\,\text{N})\frac{2.6\,\text{cm}}{12\,\text{cm}} = 8.7\,\text{N}.$$

20. Our system consists of the lower arm holding a bowling ball. As shown in the free-body diagram, the forces on the lower arm consist of $\vec{T}$ from the biceps muscle, $\vec{F}$ from the bone of the upper arm, and the gravitational forces, $m\vec{g}$ and $M\vec{g}$. Since the system is in static equilibrium, the net force acting on the system is zero:



$$0 = \sum F_{\text{net},y} = T - F - (m+M)g\,.$$

In addition, the net torque about O must also vanish:

$$0 = \sum_O \tau_{\text{net}} = (d)(T) + (0)F - (D)(mg) - L(Mg)\,.$$

(a) From the torque equation, we find the force on the lower arms by the biceps muscle to be

$$T = \frac{(mD+ML)g}{d} = \frac{[(1.8\text{ kg})(0.15\text{ m}) + (7.2\text{ kg})(0.33\text{ m})](9.8\text{ m/s}^2)}{0.040\text{ m}}$$
$$= 648\text{ N} \approx 6.5\times10^2\text{ N}.$$

(b) Substituting the above result into the force equation, we find F to be

$$F = T - (M+m)g = 648\text{ N} - (7.2\text{ kg} + 1.8\text{ kg})(9.8\text{ m/s}^2) = 560\text{ N} = 5.6\times10^2\text{ N}.$$

21. (a) We note that the angle between the cable and the strut is

$$\alpha = \theta - \phi = 45^\circ - 30^\circ = 15^\circ.$$

The angle between the strut and any vertical force (like the weights in the problem) is $\beta = 90^\circ - 45^\circ = 45^\circ$. Denoting $M = 225$ kg and $m = 45.0$ kg, and ℓ as the length of the boom, we compute torques about the hinge and find

$$T = \frac{Mg\ell\sin\beta + mg\left(\frac{\ell}{2}\right)\sin\beta}{\ell\sin\alpha} = \frac{Mg\sin\beta + mg\sin\beta/2}{\sin\alpha}.$$

The unknown length ℓ cancels out and we obtain $T = 6.63\times10^3$ N.

(b) Since the cable is at 30º from horizontal, then horizontal equilibrium of forces requires that the horizontal hinge force be

$$F_x = T\cos 30° = 5.74\times 10^3\,\text{N}.$$

(c) And vertical equilibrium of forces gives the vertical hinge force component:

$$F_y = Mg + mg + T\sin 30° = 5.96\times 10^3\,\text{N}.$$

22. (a) The problem asks for the person's pull (his force exerted on the rock) but since we are examining forces and torques *on the person*, we solve for the reaction force F_{N1} (exerted leftward on the hands by the rock). At that point, there is also an upward force of static friction on his hands, f_1, which we will take to be at its maximum value $\mu_1 F_{N1}$. We note that equilibrium of horizontal forces requires $F_{N1} = F_{N2}$ (the force exerted leftward on his feet); on his feet there is also an upward static friction force of magnitude $\mu_2 F_{N2}$. Equilibrium of vertical forces gives

$$f_1 + f_2 - mg = 0 \Rightarrow F_{N1} = \frac{mg}{\mu_1 + \mu_2} = 3.4\times 10^2\,\text{N}.$$

(b) Computing torques about the point where his feet come in contact with the rock, we find

$$mg(d+w) - f_1 w - F_{N1}h = 0 \;\Rightarrow\; h = \frac{mg(d+w) - \mu_1 F_{N1} w}{F_{N1}} = 0.88\text{ m}.$$

(c) Both intuitively and mathematically (since both coefficients are in the denominator) we see from part (a) that F_{N1} would increase in such a case.

(d) As for part (b), it helps to plug part (a) into part (b) and simplify:

$$h = (d + w)\mu_2 + d\mu_1$$

from which it becomes apparent that h should decrease if the coefficients decrease.

23. The beam is in equilibrium: the sum of the forces and the sum of the torques acting on it each vanish. As shown in the figure, the beam makes an angle of 60º with the vertical and the wire makes an angle of 30º with the vertical.

(a) We calculate the torques around the hinge. Their sum is

$$TL\sin 30° - W(L/2)\sin 60° = 0.$$

Here W is the force of gravity acting at the center of the beam, and T is the tension force of the wire. We solve for the tension:

$$T = \frac{W\sin 60°}{2\sin 30°} = \frac{(222\,\text{N})\sin 60°}{2\sin 30°} = 192\,\text{N}.$$

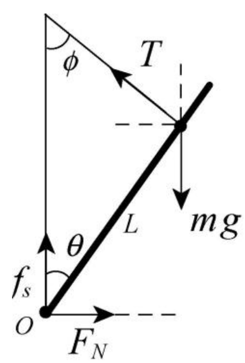
(b) Let F_h be the horizontal component of the force exerted by the hinge and take it to be positive if the force is outward from the wall. Then, the vanishing of the horizontal component of the net force on the beam yields $F_h - T\sin 30° = 0$ or

$$F_h = T\sin 30° = (192.3\,\text{N})\sin 30° = 96.1\,\text{N}.$$

(c) Let F_v be the vertical component of the force exerted by the hinge and take it to be positive if it is upward. Then, the vanishing of the vertical component of the net force on the beam yields $F_v + T\cos 30° - W = 0$ or

$$F_v = W - T\cos 30° = 222\,\text{N} - (192.3\,\text{N})\cos 30° = 55.5\,\text{N}.$$

24. As shown in the free-body diagram, the forces on the climber consist of $\vec{T}$ from the rope, normal force $\vec{F}_N$ on her feet, upward static frictional force $\vec{f}_s$, and downward gravitational force $m\vec{g}$.



Since the climber is in static equilibrium, the net force acting on her is zero. Applying Newton's second law to the vertical and horizontal directions, we have

$$0 = \sum F_{\text{net},x} = F_N - T\sin\phi$$
$$0 = \sum F_{\text{net},y} = T\cos\phi + f_s - mg\,.$$

In addition, the net torque about O (contact point between her feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = mgL\sin\theta - TL\sin(180° - \theta - \phi)$$

From the torque equation, we obtain

$$T = mg\sin\theta / \sin(180° - \theta - \phi).$$

Substituting the expression into the force equations, and noting that $f_s = \mu_s F_N$, we find the coefficient of static friction to be

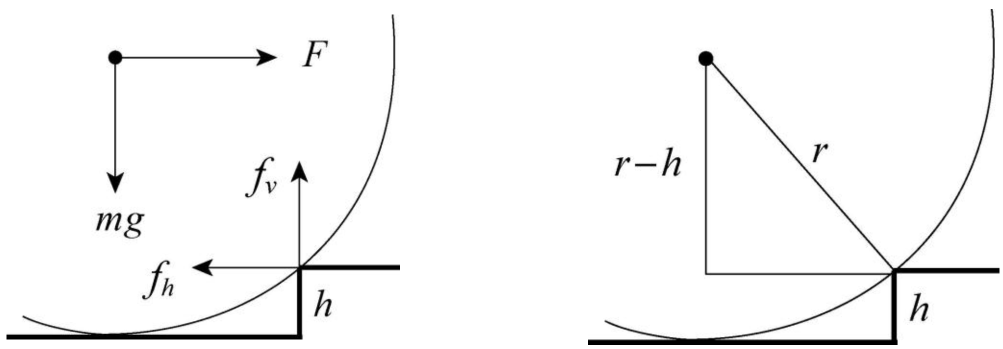
$$\mu_s = \frac{f_s}{F_N} = \frac{mg - T\cos\phi}{T\sin\phi} = \frac{mg - mg\sin\theta\cos\phi/\sin(180° - \theta - \phi)}{mg\sin\theta\sin\phi/\sin(180° - \theta - \phi)}$$
$$= \frac{1 - \sin\theta\cos\phi/\sin(180° - \theta - \phi)}{\sin\theta\sin\phi/\sin(180° - \theta - \phi)}.$$

With $\theta = 40°$ and $\phi = 30°$, the result is

$$\mu_s = \frac{1 - \sin\theta\cos\phi/\sin(180° - \theta - \phi)}{\sin\theta\sin\phi/\sin(180° - \theta - \phi)} = \frac{1 - \sin 40°\cos 30°/\sin(180° - 40° - 30°)}{\sin 40°\sin 30°/\sin(180° - 40° - 30°)}$$
$$= 1.19.$$

25. **THINK** At the moment when the wheel leaves the lower floor, the floor no longer exerts a force on it.

EXPRESS As the wheel is raised over the obstacle, the only forces acting are the force *F* applied horizontally at the axle, the force of gravity *mg* acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v.



If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.

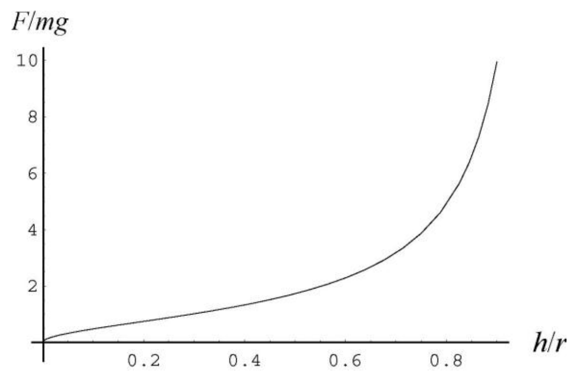
We calculate the torque around the step corner. The second diagram (above right) indicates that the distance from the line of *F* to the corner is *r* – *h*, where *r* is the radius of the wheel and *h* is the height of the step. The distance from the line of *mg* to the corner is $\sqrt{r^2 + (r-h)^2} = \sqrt{2rh - h^2}$. Thus,

$$F(r-h) - mg\sqrt{2rh - h^2} = 0.$$

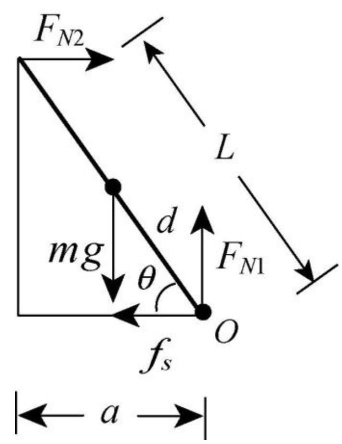
ANALYZE The solution for *F* is

$$F = \frac{\sqrt{2rh - h^2}}{r - h} mg = \frac{\sqrt{2(6.00\times10^{-2}\,\text{m})(3.00\times10^{-2}\,\text{m}) - (3.00\times10^{-2}\,\text{m})^2}}{(6.00\times10^{-2}\,\text{m}) - (3.00\times10^{-2}\,\text{m})}(0.800\text{ kg})(9.80\text{ m/s}^2)$$
$$= 13.6\text{ N}.$$

LEARN The applied force here is about 1.73 times the weight of the wheel. If the height is increased, the force that must be applied also goes up. Below we plot F/mg as a function of the ratio h/r. The required force increases rapidly as $h/r \to 1$.



26. As shown in the free-body diagram, the forces on the climber consist of the normal forces F_{N1} on his hands from the ground and F_{N2} on his feet from the wall, static frictional force f_s, and downward gravitational force mg. Since the climber is in static equilibrium, the net force acting on him is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0 = \sum F_{\text{net},x} = F_{N2} - f_s$$
$$0 = \sum F_{\text{net},y} = F_{N1} - mg .$$

In addition, the net torque about O (contact point between his feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = mgd\cos\theta - F_{N2}L\sin\theta .$$

The torque equation gives

$$F_{N2} = mgd\cos\theta / L\sin\theta = mgd\cot\theta / L .$$

On the other hand, from the force equation we have $F_{N2} = f_s$ and $F_{N1} = mg$. These expressions can be combined to yield

$$f_s = F_{N2} = F_{N1}\cot\theta \frac{d}{L} .$$

On the other hand, the frictional force can also be written as $f_s = \mu_s F_{N1}$, where μ_s is the coefficient of static friction between his feet and the ground. From the above equation and the values given in the problem statement, we find μ_s to be

$$\mu_s = \cot\theta \frac{d}{L} = \frac{a}{\sqrt{L^2 - a^2}} \frac{d}{L} = \frac{0.914 \text{ m}}{\sqrt{(2.10 \text{ m})^2 - (0.914 \text{ m})^2}} \frac{0.940 \text{ m}}{2.10 \text{ m}} = 0.216 .$$

27. (a) All forces are vertical and all distances are measured along an axis inclined at θ = 30º. Thus, any trigonometric factor cancels out and the application of torques about the contact point (referred to in the problem) leads to

$$F_{\text{tricep}} = \frac{(15\,\text{kg})(9.8\,\text{m/s}^2)(35\,\text{cm}) - (2.0\,\text{kg})(9.8\,\text{m/s}^2)(15\,\text{cm})}{2.5\,\text{cm}} = 1.9\times10^3 \text{ N}.$$

(b) The direction is upward since $F_{\text{tricep}} > 0$.

(c) Equilibrium of forces (with upward positive) leads to

$$F_{\text{tricep}} + F_{\text{humer}} + (15\,\text{kg})(9.8\,\text{m/s}^2) - (2.0\,\text{kg})(9.8\,\text{m/s}^2) = 0$$

and thus to $F_{\text{humer}} = -2.1\times10^3\,\text{N}$, or $|F_{\text{humer}}| = 2.1\times10^3\,\text{N}$.

(d) The negative sign implies that F_{humer} points downward.

28. (a) Computing torques about point A, we find

$$T_{\max} L\sin\theta = W x_{\max} + W_b \left(\frac{L}{2}\right).$$

We solve for the maximum distance:

$$x_{\max} = \left(\frac{T_{\max}\sin\theta - W_b/2}{W}\right) L = \left(\frac{(500 \text{ N})\sin 30.0° - (200 \text{ N})/2}{300 \text{ N}}\right)(3.00 \text{ m}) = 1.50\,\text{m}.$$

(b) Equilibrium of horizontal forces gives $F_x = T_{\max}\cos\theta = 433\,\text{N}$.

(c) And equilibrium of vertical forces gives $F_y = W + W_b - T_{\max}\sin\theta = 250\,\text{N}$.

29. The problem states that each hinge supports half the door's weight, so each vertical hinge force component is $F_y = mg/2 = 1.3 \times 10^2$ N. Computing torques about the top hinge, we find the horizontal hinge force component (at the bottom hinge) is

$$F_h = \frac{(27\,\text{kg})(9.8\,\text{m/s}^2)\left(0.91\text{ m}/2\right)}{2.1\,\text{m} - 2(0.30\,\text{m})} = 80\,\text{N}.$$

Equilibrium of horizontal forces demands that the horizontal component of the top hinge force has the same magnitude (though opposite direction).

(a) In unit-vector notation, the force on the door at the top hinge is

$$F_{\text{top}} = (-80\text{ N})\hat{\text{i}} + (1.3\times10^2\,\text{N})\hat{\text{j}}.$$

(b) Similarly, the force on the door at the bottom hinge is

$$F_{\text{bottom}} = (+80\text{ N})\hat{\text{i}} + (1.3\times10^2\,\text{N})\hat{\text{j}}.$$

30. (a) The sign is attached in two places: at x_1 = 1.00 m (measured rightward from the hinge) and at x_2 = 3.00 m. We assume the downward force due to the sign's weight is equal at these two attachment points, each being *half* the sign's weight of *mg*. The angle where the cable comes into contact (also at x_2) is

$$\theta = \tan^{-1}(d_v/d_h) = \tan^{-1}(4.00\text{ m}/3.00\text{ m})$$

and the force exerted there is the tension *T*. Computing torques about the hinge, we find

$$T = \frac{\frac{1}{2}mgx_1 + \frac{1}{2}mgx_2}{x_2\sin\theta} = \frac{\frac{1}{2}\left(50.0\text{ kg}\right)\left(9.8\text{ m/s}^2\right)\left(1.00\text{ m}\right) + \frac{1}{2}(50.0\text{ kg})(9.8\text{m/s}^2)(3.00\text{ m})}{\left(3.00\text{ m}\right)\left(0.800\right)}$$
$$= 408\text{ N}.$$

(b) Equilibrium of horizontal forces requires that the horizontal hinge force be

$$F_x = T\cos\theta = 245\text{ N}.$$

(c) The direction of the horizontal force is rightward.

(d) Equilibrium of vertical forces requires that the vertical hinge force be

$$F_y = mg - T\sin\theta = 163\text{ N}.$$

(e) The direction of the vertical force is upward.

31. The bar is in equilibrium, so the forces and the torques acting on it each sum to zero. Let T_l be the tension force of the left-hand cord, T_r be the tension force of the right-hand cord, and *m* be the mass of the bar. The equations for equilibrium are:

$$\begin{aligned} \text{vertical force components:} &\quad T_l \cos\theta + T_r \cos\phi - mg = 0 \\ \text{horizontal force components:} &\quad -T_l \sin\theta + T_r \sin\phi = 0 \\ \text{torques:} &\quad mgx - T_r L \cos\phi = 0. \end{aligned}$$

The origin was chosen to be at the left end of the bar for purposes of calculating the torque. The unknown quantities are T_l, T_r, and x. We want to eliminate T_l and T_r, then solve for x. The second equation yields $T_l = T_r \sin\phi/\sin\theta$ and when this is substituted into the first and solved for T_r the result is

$$T_r = \frac{mg\sin\theta}{\sin\phi\cos\theta + \cos\phi\sin\theta}.$$

This expression is substituted into the third equation and the result is solved for x:

$$x = L\frac{\sin\theta\cos\phi}{\sin\phi\cos\theta + \cos\phi\sin\theta} = L\frac{\sin\theta\cos\phi}{\sin(\theta+\phi)}.$$

The last form was obtained using the trigonometric identity

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

For the special case of this problem $\theta + \phi = 90^\circ$ and $\sin(\theta + \phi) = 1$. Thus,

$$x = L\sin\theta\cos\phi = (6.10 \text{ m})\sin 36.9^\circ \cos 53.1^\circ = 2.20 \text{ m}.$$

32. (a) With $F = ma = -\mu_k mg$ the magnitude of the deceleration is

$$|a| = \mu_k g = (0.40)(9.8 \text{ m/s}^2) = 3.92 \text{ m/s}^2.$$

(b) As hinted in the problem statement, we can use Eq. 12-9, evaluating the torques about the car's center of mass, and bearing in mind that the friction forces are acting horizontally at the bottom of the wheels; the total friction force there is $f_k = \mu_k gm = 3.92m$ (with SI units understood, and m is the car's mass), a vertical distance of 0.75 meter below the center of mass. Thus, torque equilibrium leads to

$$(3.92m)(0.75) + F_{Nr}(2.4) - F_{Nf}(1.8) = 0.$$

Equation 12-8 also holds (the acceleration is horizontal, not vertical), so we have $F_{Nr} + F_{Nf} = mg$, which we can solve simultaneously with the above torque equation. The mass is obtained from the car's weight: $m = 11000/9.8$, and we obtain $F_{Nr} = 3929 \approx 4000$ N. Since each involves two wheels then we have (roughly) 2.0×10^3 N on each rear wheel.

(c) From the above equation, we also have F_{Nf} = 7071 ≈ 7000 N, or 3.5×10^3 N on each front wheel, as the values of the individual normal forces.

(d) For friction on each rear wheel, Eq. 6-2 directly yields

$$f_{r1} = \mu_k (F_{Nr}/2) = (0.40)(3929 \text{ N}/2) = 7.9\times10^2 \text{ N} .$$

(e) Similarly, for friction on the front rear wheel, Eq. 6-2 gives

$$f_{f1} = \mu_k (F_{Nf}/2) = (0.40)(7071 \text{ N}/2) = 1.4\times10^3 \text{ N} .$$

33. (a) With the pivot at the hinge, Eq. 12-9 yields

$$TL\cos\theta - F_a y = 0 .$$

This leads to $T = (F_a/\cos\theta)(y/L)$ so that we can interpret $F_a/\cos\theta$ as the slope on the tension graph (which we estimate to be 600 in SI units). Regarding the F_h graph, we use Eq. 12-7 to get

$$F_h = T\cos\theta - F_a = (-F_a)(y/L) - F_a$$

after substituting our previous expression. The result implies that the slope on the F_h graph (which we estimate to be –300) is equal to $-F_a$, or F_a = 300 N and (plugging back in) θ = 60.0°.

(b) As mentioned in the previous part, F_a = 300 N.

34. (a) Computing torques about the hinge, we find the tension in the wire:

$$TL\sin\theta - Wx = 0 \Rightarrow T = \frac{Wx}{L\sin\theta}.$$

(b) The horizontal component of the tension is $T\cos\theta$, so equilibrium of horizontal forces requires that the horizontal component of the hinge force is

$$F_x = \left(\frac{Wx}{L\sin\theta}\right)\cos\theta = \frac{Wx}{L\tan\theta}.$$

(c) The vertical component of the tension is $T\sin\theta$, so equilibrium of vertical forces requires that the vertical component of the hinge force is

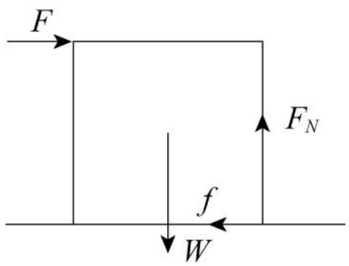
$$F_y = W - \left(\frac{Wx}{L\sin\theta}\right)\sin\theta = W\left(1 - \frac{x}{L}\right).$$

35. **THINK** We examine the box when it is about to tip. Since it will rotate about the lower right edge, this is where the normal force of the floor is exerted.

EXPRESS The free-body diagram is shown below. The normal force is labeled F_N, the force of friction is denoted by *f*, the applied force by *F*, and the force of gravity by *W*. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F - f = 0$, the sum of the vertical force components vanishes: $F_N - W = 0$, and the sum of the torques vanishes:

$$FL - WL/2 = 0.$$

Here *L* is the length of a side of the box and the origin was chosen to be at the lower right edge.



ANALYZE (a) From the torque equation, we find $F = \dfrac{W}{2} = \dfrac{890\,\text{N}}{2} = 445\,\text{N}.$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/F_N$. According to the equations of equilibrium

$$F_N = W = 890\ \text{N}$$
$$f = F = 445\ \text{N},$$

so

$$\mu_s = \frac{f}{F_N} = \frac{445\,\text{N}}{890\,\text{N}} = 0.50.$$

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes

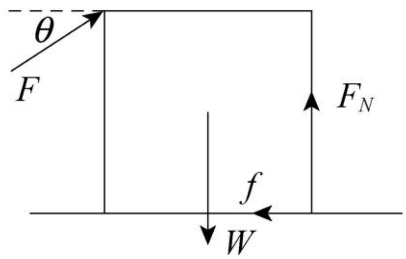
$$FL\cos\theta + FL\sin\theta - WL/2 = 0,$$

with the solution

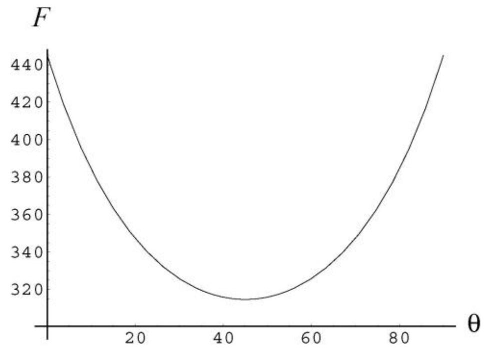
$$F = \frac{W}{2(\cos\theta + \sin\theta)}.$$

We want $\cos\theta + \sin\theta$ to have the largest possible value. This occurs if $\theta = 45°$, a result we can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

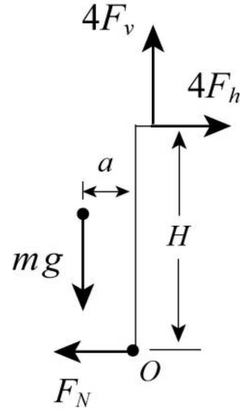
$$F = \frac{W}{2(\cos 45° + \sin 45°)} = \frac{890\,\text{N}}{2(\cos 45° + \sin 45°)} = 315\,\text{N}.$$



LEARN The applied force as a function of θ is plotted below. From the figure, we readily see that $\theta = 0°$ corresponds to a maximum and $\theta = 45°$ a minimum.



36. As shown in the free-body diagram, the forces on the climber consist of the normal force from the wall, the vertical component F_v and the horizontal component F_h of the force acting on her four fingertips, and the downward gravitational force mg.



Since the climber is in static equilibrium, the net force acting on her is zero. Applying Newton's second law to the vertical and horizontal directions, we have

$$0 = \sum F_{\text{net},x} = 4F_h - F_N$$
$$0 = \sum F_{\text{net},y} = 4F_v - mg\,.$$

In addition, the net torque about O (contact point between her feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = (mg)a - (4F_h)H\ .$$

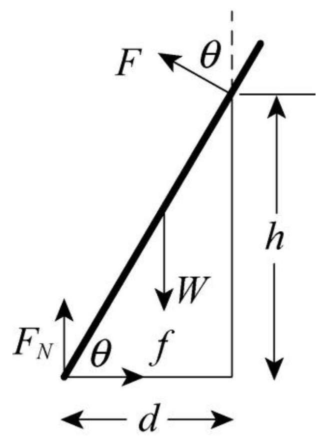
(a) From the torque equation, we find the horizontal component of the force on her fingertip to be

$$F_h = \frac{mga}{4H} = \frac{(70\ \text{kg})(9.8\ \text{m/s}^2)(0.20\ \text{m})}{4(2.0\ \text{m})} \approx 17\ \text{N}.$$

(b) From the y-component of the force equation, we obtain

$$F_v = \frac{mg}{4} = \frac{(70\ \text{kg})(9.8\ \text{m/s}^2)}{4} \approx 1.7\times 10^2\ \text{N}.$$

37. The free-body diagram below shows the forces acting on the plank. Since the roller is frictionless, the force it exerts is normal to the plank and makes the angle θ with the vertical.



Its magnitude is designated F. W is the force of gravity; this force acts at the center of the plank, a distance $L/2$ from the point where the plank touches the floor. F_N is the normal force of the floor and f is the force of friction. The distance from the foot of the plank to the wall is denoted by d. This quantity is not given directly but it can be computed using $d = h/\tan\theta$.

The equations of equilibrium are:

horizontal force components:	$F\sin\theta - f = 0$
vertical force components:	$F\cos\theta - W + F_N = 0$
torques:	$F_N d - fh - W\left(d - \frac{L}{2}\cos\theta\right) = 0.$

The point of contact between the plank and the roller was used as the origin for writing the torque equation.

When $\theta = 70^{\circ}$ the plank just begins to slip and $f = \mu_s F_N$, where μ_s is the coefficient of static friction. We want to use the equations of equilibrium to compute F_N and f for $\theta = 70^{\circ}$, then use $\mu_s = f/F_N$ to compute the coefficient of friction.

The second equation gives $F = (W - F_N)/\cos\theta$ and this is substituted into the first to obtain

$$f = (W - F_N)\sin\theta/\cos\theta = (W - F_N)\tan\theta.$$

This is substituted into the third equation and the result is solved for F_N:

$$F_N = \frac{d - (L/2)\cos\theta + h\tan\theta}{d + h\tan\theta}W = \frac{h(1+\tan^2\theta) - (L/2)\sin\theta}{h(1+\tan^2\theta)}W,$$

where we have used $d = h/\tan\theta$ and multiplied both numerator and denominator by $\tan\theta$. We use the trigonometric identity $1+\tan^2\theta = 1/\cos^2\theta$ and multiply both numerator and denominator by $\cos^2\theta$ to obtain

$$F_N = W\left(1 - \frac{L}{2h}\cos^2\theta\sin\theta\right).$$

Now we use this expression for F_N in $f = (W - F_N)\tan\theta$ to find the friction:

$$f = \frac{WL}{2h}\sin^2\theta\cos\theta.$$

Substituting these expressions for f and F_N into $\mu_s = f/F_N$ leads to

$$\mu_s = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta}.$$

Evaluating this expression for $\theta = 70^{\circ}$, $L = 6.10$ m and $h = 3.05$ m gives

$$\mu_s = \frac{(6.1\,\text{m})\sin^2 70^{\circ}\cos 70^{\circ}}{2(3.05\,\text{m}) - (6.1\,\text{m})\sin 70^{\circ}\cos^2 70^{\circ}} = 0.34.$$

38. The phrase "loosely bolted" means that there is no torque exerted by the bolt at that point (where A connects with B). The force exerted on A at the hinge has x and y components F_x and F_y. The force exerted on A at the bolt has components G_x and G_y, and those exerted on B are simply $-G_x$ and $-G_y$ by Newton's third law. The force exerted on B at its hinge has components H_x and H_y. If a horizontal force is positive, it points rightward, and if a vertical force is positive it points upward.

(a) We consider the combined $A \cup B$ system, which has a total weight of Mg where $M = 122$ kg and the line of action of that downward force of gravity is $x = 1.20$ m from the wall. The vertical distance between the hinges is y = 1.80 m. We compute torques about the bottom hinge and find

$$F_x = -\frac{Mgx}{y} = -797\,\text{N}.$$

If we examine the forces on A alone and compute torques about the bolt, we instead find

$$F_y = \frac{m_A gx}{\ell} = 265\,\text{N}$$

where m_A = 54.0 kg and ℓ = 2.40 m (the length of beam A). Thus, in unit-vector notation, we have

$$\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} = (-797\text{ N})\hat{\text{i}} + (265\text{ N})\hat{\text{j}}.$$

(b) Equilibrium of horizontal and vertical forces on beam A readily yields

$$G_x = -F_x = 797\text{ N}, \quad G_y = m_A g - F_y = 265\text{ N}.$$

In unit-vector notation, we have

$$\vec{G} = G_x\hat{\text{i}} + G_y\hat{\text{j}} = (+797\text{ N})\hat{\text{i}} + (265\text{ N})\hat{\text{j}}.$$

(c) Considering again the combined A∪B system, equilibrium of horizontal and vertical forces readily yields $H_x = -F_x$ = 797 N and $H_y = Mg - F_y$ = 931 N. In unit-vector notation, we have

$$\vec{H} = H_x\hat{\text{i}} + H_y\hat{\text{j}} = (+797\text{ N})\hat{\text{i}} + (931\text{ N})\hat{\text{j}}.$$

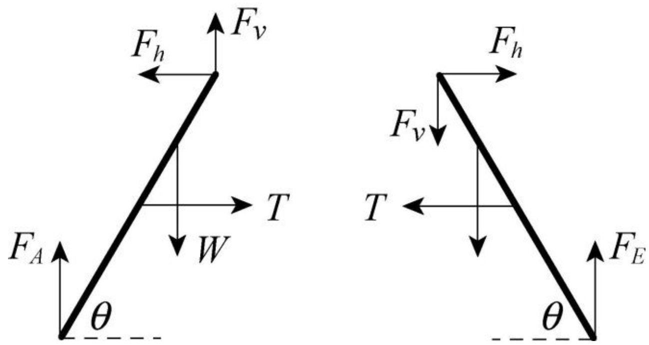
(d) As mentioned above, Newton's third law (and the results from part (b)) immediately provide $-G_x$ = – 797 N and $-G_y$ = – 265 N for the force components acting on B at the bolt. In unit-vector notation, we have

$$-\vec{G} = -G_x\hat{\text{i}} - G_y\hat{\text{j}} = (-797\text{ N})\hat{\text{i}} - (265\text{ N})\hat{\text{j}}.$$

39. The diagrams show the forces on the two sides of the ladder, separated. F_A and F_E are the forces of the floor on the two feet, T is the tension force of the tie rod, W is the force of the man (equal to his weight), F_h is the horizontal component of the force exerted by one side of the ladder on the other, and F_v is the vertical component of that force. Note that the forces exerted by the floor are normal to the floor since the floor is frictionless. Also note that the force of the left side on the right and the force of the right side on the left are equal in magnitude and opposite in direction. Since the ladder is in equilibrium, the vertical components of the forces on the left side of the ladder must sum to zero:

$$F_v + F_A - W = 0.$$

The horizontal components must sum to zero: $T - F_h = 0$.



The torques must also sum to zero. We take the origin to be at the hinge and let L be the length of a ladder side. Then

$$F_A L \cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta = 0.$$

Here we recognize that the man is a distance d from the bottom of the ladder (or $L - d$ from the top), and the tie rod is at the midpoint of the side.

The analogous equations for the right side are $F_E - F_v = 0$, $F_h - T = 0$, and $F_E L \cos\theta - T(L/2)\sin\theta = 0$. There are 5 different equations:

$$\begin{aligned} F_v + F_A - W &= 0, \\ T - F_h &= 0 \\ F_A L\cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta &= 0 \\ F_E - F_v &= 0 \\ F_E L\cos\theta - T(L/2)\sin\theta &= 0. \end{aligned}$$

The unknown quantities are F_A, F_E, F_v, F_h, and T.

(a) First we solve for T by systematically eliminating the other unknowns. The first equation gives $F_A = W - F_v$ and the fourth gives $F_v = F_E$. We use these to substitute into the remaining three equations to obtain

$$\begin{aligned} T - F_h &= 0 \\ WL\cos\theta - F_E L\cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta &= 0 \\ F_E L\cos\theta - T(L/2)\sin\theta &= 0. \end{aligned}$$

The last of these gives $F_E = T\sin\theta/2\cos\theta = (T/2)\tan\theta$. We substitute this expression into the second equation and solve for T. The result is

$$T = \frac{Wd}{L\tan\theta}.$$

To find $\tan\theta$, we consider the right triangle formed by the upper half of one side of the ladder, half the tie rod, and the vertical line from the hinge to the tie rod. The lower side of the triangle has a length of 0.381 m, the hypotenuse has a length of 1.22 m, and the vertical side has a length of $\sqrt{(1.22\text{ m})^2 - (0.381\text{ m})^2} = 1.16\text{ m}$. This means

$$\tan\theta = (1.16\text{m})/(0.381\text{m}) = 3.04.$$

Thus,

$$T = \frac{(854\text{ N})(1.80\text{ m})}{(2.44\text{ m})(3.04)} = 207\text{ N}.$$

(b) We now solve for F_A. We substitute $F_v = F_E = (T/2)\tan\theta = Wd/2L$ into the equation $F_v + F_A - W = 0$ and solve for F_A. The solution is

$$F_A = W - F_v = W\left(1 - \frac{d}{2L}\right) = (854\text{ N})\left(1 - \frac{1.80\text{ m}}{2(2.44\text{ m})}\right) = 539\text{ N}.$$

(c) Similarly, $F_E = W\frac{d}{2L} = (854\text{ N})\frac{1.80\text{ m}}{2(2.44\text{ m})} = 315\text{ N}$.

40. (a) Equation 12-9 leads to

$$TL\sin\theta - m_p g x - m_b g\left(\tfrac{L}{2}\right) = 0.$$

This can be written in the form of a straight line (in the graph) with

$$T = (\text{“slope”})\,\tfrac{x}{L} + \text{“}y\text{-intercept”}$$

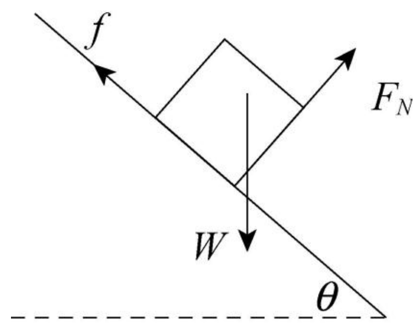
where “slope” = $m_p g/\sin\theta$ and “y-intercept” = $m_b g/2\sin\theta$. The graph suggests that the slope (in SI units) is 200 and the y-intercept is 500. These facts, combined with the given $m_p + m_b = 61.2$ kg datum, lead to the conclusion:

$$\sin\theta = 61.22g/1200 \Rightarrow \theta = 30.0°.$$

(b) It also follows that $m_p = 51.0$ kg.

(c) Similarly, $m_b = 10.2$ kg.

41. The force diagram shown depicts the situation just before the crate tips, when the normal force acts at the front edge. However, it may also be used to calculate the angle for which the crate begins to slide. W is the force of gravity on the crate, F_N is the normal force of the plane on the crate, and f is the force of friction. We take the x-axis to be down the plane and the y-axis to be in the direction of the normal force. We assume the acceleration is zero but the crate is on the verge of sliding.



(a) The x and y components of Newton's second law are

$$W\sin\theta - f = 0 \text{ and } F_N - W\cos\theta = 0$$

respectively. The y equation gives $F_N = W\cos\theta$. Since the crate is about to slide

$$f = \mu_s F_N = \mu_s W\cos\theta,$$

where μ_s is the coefficient of static friction. We substitute into the x equation and find

$$W\sin\theta - \mu_s W\cos\theta = 0 \quad\Rightarrow\quad \tan\theta = \mu_s.$$

This leads to $\theta = \tan^{-1}\mu_s = \tan^{-1}(0.60) = 31.0°$.

In developing an expression for the total torque about the center of mass when the crate is about to tip, we find that the normal force and the force of friction act at the front edge. The torque associated with the force of friction tends to turn the crate clockwise and has magnitude fh, where h is the perpendicular distance from the bottom of the crate to the center of gravity. The torque associated with the normal force tends to turn the crate counterclockwise and has magnitude $F_N\ell/2$, where ℓ is the length of an edge. Since the total torque vanishes, $fh = F_N\ell/2$. When the crate is about to tip, the acceleration of the center of gravity vanishes, so $f = W\sin\theta$ and $F_N = W\cos\theta$. Substituting these expressions into the torque equation, we obtain

$$\theta = \tan^{-1}\frac{\ell}{2h} = \tan^{-1}\frac{1.2\,\text{m}}{2(0.90\,\text{m})} = 33.7°.$$

As θ is increased from zero the crate slides before it tips.

(b) It starts to slide when $\theta = 31°$.

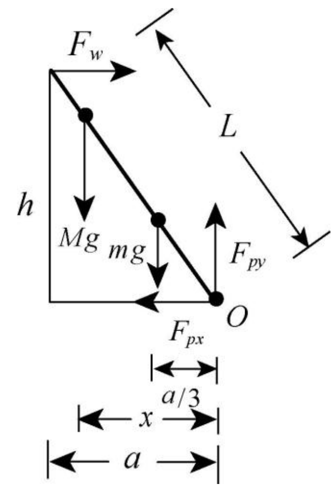
(c) The crate begins to slide when

$$\theta = \tan^{-1}\mu_s = \tan^{-1}(0.70) = 35.0°$$

and begins to tip when $\theta = 33.7°$. Thus, it tips first as the angle is increased.

(d) Tipping begins at $\theta = 33.7° \approx 34°$.

42. Let x be the horizontal distance between the firefighter and the origin O (see the figure) that makes the ladder on the verge of sliding. The forces on the firefighter + ladder system consist of the horizontal force F_w from the wall, the vertical component F_{py} and the horizontal component F_{px} of the force $\vec{F}_p$ on the ladder from the pavement, and the downward gravitational forces Mg and mg, where M and m are the masses of the firefighter and the ladder, respectively.



Since the system is in static equilibrium, the net force acting on the system is zero. Applying Newton's second law to the vertical and horizontal directions, we have

$$0 = \sum F_{\text{net},x} = F_w - F_{px}$$
$$0 = \sum F_{\text{net},y} = F_{py} - (M+m)g .$$

Since the ladder is on the verge of sliding, $F_{px} = \mu_s F_{py}$. Therefore, we have

$$F_w = F_{px} = \mu_s F_{py} = \mu_s (M+m)g .$$

In addition, the net torque about O (contact point between the ladder and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = -h(F_w) + x(Mg) + \frac{a}{3}(mg) = 0 .$$

Solving for x, we obtain

$$x = \frac{hF_w - (a/3)mg}{Mg} = \frac{h\mu_s(M+m)g - (a/3)mg}{Mg} = \frac{h\mu_s(M+m) - (a/3)m}{M}$$

Substituting the values given in the problem statement (with $a = \sqrt{L^2 - h^2} = 7.58$ m), the fraction of ladder climbed is

$$\frac{x}{a} = \frac{h\mu_s(M+m) - (a/3)m}{Ma} = \frac{(9.3\text{ m})(0.53)(72\text{ kg} + 45\text{ kg}) - (7.58\text{ m}/3)(45\text{ kg})}{(72\text{ kg})(7.58\text{ m})}$$
$$= 0.848 \approx 85\%.$$

43. **THINK** The weight of the object hung on the end provides the source of shear stress.

EXPRESS The shear stress is given by F/A, where F is the magnitude of the force applied parallel to one face of the aluminum rod and A is the cross–sectional area of the rod. In this case $F = mg$, where m is the mass of the object. The cross-sectional area is $A = \pi r^2$ where r is the radius of the rod.

ANALYZE (a) Substituting the values given, we find the shear stress to be

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1200\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(0.024\,\text{m})^2} = 6.5\times10^6\,\text{N/m}^2.$$

(b) The shear modulus G is given by

$$G = \frac{F/A}{\Delta x / L},$$

where L is the protrusion of the rod and Δx is its vertical deflection at its end. Thus,

$$\Delta x = \frac{(F/A)L}{G} = \frac{(6.5\times10^6\ \text{N/m}^2)(0.053\,\text{m})}{3.0\times10^{10}\ \text{N/m}^2} = 1.1\times10^{-5}\ \text{m}.$$

LEARN As expected, the extent of vertical deflection Δx is proportional to F, the weight of the object hung from the end. On the other hand, it is inversely proportional to the shear modulus G.

44. (a) The Young's modulus is given by

$$E = \frac{\text{stress}}{\text{strain}} = \text{slope of the stress-strain curve} = \frac{150\times10^6\text{N/m}^2}{0.002} = 7.5\times10^{10}\text{N/m}^2.$$

(b) Since the linear range of the curve extends to about 2.9×10^8 N/m^2, this is approximately the yield strength for the material.

45. (a) Since the brick is now horizontal and the cylinders were initially the same length ℓ, then both have been compressed an equal amount $\Delta\ell$. Thus,

$$\frac{\Delta\ell}{\ell} = \frac{FA}{A_A E_A} \quad \text{and} \quad \frac{\Delta\ell}{\ell} = \frac{F_B}{A_B E_B}$$

which leads to

$$\frac{F_A}{F_B} = \frac{A_A E_A}{A_B E_B} = \frac{(2A_B)(2E_B)}{A_B E_B} = 4.$$

When we combine this ratio with the equation $F_A + F_B = W$, we find $F_A/W = 4/5 = 0.80$.

(b) This also leads to the result $F_B/W = 1/5 = 0.20$.

(c) Computing torques about the center of mass, we find $F_A d_A = F_B d_B$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{1}{4} = 0.25.$$

46. Since the force is (stress × area) and the displacement is (strain × length), we can write the work integral (eq. 7-32) as

$$W = \int F dx \;\; = \int (\text{stress})\, A \;(\text{differential strain}) L \;= AL \int (\text{stress})\,(\text{differential strain})$$

which means the work is (thread cross-sectional area) × (thread length) × (graph area under curve). The area under the curve is

$$\begin{aligned}\text{graph area} &= \frac{1}{2}as_1 + \frac{1}{2}(a+b)(s_2 - s_1) + \frac{1}{2}(b+c)(s_3 - s_2) = \frac{1}{2}\left[as_2 + b(s_3 - s_1) + c(s_3 - s_2)\right] \\ &= \frac{1}{2}\left[(0.12\times 10^9 \text{ N/m}^2)(1.4) + (0.30\times 10^9 \text{ N/m}^2)(1.0) + (0.80\times 10^9 \text{ N/m}^2)(0.60)\right] \\ &= 4.74\times 10^8 \text{ N/m}^2.\end{aligned}$$

(a) The kinetic energy that would put the thread on the verge of breaking is simply equal to W:

$$\begin{aligned}K = W = AL(\text{graph area}) &= (8.0\times 10^{-12} \text{ m}^2)(8.0\times 10^{-3} \text{ m})(4.74\times 10^8 \text{ N/m}^2) \\ &= 3.03\times 10^{-5} \text{ J}.\end{aligned}$$

(b) The kinetic energy of the fruit fly of mass 6.00 mg and speed 1.70 m/s is

$$K_f = \frac{1}{2}m_f v_f^2 = \frac{1}{2}(6.00\times 10^{-6} \text{ kg})(1.70 \text{ m/s})^2 = 8.67\times 10^{-6} \text{ J}.$$

(c) Since $K_f < W$, the fruit fly will not be able to break the thread.

(d) The kinetic energy of a bumble bee of mass 0.388 g and speed 0.420 m/s is

$$K_b = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(3.99\times 10^{-4} \text{ kg})0.420 \text{ m/s})^2 = 3.42\times 10^{-5} \text{ J}.$$

(e) On the other hand, since $K_b > W$, the bumble bee will be able to break the thread.

47. The flat roof (as seen from the air) has area A = 150 m × 5.8 m = 870 m^2. The volume of material directly above the tunnel (which is at depth d = 60 m) is therefore

$$V = A \times d = (870 \text{ m}^2) \times (60 \text{ m}) = 52200 \text{ m}^3.$$

Since the density is $\rho = 2.8$ g/cm^3 = 2800 kg/m^3, we find the mass of material supported by the steel columns to be $m = \rho V = 1.46 \times 10^8$ kg.

(a) The weight of the material supported by the columns is $mg = 1.4 \times 10^9$ N.

(b) The number of columns needed is

$$n = \frac{1.43 \times 10^9 \text{ N}}{\frac{1}{2}(400 \times 10^6 \text{ N/m}^2)(960 \times 10^{-4} \text{m}^2)} = 75.$$

48. Since the force is (stress × area) and the displacement is (strain × length), we can write the work integral (Eq. 7-32) as

$$W = \int F dx = \int (\text{stress})\, A\, (\text{differential strain}) L = AL \int (\text{stress})\, (\text{differential strain})$$

which means the work is (wire area) × (wire length) × (graph area under curve). Since the area of a triangle (see the graph in the problem statement) is $\frac{1}{2}$(base)(height) then we determine the work done to be

$$W = (2.00 \times 10^{-6} \text{ m}^2)(0.800 \text{ m})\left(\tfrac{1}{2}\right)(1.0 \times 10^{-3})(7.0 \times 10^7 \text{ N/m}^2) = 0.0560 \text{ J}.$$

49. (a) Let F_A and F_B be the forces exerted by the wires on the log and let m be the mass of the log. Since the log is in equilibrium, $F_A + F_B - mg = 0$. Information given about the stretching of the wires allows us to find a relationship between F_A and F_B. If wire A originally had a length L_A and stretches by ΔL_A, then $\Delta L_A = F_A L_A / AE$, where A is the cross-sectional area of the wire and E is Young's modulus for steel (200×10^9 N/m^2). Similarly, $\Delta L_B = F_B L_B / AE$. If ℓ is the amount by which B was originally longer than A then, since they have the same length after the log is attached, $\Delta L_A = \Delta L_B + \ell$. This means

$$\frac{F_A L_A}{AE} = \frac{F_B L_B}{AE} + \ell.$$

We solve for F_B:

$$F_B = \frac{F_A L_A}{L_B} - \frac{AE\ell}{L_B}.$$

We substitute into $F_A + F_B - mg = 0$ and obtain

$$F_A = \frac{mgL_B + AE\ell}{L_A + L_B}.$$

The cross-sectional area of a wire is

$$A = \pi r^2 = \pi \left(1.20 \times 10^{-3}\,\text{m}\right)^2 = 4.52 \times 10^{-6}\,\text{m}^2 .$$

Both L_A and L_B may be taken to be 2.50 m without loss of significance. Thus

$$F_A = \frac{(103\,\text{kg})(9.8\,\text{m/s}^2)(2.50\,\text{m}) + (4.52\times10^{-6}\,\text{m}^2)(200\times10^9\,\text{N/m}^2)(2.0\times10^{-3}\,\text{m})}{2.50\,\text{m} + 2.50\,\text{m}}$$
$$= 866\,\text{N}.$$

(b) From the condition $F_A + F_B - mg = 0$, we obtain

$$F_B = mg - F_A = (103\,\text{kg})(9.8\,\text{m/s}^2) - 866\,\text{N} = 143\,\text{N}.$$

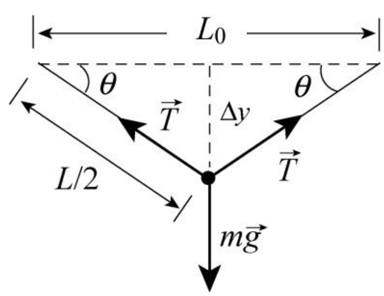
(c) The net torque must also vanish. We place the origin on the surface of the log at a point directly above the center of mass. The force of gravity does not exert a torque about this point. Then, the torque equation becomes $F_A d_A - F_B d_B = 0$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{143\,\text{N}}{866\,\text{N}} = 0.165.$$

50. On the verge of breaking, the length of the thread is

$$L = L_0 + \Delta L = L_0(1 + \Delta L / L_0) = L_0(1+2) = 3L_0 ,$$

where $L_0 = 0.020$ m is the original length, and strain $= \Delta L / L_0 = 2$, as given in the problem. The free-body diagram of the system is shown below.



The condition for equilibrium is $mg = 2T\sin\theta$, where m is the mass of the insect and $T = A(\text{stress})$. Since the volume of the thread remains constant as it is being stretched, we have $V = A_0 L_0 = AL$, or $A = A_0(L_0 / L) = A_0 / 3$. The vertical distance Δy is

$$\Delta y = \sqrt{(L/2)^2 - (L_0/2)^2} = \sqrt{\frac{9L_0^2}{4} - \frac{L_0^2}{4}} = \sqrt{2}L_0 .$$

Thus, the mass of the insect is

$$m=\frac{2T\sin\theta}{g}=\frac{2(A_0/3)(\text{stress})\sin\theta}{g}=\frac{2A_0(\text{stress})}{3g}\frac{\Delta y}{3L_0/2}=\frac{4\sqrt{2}A_0(\text{stress})}{9g}$$
$$=\frac{4\sqrt{2}(8.00\times10^{-12}\text{ m}^2)(8.20\times10^8\text{ N/m}^2)}{9(9.8\text{ m/s}^2)}=4.21\times10^{-4}\text{ kg}$$

or 0.421 g.

51. Let the forces that compress stoppers A and B be F_A and F_B, respectively. Then equilibrium of torques about the axle requires

$$FR = r_AF_A + r_BF_B.$$

If the stoppers are compressed by amounts $|\Delta y_A|$ and $|\Delta y_B|$, respectively, when the rod rotates a (presumably small) angle θ (in radians), then $|\Delta y_A| = r_A\theta$ and $|\Delta y_B| = r_B\theta$.

Furthermore, if their "spring constants" k are identical, then $k = |F/\Delta y|$ leads to the condition $F_A/r_A = F_B/r_B$, which provides us with enough information to solve.

(a) Simultaneous solution of the two conditions leads to

$$F_A=\frac{Rr_A}{r_A^2+r_B^2}F=\frac{(5.0\text{ cm})(7.0\text{ cm})}{(7.0\text{ cm})^2+(4.0\text{ cm})^2}(220\text{ N})=118\text{ N}\approx1.2\times10^2\text{ N}.$$

(b) It also yields

$$F_B=\frac{Rr_B}{r_A^2+r_B^2}F=\frac{(5.0\text{ cm})(4.0\text{ cm})}{(7.0\text{ cm})^2+(4.0\text{ cm})^2}(220\text{ N})=68\text{ N}.$$

52. (a) If L (= 1500 cm) is the unstretched length of the rope and $\Delta L = 2.8\text{ cm}$ is the amount it stretches, then the strain is

$$\Delta L/L=\left(2.8\text{ cm}\right)/\left(1500\text{ cm}\right)=1.9\times10^{-3}.$$

(b) The stress is given by F/A where F is the stretching force applied to one end of the rope and A is the cross-sectional area of the rope. Here F is the force of gravity on the rock climber. If m is the mass of the rock climber then $F = mg$. If r is the radius of the rope then $A=\pi r^2$. Thus the stress is

$$\frac{F}{A}=\frac{mg}{\pi r^2}=\frac{(95\text{ kg})(9.8\text{ m/s}^2)}{\pi(4.8\times10^{-3}\text{ m})^2}=1.3\times10^7\text{ N/m}^2.$$

(c) Young's modulus is the stress divided by the strain:

$$E = (1.3\times10^7\text{ N/m}^2)/(1.9\times10^{-3}) = 6.9\times10^9\text{ N/m}^2.$$

53. **THINK** The slab can remain in static equilibrium if the combined force of the friction and the bolts is greater than the component of the weight of the slab along the incline.

EXPRESS We denote the mass of the slab as *m*, its density as ρ, and volume as $V = LTW$. The angle of inclination is $\theta = 26°$. The component of the weight of the slab along the incline is $F_1 = mg\sin\theta = \rho Vg\sin\theta$, and the static force of friction is

$$f_s = \mu_s F_N = \mu_s mg\cos\theta = \mu_s \rho Vg\cos\theta.$$

ANALYZE (a) Substituting the values given, we find F_1 to be

$$F_1 = \rho Vg\sin\theta = (3.2\times10^3\ \text{kg/m}^3)(43\,\text{m})(2.5\,\text{m})(12\,\text{m})(9.8\,\text{m/s}^2)\sin 26° \approx 1.8\times10^7\ \text{N}.$$

(b) Similarly, the static force of friction is

$$f_s = \mu_s \rho Vg\cos\theta = (0.39)(3.2\times10^3\ \text{kg/m}^3)(43\,\text{m})(2.5\,\text{m})(12\,\text{m})(9.8\,\text{m/s}^2)\cos 26° \approx 1.4\times10^7\ \text{N}.$$

(c) The minimum force needed from the bolts to stabilize the slab is

$$F_2 = F_1 - f_s = 1.77\times10^7\,\text{N} - 1.42\times10^7\ \text{N} = 3.5\times10^6\ \text{N}.$$

If the minimum number of bolts needed is *n*, then $F_2/nA \le S_G$, where $S_G = 3.6\times10^8\ \text{N/m}^2$ is the shear stress. Solving for *n*, we find

$$n \ge \frac{3.5\times10^6\ \text{N}}{(3.6\times10^8\ \text{N/m}^2)(6.4\times10^{-4}\ \text{m}^2)} = 15.2$$

Therefore, 16 bolts are needed.

LEARN In general, the number of bolts needed to maintain static equilibrium of the slab is

$$n = \frac{F_1 - f_s}{S_G A}.$$

Thus, no bolt would be necessary if $f_s > F_1$.

54. The notation and coordinates are as shown in Fig. 12-7 in the textbook. Here, the ladder's center of mass is halfway up the ladder (unlike in the textbook figure). Also, we label the *x* and *y* forces at the ground f_s and F_N, respectively. Now, balancing forces, we have

$$\sum F_x = 0 \quad \Rightarrow \quad f_s = F_w$$
$$\sum F_y = 0 \quad \Rightarrow \quad F_N = mg\,.$$

Since $f_s = f_{s,\,\max}$, we divide the equations to obtain

$$\frac{f_{s,\max}}{F_N} = \mu_s = \frac{F_w}{mg}\,.$$

Now, from $\Sigma\ \tau_z = 0$ (with axis at the ground) we have $mg(a/2) - F_w h = 0$. But from the Pythagorean theorem, $h = \sqrt{L^2 - a^2}$, where L is the length of the ladder. Therefore,

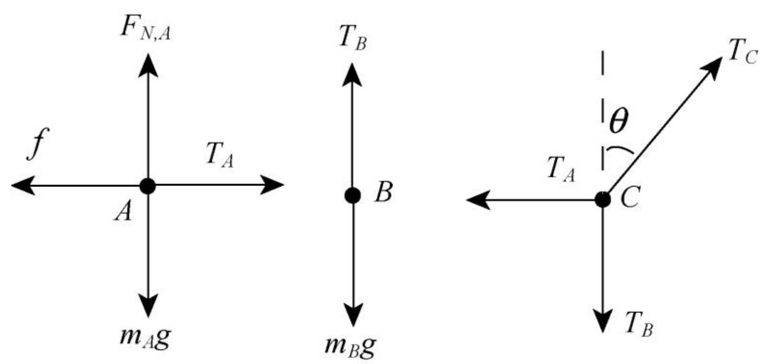
$$\frac{F_w}{mg} = \frac{a/2}{h} = \frac{a}{2\sqrt{L^2 - a^2}}\,.$$

In this way, we find

$$\mu_s = \frac{a}{2\sqrt{L^2 - a^2}} \quad \Rightarrow \quad a = \frac{2\mu_s L}{\sqrt{1 + 4\mu_s^2}} = 3.4 \text{ m.}$$

55. **THINK** Block A can be in equilibrium if friction is present between the block and the surface in contact.

EXPRESS The free-body diagrams for blocks A, B and the knot (denoted as C) are shown below.



The tensions in the three strings are denoted as T_A, T_B and T_C Analyzing forces at C, the conditions for static equilibrium are

$$T_C \cos\theta = T_B, \qquad T_C \sin\theta = T_A$$

which can be combined to give $\tan\theta = T_A / T_B$. On the other hand, equilibrium condition for block B implies $T_B = m_B g$. Similarly, for block A, the conditions are

$$F_{N,A} = m_A g, \quad f = T_A$$

For the static force to be at its maximum value, we have $f = \mu_s F_{N,A} = \mu_s m_A g$. Combining all the equations leads to

$$\tan\theta = \frac{T_A}{T_B} = \frac{\mu_s m_A g}{m_B g} = \frac{\mu_s m_A}{m_B}.$$

ANALYZE Solving for μ_s, we get

$$\mu_s = \left(\frac{m_B}{m_A}\right)\tan\theta = \left(\frac{5.0\,\text{kg}}{10\,\text{kg}}\right)\tan 30° = 0.29$$

LEARN The greater the mass of block *B*, the greater the static coefficient μ_s would be required for block *A* to be in equilibrium.

56. (a) With pivot at the hinge (at the left end), Eq. 12-9 gives

$$-mgx - Mg\frac{L}{2} + F_h h = 0$$

where *m* is the man's mass and *M* is that of the ramp; F_h is the leftward push of the right wall onto the right edge of the ramp. This equation can be written in the form (for a straight line in a graph)

$$F_h = (\text{“slope”})x + (\text{“y-intercept”}),$$

where the "slope" is *mg/h* and the "y-intercept" is *MgD*/2*h*. Since *h* = 0.480 m and $D = 4.00$ m, and the graph seems to intercept the vertical axis at 20 kN, then we find *M* = 500 kg.

(b) Since the "slope" (estimated from the graph) is (5000 N)/(4 m), then the man's mass must be *m* = 62.5 kg.

57. With the *x* axis parallel to the incline (positive uphill), then

$$\sum F_x = 0 \quad \Rightarrow \quad T\cos 25° - mg\sin 45° = 0.$$

Therefore,

$$T = mg\frac{\sin 45°}{\cos 25°} = (10\,\text{kg})(9.8\,\text{m/s}^2)\frac{\sin 45°}{\cos 25°} \approx 76\,\text{N}.$$

58. The beam has a mass *M* = 40.0 kg and a length *L* = 0.800 m. The mass of the package of tamale is *m* = 10.0 kg.

(a) Since the system is in static equilibrium, the normal force on the beam from roller *A* is equal to half of the weight of the beam:

$$F_A = Mg/2 = (40.0 \text{ kg})(9.80 \text{ m/s}^2)/2 = 196 \text{ N}.$$

(b) The normal force on the beam from roller B is equal to half of the weight of the beam plus the weight of the tamale:

$$F_B = Mg/2 + mg = (40.0 \text{ kg})(9.80 \text{ m/s}^2)/2 + (10.0 \text{ kg})(9.80 \text{ m/s}^2) = 294 \text{ N}.$$

(c) When the right-hand end of the beam is centered over roller B, the normal force on the beam from roller A is equal to the weight of the beam plus half of the weight of the tamale:

$$F_A = Mg + mg/2 = (40.0 \text{ kg})(9.8 \text{ m/s}^2) + (10.0 \text{ kg})(9.80 \text{ m/s}^2)/2 = 441 \text{ N}.$$

(d) Similarly, the normal force on the beam from roller B is equal to half of the weight of the tamale:

$$F_B = mg/2 = (10.0 \text{ kg})(9.80 \text{ m/s}^2)/2 = 49.0 \text{ N}.$$

(e) We choose the rotational axis to pass through roller B. When the beam is on the verge of losing contact with roller A, the net torque is zero. The balancing equation may be written as

$$mgx = Mg(L/4 - x) \quad \Rightarrow \quad x = \frac{L}{4}\frac{M}{M+m}.$$

Substituting the values given, we obtain $x = 0.160$ m.

59. **THINK** The bucket is in static equilibrium. The forces acting on it are the downward force of gravity and the upward tension force of cable A.

EXPRES Since the bucket is in equilibrium, the tension force of cable A is equal to the weight of the bucket: $T_A = W = mg$. To solve for T_B and T_C, we use the coordinates axes defined in the diagram. Cable A makes an angle of $\theta_2 = 66.0°$ with the negative y axis, cable B makes an angle of 27.0º with the positive y axis, and cable C is along the x axis. The y components of the forces must sum to zero since the knot is in equilibrium. This means

$$T_B \cos 27.0° - T_A \cos 66.0° = 0.$$

Similarly, the fact that the x components of forces must also sum to zero implies

$$T_C + T_B \sin 27.0° - T_A \sin 66.0° = 0 .$$

ANALYZE (a) Substituting the values given, we find the tension force of cable A to be

$$T_A = mg = (817 \text{ kg})(9.80 \text{ m/s}^2) = 8.01 \times 10^3 \text{ N}.$$

(b) Equilibrium condition for the y-components gives

$$T_B = \left(\frac{\cos 66.0°}{\cos 27.0°}\right) T_A = \left(\frac{\cos 66.0°}{\cos 27.0°}\right)(8.01\times10^3\,\text{N}) = 3.65\times10^3\,\text{N}.$$

(c) Using the equilibrium condition for the *x*-components, we have

$$T_C = T_A \sin 66.0° - T_B \sin 27.0° = (8.01\times10^3\,\text{N})\sin 66.0° - (3.65\times10^3\,\text{N})\sin 27.0°$$
$$= 5.66\times10^3\ \text{N}.$$

LEARN One may verify that the tensions obey law of sine:

$$\frac{T_A}{\sin(180° - \theta_1 - \theta_2)} = \frac{T_B}{\sin(90° + \theta_2)} = \frac{T_C}{\sin(90° + \theta_1)}\ .$$

60. (a) Equation 12-8 leads to $T_1 \sin 40^{\circ} + T_2 \sin\theta = mg$. Also, Eq. 12-7 leads to

$$T_1 \cos 40^{\circ} - T_2 \cos\theta = 0.$$

Combining these gives the expression

$$T_2 = \frac{mg}{\cos\theta \tan 40° + \sin\theta}\ .$$

To minimize this, we can plot it or set its derivative equal to zero. In either case, we find that it is at its minimum at $\theta = 50°$.

(b) At $\theta = 50°$, we find $T_2 = 0.77mg$.

61. The cable that goes around the lowest pulley is cable 1 and has tension $T_1 = F$. That pulley is supported by the cable 2 (so $T_2 = 2T_1 = 2F$) and goes around the middle pulley. The middle pulley is supported by cable 3 (so $T_3 = 2T_2 = 4F$) and goes around the top pulley. The top pulley is supported by the upper cable with tension T, so $T = 2T_3 = 8F$. Three cables are supporting the block (which has mass m = 6.40 kg):

$$T_1 + T_2 + T_3 = mg \Rightarrow F = \frac{mg}{7} = 8.96\,\text{N}.$$

Therefore, T = 8(8.96 N) = 71.7 N.

62. To support a load of $W = mg$ = (670 kg)(9.8 m/s^2) = 6566 N, the steel cable must stretch an amount proportional to its “free” length:

$$\Delta L = \left(\frac{W}{AY}\right) L \quad \text{where } A = \pi r^2$$

and r = 0.0125 m.

(a) If L = 12 m, then $\Delta L = \left(\frac{6566 \text{ N}}{\pi(0.0125 \text{ m})^2 (2.0\times10^{11} \text{ N/m}^2)}\right)(12 \text{ m}) = 8.0\times10^{-4}\text{m}.$

(b) Similarly, when L = 350 m, we find $\Delta L = 0.023\,\text{m}$.

63. (a) The center of mass of the top brick cannot be further (to the right) with respect to the brick below it (brick 2) than $L/2$; otherwise, its center of gravity is past any point of support and it will fall. So $a_1 = L/2$ in the maximum case.

(b) With brick 1 (the top brick) in the maximum situation, then the combined center of mass of brick 1 and brick 2 is halfway between the middle of brick 2 and its right edge. That point (the combined com) must be supported, so in the maximum case, it is just above the right edge of brick 3. Thus, $a_2 = L/4$.

(c) Now the total center of mass of bricks 1, 2, and 3 is one-third of the way between the middle of brick 3 and its right edge, as shown by this calculation:

$$x_{\text{com}} = \frac{2m(0) + m(-L/2)}{3m} = -\frac{L}{6}$$

where the origin is at the right edge of brick 3. This point is above the right edge of brick 4 in the maximum case, so $a_3 = L/6$.

(d) A similar calculation,

$$x'_{\text{com}} = \frac{3m(0) + m(-L/2)}{4m} = -\frac{L}{8}$$

shows that $a_4 = L/8$.

(e) We find $h = \sum_{i=1}^{4} a_i = 25L/24$.

64. Since all surfaces are frictionless, the contact force $\vec{F}$ exerted by the lower sphere on the upper one is along that 45° line, and the forces exerted by walls and floors are "normal" (perpendicular to the wall and floor surfaces, respectively). Equilibrium of forces on the top sphere leads to the two conditions

$$F_{\text{wall}} = F\cos 45° \quad \text{and} \quad F\sin 45° = mg.$$

And (using Newton's third law) equilibrium of forces on the bottom sphere leads to the two conditions

$$F'_{\text{wall}} = F\cos 45° \quad \text{and} \quad F'_{\text{floor}} = F\sin 45° + mg.$$

(a) Solving the above equations, we find $F'_{\text{floor}} = 2mg$.

(b) We obtain for the left side of the container, $F'_{\text{wall}} = mg$.

(c) We obtain for the right side of the container, $F_{\text{wall}} = mg$.

(d) We get $F = mg/\sin 45° = \sqrt{2}mg$.

65. (a) Choosing an axis through the hinge, perpendicular to the plane of the figure and taking torques that would cause counterclockwise rotation as positive, we require the net torque to vanish:

$$FL\sin 90° - Th\sin 65° = 0$$

where the length of the beam is L = 3.2 m and the height at which the cable attaches is h = 2.0 m. Note that the weight of the beam does not enter this equation since its line of action is directed towards the hinge. With F = 50 N, the above equation yields

$$T = \frac{FL}{h\sin 65°} = \frac{(50\text{ N})(3.2\text{ m})}{(2.0\text{ m})\sin 65°} = 88\text{ N}.$$

(b) To find the components of $\vec{F}_p$ we balance the forces:

$$\sum F_x = 0 \quad \Rightarrow \quad F_{px} = T\cos 25° - F$$
$$\sum F_y = 0 \quad \Rightarrow \quad F_{py} = T\sin 25° + W$$

where W is the weight of the beam (60 N). Thus, we find that the hinge force components are F_{px} = 30 N pointing rightward, and F_{py} = 97 N pointing upward. In unit-vector notation, $\vec{F}_p = (30\text{ N})\hat{\text{i}} + (97\text{ N})\hat{\text{j}}$.

66. Adopting the usual convention that torques that would produce counterclockwise rotation are positive, we have (with axis at the hinge)

$$\sum \tau_z = 0 \quad \Rightarrow \quad TL\sin 60° - Mg\left(\frac{L}{2}\right) = 0$$

where L = 5.0 m and M = 53 kg. Thus, T = 300 N. Now (with F_p for the force of the hinge)

$$\sum F_x = 0 \Rightarrow F_{px} = -T\cos\theta = -150\text{N}$$
$$\sum F_y = 0 \Rightarrow F_{py} = Mg - T\sin\theta = 260\text{N}$$

where $\theta = 60°$. Therefore, $\vec{F}_p = (-1.5\times10^2\ \text{N})\hat{\text{i}} + (2.6\times10^2\ \text{N})\hat{\text{j}}$.

67. The cube has side length l and volume $V = l^3$. We use $p = B\Delta V / V$ for the pressure p. We note that

$$\frac{\Delta V}{V} = \frac{\Delta l^3}{l^3} = \frac{(l+\Delta l)^3 - l^3}{l^3} \approx \frac{3l^2\Delta l}{l^3} = 3\frac{\Delta l}{l}.$$

Thus, the pressure required is

$$p = \frac{3B\Delta l}{l} = \frac{3(1.4\times10^{11}\ \text{N/m}^2)(85.5\,\text{cm} - 85.0\,\text{cm})}{85.5\,\text{cm}} = 2.4\times10^9\ \text{N/m}^2.$$

68. (a) The angle between the beam and the floor is

$$\sin^{-1}(d/L) = \sin^{-1}(1.5/2.5) = 37°,$$

so that the angle between the beam and the weight vector $\vec{W}$ of the beam is 53°. With L = 2.5 m being the length of the beam, and choosing the axis of rotation to be at the base,

$$\sum \tau_z = 0 \;\Rightarrow\; PL - W\left(\frac{L}{2}\right)\sin 53° = 0$$

Thus, $P = ½\, W \sin 53° = 200$ N.

(b) Note that

$$\vec{P} + \vec{W} = (200 \angle 90°) + (500 \angle -127°) = (360 \angle -146°)$$

using magnitude-angle notation (with angles measured relative to the beam, where "uphill" along the beam would correspond to 0°) with the unit newton understood. The "net force of the floor" $\vec{F}_f$ is equal and opposite to this (so that the total net force on the beam is zero), so that $|\vec{F}_f| = 360$ N and is directed 34° counterclockwise from the beam.

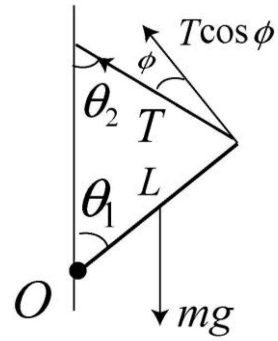
(c) Converting that angle to one measured from true horizontal, we have $\theta = 34° + 37° = 71°$. Thus, $f_s = F_f \cos\theta$ and $F_N = F_f \sin\theta$. Since $f_s = f_{s,\max}$, we divide the equations to obtain

$$\frac{F_N}{f_{s,\max}} = \frac{1}{\mu_s} = \tan\theta.$$

Therefore, $\mu_s = 0.35$.

69. **THINK** Since the rod is in static equilibrium, the net torque about the hinge must be zero.

EXPRESS The free-body diagram is shown below (not to scale). The tension in the rope is denoted as T. Since the rod is in rotational equilibrium, the net torque about the hinge, denoted as O, must be zero. This implies



$$-mg\sin\theta_1\frac{L}{2} + TL\cos\phi = 0,$$

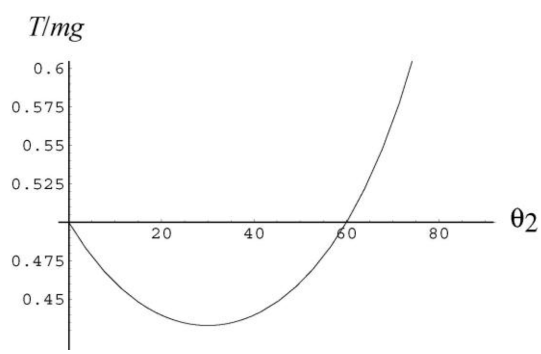
where $\phi = \theta_1 + \theta_2 - 90°$.

ANALYZE Solving for T gives

$$T = \frac{mg}{2}\frac{\sin\theta_1}{\cos(\theta_1+\theta_2-90°)} = \frac{mg}{2}\frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}.$$

With $\theta_1 = 60°$ and $T = mg/2$, we have $\sin 60° = \sin(60° + \theta_2)$, which yields $\theta_2 = 60°$.

LEARN A plot of T/mg as a function of θ_2 is shown below. The other solution, $\theta_2 = 0°$, is rejected since it corresponds to the limit where the rope becomes infinitely long.



70. (a) Setting up equilibrium of torques leads to

$$F_{\text{far end}}L = (73\,\text{kg})(9.8\,\text{m/s}^2)\frac{L}{4} + (2700\,\text{N})\frac{L}{2}$$

which yields $F_{\text{far end}} = 1.5 \times 10^3$ N.

(b) Then, equilibrium of vertical forces provides

$$F_{\text{near end}} = (73)(9.8) + 2700 - F_{\text{far end}} = 1.9 \times 10^3\,\text{N}.$$

71. **THINK** Upon applying a horizontal force, the cube may tip or slide, depending on the friction between the cube and the floor.

EXPRESS When the cube is about to move, we are still able to apply the equilibrium conditions, but (to obtain the critical condition) we set static friction equal to its maximum value and picture the normal force $\vec{F}_N$ as a concentrated force (upward) at the bottom corner of the cube, directly below the point *O* where *P* is being applied. Thus, the line of action of $\vec{F}_N$ passes through point *O* and exerts no torque about *O* (of course, a similar observation applied to the pull *P*). Since $F_N = mg$ in this problem, we have $f_{s\max} = \mu_c mg$ applied a distance *h* away from *O*. And the line of action of force of gravity (of magnitude *mg*), which is best pictured as a concentrated force at the center of the cube, is a distance *L*/2 away from *O*. Therefore, equilibrium of torques about *O* produces

$$\mu_c mgh = mg\left(\frac{L}{2}\right) \Rightarrow \mu_c = \frac{L}{2h} = \frac{(8.0\text{ cm})}{2(7.0\text{ cm})} = 0.57$$

for the critical condition we have been considering. We now interpret this in terms of a range of values for μ.

ANALYZE (a) For it to slide but not tip, a value of μ *less* than μ_c is needed, since then — static friction will be exceeded for a smaller value of *P*, before the pull is strong enough to cause it to tip. Thus, the required condition is

$$\mu < \mu_c = L/2h = 0.57.$$

(b) And for it to tip but not slide, we need μ *greater* than μ_c is needed, since now — static friction will not be exceeded even for the value of *P* which makes the cube rotate about its front lower corner. That is, we need to have $\mu > \mu_c = L/2h = 0.57$ in this case.

LEARN Note that the value μ_c depends only on the ratio L/h. The cube will tend to slide when μ is mall (think about the limit of a frictionless floor), and tend to tip over when the friction is sufficiently large.

72. We denote the tension in the upper left string (*bc*) as *T*′ and the tension in the lower right string (*ab*) as *T*. The supported weight is $W = Mg = (2.0\text{ kg})(9.8\text{ m/s}^2) = 19.6$ N. The force equilibrium conditions lead to

$$T'\cos 60° = T\cos 20° \qquad \text{horizontal forces}$$
$$T'\sin 60° = W + T\sin 20° \qquad \text{vertical forces.}$$

(a) We solve the above simultaneous equations and find

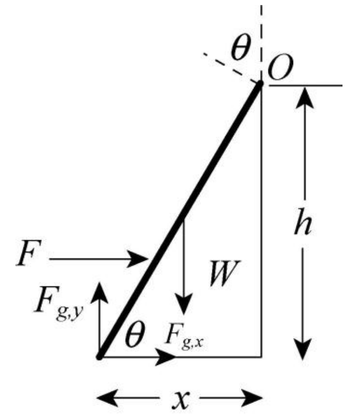
$$T = \frac{W}{\tan 60°\cos 20° - \sin 20°} = \frac{19.6\text{ N}}{\tan 60°\cos 20° - \sin 20°} = 15\text{ N}.$$

(b) Also, we obtain

$$T' = T\cos 20° / \cos 60° = 29\text{ N}.$$

73. **THINK** The force of the ground prevents the ladder from sliding.

EXPRESS The free-body diagram for the ladder is shown to the right. We choose an axis through *O*, the top (where the ladder comes into contact with the wall), perpendicular to the plane of the figure and take torques that would cause counterclockwise rotation as positive. The length of the ladder is $L = 10$ m. Given that $h = 8.0$ m, the horizontal distance to the wall is



$$x = \sqrt{L^2 - h^2} = \sqrt{(10\text{ m})^2 - (8\text{ m})^2} = 6.0\,\text{m}.$$

Note that the line of action of the applied force $\vec{F}$ intersects the wall at a height of $(8.0\text{ m})/5 = 1.6\,\text{m}$.

In other words, the *moment arm* for the applied force (in terms of where we have chosen the axis) is

$$r_\perp = (L-d)\sin\theta = (L-d)(h/L) = (8.0\text{ m})(8.0\text{ m}/10.0\text{ m}) = 6.4\,\text{m}.$$

The moment arm for the weight is $x/2 = 3.0\,\text{m}$, half the horizontal distance from the wall to the base of the ladder. Similarly, the moment arms for the *x* and *y* components of the force at the ground $(\vec{F}_g)$ are h = 8.0 m and x = 6.0 m, respectively. Thus, we have

$$\begin{aligned}\sum \tau_z &= Fr_\perp + W(x/2) + F_{g,x}h - F_{g,y}x \\ &= F(6.4\text{ m}) + W(3.0\text{ m}) + F_{g,x}(8.0\text{ m}) - F_{g,y}(6.0\text{ m}) = 0.\end{aligned}$$

In addition, from balancing the vertical forces we find that $W = F_{g,y}$ (keeping in mind that the wall has no friction). Therefore, the above equation can be written as

$$\sum \tau_z = F(6.4\text{ m}) + W(3.0\text{ m}) + F_{g,x}(8.0\text{ m}) - W(6.0\text{ m}) = 0.$$

ANALYZE (a) With F = 50 N and W = 200 N, the above equation yields $F_{g,x} = 35$ N. Thus, in unit vector notation we obtain

$$\vec{F}_g = (35\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(b) Similarly, with F = 150 N and W = 200 N, the above equation yields $F_{g,x} = -45$ N. Therefore, in unit vector notation we obtain

$$\vec{F}_g = (-45\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(c) Note that the phrase "start to move towards the wall" implies that the friction force is pointed away from the wall (in the $-\hat{\text{i}}$ direction). Now, if $f = -F_{g,x}$ and

$F_N = F_{g,y} = 200\text{ N}$ are related by the (maximum) static friction relation ($f = f_{s,\max} = \mu_s F_N$) with μ_s = 0.38, then we find $F_{g,x} = -76\text{ N}$. Returning this to the above equation, we obtain

$$F = \frac{W(x/2) + \mu_s Wh}{r_\perp} = \frac{(200\text{ N})(3.0\text{ m}) + (0.38)(200\text{N})(8.0\text{ m})}{6.4\text{ m}} = 1.9\times10^2\text{ N}.$$

LEARN The force needed to move the ladder toward the wall would decrease with a larger $r_\perp$ or a smaller μ_s.

74. One arm of the balance has length ℓ_1 and the other has length ℓ_2. The two cases described in the problem are expressed (in terms of torque equilibrium) as

$$m_1\ell_1 = m\ell_2 \quad \text{and} \quad m\ell_1 = m_2\ell_2.$$

We divide equations and solve for the unknown mass: $m = \sqrt{m_1 m_2}$.

75. Since *GA* exerts a leftward force *T* at the corner *A*, then (by equilibrium of horizontal forces at that point) the force F_{diag} in *CA* must be pulling with magnitude

$$F_{\text{diag}} = \frac{T}{\sin 45°} = T\sqrt{2}.$$

This analysis applies equally well to the force in *DB*. And these diagonal bars are pulling on the bottom horizontal bar exactly as they do to the top bar, so the bottom bar *CD* is the "mirror image" of the top one (it is also under tension *T*). Since the figure is symmetrical (except for the presence of the turnbuckle) under 90° rotations, we conclude that the side bars (*DA* and *BC*) also are under tension *T* (a conclusion that also follows from considering the vertical components of the pull exerted at the corners by the diagonal bars).

(a) Bars that are in tension are *BC*, *CD*, and *DA*.

(b) The magnitude of the forces causing tension is $T = 535\text{ N}$.

(c) The magnitude of the forces causing compression on *CA* and *DB* is

$$F_{\text{diag}} = \sqrt{2}T = (1.41)535\text{ N} = 757\text{ N}.$$

76. (a) For computing torques, we choose the axis to be at support 2 and consider torques that encourage counterclockwise rotation to be positive. Let *m* = mass of gymnast and *M* = mass of beam. Thus, equilibrium of torques leads to

$$Mg(1.96\text{ m}) - mg(0.54\text{ m}) - F_1(3.92\text{ m}) = 0.$$

Therefore, the upward force at support 1 is F_1 = 1163 N (quoting more figures than are significant — but with an eye toward using this result in the remaining calculation). In unit-vector notation, we have $\vec{F}_1 \approx (1.16\times10^3\ \text{N})\hat{\text{j}}$.

(b) Balancing forces in the vertical direction, we have $F_1 + F_2 - Mg - mg = 0$, so that the upward force at support 2 is $F_2 = 1.74\times10^3$ N. In unit-vector notation, we have $\vec{F}_2 \approx (1.74\times10^3\ \text{N})\hat{\text{j}}$.

77. (a) Let d = 0.00600 m. In order to achieve the same final lengths, wires 1 and 3 must stretch an amount d more than wire 2 stretches:

$$\Delta L_1 = \Delta L_3 = \Delta L_2 + d\ .$$

Combining this with Eq. 12-23 we obtain

$$F_1 = F_3 = F_2 + \frac{dAE}{L}\ .$$

Now, Eq. 12-8 produces $F_1 + F_3 + F_2 - mg = 0$. Combining this with the previous relation (and using Table 12-1) leads to $F_1 = 1380\ \text{N} \approx 1.38\times10^3\text{N}$.

(b) Similarly, F_2 = 180 N.

78. (a) Computing the torques about the hinge, we have

$$TL\sin 40° = W\frac{L}{2}\sin 50°,$$

where the length of the beam is L = 12 m and the tension is T = 400 N. Therefore, the weight is $W = 671\ \text{N}$, which means that the gravitational force on the beam is $\vec{F}_w = (-671\ \text{N})\hat{\text{j}}$.

(b) Equilibrium of horizontal and vertical forces yields, respectively,

$$F_{\text{hinge}\,x} = T = 400\ \text{N}$$
$$F_{\text{hinge}\,y} = W = 671\ \text{N}$$

where the hinge force components are rightward (for x) and upward (for y). In unit-vector notation, we have $\vec{F}_{\text{hinge}} = (400\ \text{N})\hat{\text{i}} + (671\ \text{N})\hat{\text{j}}$.

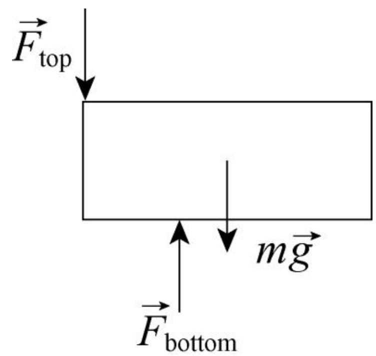
79. We locate the origin of the x axis at the edge of the table and choose rightward positive. The criterion (in part (a)) is that the center of mass of the block above another must be no further than the edge of the one below; the criterion in part (b) is more subtle

and is discussed below. Since the edge of the table corresponds to $x = 0$ then the total center of mass of the blocks must be zero.

(a) We treat this as three items: one on the upper left (composed of two bricks, one directly on top of the other) of mass $2m$ whose center is above the left edge of the bottom brick; a single brick at the upper right of mass m, which necessarily has its center over the right edge of the bottom brick (so $a_1 = L/2$ trivially); and, the bottom brick of mass m. The total center of mass is

$$\frac{(2m)(a_2 - L) + ma_2 + m(a_2 - L/2)}{4m} = 0$$

which leads to $a_2 = 5L/8$. Consequently, $h = a_2 + a_1 = 9L/8$.



(b) We have four bricks (each of mass m) where the center of mass of the top one and the center of mass of the bottom one have the same value, $x_{cm} = b_2 - L/2$. The middle layer consists of two bricks, and we note that it is possible for each of their centers of mass to be beyond the respective edges of the bottom one! This is due to the fact that the top brick is exerting downward forces (each equal to half its weight) on the middle blocks — and in the extreme case, this may be thought of as a pair of concentrated forces exerted at the innermost edges of the middle bricks. Also, in the extreme case, the support force (upward) exerted on a middle block (by the bottom one) may be thought of as a concentrated force located at the edge of the bottom block (which is the point about which we compute torques, in the following).

If (as indicated in our sketch, where $\vec{F}_{\text{top}}$ has magnitude $mg/2$) we consider equilibrium of torques on the rightmost brick, we obtain

$$mg\left(b_1 - \frac{1}{2}L\right) = \frac{mg}{2}(L - b_1)$$

which leads to $b_1 = 2L/3$. Once we conclude from symmetry that $b_2 = L/2$, then we also arrive at $h = b_2 + b_1 = 7L/6$.

80. The assumption stated in the problem (that the density does not change) is not meant to be realistic; those who are familiar with Poisson's ratio (and other topics related to the strengths of materials) might wish to think of this problem as treating a fictitious material (which happens to have the same value of E as aluminum, given in Table 12-1) whose density does not significantly change during stretching. Since the mass does not change either, then the constant-density assumption implies the volume (which is the circular area times its length) stays the same:

$$(\pi r^2 L)_{\text{new}} = (\pi r^2 L)_{\text{old}} \quad \Rightarrow \quad \Delta L = L\left[(1000/999.9)^2 - 1\right].$$

Now, Eq. 12-23 gives

$$F = \pi r^2 E\,\Delta L/L = \pi r^2(7.0 \times 10^9 \text{ N/m}^2)\left[(1000/999.9)^2 - 1\right].$$

Using either the new or old value for r gives the answer $F = 44$ N.

81. Where the crosspiece comes into contact with the beam, there is an upward force of $2F$ (where F is the upward force exerted by each man). By equilibrium of vertical forces, $W = 3F$ where W is the weight of the beam. If the beam is uniform, its center of gravity is a distance $L/2$ from the man in front, so that computing torques about the front end leads to

$$W\frac{L}{2} = 2Fx = 2\left(\frac{W}{3}\right)x$$

which yields $x = 3L/4$ for the distance from the crosspiece to the front end. It is therefore a distance $L/4$ from the rear end (the "free" end).

82. The force F exerted on the beam is $F = 7900$ N, as computed in the Sample Problem. Let $F/A = S_u/6$, where $S_u = 50\times10^6 \text{ N/m}^2$ is the ultimate strength (see Table 12-1). Then

$$A = \frac{6F}{S_u} = \frac{6(7900\text{ N})}{50\times10^6 \text{ N/m}^2} = 9.5\times10^{-4}\,\text{m}^2.$$

Thus the thickness is $\sqrt{A} = \sqrt{9.5\times10^{-4}\text{ m}^2} = 0.031\,\text{m}$.

83. (a) Because of Eq. 12-3, we can write

$$\vec{T} + (m_B\, g \;\angle -90^\circ) + (m_A\, g \;\angle -150^\circ) = 0\,.$$

Solving the equation, we obtain $\vec{T} = (106.34 \;\angle\; 63.963^\circ)$. Thus, the magnitude of the tension in the upper cord is 106 N,

(b) and its angle (measured counterclockwise from the $+x$ axis) is 64.0°.

84. (a) and (b) With $+x$ rightward and $+y$ upward (we assume the adult is pulling with force $\vec{P}$ to the right), we have

$$\sum F_y = 0 \quad\Rightarrow\quad W = T\cos\theta = 270\text{ N}$$
$$\sum F_x = 0 \quad\Rightarrow\quad P = T\sin\theta = 72\text{ N}$$

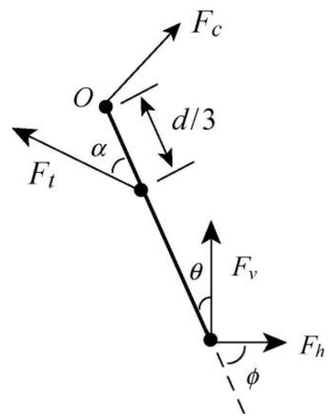
where $\theta = 15°$.

(c) Dividing the above equations leads to

$$\frac{P}{W} = \tan\theta \;.$$

Thus, with W = 270 N and P = 93 N, we find θ = 19°.

85. Our system is the second finger bone. Since the system is in static equilibrium, the net force acting on it is zero. In addition, the torque about any point must be zero. We set up the torque equation about point O where $\vec{F}_c$ act:



$$0 = \sum_O \tau_{\text{net}} = -\left(\frac{d}{3}\right)F_t \sin\alpha + (d)F_v \sin\theta + (d)F_h \sin\phi \,.$$

Solving for F_t and substituting the values given, we obtain

$$F_t = \frac{3(F_v \sin\theta + F_h \sin\phi)}{\sin\alpha} = \frac{3[(162.4\text{ N})\sin 10° + (13.4\text{ N})\sin 80°]}{\sin 45°} = 175.6\text{ N}$$
$$\approx 1.8\times 10^2\text{ N}.$$

86. (a) Setting up equilibrium of torques leads to a simple "level principle" ratio:

$$F_{\text{catch}} = (11\,\text{kg})(9.8\,\text{m/s}^2)\frac{(91/2-10)\,\text{cm}}{91\,\text{cm}} = 42\,\text{N}.$$

(b) Then, equilibrium of vertical forces provides

$$F_{\text{hinge}} = (11\,\text{kg})(9.8\,\text{m/s}^2) - F_{\text{catch}} = 66\,\text{N}.$$

87. (a) For the net force to be zero, $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0,$ we require

$$\vec{F}_3 = -\vec{F}_1 - \vec{F}_2 = -\left[(8.40\text{ N})\hat{\text{i}} - (5.70\text{ N})\hat{\text{j}}\right] - \left[(16.0\text{ N})\hat{\text{i}} + (4.10\text{ N})\hat{\text{j}}\right]$$
$$= (-24.4\text{ N})\hat{\text{i}} + (1.60\text{ N})\hat{\text{j}}$$

Thus, $F_{3x} = -24.4\text{ N}$.

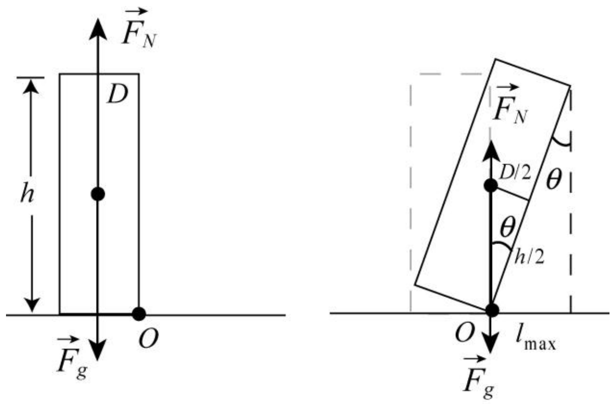
(b) Similarly, $F_{3y} = 1.60\text{ N}$.

(c) The angle $\vec{F}_3$ makes relative to the +x-axis is

$$\theta = \tan^{-1}\left(\frac{F_{3y}}{F_{3x}}\right) = \tan^{-1}\left(\frac{1.60\text{ N}}{-24.4\text{ N}}\right) = 176.25°.$$

88. We solve part (b) first.

(b) The critical tilt angle corresponds to the situation where the line of action of $\vec{F}_g$ passes through the supporting edge (point O in the figure).



At this state, the normal force also passes through the supporting edge, so the net torque is zero and the Tower is in static equilibrium. However, this equilibrium is unstable and the Tower is on the verge of falling over. From the figure, we find the critical angle to be

$$\tan\theta = \frac{D/2}{h/2} = \frac{D}{h} \qquad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{D}{h}\right)\tan^{-1}\left(\frac{7.44\text{ m}}{59.1\text{ m}}\right) = 7.18°$$

(a) From the figure, the maximum displacement is

$$l_{\max} = h\sin\theta = (59.1\text{ m})\sin 7.18° = 7.38\text{ m}$$

Thus, the additional displacement to put the Tower on the verge of toppling is

$$\Delta l = l_{\max} - l = 7.38\text{ m} - 4.01\text{ m} = 3.37\text{ m}$$

Chapter 13

1. The gravitational force between the two parts is

$$F = \frac{Gm\left(M-m\right)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to *m* and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}\left(M-2m\right) \Rightarrow M = 2m .$$

This leads to the result $m/M = 1/2$.

2. The gravitational force between you and the moon at its initial position (directly opposite of Earth from you) is

$$F_0 = \frac{GM_m m}{(R_{ME} + R_E)^2}$$

where M_m is the mass of the moon, R_{ME} is the distance between the moon and the Earth, and R_E is the radius of the Earth. At its final position (directly above you), the gravitational force between you and the moon is

$$F_1 = \frac{GM_m m}{(R_{ME} - R_E)^2} .$$

(a) The ratio of the moon's gravitational pulls at the two different positions is

$$\frac{F_1}{F_0} = \frac{GM_m m/(R_{ME} - R_E)^2}{GM_m m/(R_{ME} + R_E)^2} = \left(\frac{R_{ME} + R_E}{R_{ME} - R_E}\right)^2 = \left(\frac{3.82\times10^8 \text{ m} + 6.37\times10^6 \text{ m}}{3.82\times10^8 \text{ m} - 6.37\times10^6 \text{ m}}\right)^2 = 1.06898.$$

Therefore, the increase is 0.06898, or approximately 6.9%.

(b) The change of the gravitational pull may be approximated as

$$\begin{aligned} F_1 - F_0 &= \frac{GM_m m}{(R_{ME} - R_E)^2} - \frac{GM_m m}{(R_{ME} + R_E)^2} \approx \frac{GM_m m}{R_{ME}^2}\left(1 + 2\frac{R_E}{R_{ME}}\right) - \frac{GM_m m}{R_{ME}^2}\left(1 - 2\frac{R_E}{R_{ME}}\right) \\ &= \frac{4GM_m m R_E}{R_{ME}^3} . \end{aligned}$$

On the other hand, your weight, as measured on a scale on Earth, is

$$F_g = mg_E = \frac{GM_E m}{R_E^2}.$$

Since the moon pulls you "up," the percentage decrease of weight is

$$\frac{F_1 - F_0}{F_g} = 4\left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_{ME}}\right)^3 = 4\left(\frac{7.36\times10^{22}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)\left(\frac{6.37\times10^6\ \text{m}}{3.82\times10^8\ \text{m}}\right)^3 = 2.27\times10^{-7} \approx (2.3\times10^{-5})\%.$$

3. **THINK** The magnitude of gravitational force between two objects depends on their distance of separation.

EXPRESS The magnitude of the gravitational force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant.

ANALYZE Solve for r using the values given, we obtain

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.2\text{kg}\right)\left(2.4\text{kg}\right)}{2.3\times10^{-12}\,\text{N}}} = 19\,\text{m}.$$

LEARN The force of gravitation is inversely proportional to r^2.

4. We use subscripts s, e, and m for the Sun, Earth and Moon, respectively. Plugging in the numerical values (say, from Appendix C) we find

$$\frac{F_{sm}}{F_{em}} = \frac{Gm_sm_m/r_{sm}^2}{Gm_em_m/r_{em}^2} = \frac{m_s}{m_e}\left(\frac{r_{em}}{r_{sm}}\right)^2 = \frac{1.99\times10^{30}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\left(\frac{3.82\times10^8\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^2 = 2.16.$$

5. The gravitational force from Earth on you (with mass m) is

$$F_g = \frac{GM_E m}{R_E^2} = mg$$

where $g = GM_E/R_E^2 = 9.8\ \text{m/s}^2$. If r is the distance between you and a tiny black hole of mass $M_b = 1\times10^{11}\ \text{kg}$ that has the same gravitational pull on you as the Earth, then

$$F_g = \frac{GM_b m}{r^2} = mg.$$

Combining the two equations, we obtain

$$mg = \frac{GM_E m}{R_E^2} = \frac{GM_b m}{r^2} \Rightarrow r = \sqrt{\frac{GM_b}{g}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1\times10^{11}\ \text{kg})}{9.8\ \text{m/s}^2}} \approx 0.8\ \text{m}.$$

6. The gravitational forces on m_5 from the two 5.00 g masses m_1 and m_4 cancel each other. Contributions to the net force on m_5 come from the remaining two masses:

$$F_{\text{net}} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.50\times10^{-3}\ \text{kg}\right)\left(3.00\times10^{-3}\ \text{kg} - 1.00\times10^{-3}\ \text{kg}\right)}{\left(\sqrt{2}\times10^{-1}\ \text{m}\right)^2}$$
$$= 1.67\times10^{-14}\ \text{N}.$$

The force is directed along the diagonal between m_2 and m_3, toward m_2. In unit-vector notation, we have

$$\vec{F}_{\text{net}} = F_{\text{net}}(\cos 45°\hat{\text{i}} + \sin 45°\hat{\text{j}}) = (1.18\times10^{-14}\,\text{N})\,\hat{\text{i}} + (1.18\times10^{-14}\,\text{N})\,\hat{\text{j}}.$$

7. We require the magnitude of force (given by Eq. 13-1) exerted by particle C on A be equal to that exerted by B on A. Thus,

$$\frac{Gm_A m_C}{r^2} = \frac{Gm_A m_B}{d^2}.$$

We substitute in $m_B = 3m_A$ and $m_B = 3m_A$, and (after canceling "m_A") solve for r. We find $r = 5d$. Thus, particle C is placed on the x axis, to the left of particle A (so it is at a negative value of x), at $x = -5.00d$.

8. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with 1.9×10^{-8} N, and the rightmost mass pulls rightward on the one at the origin with 1.0×10^{-8} N. Thus, the (x, y) components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{\text{net}} = \left(1.04\times10^{-8}, 1.85\times10^{-8}\right) \Rightarrow \left(2.13\times10^{-8}\ \angle\ 60.6°\right).$$

(a) The magnitude of the force is 2.13×10^{-8} N.

(b) The direction of the force relative to the $+x$ axis is $60.6°$.

9. **THINK** Both the Sun and the Earth exert a gravitational pull on the space probe. The net force can be calculated by using superposition principle.

EXPRESS At the point where the two forces balance, we have $GM_E m/r_1^2 = GM_S m/r_2^2$, where M_E is the mass of Earth, M_S is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_E}{r_1^2} = \frac{M_S}{(d - r_1)^2}.$$

ANALYZE Using the values for M_E, M_S, and d given in Appendix C, we take the positive square root of both sides to solve for r_1. A little algebra yields

$$r_1 = \frac{d}{1 + \sqrt{M_S / M_E}} = \frac{1.50 \times 10^{11} \text{ m}}{1 + \sqrt{(1.99 \times 10^{30} \text{ kg})/(5.98 \times 10^{24} \text{ kg})}} = 2.60 \times 10^8 \text{ m}.$$

LEARN The fact that $r_1 \ll d$ indicates that the probe is much closer to the Earth than the Sun.

10. Using Eq. 13-1, we find

$$\vec{F}_{AB} = \frac{2Gm_A^2}{d^2}\,\hat{\mathrm{j}}\,, \quad \vec{F}_{AC} = -\frac{4Gm_A^2}{3d^2}\,\hat{\mathrm{i}}\ .$$

Since the vector sum of all three forces must be zero, we find the third force (using magnitude-angle notation) is

$$\vec{F}_{AD} = \frac{Gm_A^2}{d^2}\,(2.404 \ \angle \ -56.3^\circ)\ .$$

This tells us immediately the direction of the vector $\vec{r}$ (pointing from the origin to particle D), but to find its magnitude we must solve (with $m_D = 4m_A$) the following equation:

$$2.404\left(\frac{Gm_A^2}{d^2}\right) = \frac{Gm_A m_D}{r^2}\ .$$

This yields $r = 1.29d$. In magnitude-angle notation, then, $\vec{r} = (1.29 \ \angle \ -56.3^\circ)$, with SI units understood. The "exact" answer without regard to significant figure considerations is

$$\vec{r} = \left(2\sqrt{\frac{6}{13\sqrt{13}}},\ -3\sqrt{\frac{6}{13\sqrt{13}}}\right).$$

(a) In (x, y) notation, the x coordinate is $x = 0.716d$.

(b) Similarly, the y coordinate is $y = -1.07d$.

11. (a) The distance between any of the spheres at the corners and the sphere at the center is

$$r = \ell / 2\cos 30^\circ = \ell / \sqrt{3}$$

where ℓ is the length of one side of the equilateral triangle. The net (downward) contribution caused by the two bottom-most spheres (each of mass m) to the total force on m_4 has magnitude

$$2F_y = 2\left(\frac{Gm_4m}{r^2}\right)\sin 30° = 3\frac{Gm_4m}{\ell^2}.$$

This must equal the magnitude of the pull from M, so

$$3\frac{Gm_4m}{\ell^2} = \frac{Gm_4m}{(\ell/\sqrt{3})^2}$$

which readily yields $m = M$.

(b) Since m_4 cancels in that last step, then the amount of mass in the center sphere is not relevant to the problem. The net force is still zero.

12. (a) We are told the value of the force when particle C is removed (that is, as its position x goes to infinity), which is a situation in which any force caused by C vanishes (because Eq. 13-1 has r^2 in the denominator). Thus, this situation only involves the force exerted by A on B:

$$F_{\text{net},x} = F_{AB} = \frac{Gm_Am_B}{r_{AB}^2} = 4.17 \times 10^{-10}\ \text{N}.$$

Since m_B = 1.0 kg and $r_{AB} = 0.20$ m, then this yields

$$m_A = \frac{r_{AB}^2 F_{AB}}{Gm_B} = \frac{(0.20\ \text{m})^2(4.17\times10^{-10}\ \text{N})}{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.0\ \text{kg})} = 0.25\ \text{kg}.$$

(b) We note (from the graph) that the net force on B is zero when x = 0.40 m. Thus, at that point, the force exerted by C must have the same magnitude (but opposite direction) as the force exerted by A (which is the one discussed in part (a)). Therefore

$$\frac{Gm_Cm_B}{(0.40\ \text{m})^2} = 4.17 \times 10^{-10}\ \text{N} \quad \Rightarrow m_C = 1.00\ \text{kg}.$$

13. If the lead sphere were not hollowed the magnitude of the force it exerts on m would be $F_1 = GMm/d^2$. Part of this force is due to material that is removed. We calculate the force exerted on m by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere.

The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere, that is, $M_c/r^3 = M/R^3$, where M_c is the mass that fills the cavity. The common factor $4\pi/3$ has been canceled. Thus,

$$M_c = \left(\frac{r^3}{R^3}\right)M = \left(\frac{R^3}{8R^3}\right)M = \frac{M}{8}.$$

The center of the cavity is $d - r = d - R/2$ from m, so the force it exerts on m is

$$F_2 = \frac{G(M/8)m}{(d - R/2)^2}.$$

The force of the hollowed sphere on m is

$$\begin{aligned} F = F_1 - F_2 &= GMm\left(\frac{1}{d^2} - \frac{1}{8(d - R/2)^2}\right) = \frac{GMm}{d^2}\left(1 - \frac{1}{8(1 - R/2d)^2}\right) \\ &= \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(2.95\ \text{kg})(0.431\ \text{kg})}{(9.00\times10^{-2}\,\text{m})^2}\left(1 - \frac{1}{8[1-(4\times10^{-2}\,\text{m})/(2\cdot9\times10^{-2}\,\text{m})]^2}\right) \\ &= 8.31\times10^{-9}\,\text{N}. \end{aligned}$$

14. All the forces are being evaluated at the origin (since particle A is there), and all forces (except the net force) are along the location vectors $\vec{r}$, which point to particles B and C. We note that the angle for the location-vector pointing to particle B is 180º – 30.0º = 150º (measured counterclockwise from the $+x$ axis). The component along, say, the x axis of one of the force vectors $\vec{F}$ is simply Fx/r in this situation (where F is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to r^2, then the aforementioned x component would have the form $GmMx/r^3$; similarly for the other components. With m_A = 0.0060 kg, m_B = 0.0120 kg, and m_C = 0.0080 kg, we therefore have

$$F_{\text{net}\,x} = \frac{Gm_Am_Bx_B}{r_B^3} + \frac{Gm_Am_Cx_C}{r_C^3} = (2.77\times10^{-14}\,\text{N})\cos(-163.8^\circ)$$

and

$$F_{\text{net}\,y} = \frac{Gm_Am_By_B}{r_B^3} + \frac{Gm_Am_Cy_C}{r_C^3} = (2.77\times10^{-14}\,\text{N})\sin(-163.8^\circ)$$

where $r_B = d_{AB}$ = 0.50 m, and $(x_B, y_B) = (r_B\cos(150^\circ), r_B\sin(150^\circ))$ (with SI units understood). A fairly quick way to solve for r_C is to consider the vector difference between the net force and the force exerted by A, and then employ the Pythagorean theorem. This yields r_C = 0.40 m.

(a) By solving the above equations, the x coordinate of particle C is $x_C = -0.20$ m.

(b) Similarly, the y coordinate of particle C is $y_C = -0.35$ m.

15. All the forces are being evaluated at the origin (since particle *A* is there), and all forces are along the location vectors $\vec{r}$, which point to particles *B*, *C*, and *D*. In three dimensions, the Pythagorean theorem becomes $r = \sqrt{x^2 + y^2 + z^2}$. The component along, say, the *x* axis of one of the force-vectors $\vec{F}$ is simply Fx/r in this situation (where *F* is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to r^2 then the aforementioned *x* component would have the form $GmMx/r^3$; similarly for the other components. For example, the *z* component of the force exerted on particle *A* by particle *B* is

$$\frac{Gm_A m_B z_B}{r_B^{\ 3}} = \frac{Gm_A(2m_A)(2d)}{((2d)^2 + d^2 + (2d)^2)^3} = \frac{4Gm_A^2}{27d^2}.$$

In this way, each component can be written as some multiple of $Gm_A^{\ 2}/d^2$. For the *z* component of the force exerted on particle *A* by particle *C*, that multiple is $-9\sqrt{14}/196$. For the *x* components of the forces exerted on particle *A* by particles *B* and *C*, those multiples are 4/27 and $-3\sqrt{14}/196$, respectively. And for the *y* components of the forces exerted on particle *A* by particles *B* and *C*, those multiples are 2/27 and $3\sqrt{14}/98$, respectively. To find the distance *r* to particle *D* one method is to solve (using the fact that the vector add to zero)

$$\left(\frac{Gm_A m_D}{r^2}\right)^2 = \left[\left(\frac{4}{27} - \frac{3\sqrt{14}}{196}\right)^2 + \left(\frac{2}{27} + \frac{3\sqrt{14}}{98}\right)^2 + \left(\frac{4}{27} - \frac{9\sqrt{14}}{196}\right)^2\right]\left(\frac{Gm_A^2}{d^2}\right)^2 = 0.4439\left(\frac{Gm_A^2}{d^2}\right)^2$$

With $m_D = 4m_A$, we obtain

$$\left(\frac{4}{r^2}\right)^2 = \frac{0.4439}{(d^2)^2} \quad \Rightarrow \quad r = \left(\frac{16}{0.4439}\right)^{1/4} d = 4.357d\,.$$

The individual values of *x*, *y*, and *z* (locating the particle *D*) can then be found by considering each component of the Gm_Am_D/r^2 force separately.

(a) The *x* component of $\vec{r}$ would be

$$\frac{Gm_A m_D x}{r^3} = -\left(\frac{4}{27} - \frac{3\sqrt{14}}{196}\right)^2 \frac{Gm_A^2}{d^2} = -0.0909\frac{Gm_A^2}{d^2},$$

which yields $x = -0.0909\dfrac{m_A r^3}{m_D d^2} = -0.0909\dfrac{m_A(4.357d)^3}{(4m_A)d^2} = -1.88d$.

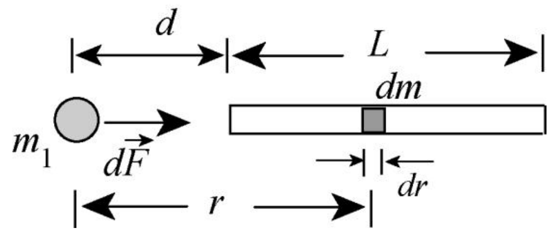
(b) Similarly, $y = -3.90d$,

(c) and $z = 0.489d$.

In this way we are able to deduce that $(x, y, z) = (1.88d, 3.90d, 0.489d)$.

16. Since the rod is an extended object, we cannot apply Equation 13-1 directly to find the force. Instead, we consider a small differential element of the rod, of mass dm of thickness dr at a distance r from m_1. The gravitational force between dm and m_1 is

$$dF = \frac{Gm_1 dm}{r^2} = \frac{Gm_1 (M/L)dr}{r^2},$$



where we have substituted $dm = (M/L)dr$ since mass is uniformly distributed. The direction of $d\vec{F}$ is to the right (see figure). The total force can be found by integrating over the entire length of the rod:

$$F = \int dF = \frac{Gm_1 M}{L}\int_d^{L+d} \frac{dr}{r^2} = -\frac{Gm_1 M}{L}\left(\frac{1}{L+d} - \frac{1}{d}\right) = \frac{Gm_1 M}{d(L+d)}.$$

Substituting the values given in the problem statement, we obtain

$$F = \frac{Gm_1 M}{d(L+d)} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(0.67\ \text{kg})(5.0\ \text{kg})}{(0.23\ \text{m})(3.0\ \text{m}+0.23\ \text{m})} = 3.0\times10^{-10}\ \text{N}.$$

17. (a) The gravitational acceleration at the surface of the Moon is $g_{\text{moon}} = 1.67\ \text{m/s}^2$ (see Appendix C). The ratio of weights (for a given mass) is the ratio of g-values, so

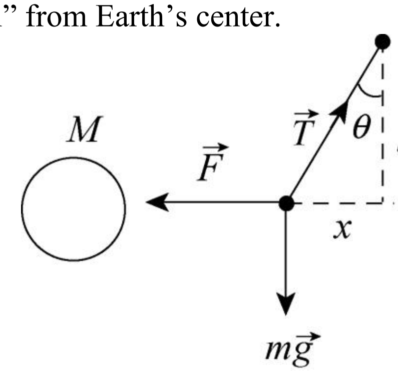
$$W_{\text{moon}} = (100\ \text{N})(1.67/9.8) = 17\ \text{N}.$$

(b) For the force on that object caused by Earth's gravity to equal 17 N, then the free-fall acceleration at its location must be $a_g = 1.67\ \text{m/s}^2$. Thus,

$$a_g = \frac{Gm_E}{r^2} \Rightarrow r = \sqrt{\frac{Gm_E}{a_g}} = 1.5\times10^7\ \text{m}$$

so the object would need to be a distance of $r/R_E = 2.4$ "radii" from Earth's center.

18. The free-body diagram of the force acting on the plumb line is shown to the right. The mass of the sphere is



$$M = \rho V = \rho\left(\frac{4\pi}{3}R^3\right) = \frac{4\pi}{3}(2.6\times10^3\ \text{kg/m}^3)(2.00\times10^3\ \text{m})^3$$
$$= 8.71\times10^{13}\ \text{kg}.$$

The force between the "spherical" mountain and the plumb line is $F = GMm/r^2$. Suppose at equilibrium the line makes an angle θ with the vertical and the net force acting on the line is zero. Therefore,

$$0 = \sum F_{\text{net},\,x} = T\sin\theta - F = T\sin\theta - \frac{GMm}{r^2}$$
$$0 = \sum F_{\text{net},\,y} = T\cos - mg$$

The two equations can be combined to give $\tan\theta = \dfrac{F}{mg} = \dfrac{GM}{gr^2}$. The distance the lower end moves toward the sphere is

$$x = l\tan\theta = l\frac{GM}{gr^2} = (0.50\text{ m})\frac{(6.67\times10^{-11}\text{ m}^3/\text{kg}\cdot\text{s}^2)(8.71\times10^{13}\text{ kg})}{(9.8)(3\times2.00\times10^3\text{ m})^2}$$
$$= 8.2\times10^{-6}\text{ m}.$$

19. **THINK** Earth's gravitational acceleration varies with altitude.

EXPRESS The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R + h$, where R is the radius of Earth and h is the altitude, to obtain

$$a_g = \frac{GM}{r^2} = \frac{GM}{(R_E + h)^2}.$$

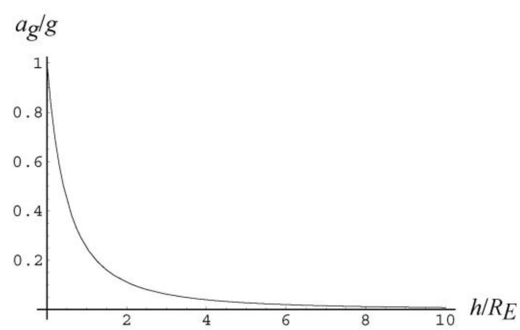
ANALYZE Solving for h, we obtain $h = \sqrt{GM/a_g} - R_E$. From Appendix C, $R_E = 6.37 \times 10^6$ m and $M = 5.98 \times 10^{24}$ kg, so

$$h = \sqrt{\frac{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\text{kg})}{(4.9\text{m/s}^2)}} - 6.37\times10^6\text{m} = 2.6\times10^6\text{m}.$$

LEARN We may rewrite a_g as

$$a_g = \frac{GM}{r^2} = \frac{GM/R_E^2}{(1+h/R_E)^2} = \frac{g}{(1+h/R_E)^2}$$

where $g = 9.83\text{ m/s}^2$ is the gravitational acceleration on the Surface of the Earth. The plot below depicts how a_g decreases with increasing altitude.



20. We follow the method shown in Sample Problem 13.2 – "Difference in acceleration at head and feet." Thus,

$$a_g = \frac{GM_E}{r^2} \Rightarrow da_g = -2\frac{GM_E}{r^3}dr$$

which implies that the change in weight is

$$W_{\text{top}} - W_{\text{bottom}} \approx m\left(da_g\right).$$

However, since $W_{\text{bottom}} = GmM_E/R^2$ (where R is Earth's mean radius), we have

$$mda_g = -2\frac{GmM_E}{R^3}dr = -2W_{\text{bottom}}\frac{dr}{R} = -2\left(600\text{ N}\right)\frac{1.61\times10^3\text{ m}}{6.37\times10^6\text{ m}} = -0.303\text{ N}$$

for the weight change (the minus sign indicating that it is a decrease in W). We are not including any effects due to the Earth's rotation (as treated in Eq. 13-13).

21. From Eq. 13-14, we see the extreme case is when "g" becomes zero, and plugging in Eq. 13-15 leads to

$$0 = \frac{GM}{R^2} - R\omega^2 \Rightarrow M = \frac{R^3\omega^2}{G}.$$

Thus, with R = 20000 m and $\omega = 2\pi$ rad/s, we find $M = 4.7 \times 10^{24}$ kg $\approx 5 \times 10^{24}$ kg.

22. (a) Plugging $R_h = 2GM_h/c^2$ into the indicated expression, we find

$$a_g = \frac{GM_h}{\left(1.001R_h\right)^2} = \frac{GM_h}{\left(1.001\right)^2\left(2GM_h/c^2\right)^2} = \frac{c^4}{\left(2.002\right)^2 G}\frac{1}{M_h}$$

which yields $a_g = (3.02 \times 10^{43}\ \text{kg}\cdot\text{m/s}^2)/M_h$.

(b) Since M_h is in the denominator of the above result, a_g decreases as M_h increases.

(c) With $M_h = (1.55 \times 10^{12})(1.99 \times 10^{30}$ kg), we obtain $a_g = 9.82$ m/s^2.

(d) This part refers specifically to the very large black hole treated in the previous part. With that mass for M in Eq. 13-16, and $r = 2.002GM/c^2$, we obtain

$$da_g = -2\frac{GM}{\left(2.002GM/c^2\right)^3}dr = -\frac{2c^6}{\left(2.002\right)^3\left(GM\right)^2}dr$$

where $dr \to 1.70$ m as in Sample Problem 13.2 – "Difference in acceleration at head and feet." This yields (in absolute value) an acceleration difference of 7.30×10^{-15} m/s^2.

(e) The miniscule result of the previous part implies that, in this case, any effects due to the differences of gravitational forces on the body are negligible.

23. (a) The gravitational acceleration is $a_g = \dfrac{GM}{R^2} = 7.6 \text{ m/s}^2$.

(b) Note that the total mass is 5*M*. Thus, $a_g = \dfrac{G(5M)}{(3R)^2} = 4.2 \text{ m/s}^2$.

24. (a) What contributes to the GmM/r^2 force on *m* is the (spherically distributed) mass *M* contained within *r* (where *r* is measured from the center of *M*). At point *A* we see that $M_1 + M_2$ is at a smaller radius than $r = a$ and thus contributes to the force:

$$|F_{\text{on } m}| = \frac{G(M_1 + M_2)m}{a^2}.$$

(b) In the case $r = b$, only M_1 is contained within that radius, so the force on *m* becomes GM_1m/b^2.

(c) If the particle is at *C*, then no other mass is at smaller radius and the gravitational force on it is zero.

25. Using the fact that the volume of a sphere is $4\pi R^3/3$, we find the density of the sphere:

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{1.0\times10^4 \text{ kg}}{\frac{4}{3}\pi(1.0 \text{ m})^3} = 2.4\times10^3 \text{ kg/m}^3.$$

When the particle of mass *m* (upon which the sphere, or parts of it, are exerting a gravitational force) is at radius *r* (measured from the center of the sphere), then whatever mass *M* is at a radius less than *r* must contribute to the magnitude of that force (GMm/r^2).

(a) At $r = 1.5$ m, all of M_{total} is at a smaller radius and thus all contributes to the force:

$$|F_{\text{on } m}| = \frac{GmM_{\text{total}}}{r^2} = m(3.0\times10^{-7} \text{ N/kg}).$$

(b) At $r = 0.50$ m, the portion of the sphere at radius smaller than that is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.3\times10^3 \text{ kg}.$$

Thus, the force on m has magnitude $GMm/r^2 = m$ (3.3 × 10^{-7} N/kg).

(c) Pursuing the calculation of part (b) algebraically, we find

$$\left|F_{\text{on } m}\right| = \frac{Gm\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = mr\left(6.7\times10^{-7}\ \frac{\text{N}}{\text{kg}\cdot\text{m}}\right).$$

26. (a) Since the volume of a sphere is $4\pi R^3/3$, the density is

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{3M_{\text{total}}}{4\pi R^3}.$$

When we test for gravitational acceleration (caused by the sphere, or by parts of it) at radius r (measured from the center of the sphere), the mass M, which is at radius less than r, is what contributes to the reading (GM/r^2). Since $M = \rho(4\pi r^3/3)$ for $r \le R$, then we can write this result as

$$\frac{G\left(\frac{3M_{\text{total}}}{4\pi R^3}\right)\left(\frac{4\pi r^3}{3}\right)}{r^2} = \frac{GM_{\text{total}}r}{R^3}$$

when we are considering points on or inside the sphere. Thus, the value a_g referred to in the problem is the case where $r = R$:

$$a_g = \frac{GM_{\text{total}}}{R^2},$$

and we solve for the case where the acceleration equals $a_g/3$:

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}r}{R^3} \;\Rightarrow\; r = \frac{R}{3}.$$

(b) Now we treat the case of an external test point. For points with $r > R$ the acceleration is GM_{total}/r^2, so the requirement that it equal $a_g/3$ leads to

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}}{r^2} \;\Rightarrow\; r = \sqrt{3}R.$$

27. (a) The magnitude of the force on a particle with mass m at the surface of Earth is given by $F = GMm/R^2$, where M is the total mass of Earth and R is Earth's radius. The acceleration due to gravity is

$$a_g = \frac{F}{m} = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m}\right)^2} = 9.83\ \text{m/s}^2.$$

(b) Now $a_g = GM/R^2$, where M is the total mass contained in the core and mantle together and R is the outer radius of the mantle (6.345×10^6 m, according to the figure). The total mass is

$$M = (1.93 \times 10^{24}\ \text{kg} + 4.01 \times 10^{24}\ \text{kg}) = 5.94 \times 10^{24}\ \text{kg}.$$

The first term is the mass of the core and the second is the mass of the mantle. Thus,

$$a_g = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.94\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.84\ \text{m/s}^2.$$

(c) A point 25 km below the surface is at the mantle–crust interface and is on the surface of a sphere with a radius of $R = 6.345 \times 10^6$ m. Since the mass is now assumed to be uniformly distributed, the mass within this sphere can be found by multiplying the mass per unit volume by the volume of the sphere: $M = (R^3/R_e^3)M_e$, where M_e is the total mass of Earth and R_e is the radius of Earth. Thus,

$$M = \left(\frac{6.345\times10^{6}\ \text{m}}{6.37\times10^{6}\ \text{m}}\right)^3 \left(5.98\times10^{24}\ \text{kg}\right) = 5.91\times10^{24}\ \text{kg}.$$

The acceleration due to gravity is

$$a_g = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.91\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.79\ \text{m/s}^2.$$

28. (a) Using Eq. 13-1, we set GmM/r^2 equal to $\frac{1}{2}GmM/R^2$, and we find $r = R\sqrt{2}$. Thus, the distance from the surface is $(\sqrt{2} - 1)R = 0.414R$.

(b) Setting the density ρ equal to M/V where $V = \frac{4}{3}\pi R^3$, we use Eq. 13-19:

$$F = \frac{4\pi Gmr\rho}{3} = \frac{4\pi Gmr}{3}\left(\frac{M}{4\pi R^3/3}\right) = \frac{GMmr}{R^3} = \frac{1}{2}\frac{GMm}{R^2} \quad \Rightarrow \quad r = R/2.$$

29. The equation immediately preceding Eq. 13-28 shows that $K = -U$ (with U evaluated at the planet's surface: -5.0×10^9 J) is required to "escape." Thus, $K = 5.0 \times 10^9$ J.

30. The gravitational potential energy is

$$U = -\frac{Gm\left(M-m\right)}{r} = -\frac{G}{r}\left(Mm - m^2\right)$$

which we differentiate with respect to m and set equal to zero (in order to minimize). Thus, we find $M - 2m = 0$, which leads to the ratio $m/M = 1/2$ to obtain the least potential energy.

Note that a second derivative of U with respect to m would lead to a positive result regardless of the value of m, which means its graph is everywhere concave upward and thus its extremum is indeed a minimum.

31. **THINK** Given the mass and diameter of Mars, we can calculate its mean density, gravitational acceleration and escape speed.

EXPRESS The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where M is its mass and R is its radius. On the other hand, the value of gravitational acceleration a_g at the surface of a planet is given by $a_g = GM/R^2$. for a particle of mass m, its escape speed is given by

$$\frac{1}{2}mv^2 = G\frac{mM}{R} \quad \Rightarrow \quad v = \sqrt{\frac{2GM}{R}}.$$

ANALYZE (a) From the definition of density above, we find the ratio of the density of Mars to the density of Earth to be

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65\times10^4 \text{ km}}{3.45\times10^3 \text{ km}}\right)^3 = 0.74.$$

(b) The value of gravitational acceleration for Mars is

$$a_{gM} = \frac{GM_M}{R_M^2} = \frac{M_M}{R_M^2}\cdot\frac{R_E^2}{M_E}\cdot\frac{GM_E}{R_E^2} = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2}a_{gE} = 0.11\left(\frac{0.65\times10^4 \text{ km}}{3.45\times10^3 \text{ km}}\right)^2\left(9.8 \text{ m/s}^2\right) = 3.8 \text{ m/s}^2.$$

(c) For Mars, the escape speed is

$$v_M = \sqrt{\frac{2GM_M}{R_M}} = \sqrt{\frac{2(6.67\times10^{-11} \text{ m}^3/\text{s}^2\cdot\text{kg})\left(0.11\right)\left(5.98\times10^{24} \text{ kg}\right)}{3.45\times10^6 \text{ m}}} = 5.0\times10^3 \text{ m/s}.$$

LEARN The ratio of the escape speeds on Mars and on Earth is

$$\frac{v_M}{v_E} = \frac{\sqrt{2GM_M/R_M}}{\sqrt{2GM_E/R_E}} = \sqrt{\frac{M_M}{M_E}\cdot\frac{R_E}{R_M}} = \sqrt{(0.11)\cdot\frac{6.5\times10^3 \text{ km}}{3.45\times10^3 \text{ km}}} = 0.455\,.$$

32. (a) The gravitational potential energy is

$$U=-\frac{GMm}{r}=-\frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.2\ \text{kg}\right)\left(2.4\ \text{kg}\right)}{19\ \text{m}}=-\ 4.4\times10^{-11}\ \text{J}.$$

(b) Since the change in potential energy is

$$\Delta U=-\frac{GMm}{3r}-\left(-\frac{GMm}{r}\right)=-\frac{2}{3}\left(-4.4\times10^{-11}\ \text{J}\right)=2.9\times10^{-11}\ \text{J},$$

the work done by the gravitational force is $W = -\ \Delta U = -2.9 \times 10^{-11}$ J.

(c) The work done by you is $W' = \Delta U = 2.9 \times 10^{-11}$ J.

33. The amount of (kinetic) energy needed to escape is the same as the (absolute value of the) gravitational potential energy at its original position. Thus, an object of mass m on a planet of mass M and radius R needs $K = GmM/R$ in order to (barely) escape.
(a) Setting up the ratio, we find

$$\frac{K_m}{K_E}=\frac{M_m}{M_E}\frac{R_E}{R_m}=0.0451$$

using the values found in Appendix C.

(b) Similarly, for the Jupiter escape energy (divided by that for Earth) we obtain

$$\frac{K_J}{K_E}=\frac{M_J}{M_E}\frac{R_E}{R_J}=28.5.$$

34. (a) The potential energy U at the surface is $U_s = -5.0 \times 10^9$ J according to the graph, since U is inversely proportional to r (see Eq. 13-21), at an r-value a factor of 5/4 times what it was at the surface then U must be 4 $U_s/5$. Thus, at $r = 1.25R_s$, $U = -\ 4.0 \times 10^9$ J. Since mechanical energy is assumed to be conserved in this problem, we have

$$K + U = -2.0 \times 10^9\ \text{J}$$

at this point. Since $U = -\ 4.0 \times 10^9$ J here, then $K = 2.0\times10^9$ J at this point.

(b) To reach the point where the mechanical energy equals the potential energy (that is, where $U = -\ 2.0 \times 10^9$ J) means that U must reduce (from its value at $r = 1.25R_s$) by a factor of 2, which means the r value must increase (relative to $r = 1.25R_s$) by a corresponding factor of 2. Thus, the turning point must be at $r = 2.5R_s$.

35. Let m = 0.020 kg and d = 0.600 m (the original edge-length, in terms of which the final edge-length is d/3). The total initial gravitational potential energy (using Eq. 13-21 and some elementary trigonometry) is

$$U_i = -\frac{4Gm^2}{d} - \frac{2Gm^2}{\sqrt{2}\, d} .$$

Since U is inversely proportional to r then reducing the size by 1/3 means increasing the magnitude of the potential energy by a factor of 3, so

$$U_f = 3U_i \;\Rightarrow\; \Delta U = 2U_i = 2(4+\sqrt{2})\left(-\frac{Gm^2}{d}\right) = -4.82 \times 10^{-13} \text{ J} .$$

36. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \;\Rightarrow\; K_1 - \frac{GmM}{r_1} = K_2 - \frac{GmM}{r_2}$$

where $M = 5.0 \times 10^{23}$ kg, $r_1 = R = 3.0 \times 10^6$ m and m = 10 kg.

(a) If $K_1 = 5.0 \times 10^7$ J and $r_2 = 4.0 \times 10^6$ m, then the above equation leads to

$$K_2 = K_1 + GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 2.2 \times 10^7 \text{ J.}$$

(b) In this case, we require $K_2 = 0$ and $r_2 = 8.0 \times 10^6$ m, and solve for K_1:

$$K_1 = K_2 + GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 6.9 \times 10^7 \text{ J.}$$

37. (a) The work done by you in moving the sphere of mass m_B equals the change in the potential energy of the three-sphere system. The initial potential energy is

$$U_i = -\frac{Gm_Am_B}{d} - \frac{Gm_Am_C}{L} - \frac{Gm_Bm_C}{L-d}$$

and the final potential energy is

$$U_f = -\frac{Gm_Am_B}{L-d} - \frac{Gm_Am_C}{L} - \frac{Gm_Bm_C}{d}.$$

The work done is

$$W = U_f - U_i = Gm_B\left[m_A\left(\frac{1}{d} - \frac{1}{L-d}\right) + m_C\left(\frac{1}{L-d} - \frac{1}{d}\right)\right]$$
$$= Gm_B\left[m_A\frac{L-2d}{d(L-d)} + m_C\frac{2d-L}{d(L-d)}\right] = Gm_B(m_A - m_C)\frac{L-2d}{d(L-d)}$$
$$= (6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.010\ \text{kg})(0.080\ \text{kg} - 0.020\ \text{kg})\frac{0.12\ \text{m} - 2(0.040\ \text{m})}{(0.040\ \text{m})(0.12 - 0.040\ \text{m})}$$
$$= +5.0\times10^{-13}\ \text{J}.$$

(b) The work done by the force of gravity is $-(U_f - U_i) = -5.0 \times 10^{-13}$ J.

38. (a) The initial gravitational potential energy is

$$U_i = -\frac{GM_AM_B}{r_i} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.80\ \text{m}}$$
$$= -1.67\times10^{-8}\ \text{J} \approx -1.7\times10^{-8}\ \text{J}.$$

(b) We use conservation of energy (with $K_i = 0$):

$$U_i = K + U \quad\Rightarrow\quad -1.7\times10^{-8} = K - \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.60\ \text{m}}$$

which yields $K = 5.6 \times 10^{-9}$ J. Note that the value of r is the difference between 0.80 m and 0.20 m.

39. **THINK** The escape speed on the asteroid is related to the gravitational acceleration at the surface of the asteroid and its size.

EXPRESS We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields

$$-GMm/R + \tfrac{1}{2}mv^2 = 0.$$

We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \tfrac{1}{2}v^2 = 0$. Solving for v, we have

$$v = \sqrt{2a_gR}\ .$$

ANALYZE (a) Given that $R = 500\,\text{km}$ and $a_g = 3.0\,\text{m/s}^2$, we find the escape speed to be

$$v = \sqrt{2a_g R} = \sqrt{2(3.0\ \text{m/s}^2)(500\times 10^3\ \text{m})} = 1.7\times 10^3\ \text{m/s}.$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = ½mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R + h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}.$$

We replace GM with a_gR^2 and cancel m in the energy equation to obtain

$$-a_g R + \frac{1}{2}v^2 = -\frac{a_g R^2}{(R+h)}.$$

The solution for h is

$$h = \frac{2a_g R^2}{2a_g R - v^2} - R = \frac{2(3.0\ \text{m/s}^2)(500\times 10^3\ \text{m})^2}{2(3.0\ \text{m/s}^2)(500\times 10^3\ \text{m}) - (1000\ \text{m/s})^2} - (500\times 10^3\ \text{m})$$
$$= 2.5\times 10^5\ \text{m}.$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R + h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2.$$

We substitute a_gR^2 for GM and cancel m, obtaining

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2.$$

The solution for v is

$$v = \sqrt{2a_g R - \frac{2a_g R^2}{R+h}} = \sqrt{2(3.0\ \text{m/s}^2)(500\times 10^3\ \text{m}) - \frac{2(3.0\ \text{m/s}^2)(500\times 10^3\ \text{m})^2}{(500\times 10^3\ \text{m}) + (1000\times 10^3\ \text{m})}}$$
$$= 1.4\times 10^3\ \text{m/s}.$$

LEARN The key idea in this problem is to realize that energy is conserved in the process:

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad \Delta K + \Delta U = 0.$$

The decrease in potential energy is equal to the gain in kinetic energy, and vice versa.

40. (a) From Eq. 13-28, we see that $v_0 = \sqrt{GM/2R_E}$ in this problem. Using energy conservation, we have

$$\frac{1}{2}mv_0^2 - GMm/R_E = -\,GMm/r$$

which yields $r = 4R_E/3$. So the multiple of R_E is 4/3 or 1.33.

(b) Using the equation in the textbook immediately preceding Eq. 13-28, we see that in this problem we have $K_i = GMm/2R_E$, and the above manipulation (using energy conservation) in this case leads to $r = 2R_E$. So the multiple of R_E is 2.00.

(c) Again referring to the equation in the textbook immediately preceding Eq. 13-28, we see that the mechanical energy = 0 for the "escape condition."

41. **THINK** The two neutron stars are attracted toward each other due to their gravitational interaction.

EXPRESS The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -GM^2/r_f$, where the final separation is $r_f = r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2.$$

ANALYZE (a) The solution for v is

$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\ \text{kg})}{10^{10}\ \text{m}}} = 8.2\times10^4\ \text{m/s}.$$

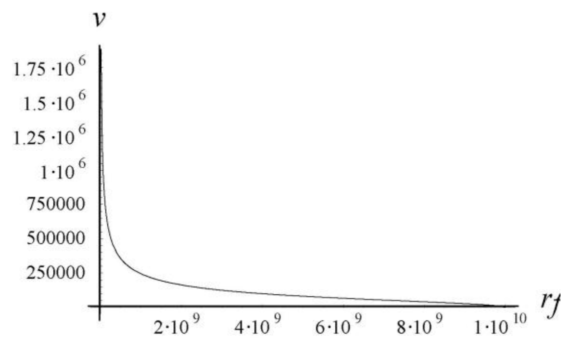
(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5$ m, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes

$$-GM^2/r_i = -GM^2/r_f + Mv^2.$$

The solution for v is

$$v=\sqrt{GM\left(\frac{1}{r_f}-\frac{1}{r_i}\right)}=\sqrt{(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\,\text{kg})\left(\frac{1}{2\times10^5\ \text{m}}-\frac{1}{10^{10}\ \text{m}}\right)}$$
$$=\ 1.8\times10^7\ \text{m/s}.$$

LEARN The speed of the stars as a function of their final separation is plotted below. The decrease in gravitational potential energy is accompanied by an increase in kinetic energy, so that the total energy of the two-star system remains conserved.



42. (a) Applying Eq. 13-21 and the Pythagorean theorem leads to

$$U=-\left(\frac{GM^2}{2D}+\frac{2GmM}{\sqrt{y^2+D^2}}\right)$$

where M is the mass of particle B (also that of particle C) and m is the mass of particle A. The value given in the problem statement (for infinitely large y, for which the second term above vanishes) determines M, since D is given. Thus $M = 0.50$ kg.

(b) We estimate (from the graph) the $y = 0$ value to be $U_o = -3.5\times10^{-10}$ J. Using this, our expression above determines m. We obtain $m = 1.5$ kg.

43. (a) If r is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by GMm/r^2, where M is the mass of Earth and m is the mass of the satellite. The magnitude of the acceleration of the satellite is given by v^2/r, where v is its speed. Newton's second law yields $GMm/r^2 = mv^2/r$. Since the radius of Earth is 6.37×10^6 m, the orbit radius is $r = (6.37\times10^6\text{ m} + 160\times10^3\text{ m}) = 6.53\times10^6$ m. The solution for v is

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\,\text{kg})}{6.53\times10^6\ \text{m}}}=7.82\times10^3\ \text{m/s}.$$

(b) Since the circumference of the circular orbit is $2\pi r$, the period is

$$T=\frac{2\pi r}{v}=\frac{2\pi(6.53\times10^6\ \text{m})}{7.82\times10^3\ \text{m/s}}=5.25\times10^3\ \text{s}.$$

This is equivalent to 87.5 min.

44. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{r_s}{r_m}\right)^3 = \left(\frac{T_s}{T_m}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{T_s}{1 \text{ lunar month}}\right)^2$$

which yields T_s = 0.35 lunar month for the period of the satellite.

45. The period T and orbit radius r are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of Mars. The period is 7 h 39 min, which is 2.754×10^4 s. We solve for M:

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})\left(2.754 \times 10^4 \text{ s}\right)^2} = 6.5 \times 10^{23} \text{ kg}.$$

46. From Eq. 13-37, we obtain $v = \sqrt{GM/r}$ for the speed of an object in circular orbit (of radius r) around a planet of mass M. In this case, $M = 5.98 \times 10^{24}$ kg and

$$r = (700 + 6370)\text{m} = 7070 \text{ km} = 7.07 \times 10^6 \text{ m}.$$

The speed is found to be $v = 7.51 \times 10^3$ m/s. After multiplying by 3600 s/h and dividing by 1000 m/km this becomes $v = 2.7 \times 10^4$ km/h.

(a) For a head-on collision, the relative speed of the two objects must be $2v = 5.4 \times 10^4$ km/h.

(b) A perpendicular collision is possible if one satellite is, say, orbiting above the equator and the other is following a longitudinal line. In this case, the relative speed is given by the Pythagorean theorem: $\sqrt{v^2 + v^2} = 3.8 \times 10^4$ km/h.

47. **THINK** The centripetal force on the Sun is due to the gravitational attraction between the Sun and the stars at the center of the Galaxy.

EXPRESS Let N be the number of stars in the galaxy, M be the mass of the Sun, and r be the radius of the galaxy. The total mass in the galaxy is $N\,M$ and the magnitude of the gravitational force acting on the Sun is

$$F_g = \frac{GM(NM)}{R^2} = \frac{GNM^2}{R^2}.$$

The force, pointing toward the galactic center, is the centripetal force on the Sun. Thus,

$$F_c = F_g \quad \Rightarrow \quad \frac{Mv^2}{R} = \frac{GNM^2}{R^2}.$$

The magnitude of the Sun's acceleration is $a = v^2/R$, where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields

$$GNM^2/R^2 = 4\pi^2 MR/T^2.$$

The solution for N is

$$N = \frac{4\pi^2 R^3}{GT^2 M}.$$

ANALYZE The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2 (2.2 \times 10^{20}\text{ m})^3}{(6.67 \times 10^{-11}\text{ m}^3/\text{s}^2 \cdot \text{kg})(7.88 \times 10^{15}\text{ s})^2 (2.0 \times 10^{30}\text{ kg})} = 5.1 \times 10^{10}.$$

LEARN The number of stars in the Milky Way is between 10^{11} to 4×10^{11}. Our simplified model provides a reasonable estimate.

48. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{a_M}{a_E}\right)^3 = \left(\frac{T_M}{T_E}\right)^2 \Rightarrow (1.52)^3 = \left(\frac{T_M}{1\text{ y}}\right)^2$$

where we have substituted the mean-distance (from Sun) ratio for the semi-major axis ratio. This yields $T_M = 1.87$ y. The value in Appendix C (1.88 y) is quite close, and the small apparent discrepancy is not significant, since a more precise value for the semi-major axis ratio is $a_M/a_E = 1.523$, which does lead to $T_M = 1.88$ y using Kepler's law. A question can be raised regarding the use of a ratio of mean distances for the ratio of semi-major axes, but this requires a more lengthy discussion of what is meant by a "mean distance" than is appropriate here.

49. (a) The period of the comet is 1420 years (and one month), which we convert to $T = 4.48 \times 10^{10}$ s. Since the mass of the Sun is 1.99×10^{30} kg, then Kepler's law of periods gives

$$(4.48 \times 10^{10}\text{ s})^2 = \left(\frac{4\pi^2}{(6.67\times10^{-11}\text{ m}^3/\text{kg}\cdot\text{s}^2)(1.99 \times 10^{30}\text{ kg})}\right) a^3 \Rightarrow a = 1.89 \times 10^{13}\text{ m}.$$

(b) Since the distance from the focus (of an ellipse) to its center is ea and the distance from center to the aphelion is a, then the comet is at a distance of

$$ea + a = (0.9932+1)\,(1.89\times10^{13}\text{ m}) = 3.767\times10^{13}\text{ m}$$

when it is farthest from the Sun. To express this in terms of Pluto's orbital radius (found in Appendix C), we set up a ratio:

$$\left(\frac{3.767\times10^{13}}{5.9\times10^{12}}\right)R_P \approx 6{,}4R_P.$$

50. To "hover" above Earth (M_E = 5.98 × 10^{24} kg) means that it has a period of 24 hours (86400 s). By Kepler's law of periods,

$$(86400)^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 \Rightarrow r = 4.225\times10^7\ \text{m}.$$

Its altitude is therefore $r - R_E$ (where R_E = 6.37 × 10^6 m), which yields 3.58 × 10^7 m.

51. **THINK** The satellite moves in an elliptical orbit about Earth. An elliptical orbit can be characterized by its semi-major axis and eccentricity.

EXPRESS The greatest distance between the satellite and Earth's center (the apogee distance) and the least distance (perigee distance) are, respectively,

$$R_a = R_E + d_a = 6.37\times10^6\ \text{m} + 360\times10^3\ \text{m} = 6.73\times10^6\ \text{m}$$
$$R_p = R_E + d_p = 6.37\times10^6\ \text{m} + 180\times10^3\ \text{m} = 6.55\times10^6\ \text{m}.$$

Here R_E = 6.37 × 10^6 m is the radius of Earth.

ANALYZE The semi-major axis is given by

$$a = \frac{R_a + R_p}{2} = \frac{6.73\times10^6\ \text{m} + 6.55\times10^6\ \text{m}}{2} = 6.64\times10^6\ \text{m}.$$

(b) The apogee and perigee distances are related to the eccentricity e by $R_a = a(1 + e)$ and $R_p = a(1 - e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73\times10^6\ \text{m} - 6.55\times10^6\ \text{m}}{6.73\times10^6\ \text{m} + 6.55\times10^6\ \text{m}} = 0.0136.$$

LEARN Since e is very small, the orbit is nearly circular. On the other hand, if e is close to unity, then the orbit would be a long, thin ellipse.

52. (a) The distance from the center of an ellipse to a focus is ae where a is the semi-major axis and e is the eccentricity. Thus, the separation of the foci (in the case of Earth's orbit) is

$$2ae = 2\left(1.50\times10^{11}\ \text{m}\right)\left(0.0167\right) = 5.01\times10^9\ \text{m}.$$

(b) To express this in terms of solar radii (see Appendix C), we set up a ratio:

$$\frac{5.01 \times 10^9 \text{ m}}{6.96 \times 10^8 \text{ m}} = 7.20.$$

53. From Kepler's law of periods (where T = (2.4 h)(3600 s/h) = 8640 s), we find the planet's mass M:

$$(8640\,\text{s})^2 = \left(\frac{4\pi^2}{GM}\right)(8.0\times10^6 \text{ m})^3 \Rightarrow M = 4.06 \times 10^{24} \text{ kg}.$$

However, we also know $a_g = GM/R^2 = 8.0 \text{ m/s}^2$ so that we are able to solve for the planet's radius:

$$R = \sqrt{\frac{GM}{a_g}} = \sqrt{\frac{(6.67\times10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)(4.06\times10^{24} \text{ kg})}{8.0 \text{ m/s}^2}} = 5.8 \times 10^6 \text{ m}.$$

54. The two stars are in circular orbits, not about each other, but about the two-star system's center of mass (denoted as O), which lies along the line connecting the centers of the two stars. The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, for the visible, Newton's second law gives

$$F = \frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r_1}$$

where r is the distance between the centers of the stars. To find the relation between r and r_1, we locate the center of mass relative to m_1. Using Equation 9-1, we obtain

$$r_1 = \frac{m_1(0) + m_2r}{m_1 + m_2} = \frac{m_2r}{m_1 + m_2} \quad \Rightarrow \quad r = \frac{m_1 + m_2}{m_2}r_1.$$

On the other hand, since the orbital speed of m_1 is $v = 2\pi r_1/T$, then $r_1 = vT/2\pi$ and the expression for r can be rewritten as

$$r = \frac{m_1 + m_2}{m_2}\frac{vT}{2\pi}.$$

Substituting r and r_1 into the force equation, we obtain

$$F = \frac{4\pi^2Gm_1m_2^3}{(m_1 + m_2)^2v^2T^2} = \frac{2\pi m_1v}{T}$$

or

$$\frac{m_2^3}{(m_1 + m_2)^2} = \frac{v^3T}{2\pi G} = \frac{(2.7\times10^5 \text{ m/s})^3(1.70 \text{ days})(86400 \text{ s/day})}{2\pi(6.67\times10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)} = 6.90\times10^{30} \text{ kg}$$
$$= 3.467M_s,$$

where $M_s = 1.99\times10^{30}$ kg is the mass of the sun. With $m_1 = 6M_s$, we write $m_2 = \alpha M_s$ and solve the following cubic equation for α :

$$\frac{\alpha^3}{(6+\alpha)^2} - 3.467 = 0 .$$

The equation has one real solution: $\alpha = 9.3$, which implies $m_2 / M_s \approx 9$.

55. (a) If we take the logarithm of Kepler's law of periods, we obtain

$$2\log(T) = \log(4\pi^2/GM) + 3\log(a) \Rightarrow \log(a) = \frac{2}{3}\log(T) - \frac{1}{3}\log(4\pi^2/GM)$$

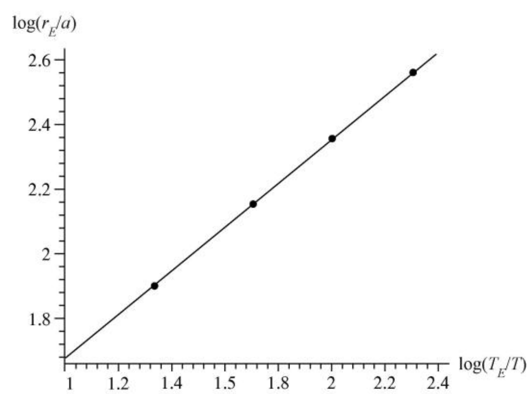
where we are ignoring an important subtlety about units (the arguments of logarithms cannot have units, since they are transcendental functions). Although the problem can be continued in this way, we prefer to set it up without units, which requires taking a ratio. If we divide Kepler's law (applied to the Jupiter–moon system, where M is mass of Jupiter) by the law applied to Earth orbiting the Sun (of mass M_o), we obtain

$$(T/T_E)^2 = \left(\frac{M_o}{M}\right)\left(\frac{a}{r_E}\right)^3$$

where T_E = 365.25 days is Earth's orbital period and $r_E = 1.50 \times 10^{11}$ m is its mean distance from the Sun. In this case, it is perfectly legitimate to take logarithms and obtain

$$\log\left(\frac{r_E}{a}\right) = \frac{2}{3}\log\left(\frac{T_E}{T}\right) + \frac{1}{3}\log\left(\frac{M_o}{M}\right)$$

(written to make each term positive), which is the way we plot the data ($\log(r_E/a)$ on the vertical axis and $\log(T_E/T)$ on the horizontal axis).



(b) When we perform a least-squares fit to the data, we obtain

$$\log (r_E/a) = 0.666 \log (T_E/T) + 1.01,$$

which confirms the expectation of slope = 2/3 based on the above equation.

(c) And the 1.01 intercept corresponds to the term 1/3 log (M_o/M), which implies

$$\frac{M_o}{M} = 10^{3.03} \Rightarrow M = \frac{M_o}{1.07 \times 10^3}.$$

Plugging in $M_o = 1.99 \times 10^{30}$ kg (see Appendix C), we obtain $M = 1.86 \times 10^{27}$ kg for Jupiter's mass. This is reasonably consistent with the value 1.90×10^{27} kg found in Appendix C.

56. (a) The period is $T = 27(3600) = 97200$ s, and we are asked to assume that the orbit is circular (of radius $r = 100000$ m). Kepler's law of periods provides us with an approximation to the asteroid's mass:

$$(97200)^2 = \left(\frac{4\pi^2}{GM}\right)(100000)^3 \Rightarrow M = 6.3 \times 10^{16} \text{ kg}.$$

(b) Dividing the mass M by the given volume yields an average density equal to

$$\rho = (6.3 \times 10^{16} \text{ kg})/(1.41 \times 10^{13} \text{ m}^3) = 4.4 \times 10^3 \text{ kg/m}^3,$$

which is about 20% less dense than Earth.

57. In our system, we have $m_1 = m_2 = M$ (the mass of our Sun, 1.99×10^{30} kg). With $r = 2r_1$ in this system (so r_1 is one-half the Earth-to-Sun distance r), and $v = \pi r/T$ for the speed, we have

$$\frac{Gm_1m_2}{r^2} = m_1 \frac{(\pi r/T)^2}{r/2} \Rightarrow T = \sqrt{\frac{2\pi^2 r^3}{GM}}.$$

With $r = 1.5 \times 10^{11}$ m, we obtain $T = 2.2 \times 10^7$ s. We can express this in terms of Earth-years, by setting up a ratio:

$$T = \left(\frac{T}{1\text{y}}\right)(1\text{y}) = \left(\frac{2.2 \times 10^7 \text{ s}}{3.156 \times 10^7 \text{ s}}\right)(1 \text{ y}) = 0.71 \text{ y}.$$

58. (a) We make use of

$$\frac{m_2^3}{(m_1 + m_2)^2} = \frac{v^3 T}{2\pi G}$$

where $m_1 = 0.9M_{Sun}$ is the estimated mass of the star. With $v = 70$ m/s and $T = 1500$ days (or $1500 \times 86400 = 1.3 \times 10^8$ s), we find

$$\frac{m_2^3}{(0.9M_{\text{Sun}}+m_2)^2} = 1.06\times10^{23}\text{ kg}\ .$$

Since $M_{\text{Sun}} \approx 2.0 \times 10^{30}$ kg, we find $m_2 \approx 7.0 \times 10^{27}$ kg. Dividing by the mass of Jupiter (see Appendix C), we obtain $m \approx 3.7m_J$.

(b) Since $v = 2\pi r_1/T$ is the speed of the star, we find

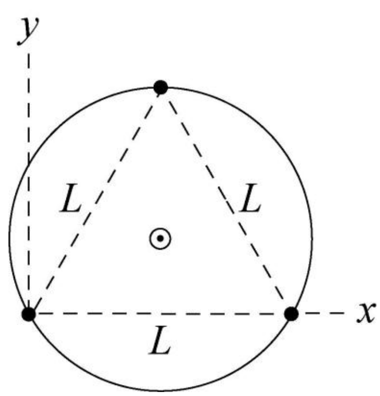
$$r_1 = \frac{vT}{2\pi} = \frac{(70\text{ m/s})(1.3\times10^8\text{ s})}{2\pi} = 1.4\times10^9\text{ m}$$

for the star's orbital radius. If r is the distance between the star and the planet, then $r_2 = r - r_1$ is the orbital radius of the planet, and is given by

$$r_2 = r_1\left(\frac{m_1+m_2}{m_2}-1\right) = r_1\frac{m_1}{m_2} = 3.7\times10^{11}\text{ m}\ .$$

Dividing this by 1.5×10^{11} m (Earth's orbital radius, r_E) gives $r_2 = 2.5r_E$.

59. Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos 30° = Mv^2/R$.



The stars rotate about their center of mass (marked by a circled dot on the diagram above) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $\left(\sqrt{3}/2\right)L$, so the stars are located at $x = 0$, $y = 0$; $x = L$, $y = 0$; and $x = L/2$, $y = \sqrt{3}L/2$. The x coordinate of the center of mass is $x_c = (L +$

L/2)/3 = L/2 and the y coordinate is $y_c = \left(\sqrt{3}L/2\right)/3 = L/2\sqrt{3}$. The distance from a star to the center of mass is

$$R = \sqrt{x_c^2 + y_c^2} = \sqrt{\left(L^2/4\right) + \left(L^2/12\right)} = L/\sqrt{3}\,.$$

Once the substitution for R is made, Newton's second law then becomes $\left(2GM^2/L^2\right)\cos 30° = \sqrt{3}Mv^2/L$. This can be simplified further by recognizing that $\cos 30° = \sqrt{3}/2$. Divide the equation by M then gives $GM/L^2 = v^2/L$, or $v = \sqrt{GM/L}$.

60. (a) From Eq. 13-40, we see that the energy of each satellite is $-GM_Em/2r$. The total energy of the two satellites is twice that result:

$$E = E_A + E_B = -\frac{GM_Em}{r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})(125\ \text{kg})}{7.87\times10^6\ \text{m}}$$
$$= -6.33\times10^9\,\text{J}.$$

(b) We note that the speed of the wreckage will be zero (immediately after the collision), so it has no kinetic energy at that moment. Replacing m with $2m$ in the potential energy expression, we therefore find the total energy of the wreckage at that instant is

$$E = -\frac{GM_E(2m)}{2r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})2(125\ \text{kg})}{2(7.87\times10^6\ \text{m})} = -6.33\times10^9\,\text{J}.$$

(c) An object with zero speed at that distance from Earth will simply fall toward the Earth, its trajectory being toward the center of the planet.

61. The energy required to raise a satellite of mass m to an altitude h (at rest) is given by

$$E_1 = \Delta U = GM_Em\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right),$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2}\,mv_{\text{orb}}^2 = \frac{GM_Em}{2\left(R_E + h\right)}.$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_Em\left[\frac{1}{R_E} - \frac{3}{2(R_E + h)}\right].$$

(a) Solving the above equation, the height h_0 at which $\Delta E = 0$ is given by

$$\frac{1}{R_E} - \frac{3}{2(R_E + h_0)} = 0 \quad \Rightarrow h_0 = \frac{R_E}{2} = 3.19 \times 10^6 \text{ m}.$$

(b) For greater height $h > h_0$, $\Delta E > 0$, implying $E_1 > E_2$. Thus, the energy of lifting is greater.

62. Although altitudes are given, it is the orbital radii that enter the equations. Thus, r_A = (6370 + 6370) km = 12740 km, and r_B = (19110 + 6370) km = 25480 km.

(a) The ratio of potential energies is

$$\frac{U_B}{U_A} = \frac{-GmM / r_B}{-GmM / r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(b) Using Eq. 13-38, the ratio of kinetic energies is

$$\frac{K_B}{K_A} = \frac{GmM / 2r_B}{GmM / 2r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(c) From Eq. 13-40, it is clear that the satellite with the largest value of *r* has the smallest value of |*E*| (since *r* is in the denominator). And since the values of *E* are negative, then the smallest value of |*E*| corresponds to the largest energy *E*. Thus, satellite *B* has the largest energy.

(d) The difference is

$$\Delta E = E_B - E_A = -\frac{GmM}{2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

Being careful to convert the *r* values to meters, we obtain $\Delta E = 1.1 \times 10^8$ J. The mass *M* of Earth is found in Appendix C.

63. **THINK** We apply Kepler's laws to analyze the motion of the asteroid.

EXPRESS We use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where *M* is the mass of the Sun (1.99×10^{30} kg) and *r* is the radius of the orbit. On the other hand, the kinetic energy of any asteroid or planet in a circular orbit of radius *r* is given by $K = GMm/2r$, where *m* is the mass of the asteroid or planet. We note that it is proportional to *m* and inversely proportional to *r*.

ANALYZE (a) The radius of the orbit is twice the radius of Earth's orbit: $r = 2r_{SE} = 2(150 \times 10^9 \text{ m}) = 300 \times 10^9$ m. Thus, the period of the asteroid is

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (300\times10^9\ \text{m})^3}{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(1.99\times10^{30}\text{kg})}} = 8.96\times10^7\ \text{s}.$$

Dividing by (365 d/y) (24 h/d) (60 min/h) (60 s/min), we obtain $T = 2.8$ y.

(b) The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is

$$\frac{K}{K_E} = \frac{GMm/(2r)}{GMM_E/(2r_{SE})} = \frac{m}{M_E}\cdot\frac{r_{SE}}{r} = (2.0\times10^{-4})\left(\frac{1}{2}\right) = 1.0\times10^{-4}.$$

LEARN An alternative way to calculate the ratio of kinetic energies is to use $K = mv^2/2$ and note that $v = 2\pi r/T$. This gives

$$\frac{K}{K_E} = \frac{mv^2/2}{M_E v_E^2/2} = \frac{m}{M_E}\left(\frac{v}{v_E}\right)^2 = \frac{m}{M_E}\left(\frac{r/T}{r_{SE}/T_E}\right)^2 = \frac{m}{M_E}\left(\frac{r}{r_{SE}}\cdot\frac{T_E}{T}\right)^2$$
$$= (2.0\times10^{-4})\left(2\cdot\frac{1.0\ \text{y}}{2.8\ \text{y}}\right)^2 = 1.0\times10^{-4}$$

in agreement with what we found in (b).

64. (a) Circular motion requires that the force in Newton's second law provide the necessary centripetal acceleration:

$$\frac{GmM}{r^2} = m\frac{v^2}{r}.$$

Since the left-hand side of this equation is the force given as 80 N, then we can solve for the combination mv^2 by multiplying both sides by $r = 2.0\times10^7$ m. Thus, $mv^2 = (2.0\times10^7$ m) (80 N) $= 1.6\times10^9$ J. Therefore,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.6\times10^9\ \text{J}\right) = 8.0\times10^8\ \text{J}.$$

(b) Since the gravitational force is inversely proportional to the square of the radius, then

$$\frac{F'}{F} = \left(\frac{r}{r'}\right)^2.$$

Thus, $F' = (80\ \text{N})\,(2/3)^2 = 36$ N.

65. (a) From Kepler's law of periods, we see that T is proportional to $r^{3/2}$.

(b) Equation 13-38 shows that K is inversely proportional to r.

(c) and (d) From the previous part, knowing that K is proportional to v^2, we find that v is proportional to $1/\sqrt{r}$. Thus, by Eq. 13-31, the angular momentum (which depends on the product rv) is proportional to $r/\sqrt{r} = \sqrt{r}$.

66. (a) The pellets will have the same speed v but opposite direction of motion, so the *relative speed* between the pellets and satellite is $2v$. Replacing v with $2v$ in Eq. 13-38 is equivalent to multiplying it by a factor of 4. Thus,

$$K_{\text{rel}} = 4\left(\frac{GM_E m}{2r}\right) = \frac{2(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)\left(5.98\times10^{24}\ \text{kg}\right)\left(0.0040\ \text{kg}\right)}{(6370+500)\times10^3\ \text{m}}$$
$$= 4.6\times10^5\ \text{J}.$$

(b) We set up the ratio of kinetic energies:

$$\frac{K_{\text{rel}}}{K_{\text{bullet}}} = \frac{4.6\times10^5\ \text{J}}{\frac{1}{2}\left(0.0040\,\text{kg}\right)\left(950\,\text{m/s}\right)^2} = 2.6\times10^2.$$

67. (a) The force acting on the satellite has magnitude GMm/r^2, where M is the mass of Earth, m is the mass of the satellite, and r is the radius of the orbit. The force points toward the center of the orbit. Since the acceleration of the satellite is v^2/r, where v is its speed, Newton's second law yields $GMm/r^2 = mv^2/r$ and the speed is given by $v = \sqrt{GM/r}$. The radius of the orbit is the sum of Earth's radius and the altitude of the satellite:

$$r = (6.37\times10^6 + 640\times10^3)\ \text{m} = 7.01\times10^6\ \text{m}.$$

Thus,

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)}{7.01\times10^6\ \text{m}}} = 7.54\times10^3\ \text{m/s}.$$

(b) The period is

$$T = 2\pi r/v = 2\pi(7.01\times10^6\ \text{m})/(7.54\times10^3\ \text{m/s}) = 5.84\times10^3\ \text{s} \approx 97\ \text{min}.$$

(c) If E_0 is the initial energy then the energy after n orbits is $E = E_0 - nC$, where $C = 1.4\times10^5$ J/orbit. For a circular orbit the energy and orbit radius are related by $E = -GMm/2r$, so the radius after n orbits is given by $r = -GMm/2E$.
The initial energy is

$$E_0 = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)\left(220\ \text{kg}\right)}{2(7.01\times10^6\ \text{m})} = -6.26\times10^9\ \text{J},$$

the energy after 1500 orbits is

$$E = E_0 - nC = -6.26 \times 10^9 \text{ J} - (1500 \text{ orbit})(1.4 \times 10^5 \text{ J/orbit}) = -6.47 \times 10^9 \text{ J},$$

and the orbit radius after 1500 orbits is

$$r = -\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})(220 \text{ kg})}{2(-6.47 \times 10^9 \text{ J})} = 6.78 \times 10^6 \text{ m}.$$

The altitude is

$$h = r - R = (6.78 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m}) = 4.1 \times 10^5 \text{ m}.$$

Here R is the radius of Earth. This torque is internal to the satellite−Earth system, so the angular momentum of that system is conserved.

(d) The speed is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{6.78 \times 10^6 \text{ m}}} = 7.67 \times 10^3 \text{ m/s} \approx 7.7 \text{ km/s}.$$

(e) The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.78 \times 10^6 \text{ m})}{7.67 \times 10^3 \text{ m/s}} = 5.6 \times 10^3 \text{ s} \approx 93 \text{ min}.$$

(f) Let F be the magnitude of the average force and s be the distance traveled by the satellite. Then, the work done by the force is $W = -Fs$. This is the change in energy: $-Fs = \Delta E$. Thus, $F = -\Delta E/s$. We evaluate this expression for the first orbit. For a complete orbit $s = 2\pi r = 2\pi(7.01 \times 10^6 \text{ m}) = 4.40 \times 10^7$ m, and $\Delta E = -1.4 \times 10^5$ J. Thus,

$$F = -\frac{\Delta E}{s} = \frac{1.4 \times 10^5 \text{ J}}{4.40 \times 10^7 \text{ m}} = 3.2 \times 10^{-3} \text{ N}.$$

(g) The resistive force exerts a torque on the satellite, so its angular momentum is not conserved.

(h) The satellite−Earth system is essentially isolated, so its momentum is very nearly conserved.

68. The orbital radius is $r = R_E + h = 6370 \text{ km} + 400 \text{ km} = 6770 \text{ km} = 6.77 \times 10^6 \text{ m}.$

(a) Using Kepler's law given in Eq. 13-34, we find the period of the ships to be

$$T_0 = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (6.77 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{kg})}} = 5.54 \times 10^3 \text{ s} \approx 92.3 \text{ min.}$$

(b) The speed of the ships is

$$v_0 = \frac{2\pi r}{T_0} = \frac{2\pi(6.77 \times 10^6 \text{ m})}{5.54 \times 10^3 \text{ s}} = 7.68 \times 10^3 \text{ m/s}^2 .$$

(c) The new kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(0.99v_0)^2 = \frac{1}{2}(2000 \text{ kg})(0.99)^2(7.68 \times 10^3 \text{ m/s})^2 = 5.78 \times 10^{10} \text{ J.}$$

(d) Immediately after the burst, the potential energy is the same as it was before the burst. Therefore,

$$U = -\frac{GMm}{r} = -\frac{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{kg})(2000 \text{ kg})}{6.77 \times 10^6 \text{ m}} = -1.18 \times 10^{11} \text{ J.}$$

(e) In the new elliptical orbit, the total energy is

$$E = K + U = 5.78 \times 10^{10} \text{ J} + (-1.18 \times 10^{11} \text{ J}) = -6.02 \times 10^{10} \text{ J.}$$

(f) For elliptical orbit, the total energy can be written as (see Eq. 13-42) $E = -GMm/2a$, where a is the semi-major axis. Thus,

$$a = -\frac{GMm}{2E} = -\frac{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{kg})(2000 \text{ kg})}{2(-6.02 \times 10^{10} \text{ J})} = 6.63 \times 10^6 \text{ m.}$$

(g) To find the period, we use Eq. 13-34 but replace r with a. The result is

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2 (6.63 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{kg})}} = 5.37 \times 10^3 \text{ s} \approx 89.5 \text{ min.}$$

(h) The orbital period T for Picard's elliptical orbit is shorter than Igor's by

$$\Delta T = T_0 - T = 5540 \text{ s} - 5370 \text{ s} = 170 \text{ s} .$$

Thus, Picard will arrive back at point P ahead of Igor by 170 s – 90 s = 80 s.

69. We define the "effective gravity" in his environment as $g_{eff} = 220/60 = 3.67 \text{ m/s}^2$. Thus, using equations from Chapter 2 (and selecting downward as the positive direction), we find the "fall-time" to be

$$\Delta y = v_0 t + \frac{1}{2} g_{eff} t^2 \Rightarrow t = \sqrt{\frac{2(2.1\text{ m})}{3.67\text{ m/s}^2}} = 1.1\text{ s.}$$

70. (a) The gravitational acceleration a_g is defined in Eq. 13-11. The problem is concerned with the difference between a_g evaluated at $r = 50R_h$ and a_g evaluated at $r = 50R_h + h$ (where h is the estimate of your height). Assuming h is much smaller than $50R_h$ then we can approximate h as the dr that is present when we consider the differential of Eq. 13-11:

$$|da_g| = \frac{2GM}{r^3}dr \approx \frac{2GM}{50^3 R_h^3} h = \frac{2GM}{50^3 (2GM/c^2)^3} h .$$

If we approximate $|da_g| = 10\text{ m/s}^2$ and $h \approx 1.5$ m, we can solve this for M. Giving our results in terms of the Sun's mass means dividing our result for M by 2×10^{30} kg. Thus, admitting some tolerance into our estimate of h we find the "critical" black hole mass should in the range of 105 to 125 solar masses.

(b) Interestingly, this turns out to be lower limit (which will surprise many students) since the above expression shows $|da_g|$ is inversely proportional to M. It should perhaps be emphasized that a distance of $50R_h$ from a small black hole is much smaller than a distance of $50R_h$ from a large black hole.

71. (a) All points on the ring are the same distance ($r = \sqrt{x^2 + R^2}$) from the particle, so the gravitational potential energy is simply $U = -GMm/\sqrt{x^2 + R^2}$, from Eq. 13-21. The corresponding force (by symmetry) is expected to be along the x axis, so we take a (negative) derivative of U (with respect to x) to obtain it (see Eq. 8-20). The result for the magnitude of the force is $GMmx(x^2 + R^2)^{-3/2}$.

(b) Using our expression for U, the change in potential energy as the particle falls to the center is

$$\Delta U = -GMm\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right)$$

By conservation of mechanical energy, this must "turn into" kinetic energy, $\Delta K = -\Delta U = mv^2/2$. We solve for the speed and obtain

$$\frac{1}{2}mv^2 = GMm\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right) \Rightarrow v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right)}.$$

72. (a) With $M = 2.0 \times 10^{30}$ kg and $r = 10000$ m, we find $a_g = \frac{GM}{r^2} = 1.3 \times 10^{12}\text{ m/s}^2$.

(b) Although a close answer may be gotten by using the constant acceleration equations of Chapter 2, we show the more general approach (using energy conservation):

$$K_o + U_o = K + U$$

where $K_o = 0$, $K = \frac{1}{2}mv^2$, and U is given by Eq. 13-21. Thus, with $r_o = 10001$ m, we find

$$v = \sqrt{2GM\left(\frac{1}{r} - \frac{1}{r_o}\right)} = 1.6 \times 10^6 \text{ m/s} .$$

73. Using energy conservation (and Eq. 13-21) we have

$$K_1 - \frac{GMm}{r_1} = K_2 - \frac{GMm}{r_2} .$$

(a) Plugging in two pairs of values (for (K_1, r_1) and (K_2, r_2)) from the graph and using the value of G and M (for Earth) given in the book, we find $m \approx 1.0 \times 10^3$ kg.

(b) Similarly, $v = (2K/m)^{1/2} \approx 1.5 \times 10^3$ m/s (at $r = 1.945 \times 10^7$ m).

74. We estimate the planet to have radius $r = 10$ m. To estimate the mass m of the planet, we require its density equal that of Earth (and use the fact that the volume of a sphere is $4\pi r^3/3$):

$$\frac{m}{4\pi r^3/3} = \frac{M_E}{4\pi R_E^3/3} \Rightarrow m = M_E\left(\frac{r}{R_E}\right)^3$$

which yields (with $M_E \approx 6 \times 10^{24}$ kg and $R_E \approx 6.4 \times 10^6$ m) $m = 2.3 \times 10^7$ kg.

(a) With the above assumptions, the acceleration due to gravity is

$$a_g = \frac{Gm}{r^2} = \frac{\left(6.7\times10^{-11} \text{ m}^3/\text{s}^2\cdot\text{kg}\right)\left(2.3\times10^7 \text{ kg}\right)}{(10 \text{ m})^2} = 1.5\times10^{-5} \text{ m/s}^2 \approx 2\times10^{-5} \text{ m/s}^2 .$$

(b) Equation 13-28 gives the escape speed: $v = \sqrt{\frac{2Gm}{r}} \approx 0.02$ m/s .

75. We use m_1 for the 20 kg of the sphere at $(x_1, y_1) = (0.5, 1.0)$ (SI units understood), m_2 for the 40 kg of the sphere at $(x_2, y_2) = (-1.0, -1.0)$, and m_3 for the 60 kg of the sphere at $(x_3, y_3) = (0, -0.5)$. The mass of the 20 kg object at the origin is simply denoted m. We note that $r_1 = \sqrt{1.25}$, $r_2 = \sqrt{2}$, and $r_3 = 0.5$ (again, with SI units understood). The force $\vec{F}_n$ that the n^{th} sphere exerts on m has magnitude $Gm_n m/r_n^2$ and is directed from the origin toward m_n, so that it is conveniently written as

$$\vec{F}_n = \frac{Gm_n m}{r_n^2}\left(\frac{x_n}{r_n}\hat{\mathrm{i}} + \frac{y_n}{r_n}\hat{\mathrm{j}}\right) = \frac{Gm_n m}{r_n^3}\left(x_n\hat{\mathrm{i}} + y_n\hat{\mathrm{j}}\right).$$

Consequently, the vector addition to obtain the net force on m becomes

$$\vec{F}_{\text{net}} = \sum_{n=1}^{3}\vec{F}_n = Gm\left(\left(\sum_{n=1}^{3}\frac{m_n x_n}{r_n^3}\right)\hat{\mathrm{i}} + \left(\sum_{n=1}^{3}\frac{m_n y_n}{r_n^3}\right)\hat{\mathrm{j}}\right) = (-9.3\times10^{-9}\ \text{N})\hat{\mathrm{i}} - (3.2\times10^{-7}\ \text{N})\hat{\mathrm{j}}\ .$$

Therefore, we find the net force magnitude is $\left|\vec{F}_{\text{net}}\right| = 3.2\times10^{-7}\,\text{N}$.

76. **THINK** We apply Newton's law of gravitation to calculate the force between the meteor and the satellite.

EXPRESS We use $F = Gm_s m_m/r^2$, where m_s is the mass of the satellite, m_m is the mass of the meteor, and r is the distance between their centers. The distance between centers is $r = R + d$ = 15 m + 3 m = 18 m. Here R is the radius of the satellite and d is the distance from its surface to the center of the meteor.

ANALYZE The gravitational force between the meteor and the satellite is

$$F = \frac{Gm_s m_s}{r^2} = \frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(20\text{kg}\right)\left(7.0\text{kg}\right)}{\left(18\text{m}\right)^2} = 2.9\times10^{-11}\,\text{N}.$$

LEARN The force of gravitation is inversely proportional to r^2.

77. We note that r_A (the distance from the origin to sphere A, which is the same as the separation between A and B) is 0.5, r_C = 0.8, and r_D = 0.4 (with SI units understood). The force $\vec{F}_k$ that the k^{th} sphere exerts on m_B has magnitude $Gm_k m_B / r_k^2$ and is directed from the origin toward m_k so that it is conveniently written as

$$\vec{F}_k = \frac{Gm_k m_B}{r_k^2}\left(\frac{x_k}{r_k}\hat{\mathrm{i}} + \frac{y_k}{r_k}\hat{\mathrm{j}}\right) = \frac{Gm_k m_B}{r_k^3}\left(x_k\hat{\mathrm{i}} + y_k\hat{\mathrm{j}}\right).$$

Consequently, the vector addition (where k equals A, B, and D) to obtain the net force on m_B becomes

$$\vec{F}_{\text{net}} = \sum_k \vec{F}_k = Gm_B\left(\left(\sum_k \frac{m_k x_k}{r_k^3}\right)\hat{\mathrm{i}} + \left(\sum_k \frac{m_k y_k}{r_k^3}\right)\hat{\mathrm{j}}\right) = (3.7\times10^{-5}\,\text{N})\hat{\mathrm{j}}.$$

78. (a) We note that r_C (the distance from the origin to sphere C, which is the same as the separation between C and B) is 0.8, r_D = 0.4, and the separation between spheres C and D is r_{CD} = 1.2 (with SI units understood). The total potential energy is therefore

$$-\frac{GM_BM_C}{r_C^2}-\frac{GM_BM_D}{r_D^2}-\frac{GM_CM_D}{r_{CD}^2}=-1.3\times10^{-4}\text{ J}$$

using the mass-values given in the previous problem.

(b) Since any gravitational potential energy term (of the sort considered in this chapter) is necessarily negative ($-GmM/r^2$ where all variables are positive) then having another mass to include in the computation can only lower the result (that is, make the result more negative).

(c) The observation in the previous part implies that the work I do in removing sphere *A* (to obtain the case considered in part (a)) must lead to an increase in the system energy; thus, I do positive work.

(d) To put sphere *A* back in, I do negative work, since I am causing the system energy to become more negative.

79. **THINK** Since the orbit is circular, the net gravitational force on the smaller star is equal to the centripetal force.

EXPRESS The magnitude of the net gravitational force on one of the smaller stars (of mass *m*) is

$$F=\frac{GMm}{r^2}+\frac{Gmm}{(2r)^2}=\frac{Gm}{r^2}\left(M+\frac{m}{4}\right).$$

This supplies the centripetal force needed for the motion of the star:

$$\frac{Gm}{r^2}\left(M+\frac{m}{4}\right)=m\frac{v^2}{r}$$

where $v=2\pi r/T$. Combining the two expressions allows us to solve for *T*.

ANALYZE Plugging in for speed *v*, we arrive at an equation for the period *T*:

$$T=\frac{2\pi r^{3/2}}{\sqrt{G(M+m/4)}}.$$

LEARN In the limit where $m\ll M$, we recover the expected result $T=\dfrac{2\pi r^{3/2}}{\sqrt{GM}}$ for two bodies.

80. If the angular velocity were any greater, loose objects on the surface would not go around with the planet but would travel out into space.

(a) The magnitude of the gravitational force exerted by the planet on an object of mass *m* at its surface is given by $F = GmM / R^2$, where *M* is the mass of the planet and *R* is its radius. According to Newton's second law this must equal mv^2 / R, where *v* is the speed of the object. Thus,

$$\frac{GM}{R^2} = \frac{v^2}{R}.$$

With $M = 4\pi\rho R^3/3$ where ρ is the density of the planet, and $v = 2\pi R/T$, where *T* is the period of revolution, we find

$$\frac{4\pi}{3}G\rho R = \frac{4\pi^2 R}{T^2}.$$

We solve for *T* and obtain

$$T = \sqrt{\frac{3\pi}{G\rho}}.$$

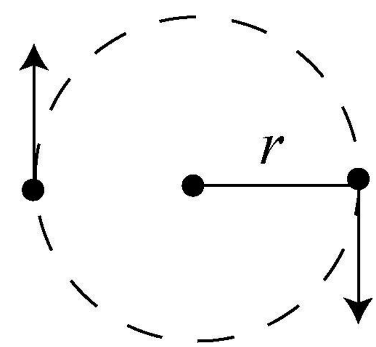
(b) The density is 3.0×10^3 kg/m³. We evaluate the equation for *T*:

$$T = \sqrt{\frac{3\pi}{\left(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(3.0\times10^3\,\text{kg/m}^3\right)}} = 6.86\times10^3\,\text{s} = 1.9\,\text{h}.$$

81. **THINK** In a two-star system, the stars rotate about their common center of mass.

EXPRESS The situation is depicted on the right. The gravitational force between the two stars (each having a mass *M*) is

$$F_g = \frac{GM^2}{(2r)^2} = \frac{GM^2}{4r^2}$$



The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, writing the centripetal acceleration as $r\omega^2$ where ω is the angular speed, we have

$$F_g = F_c \Rightarrow \frac{GM^2}{4r^2} = Mr\omega^2.$$

ANALYZE (a) Substituting the values given, we find the common angular speed to be

$$\omega = \frac{1}{2}\sqrt{\frac{GM}{r^3}} = \frac{1}{2}\sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.0\times10^{30}\ \text{kg})}{(1.0\times10^{11}\ \text{m})^3}} = 2.2\times10^{-7}\,\text{rad/s}.$$

(b) To barely escape means to have total energy equal to zero (see discussion prior to Eq. 13-28). If m is the mass of the meteoroid, then

$$\frac{1}{2}mv^2 - \frac{GmM}{r} - \frac{GmM}{r} = 0 \;\Rightarrow\; v = \sqrt{\frac{4GM}{r}} = 8.9\times10^4 \text{ m/s} .$$

LEARN Comparing with Eq. 13-28, we see that the escape speed of the two-star system is the same as that of a star with mass $2M$.

82. The key point here is that angular momentum is conserved:

$$I_p\omega_p = I_a\omega_a$$

which leads to $\omega_p = (r_a/r_p)^2\omega_a$, but $r_p = 2a - r_a$ where a is determined by Eq. 13-34 (particularly, see the paragraph after that equation in the textbook). Therefore,

$$\omega_p = \frac{r_a^{\,2}\,\omega_a}{(2(GMT^2/4\pi^2)^{1/3} - r_a)^2} = 9.24\times10^{-5} \text{ rad/s} .$$

83. **THINK** The orbit of the shuttle goes from circular to elliptical after changing its speed by firing the thrusters.

EXPRESS We first use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of the planet and r is the radius of the orbit. After the orbit of the shuttle turns elliptical by firing the thrusters to reduce its speed, the semi-major axis is $a = -GMm/2E$, where $E = K + U$ is the mechanical energy of the shuttle and its new period becomes $T' = \sqrt{4\pi^2a^3/GM}$.

ANALYZE (a) Using Kepler's law of periods, we find the period to be

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right)r^3} = \sqrt{\frac{4\pi^2(4.20\times10^7\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\text{ kg})}} = 2.15\times10^4 \text{ s} .$$

(b) The speed is constant (before she fires the thrusters), so

$$v_0 = \frac{2\pi r}{T} = \frac{2\pi(4.20\times10^7\text{m})}{2.15\times10^4\text{ s}} = 1.23\times10^4 \text{ m/s} .$$

(c) A two percent reduction in the previous value gives

$$v = 0.98v_0 = 0.98(1.23\times10^4\text{ m/s}) = 1.20\times10^4 \text{ m/s} .$$

(d) The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(3000\text{ kg})(1.20\times10^4\text{ m/s})^2 = 2.17\times10^{11}\text{ J}$.

(e) Immediately after the firing, the potential energy is the same as it was before firing the thruster:

$$U = -\frac{GMm}{r} = -\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\text{ kg})(3000\text{ kg})}{4.20\times10^7\text{ m}} = -4.53\times10^{11}\text{ J}.$$

(f) Adding these two results gives the total mechanical energy:

$$E = K + U = 2.17\times10^{11}\text{ J} + (-4.53\times10^{11}\text{ J}) = -2.35\times10^{11}\text{ J}.$$

(g) Using Eq. 13-42, we find the semi-major axis to be

$$a = -\frac{GMm}{2E} = -\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\text{ kg})(3000\text{ kg})}{2(-2.35\times10^{11}\text{ J})} = 4.04\times10^7\text{ m}.$$

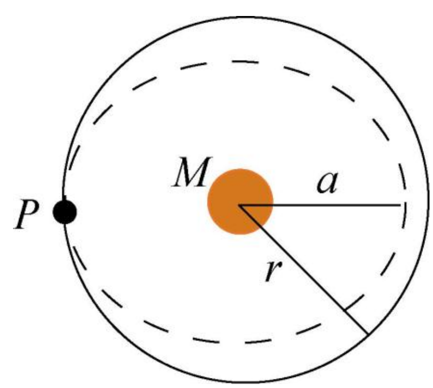
(h) Using Kepler's law of periods for elliptical orbits (using a instead of r) we find the new period to be

$$T' = \sqrt{\left(\frac{4\pi^2}{GM}\right)a^3} = \sqrt{\frac{4\pi^2(4.04\times10^7\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\text{ kg})}} = 2.03\times10^4\text{ s}.$$

This is smaller than our result for part (a) by $T - T' = 1.22\times10^3$ s.

(i) Comparing the results in (a) and (h), we see that elliptical orbit has a smaller period.

LEARN The orbits of the shuttle before and after firing the thruster are shown below. Point P corresponds to the location where the thruster was fired.



84. The difference between free-fall acceleration g and the gravitational acceleration a_g at the equator of the star is (see Equation 13.14):

$$a_g - g = \omega^2 R$$

where

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.041\,\text{s}} = 153\,\text{rad/s}$$

is the angular speed of the star. The gravitational acceleration at the equator is

$$a_g = \frac{GM}{R^2} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.98\times10^{30}\ \text{kg})}{(1.2\times10^4\ \text{m})^2} = 9.17\times10^{11}\ \text{m/s}^2.$$

Therefore, the percentage difference is

$$\frac{a_g - g}{a_g} = \frac{\omega^2 R}{a_g} = \frac{(153\,\text{rad/s})^2(1.2\times10^4\ \text{m})}{9.17\times10^{11}\ \text{m/s}^2} = 3.06\times10^{-4} \approx 0.031\%.$$

85. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \quad\Rightarrow\quad \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 5.98 \times 10^{24}$ kg, $r_1 = R = 6.37 \times 10^6$, m and $v_1 = 10000$ m/s. Setting $v_2 = 0$ to find the maximum of its trajectory, we solve the above equation (noting that m cancels in the process) and obtain $r_2 = 3.2 \times 10^7$ m. This implies that its *altitude* is

$$h = r_2 - R = 2.5 \times 10^7\ \text{m}.$$

86. We note that, since $v = 2\pi r/T$, the centripetal acceleration may be written as $a = 4\pi^2 r/T^2$. To express the result in terms of g, we divide by 9.8 m/s².

(a) The acceleration associated with Earth's spin (T = 24 h = 86400 s) is

$$a = g\,\frac{4\pi^2\,(6.37 \times 10^6\ \text{m})}{(86400\,\text{s})^2\,(9.8\,\text{m/s}^2)} = 3.4\times10^{-3}\,g\ .$$

(b) The acceleration associated with Earth's motion around the Sun ($T = 1$ y $= 3.156 \times 10^7$ s) is

$$a = g\,\frac{4\pi^2\,(1.5 \times 10^{11}\ \text{m})}{(3.156 \times 10^7\ \text{s})^2\,(9.8\,\text{m/s}^2)} = 6.1\times10^{-4}\,g\ .$$

(c) The acceleration associated with the Solar System's motion around the galactic center ($T = 2.5 \times 10^8$ y $= 7.9 \times 10^{15}$ s) is

$$a = g\frac{4\pi^2(2.2\times10^{20}\text{ m})}{(7.9\times10^{15}\text{ s})^2(9.8\,\text{m/s}^2)} = 1.4\times10^{-11}g\ .$$

87. (a) It is possible to use $v^2 = v_0^2 + 2a\,\Delta y$ as we did for free-fall problems in Chapter 2 because the acceleration can be considered approximately constant over this interval. However, our approach will not assume constant acceleration; we use energy conservation:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv^2 - \frac{GMm}{r} \Rightarrow v = \sqrt{\frac{2GM(r_0-r)}{r_0\,r}}$$

which yields $v = 1.4\times10^6$ m/s.

(b) We estimate the height of the apple to be h = 7 cm = 0.07 m. We may find the answer by evaluating Eq. 13-11 at the surface (radius r in part (a)) and at radius $r + h$, being careful not to round off, and then taking the difference of the two values, or we may take the differential of that equation — setting dr equal to h. We illustrate the latter procedure:

$$|\,da_g\,| = \left|-2\frac{GM}{r^3}dr\right| \approx 2\frac{GM}{r^3}h = 3\times10^6\text{ m/s}^2\,.$$

88. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed v_f. The corresponding increase in its kinetic energy, $\frac{1}{2}mv_f^2$, is equal to the work done on it by Earth's gravity: $\int F\,dr = \int(-Kr)dr$. Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F\,dr = \int_R^0(-Kr)\,dr = \frac{1}{2}KR^2$$

where R is the radius of Earth. Solving for the final speed, we obtain $v_f = R\sqrt{K/m}$. We note that the acceleration of gravity $a_g = g = 9.8$ m/s^2 on the surface of Earth is given by

$$a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2,$$

where ρ is Earth's average density. This permits us to write $K/m = 4\pi G\rho/3 = g/R$. Consequently,

$$v_f = R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(6.37\times10^6\text{ m})} = 7.9\times10^3\text{ m/s}\,.$$

89. **THINK** To compare the kinetic energy, potential energy, and the speed of the Earth at aphelion (farthest distance) and perihelion (closest distance), we apply both conservation of energy and conservation of angular momentum.

EXPRESS As Earth orbits about the Sun, its total energy is conserved:

$$\frac{1}{2}mv_a^2 - \frac{GM_SM_E}{R_a} = \frac{1}{2}mv_p^2 - \frac{GM_SM_E}{R_p}.$$

In addition, angular momentum conservation implies $v_aR_a = v_pR_p$.

ANALYZE (a) The total energy is conserved, so there is no difference between its values at aphelion and perihelion.

(b) The difference in potential energy is

$$\begin{aligned}\Delta U = U_a - U_p &= -GM_SM_E\left(\frac{1}{R_a} - \frac{1}{R_p}\right)\\ &= -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(5.98\times10^{24}\ \text{kg})\left(\frac{1}{1.52\times10^{11}\ \text{m}} - \frac{1}{1.47\times10^{11}\ \text{m}}\right)\\ &\approx 1.8\times10^{32}\ \text{J}.\end{aligned}$$

(c) Since $\Delta K + \Delta U = 0$, $\Delta K = K_a - K_p = -\Delta U \approx -1.8\times10^{32}\ \text{J}$.

(d) With $v_aR_a = v_pR_p$, the change in kinetic energy may be written as

$$\Delta K = K_a - K_p = \frac{1}{2}M_E\left(v_a^2 - v_p^2\right) = \frac{1}{2}M_Ev_a^2\left(1 - \frac{R_a^2}{R_p^2}\right)$$

from which we find the speed at the aphelion to be

$$v_a = \sqrt{\frac{2(\Delta K)}{M_E(1 - R_a^2/R_p^2)}} = 2.95\times10^4\ \text{m/s}.$$

Thus, the variation in speed is

$$\begin{aligned}\Delta v = v_a - v_p &= \left(1 - \frac{R_a}{R_p}\right)v_a = \left(1 - \frac{1.52\times10^{11}\ \text{m}}{1.47\times10^{11}\ \text{m}}\right)(2.95\times10^4\ \text{m/s})\\ &= -0.99\times10^3\ \text{m/s} = -0.99\ \text{km/s}.\end{aligned}$$

The speed at the aphelion is smaller than that at the perihelion.

LEARN Since the changes are small, the problem could also be solved by using differentials:

$$dU = \left(\frac{GM_E M_S}{r^2}\right) dr \approx \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(1.5\times10^{11}\ \text{m}\right)^2}\left(5\times10^{9}\ \text{m}\right).$$

This yields $\Delta U \approx 1.8 \times 10^{32}$ J. Similarly, with $\Delta K \approx dK = M_E v\, dv$, where $v \approx 2\pi R/T$, we have

$$1.8\times10^{32}\ \text{J} \approx \left(5.98\times10^{24}\ \text{kg}\right)\left(\frac{2\pi\left(1.5\times10^{11}\ \text{m}\right)}{3.156\times10^{7}\ \text{s}}\right)\Delta v$$

which yields a difference of $\Delta v \approx 0.99$ km/s in Earth's speed (relative to the Sun) between aphelion and perihelion.

90. (a) Because it is moving in a circular orbit, F/m must equal the centripetal acceleration:

$$\frac{80\ \text{N}}{50\ \text{kg}} = \frac{v^2}{r}.$$

However, $v = 2\pi r/T$, where $T = 21600$ s, so we are led to

$$1.6\,\text{m/s}^2 = \frac{4\pi^2}{T^2} r$$

which yields $r = 1.9 \times 10^7$ m.

(b) From the above calculation, we infer $v^2 = (1.6\ \text{m/s}^2)r$, which leads to $v^2 = 3.0 \times 10^7$ m^2/s^2. Thus, $K = \frac{1}{2}mv^2 = 7.6 \times 10^8$ J.

(c) As discussed in Section 13-4, F/m also tells us the gravitational acceleration:

$$a_g = 1.6\ \text{m/s}^2 = \frac{GM}{r^2}.$$

We therefore find $M = 8.6 \times 10^{24}$ kg.

91. (a) Their initial potential energy is $-Gm^2/R_i$ and they started from rest, so energy conservation leads to

$$-\frac{Gm^2}{R_i} = K_{\text{total}} - \frac{Gm^2}{0.5R_i} \Rightarrow K_{\text{total}} = \frac{Gm^2}{R_i}.$$

(b) They have equal mass, and this is being viewed in the center-of-mass frame, so their speeds are identical and their kinetic energies are the same. Thus,

$$K = \frac{1}{2} K_{\text{total}} = \frac{Gm^2}{2R_i} .$$

(c) With $K = ½\, mv^2$, we solve the above equation and find $v = \sqrt{Gm/R_i}$.

(d) Their relative speed is $2v = 2\sqrt{Gm/R_i}$. This is the (instantaneous) rate at which the gap between them is closing.

(e) The premise of this part is that we assume we are not moving (that is, that body *A* acquires no kinetic energy in the process). Thus, $K_{\text{total}} = K_B$, and the logic of part (a) leads to $K_B = Gm^2/R_i$.

(f) And $\frac{1}{2} mv_B^2 = K_B$ yields $v_B = \sqrt{2Gm/R_i}$.

(g) The answer to part (f) is incorrect, due to having ignored the accelerated motion of "our" frame (that of body *A*). Our computations were therefore carried out in a noninertial frame of reference, for which the energy equations of Chapter 8 are not directly applicable.

92. (a) We note that the altitude of the rocket is $h = R - R_E$ where $R_E = 6.37\times10^6$ m. With $M = 5.98\times10^{24}$ kg, $R_0 = R_E + h_0 = 6.57 \times 10^6$ m and $R = 7.37 \times 10^6$ m, we have

$$K_i + U_i = K + U \Rightarrow \frac{1}{2} m\,(3.70\times10^3 \text{ m/s})^2 - \frac{GmM}{R_0} = K - \frac{GmM}{R},$$

which yields $K = 3.83 \times 10^7$ J.

(b) Again, we use energy conservation.

$$K_i + U_i = K_f + U_f \Rightarrow \frac{1}{2} m\,(3.70\times10^3)^2 - \frac{GmM}{R_0} = 0 - \frac{GmM}{R_f}$$

Therefore, we find $R_f = 7.40 \times 10^6$ m. This corresponds to a distance of 1034.9 km ≈ 1.03 × 10^3 km above the Earth's surface.

93. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \;\Rightarrow\; \frac{1}{2} mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2} mv_2^2 - \frac{GmM}{r_2}$$

where $M = 7.0 \times 10^{24}$ kg, $r_2 = R = 1.6 \times 10^6$ m, and $r_1 = \infty$ (which means that $U_1 = 0$). We are told to assume the meteor starts at rest, so $v_1 = 0$. Thus, $K_1 + U_1 = 0$, and the above equation is rewritten as

$$\frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \quad \Rightarrow \quad v_2 = \sqrt{\frac{2GM}{R}} = 2.4 \times 10^4 \text{ m/s}.$$

94. The initial distance from each fixed sphere to the ball is $r_0 = \infty$, which implies the initial gravitational potential energy is zero. The distance from each fixed sphere to the ball when it is at $x = 0.30$ m is $r = 0.50$ m, by the Pythagorean theorem.

(a) With $M = 20$ kg and $m = 10$ kg, energy conservation leads to

$$K_i + U_i \;=\; K + U \;\Rightarrow\; 0 + 0 = K - 2\frac{GmM}{r}$$

which yields $K = 2GmM/r = 5.3 \times 10^{-8}$ J.

(b) Since the y-component of each force will cancel, the net force points in the $-x$ direction, with a magnitude

$$2F_x = 2\,(GmM/r^2)\cos\,\theta\,,$$

where $\theta = \tan^{-1}(4/3) = 53°$. Thus, the result is $\vec{F}_{\text{net}} = (-6.4\times10^{-8}\text{ N})\hat{\text{i}}$.

95. The magnitudes of the individual forces (acting on m_C, exerted by m_A and m_B, respectively) are

$$F_{AC} = \frac{Gm_A\,m_C}{r_{AC}^2} = 2.7 \times 10^{-8}\text{ N} \quad \text{and} \quad F_{BC} = \frac{Gm_B\,m_C}{r_{BC}^2} = 3.6 \times 10^{-8}\text{ N}$$

where $r_{AC} = 0.20$ m and $r_{BC} = 0.15$ m. With $r_{AB} = 0.25$ m, the angle $\vec{F}_A$ makes with the x axis can be obtained as

$$\theta_A = \pi + \cos^{-1}\left(\frac{r_{AC}^2 + r_{AB}^2 - r_{BC}^2}{2r_{AC}r_{AB}}\right) = \pi + \cos^{-1}(0.80) = 217°.$$

Similarly, the angle $\vec{F}_B$ makes with the x axis can be obtained as

$$\theta_B = -\cos^{-1}\left(\frac{r_{AB}^2 + r_{BC}^2 - r_{AC}^2}{2r_{AB}r_{BC}}\right) = -\cos^{-1}(0.60) = -53°.$$

The net force acting on m_C then becomes

$$\vec{F}_C = F_{AC}(\cos\theta_A\,\hat{\mathrm{i}} + \sin\theta_A\,\hat{\mathrm{j}}) + F_{BC}(\cos\theta_B\,\hat{\mathrm{i}} + \sin\theta_B\,\hat{\mathrm{j}})$$
$$= (F_{AC}\cos\theta_A + F_{BC}\cos\theta_B)\hat{\mathrm{i}} + (F_{AC}\sin\theta_A + F_{BC}\sin\theta_B)\hat{\mathrm{j}}$$
$$= (-4.4\times10^{-8}\ \mathrm{N})\hat{\mathrm{j}}.$$

96. (a) From Chapter 2, we have $v^2 = v_0^2 + 2a\Delta x$, where a may be interpreted as an average acceleration in cases where the acceleration is not uniform. With $v_0 = 0$, $v =$ 11000 m/s, and $\Delta x = 220$ m, we find $a = 2.75 \times 10^5\ \mathrm{m/s^2}$. Therefore,

$$a = \left(\frac{2.75\times10^5\ \mathrm{m/s^2}}{9.8\ \mathrm{m/s^2}}\right)g = 2.8\times10^4\,g.$$

(b) The acceleration is certainly deadly enough to kill the passengers.

(c) Again using $v^2 = v_0^2 + 2a\Delta x$, we find

$$a = \frac{(7000\ \mathrm{m/s})^2}{2(3500\ \mathrm{m})} = 7000\ \mathrm{m/s^2} = 714g.$$

(d) Energy conservation gives the craft's speed v (in the absence of friction and other dissipative effects) at altitude $h = 700$ km after being launched from $R = 6.37 \times 10^6$ m (the surface of Earth) with speed $v_0 = 7000$ m/s. That altitude corresponds to a distance from Earth's center of $r = R + h = 7.07 \times 10^6$ m.

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r}.$$

With $M = 5.98 \times 10^{24}$ kg (the mass of Earth) we find $v = 6.05 \times 10^3$ m/s. However, to orbit at that radius requires (by Eq. 13-37)

$$v' = \sqrt{GM/r} = 7.51\times10^3\ \mathrm{m/s}.$$

The difference between these two speeds is $v' - v = 1.46 \times 10^3\ \mathrm{m/s} \approx 1.5\times10^3\ \mathrm{m/s}$, which presumably is accounted for by the action of the rocket engine.

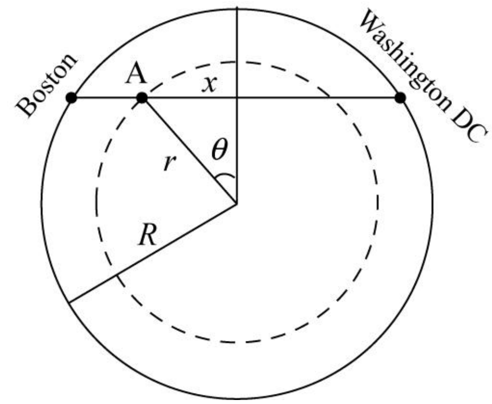
97. We integrate Eq. 13-1 with respect to r from $3R_E$ to $4R_E$ and obtain the work equal to

$$W = -\Delta U = -GM_E m\left(\frac{1}{4R_E} - \frac{1}{3R_E}\right) = \frac{GM_E m}{12R_E}.$$

98. The gravitational force at a radial distance r inside Earth (e.g., point A in the figure) is

$$F_g = -\frac{GMm}{R^3} r$$

The component of the force along the tunnel is



$$F_x = F_g \sin\theta = \left(-\frac{GMm}{R^3} r\right)\frac{x}{r} = -\frac{GMm}{R^3} x$$

which can be rewritten as

$$a_x = \frac{d^2x}{dt^2} - \frac{GM}{R^3} x = -\omega^2 x$$

where $\omega^2 = GM/R^3$. The equation is similar to Hooke's law, in that the force on the train is proportional to the displacement of the train but oppositely directed. Without exiting the tunnel, the motion of the train would be periodic would a period given by $T = 2\pi/\omega$. The travel time required from Boston to Washington DC is only half that (one-way):

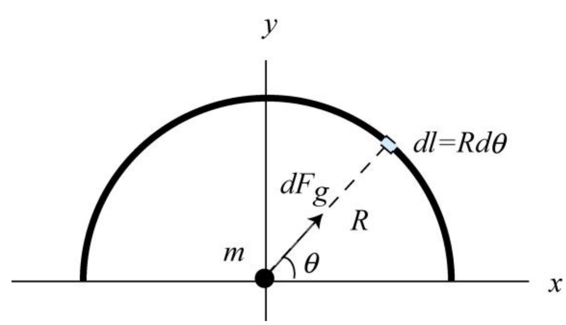
$$\Delta t = \frac{T}{2} = \frac{\pi}{\omega} = \pi\sqrt{\frac{R^3}{GM}} = \pi\sqrt{\frac{(6.37\times10^6\,\text{m})^3}{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\ \text{kg})}} = 2529\ \text{s} = 42.1\ \text{min}$$

Note that the result is independent of the distance between the two cities.

99. The gravitational force exerted on m due to a mass element dM from the thin rod is

$$dF_g = \frac{Gm(dM)}{R^2}$$

By symmetry, the force is along the y-direction. With



$$dM = \lambda dl = \left(\frac{M}{\pi R}\right) R d\theta = \frac{M}{\pi} d\theta$$

where $\lambda = M/\pi R$ is the mass density (mass per unit length), we have

$$dF_{g,y} = dF_g \sin\theta = \frac{Gm}{R^2}\left(\frac{Md\theta}{\pi}\right)\sin\theta = \frac{GMm}{\pi R^2}\sin\theta d\theta$$

Integrating over θ gives

$$F_{g,y} = \int_0^\pi \frac{GMm}{\pi R^2}\sin\theta d\theta = \frac{GMm}{\pi R^2}\int_0^\pi \sin\theta d\theta = \frac{2GMm}{\pi R^2}$$

Substituting the values given leads to

$$F_{g,y} = \frac{2GMm}{\pi R^2} = \frac{2(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.0\ \text{kg})(3.0\times10^{-3}\ \text{kg})}{\pi(0.650\ \text{m})^2} = 1.51\times10^{-12}\ \text{N}$$

If the rod were a complete circle, by symmetry, the net force on the particle would be zero.

100. The gravitational acceleration at a distance r from the center of Earth is

$$a_g = \frac{GM}{r^2}$$

Thus, the weight difference between the two objects is

$$\Delta w = m(g - a_g) = \frac{GMm}{R^2} - \frac{GMm}{(R+h)^2} = \frac{GMm}{R^2}\left[1-(1+h/R)^{-2}\right] \approx \frac{GMm}{R^2}\cdot\frac{2h}{R} = \frac{2GMmh}{R^3}$$

With $M = \frac{4}{3}\pi R^3\rho$, the above expression can be rewritten as

$$\Delta w = \frac{2GMmh}{R^3} = \frac{2Gmh}{R^3}\cdot\left(\frac{4\pi}{3}R^3\rho\right) = \frac{8\pi\rho Gmh}{3}$$

Substituting the values given, we obtain

$$\Delta w = \frac{8\pi\rho Gmh}{3} = \frac{8\pi}{3}(5.5\times10^3\ \text{kg/m}^3)(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(2.00\ \text{kg})(0.050\ \text{m})$$
$$= 3.07\times10^{-7}\ \text{N}$$

101. Let the distance from Earth to the spaceship be *r*. $R_{em} = 3.82\times10^8$ m is the distance from Earth to the moon. Thus,

$$F_m = \frac{GM_m m}{(R_{em}-r)^2} = F_E = \frac{GM_e m}{r^2},$$

where *m* is the mass of the spaceship. Solving for *r*, we obtain

$$r = \frac{R_{em}}{\sqrt{M_m/M_e}+1} = \frac{3.82\times10^8\,\text{m}}{\sqrt{(7.36\times10^{22}\,\text{kg})/(5.98\times10^{24}\,\text{kg})}+1} = 3.44\times10^8\,\text{m}.$$

Chapter 14

1. Let the volume of the expanded air sacs be V_a and that of the fish with its air sacs collapsed be V. Then

$$\rho_{\text{fish}} = \frac{m_{\text{fish}}}{V} = 1.08 \text{ g/cm}^3 \quad \text{and} \quad \rho_w = \frac{m_{\text{fish}}}{V + V_a} = 1.00 \text{ g/cm}^3$$

where ρ_w is the density of the water. This implies

$$\rho_{\text{fish}} V = \rho_w (V + V_a) \text{ or } (V + V_a)/V = 1.08/1.00,$$

which gives $V_a/(V + V_a) = 0.074 = 7.4\%$.

2. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\text{N/m}^2 = 1$ Pa, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0\times10^5 \text{ Pa} - \frac{480 \text{ N}}{77\times10^{-4} \text{ m}^2} = 3.8\times10^4 \text{ Pa}.$$

3. **THINK** The increase in pressure is equal to the applied force divided by the area.

EXPRESS The change in pressure is given by $\Delta p = F/A = F/\pi r^2$, where r is the radius of the piston.

ANALYZE substituting the values given, we obtain

$$\Delta p = (42 \text{ N})/\pi(0.011 \text{ m})^2 = 1.1 \times 10^5 \text{ Pa}.$$

This is equivalent to 1.1 atm.

LEARN The increase in pressure is proportional to the force applied. In addition, since $\Delta p \sim 1/A$, the smaller the cross-sectional area of the syringe, the greater the pressure increase under the same applied force.

4. We note that the container is cylindrical, the important aspect of this being that it has a uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquids. Using the fact that $1\text{L} = 1000 \text{ cm}^3$, we find the weight of the first liquid to be

$$W_1 = m_1 g = \rho_1 V_1 g = (2.6 \text{ g/cm}^3)(0.50 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 1.27\times10^6 \text{ g}\cdot\text{cm/s}^2$$
$$= 12.7 \text{ N}.$$

In the last step, we have converted grams to kilograms and centimeters to meters. Similarly, for the second and the third liquids, we have

$$W_2 = m_2 g = \rho_2 V_2 g = (1.0 \text{ g/cm}^3)(0.25 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 2.5 \text{ N}$$

and

$$W_3 = m_3 g = \rho_3 V_3 g = (0.80 \text{ g/cm}^3)(0.40 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 3.1 \text{ N}.$$

The total force on the bottom of the container is therefore $F = W_1 + W_2 + W_3 = 18$ N.

5. **THINK** The pressure difference between two sides of the window results in a net force acting on the window.

EXPRESS The air inside pushes outward with a force given by $p_i A$, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by $p_o A$, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$.

ANALYZE Since 1 atm = 1.013×10^5 Pa, the net force is

$$F = (p_i - p_o)A = (1.0 \text{ atm} - 0.96 \text{ atm})(1.013\times10^5 \text{ Pa/atm})(3.4 \text{ m})(2.1 \text{ m})$$
$$= 2.9 \times 10^4 \text{ N}.$$

LEARN The net force on the window vanishes when the pressure inside the office is equal to the pressure outside.

6. Knowing the standard air pressure value in several units allows us to set up a variety of conversion factors:

(a) $P = \left(28 \text{ lb/in.}^2\right)\left(\dfrac{1.01\times10^5 \text{ Pa}}{14.7 \text{ lb/in}^2}\right) = 190 \text{ kPa}$.

(b) $(120 \text{ mmHg})\left(\dfrac{1.01\times10^5 \text{Pa}}{760 \text{ mmHg}}\right) = 15.9 \text{ kPa}$, $\quad (80 \text{ mmHg})\left(\dfrac{1.01\times10^5 \text{ Pa}}{760 \text{ mmHg}}\right) = 10.6 \text{ kPa}$.

7. (a) The pressure difference results in forces applied as shown in the figure. We consider a team of horses pulling to the right. To pull the sphere apart, the team must exert a force at least as great as the horizontal component of the total force determined by "summing" (actually, integrating) these force vectors.

We consider a force vector at angle θ. Its leftward component is $\Delta p \cos \theta dA$, where dA is the area element for where the force is applied. We make use of the symmetry of the problem and let dA be that of a ring of constant θ on the surface. The radius of the ring is $r = R \sin \theta$, where R is the radius of the sphere. If the angular width of the ring is $d\theta$, in radians, then its width is $R\, d\theta$ and its area is $dA = 2\pi R^2 \sin \theta\, d\theta$. Thus the net horizontal component of the force of the air is given by

$$F_h = 2\pi R^2 \Delta p \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi R^2 \Delta p \ \sin^2 \theta \Big|_0^{\pi/2} = \pi R^2 \Delta p.$$

(b) We use 1 atm = 1.01×10^5 Pa to show that Δp = 0.90 atm = 9.09×10^4 Pa. The sphere radius is R = 0.30 m, so

$$F_h = \pi(0.30 \text{ m})^2(9.09 \times 10^4 \text{ Pa}) = 2.6 \times 10^4 \text{ N}.$$

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses.

8. Using Eq. 14-7, we find the gauge pressure to be $p_{\text{gauge}} = \rho g h$, where ρ is the density of the fluid medium, and h is the vertical distance to the point where the pressure is equal to the atmospheric pressure.

The gauge pressure at a depth of 20 m in seawater is

$$p_1 = \rho_{\text{sw}} g d = (1024 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(20 \text{ m}) = 2.00 \times 10^5 \text{ Pa}.$$

On the other hand, the gauge pressure at an altitude of 7.6 km is

$$p_2 = \rho_{\text{air}} g h = (0.87 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(7600 \text{ m}) = 6.48 \times 10^4 \text{ Pa}.$$

Therefore, the change in pressure is

$$\Delta p = p_1 - p_2 = 2.00 \times 10^5 \text{ Pa} - 6.48 \times 10^4 \text{ Pa} \approx 1.4 \times 10^5 \text{ Pa}.$$

9. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the heart of the *Argentinosaurus* is

$$p_{\text{heart}} = p_{\text{brain}} + \rho g h = 80 \text{ torr} + (1.06 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(21 \text{ m} - 9.0 \text{ m})\left(\frac{1 \text{ torr}}{133.33 \text{ Pa}}\right)$$
$$= 1.0 \times 10^3 \text{ torr}.$$

(b) The gauge pressure at the feet of the *Argentinosaurus* is

$$p_{\text{feet}} = p_{\text{brain}} + \rho g h' = 80 \text{ torr} + (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(21 \text{ m})\left(\frac{1 \text{ torr}}{133.33 \text{ Pa}}\right)$$
$$= 80 \text{ torr} + 1642 \text{ torr} = 1722 \text{ torr} \approx 1.7\times10^3 \text{ torr}.$$

10. With A = 0.000500 m^2 and $F = pA$ (with p given by Eq. 14-9), then we have ρghA = 9.80 N. This gives $h \approx 2.0$ m, which means $d + h = 2.80$ m.

11. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the brain of the giraffe is

$$p_{\text{brain}} = p_{\text{heart}} - \rho g h = 250 \text{ torr} - (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2.0 \text{ m})\left(\frac{1 \text{ torr}}{133.33 \text{ Pa}}\right)$$
$$= 94 \text{ torr}.$$

(b) The gauge pressure at the feet of the giraffe is

$$p_{\text{feet}} = p_{\text{heart}} + \rho g h = 250 \text{ torr} + (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2.0 \text{ m})\left(\frac{1 \text{ torr}}{133.33 \text{ Pa}}\right) = 406 \text{ torr}$$
$$\approx 4.1\times10^2 \text{ torr}.$$

(c) The increase in the blood pressure at the brain as the giraffe lowers its head to the level of its feet is

$$\Delta p = p_{\text{feet}} - p_{\text{brain}} = 406 \text{ torr} - 94 \text{ torr} = 312 \text{ torr} \approx 3.1\times10^2 \text{ torr}.$$

12. Note that 0.05 atm equals 5065 Pa. Application of Eq. 14-7 with the notation in this problem leads to

$$d_{\max} = \frac{p}{\rho_{\text{liquid}} g} = \frac{0.05 \text{ atm}}{\rho_{\text{liquid}} g} = \frac{5065 \text{ Pa}}{\rho_{\text{liquid}} g}.$$

Thus the difference of this quantity between fresh water (998 kg/m^3) and Dead Sea water (1500 kg/m^3) is

$$\Delta d_{\max} = \frac{5065 \text{ Pa}}{g}\left(\frac{1}{\rho_{\text{fw}}} - \frac{1}{\rho_{\text{sw}}}\right) = \frac{5065 \text{ Pa}}{9.8 \text{ m/s}^2}\left(\frac{1}{998 \text{ kg/m}^3} - \frac{1}{1500 \text{ kg/m}^3}\right) = 0.17 \text{ m}.$$

13. Recalling that 1 atm = 1.01 × 10^5 Pa, Eq. 14-8 leads to

$$\rho gh = (1024 \text{ kg/m}^3)\ (9.80 \text{ m/s}^2)\ (10.9 \times 10^3 \text{ m}) \left(\frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} \right) \approx 1.08 \times 10^3 \text{ atm}.$$

14. We estimate the pressure difference (specifically due to hydrostatic effects) as follows:

$$\Delta p = \rho gh = (1.06 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.83 \text{ m}) = 1.90 \times 10^4 \text{ Pa}.$$

15. In this case, Bernoulli's equation reduces to Eq. 14-10. Thus,

$$p_g = \rho g(-h) = -(1800 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.5 \text{ m}) = -2.6 \times 10^4 \text{ Pa}.$$

16. At a depth h without the snorkel tube, the external pressure on the diver is $p = p_0 + \rho gh$, where p_0 is the atmospheric pressure. Thus, with a snorkel tube of length h, the pressure difference between the internal air pressure and the water pressure against the body is

$$\Delta p = p = p_0 = \rho gh.$$

(a) If $h = 0.20$ m, then

$$\Delta p = \rho gh = (998 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.20 \text{ m}) \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} = 0.019 \text{ atm}.$$

(b) Similarly, if $h = 4.0$ m, then

$$\Delta p = \rho gh = (998 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(4.0 \text{ m}) \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} \approx 0.39 \text{ atm}.$$

17. **THINK** The minimum force that must be applied to open the hatch is equal to the gauge pressure times the area of the hatch.

EXPRESS The pressure p at the depth d of the hatch cover is $p_0 + \rho gd$, where ρ is the density of ocean water and p_0 is atmospheric pressure. Thus, the gauge pressure is $p_{\text{gauge}} = \rho gd$, and the minimum force that must be applied by the crew to open the hatch has magnitude $F = p_{\text{gauge}} A = (\rho gd)A$, where A is the area of the hatch.

Substituting the values given, we find the force to be

$$\begin{aligned} F &= p_{\text{gauge}} A = (\rho gd)A = (1024 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(100 \text{ m})(1.2 \text{ m})(0.60 \text{ m}) \\ &= 7.2 \times 10^5 \text{ N}. \end{aligned}$$

LEARN The downward force of the water on the hatch cover is $(p_0 + \rho gd)A$, and the air in the submarine exerts an upward force of $p_0 A$. The greater the depth of the submarine, the greater the force required to open the hatch.

18. Since the pressure (caused by liquid) at the bottom of the barrel is doubled due to the presence of the narrow tube, so is the hydrostatic force. The ratio is therefore equal to 2.0. The difference between the hydrostatic force and the weight is accounted for by the additional upward force exerted by water on the top of the barrel due to the increased pressure introduced by the water in the tube.

19. We can integrate the pressure (which varies linearly with depth according to Eq. 14-7) over the area of the wall to find out the net force on it, and the result turns out fairly intuitive (because of that linear dependence): the force is the "average" water pressure multiplied by the area of the wall (or at least the part of the wall that is exposed to the water), where "average" pressure is taken to mean $\frac{1}{2}$(pressure at surface + pressure at bottom). Assuming the pressure at the surface can be taken to be zero (in the gauge pressure sense explained in section 14-4), then this means the force on the wall is $\frac{1}{2}\rho gh$ multiplied by the appropriate area. In this problem the area is hw (where w is the 8.00 m width), so the force is $\frac{1}{2}\rho gh^2w$, and the change in force (as h is changed) is

$$\frac{1}{2}\rho gw\,(h_f^{\,2}-h_i^{\,2}) = \frac{1}{2}(998\text{ kg/m}^3)(9.80\text{ m/s}^2)(8.00\text{ m})(4.00^2-2.00^2)\text{m}^2 = 4.69\times10^5\text{ N}.$$

20. (a) The force on face A of area A_A due to the water pressure alone is

$$F_A = p_A A_A = \rho_w g h_A A_A = \rho_w g(2d)d^2 = 2\left(1.0\times10^3\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)\left(5.0\text{ m}\right)^3 = 2.5\times10^6\text{ N}.$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0\times10^5\text{ Pa})(5.0\text{ m})^2 = 2.5\times10^6\text{ N},$$

we have

$$F_A' = F_0 + F_A = 2.5\times10^6\text{ N} + 2.5\times10^6\text{ N} = 5.0\times10^6\text{ N}.$$

(b) The force on face B due to water pressure alone is

$$F_B = p_{\text{avg}B}A_B = \rho_\omega g\left(\frac{5d}{2}\right)d^2 = \frac{5}{2}\rho_w g d^3 = \frac{5}{2}\left(1.0\times10^3\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)\left(5.0\text{ m}\right)^3 = 3.1\times10^6\text{ N}.$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0\times10^5\text{ Pa})(5.0\text{ m})^2 = 2.5\times10^6\text{ N},$$

we obtain

$$F_B' = F_0 + F_B = 2.5\times10^6\text{ N} + 3.1\times10^6\text{ N} = 5.6\times10^6\text{ N}.$$

21. **THINK** Work is done to remove liquid from one vessel to another.

EXPRESS When the levels are the same, the height of the liquid is $h = (h_1 + h_2)/2$, where h_1 and h_2 are the original heights. Suppose h_1 is greater than h_2. The final situation can then be achieved by taking liquid from the first vessel with volume $V = A(h_1 - h)$ and mass $m = \rho V = \rho A(h_1 - h)$, and lowering it a distance $\Delta y = h - h_2$. The work done by the force of gravity is

$$W_g = mg\Delta y = \rho A(h_1 - h)g(h - h_2).$$

ANALYZE We substitute $h = (h_1 + h_2)/2$ to obtain

$$W_g = \frac{1}{4}\rho g A\left(h_1 - h_2\right)^2 = \frac{1}{4}(1.30\times10^3\,\text{kg/m}^3)(9.80\text{ m/s}^2)(4.00\times10^{-4}\,\text{m}^2)(1.56\text{ m} - 0.854\text{ m})^2$$
$$= 0.635\text{ J}$$

LEARN Since gravitational force is conservative, the work done only depends on the initial and final heights of the vessels, and not on how the liquid is transferred.

22. To find the pressure at the brain of the pilot, we note that the inward acceleration can be treated from the pilot's reference frame as though it is an outward gravitational acceleration against which the heart must push the blood. Thus, with $a = 4g$, we have

$$p_{\text{brain}} = p_{\text{heart}} - \rho a r = 120\text{ torr} - (1.06\times10^3\text{ kg/m}^3)(4\times9.8\text{ m/s}^2)(0.30\text{ m})\left(\frac{1\text{ torr}}{133\text{ Pa}}\right)$$
$$= 120\text{ torr} - 94\text{ torr} = 26\text{ torr}.$$

23. Letting $p_a = p_b$, we find

$$\rho_c g(6.0\text{ km} + 32\text{ km} + D) + \rho_m(y - D) = \rho_c g(32\text{ km}) + \rho_m y$$

and obtain

$$D = \frac{(6.0\,\text{km})\rho_c}{\rho_m - \rho_c} = \frac{(6.0\,\text{km})\left(2.9\,\text{g/cm}^3\right)}{3.3\,\text{g/cm}^3 - 2.9\,\text{g/cm}^3} = 44\,\text{km}.$$

24. (a) At depth y the gauge pressure of the water is $p = \rho g y$, where ρ is the density of the water. We consider a horizontal strip of width W at depth y, with (vertical) thickness dy, across the dam. Its area is $dA = W\,dy$ and the force it exerts on the dam is $dF = p\,dA = \rho g y W\,dy$. The total force of the water on the dam is

$$F = \int_0^D \rho g y W\,dy = \frac{1}{2}\rho g W D^2 = \frac{1}{2}\left(1.00\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(314\,\text{m}\right)\left(35.0\,\text{m}\right)^2$$
$$= 1.88\times10^9\,\text{N}.$$

(b) Again we consider the strip of water at depth y. Its moment arm for the torque it exerts about O is $D - y$ so the torque it exerts is

$$d\tau = dF(D-y) = \rho g y W(D-y)dy$$

and the total torque of the water is

$$\tau = \int_0^D \rho g y W(D-y)dy = \rho g W\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \frac{1}{6}\rho g W D^3$$
$$= \frac{1}{6}\left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(314\ \text{m}\right)\left(35.0\ \text{m}\right)^3 = 2.20\times10^{10}\ \text{N}\cdot\text{m}.$$

(c) We write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho g W D^3}{\frac{1}{2}\rho g W D^2} = \frac{D}{3} = \frac{35.0\ \text{m}}{3} = 11.7\ \text{m}.$$

25. As shown in Eq. 14-9, the atmospheric pressure p_0 bearing down on the barometer's mercury pool is equal to the pressure $\rho g h$ at the base of the mercury column: $p_0 = \rho g h$. Substituting the values given in the problem statement, we find the atmospheric pressure to be

$$p_0 = \rho g h = (1.3608\times10^4\ \text{kg/m}^3)(9.7835\ \text{m/s}^2)(0.74035\ \text{m})\left(\frac{1\ \text{torr}}{133.33\ \text{Pa}}\right)$$
$$= 739.26\ \text{torr}.$$

26. The gauge pressure you can produce is

$$p = -\rho g h = -\frac{\left(1000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)\left(4.0\times10^{-2}\ \text{m}\right)}{1.01\times10^5\ \text{Pa/atm}} = -3.9\times10^{-3}\ \text{atm}$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure.

27. **THINK** The atmospheric pressure at a given height depends on the density distribution of air.

EXPRESS If the air density were uniform, ρ =const., then the variation of pressure with height may be written as: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. On the other hand, if the density varies with altitude, then

$$p_2 = p_1 - \int_0^h \rho g\ dy\ .$$

For the case where the density decreases linearly with height, $\rho = \rho_0\,(1 - y/h)$, where ρ_0 is the density at Earth's surface and $g = 9.8$ m/s^2 for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g\left(1 - \frac{y}{h}\right)dy = p_1 - \frac{1}{2}\rho_0 gh.$$

ANALYZE (a) For uniform density with $\rho = 1.3$ kg/m^3, we find the height of the atmosphere to be

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01\times10^5\ \text{Pa}}{(1.3\ \text{kg/m}^3)(9.8\,\text{m/s}^2)} = 7.9\times10^3\ \text{m} = 7.9\ \text{km}.$$

(b) With density decreasing linearly with height, $p_2 = p_1 - \rho_0 gh/2$. The condition $p_2 = 0$ implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01\times10^5\ \text{Pa})}{(1.3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 16\times10^3\ \text{m} = 16\ \text{km}.$$

LEARN Actually the decrease in air density is approximately exponential, with pressure halved at a height of about 5.6 km.

28. (a) According to Pascal's principle, $F/A = f/a \rightarrow F = (A/a)f$.

(b) We obtain

$$f = \frac{a}{A}F = \frac{(3.80\ \text{cm})^2}{(53.0\ \text{cm})^2}(20.0\times10^3\ \text{N}) = 103\ \text{N}.$$

The ratio of the squares of diameters is equivalent to the ratio of the areas. We also note that the area units cancel.

29. Equation 14-13 combined with Eq. 5-8 and Eq. 7-21 (in absolute value) gives

$$mg = kx\frac{A_1}{A_2}.$$

With $A_2 = 18A_1$ (and the other values given in the problem) we find $m = 8.50$ kg.

30. Taking "down" as the positive direction, then using Eq. 14-16 in Newton's second law, we have $(5.00\ \text{kg})g - (3.00\ \text{kg})g = 5a$. This gives $a = \frac{2}{5}g = 3.92$ m/s^2, where $g = 9.8$ m/s^2. Then (see Eq. 2-15) $\frac{1}{2}at^2 = 0.0784$ m (in the downward direction).

31. **THINK** The block floats in both water and oil. We apply Archimedes' principle to analyze the problem.

EXPRESS Let V be the volume of the block. Then, the submerged volume in water is $V_s = 2V/3$. Since the block is floating, by Archimedes' principle the weight of the displaced water is equal to the weight of the block, i.e., $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block.

ANALYZE (a) We substitute $V_s = 2V/3$ to obtain the density of the block:

$$\rho_b = 2\rho_w/3 = 2(1000 \text{ kg/m}^3)/3 \approx 6.7 \times 10^2 \text{ kg/m}^3.$$

(b) Now, if ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s' = \rho_b V$. Since the volume submerged in oil is $V_s' = 0.90V$, the density of the oil is

$$\rho_o = \rho_b\left(\frac{V}{V'}\right) = (6.7\times 10^2 \text{ kg/m}^3)\frac{V}{0.90V} = 7.4\times 10^2 \text{ kg/m}^3.$$

LEARN Another way to calculate the density of the oil is to note that the mass of the block can be written as

$$m = \rho_b V = \rho_o V_s' = \rho_w V_s.$$

Therefore,

$$\rho_o = \rho_w\left(\frac{V_s}{V_s'}\right) = (1000 \text{ kg/m}^3)\frac{2V/3}{0.90V} = 7.4\times 10^2 \text{ kg/m}^3.$$

That is, by comparing the fraction submerged with that in water (or another liquid with known density), the density of the oil can be deduced.

32. (a) The pressure (including the contribution from the atmosphere) at a depth of $h_{\text{top}} = L/2$ (corresponding to the top of the block) is

$$p_{\text{top}} = p_{\text{atm}} + \rho g h_{\text{top}} = 1.01\times 10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.300 \text{ m}) = 1.04\times 10^5 \text{ Pa}$$

where the unit Pa (pascal) is equivalent to N/m^2. The force on the top surface (of area $A = L^2 = 0.36$ m^2) is

$$F_{\text{top}} = p_{\text{top}} A = 3.75 \times 10^4 \text{ N}.$$

(b) The pressure at a depth of $h_{\text{bot}} = 3L/2$ (that of the bottom of the block) is

$$p_{\text{bot}} = p_{\text{atm}} + \rho g h_{\text{bot}} = 1.01\times 10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.900 \text{ m})$$
$$= 1.10\times 10^5 \text{ Pa}$$

where we recall that the unit Pa (pascal) is equivalent to N/m^2. The force on the bottom surface is

$$F_{\text{bot}} = p_{\text{bot}} A = 3.96 \times 10^4 \text{ N}.$$

(c) Taking the difference $F_{bot} - F_{top}$ cancels the contribution from the atmosphere (including any numerical uncertainties associated with that value) and leads to

$$F_{bot} - F_{top} = \rho g\,(h_{bot} - h_{top})\; A \;=\rho g L^3 = 2.18\times10^3\ \text{N}$$

which is to be expected on the basis of Archimedes' principle. Two other forces act on the block: an upward tension T and a downward pull of gravity mg. To remain stationary, the tension must be

$$T = mg - (F_{bot} - F_{top}) = (450\ \text{kg})(9.80\ \text{m/s}^2) - 2.18\times10^3\ \text{N} = 2.23\times10^3\ \text{N}.$$

(d) This has already been noted in the previous part: $F_b = 2.18\times10^3\ \text{N}$, and $T + F_b = mg$.

33. **THINK** The iron anchor is submerged in water, so we apply Archimedes' principle to calculate its volume and weight in air.

EXPRESS The anchor is completely submerged in water of density ρ_w. Its apparent weight is $W_{app} = W - F_b$, where $W= mg$ is its actual weight and $F_b = \rho_w gV$ is the buoyant force.

ANALYZE (a) Substituting the values given, we find the volume of the anchor to be

$$V = \frac{W - W_{app}}{\rho_w g} = \frac{F_b}{\rho_w g} = \frac{200\ \text{N}}{\left(1000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)} = 2.04\times10^{-2}\ \text{m}^3.$$

(b) The mass of the anchor is $m = \rho_{Fe} g$, where ρ_{Fe} is the density of iron (found in Table 14-1). Therefore, its weight in air is

$$W = mg = \rho_{Fe} Vg = (7870\ \text{kg/m}^3)(2.04\times10^{-2}\ \text{m}^3)(9.80\ \text{m/s}^2) = 1.57\times10^3\ \text{N}.$$

LEARN In general, the apparent weight of an object of density ρ that is completely submerged in a fluid of density ρ_f can be written as $W_{app} = (\rho - \rho_f)Vg$.

34. (a) Archimedes' principle makes it clear that a body, in order to float, displaces an amount of the liquid that corresponds to the weight of the body. The problem (indirectly) tells us that the weight of the boat is W = 35.6 kN. In salt water of density ρ' = 1100 kg/m^3, it must displace an amount of liquid having weight equal to 35.6 kN.

(b) The displaced volume of salt water is equal to

$$V' = \frac{W}{\rho' g} = \frac{3.56\times10^3\,\text{N}}{(1.10\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 3.30\ \text{m}^3.$$

In freshwater, it displaces a volume of $V = W/\rho g = 3.63\ \text{m}^3$, where $\rho = 1000\ \text{kg/m}^3$. The difference is $V - V' = 0.330\ \text{m}^3$.

35. The problem intends for the children to be completely above water. The total downward pull of gravity on the system is

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV$$

where N is the (minimum) number of logs needed to keep them afloat and V is the volume of each log:

$$V = \pi(0.15\ \text{m})^2\,(1.80\ \text{m}) = 0.13\ \text{m}^3.$$

The buoyant force is $F_b = \rho_{\text{water}}gV_{\text{submerged}}$, where we require $V_{\text{submerged}} \le NV$. The density of water is 1000 kg/m^3. To obtain the minimum value of N, we set $V_{\text{submerged}} = NV$ and then round our "answer" for N up to the nearest integer:

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV = \rho_{\text{water}}gNV \;\Rightarrow\; N = \frac{3(356\,\text{N})}{gV(\rho_{\text{water}} - \rho_{\text{wood}})}$$

which yields $N = 4.28 \to 5$ logs.

36. From the "kink" in the graph it is clear that $d = 1.5$ cm. Also, the $h = 0$ point makes it clear that the (true) weight is 0.25 N. We now use Eq. 14-19 at $h = d = 1.5$ cm to obtain

$$F_b = (0.25\ \text{N} - 0.10\ \text{N}) = 0.15\ \text{N}.$$

Thus, $\rho_{\text{liquid}}\,g\,V = 0.15$, where

$$V = (1.5\ \text{cm})(5.67\ \text{cm}^2) = 8.5 \times 10^{-6}\ \text{m}^3.$$

Thus, $\rho_{\text{liquid}} = 1800\ \text{kg/m}^3 = 1.8\ \text{g/cm}^3$.

37. For our estimate of $V_{\text{submerged}}$ we interpret "almost completely submerged" to mean

$$V_{\text{submerged}} \approx \frac{4}{3}\pi r_o^3 \quad \text{where } r_o = 60\ \text{cm}.$$

Thus, equilibrium of forces (on the iron sphere) leads to

$$F_b = m_{\text{iron}}g \;\Rightarrow\; \rho_{\text{water}}gV_{\text{submerged}} = \rho_{\text{iron}}g\left(\frac{4}{3}\pi r_o^3 - \frac{4}{3}\pi r_i^3\right)$$

where r_i is the inner radius (half the inner diameter). Plugging in our estimate for $V_{\text{submerged}}$ as well as the densities of water (1.0 g/cm^3) and iron (7.87 g/cm^3), we obtain the inner diameter:

$$2r_i = 2r_o\left(1 - \frac{1.0\ \text{g/cm}^3}{7.87\ \text{g/cm}^3}\right)^{1/3} = 57.3\ \text{cm}.$$

38. (a) An object of the same density as the surrounding liquid (in which case the "object" could just be a packet of the liquid itself) is not going to accelerate up or down (and thus won't gain any kinetic energy). Thus, the point corresponding to zero K in the graph must correspond to the case where the density of the object equals ρ_{liquid}. Therefore, $\rho_{\text{ball}} = 1.5$ g/cm^3 (or 1500 kg/m^3).

(b) Consider the $\rho_{\text{liquid}} = 0$ point (where $K_{\text{gained}} = 1.6$ J). In this case, the ball is falling through perfect vacuum, so that $v^2 = 2gh$ (see Eq. 2-16) which means that $K = \frac{1}{2}mv^2 = 1.6$ J can be used to solve for the mass. We obtain $m_{\text{ball}} = 4.082$ kg. The volume of the ball is then given by

$$m_{\text{ball}}/\rho_{\text{ball}} = 2.72 \times 10^{-3}\ \text{m}^3.$$

39. **THINK** The hollow sphere is half submerged in a fluid. We apply Archimedes' principle to calculate its mass and density.

EXPRESS The downward force of gravity mg is balanced by the upward buoyant force of the liquid: $mg = \rho g V_s$. Here m is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3)r_o^3$, where r_o is the outer radius.

ANALYZE (a) Substituting the values given, we find the mass of the sphere to be

$$m = \rho V_s = \rho\left(\frac{1}{2}\cdot\frac{4\pi}{3}r_o^3\right) = \frac{2\pi}{3}\rho r_o^3 = \left(\frac{2\pi}{3}\right)(800\ \text{kg/m}^3)(0.090\ \text{m})^3 = 1.22\ \text{kg}.$$

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where m is the mass of the sphere and V is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3}(r_o^3 - r_i^3) = \frac{4\pi}{3}\left((0.090\ \text{m})^3 - (0.080\ \text{m})^3\right) = 9.09\times 10^{-4}\ \text{m}^3.$$

The density is

$$\rho_m = \frac{1.22\ \text{kg}}{9.09\times 10^{-4}\ \text{m}^3} = 1.3\times 10^3\ \text{kg/m}^3.$$

LEARN Note that $\rho_m > \rho$, i.e., the density of the material is greater that of the fluid. However, the sphere floats (and displaces its own weight of fluid) because it's hollow.

40. If the alligator floats, by Archimedes' principle the buoyancy force is equal to the alligator's weight (see Eq. 14-17). Therefore,

$$F_b = F_g = m_{H_2O}g = (\rho_{H_2O}Ah)g.$$

If the mass is to increase by a small amount $m \to m' = m + \Delta m$, then

$$F_b \to F_b' = \rho_{H_2O}A(h+\Delta h)g.$$

With $\Delta F_b = F_b' - F_b = 0.010mg$, the alligator sinks by

$$\Delta h = \frac{\Delta F_b}{\rho_{H_2O}Ag} = \frac{0.01mg}{\rho_{H_2O}Ag} = \frac{0.010(130\text{ kg})}{(998\text{ kg/m}^3)(0.20\text{ m}^2)} = 6.5\times10^{-3}\text{ m} = 6.5\text{ mm}.$$

41. Let V_i be the total volume of the iceberg. The non-visible portion is below water, and thus the volume of this portion is equal to the volume V_f of the fluid displaced by the iceberg. The fraction of the iceberg that is visible is

$$\text{frac} = \frac{V_i - V_f}{V_i} = 1 - \frac{V_f}{V_i}.$$

Since iceberg is floating, Eq. 14-18 applies:

$$F_g = m_i g = m_f g \quad \Rightarrow \quad m_i = m_f.$$

Since $m = \rho V$, the above equation implies

$$\rho_i V_i = \rho_f V_f \quad \Rightarrow \quad \frac{V_f}{V_i} = \frac{\rho_i}{\rho_f}.$$

Thus, the visible fraction is

$$\text{frac} = 1 - \frac{V_f}{V_i} = 1 - \frac{\rho_i}{\rho_f}.$$

(a) If the iceberg ($\rho_i = 917\text{ kg/m}^3$) floats in salt water with $\rho_f = 1024\text{ kg/m}^3$, then the fraction would be

$$\text{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\text{ kg/m}^3}{1024\text{ kg/m}^3} = 0.10 = 10\%.$$

(b) On the other hand, if the iceberg floats in fresh water ($\rho_f = 1000\text{ kg/m}^3$), then the fraction would be

$$\text{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\text{ kg/m}^3}{1000\text{ kg/m}^3} = 0.083 = 8.3\%.$$

42. Work is the integral of the force over distance (see Eq. 7-32). Referring to the equation immediately preceding Eq. 14-7, we see the work can be written as

$$W = \int \rho_{\text{water}}\, gA(-y)\, dy$$

where we are using $y = 0$ to refer to the water surface (and the $+y$ direction is upward). Let $h = 0.500$ m. Then, the integral has a lower limit of $-h$ and an upper limit of y_f, with

$$y_f/h = -\rho_{\text{cylinder}}/\rho_{\text{water}} = -0.400.$$

The integral leads to

$$W = \tfrac{1}{2}\rho_{\text{water}} gAh^2(1 - 0.4^2) = 4.11 \text{ kJ}.$$

43. (a) When the model is suspended (in air) the reading is F_g (its true weight, neglecting any buoyant effects caused by the air). When the model is submerged in water, the reading is lessened because of the buoyant force: $F_g - F_b$. We denote the difference in readings as Δm. Thus,

$$F_g - (F_g - F_b) = \Delta mg$$

which leads to $F_b = \Delta mg$. Since $F_b = \rho_w g V_m$ (the weight of water displaced by the model) we obtain

$$V_m = \frac{\Delta m}{\rho_w} = \frac{0.63776\,\text{kg}}{1000 \text{ kg/m}} \approx 6.378 \times 10^{-4} \text{ m}^3.$$

(b) The $\frac{1}{20}$ scaling factor is discussed in the problem (and for purposes of significant figures is treated as exact). The actual volume of the dinosaur is

$$V_{\text{dino}} = 20^3\, V_m = 5.102 \text{ m}^3.$$

(c) Using $\rho = \dfrac{m_{\text{dino}}}{V_{\text{dino}}} \approx \rho_w = 1000 \text{ kg/m}^3$, we find the mass of the *T. rex* to be

$$m_{\text{dino}} \approx \rho_w V_{\text{dino}} = (1000 \text{ kg/m}^3)\,(5.102 \text{ m}^3) = 5.102 \times 10^3 \text{ kg}.$$

44. (a) Since the lead is not displacing any water (of density ρ_w), the lead's volume is not contributing to the buoyant force F_b. If the immersed volume of wood is V_i, then

$$F_b = \rho_w V_i g = 0.900\,\rho_w V_{\text{wood}} g = 0.900\,\rho_w g \left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right),$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.900\,\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) = (m_{\text{wood}} + m_{\text{lead}})g\,.$$

Thus,

$$m_{\text{lead}} = 0.900\rho_w\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) - m_{\text{wood}} = \frac{(0.900)(1000\,\text{kg/m}^3)(3.67\,\text{kg})}{600\ \text{kg/m}^3} - 3.67\ \text{kg}$$
$$= 1.84\ \text{kg}.$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to F_b. Consequently,

$$F_b = 0.900\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) + \left(\frac{\rho_w}{\rho_{\text{lead}}}\right) m_{\text{lead}} g = (m_{\text{wood}} + m_{\text{lead}})g,$$

which leads to

$$m_{\text{lead}} = \frac{0.900(\rho_w/\rho_{\text{wood}})m_{\text{wood}} - m_{\text{wood}}}{1 - \rho_w/\rho_{\text{lead}}} = \frac{1.84\ \text{kg}}{1 - (1.00\times10^3\ \text{kg/m}^3/1.13\times10^4\ \text{kg/m}^3)}$$
$$= 2.01\ \text{kg}.$$

45. The volume V_{cav} of the cavities is the difference between the volume V_{cast} of the casting as a whole and the volume V_{iron} contained: $V_{\text{cav}} = V_{\text{cast}} - V_{\text{iron}}$. The volume of the iron is given by $V_{\text{iron}} = W/g\rho_{\text{iron}}$, where W is the weight of the casting and ρ_{iron} is the density of iron. The effective weight in water (of density ρ_w) is $W_{\text{eff}} = W - g\rho_w V_{\text{cast}}$. Thus, $V_{\text{cast}} = (W - W_{\text{eff}})/g\rho_w$ and

$$V_{\text{cav}} = \frac{W - W_{\text{eff}}}{g\rho_w} - \frac{W}{g\rho_{\text{iron}}} = \frac{6000\ \text{N} - 4000\ \text{N}}{(9.8\ \text{m/s}^2)(1000\,\text{kg/m}^3)} - \frac{6000\ \text{N}}{(9.8\ \text{m/s}^2)(7.87\times10^3\ \text{kg/m}^3)}$$
$$= 0.126\ \text{m}^3\ .$$

46. Due to the buoyant force, the ball accelerates upward (while in the water) at rate a given by Newton's second law: $\rho_{\text{water}}Vg - \rho_{\text{ball}}Vg = \rho_{\text{ball}}Va$, which yields

$$\rho_{\text{water}} = \rho_{\text{ball}}(1 + a/g)\,.$$

With $\rho_{\text{ball}} = 0.300\ \rho_{\text{water}}$, we find that

$$a = g\left(\frac{\rho_{\text{water}}}{\rho_{\text{ball}}} - 1\right) = (9.80\ \text{m/s}^2)\left(\frac{1}{0.300} - 1\right) = 22.9\ \text{m/s}^2\,.$$

Using Eq. 2-16 with $\Delta y = 0.600$ m, the speed of the ball as it emerges from the water is

$$v = \sqrt{2a\Delta y} = \sqrt{2(22.9\ \text{m/s}^2)(0.600\ \text{m})} = 5.24\ \text{m/s}\,.$$

This causes the ball to reach a maximum height h_{max} (measured above the water surface) given by $h_{max} = v^2/2g$ (see Eq. 2-16 again). Thus,

$$h_{max} = \frac{v^2}{2g} = \frac{(5.24\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 1.40\text{ m}.$$

47. (a) If the volume of the car below water is V_1 then $F_b = \rho_w V_1 g = W_{car}$, which leads to

$$V_1 = \frac{W_{car}}{\rho_w g} = \frac{(1800\text{ kg})(9.8\text{ m/s}^2)}{(1000\text{ kg/m}^3)(9.8\text{ m/s}^2)} = 1.80\text{ m}^3.$$

(b) We denote the total volume of the car as V and that of the water in it as V_2. Then

$$F_b = \rho_w V g = W_{car} + \rho_w V_2 g$$

which gives

$$V_2 = V - \frac{W_{car}}{\rho_w g} = \left(0.750\text{ m}^3 + 5.00\text{ m}^3 + 0.800\text{ m}^3\right) - \frac{1800\text{ kg}}{1000\text{ kg/m}^3} = 4.75\text{ m}^3.$$

48. Let ρ be the density of the cylinder (0.30 g/cm^3 or 300 kg/m^3) and ρ_{Fe} be the density of the iron (7.9 g/cm^3 or 7900 kg/m^3). The volume of the cylinder is

$$V_c = (6\times 12)\text{ cm}^3 = 72\text{ cm}^3 = 0.000072\text{ m}^3,$$

and that of the ball is denoted V_b. The part of the cylinder that is submerged has volume

$$V_s = (4\times 12)\text{ cm}^3 = 48\text{ cm}^3 = 0.000048\text{ m}^3.$$

Using the ideas of section 14-7, we write the equilibrium of forces as

$$\rho g V_c + \rho_{Fe} g V_b = \rho_w g V_s + \rho_w g V_b \quad \Rightarrow \quad V_b = 3.8\text{ cm}^3$$

where we have used ρ_w = 998 kg/m^3 (for water, see Table 14-1). Using $V_b = \frac{4}{3}\pi r^3$ we find r = 9.7 mm.

49. This problem involves use of continuity equation (Eq. 14-23): $A_1 v_1 = A_2 v_2$.

(a) Initially the flow speed is $v_i = 1.5$ m/s and the cross-sectional area is $A_i = HD$. At point a, as can be seen from the figure, the cross-sectional area is

$$A_a = (H-h)D - (b-h)d.$$

Thus, by continuity equation, the speed at point a is

$$v_a = \frac{A_i v_i}{A_a} = \frac{HDv_i}{(H-h)D-(b-h)d} = \frac{(14\text{ m})(55\text{ m})(1.5\text{ m/s})}{(14\text{ m}-0.80\text{ m})(55\text{ m})-(12\text{ m}-0.80\text{ m})(30\text{ m})}$$
$$= 2.96\text{ m/s} \approx 3.0\text{ m/s}.$$

(b) Similarly, at point b, the cross-sectional area is $A_b = HD - bd$, and therefore, by continuity equation, the speed at point b is

$$v_b = \frac{A_i v_i}{A_b} = \frac{HDv_i}{HD-bd} = \frac{(14\text{ m})(55\text{ m})(1.5\text{ m/s})}{(14\text{ m})(55\text{ m})-(12\text{ m})(30\text{ m})} = 2.8\text{ m/s}.$$

50. The left and right sections have a total length of 60.0 m, so (with a speed of 2.50 m/s) it takes 60.0/2.50 = 24.0 seconds to travel through those sections. Thus it takes (88.8 – 24.0) s = 64.8 s to travel through the middle section. This implies that the speed in the middle section is

$$v_{\text{mid}} = (50\text{ m})/(64.8\text{ s}) = 0.772\text{ m/s}.$$

Now Eq. 14-23 (plus that fact that $A = \pi r^2$) implies $r_{\text{mid}} = r_A\sqrt{(2.5\text{ m/s})/(0.772\text{ m/s})}$ where $r_A = 2.00$ cm. Therefore, $r_{\text{mid}} = 3.60\text{ cm}$.

51. **THINK** We use the equation of continuity to solve for the speed of water as it leaves the sprinkler hole.

EXPRESS Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. The cross-sectional area of the hose is $A_1 = \pi R^2$. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2\left(NA_2\right) \quad\Rightarrow\quad v_2 = \frac{A_1}{NA_2}v_1 = \frac{R^2}{Nr^2}v_1$$

where R is the radius of the hose and r is the radius of a hole.

ANALYZE Noting that $R/r = D/d$ (the ratio of diameters) we find the speed to be

$$v_2 = \frac{D^2}{Nd^2}v_1 = \frac{\left(1.9\text{ cm}\right)^2}{24\left(0.13\text{ cm}\right)^2}\left(0.91\text{ m/s}\right) = 8.1\text{ m/s}.$$

LEARN The equation of continuity implies that the smaller the cross-sectional area of the sprinkler hole, the greater the speed of water as it emerges from the hole.

52. We use the equation of continuity and denote the depth of the river as h. Then,

$$(8.2\,\text{m})(3.4\,\text{m})(2.3\,\text{m/s})+(6.8\,\text{m})(3.2\,\text{m})(2.6\,\text{m/s})=h(10.5\,\text{m})(2.9\,\text{m/s})$$

which leads to h = 4.0 m.

53. **THINK** The power of the pump is the rate of work done in lifting the water.

EXPRESS Suppose that a mass Δm of water is pumped in time Δt. The pump increases the potential energy of the water by $\Delta U = (\Delta m)gh$, where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\Delta K = \frac{1}{2}(\Delta m)v^2$, where v is its final speed. The work it does is

$$\Delta W = \Delta U + \Delta K = (\Delta m)gh + \frac{1}{2}(\Delta m)v^2$$

and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right).$$

The rate of mass flow is $\Delta m/\Delta t = \rho_w A v$, where ρ_w is the density of water and A is the area of the hose.

ANALYZE The area of the hose is $A = \pi r^2 = \pi(0.010\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$ and

$$\rho_w A v = (1000\text{ kg/m}^3)(3.14 \times 10^{-4}\text{ m}^2)(5.00\text{ m/s}) = 1.57\text{ kg/s}.$$

Thus, the power of the pump is

$$P = \rho A v\left(gh + \frac{1}{2}v^2\right) = (1.57\,\text{kg/s})\left((9.8\,\text{m/s}^2)(3.0\,\text{m}) + \frac{(5.0\,\text{m/s})^2}{2}\right) = 66\,\text{W}.$$

LEARN The work done by the pump is converted into both the potential energy and kinetic energy of the water.

54. (a) The equation of continuity provides (26 + 19 + 11) L/min = 56 L/min for the flow rate in the main (1.9 cm diameter) pipe.

(b) Using $v = R/A$ and $A = \pi d^2/4$, we set up ratios:

$$\frac{v_{56}}{v_{26}} = \frac{56/\pi(1.9)^2/4}{26/\pi(1.3)^2/4} \approx 1.0.$$

55. We rewrite the formula for work W (when the force is constant in a direction parallel to the displacement d) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where V is the volume of the water being forced through, and p is to be interpreted as the pressure difference between the two ends of the pipe. Thus,

$$W = (1.0 \times 10^5 \text{ Pa})\,(1.4 \text{ m}^3) = 1.4 \times 10^5 \text{ J}.$$

56. (a) The speed v of the fluid flowing out of the hole satisfies $\frac{1}{2}\rho v^2 = \rho g h$ or $v = \sqrt{2gh}$. Thus, $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$, which leads to

$$\rho_1\sqrt{2gh}A_1 = \rho_2\sqrt{2gh}A_2 \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{A_2}{A_1} = 2.$$

(b) The ratio of volume flow is

$$\frac{R_1}{R_2} = \frac{v_1 A_1}{v_2 A_2} = \frac{A_1}{A_2} = \frac{1}{2}.$$

(c) Letting $R_1/R_2 = 1$, we obtain $v_1/v_2 = A_2/A_1 = 2 = \sqrt{h_1/h_2}$. Thus,

$$h_2 = h_1/4 = (12.0 \text{ cm})/4 = 3.00 \text{ cm}.$$

57. **THINK** We use the Bernoulli equation to solve for the flow rate, and the continuity equation to relate cross-sectional area to the vertical distance from the hole.

EXPRESS According to the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2,$$

where ρ is the density of water, h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then simplifies to $\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$.

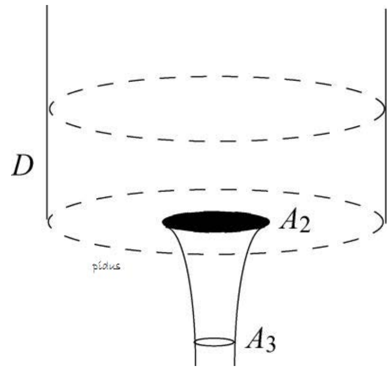
ANALYZE (a) With $D = h_1 - h_2 = 0.30$ m, the speed of water as it emerges from the hole is

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2\left(9.8 \text{ m/s}^2\right)(0.30 \text{ m})} = 2.42 \text{ m/s}.$$

Thus, the flow rate is

$$A_2v_2 = (6.5 \times 10^{-4}\ \text{m}^2)(2.42\ \text{m/s}) = 1.6 \times 10^{-3}\ \text{m}^3/\text{s}.$$

(b) We use the equation of continuity: $A_2v_2 = A_3v_3$, where $A_3 = \frac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole (see diagram below).



Thus,

$$v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84\ \text{m/s}.$$

The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\frac{1}{2}\rho v_2^2 + \rho g h_2 = \frac{1}{2}\rho v_3^2 + \rho g h_3$. Thus,

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84\ \text{m/s})^2 - (2.42\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 0.90\ \text{m}.$$

LEARN By combing the two expressions obtained from Bernoulli's equation and equation of continuity, the cross-sectional area of the stream may be related to the vertical height fallen as

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{v_2^2}{2g}\left[\left(\frac{A_2}{A_3}\right)^2 - 1\right] = \frac{v_3^2}{2g}\left[1 - \left(\frac{A_3}{A_2}\right)^2\right].$$

58. We use Bernoulli's equation:

$$p_2 - p_i = \rho g D + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

where $\rho = 1000\ \text{kg/m}^3$, $D = 180$ m, $v_1 = 0.40$ m/s, and $v_2 = 9.5$ m/s. Therefore, we find $\Delta p = 1.7 \times 10^6$ Pa, or 1.7 MPa. The SI unit for pressure is the pascal (Pa) and is equivalent to N/m^2.

59. **THINK** The elevation and cross-sectional area of the pipe are changing, so we apply the Bernoulli equation and continuity equation to analyze the flow of water through the pipe.

EXPRESS To calculate the flow speed at the lower level, we use the equation of continuity: $A_1v_1 = A_2v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. As for the pressure at the lower level, we use the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 ,$$

where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude.

ANALYZE (a) From the continuity equation, we find the speed at the lower level to be

$$v_2 = (A_1/A_2)v_1 = [(4.0 \text{ cm}^2)/(8.0 \text{ cm}^2)]\ (5.0 \text{ m/s}) = 2.5\text{m/s}.$$

(b) Similarly, from the Bernoulli equation, the pressure at the lower level is

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right) \\ &= 1.5\times10^5\,\text{Pa} + \frac{1}{2}(1000\,\text{kg/m}^3)\left[(5.0\,\text{m/s})^2 - (2.5\,\text{m/s})^2\right] + (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(10\text{ m}) \\ &= 2.6\times10^5\text{ Pa.} \end{aligned}$$

LEARN The water at the lower level has a smaller speed ($v_2 < v_1$) but higher pressure ($p_2 > p_1$).

60. (a) We use Av = const. The speed of water is

$$v = \frac{(25.0\,\text{cm})^2 - (5.00\,\text{cm})^2}{(25.0\,\text{cm})^2}(2.50\,\text{m/s}) = 2.40\,\text{m/s}.$$

(b) Since $p + \frac{1}{2}\rho v^2 = \text{const.},$ the pressure difference is

$$\Delta p = \frac{1}{2}\rho\Delta v^2 = \frac{1}{2}(1000\,\text{kg/m}^3)\left[(2.50\,\text{m/s})^2 - (2.40\,\text{m/s})^2\right] = 245\,\text{Pa}.$$

61. (a) The equation of continuity leads to

$$v_2 A_2 = v_1 A_1 \quad\Rightarrow\quad v_2 = v_1\left(\frac{r_1^2}{r_2^2}\right)$$

which gives v_2 = 3.9 m/s.

(b) With h = 7.6 m and $p_1 = 1.7 \times 10^5$ Pa, Bernoulli's equation reduces to

$$p_2 = p_1 - \rho g h + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 8.8\times 10^4\,\text{Pa}.$$

62. (a) Bernoulli's equation gives $p_A = p_B + \frac{1}{2}\rho_{\text{air}}v^2\cdot$ However, $\Delta p = p_A - p_B = \rho g h$ in order to balance the pressure in the two arms of the U-tube. Thus $\rho g h = \frac{1}{2}\rho_{\text{air}}v^2$, or

$$v = \sqrt{\frac{2\rho g h}{\rho_{\text{air}}}}.$$

(b) The plane's speed relative to the air is

$$v = \sqrt{\frac{2\rho g h}{\rho_{\text{air}}}} = \sqrt{\frac{2\left(810\,\text{kg/m}^3\right)(9.8\,\text{m/s}^2)(0.260\,\text{m})}{1.03\,\text{kg/m}^3}} = 63.3\,\text{m/s}.$$

63. We use the formula for v obtained in the previous problem:

$$v = \sqrt{\frac{2\Delta p}{\rho_{\text{air}}}} = \sqrt{\frac{2(180\,\text{Pa})}{0.031\,\text{kg/m}^3}} = 1.1\times 10^2\,\text{m/s}.$$

64. (a) The volume of water (during 10 minutes) is

$$V = \left(v_1 t\right)A_1 = \left(15\,\text{m/s}\right)\left(10\,\text{min}\right)\left(60\,\text{s/min}\right)\left(\frac{\pi}{4}\right)\left(0.03\,\text{m}\right)^2 = 6.4\,\text{m}^3.$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = \left(15\,\text{m/s}\right)\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\,\text{m/s}.$$

(c) Since

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

and $h_1 = h_2, p_1 = p_0$, which is the atmospheric pressure,

$$p_2 = p_0 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 1.01\times 10^5\,\text{Pa} + \frac{1}{2}\left(1.0\times 10^3\,\text{kg/m}^3\right)\left[\left(15\,\text{m/s}\right)^2 - \left(5.4\,\text{m/s}\right)^2\right]$$
$$= 1.99\times 10^5\,\text{Pa} = 1.97\,\text{atm}.$$

Thus, the gauge pressure is (1.97 atm – 1.00 atm) = 0.97 atm = 9.8×10^4 Pa.

65. **THINK** The design principles of the Venturi meter, a device that measures the flow speed of a fluid in a pipe, involve both the continuity equation and Bernoulli's equation.

EXPRESS The continuity equation yields $AV = av$, and Bernoulli's equation yields $\frac{1}{2}\rho V^2 = \Delta p + \frac{1}{2}\rho v^2$, where $\Delta p = p_2 - p_1$ with p_2 equal to the pressure in the throat and p_1 the pressure in the pipe. The first equation gives $v = (A/a)V$. We use this to substitute for v in the second equation and obtain

$$\frac{1}{2}\rho V^2 = \Delta p + \frac{1}{2}\rho\left(A/a\right)^2 V^2 .$$

The equation can be used to solve for V.

ANALYZE (a) The above equation gives the following expression for V:

$$V = \sqrt{\frac{2\Delta p}{\rho\left(1-(A/a)^2\right)}} = \sqrt{\frac{2a^2\Delta p}{\rho\left(a^2 - A^2\right)}}.$$

(b) We substitute the values given to obtain

$$V = \sqrt{\frac{2a^2\Delta p}{\rho\left(a^2 - A^2\right)}} = \sqrt{\frac{2(32\times 10^{-4}\,\text{m}^2)^2(41\times 10^3\,\text{Pa} - 55\times 10^3\,\text{Pa})}{(1000\,\text{kg/m}^3)\left((32\times 10^{-4}\,\text{m}^2)^2 - (64\times 10^{-4}\,\text{m}^2)^2\right)}} = 3.06\,\text{m/s}.$$

Consequently, the flow rate is

$$R = AV = (64\times 10^{-4}\,\text{m}^2)(3.06\text{ m/s}) = 2.0\times 10^{-2}\,\text{m}^3/\text{s}.$$

LEARN The pressure difference Δp between points 1 and 2 is what causes the height difference of the fluid in the two arms of the manometer. Note that $\Delta p = p_2 - p_1 < 0$ (pressure in throat less than that in the pipe), but $a < A$, so the expression inside the square root is positive.

66. We use the result of part (a) in the previous problem.

(a) In this case, we have $\Delta p = p_1 = 2.0$ atm. Consequently,

$$v = \sqrt{\frac{2\Delta p}{\rho((A/a)^2 - 1)}} = \sqrt{\frac{4(1.01\times 10^5\text{ Pa})}{(1000\text{ kg/m}^3)\,[(5a/a)^2 - 1]}} = 4.1\,\text{m/s}.$$

(b) And the equation of continuity yields $V = (A/a)v = (5a/a)v = 5v = 21$ m/s.

(c) The flow rate is given by

$$Av = \frac{\pi}{4}\,(5.0\times 10^{-4}\text{ m}^2)\,(4.1\text{ m/s}) = 8.0\times 10^{-3}\,\text{m}^3/\text{s}.$$

67. (a) The friction force is

$$f = A\Delta p = \rho_\omega g d A = (1.0\times10^3\ \text{kg/m}^3)\ (9.8\ \text{m/s}^2)\ (6.0\text{m})\left(\frac{\pi}{4}\right)(0.040\ \text{m})^2 = 74\ \text{N}.$$

(b) The speed of water flowing out of the hole is $v = \sqrt{2gd}$. Thus, the volume of water flowing out of the pipe in t = 3.0 h is

$$V = Avt = \frac{\pi^2}{4}(0.040\ \text{m})^2\sqrt{2(9.8\ \text{m/s}^2)\ (6.0\ \text{m})}\ (3.0\ \text{h})\ (3600\ \text{s/h}) = 1.5\times10^2\,\text{m}^3.$$

68. (a) We note (from the graph) that the pressures are equal when the value of inverse-area-squared is 16 (in SI units). This is the point at which the areas of the two pipe sections are equal. Thus, if $A_1 = 1/\sqrt{16}$ when the pressure difference is zero, then A_2 is 0.25 m^2.

(b) Using Bernoulli's equation (in the form Eq. 14-30) we find the pressure difference may be written in the form of a straight line: $mx + b$ where x is inverse-area-squared (the horizontal axis in the graph), m is the slope, and b is the intercept (seen to be –300 kN/m^2). Specifically, Eq. 14-30 predicts that b should be $-\frac{1}{2}\rho v_2^2$. Thus, with ρ = 1000 kg/m^3 we obtain $v_2 = \sqrt{600}$ m/s. Then the volume flow rate (see Eq. 14-24) is

$$R = A_2\, v_2 = (0.25\ \text{m}^2)(\sqrt{600}\ \text{m/s}) = 6.12\ \text{m}^3/\text{s}.$$

If the more accurate value (see Table 14-1) ρ = 998 kg/m^3 is used, then the answer is 6.13 m^3/s.

69. (a) Combining Eq. 14-35 and Eq. 14-36 in a manner very similar to that shown in the textbook, we find

$$R = A_1 A_2\sqrt{\frac{2\Delta p}{\rho\left(A_1^2 - A_2^2\right)}}$$

for the flow rate expressed in terms of the pressure difference and the cross-sectional areas. Note that $\Delta p = p_1 - p_2 = -7.2\times10^3$ Pa and $A_1^2 - A_2^2 = -8.66\times10^{-3}\ \text{m}^4$, so that the square root is well defined. Therefore, we obtain R = 0.0776 m^3/s.

(b) The mass rate of flow is $\rho R = (900\ \text{kg/m}^3)(0.0776\ \text{m}^3/\text{s}) = 69.8\ \text{kg/s}$.

70. By Eq. 14-23, the speeds in the left and right sections are $\frac{1}{4}v_{\text{mid}}$ and $\frac{1}{9}v_{\text{mid}}$, respectively, where v_{mid} = 0.500 m/s. We also note that 0.400 m^3 of water has a mass of 399 kg (see Table 14-1). Then Eq. 14-31 (and the equation below it) gives

$$W=\frac{1}{2}mv_{\text{mid}}^2\left(\frac{1}{9^2}-\frac{1}{4^2}\right)=\frac{1}{2}(399\text{ kg})(0.50\text{ m/s})^2\left(\frac{1}{9^2}-\frac{1}{4^2}\right)=-2.50\text{ J}.$$

71. (a) The stream of water emerges horizontally (θ_0 = 0° in the notation of Chapter 4) with $v_0=\sqrt{2gh}$. Setting $y - y_0 = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t=\sqrt{\frac{-2(H-h)}{-g}}=\sqrt{\frac{2}{g}(H-h)}.$$

Using this in Eq. 4-21, where x_0 = 0 by choice of coordinate origin, we find

$$x=v_0t=\sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}=2\sqrt{h(H-h)}=2\sqrt{(10\text{ cm})(40\text{ cm}-10\text{ cm})}=35\text{ cm}.$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H - h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2-Hh+\frac{x^2}{4}=0 \;\;\Rightarrow\;\; h=\frac{H\pm\sqrt{H^2-x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1+h_2=\frac{H+\sqrt{H^2-x^2}}{2}+\frac{H-\sqrt{H^2-x^2}}{2}=H.$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$. Its numerical value is $h'=40\text{cm}-10\text{ cm}=30\text{ cm}.$

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh}=4H-8h=0\Rightarrow h=\frac{H}{2}$$

or h = (40 cm)/2 = 20 cm, as the depth from which an emerging stream of water will travel the maximum horizontal distance.

72. We use Bernoulli's equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 .$$

When the water level rises to height h_2, just on the verge of flooding, v_2, the speed of water in pipe M is given by

$$\rho g(h_1 - h_2) = \frac{1}{2}\rho v_2^2 \;\Rightarrow\; v_2 = \sqrt{2g(h_1 - h_2)} = 13.86 \text{ m/s}.$$

By the continuity equation, the corresponding rainfall rate is

$$v_1 = \left(\frac{A_2}{A_1}\right) v_2 = \frac{\pi(0.030 \text{ m})^2}{(30 \text{ m})(60 \text{ m})}(13.86 \text{ m/s}) = 2.177\times 10^{-5} \text{ m/s} \approx 7.8 \text{ cm/h}.$$

73. Equilibrium of forces (on the floating body) is expressed as

$$F_b = m_{\text{body}}\, g \Rightarrow \rho_{\text{liquid}}\, g V_{\text{submerged}} = \rho_{\text{body}} g V_{\text{total}}$$

which leads to

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{liquid}}}.$$

We are told (indirectly) that two-thirds of the body is below the surface, so the fraction above is 2/3. Thus, with $\rho_{\text{body}} = 0.98$ g/cm^3, we find $\rho_{\text{liquid}} \approx 1.5$ g/cm^3 — certainly much more dense than normal seawater (the Dead Sea is about seven times saltier than the ocean due to the high evaporation rate and low rainfall in that region).

74. If the mercury level in one arm of the tube is lowered by an amount x, it will rise by x in the other arm. Thus, the net difference in mercury level between the two arms is $2x$, causing a pressure difference of $\Delta p = 2\rho_{\text{Hg}} g x$, which should be compensated for by the water pressure $p_w = \rho_w g h$, where $h = 11.2$ cm. In these units, $\rho_w = 1.00$ g/cm^3 and $\rho_{\text{Hg}} =$ 13.6 g/cm^3 (see Table 14-1). We obtain

$$x = \frac{\rho_w g h}{2\rho_{\text{Hg}} g} = \frac{(1.00 \text{ g/cm}^3)\ (11.2 \text{ cm})}{2(13.6 \text{ g/cm}^3)} = 0.412 \text{ cm}.$$

75. Using $m = \rho V$, Newton's second law becomes

$$\rho_{\text{water}} V g - \rho_{\text{bubble}} V g = \rho_{\text{bubble}} V a,$$

or

$$\rho_{\text{water}} = \rho_{\text{bubble}} (1 + a/g)$$

With $\rho_{\text{water}} = 998$ kg/m^3 (see Table 14-1), we find

$$\rho_{\text{bubble}} = \frac{\rho_{\text{water}}}{1 + a/g} = \frac{998\ \text{kg/m}^3}{1 + (0.225\ \text{m/s}^2)/(9.80\ \text{m/s}^2)} = 975.6\ \text{kg/m}^3.$$

Using volume $V = \frac{4}{3}\pi r^3$ with $r = 5.00\times10^{-4}$ m for the bubble, we then find its mass: $m_{\text{bubble}} = 5.11 \times 10^{-7}$ kg.

76. To be as general as possible, we denote the ratio of body density to water density as f (so that $f = \rho/\rho_w = 0.95$ in this problem). Floating involves equilibrium of vertical forces acting on the body (Earth's gravity pulls down and the buoyant force pushes up). Thus,

$$F_b = F_g \Rightarrow \rho_w g V_w = \rho g V$$

where V is the total volume of the body and V_w is the portion of it that is submerged.

(a) We rearrange the above equation to yield

$$\frac{V_w}{V} = \frac{\rho}{\rho_w} = f$$

which means that 95% of the body is submerged and therefore 5.0% is above the water surface.

(b) We replace ρ_w with $1.6\rho_w$ in the above equilibrium of forces relationship, and find

$$\frac{V_w}{V} = \frac{\rho}{1.6\rho_w} = \frac{f}{1.6}$$

which means that 59% of the body is submerged and thus 41% is above the quicksand surface.

(c) The answer to part (b) suggests that a person in that situation is able to breathe.

77. The normal force $\vec{F}_N$ exerted (upward) on the glass ball of mass m has magnitude 0.0948 N. The buoyant force exerted by the milk (upward) on the ball has magnitude

$$F_b = \rho_{\text{milk}}\, g\, V$$

where $V = \frac{4}{3}\pi r^3$ is the volume of the ball. Its radius is $r = 0.0200$ m. The milk density is $\rho_{\text{milk}} = 1030$ kg/m^3. The (actual) weight of the ball is, of course, downward, and has magnitude $F_g = m_{\text{glass}}\, g$. Application of Newton's second law (in the case of zero acceleration) yields

$$F_N + \rho_{\text{milk}}\, g\, V - m_{\text{glass}}\, g = 0$$

which leads to $m_{\text{glass}} = 0.0442$ kg.

78. Since $F_g = mg = \rho_{skier} g V$ and the buoyant force is $F_b = \rho_{snow} g V$, then their ratio is

$$\frac{F_b}{F_g} = \frac{\rho_{snow} g V}{\rho_{skier} g V} = \frac{\rho_{snow}}{\rho_{skier}} = \frac{96}{1020} = 0.094 \text{ (or 9.4\%)}.$$

79. Neglecting the buoyant force caused by air, then the 30 N value is interpreted as the true weight W of the object. The buoyant force of the water on the object is therefore (30 – 20) N = 10 N, which means

$$F_b = \rho_w V g \Rightarrow V = \frac{10\,\text{N}}{(1000\ \text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.02 \times 10^{-3}\ \text{m}^3$$

is the volume of the object. When the object is in the second liquid, the buoyant force is (30 – 24) N = 6.0 N, which implies

$$\rho_2 = \frac{6.0\ \text{N}}{(9.8\ \text{m/s}^2)(1.02 \times 10^{-3}\ \text{m}^3)} = 6.0\times10^2\ \text{kg/m}^3 .$$

80. An object of mass $m = \rho V$ floating in a liquid of density ρ_{liquid} is able to float if the downward pull of gravity mg is equal to the upward buoyant force $F_b = \rho_{liquid} g V_{sub}$ where V_{sub} is the portion of the object that is submerged. This readily leads to the relation:

$$\frac{\rho}{\rho_{liquid}} = \frac{V_{sub}}{V}$$

for the fraction of volume submerged of a floating object. When the liquid is water, as described in this problem, this relation leads to

$$\frac{\rho}{\rho_w} = 1$$

since the object "floats fully submerged" in water (thus, the object has the same density as water). We assume the block maintains an "upright" orientation in each case (which is not necessarily realistic).

(a) For liquid A, $\frac{\rho}{\rho_A} = \frac{1}{2}$, so that, in view of the fact that $\rho = \rho_w$, we obtain $\rho_A/\rho_w = 2$.

(b) For liquid B, noting that two-thirds *above* means one-third *below*, $\frac{\rho}{\rho_B} = \frac{1}{3}$, so that $\rho_B/\rho_w = 3$.

(c) For liquid *C*, noting that one-fourth *above* means three-fourths *below*, $\dfrac{\rho}{\rho_C} = \dfrac{3}{4}$, so that $\rho_C/\rho_w = 4/3$.

81. **THINK** The U-tube contains two types of liquid in static equilibrium. The pressures at the interface level on both sides of the tube must be the same.

EXPRESS If we examine both sides of the U-tube at the level where the low-density liquid (with $\rho = 0.800\ \text{g/cm}^3 = 800\ \text{kg/m}^3$) meets the water (with $\rho_w = 0.998\ \text{g/cm}^3 = 998\ \text{kg/m}^3$), then the pressures there on either side of the tube must agree:

$$\rho g h = \rho_w g h_w$$

where $h = 8.00$ cm = 0.0800 m, and Eq. 14-9 has been used. Thus, the height of the water column (as measured from that level) is $h_w = (800/998)(8.00\ \text{cm}) = 6.41$ cm.

ANALYZE The volume of water in that column is

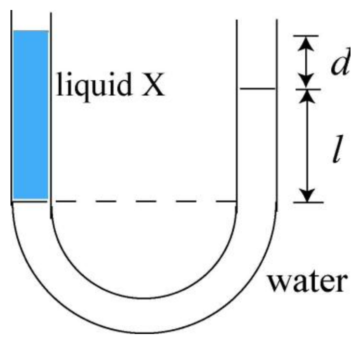
$$V = \pi r^2 h_w = \pi(1.50\ \text{cm})^2(6.41\ \text{cm}) = 45.3\ \text{cm}^3.$$

This is the amount of water that flows out of the right arm.

LEARN As discussed in the Sample Problem 14.3 – Balancing of pressure in a U-tube, the relationship between the densities of the two liquids can be written as

$$\rho_X = \rho_w \frac{l}{l+d}$$

The liquid in the left arm is higher than the water in the right because the liquid is less dense than water $\rho_X < \rho_w$.



82. The downward force on the balloon is *mg* and the upward force is $F_b = \rho_{out} V g$. Newton's second law (with $m = \rho_{in} V$) leads to

$$\rho_{out} V g - \rho_{in} V g = \rho_{in} V a \quad \Rightarrow \left(\frac{\rho_{out}}{\rho_{in}} - 1\right) g = a.$$

The problem specifies $\rho_{out} / \rho_{in} = 1.39$ (the outside air is cooler and thus more dense than the hot air inside the balloon). Thus, the upward acceleration is

$$a = (1.39 - 1.00)(9.80\ \text{m/s}^2) = 3.82\ \text{m/s}^2.$$

83. (a) We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B, and C. Applying Bernoulli's equation to points D and C, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

which leads to

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \approx \sqrt{2g(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{air}$ and $v_D/v_C \approx 0$. Plugging in the values, we obtain

$$v_C = \sqrt{2(9.8\text{ m/s}^2)(0.40\text{ m} + 0.12\text{ m})} = 3.2\text{ m/s}.$$

(b) We now consider points B and C:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C\ .$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{air}$, Bernoulli's equation becomes

$$\begin{aligned} p_B &= p_C + \rho g(h_C - h_B) = p_{air} - \rho g(h_1 + h_2 + d) \\ &= 1.0\times10^5\text{ Pa} - (1.0\times10^3\text{kg/m}^3)(9.8\text{ m/s}^2)(0.25\text{ m} + 0.40\text{ m} + 0.12\text{ m}) \\ &= 9.2\times10^4\text{ Pa}. \end{aligned}$$

(c) Since $p_B \geq 0$, we must let

$$p_{air} - \rho g(h_1 + d + h_2) \geq 0,$$

which yields

$$h_1 \leq h_{1,max} = \frac{p_{air}}{\rho} - d - h_2 \leq \frac{p_{air}}{\rho} = 10.3\text{ m}.$$

84. The volume rate of flow is $R = vA$ where $A = \pi r^2$ and $r = d/2$. Solving for speed, we obtain

$$v = \frac{R}{A} = \frac{R}{\pi(d/2)^2} = \frac{4R}{\pi d^2}.$$

(a) With $R = 7.0 \times 10^{-3}$ m^3/s and $d = 14 \times 10^{-3}$ m, our formula yields $v = 45$ m/s, which is about 13% of the speed of sound (which we establish by setting up a ratio: v/v_s where v_s = 343 m/s).

(b) With the contracted trachea ($d = 5.2 \times 10^{-3}$ m) we obtain $v = 330$ m/s, or 96% of the speed of sound.

85. We consider the can with nearly its total volume submerged, and just the rim above water. For calculation purposes, we take its submerged volume to be V = 1200 cm^3. To float, the total downward force of gravity (acting on the tin mass m_t and the lead mass m_ℓ) must be equal to the buoyant force upward:

$$(m_t + m_\ell)\, g = \rho_w V g \Rightarrow m_\ell = (1\text{g/cm}^3)\ (1200\ \text{cm}^3) - 130\ \text{g}$$

which yields 1.07×10^3 g for the (maximum) mass of the lead (for which the can still floats). The given density of lead is not used in the solution.

86. Before undergoing acceleration, the net force exerted on the block is zero, and Newton's second law gives

$$F_b - mg - T_0 = 0 \quad \Rightarrow \quad T_0 = F_b - mg$$

where $F_b = \rho V g$ is the buoyant force from the fluid of density ρ. When the container is given an upward acceleration a, the apparent weight of the block becomes $m(g+a)$, and the corresponding buoyant force is $F_b' = \rho V(g+a)$. In this case, Newton's second-law equation is

$$F_b' - m(g+a) - T = 0$$

which gives

$$T = F_b' - m(g+a) = \rho V(g+a) - m(g+a) = (\rho V - m)g(1 + a/g) = T_0(1 + a/g)\,.$$

With $a = 0.25g$, we have $T/T_0 = 1 + a/g = 1.25$.

87. We assume that the top surface of the slab is at the surface of the water and that the automobile is at the center of the ice surface. Let M be the mass of the automobile, ρ_i be the density of ice, and ρ_w be the density of water. Suppose the ice slab has area A and thickness h. Since the volume of ice is Ah, the downward force of gravity on the automobile and ice is $(M + \rho_i Ah)g$. The buoyant force of the water is $\rho_w Ahg$, so the condition of equilibrium is $(M + \rho_i Ah)g - \rho_w Ahg = 0$ and

$$A = \frac{M}{(\rho_w - \rho_i)h} = \frac{938\,\text{kg}}{\left(998\,\text{kg/m}^3 - 917\,\text{kg/m}^3\right)(0.441\text{m})} = 26.3\ \text{m}^2.$$

88. (a) Using Eq. 14-10, we have

$$p_g = \rho g h = (1025\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(2.22\times10^3\,\text{m}) = 2.23\times10^7\,\text{Pa}\,.$$

(b) By definition, the total pressure is

$$p = p_0 + p_g = 1.01\times10^5\,\text{Pa} + 2.23\times10^7\,\text{Pa} = 2.24\times10^7\,\text{Pa}\,.$$

(c) The net force compressing the sphere's surface is

$$F = pA = p(4\pi R^2) = (2.24\times10^7\,\text{Pa})4\pi(6.22\times10^{-2}\,\text{m})^2 = 1.09\times10^6\,\text{N}.$$

(d) The upward buoyant force exerted on the sphere by the seawater is

$$F_b = \rho g V = \rho g\left(\frac{4\pi}{3}R^3\right) = (1025\,\text{kg/m}^3)(9.8\,\text{m/s}^2)\frac{4\pi}{3}(6.22\times10^{-2}\,\text{m})^3 = 10.1\,\text{N}.$$

(e) Newton's second law applied to the sphere of mass m = 6.80 kg yields

$$F_b - mg = ma \quad\Rightarrow\quad a = \frac{F_b}{m} - g = \frac{10.1\,\text{N}}{8.60\,\text{kg}} - 9.8\,\text{m/s}^2 = -8.62\,\text{m/s}^2.$$

The acceleration vector has a magnitude of 8.62 m/s^2 and the direction is downward.

89. (a) The total weight is

$$W = \rho g V = \rho g h A = (1030\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(255\,\text{m})(2200\,\text{m}^2) = 5.66\times10^9\,\text{N}.$$

(b) The gauge pressure at this depth is

$$p_g = \rho g h = (1030\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(255\,\text{m})\left(\frac{1\,\text{atm}}{1.01\times10^5\,\text{Pa}}\right) = 25.5\,\text{atm}.$$

90. Using Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2,$$

we find the minimum pressure to be (setting $v_1 = v_2$)

$$\Delta p = p_2 - p_1 = \rho g(y_1 - y_2) = (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(6.59\text{ m} - 2.16\text{ m}) = 4.34\times10^4\ \text{Pa}.$$

Chapter 15

1. (a) During simple harmonic motion, the speed is (momentarily) zero when the object is at a "turning point" (that is, when $x = +x_m$ or $x = -x_m$). Consider that it starts at $x = +x_m$ and we are told that $t = 0.25$ second elapses until the object reaches $x = -x_m$. To execute a full cycle of the motion (which takes a period T to complete), the object which started at $x = +x_m$, must return to $x = +x_m$ (which, by symmetry, will occur 0.25 second *after* it was at $x = -x_m$). Thus, $T = 2t = 0.50$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T = 2.0$ Hz.

(c) The 36 cm distance between $x = +x_m$ and $x = -x_m$ is $2x_m$. Thus, $x_m = 36/2 = 18$ cm.

2. (a) The acceleration amplitude is related to the maximum force by Newton's second law: $F_{max} = ma_m$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). The frequency is the reciprocal of the period: $f = 1/T = 1/0.20 = 5.0$ Hz, so the angular frequency is $\omega = 10\pi$ (understood to be valid to two significant figures). Therefore,

$$F_{max} = m\omega^2 x_m = (0.12 \text{ kg})(10\pi \text{ rad/s})^2 (0.085 \text{ m}) = 10 \text{ N}.$$

(b) Using Eq. 15-12, we obtain

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = (0.12\text{kg})(10\pi \text{ rad/s})^2 = 1.2\times10^2 \text{ N/m}.$$

3. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = \left(2\pi(6.60 \text{ Hz})\right)^2 (0.0220 \text{ m}) = 37.8 \text{ m/s}^2.$$

4. (a) Since the problem gives the frequency $f = 3.00$ Hz, we have $\omega = 2\pi f = 6\pi$ rad/s (understood to be valid to three significant figures). Each spring is considered to support one fourth of the mass m_{car} so that Eq. 15-12 leads to

$$\omega = \sqrt{\frac{k}{m_{car}/4}} \Rightarrow k = \frac{1}{4}(1450\text{kg})(6\pi \text{ rad/s})^2 = 1.29\times10^5 \text{ N/m}.$$

(b) If the new mass being supported by the four springs is m_{total} = [1450 + 5(73)] kg = 1815 kg, then Eq. 15-12 leads to

$$\omega_{\text{new}} = \sqrt{\frac{k}{m_{\text{total}}/4}} \quad \Rightarrow \quad f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{1.29\times10^5 \text{ N/m}}{(1815/4) \text{ kg}}} = 2.68\text{Hz}.$$

5. **THINK** The blade of the shaver undergoes simple harmonic motion. We want to find its amplitude, maximum speed and maximum acceleration.

EXPRESS The amplitude x_m is half the range of the displacement D. Once the amplitude is known, the maximum speed v_m is related to the amplitude by $v_m = \omega x_m$, where ω is the angular frequency. Similarly, the maximum acceleration is $a_m = \omega^2 x_m$.

ANALYZE (a) The amplitude is $x_m = D/2$ = (2.0 mm)/2 = 1.0 mm.

(b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi(120 \text{ Hz})(1.0\times10^{-3} \text{ m}) = 0.75 \text{ m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi(120 \text{ Hz}))^2 (1.0\times10^{-3} \text{ m}) = 5.7\times10^2 \text{ m/s}^2.$$

LEARN In SHM, acceleration is proportional to the displacement x_m.

6. (a) The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period. The relationship $f = 1/T$ was used to obtain the last form. Thus

$$\omega = 2\pi/(1.00 \times 10^{-5} \text{ s}) = 6.28 \times 10^5 \text{ rad/s}.$$

(b) The maximum speed v_m and maximum displacement x_m are related by $v_m = \omega x_m$, so

$$x_m = \frac{v_m}{\omega} = \frac{1.00\times10^3 \text{ m/s}}{6.28\times10^5 \text{ rad/s}} = 1.59\times10^{-3} \text{ m}.$$

7. **THINK** This problem compares the magnitude of the acceleration of an oscillating diaphragm in a loudspeaker to gravitational acceleration g.

EXPRESS The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude.

ANALYZE (a) The angular frequency for which the maximum acceleration has a magnitude g is given by $\omega = \sqrt{g/x_m}$, so the corresponding frequency is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\ \text{m/s}^2}{1.0\times10^{-6}\,\text{m}}} = 498\ \text{Hz}.$$

(b) For frequencies greater than 498 Hz, the acceleration exceeds g for some part of the motion.

LEARN The acceleration a_m of the diaphragm in a loudspeaker increases with ω^2, or equivalently, with f^2.

8. We note (from the graph in the text) that $x_m = 6.00$ cm. Also the value at $t = 0$ is $x_o = -2.00$ cm. Then Eq. 15-3 leads to

$$\phi = \cos^{-1}(-2.00/6.00) = +1.91 \text{ rad or } -4.37 \text{ rad}.$$

The other "root" (+4.37 rad) can be rejected on the grounds that it would lead to a positive slope at $t = 0$.

9. (a) Making sure our calculator is in radians mode, we find

$$x = 6.0\cos\left[3\pi(2.0) + \frac{\pi}{3}\right] = 3.0 \text{ m}.$$

(b) Differentiating with respect to time and evaluating at $t = 2.0$ s, we find

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin\left[3\pi(2.0) + \frac{\pi}{3}\right] = -49 \text{ m/s}.$$

(c) Differentiating again, we obtain

$$a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos\left[3\pi(2.0) + \frac{\pi}{3}\right] = -2.7\times10^2 \text{ m/s}^2.$$

(d) In the second paragraph after Eq. 15-3, the textbook defines the phase of the motion. In this case (with $t = 2.0$ s) the phase is $3\pi(2.0) + \pi/3 \approx 20$ rad.

(e) Comparing with Eq. 15-3, we see that $\omega = 3\pi$ rad/s. Therefore, $f = \omega/2\pi = 1.5$ Hz.

(f) The period is the reciprocal of the frequency: $T = 1/f \approx 0.67$ s.

10. (a) The problem describes the time taken to execute one cycle of the motion. The period is $T = 0.75$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T \approx 1.3$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second.

(c) Since 2π radians are equivalent to a cycle, the angular frequency ω (in radians-per-second) is related to frequency f by $\omega = 2\pi f$ so that $\omega \approx 8.4$ rad/s.

11. When displaced from equilibrium, the net force exerted by the springs is $-2kx$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -2kx.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find $\omega^2 = 2k/m$, where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2(7580\ \text{N/m})}{0.245\ \text{kg}}} = 39.6\ \text{Hz}.$$

12. We note (from the graph) that $v_m = \omega x_m = 5.00$ cm/s. Also the value at $t = 0$ is $v_o = 4.00$ cm/s. Then Eq. 15-6 leads to

$$\phi = \sin^{-1}(-4.00/5.00) = -0.927 \text{ rad or } +5.36 \text{ rad}.$$

The other "root" (+4.07 rad) can be rejected on the grounds that it would lead to a positive slope at $t = 0$.

13. **THINK** The mass-spring system undergoes simple harmonic motion. Given the amplitude and the period, we can determine the corresponding frequency, angular frequency, spring constant, maximum speed and maximum force.

EXPRESS The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period, with $f = 1/T$. The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$.

ANALYZE (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500\ \text{s}) = 2.00$ Hz.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(2.00\ \text{Hz}) = 12.6$ rad/s.

(d) We solve for the spring constant k and obtain

$$k = m\omega^2 = (0.500 \text{ kg})(12.6 \text{ rad/s})^2 = 79.0 \text{ N/m}.$$

(e) The amplitude is x_m=35.0 cm = 0.350 m, so the maximum speed is

$$v_m = \omega x_m = (12.6 \text{ rad/s})(0.350 \text{ m}) = 4.40 \text{ m/s}.$$

(f) The maximum force is exerted when the displacement is a maximum. Thus, we have

$$F_m = kx_m = (79.0 \text{ N/m})(0.350 \text{ m}) = 27.6 \text{ N}.$$

LEARN With the maximum acceleration given by $a_m = \omega^2 x_m$, we see that the magnitude of the maximum force can also be written as $F_m = kx_m = m\omega^2 x_m = ma_m$. Maximum acceleration occurs at the endpoints of the path of the block.

14. Equation 15-12 gives the angular velocity:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N/m}}{2.00 \text{ kg}}} = 7.07 \text{ rad/s}.$$

Energy methods (discussed in Section 15-4) provide one method of solution. Here, we use trigonometric techniques based on Eq. 15-3 and Eq. 15-6.

(a) Dividing Eq. 15-6 by Eq. 15-3, we obtain

$$\frac{v}{x} = -\omega \tan(\omega t + \phi)$$

so that the phase ($\omega t + \phi$) is found from

$$\omega t + \phi = \tan^{-1}\left(\frac{-v}{\omega x}\right) = \tan^{-1}\left(\frac{-3.415 \text{ m/s}}{(7.07 \text{ rad/s})(0.129 \text{ m})}\right).$$

With the calculator in radians mode, this gives the phase equal to –1.31 rad. Plugging this back into Eq. 15-3 leads to $0.129 \text{ m} = x_m \cos(-1.31) \Rightarrow x_m = 0.500 \text{ m}$.

(b) Since $\omega t + \phi = -1.31$ rad at t = 1.00 s, we can use the above value of ω to solve for the phase constant ϕ. We obtain $\phi = -8.38$ rad (though this, as well as the previous result, can have 2π or 4π (and so on) added to it without changing the physics of the situation). With this value of ϕ, we find $x_o = x_m \cos \phi = -0.251$ m.

(c) And we obtain $v_o = -x_m \omega \sin\phi = 3.06$ m/s.

15. **THINK** Our system consists of two particles undergoing SHM along a common straight-line segment. Their oscillations are out of phase.

EXPRESS Let

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A, the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period.

ANALYZE (a) The coordinates of the particles 0.50 s later (that is, at $t = 0.50$ s) are

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\text{ s}}{1.5\text{ s}}\right) = -0.25A$$

and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\text{ s}}{1.5\text{ s}} + \frac{\pi}{6}\right) = -0.43A.$$

Their separation at that time is $\Delta x = x_1 - x_2 = -0.25A + 0.43A = 0.18A$.

(b) The velocities of the particles are given by

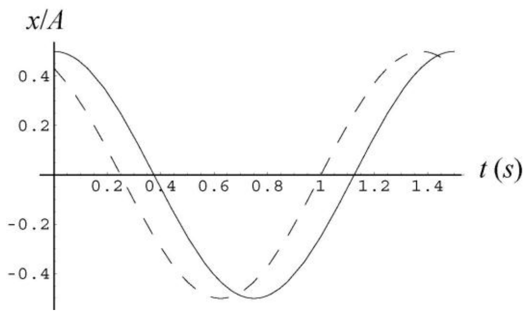
$$v_1 = \frac{dx_1}{dt} = -\frac{\pi A}{T}\sin\left(\frac{2\pi t}{T}\right)$$

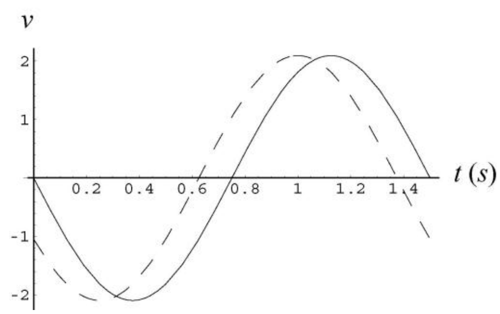
and

$$v_2 = \frac{dx_2}{dt} = -\frac{\pi A}{T}\sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right).$$

We evaluate these expressions for $t = 0.50$ s and find they are both negative-valued, indicating that the particles are moving in the same direction.

LEARN The plots of x and v as a function of time for particle 1 (so1id) and particle 2 (dashed line) are given below.





16. They pass each other at time t, at $x_1 = x_2 = \frac{1}{2} x_m$ where

$$x_1 = x_m \cos(\omega t + \phi_1) \quad \text{and} \quad x_2 = x_m \cos(\omega t + \phi_2).$$

From this, we conclude that $\cos(\omega t + \phi_1) = \cos(\omega t + \phi_2) = \frac{1}{2}$, and therefore that the phases (the arguments of the cosines) are either both equal to $\pi/3$ or one is $\pi/3$ while the other is $-\pi/3$. Also at this instant, we have $v_1 = -v_2 \neq 0$ where

$$v_1 = -x_m \omega \sin(\omega t + \phi_1) \quad \text{and} \quad v_2 = -x_m \omega \sin(\omega t + \phi_2).$$

This leads to $\sin(\omega t + \phi_1) = -\sin(\omega t + \phi_2)$. This leads us to conclude that the phases have opposite sign. Thus, one phase is $\pi/3$ and the other phase is $-\pi/3$; the *wt* term cancels if we take the phase difference, which is seen to be $\pi/3 - (-\pi/3) = 2\pi/3$.

17. (a) Equation 15-8 leads to

$$a = -\omega^2 x \Rightarrow \omega = \sqrt{\frac{-a}{x}} = \sqrt{\frac{123 \text{ m/s}^2}{0.100 \text{ m}}} = 35.07 \text{ rad/s}.$$

Therefore, $f = \omega/2\pi = 5.58$ Hz.

(b) Equation 15-12 provides a relation between ω (found in the previous part) and the mass:

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow m = \frac{400 \text{ N/m}}{(35.07 \text{ rad/s})^2} = 0.325 \text{kg}.$$

(c) By energy conservation, $\frac{1}{2} k x_m^2$ (the energy of the system at a turning point) is equal to the sum of kinetic and potential energies at the time t described in the problem.

$$\frac{1}{2} k x_m^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \Rightarrow x_m = \frac{m}{k} v^2 + x^2.$$

Consequently, $x_m = \sqrt{(0.325 \text{ kg} / 400 \text{ N/m})(13.6 \text{ m/s})^2 + (0.100 \text{ m})^2} = 0.400 \text{ m}.$

18. From highest level to lowest level is twice the amplitude x_m of the motion. The period is related to the angular frequency by Eq. 15-5. Thus, $x_m = \frac{1}{2}d$ and $\omega = 0.503$ rad/h. The phase constant ϕ in Eq. 15-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for t when x is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek t when the ocean surface is at $x = \frac{1}{2}x_m = \frac{1}{4}d$. With $x = x_m\cos(\omega t + \phi)$, we obtain

$$\frac{1}{4}d = \left(\frac{1}{2}d\right)\cos\left(0.503t + 0\right) \;\Rightarrow\; \frac{1}{2} = \cos(0.503t)$$

which has $t = 2.08$ h as the smallest positive root. The calculator is in radians mode during this calculation.

19. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency; this is the expression we set equal to $g = 9.8$ m/s^2.

(a) Using Eq. 15-5 and $T = 1.0$ s, we have

$$\left(\frac{2\pi}{T}\right)^2 x_m = g \Rightarrow x_m = \frac{gT^2}{4\pi^2} = 0.25 \text{ m.}$$

(b) Since $\omega = 2\pi f$, and $x_m = 0.050$ m is given, we find

$$\left(2\pi f\right)^2 x_m = g \quad \Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = 2.2 \text{ Hz.}$$

20. We note that the ratio of Eq. 15-6 and Eq. 15-3 is $v/x = -\omega\tan(\omega t + \phi)$ where $\omega = 1.20$ rad/s in this problem. Evaluating this at $t = 0$ and using the values from the graphs shown in the problem, we find

$$\phi = \tan^{-1}\left(\frac{-v_0}{x_0\omega}\right) = \tan^{-1}\left(\frac{+4.00 \text{ cm/s}}{(2.0 \text{ cm})(1.20 \text{ rad/s})}\right) = 1.03 \text{ rad (or } -5.25 \text{ rad).}$$

One can check that the other "root" (4.17 rad) is unacceptable since it would give the wrong signs for the individual values of v_0 and x_0.

21. Let the spring constants be k_1 and k_2. When displaced from equilibrium, the magnitude of the net force exerted by the springs is $|k_1 x + k_2 x|$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/d^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -k_1x - k_2x.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{k_1 + k_2}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}.$$

The single springs each acting alone would produce simple harmonic motions of frequency

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = 30 \text{ Hz}, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = 45 \text{ Hz},$$

respectively. Comparing these expressions, it is clear that

$$f = \sqrt{f_1^2 + f_2^2} = \sqrt{(30 \text{ Hz})^2 + (45 \text{ Hz})^2} = 54 \text{ Hz}.$$

22. The statement that "the spring does not affect the collision" justifies the use of elastic collision formulas in section 10-5. We are told the period of SHM so that we can find the mass of block 2:

$$T = 2\pi\sqrt{\frac{m_2}{k}} \quad \Rightarrow \quad m_2 = \frac{kT^2}{4\pi^2} = 0.600 \text{ kg}.$$

At this point, the rebound speed of block 1 can be found from Eq. 10-30:

$$|v_{1f}| = \left|\frac{0.200 \text{ kg} - 0.600 \text{ kg}}{0.200 \text{ kg} + 0.600 \text{ kg}}\right|(8.00 \text{ m/s}) = 4.00 \text{ m/s}.$$

This becomes the initial speed v_0 of the projectile motion of block 1. A variety of choices for the positive axis directions are possible, and we choose left as the $+x$ direction and down as the $+y$ direction, in this instance. With the "launch" angle being zero, Eq. 4-21 and Eq. 4-22 (with $-g$ replaced with $+g$) lead to

$$x - x_0 = v_0t = v_0\sqrt{\frac{2h}{g}} = (4.00 \text{ m/s})\sqrt{\frac{2(4.90 \text{ m})}{9.8 \text{ m/s}^2}}.$$

Since $x - x_0 = d$, we arrive at $d = 4.00$ m.

23. **THINK** The maximum force that can be exerted by the surface must be less than the static frictional force or else the block will not follow the surface in its motion.

EXPRESS The static frictional force is given by $f_s = \mu_s F_N$, where μ_s is the coefficient of static friction and F_N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by

$$F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m,$$

where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form.

ANALYZE We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\ \text{m/s}^2)}{(2\pi \times 2.0\ \text{Hz})^2} = 0.031\ \text{m}.$$

LEARN A larger amplitude would require a larger force at the end points of the motion. The block slips if the surface cannot supply a larger force.

24. We wish to find the effective spring constant for the combination of springs shown in the figure. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r. The left-hand spring exerts a force of magnitude $k\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same, and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k\Delta x_\ell$. Thus,

$$k_{\text{eff}} = k\Delta x_\ell / 2\Delta x_r = k/2.$$

The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion, replace k_{eff} in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}.$$

With $m = 0.245$ kg and $k = 6430$ N/m, the frequency is $f = 18.2$ Hz.

25. (a) We interpret the problem as asking for the equilibrium position; that is, the block is gently lowered until forces balance (as opposed to being suddenly released and allowed to oscillate). If the amount the spring is stretched is x, then we examine force-components along the incline surface and find

$$kx = mg\sin\theta \;\Rightarrow\; x = \frac{mg\sin\theta}{k} = \frac{(14.0\text{ N})\sin 40.0°}{120\text{ N/m}} = 0.0750\text{ m}$$

at equilibrium. The calculator is in degrees mode in the above calculation. The distance from the top of the incline is therefore (0.450 + 0.75) m = 0.525 m.

(b) Just as with a vertical spring, the effect of gravity (or one of its components) is simply to shift the equilibrium position; it does not change the characteristics (such as the period) of simple harmonic motion. Thus, Eq. 15-13 applies, and we obtain

$$T = 2\pi\sqrt{\frac{14.0\text{ N}/9.80\text{ m/s}^2}{120\text{ N/m}}} = 0.686\text{ s}.$$

26. To be on the verge of slipping means that the force exerted on the smaller block (at the point of maximum acceleration) is $f_{\max} = \mu_s mg$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega = \sqrt{k/(m+M)}$ is the angular frequency (from Eq. 15-12). Therefore, using Newton's second law, we have

$$ma_m = \mu_s mg \Rightarrow \frac{k}{m+M}x_m = \mu_s g$$

which leads to

$$x_m = \frac{\mu_s g(m+M)}{k} = \frac{(0.40)(9.8\text{ m/s}^2)(1.8\text{ kg}+10\text{ kg})}{200\text{ N/m}} = 0.23\text{ m} = 23\text{ cm}.$$

27. **THINK** This problem explores the relationship between energies, both kinetic and potential, with amplitude in SHM.

EXPRESS In simple harmonic motion, let the displacement be

$$x(t) = x_m\cos(\omega t + \phi).$$

The corresponding velocity is

$$v(t) = dx/dt = -\omega x_m\sin(\omega t + \phi).$$

Using the expressions for $x(t)$ and $v(t)$, we find the potential and kinetic energies to be

$$U(t)=\frac{1}{2}kx^2(t)=\frac{1}{2}kx_m^2\cos^2(\omega t+\phi)$$
$$K(t)=\frac{1}{2}mv^2(t)=\frac{1}{2}m\omega^2x_m^2\sin^2(\omega t+\phi)=\frac{1}{2}kx_m^2\sin^2(\omega t+\phi)$$

where $k = m\omega^2$ is the spring constant and x_m is the amplitude. The total energy is

$$E=U(t)+K(t)=\frac{1}{2}kx_m^2\left[\cos^2(\omega t+\phi)+\sin^2(\omega t+\phi)\right]=\frac{1}{2}kx_m^2.$$

ANALYZE (a) The condition $x(t)=x_m/2$ implies that $\cos(\omega t+\phi)=1/2$, or $\sin(\omega t+\phi)=\sqrt{3}/2$. Thus, the fraction of energy that is kinetic is

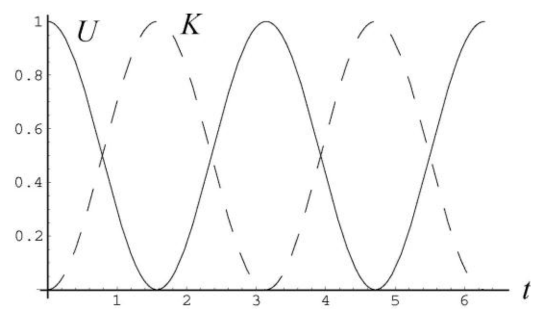
$$\frac{K}{E}=\sin^2(\omega t+\phi)=\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3}{4}.$$

(b) Similarly, we have $\dfrac{U}{E}=\cos^2(\omega t+\phi)=\left(\dfrac{1}{2}\right)^2=\dfrac{1}{4}$.

(c) Since $E=\frac{1}{2}kx_m^2$ and $U=\frac{1}{2}kx(t)^2$, $U/E = x^2/x_m^2$. Solving x^2/x_m^2 = 1/2 for x, we get $x=x_m/\sqrt{2}$.

LEARN The figure to the right depicts the potential energy (solid line) and kinetic energy (dashed line) as a function of time, assuming $x(0)=x_m$. The curves intersect when $K=U=E/2$, or equivalently,

$$\cos^2\omega t=\sin^2\omega t=1/2.$$



28. The total mechanical energy is equal to the (maximum) kinetic energy as it passes through the equilibrium position (x = 0):

$$\tfrac{1}{2}mv^2=\tfrac{1}{2}(2.0\text{ kg})(0.85\text{ m/s})^2=0.72\text{ J}.$$

Looking at the graph in the problem, we see that $U(x = 10) = 0.5$ J. Since the potential function has the form $U(x)=bx^2$, the constant is $b=5.0\times10^{-3}\text{ J/cm}^2$. Thus, $U(x)$ = 0.72 J when x = 12 cm.

(a) Thus, the mass does turn back before reaching x = 15 cm.

(b) It turns back at x = 12 cm.

29. **THINK** Knowing the amplitude and the spring constant, we can calculate the mechanical energy of the mass-spring system in simple harmonic motion.

EXPRESS In simple harmonic motion, let the displacement be $x(t) = x_m \cos(\omega t + \phi)$. The corresponding velocity is

$$v(t) = dx/dt = -\omega x_m \sin(\omega t + \phi).$$

Using the expressions for $x(t)$ and $v(t)$, we find the potential and kinetic energies to be

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi)$$
$$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}m\omega^2 x_m^2 \sin^2(\omega t + \phi) = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

where $k = m\omega^2$ is the spring constant and x_m is the amplitude. The total energy is

$$E = U(t) + K(t) = \frac{1}{2}kx_m^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right] = \frac{1}{2}kx_m^2.$$

ANALYZE With $k = 1.3\ \text{N/cm} = 130\ \text{N/m}$ and $x_m = 2.4\ \text{cm} = 0.024\ \text{m}$, the mechanical energy is

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2}\left(1.3\times10^2\ \text{N/m}\right)\left(0.024\ \text{m}\right)^2 = 3.7\times10^{-2}\ \text{J}.$$

LEARN An alternative to calculate E is to note that when the block is at the end of its path and is momentarily stopped ($v = 0 \Rightarrow K = 0$), its displacement is equal to the amplitude and all the energy is potential in nature ($E = U + K = U$). With the spring potential energy taken to be zero when the block is at its equilibrium position, we recover the expression $E = kx_m^2/2$.

30. (a) The energy at the turning point is all potential energy: $E = \frac{1}{2}kx_m^2$ where E = 1.00 J and x_m = 0.100 m. Thus,

$$k = \frac{2E}{x_m^2} = 200\ \text{N/m}.$$

(b) The energy as the block passes through the equilibrium position (with speed v_m = 1.20 m/s) is purely kinetic:

$$E = \frac{1}{2}mv_m^2 \quad \Rightarrow \quad m = \frac{2E}{v_m^2} = 1.39\ \text{kg}.$$

(c) Equation 15-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.91\,\text{Hz}.$$

31. (a) Equation 15-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000\ \text{N/m}}{5.00\ \text{kg}}} = 2.25\,\text{Hz}.$$

(b) With x_0 = 0.500 m, we have $U_0 = \frac{1}{2}kx_0^2 = 125\ \text{J}$.

(c) With v_0 = 10.0 m/s, the initial kinetic energy is $K_0 = \frac{1}{2}mv_0^2 = 250\ \text{J}$.

(d) Since the total energy $E = K_0 + U_0$ = 375 J is conserved, then consideration of the energy at the turning point leads to

$$E = \frac{1}{2}kx_m^2 \Rightarrow x_m = \sqrt{\frac{2E}{k}} = 0.866\ \text{m}.$$

32. We infer from the graph (since mechanical energy is conserved) that the *total* energy in the system is 6.0 J; we also note that the amplitude is apparently x_m = 12 cm = 0.12 m. Therefore we can set the maximum *potential* energy equal to 6.0 J and solve for the spring constant k:

$$\tfrac{1}{2}k\,x_m^2 = 6.0\ \text{J} \quad \Rightarrow \quad k = 8.3\times 10^2\ \text{N/m}\,.$$

33. The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m + M$ attached to a spring of spring constant k).

(a) Momentum conservation readily yields $v' = mv/(m + M)$. With m = 9.5 g, M = 5.4 kg, and v = 630 m/s, we obtain $v' = 1.1$ m/s.

(b) Since v' occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for x_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\frac{1}{2}(m+M)v'^2 = \frac{1}{2}kx_m^2 \quad \Rightarrow \quad \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} = \frac{1}{2}kx_m^2$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} = \frac{(9.5\times10^{-3}\text{kg})(630\ \text{m/s})}{\sqrt{(6000\ \text{N/m})(9.5\times10^{-3}\text{kg} +\ 5.4\text{kg})}} = 3.3\times10^{-2}\ \text{m}.$$

34. We note that the spring constant is

$$k = 4\pi^2 m_1/T^2 = 1.97 \times 10^5 \text{ N/m}.$$

It is important to determine where in its simple harmonic motion (which "phase" of its motion) block 2 is when the impact occurs. Since $\omega = 2\pi/T$ and the given value of t (when the collision takes place) is one-fourth of T, then $\omega t = \pi/2$ and the location then of block 2 is $x = x_m\cos(\omega t + \phi)$ where $\phi = \pi/2$ which gives

$$x = x_m\cos(\pi/2 + \pi/2) = -x_m.$$

This means block 2 is at a turning point in its motion (and thus has zero speed right before the impact occurs); this means, too, that the spring is stretched an amount of 1 cm = 0.01 m at this moment. To calculate its after-collision speed (which will be the same as that of block 1 right after the impact, since they stick together in the process) we use momentum conservation and obtain

$$v = (4.0 \text{ kg})(6.0 \text{ m/s})/(6.0 \text{ kg}) = 4.0 \text{ m/s}.$$

Thus, at the end of the impact itself (while block 1 is still at the same position as before the impact) the system (consisting now of a total mass $M = 6.0$ kg) has kinetic energy

$$K = \frac{1}{2}(6.0 \text{ kg})(4.0 \text{ m/s})^2 = 48 \text{ J}$$

and potential energy

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(1.97 \times 10^5 \text{ N/m})(0.010 \text{ m})^2 \approx 10 \text{ J},$$

meaning the total mechanical energy in the system at this stage is approximately $E = K + U = 58$ J. When the system reaches its new turning point (at the new amplitude X) then this amount must equal its (maximum) potential energy there: $E = \frac{1}{2}(1.97 \times 10^5 \text{ N/m})\, X^2$. Therefore, we find

$$X = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(58 \text{ J})}{1.97\times10^5 \text{ N/m}}} = 0.024 \text{ m} .$$

35. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency and $x_m = 0.0020$ m is the amplitude. Thus, $a_m = 8000 \text{ m/s}^2$ leads to $\omega = 2000$ rad/s. Using Newton's second law with $m = 0.010$ kg, we have

$$F = ma = m\left(-a_m\cos(\omega t + \phi)\right) = -(80 \text{ N})\cos\left(2000t - \frac{\pi}{3}\right)$$

where t is understood to be in seconds.

(a) Equation 15-5 gives $T = 2\pi/\omega = 3.1 \times 10^{-3}$ s.

(b) The relation $v_m = \omega x_m$ can be used to solve for v_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter. By Eq. 15-12, the spring constant is $k = \omega^2 m = 40000$ N/m. Then, energy conservation leads to

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \quad \Rightarrow \quad v_m = x_m\sqrt{\frac{k}{m}} = 4.0 \text{ m/s}.$$

(c) The total energy is $\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 = 0.080$ J.

(d) At the maximum displacement, the force acting on the particle is

$$F = kx = (4.0\times10^4 \text{ N/m})(2.0\times10^{-3}\text{m}) = 80 \text{ N}.$$

(e) At half of the maximum displacement, $x = 1.0$ mm, and the force is

$$F = kx = (4.0\times10^4 \text{ N/m})(1.0\times10^{-3}\text{m}) = 40 \text{ N}.$$

36. We note that the ratio of Eq. 15-6 and Eq. 15-3 is $v/x = -\omega\tan(\omega t + \phi)$ where ω is given by Eq. 15-12. Since the kinetic energy is $\frac{1}{2}mv^2$ and the potential energy is $\frac{1}{2}kx^2$ (which may be conveniently written as $\frac{1}{2}m\omega^2x^2$) then the ratio of kinetic to potential energy is simply

$$(v/x)^2/\omega^2 = \tan^2(\omega t + \phi),$$

which at $t = 0$ is $\tan^2\phi$. Since $\phi = \pi/6$ in this problem, then the ratio of kinetic to potential energy at $t = 0$ is $\tan^2(\pi/6) = 1/3$.

37. (a) The object oscillates about its equilibrium point, where the downward force of gravity is balanced by the upward force of the spring. If ℓ is the elongation of the spring at equilibrium, then $k\ell = mg$, where k is the spring constant and m is the mass of the object. Thus $k/m = g/\ell$ and

$$f = \omega/2\pi = (1/2\pi)\sqrt{k/m} = (1/2\pi)\sqrt{g/\ell}.$$

Now the equilibrium point is halfway between the points where the object is momentarily at rest. One of these points is where the spring is unstretched and the other is the lowest point, 10 cm below. Thus $\ell = 5.0\,\text{cm} = 0.050\,\text{m}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8 \text{ m/s}^2}{0.050 \text{ m}}} = 2.2 \text{ Hz}.$$

(b) Use conservation of energy. We take the zero of gravitational potential energy to be at the initial position of the object, where the spring is unstretched. Then both the initial potential and kinetic energies are zero. We take the *y*-axis to be positive in the downward direction and let y = 0.080 m. The potential energy when the object is at this point is $U = \frac{1}{2}ky^2 - mgy$. The energy equation becomes

$$0 = \tfrac{1}{2}ky^2 - mgy + \tfrac{1}{2}mv^2 .$$

We solve for the speed:

$$v = \sqrt{2gy - \frac{k}{m}y^2} = \sqrt{2gy - \frac{g}{\ell}y^2} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.080\,\text{m}\right) - \left(\frac{9.8\,\text{m/s}^2}{0.050\,\text{m}}\right)\left(0.080\,\text{m}\right)^2}$$
$$= 0.56\,\text{m/s}$$

(c) Let m be the original mass and Δm be the additional mass. The new angular frequency is $\omega' = \sqrt{k/(m+\Delta m)}$. This should be half the original angular frequency, or $\frac{1}{2}\sqrt{k/m}$. We solve

$$\sqrt{k/(m+\Delta m)} = \tfrac{1}{2}\sqrt{k/m}$$

for m. Square both sides of the equation, then take the reciprocal to obtain $m + \Delta m = 4m$. This gives

$$m = \Delta m/3 = (300\text{ g})/3 = 100\text{ g} = 0.100\text{ kg}.$$

(d) The equilibrium position is determined by the balancing of the gravitational and spring forces: $ky = (m + \Delta m)g$. Thus $y = (m + \Delta m)g/k$. We will need to find the value of the spring constant k using $k = m\omega^2 = m(2\pi f)^2$. Then

$$y\frac{(m+\Delta m)g}{m(2\pi f)^2} = \frac{(0.100\text{ kg} + 0.300\text{ kg})(9.80\text{ m/s}^2)}{(0.100\text{ kg})(2\pi \times 2.24\text{ Hz})^2} = 0.200\text{ m}.$$

This is measured from the initial position.

38. From Eq. 15-23 (in absolute value) we find the torsion constant:

$$\kappa = \left|\frac{\tau}{\theta}\right| = \frac{0.20\text{ N}\cdot\text{m}}{0.85\text{ rad}} = 0.235\ \text{N}\cdot\text{m/rad}.$$

With $I = 2mR^2/5$ (the rotational inertia for a solid sphere — from Chapter 11), Eq. 15–23 leads to

$$T = 2\pi\sqrt{\frac{\frac{2}{5}mR^2}{\kappa}} = 2\pi\sqrt{\frac{\frac{2}{5}(95\text{ kg})(0.15\text{ m})^2}{0.235\ \text{N}\cdot\text{m/rad}}} = 12\text{ s}.$$

39. **THINK** The balance wheel in the watch undergoes angular simple harmonic oscillation. From the amplitude and period, we can calculate the corresponding angular velocity and angular acceleration.

EXPRESS We take the angular displacement of the wheel to be $\theta(t) = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. We differentiate with respect to time to find the angular velocity:

$$\Omega = d\theta/dt = -(2\pi/T)\theta_m \sin(2\pi t/T).$$

The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency.

ANALYZE (a) The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi \text{ rad})}{0.500 \text{ s}} = 39.5 \text{ rad/s}.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1-\cos^2(2\pi t/T)} = \sqrt{1-(1/2)^2} = \sqrt{3/2}$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500 \text{ s}}\right)(\pi \text{ rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2 \text{ rad / s}.$$

During another portion of the cycle its angular speed is +34.2 rad/s when its angular displacement is $\pi/2$ rad.

(c) The angular acceleration is

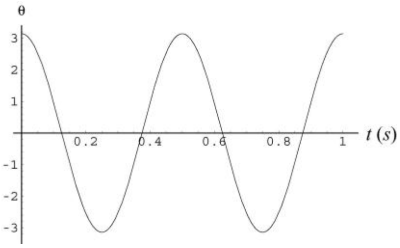
$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta.$$

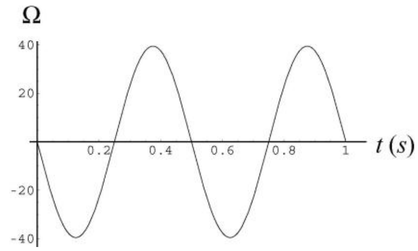
When $\theta = \pi/4$,

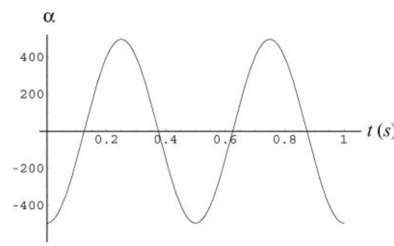
$$\alpha = -\left(\frac{2\pi}{0.500 \text{ s}}\right)^2 \left(\frac{\pi}{4}\right) = -124 \text{ rad/s}^2,$$

or $|\alpha| = 124 \text{ rad/s}^2$.

LEARN The angular displacement, angular velocity and angular acceleration as a function of time are plotted next.







40. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$, the unknown. For a meter stick of mass m, the rotational inertia about its center of mass is $I_{cm} = mL^2/12$ where L = 1.0 m. Thus, for T = 2.5 s, we obtain

$$T = 2\pi\sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi\sqrt{\frac{L^2}{12gd} + \frac{d}{g}}.$$

Squaring both sides and solving for d leads to the quadratic formula:

$$d = \frac{g(T/2\pi)^2 \pm \sqrt{d^2(T/2\pi)^4 - L^2/3}}{2}.$$

Choosing the plus sign leads to an impossible value for d ($d = 1.5 > L$). If we choose the minus sign, we obtain a physically meaningful result: d = 0.056 m.

41. **THINK** Our physical pendulum consists of a disk and a rod. To find the period of oscillation, we first calculate the moment of inertia and the distance between the center-of-mass of the disk-rod system to the pivot.

EXPRESS A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}Mr^2$, where M is its mass and r is its radius. The disk of this problem rotates about a point that is displaced from its center by $r + L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}Mr^2 + \frac{1}{2}M(L+r)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass.

ANALYZE (a) The total rotational inertia of the disk and rod is

$$\begin{aligned} I &= \frac{1}{2}Mr^2 + M(L+r)^2 + \frac{1}{3}mL^2 \\ &= \frac{1}{2}(0.500\text{kg})(0.100\text{m})^2 + (0.500\text{kg})(0.500\text{m} + 0.100\text{m})^2 + \frac{1}{3}(0.270\text{kg})(0.500\text{m})^2 \\ &= 0.205\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + r = 0.500\ \text{m} + 0.100\ \text{m} = 0.600\ \text{m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\ \text{m})/2 = 0.250\ \text{m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M+m} = \frac{(0.500\ \text{kg})(0.600\ \text{m}) + (0.270\ \text{kg})(0.250\ \text{m})}{0.500\ \text{kg} + 0.270\ \text{kg}} = 0.477\ \text{m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\ \text{kg}\cdot\text{m}^2}{(0.500\ \text{kg} + 0.270\ \text{kg})(9.80\ \text{m/s}^2)(0.447\ \text{m})}} = 1.50\ \text{s}.$$

LEARN Consider the limit where $M \to 0$ (i.e., uniform disk removed). In this case, $I = mL^2/3$, $d = \ell_r = L/2$ and the period of oscillation becomes

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{mL^2/3}{mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}}$$

which is the result given in Eq. 15-32.

42. (a) Comparing the given expression to Eq. 15-3 (after changing notation $x \to \theta$), we see that $\omega = 4.43$ rad/s. Since $\omega = \sqrt{g/L}$ then we can solve for the length: $L = 0.499$ m.

(b) Since $v_m = \omega x_m = \omega L\theta_m = (4.43\ \text{rad/s})(0.499\ \text{m})(0.0800\ \text{rad})$ and $m = 0.0600$ kg, then we can find the maximum kinetic energy: $\frac{1}{2}mv_m^2 = 9.40 \times 10^{-4}$ J.

43. (a) Referring to Sample Problem 15.5 – "Physical pendulum, period and length," we see that the distance between P and C is $h = \frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. The parallel axis theorem (see Eq. 15–30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = \left(\frac{1}{12} + \frac{1}{36}\right)mL^2 = \frac{1}{9}mL^2.$$

Equation 15-29 then gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{L^2/9}{gL/6}} = 2\pi\sqrt{\frac{2L}{3g}}$$

which yields $T = 1.64$ s for $L = 1.00$ m.

(b) We note that this T is identical to that computed in Sample Problem 15.5 – "Physical pendulum, period and length." As far as the characteristics of the periodic motion are concerned, the center of oscillation provides a pivot that is equivalent to that chosen in the Sample Problem (pivot at the edge of the stick).

44. To use Eq. 15-29 we need to locate the center of mass and we need to compute the rotational inertia about A. The center of mass of the stick shown horizontal in the figure is at A, and the center of mass of the other stick is 0.50 m below A. The two sticks are of equal mass, so the center of mass of the system is $h = \frac{1}{2}(0.50 \text{ m}) = 0.25\text{m}$ below A, as shown in the figure. Now, the rotational inertia of the system is the sum of the rotational inertia I_1 of the stick shown horizontal in the figure and the rotational inertia I_2 of the stick shown vertical. Thus, we have

$$I = I_1 + I_2 = \frac{1}{12}ML^2 + \frac{1}{3}ML^2 = \frac{5}{12}ML^2$$

where L = 1.00 m and M is the mass of a meter stick (which cancels in the next step). Now, with $m = 2M$ (the total mass), Eq. 15-29 yields

$$T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mgh}} = 2\pi\sqrt{\frac{5L}{6g}}$$

where $h = L/4$ was used. Thus, T = 1.83 s.

45. From Eq. 15-28, we find the length of the pendulum when the period is T = 8.85 s:

$$L = \frac{gT^2}{4\pi^2}.$$

The new length is $L' = L - d$ where d = 0.350 m. The new period is

$$T' = 2\pi\sqrt{\frac{L'}{g}} = 2\pi\sqrt{\frac{L}{g} - \frac{d}{g}} = 2\pi\sqrt{\frac{T^2}{4\pi^2} - \frac{d}{g}}$$

which yields T' = 8.77 s.

46. We require

$$T = 2\pi\sqrt{\frac{L_o}{g}} = 2\pi\sqrt{\frac{I}{mgh}}$$

similar to the approach taken in part (b) of Sample Problem 15.5 – "Physical pendulum, period and length," but treating in our case a more general possibility for I. Canceling 2π, squaring both sides, and canceling g leads directly to the result; $L_o = I/mh$.

47. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{cm} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + md^2}{mgd}} = 2\pi\sqrt{\frac{R^2 + 2d^2}{2gd}} = 2\pi\sqrt{\frac{(2.35\text{ cm})^2 + 2(1.75\text{ cm})^2}{2(980\text{ cm/s}^2)(1.75\text{ cm})}} = 0.366\text{ s.}$$

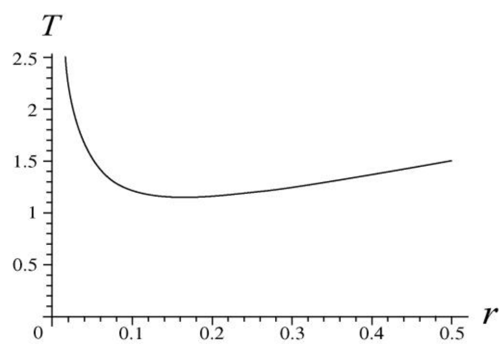
48. (a) For the "physical pendulum" we have

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{I_{com} + mh^2}{mgh}}\ .$$

If we substitute r for h and use item (i) in Table 10-2, we have

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{a^2 + b^2}{12r} + r}\ .$$

In the figure below, we plot T as a function of r, for $a = 0.35$ m and $b = 0.45$ m.



(b) The minimum of T can be located by setting its derivative to zero, $dT/dr = 0$. This yields

$$r = \sqrt{\frac{a^2 + b^2}{12}} = \sqrt{\frac{(0.35\text{ m})^2 + (0.45\text{ m})^2}{12}} = 0.16\text{ m.}$$

(c) The direction from the center does not matter, so the locus of points is a circle around the center, of radius $[(a^2 + b^2)/12]^{1/2}$.

49. Replacing x and v in Eq. 15-3 and Eq. 15-6 with θ and $d\theta/dt$, respectively, we identify 4.44 rad/s as the angular frequency ω. Then we evaluate the expressions at $t = 0$ and divide the second by the first:

$$\left(\frac{d\theta/dt}{\theta}\right)_{\text{at } t=0} = -\,\omega\tan\phi\,.$$

(a) The value of θ at $t = 0$ is 0.0400 rad, and the value of $d\theta/dt$ then is –0.200 rad/s, so we are able to solve for the phase constant:

$$\phi = \tan^{-1}[0.200/(0.0400 \times 4.44)] = 0.845 \text{ rad}.$$

(b) Once ϕ is determined we can plug back in to $\theta_o = \theta_m \cos\phi$ to solve for the angular amplitude. We find $\theta_m = 0.0602$ rad.

50. (a) The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = 1/3ML^2$. Therefore, Eq. 15-29 leads to

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} \quad \Rightarrow \quad L = \frac{3gT^2}{8\pi^2} = \frac{3(9.8 \text{ m/s}^2)(1.5 \text{ s})^2}{8\pi^2} = 0.84 \text{ m}.$$

(b) By energy conservation

$$E_{\text{bottom of swing}} = E_{\text{end of swing}} \quad \Rightarrow \quad K_m = U_m$$

where $U = Mg\ell(1-\cos\theta)$ with ℓ being the distance from the axis of rotation to the center of mass. If we use the small-angle approximation ($\cos\theta \approx 1 - \frac{1}{2}\theta^2$ with θ in radians (Appendix E)), we obtain

$$U_m = (0.5 \text{ kg})(9.8 \text{ m/s}^2)\left(\frac{L}{2}\right)\left(\frac{1}{2}\theta_m^2\right)$$

where $\theta_m = 0.17$ rad. Thus, $K_m = U_m = 0.031$ J. If we calculate $(1 - \cos\theta)$ directly (without using the small angle approximation) then we obtain within 0.3% of the same answer.

51. This is similar to the situation treated in Sample Problem 15.5 — "Physical pendulum, period and length," except that O is no longer at the end of the stick. Referring to the center of mass as C (assumed to be the geometric center of the stick), we see that the distance between O and C is $h = x$. The parallel axis theorem (see Eq. 15-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{L^2}{12} + x^2\right).$$

Equation 15-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{L^2}{12} + x^2\right)}{gx}} = 2\pi\sqrt{\frac{\left(L^2 + 12x^2\right)}{12gx}}.$$

(a) Minimizing T by graphing (or special calculator functions) is straightforward, but the standard calculus method (setting the derivative equal to zero and solving) is somewhat awkward. We pursue the calculus method but choose to work with $12gT^2/2\pi$ instead of T (it should be clear that $12gT^2/2\pi$ is a minimum whenever T is a minimum). The result is

$$\frac{d\left(\frac{12gT^2}{2\pi}\right)}{dx} = 0 = \frac{d\left(\frac{L^2}{x} + 12x\right)}{dx} = -\frac{L^2}{x^2} + 12$$

which yields $x = L/\sqrt{12} = (1.85 \text{ m})/\sqrt{12} = 0.53$ m as the value of x that should produce the smallest possible value of T.

(b) With L = 1.85 m and x = 0.53 m, we obtain T = 2.1 s from the expression derived in part (a).

52. Consider that the length of the spring as shown in the figure (with one of the block's corners lying directly above the block's center) is some value L (its rest length). If the (constant) distance between the block's center and the point on the wall where the spring attaches is a distance r, then $r\cos\theta = d/\sqrt{2}$, and $r\cos\theta = L$ defines the angle θ measured from a line on the block drawn from the center to the top corner to the line of r (a straight line from the center of the block to the point of attachment of the spring on the wall). In terms of this angle, then, the problem asks us to consider the dynamics that results from increasing θ from its original value θ_o to $\theta_o + 3°$ and then releasing the system and letting it oscillate. If the new (stretched) length of spring is L' (when $\theta = \theta_o + 3°$), then it is a straightforward trigonometric exercise to show that

$$(L')^2 = r^2 + (d/\sqrt{2}\,)^2 - 2r(d/\sqrt{2}\,)\cos(\theta_o + 3°) = L^2 + d^2 - d^2\cos(3°) + \sqrt{2}\,Ld\sin(3°)$$

since $\theta_o = 45°$. The difference between L' (as determined by this expression) and the original spring length L is the amount the spring has been stretched (denoted here as x_m). If one plots x_m versus L over a range that seems reasonable considering the figure shown in the problem (say, from L = 0.03 m to L = 0.10 m) one quickly sees that $x_m \approx 0.00222$ m is an excellent approximation (and is very close to what one would get by approximating x_m as the arc length of the path made by that upper block corner as the block is turned through 3°, even though this latter procedure should in principle overestimate x_m). Using this value of x_m with the given spring constant leads to a potential energy of $U = \frac{1}{2}k\,x_m^{\,2}$ = 0.00296 J. Setting this equal to the kinetic energy the block has as it passes back through the initial position, we have

$$K = 0.00296 \text{ J} = \tfrac{1}{2}\, I\, \omega_m^{\,2}$$

where ω_m is the maximum angular speed of the block (and is not to be confused with the angular frequency ω of the oscillation, though they are related by $\omega_m = \theta_o\omega$ if θ_o is expressed in radians). The rotational inertia of the block is $I = \frac{1}{6}Md^2$ = 0.0018 kg·m^2. Thus, we can solve the above relation for the maximum angular speed of the block:

$$\omega_m = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(0.00296\text{ J})}{0.0018\text{ kg}\cdot\text{m}^2}} = 1.81\text{ rad/s}.$$

Therefore the angular frequency of the oscillation is $\omega = \omega_m/\theta_o = 34.6$ rad/s. Using Eq. 15-5, then, the period is $T = 0.18$ s.

53. **THINK** By assuming that the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and that the torque tends to pull the rod toward its equilibrium orientation, we see that the rod will oscillate in simple harmonic motion.

EXPRESS Let $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}},$$

where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved a distance $(L/2)(1 - \cos\theta)$ to the right. The length of the spring is now

$$\ell = \sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}\ .$$

If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell \approx \ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus, $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$ (see Table 10-2), where m is its mass.

ANALYZE Substituting the expressions for C and I, we find the period of oscillation to be

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}.$$

With $m = 0.600$ kg and $k = 1850$ N/m, we obtain $T = 0.0653$ s.

LEARN As in the case of a simple linear harmonic oscillator formed by a mass and a spring, the period of the rotating rod is inversely proportional to $\sqrt{k}$. Our result indicates that the rod oscillates very rapidly, with a frequency $f = 1/T = 15.3\,\text{Hz}$, i.e., about 15 times in one second.

54. We note that the initial angle is θ_o = 7º = 0.122 rad (though it turns out this value will cancel in later calculations). If we approximate the initial stretch of the spring as the arc-length that the corresponding point on the plate has moved through ($x = r\,\theta_o$ where r = 0.025 m) then the initial potential energy is approximately $\frac{1}{2}kx^2 =$ 0.0093 J. This should equal to the kinetic energy of the plate ($\frac{1}{2}I\,\omega_m{}^2$ where this ω_m is the maximum angular speed of the plate, not the angular frequency ω). Noting that the maximum angular speed of the plate is $\omega_m = \omega\theta_o$ where $\omega = 2\pi/T$ with T = 20 ms = 0.02 s as determined from the graph, then we can find the rotational inertial from $\frac{1}{2}\,I\omega_m{}^2$ = 0.0093 J. Thus, $I = 1.3\times10^{-5}\ \text{kg}\cdot\text{m}^2$.

55. (a) The period of the pendulum is given by $T = 2\pi\sqrt{I/mgd}$, where I is its rotational inertia, m = 22.1 g is its mass, and d is the distance from the center of mass to the pivot point. The rotational inertia of a rod pivoted at its center is $mL^2/12$ with L = 2.20 m. According to the parallel-axis theorem, its rotational inertia when it is pivoted a distance d from the center is $I = mL^2/12 + md^2$. Thus,

$$T = 2\pi\sqrt{\frac{m(L^2/12 + d^2)}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}}.$$

Minimizing T with respect to d, $dT/d(d)$ = 0, we obtain $d = L/\sqrt{12}$. Therefore, the minimum period T is

$$T_{\min} = 2\pi\sqrt{\frac{L^2 + 12(L/\sqrt{12})^2}{12g(L/\sqrt{12})}} = 2\pi\sqrt{\frac{2L}{\sqrt{12}g}} = 2\pi\sqrt{\frac{2(2.20\text{ m})}{\sqrt{12}(9.80\text{ m/s}^2)}} = 2.26\text{ s}.$$

(b) If d is chosen to minimize the period, then as L is increased the period will increase as well.

(c) The period does not depend on the mass of the pendulum, so T does not change when m increases.

56. The table of moments of inertia in Chapter 11, plus the parallel axis theorem found in that chapter, leads to

$$I_P = \tfrac{1}{2}MR^2 + Mh^2 = \tfrac{1}{2}(2.5\text{ kg})(0.21\text{ m})^2 + (2.5\text{ kg})(0.97\text{ m})^2 = 2.41\text{ kg}\cdot\text{m}^2$$

where P is the hinge pin shown in the figure (the point of support for the physical pendulum), which is a distance h = 0.21 m + 0.76 m away from the center of the disk.

(a) Without the torsion spring connected, the period is

$$T = 2\pi\sqrt{\frac{I_P}{Mgh}} = 2.00 \text{ s}.$$

(b) Now we have two "restoring torques" acting in tandem to pull the pendulum back to the vertical position when it is displaced. The magnitude of the torque-sum is $(Mgh + \kappa)\theta = I_P\,\alpha$, where the small-angle approximation ($\sin\theta \approx \theta$ in radians) and Newton's second law (for rotational dynamics) have been used. Making the appropriate adjustment to the period formula, we have

$$T' = 2\pi\sqrt{\frac{I_P}{Mgh + \kappa}}\,.$$

The problem statement requires $T = T' + 0.50$ s. Thus, $T' = (2.00 - 0.50)\text{s} = 1.50$ s. Consequently,

$$\kappa = \frac{4\pi^2}{T'^2} I_P - Mgh = 18.5 \text{ N}\cdot\text{m/rad}\,.$$

57. Since the energy is proportional to the amplitude squared (see Eq. 15-21), we find the fractional change (assumed small) is

$$\frac{E' - E}{E} \approx \frac{dE}{E} = \frac{dx_m^2}{x_m^2} = \frac{2x_m dx_m}{x_m^2} = 2\frac{dx_m}{x_m}.$$

Thus, if we approximate the fractional change in x_m as dx_m/x_m, then the above calculation shows that multiplying this by 2 should give the fractional energy change. Therefore, if x_m decreases by 3%, then E must decrease by 6.0%.

58. Referring to the numbers in Sample Problem 15.6 – "Damped harmonic oscillator, time to decay, energy," we have $m = 0.25$ kg, $b = 0.070$ kg/s, and $T = 0.34$ s. Thus, when $t = 20T$, the damping factor becomes

$$e^{-bt/2m} = e^{-(0.070)(20)(0.34)/2(0.25)} = 0.39.$$

59. **THINK** In the presence of a damping force, the amplitude of oscillation of the mass-spring system decreases with time.

EXPRESS As discussed in 15-8, when a damping force is present, we have

$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi)$$

where b is the damping constant and the angular frequency is given by

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\,.$$

ANALYZE (a) We want to solve $e^{-bt/2m} = 1/3$ for t. We take the natural logarithm of both sides to obtain $-bt/2m = \ln(1/3)$. Therefore,

$$t = -(2m/b)\ln(1/3) = (2m/b)\ln 3.$$

Thus,

$$t = \frac{2(1.50\text{ kg})}{0.230\text{ kg/s}}\ln 3 = 14.3\text{ s}.$$

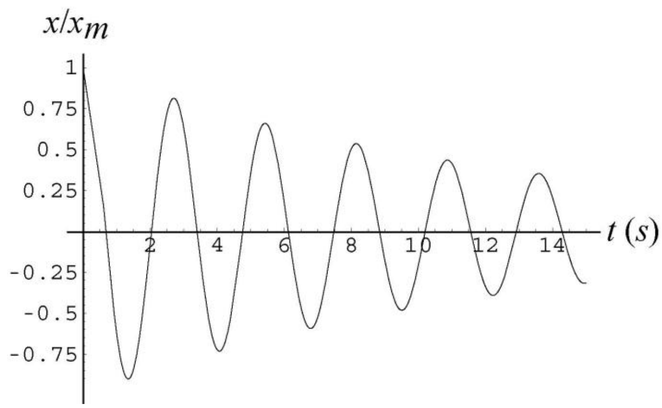
(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00\text{ N/m}}{1.50\text{ kg}} - \frac{(0.230\text{ kg/s})^2}{4(1.50\text{ kg})^2}} = 2.31\text{ rad/s}.$$

The period is $T = 2\pi/\omega' = (2\pi)/(2.31\text{ rad/s}) = 2.72$ s and the number of oscillations is

$$t/T = (14.3\text{ s})/(2.72\text{ s}) = 5.27.$$

LEARN The displacement $x(t)$ as a function of time is shown below. The amplitude, $x_m e^{-bt/2m}$, decreases exponentially with time.



60. (a) From Hooke's law, we have

$$k = \frac{(500\text{ kg})(9.8\text{ m/s}^2)}{10\text{cm}} = 4.9\times10^2\text{ N/cm}.$$

(b) The amplitude decreasing by 50% during one period of the motion implies

$$e^{-bT/2m} = \frac{1}{2} \quad \text{where} \quad T = \frac{2\pi}{\omega'}.$$

Since the problem asks us to estimate, we let $\omega' \approx \omega = \sqrt{k/m}$. That is, we let

$$\omega' \approx \sqrt{\frac{49000\text{ N/m}}{500\text{ kg}}} \approx 9.9\text{ rad/s},$$

so that $T \approx 0.63$ s. Taking the (natural) log of both sides of the above equation, and rearranging, we find

$$b = \frac{2m}{T}\ln 2 \approx \frac{2(500\text{ kg})}{0.63\text{ s}}(0.69) = 1.1\times 10^3\text{ kg/s}.$$

Note: if one worries about the $\omega' \approx \omega$ approximation, it is quite possible (though messy) to use Eq. 15-43 in its full form and solve for b. The result would be (quoting more figures than are significant)

$$b = \frac{2\ln 2\sqrt{mk}}{\sqrt{(\ln 2)^2 + 4\pi^2}} = 1086\text{ kg/s}$$

which is in good agreement with the value gotten "the easy way" above.

61. (a) We set $\omega = \omega_d$ and find that the given expression reduces to $x_m = F_m/b\omega$ at resonance.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. Thus, at resonance, we have $v_m = \omega F_m/b\omega = F_m/b$.

62. With $\omega = 2\pi/T$ then Eq. 15-28 can be used to calculate the angular frequencies for the given pendulums. For the given range of $2.00 < \omega < 4.00$ (in rad/s), we find only two of the given pendulums have appropriate values of ω: pendulum (d) with length of 0.80 m (for which $\omega = 3.5$ rad/s) and pendulum (e) with length of 1.2 m (for which $\omega = 2.86$ rad/s).

63. With $M = 1000$ kg and $m = 82$ kg, we adapt Eq. 15-12 to this situation by writing

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{M+4m}}\ .$$

If $d = 4.0$ m is the distance traveled (at constant car speed v) between impulses, then we may write $T = v/d$, in which case the above equation may be solved for the spring constant:

$$\frac{2\pi v}{d} = \sqrt{\frac{k}{M+4m}} \ \Rightarrow\ k = (M+4m)\left(\frac{2\pi v}{d}\right)^2 .$$

Before the people got out, the equilibrium compression is $x_i = (M + 4m)g/k$, and afterward it is $x_f = Mg/k$. Therefore, with $v = 16000/3600 = 4.44$ m/s, we find the rise of the car body on its suspension is

$$x_i - x_f = \frac{4mg}{k} = \frac{4mg}{M+4m}\left(\frac{d}{2\pi v}\right)^2 = 0.050\text{ m}.$$

64. Since $\omega = 2\pi f$ where $f = 2.2$ Hz, we find that the angular frequency is $\omega = 13.8$ rad/s. Thus, with $x = 0.010$ m, the acceleration amplitude is $a_m = x_m\,\omega^2 = 1.91$ m/s^2. We set up a ratio:

$$a_m = \left(\frac{a_m}{g}\right)g = \left(\frac{1.91}{9.8}\right)g = 0.19g.$$

65. (a) The problem gives the frequency $f = 440$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second. The angular frequency ω is similar to frequency except that ω is in radians-per-second. Recalling that 2π radians are equivalent to a cycle, we have $\omega = 2\pi f \approx 2.8\times10^3$ rad/s.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. With $x_m = 0.00075$ m and the above value for ω, this expression yields $v_m = 2.1$ m/s.

(c) In the discussion immediately after Eq. 15-7, the book introduces the acceleration amplitude $a_m = \omega^2 x_m$, which (if the more precise value $\omega = 2765$ rad/s is used) yields $a_m =$ 5.7 km/s.

66. (a) First consider a single spring with spring constant k and unstretched length L. One end is attached to a wall and the other is attached to an object. If it is elongated by Δx the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants k_1 and k_2, arranged so spring 1 is attached to the object. If spring 1 is elongated by Δx_1 then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as the force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between Δx and Δx_1. The springs are uniform so equal unstretched lengths are elongated by the same amount and the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by $\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where C is a constant of proportionality. The total elongation is

$$\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_2(n + 1),$$

where $L_1 = nL_2$ was used to obtain the last form. Since $L_2 = L_1/n$, this can also be written $\Delta x = CL_1(n + 1)/n$. We substitute $\Delta x_1 = CL_1$ and $\Delta x = CL_1(n + 1)/n$ into $k\,\Delta x = k_1\,\Delta x_1$ and solve for k_1. With $k = 8600$ N/m and $n = L_1/L_2 = 0.70$, we obtain

$$k_1 = \left(\frac{n+1}{n}\right)k = \left(\frac{0.70+1.0}{0.70}\right)(8600\text{ N/m}) = 20886\text{ N/m} \approx 2.1\times10^4\text{N/m}.$$

(b) Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a force on it. Now $k\,\Delta x = k_2\,\Delta x_2$. We use $\Delta x_2 = CL_2$ and $\Delta x = CL_2(n + 1)$, then solve for k_2. The result is $k_2 = k(n + 1)$.

$$k_2=(n+1)k=(0.70+1.0)(8600\text{ N/m})=14620\text{ N/m}\approx 1.5\times 10^4\text{N/m}$$

(c) To find the frequency when spring 1 is attached to mass m, we replace k in $(1/2\pi)\sqrt{k/m}$ with $k(n+1)/n$. With $f=(1/2\pi)\sqrt{k/m}$, we obtain, for $f=200$ Hz and n = 0.70,

$$f_1=\frac{1}{2\pi}\sqrt{\frac{(n+1)k}{nm}}=\sqrt{\frac{n+1}{n}}f=\sqrt{\frac{0.70+1.0}{0.70}}(200\text{ Hz})=3.1\times 10^2\text{ Hz.}$$

(d) To find the frequency when spring 2 is attached to the mass, we replace k with $k(n+1)$ to obtain

$$f_2=\frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}}=\sqrt{n+1}f=\sqrt{0.70+1.0}(200\text{ Hz})=2.6\times 10^2\text{Hz.}$$

67. The magnitude of the downhill component of the gravitational force acting on each ore car is

$$w_x=(10000\text{ kg})(9.8\text{ m/s}^2)\sin\theta$$

where θ = 30° (and it is important to have the calculator in degrees mode during this problem). We are told that a downhill pull of $3\omega_x$ causes the cable to stretch x = 0.15 m. Since the cable is expected to obey Hooke's law, its spring constant is

$$k=\frac{3w_x}{x}=9.8\times 10^5\text{ N/m.}$$

(a) Noting that the oscillating mass is that of *two* of the cars, we apply Eq. 15-12 (divided by 2π).

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{9.8\times 10^5\text{ N/m}}{20000\text{ kg}}}=1.1\text{ Hz.}$$

(b) The difference between the equilibrium positions of the end of the cable when supporting two as opposed to three cars is

$$\Delta x=\frac{3w_x-2w_x}{k}=0.050\text{ m.}$$

68. (a) Hooke's law readily yields (0.300 kg)(9.8 m/s^2)/(0.0200 m) = 147 N/m.

(b) With m = 2.00 kg, the period is $T=2\pi\sqrt{\frac{m}{k}}=0.733\text{ s}$.

69. **THINK** The piston undergoes simple harmonic motion. Given the amplitude and frequency of oscillation, its maximum speed can be readily calculated.

EXPRESS Let the amplitude be x_m. The maximum speed v_m is related to the amplitude by $v_m = \omega x_m$, where ω is the angular frequency.

ANALYZE We use $v_m = \omega x_m = 2\pi f x_m$, where the frequency is f = (180 rev)/(60 s) = 3.0 Hz and the amplitude is half the stroke, or x_m = 0.38 m. Thus,

$$v_m = 2\pi(3.0\text{ Hz})(0.38\text{ m}) = 7.2\text{ m/s}.$$

LEARN In a similar manner, the maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi(3.0\text{ Hz}))^2(0.38\text{ m}) = 135\text{ m/s}^2.$$

Acceleration is proportional to the displacement x_m in SHM.

70. (a) The rotational inertia of a hoop is $I = mR^2$, and the energy of the system becomes

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

and θ is in radians. We note that $r\omega = v$ (where $v = dx/dt$). Thus, the energy becomes

$$E = \frac{1}{2}\left(\frac{mR^2}{r^2}\right)v^2 + \frac{1}{2}kx^2$$

which looks like the energy of the simple harmonic oscillator discussed in Section15-4 if we identify the mass m in that section with the term mR^2/r^2 appearing in this problem. Making this identification, Eq. 15-12 yields

$$\omega = \sqrt{\frac{k}{mR^2/r^2}} = \frac{r}{R}\sqrt{\frac{k}{m}}.$$

(b) If $r = R$ the result of part (a) reduces to $\omega = \sqrt{k/m}$.

(c) And if $r = 0$ then $\omega = 0$ (the spring exerts no restoring torque on the wheel so that it is not brought back toward its equilibrium position).

71. Since T = 0.500 s, we note that $\omega = 2\pi/T = 4\pi$ rad/s. We work with SI units, so m = 0.0500 kg and v_m = 0.150 m/s.

(a) Since $\omega = \sqrt{k/m}$, the spring constant is

$$k = \omega^2 m = (4\pi\text{ rad/s})^2(0.0500\text{ kg}) = 7.90\text{ N/m}.$$

(b) We use the relation $v_m = x_m\omega$ and obtain

$$x_m = \frac{v_m}{\omega} = \frac{0.150}{4\pi} = 0.0119 \text{ m.}$$

(c) The frequency is $f = \omega/2\pi = 2.00$ Hz (which is equivalent to $f = 1/T$).

72. (a) We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = R = 0.126$ m. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{cm} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} = 0.873 \text{s.}$$

(b) We seek a value of $r \neq R$ such that

$$2\pi\sqrt{\frac{R^2 + 2r^2}{2gr}} = 2\pi\sqrt{\frac{3R}{2g}}$$

and are led to the quadratic formula:

$$r = \frac{3R \pm \sqrt{(3R)^2 - 8R^2}}{4} = R \quad \text{or} \quad \frac{R}{2}.$$

Thus, our result is $r = 0.126/2 = 0.0630$ m.

73. **THINK** A mass attached to the end of a vertical spring undergoes simple harmonic motion. Energy is conserved in the process.

EXPRESS The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky_0 = mg$, where $y_0 = 0.096$ m is the elongation of the spring at equilibrium, k is the spring constant, and $m = 1.3$ kg is the mass of the block. As the block oscillate, its speed is a maximum as it passes the equilibrium point, and zero at the endpoints.

ANALYZE (a) The spring constant is

$$k = mg/y_0 = (1.3 \text{ kg})(9.8 \text{ m/s}^2)/(0.096 \text{ m}) = 1.33 \times 10^2 \text{ N/m.}$$

(b) The period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.3 \text{ kg}}{133 \text{ N/m}}} = 0.62 \text{ s.}$$

(c) The frequency is $f = 1/T = 1/0.62 \text{ s} = 1.6$ Hz.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest Δy = 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) At the initial position,

$$y_i = y_0 + \Delta y = 9.6\text{ cm} + 5.0\text{ cm} = 14.6\text{ cm} = 0.146\text{ m},$$

the block is not moving but it has potential energy

$$U_i = -mgy_i + \frac{1}{2}ky_i^2 = -(1.3\text{ kg})(9.8\text{ m/s}^2)(0.146\text{ m}) + \frac{1}{2}(133\text{ N/m})(0.146\text{ m})^2 = -0.44\text{ J}.$$

When the block is at the equilibrium point, the elongation of the spring is y_0 = 9.6 cm and the potential energy is

$$U_f = -mgy_0 + \frac{1}{2}ky_0^2 = -(1.3\text{ kg})(9.8\text{ m/s}^2)(0.096\text{ m}) + \frac{1}{2}(133\text{ N/m})(0.096\text{ m})^2$$
$$= -0.61\text{ J}.$$

We write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for v:

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44\text{ J} + 0.61\text{ J})}{1.3\text{ kg}}} = 0.51\text{ m/s}.$$

LEARN Both the gravitational force and the spring force are conservative, so the work done by the forces is independent of path. By energy conservation, the kinetic energy of the block is equal to the negative of the change in potential energy of the system:

$$\begin{aligned}\Delta K = -\Delta U = -(U_f - U_i) = U_i - U_f &= -mg(y_i - y_0) + \frac{1}{2}k(y_i^2 - y_0^2)\\ &= -mg\Delta y + \frac{1}{2}k\left[(y_0 + \Delta y)^2 - y_0^2\right] = -mg\Delta y + \frac{1}{2}k\left[(\Delta y)^2 + 2y_0\Delta y\right]\\ &= \Delta y(-mg + ky_0) + \frac{1}{2}k(\Delta y)^2\\ &= \frac{1}{2}k(\Delta y)^2\end{aligned}$$

where the relation $ky_0 = mg$ was used.

74. The distance from the relaxed position of the bottom end of the spring to its equilibrium position when the body is attached is given by Hooke's law:

$$\Delta x = F/k = (0.20 \text{ kg})(9.8 \text{ m/s}^2)/(19 \text{ N/m}) = 0.103 \text{ m}.$$

(a) The body, once released, will not only fall through the Δx distance but continue through the equilibrium position to a "turning point" equally far on the other side. Thus, the total descent of the body is $2\Delta x = 0.21$ m.

(b) Since $f = \omega/2\pi$, Eq. 15-12 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.6 \text{ Hz}.$$

(c) The maximum distance from the equilibrium position gives the amplitude:

$$x_m = \Delta x = 0.10 \text{ m}.$$

75. (a) Assume the bullet becomes embedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet–block system is conserved during the collision. Let m be the mass of the bullet, M be the mass of the block, v_0 be the initial speed of the bullet, and v be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m + M)v$, so

$$v = \frac{mv_0}{m+M} = \frac{(0.050 \text{ kg})(150 \text{ m/s})}{0.050 \text{ kg} + 4.0 \text{ kg}} = 1.85 \text{ m/s}.$$

When the block is in its initial position the spring and gravitational forces balance, so the spring is elongated by Mg/k. After the collision, however, the block oscillates with simple harmonic motion about the point where the spring and gravitational forces balance with the bullet embedded. At this point the spring is elongated a distance

$$\ell = (M+m)g/k,$$

somewhat different from the initial elongation. Mechanical energy is conserved during the oscillation. At the initial position, just after the bullet is embedded, the kinetic energy is $\frac{1}{2}(M+m)v^2$ and the elastic potential energy is $\frac{1}{2}k(Mg/k)^2$. We take the gravitational potential energy to be zero at this point. When the block and bullet reach the highest point in their motion the kinetic energy is zero. The block is then a distance y_m above the position where the spring and gravitational forces balance. Note that y_m is the amplitude of the motion. The spring is compressed by $y_m - \ell$, so the elastic potential energy is $\frac{1}{2}k(y_m - \ell)^2$. The gravitational potential energy is $(M + m)gy_m$. Conservation of mechanical energy yields

$$\frac{1}{2}(M+m)v^2 + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 = \frac{1}{2}k(y_m - \ell)^2 + (M+m)gy_m.$$

We substitute $\ell = (M+m)g/k$. Algebraic manipulation leads to

$$y_m = \sqrt{\frac{(m+M)v^2}{k} - \frac{mg^2}{k^2}(2M+m)}$$

$$= \sqrt{\frac{(0.050 \text{ kg} + 4.0 \text{ kg})(1.85 \text{ m/s})^2}{500 \text{ N/m}} - \frac{(0.050 \text{ kg})(9.8 \text{ m/s}^2)^2}{(500 \text{ N/m})^2}\,[2(4.0 \text{ kg}) + 0.050 \text{ kg}]}$$

$= 0.166$ m.

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050 \text{ kg})(150 \text{ m/s})^2 = 563 \text{ J}$. The kinetic energy of the bullet–block system just after the collision is

$$E = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.050 \text{ kg} + 4.0 \text{ kg})(1.85 \text{ m/s})^2 = 6.94 \text{ J}.$$

Since the block does not move significantly during the collision, the elastic and gravitational potential energies do not change. Thus, E is the energy that is transferred. The ratio is

$$E/E_0 = (6.94 \text{ J})/(563 \text{ J}) = 0.0123 \text{ or } 1.23\%.$$

76. (a) We note that

$$\omega = \sqrt{k/m} = \sqrt{1500/0.055} = 165.1 \text{ rad/s}.$$

We consider the most direct path in each part of this problem. That is, we consider in part (a) the motion directly from $x_1 = +0.800x_m$ at time t_1 to $x_2 = +0.600x_m$ at time t_2 (as opposed to, say, the block moving from $x_1 = +0.800x_m$ through $x = +0.600x_m$, through $x = 0$, reaching $x = -x_m$ and after returning back through $x = 0$ then getting to $x_2 = +0.600x_m$). Equation 15-3 leads to

$$\omega t_1 + \phi = \cos^{-1}(0.800) = 0.6435 \text{ rad}$$

$$\omega t_2 + \phi = \cos^{-1}(0.600) = 0.9272 \text{ rad} .$$

Subtracting the first of these equations from the second leads to

$$\omega(t_2 - t_1) = 0.9272 - 0.6435 = 0.2838 \text{ rad} .$$

Using the value for ω computed earlier, we find $t_2 - t_1 = 1.72 \times 10^{-3}$ s.

(b) Let t_3 be when the block reaches $x = -0.800x_m$ in the direct sense discussed above. Then the reasoning used in part (a) leads here to

$$\omega(t_3 - t_1) = (\,2.4981 - 0.6435) \text{ rad} = 1.8546 \text{ rad}$$

and thus to $t_3 - t_1 = 11.2 \times 10^{-3}$ s.

77. (a) From the graph, we find $x_m = 7.0$ cm = 0.070 m, and $T = 40$ ms = 0.040 s. Thus, the angular frequency is $\omega = 2\pi/T = 157$ rad/s. Using $m = 0.020$ kg, the maximum kinetic energy is then $\frac{1}{2}mv^2 = \frac{1}{2}m\,\omega^2 x_m^2 = 1.2$ J.

(b) Using Eq. 15-5, we have $f = \omega/2\pi = 50$ oscillations per second. Of course, Eq. 15-2 can also be used for this.

78. (a) From the graph we see that $x_m = 7.0$ cm = 0.070 m and $T = 40$ ms = 0.040 s. The maximum speed is $x_m\omega = x_m 2\pi/T = 11$ m/s.

(b) The maximum acceleration is $x_m\omega^2 = x_m(2\pi/T)^2 = 1.7 \times 10^3$ m/s^2.

79. Setting 15 mJ (0.015 J) equal to the maximum kinetic energy leads to $v_{\max} = 0.387$ m/s. Then one can use either an "exact" approach using $v_{\max} = \sqrt{2gL(1-\cos\theta_{\max})}$ or the "SHM" approach where

$$v_{\max} = L\omega_{\max} = L\omega\theta_{\max} = L\sqrt{g/L}\;\theta_{\max}$$

to find L. Both approaches lead to $L = 1.53$ m.

80. Its total mechanical energy is equal to its maximum potential energy $\frac{1}{2}kx_m^2$, and its potential energy at $t = 0$ is $\frac{1}{2}kx_o^2$ where $x_o = x_m\cos(\pi/5)$ in this problem. The ratio is therefore $\cos^2(\pi/5) = 0.655 = 65.5\%$.

81. (a) From the graph, it is clear that $x_m = 0.30$ m.

(b) With $F = -kx$, we see k is the (negative) slope of the graph — which is 75/0.30 = 250 N/m. Plugging this into Eq. 15-13 yields

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.50\text{ kg}}{250\text{ N/m}}} = 0.28\text{ s}.$$

(c) As discussed in Section 15-2, the maximum acceleration is

$$a_m = \omega^2 x_m = \left(\frac{k}{m}\right)x_m = \left(\frac{250\text{ N/m}}{0.50\text{ kg}}\right)(0.30\text{ m}) = 1.5\times10^2\text{ m/s}^2.$$

Alternatively, we could arrive at this result using $a_m = (2\pi/T)^2\,x_m$.

(d) Also in Section 15-2 is $v_m = \omega x_m$ so that the maximum kinetic energy is

$$K_m = \frac{1}{2}mv_m^2 = \frac{1}{2}m\omega^2 x_m^2 = \frac{1}{2}kx_m^2 = \frac{1}{2}(250 \text{ N/m})(0.30 \text{ m})^2 = 11.3 \text{ J} \approx 11 \text{ J}.$$

We note that the above manipulation reproduces the notion of energy conservation for this system (maximum kinetic energy being equal to the maximum potential energy).

82. Since the centripetal acceleration is horizontal and Earth's gravitational $\vec{g}$ is downward, we can define the magnitude of an "effective" gravitational acceleration using the Pythagorean theorem:

$$g_{eff} = \sqrt{g^2 + (v^2/R)^2}.$$

Then, since frequency is the reciprocal of the period, Eq. 15-28 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{eff}}{L}} = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + v^4/R^2}}{L}}.$$

With v = 70 m/s, R = 50 m, and L = 0.20 m, we have $f \approx 3.5 \text{ s}^{-1} = 3.5 \text{ Hz}$.

83. (a) Hooke's law readily yields

$$k = (15 \text{ kg})(9.8 \text{ m/s}^2)/(0.12 \text{ m}) = 1225 \text{ N/m}.$$

Rounding to three significant figures, the spring constant is therefore 1.23 kN/m.

(b) We are told f = 2.00 Hz = 2.00 cycles/sec. Since a cycle is equivalent to 2π radians, we have ω = 2π(2.00) = 4π rad/s (understood to be valid to three significant figures). Using Eq. 15-12, we find

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow m = \frac{1225 \text{ N/m}}{(4\pi \text{ rad/s})^2} = 7.76 \text{ kg}.$$

Consequently, the weight of the package is mg = 76.0 N.

84. (a) Comparing with Eq. 15-3, we see ω = 10 rad/s in this problem. Thus, $f = \omega/2\pi$ = 1.6 Hz.

(b) Since $v_m = \omega x_m$ and x_m = 10 cm (see Eq. 15-3), then v_m = (10 rad/s)(10 cm) = 100 cm/s or 1.0 m/s.

(c) The maximum occurs at t = 0.

(d) Since $a_m = \omega^2 x_m$, then $v_m = (10 \text{ rad/s})^2(10 \text{ cm}) = 1000 \text{ cm/s}^2$ or 10 m/s^2.

(e) The acceleration extremes occur at the displacement extremes: $x = \pm x_m$ or $x = \pm 10$ cm.

(f) Using Eq. 15-12, we find

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = (0.10 \text{ kg})(10 \text{ rad/s})^2 = 10 \text{ N/m}.$$

Thus, Hooke's law gives $F = -kx = -10x$ in SI units.

85. Using $\Delta m = 2.0$ kg, $T_1 = 2.0$ s and $T_2 = 3.0$ s, we write

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m+\Delta m}{k}}.$$

Dividing one relation by the other, we obtain

$$\frac{T_2}{T_1} = \sqrt{\frac{m+\Delta m}{m}}$$

which (after squaring both sides) simplifies to $m = \frac{\Delta m}{(T_2/T_1)^2 - 1} = 1.6 \text{kg}.$

86. (a) The amplitude of the acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \left(2\pi(1000 \text{ Hz})\right)^2 (0.00040 \text{ m}) = 1.6 \times 10^4 \text{ m/s}^2.$$

(b) Similarly, in the discussion after Eq. 15-6, we find $v_m = \omega x_m$ so that

$$v_m = \left(2\pi(1000 \text{ Hz})\right)(0.00040 \text{ m}) = 2.5 \text{ m/s}.$$

(c) From Eq. 15-8, we have (in absolute value)

$$|a| = \left(2\pi(1000 \text{ Hz})\right)^2 (0.00020 \text{ m}) = 7.9 \times 10^3 \text{ m/s}^2.$$

(d) This can be approached with the energy methods of Section 15-4, but here we will use trigonometric relations along with Eq. 15-3 and Eq. 15-6. Thus, allowing for both roots stemming from the square root,

$$\sin(\omega t + \phi) = \pm\sqrt{1 - \cos^2(\omega t + \phi)} \quad \Rightarrow \quad -\frac{v}{\omega x_m} = \pm\sqrt{1 - \frac{x^2}{x_m^2}}.$$

Taking absolute values and simplifying, we obtain

$$|v| = 2\pi f\sqrt{x_m^2 - x^2} = 2\pi\left(1000\right)\sqrt{0.00040^2 - 0.00020^2} = 2.2 \text{ m/s}.$$

87. (a) The rotational inertia is $I = \frac{1}{2}MR^2 = \frac{1}{2}(3.00 \text{ kg})(0.700 \text{ m})^2 = 0.735 \text{ kg}\cdot\text{m}^2$.

(b) Using Eq. 15-22 (in absolute value), we find

$$\kappa = \frac{\tau}{\theta} = \frac{0.0600 \text{ N}\cdot\text{m}}{2.5 \text{ rad}} = 0.0240 \text{ N}\cdot\text{m/rad}.$$

(c) Using Eq. 15-5, Eq. 15-23 leads to

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{0.024\text{N}\cdot\text{m/rad}}{0.735\text{kg}\cdot\text{m}^2}} = 0.181 \text{ rad/s}.$$

88. (a) The Hooke's law force (of magnitude (100)(0.30) = 30 N) is directed upward and the weight (20 N) is downward. Thus, the net force is 10 N upward.

(b) The equilibrium position is where the upward Hooke's law force balances the weight, which corresponds to the spring being stretched (from unstretched length) by 20 N/100 N/m = 0.20 m. Thus, relative to the equilibrium position, the block (at the instant described in part (a)) is at what one might call *the bottom turning point* (since $v = 0$) at $x = -x_m$ where the amplitude is $x_m = 0.30 - 0.20 = 0.10$ m.

(c) Using Eq. 15-13, we have

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(20 \text{ N})/(9.8 \text{ m/s}^2)}{100 \text{ N/m}}} = 0.90 \text{ s}.$$

(d) The maximum kinetic energy is equal to the maximum potential energy $\frac{1}{2}kx_m^2$. Thus,

$$K_m = U_m = \frac{1}{2}\left(100 \text{ N/m}\right)\left(0.10 \text{ m}\right)^2 = 0.50 \text{ J}.$$

89. (a) We require $U = \frac{1}{2}E$ at some value of x. Using Eq. 15-21, this becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kx_m^2\right) \Rightarrow x = \frac{x_m}{\sqrt{2}}.$$

We compare the given expression x as a function of t with Eq. 15-3 and find $x_m = 5.0$ m. Thus, the value of x we seek is $x = 5.0/\sqrt{2} \approx 3.5$ m.

(b) We solve the given expression (with $x = 5.0/\sqrt{2}$), making sure our calculator is in radians mode:

$$t = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 1.54 \text{ s.}$$

Since we are asked for the interval $t_{eq} - t$ where t_{eq} specifies the instant the particle passes through the equilibrium position, then we set $x = 0$ and find

$$t_{eq} = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}(0) = 2.29 \text{ s.}$$

Consequently, the time interval is $t_{eq} - t = 0.75$ s.

90. Since the particle has zero speed (momentarily) at $x \neq 0$, then it must be at its turning point; thus, $x_o = x_m = 0.37$ cm. It is straightforward to infer from this that the phase constant ϕ in Eq. 15-2 is zero. Also, $f = 0.25$ Hz is given, so we have $\omega = 2\pi f = \pi/2$ rad/s. The variable t is understood to take values in seconds.

(a) The period is $T = 1/f = 4.0$ s.

(b) As noted above, $\omega = \pi/2$ rad/s.

(c) The amplitude, as observed above, is 0.37 cm.

(d) Equation 15-3 becomes $x = (0.37 \text{ cm}) \cos(\pi t/2)$.

(e) The derivative of x is $v = -(0.37 \text{ cm/s})(\pi/2) \sin(\pi t/2) \approx (-0.58 \text{ cm/s}) \sin(\pi t/2)$.

(f) From the previous part, we conclude $v_m = 0.58$ cm/s.

(g) The acceleration-amplitude is $a_m = \omega^2 x_m = 0.91 \text{ cm/s}^2$.

(h) Making sure our calculator is in radians mode, we find $x = (0.37) \cos(\pi(3.0)/2) = 0$. It is important to avoid rounding off the value of π in order to get precisely zero, here.

(i) With our calculator still in radians mode, we obtain $v = -(0.58 \text{ cm/s})\sin(\pi(3.0)/2) = 0.58$ cm/s.

91. **THINK** This problem explores the oscillation frequency of a pendulum under various accelerating conditions.

EXPRESS In a room, the frequency for small amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum. Inside an elevator, the forces acting on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $m\vec{g}$. Newton's second law yields $\vec{T} + m\vec{g} = m\vec{a}$, where m is the mass and $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a} = \vec{a}_e + \vec{a}'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g} - \vec{a}_e) + \vec{T} = m\vec{a}'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}_e$.

ANALYZE (a) With $L = 2.0$ m, we find the frequency of the pendulum in a room to be

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\ \text{m/s}^2}{2.0\ \text{m}}} = 0.35\ \text{Hz}.$$

(b) With the elevator accelerating upward, $\vec{g}$ and $\vec{a}_e$ are along the same line but in opposite directions, we can find the frequency for small amplitude oscillations by replacing g with the effective gravitational acceleration $g_{\text{eff}} = g + a_e$ in the expression $f = (1/2\pi)\sqrt{g/L}$. Thus,

$$f = \frac{1}{2\pi}\sqrt{\frac{g + a_e}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8\ \text{m/s}^2 + 2.0\ \text{m/s}^2}{2.0\ \text{m}}} = 0.39\ \text{Hz}.$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is, $\vec{g} - \vec{a}_e = 0$. To find the frequency for small amplitude oscillations, replace g with zero in $f = (1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

LEARN The frequency of the pendulum increases as g_{eff} increases.

92. The period formula, Eq. 15-29, requires knowing the distance h from the axis of rotation and the center of mass of the system. We also need the rotational inertia I about the axis of rotation. From the figure, we see $h = L + R$ where $R = 0.15$ m. Using the parallel-axis theorem, we find

$$I = \frac{1}{2}MR^2 + M\left(L + R\right)^2,$$

where $M = 1.0$ kg. Thus, Eq. 15-29, with $T = 2.0$ s, leads to

$$2.0 = 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + M(L+R)^2}{Mg(L+R)}}$$

which leads to $L = 0.8315$ m.

93. (a) Hooke's law provides the spring constant:

$$k = (4.00\ \text{kg})(9.8\ \text{m/s}^2)/(0.160\ \text{m}) = 245\ \text{N/m}.$$

(b) The attached mass is m = 0.500 kg. Consequently, Eq. 15-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.500}{245}} = 0.284 \text{ s}.$$

94. We note (from the graph) that $a_m = \omega^2 x_m$ = 4.00 cm/s^2. Also, the value at t = 0 is a_o = 1.00 cm/s^2. Then Eq. 15-7 leads to

$$\phi = \cos^{-1}(-1.00/4.00) = +1.82 \text{ rad or } -4.46 \text{ rad}.$$

The other "root" (+4.46 rad) can be rejected on the grounds that it would lead to a negative slope at t = 0.

95. The time for one cycle is T = (50 s)/20 = 2.5 s. Thus, from Eq. 15-23, we find

$$I = \kappa\left(\frac{T}{2\pi}\right)^2 = (0.50)\left(\frac{2.5}{2\pi}\right)^2 = 0.079 \text{ kg}\cdot\text{m}^2.$$

96. The angular frequency of the simple harmonic oscillation is given by Eq. 15-13:

$$\omega = \sqrt{\frac{k}{m}}.$$

Thus, for two different masses m_1 and m_2, with the same spring constant k, the ratio of the frequencies would be

$$\frac{\omega_1}{\omega_2} = \frac{\sqrt{k/m_1}}{\sqrt{k/m_2}} = \sqrt{\frac{m_2}{m_1}}.$$

In our case, with $m_1 = m$ and $m_2 = 2.5m$, the ratio is $\frac{\omega_1}{\omega_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{2.5} = 1.58$.

97. (a) The graphs suggest that T = 0.40 s and κ = 4/0.2 = 0.02 N·m/rad. With these values, Eq. 15-23 can be used to determine the rotational inertia:

$$I = \kappa T^2/4\pi^2 = 8.11\times 10^{-5} \text{ kg}\cdot\text{m}^2.$$

(b) We note (from the graph) that θ_{max} = 0.20 rad. Setting the maximum kinetic energy ($\frac{1}{2}I\omega_{max}{}^2$) equal to the maximum potential energy (see the hint in the problem) leads to $\omega_{max} = \theta_{max}\sqrt{\kappa/I}$ = 3.14 rad/s.

98. (a) Hooke's law provides the spring constant: k = (20 N)/(0.20 m) = 1.0×10^2 N/m.

(b) The attached mass is $m = (5.0 \text{ N})/(9.8 \text{ m/s}^2) = 0.51$ kg. Consequently, Eq. 15-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.51\,\text{kg}}{100\,\text{N/m}}} = 0.45\,\text{s}.$$

99. For simple harmonic motion, Eq. 15-24 must reduce to

$$\tau = -L(F_g \sin\theta) \to -L(F_g\theta)$$

where θ is in radians. We take the percent difference (in absolute value)

$$\left|\frac{(-LF_g\sin\theta) - (-LF_g\theta)}{-LF_g\sin\theta}\right| = \left|1 - \frac{\theta}{\sin\theta}\right|$$

and set this equal to 0.010 (corresponding to 1.0%). In order to solve for θ (since this is not possible "in closed form"), several approaches are available. Some calculators have built-in numerical routines to facilitate this, and most math software packages have this capability. Alternatively, we could expand $\sin\theta \approx \theta - \theta^3/6$ (valid for small θ) and thereby find an approximate solution (which, in turn, might provide a seed value for a numerical search). Here we show the latter approach:

$$\left|1 - \frac{\theta}{\theta - \theta^3/6}\right| \approx 0.010 \Rightarrow \frac{1}{1-\theta^2/6} \approx 1.010$$

which leads to $\theta \approx \sqrt{6(0.01/1.01)} = 0.24 \text{ rad} = 14.0°$. A more accurate value (found numerically) for θ that results in a 1.0% deviation is 13.986°.

100. (a) The potential energy at the turning point is equal (in the absence of friction) to the total kinetic energy (translational plus rotational) as it passes through the equilibrium position:

$$\frac{1}{2}kx_m^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}^2\omega^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$$
$$= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{4}Mv_{\text{cm}}^2 = \frac{3}{4}Mv_{\text{cm}}^2$$

which leads to $Mv_{\text{cm}}^2 = 2kx_m^2/3 = 0.125$ J. The translational kinetic energy is therefore $\frac{1}{2}Mv_{\text{cm}}^2 = kx_m^2/3 = 0.0625\,\text{J}$.

(b) And the rotational kinetic energy is $\frac{1}{4}Mv_{\text{cm}}^2 = kx_m^2/6 = 0.03125\text{J} \approx 3.13\times10^{-2}$ J .

(c) In this part, we use v_{cm} to denote the speed at any instant (and not just the maximum speed as we had done in the previous parts). Since the energy is constant, then

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{3}{4}Mv_{cm}^2\right) + \frac{d}{dt}\left(\frac{1}{2}kx^2\right) = \frac{3}{2}Mv_{cm}a_{cm} + kxv_{cm} = 0$$

which leads to

$$a_{cm} = -\left(\frac{2k}{3M}\right)x.$$

Comparing with Eq. 15-8, we see that $\omega = \sqrt{2k/3M}$ for this system. Since $\omega = 2\pi/T$, we obtain the desired result: $T = 2\pi\sqrt{3M/2k}$.

101. **THINK** The block is in simple harmonic motion, so its position relative to the equilibrium position can be written as $x(t) = x_m \cos(\omega t + \phi)$.

EXPRESS The speed of the block is

$$v(t) = dx/dt = -\omega x_m \sin(\omega t + \phi).$$

For a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given v = 5.2 m/s at x = 0 is the maximum value $v_m = \omega x_m$ where

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{480\text{ N/m}}{1.2\text{ kg}}} = 20\text{ rad/s}.$$

ANALYZE (a) Since $\omega = 2\pi f$, we find f = 3.2 Hz.

(b) We have v_m = 5.2 m/s = ωx_m = (20 rad/s)x_m, which leads to x_m = 0.26 m.

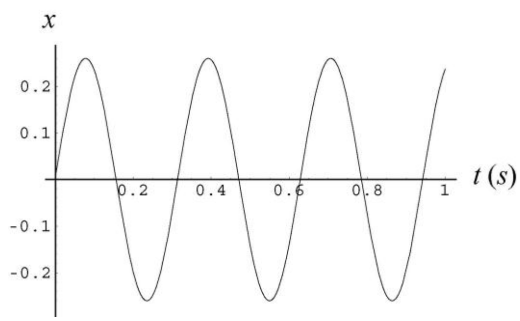
(c) With meters, seconds and radians understood,

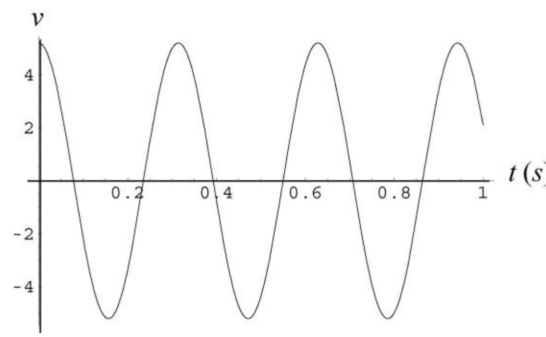
$$x = (0.26\text{ m})\cos(20t + \phi)$$
$$v = -(5.2\text{ m/s})\sin(20t + \phi).$$

The requirement that x = 0 at t = 0 implies (from the first equation above) that either ϕ = $+\pi/2$ or $\phi = -\pi/2$. Only one of these choices meets the further requirement that $v > 0$ when t = 0; that choice is $\phi = -\pi/2$. Therefore,

$$x = (0.26\text{ m})\cos\left(20t - \frac{\pi}{2}\right) = (0.26\text{ m})\sin(20t).$$

LEARN The plots of x and v as a function of time are given next:





102. (a) Equation 15-21 leads to

$$E = \frac{1}{2}kx_m^2 \;\Rightarrow\; x_m = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(4.0\text{ J})}{200\text{ N/m}}} = 0.20\text{ m}.$$

(b) Since $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.80\text{ kg}/200\text{ N/m}} \approx 0.4\text{ s}$, then the block completes 10/0.4 = 25 cycles during the specified interval.

(c) The maximum kinetic energy is the total energy, 4.0 J.

(d) This can be approached more than one way; we choose to use energy conservation:

$$E = K + U \Rightarrow 4.0 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

Therefore, when x = 0.15 m, we find v = 2.1 m/s.

103. (a) By Eq. 15-13, the mass of the block is

$$m_b = \frac{kT_0^2}{4\pi^2} = 2.43\text{ kg}.$$

Therefore, with m_p = 0.50 kg, the new period is

$$T = 2\pi\sqrt{\frac{m_p + m_b}{k}} = 0.44\text{ s}.$$

(b) The speed before the collision (since it is at its maximum, passing through equilibrium) is $v_0 = x_m\omega_0$ where $\omega_0 = 2\pi/T_0$; thus, v_0 = 3.14 m/s. Using momentum conservation (along the horizontal direction) we find the speed after the collision:

$$V = v_0\frac{m_b}{m_p + m_b} = 2.61\text{ m/s}.$$

The equilibrium position has not changed, so (for the new system of greater mass) this represents the maximum speed value for the subsequent harmonic motion: $V = x'_m\omega$ where $\omega = 2\pi/T = 14.3$ rad/s. Therefore, $x'_m = 0.18$ m.

104. (a) We are told that when $t = 4T$, with $T = 2\pi/\omega' \approx 2\pi\sqrt{m/k}$ (neglecting the second term in Eq. 15-43),

$$e^{-bt/2m} = \frac{3}{4}.$$

Thus,

$$T \approx 2\pi\sqrt{(2.00\text{kg})/(10.0\,\text{N/m})} = 2.81\,\text{s}$$

and we find

$$\frac{b(4T)}{2m} = \ln\left(\frac{4}{3}\right) = 0.288 \quad\Rightarrow\quad b = \frac{2(2.00\text{ kg})(0.288)}{4(2.81\,\text{s})} = 0.102\text{ kg/s}.$$

(b) Initially, the energy is $E_o = \frac{1}{2}kx_{mo}^2 = \frac{1}{2}(10.0)(0.250)^2 = 0.313$ J. At $t = 4T$,

$$E = \tfrac{1}{2}k(\tfrac{3}{4}x_{mo})^2 = 0.176\text{ J}.$$

Therefore, $E_o - E = 0.137$ J.

105. (a) From Eq. 16-12, $T = 2\pi\sqrt{m/k} = 0.45$ s.

(b) For a vertical spring, the distance between the unstretched length and the equilibrium length (with a mass m attached) is mg/k, where in this problem $mg = 10$ N and $k = 200$ N/m (so that the distance is 0.05 m). During simple harmonic motion, the convention is to establish $x = 0$ at the equilibrium length (the middle level for the oscillation) and to write the total energy without any gravity term; that is, $E = K + U$, where $U = kx^2/2$. Thus, as the block passes through the unstretched position, the energy is

$$E = 2.0 + \tfrac{1}{2}k(0.05)^2 = 2.25\text{ J}.$$

At its topmost and bottommost points of oscillation, the energy (using this convention) is all elastic potential: $\frac{1}{2}kx_m^2$. Therefore, by energy conservation,

$$2.25 = \frac{1}{2}kx_m^2 \Rightarrow x_m = \pm 0.15\text{ m}.$$

This gives the amplitude of oscillation as 0.15 m, but how far are these points from the *unstretched* position? We add (or subtract) the 0.05 m value found above and obtain 0.10 m for the top-most position and 0.20 m for the bottom-most position.

(c) As noted in part (b), $x_m = \pm 0.15$ m.

(d) The maximum kinetic energy equals the maximum potential energy (found in part (b)) and is equal to 2.25 J.

106. (a) The graph makes it clear that the period is $T = 0.20$ s.

(b) The period of the simple harmonic oscillator is given by Eq. 15-13: $T = 2\pi\sqrt{\frac{m}{k}}$.

Thus, using the result from part (a) with $k = 200$ N/m, we obtain $m = 0.203 \approx 0.20$ kg.

(c) The graph indicates that the speed is (momentarily) zero at $t = 0$, which implies that the block is at $x_0 = \pm x_m$. From the graph we also note that the slope of the velocity curve (hence, the acceleration) is positive at $t = 0$, which implies (from $ma = -kx$) that the value of x is negative. Therefore, with $x_m = 0.20$ m, we obtain $x_0 = -0.20$ m.

(d) We note from the graph that $v = 0$ at $t = 0.10$ s, which implied $a = \pm a_m = \pm\omega^2 x_m$. Since acceleration is the instantaneous slope of the velocity graph, then (looking again at the graph) we choose the negative sign. Recalling $\omega^2 = k/m$ we obtain $a = -197 \approx -2.0 \times 10^2$ m/s^2.

(e) The graph shows $v_m = 6.28$ m/s, so $K_m = \frac{1}{2}mv_m^2 = \frac{1}{2}(0.20\text{ kg})(6.28\text{ m/s})^2 = 4.0$ J.

107. The mass is $m = \frac{0.108\text{ kg}}{6.02\times10^{23}} = 1.8\times10^{-25}$ kg. Using Eq. 15-12 and the fact that $f = \omega/2\pi$, we have

$$1\times10^{13}\text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = \left(2\pi\times10^{13}\right)^2\left(1.8\times10^{-25}\right) \approx 7\times10^2\text{ N/m}.$$

108. Using Hooke's law, we have $mg = k\Delta y = kh.$ The frequency of oscillation for the mass-spring system is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Similarly, the frequency of oscillation for a simple pendulum is

$$f' = \frac{1}{T'} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

If $f = f'$, then $\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, which gives

$$L = \frac{mg}{k} = \frac{kh}{k} = h = 2.00\text{ cm}.$$

109. The rotational inertia for an axis through A is $I_A = I_{cm} + mh_A^2$ and that for an axis through B is $I_B = I_{cm} + mh_B^2$, where h_A and h_B are distances from A and B to the center of mass. Using Eq. 15-29, $T = 2\pi\sqrt{I/mgh}$, we require

$$T_A = T_B \qquad \Rightarrow \qquad 2\pi\sqrt{\frac{I_{cm} + mh_A^2}{mgh_A}} = 2\pi\sqrt{\frac{I_{cm} + mh_B^2}{mgh_B}}$$

which (after canceling 2π and squaring both sides) becomes

$$\frac{I_{cm} + mh_A^2}{mgh_A} = \frac{I_{cm} + mh_B^2}{mgh_B}.$$

Cross-multiplying and rearranging, we obtain

$$I_{cm}(h_B - h_A) = m(h_A h_B^2 - h_B h_A^2) = mh_A h_B (h_B - h_A)$$

which simplifies to $I_{cm} = mh_A h_B$. We plug this back into the first period formula above and obtain

$$T = 2\pi\sqrt{\frac{mh_A h_B + mh_A^2}{mgh_A}} = 2\pi\sqrt{\frac{h_B + h_A}{g}}.$$

From the figure, we see that $h_B + h_A = L$, and (after squaring both sides) we can solve the above equation for L:

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.8\ \text{m/s}^2)(1.80\ \text{s})^2}{4\pi^2} = 0.804\ \text{m}.$$

110. Since d_m is the amplitude of oscillation, then the maximum acceleration being set to $0.2g$ provides the condition: $\omega^2 d_m = 0.2g$. Since d_s is the amount the spring stretched in order to achieve vertical equilibrium of forces, then we have the condition $kd_s = mg$. Since we can write this latter condition as $m\omega^2 d_s = mg$, then $\omega^2 = g/d_s$. Plugging this into our first condition, we obtain

$$d_s = d_m/0.2 = (10\ \text{cm})/0.2 = 50\ \text{cm}.$$

111. Using Eq. 15-12, we find $\omega = \sqrt{k/m} = 10\ \text{rad/s}$. We also use $v_m = x_m\omega$ and $a_m = x_m\omega^2$.

(a) The amplitude (meaning "displacement amplitude") is $x_m = v_m/\omega = 3/10 = 0.30$ m.

(b) The acceleration-amplitude is $a_m = (0.30\ \text{m})(10\ \text{rad/s})^2 = 30\ \text{m/s}^2$.

(c) One interpretation of this question is "what is the most negative value of the acceleration?" in which case the answer is $-a_m = -30$ m/s^2. Another interpretation is "what is the smallest value of the absolute-value of the acceleration?" in which case the answer is zero.

(d) Since the period is $T = 2\pi/\omega = 0.628$ s. Therefore, seven cycles of the motion requires $t = 7T = 4.4$ s.

112. (a) Eq. 15-28 gives

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{17m}{9.8\,\mathrm{m/s^2}}} = 8.3\ \mathrm{s}.$$

(b) Plugging $I = mL^2$ into Eq. 15-25, we see that the mass m cancels out. Thus, the characteristics (such as the period) of the periodic motion do not depend on the mass.

113. (a) The net horizontal force is F since the batter is assumed to exert no horizontal force on the bat. Thus, the horizontal acceleration (which applies as long as F acts on the bat) is $a = F/m$.

(b) The only torque on the system is that due to F, which is exerted at P, at a distance $L_o - \frac{1}{2}L$ from C. Since $L_o = 2L/3$ (see Sample Problem 15-5), then the distance from C to P is $\frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. Since the net torque is equal to the rotational inertia ($I = 1/12mL^2$ about the center of mass) multiplied by the angular acceleration, we obtain

$$\alpha = \frac{\tau}{I} = \frac{F\left(\frac{1}{6}L\right)}{\frac{1}{12}mL^2} = \frac{2F}{mL}.$$

(c) The distance from C to O is $r = L/2$, so the contribution to the acceleration at O stemming from the angular acceleration (in the counterclockwise direction of Fig. 15-13) is $\alpha r = \frac{1}{2}\alpha L$ (leftward in that figure). Also, the contribution to the acceleration at O due to the result of part (a) is F/m (rightward in that figure). Thus, if we choose rightward as positive, then the net acceleration of O is

$$a_O = \frac{F}{m} - \frac{1}{2}\alpha L = \frac{F}{m} - \frac{1}{2}\left(\frac{2F}{mL}\right)L = 0.$$

(d) Point O stays relatively stationary in the batting process, and that might be possible due to a force exerted by the batter or due to a finely tuned cancellation such as we have shown here. We assumed that the batter exerted no force, and our first expectation is that the impulse delivered by the impact would make all points on the bat go into motion, but for this particular choice of impact point, we have seen that the point being held by the batter is naturally stationary and exerts no force on the batter's hands which would otherwise have to "fight" to keep a good hold of it.

114. (a) By energy conservation, the required elastic potential energy stored in the spring is $\frac{1}{2}k(\Delta y)^2 = \frac{1}{2}mv_{\text{esc}}^2$. Solving for k, we obtain

$$k = \frac{mv_{\text{esc}}^2}{(\Delta y)^2} = \frac{(0.170\text{ kg})(11.2\times 10^3\text{ m/s})}{(2.30\text{ m})^2} = 4.03\times 10^6\text{ N/m}.$$

(b) The total applied force on the spring is

$$F_a = k(\Delta y) = (4.03\times 10^6\text{ N/m})(2.30\text{ m}) = 9.27\times 10^6\text{ N}.$$

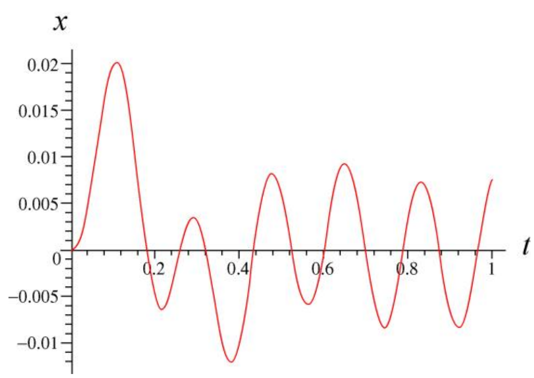
Thus, the number of people needed to exert this force is

$$\frac{F_a}{F_1} = \frac{9.27\times 10^6\text{ N}}{490\text{ N}} = 1.89\times 10^4.$$

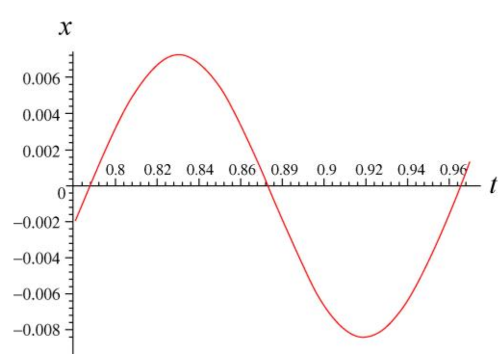
115. The period of oscillation is $T = 2\pi\sqrt{L/g} = 3.2$ s. Thus, the length for this simple pendulum is

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\text{ m})(3.20\text{ s})^2}{4\pi^2} = 2.54\text{ m}.$$

116. (a) A plot of x versus t (in SI units) is shown below:

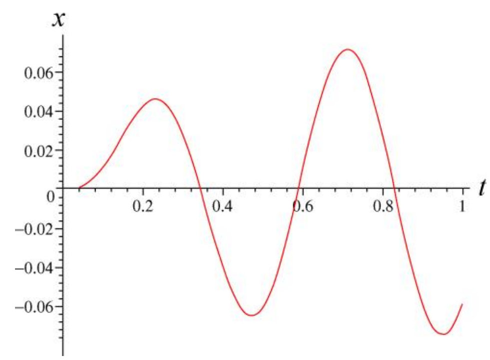


If we expand the plot near the end of that time interval we have



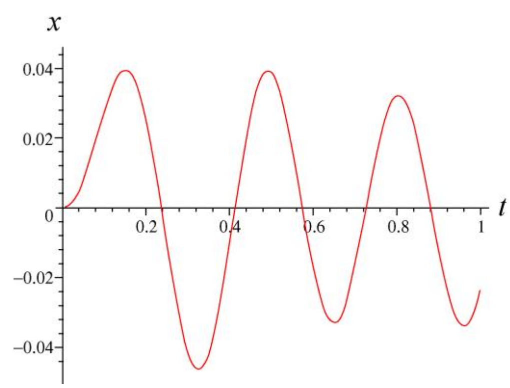
This is close enough to a regular sine wave cycle that we can estimate its period (T = 0.18 s, so ω = 35 rad/s) and its amplitude (y_m = 0.008 m).

(b) Now, with the new driving frequency (ω_d = 13.2 rad/s), the x versus t graph (for the first one second of motion) is as shown below:



It is a little more difficult in this case to estimate a regular sine-curve-like amplitude and period (for the part of the above graph near the end of that time interval), but we arrive at roughly y_m = 0.07 m, T = 0.48 s, and ω = 13 rad/s.

(c) Now, with ω_d = 20 rad/s, we obtain (for the behavior of the graph, below, near the end of the interval) the estimates: y_m = 0.03 m, T = 0.31 s, and ω = 20 rad/s.



Chapter 16

1. Let y_1 = 2.0 mm (corresponding to time t_1) and y_2 = –2.0 mm (corresponding to time t_2). Then we find

$$kx + 600t_1 + \phi = \sin^{-1}(2.0/6.0)$$

and

$$kx + 600t_2 + \phi = \sin^{-1}(-2.0/6.0) \; .$$

Subtracting equations gives

$$600(t_1 - t_2) = \sin^{-1}(2.0/6.0) - \sin^{-1}(-2.0/6.0).$$

Thus we find $t_1 - t_2$ = 0.011 s (or 1.1 ms).

2. (a) The speed of the wave is the distance divided by the required time. Thus,

$$v = \frac{853 \text{ seats}}{39 \text{ s}} = 21.87 \text{ seats/s} \approx 22 \text{ seats/s} \, .$$

(b) The width w is equal to the distance the wave has moved during the average time required by a spectator to stand and then sit. Thus,

$$w = vt = (21.87 \text{ seats/s})(1.8 \text{ s}) \approx 39 \text{ seats} \, .$$

3. (a) The angular wave number is $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{1.80\,\text{m}} = 3.49\,\text{m}^{-1}$.

(b) The speed of the wave is $v = \lambda f = \dfrac{\lambda\omega}{2\pi} = \dfrac{(1.80\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\,\text{m/s}$.

4. The distance d between the beetle and the scorpion is related to the transverse speed v_t and longitudinal speed v_ℓ as

$$d = v_t t_t = v_\ell t_\ell$$

where t_t and t_ℓ are the arrival times of the wave in the transverse and longitudinal directions, respectively. With $v_t = 50$ m/s and $v_\ell = 150$ m/s, we have

$$\frac{t_t}{t_\ell} = \frac{v_\ell}{v_t} = \frac{150\text{ m/s}}{50\text{ m/s}} = 3.0\,.$$

Thus, if

$$\Delta t = t_t - t_\ell = 3.0t_\ell - t_\ell = 2.0t_\ell = 4.0\times10^{-3}\text{ s} \;\;\Rightarrow\;\; t_\ell = 2.0\times10^{-3}\text{ s}\,,$$

then $d = v_\ell t_\ell = (150\text{ m/s})(2.0\times10^{-3}\text{ s}) = 0.30\text{ m} = 30\text{ cm}.$

5. (a) The motion from maximum displacement to zero is one-fourth of a cycle. One-fourth of a period is 0.170 s, so the period is $T = 4(0.170\text{ s}) = 0.680\text{ s}$.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\,\text{Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\,\text{m/s}.$$

6. The slope that they are plotting is the physical slope of the sinusoidal waveshape (not to be confused with the more abstract "slope" of its time development; the physical slope is an x-derivative, whereas the more abstract "slope" would be the t-derivative). Thus, where the figure shows a maximum slope equal to 0.2 (with no unit), it refers to the maximum of the following function:

$$\frac{dy}{dx} = \frac{d}{dx}\left[y_m \sin(kx - \omega t)\right] = y_m k\cos(kx - \omega t)\,.$$

The problem additionally gives $t = 0$, which we can substitute into the above expression if desired. In any case, the maximum of the above expression is $y_m k$, where

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.40\text{ m}} = 15.7\text{ rad/m}\,.$$

Therefore, setting $y_m k$ equal to 0.20 allows us to solve for the amplitude y_m. We find

$$y_m = \frac{0.20}{15.7\text{ rad/m}} = 0.0127\text{ m} \approx 1.3\text{ cm}\,.$$

7. (a) From the simple harmonic motion relation $u_m = y_m\omega$, we have

$$\omega = \frac{16 \text{ m/s}}{0.040 \text{ m}} = 400 \text{ rad/s}.$$

Since $\omega = 2\pi f$, we obtain f = 64 Hz.

(b) Using $v = f\lambda$, we find λ = (80 m/s)/(64 Hz) = 1.26 m ≈ 1.3 m.

(c) The amplitude of the transverse displacement is $y_m = 4.0 \text{ cm} = 4.0\times10^{-2}\text{ m}$.

(d) The wave number is $k = 2\pi/\lambda$ = 5.0 rad/m.

(e) As shown in (a), the angular frequency is $\omega = (16 \text{ m/s})/(0.040 \text{ m}) = 4.0\times10^2$ rad/s.

(f) The function describing the wave can be written as

$$y = 0.040\sin\left(5x - 400t + \phi\right)$$

where distances are in meters and time is in seconds. We adjust the phase constant ϕ to satisfy the condition y = 0.040 at $x = t = 0$. Therefore, sin ϕ = 1, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.040\sin\left(5x - 400t + \frac{\pi}{2}\right).$$

(g) The sign in front of ω is minus.

8. Setting $x = 0$ in $u = -\omega y_m \cos(kx - \omega t + \phi)$ (see Eq. 16-21 or Eq. 16-28) gives

$$u = -\omega y_m \cos(-\omega t + \phi)$$

as the function being plotted in the graph. We note that it has a positive "slope" (referring to its t-derivative) at t = 0, or

$$\frac{du}{dt} = \frac{d}{dt}\left[-\omega y_m \cos(-\omega t + \phi)\right] = -y_m\omega^2 \sin(-\omega t + \phi) > 0$$

at t = 0. This implies that $-\sin\phi > 0$ and consequently that ϕ is in either the third or fourth quadrant. The graph shows (at t = 0) u = –4 m/s, and (at some later t) u_{max} = 5 m/s. We note that $u_{max} = y_m\,\omega$. Therefore,

$$u = -u_{max}\cos(-\omega t + \phi)\Big|_{t=0} \implies \phi = \cos^{-1}\left(\tfrac{4}{5}\right) = \pm 0.6435 \text{ rad}$$

(bear in mind that $\cos\theta = \cos(-\theta)$), and we must choose $\phi = -0.64$ rad (since this is about $-37°$ and is in fourth quadrant). Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = -0.64 + 2\pi = 5.64$ rad is also an acceptable result.

9. (a) The amplitude y_m is half of the 6.00 mm vertical range shown in the figure, that is, $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t = 15$ m/s, where $d = 0.060$ m and $t = 0.0040$ s. The angular wave number is $k = 2\pi/\lambda$ where $\lambda = 0.40$ m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m} .$$

(c) The angular frequency is found from

$$\omega = k\,v = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4\times10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between kx and ωt) in the argument of the sine function because the wave is shown traveling to the right (in the $+x$ direction, see Section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16\,x - 2.4\times10^2\; t) .$$

10. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0\times10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of y) is

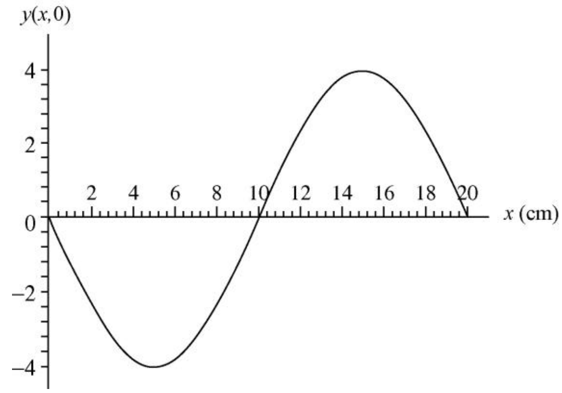
$$u_{\text{max}} = 2\pi\, f y_m = \left(4.0\,\pi\,\text{s}^{-1}\right)\left(6.0\,\text{cm}\right) = 75\,\text{cm/s}.$$

(g) $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0$ cm.

11. From Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi) .$$

(a) The figure shows that at $x = 0$, $y(0,t) = y_m \sin(-\omega t + \phi)$ is a positive sine function, that is, $y(0,t) = +y_m \sin \omega t$. Therefore, the phase constant must be $\phi = \pi$. At $t = 0$, we then have



$$y(x,0) = y_m \sin(kx + \pi) = -y_m \sin kx$$

which is a negative sine function. A plot of $y(x, 0)$ is depicted on the right.

(b) From the figure we see that the amplitude is $y_m = 4.0$ cm.

(c) The angular wave number is given by $k = 2\pi/\lambda = \pi/10 = 0.31$ rad/cm.

(d) The angular frequency is $\omega = 2\pi/T = \pi/5 = 0.63$ rad/s.

(e) As found in part (a), the phase is $\phi = \pi$.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

(g) Since the frequency is $f = 1/T = 0.10$ s, the speed of the wave is $v = f\lambda = 2.0$ cm/s.

(h) From the results above, the wave may be expressed as

$$y(x,t) = 4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5} + \pi\right) = -4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right).$$

Taking the derivative of y with respect to t, we find

$$u(x,t) = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{t}\right)\cos\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

which yields $u(0, 5.0) = -2.5$ cm/s.

12. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = (225\pi) \sin(\pi x - 15\pi t) .$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi\sqrt{15^2 - y^2}.$$

Therefore, where y = 12, u must be ± 135π. Consequently, the *speed* there is 424 cm/s = 4.24 m/s.

13. Using $v = f\lambda$, we find the length of one cycle of the wave is

$$\lambda = 350/500 = 0.700 \text{ m} = 700 \text{ mm}.$$

From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}$ s = 2.00 ms.

(a) A cycle is equivalent to 2π radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6$ = (700 mm)/6 = 117 mm.

(b) The interval 1.00 ms is half of T and thus corresponds to half of one cycle, or half of 2π rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

14. (a) Comparing with Eq. 16-2, we see that k = 20/m and ω = 600 rad/s. Therefore, the speed of the wave is (see Eq. 16-13) $v = \omega/k$ = 30 m/s.

(b) From Eq. 16–26, we find

$$\mu = \frac{\tau}{v^2} = \frac{15}{30^2} = 0.017 \text{ kg/m} = 17 \text{ g/m}.$$

15. **THINK** Numerous physical properties of a traveling wave can be deduced from its wave function.

EXPRESS We first recall that from Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

where y_m is the amplitude, $k = 2\pi/\lambda$ is the angular wave number, $\omega = 2\pi/T$ is the angular frequency and ϕ is the phase constant. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string.

ANALYZE (a) The amplitude of the wave is y_m=0.120 mm.

(b) The wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}.$$

(c) The frequency is f = 100 Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100\text{ Hz}) = 628\text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction.

LEARN In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[\left(141\,\text{m}^{-1}\right)x + \left(628\,\text{s}^{-1}\right)t\right].$$

16. We use $v = \sqrt{\tau/\mu} \propto \sqrt{\tau}$ to obtain

$$\tau_2 = \tau_1\left(\frac{v_2}{v_1}\right)^2 = (120\,\text{N})\left(\frac{180\,\text{m/s}}{170\,\text{m/s}}\right)^2 = 135\text{N}.$$

17. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency ($2\pi/T$), and k is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\text{ m}^{-1}$ and $\omega = 30$ rad/s. Thus

$$v = (30\text{ rad/s})/(2.0\text{ m}^{-1}) = 15\text{ m/s}.$$

(b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6\times10^{-4}\,\text{kg/m}\right)(15\,\text{m/s})^2 = 0.036\,\text{N}.$$

18. The volume of a cylinder of height ℓ is $V = \pi r^2\ell = \pi d^2\ell/4$. The strings are long, narrow cylinders, one of diameter d_1 and the other of diameter d_2 (and corresponding linear densities μ_1 and μ_2). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho \pi d^2 \ \ell/4}{\ell} = \frac{\pi \rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi \rho\, d_1^2/4}{\pi \rho\, d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2.$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2.$$

19. **THINK** The speed of a transverse wave in a rope is related to the tension in the rope and the linear mass density of the rope.

EXPRESS The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the rope's linear mass density, which is defined as the mass per unit length of rope $\mu = m/L$.

ANALYZE With a linear mass density of

$$\mu = m/L = (0.0600 \text{ kg})/(2.00 \text{ m}) = 0.0300 \text{ kg/m},$$

we find the wave speed to be

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129\,\text{m/s}.$$

LEARN Since $v \sim 1/\sqrt{\mu}$, the thicker the rope (larger μ), the slower the speed of the rope under the same tension τ.

20. From $v = \sqrt{\tau/\mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}} = \sqrt{2}.$$

21. The pulses have the same speed v. Suppose one pulse starts from the left end of the wire at time $t = 0$. Its coordinate at time t is $x_1 = vt$. The other pulse starts from the right end, at $x = L$, where L is the length of the wire, at time $t = 30$ ms. If this time is denoted by t_0, then the coordinate of this wave at time t is $x_2 = L - v(t - t_0)$. They meet when $x_1 = x_2$, or, what is the same, when $vt = L - v(t - t_0)$. We solve for the time they meet: $t = (L + vt_0)/2v$ and the coordinate of the meeting point is $x = vt = (L + vt_0)/2$. Now, we calculate the wave speed:

$$v=\sqrt{\frac{\tau L}{m}}=\sqrt{\frac{(250\,\text{N})(10.0\,\text{m})}{0.100\,\text{kg}}}=158\,\text{m/s}.$$

Here τ is the tension in the wire and L/m is the linear mass density of the wire. The coordinate of the meeting point is

$$x=\frac{10.0\,\text{m}+(158\,\text{m/s})(30.0\times10^{-3}\,\text{s})}{2}=7.37\,\text{m}.$$

This is the distance from the left end of the wire. The distance from the right end is $L-x$ = (10.0 m – 7.37 m) = 2.63 m.

22. (a) The general expression for $y(x, t)$ for the wave is $y(x, t) = y_m \sin(kx - \omega t)$, which, at x = 10 cm, becomes $y(x = 10\text{ cm}, t) = y_m \sin[k(10\text{ cm} - \omega t)]$. Comparing this with the expression given, we find ω = 4.0 rad/s, or $f = \omega/2\pi$ = 0.64 Hz.

(b) Since k(10 cm) = 1.0, the wave number is k = 0.10/cm. Consequently, the wavelength is $\lambda = 2\pi/k$ = 63 cm.

(c) The amplitude is $y_m = 5.0$ cm.

(d) In part (b), we have shown that the angular wave number is k = 0.10/cm.

(e) The angular frequency is ω = 4.0 rad/s.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

Summarizing the results obtained above by substituting the values of k and ω into the general expression for $y(x, t)$, with centimeters and seconds understood, we obtain

$$y(x,t)=5.0\sin(0.10x-4.0t).$$

(g) Since $v=\omega/k=\sqrt{\tau/\mu}$, the tension is

$$\tau=\frac{\omega^2\mu}{k^2}=\frac{(4.0\,\text{g/cm})(4.0\,\text{s}^{-1})^2}{(0.10\,\text{cm}^{-1})^2}=6400\,\text{g}\cdot\text{cm/s}^2=0.064\,\text{N}.$$

23. **THINK** Various properties of the sinusoidal wave can be deduced from the plot of its displacement as a function of position.

EXPRESS In analyzing the properties of the wave, we first recall that from Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

where y_m is the amplitude, $k = 2\pi/\lambda$ is the angular wave number, $\omega = 2\pi/T$ is the angular frequency and ϕ is the phase constant. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string.

ANALYZE (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15$ cm and again with the same slope at about $x = 55$ cm, so

$$\lambda = (55\text{ cm} - 15\text{ cm}) = 40\text{ cm} = 0.40\text{ m}.$$

(c) The wave speed is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{3.6\,\text{N}}{25\times 10^{-3}\,\text{kg/m}}} = 12\text{ m/s}.$$

(d) The frequency is $f = v/\lambda = (12\text{ m/s})/(0.40\text{ m}) = 30$ Hz and the period is

$$T = 1/f = 1/(30\text{ Hz}) = 0.033\text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30\text{ Hz})\,(5.0\text{ cm}) = 940\text{ cm/s} = 9.4\text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40\text{ m}) = 16\text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30\text{ Hz}) = 1.9\times 10^2$ rad/s .

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is 4.0×10^{-2} m. The formula for the displacement gives $y(0, 0) = y_m \sin\phi$. We wish to select ϕ so that

$$(5.0 \times 10^{-2}\text{ m})\sin\phi = (4.0 \times 10^{-2}\text{ m}).$$

The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction.

LEARN Summarizing the results obtained above, the wave function of the traveling wave can be written as

$$y(x,t)=\left(5.0\times10^{-2}\,\text{m}\right)\sin\left[(16\,\text{m}^{-1})x+(190\,\text{s}^{-1})t+0.93\right].$$

24. (a) The tension in each string is given by $\tau = Mg/2$. Thus, the wave speed in string 1 is

$$v_1=\sqrt{\frac{\tau}{\mu_1}}=\sqrt{\frac{Mg}{2\mu_1}}=\sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(3.00\,\text{g/m})}}=28.6\,\text{m/s}.$$

(b) And the wave speed in string 2 is

$$v_2=\sqrt{\frac{Mg}{2\mu_2}}=\sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(5.00\,\text{g/m})}}=22.1\,\text{m/s}.$$

(c) Let $v_1=\sqrt{M_1g/(2\mu_1)}=v_2=\sqrt{M_2g/(2\mu_2)}$ and $M_1 + M_2 = M$. We solve for M_1 and obtain

$$M_1=\frac{M}{1+\mu_2/\mu_1}=\frac{500\,\text{g}}{1+5.00/3.00}=187.5\,\text{g}\approx188\,\text{g}.$$

(d) And we solve for the second mass: $M_2 = M - M_1 = (500\text{ g} - 187.5\text{ g}) \approx 313\text{ g}$.

25. (a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where τ is the tension at that point and μ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance y from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by μgy, so the tension is $\tau = \mu gy$. The wave speed is $v=\sqrt{\mu gy/\mu}=\sqrt{gy}.$

(b) The time dt for the wave to move past a length dy, a distance y from the bottom end, is $dt=dy/v=dy\big/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t=\int_0^L\frac{dy}{\sqrt{gy}}=2\sqrt{\frac{y}{g}}\,\Bigg|_0^L=2\sqrt{\frac{L}{g}}\,.$$

26. Using Eq. 16–33 for the average power and Eq. 16–26 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$f = \frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}} = \frac{1}{2\pi(7.70\times10^{-3}\,\text{m})}\sqrt{\frac{2(85.0\,\text{W})}{\sqrt{(36.0\,\text{N})(0.260\,\text{kg}/2.70\,\text{m})}}} = 198\ \text{Hz}.$$

27. We note from the graph (and from the fact that we are dealing with a cosine-squared, see Eq. 16-30) that the wave frequency is $f = \frac{1}{2\text{ ms}}$ = 500 Hz, and that the wavelength λ = 0.20 m. We also note from the graph that the maximum value of *dK/dt* is 10 W. Setting this equal to the maximum value of Eq. 16-29 (where we just set that cosine term equal to 1) we find

$$\tfrac{1}{2}\mu v\,\omega^2 y_m^{\ 2} = 10$$

with SI units understood. Substituting in $\mu = 0.002$ kg/m, $\omega = 2\pi f$ and $v = f\lambda$, we solve for the wave amplitude:

$$y_m = \sqrt{\frac{10}{2\pi^2\mu\lambda f^3}} = 0.0032\ \text{m}.$$

28. Comparing

$$y(x,t) = (3.00\ \text{mm})\sin[(4.00\ \text{m}^{-1})x - (7.00\ \text{s}^{-1})t]$$

to the general expression $y(x,t) = y_m\sin(kx - \omega t)$, we see that $k = 4.00\ \text{m}^{-1}$ and $\omega = 7.00$ rad/s. The speed of the wave is

$$v = \omega/k = (7.00\ \text{rad/s})/(4.00\ \text{m}^{-1}) = 1.75\ \text{m/s}.$$

29. The wave

$$y(x,t) = (2.00\ \text{mm})[(20\ \text{m}^{-1})x - (4.0\ \text{s}^{-1})t]^{1/2}$$

is of the form $h(kx - \omega t)$ with angular wave number $k = 20\ \text{m}^{-1}$ and angular frequency $\omega = 4.0$ rad/s. Thus, the speed of the wave is

$$v = \omega/k = (4.0\ \text{rad/s})/(20\ \text{m}^{-1}) = 0.20\ \text{m/s}.$$

30. The wave $y(x,t) = (4.00\ \text{mm})\,h[(30\ \text{m}^{-1})x + (6.0\ \text{s}^{-1})t]$ is of the form $h(kx - \omega t)$ with angular wave number $k = 30\ \text{m}^{-1}$ and angular frequency $\omega = 6.0$ rad/s. Thus, the speed of the wave is

$$v = \omega/k = (6.0\ \text{rad/s})/(30\ \text{m}^{-1}) = 0.20\ \text{m/s}.$$

31. **THINK** By superposition principle, the resultant wave is the algebraic sum of the two interfering waves.

EXPRESS The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \;= 2y_m \cos\left(\tfrac{1}{2}\phi\right)\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right),$$

where we have used

$$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta).$$

ANALYZE The two waves are out of phase by $\phi = \pi/2$, so the amplitude is

$$A = 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41y_m .$$

LEARN The interference between two waves can be constructive or destructive, depending on their phase difference.

32. (a) Let the phase difference be ϕ. Then from Eq. 16-52, $2y_m \cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8°.$$

(b) Converting to radians, we have $\phi = 1.45$ rad.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to 2π rad), this is equivalent to 1.45 rad/2π = 0.230 wavelength.

33. (a) The amplitude of the second wave is $y_m = 9.00$ mm, as stated in the problem.

(b) The figure indicates that λ = 40 cm = 0.40 m, which implies that the angular wave number is $k = 2\pi/0.40 = 16$ rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is $v = dx/t$ = (56.0 cm)/(8.0 ms) = 70 m/s. This, in turn, implies that the angular frequency is

$$\omega = kv = 1100 \text{ rad/s} = 1.1\times10^3 \text{ rad/s}.$$

(d) The figure depicts two traveling waves (both going in the $-x$ direction) of equal amplitude y_m. The amplitude of their resultant wave, as shown in the figure, is $y'_m = 4.00$ mm. Equation 16-52 applies:

$$y'_m = 2y_m \cos\left(\tfrac{1}{2}\phi_2\right) \quad\Rightarrow\quad \phi_2 = 2\cos^{-1}(2.00/9.00) = 2.69 \text{ rad}.$$

(e) In making the plus-or-minus sign choice in $y = y_m \sin(kx \pm \omega t + \phi)$, we recall the discussion in section 16-5, where it was shown that sinusoidal waves traveling in the $-x$ direction are of the form $y = y_m \sin(kx + \omega t + \phi)$. Here, ϕ should be thought of as the phase *difference* between the two waves (that is, $\phi_1 = 0$ for wave 1 and $\phi_2 = 2.69$ rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16\,x + 1100\,t) \quad \text{and} \quad y_2 = (0.00900)\sin(16\,x + 1100\,t + 2.7\,) .$$

34. (a) We use Eq. 16-26 and Eq. 16-33 with μ = 0.00200 kg/m and $y_m = 0.00300$ m. These give $v = \sqrt{\tau/\mu} = 775$ m/s and

$$P_{\text{avg}} = \frac{1}{2}\ \mu v\ \omega^2 y_m^{\ 2} = 10 \text{ W}.$$

(b) In this situation, the waves are two separate string (no superposition occurs). The answer is clearly twice that of part (a); $P = 20$ W.

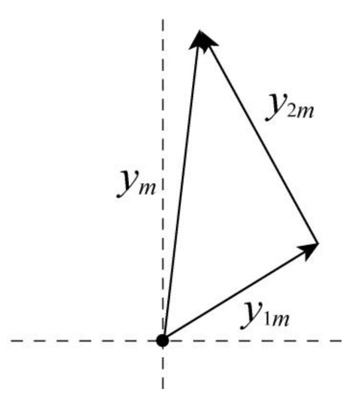
(c) Now they are on the same string. If they are interfering constructively (as in Fig. 16-13(a)) then the amplitude y_m is doubled, which means its square $y_m^{\ 2}$ increases by a factor of 4. Thus, the answer now is four times that of part (a); $P = 40$ W.

(d) Equation 16-52 indicates in this case that the amplitude (for their superposition) is $2\,y_m\cos(0.2\pi) = 1.618$ times the original amplitude y_m. Squared, this results in an increase in the power by a factor of 2.618. Thus, $P = 26$ W in this case.

(e) Now the situation depicted in Fig. 16-13(b) applies, so $P = 0$.

35. **THINK** We use phasors to add the two waves and calculate the amplitude of the resultant wave.

EXPRESS The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle.



ANALYZE The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

Thus, the amplitude of the resultant wave is y_m = 5.0 cm.

LEARN When adding two waves, it is convenient to represent each wave with a phasor, which is a vector whose magnitude is equal to the amplitude of the wave. The same result, however, could also be obtained as follows: Writing the two waves as $y_1 = 3\sin(kx - \omega t)$ and $y_2 = 4\sin(kx - \omega t + \pi/2) = 4\cos(kx - \omega t)$, we have, after a little algebra,

$$y = y_1 + y_2 = 3\sin(kx - \omega t) + 4\cos(kx - \omega t) = 5\left[\frac{3}{5}\sin(kx - \omega t) + \frac{4}{5}\cos(kx - \omega t)\right]$$
$$= 5\sin(kx - \omega t + \phi)$$

where $\phi = \tan^{-1}(4/3)$. In deducing the phase ϕ, we set $\cos\phi = 3/5$ and $\sin\phi = 4/5$, and use the relation $\cos\phi\sin\theta + \sin\phi\cos\theta = \sin(\theta + \phi)$.

36. We see that y_1 and y_3 cancel (they are 180º) out of phase, and y_2 cancels with y_4 because their phase difference is also equal to π rad (180º). There is no resultant wave in this case.

37. (a) Using the phasor technique, we think of these as two "vectors" (the first of "length" 4.6 mm and the second of "length" 5.60 mm) separated by an angle of $\phi = 0.8\pi$ radians (or 144º). Standard techniques for adding vectors then lead to a resultant vector of length 3.29 mm.

(b) The angle (relative to the first vector) is equal to 88.8º (or 1.55 rad).

(c) Clearly, it should in "in phase" with the result we just calculated, so its phase angle relative to the first phasor should be also 88.8º (or 1.55 rad).

38. (a) As shown in Figure 16-13(b) in the textbook, the least-amplitude resultant wave is obtained when the phase difference is π rad.

(b) In this case, the amplitude is (8.0 mm – 5.0 mm) = 3.0 mm.

(c) As shown in Figure 16-13(a) in the textbook, the greatest-amplitude resultant wave is obtained when the phase difference is 0 rad.

(d) In the part (c) situation, the amplitude is (8.0 mm + 5.0 mm) = 13 mm.

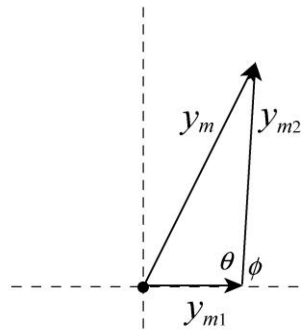
(e) Using phasor terminology, the angle "between them" in this case is π/2 rad (90º), so the Pythagorean theorem applies:

$$\sqrt{(8.0\text{ mm})^2+(5.0\text{ mm})^2} = 9.4\text{ mm}.$$

39. The phasor diagram is shown to the right. We use the cosine theorem:

$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos\theta = y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos\phi.$$

We solve for $\cos\phi$:



$$\cos\phi = \frac{y_m^2 - y_{m1}^2 - y_{m2}^2}{2y_{m1}\,y_{m2}} = \frac{(9.0\,\text{mm})^2-(5.0\,\text{mm})^2-(7.0\,\text{mm})^2}{2(5.0\,\text{mm})(7.0\,\text{mm})} = 0.10.$$

The phase constant is therefore $\phi = 84°$.

40. The string is flat each time the particle passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes *half* of one complete cycle, so we conclude $T = 2(0.50\text{ s}) = 1.0\text{ s}$. Thus, $f = 1/T = 1.0$ Hz, and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\,\text{cm/s}}{1.0\,\text{Hz}} = 10\text{ cm}.$$

41. **THINK** A string clamped at both ends can be made to oscillate in standing wave patterns.

EXPRESS The wave speed is given by $v=\sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. The possible wavelengths of a standing wave are given by $\lambda_n = 2L/n$, where L is the length of the string and n is an integer.

ANALYZE (a) The wave speed is

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0\text{ N})\,(8.40\text{ m})}{0.120\text{ kg}}} = 82.0\text{ m/s}.$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda_1/2$ ($n = 1$), so $\lambda_1 = 2L = 2(8.40\text{ m}) = 16.8$ m.

(c) The corresponding frequency is $f_1 = v/\lambda_1 = (82.0\text{ m/s})/(16.8\text{ m}) = 4.88$ Hz.

LEARN The resonant frequencies are given by

$$f_n = \frac{v}{\lambda} = \frac{v}{2L/n} = n\frac{v}{2L} = nf_1,$$

where $f_1 = v/\lambda_1 = v/2L$. The oscillation mode with $n = 1$ is called the fundamental mode or the first harmonic.

42. Use Eq. 16-66 (for the resonant frequencies) and Eq. 16-26 $(v = \sqrt{\tau/\mu})$ to find f_n:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$.

(a) When $\tau_f = 4\tau_i$, we get the new frequency

$$f_3' = \frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}} = 2f_3.$$

(b) And we get the new wavelength $\lambda'_3 = \dfrac{v'}{f_3'} = \dfrac{2L}{3} = \lambda_3$.

43. **THINK** A string clamped at both ends can be made to oscillate in standing wave patterns.

EXPRESS Possible wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus,

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\text{ N}}{(10.0\text{ m})\,(0.100\text{ kg})}} = n\ (7.91\text{ Hz}).$$

ANALYZE (a) The lowest frequency is $f_1 = 7.91$ Hz.

(b) The second lowest frequency is $f_2 = 2(7.91\text{ Hz}) = 15.8$ Hz.

(c) The third lowest frequency is $f_3 = 3(7.91\text{ Hz}) = 23.7$ Hz.

LEARN The frequencies are integer multiples of the fundamental frequency f_1. This means that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency f_1.

44. (a) The wave speed is given by $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{7.00\,\text{N}}{2.00 \times 10^{-3}\,\text{kg}/1.25\,\text{m}}} = 66.1\,\text{m/s}.$

(b) The wavelength of the wave with the lowest resonant frequency f_1 is $\lambda_1 = 2L$, where L = 125 cm. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{66.1\text{ m/s}}{2(1.25\text{ m})} = 26.4\text{ Hz}.$$

45. **THINK** The difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency.

EXPRESS The resonant wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are

$$f_n = v/\lambda = nv/2L = nf_1,$$

where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with n + 1. The frequency difference between successive modes is

$$\Delta f = f_{n+1} - f_n = \frac{v}{2L} = f_1.$$

ANALYZE (a) The lowest possible resonant frequency is

$$f_1 = \Delta f = f_{n+1} - f_n = 420\text{ Hz} - 315\text{ Hz} = 105\text{ Hz}.$$

(b) The longest possible wavelength is $\lambda_1 = 2L$. If f_1 is the lowest possible frequency then

$$v = \lambda_1 f_1 = (2L)f_1 = 2(0.75\text{ m})(105\text{ Hz}) = 158\text{ m/s}.$$

LEARN Since 315 Hz = 3(105 Hz) and 420 Hz = 4(105 Hz), the two frequencies correspond to n = 3 and n = 4, respectively.

46. The nth resonant frequency of string A is

$$f_{n,A} = \frac{v_A}{2l_A}n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}},$$

while for string B it is

$$f_{n,B} = \frac{v_B}{2l_B}n = \frac{n}{8L}\sqrt{\frac{\tau}{\mu}} = \frac{1}{4}f_{n,A}.$$

(a) Thus, we see $f_{1,A} = f_{4,B}$. That is, the fourth harmonic of *B* matches the frequency of *A*'s first harmonic.

(b) Similarly, we find $f_{2,A} = f_{8,B}$.

(c) No harmonic of *B* would match $f_{3,A} = \dfrac{3v_A}{2l_A} = \dfrac{3}{2L}\sqrt{\dfrac{\tau}{\mu}}$.

47. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus,

$$f_1 = (390 \text{ Hz} - 325 \text{ Hz}) = 65 \text{ Hz}.$$

This further implies that the next higher resonance above 195 Hz should be (195 Hz + 65 Hz) = 260 Hz.

48. Using Eq. 16-26, we find the wave speed to be

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{65.2\times10^6 \text{ N}}{3.35 \text{ kg/m}}} = 4412 \text{ m/s}.$$

The corresponding resonant frequencies are

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

(a) The wavelength of the wave with the lowest (fundamental) resonant frequency f_1 is $\lambda_1 = 2L$, where L = 347 m. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{4412 \text{ m/s}}{2(347 \text{ m})} = 6.36 \text{ Hz}.$$

(b) The frequency difference between successive modes is

$$\Delta f = f_n - f_{n-1} = \frac{v}{2L} = \frac{4412 \text{ m/s}}{2(347 \text{ m})} = 6.36 \text{ Hz}.$$

49. (a) Equation 16-26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150 \text{ N}}{7.20\times10^{-3} \text{ kg/m}}} = 144.34 \text{ m/s} \approx 1.44\times10^2 \text{ m/s}.$$

(b) From the figure, we find the wavelength of the standing wave to be

$$\lambda = (2/3)(90.0 \text{ cm}) = 60.0 \text{ cm}.$$

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times10^2 \text{ m/s}}{0.600\,\text{m}} = 241\,\text{Hz}.$$

50. From the $x = 0$ plot (and the requirement of an anti-node at $x = 0$), we infer a standing wave function of the form

$$y(x,t) = -(0.04)\cos(kx)\sin(\omega t),$$

where $\omega = 2\pi/T = \pi$ rad/s, with length in meters and time in seconds. The parameter k is determined by the existence of the node at $x = 0.10$ (presumably the *first* node that one encounters as one moves from the origin in the positive x direction). This implies $k(0.10) = \pi/2$ so that $k = 5\pi$ rad/m.

(a) With the parameters determined as discussed above and $t = 0.50$ s, we find

$$y(0.20 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0.040\,\text{m}.$$

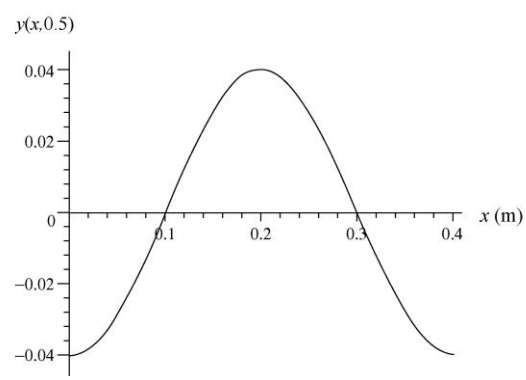
(b) The above equation yields $y(0.30 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0$.

(c) We take the derivative with respect to time and obtain, at $t = 0.50$ s and $x = 0.20$ m,

$$u = \frac{dy}{dt} = -0.04\omega\cos(kx)\cos(\omega t) = 0.$$

d) The above equation yields $u = -0.13$ m/s at $t = 1.0$ s.

(e) The sketch of this function at $t = 0.50$ s for $0 \le x \le 0.40$ m is shown next:



51. **THINK** In this problem, in order to produce the standing wave pattern, the two waves must have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions.

EXPRESS We take the two waves to be

$$y_1 = y_m \sin(kx - \omega t), \quad y_2 = y_m \sin(kx + \omega t).$$

The superposition principle gives

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) = [2y_m \sin kx]\cos \omega t \ .$$

ANALYZE (a) The amplitude y_m is half the maximum displacement of the standing wave, or (0.01 m)/2 = 5.0×10^{-3} m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With L = 3.0m, λ = 2.0 m. The angular wave number is

$$k = 2\pi/\lambda = 2\pi/(2.0 \text{ m}) = 3.1 \text{ m}^{-1}.$$

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100 \text{ m/s})}{2(3.0 \text{ m})} = 50 \text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or

$$\omega = 2\pi f = 2\pi(50 \text{ Hz}) = 314 \text{ rad/s}.$$

(d) If one of the waves has the form $y_2(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y_1(x,t) = y_m \sin(kx - \omega t)$. The sign in front of ω for $y'(x,t)$ is minus.

LEARN Using the results above, the two waves can be written as

$$y_1 = (5.0 \times 10^{-3} \text{ m}) \sin\left[(3.14 \text{ m}^{-1})x - (314 \text{ s}^{-1})t\right]$$

and

$$y_2 = (5.0 \times 10^{-3} \text{ m}) \sin\left[(3.14 \text{ m}^{-1})x + (314 \text{ s}^{-1})t\right].$$

52. Since the rope is fixed at both ends, then the phrase "second-harmonic standing wave pattern" describes the oscillation shown in Figure 16-20(b), where (see Eq. 16-65)

$$\lambda = L, \quad f = \frac{v}{L}.$$

(a) Comparing the given function with Eq. 16-60, we obtain $k = \pi/2$ and $\omega = 12\pi$ rad/s. Since $k = 2\pi/\lambda$, then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \quad \Rightarrow \quad \lambda = 4.0\,\text{m} \quad \Rightarrow \quad L = 4.0\,\text{m}.$$

(b) Since $\omega = 2\pi f$, then $2\pi f = 12\pi$ rad/s, which yields

$$f = 6.0\,\text{Hz} \quad \Rightarrow \quad v = f\lambda = 24\,\text{m/s}.$$

(c) Using Eq. 16-26, we have

$$v = \sqrt{\frac{\tau}{\mu}} \quad \Rightarrow \quad 24\ \text{m/s} = \sqrt{\frac{200\ \text{N}}{m/(4.0\ \text{m})}}$$

which leads to $m = 1.4$ kg.

(d) With

$$f = \frac{3v}{2L} = \frac{3(24\ \text{m/s})}{2(4.0\ \text{m})} = 9.0\,\text{Hz}$$

the period is $T = 1/f = 0.11$ s.

53. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3\ \text{cm}^{-1}$. The wave speed is

$$v = \omega/k = (40\pi\ \text{rad/s})/(\pi/3\ \text{cm}^{-1}) = 1.2\times10^2\ \text{cm/s}.$$

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3\ \text{cm}^{-1}) = 3.0$ cm. Here $2\pi/k$ was substituted for λ.

(d) The string speed is given by

$$u(x, t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t).$$

For the given coordinate and time,

$$u = -(40\pi\ \text{rad/s})\ (0.50\,\text{cm})\ \sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)(1.5\,\text{cm})\right]\sin\ \left[\left(40\pi\ \text{s}^{-1}\right)\left(\frac{9}{8}\ \text{s}\right)\right] = 0.$$

54. Reference to point A as an anti-node suggests that this is a standing wave pattern and thus that the waves are traveling in opposite directions. Thus, we expect one of them to be of the form $y = y_m \sin(kx + \omega t)$ and the other to be of the form $y = y_m \sin(kx - \omega t)$.

(a) Using Eq. 16-60, we conclude that $y_m = \frac{1}{2}(9.0 \text{ mm}) = 4.5$ mm, due to the fact that the amplitude of the standing wave is $\frac{1}{2}(1.80 \text{ cm}) = 0.90$ cm = 9.0 mm.

(b) Since one full cycle of the wave (one wavelength) is 40 cm, $k = 2\pi/\lambda \approx 16 \text{ m}^{-1}$.

(c) The problem tells us that the time of half a full period of motion is 6.0 ms, so $T = 12$ ms and Eq. 16-5 gives $\omega = 5.2 \times 10^2$ rad/s.

(d) The two waves are therefore

$$y_1(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x + (520 \text{ s}^{-1})t]$$

and

$$y_2(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x - (520 \text{ s}^{-1})t] .$$

If one wave has the form $y(x,t) = y_m \sin(kx + \omega t)$ as in y_1, then the other wave must be of the form $y'(x,t) = y_m \sin(kx - \omega t)$ as in y_2. Therefore, the sign in front of ω is minus.

55. Recalling the discussion in section 16-12, we observe that this problem presents us with a standing wave condition with amplitude 12 cm. The angular wave number and frequency are noted by comparing the given waves with the form $y = y_m \sin(kx \pm \omega t)$. The anti-node moves through 12 cm in simple harmonic motion, just as a mass on a vertical spring would move from its upper turning point to its lower turning point, which occurs during a half-period. Since the period T is related to the angular frequency by Eq. 15-5, we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.00\pi} = 0.500 \text{ s}.$$

Thus, in a time of $t = \frac{1}{2}T = 0.250$ s, the wave moves a distance $\Delta x = vt$ where the speed of the wave is $v = \omega / k = 1.00$ m/s. Therefore, $\Delta x = (1.00 \text{ m/s})(0.250 \text{ s}) = 0.250$ m.

56. The nodes are located from vanishing of the spatial factor $\sin 5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \cdots$$

(a) The smallest value of x that corresponds to a node is $x = 0$.

(b) The second smallest value of x that corresponds to a node is $x = 0.20$ m.

(c) The third smallest value of x that corresponds to a node is $x = 0.40$ m.

(d) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(e) Comparing the given function with Eq. 16-58 through Eq. 16-60, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(f) And we see the amplitude is $y_m = 0.020$ m.

(g) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -(0.040)(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all x) at times such as $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \cdots$$

Thus, the first time in which all points on the string have zero transverse velocity is when $t = 0$ s.

(h) The second time in which all points on the string have zero transverse velocity is when $t = 1/40$ s = 0.025 s.

(i) The third time in which all points on the string have zero transverse velocity is when $t = 2/40$ s = 0.050 s.

57. (a) The angular frequency is $\omega = 8.00\pi/2 = 4.00\pi$ rad/s, so the frequency is

$$f = \omega/2\pi = (4.00\pi \text{ rad/s})/2\pi = 2.00 \text{ Hz}.$$

(b) The angular wave number is $k = 2.00\pi/2 = 1.00\pi$ m^{-1}, so the wavelength is

$$\lambda = 2\pi/k = 2\pi/(1.00\pi \text{ m}^{-1}) = 2.00 \text{ m}.$$

(c) The wave speed is

$$v = \lambda f = (2.00\,\text{m})(2.00\,\text{Hz}) = 4.00 \text{ m/s}.$$

(d) We need to add two cosine functions. First convert them to sine functions using cos α = sin ($\alpha + \pi/2$), then apply

$$\cos\alpha + \cos\beta = \sin\left(\alpha + \frac{\pi}{2}\right) + \sin\left(\beta + \frac{\pi}{2}\right) = 2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right).$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m\cos(kx+\omega t) + y_m\cos(kx-\omega t) = 2y_m\cos(kx)\cos(\omega t).$$

Nodes occur where cos(kx) = 0 or $kx = n\pi + \pi/2$, where n is an integer (including zero). Since $k = 1.0\pi\ \text{m}^{-1}$, this means $x = \left(n+\frac{1}{2}\right)(1.00\,\text{m})$. Thus, the smallest value of x that corresponds to a node is x = 0.500 m (n = 0).

(e) The second smallest value of x that corresponds to a node is x = 1.50 m (n = 1).

(f) The third smallest value of x that corresponds to a node is x = 2.50 m (n = 2).

(g) The displacement is a maximum where cos(kx) = ±1. This means $kx = n\pi$, where n is an integer. Thus, $x = n$(1.00 m). The smallest value of x that corresponds to an anti-node (maximum) is x = 0 (n = 0).

(h) The second smallest value of x that corresponds to an anti-node (maximum) is $x = 1.00$ m (n = 1).

(i) The third smallest value of x that corresponds to an anti-node (maximum) is $x = 2.00$ m (n = 2).

58. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

(a) The mass that allows the oscillator to set up the 4th harmonic ($n = 4$) on the string is

$$m = \left.\frac{4L^2f^2\mu}{n^2g}\right|_{n=4} = \frac{4(1.20\text{ m})^2(120\text{ Hz})^2(0.00160\text{ kg/m})}{(4)^2(9.80\text{ m/s}^2)} = 0.846\text{ kg}$$

(b) If the mass of the block is $m = 1.00$ kg, the corresponding n is

$$n = \sqrt{\frac{4L^2 f^2 \mu}{g}} = \sqrt{\frac{4(1.20\text{ m})^2(120\text{ Hz})^2(0.00160\text{ kg/m})}{9.80\text{ m/s}^2}} = 3.68$$

which is not an integer. Therefore, the mass cannot set up a standing wave on the string.

59. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are n_1 loops in the aluminum section of the wire. Then,

$$L_1 = n_1\lambda_1/2 = n_1 v_1/2f,$$

where λ_1 is the wavelength and v_1 is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where f is the frequency. Thus $f = n_1v_1/2L_1$. A similar expression holds for the steel section: $f = n_2v_2/2L_2$. Since the frequency is the same for the two sections, $n_1v_1/L_1 = n_2v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\tau/\mu_1}$, where μ_1 is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1 A L_1$, where ρ_1 is the mass density (mass per unit volume) for aluminum and A is the cross-sectional area of the wire. Thus

$$\mu_1 = \rho_1 A L_1/L_1 = \rho_1 A$$

and $v_1 = \sqrt{\tau/\rho_1 A}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\tau/\rho_2 A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$, where A has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\text{m})\sqrt{7.80\times10^3\text{ kg/m}^3}}{(0.600\,\text{m})\sqrt{2.60\times10^3\text{ kg/m}^3}} = 2.50.$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is

$$f = n_1v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1 A}.$$

The tension is provided by the hanging block and is $\tau = mg$, where m is the mass of the block. Thus,

$$f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{(10.0\,\text{kg})(9.80\,\text{m/s}^2)}{(2.60\times10^3\text{kg/m}^3)(1.00\times10^{-6}\text{m}^2)}} = 324\,\text{Hz}.$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

60. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

The mass that allows the oscillator to set up the nth harmonic on the string is

$$m = \frac{4L^2 f^2 \mu}{n^2 g}.$$

Thus, we see that the block mass is inversely proportional to the harmonic number squared. Thus, if the 447 gram block corresponds to harmonic number n, then

$$\frac{447}{286.1} = \frac{(n+1)^2}{n^2} = \frac{n^2 + 2n + 1}{n^2} = 1 + \frac{2n+1}{n^2}.$$

Therefore, $\frac{447}{286.1} - 1 = 0.5624$ must equal an odd integer $(2n + 1)$ divided by a squared integer (n^2). That is, multiplying 0.5624 by a square (such as 1, 4, 9, 16, etc.) should give us a number very close (within experimental uncertainty) to an odd number (1, 3, 5, …). Trying this out in succession (starting with multiplication by 1, then by 4, …), we find that multiplication by 16 gives a value very close to 9; we conclude $n = 4$ (so $n^2 = 16$ and $2n + 1 = 9$). Plugging in $m = 0.447$ kg, $n = 4$, and the other values given in the problem, we find

$$\mu = 0.000845 \text{ kg/m} = 0.845 \text{ g/m}.$$

61. To oscillate in four loops means $n = 4$ in Eq. 16-65 (treating both ends of the string as effectively "fixed"). Thus, $\lambda = 2(0.90 \text{ m})/4 = 0.45$ m. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is

$$\mu = m/L = (0.044 \text{ kg})/(0.90 \text{ m}) = 0.049 \text{ kg/m}.$$

Thus, using Eq. 16-26, we obtain the tension:

$$\tau = v^2 \mu = (27 \text{ m/s})^2(0.049 \text{ kg/m}) = 36 \text{ N}.$$

62. We write the expression for the displacement in the form $y(x, t) = y_m \sin(kx - \omega t)$.

(a) The amplitude is $y_m = 2.0$ cm $= 0.020$ m, as given in the problem.

(b) The angular wave number k is $k = 2\pi/\lambda = 2\pi/(0.10 \text{ m}) = 63 \text{ m}^{-1}$.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(400 \text{ Hz}) = 2510 \text{ rad/s} = 2.5\times10^3 \text{ rad/s}$.

(d) A minus sign is used before the ωt term in the argument of the sine function because the wave is traveling in the positive x direction.

Using the results above, the wave may be written as

$$y(x,t) = (2.00\,\text{cm})\sin\left(\left(62.8\,\text{m}^{-1}\right)x - \left(2510\,\text{s}^{-1}\right)t\right).$$

(e) The (transverse) speed of a point on the cord is given by taking the derivative of y:

$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

which leads to a maximum speed of $u_m = \omega y_m = (2510 \text{ rad/s})(0.020 \text{ m}) = 50 \text{ m/s}$.

(f) The speed of the wave is

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2510\,\text{rad/s}}{62.8\,\text{rad/m}} = 40\,\text{m/s}.$$

63. (a) Using $v = f\lambda$, we obtain

$$f = \frac{240\,\text{m/s}}{3.2\,\text{m}} = 75 \text{ Hz}.$$

(b) Since frequency is the reciprocal of the period, we find

$$T = \frac{1}{f} = \frac{1}{75\,\text{Hz}} = 0.0133\,\text{s} \approx 13\,\text{ms}.$$

64. (a) At $x = 2.3$ m and $t = 0.16$ s the displacement is

$$y(x,t) = 0.15\sin\left[(0.79)(2.3) - 13(0.16)\right]\text{m} = -0.039\,\text{m}.$$

(b) We choose $y_m = 0.15$ m, so that there would be nodes (where the wave amplitude is zero) in the string as a result.

(c) The second wave must be traveling with the same speed and frequency. This implies $k = 0.79 \text{ m}^{-1}$,

(d) and $\omega = 13 \text{ rad/s}$.

(e) The wave must be traveling in the –x direction, implying a plus sign in front of ω.

Thus, its general form is $y'(x,t) = (0.15 \text{ m})\sin(0.79x + 13t)$.

(f) The displacement of the standing wave at $x = 2.3$ m and $t = 0.16$ s is

$$y(x,t) = -0.039\,\text{m} + (0.15\,\text{m})\sin[(0.79)(2.3) + 13(0.16)] = -0.14\,\text{m}.$$

65. We use Eq. 16-2, Eq. 16-5, Eq. 16-9, Eq. 16-13, and take the derivative to obtain the transverse speed u.

(a) The amplitude is $y_m = 2.0$ mm.

(b) Since $\omega = 600$ rad/s, the frequency is found to be $f = 600/2\pi \approx 95$ Hz.

(c) Since $k = 20$ rad/m, the velocity of the wave is $v = \omega/k = 600/20 = 30$ m/s in the $+x$ direction.

(d) The wavelength is $\lambda = 2\pi/k \approx 0.31$ m, or 31 cm.

(e) We obtain

$$u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t) \Rightarrow u_m = \omega y_m$$

so that the maximum transverse speed is $u_m = (600)(2.0) = 1200$ mm/s, or 1.2 m/s.

66. Setting $x = 0$ in $y = y_m \sin(kx - \omega t + \phi)$ gives $y = y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a positive "slope" (referring to its t-derivative) at $t = 0$, or

$$\frac{dy}{dt} = \frac{d}{dt}\left[y_m \sin(-\omega t + \phi)\right] = -y_m \omega \cos(-\omega t + \phi) > 0$$

at $t = 0$. This implies that $-\cos\phi > 0$ and consequently that ϕ is in either the second or third quadrant. The graph shows (at $t = 0$) $y = 2.00$ mm, and (at some later t) $y_m = 6.00$ mm. Therefore,

$$y = y_m \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \sin^{-1}\left(\tfrac{1}{3}\right) = 0.34 \text{ rad} \quad \text{or} \quad 2.8 \text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi = 2.8$ rad because this is about 161° and is in second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.8 - 2\pi = -3.48$ rad is also an acceptable result.

67. We compare the resultant wave given with the standard expression (Eq. 16–52) to obtain $k = 20\,\text{m}^{-1} = 2\pi/\lambda, 2y_m \cos\left(\frac{1}{2}\phi\right) = 3.0\,\text{mm}$, and $\frac{1}{2}\phi = 0.820\,\text{rad}$.

(a) Therefore, $\lambda = 2\pi/k = 0.31$ m.

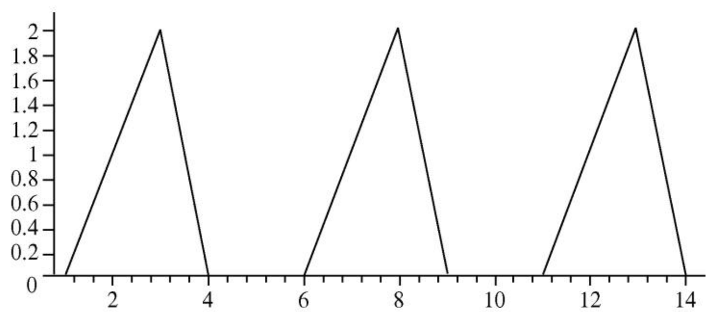
(b) The phase difference is $\phi = 1.64$ rad.

(c) And the amplitude is $y_m = 2.2$ mm.

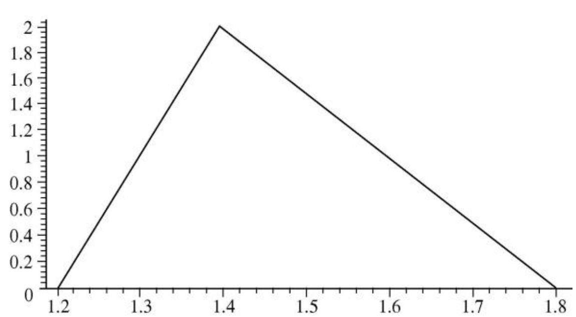
68. (a) Recalling the discussion in Section 16-5, we see that the speed of the wave given by a function with argument $x - 5.0t$ (where x is in centimeters and t is in seconds) must be 5.0 cm/s.

(b) In part (c), we show several "snapshots" of the wave: the one on the left is as shown in Figure 16-44 (at $t = 0$), the middle one is at $t = 1.0$ s, and the rightmost one is at $t = 2.0$ s. It is clear that the wave is traveling to the right (the $+x$ direction).

(c) The third picture in the sequence below shows the pulse at 2.0 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.

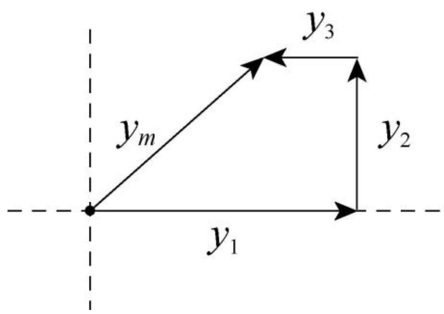


(d) The leading edge of the pulse reaches $x = 10$ cm at $t = (10 - 4.0)/5 = 1.2$ s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement $h = 2$ cm at $t = (10 - 3.0)/5 = 1.4$ s. Finally, the trailing edge of the pulse departs from $x = 10$ cm at $t = (10 - 1.0)/5 = 1.8$ s. Thus, we find for $h(t)$ at $x = 10$ cm (with the horizontal axis, t, in seconds):



69. **THINK** We use phasors to add the three waves and calculate the amplitude of the resultant wave.

EXPRESS The phasor diagram is shown here: y_1, y_2, and y_3 represent the original waves and y_m represents the resultant wave.



The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$.

ANALYZE (a) The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1.$$

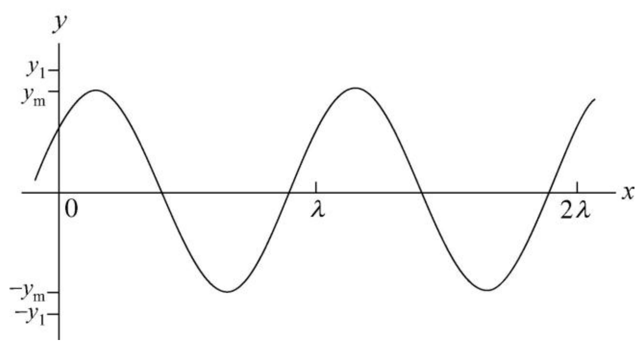
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\left(\frac{y_{mv}}{y_{mh}}\right) = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 0.644 \text{ rad} = 37°.$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644 \text{ rad}).$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



LEARN In adding the three sinusoidal waves, it is convenient to represent each wave with a phasor, which is a vector whose magnitude is equal to the amplitude of the wave. However, adding the three terms explicitly gives, after a little algebra,

$$\begin{aligned}
y_1 + y_2 + y_3 &= y_1 \sin(kx-\omega t) + \frac{1}{2} y_1 \sin(kx-\omega t+\pi/2) + \frac{1}{3} y_1 \sin(kx-\omega t+\pi) \\
&= y_1 \sin(kx-\omega t) + \frac{1}{2} y_1 \cos(kx-\omega t) - \frac{1}{3} y_1 \sin(kx-\omega t) \\
&= \frac{2}{3} y_1 \sin(kx-\omega t) + \frac{1}{2} y_1 \cos(kx-\omega t) \\
&= \frac{5}{6} y_1 \left[\frac{4}{5} \sin(kx-\omega t) + \frac{3}{5} \cos(kx-\omega t) \right] \\
&= \frac{5}{6} y_1 \sin(kx-\omega t+\phi)
\end{aligned}$$

where $\phi = \tan^{-1}(3/4) = 0.644$ rad. In deducing the phase ϕ, we set $\cos\phi = 4/5$ and $\sin\phi = 3/5$, and use the relation $\cos\phi\sin\theta + \sin\phi\cos\theta = \sin(\theta+\phi)$. The result indeed agrees with that obtained in (c).

70. Setting $x = 0$ in $a_y = -\omega^2 y$, where $y = y_m \sin(kx - \omega t + \phi)$ gives

$$a_y = -\omega^2 y_m \sin(-\omega t + \phi)$$

as the function being plotted in the graph. We note that it has a negative "slope" (referring to its t-derivative) at $t = 0$, or

$$\frac{da_y}{dt} = \frac{d}{dt}\left[-\omega^2 y_m \sin(-\omega t+\phi)\right] = \omega^3 y_m \cos(-\omega t+\phi) < 0$$

at $t = 0$. This implies that $\cos\phi < 0$ and consequently that ϕ is in either the second or third quadrant. The graph shows (at $t = 0$) $a_y = -100$ m/s², and (at another t) $a_{max} = 400$ m/s². Therefore,

$$a_y = -a_{max} \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \sin^{-1}(\tfrac{1}{4}) = 0.25 \text{ rad} \quad \text{or} \quad 2.9 \text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi = 2.9$ rad because this is about 166° and is in the second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.9 - 2\pi = -3.4$ rad is also an acceptable result.

71. (a) Let the displacement of the string be of the form $y(x, t) = y_m \sin(kx - \omega t)$. The velocity of a point on the string is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and its maximum value is $u_m = \omega y_m$. For this wave the frequency is f = 120 Hz and the angular frequency is $\omega = 2\pi f = 2\pi$ (120 Hz) = 754 rad/s. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00 \times 10^{-3}$ m. The maximum speed is

$$u_m = (754 \text{ rad/s})(5.00 \times 10^{-3} \text{ m}) = 3.77 \text{ m/s}.$$

(b) Consider the string at coordinate x and at time t and suppose it makes the angle θ with the x axis. The tension is along the string and makes the same angle with the x axis. Its transverse component is $\tau_{\text{trans}} = \tau \sin \theta$. Now θ is given by $\tan \theta = \partial y/\partial x = ky_m \cos(kx - \omega t)$ and its maximum value is given by $\tan \theta_m = ky_m$. We must calculate the angular wave number k. It is given by $k = \omega/v$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1}.$$

Thus,

$$\tan \theta_m = (27.5\,\text{m}^{-1})(5.00 \times 10^{-3}\text{m}) = 0.138$$

and $\theta = 7.83°$. The maximum value of the transverse component of the tension in the string is

$$\tau_{\text{trans}} = (90.0 \text{ N}) \sin 7.83° = 12.3 \text{ N}.$$

We note that $\sin \theta$ is nearly the same as $\tan \theta$ because θ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at x. The transverse component of the tension pulling on it due to the string to the left is $-\tau(\partial y/\partial x) = -\tau k y_m \cos(kx - \omega t)$ and it reaches its maximum value when $\cos(kx - \omega t) = -1$. The wave speed is

$$u = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and it also reaches its maximum value when $\cos(kx - \omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx - \omega t) = -1$ the value of $\sin(kx - \omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement Δy, the tension does work $\Delta W = \tau_{\text{trans}}\,\Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}}\frac{\Delta y}{\Delta t} = \tau_{\text{trans}}u.$$

P has its maximum value when the transverse component τ_{trans} of the tension and the string speed u have their maximum values. Hence the maximum power is (12.3 N)(3.77 m/s) = 46.4 W.

(e) As shown above, $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx - \omega t) = 0$ and $\sin(kx - \omega t) = \pm 1$ at that time. The string displacement is $y = \pm y_m = \pm 0.50$ cm.

72. We use Eq. 16-52 in interpreting the figure.

(a) Since $y' = 6.0$ mm when $\phi = 0$, then Eq. 16-52 can be used to determine $y_m = 3.0$ mm.

(b) We note that $y' = 0$ when the shift distance is 10 cm; this occurs because $\cos(\phi/2) = 0$ there $\Rightarrow$ $\phi = \pi$ rad or ½ cycle. Since a full cycle corresponds to a distance of one full wavelength, this ½ cycle shift corresponds to a distance of $\lambda/2$. Therefore, $\lambda = 20$ cm $\Rightarrow$ $k = 2\pi/\lambda = 31\ \text{m}^{-1}$.

(c) Since $f = 120$ Hz, $\omega = 2\pi f = 754\ \text{rad/s} \approx 7.5\times 10^{2}\ \text{rad/s}$.

(d) The sign in front of ω is minus since the waves are traveling in the $+x$ direction.

The results may be summarized as $y = (3.0\ \text{mm})\sin[(31.4\ \text{m}^{-1})x - (754\ \text{s}^{-1})t]]$ (this applies to each wave when they are in phase).

73. We note that

$$dy/dt = -\omega\cos(kx - \omega t + \phi),$$

which we will refer to as $u(x,t)$. so that the ratio of the function $y(x,t)$ divided by $u(x,t)$ is $-\tan(kx - \omega t + \phi)/\omega$. With the given information (for $x = 0$ and $t = 0$) then we can take the inverse tangent of this ratio to solve for the phase constant:

$$\phi = \tan^{-1}\left(\frac{-\omega y(0,0)}{u(0,0)}\right) = \tan^{-1}\left(\frac{-(440)(0.0045)}{-0.75}\right) = 1.2\ \text{rad}.$$

74. We use $P = \frac{1}{2}\mu v\omega^2 y_m^2 \propto vf^2 \propto \sqrt{\tau} f^2$.

(a) If the tension is quadrupled, then $P_2 = P_1\sqrt{\dfrac{\tau_2}{\tau_1}} = P_1\sqrt{\dfrac{4\tau_1}{\tau_1}} = 2P_1$.

(b) If the frequency is halved, then $P_2 = P_1\left(\dfrac{f_2}{f_1}\right)^2 = P_1\left(\dfrac{f_1/2}{f_1}\right)^2 = \dfrac{1}{4}P_1$.

75. (a) Let the cross-sectional area of the wire be A and the density of steel be ρ. The tensile stress is given by τ/A where τ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$v_{\max} = \sqrt{\frac{\tau_{\max}}{\mu}} = \sqrt{\frac{\tau_{\max}/A}{\rho}} = \sqrt{\frac{7.00\times10^8\ \text{N/m}^2}{7800\,\text{kg/m}^3}} = 3.00\times10^2\ \text{m/s}\,.$$

(b) The result does not depend on the diameter of the wire.

76. Repeating the steps of Eq. 16-47 → Eq. 16-53, but applying

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

(see Appendix E) instead of Eq. 16-50, we obtain $y' = [0.10\cos\pi x]\cos 4\pi t$, with SI units understood.

(a) For non-negative x, the smallest value to produce $\cos \pi x = 0$ is $x = 1/2$, so the answer is $x = 0.50$ m.

(b) Taking the derivative,

$$u' = \frac{dy'}{dt} = \left[0.10\cos\pi x\right]\left(-4\pi\sin 4\pi t\right).$$

We observe that the last factor is zero when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ Thus, the value of the first time the particle at $x = 0$ has zero velocity is $t = 0$.

(c) Using the result obtained in (b), the second time where the velocity at $x = 0$ vanishes would be $t = 0.25$ s,

(d) and the third time is $t = 0.50$ s.

77. **THINK** The speed of a transverse wave in the stretched rubber band is related to the tension in the band and the linear mass density of the band.

EXPRESS The wave speed v is given by $v = \sqrt{F/\mu}$, where F is the tension in the rubber band and μ is the band's linear mass density, which is defined as the mass per unit length $\mu = m/L$. The fact that the band obeys Hooke's law implies $F = k\Delta\ell$, where k is the spring constant and $\Delta\ell$ is the elongation. Thus, when a force F is applied, the rubber band has a length $L = \ell + \Delta\ell$, where ℓ is the unstretched length, resulting in a linear mass density $\mu = m/(\ell + \Delta\ell)$.

ANALYZE (a) The wave speed is $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{k\Delta\ell}{m/(\ell+\Delta\ell)}} = \sqrt{\dfrac{k\Delta\ell(\ell+\Delta\ell)}{m}}$.

(b) The time required for the pulse to travel the length of the rubber band is

$$t = \frac{2\pi(\ell+\Delta\ell)}{v} = \frac{2\pi(\ell+\Delta\ell)}{\sqrt{k\Delta\ell(\ell+\Delta\ell)/m}} = 2\pi\sqrt{\frac{m}{k}}\sqrt{1+\frac{\ell}{\Delta\ell}}.$$

Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$. On the other hand, if $\ell/\Delta\ell \ll 1$, then we have $t \simeq 2\pi\sqrt{m/k} = \text{const}$.

LEARN When $\Delta\ell \ll \ell$, the applied force $F = k\Delta\ell$ is small while $\mu \approx m/\ell = \text{constant}$, leading to a small wave speed. On the other hand, when $\Delta\ell \gg \ell$, $\mu \approx m/\Delta\ell$ and $v = \sqrt{F/\mu} \propto \Delta\ell$, so that $t \simeq 2\pi\sqrt{m/k}$, which is a constant.

78. (a) For visible light

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{700\times10^{-9}\,\text{m}} = 4.3\times10^{14}\ \text{Hz}$$

and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{400\times10^{-9}\,\text{m}} = 7.5\times10^{14}\ \text{Hz}.$$

(b) For radio waves

$$\lambda_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{300\times10^{6}\,\text{Hz}} = 1.0\,\text{m}$$

and

$$\lambda_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.5\times10^{6}\,\text{Hz}} = 2.0\times10^{2}\,\text{m}.$$

(c) For X rays

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{5.0\times10^{-9}\ \text{m}} = 6.0\times10^{16}\ \text{Hz}$$

and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.0\times10^{-11}\ \text{m}} = 3.0\times10^{19}\ \text{Hz}.$$

79. **THINK** A wire held rigidly at both ends can be made to oscillate in standing wave patterns.

EXPRESS Possible wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$ where τ is the tension in the wire and μ is the linear mass density of the wire.

ANALYZE (a) The wave speed is $v = \sqrt{\dfrac{\tau}{\mu}} = \sqrt{\dfrac{120\ \text{N}}{8.70\times10^{-3}\ \text{kg}/1.50\ \text{m}}} = 144\ \text{m/s}.$

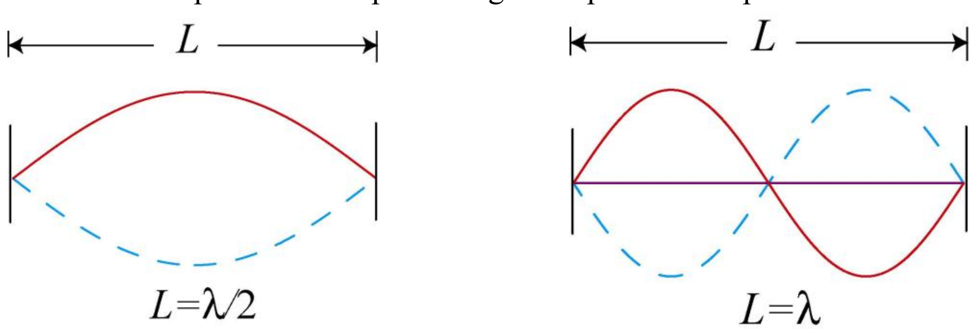
(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50\ \text{m}) = 3.00\ \text{m}$.

(c) For the two-loop standing wave $\lambda_2 = L = 1.50\ \text{m}$.

(d) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144\ \text{m/s})/(3.00\ \text{m}) = 48.0\ \text{Hz}$.

(e) The frequency for the two-loop wave is $f_2 = v/\lambda_2 = (144\ \text{m/s})/(1.50\ \text{m}) = 96.0\ \text{Hz}$.

LEARN The one-loop and two-loop standing wave patterns are plotted below:



80. By Eq. 16–66, the higher frequencies are integer multiples of the lowest (the fundamental).

(a) The frequency of the second harmonic is $f_2 = 2(440) = 880\ \text{Hz}$.

(b) The frequency of the third harmonic is $f_3 = 3(440) = 1320\ \text{Hz}$.

81. (a) The amplitude is y_m = 1.00 cm = 0.0100 m, as given in the problem.

(b) Since the frequency is f = 550 Hz, the angular frequency is $\omega = 2\pi f = 3.46\times10^3$ rad/s.

(c) The angular wave number is $k = \omega / v = (3.46\times10^3 \text{ rad/s})/(330 \text{ m/s}) = 10.5 \text{ rad/m}$.

(d) Since the wave is traveling in the $-x$ direction, the sign in front of ω is plus and the argument of the trig function is $kx + \omega t$.

The results may be summarized as

$$\begin{aligned} y(x,t) &= y_m \sin(kx+\omega t) = y_m \sin\left[2\pi f\left(\frac{x}{v}+t\right)\right] \\ &= (0.010\,\text{m})\sin\left[2\pi(550\,\text{Hz})\left(\frac{x}{330\,\text{m/s}}+t\right)\right] \\ &= (0.010\,\text{m})\sin[(10.5\text{ rad/s})\,x + (3.46\times10^3\text{ rad/s})t]. \end{aligned}$$

82. We orient one phasor along the x axis with length 3.0 mm and angle 0 and the other at 70° (in the first quadrant) with length 5.0 mm. Adding the components, we obtain

$$\begin{aligned} (3.0\text{ mm}) + (5.0\text{ mm})\cos(70°) &= 4.71\,\text{mm along } x \text{ axis} \\ (5.0\text{ mm})\sin(70°) &= 4.70\text{ mm along } y \text{ axis.} \end{aligned}$$

(a) Thus, amplitude of the resultant wave is $\sqrt{(4.71\text{ mm})^2 + (4.70\text{ mm})^2} = 6.7\,\text{mm}$.

(b) And the angle (phase constant) is $\tan^{-1}(4.70/4.71) = 45°$.

83. **THINK** The speed of a point on the cord is given by $u(x, t) = \partial y/\partial t$, where $y(x, t)$ is displacement.

EXPRESS We take the form of the displacement to be

$$y(x, t) = y_m \sin(kx - \omega t).$$

The speed of a point on the cord is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t),$$

and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$.

(a) The ratio of the maximum particle speed to the wave speed is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on y_m/λ, the ratio of the amplitude to the wavelength.

LEARN Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

84. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where λ is the wavelength and L is the length of the string. The wavelength is related to the frequency f and wave speed v by $\lambda = v/f$, so $L/2 = v/f$ and

$$L = 2v/f = 2(400 \text{ m/s})/(600 \text{ Hz}) = 1.3 \text{ m}.$$

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx) \cos(\omega t)$, where y_m is the maximum displacement, k is the angular wave number, and ω is the angular frequency. The angular wave number is

$$k = 2\pi/\lambda = 2\pi f/v = 2\pi(600 \text{ Hz})/(400 \text{ m/s}) = 9.4\text{m}^{-1}$$

and the angular frequency is

$$\omega = 2\pi f = 2\pi(600 \text{ Hz}) = 3800 \text{ rad/s}.$$

With $y_m = 2.0$ mm, the displacement is given by

$$y(x,t) = (2.0\,\text{mm})\sin[(9.4\,\text{m}^{-1})x]\cos[(3800\,\text{s}^{-1})t].$$

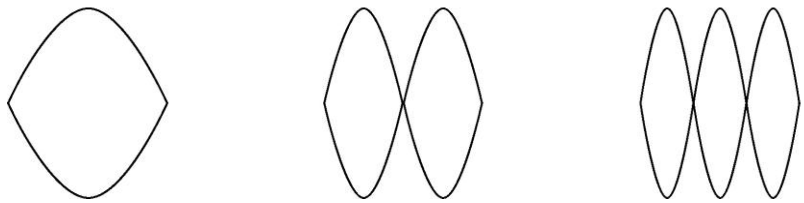
85. We make use of Eq. 16-65 with $L = 120$ cm.

(a) The longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_1 = 2L/1 = 240$ cm.

(b) The second longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_2 = 2L/2 = 120$ cm.

(c) The third longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_3 = 2L/3 = 80.0$ cm.

The three standing waves are shown next:



86. (a) Let the displacements of the wave at (y,t) be $z(y,t)$. Then

$$z(y,t) = z_m \sin(ky - \omega t),$$

where $z_m = 3.0$ mm, $k = 60\ \text{cm}^{-1}$, and $\omega = 2\pi/T = 2\pi/0.20\ \text{s} = 10\pi\ \text{s}^{-1}$. Thus

$$z(y,t) = (3.0\,\text{mm})\sin\left[\left(60\,\text{cm}^{-1}\right)y - \left(10\pi\,\text{s}^{-1}\right)t\right].$$

(b) The maximum transverse speed is $u_m = \omega z_m = (2\pi/0.20\,\text{s})(3.0\,\text{mm}) = 94\,\text{mm/s}$.

87. (a) With length in centimeters and time in seconds, we have

$$u = \frac{dy}{dt} = -60\pi\cos\left(\frac{\pi x}{8} - 4\pi t\right).$$

Thus, when $x = 6$ and $t = \frac{1}{4}$, we obtain

$$u = -60\pi\cos\frac{-\pi}{4} = \frac{-60\pi}{\sqrt{2}} = -133$$

so that the *speed* there is 1.33 m/s.

(b) The numerical coefficient of the cosine in the expression for u is -60π. Thus, the maximum *speed* is 1.88 m/s.

(c) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2\sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

so that when $x = 6$ and $t = \frac{1}{4}$ we obtain $a = -240\pi^2 \sin(-\pi/4)$, which yields $a = 16.7\ \text{m/s}^2$.

(d) The numerical coefficient of the sine in the expression for a is $-240\pi^2$. Thus, the maximum acceleration is $23.7\ \text{m/s}^2$.

88. (a) This distance is determined by the longitudinal speed:

$$d_\ell = v_\ell t = \left(2000\,\text{m/s}\right)\left(40\times10^{-6}\,\text{s}\right) = 8.0\times10^{-2}\,\text{m}.$$

(b) Assuming the acceleration is constant (justified by the near-straightness of the curve $a = 300/40 \times 10^{-6}$) we find the stopping distance d:

$$v^2 = v_o^2 + 2ad \Rightarrow d = \frac{(300)^2 (40\times10^{-6})}{2(300)}$$

which gives $d = 6.0\times10^{-3}$ m. This and the radius r form the legs of a right triangle (where r is opposite from $\theta = 60°$). Therefore,

$$\tan 60° = \frac{r}{d} \Rightarrow r = d \tan 60° = 1.0\times10^{-2}\,\text{m}.$$

89. Using Eq. 16-50, we have

$$y' = \left(0.60\cos\frac{\pi}{6}\right)\sin\left(5\pi x - 200\pi t + \frac{\pi}{6}\right)$$

with length in meters and time in seconds (see Eq. 16-55 for comparison).

(a) The amplitude is seen to be $0.60\cos\frac{\pi}{6} = 0.3\sqrt{3} = 0.52\,\text{m}$.

(b) Since $k = 5\pi$ and $\omega = 200\pi$, then (using Eq. 16-12), $v = \frac{\omega}{k} = 40\,\text{m/s}$.

(c) $k = 2\pi/\lambda$ leads to $\lambda = 0.40$ m.

90. (a) The frequency is $f = 1/T = 1/4$ Hz, so $v = f\lambda = 5.0$ cm/s.

(b) We refer to the graph to see that the maximum transverse speed (which we will refer to as u_m) is 5.0 cm/s. Using the simple harmonic motion relation $u_m = y_m\omega = y_m 2\pi f$, we have

$$5.0 = y_m\left(2\pi\frac{1}{4}\right) \quad \Rightarrow \quad y_m = 3.2 \text{ cm}.$$

(c) As already noted, $f = 0.25$ Hz.

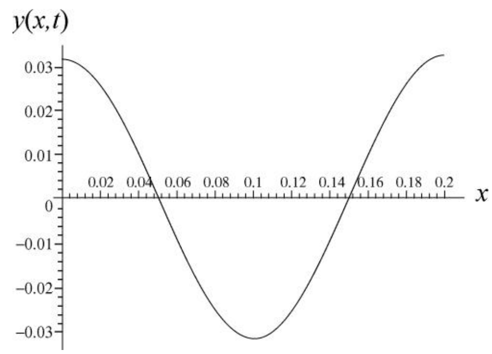
(d) Since $k = 2\pi/\lambda$, we have $k = 10\pi$ rad/m. There must be a sign difference between the t and x terms in the argument in order for the wave to travel to the right. The figure shows that at $x = 0$, the transverse velocity function is $0.050 \sin \pi t/2$. Therefore, the function $u(x,t)$ is

$$u(x,t) = 0.050\sin\left(\frac{\pi}{2}t - 10\pi x\right)$$

with lengths in meters and time in seconds. Integrating this with respect to time yields

$$y(x,t) = -\frac{2(0.050)}{\pi}\cos\left(\frac{\pi}{2}t - 10\pi x\right) + C$$

where C is an integration constant (which we will assume to be zero). The sketch of this function at t = 2.0 s for $0 \le x \le 0.20$ m is shown below.



91. **THINK** The rope with both ends fixed and made to oscillate in fundamental mode has wavelength $\lambda = 2L$, where L is the length of the rope.

EXPRESS We first observe that the anti-node at x = 1.0 m having zero displacement at t = 0 suggests the use of sine instead of cosine for the simple harmonic motion factor. We take the form of the displacement to be

$$y(x, t) = y_m \sin(kx)\sin(\omega t).$$

A point on the rope undergoes simple harmonic motion with a speed

$$u(x, t) = \partial y/\partial t = \omega y_m \sin(kx)\cos(\omega t).$$

It has maximum speed $u_m = \omega y_m$ as it passes through its "middle" point. On the other hand, the wave speed is $v = \sqrt{\tau/\mu}$ where τ is the tension in the rope and μ is the linear mass density of the rope. For standing waves, possible wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the rope and n is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where v is the wave speed. For fundamental mode, we set n = 1.

ANALYZE (a) With f = 5.0 Hz, we find the angular frequency to be $\omega = 2\pi f = 10\pi$ rad/s. Thus, if the maximum speed of a point on the rope is u_m = 5.0 m/s, then its amplitude is

$$y_m = \frac{u_m}{\omega} = \frac{5.0\text{ m/s}}{10\pi\text{ rad/s}} = 0.16\text{ m}.$$

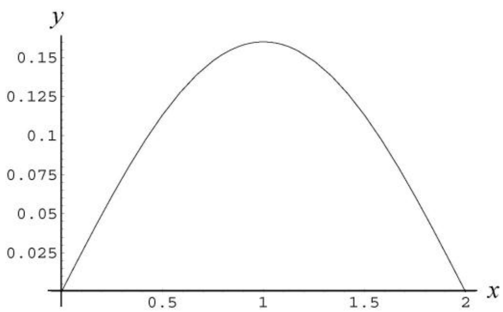
(b) Since the oscillation is in the *fundamental* mode, we have $\lambda = 2L = 4.0$ m. Therefore, the speed of waves along the rope is $v = f\lambda = 20$ m/s. Then, with $\mu = m/L = 0.60$ kg/m, Eq. 16-26 leads to

$$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = \mu v^2 = 240 \text{ N} \approx 2.4\times10^2 \text{ N}.$$

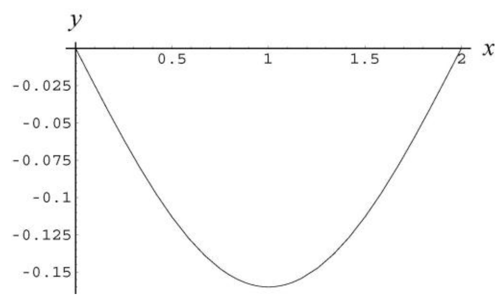
(c) We note that for the fundamental, $k = 2\pi/\lambda = \pi/L$. Now, *if* the fundamental mode is the only one present (so the amplitude calculated in part (a) is indeed the amplitude of the fundamental wave pattern) then we have

$$y = (0.16 \text{ m}) \sin\left(\frac{\pi x}{2}\right) \sin(10\pi t) = (0.16 \text{ m})\sin[(1.57 \text{ m}^{-1})x]\sin[(31.4 \text{ rad/s})t]$$

LEARN The period of oscillation is $T = 1/f = 0.20$ s. The snapshots of the patterns at $t = T/4 = 0.05$ s and $t = 3T/4 = 0.15$ s are given below. At $t = T/2$ and T, the displacement is zero everywhere.

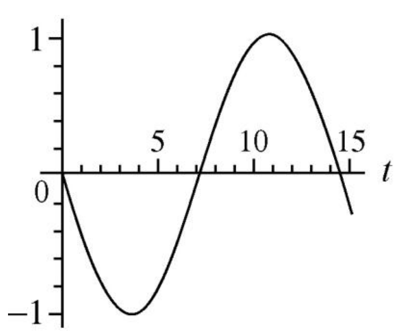


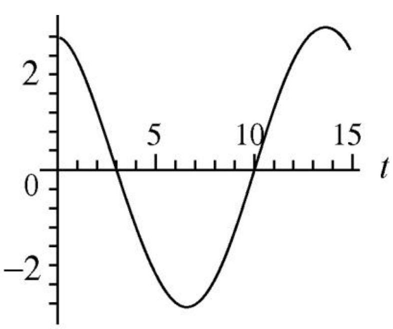
$t = T/4 = 0.05$ s



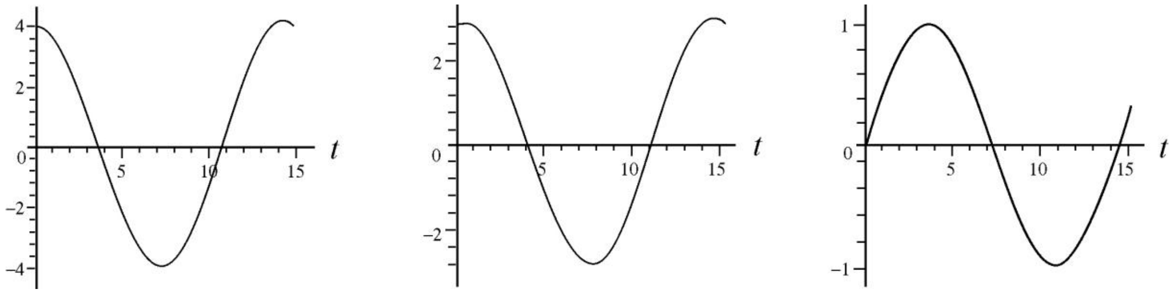
$t = 3T/4 = 0.15$ s

92. (a) The wave number for each wave is $k = 25.1$/m, which means $\lambda = 2\pi/k = 250.3$ mm. The angular frequency is $\omega = 440$/s; therefore, the period is $T = 2\pi/\omega = 14.3$ ms. We plot the superposition of the two waves $y = y_1 + y_2$ over the time interval $0 \le t \le 15$ ms. The first two graphs below show the oscillatory behavior at $x = 0$ (the graph on the left) and at $x = \lambda/8 \approx 31$ mm. The time unit is understood to be the millisecond and vertical axis (y) is in millimeters.





The following three graphs show the oscillation at $x = \lambda/4$ =62.6 mm ≈ 63 mm (graph on the left), at $x = 3\lambda/8$ ≈ 94 mm (middle graph), and at $x = \lambda/2$ ≈ 125 mm.



(b) We can think of wave y_1 as being made of two smaller waves going in the same direction, a wave y_{1a} of amplitude 1.50 mm (the same as y_2) and a wave y_{1b} of amplitude 1.00 mm. It is made clear in Section 16-12 that two equal-magnitude oppositely-moving waves form a standing wave pattern. Thus, waves y_{1a} and y_2 form a standing wave, which leaves y_{1b} as the remaining traveling wave. Since the argument of y_{1b} involves the subtraction $kx - \omega t$, then y_{1b} travels in the $+x$ direction.

(c) If y_2 (which travels in the $-x$ direction, which for simplicity will be called "leftward") had the larger amplitude, then the system would consist of a standing wave plus a leftward moving wave. A simple way to obtain such a situation would be to interchange the amplitudes of the given waves.

(d) Examining carefully the vertical axes, the graphs above certainly suggest that the largest amplitude of oscillation is y_{max} = 4.0 mm and occurs at $x = \lambda/4$ = 62.6 mm.

(e) The smallest amplitude of oscillation is y_{min} = 1.0 mm and occurs at $x = 0$ and at

$$x = \lambda/2 = 125 \text{ mm}.$$

(f) The largest amplitude can be related to the amplitudes of y_1 and y_2 in a simple way:

$$y_{max} = y_{1m} + y_{2m},$$

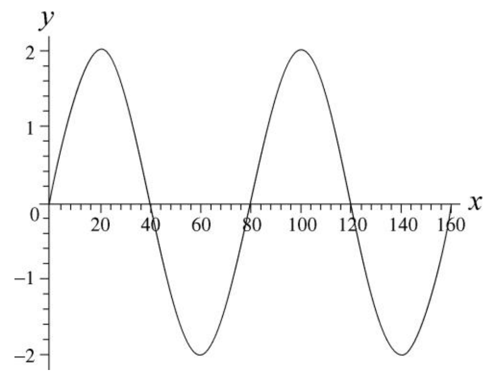
where y_{1m} = 2.5 mm and y_{2m} = 1.5 mm are the amplitudes of the original traveling waves.

(g) The smallest amplitudes is

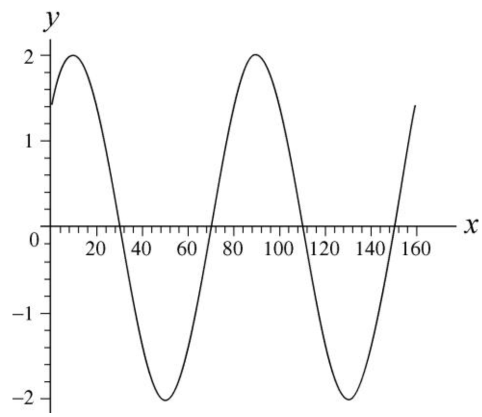
$$y_{min} = y_{1m} - y_{2m},$$

where y_{1m} = 2.5 mm and y_{2m} = 1.5 mm are the amplitudes of the original traveling waves.

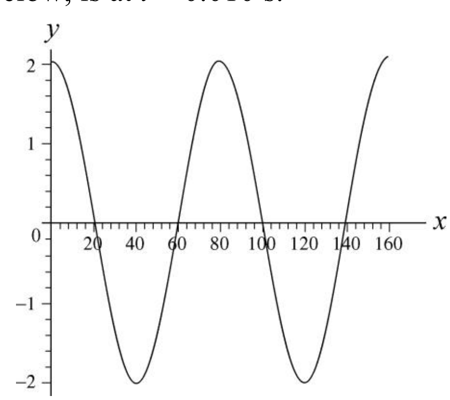
93. (a) Centimeters are to be understood as the length unit and seconds as the time unit. Making sure our (graphing) calculator is in radians mode, we find



(b) The previous graph is at t = 0, and this next one is at t = 0.050 s.



And the final one, shown below, is at t = 0.010 s.



(c) The wave can be written as $y(x,t) = y_m \sin(kx + \omega t)$, where $v = \omega / k$ is the speed of propagation. From the problem statement, we see that $\omega = 2\pi / 0.40 = 5\pi$ rad/s and $k = 2\pi / 80 = \pi / 40$ rad/cm. This yields $v = 2.0 \times 10^2$ cm/s $= 2.0$ m/s.

(d) These graphs (as well as the discussion in the textbook) make it clear that the wave is traveling in the –x direction.

94. The speed of the transverse wave along the string is given by Eq. 16-26: $v=\sqrt{\tau/\mu}$, where τ is the tension and μ is the linear mass density of the string. Applying Newton's second law to a small segment of the string, the radial restoring force is (see Eq. 16-23)

$$F = 2(\tau\sin\theta) \approx \tau\frac{\Delta l}{R}$$

Since $F=(\Delta m)v_T^2/R$, where v_T is the tangential speed of the segment of mass $\Delta m=\mu\Delta l$, and R is the radius of the circle, we have

$$\tau\frac{\Delta l}{R} = (\mu\Delta l)\frac{v_T^2}{R} \quad\Rightarrow\quad \tau = \mu v_T^2$$

On the other hand, the fact that $v=\sqrt{\tau/\mu}$ implies $\tau=\mu v^2$. Thus, we must have $v=v_T$, which in this case, is equal to 5.00 cm/s. Note that v is independent of the radius of the circular loop.

95. (a) With total reflection, $A=B$, and $\text{SWR}=\dfrac{A+B}{A-B}\to\infty$.

(b) With no reflection, $B=0$, and $\text{SWR}=\dfrac{A+B}{A-B}=\dfrac{A}{A}=1$.

(c) In terms of $R=(B/A)^2$, we can rewrite SWR as

$$\text{SWR}=\frac{A+B}{A-B}=\frac{1+(B/A)}{1-(B/A)}=\frac{1+\sqrt{R}}{1-\sqrt{R}} \quad\Rightarrow\quad R=\left(\frac{\text{SWR}-1}{\text{SWR}+1}\right)^2$$

With SWR = 1.50, we obtain

$$R=\left(\frac{\text{SWR}-1}{\text{SWR}+1}\right)^2=\left(\frac{1.50-1}{1.50+1}\right)^2=0.040=4.0\%.$$

96. (a) The speed of each individual wave is

$$v=\sqrt{\frac{\tau}{\mu}}=\sqrt{\frac{40\text{ N}}{(0.125\text{ kg})/(2.25\text{ m})}}=26.83\text{ m/s}.$$

The average rate at which energy is transmitted from one side is

$$P_{\text{avg},1}=\frac{1}{2}\mu v\omega^2 y_m^2=\frac{1}{2}\left(\frac{0.125\text{ kg}}{2.25\text{ m}}\right)(26.83\text{ m/s})(2\pi\times 120\text{ Hz})^2(5.0\times 10^{-3}\text{ m})^2=10.6\text{ W}.$$

(b) From both sides, $P_{\text{avg}}=2P_{\text{avg},1}=2(10.6\text{ W})=21.2\text{ W}.$

(c) The rate of change of kinetic energy from one side is given by Eq. 16-30:

$$\frac{dK_1}{dt}=\frac{1}{2}\mu v\omega^2 y_m^2\cos^2(kx-\omega t).$$

Integrating over one period for both sides, we obtain

$$K=\int\left(2\frac{dK_1}{dt}\right)dt=\mu v\omega^2 y_m^2\int_0^T\cos^2(kx-\omega t)dt=\frac{T}{2}\mu v\omega^2 y_m^2=\frac{P_{\text{avg}}}{2f}$$
$$=\frac{21.2\text{ W}}{2(120\text{ Hz})}=8.83\times 10^{-2}\text{ J}.$$

Chapter 17

1. (a) The time for the sound to travel from the kicker to a spectator is given by d/v, where d is the distance and v is the speed of sound. The time for light to travel the same distance is given by d/c, where c is the speed of light. The delay between seeing and hearing the kick is $\Delta t = (d/v) - (d/c)$. The speed of light is so much greater than the speed of sound that the delay can be approximated by $\Delta t = d/v$. This means $d = v\,\Delta t$. The distance from the kicker to spectator A is

$$d_A = v\,\Delta t_A = (343\text{ m/s})(0.23\text{ s}) = 79\text{ m}.$$

(b) The distance from the kicker to spectator B is $d_B = v\,\Delta t_B = (343\text{ m/s})(0.12\text{ s}) = 41\text{ m}$.

(c) Lines from the kicker to each spectator and from one spectator to the other form a right triangle with the line joining the spectators as the hypotenuse, so the distance between the spectators is

$$D = \sqrt{d_A^2 + d_B^2} = \sqrt{(79\,\text{m})^2 + (41\,\text{m})^2} = 89\,\text{m}.$$

2. The density of oxygen gas is

$$\rho = \frac{0.0320\,\text{kg}}{0.0224\,\text{m}^3} = 1.43\,\text{kg/m}^3.$$

From $v = \sqrt{B/\rho}$ we find

$$B = v^2\rho = (317\,\text{m/s})^2\left(1.43\,\text{kg/m}^3\right) = 1.44\times10^5\text{ Pa}.$$

3. (a) When the speed is constant, we have $v = d/t$ where $v = 343$ m/s is assumed. Therefore, with $t = 15/2$ s being the time for sound to travel to the far wall we obtain $d = (343\text{ m/s}) \times (15/2\text{ s})$, which yields a distance of 2.6 km.

(b) Just as the $\frac{1}{2}$ factor in part (a) was $1/(n + 1)$ for $n = 1$ reflection, so also can we write

$$d = (343\,\text{m/s})\left(\frac{15\,\text{s}}{n+1}\right) \quad\Rightarrow\quad n = \frac{(343)(15)}{d} - 1$$

for multiple reflections (with d in meters). For $d = 25.7$ m, we find $n = 199 \approx 2.0\times10^2$.

4. The time it takes for a soldier in the rear end of the column to switch from the left to the right foot to stride forward is $t = 1$ min/120 = 1/120 min = 0.50 s. This is also the time

for the sound of the music to reach from the musicians (who are in the front) to the rear end of the column. Thus the length of the column is

$$l = vt = (343\,\text{m/s})(0.50\,\text{s}) = 1.7 \times 10^2\text{m}.$$

5. **THINK** The S and P waves generated by the earthquake travel at different speeds. Knowing the speeds of the waves and the time difference of their arrival to the seismograph allows us to determine the location of the earthquake.

EXPRESS Let d be the distance from the location of the earthquake to the seismograph. If v_s is the speed of the S waves, then the time for these waves to reach the seismograph is t_s. = d/v_s. Similarly, the time for P waves to reach the seismograph is $t_p = d/v_p$. The time delay is

$$\Delta t = (d/v_s) - (d/v_p) = d(v_p - v_s)/v_s v_p,$$

ANALYZE With $v_s = 4.5\,\text{km/s}$, $v_p = 8.0\,\text{km/s}$ and $\Delta t = 3.0\ \text{min} = 180\,\text{s}$, we find the distance to be

$$d = \frac{v_s v_p \Delta t}{(v_p - v_s)} = \frac{(4.5\ \text{km/s})(8.0\,\text{km/s})(180\ \text{s})}{8.0\,\text{km/s} - 4.5\,\text{km/s}} = 1.9 \times 10^3\ \text{km}.$$

LEARN The distance to the earthquake is proportional to the difference in the arrival times of the P and S waves.

6. Let ℓ be the length of the rod. Then the time of travel for sound in air (speed v_s) will be $t_s = \ell / v_s$. And the time of travel for compression waves in the rod (speed v_r) will be $t_r = \ell / v_r$. In these terms, the problem tells us that

$$t_s - t_r = 0.12\,\text{s} = \ell\left(\frac{1}{v_s} - \frac{1}{v_r}\right).$$

Thus, with v_s = 343 m/s and $v_r = 15v_s$ = 5145 m/s, we find $\ell = 44\,\text{m}$.

7. **THINK** The time elapsed before hearing the splash is the sum of the time it takes for the stone to hit the water in the well, and the time it takes for the sound wave to travel back to the listener.

EXPRESS Let t_f be the time for the stone to fall to the water and t_s be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If d is the depth of the well, then the kinematics of free fall gives

$$d = \frac{1}{2} g t_f^2 \quad \Rightarrow \quad t_f = \sqrt{2d / g}.$$

The sound travels at a constant speed v_s, so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = \sqrt{2d/g} + d/v_s$. This equation is to be solved for d.

ANALYZE Rewriting the above expression as $\sqrt{2d/g} = t - d/v_s$ and squaring both sides, we obtain

$$2d/g = t^2 - 2(t/v_s)d + (1 + v_s^2)d^2.$$

Now multiply by $g v_s^2$ and rearrange to get

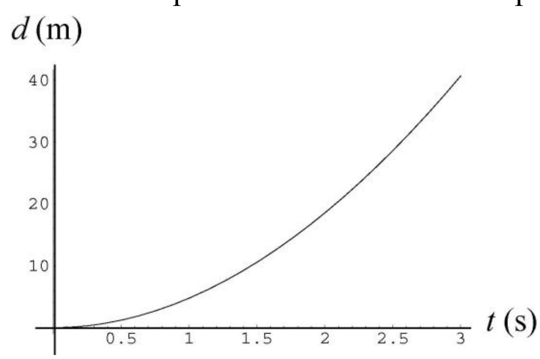
$$gd^2 - 2v_s(gt + v_s)d + g v_s^2 t^2 = 0.$$

This is a quadratic equation for d. Its solutions are

$$d = \frac{2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2 v_s^2 t^2}}{2g}.$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7$ m is obtained.

LEARN The relation between the depth of the well and time is plotted below:



8. Using Eqs. 16-13 and 17-3, the speed of sound can be expressed as

$$v = \lambda f = \sqrt{\frac{B}{\rho}},$$

where $B = -(dp/dV)/V$. Since $V, \lambda,$ and ρ are not changed appreciably, the frequency ratio becomes

$$\frac{f_s}{f_i} = \frac{v_s}{v_i} = \sqrt{\frac{B_s}{B_i}} = \sqrt{\frac{(dp/dV)_s}{(dp/dV)_i}} .$$

Thus, we have

$$\frac{(dV/dp)_s}{(dV/dp)_i} = \frac{B_i}{B_s} = \left(\frac{f_i}{f_s}\right)^2 = \left(\frac{1}{0.333}\right)^2 = 9.00 .$$

9. Without loss of generality we take $x = 0$, and let $t = 0$ be when $s = 0$. This means the phase is $\phi = -\pi/2$ and the function is $s = (6.0 \text{ nm})\sin(\omega t)$ at $x = 0$. Noting that $\omega = 3000$ rad/s, we note that at $t = \sin^{-1}(1/3)/\omega = 0.1133$ ms the displacement is $s = +2.0$ nm. Doubling that time (so that we consider the excursion from –2.0 nm to +2.0 nm) we conclude that the time required is 2(0.1133 ms) = 0.23 ms.

10. The key idea here is that the time delay Δt is due to the distance d that each wavefront must travel to reach your left ear (L) after it reaches your right ear (R).

(a) From the figure, we find $\Delta t = \dfrac{d}{v} = \dfrac{D\sin\theta}{v}$.

(b) Since the speed of sound in water is now v_w, with $\theta = 90°$, we have

$$\Delta t_w = \frac{D\sin 90°}{v_w} = \frac{D}{v_w} .$$

(c) The apparent angle can be found by substituting D/v_w for Δt:

$$\Delta t = \frac{D\sin\theta}{v} = \frac{D}{v_w} .$$

Solving for θ with $v_w = 1482$ m/s (see Table 17-1), we obtain

$$\theta = \sin^{-1}\left(\frac{v}{v_w}\right) = \sin^{-1}\left(\frac{343 \text{ m/s}}{1482 \text{ m/s}}\right) = \sin^{-1}(0.231) = 13° .$$

11. **THINK** The speed of sound in a medium is the product of the wavelength and frequency.

EXPRESS The wavelength of the sound wave is given by $\lambda = v/f$, where v is the speed of sound in the medium and f is the frequency,

ANALYZE (a) The speed of sound in air (at $20°\text{C}$) is $v = 343 \text{ m/s}$. Thus, we find

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{4.50\times10^{6}\ \text{Hz}} = 7.62\times10^{-5}\ \text{m}.$$

(b) The frequency of sound is the same for air and tissue. Now the speed of sound in tissue is $v = 1500\ \text{m/s}$, the corresponding wavelength is

$$\lambda = \frac{v}{f} = \frac{1500\,\text{m/s}}{4.50\times10^{6}\ \text{Hz}} = 3.33\times10^{-4}\ \text{m}.$$

LEARN The speed of sound depends on the medium through which it propagates. Table 17-1 provides a list of sound speed in various media.

12. (a) The amplitude of a sinusoidal wave is the numerical coefficient of the sine (or cosine) function: p_m = 1.50 Pa.

(b) We identify $k = 0.9\pi$ and $\omega = 315\pi$ (in SI units), which leads to $f = \omega/2\pi$ = 158 Hz.

(c) We also obtain $\lambda = 2\pi/k$ = 2.22 m.

(d) The speed of the wave is $v = \omega/k$ = 350 m/s.

13. The problem says "At one instant..." and we choose that instant (without loss of generality) to be $t = 0$. Thus, the displacement of "air molecule A" at that instant is

$$s_A = +s_m = s_m\cos(kx_A - \omega t + \phi)\big|_{t=0} = s_m\cos(kx_A + \phi),$$

where x_A = 2.00 m. Regarding "air molecule B" we have

$$s_B = +\tfrac{1}{3}s_m = s_m\cos(kx_B - \omega t + \phi)\big|_{t=0} = s_m\cos(kx_B + \phi).$$

These statements lead to the following conditions:

$$kx_A + \phi = 0$$
$$kx_B + \phi = \cos^{-1}(1/3) = 1.231$$

where x_B = 2.07 m. Subtracting these equations leads to

$$k(x_B - x_A) = 1.231 \quad\Rightarrow\quad k = 17.6\ \text{rad/m}.$$

Using the fact that $k = 2\pi/\lambda$ we find λ = 0.357 m, which means

$$f = v/\lambda = 343/0.357 = 960\ \text{Hz}.$$

Another way to complete this problem (once k is found) is to use $kv = \omega$ and then the fact that $\omega = 2\pi f$.

14. (a) The period is $T = 2.0$ ms (or 0.0020 s) and the amplitude is $\Delta p_m = 8.0$ mPa (which is equivalent to 0.0080 N/m^2). From Eq. 17-15 we get

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{\Delta p_m}{v\rho(2\pi/T)} = 6.1\times10^{-9}\ \text{m}$$

where $\rho = 1.21$ kg/m^3 and $v = 343$ m/s.

(b) The angular wave number is $k = \omega/v = 2\pi/vT = 9.2$ rad/m.

(c) The angular frequency is $\omega = 2\pi/T = 3142$ rad/s $\approx 3.1\times10^3$ rad/s.

The results may be summarized as $s(x, t) = (6.1\ \text{nm})\cos[(9.2\ \text{m}^{-1})x - (3.1\times10^3\ \text{s}^{-1})t]$.

(d) Using similar reasoning, but with the new values for density ($\rho' = 1.35$ kg/m^3) and speed ($v' = 320$ m/s), we obtain

$$s_m = \frac{\Delta p_m}{v'\rho'\omega} = \frac{\Delta p_m}{v'\rho'(2\pi/T)} = 5.9\times10^{-9}\ \text{m}.$$

(e) The angular wave number is $k = \omega/v' = 2\pi/v'T = 9.8$ rad/m.

(f) The angular frequency is $\omega = 2\pi/T = 3142$ rad/s $\approx 3.1\times10^3$ rad/s.

The new displacement function is $s(x, t) = (5.9\ \text{nm})\cos[(9.8\ \text{m}^{-1})x - (3.1\times10^3\ \text{s}^{-1})t]$.

15. (a) Consider a string of pulses returning to the stage. A pulse that came back just before the previous one has traveled an extra distance of $2w$, taking an extra amount of time $\Delta t = 2w/v$. The frequency of the pulse is therefore

$$f = \frac{1}{\Delta t} = \frac{v}{2w} = \frac{343\,\text{m/s}}{2(0.75\,\text{m})} = 2.3\times10^2\ \text{Hz}.$$

(b) Since $f \propto 1/w$, the frequency would be higher if w were smaller.

16. Let the separation between the point and the two sources (labeled 1 and 2) be x_1 and x_2, respectively. Then the phase difference is

$$\Delta\phi = \phi_1 - \phi_2 = 2\pi\left(\frac{x_1}{\lambda} + ft\right) - 2\pi\left(\frac{x_2}{\lambda} + ft\right) = \frac{2\pi(x_1 - x_2)}{\lambda} = \frac{2\pi(4.40\,\text{m} - 4.00\,\text{m})}{(330\,\text{m/s})/540\,\text{Hz}}$$

$$= 4.12\,\text{rad}.$$

17. Building on the theory developed in Section 17-5, we set $\Delta L/\lambda = n - 1/2,\ n = 1, 2, ...$ in order to have destructive interference. Since $v = f\lambda$, we can write this in terms of frequency:

$$f_{\min,n} = \frac{(2n-1)v}{2\Delta L} = (n - 1/2)(286\ \text{Hz})$$

where we have used v = 343 m/s (note the remarks made in the textbook at the beginning of the exercises and problems section) and ΔL = (19.5 – 18.3) m = 1.2 m.

(a) The lowest frequency that gives destructive interference is (n = 1)

$$f_{\min,1} = (1 - 1/2)(286\ \text{Hz}) = 143\ \text{Hz}.$$

(b) The second lowest frequency that gives destructive interference is (n = 2)

$$f_{\min,2} = (2 - 1/2)(286\ \text{Hz}) = 429\ \text{Hz} = 3(143\ \text{Hz}) = 3f_{\min,1}.$$

So the factor is 3.

(c) The third lowest frequency that gives destructive interference is (n = 3)

$$f_{\min,3} = (3 - 1/2)(286\ \text{Hz}) = 715\ \text{Hz} = 5(143\ \text{Hz}) = 5f_{\min,1}.$$

So the factor is 5.

Now we set $\Delta L/\lambda = \frac{1}{2}$ (even numbers) — which can be written more simply as "(all integers n = 1, 2,…)" — in order to establish constructive interference. Thus,

$$f_{\max,n} = \frac{nv}{\Delta L} = n(286\ \text{Hz}).$$

(d) The lowest frequency that gives constructive interference is (n =1) $f_{\max,1} = (286\ \text{Hz})$.

(e) The second lowest frequency that gives constructive interference is (n = 2)

$$f_{\max,2} = 2(286\ \text{Hz}) = 572\ \text{Hz} = 2f_{\max,1}.$$

Thus, the factor is 2.

(f) The third lowest frequency that gives constructive interference is (n = 3)

$$f_{\max,3} = 3(286 \text{ Hz}) = 858 \text{ Hz} = 3 f_{\max,1}.$$

Thus, the factor is 3.

18. (a) To be out of phase (and thus result in destructive interference if they superpose) means their path difference must be $\lambda/2$ (or $3\lambda/2$ or $5\lambda/2$ or …). Here we see their path difference is L, so we must have (in the least possibility) $L = \lambda/2$, or $q = L/\lambda = 0.5$.

(b) As noted above, the next possibility is $L = 3\lambda/2$, or $q = L/\lambda = 1.5$.

19. (a) The problem is asking at how many angles will there be "loud" resultant waves, and at how many will there be "quiet" ones? We note that at all points (at large distance from the origin) along the x axis there will be quiet ones; one way to see this is to note that the path-length difference (for the waves traveling from their respective sources) divided by wavelength gives the (dimensionless) value 3.5, implying a half-wavelength (180º) phase difference (destructive interference) between the waves. To distinguish the destructive interference along the $+x$ axis from the destructive interference along the $-x$ axis, we label one with +3.5 and the other –3.5. This labeling is useful in that it suggests that the complete enumeration of the quiet directions in the upper-half plane (including the x axis) is: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5. Similarly, the complete enumeration of the loud directions in the upper-half plane is: –3, –2, –1, 0, +1, +2, +3. Counting also the "other" –3, –2, –1, 0, +1, +2, +3 values for the *lower*-half plane, then we conclude there are a total of 7 + 7 = 14 "loud" directions.

(b) The discussion about the "quiet" directions was started in part (a). The number of values in the list: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5 along with –2.5, –1.5, –0.5, +0.5, +1.5, +2.5 (for the lower-half plane) is 14. There are 14 "quiet" directions.

20. (a) The problem indicates that we should ignore the decrease in sound amplitude, which means that all waves passing through point P have equal amplitude. Their superposition at P if $d = \lambda/4$ results in a net effect of zero there since there are four sources (so the first and third are $\lambda/2$ apart and thus interfere destructively; similarly for the second and fourth sources).

(b) Their superposition at P if $d = \lambda/2$ also results in a net effect of zero there since there are an even number of sources (so the first and second being $\lambda/2$ apart will interfere destructively; similarly for the waves from the third and fourth sources).

(c) If $d = \lambda$ then the waves from the first and second sources will arrive at P in phase; similar observations apply to the second and third, and to the third and fourth sources. Thus, four waves interfere constructively there with net amplitude equal to $4s_m$.

21. **THINK** The sound waves from the two speakers undergo interference. Whether the interference is constructive or destructive depends on the path length difference, or the phase difference.

EXPRESS From the figure, we see that the distance from the closer speaker to the listener is $L = d_2$, and the distance from the other speaker to the listener is $L' = \sqrt{d_1^2 + d_2^2}$, where d_1 is the distance between the speakers. The phase difference at the location of the listener is $\phi = 2\pi(L' - L)/\lambda$, where λ is the wavelength. For a minimum in intensity at the listener, $\phi = (2n + 1)\pi$, where n is an integer. Thus,

$$\phi = \frac{2\pi(L'-L)}{\lambda_{\text{min}}} = (2n+1)\pi \quad \Rightarrow \quad \lambda_{\text{min}} = \frac{2(L'-L)}{2n+1},$$

and the frequency is

$$f_{\text{min}} = \frac{v}{\lambda_{\text{min}}} = \frac{(2n+1)v}{2\left(\sqrt{d_1^2+d_2^2}-d_2\right)} = \frac{(2n+1)(343\,\text{m/s})}{2\left(\sqrt{(2.00\,\text{m})^2+(3.75\,\text{m})^2}-3.75\,\text{m}\right)} = (2n+1)(343\,\text{Hz}).$$

Now 20,000/343 = 58.3, so $2n + 1$ must range from 0 to 57 for the frequency to be in the audible range (20 Hz to 20 kHz). This means n ranges from 0 to 28.

On the other hand, for a maximum in intensity at the listener, $\phi = 2n\pi$, where n is any positive integer. Thus $\lambda_{\text{max}} = (1/n)\left(\sqrt{d_1^2+d_2^2}-d_2\right)$ and

$$f_{\text{max}} = \frac{v}{\lambda_{\text{max}}} = \frac{nv}{\sqrt{d_1^2+d_2^2}-d_2} = \frac{n(343\,\text{m/s})}{\sqrt{(2.00\,\text{m})^2+(3.75\,\text{m})^2}-3.75\,\text{m}} = n(686\,\text{Hz}).$$

Since 20,000/686 = 29.2, n must be in the range from 1 to 29 for the frequency to be audible.

ANALYZE (a) The lowest frequency that gives minimum signal is (n = 0) $f_{\text{min},1} = 343$ Hz.

(b) The second lowest frequency is (n = 1) $f_{\text{min},2} = [2(1)+1](343\text{ Hz}) = 1029\text{ Hz} = 3f_{\text{min},1}$. Thus, the factor is 3.

(c) The third lowest frequency is (n = 2) $f_{\text{min},3} = [2(2)+1](343\text{ Hz}) = 1715\text{ Hz} = 5f_{\text{min},1}$. Thus, the factor is 5.

(d) The lowest frequency that gives maximum signal is (n =1) $f_{\text{max},1} = 686$ Hz.

(e) The second lowest frequency is (n = 2) $f_{\text{max},2} = 2(686\text{ Hz}) = 1372\text{ Hz} = 2f_{\text{max},1}$. Thus, the factor is 2.

(f) The third lowest frequency is (n = 3) $f_{\max,3} = 3(686\text{ Hz}) = 2058\text{ Hz} = 3f_{\max,1}$. Thus, the factor is 3.

LEARN We see that the interference of the two sound waves depends on their phase difference $\phi = 2\pi(L' - L)/\lambda$. The interference is fully constructive when ϕ is a multiple of 2π, but fully destructive when ϕ is an odd multiple of π.

22. At the location of the detector, the phase difference between the wave that traveled straight down the tube and the other one, which took the semi-circular detour, is

$$\Delta\phi = k\Delta d = \frac{2\pi}{\lambda}(\pi r - 2r).$$

For $r = r_{\min}$ we have $\Delta\phi = \pi$, which is the smallest phase difference for a destructive interference to occur. Thus,

$$r_{\min} = \frac{\lambda}{2(\pi-2)} = \frac{40.0\text{ cm}}{2(\pi-2)} = 17.5\text{ cm}.$$

23. (a) If point P is infinitely far away, then the small distance d between the two sources is of no consequence (they seem effectively to be the same distance away from P). Thus, there is no perceived phase difference.

(b) Since the sources oscillate in phase, then the situation described in part (a) produces fully constructive interference.

(c) For finite values of x, the difference in source positions becomes significant. The path lengths for waves to travel from S_1 and S_2 become now different. We interpret the question as asking for the behavior of the absolute value of the phase difference $|\Delta\phi|$, in which case any change from zero (the answer for part (a)) is certainly an increase.

The path length difference for waves traveling from S_1 and S_2 is

$$\Delta\ell = \sqrt{d^2 + x^2} - x \qquad \text{for} \quad x > 0.$$

The phase difference in "cycles" (in absolute value) is therefore

$$|\Delta\phi| = \frac{\Delta\ell}{\lambda} = \frac{\sqrt{d^2+x^2} - x}{\lambda}.$$

Thus, in terms of λ, the phase difference is identical to the path length difference: $|\Delta\phi| = \Delta\ell > 0$. Consider $\Delta\ell = \lambda/2$. Then $\sqrt{d^2+x^2} = x + \lambda/2$. Squaring both sides, rearranging, and solving, we find

$$x = \frac{d^2}{\lambda} - \frac{\lambda}{4}.$$

In general, if $\Delta\ell = \xi\lambda$ for some multiplier $\xi > 0$, we find

$$x = \frac{d^2}{2\xi\lambda} - \frac{1}{2}\xi\lambda = \frac{64.0}{\xi} - \xi$$

where we have used d = 16.0 m and λ = 2.00 m.

(d) For $\Delta\ell = 0.50\lambda$, or $\xi = 0.50$, we have $x = (64.0/0.50 - 0.50)\text{ m} = 127.5\text{ m} \approx 128\text{ m}$.

(e) For $\Delta\ell = 1.00\lambda$, or $\xi = 1.00$, we have $x = (64.0/1.00 - 1.00)\text{ m} = 63.0\text{ m}$.

(f) For $\Delta\ell = 1.50\lambda$, or $\xi = 1.50$, we have $x = (64.0/1.50 - 1.50)\text{ m} = 41.2\text{ m}$.

Note that since whole cycle phase differences are equivalent (as far as the wave superposition goes) to zero phase difference, then the ξ = 1, 2 cases give constructive interference. A shift of a half-cycle brings "troughs" of one wave in superposition with "crests" of the other, thereby canceling the waves; therefore, the $\xi = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ cases produce destructive interference.

24. (a) Equation 17-29 gives the relation between sound level β and intensity I, namely

$$I = I_0 10^{(\beta/10\text{dB})} = (10^{-12}\text{ W/m}^2)10^{(\beta/10\text{dB})} = 10^{-12+(\beta/10\text{dB})}\text{ W/m}^2$$

Thus we find that for a β = 70 dB level we have a high intensity value of I_{high} = 10 μW/m^2.

(b) Similarly, for a β = 50 dB level we have a low intensity value of I_{low} = 0.10 μW/m^2.

(c) Equation 17-27 gives the relation between the displacement amplitude and I. Using the values for density and wave speed, we find s_m = 70 nm for the high intensity case.

(d) Similarly, for the low intensity case we have s_m = 7.0 nm.

We note that although the intensities differed by a factor of 100, the amplitudes differed by only a factor of 10.

25. The intensity is given by $I = \frac{1}{2}\rho v \omega^2 s_m^2$, where ρ is the density of air, v is the speed of sound in air, ω is the angular frequency, and s_m is the displacement amplitude for the sound wave. Replace ω with $2\pi f$ and solve for s_m:

$$s_m = \sqrt{\frac{I}{2\pi^2 \rho v f^2}} = \sqrt{\frac{1.00\times10^{-6}\ \text{W/m}^2}{2\pi^2(1.21\,\text{kg/m}^3)(343\,\text{m/s})(300\,\text{Hz})^2}} = 3.68\times10^{-8}\ \text{m}.$$

26. (a) Since intensity is power divided by area, and for an isotropic source the area may be written $A = 4\pi r^2$ (the area of a sphere), then we have

$$I = \frac{P}{A} = \frac{1.0\,\text{W}}{4\pi(1.0\,\text{m})^2} = 0.080\,\text{W/m}^2.$$

(b) This calculation may be done exactly as shown in part (a) (but with r = 2.5 m instead of r = 1.0 m), or it may be done by setting up a ratio. We illustrate the latter approach. Thus,

$$\frac{I'}{I} = \frac{P/4\pi(r')^2}{P/4\pi r^2} = \left(\frac{r}{r'}\right)^2$$

leads to I' = (0.080 W/m^2)(1.0/2.5)^2 = 0.013 W/m^2.

27. **THINK** The sound level increases by 10 dB when the intensity increases by a factor of 10.

EXPRESS The sound level β is defined as (see Eq. 17-29):

$$\beta = (10\ \text{dB})\log\frac{I}{I_0}$$

where $I_0 = 10^{-12}\ \text{W/m}^2$ is the standard reference intensity. In this problem, let I_1 be the original intensity and I_2 be the final intensity. The original sound level is $\beta_1 = (10\ \text{dB})\log(I_1/I_0)$ and the final sound level is β_2 = (10 dB) log(I_2/I_0). With $\beta_2 = \beta_1 + 30\ \text{dB}$, we have

$$(10\ \text{dB})\log(I_2/I_0) = (10\ \text{dB})\log(I_1/I_0) + 30\ \text{dB},$$

or

$$(10\ \text{dB})\log(I_2/I_0) - (10\ \text{dB})\log(I_1/I_0) = 30\ \text{dB}.$$

The above equation allows us to solve for the ratio I_2/I_1. On the other hand, combing Eqs. 17-15 and 17-27 leads to the following relation between the intensity I and the pressure amplitude Δp_m: $I = \frac{1}{2}\frac{(\Delta p_m)^2}{\rho v}$.

ANALYZE (a) Divide by 10 dB and use $\log(I_2/I_0) - \log(I_1/I_0) = \log(I_2/I_1)$ to obtain $\log(I_2/I_1) = 3$. Now use each side as an exponent of 10 and recognize that

$10^{\log(I_2/I_1)} = I_2/I_1$. The result is $I_2/I_1 = 10^3$. The intensity is increased by a factor of 1.0×10^3.

(b) The pressure amplitude is proportional to the square root of the intensity so it is increased by a factor of $\sqrt{1000} \approx 32$.

LEARN From the definition of β, we see that doubling sound intensity increases the sound level by $\Delta\beta = (10 \text{ dB})\log 2 = 3.01 \text{ dB}$.

28. The sound level β is defined as (see Eq. 17-29):

$$\beta = (10 \text{ dB})\log\frac{I}{I_0}$$

where $I_0 = 10^{-12} \text{ W/m}^2$ is the standard reference intensity. In this problem, let the two intensities be I_1 and I_2 such that $I_2 > I_1$. The sound levels are $\beta_1 = (10 \text{ dB})\log(I_1/I_0)$ and β_2 = (10 dB) log(I_2/I_0). With $\beta_2 = \beta_1 + 1.0 \text{ dB}$, we have

$$(10 \text{ dB}) \log(I_2/I_0) = (10 \text{ dB}) \log(I_1/I_0) + 1.0 \text{ dB},$$

or

$$(10 \text{ dB}) \log(I_2/I_0) - (10 \text{ dB}) \log(I_1/I_0) = 1.0 \text{ dB}.$$

Divide by 10 dB and use $\log(I_2/I_0) - \log(I_1/I_0) = \log(I_2/I_1)$ to obtain $\log(I_2/I_1) = 0.1$. Now use each side as an exponent of 10 and recognize that $10^{\log(I_2/I_1)} = I_2/I_1$. The result is

$$\frac{I_2}{I_1} = 10^{0.1} = 1.26\,.$$

29. **THINK** Power is the time rate of energy transfer, and intensity is the rate of energy flow per unit area perpendicular to the flow.

EXPRESS The rate at which energy flow across every sphere centered at the source is the same, regardless of the sphere radius, and is the same as the power output of the source. If P is the power output and I is the intensity a distance r from the source, then $P = IA = 4\pi r^2 I$, where $A = 4\pi r^2$ is the surface area of a sphere of radius r.

ANALYZE With $r = 2.50$ m and $I = 1.91\times10^{-4} \text{ W/m}^2$, we find the power of the source to be

$$P = 4\pi(2.50 \text{ m})^2 (1.91 \times 10^{-4} \text{ W/m}^2) = 1.50 \times 10^{-2} \text{ W}.$$

LEARN Since intensity falls off as $1/r^2$, the further away from the source, the weaker the intensity.

30. (a) The intensity is given by $I = P/4\pi r^2$ when the source is "point-like." Therefore, at r = 3.00 m,

$$I = \frac{1.00\times10^{-6}\text{ W}}{4\pi(3.00\text{ m})^2} = 8.84\times10^{-9}\text{ W/m}^2.$$

(b) The sound level there is

$$\beta = 10\log\left(\frac{8.84\times10^{-9}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = 39.5\text{ dB}.$$

31. We use $\beta = 10 \log (I/I_o)$ with $I_o = 1 \times 10^{-12}$ W/m^2 and $I = P/4\pi r^2$ (an assumption we are asked to make in the problem). We estimate $r \approx 0.3$ m (distance from knuckle to ear) and find

$$P \approx 4\pi(0.3\text{ m})^2\left(1\times10^{-12}\text{ W/m}^2\right)10^{6.2} = 2\times10^{-6}\text{ W} = 2\ \mu\text{W}.$$

32. (a) Since $\omega = 2\pi f$, Eq. 17-15 leads to

$$\Delta p_m = v\rho(2\pi f)s_m \;\Rightarrow\; s_m = \frac{1.13\times10^{-3}\text{ Pa}}{2\pi(1665\text{ Hz})(343\text{ m/s})(1.21\text{ kg/m}^3)}$$

which yields s_m = 0.26 nm. The nano prefix represents 10^{-9}. We use the speed of sound and air density values given at the beginning of the exercises and problems section in the textbook.

(b) We can plug into Eq. 17–27 or into its equivalent form, rewritten in terms of the pressure amplitude:

$$I = \frac{1}{2}\frac{(\Delta p_m)^2}{\rho v} = \frac{1}{2}\frac{(1.13\times10^{-3}\text{ Pa})^2}{(1.21\text{ kg/m}^3)(343\text{ m/s})} = 1.5\ \text{nW/m}^2.$$

33. We use $\beta = 10 \log(I/I_o)$ with $I_o = 1 \times 10^{-12}$ W/m^2 and Eq. 17–27 with

$$\omega = 2\pi f = 2\pi(260\text{ Hz}),$$

v = 343 m/s and ρ = 1.21 kg/m^3.

$$I = I_o\left(10^{8.5}\right) = \frac{1}{2}\rho v(2\pi f)^2 s_m^2 \quad\Rightarrow\quad s_m = 7.6\times10^{-7}\text{ m} = 0.76\ \mu\text{m}.$$

34. Combining Eqs.17-28 and 17-29 we have $\beta = 10\log\left(\frac{P}{I_o 4\pi r^2}\right)$. Taking differences (for sounds A and B) we find

$$\Delta\beta = 10\log\left(\frac{P_A}{I_o 4\pi r^2}\right) - 10\log\left(\frac{P_B}{I_o 4\pi r^2}\right) = 10\log\left(\frac{P_A}{P_B}\right)$$

using well-known properties of logarithms. Thus, we see that $\Delta\beta$ is independent of r and can be evaluated anywhere.

(a) We can solve the above relation (once we know $\Delta\beta = 5.0$) for the ratio of powers; we find $P_A/P_B \approx 3.2$.

(b) At $r = 1000$ m it is easily seen (in the graph) that $\Delta\beta = 5.0$ dB. This is the same Δβ we expect to find, then, at $r = 10$ m.

35. (a) The intensity is

$$I = \frac{P}{4\pi r^2} = \frac{30.0\,\text{W}}{(4\pi)(200\,\text{m})^2} = 5.97\times 10^{-5}\ \text{W/m}^2.$$

(b) Let A (= 0.750 cm^2) be the cross-sectional area of the microphone. Then the power intercepted by the microphone is

$$P' = IA = 0 = (6.0\times 10^{-5}\ \text{W/m}^2)(0.750\,\text{cm}^2)(10^{-4}\ \text{m}^2/\text{cm}^2) = 4.48\times 10^{-9}\ \text{W}.$$

36. The difference in sound level is given by Eq. 17-37:

$$\Delta\beta = \beta_f - \beta_i = (10\ \text{db})\log\left(\frac{I_f}{I_i}\right).$$

Thus, if $\Delta\beta = 5.0$ db, then $\log(I_f/I_i) = 1/2$, which implies that $I_f = \sqrt{10}I_i$. On the other hand, the intensity at a distance r from the source is $I = \dfrac{P}{4\pi r^2}$, where P is the power of the source. A fixed P implies that $I_i r_i^2 = I_f r_f^2$. Thus, with $r_i = 1.2$ m, we obtain

$$r_f = \left(\frac{I_i}{I_f}\right)^{1/2} r_i = \left(\frac{1}{10}\right)^{1/4}(1.2\ \text{m}) = 0.67\ \text{m}.$$

37. (a) The average potential energy transport rate is the same as that of the kinetic energy. This implies that the (average) rate for the total energy is

$$\left(\frac{dE}{dt}\right)_{\text{avg}} = 2\left(\frac{dK}{dt}\right)_{\text{avg}} = 2\left(\tfrac{1}{4}\rho A v \omega^2 s_m^2\right)$$

using Eq. 17-44. In this equation, we substitute $\rho = 1.21$ kg/m^3, $A = \pi r^2 = \pi(0.020\ \text{m})^2$, $v = 343$ m/s, $\omega = 3000$ rad/s, $s_m = 12\times 10^{-9}$ m, and obtain the answer 3.4×10^{-10} W.

(b) The second string is in a separate tube, so there is no question about the waves superposing. The total rate of energy, then, is just the addition of the two: $2(3.4 \times 10^{-10}$ W) $= 6.8 \times 10^{-10}$ W.

(c) Now we *do* have superposition, with $\phi = 0$, so the resultant amplitude is twice that of the individual wave, which leads to the energy transport rate being four times that of part (a). We obtain $4(3.4 \times 10^{-10}$ W) $= 1.4 \times 10^{-9}$ W.

(d) In this case $\phi = 0.4\pi$, which means (using Eq. 17-39)

$$s_m' = 2\, s_m \cos(\phi/2) = 1.618 s_m.$$

This means the energy transport rate is $(1.618)^2 = 2.618$ times that of part (a). We obtain $2.618(3.4 \times 10^{-10}$ W) $= 8.8 \times 10^{-10}$ W.

(e) The situation is as shown in Fig. 17-14(b). The answer is zero.

38. The frequency is $f = 686$ Hz and the speed of sound is $v_{\text{sound}} = 343$ m/s. If L is the length of the air-column, then using Eq. 17–41, the water height is (in unit of meters)

$$h = 1.00 - L = 1.00 - \frac{nv}{4f} = 1.00 - \frac{n(343)}{4(686)} = (1.00 - 0.125n) \text{ m}$$

where $n = 1, 3, 5, \ldots$ with only one end closed.

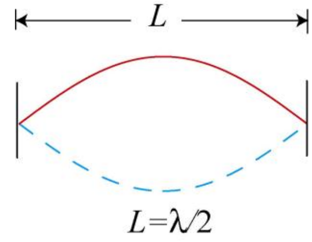
(a) There are 4 values of n ($n = 1, 3, 5, 7$) which satisfies $h > 0$.

(b) The smallest water height for resonance to occur corresponds to $n = 7$ with $h = 0.125$ m.

(c) The second smallest water height corresponds to $n = 5$ with $h = 0.375$ m.

39. **THINK** Violin strings are fixed at both ends. A string clamped at both ends can be made to oscillate in standing wave patterns.

EXPRESS When the string is fixed at both ends and set to vibrate at its fundamental, lowest resonant frequency, exactly one-half of a wavelength fits between the ends (see figure to the right). The wave speed is given by $v = \lambda f = \sqrt{\tau / \mu}$, where τ is the tension in the string and μ is the linear mass density of the string.



ANALYZE (a) With $\lambda = 2L$, we find the wave speed to be

$$v = f\lambda = 2Lf = 2(0.220 \text{ m})(920 \text{ Hz}) = 405 \text{ m/s}.$$

(b) If M is the mass of the (uniform) string, then $\mu = M/L$. Thus, the string tension is

$$\tau = \mu v^2 = (M/L)v^2 = [(800 \times 10^{-6} \text{ kg})/(0.220 \text{ m})]\,(405 \text{ m/s})^2 = 596 \text{ N}.$$

(c) The wavelength is $\lambda = 2L = 2(0.220 \text{ m}) = 0.440$ m.

(d) If v_a is the speed of sound in air, then the wavelength in air is

$$\lambda_a = v_a/f = (343 \text{ m/s})/(920 \text{ Hz}) = 0.373 \text{ m}.$$

LEARN The frequency of the sound wave in air is the same as the frequency of oscillation of the string. However, the wavelengths of the wave on the string and the sound waves emitted by the string are different because their wave speeds are not the same.

40. At the beginning of the exercises and problems section in the textbook, we are told to assume $v_{\text{sound}} = 343$ m/s unless told otherwise. The second harmonic of pipe A is found from Eq. 17-39 with $n = 2$ and $L = L_A$, and the third harmonic of pipe B is found from Eq. 17-41 with $n = 3$ and $L = L_B$. Since these frequencies are equal, we have

$$\frac{2v_{\text{sound}}}{2L_A} = \frac{3v_{\text{sound}}}{4L_B} \Rightarrow L_B = \frac{3}{4}L_A.$$

(a) Since the fundamental frequency for pipe A is 300 Hz, we immediately know that the second harmonic has $f = 2(300 \text{ Hz}) = 600$ Hz. Using this, Eq. 17-39 gives

$$L_A = (2)(343 \text{ m/s})/2(600 \text{ s}^{-1}) = 0.572 \text{ m}.$$

(b) The length of pipe B is $L_B = \frac{3}{4}L_A = 0.429\,\text{m}.$

41. (a) From Eq. 17–53, we have

$$f = \frac{nv}{2L} = \frac{(1)(250\,\text{m/s})}{2(0.150\,\text{m})} = 833\,\text{Hz}.$$

(b) The frequency of the wave on the string is the same as the frequency of the sound wave it produces during its vibration. Consequently, the wavelength in air is

$$\lambda = \frac{v_{\text{sound}}}{f} = \frac{348\,\text{m/s}}{833\,\text{Hz}} = 0.418\,\text{m}.$$

42. The distance between nodes referred to in the problem means that $\lambda/2 = 3.8$ cm, or $\lambda = 0.076$ m. Therefore, the frequency is

$$f = v/\lambda = (1500 \text{ m/s})/(0.076 \text{ m}) \approx 20 \times 10^3 \text{ Hz}.$$

43. **THINK** The pipe is open at both ends so there are displacement antinodes at both ends.

EXPRESS If L is the pipe length and λ is the wavelength then $\lambda = 2L/n$, where n is an integer. That is, an integer number of half-wavelengths fit into the length of the pipe. If v is the speed of sound then the resonant frequencies are given by $f = v/\lambda = nv/2L$. Now L = 0.457 m, so

$$f = \frac{nv}{2L} = \frac{n(344 \text{ m/s})}{2(0.457 \text{ m})} = (376.4 \text{ Hz})n .$$

ANALYZE (a) To find the resonant frequencies that lie between 1000 Hz and 2000 Hz, first set f = 1000 Hz and solve for n, then set f = 2000 Hz and again solve for n. The results are 2.66 and 5.32, which imply that n = 3, 4, and 5 are the appropriate values of n. Thus, there are 3 frequencies.

(b) The lowest frequency at which resonance occurs corresponds to n = 3, or

$$f = 3(376.4 \text{ Hz}) = 1129 \text{ Hz} .$$

(c) The second lowest frequency at which resonance occurs corresponds to n = 4, or

$$f = 4(376.4 \text{ Hz}) = 1506 \text{ Hz} .$$

LEARN The third lowest frequency at which resonance occurs corresponds to n = 5, or

$$f = 5(376.4 \text{ Hz}) = 1882 \text{ Hz} .$$

Changing the length of the pipe can affect the number of resonant frequencies.

44. (a) Using Eq. 17-39 with v = 343 m/s and n = 1, we find $f = nv/2L$ = 86 Hz for the fundamental frequency in a nasal passage of length L = 2.0 m (subject to various assumptions about the nature of the passage as a "bent tube open at both ends").

(b) The sound would be perceptible as *sound* (as opposed to just a general vibration) of very low frequency.

(c) Smaller L implies larger f by the formula cited above. Thus, the female's sound is of higher pitch (frequency).

45. (a) We note that 1.2 = 6/5. This suggests that both even and odd harmonics are present, which means the pipe is open at both ends (see Eq. 17-39).

(b) Here we observe 1.4 = 7/5. This suggests that only odd harmonics are present, which means the pipe is open at only one end (see Eq. 17-41).

46. We observe that "third lowest … frequency" corresponds to harmonic number $n_A = 3$ for pipe *A*, which is open at both ends. Also, "second lowest … frequency" corresponds to harmonic number $n_B = 3$ for pipe *B*, which is closed at one end.

(a) Since the frequency of *B* matches the frequency of A, using Eqs. 17-39 and 17-41, we have

$$f_A = f_B \quad \Rightarrow \frac{3v}{2L_A} = \frac{3v}{4L_B}$$

which implies $L_B = L_A / 2 = (1.20 \text{ m})/2 = 0.60 \text{ m}$. Using Eq. 17-40, the corresponding wavelength is

$$\lambda = \frac{4L_B}{3} = \frac{4(0.60 \text{ m})}{3} = 0.80 \text{ m}.$$

The change from node to anti-node requires a distance of $\lambda/4$ so that every increment of 0.20 m along the *x*-axis involves a switch between node and anti-node. Since the closed end is a node, the next node appears at x = 0.40 m, so there are 2 nodes. The situation corresponds to that illustrated in Fig. 17-14(b) with $n = 3$.

(b) The smallest value of x where a node is present is $x = 0$.

(c) The second smallest value of x where a node is present is x = 0.40 m.

(d) Using v = 343 m/s, we find $f_3 = v/\lambda$ = 429 Hz. Now, we find the fundamental resonant frequency by dividing by the harmonic number, $f_1 = f_3/3$ = 143 Hz.

47. The top of the water is a displacement node and the top of the well is a displacement anti-node. At the lowest resonant frequency exactly one-fourth of a wavelength fits into the depth of the well. If d is the depth and λ is the wavelength, then $\lambda = 4d$. The frequency is $f = v/\lambda = v/4d$, where v is the speed of sound. The speed of sound is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of air in the well. Thus $f = (1/4d)\sqrt{B/\rho}$ and

$$d = \frac{1}{4f}\sqrt{\frac{B}{\rho}} = \frac{1}{4(7.00\,\text{Hz})}\sqrt{\frac{1.33\times10^5 \text{ Pa}}{1.10\,\text{kg/m}^3}} = 12.4 \text{ m}.$$

48. (a) Since the difference between consecutive harmonics is equal to the fundamental frequency (see section 17-6) then f_1 = (390 – 325) Hz = 65 Hz. The next harmonic after 195 Hz is therefore (195 + 65) Hz = 260 Hz.

(b) Since $f_n = nf_1$, then n = 260/65 = 4.

(c) Only *odd* harmonics are present in tube *B*, so the difference between consecutive harmonics is equal to *twice* the fundamental frequency in this case (consider taking differences of Eq. 17-41 for various values of *n*). Therefore,

$$f_1 = \tfrac{1}{2}(1320 - 1080)\ \text{Hz} = 120\ \text{Hz}.$$

The next harmonic after 600 Hz is consequently [600 + 2(120)] Hz = 840 Hz.

(d) Since $f_n = nf_1$ (for *n* odd), then n = 840/120 = 7.

49. **THINK** Violin strings are fixed at both ends. A string clamped at both ends can be made to oscillate in standing wave patterns.

EXPRESS The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer. The resonant frequencies are given by $f_n = v/\lambda = nv/2L$, where v is the wave speed on the string. Now $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus $f_n = (n/2L)\sqrt{\tau/\mu}$.

ANALYZE Suppose the lower frequency is associated with n_1 and the higher frequency is associated with $n_2 = n_1 + 1$. There are no resonant frequencies between so you know that the integers associated with the given frequencies differ by 1. Thus, $f_{n_1} = (n_1/2L)\sqrt{\tau/\mu}$ and

$$f_{n_2} = \frac{n_1+1}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n_1}{2L}\sqrt{\frac{\tau}{\mu}} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}} = f_{n_1} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}.$$

This means $f_{n_2} - f_{n_1} = (1/2L)\sqrt{\tau/\mu}$ and

$$\begin{aligned}\tau &= 4L^2\mu(f_{n_2} - f_{n_1})^2 = 4(0.300\,\text{m})^2(0.650\times10^{-3}\ \text{kg/m})(1320\,\text{Hz} - 880\,\text{Hz})^2 \\ &= 45.3\,\text{N}.\end{aligned}$$

LEARN Since the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency: $\Delta f = f_{n+1} - f_n = \dfrac{v}{2L} = f_1$, we find

$$f_1 = 1320\,\text{Hz} - 880\,\text{Hz} = 440\,\text{Hz}.$$

Since 880 Hz = 2(440 Hz) and 1320 Hz = 3(440 Hz), the two frequencies correspond to $n_1 = 2$ and $n_2 = 3$, respectively.

50. (a) Using Eq. 17-39 with n = 1 (for the fundamental mode of vibration) and 343 m/s for the speed of sound, we obtain

$$f = \frac{(1)v_{\text{sound}}}{4L_{\text{tube}}} = \frac{343\,\text{m/s}}{4(1.20\,\text{m})} = 71.5\,\text{Hz}.$$

(b) For the wire (using Eq. 17-53) we have

$$f' = \frac{nv_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}}\sqrt{\frac{\tau}{\mu}}$$

where $\mu = m_{\text{wire}}/L_{\text{wire}}$. Recognizing that $f = f'$ (both the wire and the air in the tube vibrate at the same frequency), we solve this for the tension τ.

$$\tau = (2L_{\text{wire}}\,f)^2\left(\frac{m_{\text{wire}}}{L_{\text{wire}}}\right) = 4f^2 m_{\text{wire}}L_{\text{wire}} = 4(71.5\,\text{Hz})^2(9.60\times10^{-3}\,\text{kg})(0.330\,\text{m}) = 64.8\,\text{N}.$$

51. Let the period be T. Then the beat frequency is $1/T - 440\,\text{Hz} = 4.00\,\text{beats/s}.$ Therefore, $T = 2.25 \times 10^{-3}$ s. The string that is "too tightly stretched" has the higher tension and thus the higher (fundamental) frequency.

52. Since the beat frequency equals the difference between the frequencies of the two tuning forks, the frequency of the first fork is either 381 Hz or 387 Hz. When mass is added to this fork its frequency decreases (recall, for example, that the frequency of a mass–spring oscillator is proportional to $1/\sqrt{m}$). Since the beat frequency also decreases, the frequency of the first fork must be greater than the frequency of the second. It must be 387 Hz.

53. **THINK** Beat arises when two waves detected have slightly different frequencies: $f_{\text{beat}} = f_2 - f_1$.

EXPRESS Each wire is vibrating in its fundamental mode so the wavelength is twice the length of the wire ($\lambda = 2L$) and the frequency is

$$f = v/\lambda = (1/2L)\sqrt{\tau/\mu}\,,$$

where $v=\sqrt{\tau/\mu}$ is the wave speed for the wire, τ is the tension in the wire, and μ is the linear mass density of the wire. Suppose the tension in one wire is τ and the oscillation frequency of that wire is f_1. The tension in the other wire is $\tau+\Delta\tau$ and its frequency is f_2. You want to calculate $\Delta\tau/\tau$ for f_1 = 600 Hz and f_2 = 606 Hz. Now, $f_1=(1/2L)\sqrt{\tau/\mu}$ and $f_2=(1/2L)\sqrt{(\tau+\Delta\tau)/\mu}$, so

$$f_2/f_1=\sqrt{(\tau+\Delta\tau)/\tau}=\sqrt{1+(\Delta\tau/\tau)}.$$

ANALYZE The fractional increase in tension is

$$\Delta\tau/\tau=(f_2/f_1)^2-1=[(606\,\text{Hz})/(600\,\text{Hz})]^2-1=0.020.$$

LEARN Beat frequency $f_{\text{beat}}=f_2-f_1$ is zero when $\Delta\tau=0$. The beat phenomenon is used by musicians to tune musical instruments. The instrument tuned is sounded against a standard frequency until beat disappears.

54. (a) The number of different ways of picking up a pair of tuning forks out of a set of five is 5!/(2!3!) = 10. For each of the pairs selected, there will be one beat frequency. If these frequencies are all different from each other, we get the maximum possible number of 10.

(b) First, we note that the minimum number occurs when the frequencies of these forks, labeled 1 through 5, increase in equal increments: $f_n = f_1 + n\Delta f$, where n = 2, 3, 4, 5. Now, there are only 4 different beat frequencies: $f_{\text{beat}} = n\Delta f$, where n = 1, 2, 3, 4.

55. We use $v_S = r\omega$ (with r = 0.600 m and ω = 15.0 rad/s) for the linear speed during circular motion, and Eq. 17-47 for the Doppler effect (where f = 540 Hz, and v = 343 m/s for the speed of sound).

(a) The lowest frequency is

$$f'=f\left(\frac{v+0}{v+v_S}\right)=526\text{ Hz}.$$

(b) The highest frequency is

$$f'=f\left(\frac{v+0}{v-v_S}\right)=555\text{ Hz}.$$

56. The Doppler effect formula, Eq. 17-47, and its accompanying rule for choosing ± signs, are discussed in Section 17-10. Using that notation, we have v = 343 m/s, v_D = 2.44 m/s, f' = 1590 Hz, and f = 1600 Hz. Thus,

$$f'=f\left(\frac{v+v_D}{v+v_S}\right)\;\Rightarrow\; v_S=\frac{f}{f'}\,(v+v_D)-v=4.61\,\text{m/s}.$$

57. In the general Doppler shift equation, the trooper's speed is the source speed and the speeder's speed is the detector's speed. The Doppler effect formula, Eq. 17-47, and its accompanying rule for choosing ± signs, are discussed in Section 17-10. Using that notation, we have v = 343 m/s,

$$v_D = v_S = 160 \text{ km/h} = (160000 \text{ m})/(3600 \text{ s}) = 44.4 \text{ m/s},$$

and f = 500 Hz. Thus,

$$f' = (500 \text{ Hz})\left(\frac{343 \text{ m/s} - 44.4 \text{ m/s}}{343 \text{ m/s} - 44.4 \text{ m/s}}\right) = 500 \text{ Hz} \Rightarrow \Delta f = 0.$$

58. We use Eq. 17-47 with f = 1200 Hz and v = 329 m/s.

(a) In this case, v_D = 65.8 m/s and v_S = 29.9 m/s, and we choose signs so that f' is larger than f:

$$f' = f\left(\frac{329 \text{ m/s} + 65.8 \text{ m/s}}{329 \text{ m/s} - 29.9 \text{ m/s}}\right) = 1.58 \times 10^3 \text{ Hz}.$$

(b) The wavelength is $\lambda = v/f'$ = 0.208 m.

(c) The wave (of frequency f') "emitted" by the moving reflector (now treated as a "source," so v_S = 65.8 m/s) is returned to the detector (now treated as a detector, so v_D = 29.9 m/s) and registered as a new frequency f'':

$$f'' = f'\left(\frac{329 \text{ m/s} + 29.9 \text{ m/s}}{329 \text{ m/s} - 65.8 \text{ m/s}}\right) = 2.16 \times 10^3 \text{ Hz}.$$

(d) This has wavelength v/f'' = 0.152 m.

59. We denote the speed of the French submarine by u_1 and that of the U.S. sub by u_2.

(a) The frequency as detected by the U.S. sub is

$$f_1' = f_1\left(\frac{v + u_2}{v - u_1}\right) = (1.000 \times 10^3 \text{ Hz})\left(\frac{5470 \text{ km/h} + 70.00 \text{ km/h}}{5470 \text{ km/h} - 50.00 \text{ km/h}}\right) = 1.022 \times 10^3 \text{ Hz}.$$

(b) If the French sub were stationary, the frequency of the reflected wave would be

$$f_r = f_1(v + u_2)/(v - u_2).$$

Since the French sub is moving toward the reflected signal with speed u_1, then

$$f_r' = f_r\left(\frac{v+u_1}{v}\right) = f_1\frac{(v+u_1)(v+u_2)}{v(v-u_2)} = \frac{(1.000\times10^3\text{ Hz})(5470+50.00)(5470+70.00)}{(5470)(5470-70.00)}$$
$$=1.045\times10^3\text{ Hz}.$$

60. We are combining two effects: the reception of a moving object (the truck of speed u = 45.0 m/s) of waves emitted by a stationary object (the motion detector), and the subsequent emission of those waves by the moving object (the truck), which are picked up by the stationary detector. This could be figured in two steps, but is more compactly computed in one step as shown here:

$$f_{\text{final}} = f_{\text{initial}}\left(\frac{v+u}{v-u}\right) = (0.150\,\text{MHz})\left(\frac{343\,\text{m/s} + 45\,\text{m/s}}{343\,\text{m/s} - 45\,\text{m/s}}\right) = 0.195\,\text{MHz}.$$

61. As a result of the Doppler effect, the frequency of the reflected sound as heard by the bat is

$$f_r = f'\left(\frac{v+u_{\text{bat}}}{v-u_{\text{bat}}}\right) = (3.9\times10^4\text{ Hz})\left(\frac{v+v/40}{v-v/40}\right) = 4.1\times10^4\text{ Hz}.$$

62. The "third harmonic" refers to a resonant frequency f_3 = 3 f_1, where f_1 is the fundamental lowest resonant frequency. When the source is stationary, with respect to the air, then Eq. 17-47 gives

$$f' = f\left(1-\frac{v_d}{v}\right)$$

where v_d is the speed of the detector (assumed to be moving away from the source, in the way we've written it, above). The problem, then, wants us to find v_d such that $f' = f_1$ when the emitted frequency is $f = f_3$. That is, we require $1 - v_d/v = 1/3$. Clearly, the solution to this is $v_d/v = 2/3$ (independent of length and whether one or both ends are open [the latter point being due to the fact that the odd harmonics occur in both systems]). Thus,

(a) For tube 1, $v_d = 2v/3$.

(b) For tube 2, $v_d = 2v/3$.

(c) For tube 3, $v_d = 2v/3$.

(d) For tube 4, $v_d = 2v/3$.

63. In this case, the intruder is moving *away* from the source with a speed u satisfying $u/v \ll 1$. The Doppler shift (with $u = -0.950$ m/s) leads to

$$f_{\text{beat}} = |f_r - f_s| \approx \frac{2|u|}{v} f_s = \frac{2(0.95\,\text{m/s})(28.0\,\text{kHz})}{343\,\text{m/s}}) = 155\,\text{Hz}\,.$$

64. When the detector is stationary (with respect to the air) then Eq. 17-47 gives

$$f' = \frac{f}{1 - v_s / v}$$

where v_s is the speed of the source (assumed to be approaching the detector in the way we've written it, above). The difference between the approach and the recession is

$$f' - f'' = f \left(\frac{1}{1 - v_s / v} - \frac{1}{1 + v_s / v} \right) = f \left(\frac{2v_s / v}{1 - (v_s / v)^2} \right)$$

which, after setting $(f' - f'')/f = 1/2$, leads to an equation that can be solved for the ratio v_s/v. The result is $\sqrt{5} - 2 = 0.236$. Thus, $v_s/v = 0.236$.

65. The Doppler shift formula, Eq. 17-47, is valid only when both u_S and u_D are measured with respect to a stationary medium (i.e., no wind). To modify this formula in the presence of a wind, we switch to a new reference frame in which there is no wind.

(a) When the wind is blowing from the source to the observer with a speed w, we have $u'_S = u'_D = w$ in the new reference frame that moves together with the wind. Since the observer is now approaching the source while the source is backing off from the observer, we have, in the new reference frame,

$$f' = f \left(\frac{v + u'_D}{v + u'_S} \right) = f \left(\frac{v + w}{v + w} \right) = 2.0 \times 10^3 \text{ Hz}.$$

In other words, there is no Doppler shift.

(b) In this case, all we need to do is to reverse the signs in front of both u'_D and u'_S. The result is that there is still no Doppler shift:

$$f' = f \left(\frac{v - u'_D}{v - u'_S} \right) = f \left(\frac{v - w}{v - w} \right) = 2.0 \times 10^3 \text{ Hz}.$$

In general, there will always be no Doppler shift as long as there is no relative motion between the observer and the source, regardless of whether a wind is present or not.

66. We use Eq. 17-47 with f = 500 Hz and v = 343 m/s. We choose signs to produce $f' > f$.

(a) The frequency heard in still air is

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 30.5\text{ m/s}}{343\text{ m/s} - 30.5\text{ m/s}}\right) = 598\text{ Hz}.$$

(b) In a frame of reference where the air seems still, the velocity of the detector is 30.5 – 30.5 = 0, and that of the source is 2(30.5). Therefore,

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 0}{343\text{ m/s} - 2(30.5\text{ m/s})}\right) = 608\text{ Hz}.$$

(c) We again pick a frame of reference where the air seems still. Now, the velocity of the source is 30.5 – 30.5 = 0, and that of the detector is 2(30.5). Consequently,

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 2(30.5\text{ m/s})}{343\text{ m/s} - 0}\right) = 589\text{ Hz}.$$

67. **THINK** The girl and her uncle hear different frequencies because of Doppler effect.

EXPRESS The Doppler shifted frequency is given by

$$f' = f\,\frac{v \pm v_D}{v \mp v_S},$$

where f is the unshifted frequency, v is the speed of sound, v_D is the speed of the detector (the uncle), and v_S is the speed of the source (the locomotive). All speeds are relative to the air.

ANALYZE (a) The uncle is at rest with respect to the air, so $v_D = 0$. The speed of the source is v_S = 10 m/s. Since the locomotive is moving away from the uncle the frequency decreases and we use the plus sign in the denominator. Thus

$$f' = f\,\frac{v}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} + 10.00\,\text{m/s}}\right) = 485.8\,\text{Hz}.$$

(b) The girl is now the detector. Relative to the air she is moving with speed v_D = 10.00 m/s toward the source. This tends to increase the frequency and we use the plus sign in the numerator. The source is moving at v_S = 10.00 m/s away from the girl. This tends to decrease the frequency and we use the plus sign in the denominator. Thus, $(v + v_D)$ =

$(v + v_S)$ and $f' = f = 500.0$ Hz.

(c) Relative to the air the locomotive is moving at $v_S = 20.00$ m/s away from the uncle. Use the plus sign in the denominator. Relative to the air the uncle is moving at v_D = 10.00 m/s toward the locomotive. Use the plus sign in the numerator. Thus

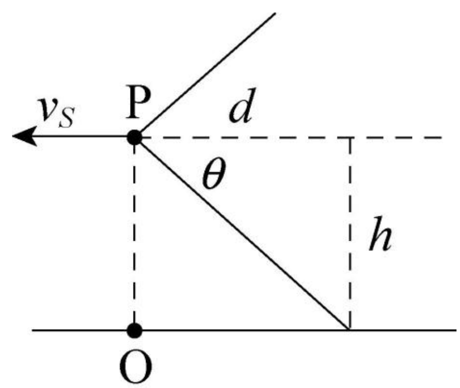
$$f' = f\frac{v+v_D}{v+v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s} + 10.00\,\text{m/s}}{343\,\text{m/s} + 20.00\,\text{m/s}}\right) = 486.2\,\text{Hz}.$$

(d) Relative to the air the locomotive is moving at $v_S = 20.00$ m/s away from the girl and the girl is moving at $v_D = 20.00$ m/s toward the locomotive. Use the plus signs in both the numerator and the denominator. Thus, $(v + v_D) = (v + v_S)$ and $f' = f = 500.0$ Hz.

LEARN The uncle, standing near the track, hears different frequencies, depending on the direction of the wind. On other hand, since the girl (a detector) is sitting in the train and there's no relative motion between her and the source (locomotive whistle), she hears the same frequency as the source regardless of the wind direction.

68. We note that 1350 km/h is v_S = 375 m/s. Then, with $\theta = 60º$, Eq. 17-57 gives $v = 3.3\times10^2$ m/s.

69. **THINK** Mach number is the ratio v_S / v, where v_S is the speed of the source and v is the sound speed. A mach number of 1.5 means that the jet plane moves at a supersonic speed.



EXPRESS The half angle θ of the Mach cone is given by sin $\theta = v/v_S$, where v is the speed of sound and v_S is the speed of the plane. To calculate the time it takes for the shock wave to each you after the plane has passed directly overhead, let h be the altitude of the plane and suppose the Mach cone intersects Earth's surface a distance d behind the plane. The situation is shown in the diagram below, with P indicating the plane and O indicating the observer.

The cone angle is related to h and d by tan $\theta = h/d$, so $d = h/\tan\theta$. The shock wave reaches O in the time the plane takes to fly the distance d.

ANALYZE (a) Since $v_S = 1.5v$, sin $\theta = v/1.5v = 1/1.5$. This means $\theta = 42°$.

(b) The time required for the shock wave to reach you is

$$t = \frac{d}{v} = \frac{h}{v\tan\theta} = \frac{5000\text{ m}}{1.5(331\text{ m/s})\tan 42°} = 11\,\text{s}.$$

LEARN The shock wave generated by the supersonic jet produces an explosive sound called sonic boom, in which the air pressure first increases suddenly, and then drops suddenly below normal before returning to normal.

70. The altitude H and the horizontal distance x for the legs of a right triangle, so we have

$$H = x\tan\theta = v_p t\tan\theta = 1.25vt\sin\theta$$

where v is the speed of sound, v_p is the speed of the plane, and

$$\theta = \sin^{-1}\left(\frac{v}{v_p}\right) = \sin^{-1}\left(\frac{v}{1.25v}\right) = 53.1°.$$

Thus the altitude is

$$H = x\tan\theta = (1.25)(330\,\text{m/s})(60\,\text{s})(\tan 53.1°) = 3.30\times10^4\ \text{m}.$$

71. The source being a "point source" means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2.$$

From the discussion in Section 17-5, we know that the intensity ratio between "barely audible" and the "painful threshold" is $10^{-12} = I_2/I_1$. Thus, with $r_2 = 10000$ m, we find

$$r_1 = r_2\sqrt{10^{-12}} = 0.01\,\text{m} = 1\ \text{cm}.$$

72. The angle is $\sin^{-1}(v/v_s) = \sin^{-1}(343/685) = 30°$.

73. The round-trip time is $t = 2L/v$, where we estimate from the chart that the time between clicks is 3 ms. Thus, with $v = 1372$ m/s, we find $L = \frac{1}{2}vt = 2.1$ m.

74. We use $v = \sqrt{B/\rho}$ to find the bulk modulus B:

$$B = v^2\rho = \left(5.4\times10^3\ \text{m/s}\right)^2\left(2.7\times10^3\ \text{kg/m}^3\right) = 7.9\times10^{10}\ \text{Pa}.$$

75. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 .$$

(a) With $I_1 = 9.60 \times 10^{-4}$ W/m^2, $r_1 = 6.10$ m, and $r_2 = 30.0$ m, we find

$$I_2 = (9.60 \times 10^{-4}\ \text{W/m}^2)(6.10/30.0)^2 = 3.97 \times 10^{-5}\ \text{W/m}^2.$$

(b) Using Eq. 17-27 with $I_1 = 9.60 \times 10^{-4}$ W/m^2, $\omega = 2\pi(2000\ \text{Hz})$, $v = 343$ m/s, and $\rho =$ 1.21 kg/m^3, we obtain

$$s_m = \sqrt{\frac{2I}{\rho v \omega^2}} = 1.71\times 10^{-7}\ \text{m}.$$

(c) Equation 17-15 gives the pressure amplitude:

$$\Delta p_m = \rho v \omega s_m = 0.893\ \text{Pa}.$$

76. We use $\Delta\beta_{12} = \beta_1 - \beta_2 = (10\ \text{dB})\log(I_1/I_2)$.

(a) Since $\Delta\beta_{12} = (10\ \text{dB})\log(I_1/I_2) = 37$ dB, we get

$$I_1/I_2 = 10^{37\ \text{dB}/10\ \text{dB}} = 10^{3.7} = 5.0 \times 10^3.$$

(b) Since $\Delta p_m \propto s_m \propto \sqrt{I}$, we have $\Delta p_{m1}/\Delta p_{m2} = \sqrt{I_1/I_2} = \sqrt{5.0\times 10^3} = 71.$

(c) The displacement amplitude ratio is $s_{m1}/s_{m2} = \sqrt{I_1/I_2} = 71.$

77. Any phase changes associated with the reflections themselves are rendered inconsequential by the fact that there is an even number of reflections. The additional path length traveled by wave A consists of the vertical legs in the zig-zag path: $2L$. To be (minimally) out of phase means, therefore, that $2L = \lambda/2$ (corresponding to a half-cycle, or 180°, phase difference). Thus, $L = \lambda/4$, or $L/\lambda = 1/4 = 0.25$.

78. Since they are approaching each other, the sound produced (of emitted frequency f) by the flatcar-trumpet received by an observer on the ground will be of higher pitch f'. In these terms, we are told $f' - f = 4.0$ Hz, and consequently that $f'/f = 444/440 = 1.0091$. With v_S designating the speed of the flatcar and $v = 343$ m/s being the speed of sound, the Doppler equation leads to

$$\frac{f'}{f} = \frac{v+0}{v-v_S} \;\Rightarrow\; v_S = (343\ \text{m/s})\frac{1.0091-1}{1.0091} = 3.1\,\text{m/s}.$$

79. (a) Incorporating a term ($\lambda/2$) to account for the phase shift upon reflection, then the path difference for the waves (when they come back together) is

$$\sqrt{L^2 + (2d)^2} - L + \lambda/2 = \Delta(\text{path}) .$$

Setting this equal to the condition needed to destructive interference ($\lambda/2$, $3\lambda/2$, $5\lambda/2$ …) leads to $d = 0$, 2.10 m, … Since the problem explicitly excludes the $d = 0$ possibility, then our answer is $d = 2.10$ m.

(b) Setting this equal to the condition needed to constructive interference (λ, 2λ, 3λ …) leads to $d = 1.47$ m, … Our answer is $d = 1.47$ m.

80. When the source is stationary (with respect to the air) then Eq. 17-47 gives

$$f' = f\left(1 - \frac{v_d}{v}\right),$$

where v_d is the speed of the detector (assumed to be moving away from the source, in the way we've written it, above). The difference between the approach and the recession is

$$f'' - f' = f\left[\left(1 + \frac{v_d}{v}\right) - \left(1 - \frac{v_d}{v}\right)\right] = f\left(2\frac{v_d}{v}\right)$$

which, after setting $(f'' - f')/f = 1/2$, leads to an equation that can be solved for the ratio v_d/v. The result is 1/4. Thus, $v_d/v = 0.250$.

81. **THINK** The pressure amplitude of the sound wave depends on the medium it propagates through.

EXPRESS The intensity of a sound wave is given by $I = \frac{1}{2}\rho v\omega^2 s_m^2$, where ρ is the density of the medium, v is the speed of sound, ω is the angular frequency, and s_m is the displacement amplitude. The displacement and pressure amplitudes are related by $\Delta p_m = \rho v\omega s_m$, so $s_m = \Delta p_m/\rho v\omega$ and $I = (\Delta p_m)^2/2\rho v$. For waves of the same frequency the ratio of the intensity for propagation in water to the intensity for propagation in air is

$$\frac{I_w}{I_a} = \left(\frac{\Delta p_{mw}}{\Delta p_{ma}}\right)^2 \frac{\rho_a v_a}{\rho_w v_w},$$

where the subscript a denotes air and the subscript w denotes water.

ANALYZE (a) In case where the intensities are equal, $I_a = I_w$, the ratio of the pressure amplitude is

$$\frac{\Delta p_{mw}}{\Delta p_{ma}} = \sqrt{\frac{\rho_w v_w}{\rho_a v_a}} = \sqrt{\frac{(0.998\times10^3\ \text{kg/m}^3)(1482\ \text{m/s})}{(1.21\ \text{kg/m}^3)(343\ \text{m/s})}} = 59.7.$$

The speeds of sound are given in Table 17-1 and the densities are given in Table 14-1.

(b) Now, if the pressure amplitudes are equal: $\Delta p_{mw} = \Delta p_{ma}$, then the ratio of the intensities is

$$\frac{I_w}{I_a} = \frac{\rho_a v_a}{\rho_w v_w} = \frac{(1.21\ \text{kg/m}^3)(343\ \text{m/s})}{(0.998\times10^3\ \text{kg/m}^3)(1482\ \text{m/s})} = 2.81\times10^{-4}.$$

LEARN The pressure amplitude of sound wave and the intensity depend on the density of the medium and the sound speed in the medium.

82. The wave is written as $s(x,t) = s_m \cos(kx \pm \omega t)$.

(a) The amplitude s_m is equal to the maximum displacement: $s_m = 0.30\ \text{cm}$.

(b) Since λ = 24 cm, the angular wave number is $k = 2\pi/\lambda = 0.26\ \text{cm}^{-1}$.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(25\ \text{Hz}) = 1.6\times10^2\ \text{rad/s}$.

(d) The speed of the wave is $v = \lambda f$ = (24 cm)(25 Hz) = 6.0×10^2 cm/s.

(e) Since the direction of propagation is $-x$, the sign is plus, so $s(x,t) = s_m \cos(kx + \omega t)$.

83. **THINK** This problem deals with the principle of Doppler ultrasound. The technique can be used to measure blood flow and blood pressure by reflecting high-frequency, ultrasound sound waves off blood cells.

EXPRESS The direction of blood flow can be determined by the Doppler shift in frequency. The reception of the ultrasound by the blood and the subsequent remitting of the signal by the blood back toward the detector is a two-step process which may be compactly written as

$$f + \Delta f = f\left(\frac{v + v_x}{v - v_x}\right)$$

where $v_x = v_{\text{blood}} \cos\theta$. If we write the ratio of frequencies as $R = (f + \Delta f)/f$, then the solution of the above equation for the speed of the blood is

$$v_{\text{blood}} = \frac{(R-1)v}{(R+1)\cos\theta}.$$

ANALYZE (a) The blood is moving towards the right (towards the detector), because the Doppler shift in frequency is an *increase*: $\Delta f > 0$.

(b) With v = 1540 m/s, θ = 20°, and R = 1 + (5495 Hz)/(5 × 10^6 Hz) =1.0011, using the expression above, we find the speed of the blood to be

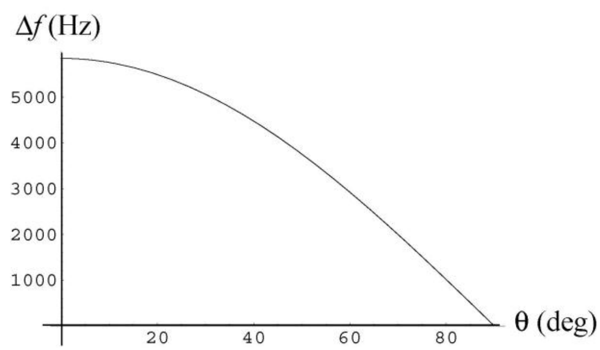
$$v_{\text{blood}} = \frac{(R-1)v}{(R+1)\cos\theta} = 0.90\,\text{m/s}.$$

(c) We interpret the question as asking how Δf (still taken to be positive, since the detector is in the "forward" direction) changes as the detection angle θ changes. Since larger θ means smaller horizontal component of velocity v_x then we expect Δf to decrease towards zero as θ is increased towards 90°.

LEARN The expression for v_{blood} can be inverted to give

$$\Delta f = \left(\frac{2v_{\text{blood}}\cos\theta}{v - v_{\text{blood}}\cos\theta}\right) f .$$

The plot of the frequency shift Δf as a function of θ is given below. Indeed we find Δf to decrease with increasing θ.



84. (a) The time it takes for sound to travel in air is $t_a = L/v$, while it takes $t_m = L/v_m$ for the sound to travel in the metal. Thus,

$$\Delta t = t_a - t_m = \frac{L}{v} - \frac{L}{v_m} = \frac{L(v_m - v)}{v_m v}.$$

(b) Using the values indicated (see Table 17-1), we obtain

$$L = \frac{\Delta t}{1/v - 1/v_m} = \frac{1.00\,\text{s}}{1/(343\,\text{m/s}) \; - \; 1/(5941\,\text{m/s})} = 364\,\text{m}.$$

85. (a) The period is the reciprocal of the frequency: $T = 1/f = 1/(90\text{ Hz}) = 1.1 \times 10^{-2}$ s.

(b) Using $v = 343$ m/s, we find $\lambda = v/f = 3.8$ m.

86. Let r stand for the ratio of the source speed to the speed of sound. Then, Eq. 17-55 (plus the fact that frequency is inversely proportional to wavelength) leads to

$$2\left(\frac{1}{1+r}\right) = \frac{1}{1-r} .$$

Solving, we find $r = 1/3$. Thus, $v_s/v = 0.33$.

87. **THINK** The siren is between you and the cliff, moving away from you and towards the cliff. You hear two frequencies, one directly from the siren and the other from the sound reflected off the cliff.

EXPRESS The Doppler shifted frequency is given by

$$f' = f\,\frac{v \pm v_D}{v \mp v_S},$$

where f is the unshifted frequency, v is the speed of sound, v_D is the speed of the detector, and v_S is the speed of the source. All speeds are relative to the air. Both "detectors" (you and the cliff) are stationary, so $v_D = 0$ in Eq. 17–47. The source is the siren with $v_S = 10\text{ m/s}$. The problem asks us to use $v = 330$ m/s for the speed of sound.

ANALYZE (a) With $f = 1000$ Hz, the frequency f_y you hear becomes

$$f_y = f\left(\frac{v+0}{v+v_S}\right) = (1000\text{ Hz})\left(\frac{330\text{ m/s}}{330\text{ m/s}+10\text{ m/s}}\right) = 970.6\text{ Hz} \approx 9.7\times10^2\text{ Hz}.$$

(b) The frequency heard by an observer at the cliff (and thus the frequency of the sound reflected by the cliff, ultimately reaching your ears at some distance from the cliff) is

$$f_c = f\left(\frac{v+0}{v-v_S}\right) = (1000\text{ Hz})\left(\frac{330\text{ m/s}}{330\text{ m/s}-10\text{ m/s}}\right) = 1031.3\text{ Hz} \approx 1.0\times10^3\text{ Hz}.$$

(c) The beat frequency is $f_{\text{beat}} = f_c - f_y = 60$ beats/s (which, due to specific features of the human ear, is too large to be perceptible).

LEARN The beat frequency in this case can be written as

$$f_{\text{beat}} = f_c - f_y = f\left(\frac{v}{v-v_S}\right) - f\left(\frac{v}{v+v_S}\right) = \frac{2vv_S}{v^2 - v_S^2} f$$

Solving for the source speed, we obtain

$$v_S = \left(\frac{-f + \sqrt{f^2 + f_{\text{beat}}^2}}{f_{\text{beat}}}\right) v$$

For the beat frequency to be perceptible ($f_{\text{beat}} < 20\text{ Hz}$), the source speed would have to be less than 3.3 m/s.

88. When $\phi = 0$ it is clear that the superposition wave has amplitude $2\Delta p_m$. For the other cases, it is useful to write

$$\Delta p_1 + \Delta p_2 = \Delta p_m \left(\sin(\omega t) + \sin(\omega t - \phi)\right) = \left(2\Delta p_m \cos\frac{\phi}{2}\right)\sin\left(\omega t - \frac{\phi}{2}\right).$$

The factor in front of the sine function gives the amplitude Δp_r. Thus, $\Delta p_r / \Delta p_m = 2\cos(\phi/2)$.

(a) When $\phi = 0$, $\Delta p_r / \Delta p_m = 2\cos(0) = 2.00$.

(b) When $\phi = \pi/2$, $\Delta p_r / \Delta p_m = 2\cos(\pi/4) = \sqrt{2} = 1.41$.

(c) When $\phi = \pi/3$, $\Delta p_r / \Delta p_m = 2\cos(\pi/6) = \sqrt{3} = 1.73$.

(d) When $\phi = \pi/4$, $\Delta p_r / \Delta p_m = 2\cos(\pi/8) = 1.85$.

89. (a) Adapting Eq. 17-39 to the notation of this chapter, we have

$$s_m' = 2\, s_m \cos(\phi/2) = 2(12\text{ nm}) \cos(\pi/6) = 20.78\text{ nm}.$$

Thus, the amplitude of the resultant wave is roughly 21 nm.

(b) The wavelength (λ = 35 cm) does not change as a result of the superposition.

(c) Recalling Eq. 17-47 (and the accompanying discussion) from the previous chapter, we conclude that the standing wave amplitude is 2(12 nm) = 24 nm when they are traveling in opposite directions.

(d) Again, the wavelength (λ = 35 cm) does not change as a result of the superposition.

90. (a) The separation distance between points A and B is one-quarter of a wavelength; therefore, $\lambda = 4(0.15\text{ m}) = 0.60\text{ m}$. The frequency, then, is

$$f = v/\lambda = (343\text{ m/s})/(0.60\text{ m}) = 572\text{ Hz}.$$

(b) The separation distance between points C and D is one-half of a wavelength; therefore, $\lambda = 2(0.15\text{ m}) = 0.30\text{ m}$. The frequency, then, is

$$f = v/\lambda = (343\text{ m/s})/(0.30\text{ m}) = 1144\text{ Hz},$$

or approximately 1.14 kHz.

91. Let the frequencies of sound heard by the person from the left and right forks be f_l and f_r, respectively.

92. If the speeds of both forks are u, then $f_{l,r} = fv/(v \pm u)$ and

$$f_{\text{beat}} = |f_r - f_l| = fv\left(\frac{1}{v-u} - \frac{1}{v+u}\right) = \frac{2fuv}{v^2 - u^2} = \frac{2(440\text{ Hz})(3.00\text{ m/s})(343\text{ m/s})}{(343\text{ m/s})^2 - (3.00\text{ m/s})^2} = 7.70\text{ Hz}.$$

(b) If the speed of the listener is u, then $f_{l,r} = f(v \pm u)/v$ and

$$f_{\text{beat}} = |f_l - f_r| = 2f\left(\frac{u}{v}\right) = 2(440\text{ Hz})\left(\frac{3.00\text{ m/s}}{343\text{ m/s}}\right) = 7.70\text{ Hz}.$$

92. The rule: if you divide the time (in seconds) by 3, then you get (approximately) the straight-line distance d. We note that the speed of sound we are to use is given at the beginning of the problem section in the textbook, and that the speed of light is very much larger than the speed of sound. The proof of our rule is as follows:

$$t = t_{\text{sound}} - t_{\text{light}} \approx t_{\text{sound}} = \frac{d}{v_{\text{sound}}} = \frac{d}{343\text{ m/s}} = \frac{d}{0.343\text{ km/s}}.$$

Cross-multiplying yields (approximately) $(0.3\text{ km/s})t = d$, which (since $1/3 \approx 0.3$) demonstrates why the rule works fairly well.

93. **THINK** Acoustic interferometer can be used to demonstrate the interference of sound waves.

EXPRESS When the right side of the instrument is pulled out a distance d the path length for sound waves increases by $2d$. Since the interference pattern changes from a minimum to the next maximum, this distance must be half a wavelength of the sound. So $2d = \lambda/2$, where λ is the wavelength. Thus $\lambda = 4d$.

On the other hand, the intensity is given by $I = \frac{1}{2}\rho v\omega^2 s_m^2$, where ρ is the density of the medium, v is the speed of sound, ω is the angular frequency, and s_m is the displacement amplitude. Thus, s_m is proportional to the square root of the intensity, and we write $\sqrt{I} = Cs_m$, where C is a constant of proportionality. At the minimum, interference is destructive and the displacement amplitude is the difference in the amplitudes of the individual waves: $s_m = s_{SAD} - s_{SBD}$, where the subscripts indicate the paths of the waves. At the maximum, the waves interfere constructively and the displacement amplitude is the sum of the amplitudes of the individual waves: $s_m = s_{SAD} + s_{SBD}$.

ANALYZE (a) The speed of sound is v = 343 m/s, so the frequency is

$$f = v/\lambda = v/4d = (343 \text{ m/s})/4(0.0165 \text{ m}) = 5.2 \times 10^3 \text{ Hz}.$$

(b) At intensity minimum, we have $\sqrt{100} = C(s_{SAD} - s_{SBD})$, and $\sqrt{900} = C(s_{SAD} + s_{SBD})$ at the maximum. Adding the equations give

$$s_{SAD} = (\sqrt{100} + \sqrt{900} / 2C = 20/C,$$

while subtracting them yields

$$s_{SBD} = (\sqrt{900} - \sqrt{100})/2C = 10/C.$$

Thus, the ratio of the amplitudes is $s_{SAD}/s_{SBD} = 2$.

(c) Any energy losses, such as might be caused by frictional forces of the walls on the air in the tubes, result in a decrease in the displacement amplitude. Those losses are greater on path B since it is longer than path A.

LEARN We see that the sound waves propagated along the two paths in the interferometer can interfere constructively or destructively, depending on their path length difference.

94. (a) Using $m = 7.3 \times 10^7$ kg, the initial gravitational potential energy is $U = mgy = 3.9 \times 10^{11}$ J, where h = 550 m. Assuming this converts primarily into kinetic energy during the fall, then $K = 3.9 \times 10^{11}$ J just before impact with the ground. Using instead the mass estimate $m = 1.7 \times 10^8$ kg, we arrive at $K = 9.2 \times 10^{11}$ J.

(b) The process of converting this kinetic energy into other forms of energy (during the impact with the ground) is assumed to take Δt = 0.50 s (and in the average sense, we take the "power" P to be wave-energy/Δt). With 20% of the energy going into creating a seismic wave, the intensity of the body wave is estimated to be

$$I = \frac{P}{A_{\text{hemisphere}}} = \frac{(0.20)K/\Delta t}{\frac{1}{2}(4\pi r^2)} = 0.63\,\text{W/m}^2$$

using $r = 200 \times 10^3$ m and the smaller value for K from part (a). Using instead the larger estimate for K, we obtain $I = 1.5$ W/m^2.

(c) The surface area of a cylinder of "height" d is $2\pi rd$, so the intensity of the surface wave is

$$I = \frac{P}{A_{\text{cylinder}}} = \frac{(0.20)K/\Delta t}{(2\pi rd)} = 25\times10^3\ \text{W/m}^2$$

using $d = 5.0$ m, $r = 200 \times 10^3$ m, and the smaller value for K from part (a). Using instead the larger estimate for K, we obtain $I = 58$ kW/m^2.

(d) Although several factors are involved in determining which seismic waves are most likely to be detected, we observe that on the basis of the above findings we should expect the more intense waves (the surface waves) to be more readily detected.

95. **THINK** Intensity is power divided by area. For an isotropic source the area is the surface area of a sphere.

EXPRESS If P is the power output and I is the intensity a distance r from the source, then $P = IA = 4\pi r^2 I$, where $A = 4\pi r^2$ is the surface area of a sphere of radius r. On the other hand, the sound level β can be calculated using Eq. 17-29:

$$\beta = (10\ \text{dB})\log\frac{I}{I_0}$$

where $I_0 = 10^{-12}\ \text{W/m}^2$ is the standard reference intensity.

ANALYZE (a) With $r = 10$ m and $I = 8.0\times10^{-3}\ \text{W/m}^2$, we have

$$P = 4\pi r^2 I = 4\pi(10)^2(8.0\times10^{-3}\ \text{W/m}^2) = 10\,\text{W}.$$

(b) Using the value of P obtained in (a), we find the intensity at $r' = 5.0$ m to be

$$I' = \frac{P}{4\pi r'^2} = \frac{10\ \text{W}}{4\pi(5.0\ \text{m})^2} = 0.032\ \text{W/m}^2.$$

(c) Using Eq. 17–29 with $I = 0.0080$ W/m^2, we find the sound level to be

$$\beta = (10\ \text{dB})\log\left(\frac{8.0\times10^{-3}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = 99\,\text{dB}.$$

LEARN The ratio of the sound intensities at two different locations can be written as

$$\frac{I}{I'}=\frac{P/4\pi r^2}{P/4\pi r'^2}=\left(\frac{r'}{r}\right)^2.$$

Similarly, the difference in sound level is given by $\Delta\beta=\beta-\beta'=(10\text{ dB})\log\left(\frac{I}{I'}\right)$.

96. We note that waves 1 and 3 differ in phase by π radians (so they cancel upon superposition). Waves 2 and 4 also differ in phase by π radians (and also cancel upon superposition). Consequently, there is no resultant wave.

97. Since they oscillate out of phase, then their waves will cancel (producing a node) at a point exactly midway between them (the midpoint of the system, where we choose $x = 0$). We note that Figure 17-13, and the $n = 3$ case of Figure 17-14(a) have this property (of a node at the midpoint). The distance Δx between nodes is $\lambda/2$, where $\lambda = v/f$ and $f = 300$ Hz and $v = 343$ m/s. Thus, $\Delta x = v/2f = 0.572$ m.

Therefore, nodes are found at the following positions:

$$x=n\Delta x=n(0.572\,\text{m}),\;\; n=0,\pm1,\pm2,...$$

(a) The shortest distance from the midpoint where nodes are found is $\Delta x = 0$.

(b) The second shortest distance from the midpoint where nodes are found is Δx =0.572 m.

(c) The third shortest distance from the midpoint where nodes are found is $2\Delta x = 1.14$ m.

98. (a) With $f = 686$ Hz and $v = 343$ m/s, then the "separation between adjacent wavefronts" is $\lambda = v/f = 0.50$ m.

(b) This is one of the effects that are part of the Doppler phenomena. Here, the wavelength shift (relative to its "true" value in part (a)) equals the source speed v_s (with appropriate ± sign) relative to the speed of sound v:

$$\frac{\Delta\lambda}{\lambda}=\pm\frac{v_s}{v}.$$

In front of the source, the shift in wavelength is –(0.50 m)(110 m/s)/(343 m/s) = –0.16 m, and the wavefront separation is 0.50 m – 0.16 m = 0.34 m.

(c) Behind the source, the shift in wavelength is +(0.50 m)(110 m/s)/(343 m/s) = +0.16 m, and the wavefront separation is 0.50 m + 0.16 m = 0.66 m.

99. We use $I \propto r^{-2}$ appropriate for an isotropic source. We have

$$\frac{I_{r=d}}{I_{r=D-d}} = \frac{(D-d)^2}{D^2} = \frac{1}{2},$$

where d = 50.0 m. We solve for

$$D: D = \sqrt{2}d/\left(\sqrt{2}-1\right) = \sqrt{2}\,(50.0\,\text{m})/\left(\sqrt{2}-1\right) = 171\,\text{m}.$$

100. Pipe A (which can only support odd harmonics – see Eq. 17-41) has length L_A. Pipe B (which supports both odd and even harmonics [any value of n] – see Eq. 17-39) has length $L_B = 4L_A$. Taking ratios of these equations leads to the condition:

$$\left(\frac{n}{2}\right)_B = (n_{\text{odd}})_A\ .$$

Solving for n_B we have $n_B = 2n_{\text{odd}}$.

(a) Thus, the smallest value of n_B at which a harmonic frequency of B matches that of A is $n_B = 2(1) = 2$.

(b) The second smallest value of n_B at which a harmonic frequency of B matches that of A is $n_B = 2(3) = 6$.

(c) The third smallest value of n_B at which a harmonic frequency of B matches that of A is $n_B = 2(5) = 10$.

101. (a) We observe that "third lowest … frequency" corresponds to harmonic number n = 5 for such a system. Using Eq. 17-41, we have

$$f = \frac{nv}{4L} \quad \Rightarrow \quad 750\,\text{Hz} = \frac{5v}{4(0.60\ \text{m})}$$

so that $v = 3.6\times10^2$ m/s.

(b) As noted, n = 5; therefore, f_1 = 750/5 = 150 Hz.

102. (a) Let P be the power output of the source. This is the rate at which energy crosses the surface of any sphere centered at the source and is therefore equal to the product of the intensity I at the sphere surface and the area of the sphere. For a sphere of radius r, $P = 4\pi r^2 I$ and $I = P/4\pi r^2$. The intensity is proportional to the square of the displacement amplitude s_m. If we write $I = Cs_m^2$, where C is a constant of proportionality, then $Cs_m^2 = P/4\pi r^2$. Thus,

$$s_m = \sqrt{P/4\pi r^2 C} = \left(\sqrt{P/4\pi C}\right)(1/r).$$

The displacement amplitude is proportional to the reciprocal of the distance from the source. We take the wave to be sinusoidal. It travels radially outward from the source, with points on a sphere of radius r in phase. If ω is the angular frequency and k is the angular wave number, then the time dependence is $\sin(kr - \omega t)$. Letting $b = \sqrt{P/4\pi C}$, the displacement wave is then given by

$$s(r,t) = \sqrt{\frac{P}{4\pi C}}\frac{1}{r}\sin(kr - \omega t) = \frac{b}{r}\sin(kr - \omega t).$$

(b) Since s and r both have dimensions of length and the trigonometric function is dimensionless, the dimensions of b must be length squared.

103. Using Eq. 17-47 with great care (regarding its ± sign conventions), we have

$$f' = (440\text{ Hz})\left(\frac{340\text{ m/s } - 80.0\text{ m/s}}{340\text{ m/s} - 54.0\text{ m/s}}\right) = 400\text{ Hz}.$$

104. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$. Since intensity is proportional to the square of the amplitude (see Eq. 17-27), this further implies

$$\frac{I_2}{I_1} = \left(\frac{s_{m2}}{s_{m1}}\right)^2 = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2$$

or $s_{m2}/s_{m1} = r_1/r_2$.

(a) $I = P/4\pi r^2 = (10\text{ W})/4\pi(3.0\text{ m})^2 = 0.088\text{ W/m}^2$.

(b) Using the notation A instead of s_m for the amplitude, we find

$$\frac{A_4}{A_3} = \frac{3.0\,\text{m}}{4.0\,\text{m}} = 0.75\,.$$

105. (a) The problem is asking at how many angles will there be "loud" resultant waves, and at how many will there be "quiet" ones? We consider the resultant wave (at large distance from the origin) along the $+x$ axis; we note that the path-length difference (for the waves traveling from their respective sources) divided by wavelength gives the (dimensionless) value $n = 3.2$, implying a sort of intermediate condition between constructive interference (which would follow if, say, $n = 3$) and destructive interference (such as the $n = 3.5$ situation found in the solution to the previous problem) between the waves. To distinguish this resultant along the $+x$ axis from the similar one along the $-x$ axis, we label one with $n = +3.2$ and the other $n = -3.2$. This labeling facilitates the complete enumeration of the loud directions in the upper-half plane: $n = -3, -2, -1, 0, +1,$

+2, +3. Counting also the "other" –3, –2, –1, 0, +1, +2, +3 values for the *lower*-half plane, then we conclude there are a total of 7 + 7 = 14 "loud" directions.

(b) The labeling also helps us enumerate the quiet directions. In the upper-half plane we find: $n =$ –2.5, –1.5, –0.5, +0.5, +1.5, +2.5. This is duplicated in the lower half plane, so the total number of quiet directions is 6 + 6 = 12.

106. We are combining two effects: the reception by a moving target with speed u of waves emitted by the stationary transmitter/detector, and the subsequent emission of those waves by the moving target, which are picked up by the stationary transmitter/detector. The first step gives

$$f_S' = f_s \frac{v+u}{v}$$

and the second step leads to

$$f_r = f_S' \frac{v}{v-u} = f_s \frac{v+u}{v} \cdot \frac{v}{v-u} = f_s \left(\frac{v+u}{v-u} \right)$$

Solving for u, we get

$$u = \left(\frac{f_r - f_s}{f_r + f_s} \right) v = \left(\frac{22.2 \text{ kHz} - 18.0 \text{ kHz}}{22.2 \text{ kHz} + 18.0 \text{ kHz}} \right) (343 \text{ m/s}) = 35.84 \text{ m/s}$$

107. The cork fillings are collected at the pressure anti-nodes when the standing waves are set up. The anti-nodes are separated by half a wavelength, $d = \lambda / 2$. Thus, the speed of the sound in the gas is

$$v = f\lambda = f(2d) = 2fd = 2(4.46 \times 10^3 \text{ Hz})(0.0920 \text{ m}) = 821 \text{ m/s}$$

108. When the layer is at height H, a constructive interference implies that the path length difference must be an integer multiple of the wavelength:

$$n\lambda = L_1 - d = 2\sqrt{H^2 + (d/2)^2} - d = \sqrt{4H^2 + d^2} - d$$

On the other hand, when the layer is at height $H + h$, a destructive interference implies that the path length difference must be an odd multiple of half the wavelength:

$$\left(n + \frac{1}{2} \right) \lambda = L_2 - d = 2\sqrt{(H+h)^2 + (d/2)^2} - d = \sqrt{4(H+h)^2 + d^2} - d$$

Subtracting the first equation from the second, we obtain

$$\frac{1}{2}\lambda = \sqrt{4(H+h)^2 + d^2} - \sqrt{4H^2 + d^2}$$

or

$$\lambda = 2\left(\sqrt{4(H+h)^2 + d^2} - \sqrt{4H^2 + d^2}\right).$$

109. The difference between the sound waves that travel along R_1 and thus that bounce and travel along R_2 is

$$\Delta d = \sqrt{25.0^2 + 12.5^2} - \sqrt{20.0^2 + 12.5^2} + \tfrac{1}{2}\lambda$$

where the last term is included for the reflection effect (mentioned in the problem). To produce constructive interference at D then we require $\Delta d = m\lambda$ where m is an integer. Since λ relates to frequency by the relation $\lambda = v/f$ (with v = 343 m/s) then we have an equation for a set of values (depending on m) for the frequency. We find

$$f = 39.3 \text{ Hz} \quad \text{for} \quad m = 1$$
$$f = 118 \text{ Hz} \quad \text{for} \quad m = 2$$
$$f = 196 \text{ Hz} \quad \text{for} \quad m = 3$$
$$f = 275 \text{ Hz} \quad \text{for} \quad m = 4$$

and so on.

(a) The lowest frequency is f = 39.3 Hz.

(b) The second lowest frequency is f = 118 Hz.

110. (a) Since the source is moving toward the wall, the frequency of the sound as received at the wall is

$$f' = f\left(\frac{v}{v - v_S}\right) = (440\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} - 20.0\,\text{m/s}}\right) = 467\,\text{Hz}.$$

(b) Since the person is moving with a speed u toward the reflected sound with frequency f', the frequency registered at the source is

$$f_r = f'\left(\frac{v+u}{v}\right) = (467\,\text{Hz})\left(\frac{343\,\text{m/s} + 20.0\,\text{m/s}}{343\,\text{m/s}}\right) = 494\,\text{Hz}.$$

111. We find the difference in the two applications of the Doppler formula:

$$f_2 - f_1 = 37\ \text{Hz} = f\left(\frac{340\ \text{m/s} + 25\ \text{m/s}}{340\,\text{m/s} - 15\ \text{m/s}} - \frac{340\ \text{m/s}}{340\ \text{m/s} - 15\ \text{m/s}}\right) = f\left(\frac{25\ \text{m/s}}{340\ \text{m/s} - 15\ \text{m/s}}\right)$$

which leads to $f = 4.8 \times 10^2\ \text{Hz}$.

Chapter 18

1. From Eq. 18-6, we see that the limiting value of the pressure ratio is the same as the absolute temperature ratio: (373.15 K)/(273.16 K) = 1.366.

2. We take p_3 to be 80 kPa for both thermometers. According to Fig. 18-6, the nitrogen thermometer gives 373.35 K for the boiling point of water. Use Eq. 18-5 to compute the pressure:

$$p_{\mathrm{N}} = \frac{T}{273.16\,\mathrm{K}}\,p_3 = \left(\frac{373.35\,\mathrm{K}}{273.16\,\mathrm{K}}\right)(80\,\mathrm{kPa}) = 109.343\,\mathrm{kPa}.$$

The hydrogen thermometer gives 373.16 K for the boiling point of water and

$$p_{\mathrm{H}} = \left(\frac{373.16\,\mathrm{K}}{273.16\,\mathrm{K}}\right)(80\,\mathrm{kPa}) = 109.287\,\mathrm{kPa}.$$

(a) The difference is $p_{\mathrm{N}} - p_{\mathrm{H}} = 0.056$ kPa ≈ 0.06 kPa.

(b) The pressure in the nitrogen thermometer is higher than the pressure in the hydrogen thermometer.

3. Let T_L be the temperature and p_L be the pressure in the left-hand thermometer. Similarly, let T_R be the temperature and p_R be the pressure in the right-hand thermometer. According to the problem statement, the pressure is the same in the two thermometers when they are both at the triple point of water. We take this pressure to be p_3. Writing Eq. 18-5 for each thermometer,

$$T_L = (273.16\,\mathrm{K})\left(\frac{p_L}{p_3}\right) \quad \text{and} \quad T_R = (273.16\,\mathrm{K})\left(\frac{p_R}{p_3}\right),$$

we subtract the second equation from the first to obtain

$$T_L - T_R = (273.16\,\mathrm{K})\left(\frac{p_L - p_R}{p_3}\right).$$

First, we take T_L = 373.125 K (the boiling point of water) and T_R = 273.16 K (the triple point of water). Then, $p_L - p_R$ = 120 torr. We solve

$$373.125\,\mathrm{K} - 273.16\,\mathrm{K} = (273.16\,\mathrm{K})\left(\frac{120\,\mathrm{torr}}{p_3}\right)$$

for p_3. The result is p_3 = 328 torr. Now, we let T_L = 273.16 K (the triple point of water) and T_R be the unknown temperature. The pressure difference is $p_L - p_R$ = 90.0 torr. Solving the equation

$$273.16\,\text{K} - T_R = (273.16\,\text{K})\left(\frac{90.0\,\text{torr}}{328\,\text{torr}}\right)$$

for the unknown temperature, we obtain T_R = 348 K.

4. (a) Let the reading on the Celsius scale be x and the reading on the Fahrenheit scale be y. Then $y = \frac{9}{5}x + 32$. For x = –71°C, this gives y = –96°F.

(b) The relationship between y and x may be inverted to yield $x = \frac{5}{9}(y - 32)$. Thus, for y = 134 we find $x \approx 56.7$ on the Celsius scale.

5. (a) Let the reading on the Celsius scale be x and the reading on the Fahrenheit scale be y. Then $y = \frac{9}{5}x + 32$. If we require $y = 2x$, then we have

$$2x = \frac{9}{5}x + 32 \quad \Rightarrow \quad x = (5)(32) = 160°\text{C}$$

which yields $y = 2x$ = 320°F.

(b) In this case, we require $y = \frac{1}{2}x$ and find

$$\frac{1}{2}x = \frac{9}{5}x + 32 \quad \Rightarrow \quad x = -\frac{(10)(32)}{13} \approx -24.6°\text{C}$$

which yields $y = x/2$ = –12.3°F.

6. We assume scales X and Y are linearly related in the sense that reading x is related to reading y by a linear relationship $y = mx + b$. We determine the constants m and b by solving the simultaneous equations:

$$-70.00 = m(-125.0) + b$$
$$-30.00 = m(375.0) + b$$

which yield the solutions $m = 40.00/500.0 = 8.000 \times 10^{-2}$ and b = –60.00. With these values, we find x for y = 50.00:

$$x = \frac{y - b}{m} = \frac{50.00 + 60.00}{0.08000} = 1375°\text{X}.$$

7. We assume scale X is a linear scale in the sense that if its reading is x then it is related to a reading y on the Kelvin scale by a linear relationship $y = mx + b$. We determine the constants m and b by solving the simultaneous equations:

$$373.15 = m(-53.5) + b$$
$$273.15 = m(-170) + b$$

which yield the solutions $m = 100/(170 - 53.5) = 0.858$ and $b = 419$. With these values, we find x for $y = 340$:

$$x = \frac{y-b}{m} = \frac{340-419}{0.858} = -92.1°\text{X}.$$

8. The increase in the surface area of the brass cube (which has six faces), which had side length L at 20°, is

$$\Delta A = 6(L+\Delta L)^2 - 6L^2 \approx 12L\Delta L = 12\alpha_b L^2 \Delta T = 12\ (19\times10^{-6}/\text{C}°)\ (30\,\text{cm})^2(75°\text{C} - 20°\text{C})$$
$$= 11\,\text{cm}^2.$$

9. The new diameter is

$$D = D_0(1+\alpha_{Al}\Delta T) = (2.725\,\text{cm})[1+(23\times10^{-6}/\text{C}°)(100.0°\text{C} - 0.000°\text{C})] = 2.731\,\text{cm}.$$

10. The change in length for the aluminum pole is

$$\Delta\ell = \ell_0\alpha_{Al}\Delta T = (33\,\text{m})(23\times10^{-6}/\text{C}°)(15\ °\text{C}) = 0.011\,\text{m}.$$

11. The volume at 30°C is given by

$$V' = V(1+\beta\Delta T) = V(1+3\alpha\Delta T) = (50.00\,\text{cm}^3)[1+3(29.00\times10^{-6}/\text{C}°)\ (30.00°\text{C} - 60.00°\text{C})]$$
$$= 49.87\,\text{cm}^3$$

where we have used $\beta = 3\alpha$.

12. (a) The coefficient of linear expansion α for the alloy is

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{10.015\,\text{cm} - 10.000\,\text{cm}}{(10.01\,\text{cm})(100°\text{C} - 20.000°\text{C})} = 1.88\times10^{-5}/\text{C}°.$$

Thus, from 100°C to 0°C we have

$$\Delta L = L\alpha\Delta T = (10.015\,\text{cm})(1.88\times10^{-5}/\text{C}°)(0°\text{C} - 100°\text{C}) = -1.88\ \times\ 10^{-2}\,\text{cm}.$$

The length at 0°C is therefore $L' = L + \Delta L = (10.015\text{ cm} - 0.0188\text{ cm}) = 9.996\text{ cm}$.

(b) Let the temperature be T_x. Then from 20°C to T_x we have

$$\Delta L = 10.009\,\text{cm} - 10.000\,\text{cm} = \alpha L \Delta T = (1.88\times10^{-5}/\text{C°})(10.000\,\text{cm})\,\Delta T,$$

giving ΔT = 48 °C. Thus, T_x = (20°C + 48 °C) = 68°C.

13. **THINK** The aluminum sphere expands thermally when being heated, so its volume increases.

EXPRESS Since a volume is the product of three lengths, the change in volume due to a temperature change ΔT is given by $\Delta V = 3\alpha V\,\Delta T$, where V is the original volume and α is the coefficient of linear expansion (see Eq. 18-11).

ANALYZE With the volume of the sphere given by $V = (4\pi/3)R^3$, where R= 10 cm is the original radius of the sphere and $\alpha = 23\times10^{-6}/\text{C°}$, then

$$\Delta V = 3\alpha\left(\frac{4\pi}{3}R^3\right)\Delta T = \left(23\times10^{-6}/\text{C°}\right)(4\pi)(10\,\text{cm})^3(100\,\text{°C}) = 29\,\text{cm}^3.$$

The value for the coefficient of linear expansion is found in Table 18-2.

LEARN The change in volume can be expressed as $\Delta V/V = \beta\Delta T$, where $\beta = 3\alpha$ is the coefficient of volume expansion. For aluminum, we have $\beta = 3\alpha = 69\times10^{-6}/\text{C°}$.

14. (a) Since $A = \pi D^2/4$, we have the differential $dA = 2(\pi D/4)dD$. Dividing the latter relation by the former, we obtain $dA/A = 2\,dD/D$. In terms of Δ's, this reads

$$\frac{\Delta A}{A} = 2\frac{\Delta D}{D} \qquad \text{for} \quad \frac{\Delta D}{D} \ll 1.$$

We can think of the factor of 2 as being due to the fact that area is a two-dimensional quantity. Therefore, the area increases by 2(0.18%) = 0.36%.

(b) Assuming that all dimensions are allowed to freely expand, then the thickness increases by 0.18%.

(c) The volume (a three-dimensional quantity) increases by 3(0.18%) = 0.54%.

(d) The mass does not change.

(e) The coefficient of linear expansion is

$$\alpha = \frac{\Delta D}{D\Delta T} = \frac{0.18\times10^{-2}}{100\text{°C}} = 1.8\times10^{-5}/\text{C°}.$$

15. After the change in temperature the diameter of the steel rod is $D_s = D_{s0} + \alpha_s D_{s0}\,\Delta T$ and the diameter of the brass ring is $D_b = D_{b0} + \alpha_b D_{b0}\,\Delta T$, where D_{s0} and D_{b0} are the original diameters, α_s and α_b are the coefficients of linear expansion, and ΔT is the change in temperature. The rod just fits through the ring if $D_s = D_b$. This means

$$D_{s0} + \alpha_s D_{s0}\,\Delta T = D_{b0} + \alpha_b D_{b0}\,\Delta T.$$

Therefore,

$$\Delta T = \frac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}} = \frac{3.000\,\text{cm} - 2.992\,\text{cm}}{(19.00\ \times 10^{-6}/\text{C°})(2.992\,\text{cm}) - (11.00\times 10^{-6}/\text{C°})(3.000\,\text{cm})}$$
$$= 335.0\,°\text{C}.$$

The temperature is $T = (25.00°\text{C} + 335.0\ °\text{C}) = 360.0°\text{C}$.

16. (a) We use $\rho = m/V$ and

$$\Delta\rho = \Delta(m/V) = m\Delta(1/V) \simeq -m\Delta V/V^2 = -\rho(\Delta V/V) = -3\rho(\Delta L/L).$$

The percent change in density is

$$\frac{\Delta\rho}{\rho} = -3\frac{\Delta L}{L} = -3(0.23\%) = -0.69\%.$$

(b) Since $\alpha = \Delta L/(L\Delta T) = (0.23 \times 10^{-2}) / (100°\text{C} - 0.0°\text{C}) = 23 \times 10^{-6}\ /\text{C°}$, the metal is aluminum (using Table 18-2).

17. **THINK** Since the aluminum cup and the glycerin have different coefficients of thermal expansion, their volumes would change by a different amount under the same ΔT.

EXPRESS If V_c is the original volume of the cup, α_a is the coefficient of linear expansion of aluminum, and ΔT is the temperature increase, then the change in the volume of the cup is $\Delta V_c = 3\alpha_a V_c\,\Delta T$ (See Eq. 18-11).

On the other hand, if β is the coefficient of volume expansion for glycerin, then the change in the volume of glycerin is $\Delta V_g = \beta V_c\,\Delta T$. Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\Delta V_g - \Delta V_c = (\beta - 3\alpha_a)V_c\Delta T = \left[(5.1\times 10^{-4}/\text{C°}) - 3(23\times 10^{-6}/\text{C°})\right](100\,\text{cm}^3)(6.0\,°\text{C})$$
$$= 0.26\,\text{cm}^3.$$

LEARN Glycerin spills over because $\beta > 3\alpha$, which gives $\Delta V_g - \Delta V_c > 0$. Note that since liquids in general have greater coefficients of thermal expansion than solids, heating a cup filled with liquid generally will cause the liquid to spill out.

18. The change in length for the section of the steel ruler between its 20.05 cm mark and 20.11 cm mark is

$$\Delta L_s = L_s \alpha_s \Delta T = (20.11\,\text{cm})(11\times10^{-6}/\text{C°})(270°\text{C} - 20°\text{C}) = 0.055\,\text{cm}.$$

Thus, the actual change in length for the rod is

$$\Delta L = (20.11\text{ cm} - 20.05\text{ cm}) + 0.055\text{ cm} = 0.115\text{ cm}.$$

The coefficient of thermal expansion for the material of which the rod is made is then

$$\alpha = \frac{\Delta L}{\Delta T} = \frac{0.115\text{ cm}}{270°\text{C} - 20°\text{C}} = 23\times10^{-6}/\text{C°}.$$

19. The initial volume V_0 of the liquid is $h_0 A_0$ where A_0 is the initial cross-section area and h_0 = 0.64 m. Its final volume is $V = hA$ where $h - h_0$ is what we wish to compute. Now, the area expands according to how the glass expands, which we analyze as follows. Using $A = \pi r^2$, we obtain

$$dA = 2\pi r\,dr = 2\pi r\left(r\alpha\,dT\right) = 2\alpha(\pi r^2)dT = 2\alpha A\,dT\ .$$

Therefore, the height is

$$h = \frac{V}{A} = \frac{V_0\left(1+\beta_{\text{liquid}}\Delta T\right)}{A_0\left(1+2\alpha_{\text{glass}}\Delta T\right)}.$$

Thus, with $V_0/A_0 = h_0$ we obtain

$$h - h_0 = h_0\left(\frac{1+\beta_{\text{liquid}}\Delta T}{1+2\alpha_{\text{glass}}\Delta T} - 1\right) = (0.64)\left(\frac{1+\left(4\times10^{-5}\right)\left(10°\right)}{1+2\left(1\times10^{-5}\right)\left(10°\right)}\right) = 1.3\times10^{-4}\text{ m}.$$

20. We divide Eq. 18-9 by the time increment Δt and equate it to the (constant) speed v = 100×10^{-9} m/s.

$$v = \alpha L_0 \frac{\Delta T}{\Delta t}$$

where L_0 = 0.0200 m and $\alpha = 23 \times 10^{-6}$/C°. Thus, we obtain

$$\frac{\Delta T}{\Delta t} = 0.217\frac{\text{C°}}{\text{s}} = 0.217\frac{\text{K}}{\text{s}}.$$

21. **THINK** The bar expands thermally when heated. Since its two ends are held fixed, the bar buckles upward.

EXPRESS Consider half the bar. Its original length is $\ell_0 = L_0/2$ and its length after the temperature increase is $\ell = \ell_0 + \alpha\ell_0\Delta T$. The old position of the half-bar, its new position, and the distance *x* that one end is displaced form a right triangle, with a hypotenuse of length ℓ, one side of length ℓ_0, and the other side of length *x*. The Pythagorean theorem yields

$$x^2 = \ell^2 - \ell_0^2 = \ell_0^2(1+\alpha\Delta T)^2 - \ell_0^2.$$

Since the change in length is small we may approximate $(1 + \alpha\,\Delta T)^2$ by $1 + 2\alpha\,\Delta T$, where the small term $(\alpha\,\Delta T\,)^2$ was neglected. Then,

$$x^2 = \ell_0^2 + 2\ell_0^2\alpha\,\Delta T - \ell_0^2 = 2\ell_0^2\alpha\,\Delta T$$

and $x \approx \ell_0\sqrt{2\alpha\,\Delta T}$.

ANALYZE Substituting the values given, we obtain

$$x = \ell_0\sqrt{2\alpha\,\Delta T} = \frac{3.77\,\text{m}}{2}\sqrt{2\left(25\times10^{-6}/\text{C°}\right)\left(32°\,\text{C}\right)} = 7.5\times10^{-2}\ \text{m}.$$

LEARN The length of the bar changes by $\Delta\ell = \alpha\ell_0\Delta T \sim \alpha\Delta T$. However, to the leading order, the vertical distance the bar has risen is proportional to $(\alpha\Delta T)^{1/2}$.

22. (a) The water (of mass *m*) releases energy in two steps, first by lowering its temperature from 20°C to 0°C, and then by freezing into ice. Thus the total energy transferred from the water to the surroundings is

$$Q = c_w m\Delta T + L_F m = \left(4190\,\text{J/kg}\cdot\text{K}\right)\left(125\,\text{kg}\right)\left(20°\text{C}\right) + \left(333\,\text{kJ/kg}\right)\left(125\,\text{kg}\right) = 5.2\times10^7\ \text{J}.$$

(b) Before all the water freezes, the lowest temperature possible is 0°C, below which the water must have already turned into ice.

23. **THINK** Electrical energy is supplied and converted into thermal energy to raise the water temperature.

EXPRESS The water has a mass m = 0.100 kg and a specific heat c = 4190 J/kg·K. When raised from an initial temperature T_i = 23°C to its boiling point T_f = 100°C, the heat input is given by $Q = cm(T_f - T_i)$. This must be the power output of the heater *P* multiplied by the time *t*: $Q = Pt$.

ANALYZE The time it takes to heat up the water is

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P} = \frac{(4190\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(100°\text{C} - 23°\text{C})}{200\,\text{J/s}} = 160\,\text{s}.$$

LEARN With a fixed power output, the time required is proportional to Q, which is proportional to $\Delta T = T_f - T_i$. In real life, it would take longer because of heat loss.

24. (a) The specific heat is given by $c = Q/m(T_f - T_i)$, where Q is the heat added, m is the mass of the sample, T_i is the initial temperature, and T_f is the final temperature. Thus, recalling that a change in Celsius degrees is equal to the corresponding change on the Kelvin scale,

$$c = \frac{314\,\text{J}}{(30.0\times 10^{-3}\,\text{kg})(45.0°\text{C} - 25.0°\text{C})} = 523\,\text{J/kg}\cdot\text{K}.$$

(b) The molar specific heat is given by

$$c_m = \frac{Q}{N(T_f - T_i)} = \frac{314\,\text{J}}{(0.600\,\text{mol})(45.0°\text{C} - 25.0°\text{C})} = 26.2\,\text{J/mol}\cdot\text{K}.$$

(c) If N is the number of moles of the substance and M is the mass per mole, then $m = NM$, so

$$N = \frac{m}{M} = \frac{30.0\times 10^{-3}\,\text{kg}}{50\times 10^{-3}\,\text{kg/mol}} = 0.600\,\text{mol}.$$

25. We use $Q = cm\Delta T$. The textbook notes that a nutritionist's "Calorie" is equivalent to 1000 cal. The mass m of the water that must be consumed is

$$m = \frac{Q}{c\Delta T} = \frac{3500\times 10^{3}\,\text{cal}}{(1\,\text{g/cal}\cdot\text{C°})(37.0°\text{C} - 0.0°\text{C}\ )} = 94.6\times 10^{4}\,\text{g},$$

which is equivalent to 9.46×10^4 g/(1000 g/liter) = 94.6 liters of water. This is certainly too much to drink in a single day!

26. The work the man has to do to climb to the top of Mt. Everest is given by

$$W = mgy = (73.0\text{ kg})(9.80\text{ m/s}^2)(8840\text{ m}) = 6.32 \times 10^6\text{ J}.$$

Thus, the amount of butter needed is

$$m = \frac{(6.32\times 10^{6}\text{ J})\left(\frac{1.00\,\text{cal}}{4.186\,\text{J}}\right)}{6000\,\text{cal/g}} \approx 250\,\text{g} = 0.25\,\text{kg}.$$

27. **THINK** Silver is solid at 15.0° C. To melt the sample, we must first raise its temperature to the melting point, and then supply heat of fusion.

EXPRESS The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0° C (= 288 K) to 1235 K. This requires heat

$$Q_1 = cm(T_f - T_i) = (236\,\text{J/kg}\cdot\text{K})(0.130\,\text{kg})(1235°\text{C} - 288°\text{C}) = 2.91\times10^4\ \text{J}.$$

Now the silver at its melting point must be melted. If L_F is the heat of fusion for silver this requires

$$Q_2 = mL_F = (0.130\,\text{kg})(105\times10^3\ \text{J/kg}) = 1.36\times10^4\ \text{J}.$$

ANALYZE The total heat required is

$$Q = Q_1 + Q_2 = 2.91\times10^4\ \text{J} + 1.36\times10^4\ \text{J} = 4.27\times10^4\ \text{J}.$$

LEARN The heating process is associated with the specific heat of silver, while the melting process involves heat of fusion. Both the specific heat and the heat of fusion are chemical properties of the material itself.

28. The amount of water m that is frozen is

$$m = \frac{Q}{L_F} = \frac{50.2\,\text{kJ}}{333\,\text{kJ/kg}} = 0.151\,\text{kg} = 151\,\text{g}.$$

Therefore the amount of water that remains unfrozen is 260 g – 151 g = 109 g.

29. The power consumed by the system is

$$P = \left(\frac{1}{20\%}\right)\frac{cm\Delta T}{t} = \left(\frac{1}{20\%}\right)\frac{(4.18\,\text{J/g}\cdot°\text{C})(200\times10^3\,\text{cm}^3)(1\,\text{g/cm}^3)(40°\text{C} - 20°\text{C})}{(1.0\,\text{h})(3600\,\text{s/h})}$$
$$= 2.3\times10^4\ \text{W}.$$

The area needed is then $A = \dfrac{2.3\times10^4\ \text{W}}{700\,\text{W/m}^2} = 33\,\text{m}^2.$

30. While the sample is in its liquid phase, its temperature change (in absolute values) is $|\Delta T| = 30\ °\text{C}$. Thus, with $m = 0.40$ kg, the absolute value of Eq. 18-14 leads to

$$|Q| = c\,m\,|\Delta T| = (3000\ \text{J/kg}\cdot°\text{C})(0.40\ \text{kg})(30\,°\text{C}) = 36000\ \text{J}.$$

The rate (which is constant) is

$$P = |Q|\,/\,t = (36000\ \text{J})/(40\ \text{min}) = 900\ \text{J/min},$$

which is equivalent to 15 W.

(a) During the next 30 minutes, a phase change occurs that is described by Eq. 18-16:

$$|Q| = P\,t = (900 \text{ J/min})(30 \text{ min}) = 27000 \text{ J} = L\,m\,.$$

Thus, with m = 0.40 kg, we find L = 67500 J/kg $\approx$ 68 kJ/kg.

(b) During the final 20 minutes, the sample is solid and undergoes a temperature change (in absolute values) of $|\Delta T| = 20\,\text{C°}$. Now, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m\,|\Delta T|} = \frac{P\,t}{m\,|\Delta T|} = \frac{(900)(20)}{(0.40)(20)} = 2250\ \frac{\text{J}}{\text{kg·C°}} \approx 2.3\ \frac{\text{kJ}}{\text{kg·C°}}\ .$$

31. Let the mass of the steam be m_s and that of the ice be m_i. Then

$$L_F m_c + c_w m_c (T_f - 0.0°\text{C}) = m_s L_s + m_s c_w (100°\text{C} - T_f)\,,$$

where T_f = 50°C is the final temperature. We solve for m_s:

$$m_s = \frac{L_F m_c + c_w m_c (T_f - 0.0°\text{C})}{L_s + c_w (100°\text{C} - T_f)} = \frac{(79.7\,\text{cal/g})(150\,\text{g}) + (1\,\text{cal/g·°C})(150\text{g})(50°\text{C} - 0.0°\text{C})}{539\,\text{cal/g} + (1\,\text{cal/g·C°})(100°\text{C} - 50°\text{C})}$$
$$= 33\,\text{g}.$$

32. The heat needed is found by integrating the heat capacity:

$$Q = \int_{T_i}^{T_f} cm\ dT = m\int_{T_i}^{T_f} c dT = (2.09)\int_{5.0°\text{C}}^{15.0°\text{C}} (0.20 + 0.14T + 0.023T^2)\,dT$$
$$= (2.0)\,(0.20T + 0.070T^2 + 0.00767T^3)\Big|_{5.0}^{15.0}\ (\text{cal})$$
$$= 82\,\text{cal}.$$

33. We note from Eq. 18-12 that 1 Btu = 252 cal. The heat relates to the power, and to the temperature change, through

$$Q = Pt = cm\Delta T.$$

Therefore, the time t required is

$$t = \frac{cm\Delta T}{P} = \frac{(1000\,\text{cal/kg·C°})(40\,\text{gal})(1000\,\text{kg}/264\,\text{gal})(100°\text{F} - 70°\text{F})(5°\text{C}/9°\text{F})}{(2.0\times10^5\ \text{Btu/h})(252.0\ \text{cal/Btu})(1\,\text{h}/60\,\text{min})}$$
$$= 3.0\,\text{min}.$$

The metric version proceeds similarly:

$$t = \frac{c\rho V \Delta T}{P} = \frac{(4190 \text{ J/kg·C°})(1000 \text{ kg/m}^3)(150 \text{ L})(1 \text{ m}^3/1000 \text{ L})(38°\text{C} - 21°\text{C})}{(59000 \text{ J/s})(60 \text{ s}/1 \text{min})}$$
$$= 3.0 \text{min}.$$

34. We note that the heat capacity of sample B is given by the reciprocal of the slope of the line in Figure 18-34(b) (compare with Eq. 18-14). Since the reciprocal of that slope is 16/4 = 4 kJ/kg·C°, then c_B = 4000 J/kg·C° = 4000 J/kg·K (since a change in Celsius is equivalent to a change in Kelvins). Now, following the same procedure as shown in Sample Problem 18.03 —"Hot slug in water, coming to equilibrium," we find

$$c_A\, m_A\, (T_f - T_A) + c_B\, m_B\, (T_f - T_B) = 0$$

$$c_A\, (5.0 \text{ kg})(40°\text{C} - 100°\text{C}) + (4000 \text{ J/kg·C°})(1.5 \text{ kg})(40°\text{C} - 20°\text{C}) = 0$$

which leads to $c_A = 4.0\times10^2$ J/kg·K.

35. We denote the ice with subscript I and the coffee with c, respectively. Let the final temperature be T_f. The heat absorbed by the ice is

$$Q_I = \lambda_F m_I + m_I c_w\, (T_f - 0°\text{C}),$$

and the heat given away by the coffee is $|Q_c| = m_w c_w\, (T_I - T_f)$. Setting $Q_I = |Q_c|$, we solve for T_f:

$$T_f = \frac{m_w c_w T_I - \lambda_F m_I}{(m_I + m_c)c_w} = \frac{(130\,\text{g})(4190\,\text{J/kg}\cdot\text{C°})\,(80.0°\text{C}) - (333\times10^3\ \text{J/g})\,(12.0\,\text{g})}{(12.0\,\text{g}+130\,\text{g})\,(4190\,\text{J/kg}\cdot\text{C°})}$$
$$= 66.5°\text{C}.$$

Note that we work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. Therefore, the temperature of the coffee will cool by $|\Delta T|$ = 80.0°C – 66.5°C = 13.5C°.

36. (a) Using Eq. 18-17, the heat transferred to the water is

$$Q_w = c_w m_w \Delta T + L_V m_s = (1\,\text{cal/g}\cdot\text{C°})(220\,\text{g})(100°\text{C} - 20.0°\text{C}) + (539\,\text{cal/g})(5.00\,\text{g})$$
$$= 20.3\,\text{kcal}.$$

(b) The heat transferred to the bowl is

$$Q_b = c_b m_b \Delta T = (0.0923\,\text{cal/g}\cdot\text{C°})(150\,\text{g})(100°\,\text{C} - 20.0°\,\text{C}) = 1.11\,\text{kcal}.$$

(c) If the original temperature of the cylinder be T_i, then $Q_w + Q_b = c_c m_c(T_i - T_f)$, which leads to

$$T_i = \frac{Q_w + Q_b}{c_c m_c} + T_f = \frac{20.3\,\text{kcal} + 1.11\,\text{kcal}}{(0.0923\,\text{cal/g}\cdot\text{C}°)(300\,\text{g})} + 100°\text{C} = 873°\text{C}.$$

37. We compute with Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. If the equilibrium temperature is T_f, then the energy absorbed as heat by the ice is

$$Q_I = L_F m_I + c_w m_I(T_f - 0°\text{C}),$$

while the energy transferred as heat from the water is $Q_w = c_w m_w(T_f - T_i)$. The system is insulated, so $Q_w + Q_I = 0$, and we solve for T_f:

$$T_f = \frac{c_w m_w T_i - L_F m_I}{(m_I + m_C)c_w}.$$

(a) Now $T_i = 90°\text{C}$ so

$$T_f = \frac{(4190\,\text{J/kg}\cdot\text{C}°)(0.500\,\text{kg})(90°\text{C}) - (333\times10^3\,\text{J/kg})(0.500\,\text{kg})}{(0.500\,\text{kg} + 0.500\,\text{kg})(4190\,\text{J/kg}\cdot\text{C}°)} = 5.3°\text{C}.$$

(b) Since no ice has remained at $T_f = 5.3°C$, we have $m_f = 0$.

(c) If we were to use the formula above with $T_i = 70°\text{C}$, we would get $T_f < 0$, which is impossible. In fact, not all the ice has melted in this case, and the equilibrium temperature is $T_f = 0°\text{C}$.

(d) The amount of ice that melts is given by

$$m_I' = \frac{c_w m_w(T_i - 0°\text{C})}{L_F} = \frac{(4190\,\text{J/kg}\cdot\text{C}°)(0.500\,\text{kg})(70\,\text{C}°)}{333\times10^3\,\text{J/kg}} = 0.440\,\text{kg}.$$

Therefore, the amount of (solid) ice remaining is $m_f = m_I - m'_I = 500\text{ g} - 440\text{ g} = 60.0\text{ g}$, and (as mentioned) we have $T_f = 0°\text{C}$ (because the system is an ice-water mixture in thermal equilibrium).

38. (a) Equation 18-14 (in absolute value) gives

$$|Q| = (4190\ \text{J/kg}\cdot°\text{C})(0.530\text{ kg})(40\ °\text{C}) = 88828\text{ J}.$$

Since dQ/dt is assumed constant (we will call it P) then we have

$$P = \frac{88828 \text{ J}}{40 \text{ min}} = \frac{88828 \text{ J}}{2400 \text{ s}} = 37 \text{ W} .$$

(b) During that same time (used in part (a)) the ice warms by 20 C°. Using Table 18-3 and Eq. 18-14 again we have

$$m_{\text{ice}} = \frac{Q}{c_{\text{ice}} \Delta T} = \frac{88828}{(2220)(20°)} = 2.0 \text{ kg} .$$

(c) To find the ice produced (by freezing the water that has already reached 0°C, so we concerned with the $40 \text{ min} < t < 60 \text{ min}$ time span), we use Table 18-4 and Eq. 18-16:

$$m_{\text{water becoming ice}} = \frac{Q_{20 \text{ min}}}{L_F} = \frac{44414}{333000} = 0.13 \text{ kg}.$$

39. To accomplish the phase change at 78°C,

$$Q = L_V m = (879 \text{ kJ/kg})\,(0.510 \text{ kg}) = 448.29 \text{ kJ}$$

must be removed. To cool the liquid to –114°C,

$$Q = cm|\Delta T| = (2.43 \text{ kJ/ kg}\cdot\text{K})\,(0.510 \text{ kg})\,(192 \text{ K}) = 237.95 \text{ kJ}$$

must be removed. Finally, to accomplish the phase change at –114°C,

$$Q = L_F m = (109 \text{ kJ/kg})\,(0.510 \text{ kg}) = 55.59 \text{ kJ}$$

must be removed. The grand total of heat removed is therefore (448.29 + 237.95 + 55.59) kJ = 742 kJ.

40. Let m_w = 14 kg, m_c = 3.6 kg, m_m = 1.8 kg, T_{i1} = 180°C, T_{i2} = 16.0°C, and T_f = 18.0°C. The specific heat c_m of the metal then satisfies

$$\left(m_w c_w + m_c c_m\right)\left(T_f - T_{i2}\right) + m_m c_m\left(T_f - T_{i1}\right) = 0$$

which we solve for c_m:

$$c_m = \frac{m_w c_w\left(T_{i2} - T_f\right)}{m_c\left(T_f - T_{i2}\right) + m_m\left(T_f - T_{i1}\right)} = \frac{(14\,\text{kg})(4.18\,\text{kJ/kg}\cdot\text{K})(16.0°\text{C} - 18.0°\text{C})}{(3.6\,\text{kg})(18.0°\text{C} - 16.0°\text{C}) + (1.8\,\text{kg})(18.0°\text{C} - 180°\text{C})}$$
$$= 0.41\,\text{kJ/kg}\cdot\text{C°} = 0.41\,\text{kJ/kg}\cdot\text{K}.$$

41. **THINK** Our system consists of both water and ice cubes. Initially the ice cubes are at $-15°\text{C}$ (below freezing temperatures), so they must first absorb heat until $0°\text{C}$ is reached. The final equilibrium temperature reached is related to the amount of ice melted.

EXPRESS There are three possibilities:

• None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.

• The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.

• All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale.

First, suppose that no ice melts. The temperature of the water decreases from T_{Wi} = 25°C to some final temperature T_f and the temperature of the ice increases from T_{Ii} = –15°C to T_f. If m_W is the mass of the water and c_W is its specific heat then the water rejects heat

$$|Q| = c_W m_W (T_{Wi} - T_f).$$

If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{Ii}).$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (T_f - T_{Ii}).$$

The solution for the equilibrium temperature is

$$\begin{aligned} T_f &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{c_W m_W + c_I m_I} \\ &= \frac{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg})(25°\text{C}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(-15°\text{C})}{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})} \\ &= 16.6°\text{C}. \end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at T_f = 0°C, with mass m (< m_I) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_W m_W T_{Wi},$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{Ii}) + mL_F,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + mL_F.$$

This equation can be solved for the mass m of ice melted.

ANALYZE (a) Solving for m and substituting the values given, we find the amount of ice melted to be

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F} \\ &= \frac{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg})(25°\text{C}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(-15°\text{C})}{333\times 10^3\,\text{J/kg}} \\ &= 5.3\times 10^{-2}\,\text{kg} = 53\,\text{g}. \end{aligned}$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C. This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$|Q| = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{Ii}) + c_W m_I (T_f - 0) + m_I L_F.$$

The first term is the energy required to raise the temperature of the ice to 0°C, the second term is the energy required to raise the temperature of the melted ice from 0°C to T_f, and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{Ii}) + c_W m_I T_f + m_I L_F.$$

The solution for T_f is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii} - m_I L_F}{c_W (m_W + m_I)}.$$

Inserting the given values, we obtain $T_f = 2.5°\text{C}$.

LEARN In order to melt some ice, the energy released by the water must be sufficient to first raise the temperature of the ice to the melting point ($-c_I m_I T_{Ii}$ required, $T_{Ii} < 0$), with the remaining energy contributing to the heat of fusion. If the remaining energy is greater than $m_I L_F$, then all ice will be melted and the final temperature will be above 0°C.

42. If the ring diameter at 0.000°C is D_{r0}, then its diameter when the ring and sphere are in thermal equilibrium is

$$D_r = D_{r0}(1 + \alpha_c T_f),$$

where T_f is the final temperature and α_c is the coefficient of linear expansion for copper. Similarly, if the sphere diameter at T_i (= 100.0°C) is D_{s0}, then its diameter at the final temperature is

$$D_s = D_{s0}[1 + \alpha_a(T_f - T_i)],$$

where α_a is the coefficient of linear expansion for aluminum. At equilibrium the two diameters are equal, so

$$D_{r0}(1 + \alpha_c T_f) = D_{s0}[1 + \alpha_a(T_f - T_i)].$$

The solution for the final temperature is

$$\begin{aligned} T_f &= \frac{D_{r0} - D_{s0} + D_{s0}\alpha_a T_i}{D_{s0}\alpha_a - D_{r0}\alpha_c} \\ &= \frac{2.54000\,\text{cm} - 2.54508\,\text{cm} + (2.54508\,\text{cm})(23\times10^{-6}/\text{C°})(100.0°\text{C})}{(2.54508\,\text{cm})(23\times10^{-6}/\text{C°}) - (2.54000\,\text{cm})\,(17\times10^{-6}/\text{C°})} \\ &= 50.38°\text{C}. \end{aligned}$$

The expansion coefficients are from Table 18-2 of the text. Since the initial temperature of the ring is 0°C, the heat it absorbs is $Q = c_c m_r T_f$, where c_c is the specific heat of copper and m_r is the mass of the ring. The heat released by the sphere is

$$|Q| = c_a m_s (T_i - T_f)$$

where c_a is the specific heat of aluminum and m_s is the mass of the sphere. Since these two heats are equal,

$$c_c m_r T_f = c_a m_s (T_i - T_f),$$

we use specific heat capacities from the textbook to obtain

$$m_s = \frac{c_c m_r T_f}{c_a (T_i - T_f)} = \frac{(386\,\text{J/kg}\cdot\text{K})(0.0200\,\text{kg})(50.38°\text{C})}{(900\,\text{J/kg}\cdot\text{K})(100°\text{C} - 50.38°\text{C})} = 8.71\times10^{-3}\,\text{kg}.$$

43. (a) One part of path A represents a constant pressure process. The volume changes from 1.0 m^3 to 4.0 m^3 while the pressure remains at 40 Pa. The work done is

$$W_A = p\Delta V = (40\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 1.2\times10^2\ \text{J}.$$

(b) The other part of the path represents a constant volume process. No work is done during this process. The total work done over the entire path is 120 J. To find the work done over path B we need to know the pressure as a function of volume. Then, we can evaluate the integral $W = \int p\,dV$. According to the graph, the pressure is a linear function of the volume, so we may write $p = a + bV$, where a and b are constants. In order for the pressure to be 40 Pa when the volume is 1.0 m^3 and 10 Pa when the volume is 4.00 m^3 the values of the constants must be a = 50 Pa and b = –10 Pa/m^3. Thus,

$$p = 50\ \text{Pa} - (10\ \text{Pa/m}^3)V$$

and

$$W_B = \int_1^4 p\,dV = \int_1^4 (50 - 10V)\,dV = \left(50V - 5V^2\right)\Big|_1^4 = 200\,\text{J} - 50\,\text{J} - 80\,\text{J}\ +\ 5.0\,\text{J} = 75\,\text{J}.$$

(c) One part of path C represents a constant pressure process in which the volume changes from 1.0 m^3 to 4.0 m^3 while p remains at 10 Pa. The work done is

$$W_C = p\Delta V = (10\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 30\,\text{J}.$$

The other part of the process is at constant volume and no work is done. The total work is 30 J. We note that the work is different for different paths.

44. During process $A \to B$, the system is expanding, doing work on its environment, so $W > 0$, and since $\Delta E_{\text{int}} > 0$ is given then $Q = W + \Delta E_{\text{int}}$ must also be positive.

(a) $Q > 0$.

(b) $W > 0$.

During process $B \to C$, the system is neither expanding nor contracting. Thus,

(c) $W = 0$.

(d) The sign of ΔE_{int} must be the same (by the first law of thermodynamics) as that of Q, which is given as positive. Thus, $\Delta E_{\text{int}} > 0$.

During process $C \to A$, the system is contracting. The environment is doing work on the system, which implies $W < 0$. Also, $\Delta E_{\text{int}} < 0$ because $\sum \Delta E_{\text{int}} = 0$ (for the whole cycle) and the other values of ΔE_{int} (for the other processes) were positive. Therefore, $Q = W + \Delta E_{\text{int}}$ must also be negative.

(e) $Q < 0$.

(f) $W < 0$.

(g) $\Delta E_{\text{int}} < 0$.

(h) The area of a triangle is $\frac{1}{2}$ (base)(height). Applying this to the figure, we find

$$|W_{\text{net}}| = \tfrac{1}{2}(2.0\,\text{m}^3)(20\,\text{Pa}) = 20\,\text{J}\,.$$

Since process $C \to A$ involves larger negative work (it occurs at higher average pressure) than the positive work done during process $A \to B$, then the net work done during the cycle must be negative. The answer is therefore $W_{\text{net}} = -20$ J.

45. **THINK** Over a complete cycle, the internal energy is the same at the beginning and end, so the heat Q absorbed equals the work done: $Q = W$.

EXPRESS Over the portion of the cycle from A to B the pressure p is a linear function of the volume V and we may write $p = a + bV$. The work done over this portion of the cycle is

$$W_{AB} = \int_{V_A}^{V_B} p\,dV = \int_{V_A}^{V_B} (a + bV)\,dV = a(V_B - V_A) + \frac{1}{2}b\left(V_B^2 - V_A^2\right).$$

The BC portion of the cycle is at constant pressure and the work done by the gas is

$$W_{BC} = p_B \Delta V_{BC} = p_B(V_C - V_B)\,.$$

The CA portion of the cycle is at constant volume, so no work is done. The total work done by the gas is

$$W = W_{AB} + W_{BC} + W_{CA}\,.$$

ANALYZE The pressure function can be written as

$$p = \frac{10}{3}\ \text{Pa} + \left(\frac{20}{3}\ \text{Pa/m}^3\right)V,$$

where the coefficients a and b were chosen so that $p = 10$ Pa when $V = 1.0\ \text{m}^3$ and $p = 30$ Pa when $V = 4.0\ \text{m}^3$. Therefore, the work done going from A to B is

$$W_{AB} = a(V_B - V_A) + \frac{1}{2}b\left(V_B^2 - V_A^2\right)$$
$$= \left(\frac{10}{3}\text{Pa}\right)\left(4.0\text{ m}^3 - 1.0\text{ m}^3\right) + \frac{1}{2}\left(\frac{20}{3}\text{ Pa/m}^3\right)\left[(4.0\text{ m}^3)^2 - (1.0\text{ m}^3)^2\right]$$
$$= 10\text{ J} + 50\text{ J} = 60\text{ J}$$

Similarly, with $p_B = p_C = 30\text{ Pa}$, $V_C = 1.0\text{ m}^3$ and $V_B = 4.0\text{ m}^3$, we have

$$W_{BC} = p_B(V_C - V_B) = (30\text{ Pa})(1.0\text{ m}^3 - 4.0\text{ m}^3) = -90\text{ J}.$$

Adding up all contributions, we find the total work done by the gas to be

$$W = W_{AB} + W_{BC} + W_{CA} = 60\text{ J} - 90\text{ J} + 0 = -30\text{ J}.$$

Thus, the total heat absorbed is $Q = W = -30$ J. This means the gas loses 30 J of energy in the form of heat.

LEARN Notice that in calculating the work done by the gas, we always start with Eq. 18-25: $W = \int p\,dV$. For isobaric process where p = constant, $W = p\Delta V$, and for isochoric process where V = constant, $W = 0$.

46. (a) Since work is done *on* the system (perhaps to compress it) we write $W = -200$ J.

(b) Since heat leaves the system, we have $Q = -70.0$ cal $= -293$ J.

(c) The change in internal energy is $\Delta E_{\text{int}} = Q - W = -293\text{ J} - (-200\text{ J}) = -93$ J.

47. **THINK** Since the change in internal energy ΔE_{int} only depends on the initial and final states, it is the same for path *iaf* and path *ibf*.

EXPRESS According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, where Q is the heat absorbed and W is the work done by the system. Along *iaf* , we have

$$\Delta E_{\text{int}} = Q - W = 50\text{ cal} - 20\text{ cal} = 30\text{ cal}.$$

ANALYZE (a) The work done along path *ibf* is given by

$$W = Q - \Delta E_{\text{int}} = 36\text{ cal} - 30\text{ cal} = 6.0\text{ cal}.$$

(b) Since the curved path is traversed from f to i the change in internal energy is $\Delta E_{\text{int}} = -30\text{ cal}$, and

$$Q = \Delta E_{\text{int}} + W = -30\text{ cal} - 13\text{ cal} = -43\text{ cal}.$$

(c) Let $\Delta E_{\text{int}} = E_{\text{int},f} - E_{\text{int},i}$. We then have

$$E_{\text{int},f} = \Delta E_{\text{int}} + E_{\text{int},i} = 30\text{ cal} + 10\text{ cal} = 40\text{ cal}.$$

(d) The work W_{bf} for the path *bf* is zero, so

$$Q_{bf} = E_{\text{int},f} - E_{\text{int},b} = 40\text{ cal} - 22\text{ cal} = 18\text{ cal}.$$

(e) For the path *ibf*, $Q = 36$ cal so $Q_{ib} = Q - Q_{bf} = 36\text{ cal} - 18\text{ cal} = 18\text{ cal}$.

LEARN Work W and heat Q in general are path-dependent quantities, i.e., they depend on how the finial state is reached. However, the combination $\Delta E_{\text{int}} = Q - W$ is path independent; it is a *state function*.

48. Since the process is a complete cycle (beginning and ending in the same thermodynamic state) the change in the internal energy is zero, and the heat absorbed by the gas is equal to the work done by the gas: $Q = W$. In terms of the contributions of the individual parts of the cycle $Q_{AB} + Q_{BC} + Q_{CA} = W$ and

$$Q_{CA} = W - Q_{AB} - Q_{BC} = +15.0\text{ J} - 20.0\text{ J} - 0 = -5.0\text{ J}.$$

This means 5.0 J of energy leaves the gas in the form of heat.

49. We note that there is no work done in the process going from *d* to *a*, so $Q_{da} = \Delta E_{\text{int}\ da} = 80$ J. Also, since the total change in internal energy around the cycle is zero, then

$$\Delta E_{\text{int}\ ac} + \Delta E_{\text{int}\ cd} + \Delta E_{\text{int}\ da} = 0$$

$$-200\text{ J} + \Delta E_{\text{int}\ cd} + 80\text{ J} = 0$$

which yields $\Delta E_{\text{int}\ cd} = 120$ J. Thus, applying the first law of thermodynamics to the *c* to *d* process gives the work done as

$$W_{cd} = Q_{cd} - \Delta E_{\text{int}\ cd} = 180\text{ J} - 120\text{ J} = 60\text{ J}.$$

50. (a) We note that process *a* to *b* is an expansion, so $W > 0$ for it. Thus, $W_{ab} = +5.0$ J. We are told that the change in internal energy during that process is +3.0 J, so application of the first law of thermodynamics for that process immediately yields $Q_{ab} = +8.0$ J.

(b) The net work (+1.2 J) is the same as the net heat ($Q_{ab} + Q_{bc} + Q_{ca}$), and we are told that $Q_{ca} = +2.5$ J. Thus we readily find $Q_{bc} = (1.2 - 8.0 - 2.5)\text{ J} = -9.3$ J.

51. We use Eqs. 18-38 through 18-40. Note that the surface area of the sphere is given by $A = 4\pi r^2$, where $r = 0.500$ m is the radius.

(a) The temperature of the sphere is $T = (273.15 + 27.00)\text{ K} = 300.15$ K. Thus

$$P_r = \sigma\varepsilon AT^4 = \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(0.850\right)\left(4\pi\right)\left(0.500\,\text{m}\right)^2\left(300.15\,\text{K}\right)^4 = 1.23\times10^3\ \text{W}.$$

(b) Now, T_{env} = 273.15 + 77.00 = 350.15 K so

$$P_a = \sigma\varepsilon AT_{\text{env}}^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.850)(4\pi)\left(0.500\,\text{m}\right)^2\left(350.15\,\text{K}\right)^4 = 2.28\times10^3\ \text{W}.$$

(c) From Eq. 18-40, we have

$$P_n = P_a - P_r = 2.28\times10^3\ \text{W} - 1.23\times10^3\ \text{W} = 1.05\times10^3\ \text{W}.$$

52. We refer to the polyurethane foam with subscript p and silver with subscript s. We use Eq. 18-32 to find $L = kR$.

(a) From Table 18-6 we find k_p = 0.024 W/m·K, so

$$\begin{aligned} L_p &= k_p R_p \\ &= \left(0.024\,\text{W/m}\cdot\text{K}\right)\left(30\,\text{ft}^2\cdot\text{F°}\cdot\text{h/Btu}\right)\left(1\,\text{m}/3.281\,\text{ft}\right)^2\left(5\text{C°}/9\text{F°}\right)\left(3600\,\text{s/h}\right)\left(1\,\text{Btu}/1055\,\text{J}\right) \\ &= 0.13\,\text{m}. \end{aligned}$$

(b) For silver k_s = 428 W/m·K, so

$$L_s = k_s R_s = \left(\frac{k_s R_s}{k_p R_p}\right) L_p = \left[\frac{428(30)}{0.024(30)}\right]\left(0.13\,\text{m}\right) = 2.3\times10^3\ \text{m}.$$

53. **THINK** Energy is transferred as heat from the hot reservoir at temperature T_H to the cold reservoir at temperature T_C. The conduction rate is the amount of energy transferred per unit time.

EXPRESS The rate of heat flow is given by

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L},$$

where k is the thermal conductivity of copper (401 W/m·K), A is the cross-sectional area (in a plane perpendicular to the flow), L is the distance along the direction of flow between the points where the temperature is T_H and T_C. The thermal conductivity is found in Table 18-6 of the text. Recall that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale.

ANALYZE Substituting the values given, we find the rate to be

$$P_{\text{cond}} = \frac{(401\,\text{W/m}\cdot\text{K})(90.0\times10^{-4}\,\text{m}^2)(125°\text{C}-10.0°\text{C})}{0.250\,\text{m}} = 1.66\times10^3\ \text{J/s}.$$

LEARN The thermal resistance (R-value) of the copper slab is

$$R = \frac{L}{k} = \frac{0.250\,\text{m}}{401\,\text{W/m}\cdot\text{K}} = 6.23\times10^{-4}\ \text{m}^2\cdot\text{K/W}\,.$$

The low value of R is an indication that the copper slab is a good conductor.

54. (a) We estimate the surface area of the average human body to be about 2 m^2 and the skin temperature to be about 300 K (somewhat less than the internal temperature of 310 K). Then from Eq. 18-37

$$P_r = \sigma\varepsilon AT^4 \approx (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.9)(2.0\,\text{m}^2)(300\,\text{K})^4 = 8\times10^2\ \text{W}.$$

(b) The energy lost is given by $\Delta E = P_r\Delta t = (8\times10^2\ \text{W})(30\,\text{s}) = 2\times10^4\ \text{J}.$

55. (a) Recalling that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale, we find that the rate of heat conduction is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(401\,\text{W/m}\cdot\text{K})(4.8\times10^{-4}\ \text{m}^2)(100\,°\text{C})}{1.2\,\text{m}} = 16\ \text{J/s}.$$

(b) Using Table 18-4, the rate at which ice melts is

$$\left|\frac{dm}{dt}\right| = \frac{P_{\text{cond}}}{L_F} = \frac{16\,\text{J/s}}{333\,\text{J/g}} = 0.048\,\text{g/s}.$$

56. The surface area of the ball is $A = 4\pi R^2 = 4\pi(0.020\ \text{m})^2 = 5.03\times10^{-3}\ \text{m}^2$. Using Eq. 18-37 with $T_i = 35+273 = 308\ \text{K}$ and $T_f = 47+273 = 320\ \text{K}$, the power required to maintain the temperature is

$$P_r = \sigma\varepsilon A(T_f^4 - T_i^4) \approx (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.80)(5.03\times10^{-3}\ \text{m}^2)\left[(320\ \text{K})^4 - (308\ \text{K})^4\right]$$
$$= 0.34\,\text{W}.$$

Thus, the heat each bee must produce during the 20-minute interval is

$$\frac{Q}{N} = \frac{P_r t}{N} = \frac{(0.34\ \text{W})(20\ \text{min})(60\ \text{s/min})}{500} = 0.81\ \text{J}\,.$$

57. (a) We use

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L}$$

with the conductivity of glass given in Table 18-6 as 1.0 W/m·K. We choose to use the Celsius scale for the temperature: a temperature difference of

$$T_H - T_C = 72°\text{F} - (-20°\text{F}) = 92\ °\text{F}$$

is equivalent to $\frac{5}{9}(92) = 51.1\text{C}°$. This, in turn, is equal to 51.1 K since a change in Kelvin temperature is entirely equivalent to a Celsius change. Thus,

$$\frac{P_{\text{cond}}}{A} = k\frac{T_H - T_C}{L} = (1.0\ \text{W/m}\cdot\text{K})\left(\frac{51.1°\text{C}}{3.0\times10^{-3}\text{m}}\right) = 1.7\times10^4\ \text{W/m}^2.$$

(b) The energy now passes in succession through 3 layers, one of air and two of glass. The heat transfer rate P is the same in each layer and is given by

$$P_{\text{cond}} = \frac{A(T_H - T_C)}{\sum L/k}$$

where the sum in the denominator is over the layers. If L_g is the thickness of a glass layer, L_a is the thickness of the air layer, k_g is the thermal conductivity of glass, and k_a is the thermal conductivity of air, then the denominator is

$$\sum\frac{L}{k} = \frac{2L_g}{k_g} + \frac{L_a}{k_a} = \frac{2L_g k_a + L_a k_g}{k_a k_g}.$$

Therefore, the heat conducted per unit area occurs at the following rate:

$$\frac{P_{\text{cond}}}{A} = \frac{(T_H - T_C)k_a k_g}{2L_g k_a + L_a k_g} = \frac{(51.1°\text{C})(0.026\ \text{W/m}\cdot\text{K})(1.0\ \text{W/m}\cdot\text{K})}{2(3.0\times10^{-3}\ \text{m})(0.026\ \text{W/m}\cdot\text{K}) + (0.075\ \text{m})(1.0\ \text{W/m}\cdot\text{K})}$$
$$= 18\ \text{W/m}^2.$$

58. (a) The surface area of the cylinder is given by

$$A_1 = 2\pi r_1^2 + 2\pi r_1 h_1 = 2\pi(2.5\times10^{-2}\ \text{m})^2 + 2\pi(2.5\times10^{-2}\ \text{m})(5.0\times10^{-2}\ \text{m}) = 1.18\times10^{-2}\ \text{m}^2,$$

its temperature is T_1 = 273 + 30 = 303 K, and the temperature of the environment is T_{env} = 273 + 50 = 323 K. From Eq. 18-39 we have

$$P_1 = \sigma\varepsilon A_1\left(T_{\text{env}}^4 - T^4\right) = (0.85)\left(1.18\times10^{-2}\text{ m}^2\right)\left((323\text{ K})^4 - (303\text{ K})^4\right) = 1.4\text{ W}.$$

(b) Let the new height of the cylinder be h_2. Since the volume V of the cylinder is fixed, we must have $V = \pi r_1^2 h_1 = \pi r_2^2 h_2$. We solve for h_2:

$$h_2 = \left(\frac{r_1}{r_2}\right)^2 h_1 = \left(\frac{2.5\text{ cm}}{0.50\text{ cm}}\right)^2 (5.0\text{ cm}) = 125\text{ cm} = 1.25\text{ m}.$$

The corresponding new surface area A_2 of the cylinder is

$$A_2 = 2\pi r_2^2 + 2\pi r_2 h_2 = 2\pi(0.50\times10^{-2}\text{ m})^2 + 2\pi(0.50\times10^{-2}\text{ m})(1.25\text{ m}) = 3.94\times10^{-2}\text{ m}^2.$$

Consequently,

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} = \frac{3.94\times10^{-2}\text{ m}^2}{1.18\times10^{-2}\text{ m}^2} = 3.3.$$

59. We use $P_{\text{cond}} = kA\Delta T/L \propto A/L$. Comparing cases (*a*) and (*b*) in Fig. 18-45, we have

$$P_{\text{cond }b} = \left(\frac{A_b L_a}{A_a L_b}\right) P_{\text{cond }a} = 4P_{\text{cond }a}.$$

Consequently, it would take 2.0 min/4 = 0.50 min for the same amount of heat to be conducted through the rods welded as shown in Fig. 18-45(*b*).

60. (a) As in Sample Problem 18.06 — "Thermal conduction through a layered wall," we take the rate of conductive heat transfer through each layer to be the same. Thus, the rate of heat transfer across the entire wall P_w is equal to the rate across layer 2 (P_2). Using Eq. 18-37 and canceling out the common factor of area A, we obtain

$$\frac{T_H - T_c}{(L_1/k_1 + L_2/k_2 + L_3/k_3)} = \frac{\Delta T_2}{(L_2/k_2)} \quad\Rightarrow\quad \frac{45\text{ C}^\circ}{(1 + 7/9 + 35/80)} = \frac{\Delta T_2}{(7/9)}$$

which leads to $\Delta T_2 = 15.8$ °C.

(b) We expect (and this is supported by the result in the next part) that greater conductivity should mean a larger rate of conductive heat transfer.

(c) Repeating the calculation above with the new value for k_2, we have

$$\frac{45\text{ C}^\circ}{(1 + 7/11 + 35/80)} = \frac{\Delta T_2}{(7/11)}$$

which leads to ΔT_2 = 13.8 °C. This is less than our part (a) result, which implies that the temperature gradients across layers 1 and 3 (the ones where the parameters did not change) are greater than in part (a); those larger temperature gradients lead to larger conductive heat currents (which is basically a statement of "Ohm's law as applied to heat conduction").

61. **THINK** As heat continues to leave the water via conduction, more ice is formed and the ice slab gets thicker.

EXPRESS Let h be the thickness of the ice slab and A be its area. Then, the rate of heat flow through the slab is

$$P_{\text{cond}} = \frac{kA\left(T_H - T_C\right)}{h},$$

where k is the thermal conductivity of ice, T_H is the temperature of the water (0°C), and T_C is the temperature of the air above the ice (–10°C). The heat leaving the water freezes it, the heat required to freeze mass m of water being $Q = L_F m$, where L_F is the heat of fusion for water. Differentiate with respect to time and recognize that $dQ/dt = P_{\text{cond}}$ to obtain

$$P_{\text{cond}} = L_F \frac{dm}{dt}.$$

Now, the mass of the ice is given by $m = \rho A h$, where ρ is the density of ice and h is the thickness of the ice slab, so $dm/dt = \rho A(dh/dt)$ and

$$P_{\text{cond}} = L_F \rho A \frac{dh}{dt}.$$

We equate the two expressions for P_{cond} and solve for dh/dt:

$$\frac{dh}{dt} = \frac{k\left(T_H - T_C\right)}{L_F \rho h}.$$

ANALYZE Since 1 cal = 4.186 J and 1 cm = 1×10^{-2} m, the thermal conductivity of ice has the SI value

$$k = (0.0040 \text{ cal/s}\cdot\text{cm}\cdot\text{K})\,(4.186 \text{ J/cal})/(1 \times 10^{-2} \text{ m/cm}) = 1.674 \text{ W/m}\cdot\text{K}.$$

The density of ice is $\rho = 0.92 \text{ g/cm}^3 = 0.92 \times 10^3 \text{ kg/m}^3$. Thus, we obtain

$$\frac{dh}{dt} = \frac{\left(1.674\,\text{W/m}\cdot\text{K}\right)\left(0°\text{C} + 10°\text{C}\right)}{\left(333\times10^3 \text{ J/kg}\right)\left(0.92\times10^3 \text{ kg/m}^3\right)\left(0.050\,\text{m}\right)} = 1.1\times10^{-6} \text{ m/s} = 0.40\,\text{cm/h}.$$

LEARN The rate of ice formation is proportional to the conduction rate – the faster the energy leaves the water, the faster the water freezes.

62. (a) Using Eq. 18-32, the rate of energy flow through the surface is

$$P_{\text{cond}} = \frac{kA(T_s - T_w)}{L} = (0.026\ \text{W/m}\cdot\text{K})(4.00\times10^{-6}\ \text{m}^2)\frac{300°\text{C} - 100°\text{C}}{1.0\times10^{-4}\ \text{m}} = 0.208\text{W} \approx 0.21\ \text{W}.$$

(Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.)

(b) With $P_{\text{cond}}t = L_V m = L_V(\rho V) = L_V(\rho A h)$, the drop will last a duration of

$$t = \frac{L_V \rho A h}{P_{\text{cond}}} = \frac{(2.256\times10^6\ \text{J/kg})(1000\ \text{kg/m}^3)(4.00\times10^{-6}\ \text{m}^2)(1.50\times10^{-3}\ \text{m})}{0.208\text{W}} = 65\ \text{s}.$$

63. We divide both sides of Eq. 18-32 by area A, which gives us the (uniform) rate of heat conduction per unit area:

$$\frac{P_{\text{cond}}}{A} = k_1\frac{T_H - T_1}{L_1} = k_4\frac{T - T_C}{L_4}$$

where $T_H = 30°\text{C}$, $T_1 = 25°\text{C}$ and $T_C = -10°\text{C}$. We solve for the unknown T.

$$T = T_C + \frac{k_1 L_4}{k_4 L_1}(T_H - T_1) = -4.2°\text{C}.$$

64. (a) For each individual penguin, the surface area that radiates is the sum of the top surface area and the sides:

$$A_r = a + 2\pi r h = a + 2\pi\sqrt{\frac{a}{\pi}}h = a + 2h\sqrt{\pi a}\ ,$$

where we have used $r = \sqrt{a/\pi}$ (from $a = \pi r^2$) for the radius of the cylinder. For the huddled cylinder, the radius is $r' = \sqrt{Na/\pi}$ (since $Na = \pi r'^2$), and the total surface area is

$$A_h = Na + 2\pi r' h = Na + 2\pi\sqrt{\frac{Na}{\pi}}h = Na + 2h\sqrt{N\pi a}\ .$$

Since the power radiated is proportional to the surface area, we have

$$\frac{P_h}{NP_r} = \frac{A_h}{NA_r} = \frac{Na + 2h\sqrt{N\pi a}}{N(a + 2h\sqrt{\pi a})} = \frac{1 + 2h\sqrt{\pi/Na}}{1 + 2h\sqrt{\pi/a}}\ .$$

With $N = 1000$, $a = 0.34\ \text{m}^2$, and $h = 1.1$ m, the ratio is

$$\frac{P_h}{NP_r} = \frac{1+2h\sqrt{\pi/Na}}{1+2h\sqrt{\pi/a}} = \frac{1+2(1.1\ \text{m})\sqrt{\pi/(1000\cdot 0.34\ \text{m}^2)}}{1+2(1.1\ \text{m})\sqrt{\pi/(0.34\ \text{m}^2)}} = 0.16\,.$$

(b) The total radiation loss is reduced by $1.00 - 0.16 = 0.84$, or 84%.

65. We assume (although this should be viewed as a "controversial" assumption) that the top surface of the ice is at T_C = –5.0°C. Less controversial are the assumptions that the bottom of the body of water is at T_H = 4.0°C and the interface between the ice and the water is at T_X = 0.0°C. The primary mechanism for the heat transfer through the total distance L = 1.4 m is assumed to be conduction, and we use Eq. 18-34:

$$\frac{k_{\text{water}}A(T_H - T_X)}{L - L_{\text{ice}}} = \frac{k_{\text{ice}}A(T_X - T_C)}{L_{\text{ice}}} \quad\Rightarrow\quad \frac{(0.12)A\left(4.0° - 0.0°\right)}{1.4 - L_{\text{ice}}} = \frac{(0.40)A\left(0.0° + 5.0°\right)}{L_{\text{ice}}}.$$

We cancel the area A and solve for thickness of the ice layer: L_{ice} = 1.1 m.

66. The condition that the energy lost by the beverage can be due to evaporation equals the energy gained via radiation exchange implies

$$L_V \frac{dm}{dt} = P_{\text{rad}} = \sigma\varepsilon A(T_{\text{env}}^4 - T^4)\,.$$

The total area of the top and side surfaces of the can is

$$A = \pi r^2 + 2\pi rh = \pi(0.022\ \text{m})^2 + 2\pi(0.022\ \text{m})(0.10\ \text{m}) = 1.53\times10^{-2}\ \text{m}^2\,.$$

With $T_{\text{env}} = 32°\text{C} = 305\ \text{K}$, $T = 15°\text{C} = 288\ \text{K}$, and $\varepsilon = 1$, the rate of water mass loss is

$$\frac{dm}{dt} = \frac{\sigma\varepsilon A}{L_V}(T_{\text{env}}^4 - T^4) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.0)(1.53\times10^{-2}\ \text{m}^2)}{2.256\times10^6\ \text{J/kg}}\left[(305\ \text{K})^4 - (288\ \text{K})^4\right]$$
$$= 6.82\times10^{-7}\ \text{kg/s} \approx 0.68\ \text{mg/s}.$$

67. We denote the total mass M and the melted mass m. The problem tells us that work/M = p/ρ, and that all the work is assumed to contribute to the phase change $Q = Lm$ where L = 150 × 10^3 J/kg. Thus,

$$\frac{p}{\rho}M = Lm \quad\Rightarrow\quad m = \frac{5.5\times10^6}{1200}\,\frac{M}{150\times10^3}$$

which yields m = 0.0306M. Dividing this by 0.30 M (the mass of the fats, which we are told is equal to 30% of the total mass), leads to a percentage 0.0306/0.30 = 10%.

68. The heat needed is

$$Q = (10\%)mL_F = \left(\frac{1}{10}\right)(200{,}000\,\text{metric tons})\ (1000\,\text{kg / metric ton})\ (333\,\text{kJ/kg}) = 6.7\times10^{12}\ \text{J}.$$

69. (a) Regarding part (a), it is important to recognize that the problem is asking for the total work done during the two-step "path": $a \to b$ followed by $b \to c$. During the latter part of this "path" there is no volume change and consequently no work done. Thus, the answer to part (b) is also the answer to part (a). Since ΔU for process $c \to a$ is –160 J, then $U_c - U_a$ = 160 J. Therefore, using the First Law of Thermodynamics, we have

$$\begin{aligned}160 &= U_c - U_b + U_b - U_a\\ &= Q_{b\to c} - W_{b\to c} + Q_{a\to b} - W_{a\to b}\\ &= 40 - 0 + 200 - W_{a\to b}.\end{aligned}$$

Therefore, $W_{a\to b\to c} = W_{a\to b}$ = 80 J.

(b) $W_{a\to b}$ = 80 J.

70. We use $Q = cm\Delta T$ and $m = \rho V$. The volume of water needed is

$$V = \frac{m}{\rho} = \frac{Q}{\rho C\Delta T} = \frac{(1.00\times10^6\ \text{kcal/day})(5\,\text{days})}{(1.00\times10^3\ \text{kg/m}^3)(1.00\,\text{kcal/kg})(50.0°\text{C} - 22.0°\text{C})} = 35.7\,\text{m}^3.$$

71. The graph shows that the absolute value of the temperature change is $|\Delta T|$ = 25 °C. Since a watt is a joule per second, we reason that the energy removed is

$$|Q| = (2.81\ \text{J/s})(20\ \text{min})(60\ \text{s/min}) = 3372\ \text{J}.$$

Thus, with m = 0.30 kg, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m\,|\Delta T|} = 4.5\times10^2\ \text{J/kg}\cdot\text{K}.$$

72. We use $P_{\text{cond}} = kA(T_H - T_C)/L$. The temperature T_H at a depth of 35.0 km is

$$T_H = \frac{P_{\text{cond}}L}{kA} + T_C = \frac{(54.0\times10^{-3}\ \text{W/m}^2)(35.0\times10^3\ \text{m})}{2.50\,\text{W/m}\cdot\text{K}} + 10.0°\text{C} = 766°\text{C}.$$

73. Its initial volume is $5^3 = 125\ \text{cm}^3$, and using Table 18-2, Eq. 18-10, and Eq. 18-11, we find

$$\Delta V = (125\,\text{m}^3)\,(3\times 23\times 10^{-6}/\text{C}°)\,(50.0\,\text{C}°) = 0.432\,\text{cm}^3.$$

74. As is shown Sample Problem 18.03 — "Hot slug in water, coming to equilibrium," we can express the final temperature in the following way:

$$T_f \;=\; \frac{m_A c_A T_A + m_B c_B T_B}{m_A c_A + m_B c_B} \;=\; \frac{c_A T_A + c_B T_B}{c_A + c_B}$$

where the last equality is made possible by the fact that $m_A = m_B$. Thus, in a graph of T_f versus T_A, the "slope" must be $c_A/(c_A + c_B)$, and the "y intercept" is $c_B/(c_A + c_B)T_B$. From the observation that the "slope" is equal to 2/5 we can determine, then, not only the ratio of the heat capacities but also the coefficient of T_B in the "y intercept"; that is,

$$c_B/(c_A + c_B)T_B = (1 - \text{“slope”})T_B\,.$$

(a) We observe that the "y intercept" is 150 K, so

$$T_B = 150/(1 - \text{“slope”}) = 150/(3/5)$$

which yields $T_B = 2.5\times 10^2$ K.

(b) As noted already, $c_A/(c_A + c_B) = \frac{2}{5}$, so $5\,c_A = 2c_A + 2c_B$, which leads to $c_B/c_A = \frac{3}{2} = 1.5$.

75. We note that there is no work done in process $c \to b$, since there is no change of volume. We also note that the *magnitude* of work done in process $b \to c$ is given, but not its sign (which we identify as negative as a result of the discussion in Section 18-8). The total (or *net*) heat transfer is $Q_{\text{net}} = [(-40) + (-130) + (+400)]$ J = 230 J. By the First Law of Thermodynamics (or, equivalently, conservation of energy), we have $Q_{\text{net}} = W_{\text{net}}$, or

$$230\,\text{J} = W_{a\to c} + W_{c\to b} + W_{b\to a} = W_{a\to c} + 0 + (-80\,\text{J}).$$

Therefore, $W_{a\to c} = 3.1\times 10^2$ J.

76. From the law of cosines, with $\phi = 59.95°$, we have

$$L_{\text{Invar}}^2 = L_{\text{alum}}^2 + L_{\text{steel}}^2 - 2L_{\text{alum}}L_{\text{steel}}\cos\phi$$

Plugging in $L = L_0\,(1 + \alpha\Delta T)$, dividing by L_0 (which is the same for all sides) and ignoring terms of order $(\Delta T)^2$ or higher, we obtain

$$1 + 2\alpha_{\text{Invar}}\Delta T = 2 + 2\,(\alpha_{\text{alum}} + \alpha_{\text{steel}})\,\Delta T - 2\,(1 + (\alpha_{\text{alum}} + \alpha_{\text{steel}})\,\Delta T)\cos\phi\,.$$

This is rearranged to yield

$$\Delta T = \frac{\cos\phi - ½}{(\alpha_{\text{alum}} + \alpha_{\text{steel}})(1 - \cos\phi) - \alpha_{\text{Invar}}} = \approx 46°\text{C},$$

so that the final temperature is $T = 20.0° + \Delta T = 66°$ C. Essentially the same argument, but arguably more elegant, can be made in terms of the differential of the above cosine law expression.

77. **THINK** The heat absorbed by the ice not only raises its temperature but could also change its phase – to water.

EXPRESS Let m_I be the mass of the ice cube and c_I be its specific heat. The energy required to bring the ice cube to the melting temperature (0 C°) is

$$Q_1 = c_I m_I (0\ \text{C}° - T_{Ii}) = (2220\ \text{J/kg}\cdot\text{K})(0.700\ \text{kg})(150\ \text{K}) = 2.331\times10^5\ \text{J}.$$

Since the total amount of energy transferred to the ice is $Q = 6.993\times10^5$ J, and $Q_1 < Q$, some or all the ice will melt. The energy required to melt all the ice is

$$Q_2 = m_I L_F = (0.700\,\text{kg})(3.33\times10^5\ \text{J/kg}) = 2.331\times10^5\ \text{J}.$$

However, since

$$Q_1 + Q_2 = 4.662\times10^5\ \text{J} < Q = 6.993\times10^5\ \text{J},$$

this means that all the ice will melt and the extra energy

$$\Delta Q = Q - (Q_1 + Q_2) = 6.993\times10^5\ \text{J} - 4.662\times10^5\ \text{J} = 2.331\times10^5\ \text{J}$$

would be used to raise the temperature of the water.

ANALYZE The final temperature of the water is given by $\Delta Q = m_I c_{\text{water}} T_f$. Substituting the values given, we have

$$T_f = \frac{\Delta Q}{m_I c_{\text{water}}} = \frac{2.331\times10^5\ \text{J}}{(0.700\ \text{kg})(4186.8\ \text{J/kg}\cdot\text{K})} = 79.5°\text{C}$$

LEARN The key concepts in this problem are outlined in the Sample Problem 18.04 – "Heat to change temperature and state." An important difference with part (b) of the sample problem is that, in our case, the final state of the H_2O is *all liquid* at $T_f > 0$. As discussed in part (a) of that sample problem, there are three steps to the total process.

78. (a) Using Eq. 18-32, we find the rate of energy conducted upward to be

$$P_{\text{cond}} = \frac{Q}{t} = kA\frac{T_H - T_C}{L} = (0.400\ \text{W/m}\cdot°\text{C})A\frac{5.0\ °\text{C}}{0.12\ \text{m}} = (16.7A)\ \text{W}.$$

Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.

(b) The heat of fusion in this process is $Q = L_F m$, where $L_F = 3.33\times10^5$ J/kg. Differentiating the expression with respect to t and equating the result with P_{cond}, we have

$$P_{\text{cond}} = \frac{dQ}{dt} = L_F \frac{dm}{dt}.$$

Thus, the rate of mass converted from liquid to ice is

$$\frac{dm}{dt} = \frac{P_{\text{cond}}}{L_F} = \frac{16.7A\ \text{W}}{3.33\times10^5\ \text{J/kg}} = (5.02\times10^{-5} A)\ \text{kg/s}.$$

(c) Since $m = \rho V = \rho A h$, differentiating both sides of the expression gives

$$\frac{dm}{dt} = \frac{d}{dt}\left(\rho A h\right) = \rho A \frac{dh}{dt}.$$

Thus, the rate of change of the icicle length is

$$\frac{dh}{dt} = \frac{1}{\rho A}\frac{dm}{dt} = \frac{5.02\times10^{-5}\,\text{kg/m}^2\cdot\text{s}}{1000\ \text{kg/m}^3} = 5.02\times10^{-8}\ \text{m/s}$$

79. **THINK** The work done by the expanding gas is given by Eq. 18-24: $W = \int p\,dV$.

EXPRESS Let V_i and V_f be the initial and final volumes, respectively. With $p = aV^2$, the work done by the gas is

$$W = \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} aV^2 dV = \frac{1}{3}a\left(V_f^3 - V_i^3\right).$$

ANALYZE With $a = 10\ \text{N/m}^8$, $V_i = 1.0\ \text{m}^3$ and $V_f = 2.0\ \text{m}^3$, we obtain

$$W = \frac{1}{3}a\left(V_f^3 - V_i^3\right) = \frac{1}{3}\left(10\ \text{N/m}^8\right)\left[(2.0\ \text{m}^3)^3 - (1.0\ \text{m}^3)^3\right] = 23\ \text{J}.$$

LEARN In this problem, the initial and final pressures are

$$p_i = aV_i^2 = (10\ \text{N/m}^8)(1.0\ \text{m}^3)^2 = 10\ \text{N/m}^2 = 10\ \text{Pa}$$
$$p_f = aV_f^2 = (10\ \text{N/m}^8)(2.0\ \text{m}^3)^2 = 40\ \text{N/m}^2 = 40\ \text{Pa}$$

In this case, since $p \sim V^2$, the work done would be proportional to V^3 after volume integration.

80. We use $Q = -\lambda_F m_{ice} = W + \Delta E_{\text{int}}$. In this case $\Delta E_{\text{int}} = 0$. Since $\Delta T = 0$ for the ideal gas, then the work done on the gas is

$$W' = -W = \lambda_F m_i = (333\,\text{J/g})(100\,\text{g}) = 33.3\,\text{kJ}.$$

81. **THINK** The work done is the "area under the curve:" $W = \int p\,dV$.

EXPRESS According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, where Q is the heat absorbed and W is the work done by the system. For process 1,

$$W_1 = p_i(V_b - V_i) = p_i(5.0V_i - V_i) = 4.0p_iV_i$$

so that

$$\Delta E_{\text{int}} = Q - W_1 = 10p_iV_i - 4.0p_iV_i = 6.0p_iV_i\ .$$

Path 2 involves more work than path 1 (note the triangle in the figure of area $\frac{1}{2}(4V_i)(p_i/2) = p_iV_i$). Thus, $W_2 = W_1 + p_iV_i = 5.0p_iV_i$. Note that $\Delta E_{\text{int}} = 6.0p_iV_i$ is the same for all three paths.

ANALYZE (a) The energy transferred to the gas as heat in process 2 is

$$Q_2 = \Delta E_{\text{int}} + W_2 = 6.0p_iV_i + 5.0p_iV_i = 11p_iV_i.$$

(b) Path 3 starts at a and ends at b (same as paths 1 and 2), so $\Delta E_{\text{int}} = 6.0p_iV_i$.

LEARN Work W and heat Q in general are path-dependent quantities, i.e., they depend on how the finial state is reached. However, the combination $\Delta E_{\text{int}} = Q - W$ is path independent; it is a *state function*.

82. (a) We denote T_H = 100°C, T_C = 0°C, the temperature of the copper–aluminum junction by T_1. and that of the aluminum-brass junction by T_2. Then,

$$P_{\text{cond}} = \frac{k_cA}{L}(T_H - T_1) = \frac{k_aA}{L}(T_1 - T_2) = \frac{k_bA}{L}(T_2 - T_c).$$

We solve for T_1 and T_2 to obtain

$$T_1 = T_H + \frac{T_C - T_H}{1 + k_c(k_a + k_b)/k_ak_b} = 100°\text{C} + \frac{0.00°\text{C} - 100°\text{C}}{1 + 401(235 + 109)/[(235)(109)]} = 84.3°\text{C}$$

(b) and

$$T_2 = T_c + \frac{T_H - T_C}{1 + k_b(k_c + k_a)/k_c k_a} = 0.00°\text{C} + \frac{100°\text{C} - 0.00°\text{C}}{1 + 109(235 + 401)/[(235)(401)]}$$
$$= 57.6°\text{C}.$$

83. **THINK** The Pyrex disk expands as a result of heating, so we expect $\Delta V > 0$.

EXPRESS The initial volume of the disk (thought of as a short cylinder) is $V_0 = \pi r^2 L$ where L = 0.50 cm is its thickness and r = 8.0 cm is its radius. After heating, the volume becomes

$$V = \pi(r + \Delta r)^2 (L + \Delta L) = \pi r^2 L + \pi r^2 \Delta L + 2\pi r L \Delta r + ...$$

where we ignore higher-order terms. Thus, the change in volume of the disk is

$$\Delta V = V - V_0 \approx \pi r^2 \Delta L + 2\pi r L \Delta r$$

ANALYZE With $\Delta L = L\alpha\Delta T$ and $\Delta r = r\alpha\Delta T$, the above expression becomes

$$\Delta V = \pi r^2 L\alpha\Delta T + 2\pi r^2 L\alpha\Delta T = 3\pi r^2 L\alpha\Delta T\ .$$

Substituting the values given ($\alpha = 3.2\ \times 10^{-6}$/C° from Table 18-2), we obtain

$$\Delta V = 3\pi r^2 L\alpha\Delta T = 3\pi(0.080\ \text{m})^2(0.0050\ \text{m})(3.2\times 10^{-6}\ /\ °\text{C})(60°\text{C} - 10°\text{C})$$
$$= 4.83\times 10^{-8}\ \text{m}^3$$

LEARN All dimensions of the disk expand when heated. So we must take into consideration the change in radius as well as the thickness.

84. (a) The rate of heat flow is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(0.040\ \text{W/m}\cdot\text{K})(1.8\ \text{m}^2)(33°\text{C} - 1.0°\text{C})}{1.0\times 10^{-2}\ \text{m}} = 2.3\times 10^2\ \text{J/s}.$$

(b) The new rate of heat flow is

$$P'_{\text{cond}} = \frac{k' P_{\text{cond}}}{k} = \frac{(0.60\ \text{W/m}\cdot\text{K})(230\ \text{J/s})}{0.040\ \text{W/m}\cdot\text{K}} = 3.5\times 10^3\ \text{J/s},$$

which is about 15 times as fast as the original heat flow.

85. **THINK** Since the system remains thermally insulated, the total energy remains unchanged. The energy released by the aluminum lump raises the water temperature.

EXPRESS Let T_f be the final temperature of the aluminum lump-water system. The energy transferred from the aluminum is $Q_{Al} = m_{Al}c_{Al}(T_{i,Al} - T_f)$. Similarly, the energy transferred as heat into water is $Q_{\text{water}} = m_{\text{water}}c_{\text{water}}(T_f - T_{i,\text{ water}})$. Equating Q_{Al} with Q_{water} allows us to solve for T_f.

ANALYZE With

$$m_{Al}c_{Al}(T_{i,Al} - T_f) = m_{\text{water}}c_{\text{water}}(T_f - T_{i,\text{ water}}),$$

we find the final equilibrium temperature to be

$$\begin{aligned} T_f &= \frac{m_{Al}c_{Al}T_{i,Al} + m_{\text{water}}c_{\text{water}}T_{i,\text{water}}}{m_{Al}c_{Al} + m_{\text{water}}c_{\text{water}}} \\ &= \frac{(2.50\text{ kg})(900\text{ J/kg}\cdot\text{K})(92°\text{C}) + (8.00\text{ kg})(4186.8\text{ J/kg}\cdot\text{K})(5.0°\text{C})}{(2.50\text{ kg})(900\text{ J/kg}\cdot\text{K}) + (8.00\text{ kg})(4186.8\text{ J/kg}\cdot\text{K})} \\ &= 10.5°\text{C}. \end{aligned}$$

LEARN No phase change is involved in this problem, so the thermal energy transferred from the aluminum can only change the water temperature.

86. If the window is L_1 high and L_2 wide at the lower temperature and $L_1 + \Delta L_1$ high and $L_2 + \Delta L_2$ wide at the higher temperature, then its area changes from $A_1 = L_1L_2$ to

$$A_2 = \left(L_1 + \Delta L_1\right)\left(L_2 + \Delta L_2\right) \approx L_1L_2 + L_1\ \Delta L_2 + L_2\ \Delta L_1$$

where the term $\Delta L_1\ \Delta L_2$ has been omitted because it is much smaller than the other terms, if the changes in the lengths are small. Consequently, the change in area is

$$\Delta A = A_2 - A_1 = L_1\ \Delta L_2 + L_2\ \Delta L_1.$$

If ΔT is the change in temperature then $\Delta L_1 = \alpha L_1\ \Delta T$ and $\Delta L_2 = \alpha L_2\ \Delta T$, where α is the coefficient of linear expansion. Thus

$$\Delta A = \alpha(L_1L_2 + L_1L_2)\ \Delta T = 2\alpha L_1L_2\Delta T = 2\left(9\times10^{-6}/\text{C}°\right)(30\,\text{cm})\,(20\,\text{cm})\,(30°\text{C}) = 0.32\,\text{cm}^2.$$

87. For a cylinder of height h, the surface area is $A_c = 2\pi rh$, and the area of a sphere is $A_o = 4\pi R^2$. The net radiative heat transfer is given by Eq. 18-40.

(a) We estimate the surface area A of the body as that of a cylinder of height 1.8 m and radius $r = 0.15$ m plus that of a sphere of radius $R = 0.10$ m. Thus, we have $A \approx A_c + A_o = 1.8\text{ m}^2$. The emissivity $\varepsilon = 0.80$ is given in the problem, and the Stefan-Boltzmann constant is found in Section 18-11: $\sigma = 5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4$. The "environment" temperature is $T_{\text{env}} = 303$ K, and the skin temperature is $T = \frac{5}{9}(102 - 32) + 273 = 312$ K. Therefore,

$$P_{\text{net}} = \sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right) = -86\,\text{W}.$$

The corresponding sign convention is discussed in the textbook immediately after Eq. 18-40. We conclude that heat is being lost by the body at a rate of roughly 90 W.

(b) Half the body surface area is roughly $A = 1.8/2 = 0.9$ m^2. Now, with $T_{\text{env}} = 248$ K, we find

$$|P_{\text{net}}| = |\sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right)| \approx 2.3\times10^2\ \text{W}.$$

(c) Finally, with $T_{\text{env}} = 193$ K (and still with $A = 0.9$ m^2) we obtain $|P_{\text{net}}| = 3.3\times10^2$ W.

88. We take absolute values of Eq. 18-9 and Eq. 12-25:

$$|\Delta L| = L\alpha\,|\Delta T| \quad \text{and} \quad \left|\frac{F}{A}\right| = E\left|\frac{\Delta L}{L}\right|.$$

The ultimate strength for steel is $(F/A)_{\text{rupture}} = S_u = 400 \times 10^6$ N/m^2 from Table 12-1. Combining the above equations (eliminating the ratio $\Delta L/L$), we find the rod will rupture if the temperature change exceeds

$$|\Delta T| = \frac{S_u}{E\alpha} = \frac{400\times10^6\ \text{N/m}^2}{\left(200\times10^9\ \text{N/m}^2\right)\left(11\times10^{-6}\,/\,\text{C}°\right)} = 182°\text{C}.$$

Since we are dealing with a temperature decrease, then, the temperature at which the rod will rupture is $T = 25.0°\text{C} - 182°\text{C} = -157°\text{C}$.

89. (a) Let the number of weight lift repetitions be N. Then $Nmgh = Q$, or (using Eq. 18-12 and the discussion preceding it)

$$N = \frac{Q}{mgh} = \frac{(3500\,\text{Cal})(4186\,\text{J/Cal})}{(80.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)(1.00\,\text{m})} \approx 1.87\times10^4.$$

(b) The time required is

$$t = (18700)(2.00\,\text{s})\left(\frac{1.00\,\text{h}}{3600\,\text{s}}\right) = 10.4\,\text{h}.$$

90. For isotropic materials, the coefficient of linear expansion α is related to that for volume expansion by $\alpha = \frac{1}{3}\beta$ (Eq. 18-11). The radius of Earth may be found in the Appendix. With these assumptions, the radius of the Earth should have increased by approximately

$$\Delta R_E = R_E \alpha \Delta T = \left(6.4\times 10^3\ \text{km}\right)\left(\frac{1}{3}\right)\left(3.0\times 10^{-5}\ /\text{K}\right)\ (3000\,\text{K} - 300\,\text{K}) = 1.7\times 10^2\ \text{km}.$$

91. We assume the ice is at 0°C to begin with, so that the only heat needed for melting is that described by Eq. 18-16 (which requires information from Table 18-4). Thus,

$$Q = Lm = (333\ \text{J/g})(1.00\ \text{g}) = 333\ \text{J}.$$

92. One method is to simply compute the change in length in each edge (x_0 = 0.200 m and y_0 = 0.300 m) from Eq. 18-9 ($\Delta x = 3.6 \times 10^{-5}$ m and $\Delta y = 5.4 \times 10^{-5}$ m) and then compute the area change:

$$A - A_0 = \left(x_0 + \Delta x\right)\left(y_0 + \Delta y\right) - x_0 y_0 = 2.16\times 10^{-5}\ \text{m}^2.$$

Another (though related) method uses $\Delta A = 2\alpha A_0 \Delta T$ (valid for $\Delta A/A \ll 1$) which can be derived by taking the differential of $A = xy$ and replacing d's with Δ's.

93. The problem asks for 0.5% of E, where $E = Pt$ with t = 120 s and P given by Eq. 18-38. Therefore, with $A = 4\pi r^2 = 5.0 \times 10^{-3}$ m^2, we obtain

$$\left(0.005\right) Pt = \left(0.005\right) \sigma \varepsilon A T^4 t = 8.6\ \text{J}.$$

94. Let the initial water temperature be T_{wi} and the initial thermometer temperature be T_{ti}. Then, the heat absorbed by the thermometer is equal (in magnitude) to the heat lost by the water:

$$c_t m_t \left(T_f - T_{ti}\right) = c_w m_w \left(T_{wi} - T_f\right).$$

We solve for the initial temperature of the water:

$$T_{wi} = \frac{c_t m_t \left(T_f - T_{ti}\right)}{c_w m_w} + T_f = \frac{\left(0.0550\,\text{kg}\right)\left(0.837\,\text{kJ/kg}\cdot\text{K}\right)\left(44.4 - 15.0\right)\text{K}}{\left(4.18\,\text{kJ/kg}\cdot\text{C}^\circ\right)\left(0.300\,\text{kg}\right)} + 44.4^\circ\text{C} = 45.5^\circ\text{C}.$$

95. The net work may be computed as a sum of works (for the individual processes involved) or as the "area" (with appropriate ± sign) inside the figure (representing the cycle). In this solution, we take the former approach (sum over the processes) and will need the following fact related to processes represented in pV diagrams:

$$\text{for a straight line: Work} = \frac{p_i + p_f}{2}\Delta V$$

which is easily verified using the definition Eq. 18-25. The cycle represented by the "triangle" BC consists of three processes:

• "tilted" straight line from (1.0 m^3, 40 Pa) to (4.0 m^3, 10 Pa), with

$$\text{Work} = \frac{40\,\text{Pa} + 10\,\text{Pa}}{2}\left(4.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 75\,\text{J}$$

• horizontal line from (4.0 m^3, 10 Pa) to (1.0 m^3, 10 Pa), with

$$\text{Work} = \left(10\,\text{Pa}\right)\left(1.0\,\text{m}^3 - 4.0\,\text{m}^3\right) = -30\,\text{J}$$

• vertical line from (1.0 m^3, 10 Pa) to (1.0 m^3, 40 Pa), with

$$\text{Work} = \frac{10\,\text{Pa} + 40\,\text{Pa}}{2}\left(1.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 0$$

(a) and (b) Thus, the total work during the *BC* cycle is (75 – 30) J = 45 J. During the *BA* cycle, the "tilted" part is the same as before, and the main difference is that the horizontal portion is at higher pressure, with Work = (40 Pa)(–3.0 m^3) = –120 J. Therefore, the total work during the *BA* cycle is (75 – 120) J = – 45 J.

96. (a) The total length change of the composite bar is

$$\Delta L = \Delta L_1 + \Delta L_2 = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T = (\alpha_1 L_1 + \alpha_2 L_2)\Delta T.$$

Writing $\Delta L = \alpha L \Delta T$ and equating the two expressions leads to $\alpha = \dfrac{\alpha_1 L_1 + \alpha_2 L_2}{L}$.

(b) The coefficients of thermal expansions are $\alpha_1 = 11\times 10^{-6}/\text{C}°$ for steel and $\alpha_2 = 19\times 10^{-6}/\text{C}°$ for brass. We solve the system of equations

$$\alpha = 13\times 10^{-6}/\text{C}° = \frac{(11\times 10^{-6}/\text{C}°)L_1 + (19\times 10^{-6}/\text{C}°)L_2}{L_1 + L_2}$$
$$L = L_1 + L_2 = 52.4\ \text{cm}$$

and obtain $L_1 = 39.3$ cm, and

(c) $L_2 = 13.1$ cm.

97. The heat required to raise the water of mass *m* from an initial temperature T_i to final temperature T_f is $Q = cm(T_f - T_i)$, where *c* is the specific heat of water. On the other hand, each shake supplies an energy $\Delta U_1 = mgh$, where *h* is the vertical distance the water has moved during each shake. Thus, with 27 shakes/min, the time required to raise the water temperature to T_f is

$$\Delta t = \frac{Q}{R(\Delta U_1)} = \frac{cm(T_f - T_i)}{Rmgh} = \frac{c(T_f - T_i)}{Rgh} = \frac{(4186.8\,\text{J/kg}\cdot\text{C}°)(100°\text{C} - 19°\text{C})}{(27\,\text{shakes/min})(9.8\,\text{m/s}^2)(0.32\,\text{m})}$$
$$= 4.0\times 10^3\ \text{min}.$$

98. Since the combination "p_1V_1" appears frequently in this derivation we denote it as "x. Thus for process 1, the heat transferred is $Q_1 = 5x = \Delta E_{\text{int 1}} + W_1$, and for path 2 (which consists of two steps, one at constant volume followed by an expansion accompanied by a linear pressure decrease) it is $Q_2 = 5.5x = \Delta E_{\text{int 2}} + W_2$. If we subtract these two expressions and make use of the fact that internal energy is state function (and thus has the same value for path 1 as for path 2) then we have

$$5.5x - 5x = W_2 - W_1 = \text{"area" inside the triangle} = \frac{1}{2}(2V_1)(p_2 - p_1).$$

Thus, dividing both sides by x (= p_1V_1), we find $0.5 = (p_2/p_1) - 1$, which leads immediately to the result: $p_2/p_1 = 1.5$.

99. The cube has six faces, each of which has an area of $(6.0 \times 10^{-6}\ \text{m})^2$. Using Kelvin temperatures and Eq. 18-40, we obtain

$$P_{\text{net}} = \sigma\varepsilon A\,(T_{\text{env}}^4 - T^4)$$
$$= \left(5.67\times 10^{-8}\,\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(0.75)\left(2.16\times 10^{-10}\ \text{m}^2\right)\left((123.15\ \text{K})^4 - (173.15\ \text{K})^4\right)$$
$$= -6.1\times 10^{-9}\ \text{W}.$$

100. We denote the density of the liquid as ρ, the rate of liquid flowing in the calorimeter as μ, the specific heat of the liquid as c, the rate of heat flow as P, and the temperature change as ΔT. Consider a time duration dt, during this time interval, the amount of liquid being heated is $dm = \mu\rho dt$. The energy required for the heating is

$$dQ = Pdt = c(dm)\,\Delta T = c\mu\Delta T dt.$$

Thus,

$$c = \frac{P}{\rho\mu\Delta T} = \frac{250\,\text{W}}{(8.0\times 10^{-6}\ \text{m}^3/\text{s})(0.85\times 10^3\ \text{kg/m}^3)(15°\text{C})}$$
$$= 2.5\times 10^3\ \text{J/kg}\cdot\text{C}° = 2.5\times 10^3\ \text{J/kg}\cdot\text{K}.$$

101. Consider the object of mass m_1 falling through a distance h. The loss of its mechanical energy is $\Delta E = m_1gh$. This amount of energy is then used to heat up the temperature of water of mass m_2: $\Delta E = m_1gh = Q = m_2c\Delta T$. Thus, the maximum possible rise in water temperature is

$$\Delta T = \frac{m_1 gh}{m_2 c} = \frac{(6.00\,\text{kg})(9.8\,\text{m/s}^2)(50.0\,\text{m})}{(0.600\,\text{kg})(4190\,\text{J/kg}\cdot\text{C°})} = 1.17\,°\text{C}.$$

102. When the temperature changes from T to $T + \Delta T$ the diameter of the mirror changes from D to $D + \Delta D$, where $\Delta D = \alpha D\,\Delta T$. Here $\alpha = 3.2 \times 10^{-6}$/C° is the coefficient of linear expansion for Pyrex glass. The range of values for the diameters can be found by setting ΔT equal to the temperature range. Thus

$$\Delta L = \alpha D \Delta T = (3.2\times 10^{-6}/\text{C°})\left(170 \text{ in.}\cdot\frac{0.0254\text{ m}}{1\text{ in.}}\right)\big(32\,°\text{C} - (-16\,°\text{C})\big)$$
$$= 6.63\times 10^{-4}\,\text{m} \approx 660\ \mu\text{m}.$$

103. The change in area for the plate is

$$\Delta A = (a+\Delta a)(b+\Delta b) - ab \approx a\Delta b + b\Delta a = 2ab\alpha\Delta T = 2\alpha A\Delta T$$
$$= 2(32\times 10^{-6}/\text{C°})(1.4\text{ m}^2)(89\,°\text{C}) = 7.97\times 10^{-3}\,\text{m}^2 \approx 8.0\times 10^{-3}\,\text{m}^2.$$

104. The relative volume change is

$$\frac{\Delta V}{V} = \beta\Delta T = (6.6\times 10^{-4}/\text{C°})(12\,°\text{C}) = 7.92\times 10^{-3}.$$

Since the expansion the glass tube can be ignored, the cross-sectional area of the liquid remains unchanged, and we have $\frac{\Delta h}{h} = \frac{\Delta V}{V} = 7.92\times 10^{-3}$.

105. (a) We note that if the pendulum shortens, its frequency of oscillation will increase, thereby causing it to record more units of time ("ticks") than have actually passed during an interval. Thus, as the pendulum contracts (this problem involves cooling the brass wire), the pendulum will "run fast."

(b) The period of the pendulum is $\tau = 2\pi\sqrt{L/g}$ (so not to be confused with temperature T). Differentiating τ with respect to L gives

$$\frac{d\tau}{dL} = \frac{d}{dL}\left(2\pi\sqrt{\frac{L}{g}}\right) = \pi\frac{1}{\sqrt{Lg}} = \frac{1}{2L}\left(2\pi\sqrt{\frac{L}{g}}\right) = \frac{\tau}{2L}.$$

Thus,

$$\Delta\tau = \frac{\tau\Delta L}{2L} = \frac{1}{2}\tau\alpha\Delta T.$$

Substituting the values given, the change in period is

$$\Delta\tau = \frac{1}{2}\tau\alpha\Delta T = \frac{1}{2}\left(\frac{3600\text{ s}}{1\text{ h}}\right)(19\times10^{-6}/\text{C°})(23\,\text{C°}) = 0.787\text{ s/h}.$$

106. Recalling that 1 W = 1 J/s, the heat Q which is added to the room in 6.9 h is

$$Q = 4(100\,\text{W})(0.73)(6.9\,\text{h})\left(\frac{3600\,\text{s}}{1.00\,\text{h}}\right) = 7.25\times10^{6}\text{ J}.$$

107. With 1 Calorie = 1000 cal, we find the athlete's rate of dissipating energy to be

$$P = 4000\,\text{Cal/day} = \frac{\left(4000\times10^{3}\text{ cal}\right)\left(4.1868\,\text{J/cal}\right)}{\left(1\,\text{day}\right)\left(86400\,\text{s/day}\right)} = 193.83\,\text{W},$$

which is about 1.9 times as much as the power of a 100 W light bulb.

108. The initial speed of the car is $v_i = 83\text{ km/h} = (83\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = 23.056\text{ m/s}.$ The deceleration a of the car is given by $v_f^2 - v_i^2 = -v_i^2 = 2ad$, or

$$a = -\frac{(23.056\text{ m/s})^2}{2(93\,\text{m})} = -2.86\,\text{m/s}^2.$$

The time Δt it takes for the car to stop is then

$$\Delta t = \frac{v_f - v_i}{a} = \frac{-23.056\text{ m/s}}{-2.86\,\text{m/s}^2} = 8.07\,\text{s}.$$

The change in kinetic energy of the car is

$$\Delta K = -\frac{1}{2}mv_i^2 = -\frac{1}{2}(1700\text{ kg})(23.056\text{ m/s})^2 = -4.52\times10^{5}\text{ J}.$$

Thus, the average rate at which mechanical energy is transferred to thermal energy is

$$P = \frac{\Delta E_{\text{th}}}{\Delta t} = \frac{-\Delta K}{\Delta t} = \frac{4.52\times10^{5}\text{ J}}{8.07\text{ s}} = 5.6\times10^{4}\,\text{W}.$$

Chapter 19

1. Each atom has a mass of $m = M/N_A$, where M is the molar mass and N_A is the Avogadro constant. The molar mass of arsenic is 74.9 g/mol or 74.9×10^{-3} kg/mol. Therefore, 7.50×10^{24} arsenic atoms have a total mass of

$$(7.50 \times 10^{24})\ (74.9 \times 10^{-3}\ \text{kg/mol})/(6.02 \times 10^{23}\ \text{mol}^{-1}) = 0.933\ \text{kg}.$$

2. (a) Equation 19-3 yields $n = M_{sam}/M = 2.5/197 = 0.0127$ mol.

(b) The number of atoms is found from Eq. 19-2:

$$N = nN_A = (0.0127)(6.02 \times 10^{23}) = 7.64 \times 10^{21}.$$

3. **THINK** We treat the oxygen gas in this problem as ideal and apply the ideal-gas law.

EXPRESS In solving the ideal-gas law equation $pV = nRT$ for n, we first convert the temperature to the Kelvin scale: $T_i = (40.0 + 273.15)\,\text{K} = 313.15\ \text{K}$, and the volume to SI units: $V_i = 1000\ \text{cm}^3 = 10^{-3}\,\text{m}^3$.

ANALYZE (a) The number of moles of oxygen present is

$$n = \frac{pV_i}{RT_i} = \frac{\left(1.01 \times 10^5\ \text{Pa}\right)\left(1.000 \times 10^{-3}\ \text{m}^3\right)}{\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(313.15\,\text{K}\right)} = 3.88 \times 10^{-2}\ \text{mol}.$$

(b) Similarly, the ideal gas law $pV = nRT$ leads to

$$T_f = \frac{pV_f}{nR} = \frac{\left(1.06 \times 10^5\ \text{Pa}\right)\left(1.500 \times 10^{-3}\ \text{m}^3\right)}{\left(3.88 \times 10^{-2}\ \text{mol}\right)\left(8.31\,\text{J/mol}\cdot\text{K}\right)} = 493\,\text{K}.$$

We note that the final temperature may be expressed in degrees Celsius as 220°C.

LEARN The final temperature can also be calculated by noting that $\dfrac{p_iV_i}{T_i} = \dfrac{p_fV_f}{T_f}$, or

$$T_f = \left(\frac{p_f}{p_i}\right)\left(\frac{V_f}{V_i}\right)T_i = \left(\frac{1.06 \times 10^5\ \text{Pa}}{1.01 \times 10^5\ \text{Pa}}\right)\left(\frac{1500\ \text{cm}^3}{1000\ \text{cm}^3}\right)(313.15\ \text{K}) = 493\ \text{K}.$$

4. (a) With T = 283 K, we obtain

$$n = \frac{pV}{RT} = \frac{100\times 10^3\ \text{Pa}\quad 2.50\,\text{m}^3}{8.31\,\text{J/mol}\cdot\text{K}\quad 283\,\text{K}} = 106\,\text{mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and temperature, and solve the ideal gas law for the new volume, or we could set up the gas law in ratio form as:

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

(where $n_i = n_f$ and thus cancels out), which yields a final volume of

$$V_f = V_i\left(\frac{p_i}{p_f}\right)\left(\frac{T_f}{T_i}\right) = \left(2.50\,\text{m}^3\right)\left(\frac{100\,\text{kPa}}{300\,\text{kPa}}\right)\left(\frac{303\,\text{K}}{283\,\text{K}}\right) = 0.892\ \text{m}^3.$$

5. With $V = 1.0 \times 10^{-6}$ m^3, $p = 1.01 \times 10^{-13}$ Pa, and T = 293 K, the ideal gas law gives

$$n = \frac{pV}{RT} = \frac{\left(1.01\times 10^{-13}\ \text{Pa}\right)\left(1.0\times 10^{-6}\ \text{m}^3\right)}{\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(293\ \text{K}\right)} = 4.1\times 10^{-23}\ \text{mole}.$$

Consequently, Eq. 19-2 yields $N = nN_A$ = 25 molecules. We can express this as a ratio (with V now written as 1 cm^3) N/V = 25 molecules/cm^3.

6. The initial and final temperatures are $T_i = 5.00°\text{C} = 278$ K and $T_f = 75.0°\text{C} = 348$ K, respectively. Using the ideal gas law with $V_i = V_f$, we find the final pressure to be

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \quad\Rightarrow\quad p_f = \frac{T_f}{T_i}\,p_i = \left(\frac{348\,\text{K}}{278\,\text{K}}\right)\left(1.00\ \text{atm}\right) = 1.25\ \text{atm}.$$

7. (a) Equation 19-45 (which gives 0) implies $Q = W$. Then Eq. 19-14, with T = (273 + 30.0)K leads to gives $Q = -3.14 \times 10^3$ J, or $|Q| = 3.14 \times 10^3$ J.

(b) That negative sign in the result of part (a) implies the transfer of heat is *from* the gas.

8. (a) We solve the ideal gas law $pV = nRT$ for n:

$$n = \frac{pV}{RT} = \frac{\left(100\,\text{Pa}\right)\left(1.0\times 10^{-6}\,\text{m}^3\right)}{\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(220\,\text{K}\right)} = 5.47\times 10^{-8}\,\text{mol}.$$

(b) Using Eq. 19-2, the number of molecules *N* is

$$N = nN_A = \left(5.47 \times 10^{-6}\text{ mol}\right)\left(6.02 \times 10^{23}\text{ mol}^{-1}\right) = 3.29 \times 10^{16}\text{ molecules.}$$

9. Since (standard) air pressure is 101 kPa, then the initial (absolute) pressure of the air is p_i = 266 kPa. Setting up the gas law in ratio form (where $n_i = n_f$ and thus cancels out), we have

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

which yields

$$p_f = p_i\left(\frac{V_i}{V_f}\right)\left(\frac{T_f}{T_i}\right) = (266\text{ kPa})\left(\frac{1.64 \times 10^{-2}\text{ m}^3}{1.67 \times 10^{-2}\text{ m}^3}\right)\left(\frac{300\text{ K}}{273\text{ K}}\right) = 287\text{ kPa}\,.$$

Expressed as a gauge pressure, we subtract 101 kPa and obtain 186 kPa.

10. The pressure p_1 due to the first gas is $p_1 = n_1RT/V$, and the pressure p_2 due to the second gas is $p_2 = n_2RT/V$. So the total pressure on the container wall is

$$p = p_1 + p_2 = \frac{n_1RT}{V} + \frac{n_2RT}{V} = (n_1 + n_2)\frac{RT}{V}.$$

The fraction of *P* due to the second gas is then

$$\frac{p_2}{p} = \frac{n_2RT/V}{(n_1+n_2)(RT/V)} = \frac{n_2}{n_1+n_2} = \frac{0.5}{2+0.5} = 0.2.$$

11. **THINK** The process consists of two steps: isothermal expansion, followed by isobaric (constant-pressure) compression. The total work done by the air is the sum of the works done for the two steps.

EXPRESS Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W_1 = \int_{V_i}^{V_f} p\,dV = nRT\int_{V_i}^{V_f}\frac{dV}{V} = nRT\ln\frac{V_f}{V_i},$$

where the ideal gas law $pV = nRT$ was used to replace *p* with nRT/V. Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with p_iV_i to obtain

$$W_1 = p_iV_i\ln\frac{p_i}{p_f}.$$

During the constant-pressure portion of the process the work done by the gas is $W_2 = p_f(V_i - V_f)$. The gas starts in a state with pressure p_f, so this is the pressure throughout this portion of the process. We also note that the volume decreases from V_f to V_i. Now $V_f = p_iV_i/p_f$, so

$$W_2 = p_f\left(V_i - \frac{p_iV_i}{p_f}\right) = \left(p_f - p_i\right)V_i .$$

ANALYZE For the first portion, since the initial gauge pressure is 1.03×10^5 Pa,

$$p_i = 1.03 \times 10^5 \text{ Pa} + 1.013 \times 10^5 \text{ Pa} = 2.04 \times 10^5 \text{ Pa}.$$

The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus,

$$W_1 = \left(2.04 \times 10^5 \text{ Pa}\right)\left(0.14 \text{ m}^3\right) \ln\left(\frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}}\right) = 2.00 \times 10^4 \text{ J}.$$

Similarly, for the second portion, we have

$$W_2 = (p_f - p_i)V_i = (1.013\times10^5 \text{ Pa} - 2.04\times10^5 \text{ Pa})(0.14 \text{ m}^3) = -1.44\times10^4 \text{ J}.$$

The total work done by the gas over the entire process is

$$W = W_1 + W_2 = 2.00 \times 10^4 \text{ J} + (-1.44 \times 10^4 \text{ J}) = 5.60 \times 10^3 \text{ J}.$$

LEARN The work done by the gas is positive when it expands, and negative when it contracts.

12. (a) At the surface, the air volume is

$$V_1 = Ah = \pi(1.00 \text{ m})^2(4.00 \text{ m}) = 12.57 \text{ m}^3 \approx 12.6 \text{ m}^3 .$$

(b) The temperature and pressure of the air inside the submarine at the surface are $T_1 = 20°\text{C} = 293 \text{ K}$ and $p_1 = p_0 = 1.00 \text{ atm}$. On the other hand, at depth $h = 80$ m, we have $T_2 = -30°\text{C} = 243 \text{ K}$ and

$$p_2 = p_0 + \rho gh = 1.00 \text{ atm} + (1024 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(80.0 \text{ m})\frac{1.00 \text{ atm}}{1.01\times10^5 \text{ Pa}}$$
$$= 1.00 \text{ atm} + 7.95 \text{ atm} = 8.95 \text{ atm}.$$

Therefore, using the ideal gas law, $pV = NkT$, the air volume at this depth would be

$$\frac{p_1V_1}{p_2V_2} = \frac{T_1}{T_2} \quad \Rightarrow \quad V_2 = \left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)V_1 = \left(\frac{1.00 \text{ atm}}{8.95 \text{ atm}}\right)\left(\frac{243\,\text{K}}{293\,\text{K}}\right)\left(12.57 \text{ m}^3\right) = 1.16 \text{ m}^3.$$

(c) The decrease in volume is $\Delta V = V_1 - V_2 = 11.44 \text{ m}^3$. Using Eq. 19-5, the amount of air this volume corresponds to is

$$n = \frac{p\Delta V}{RT} = \frac{(8.95 \text{ atm})\left(1.01\times10^5 \text{ Pa/atm}\right)\left(11.44\,\text{m}^3\right)}{\left(8.31 \text{ J/mol}\cdot\text{K}\right)\left(243\,\text{K}\right)} = 5.10\times10^3 \text{ mol}.$$

Thus, in order for the submarine to maintain the original air volume in the chamber, 5.10×10^3 mol of air must be released.

13. (a) At point *a*, we know enough information to compute *n*:

$$n = \frac{pV}{RT} = \frac{\left(2500\,\text{Pa}\right)\left(1.0\,\text{m}^3\right)}{\left(8.31 \text{ J/mol}\cdot\text{K}\right)\left(200\,\text{K}\right)} = 1.5\,\text{mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and volume, and solve the ideal gas law for the new temperature, or we could set up the gas law in terms of ratios (note: $n_a = n_b$ and cancels out):

$$\frac{p_bV_b}{p_aV_a} = \frac{T_b}{T_a} \Rightarrow T_b = \left(200\,\text{K}\right)\left(\frac{7.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at *b* of $T_b = 1.8\times10^3$ K.

(c) As in the previous part, we choose to approach this using the gas law in ratio form:

$$\frac{p_cV_c}{p_aV_a} = \frac{T_c}{T_a} \Rightarrow T_c = \left(200\,\text{K}\right)\left(\frac{2.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at *c* of $T_c = 6.0\times10^2$ K.

(d) The net energy added to the gas (as heat) is equal to the net work that is done as it progresses through the cycle (represented as a right triangle in the *pV* diagram shown in Fig. 19-20). This, in turn, is related to ± "area" inside that triangle (with area $= \frac{1}{2}$(base)(height)), where we choose the plus sign because the volume change at the largest pressure is an *increase*. Thus,

$$Q_{\text{net}} = W_{\text{net}} = \frac{1}{2}\left(2.0\,\text{m}^3\right)\left(5.0\times10^3\ \text{Pa}\right) = 5.0\times10^3\ \text{J}.$$

14. Since the pressure is constant the work is given by $W = p(V_2 - V_1)$. The initial volume is $V_1 = (AT_1 - BT_1^2)/p$, where $T_1 = 315$ K is the initial temperature, A =24.9 J/K and B = 0.00662 J/K^2. The final volume is $V_2 = (AT_2 - BT_2^2)/p$, where $T_2 = 315$ K. Thus

$$\begin{aligned} W &= A(T_2 - T_1) - B(T_2^2 - T_1^2) \\ &= (24.9\ \text{J/K})(325\ \text{K} - 315\ \text{K}) - (0.00662\ \text{J/K}^2)[(325\ \text{K})^2 - (315\ \text{K})^2] = 207\ \text{J}. \end{aligned}$$

15. Using Eq. 19-14, we note that since it is an isothermal process (involving an ideal gas) then $Q = W = nRT\ln(V_f/V_i)$ applies at any point on the graph. An easy one to read is Q = 1000 J and V_f = 0.30 m^3, and we can also infer from the graph that V_i = 0.20 m^3. We are told that n = 0.825 mol, so the above relation immediately yields T = 360 K.

16. We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If d is the depth of the lake and ρ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho gd$, where p_0 is atmospheric pressure. Since $p_1V_1 = nRT_1$, the number of moles of gas in the bubble is

$$n = p_1V_1/RT_1 = (p_0 + \rho gd)V_1/RT_1,$$

where V_1 is the volume of the bubble at the bottom of the lake and T_1 is the temperature there. At the surface of the lake the pressure is p_0 and the volume of the bubble is $V_2 = nRT_2/p_0$. We substitute for n to obtain

$$\begin{aligned} V_2 &= \frac{T_2}{T_1}\,\frac{p_0 + \rho gd}{p_0}\,V_1 \\ &= \left(\frac{293\,\text{K}}{277\,\text{K}}\right)\left(\frac{1.013\times10^5\ \text{Pa} + \left(0.998\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(40\,\text{m}\right)}{1.013\times10^5\ \text{Pa}}\right)\left(20\,\text{cm}^3\right) \\ &= 1.0\times10^2\ \text{cm}^3. \end{aligned}$$

17. When the valve is closed the number of moles of the gas in container A is $n_A = p_AV_A/RT_A$ and that in container B is $n_B = 4p_BV_A/RT_B$. The total number of moles in both containers is then

$$n = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right) = \text{const.}$$

After the valve is opened, the pressure in container A is $p'_A = Rn'_AT_A/V_A$ and that in container B is $p'_B = Rn'_BT_B/4V_A$. Equating p'_A and p'_B, we obtain $Rn'_AT_A/V_A = Rn'_BT_B/4V_A$, or $n'_B = (4T_A/T_B)n'_A$. Thus,

$$n = n'_A + n'_B = n'_A\left(1 + \frac{4T_A}{T_B}\right) = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right).$$

We solve the above equation for n'_A:

$$n'_A = \frac{V}{R}\,\frac{p_A/T_A + 4p_B/T_B}{1 + 4T_A/T_B}.$$

Substituting this expression for n'_A into $p'V_A = n'_A RT_A$, we obtain the final pressure:

$$p' = \frac{n'_A RT_A}{V_A} = \frac{p_A + 4p_B T_A/T_B}{1 + 4T_A/T_B} = 2.0 \times 10^5 \text{ Pa}.$$

18. First we rewrite Eq. 19-22 using Eq. 19-4 and Eq. 19-7:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(kN_A)T}{(mN_A)}} = \sqrt{\frac{3kT}{m}}.$$

The mass of the electron is given in the problem, and $k = 1.38 \times 10^{-23}$ J/K is given in the textbook. With $T = 2.00 \times 10^6$ K, the above expression gives $v_{\text{rms}} = 9.53 \times 10^6$ m/s. The pressure value given in the problem is not used in the solution.

19. Table 19-1 gives $M = 28.0$ g/mol for nitrogen. This value can be used in Eq. 19-22 with T in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using v for v_{rms}:

$$\frac{v_2}{v_1} = \frac{\sqrt{3RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}.$$

(a) With $T_2 = (20.0 + 273.15)$ K ≈ 293 K, we obtain $v_2 = (517\text{ m/s})\sqrt{\dfrac{293\text{ K}}{300\text{ K}}} = 511\text{ m/s}.$

(b) In this case, we set $v_3 = \frac{1}{2}v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for T_3:

$$T_3 = T_2\left(\frac{v_3}{v_2}\right)^2 = (293\text{ K})\left(\frac{1}{2}\right)^2 = 73.0\text{ K}$$

which we write as 73.0 – 273 = – 200°C.

(c) Now we have $v_4 = 2v_2$ and obtain

$$T_4 = T_2\left(\frac{v_4}{v_2}\right)^2 = (293\,\text{K})(4) = 1.17\times10^3\ \text{K}$$

which is equivalent to 899°C.

20. Appendix F gives $M = 4.00 \times 10^{-3}$ kg/mol (Table 19-1 gives this to fewer significant figures). Using Eq. 19-22, we obtain

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\ \ 8.31\ \text{J/mol}\cdot\text{K}\ \ \ 1000\,\text{K}}{4.00\times10^{-3}\ \text{kg/mol}}} = 2.50\times10^3\ \text{m/s}.$$

21. **THINK** According to kinetic theory, the rms speed is (see Eq. 19-34) $v_{\text{rms}} = \sqrt{3RT/M}$, where T is the temperature and M is the molar mass.

EXPRESS The rms speed is defined as $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$, where $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v)dv$, with the Maxwell's speed distribution function given by

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}\ .$$

According to Table 19-1, the molar mass of molecular hydrogen is 2.02 g/mol = 2.02×10^{-3} kg/mol.

ANALYZE At $T = 2.7\ \text{K}$, we find the rms speed to be

$$v_{\text{rms}} = \sqrt{\frac{3\,(8.31\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{2.02\times10^{-3}\ \text{kg/mol}}} = 1.8\times10^2\ \text{m/s}.$$

LEARN The corresponding average speed and most probable speed are

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.31\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{\pi(2.02\times10^{-3}\ \text{kg/mol})}} = 1.7\times10^2\ \text{m/s}$$

and

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2(8.31\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{2.02\times10^{-3}\ \text{kg/mol}}} = 1.5\times10^2\ \text{m/s}\,,$$

respectively.

22. The molar mass of argon is 39.95 g/mol. Eq. 19-22 gives

$$v_{\rm rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31\,{\rm J/mol\cdot K})(313\,{\rm K})}{39.95\times10^{-3}\,{\rm kg/mol}}} = 442\,{\rm m/s}.$$

23. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is $2mv\cos\theta$, where m is the mass of the molecule, v is its speed, and θ is the angle between its velocity and the normal to the wall. If N molecules collide with the wall, then the change in their total momentum is $2Nmv\cos\theta$, and if the total time taken for the collisions is Δt, then the average rate of change of the total momentum is $2(N/\Delta t)mv\cos\theta$. This is the average force exerted by the N molecules on the wall, and the pressure is the average force per unit area:

$$p = \frac{2}{A}\left(\frac{N}{\Delta t}\right) mv\cos\theta = \left(\frac{2}{2.0\times10^{-4}\,{\rm m}^2}\right)\left(1.0\times10^{23}\,{\rm s}^{-1}\right)\left(3.3\times10^{-27}\,{\rm kg}\right)\left(1.0\times10^{3}\,{\rm m/s}\right)\cos 55^\circ$$
$$= 1.9\times10^{3}\,{\rm Pa}.$$

We note that the value given for the mass was converted to kg and the value given for the area was converted to m^2.

24. We can express the ideal gas law in terms of density using $n = M_{\rm sam}/M$:

$$pV = \frac{M_{\rm sam}RT}{M} \Rightarrow \rho = \frac{pM}{RT}.$$

We can also use this to write the rms speed formula in terms of density:

$$v_{\rm rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(pM/\rho)}{M}} = \sqrt{\frac{3p}{\rho}}.$$

(a) We convert to SI units: $\rho = 1.24\times10^{-2}$ kg/m^3 and $p = 1.01\times10^{3}$ Pa. The rms speed is $\sqrt{3(1010)/0.0124} = 494$ m/s.

(b) We find M from $\rho = pM/RT$ with $T = 273$ K.

$$M = \frac{\rho RT}{p} = \frac{(0.0124\,{\rm kg/m^3})\ \ 8.31\,{\rm J/mol\cdot K}\ \ (273\,{\rm K})}{1.01\times10^{3}\,{\rm Pa}} = 0.0279\ {\rm kg/mol} = 27.9\ {\rm g/mol}.$$

(c) From Table 19.1, we identify the gas to be N_2.

25. (a) Equation 19-24 gives $K_{\rm avg} = \frac{3}{2}\left(1.38\times10^{-23}\ {\rm J/K}\right)(273\,{\rm K}) = 5.65\times10^{-21}\ {\rm J}$.

(b) For $T = 373$ K, the average translational kinetic energy is $K_{\text{avg}} = 7.72 \times 10^{-21}$ J .

(c) The unit mole may be thought of as a (large) collection: 6.02×10^{23} molecules of ideal gas, in this case. Each molecule has energy specified in part (a), so the large collection has a total kinetic energy equal to

$$K_{\text{mole}} = N_A K_{\text{avg}} = (6.02\times10^{23})(5.65\times10^{-21}\text{ J}) = 3.40\times10^{3}\text{ J}.$$

(d) Similarly, the result from part (b) leads to

$$K_{\text{mole}} = (6.02\times10^{23})(7.72\times10^{-21}\text{ J}) = 4.65\times10^{3}\text{ J}.$$

26. The average translational kinetic energy is given by $K_{\text{avg}} = \frac{3}{2}kT$, where k is the Boltzmann constant (1.38×10^{-23} J/K) and T is the temperature on the Kelvin scale. Thus

$$K_{\text{avg}} = \frac{3}{2}(1.38\times10^{-23}\text{ J/K})(1600\text{ K}) = 3.31\times10^{-20}\text{ J}.$$

27. (a) We use $\varepsilon = L_V/N$, where L_V is the heat of vaporization and N is the number of molecules per gram. The molar mass of atomic hydrogen is 1 g/mol and the molar mass of atomic oxygen is 16 g/mol, so the molar mass of H_2O is (1.0 + 1.0 + 16) = 18 g/mol. There are $N_A = 6.02 \times 10^{23}$ molecules in a mole, so the number of molecules in a gram of water is $(6.02 \times 10^{23}\text{ mol}^{-1})/(18\text{ g/mol}) = 3.34 \times 10^{22}$ molecules/g. Thus

$$\varepsilon = (539\text{ cal/g})/(3.34\times10^{22}\text{/g}) = 1.61\times10^{-20}\text{ cal} = 6.76\times10^{-20}\text{ J}.$$

(b) The average translational kinetic energy is

$$K_{\text{avg}} = \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23}\text{ J/K})[(32.0+273.15)\text{ K}] = 6.32\times10^{-21}\text{ J}.$$

The ratio $\varepsilon/K_{\text{avg}}$ is $(6.76 \times 10^{-20}\text{ J})/(6.32 \times 10^{-21}\text{ J}) = 10.7$.

28. Using $v = f\lambda$ with $v = 331$ m/s (see Table 17-1) with Eq. 19-2 and Eq. 19-25 leads to

$$\begin{aligned} f &= \frac{v}{\left(\dfrac{1}{\sqrt{2}\pi d^2 (N/V)}\right)} = (331\text{ m/s})\pi\sqrt{2}\,(3.0\times10^{-10}\text{ m})^2\left(\frac{nN_A}{V}\right) \\ &= \left(8.0\times10^{7}\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{n}{V}\right) = \left(8.0\times10^{7}\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{1.01\times10^{5}\text{ Pa}}{(8.31\text{ J/mol}\cdot\text{K})(273.15\text{ K})}\right) \\ &= 3.5\times10^{9}\text{ Hz} \end{aligned}$$

where we have used the ideal gas law and substituted $n/V = p/RT$. If we instead use v = 343 m/s (the "default value" for speed of sound in air, used repeatedly in Ch. 17), then the answer is 3.7×10^9 Hz.

29. **THINK** Mean free path is the average distance traveled by a molecule between successive collisions.

EXPRESS According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where d is the diameter of a molecule and N is the number of molecules in volume V.

ANALYZE (a) Substituting $d = 2.0 \times 10^{-10}$ m and $N/V = 1 \times 10^6$ molecules/m^3, we obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10}\text{ m})^2(1 \times 10^6\text{ m}^{-3})} = 6 \times 10^{12}\text{ m}.$$

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

LEARN Mean free path is inversely proportional to the number density, N/V. The typical value of N/V at room temperature and atmospheric pressure for ideal gas is

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.01\times10^5\text{ Pa}}{(1.38\times10^{-23}\text{ J/K})(298\text{ K})} = 2.46\times10^{25}\text{ molecules/m}^3 = 2.46\times10^{19}\text{ molecules/cm}^3.$$

This is much higher than that in the outer space.

30. We solve Eq. 19-25 for d:

$$d = \sqrt{\frac{1}{\lambda\pi\sqrt{2}(N/V)}} = \sqrt{\frac{1}{(0.80 \times 10^5\text{ cm})\,\pi\sqrt{2}\,(2.7 \times 10^{19}/\text{cm}^3)}}$$

which yields $d = 3.2 \times 10^{-8}$ cm, or 0.32 nm.

31. (a) We use the ideal gas law $pV = nRT = NkT$, where p is the pressure, V is the volume, T is the temperature, n is the number of moles, and N is the number of molecules. The substitutions $N = nN_A$ and $k = R/N_A$ were made. Since 1 cm of mercury = 1333 Pa, the pressure is $p = (10^{-7})(1333\text{ Pa}) = 1.333 \times 10^{-4}$ Pa. Thus,

$$\frac{N}{V}=\frac{p}{kT}=\frac{1.333\times10^{-4}\text{ Pa}}{(1.38\times10^{-23}\text{ J/K})(295\text{ K})}=3.27\times10^{16}\text{ molecules/m}^3=3.27\times10^{10}\text{ molecules/cm}^3.$$

(b) The molecular diameter is $d = 2.00 \times 10^{-10}$ m, so, according to Eq. 19-25, the mean free path is

$$\lambda=\frac{1}{\sqrt{2}\pi d^2 N/V}=\frac{1}{\sqrt{2}\pi(2.00\times10^{-10}\text{ m})^2(3.27\times10^{16}\text{ m}^{-3})}=172\text{ m}.$$

32. (a) We set up a ratio using Eq. 19-25:

$$\frac{\lambda_{\text{Ar}}}{\lambda_{\text{N}_2}}=\frac{1/\left(\pi\sqrt{2}d_{\text{Ar}}^2(N/V)\right)}{1/\left(\pi\sqrt{2}d_{\text{N}_2}^2(N/V)\right)}=\left(\frac{d_{\text{N}_2}}{d_{\text{Ar}}}\right)^2.$$

Therefore, we obtain

$$\frac{d_{\text{Ar}}}{d_{\text{N}_2}}=\sqrt{\frac{\lambda_{\text{N}_2}}{\lambda_{\text{Ar}}}}=\sqrt{\frac{27.5\times10^{-6}\text{ cm}}{9.9\times10^{-6}\text{ cm}}}=1.7.$$

(b) Using Eq. 19-2 and the ideal gas law, we substitute $N/V = N_A n/V = N_A p/RT$ into Eq. 19-25 and find

$$\lambda=\frac{RT}{\pi\sqrt{2}d^2 pN_A}.$$

Comparing (for the same species of molecule) at two different pressures and temperatures, this leads to

$$\frac{\lambda_2}{\lambda_1}=\left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right).$$

With $\lambda_1 = 9.9 \times 10^{-6}$ cm, $T_1 = 293$ K (the same as T_2 in this part), $p_1 = 750$ torr, and $p_2 = 150$ torr, we find $\lambda_2 = 5.0 \times 10^{-5}$ cm.

(c) The ratio set up in part (b), using the same values for quantities with subscript 1, leads to $\lambda_2 = 7.9 \times 10^{-6}$ cm for $T_2 = 233$ K and $p_2 = 750$ torr.

33. **THINK** We're given the speeds of 10 molecules. The speed distribution is discrete.

EXPRESS The average speed is $\bar{v}=\frac{\sum v}{N}$, where the sum is over the speeds of the particles and N is the number of particles. Similarly, the rms speed is given by $v_{\text{rms}}=\sqrt{\frac{\sum v^2}{N}}$.

ANALYZE (a) From the equation above, we find the average speed to be

$$\bar{v} = \frac{(2.0+3.0+4.0+5.0+6.0+7.0+8.0+9.0+10.0+11.0)\,\text{km/s}}{10} = 6.5\,\text{km/s}.$$

(b) With

$$\sum v^2 = [(2.0)^2 + (3.0)^2 + (4.0)^2 + (5.0)^2 + (6.0)^2 + (7.0)^2 + (8.0)^2 + (9.0)^2 + (10.0)^2 + (11.0)^2]\ \text{km}^2/\text{s}^2 = 505\ \text{km}^2/\text{s}^2$$

the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{505\ \text{km}^2/\text{s}^2}{10}} = 7.1\,\text{km/s}.$$

LEARN Each speed is weighted equally in calculating the average and the rms values.

34. (a) The average speed is

$$v_{\text{avg}} = \frac{\sum n_i v_i}{\sum n_i} = \frac{[2(1.0)+4(2.0)+6(3.0)+8(4.0)+2(5.0)]\ \text{cm/s}}{2+4+6+8+2} = 3.2\,\text{cm/s}.$$

(b) From $v_{\text{rms}} = \sqrt{\sum n_i v_i^2 / \sum n_i}$ we get

$$v_{\text{rms}} = \sqrt{\frac{2(1.0)^2 + 4(2.0)^2 + 6(3.0)^2 + 8(4.0)^2 + 2(5.0)^2}{2+4+6+8+2}}\ \text{cm/s} = 3.4\,\text{cm/s}.$$

(c) There are eight particles at v = 4.0 cm/s, more than the number of particles at any other single speed. So 4.0 cm/s is the most probable speed.

35. (a) The average speed is

$$v_{\text{avg}} = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{10}[4(200\ \text{m/s}) + 2(500\ \text{m/s}) + 4(600\ \text{m/s})] = 420\ \text{m/s}.$$

(b) The rms speed is

$$v_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200\ \text{m/s})^2 + 2(500\ \text{m/s})^2 + 4(600\ \text{m/s})^2]} = 458\ \text{m/s}$$

(c) Yes, $v_{\text{rms}} > v_{\text{avg}}$.

36. We divide Eq. 19-35 by Eq. 19-22:

$$\frac{v_P}{v_{\text{rms}}} = \frac{\sqrt{2RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{2T_2}{3T_1}}$$

which, for $v_P = v_{\text{rms}}$, leads to

$$\frac{T_2}{T_1} = \frac{3}{2}\left(\frac{v_P}{v_{\text{rms}}}\right)^2 = \frac{3}{2} .$$

37. **THINK** From the distribution function $P(v)$, we can calculate the average and rms speeds.

EXPRESS The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v)\, dv = 1$. The average speed is defined as $v_{\text{avg}} = \int_0^\infty vP(v)dv$. Similarly, the rms speed is given by $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$, where $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v)dv$.

ANALYZE (a) Evaluate the integral by calculating the area under the curve in Fig. 19-23. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0. Thus,

$$\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0 ,$$

so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3 = 0.67$.

(b) For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2 dv = \frac{a}{3v_0}v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0,$$

where $2/3v_0$ was substituted for a. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\, dv = \frac{a}{2}\left(4v_0^2 - v_0^2\right) = \frac{3a}{2}v_0^2 = v_0.$$

Therefore, we have

$$v_{\text{avg}} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \quad \Rightarrow \quad \frac{v_{\text{avg}}}{v_0} = 1.22 .$$

(c) In calculating $v_{\text{avg}}^2 = \int v^2 P(v)\,dv$, we note that the contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3 dv = \frac{a}{4v_0}v_0^4 = \frac{1}{6}v_0^2.$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2 dv = \frac{a}{3}\left(8v_0^3 - v_0^3\right) = \frac{7a}{3}v_0^3 = \frac{14}{9}v_0^2.$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0 \;\Rightarrow\; \frac{v_{\text{rms}}}{v_0} = 1.31\ .$$

(d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is

$$Na(2.0v_0 - 1.5v_0) = 0.5N\,av_0 = N/3,$$

where $2/3v_0$ was substituted for a. In other words, the fraction of particles in this range is 1/3 or 0.33.

LEARN From the distribution function shown in Fig. 19-23, it is clear that there are more particles with a speed in the range $v_0 < v < 2v_0$ than $0 < v < v_0$. In fact, straightforward calculation shows that the fraction of particles with speeds between $1.0v_0$ and $2v_0$ is

$$\int_{1.0v_0}^{2v_0} P(v)dv = a(2v_0 - 1.0v_0) = av_0 = \frac{2}{3}.$$

38. (a) From the graph we see that $v_p = 400$ m/s. Using the fact that $M = 28$ g/mol = 0.028 kg/mol for nitrogen (N_2) gas, Eq. 19-35 can then be used to determine the absolute temperature. We obtain $T = \frac{1}{2}Mv_p^2/R = 2.7\times10^2$ K.

(b) Comparing with Eq. 19-34, we conclude $v_{\text{rms}} = \sqrt{3/2}\; v_p = 4.9\times10^2$ m/s.

39. The rms speed of molecules in a gas is given by $v_{rms} = \sqrt{3RT/M}$, where T is the temperature and M is the molar mass of the gas. See Eq. 19-34. The speed required for escape from Earth's gravitational pull is $v = \sqrt{2gr_e}$, where g is the acceleration due to gravity at Earth's surface and r_e (= 6.37×10^6 m) is the radius of Earth. To derive this

expression, take the zero of gravitational potential energy to be at infinity. Then, the gravitational potential energy of a particle with mass *m* at Earth's surface is

$$U = -GMm/r_e^2 = -mgr_e,$$

where $g = GM/r_e^2$ was used. If v is the speed of the particle, then its total energy is $E = -mgr_e + \frac{1}{2}mv^2$. If the particle is just able to travel far away, its kinetic energy must tend toward zero as its distance from Earth becomes large without bound. This means E = 0 and $v = \sqrt{2gr_e}$. We equate the expressions for the speeds to obtain $\sqrt{3RT/M} = \sqrt{2gr_e}$. The solution for T is $T = 2gr_eM/3R$.

(a) The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so for that gas

$$T = \frac{2(9.8\,\text{m/s}^2)(6.37\times10^6\,\text{m})(2.02\times10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 1.0\times10^4\,\text{K}.$$

(b) The molar mass of oxygen is 32.0×10^{-3} kg/mol, so for that gas

$$T = \frac{2(9.8\,\text{m/s}^2)(6.37\times10^6\,\text{m})(32.0\times10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 1.6\times10^5\,\text{K}.$$

(c) Now, $T = 2g_mr_mM / 3R$, where $r_m = 1.74 \times 10^6$ m is the radius of the Moon and g_m = 0.16g is the acceleration due to gravity at the Moon's surface. For hydrogen, the temperature is

$$T = \frac{2(0.16)(9.8\,\text{m/s}^2)(1.74\times10^6\,\text{m})(2.02\times10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 4.4\times10^2\,\text{K}.$$

(d) For oxygen, the temperature is

$$T = \frac{2(0.16)(9.8\,\text{m/s}^2)(1.74\times10^6\,\text{m})(32.0\times10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 7.0\times10^3\,\text{K}.$$

(e) The temperature high in Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result, the atmosphere is depleted of hydrogen.

(f) On the other hand, very few oxygen atoms escape. So there should be much oxygen high in Earth's upper atmosphere.

40. We divide Eq. 19-31 by Eq. 19-22:

$$\frac{v_{\text{avg2}}}{v_{\text{rms1}}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}$$

which, for $v_{\text{avg2}} = 2v_{\text{rms1}}$, leads to

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8}\left(\frac{v_{\text{avg2}}}{v_{\text{rms1}}}\right)^2 = \frac{3\pi}{2} = 4.7\,.$$

41. (a) The root-mean-square speed is given by $v_{\text{rms}} = \sqrt{3RT/M}$. See Eq. 19-34. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31\,\text{J/mol}\cdot\text{K})(4000\,\text{K})}{2.02\times10^{-3}\,\text{kg/mol}}} = 7.0\times10^3\ \text{m/s}.$$

(b) When the surfaces of the spheres that represent an H_2 molecule and an Ar atom are touching, the distance between their centers is the sum of their radii:

$$d = r_1 + r_2 = 0.5 \times 10^{-8}\ \text{cm} + 1.5 \times 10^{-8}\text{cm} = 2.0 \times 10^{-8}\ \text{cm}.$$

(c) The argon atoms are essentially at rest so in time t the hydrogen atom collides with all the argon atoms in a cylinder of radius d, and length vt, where v is its speed. That is, the number of collisions is $\pi d^2 vtN/V$, where N/V is the concentration of argon atoms. The number of collisions per unit time is

$$\frac{\pi d^2 vN}{V} = \pi\left(2.0\times10^{-10}\ \text{m}\right)^2\left(7.0\times10^3\ \text{m/s}\right)\left(4.0\times10^{25}\ \text{m}^{-3}\right) = 3.5\times10^{10}\,\text{collisions/s}.$$

42. The internal energy is

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(1.0\,\text{mol})(8.31\ \text{J/mol}\cdot\text{K})(273\,\text{K}) = 3.4\times10^3\ \text{J}.$$

43. (a) From Table 19-3, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. Thus, Eq. 19-46 yields

$$Q = nC_p\Delta T = (3.00)\left(\frac{7}{2}(8.31)\right)(40.0) = 3.49\times10^3\ \text{J}.$$

(b) Equation 19-45 leads to

$$\Delta E_{\text{int}} = nC_V\Delta T = (3.00)\left(\frac{5}{2}(8.31)\right)(40.0) = 2.49\times10^3 \text{ J}.$$

(c) From either $W = Q - \Delta E_{\text{int}}$ or $W = p\Delta T = nR\Delta T$, we find $W = 997$ J.

(d) Equation 19-24 is written in more convenient form (for this problem) in Eq. 19-38. Thus, the increase in kinetic energy is

$$\Delta K_{\text{trans}} = \Delta\left(NK_{\text{avg}}\right) = n\left(\frac{3}{2}R\right)\Delta T \approx 1.49\times10^3 \text{ J}.$$

Since $\Delta E_{\text{int}} = \Delta K_{\text{trans}} + \Delta K_{\text{rot}}$, the increase in rotational kinetic energy is

$$\Delta K_{\text{rot}} = \Delta E_{\text{int}} - \Delta K_{\text{trans}} = 2.49\times10^3 \text{ J} - 1.49\times10^3 \text{ J} = 1.00\times10^3 \text{ J}.$$

Note that had there been no rotation, all the energy would have gone into the translational kinetic energy.

44. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a *straight line* on the pV diagram, the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Further, Eq. 19-44 with Eq. 19-51 gives

$$E_{\text{int}} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in joules, pressure should be in pascals (N/m^2) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$\begin{aligned} E_{\text{int}\,c} - E_{\text{int}\,a} &= \frac{5}{2}\left(p_cV_c - p_aV_a\right) = \frac{5}{2}\left(\left(2.0\times10^3 \text{ Pa}\right)\left(4.0\,\text{m}^3\right) - \left(5.0\times10^3 \text{ Pa}\right)\left(2.0\,\text{m}^3\right)\right) \\ &= -5.0\times10^3 \text{ J}. \end{aligned}$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a) = \left(3.5\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right)$$

which yields $W_{\text{diag}} = 7.0\times10^3$ J. Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = (-5.0\times10^3 + 7.0\times10^3)\ \text{J} = 2.0\times10^3\ \text{J}.$$

(c) The fact that ΔE_{int} only depends on the initial and final states, and not on the details of the "path" between them, means we can write $\Delta E_{\text{int}} = E_{\text{int}\,c} - E_{\text{int}\,a} = -5.0\times10^3$ J for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = \left(5.0\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right) + 0 = 1.0\times10^4\ \text{J}.$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = (-5.0\times10^3 + 1.0\times10^4)\ \text{J} = 5.0\times10^3\ \text{J}.$$

45. Argon is a monatomic gas, so $f = 3$ in Eq. 19-51, which provides

$$C_V = \frac{3}{2}R = \frac{3}{2}\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(\frac{1\ \text{cal}}{4.186\ \text{J}}\right) = 2.98\,\frac{\text{cal}}{\text{mol}\cdot\text{C}^\circ}$$

where we have converted joules to calories, and taken advantage of the fact that a Celsius degree is equivalent to a unit change on the Kelvin scale. Since (for a given substance) M is effectively a conversion factor between grams and moles, we see that c_V (see units specified in the problem statement) is related to C_V by $C_V = c_V\, M$ where $M = mN_{\text{A}}$, and m is the mass of a single atom (see Eq. 19-4).

(a) From the above discussion, we obtain

$$m = \frac{M}{N_{\text{A}}} = \frac{C_V / c_V}{N_{\text{A}}} = \frac{2.98/0.075}{6.02\times10^{23}} = 6.6\times10^{-23}\ \text{g} = 6.6\times10^{-26}\ \text{kg}.$$

(b) The molar mass is found to be

$$M = C_V/c_V = 2.98/0.075 = 39.7\ \text{g/mol}$$

which should be rounded to 40 g/mol since the given value of c_V is specified to only two significant figures.

46. (a) Since the process is a constant-pressure expansion,

$$W = p\Delta V = nR\Delta T = (2.02\,\text{mol})(8.31\ \text{J/mol}\cdot\text{K})(15\,\text{K}) = 249\,\text{J}.$$

(b) Now, $C_p = \frac{5}{2}R$ in this case, so $Q = nC_p\Delta T =$ +623 J by Eq. 19-46.

(c) The change in the internal energy is $\Delta E_{\text{int}} = Q - W =$ +374 J.

(d) The change in the average kinetic energy per atom is

$$\Delta K_{\text{avg}} = \Delta E_{\text{int}}/N = +3.11 \times 10^{-22}\ \text{J}.$$

47. (a) The work is zero in this process since volume is kept fixed.

(b) Since $C_V = \frac{3}{2}R$ for an ideal monatomic gas, then Eq. 19-39 gives Q = +374 J.

(c) $\Delta E_{\text{int}} = Q - W$ = +374 J.

(d) Two moles are equivalent to $N = 12 \times 10^{23}$ particles. Dividing the result of part (c) by N gives the average translational kinetic energy change per atom: 3.11×10^{-22} J.

48. (a) According to the first law of thermodynamics $Q = \Delta E_{\text{int}} + W$. When the pressure is a constant $W = p\,\Delta V$. So

$$\Delta E_{\text{int}} = Q - p\Delta V = 20.9\ \text{J} - (1.01\times10^5\ \text{Pa})(100\ \text{cm}^3 - 50\ \text{cm}^3)\left(\frac{1\times10^{-6}\ \text{m}^3}{1\ \text{cm}^3}\right) = 15.9\ \text{J}.$$

(b) The molar specific heat at constant pressure is

$$C_p = \frac{Q}{n\Delta T} = \frac{Q}{n(p\Delta V/nR)} = \frac{R}{p}\frac{Q}{\Delta V} = \frac{(8.31\ \text{J/mol}\cdot\text{K})(20.9\,\text{J})}{(1.01\times10^5\ \text{Pa})(50\times10^{-6}\,\text{m}^3)} = 34.4\,\text{J/mol}\cdot\text{K}.$$

(c) Using Eq. 19-49, $C_V = C_p - R$ = 26.1 J/mol·K.

49. **THINK** The molar specific heat at constant volume for a gas is given by Eq. 19-41: $C_V = \Delta E_{\text{int}}/n\Delta T$. Our system consists of three non-interacting gases.

EXPRESS When the temperature changes by ΔT the internal energy of the first gas changes by $n_1\,C_{V1}\,\Delta T$, the internal energy of the second gas changes by $n_2C_{V2}\,\Delta T$, and the internal energy of the third gas changes by $n_3C_{V3}\,\Delta T$. The change in the internal energy of the composite gas is

$$\Delta E_{\text{int}} = (n_1\, C_{V1} + n_2\, C_{V2} + n_3\, C_{V3})\, \Delta T.$$

This must be $(n_1 + n_2 + n_3)\, C_V\, \Delta T$, where C_V is the molar specific heat of the mixture. Thus,

$$C_V = \frac{n_1 C_{V1} + n_2 C_{V2} + n_3 C_{V3}}{n_1 + n_2 + n_3}.$$

ANALYZE With n_1=2.40 mol, C_{V1}=12.0 J/mol·K for gas 1, n_2=1.50 mol, C_{V2}=12.8 J/mol·K for gas 2, and n_3=3.20 mol, C_{V3}=20.0 J/mol·K for gas 3, we obtain

$$C_V = \frac{(2.40\text{ mol})(12.0\text{ J/mol}\cdot\text{K}) + (1.50\text{ mol})(12.8\text{ J/mol}\cdot\text{K}) + (3.20\text{ mol})(20.0\text{ J/mol}\cdot\text{K})}{2.40\text{ mol} + 1.50\text{ mol} + 3.20\text{ mol}}$$
$$= 15.8\text{ J/mol}\cdot\text{K}$$

for the mixture.

LEARN The molar specific heat of the mixture C_V is the sum of each individual C_{Vi} weighted by the molar fraction.

50. Referring to Table 19-3, Eq. 19-45 and Eq. 19-46, we have

$$\Delta E_{\text{int}} = nC_V \Delta T = \frac{5}{2} nR\Delta T, \quad Q = nC_p \Delta T = \frac{7}{2} nR\Delta T.$$

Dividing the equations, we obtain

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{5}{7}.$$

Thus, the given value Q = 70 J leads to $\Delta E_{\text{int}} = 50$ J.

51. The fact that they rotate but do not oscillate means that the value of f given in Table 19-3 is relevant. Thus, Eq. 19-46 leads to

$$Q = nC_p \Delta T = n\left(\frac{7}{2} R\right)\left(T_f - T_i\right) = nRT_i\left(\frac{7}{2}\right)\left(\frac{T_f}{T_i} - 1\right)$$

where T_i = 273 K and n = 1.0 mol. The ratio of absolute temperatures is found from the gas law in ratio form. With $p_f = p_i$ we have

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = 2.$$

Therefore, the energy added as heat is

$$Q = (1.0\,\text{mol})(8.31\ \text{J/mol}\cdot\text{K})(273\,\text{K})\left(\frac{7}{2}\right)(2-1) \approx 8.0\times10^3\ \text{J}.$$

52. (a) Using M = 32.0 g/mol from Table 19-1 and Eq. 19-3, we obtain

$$n = \frac{M_{\text{sam}}}{M} = \frac{12.0\ \text{g}}{32.0\ \text{g/mol}} = 0.375\ \text{mol}.$$

(b) This is a constant pressure process with a diatomic gas, so we use Eq. 19-46 and Table 19-3. We note that a change of Kelvin temperature is numerically the same as a change of Celsius degrees.

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)\Delta T = (0.375\ \text{mol})\left(\frac{7}{2}\right)(8.31\ \text{J/mol}\cdot\text{K})(100\,\text{K}) = 1.09\times10^3\ \text{J}.$$

(c) We could compute a value of ΔE_{int} from Eq. 19-45 and divide by the result from part (b), or perform this manipulation algebraically to show the generality of this answer (that is, many factors will be seen to cancel). We illustrate the latter approach:

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{n\left(\frac{5}{2}R\right)\ \Delta T}{n\left(\frac{7}{2}R\right)\ \Delta T} = \frac{5}{7} \approx 0.714.$$

53. **THINK** The molecules are diatomic, with translational and rotational degrees of freedom. The temperature change is under constant pressure.

EXPRESS Since the process is at constant pressure, energy transferred as heat to the gas is given by $Q = nC_p\ \Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. Similarly, the change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\ \Delta T$, where C_V is the specific heat at constant volume. For a diatomic ideal gas, $C_p = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$ (see Table 19-3).

ANALYZE (a) The heat transferred is

$$Q = nC_p\Delta T = n\left(\frac{7R}{2}\right)\Delta T = \frac{7}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 6.98\times10^3\ \text{J}.$$

(b) From the above, we find the change in the internal energy to be

$$\Delta E_{\text{int}} = nC_V\Delta T = n\left(\frac{5R}{2}\right)\Delta T = \frac{5}{2}(4.00\,\text{mol})(8.31\,\text{J/mol.K})(60.0\,\text{K}) = 4.99\times10^3\ \text{J}.$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = 6.98\times10^3\text{ J} - 4.99\times10^3\text{ J} = 1.99\times10^3\text{ J}.$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2}nR\Delta T = \frac{3}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 2.99\times10^3\text{ J}.$$

LEARN The diatomic gas has three translational and two rotational degrees of freedom (making $f = 3+2 = 5$). By equipartition theorem, each degree of freedom accounts for an energy of $RT/2$ per mole. Thus, $C_V = (f/2)R = 5R/2$ and $C_p = C_V + R = 7R/2$.

54. The fact that they rotate but do not oscillate means that the value of f given in Table 19-3 is relevant. In Section 19-11, it is noted that $\gamma = C_p/C_V$ so that we find $\gamma = 7/5$ in this case. In the state described in the problem, the volume is

$$V = \frac{nRT}{p} = \frac{(2.0\,\text{mol})(8.31\text{ J/mol}\cdot\text{K})(300\,\text{K})}{1.01\times10^5\text{ N/m}^2} = 0.049\text{ m}^3.$$

Consequently,

$$pV^\gamma = (1.01\times10^5\text{ N/m}^2)(0.049\,\text{m}^3)^{1.4} = 1.5\times10^3\text{ N}\cdot\text{m}^{2.2}.$$

55. (a) Let p_i, V_i, and T_i represent the pressure, volume, and temperature of the initial state of the gas. Let p_f, V_f, and T_f represent the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_iV_i^\gamma = p_fV_f^\gamma$, so

$$p_f = \left(\frac{V_i}{V_f}\right)^\gamma p_i = \left(\frac{4.3\text{ L}}{0.76\text{ L}}\right)^{1.4}(1.2\,\text{atm}) = 13.6\,\text{atm} \approx 14\text{ atm}.$$

We note that since V_i and V_f have the same units, their units cancel and p_f has the same units as p_i.

(b) The gas obeys the ideal gas law $pV = nRT$, so $p_iV_i/p_fV_f = T_i/T_f$ and

$$T_f = \frac{p_fV_f}{p_iV_i}T_i = \left[\frac{(13.6\,\text{atm})(0.76\,\text{L})}{(1.2\,\text{atm})(4.3\,\text{L})}\right](310\,\text{K}) = 6.2\times10^2\text{ K}.$$

56. (a) We use Eq. 19-54 with $V_f/V_i = \frac{1}{2}$ for the gas (assumed to obey the ideal gas law).

$$p_i V_i^\gamma = p_f V_f^\gamma \Rightarrow \frac{p_f}{p_i} = \left(\frac{V_i}{V_f}\right)^\gamma = (2.00)^{1.3}$$

which yields $p_f = (2.46)(1.0 \text{ atm}) = 2.46$ atm.

(b) Similarly, Eq. 19-56 leads to

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = (273\,\text{K})(1.23) = 336\,\text{K}.$$

(c) We use the gas law in ratio form and note that when $p_1 = p_2$ then the ratio of volumes is equal to the ratio of (absolute) temperatures. Consequently, with the subscript 1 referring to the situation (of small volume, high pressure, and high temperature) the system is in at the end of part (a), we obtain

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{273\,\text{K}}{336\,\text{K}} = 0.813.$$

The volume V_1 is half the original volume of one liter, so

$$V_2 = 0.813(0.500\,\text{L}) = 0.406\,\text{L}.$$

57. (a) Equation 19-54, $p_i V_i^\gamma = p_f V_f^{\,\gamma}$, leads to

$$p_f = p_i \left(\frac{V_i}{V_f}\right)^\gamma \Rightarrow \quad 4.00\text{ atm} = (1.00\,\text{atm})\left(\frac{200\,\text{L}}{74.3\,\text{L}}\right)^\gamma$$

which can be solved to yield

$$\gamma = \frac{\ln(p_f/p_i)}{\ln(V_i/V_f)} = \frac{\ln(4.00\,\text{atm}/1.00\,\text{atm})}{\ln(200\,\text{L}/74.3\,\text{L})} = 1.4 = \frac{7}{5}.$$

This implies that the gas is diatomic (see Table 19-3).

(b) One can now use either Eq. 19-56 or use the ideal gas law itself. Here we illustrate the latter approach:

$$\frac{P_f V_f}{P_i V_i} = \frac{nRT_f}{nRT_i} \quad \Rightarrow \quad T_f = 446\text{ K}.$$

(c) Again using the ideal gas law: $n = P_i V_i /RT_i$ = 8.10 moles. The same result would, of course, follow from $n = P_f V_f /RT_f$.

58. Let p_i, V_i, and T_i represent the pressure, volume, and temperature of the initial state of the gas, and let p_f, V_f, and T_f be the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_iV_i^\gamma = p_fV_f^\gamma$. Combining with the ideal gas law, $pV = NkT$, we obtain

$$p_iV_i^\gamma = p_i(T_i/p_i)^\gamma = p_i^{1-\gamma}T_i^\gamma = \text{constant} \quad \Rightarrow \quad p_i^{1-\gamma}T_i^\gamma = p_f^{1-\gamma}T_f^\gamma$$

With $\gamma = 4/3$, which gives $(1-\gamma)/\gamma = -1/4$, the temperature at the end of the adiabatic expansion is

$$T_f = \left(\frac{p_i}{p_f}\right)^{\frac{1-\gamma}{\gamma}} T_i = \left(\frac{5.00\text{ atm}}{1.00\text{ atm}}\right)^{-1/4} (278\text{ K}) = 186\text{ K} = -87°\text{C}.$$

59. Since ΔE_{int} does not depend on the type of process,

$$\left(\Delta E_{\text{int}}\right)_{\text{path 2}} = \left(\Delta E_{\text{int}}\right)_{\text{path 1}}.$$

Also, since (for an ideal gas) it only depends on the temperature variable (so $\Delta E_{\text{int}} = 0$ for isotherms), then

$$\left(\Delta E_{\text{int}}\right)_{\text{path1}} = \sum\left(\Delta E_{\text{int}}\right)_{\text{adiabat}}.$$

Finally, since $Q = 0$ for adiabatic processes, then (for path 1)

$$\left(\Delta E_{\text{int}}\right)_{\text{adiabatic expansion}} = -W = -40\text{ J}$$
$$\left(\Delta E_{\text{int}}\right)_{\text{adiabatic compression}} = -W = -(-25)\text{ J} = 25\text{ J}.$$

Therefore, $\left(\Delta E_{\text{int}}\right)_{\text{path 2}} = -40\text{ J} + 25\text{ J} = -15\text{ J}$.

60. Let p_1, V_1, and T_1 represent the pressure, volume, and temperature of the air at $y_1 = 4267$ m. Similarly, let p, V, and T be the pressure, volume, and temperature of the air at $y = 1567$ m. Since the process is adiabatic, $p_1V_1^\gamma = pV^\gamma$. Combining with the ideal gas law, $pV = NkT$, we obtain

$$pV^\gamma = p(T/p)^\gamma = p^{1-\gamma}T^\gamma = \text{constant} \quad \Rightarrow \quad p^{1-\gamma}T^\gamma = p_1^{1-\gamma}T_1^\gamma.$$

With $p = p_0e^{-ay}$ and $\gamma = 4/3$ (which gives $(1-\gamma)/\gamma = -1/4$), the temperature at the end of the descent is

$$T = \left(\frac{p_1}{p}\right)^{\frac{1-\gamma}{\gamma}} T_1 = \left(\frac{p_0 e^{-ay_1}}{p_0 e^{-ay}}\right)^{\frac{1-\gamma}{\gamma}} T_1 = e^{-a(y-y_1)/4} T_1 = e^{-(1.16\times10^{-4}/\text{m})(1567\text{ m}-4267\text{ m})/4}(268\text{ K})$$
$$= (1.08)(268\text{ K}) = 290\text{ K} = 17°\text{C}.$$

61. The aim of this problem is to emphasize what it means for the internal energy to be a state function. Since path 1 and path 2 start and stop at the same places, then the internal energy change along path 1 is equal to that along path 2. Now, during isothermal processes (involving an ideal gas) the internal energy change is zero, so the only step in path 1 that we need to examine is step 2. Equation 19-28 then immediately yields –20 J as the answer for the internal energy change.

62. Using Eq. 19-53 in Eq. 18-25 gives

$$W = p_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^\gamma \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma}.$$

Using Eq. 19-54 we can write this as

$$W = p_i V_i \frac{1-(p_f/p_i)^{1-1/\gamma}}{1-\gamma}$$

In this problem, $\gamma = 7/5$ (see Table 19-3) and $P_f/P_i = 2$. Converting the initial pressure to pascals we find $P_i V_i = 24240$ J. Plugging in, then, we obtain $W = -1.33 \times 10^4$ J.

63. In the following, $C_V = \frac{3}{2}R$ is the molar specific heat at constant volume, $C_p = \frac{5}{2}R$ is the molar specific heat at constant pressure, ΔT is the temperature change, and n is the number of moles.

The process $1 \to 2$ takes place at constant volume.

(a) The heat added is

$$Q = nC_V \Delta T = \frac{3}{2} nR\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = 3.74\times10^3\text{ J}.$$

(b) Since the process takes place at constant volume, the work W done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74\times10^3\text{ J}.$$

(c) The work W done by the gas is zero.

The process $2 \to 3$ is adiabatic.

(d) The heat added is zero.

(e) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K} - 600\,\text{K}) = -1.81\times10^3\ \text{J}.$$

(f) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81\times10^3\,\text{J}.$$

The process $3 \to 1$ takes place at constant pressure.

(g) The heat added is

$$Q = nC_p\Delta T = \frac{5}{2}nR\Delta T = \frac{5}{2}(1.00\ \text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -3.22\times10^3\ \text{J}.$$

(h) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -1.93\times10^3\ \text{J}.$$

(i) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22\times10^3\ \text{J} + 1.93\times10^3\ \text{J} = -1.29\times10^3\ \text{J}.$$

(j) For the entire process the heat added is

$$Q = 3.74\times10^3\ \text{J} + 0 - 3.22\times10^3\ \text{J} = 520\ \text{J}.$$

(k) The change in the internal energy is

$$\Delta E_{\text{int}} = 3.74\times10^3\ \text{J} - 1.81\times10^3\ \text{J} - 1.93\times10^3\ \text{J} = 0.$$

(l) The work done by the gas is

$$W = 0 + 1.81\times10^3\ \text{J} - 1.29\times10^3\ \text{J} = 520\ \text{J}.$$

(m) We first find the initial volume. Use the ideal gas law $p_1V_1 = nRT_1$ to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{(1.013\times10^5\,\text{Pa})} = 2.46\times10^{-2}\,\text{m}^3.$$

(n) Since 1 → 2 is a constant volume process, $V_2 = V_1 = 2.46 \times 10^{-2}$ m^3. The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K})}{2.46\times10^{-2}\,\text{m}^3} = 2.02\times10^5\,\text{Pa}\,.$$

This is approximately equal to 2.00 atm.

(o) 3 → 1 is a constant pressure process. The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K})}{1.013\times10^5\,\text{Pa}} = 3.73\times10^{-2}\,\text{m}^3.$$

(p) The pressure for state 3 is the same as the pressure for state 1: $p_3 = p_1 = 1.013 \times 10^5$ Pa (1.00 atm)

64. We write T = 273 K and use Eq. 19-14:

$$W = (1.00\,\text{mol})\,(8.31\ \ \text{J/mol}\cdot\text{K})\,(273\,\text{K})\ \ln\left(\frac{16.8}{22.4}\right)$$

which yields W = –653 J. Recalling the sign conventions for work stated in Chapter 18, this means an external agent does 653 J of work *on* the ideal gas during this process.

65. (a) We use $p_iV_i^\gamma = p_fV_f^\gamma$ to compute γ:

$$\gamma = \frac{\ln(p_i/p_f)}{\ln(V_f/V_i)} = \frac{\ln(1.0\,\text{atm}/1.0\times10^5\,\text{atm})}{\ln(1.0\times10^3\,\text{L}/1.0\times10^6\,\text{L})} = \frac{5}{3}.$$

Therefore the gas is monatomic.

(b) Using the gas law in ratio form, the final temperature is

$$T_f = T_i\frac{p_fV_f}{p_iV_i} = (273\,\text{K})\frac{(1.0\times10^5\,\text{atm})(1.0\times10^3\,\text{L})}{(1.0\,\text{atm})(1.0\times10^6\,\text{L})} = 2.7\times10^4\,\text{K}.$$

(c) The number of moles of gas present is

$$n = \frac{p_i V_i}{RT_i} = \frac{(1.01\times10^5\ \text{Pa})(1.0\times10^3\ \text{cm}^3)}{(8.31\ \text{J/mol}\cdot\text{K})(273\,\text{K})} = 4.5\times10^4\ \text{mol}.$$

(d) The total translational energy per mole before the compression is

$$K_i = \frac{3}{2}RT_i = \frac{3}{2}(8.31\ \text{J/mol}\cdot\text{K})(273\,\text{K}) = 3.4\times10^3\ \text{J}.$$

(e) After the compression,

$$K_f = \frac{3}{2}RT_f = \frac{3}{2}(8.31\ \text{J/mol}\cdot\text{K})(2.7\times10^4\ \text{K}) = 3.4\times10^5\ \text{J}.$$

(f) Since $v_{\text{rms}}^2 \propto T$, we have

$$\frac{v_{\text{rms},i}^2}{v_{\text{rms},f}^2} = \frac{T_i}{T_f} = \frac{273\,\text{K}}{2.7\times10^4\ \text{K}} = 0.010.$$

66. Equation 19-25 gives the mean free path:

$$\lambda = \frac{1}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, (N/V)} = \frac{n\,R\,T}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, P\,N}$$

where we have used the ideal gas law in that last step. Thus, the change in the mean free path is

$$\Delta\lambda = \frac{n\,R\,\Delta T}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, P\,N} = \frac{R\,Q}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, P\,N\,C_p}$$

where we have used Eq. 19-46. The constant pressure molar heat capacity is (7/2)R in this situation, so (with $N = 9 \times 10^{23}$ and $d = 250 \times 10^{-12}$ m) we find

$$\Delta\lambda = 1.52 \times 10^{-9}\ \text{m} = 1.52\ \text{nm}.$$

67. (a) The volume has increased by a factor of 3, so the pressure must decrease accordingly (since the temperature does not change in this process). Thus, the final pressure is one-third of the original 6.00 atm. The answer is 2.00 atm.

(b) We note that Eq. 19-14 can be written as $P_i V_i \ln(V_f/V_i)$. Converting "atm" to "Pa" (a pascal is equivalent to a N/m^2) we obtain $W = 333$ J.

(c) The gas is monatomic so $\gamma = 5/3$. Equation 19-54 then yields $P_f = 0.961$ atm.

(d) Using Eq. 19-53 in Eq. 18-25 gives

$$W = p_i V_i^{\gamma} \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^{\gamma} \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma} = \frac{p_f V_f - p_i V_i}{1-\gamma}$$

where in the last step Eq. 19-54 has been used. Converting “atm” to “Pa,” we obtain $W = 236$ J.

68. Using the ideal gas law, one mole occupies a volume equal to

$$V = \frac{nRT}{p} = \frac{(1)(8.31)(50.0)}{1.00\times10^{-8}} = 4.16\times10^{10}\ \text{m}^3.$$

Therefore, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_\text{A}}{V} = \frac{(1)\left(6.02\times10^{23}\right)}{4.16\times10^{10}} = 1.45\times10^{13}\ \frac{\text{molecules}}{\text{m}^3}.$$

Using $d = 20.0 \times 10^{-9}$ m, Eq. 19-25 yields

$$\lambda = \frac{1}{\sqrt{2}\pi d^2\left(\frac{N}{V}\right)} = 38.8\ \text{m}.$$

69. **THINK** The net upward force is the difference between the buoyant force and the weight of the balloon with air inside.

EXPRESS Let ρ_c be the density of the cool air surrounding the balloon and ρ_h be the density of the hot air inside the balloon. The magnitude of the buoyant force on the balloon is $F_b = \rho_c gV$, where V is the volume of the envelope. The force of gravity is $Fg = W + \rho_h gV$, where W is the combined weight of the basket and the envelope. Thus, the net upward force is

$$F_\text{net} = F_b - F_g = \rho_c gV - W - \rho_h gV\ .$$

ANALYZE With F_net = 2.67 × 10^3 N, W = 2.45 × 10^3 N, V = 2.18 × 10^3 m^3, and $\rho_c g = 11.9\ \text{N/m}^3$, we obtain

$$\rho_h g = \frac{\rho_c gV - W - F_\text{net}}{V} = \frac{(11.9\ \text{N/m}^3)(2.18\times10^3\ \text{m}^3) - 2.45\times10^3\ \text{N} - 2.67\times10^3\ \text{N}}{2.18\times10^3\ \text{m}^3} = 9.55\ \text{N/m}^3$$

The ideal gas law gives $p/RT = n/V$. Multiplying both sides by the "molar weight" Mg then leads to

$$\frac{pMg}{RT} = \frac{nMg}{V} = \rho_h g .$$

With $p = 1.01\times10^5$ Pa and M = 0.028 kg/m^3, we find the temperature to be

$$T = \frac{pMg}{R\rho_h g} = \frac{(1.01\times10^5\ \text{Pa})(0.028\ \text{kg/mol})(9.8\ \text{m/s}^2)}{(8.31\ \text{J/mol}\cdot\text{K})(9.55\ \text{N/m}^3)} = 349\ \text{K} .$$

LEARN As can be seen from the results above, increasing the temperature of the gas inside the balloon increases the value of F_{net}, i.e., the lifting capacity.

70. We label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with V_2 = 4 m^3; then continues on to state 3 isothermally, with V_3 = 10 m^3; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process $1\to2$ $p_1V_1^\gamma = p_2V_2^\gamma$, for the isothermal process 2 $\to$ 3 $p_2V_2 = p_3V_3$, and finally for the adiabatic process $3\to4$ $p_3V_3^\gamma = p_4V_4^\gamma$. These equations yield

$$p_4 = p_3\left(\frac{V_3}{V_4}\right)^\gamma = p_2\left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma = p_1\left(\frac{V_1}{V_2}\right)^\gamma\left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma .$$

We substitute this expression for p_4 into the equation $p_1V_1 = p_4V_4$ (since $T_1 = T_4$) to obtain $V_1V_3 = V_2V_4$. Solving for V_4 we obtain

$$V_4 = \frac{V_1V_3}{V_2} = \frac{\left(2.0\,\text{m}^3\right)\left(10\,\text{m}^3\right)}{4.0\,\text{m}^3} = 5.0\,\text{m}^3.$$

71. **THINK** An adiabatic process is a process in which the energy transferred as heat is zero.

EXPRESS The change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\,\Delta T$, where C_V is the specific heat at constant volume, n is the number of moles in the gas, and ΔT is the change in temperature. According to the first law of thermodynamics, the work done by the gas is $W = Q - \Delta E_{\text{int}}$. For an adiabatic process, $Q = 0$, and $W = -\Delta E_{\text{int}}$.

ANALYZE (a) The work done by the gas is

$$W = -\Delta E_{\text{int}} = -nC_V\Delta T = -\frac{3}{2}nR\Delta T = -\frac{3}{2}(2.0\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(15.0\ \text{K}) = -374\ \text{J} .$$

(b) $Q = 0$ since the process is adiabatic.

(c) The change in internal energy is $\Delta E_{\text{int}} = \frac{3}{2} nR\Delta T = 374\text{ J}$.

(d) The number of atoms in the gas is $N = nN_A$, where N_A is the Avogadro's number. Thus, the change in the average kinetic energy per atom is

$$\Delta K_1 = \frac{\Delta E_{\text{int}}}{N} = \frac{\Delta E_{\text{int}}}{nN_A} = \frac{374\text{ J}}{(2.00)(6.02\times10^{23}/\text{mol})} = 3.11\times10^{-22}\text{ J}.$$

LEARN The work done *on* the system is the negative of the work done *by* the system: $W_{on} = -W = \Delta E_{\text{int}} = +374\text{ J}$. By work-kinetic energy theorem: $\Delta K = \Delta W_{\text{on}} = \Delta E_{\text{int}}$.

72. We solve

$$\sqrt{\frac{3RT}{M_{\text{helium}}}} = \sqrt{\frac{3R(293\text{ K})}{M_{\text{hydrogen}}}}$$

for T. With the molar masses found in Table 19-1, we obtain

$$T = (293\text{ K})\left(\frac{4.0}{2.02}\right) = 580\text{ K}$$

which is equivalent to 307°C.

73. **THINK** The collision frequency is related to the mean free path and average speed of the molecules.

EXPRESS According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where d is the diameter of a molecule and N is the number of molecules in volume V. Using ideal gas law, the number density can be written as $N/V = p/kT$, where p is the pressure, T is the temperature on the Kelvin scale and k is the Boltzmann constant. The average time between collisions is $\tau = \lambda / v_{\text{avg}}$, where $v_{\text{avg}} = \sqrt{8RT/\pi M}$, where R is the universal gas constant and M is the molar mass. The collision frequency is simply given by $f = 1/\tau$.

ANALYZE With $p = 2.02 \times 10^3$ Pa and $d = 290 \times 10^{-12}$ m, we find the mean free path to be

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 (p/kT)} = \frac{kT}{\sqrt{2}\pi d^2 p} = \frac{(1.38\times10^{-23}\ \text{J/K})(400\ \text{K})}{\sqrt{2}\pi(290\times10^{-12}\ \text{m})^2(1.01\times10^5\ \text{Pa})} = 7.31\times10^{-8}\ \text{m}.$$

Similarly, with $M = 0.032$ kg/mol, we find the average speed to be

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.31\,\text{J/mol}\cdot\text{K})(400\,\text{K})}{\pi(32\times10^{-3}\,\text{kg/mol})}} = 514\,\text{m/s}.$$

Thus, the collision frequency is $f = \dfrac{v_{\text{avg}}}{\lambda} = \dfrac{514\ \text{m/s}}{7.31\times10^{-8}\ \text{m}} = 7.04\times10^9\ \text{collisions/s}$.

LEARN This is very similar to the Sample Problem 19.04 – "Mean free path, average speed and collision frequency." A general expression for f is

$$f = \frac{\text{speed}}{\text{distance}} = \frac{v_{\text{avg}}}{\lambda} = \frac{pd^2}{k}\sqrt{\frac{16\pi R}{MT}}.$$

74. (a) Since $n/V = p/RT$, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_A}{V} = N_A\left(\frac{p}{RT}\right)(6.02\times10^{23})\frac{1.01\times10^5\ \text{Pa}}{\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(293\,\text{K})} = 2.5\times10^{25}\ \frac{\text{molecules}}{\text{m}^3}.$$

(b) Three-fourths of the 2.5×10^{25} value found in part (a) are nitrogen molecules with $M = 28.0$ g/mol (using Table 19-1), and one-fourth of that value are oxygen molecules with $M = 32.0$ g/mol. Consequently, we generalize the $M_{\text{sam}} = NM/N_A$ expression for these two species of molecules and write

$$\frac{3}{4}(2.5\times10^{25})\frac{28.0}{6.02\times10^{23}} + \frac{1}{4}(2.5\times10^{25})\frac{32.0}{6.02\times10^{23}} = 1.2\times10^3\ \text{g} = 1.2\ \text{kg}.$$

75. We note that $\Delta K = n\left(\frac{3}{2}R\right)\Delta T$ according to the discussion in Sections 19-5 and 19-9. Also, $\Delta E_{\text{int}} = nC_V\Delta T$ can be used for each of these processes (since we are told this is an ideal gas). Finally, we note that Eq. 19-49 leads to $C_p = C_V + R \approx 8.0$ cal/mol·K after we convert joules to calories in the ideal gas constant value (Eq. 19-6): $R \approx 2.0$ cal/mol·K. The first law of thermodynamics $Q = \Delta E_{\text{int}} + W$ applies to each process.

• Constant volume process with $\Delta T = 50$ K and $n = 3.0$ mol.

(a) Since the change in the internal energy is $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal, and the work done by the gas is $W = 0$ for constant volume processes, the first law gives $Q = 900 + 0 = 900$ cal.

(b) As shown in part (a), $W = 0$.

(c) The change in the internal energy is, from part (a), $\Delta E_{int} = (3.0)(6.00)(50) = 900$ cal.

(d) The change in the total translational kinetic energy is

$$\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$$

• Constant pressure process with $\Delta T = 50$ K and $n = 3.0$ mol.

(e) $W = p\Delta V$ for constant pressure processes, so (using the ideal gas law)

$$W = nR\Delta T = (3.0)(2.0)(50) = 300 \text{ cal}.$$

The first law gives $Q = (900 + 300)$ cal $= 1200$ cal.

(f) From (e), we have $W = 300$ cal.

(g) The change in the internal energy is $\Delta E_{int} = (3.0)(6.00)(50) = 900$ cal.

(h) The change in the translational kinetic energy is $\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}$.

• Adiabiatic process with $\Delta T = 50$ K and $n = 3.0$ mol.

(i) $Q = 0$ by definition of "adiabatic."

(j) The first law leads to $W = Q - E_{int} = 0 - 900$ cal $= -900$ cal.

(k) The change in the internal energy is $\Delta E_{int} = (3.0)(6.00)(50) = 900$ cal.

(l) As in part (d) and (h), $\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}$.

76. (a) With work being given by

$$W = p\Delta V = (250)(-0.60) \text{ J} = -150 \text{ J},$$

and the heat transfer given as –210 J, then the change in internal energy is found from the first law of thermodynamics to be [–210 – (–150)] J = –60 J.

(b) Since the pressures (and also the number of moles) don't change in this process, then the volume is simply proportional to the (absolute) temperature. Thus, the final temperature is ¼ of the initial temperature. The answer is 90 K.

77. **THINK** From the distribution function $P(v)$, we can calculate the average and rms speeds of the gas.

EXPRESS The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v)\, dv = 1$. The average speed is defined as $v_{\text{avg}} = \int_0^\infty vP(v)dv$. Similarly, the rms speed is given by $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$, where $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v) dv$.

ANALYZE (a) By normalizing the distribution function:

$$1 = \int_0^{v_o} P(v)\, dv = \int_0^{v_o} Cv^2\, dv = \frac{C}{3} v_0^3$$

we find the constant C to be $C = 3/v_0^3$.

(b) The average speed is

$$v_{\text{avg}} = \int_0^{v_o} vP(v)\, dv = \int_0^{v_o} v\left(\frac{3v^2}{v_o^3}\right) dv = \frac{3}{v_o^3}\int_0^{v_o} v^3 dv = \frac{3}{4} v_o .$$

(c) Similarly, the rms speed is the square root of

$$\int_0^{v_o} v^2 P(v)\, dv = \int_0^{v_o} v^2\left(\frac{3v^2}{v_o^3}\right) dv = \frac{3}{v_o^3}\int_0^{v_o} v^4 dv = \frac{3}{5} v_o^2 .$$

Therefore, $v_{\text{rms}} = \sqrt{3/5}\, v_o \approx 0.775 v_o$.

LEARN The maximum speed of the gas is $v_{\max} = v_0$, as indicated by the distribution function. Using Eq. 19-29, we find the fraction of molecules with speed between v_1 and v_2 to be

$$\text{frac} = \int_{v_1}^{v_2} P(v)\, dv = \int_{v_1}^{v_2} \left(\frac{3v^2}{v_o^3}\right) dv = \frac{3}{v_o^3}\int_{v_1}^{v_2} v^2 dv = \frac{v_2^3 - v_1^3}{v_o^3}.$$

78. (a) In the free expansion from state 0 to state 1 we have $Q = W = 0$, so $\Delta E_{\text{int}} = 0$, which means that the temperature of the ideal gas has to remain unchanged. Thus the final pressure is

$$p_1 = \frac{p_0 V_0}{V_1} = \frac{p_0 V_0}{3.00 V_0} = \frac{1}{3.00} p_0 \;\Rightarrow\; \frac{p_1}{p_0} = \frac{1}{3.00} = 0.333.$$

(b) For the adiabatic process from state 1 to 2 we have $p_1 V_1^\gamma = p_2 V_2^\gamma$, that is,

$$\frac{1}{3.00} p_0 (3.00V_0)^\gamma = (3.00)^{\frac{1}{3}} p_0 V_0^\gamma$$

which gives $\gamma = 4/3$. The gas is therefore polyatomic.

(c) From $T = pV/nR$ we get

$$\frac{\bar{K}_2}{\bar{K}_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1} = (3.00)^{1/3} = 1.44.$$

79. **THINK** The compression is isothermal so $\Delta T = 0$. In addition, since the gas is ideal, we can use the ideal gas law: $pV = nRT$.

EXPRESS The work done by the gas during the isothermal compression process from volume V_i to volume V_f is given by

$$W = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right),$$

where we use the ideal gas law to replace p with nRT/V.

ANALYZE (a) The temperature is T = 10.0°C = 283 K. Then, with n = 3.50 mol, we obtain

$$W = nRT \ln\left(\frac{V_f}{V_0}\right) = (3.50\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(283\text{ K})\ln\left(\frac{3.00\text{ m}^3}{4.00\text{ m}^3}\right) = -2.37\times10^3\text{ J}.$$

(b) The internal energy change ΔE_{int} vanishes (for an ideal gas) when $\Delta T = 0$ so that the First Law of Thermodynamics leads to $Q = W = -2.37$ kJ.

LEARN The work done by the gas is negative since $V_f < V_i$. Also, the negative value in Q implies that the heat transfer is from the sample to its environment.

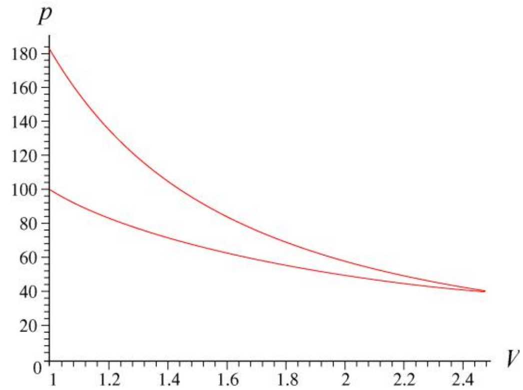
80. The ratio is

$$\frac{mgh}{mv_{\text{rms}}^2/2} = \frac{2gh}{v_{\text{rms}}^2} = \frac{2Mgh}{3RT}$$

where we have used Eq. 19-22 in that last step. With T = 273 K, h = 0.10 m and M = 32 g/mol = 0.032 kg/mol, we find the ratio equals 9.2×10^{-6}.

81. (a) The p-V diagram is shown next. Note that to obtain the graph, we have chosen n = 0.37 moles for concreteness, in which case the horizontal axis (which we note starts not at zero but at 1) is to be interpreted in units of cubic centimeters, and the vertical axis (the absolute pressure) is in kilopascals. However, the constant volume temperature-increase

process described in the third step (see the problem statement) is difficult to see in this graph since it coincides with the pressure axis.



(b) We note that the change in internal energy is zero for an ideal gas isothermal process, so (since the net change in the internal energy must be zero for the entire cycle) the increase in internal energy in step 3 must equal (in magnitude) its decease in step 1. By Eq. 19-28, we see this number must be 125 J.

(c) As implied by Eq. 19-29, this is equivalent to heat being added *to the gas*.

82. (a) The ideal gas law leads to

$$V = \frac{nRT}{p} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})\,(273\,\text{K})}{1.01\times10^5\ \text{Pa}}$$

which yields $V = 0.0225\ \text{m}^3 = 22.5$ L. If we use the standard pressure value given in Appendix D, 1 atm = 1.013×10^5 Pa, then our answer rounds more properly to 22.4 L.

(b) From Eq. 19-2, we have $N = 6.02 \times 10^{23}$ molecules in the volume found in part (a) (which may be expressed as $V = 2.24 \times 10^4\ \text{cm}^3$), so that

$$\frac{N}{V} = \frac{6.02\times10^{23}}{2.24\times10^4\ \text{cm}^3} = 2.69\times10^{19}\ \text{molecules/cm}^3 .$$

83. **THINK** For an isothermal expansion, $\Delta T = 0$. However, if the expansion is adiabatic, then $\Delta Q = 0$.

EXPRESS Using ideal gas law: $pV = nRT$, we have $\dfrac{p_f V_f}{p_i V_i} = \dfrac{T_f}{T_i}$. For isothermal process, $T_f = T_i$, which gives $p_f = \dfrac{p_i V_i}{V_f}$. The work done by the gas is

$$W = \int_{V_i}^{V_f} p\,dV = nRT\int_{V_i}^{V_f} \frac{dV}{V} = nRT\ln\left(\frac{V_f}{V_i}\right).$$

Now, for an adiabatic process, $p_i V_i^\gamma = p_f V_f^\gamma$. The final pressures and temperatures are

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^\gamma, \qquad T_f = \frac{p_f V_f T_i}{p_i V_i}$$

The work done is $W = Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}}$.

ANALYZE (a) For the isothermal process, the final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32\,\text{atm})(1.0\,\text{L})}{4.0\,\text{L}} = 8.0\,\text{atm}.$$

(b) The final temperature of the gas is the same as the initial temperature: $T_f = T_i = 300$ K.

(c) The work done is

$$W = nRT_i\ln\left(\frac{V_f}{V_i}\right) = p_i V_i\ln\left(\frac{V_f}{V_i}\right) = (32\,\text{atm})(1.01\times10^5\,\text{Pa/atm})(1.0\times10^{-3}\,\text{m}^3)\ln\left(\frac{4.0\,\text{L}}{1.0\,\text{L}}\right)$$
$$= 4.4\times10^3\,\text{J}.$$

(d) For the adiabatic process, the final pressure is ($\gamma = 5/3$ for monatomic gas)

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{5/3} = 3.2\,\text{atm}.$$

(e) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 120\,\text{K}.$$

(f) The work done is

$$W = -\Delta E_{\text{int}} = -\frac{3}{2}nR\Delta T = -\frac{3}{2}(p_f V_f - p_i V_i)$$
$$= -\frac{3}{2}\left[(3.2\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right](1.01\times10^5\,\text{Pa/atm})(10^{-3}\,\text{m}^3/\text{L})$$
$$= 2.9\times10^3\,\text{J}.$$

(g) If the gas is diatomic, then $\gamma = 1.4$, and the final pressure is

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^{\gamma} = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{1.4} = 4.6\,\text{atm}.$$

(h) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 170\,\text{K}.$$

(i) The work done is

$$W = Q - \Delta E_{\text{int}} = -\frac{5}{2}nR\Delta T = -\frac{5}{2}\left(p_f V_f - p_i V_i\right)$$
$$= -\frac{5}{2}\left[(4.6\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right]\left(1.01\times10^5\,\text{Pa/atm}\right)\left(10^{-3}\,\text{m}^3/\text{L}\right)$$
$$= 3.4\times10^3\,\text{J}.$$

LEARN Comparing (c) with (f), we see that more work is done by the gas if the expansion is isothermal rather than adiabatic.

84. (a) With $P_1 = (20.0)(1.01 \times 10^5\ \text{Pa})$ and $V_1 = 0.0015\ \text{m}^3$, the ideal gas law gives

$$P_1V_1 = nRT_1 \quad \Rightarrow \quad T_1 = 121.54\ \text{K} \approx 122\ \text{K}.$$

(b) From the information in the problem, we deduce that $T_2 = 3T_1 = 365$ K.

(c) We also deduce that $T_3 = T_1$, which means $\Delta T = 0$ for this process. Since this involves an ideal gas, this implies the change in internal energy is zero here.

85. (a) We use $pV = nRT$. The volume of the tank is

$$V = \frac{nRT}{p} = \frac{\left(\frac{300\text{g}}{17\ \text{g/mol}}\right)(8.31\ \text{J/mol}\cdot\text{K})(350\,\text{K})}{1.35\times10^6\ \text{Pa}} = 3.8\times10^{-2}\ \text{m}^3 = 38\,\text{L}.$$

(b) The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{(8.7\times10^5\ \text{Pa})(3.8\times10^{-2}\ \text{m}^3)}{(8.31\ \text{J/mol}\cdot\text{K})(293\,\text{K})} = 13.5\,\text{mol}.$$

The mass of the gas that leaked out is then

$$\Delta m = 300\ \text{g} - (13.5\ \text{mol})(17\ \text{g/mol}) = 71\ \text{g}.$$

86. To model the "uniform rates" described in the problem statement, we have expressed the volume and the temperature functions as follows:

$$V = V_i + \left(\frac{V_f - V_i}{\tau_f}\right)t, \qquad T = T_i + \left(\frac{T_f - T_i}{\tau_f}\right)t$$

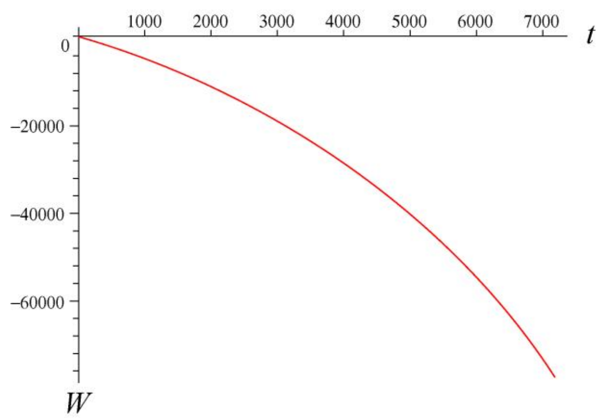
where $V_i = 0.616\ \text{m}^3$, $V_f = 0.308\ \text{m}^3$, $\tau_f = 7200$ s, $T_i = 300$ K, and $T_f = 723$ K.

(a) We can take the derivative of V with respect to t and use that to evaluate the cumulative work done (from $t = 0$ until $t = \tau$):

$$W = \int p\,dV = \int\left(\frac{nRT}{V}\right)\left(\frac{dV}{dt}\right)dt = 12.2\,\tau + 238113\,\ln(14400 - \tau) - 2.28\times 10^6$$

with SI units understood. With $\tau = \tau_f$ our result is $W = -77169$ J ≈ -77.2 kJ, or $|W| \approx$ 77.2 kJ.

The graph of cumulative work is shown below. The graph for work done is purely negative because the gas is being compressed (work is being done *on* the gas).

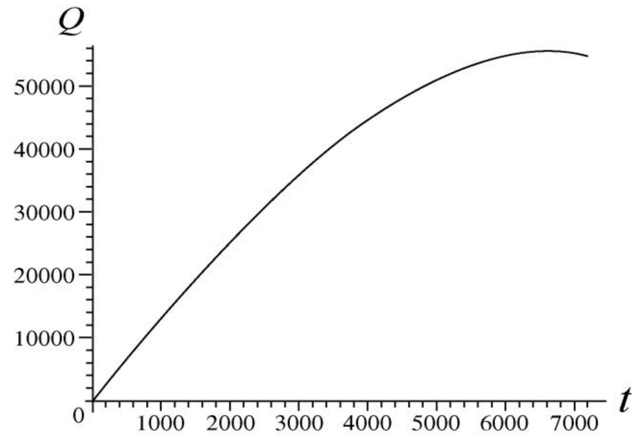


(b) With $C_V = \frac{3}{2}R$ (since it's a monatomic ideal gas) then the (infinitesimal) change in internal energy is $nC_V\,dT = \frac{3}{2}nR\left(\frac{dT}{dt}\right)dt$, which involves taking the derivative of the temperature expression listed above. Integrating this and adding this to the work done gives the cumulative heat absorbed (from $t = 0$ until $t = \tau$):

$$Q = \int \left(\frac{nRT}{V}\right)\left(\frac{dV}{dt}\right) + \frac{3}{2}nR\left(\frac{dT}{dt}\right)dt = 30.5\,\tau + 238113\,\ln(14400 - \tau) - 2.28\times 10^6$$

with SI units understood. With $\tau = \tau_f$ our result is $Q_{\text{total}} = 54649$ J $\approx 5.46\times 10^4$ J.

The graph cumulative heat is shown below. We see that $Q > 0$, since the gas is absorbing heat.



(c) Defining $C = \frac{Q_{\text{total}}}{n(T_f - T_i)}$, we obtain C = 5.17 J/mol·K. We note that this is considerably smaller than the constant-volume molar heat C_V.

We are now asked to consider this to be a two-step process (time dependence is no longer an issue) where the first step is isothermal and the second step occurs at constant volume (the ending values of pressure, volume, and temperature being the same as before).

(d) Equation 19-14 readily yields $W = -43222$ J $\approx -4.32 \times 10^4$ J (or $|W| \approx 4.32 \times 10^4$ J), where it is important to keep in mind that no work is done in a process where the volume is held constant.

(e) In step 1 the heat is equal to the work (since the internal energy does not change during an isothermal ideal gas process), and in step 2 the heat is given by Eq. 19-39. The total heat is therefore 88595 $\approx 8.86 \times 10^4$ J.

(f) Defining a molar heat capacity in the same manner as we did in part (c), we now arrive at C = 8.38 J/ mol·K.

87. For convenience, the "int" subscript for the internal energy will be omitted in this solution. Recalling Eq. 19-28, we note that $\sum_{\text{cycle}} E = 0$, which gives

$$\Delta E_{A\to B} + \Delta E_{B\to C} + \Delta E_{C\to D} + \Delta E_{D\to E} + \Delta E_{E\to A} = 0.$$

Since a gas is involved (assumed to be ideal), then the internal energy does not change when the temperature does not change, so

$$\Delta E_{A\to B} = \Delta E_{D\to E} = 0.$$

Now, with $\Delta E_{E\to A}$ = 8.0 J given in the problem statement, we have

$$\Delta E_{B\to C} + \Delta E_{C\to D} + 8.0\ \text{J} = 0.$$

In an adiabatic process, $\Delta E = -W$, which leads to

$$-5.0\ \text{J} + \Delta E_{C\to D} + 8.0\ \text{J} = 0,$$

and we obtain $\Delta E_{C\to D} = -3.0$ J.

88. (a) The work done in a constant-pressure process is $W = p\Delta V$. Therefore,

$$W = \left(25\,\text{N/m}^2\right)(1.8\,\text{m}^3 - 3.0\,\text{m}^3) = -30\,\text{J}.$$

The sign conventions discussed in the textbook for Q indicate that we should write –75 J for the energy that leaves the system in the form of heat. Therefore, the first law of thermodynamics leads to

$$\Delta E_{\text{int}} = Q - W = (-75\ \text{J}) - (-30\ \text{J}) = -45\ \text{J}.$$

(b) Since the pressure is constant (and the number of moles is presumed constant), the ideal gas law in ratio form leads to

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (300\,\text{K})\left(\frac{1.8\,\text{m}^3}{3.0\,\text{m}^3}\right) = 1.8\times 10^2\ \text{K}.$$

It should be noted that this is consistent with the gas being monatomic (that is, if one assumes $C_V = \frac{3}{2}R$ and uses Eq. 19-45, one arrives at this same value for the final temperature).

89. Consider the open end of the pipe. The balance of the pressures inside and outside the pipe requires that $p + \rho_w g(L/2) = p_0 + \rho_w gh$, where p_0 is the atmospheric pressure, and p is the pressure of the air inside the pipe, which satisfies $p(L/2) = p_0 L$, or $p = 2p_0$. We solve for h:

$$h = \frac{p - p_0}{\rho_w g} + \frac{L}{2} = \frac{1.01\times 10^5\ \text{Pa}}{1.00\times 10^3\ \text{kg/m}^3\ \ 9.8\,\text{m/s}^2} + \frac{25.0\,\text{m}}{2} = 22.8\,\text{m}.$$

90. (a) For diatomic gas, $\gamma = 7/5$. Using $pV^\gamma = \text{constant}$, we find the final gas pressure to be

$$p_f = \left(\frac{V_i}{V_f}\right)^\gamma p_i = \left(\frac{50\ \text{cm}^3}{250\ \text{cm}^3}\right)^{7/5}(15\ \text{atm}) = 1.58\ \text{atm}.$$

The work done by the gas during the adiabatic expansion process is

$$W = p_i V_i^{\gamma} \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^{\gamma} \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma} = \frac{p_f V_f - p_i V_i}{1-\gamma}$$

$$= \frac{(1.58 \text{ atm})(1.01\times10^5 \text{ Pa/atm})(250\times10^{-6}\text{m}^3) - (15 \text{ atm})(1.01\times10^5 \text{ Pa/atm})(50\times10^{-6}\text{m}^3)}{1-(7/5)}$$

$$= 89.64 \text{ J}$$

The period for each cycle is $\tau = (60\text{ s})/(4000) = 0.015\text{ s}$. Since the time involved in the expansion is one-half of the total cycle: $\Delta t = \tau/2 = 7.5\times10^{-3}\text{s}$, the average power for the expansion is

$$P = \frac{W}{\Delta t} = \frac{89.64 \text{ J}}{7.5\times10^{-3}\text{s}} = 1.2\times10^4 \text{ W}.$$

(b) Using the conversion factor $1 \text{ hp} = 746 \text{ W}$, the power can also be expressed as 16 hp.

91. (a) For adiabatic process, $pV^{\gamma} = \text{constant}$, or $p = CV^{-\gamma}$. Thus,

$$B = -V\frac{dp}{dV} = -V\frac{d}{dV}\left(CV^{-\gamma}\right) = \gamma CV^{-\gamma} = \gamma p\,.$$

(b) Using $p = nRT/V = (m/M)RT/V$ with $\rho = m/V$, we find the speed of sound in an ideal gas to be

$$v_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma(m/M)RT/V}{m/V}} = \sqrt{\frac{\gamma RT}{M}}.$$

92. With $p = 1.01 \times 10^5$ Pa and $\rho = 1.29$ kg/m^3, we use the result of part (b) of the previous problem to obtain

$$\gamma = \frac{\rho v^2}{p} = \frac{\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)^2}{1.01\times10^5 \text{ Pa}} = 1.40.$$

93. Using $v_s = \sqrt{\gamma RT/M}$, the result obtained in part (b) of problem 91, we find the ratio to be

$$\frac{v_1}{v_2} = \frac{\sqrt{\gamma RT/M_1}}{\sqrt{\gamma RT/M_2}} = \sqrt{\frac{M_2}{M_1}}.$$

94. The speed of sound in the gas is $v_s = \sqrt{\gamma RT/M}$, and the rms speed of the gas is $v_{\text{rms}} = \sqrt{3RT/M}$. Thus, the ratio is

$$\frac{v_s}{v_{\text{rms}}} = \frac{\sqrt{\gamma RT/M}}{\sqrt{3RT/M}} = \sqrt{\frac{\gamma}{3}} = \sqrt{\frac{C_p}{3C_V}} = \sqrt{\frac{C_V + R}{3C_V}} = \sqrt{\frac{5.0R + R}{3(5.0R)}} = \sqrt{\frac{2}{5}} = 0.63.$$

95. The speed of sound is an ideal gas is $v_s = \sqrt{\gamma RT/M}$, which gives

$$\gamma = \frac{Mv_s^2}{RT}.$$

Since the nodes of the standing waves are separated by half a wavelength, we have $\lambda = 2(9.57\text{ cm}) = 19.14\text{ cm} = 0.1914\text{ m},$ and the corresponding speed of sound is

$$v_s = \lambda f = (0.1914\text{ m})(1000\text{ Hz}) = 191.4\text{ m/s}.$$

Thus,

$$\gamma = \frac{Mv_s^2}{RT} = \frac{(0.127\text{ kg/mol})(191.4\text{ m/s})^2}{(8.314\text{ J/mol}\cdot\text{K})(400\text{ K})} = 1.40.$$

96. The speed of sound is an ideal gas is $v_s = \sqrt{\gamma RT/M}$. Differentiating v_s with respect to T, we obtain

$$\frac{dv_s}{dT} = \frac{1}{2}\sqrt{\frac{\gamma R}{M}}T^{-1/2} = \frac{1}{2T}\sqrt{\frac{\gamma RT}{M}} = \frac{v_s}{2T}$$

Near $T = 0\text{ °C} = 273\text{ K}$, the speed of sound is 331 m/s. Thus, with $\Delta T = 1\text{ °C} = 1\text{ K}$, the change in speed is

$$\Delta v_s = \frac{\Delta T}{2T}v_s = \frac{1\text{ K}}{2(273\text{ K})}(331\text{ m/s}) = 0.606\text{ m/s} \approx 0.61\text{ m/s}.$$

97. The average speed and rms speed of an ideal gas are given by $v_{\text{avg}} = \sqrt{8RT/\pi M}$ and $v_{\text{rms}} = \sqrt{3RT/M}$, respectively. Thus,

$$\frac{v_{\text{avg}\,2}}{v_{\text{rms}\,1}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}.$$

If $v_{\text{avg2}} = 2v_{\text{rms1}}$, then

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8}\left(\frac{v_{\text{avg}\,2}}{v_{\text{rms}\,1}}\right)^2 = \frac{3\pi}{2} = 4.71.$$

Chapter 20

1. **THINK** If the expansion of the gas is reversible and isothermal, then there's no change in internal energy. However, if the process is reversible and adiabatic, then there would be no change in entropy.

EXPRESS Since the gas is ideal, its pressure p is given in terms of the number of moles n, the volume V, and the temperature T by $p = nRT/V$. If the expansion is isothermal, the work done by the gas is

$$W = \int_{V_1}^{V_2} p\,dV = nRT\int_{V_1}^{V_2} \frac{dV}{V} = nRT\ln\frac{V_2}{V_1} ,$$

and the corresponding change in entropy is $\Delta S = \int (1/T)\,dQ = Q/T$, where Q is the heat absorbed (see Eq. 20-2).

ANALYZE (a) With $V_2 = 2.00V_1$ and $T = 400$ K, we obtain

$$W = nRT\ln 2.00 = (4.00\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(400\text{ K})\ln 2.00 = 9.22\times10^3\text{ J}.$$

(b) According to the first law of thermodynamics, $\Delta E_{int} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{int} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22\times10^3\text{ J}}{400\text{ K}} = 23.1\text{ J/K}.$$

(c) The change in entropy ΔS is zero for all reversible adiabatic processes.

LEARN The general expression for ΔS for reversible processes is given by Eq. 20-4:

$$\Delta S = S_f - S_i = nR\ln\left(\frac{V_f}{V_i}\right) + nC_V\ln\left(\frac{T_f}{T_i}\right).$$

Note that ΔS does not depend on how the gas changes from its initial state i to the final state f.

2. An isothermal process is one in which $T_i = T_f$, which implies $\ln(T_f/T_i) = 0$. Therefore, Eq. 20-4 leads to

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) \Rightarrow n = \frac{22.0}{(8.31)\ln(3.4/1.3)} = 2.75\text{ mol}.$$

3. An isothermal process is one in which $T_i = T_f$, which implies $\ln(T_f/T_i) = 0$. Therefore, with $V_f/V_i = 2.00$, Eq. 20-4 leads to

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (2.50\text{ mol})(8.31\text{ J/mol}\cdot\text{K})\ln(2.00) = 14.4\text{ J/K}.$$

4. From Eq. 20-2, we obtain $Q = T\Delta S = (405\text{ K})(46.0\text{ J/K}) = 1.86\times10^4\text{ J}$.

5. We use the following relation derived in Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium:"

$$\Delta S = mc\ln(T_f/T_i).$$

(a) The energy absorbed as heat is given by Eq. 19-14. Using Table 19-3, we find

$$Q = cm\Delta T = \left(386\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(2.00\text{ kg})(75\text{ K}) = 5.79\times10^4\text{ J}$$

where we have used the fact that a change in Kelvin temperature is equivalent to a change in Celsius degrees.

(b) With $T_f = 373.15$ K and $T_i = 298.15$ K, we obtain

$$\Delta S = (2.00\text{ kg})\left(386\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{373.15}{298.15}\right) = 173\text{ J/K}.$$

6. (a) This may be considered a reversible process (as well as isothermal), so we use $\Delta S = Q/T$ where $Q = Lm$ with $L = 333$ J/g from Table 19-4. Consequently,

$$\Delta S = \frac{(333\text{ J/g})(12.0\text{ g})}{273\text{ K}} = 14.6\text{ J/K}.$$

(b) The situation is similar to that described in part (a), except with $L = 2256$ J/g, $m = 5.00$ g, and $T = 373$ K. We therefore find $\Delta S = 30.2$ J/K.

7. (a) We refer to the copper block as block 1 and the lead block as block 2. The equilibrium temperature T_f satisfies $m_1c_1(T_f - T_{i,1}) + m_2c_2(T_f - T_{i2}) = 0$, which we solve for T_f:

$$T_f = \frac{m_1c_1T_{i,1} + m_2c_2T_{i,2}}{m_1c_1 + m_2c_2} = \frac{(50.0\text{ g})(386\text{ J/kg}\cdot\text{K})(400\text{ K}) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})(200\text{ K})}{(50.0\text{ g})(386\text{ J/kg}\cdot\text{K}) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})}$$
$$= 320\text{ K}.$$

(b) Since the two-block system in thermally insulated from the environment, the change in internal energy of the system is zero.

(c) The change in entropy is

$$\Delta S = \Delta S_1 + \Delta S_2 = m_1 c_1 \ln\left(\frac{T_f}{T_{i,1}}\right) + m_2 c_2 \ln\left(\frac{T_f}{T_{i,2}}\right)$$

$$= (50.0\text{ g})(386\text{ J/kg}\cdot\text{K})\ln\left(\frac{320\text{ K}}{400\text{ K}}\right) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})\ln\left(\frac{320\text{ K}}{200\text{ K}}\right)$$

$$= +1.72\text{ J/K}.$$

8. We use Eq. 20-1:

$$\Delta S = \int \frac{nC_V\,dT}{T} = nA\int_{5.00}^{10.0} T^2 dT = \frac{nA}{3}\left[(10.0)^3 - (5.00)^3\right] = 0.0368\text{ J/K}.$$

9. The ice warms to 0°C, then melts, and the resulting water warms to the temperature of the lake water, which is 15°C. As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I\,dT$, where m is the mass of the ice and c_I is the specific heat of ice. If T_i (= 263 K) is the initial temperature and T_f (= 273 K) is the final temperature, then the change in its entropy is

$$\Delta S = \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln\frac{T_f}{T_i} = (0.010\text{ kg})(2220\text{ J/kg}\cdot\text{K})\ln\left(\frac{273\text{ K}}{263\text{ K}}\right) = 0.828\text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F, where L_F is the heat of fusion for ice. Thus,

$$\Delta S = Q/T = mL_F/T = (0.010\text{ kg})(333 \times 10^3\text{ J/kg})/(273\text{ K}) = 12.20\text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln\frac{T_f}{T_i},$$

where c_w is the specific heat of water (4190 J/kg · K). Thus,

$$\Delta S = (0.010\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\frac{288\text{ K}}{273\text{ K}}\right) = 2.24\text{ J/K}.$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828\text{ J/K} + 12.20\text{ J/K} + 2.24\text{ J/K} = 15.27\text{ J/K}.$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is $\Delta S = Q/T$, where Q is the energy it receives as heat (the negative of the energy it supplies the ice) and T is its temperature. When the ice warms to 0°C,

$$Q = -mc_I\left(T_f - T_i\right) = -(0.010\text{ kg})(2220\text{ J/kg}\cdot\text{K})(10\text{ K}) = -222\text{ J}.$$

When the ice melts,

$$Q = -mL_F = -(0.010\text{ kg})(333\times10^3\text{ J/kg}) = -3.33\times10^3\text{ J}.$$

When the water from the ice warms,

$$Q = -mc_w(T_f - T_i) = -(0.010\text{ kg})(4190\text{ J/kg}\cdot\text{K})(15\text{ K}) = -629\text{ J}.$$

The total energy leaving the lake water is

$$Q = -222\text{ J} - 3.33\times10^3\text{ J} - 6.29\times10^2\text{ J} = -4.18\times10^3\text{ J}.$$

The change in entropy is

$$\Delta S = -\frac{4.18\times10^3\text{ J}}{288\text{ K}} = -14.51\text{ J/K}.$$

The change in the entropy of the ice–lake system is $\Delta S = (15.27 - 14.51)$ J/K = 0.76 J/K.

10. We follow the method shown in Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium." Since

$$\Delta S = mc\int_{T_i}^{T_f}\frac{dT}{T} = mc\ln(T_f/T_i),$$

then with ΔS = 50 J/K, T_f = 380 K, T_i = 280 K, and m = 0.364 kg, we obtain $c = 4.5\times10^2$ J/kg·K.

11. **THINK** The aluminum sample gives off energy as heat to water. Thermal equilibrium is reached when both the aluminum and the water come to a common final temperature T_f.

EXPRESS The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a(T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w(T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a \left(T_{ai} - T_f\right) = m_w c_w \left(T_f - T_{wi}\right) \Rightarrow T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}.$$

The change in entropy is $\Delta S = \int dQ/T$.

ANALYZE (a) The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$T_f = \frac{(0.200\text{ kg})(900\text{ J/kg}\cdot\text{K})(100°\text{C}) + (0.0500\text{ kg})(4190\text{ J/kg}\cdot\text{K})(20°\text{C})}{(0.200\text{ kg})(900\text{ J/kg}\cdot\text{K}) + (0.0500\text{ kg})(4190\text{ J/kg}\cdot\text{K})}$$
$$= 57.0°\text{C} = 330\text{ K}.$$

(b) Now temperatures must be given in Kelvins: T_{ai} = 393 K, T_{wi} = 293 K, and T_f = 330 K. For the aluminum, $dQ = m_a c_a dT$ and the change in entropy is

$$\Delta S_a = \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln\left(\frac{T_f}{T_{ai}}\right) = (0.200\text{ kg})(900\text{ J/kg}\cdot\text{K})\ln\left(\frac{330\text{ K}}{373\text{ K}}\right)$$
$$= -22.1\text{ J/K}.$$

(c) The entropy change for the water is

$$\Delta S_w = \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln\left(\frac{T_f}{T_{wi}}\right) = (0.0500\text{ kg})(4190\text{ J/kg.K})\ln\left(\frac{330\text{K}}{293\text{K}}\right)$$
$$= +24.9\text{ J/K}.$$

(d) The change in the total entropy of the aluminum-water system is

$$\Delta S = \Delta S_a + \Delta S_w = -22.1\text{ J/K} + 24.9\text{ J/K} = +2.8\text{ J/K}.$$

LEARN The system is closed and the process is irreversible. For aluminum the entropy change is negative ($\Delta S_a < 0$) since $T_f < T_{ai}$. However, for water, entropy increases because $T_f > T_{wi}$. The overall entropy change for the aluminum-water system is positive, in accordance with the second law of thermodynamics.

12. We concentrate on the first term of Eq. 20-4 (the second term is zero because the final and initial temperatures are the same, and because ln(1) = 0). Thus, the entropy change is

$$\Delta S = nR\ln(V_f/V_i).$$

Noting that $\Delta S = 0$ at V_f = 0.40 m^3, we are able to deduce that V_i = 0.40 m^3. We now examine the point in the graph where ΔS = 32 J/K and V_f = 1.2 m^3; the above expression can now be used to solve for the number of moles. We obtain n = 3.5 mol.

13. This problem is similar to Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium." The only difference is that we need to find the mass m of each of the blocks. Since the two blocks are identical, the final temperature T_f is the average of the initial temperatures:

$$T_f = \frac{1}{2}(T_i + T_f) = \frac{1}{2}(305.5\text{ K} + 294.5\text{ K}) = 300.0\text{ K}.$$

Thus from $Q = mc\Delta T$ we find the mass m:

$$m = \frac{Q}{c\Delta T} = \frac{215\text{ J}}{(386\text{ J/kg}\cdot\text{K})(300.0\text{ K} - 294.5\text{ K})} = 0.101\text{ kg}.$$

(a) The change in entropy for block L is

$$\Delta S_L = mc\ln\left(\frac{T_f}{T_{iL}}\right) = (0.101\text{ kg})(386\text{ J/kg}\cdot\text{K})\ln\left(\frac{300.0\text{ K}}{305.5\text{ K}}\right) = -0.710\text{ J/K}.$$

(b) Since the temperature of the reservoir is virtually the same as that of the block, which gives up the same amount of heat as the reservoir absorbs, the change in entropy $\Delta S'_L$ of the reservoir connected to the left block is the opposite of that of the left block: $\Delta S'_L = -\Delta S_L = +0.710$ J/K.

(c) The entropy change for block R is

$$\Delta S_R = mc\ln\left(\frac{T_f}{T_{iR}}\right) = (0.101\text{ kg})(386\text{ J/kg}\cdot\text{K})\ln\left(\frac{300.0\text{ K}}{294.5\text{ K}}\right) = +0.723\text{ J/K}.$$

(d) Similar to the case in part (b) above, the change in entropy $\Delta S'_R$ of the reservoir connected to the right block is given by $\Delta S'_R = -\Delta S_R = -0.723$ J/K.

(e) The change in entropy for the two-block system is

$$\Delta S_L + \Delta S_R = -0.710\text{ J/K} + 0.723\text{ J/K} = +0.013\text{ J/K}.$$

(f) The entropy change for the entire system is given by

$$\Delta S = \Delta S_L + \Delta S'_L + \Delta S_R + \Delta S'_R = \Delta S_L - \Delta S_L + \Delta S_R - \Delta S_R = 0,$$

which is expected of a reversible process.

14. (a) Work is done only for the ab portion of the process. This portion is at constant pressure, so the work done by the gas is

$$W=\int_{V_0}^{4V_0} p_0\,dV = p_0(4.00V_0-1.00V_0)=3.00p_0V_0 \;\Rightarrow\; \frac{W}{p_0V}=3.00.$$

(b) We use the first law: $\Delta E_{\text{int}} = Q - W$. Since the process is at constant volume, the work done by the gas is zero and $E_{\text{int}} = Q$. The energy Q absorbed by the gas as heat is $Q = nC_V\,\Delta T$, where C_V is the molar specific heat at constant volume and ΔT is the change in temperature. Since the gas is a monatomic ideal gas, $C_V = 3R/2$. Use the ideal gas law to find that the initial temperature is

$$T_b=\frac{p_bV_b}{nR}=\frac{4p_0V_0}{nR}$$

and that the final temperature is

$$T_c=\frac{p_cV_c}{nR}=\frac{(2p_0)(4V_0)}{nR}=\frac{8p_0V_0}{nR}.$$

Thus,

$$Q=\frac{3}{2}nR\left(\frac{8p_0V_0}{nR}-\frac{4p_0V_0}{nR}\right)=6.00p_0V_0.$$

The change in the internal energy is $\Delta E_{\text{int}} = 6p_0V_0$ or $\Delta E_{\text{int}}/p_0V_0 = 6.00$. Since n = 1 mol, this can also be written $Q = 6.00RT_0$.

(c) For a complete cycle, $\Delta E_{\text{int}} = 0$.

(d) Since the process is at constant volume, use $dQ = nC_V\,dT$ to obtain

$$\Delta S=\int\frac{dQ}{T}=nC_V\int_{T_b}^{T_c}\frac{dT}{T}=nC_V\ln\frac{T_c}{T_b}.$$

Substituting $C_V=\frac{3}{2}R$ and using the ideal gas law, we write

$$\frac{T_c}{T_b}=\frac{p_cV_c}{p_bV_b}=\frac{(2p_0)(4V_0)}{p_0(4V_0)}=2.$$

Thus, $\Delta S=\frac{3}{2}nR\ln 2$. Since n = 1, this is $\Delta S=\frac{3}{2}R\ln 2=8.64$ J/K.

(e) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $\Delta S = 0$.

15. (a) The final mass of ice is (1773 g + 227 g)/2 = 1000 g. This means 773 g of water froze. Energy in the form of heat left the system in the amount mL_F, where m is the mass of the water that froze and L_F is the heat of fusion of water. The process is isothermal, so the change in entropy is

$$\Delta S = Q/T = -mL_F/T = -(0.773 \text{ kg})(333 \times 10^3 \text{ J/kg})/(273 \text{ K}) = -943 \text{ J/K}.$$

(b) Now, 773 g of ice is melted. The change in entropy is

$$\Delta S = \frac{Q}{T} = \frac{mL_F}{T} = +943 \text{ J/K}.$$

(c) Yes, they are consistent with the second law of thermodynamics. Over the entire cycle, the change in entropy of the water–ice system is zero even though part of the cycle is irreversible. However, the system is not closed. To consider a closed system, we must include whatever exchanges energy with the ice and water. Suppose it is a constant-temperature heat reservoir during the freezing portion of the cycle and a Bunsen burner during the melting portion. During freezing the entropy of the reservoir increases by 943 J/K. As far as the reservoir–water–ice system is concerned, the process is adiabatic and reversible, so its total entropy does not change. The melting process is irreversible, so the total entropy of the burner–water–ice system increases. The entropy of the burner either increases or else decreases by less than 943 J/K.

16. In coming to equilibrium, the heat lost by the 100 cm^3 of liquid water (of mass m_w = 100 g and specific heat capacity c_w = 4190 J/kg·K) is absorbed by the ice (of mass m_i, which melts and reaches $T_f > 0$ °C). We begin by finding the equilibrium temperature:

$$\sum Q = 0$$

$$Q_{\text{warm water cools}} + Q_{\text{ice warms to }0^\circ} + Q_{\text{ice melts}} + Q_{\text{melted ice warms}} = 0$$

$$c_w m_w \left(T_f - 20^\circ\right) + c_i m_i \left(0^\circ - (-10^\circ)\right) + L_F m_i + c_w m_i \left(T_f - 0^\circ\right) = 0$$

which yields, after using L_F = 333000 J/kg and values cited in the problem, T_f = 12.24 ° which is equivalent to T_f = 285.39 K. Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium" shows that

$$\Delta S_{\text{temp change}} = mc \ln\left(\frac{T_2}{T_1}\right)$$

for processes where $\Delta T = T_2 - T_1$, and Eq. 20-2 gives $\Delta S_{\text{melt}} = L_F\, m / T_{\text{o}}$ for the phase change experienced by the ice (with T_{o} = 273.15 K). The total entropy change is (with T in Kelvins)

$$\Delta S_{\text{system}} = m_w c_w \ln\left(\frac{285.39}{293.15}\right) + m_i c_i \ln\left(\frac{273.15}{263.15}\right) + m_i c_w \ln\left(\frac{285.39}{273.15}\right) + \frac{L_F\, m_i}{273.15}$$
$$= (-11.24 + 0.66 + 1.47 + 9.75)\text{J/K} = 0.64 \text{ J/K}.$$

17. The connection between molar heat capacity and the degrees of freedom of a diatomic gas is given by setting f = 5 in Eq. 19-51. Thus, $C_V = 5R/2$, $C_p = 7R/2$, and

$\gamma = 7/5$. In addition to various equations from Chapter 19, we also make use of Eq. 20-4 of this chapter. We note that we are asked to use the ideal gas constant as R and not plug in its numerical value. We also recall that isothermal means constant temperature, so $T_2 = T_1$ for the $1 \to 2$ process. The statement (at the end of the problem) regarding "per mole" may be taken to mean that n may be set identically equal to 1 wherever it appears.

(a) The gas law in ratio form is used to obtain

$$p_2 = p_1\left(\frac{V_1}{V_2}\right) = \frac{p_1}{3} \;\Rightarrow\; \frac{p_2}{p_1} = \frac{1}{3} = 0.333\,.$$

(b) The adiabatic relations Eq. 19-54 and Eq. 19-56 lead to

$$p_3 = p_1\left(\frac{V_1}{V_3}\right)^{\gamma} = \frac{p_1}{3^{1.4}} \;\Rightarrow\; \frac{p_3}{p_1} = \frac{1}{3^{1.4}} = 0.215\,.$$

(c) Similarly, we find

$$T_3 = T_1\left(\frac{V_1}{V_3}\right)^{\gamma-1} = \frac{T_1}{3^{0.4}} \;\Rightarrow\; \frac{T_3}{T_1} = \frac{1}{3^{0.4}} = 0.644.$$

• process $1 \to 2$

(d) The work is given by Eq. 19-14:

$$W = nRT_1 \ln (V_2/V_1) = RT_1 \ln 3 = 1.10RT_1.$$

Thus, $W/nRT_1 = \ln 3 = 1.10$.

(e) The internal energy change is $\Delta E_{\text{int}} = 0$, since this is an ideal gas process without a temperature change (see Eq. 19-45). Thus, the energy absorbed as heat is given by the first law of thermodynamics: $Q = \Delta E_{\text{int}} + W \approx 1.10RT_1$, or $Q/nRT_1 = \ln 3 = 1.10$.

(f) $\Delta E_{\text{int}} = 0$ or $\Delta E_{\text{int}} / nRT_1 = 0$

(g) The entropy change is $\Delta S = Q/T_1 = 1.10R$, or $\Delta S/R = 1.10$.

• process $2 \to 3$

(h) The work is zero, since there is no volume change. Therefore, $W/nRT_1 = 0$.

(i) The internal energy change is

$$\Delta E_{\text{int}} = nC_V\left(T_3 - T_2\right) = (1)\left(\frac{5}{2}R\right)\left(\frac{T_1}{3^{0.4}} - T_1\right) \approx -0.889\,RT_1 \;\Rightarrow\; \frac{\Delta E_{\text{int}}}{nRT_1} \approx -0.889.$$

This ratio (–0.889) is also the value for Q/nRT_1 (by either the first law of thermodynamics or by the definition of C_V).

(j) $\Delta E_{\text{int}}/nRT_1 = -0.889$.

(k) For the entropy change, we obtain

$$\frac{\Delta S}{R} = n\ln\left(\frac{V_3}{V_1}\right) + n\frac{C_V}{R}\ln\left(\frac{T_3}{T_1}\right) = (1)\ \ln(1) + (1)\left(\frac{5}{2}\right)\ln\left(\frac{T_1/3^{0.4}}{T_1}\right) = 0 + \frac{5}{2}\ln(3^{-0.4}) \approx -1.10\ .$$

• process 3 → 1

(l) By definition, $Q = 0$ in an adiabatic process, which also implies an absence of entropy change (taking this to be a reversible process). The internal change must be the negative of the value obtained for it in the previous process (since all the internal energy changes must add up to zero, for an entire cycle, and its change is zero for process 1 → 2), so $\Delta E_{\text{int}} = +0.889RT_1$. By the first law of thermodynamics, then,

$$W = Q - \Delta E_{\text{int}} = -0.889RT_1,$$

or $W/nRT_1 = -0.889$.

(m) $Q = 0$ in an adiabatic process.

(n) $\Delta E_{\text{int}}/nRT_1 = +0.889$.

(o) $\Delta S/nR = 0$.

18. (a) It is possible to motivate, starting from Eq. 20-3, the notion that heat may be found from the integral (or "area under the curve") of a curve in a *TS* diagram, such as this one. Either from calculus, or from geometry (area of a trapezoid), it is straightforward to find the result for a "straight-line" path in the *TS* diagram:

$$Q_{\text{straight}} = \left(\frac{T_i + T_f}{2}\right)\Delta S$$

which could, in fact, be *directly* motivated from Eq. 20-3 (but it is important to bear in mind that this is rigorously true only for a process that forms a straight line in a graph that plots T versus S). This leads to

$$Q = (300\text{ K})\,(15\text{ J/K}) = 4.5\times10^3\text{ J}$$

for the energy absorbed as heat by the gas.

(b) Using Table 19-3 and Eq. 19-45, we find

$$\Delta E_{\text{int}} = n\left(\frac{3}{2}R\right)\Delta T = (2.0\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(200\text{ K} - 400\text{ K}) = -5.0\times10^3\text{ J.}$$

(c) By the first law of thermodynamics, $W = Q - \Delta E_{\text{int}} = 4.5\text{ kJ} - (-5.0\text{ kJ}) = 9.5\text{ kJ.}$

19. We note that the connection between molar heat capacity and the degrees of freedom of a monatomic gas is given by setting f = 3 in Eq. 19-51. Thus, $C_V = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$.

(a) Since this is an ideal gas, Eq. 19-45 holds, which implies $\Delta E_{\text{int}} = 0$ for this process. Equation 19-14 also applies, so that by the first law of thermodynamics,

$$Q = 0 + W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln 2 \quad \rightarrow \quad Q/p_1V_1 = \ln 2 = 0.693.$$

(b) The gas law in ratio form implies that the pressure decreased by a factor of 2 during the isothermal expansion process to $V_2 = 2.00V_1$, so that it needs to increase by a factor of 4 in this step in order to reach a final pressure of $p_2 = 2.00p_1$. That same ratio form now applied to this constant-volume process, yielding $4.00 = T_2T_1$, which is used in the following:

$$Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)(T_2 - T_1) = \frac{3}{2}nRT_1\left(\frac{T_2}{T_1} - 1\right) = \frac{3}{2}p_1V_1(4-1) = \frac{9}{2}p_1V_1$$

or $Q/p_1V_1 = 9/2 = 4.50$.

(c) The work done during the isothermal expansion process may be obtained by using Eq. 19-14:

$$W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln 2.00 \rightarrow W/p_1V_1 = \ln 2 = 0.693.$$

(d) In step 2 where the volume is kept constant, $W = 0$.

(e) The change in internal energy can be calculated by combining the above results and applying the first law of thermodynamics:

$$\Delta E_{\text{int}} = Q_{\text{total}} - W_{\text{total}} = \left(p_1V_1 \ln 2 + \frac{9}{2}p_1V_1\right) - (p_1V_1 \ln 2 + 0) = \frac{9}{2}p_1V_1$$

or $\Delta E_{\text{int}}/p_1V_1 = 9/2 = 4.50$.

(f) The change in entropy may be computed by using Eq. 20-4:

$$\Delta S = R\ln\left(\frac{2.00V_1}{V_1}\right) + C_V \ln\left(\frac{4.00T_1}{T_1}\right) = R\ln 2.00 + \left(\frac{3}{2}R\right)\ln(2.00)^2$$
$$= R\ln 2.00 + 3R\ln 2.00 = 4R\ln 2.00 = 23.0\text{ J/K.}$$

The second approach consists of an isothermal (constant T) process in which the volume halves, followed by an isobaric (constant p) process.

(g) Here the gas law applied to the first (isothermal) step leads to a volume half as big as the original. Since $\ln(1/2.00) = -\ln 2.00$, the reasoning used above leads to

$$Q = -p_1V_1 \ln 2.00 \Rightarrow Q/p_1V_1 = -\ln 2.00 = -0.693.$$

(h) To obtain a final volume twice as big as the original, in this step we need to increase the volume by a factor of 4.00. Now, the gas law applied to this isobaric portion leads to a temperature ratio $T_2/T_1 = 4.00$. Thus,

$$Q = C_p\Delta T = \frac{5}{2}R(T_2 - T_1) = \frac{5}{2}RT_1\left(\frac{T_2}{T_1} - 1\right) = \frac{5}{2}p_1V_1(4-1) = \frac{15}{2}p_1V_1$$

or $Q/p_1V_1 = 15/2 = 7.50$.

(i) During the isothermal compression process, Eq. 19-14 gives

$$W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln(-1/2.00) = -p_1V_1 \ln 2.00 \Rightarrow W/p_1V_1 = -\ln 2 = -0.693.$$

(j) The initial value of the volume, for this part of the process, is $V_i = V_1/2$, and the final volume is $V_f = 2V_1$. The pressure maintained during this process is $p' = 2.00p_1$. The work is given by Eq. 19-16:

$$W = p'\Delta V = p'(V_f - V_i) = (2.00p_1)\left(2.00V_1 - \frac{1}{2}V_1\right) = 3.00p_1V_1 \Rightarrow W/p_1V_1 = 3.00.$$

(k) Using the first law of thermodynamics, the change in internal energy is

$$\Delta E_{\text{int}} = Q_{\text{total}} - W_{\text{total}} = \left(\frac{15}{2}p_1V_1 - p_1V_1\ln 2.00\right) - (3p_1V_1 - p_1V_1\ln 2.00) = \frac{9}{2}p_1V_1$$

or $\Delta E_{\text{int}}/p_1V_1 = 9/2 = 4.50$. The result is the same as that obtained in part (e).

(l) Similarly, $\Delta S = 4R\ln 2.00 = 23.0$ J/K. the same as that obtained in part (f).

20. (a) The final pressure is

$$p_f = (5.00\text{ kPa})e^{(V_i - V_f)/a} = (5.00\text{ kPa})e^{(1.00\text{ m}^3 - 2.00\text{ m}^3)/1.00\text{ m}^3} = 1.84\text{ kPa}.$$

(b) We use the ratio form of the gas law to find the final temperature of the gas:

$$T_f = T_i\left(\frac{p_f V_f}{p_i V_i}\right) = (600\text{ K})\frac{(1.84\text{ kPa})(2.00\text{ m}^3)}{(5.00\text{ kPa})(1.00\text{ m}^3)} = 441\text{ K}.$$

For later purposes, we note that this result can be written "exactly" as $T_f = T_i\,(2e^{-1})$. In our solution, we are avoiding using the "one mole" datum since it is not clear how precise it is.

(c) The work done by the gas is

$$\begin{aligned}W &= \int_i^f p\,dV = \int_{V_i}^{V_f}(5.00\text{ kPa})\,e^{(V_i - V)/a}\,dV = (5.00\text{ kPa})\,e^{V_i/a}\cdot\left[-ae^{-V/a}\right]_{V_i}^{V_f}\\ &= (5.00\text{ kPa})\,e^{1.00}\left(1.00\text{ m}^3\right)\left(e^{-1.00} - e^{-2.00}\right)\\ &= 3.16\text{ kJ}.\end{aligned}$$

(d) Consideration of a two-stage process, as suggested in the hint, brings us simply to Eq. 20-4. Consequently, with $C_V = \frac{3}{2}R$ (see Eq. 19-43), we find

$$\begin{aligned}\Delta S &= nR\ln\left(\frac{V_f}{V_i}\right) + n\left(\frac{3}{2}R\right)\ln\left(\frac{T_f}{T_i}\right) = nR\left(\ln 2 + \frac{3}{2}\ln\left(2e^{-1}\right)\right) = \frac{p_i V_i}{T_i}\left(\ln 2 + \frac{3}{2}\ln 2 + \frac{3}{2}\ln e^{-1}\right)\\ &= \frac{(5000\text{ Pa})(1.00\text{ m}^3)}{600\text{ K}}\left(\frac{5}{2}\ln 2 - \frac{3}{2}\right)\\ &= 1.94\text{ J/K}.\end{aligned}$$

21. We consider a three-step reversible process as follows: the supercooled water drop (of mass m) starts at state 1 ($T_1 = 268$ K), moves on to state 2 (still in liquid form but at $T_2 = 273$ K), freezes to state 3 ($T_3 = T_2$), and then cools down to state 4 (in solid form, with $T_4 = T_1$). The change in entropy for each of the stages is given as follows:

$$\Delta S_{12} = mc_w \ln (T_2/T_1),$$

$$\Delta S_{23} = -mL_F/T_2,$$

$$\Delta S_{34} = mc_I \ln (T_4/T_3) = mc_I \ln (T_1/T_2) = -mc_I \ln (T_2/T_1).$$

Thus the net entropy change for the water drop is

$$\begin{aligned}\Delta S &= \Delta S_{12} + \Delta S_{23} + \Delta S_{34} = m\left(c_w - c_I\right)\ln\left(\frac{T_2}{T_1}\right) - \frac{mL_F}{T_2}\\ &= (1.00\text{ g})\left(4.19\text{ J/g}\cdot\text{K} - 2.22\text{ J/g}\cdot\text{K}\right)\ln\left(\frac{273\text{ K}}{268\text{ K}}\right) - \frac{(1.00\text{ g})(333\text{ J/g})}{273\text{ K}}\\ &= -1.18\text{ J/K}.\end{aligned}$$

22. (a) We denote the mass of the ice (which turns to water and warms to T_f) as m and the mass of original water (which cools from 80º down to T_f) as m'. From $\Sigma Q = 0$ we have

$$L_F\, m + cm\,(T_f - 0^\circ) + cm'\,(T_f - 80^\circ) = 0.$$

Since $L_F = 333 \times 10^3$ J/kg, $c = 4190$ J/(kg·Cº), $m' = 0.13$ kg, and $m = 0.012$ kg, we find T_f = 66.5ºC, which is equivalent to 339.67 K.

(b) Using Eq. 20-2, the process of ice at 0º C turning to water at 0º C involves an entropy change of

$$\frac{Q}{T} = \frac{L_F\, m}{273.15\text{ K}} = 14.6\text{ J/K}.$$

(c) Using Eq. 20-1, the process of $m = 0.012$ kg of water warming from 0º C to 66.5º C involves an entropy change of

$$\int_{273.15}^{339.67} \frac{cm dT}{T} = cm \ln\left(\frac{339.67}{273.15}\right) = 11.0\text{ J/K}.$$

(d) Similarly, the cooling of the original water involves an entropy change of

$$\int_{353.15}^{339.67} \frac{cm' dT}{T} = cm' \ln\left(\frac{339.67}{353.15}\right) = -21.2\text{ J/K}.$$

(e) The net entropy change in this calorimetry experiment is found by summing the previous results; we find (by using more precise values than those shown above) ΔS_{net} = 4.39 J/K.

23. With $T_L = 290$ k, we find

$$\varepsilon = 1 - \frac{T_L}{T_H} \;\Rightarrow\; T_H = \frac{T_L}{1-\varepsilon} = \frac{290\text{ K}}{1 - 0.40}$$

which yields the (initial) temperature of the high-temperature reservoir: T_H = 483 K. If we replace ε = 0.40 in the above calculation with ε = 0.50, we obtain a (final) high temperature equal to $T'_H = 580$ K. The difference is

$$T'_H - T_H = 580\text{ K} - 483\text{ K} = 97\text{ K}.$$

24. The answers to this exercise do not depend on the engine being of the Carnot design. Any heat engine that intakes energy as heat (from, say, consuming fuel) equal to $|Q_H|$ = 52 kJ and exhausts (or discards) energy as heat equal to $|Q_L|$ = 36 kJ will have these values of efficiency ε and net work W.

(a) Equation 20-12 gives $\varepsilon = 1 - \left|\frac{Q_L}{Q_H}\right| = 0.31 = 31\%$.

(b) Equation 20-8 gives $W = |Q_H| - |Q_L| = 16$ kJ .

25. We solve (b) first.

(b) For a Carnot engine, the efficiency is related to the reservoir temperatures by Eq. 20-13. Therefore,

$$T_H = \frac{T_H - T_L}{\varepsilon} = \frac{75\text{ K}}{0.22} = 341\text{ K}$$

which is equivalent to 68°C.

(a) The temperature of the cold reservoir is $T_L = T_H - 75 = 341$ K – 75 K = 266 K.

26. Equation 20-13 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{373\text{ K}}{7\times10^8\text{ K}} = 0.9999995$$

quoting more figures than are significant. As a percentage, this is ε = 99.99995%.

27. **THINK** The thermal efficiency of the Carnot engine depends on the temperatures of the reservoirs.

EXPRESS The efficiency of the Carnot engine is given by

$$\varepsilon_C = \frac{T_H - T_L}{T_H},$$

where T_H is the temperature of the higher-temperature reservoir, and T_L the temperature of the lower-temperature reservoir, in kelvin scale. The work done by the engine is $|W| = \varepsilon |Q_H|$.

ANALYZE (a) The efficiency of the engine is

$$\varepsilon_c = \frac{T_H - T_L}{T_H} = \frac{(235 - 115)\text{ K}}{(235+273)\text{ K}} = 0.236 = 23.6\% .$$

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_H|$, the work done is given by

$$|W| = \varepsilon |Q_{\rm H}| = 0.236(6.30\times10^4\ \text{J}) = 1.49\times10^4\ \text{J}\ .$$

LEARN Expressing the efficiency as $\varepsilon_c = 1 - T_{\rm L}/T_{\rm H}$, we see that ε_c approaches unity (100% efficiency) in the limit $T_L/T_H \to 0$. This is an impossible dream. An alternative version of the second law of thermodynamics is: *there are no perfect engines*.

28. All terms are assumed to be positive. The total work done by the two-stage system is $W_1 + W_2$. The heat-intake (from, say, consuming fuel) of the system is Q_1, so we have (by Eq. 20-11 and Eq. 20-8)

$$\varepsilon = \frac{W_1 + W_2}{Q_1} = \frac{(Q_1 - Q_2) + (Q_2 - Q_3)}{Q_1} = 1 - \frac{Q_3}{Q_1}.$$

Now, Eq. 20-10 leads to

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3}$$

where we assume Q_2 is absorbed by the second stage at temperature T_2. This implies the efficiency can be written

$$\varepsilon = 1 - \frac{T_3}{T_1} = \frac{T_1 - T_3}{T_1}.$$

29. (a) The net work done is the rectangular "area" enclosed in the pV diagram:

$$W = (V - V_0)(p - p_0) = (2V_0 - V_0)(2p_0 - p_0) = V_0 p_0.$$

Inserting the values stated in the problem, we obtain $W = 2.27$ kJ.

(b) We compute the energy added as heat during the "heat-intake" portions of the cycle using Eq. 19-39, Eq. 19-43, and Eq. 19-46:

$$\begin{aligned} Q_{abc} &= nC_V(T_b - T_a) + nC_p(T_c - T_b) = n\left(\frac{3}{2}R\right)T_a\left(\frac{T_b}{T_a} - 1\right) + n\left(\frac{5}{2}R\right)T_a\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right) \\ &= nRT_a\left(\frac{3}{2}\left(\frac{T_b}{T_a} - 1\right) + \frac{5}{2}\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right)\right) = p_0V_0\left(\frac{3}{2}(2-1) + \frac{5}{2}(4-2)\right) \\ &= \frac{13}{2}p_0V_0 \end{aligned}$$

where, to obtain the last line, the gas law in ratio form has been used. Therefore, since $W = p_0V_0$, we have $Q_{abc} = 13W/2 = 14.8$ kJ.

(c) The efficiency is given by Eq. 20-11:

$$\varepsilon = \frac{W}{|Q_\text{H}|} = \frac{2}{13} = 0.154 = 15.4\%.$$

(d) A Carnot engine operating between T_c and T_a has efficiency equal to

$$\varepsilon = 1 - \frac{T_a}{T_c} = 1 - \frac{1}{4} = 0.750 = 75.0\%$$

where the gas law in ratio form has been used.

(e) This is greater than our result in part (c), as expected from the second law of thermodynamics.

30. (a) Equation 20-13 leads to

$$\varepsilon = 1 - \frac{T_\text{L}}{T_\text{H}} = 1 - \frac{333\text{ K}}{373\text{ K}} = 0.107.$$

We recall that a watt is joule-per-second. Thus, the (net) work done by the cycle per unit time is the given value 500 J/s. Therefore, by Eq. 20-11, we obtain the heat input per unit time:

$$\varepsilon = \frac{W}{|Q_\text{H}|} \quad \Rightarrow \quad \frac{0.500\text{ kJ/s}}{0.107} = 4.67\text{ kJ/s}\ .$$

(b) Considering Eq. 20-8 on a per unit time basis, we find (4.67 – 0.500) kJ/s = 4.17 kJ/s for the rate of heat exhaust.

31. (a) We use $\varepsilon = |W/Q_H|$. The heat absorbed is $|Q_\text{H}| = \dfrac{|W|}{\varepsilon} = \dfrac{8.2\,\text{kJ}}{0.25} = 33\,\text{kJ}.$

(b) The heat exhausted is then $|Q_\text{L}| = |Q_\text{H}| - |W| = 33\,\text{kJ} - 8.2\,\text{kJ} = 25\,\text{kJ}.$

(c) Now we have $|Q_\text{H}| = \dfrac{|W|}{\varepsilon} = \dfrac{8.2\,\text{kJ}}{0.31} = 26\,\text{kJ}.$

(d) Similarly, $|Q_\text{C}| = |Q_\text{H}| - |W| = 26\,\text{kJ} - 8.2\,\text{kJ} = 18\,\text{kJ}\,.$

32. From Fig. 20-28, we see Q_H = 4000 J at T_H = 325 K. Combining Eq. 20-11 with Eq. 20-13, we have

$$\frac{W}{Q_H} = 1 - \frac{T_C}{T_H} \quad \Rightarrow \quad W = 923\text{ J}\ .$$

Now, for T'_H = 550 K, we have

$$\frac{W}{Q'_H} = 1 - \frac{T_C}{T'_H} \quad \Rightarrow \quad Q'_H = 1692 \text{ J} \approx 1.7 \text{ kJ}.$$

33. **THINK** Our engine cycle consists of three steps: isochoric heating (*a* to *b*), adiabatic expansion (*b* to *c*), and isobaric compression (*c* to *a*).

EXPRESS Energy is added as heat during the portion of the process from *a* to *b*. This portion occurs at constant volume (V_b), so $Q_H = nC_V\, \Delta T$. The gas is a monatomic ideal gas, so $C_V = 3R/2$ and the ideal gas law gives

$$\Delta T = (1/nR)(p_b V_b - p_a V_a) = (1/nR)(p_b - p_a)V_b.$$

Thus, $Q_H = \frac{3}{2}(p_b - p_a)V_b$. On the other hand, energy leaves the gas as heat during the portion of the process from *c* to *a*. This is a constant pressure process, so

$$Q_L = nC_p \Delta T = nC_p (T_a - T_c) = nC_p \left(\frac{p_a V_a}{nR} - \frac{p_c V_c}{nR} \right) = \frac{C_p}{R} p_a (V_a - V_c).$$

where C_p is the molar specific heat for constant-pressure process.

ANALYZE (a) V_b and p_b are given. We need to find p_a. Now p_a is the same as p_c and points *c* and *b* are connected by an adiabatic process. With $p_c V_c^\gamma = p_b V_b^\gamma$ for the adiabat, we have ($\gamma = 5/3$ for monatomic gas)

$$p_a = p_c = \left(\frac{V_b}{V_c} \right)^\gamma p_b = \left(\frac{1}{8.00} \right)^{5/3} \left(1.013 \times 10^6 \text{ Pa}\right) = 3.167 \times 10^4 \text{ Pa}.$$

Thus, the energy added as heat is

$$Q_H = \frac{3}{2}(p_b - p_a)V_b = \frac{3}{2}\left(1.013\times10^6 \text{ Pa} - 3.167\times10^4 \text{ Pa}\right)\left(1.00\times10^{-3} \text{ m}^3\right) = 1.47\times10^3 \text{ J}.$$

(b) The energy leaving the gas as heat going from *c* to *a* is

$$Q_L = \frac{5}{2} p_a (V_a - V_c) = \frac{5}{2}\left(3.167\times10^4 \text{ Pa}\right)(-7.00)\left(1.00\times10^{-3} \text{ m}^3\right) = -5.54\times10^2 \text{ J},$$

or $|Q_L| = 5.54\times10^2$ J. The substitutions $V_a - V_c = V_a - 8.00\, V_a = -7.00\, V_a$ and $C_p = \frac{5}{2}R$ were made.

(c) For a complete cycle, the change in the internal energy is zero and

$$W = Q = Q_H - Q_L = 1.47 \times 10^3 \text{ J} - 5.54 \times 10^2 \text{ J} = 9.18 \times 10^2 \text{ J}.$$

(d) The efficiency is

$$\varepsilon = W/Q_H = (9.18 \times 10^2 \text{ J})/(1.47 \times 10^3 \text{ J}) = 0.624 = 62.4\%.$$

LEARN To summarize, the heat engine in this problem intakes energy as heat (from, say, consuming fuel) equal to $|Q_H| = 1.47$ kJ and exhausts energy as heat equal to $|Q_L| = 554$ J; its efficiency and net work are $\varepsilon = 1 - |Q_L|/|Q_H|$ and $W = |Q_H| - |Q_L|$. The less the exhaust heat $|Q_L|$, the more efficient is the engine.

34. (a) Using Eq. 19-54 for process $D \to A$ gives

$$p_D V_D^{\gamma} = p_A V_A^{\gamma} \quad \Rightarrow \quad \frac{p_0}{32}(8V_0)^{\gamma} = p_0 V_0^{\gamma}$$

which leads to $8^{\gamma} = 32 \Rightarrow \gamma = 5/3$. The result (see Sections 19-9 and 19-11) implies the gas is monatomic.

(b) The input heat is that absorbed during process $A \to B$:

$$Q_H = nC_p\Delta T = n\left(\frac{5}{2}R\right)T_A\left(\frac{T_B}{T_A} - 1\right) = nRT_A\left(\frac{5}{2}\right)(2-1) = p_0V_0\left(\frac{5}{2}\right)$$

and the exhaust heat is that liberated during process $C \to D$:

$$Q_L = nC_p\Delta T = n\left(\frac{5}{2}R\right)T_D\left(1 - \frac{T_L}{T_D}\right) = nRT_D\left(\frac{5}{2}\right)(1-2) = -\frac{1}{4}p_0V_0\left(\frac{5}{2}\right)$$

where in the last step we have used the fact that $T_D = \frac{1}{4}T_A$ (from the gas law in ratio form). Therefore, Eq. 20-12 leads to

$$\varepsilon = 1 - \left|\frac{Q_L}{Q_H}\right| = 1 - \frac{1}{4} = 0.75 = 75\%.$$

35. (a) The pressure at 2 is $p_2 = 3.00p_1$, as given in the problem statement. The volume is $V_2 = V_1 = nRT_1/p_1$. The temperature is

$$T_2 = \frac{p_2V_2}{nR} = \frac{3.00\,p_1V_1}{nR} = 3.00T_1 \quad \Rightarrow \quad \frac{T_2}{T_1} = 3.00.$$

(b) The process 2 → 3 is adiabatic, so $T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$. Using the result from part (a), V_3 = $4.00V_1$, $V_2 = V_1$, and γ=1.30, we obtain

$$\frac{T_3}{T_1} = \frac{T_3}{T_2/3.00} = 3.00\left(\frac{V_2}{V_3}\right)^{\gamma-1} = 3.00\left(\frac{1}{4.00}\right)^{0.30} = 1.98\,.$$

(c) The process 4 → 1 is adiabatic, so $T_4V_4^{\gamma-1} = T_1V_1^{\gamma-1}$. Since $V_4 = 4.00V_1$, we have

$$\frac{T_4}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{1}{4.00}\right)^{0.30} = 0.660.$$

(d) The process 2 → 3 is adiabatic, so $p_2V_2^{\gamma} = p_3V_3^{\gamma}$ or $p_3 = \left(V_2/V_3\right)^{\gamma} p_2$. Substituting V_3 = $4.00V_1$, $V_2 = V_1$, $p_2 = 3.00p_1$, and γ = 1.30, we obtain

$$\frac{p_3}{p_1} = \frac{3.00}{(4.00)^{1.30}} = 0.495.$$

(e) The process 4 → 1 is adiabatic, so $p_4V_4^{\gamma} = p_1V_1^{\gamma}$ and

$$\frac{p_4}{p_1} = \left(\frac{V_1}{V_4}\right)^{\gamma} = \frac{1}{(4.00)^{1.30}} = 0.165,$$

where we have used $V_4 = 4.00V_1$.

(f) The efficiency of the cycle is $\varepsilon = W/Q_{12}$, where W is the total work done by the gas during the cycle and Q_{12} is the energy added as heat during the 1 → 2 portion of the cycle, the only portion in which energy is added as heat. The work done during the portion of the cycle from 2 to 3 is $W_{23} = \int p\,dV$. Substitute $p = p_2V_2^{\gamma}/V^{\gamma}$ to obtain

$$W_{23} = p_2V_2^{\gamma}\int_{V_2}^{V_3} V^{-\gamma}dV = \left(\frac{p_2V_2^{\gamma}}{\gamma-1}\right)\left(V_2^{1-\gamma} - V_3^{1-\gamma}\right).$$

Substitute $V_2 = V_1$, $V_3 = 4.00V_1$, and $p_3 = 3.00p_1$ to obtain

$$W_{23} = \left(\frac{3p_1V_1}{1-\gamma}\right)\left(1-\frac{1}{4^{\gamma-1}}\right) = \left(\frac{3nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

Similarly, the work done during the portion of the cycle from 4 to 1 is

$$W_{41} = \left(\frac{p_1V_1^{\gamma}}{\gamma-1}\right)\left(V_4^{1-\gamma} - V_1^{1-\gamma}\right) = -\left(\frac{p_1V_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right) = -\left(\frac{nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

No work is done during the $1 \to 2$ and $3 \to 4$ portions, so the total work done by the gas during the cycle is

$$W = W_{23} + W_{41} = \left(\frac{2nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

The energy added as heat is

$$Q_{12} = nC_V(T_2 - T_1) = nC_V(3T_1 - T_1) = 2nC_VT_1,$$

where C_V is the molar specific heat at constant volume. Now

$$\gamma = C_p/C_V = (C_V + R)/C_V = 1 + (R/C_V),$$

so $C_V = R/(\gamma - 1)$. Here C_p is the molar specific heat at constant pressure, which for an ideal gas is $C_p = C_V + R$. Thus, $Q_{12} = 2nRT_1/(\gamma - 1)$. The efficiency is

$$\varepsilon = \frac{2nRT_1}{\gamma-1}\left(1-\frac{1}{4^{\gamma-1}}\right)\frac{\gamma-1}{2nRT_1} = 1-\frac{1}{4^{\gamma-1}}.$$

With $\gamma = 1.30$, the efficiency is $\varepsilon = 0.340$ or 34.0%.

36. (a) Using Eq. 20-14 and Eq. 20-16, we obtain

$$|W| = \frac{|Q_L|}{K_C} = (1.0\text{ J})\left(\frac{300\text{ K} - 280\text{ K}}{280\text{ K}}\right) = 0.071\text{ J}.$$

(b) A similar calculation (being sure to use absolute temperature) leads to 0.50 J in this case.

(c) With $T_L = 100$ K, we obtain $|W| = 2.0$ J.

(d) Finally, with the low temperature reservoir at 50 K, an amount of work equal to $|W| =$ 5.0 J is required.

37. **THINK** The performance of the refrigerator is related to its rate of doing work.

EXPRESS The coefficient of performance for a refrigerator is given by

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|},$$

where Q_L is the energy absorbed from the cold reservoir as heat and W is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields

$Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected to the hot reservoir as heat. Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than W, so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|W|$ is $|W| = |Q_H|/(K + 1)$.

ANALYZE In one hour, $|Q_H| = 7.54\ \text{MJ}$. With $K = 3.8$, the work done is

$$|W| = \frac{7.54\,\text{MJ}}{3.8 + 1} = 1.57\,\text{MJ}.$$

The rate at which work is done is $P = |W|/\Delta t = (1.57 \times 10^6\ \text{J})/(3600\ \text{s}) = 440\ \text{W}$.

LEARN The greater the value of K, the less the amount of work $|W|$ required to transfer the heat.

38. Equation 20-10 still holds (particularly due to its use of absolute values), and energy conservation implies $|W| + Q_L = Q_H$. Therefore, with $T_L = 268.15$ K and $T_H = 290.15$ K, we find

$$|Q_H| = |Q_L|\left(\frac{T_H}{T_L}\right) = \left(|Q_H| - |W|\right)\left(\frac{290.15}{268.15}\right)$$

which (with $|W| = 1.0$ J) leads to $|Q_H| = |W|\left(\dfrac{1}{1 - 268.15/290.15}\right) = 13\,\text{J}.$

39. **THINK** A large (small) value of coefficient of performance K means that less (more) work would be required to transfer the heat

EXPRESS A Carnot refrigerator working between a hot reservoir at temperature T_H and a cold reservoir at temperature T_L has a coefficient of performance K that is given by

$$K = \frac{T_L}{T_H - T_L},$$

where T_H is the temperature of the higher-temperature reservoir, and T_L the temperature of the lower-temperature reservoir, in Kelvin scale. Equivalently, the coefficient of performance is the energy Q_L drawn from the cold reservoir as heat divided by the work done: $K = |Q_L|/|W|$.

ANALYZE For the refrigerator of this problem, $T_H = 96°$ F $= 309$ K and $T_L = 70°$ F $= 294$ K, so

$$K = (294\ \text{K})/(309\ \text{K} - 294\ \text{K}) = 19.6.$$

Thus, with $|W| = 1.0\text{ J}$, the amount of heat removed from the room is

$$|Q_L| = K|W| = (19.6)(1.0\text{ J}) = 20\text{ J}.$$

LEARN The Carnot air conditioner in this problem (with K = 19.6) are much more efficient than that of the typical room air conditioners ($K \approx 2.5$).

40. (a) Equation 20-15 provides

$$K_C = \frac{|Q_L|}{|Q_H| - |Q_L|} \Rightarrow |Q_H| = |Q_L|\left(\frac{1+K_C}{K_C}\right)$$

which yields $|Q_H|$ = 49 kJ when K_C = 5.7 and $|Q_L|$ = 42 kJ.

(b) From Section 20-5 we obtain

$$|W| = |Q_H| - |Q_L| = 49.4\text{ kJ} - 42.0\text{ kJ} = 7.4\text{ kJ}$$

if we take the initial 42 kJ datum to be accurate to three figures. The given temperatures are not used in the calculation; in fact, it is possible that the given room temperature value is not meant to be the high temperature for the (reversed) Carnot cycle — since it does not lead to the given K_C using Eq. 20-16.

41. We are told $K = 0.27K_C$, where

$$K_C = \frac{T_L}{T_H - T_L} = \frac{294\text{ K}}{307\text{ K} - 294\text{ K}} = 23$$

where the Fahrenheit temperatures have been converted to Kelvins. Expressed on a per unit time basis, Eq. 20-14 leads to

$$\frac{|W|}{t} = \frac{|Q_L|/t}{K} = \frac{4000\text{ Btu/h}}{(0.27)(23)} = 643\text{ Btu/h}.$$

Appendix D indicates 1 But/h = 0.0003929 hp, so our result may be expressed as $|W|/t$ = 0.25 hp.

42. The work done by the motor in t = 10.0 min is $|W| = Pt$ = (200 W)(10.0 min)(60 s/min) = 1.20×10^5 J. The heat extracted is then

$$|Q_L| = K|W| = \frac{T_L|W|}{T_H - T_L} = \frac{(270\text{ K})(1.20\times10^5\text{ J})}{300\text{ K} - 270\text{ K}} = 1.08\times10^6\text{ J}.$$

43. The efficiency of the engine is defined by $\varepsilon = W/Q_1$ and is shown in the text to be

$$\varepsilon = \frac{T_1 - T_2}{T_1} \Rightarrow \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}.$$

The coefficient of performance of the refrigerator is defined by $K = Q_4/W$ and is shown in the text to be

$$K = \frac{T_4}{T_3 - T_4} \Rightarrow \frac{Q_4}{W} = \frac{T_4}{T_3 - T_4}.$$

Now $Q_4 = Q_3 - W$, so

$$(Q_3 - W)/W = T_4/(T_3 - T_4).$$

The work done by the engine is used to drive the refrigerator, so W is the same for the two. Solve the engine equation for W and substitute the resulting expression into the refrigerator equation. The engine equation yields $W = (T_1 - T_2)Q_1/T_1$ and the substitution yields

$$\frac{T_4}{T_3 - T_4} = \frac{Q_3}{W} - 1 = \frac{Q_3 T_1}{Q_1 (T_1 - T_2)} - 1.$$

Solving for Q_3/Q_1, we obtain

$$\frac{Q_3}{Q_1} = \left(\frac{T_4}{T_3 - T_4} + 1\right)\left(\frac{T_1 - T_2}{T_1}\right) = \left(\frac{T_3}{T_3 - T_4}\right)\left(\frac{T_1 - T_2}{T_1}\right) = \frac{1 - (T_2/T_1)}{1 - (T_4/T_3)}.$$

With T_1 = 400 K, T_2 = 150 K, T_3 = 325 K, and T_4 = 225 K, the ratio becomes Q_3/Q_1 = 2.03.

44. (a) Equation 20-13 gives the Carnot efficiency as $1 - T_L/T_H$. This gives 0.222 in this case. Using this value with Eq. 20-11 leads to W = (0.222)(750 J) = 167 J.

(b) Now, Eq. 20-16 gives $K_C = 3.5$. Then, Eq. 20-14 yields $|W|$ = 1200/3.5 = 343 J.

45. We need nine labels:

Label	Number of molecules on side 1	Number of molecules on side 2
I	8	0
II	7	1
III	6	2
IV	5	3
V	4	4
VI	3	5
VII	2	6
VIII	1	7
IX	0	8

The multiplicity W is computing using Eq. 20-20. For example, the multiplicity for label IV is

$$W = \frac{8!}{(5!)(3!)} = \frac{40320}{(120)(6)} = 56$$

and the corresponding entropy is (using Eq. 20-21)

$$S = k \ln W = \left(1.38\times10^{-23}\ \text{J/K}\right) \ln\left(56\right) = 5.6\times10^{-23}\ \text{J/K}.$$

In this way, we generate the following table:

Label	W	S
I	1	0
II	8	2.9×10^{-23} J/K
III	28	4.6×10^{-23} J/K
IV	56	5.6×10^{-23} J/K
V	70	5.9×10^{-23} J/K
VI	56	5.6×10^{-23} J/K
VII	28	4.6×10^{-23} J/K
VIII	8	2.9×10^{-23} J/K
IX	1	0

46. (a) We denote the configuration with n heads out of N trials as (n; N). We use Eq. 20-20:

$$W\left(25;50\right) = \frac{50!}{(25!)(50-25)!} = 1.26\times10^{14}.$$

(b) There are 2 possible choices for each molecule: it can either be in side 1 or in side 2 of the box. If there are a total of N independent molecules, the total number of available states of the N-particle system is

$$N_{\text{total}} = 2\times2\times2\times\cdots\times2 = 2^N.$$

With $N = 50$, we obtain $N_{\text{total}} = 2^{50} = 1.13 \times 10^{15}$.

(c) The percentage of time in question is equal to the probability for the system to be in the central configuration:

$$p\left(25;50\right) = \frac{W\left(25;50\right)}{2^{50}} = \frac{1.26\times10^{14}}{1.13\times10^{15}} = 11.1\%.$$

With $N = 100$, we obtain

(d) $W(N/2, N) = N!/[(N/2)!]^2 = 1.01 \times 10^{29}$,

(e) $N_{\text{total}} = 2^N = 1.27 \times 10^{30}$,

(f) and $p(N/2; N) = W(N/2, N)/N_{\text{total}} = 8.0\%$.

Similarly, for $N = 200$, we obtain

(g) $W(N/2, N) = 9.25 \times 10^{58}$,

(h) $N_{\text{total}} = 1.61 \times 10^{60}$,

(i) and $p(N/2; N) = 5.7\%$.

(j) As N increases, the number of available microscopic states increases as 2^N, so there are more states to be occupied, leaving the probability less for the system to remain in its central configuration. Thus, the time spent there decreases with an increase in N.

47. **THINK** The gas molecules inside a box can be distributed in many different ways. The number of microstates associated with each distinct configuration is called the multiplicity.

EXPRESS Given N molecules, if the box is divided into m equal parts, with n_1 molecules in the first, n_2 in the second,…, such that $n_1 + n_2 + ... n_m = N$. There are $N!$ arrangements of the N molecules, but $n_1!$ are simply rearrangements of the n_1 molecules in the first part, $n_2!$ are rearrangements of the n_2 molecules in the second,… These rearrangements do not produce a new configuration. Therefore, the multiplicity factor associated with this is

$$W = \frac{N!}{n_1! n_2! n_3! \ldots n_m!}.$$

ANALYZE (a) Suppose there are n_L molecules in the left third of the box, n_C molecules in the center third, and n_R molecules in the right third. Using the argument above, we find the multiplicity to be

$$W = \frac{N!}{n_L! n_C! n_R!}.$$

Note that $n_L + n_C + n_R = N$.

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!(N/2)!}.$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!(N/3)!(N/3)!}.$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!(N/2)!}{(N/3)!(N/3)!(N/3)!}.$$

(c) For $N = 100$,

$$\frac{W_A}{W_B} = \frac{50!\,50!}{33!\,33!\,34!} = 4.2\times10^{16}.$$

LEARN The more parts the box is divided into, the greater the number of configurations. This exercise illustrates the statistical view of entropy, which is related to W as $S = k\ln W$.

48. (a) A good way to (mathematically) think of this is to consider the terms when you expand:

$$(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4.$$

The coefficients correspond to the multiplicities. Thus, the smallest coefficient is 1.

(b) The largest coefficient is 6.

(c) Since the logarithm of 1 is zero, then Eq. 20-21 gives $S = 0$ for the least case.

(d) $S = k\ln(6) = 2.47 \times 10^{-23}$ J/K.

49. From the formula for heat conduction, Eq. 19-32, using Table 19-6, we have

$$H = kA\frac{T_H - T_C}{L} = (401)\left(\pi(0.02)^2\right) 270/1.50$$

which yields $H = 90.7$ J/s. Using Eq. 20-2, this is associated with an entropy rate-of-decrease of the high temperature reservoir (at 573 K) equal to

$$\Delta S/t = -90.7/573 = -0.158 \text{ (J/K)/s}.$$

And it is associated with an entropy rate-of-increase of the low temperature reservoir (at 303 K) equal to

$$\Delta S/t = +90.7/303 = 0.299 \text{ (J/K)/s}.$$

The net result is (0.299 – 0.158) (J/K)/s = 0.141 (J/K)/s.

50. For an isothermal ideal gas process, we have $Q = W = nRT\ln(V_f/V_i)$. Thus,

$$\Delta S = Q/T = W/T = nR\ln(V_f/V_i)$$

(a) V_f/V_i = (0.800)/(0.200) = 4.00, ΔS = (0.55)(8.31)ln(4.00) = 6.34 J/K.

(b) V_f/V_i = (0.800)/(0.200) = 4.00, ΔS = (0.55)(8.31)ln(4.00) = 6.34 J/K.

(c) V_f/V_i = (1.20)/(0.300) = 4.00, ΔS = (0.55)(8.31)ln(4.00) = 6.34 J/K.

(d) V_f/V_i = (1.20)/(0.300) = 4.00, ΔS = (0.55)(8.31)ln(4.00) = 6.34 J/K.

51. **THINK** Increasing temperature causes a shift of the probability distribution function $P(v)$ toward higher speed.

EXPRESS According to kinetic theory, the rms speed and the most probable speed are (see Eqs. 19-34 and 19035) $v_{\rm rms}=\sqrt{3RT/M}$, $v_P=\sqrt{2RT/M}$ and where T is the temperature and M is the molar mass. The rms speed is defined as $v_{\rm rms}=\sqrt{(v^2)_{\rm avg}}$, where $(v^2)_{\rm avg}=\int_0^\infty v^2P(v)dv$, with the Maxwell's speed distribution function given by

$$P(v)=4\pi\left(\frac{M}{2\pi RT}\right)^{3/2}v^2e^{-Mv^2/2RT}.$$

Thus, the difference between the two speeds is

$$\Delta v=v_{\rm rms}-v_P=\sqrt{\frac{3RT}{M}}-\sqrt{\frac{2RT}{M}}=\left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{RT}{M}}.$$

ANALYZE (a) With M = 28 g/mol = 0.028 kg/mol (see Table 19-1), and T_i = 250 K, we have

$$\Delta v_i=\left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{RT_i}{M}}=\left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{(8.31\text{ J/mol}\cdot\text{K})(250\text{ K})}{0.028\text{ kg/mol}}}=87\text{ m/s}.$$

(b) Similarly, at T_f = 500 K,

$$\Delta v_f=\left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{RT_f}{M}}=\left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{(8.31\text{ J/mol}\cdot\text{K})(500\text{ K})}{0.028\text{ kg/mol}}}=122\text{ m/s}\approx1.2\times10^2\text{ m/s}.$$

(c) From Table 19-3 we have $C_V = 5R/2$ (see also Table 19-2). For n = 1.5 mol, using Eq. 20-4, we find the change in entropy to be

$$\Delta S=nR\ln\left(\frac{V_f}{V_i}\right)+nC_V\ln\left(\frac{T_f}{T_i}\right)=0+(1.5\text{ mol})(5/2)(8.31\text{ J/mol}\cdot\text{K})\ln\left(\frac{500\text{ K}}{250\text{ K}}\right)$$
$$=22\text{ J/K}$$

LEARN Notice that the expression for Δv implies $T = \dfrac{M}{R(\sqrt{3}-\sqrt{2})^2}(\Delta v)^2$. Thus, one may also express ΔS as

$$\Delta S = n\,C_V \ln\left(\frac{T_f}{T_i}\right) = n\,C_V \ln\left(\frac{(\Delta v_f)^2}{(\Delta v_i)^2}\right) = 2n\,C_V \ln\left(\frac{\Delta v_f}{\Delta v_i}\right).$$

The entropy of the gas increases as the result of temperature increase.

52. (a) The most obvious input-heat step is the constant-volume process. Since the gas is monatomic, we know from Chapter 19 that $C_V = \frac{3}{2}R$. Therefore,

$$Q_V = nC_V\Delta T = (1.0 \text{ mol})\left(\frac{3}{2}\right)\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600 \text{ K} - 300 \text{ K}) = 3740 \text{ J}.$$

Since the heat transfer during the isothermal step is positive, we may consider it also to be an input-heat step. The isothermal Q is equal to the isothermal work (calculated in the next part) because $\Delta E_{\text{int}} = 0$ for an ideal gas isothermal process (see Eq. 19-45). Borrowing from the part (b) computation, we have

$$Q_{\text{isotherm}} = nRT_{\text{H}}\ln 2 = (1 \text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600 \text{ K})\ln 2 = 3456 \text{ J}.$$

Therefore, $Q_{\text{H}} = Q_V + Q_{\text{isotherm}} = 7.2 \times 10^3$ J.

(b) We consider the sum of works done during the processes (noting that no work is done during the constant-volume step). Using Eq. 19-14 and Eq. 19-16, we have

$$W = nRT_{\text{H}}\ln\left(\frac{V_{\max}}{V_{\min}}\right) + p_{\min}\left(V_{\min} - V_{\max}\right)$$

where, by the gas law in ratio form, the volume ratio is $\dfrac{V_{\max}}{V_{\min}} = \dfrac{T_{\text{H}}}{T_{\text{L}}} = \dfrac{600 \text{ K}}{300 \text{ K}} = 2.$

Thus, the net work is

$$\begin{aligned} W &= nRT_{\text{H}}\ln 2 + p_{\min}V_{\min}\left(1 - \frac{V_{\max}}{V_{\min}}\right) = nRT_{\text{H}}\ln 2 + nRT_{\text{L}}(1-2) = nR\left(T_{\text{H}}\ln 2 - T_{\text{L}}\right) \\ &= (1 \text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\left((600 \text{ K})\ln 2 - (300 \text{ K})\right) \\ &= 9.6\times 10^2 \text{ J}. \end{aligned}$$

(c) Equation 20-11 gives $\varepsilon = \dfrac{W}{Q_{\text{H}}} = 0.134 \approx 13\%$.

53. (a) If T_{H} is the temperature of the high-temperature reservoir and T_{L} is the temperature of the low-temperature reservoir, then the maximum efficiency of the engine is

$$\varepsilon = \frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}} = \frac{(800+40)\ \text{K}}{(800+273)\ \text{K}} = 0.78 \ \text{ or } \ 78\%.$$

(b) The efficiency is defined by $\varepsilon = |W|/|Q_{\text{H}}|$, where W is the work done by the engine and Q_{H} is the heat input. W is positive. Over a complete cycle, $Q_{\text{H}} = W + |Q_{\text{L}}|$, where Q_{L} is the heat output, so $\varepsilon = W/(W + |Q_{\text{L}}|)$ and $|Q_{\text{L}}| = W[(1/\varepsilon) - 1]$. Now $\varepsilon = (T_{\text{H}} - T_{\text{L}})/T_{\text{H}}$, where T_{H} is the temperature of the high-temperature heat reservoir and T_{L} is the temperature of the low-temperature reservoir. Thus,

$$\frac{1}{\varepsilon} - 1 = \frac{T_{\text{L}}}{T_{\text{H}} - T_{\text{L}}} \ \text{ and } \ |Q_{\text{L}}| = \frac{WT_{\text{L}}}{T_{\text{H}} - T_{\text{L}}}.$$

The heat output is used to melt ice at temperature $T_i = -40°\text{C}$. The ice must be brought to 0°C, then melted, so

$$|Q_{\text{L}}| = mc(T_f - T_i) + mL_F,$$

where m is the mass of ice melted, T_f is the melting temperature (0°C), c is the specific heat of ice, and L_F is the heat of fusion of ice. Thus,

$$WT_{\text{L}}/(T_{\text{H}} - T_{\text{L}}) = mc(T_f - T_i) + mL_F.$$

We differentiate with respect to time and replace dW/dt with P, the power output of the engine, and obtain

$$PT_{\text{L}}/(T_{\text{H}} - T_{\text{L}}) = (dm/dt)[c(T_f - T_i) + L_F].$$

Therefore,

$$\frac{dm}{dt} = \left(\frac{PT_{\text{L}}}{T_{\text{H}} - T_{\text{L}}}\right)\left(\frac{1}{c(T_f - T_i) + L_F}\right).$$

Now, $P = 100 \times 10^6$ W, $T_{\text{L}} = 0 + 273 = 273$ K, $T_{\text{H}} = 800 + 273 = 1073$ K, $T_i = -40 + 273 = 233$ K, $T_f = 0 + 273 = 273$ K, $c = 2220$ J/kg·K, and $L_F = 333 \times 10^3$ J/kg, so

$$\frac{dm}{dt} = \left[\frac{(100\times10^6\ \text{J/s})(273\ \text{K})}{1073\ \text{K} - 273\ \text{K}}\right]\left[\frac{1}{(2220\ \text{J/kg}\cdot\text{K})(273\ \text{K} - 233\ \text{K}) + 333\times10^3\ \text{J/kg}}\right]$$
$$= 82\ \text{kg/s}.$$

We note that the engine is now operated between 0°C and 800°C.

54. Equation 20-4 yields

$$\Delta S = nR\ln(V_f/V_i) + nC_V\ln(T_f/T_i) = 0 + nC_V\ln(425/380)$$

where n = 3.20 and $C_V = \frac{3}{2}R$ (Eq. 19-43). This gives 4.46 J/K.

55. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = 40.9°\text{C},$$

which is equivalent to 314 K.

(b) From Eq. 20-1, we have

$$\Delta S_{\text{copper}} = \int_{353}^{314} \frac{cm\,dT}{T} = (386)(0.600)\ln\left(\frac{314}{353}\right) = -27.1 \text{ J/K}.$$

(c) For water, the change in entropy is

$$\Delta S_{\text{water}} = \int_{283}^{314} \frac{cm\,dT}{T} = (4187)(0.0700)\ln\left(\frac{314}{283.15}\right) = 30.3 \text{ J/K}.$$

(d) The net result for the system is (30.3 – 27.1) J/K = 3.2 J/K. (Note: These calculations are fairly sensitive to round-off errors. To arrive at this final answer, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

56. Using Hooke's law, we find the spring constant to be

$$k = \frac{F_s}{x_s} = \frac{1.50 \text{ N}}{0.0350 \text{ m}} = 42.86 \text{ N/m}.$$

To find the rate of change of entropy with a small additional stretch, we use Eq. 20-7 and obtain

$$\left|\frac{dS}{dx}\right| = \frac{k\,|x|}{T} = \frac{(42.86 \text{ N/m})(0.0170 \text{ m})}{275 \text{ K}} = 2.65\times10^{-3} \text{ J/K}\cdot\text{m}.$$

57. Since the volume of the monatomic ideal gas is kept constant, it does not do any work in the heating process. Therefore the heat Q it absorbs is equal to the change in its inertial energy: $dQ = dE_{\text{int}} = \frac{3}{2}nR\,dT$. Thus,

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{(3nR/2)\,dT}{T} = \frac{3}{2} nR \ln\left(\frac{T_f}{T_i}\right) = \frac{3}{2}(1.00 \text{ mol})\left(8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln\left(\frac{400 \text{ K}}{300 \text{ K}}\right)$$
$$= 3.59 \text{ J/K}.$$

58. With the pressure kept constant,

$$dQ = nC_p dT = n(C_V + R)dT = \left(\frac{3}{2}nR + nR\right)dT = \frac{5}{2}nRdT,$$

so we need to replace the factor 3/2 in the last problem by 5/2. The rest is the same. Thus the answer now is

$$\Delta S = \frac{5}{2} nR \ln\left(\frac{T_f}{T_i}\right) = \frac{5}{2}(1.00 \text{ mol})\left(8.31 \frac{\text{J}}{\text{mol}\cdot\text{ K}}\right)\ln\left(\frac{400 \text{ K}}{300 \text{ K}}\right) = 5.98 \text{ J/K}.$$

59. **THINK** The temperature of the ice is first raised to 0°C, then the ice melts and the temperature of the resulting water is raised to 40°C. We want to calculate the entropy change in this process.

EXPRESS As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I dT$, where m is the mass of the ice and c_I is the specific heat of ice. If T_i (= –20°C = 253 K) is the initial temperature and T_f (= 273 K) is the final temperature, then the change in its entropy is

$$\Delta S_1 = \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln\left(\frac{T_f}{T_i}\right) = (0.60 \text{ kg})(2220 \text{ J/kg}\cdot\text{K})\ln\left(\frac{273 \text{ K}}{253 \text{ K}}\right) = 101 \text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F, where L_F is the heat of fusion for ice. Thus,

$$\Delta S_2 = \frac{Q}{T} = \frac{mL_F}{T} = \frac{(0.60 \text{ kg})(333\times10^3 \text{ J/kg})}{273 \text{ K}} = 732 \text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S_3 = mc_w \ln\left(\frac{T_f}{T_i}\right) = (0.600 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})\ln\left(\frac{313 \text{ K}}{273 \text{ K}}\right) = 344 \text{ J/K},$$

where c_w = 4190 J/kg · K is the specific heat of water.

ANALYZE The total change in entropy for the ice and the water it becomes is

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 = 101 \text{ J/K} + 732 \text{ J/K} + 344 \text{ J/K} = 1.18\times 10^3 \text{ J/K}.$$

LEARN From the above, we readily see that the biggest increase in entropy comes from ΔS_2, which accounts for the melting process.

60. The net work is figured from the (positive) isothermal expansion (Eq. 19-14) and the (negative) constant-pressure compression (Eq. 19-48). Thus,

$$W_{net} = nRT_H \ln(V_{max}/V_{min}) + nR(T_L - T_H)$$

where n = 3.4, T_H = 500 K, T_L = 200 K, and V_{max}/V_{min} = 5/2 (same as the ratio T_H/T_L). Therefore, W_{net} = 4468 J. Now, we identify the "input heat" as that transferred in steps 1 and 2:

$$Q_{in} = Q_1 + Q_2 = nC_V(T_H - T_L) + nRT_H \ln(V_{max}/V_{min})$$

where C_V = 5R/2 (see Table 19-3). Consequently, Q_{in} = 34135 J. Dividing these results gives the efficiency: W_{net}/Q_{in} = 0.131 (or about 13.1%).

61. Since the inventor's claim implies that less heat (typically from burning fuel) is needed to operate his engine than, say, a Carnot engine (for the same magnitude of net work), then $Q_H' < Q_H$ (see Fig. 20-34(a)) which implies that the Carnot (ideal refrigerator) unit is delivering more heat to the high temperature reservoir than engine X draws from it. This (using also energy conservation) immediately implies Fig. 20-34(b), which violates the second law.

62. (a) From Eq. 20-1, we infer $Q = \int T\,dS$, which corresponds to the "area under the curve" in a T-S diagram. Thus, since the area of a rectangle is (height)×(width), we have

$$Q_{1\to 2} = (350)(2.00) = 700 \text{ J}.$$

(b) With no "area under the curve" for process 2 → 3, we conclude $Q_{2\to 3} = 0$.

(c) For the cycle, the (net) heat should be the "area inside the figure," so using the fact that the area of a triangle is ½ (base) × (height), we find

$$Q_{net} = \frac{1}{2}(2.00)(50) = 50 \text{ J}.$$

(d) Since we are dealing with an ideal gas (so that $\Delta E_{int} = 0$ in an isothermal process), then

$$W_{1\to 2} = Q_{1\to 2} = 700 \text{ J}.$$

(e) Using Eq. 19-14 for the isothermal work, we have

$$W_{1\to2} = nRT \ln \frac{V_2}{V_1} .$$

where $T = 350$ K. Thus, if $V_1 = 0.200$ m^3, then we obtain

$$V_2 = V_1 \exp(W/nRT) = (0.200)\, e^{0.12} = 0.226 \text{ m}^3.$$

(f) Process 2 → 3 is adiabatic; Eq. 19-56 applies with γ = 5/3 (since only translational degrees of freedom are relevant here):

$$T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1} .$$

This yields $V_3 = 0.284$ m^3.

(g) As remarked in part (d), $\Delta E_{\text{int}} = 0$ for process 1 → 2.

(h) We find the change in internal energy from Eq. 19-45 (with $C_V = \frac{3}{2}R$):

$$\Delta E_{\text{int}} = nC_V (T_3 - T_2) = -1.25 \times 10^3 \text{ J}.$$

(i) Clearly, the net change of internal energy for the entire cycle is zero. This feature of a closed cycle is as true for a *T-S* diagram as for a *p-V* diagram.

(j) For the adiabatic (2 → 3) process, we have $W = -\Delta E_{\text{int}}$. Therefore, $W = 1.25 \times 10^3$ J. Its positive value indicates an expansion.

63. (a) It is a reversible set of processes returning the system to its initial state; clearly, $\Delta S_{\text{net}} = 0$.

(b) Process 1 is adiabatic and reversible (as opposed to, say, a free expansion) so that Eq. 20-1 applies with $dQ = 0$ and yields $\Delta S_1 = 0$.

(c) Since the working substance is an ideal gas, then an isothermal process implies $Q = W$, which further implies (regarding Eq. 20-1) $dQ = p\, dV$. Therefore,

$$\int \frac{dQ}{T} = \int \frac{p\, dV}{\left(\frac{pV}{nR}\right)} = nR \int \frac{dV}{V}$$

which leads to $\Delta S_3 = nR \ln(1/2) = -23.0$ J/K.

(d) By part (a), $\Delta S_1 + \Delta S_2 + \Delta S_3 = 0$. Then, part (b) implies $\Delta S_2 = -\Delta S_3$. Therefore, $\Delta S_2 =$ 23.0 J/K.

64. (a) Combining Eq. 20-11 with Eq. 20-13, we obtain

$$|W| = |Q_H|\left(1 - \frac{T_L}{T_H}\right) = (500\,\text{J})\left(1 - \frac{260\,\text{K}}{320\,\text{K}}\right) = 93.8\,\text{J}.$$

(b) Combining Eq. 20-14 with Eq. 20-16, we find

$$|W| = \frac{|Q_L|}{\left(\frac{T_L}{T_H - T_L}\right)} = \frac{1000\,\text{J}}{\left(\frac{260\text{K}}{320\text{K} - 260\text{K}}\right)} = 231\ \text{J}.$$

65. (a) Processes 1 and 2 both require the input of heat, which is denoted Q_H. Noting that rotational degrees of freedom are not involved, then, from the discussion in Chapter 19, $C_V = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$. We further note that since the working substance is an ideal gas, process 2 (being isothermal) implies $Q_2 = W_2$. Finally, we note that the volume ratio in process 2 is simply 8/3. Therefore,

$$Q_H = Q_1 + Q_2 = nC_V\left(T' - T\right) + nRT' \ln\frac{8}{3}$$

which yields (for T = 300 K and T' = 800 K) the result $Q_H = 25.5 \times 10^3$ J.

(b) The net work is the net heat ($Q_1 + Q_2 + Q_3$). We find Q_3 from

$$nC_p\,(T - T') = -20.8 \times 10^3\ \text{J}.$$

Thus, $W = 4.73 \times 10^3$ J.

(c) Using Eq. 20-11, we find that the efficiency is

$$\varepsilon = \frac{|W|}{|Q_H|} = \frac{4.73 \times 10^3}{25.5 \times 10^3} = 0.185 \text{ or } 18.5\%.$$

66. (a) Equation 20-14 gives K = 560/150 = 3.73.

(b) Energy conservation requires the exhaust heat to be 560 + 150 = 710 J.

67. The change in entropy in transferring a certain amount of heat Q from a heat reservoir at T_1 to another one at T_2 is $\Delta S = \Delta S_1 + \Delta S_2 = Q(1/T_2 - 1/T_1)$.

(a) ΔS = (260 J)(1/100 K – 1/400 K) = 1.95 J/K.

(b) ΔS = (260 J)(1/200 K – 1/400 K) = 0.650 J/K.

(c) ΔS = (260 J)(1/300 K – 1/400 K) = 0.217 J/K.

(d) ΔS = (260 J)(1/360 K – 1/400 K) = 0.072 J/K.

(e) We see that as the temperature difference between the two reservoirs decreases, so does the change in entropy.

68. Equation 20-10 gives

$$\left|\frac{Q_{\text{to}}}{Q_{\text{from}}}\right| = \frac{T_{\text{to}}}{T_{\text{from}}} = \frac{300\,\text{K}}{4.0\,\text{K}} = 75.$$

69. (a) Equation 20-2 gives the entropy change for each reservoir (each of which, by definition, is able to maintain constant temperature conditions within itself). The net entropy change is therefore

$$\Delta S = \frac{+|Q|}{273 + 24} + \frac{-|Q|}{273 + 130} = 4.45 \text{ J/K}$$

where we set $|Q|$ = 5030 J.

(b) We have assumed that the conductive heat flow in the rod is "steady-state"; that is, the situation described by the problem has existed and will exist for "long times." Thus there are no entropy change terms included in the calculation for elements of the rod itself.

70. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = -44.2°\text{C},$$

which is equivalent to 229 K.

(b) From Eq. 20-1, we have

$$\Delta S_{\text{tungsten}} = \int_{303}^{229} \frac{cm\,dT}{T} = (134)(0.045)\ln\left(\frac{229}{303}\right) = -1.69 \text{ J/K}.$$

(c) Also,

$$\Delta S_{\text{silver}} = \int_{153}^{229} \frac{cm\,dT}{T} = (236)(0.0250)\ln\left(\frac{229}{153}\right) = 2.38 \text{ J/K}.$$

(d) The net result for the system is (2.38 – 1.69) J/K = 0.69 J/K. (Note: These calculations are fairly sensitive to round-off errors. To arrive at this final answer, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

71. (a) We use Eq. 20-16. For configuration *A*

$$W_A = \frac{N!}{(N/2)!(N/2)!} = \frac{50!}{(25!)(25!)} = 1.26\times10^{14}.$$

(b) For configuration B

$$W_B = \frac{N!}{(0.6N)!(0.4N)!} = \frac{50!}{[0.6(50)]![0.4(50)]!} = 4.71\times10^{13}.$$

(c) Since all microstates are equally probable,

$$f = \frac{W_B}{W_A} = \frac{1265}{3393} \approx 0.37.$$

We use these formulas for $N = 100$. The results are

(d) $W_A = \frac{N!}{(N/2)!(N/2)!} = \frac{100!}{(50!)(50!)} = 1.01\times10^{29}$,

(e) $W_B = \frac{N!}{(0.6N)!(0.4N)!} = \frac{100!}{[0.6(100)]![0.4(100)]!} = 1.37\times10^{28}$,

(f) and $f \;\; W_B/W_A \approx 0.14$.

Similarly, using the same formulas for $N = 200$, we obtain

(g) $W_A = 9.05 \times 10^{58}$,

(h) $W_B = 1.64 \times 10^{57}$,

(i) and $f = 0.018$.

(j) We see from the calculation above that f decreases as N increases, as expected.

72. A metric ton is 1000 kg, so that the heat generated by burning 380 metric tons during one hour is $(380000 \text{ kg})(28 \text{ MJ/kg}) = 10.6\times10^6 \text{ MJ}$. The work done in one hour is

$$W = (750 \text{ MJ/s})(3600 \text{ s}) = 2.7\times10^6 \text{ MJ}$$

where we use the fact that a watt is a joule-per-second. By Eq. 20-11, the efficiency is

$$\varepsilon = \frac{2.7\times10^6 \text{ MJ}}{10.6\times10^6 \text{ MJ}} = 0.253 = 25\%.$$

73. **THINK** The performance of the Carnot refrigerator is related to its rate of doing work.

EXPRESS The coefficient of performance for a refrigerator is defined as

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|},$$

where Q_L is the energy absorbed from the cold reservoir (interior of refrigerator) as heat and W is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields $Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected as heat to the hot reservoir (the room). Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than W, so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|Q_H| = |W|(1+K) = |Q_L|(1+K)/K$.

ANALYZE (a) From the expression above, the energy per cycle transferred as heat to the room is

$$|Q_H| = |Q_L|\frac{1+K}{K} = (35.0\text{ kJ})\frac{1+4.60}{4.60} = 42.6\text{ kJ}.$$

(b) Similarly, the work done per cycle is $|W| = \frac{|Q_L|}{K} = \frac{35.0\text{ kJ}}{4.60} = 7.61\text{ kJ}$.

LEARN A Carnot refrigerator is a Carnot engine operating in reverse. Its coefficient of performance can also be written as

$$K = \frac{T_L}{T_H - T_L}$$

The value of K is higher when the temperatures of the two reservoirs are close to each other.

74. The Carnot efficiency (Eq. 20-13) depends linearly on T_L so that we can take a derivative

$$\varepsilon = 1 - \frac{T_L}{T_H} \Rightarrow \frac{d\varepsilon}{dT_L} = -\frac{1}{T_H}$$

and quickly get to the result. With $d\varepsilon \to \Delta\varepsilon = 0.100$ and $T_H = 400$ K, we find $dT_L \to \Delta T_L = -40$ K.

75. **THINK** The gas molecules inside a box can be distributed in many different ways. The number of microstates associated with each distinct configuration is called the multiplicity.

EXPRESS In general, if there are N molecules and if the box is divided into two halves, with n_L molecules in the left half and n_R in the right half, such that $n_L + n_R = N$. There are $N!$ arrangements of the N molecules, but $n_L!$ are simply rearrangements of the n_L molecules in the left half, and $n_R!$ are rearrangements of the n_R molecules in the right half. These rearrangements do not produce a new configuration. Therefore, the multiplicity factor associated with this is

$$W = \frac{N!}{n_L!\,n_R!}.$$

The entropy is given by $S = k\ln W$.

ANALYZE (a) The least multiplicity configuration is when all the particles are in the same half of the box. In this case, for system A with with $N = 3$, we have

$$W = \frac{3!}{3!0!} = 1.$$

(b) Similarly for box B, with $N = 5$, $W = 5!/(5!0!) = 1$ in the “least” case.

(c) The most likely configuration in the 3 particle case is to have 2 on one side and 1 on the other. Thus,

$$W = \frac{3!}{2!1!} = 3.$$

(d) The most likely configuration in the 5 particle case is to have 3 on one side and 2 on the other. Therefore,

$$W = \frac{5!}{3!2!} = 10.$$

(e) We use Eq. 20-21 with our result in part (c) to obtain

$$S = k\ln W = \left(1.38\times10^{-23}\right)\ln 3 = 1.5\times10^{-23}\ \text{J/K}.$$

(f) Similarly for the 5 particle case (using the result from part (d)), we find

$$S = k \ln 10 = 3.2 \times 10^{-23}\ \text{J/K}.$$

LEARN The least multiplicity is $W = 1$; this happens when $n_L = N$ or $n_L = 0$. On the other hand, the greatest multiplicity occurs when $n_L = (N-1)/2$ or $n_L = (N+1)/2$.

76. (a) Using $Q = T\Delta S$, we note that heat enters the cycle along the top path at 400 K, and leaves along the bottom path at 250 K. Thus,

$$Q_{\text{in}} = (400\text{ K})(0.60\text{ J/K} - 0.10\text{ J/K}) = 200\text{ J}$$
$$Q_{\text{out}} = (250\text{ K})(0.10\text{ J/K} - 0.60\text{ J/K}) = -125\text{ J}$$

and the net heat transfer is $Q = Q_{\text{in}} + Q_{\text{out}} = 200\text{ J} - 125\text{ J} = 75\text{ J}$.

(b) For cyclic path, $\Delta E_{\text{int}} = Q - W = 0$. Therefore, the work done by the system is $W = Q = 75\text{ J}$.

77. The efficiency of an ideal heat engine and coefficient of performance of a reversible refrigerator are

$$\varepsilon = \frac{|W|}{|Q_{\text{H}}|}, \qquad K = \frac{|Q_{\text{H}}| - |W|}{|W|}.$$

Thus,

$$K = \frac{|Q_{\text{H}}| - |W|}{|W|} = \frac{|Q_{\text{H}}|}{|W|} - 1 = \frac{1}{\varepsilon} - 1 \quad \Rightarrow \quad \varepsilon = \frac{1}{K+1}$$

78. (a) The efficiency is defined by $\varepsilon = |W|/|Q_{\text{H}}|$, where W is the work done by the engine and Q_{H} is the heat input. In our case, the temperatures of the hot and cold reservoirs are $T_{\text{H}} = 100\text{ °C} = 373\text{ K}$ and $T_{\text{L}} = 60\text{ °C} = 333\text{ K}$, respectively. The maximum efficiency of the engine is

$$\varepsilon = \frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{333\text{ K}}{373\text{ K}} = 0.107.$$

Thus, the rate of heat input is

$$\frac{dQ_{\text{H}}}{dt} = \frac{1}{\varepsilon}\frac{dW}{dt} = \frac{1}{0.107}(500\text{ W}) = 4.66 \times 10^{3}\text{ W}.$$

(b) The rate of exhaust heat output is

$$\frac{dQ_{\text{L}}}{dt} = \frac{dQ_{\text{H}}}{dt} - \frac{dW}{dt} = 4.66 \times 10^{3}\text{ W} - 500\text{ W} = 4.16 \times 10^{3}\text{ W}.$$

79. The temperatures of the hot and cold reservoirs are $T_{\text{H}} = 26\text{ °C} = 299\text{ K}$ and $T_{\text{L}} = -13\text{ °C} = 260\text{ K}$, respectively. Therefore, the theoretical coefficient of performance of the refrigerator is

$$K = \frac{T_{\text{L}}}{T_{\text{H}} - T_{\text{L}}} = \frac{260\text{ K}}{299\text{ K} - 260\text{ K}} = 6.67.$$

Chapter 21

1. **THINK** After the transfer, the charges on the two spheres are $Q-q$ and q.

EXPRESS The magnitude of the electrostatic force between two charges q_1 and q_2 separated by a distance r is given by the Coulomb's law (see Eq. 21-1):

$$F = k\frac{q_1 q_2}{r^2},$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$. In our case, $q_1 = Q-q$ and $q_2 = q$, so the magnitude of the force of either of the charges on the other is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}.$$

We want the value of q that maximizes the function $f(q) = q(Q-q)$.

ANALYZE Setting the derivative df/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$. Thus, $q/Q = 0.500$.

LEARN The force between the two spheres is a maximum when charges are distributed evenly between them.

2. The fact that the spheres are identical allows us to conclude that when two spheres are in contact, they share equal charge. Therefore, when a charged sphere (q) touches an uncharged one, they will (fairly quickly) each attain half that charge ($q/2$). We start with spheres 1 and 2, each having charge q and experiencing a mutual repulsive force $F = kq^2/r^2$. When the neutral sphere 3 touches sphere 1, sphere 1's charge decreases to $q/2$. Then sphere 3 (now carrying charge $q/2$) is brought into contact with sphere 2; a total amount of $q/2 + q$ becomes shared equally between them. Therefore, the charge of sphere 3 is $3q/4$ in the final situation. The repulsive force between spheres 1 and 2 is finally

$$F' = k\frac{(q/2)(3q/4)}{r^2} = \frac{3}{8}k\frac{q^2}{r^2} = \frac{3}{8}F \;\Rightarrow\; \frac{F'}{F} = \frac{3}{8} = 0.375.$$

3. **THINK** The magnitude of the electrostatic force between two charges q_1 and q_2 separated by a distance r is given by Coulomb's law.

EXPRESS Equation 21-1 gives Coulomb's law, $F = k\frac{|q_1||q_2|}{r^2}$, which can be used to solve for the distance:

$$r = \sqrt{\frac{k\,|q_1||q_2|}{F}}\,.$$

ANALYZE With $F = 5.70\text{ N}$, $q_1 = 2.60\times10^{-6}\text{ C}$ and $q_2 = -47.0\times10^{-6}\text{ C}$, the distance between the two charges is

$$r = \sqrt{\frac{k\,|q_1||q_2|}{F}} = \sqrt{\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(26.0\times10^{-6}\text{C}\right)\left(47.0\times10^{-6}\text{C}\right)}{5.70\text{N}}} = 1.39\text{ m}.$$

LEARN The electrostatic force between two charges falls as $1/r^2$. The same inverse-square nature is also seen in the gravitational force between two masses.

4. The unit ampere is discussed in Section 21-4. Using i for current, the charge transferred is

$$q = it = \left(2.5\times10^4\text{ A}\right)\left(20\times10^{-6}\text{s}\right) = 0.50\text{ C}.$$

5. The magnitude of the mutual force of attraction at r = 0.120 m is

$$F = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times10^{-6}\text{C}\right)\left(1.50\times10^{-6}\text{C}\right)}{(0.120\text{ m})^2} = 2.81\text{N}.$$

6. (a) With a understood to mean the magnitude of acceleration, Newton's second and third laws lead to

$$m_2a_2 = m_1a_1 \Rightarrow m_2 = \frac{\left(6.3\times10^{-7}\text{ kg}\right)\left(7.0\text{ m/s}^2\right)}{9.0\text{ m/s}^2} = 4.9\times10^{-7}\text{ kg}.$$

(b) The magnitude of the (only) force on particle 1 is

$$F = m_1a_1 = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{|q|^2}{(0.0032\text{ m})^2}.$$

Inserting the values for m_1 and a_1 (see part (a)) we obtain $|q| = 7.1\times10^{-11}$ C.

7. With rightward positive, the net force on q_3 is

$$F_3 = F_{13} + F_{23} = k\frac{q_1q_3}{\left(L_{12}+L_{23}\right)^2} + k\frac{q_2q_3}{L_{23}^2}\,.$$

We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on q_3 by q_1) is negative if they are unlike charges, indicating that q_3 is being pulled toward q_1, and it is positive if they are like charges (so q_3 would be repelled from q_1). Setting the net force equal to zero $L_{23}= L_{12}$ and canceling k, q_3, and L_{12} leads to

$$\frac{q_1}{4.00} + q_2 = 0 \quad \Rightarrow \quad \frac{q_1}{q_2} = -\,4.00.$$

8. In experiment 1, sphere C first touches sphere A, and they divided up their total charge (Q/2 plus Q) equally between them. Thus, sphere A and sphere C each acquired charge 3Q/4. Then, sphere C touches B and those spheres split up their total charge (3Q/4 plus –Q/4) so that B ends up with charge equal to Q/4. The force of repulsion between A and B is therefore

$$F_1 = k\frac{(3Q/4)(Q/4)}{d^2}$$

at the end of experiment 1. Now, in experiment 2, sphere C first touches B, which leaves each of them with charge Q/8. When C next touches A, sphere A is left with charge 9Q/16. Consequently, the force of repulsion between A and B is

$$F_2 = k\frac{(9Q/16)(Q/8)}{d^2}$$

at the end of experiment 2. The ratio is

$$\frac{F_2}{F_1} = \frac{(9/16)(1/8)}{(3/4)(1/4)} = 0.375.$$

9. **THINK** Since opposite charges attract, the initial charge configurations must be of opposite signs. Similarly, since like charges repel, the final charge configurations must carry the same sign.

EXPRESS We assume that the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges. We choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Then the force on q_2 is

$$F_a = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = -k\frac{q_1 q_2}{r^2}$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\mathrm{N\cdot m^2/C^2}$ and r = 0.500 m. The negative sign indicates that the spheres attract each other. After the wire is connected, the spheres, being identical, acquire the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is (q_1 + q_2)/2. The force is now repulsive and is given by

$$F_b = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{r^2} = k\frac{(q_1+q_2)^2}{4r^2}.$$

We solve the two force equations simultaneously for q_1 and q_2.

ANALYZE The first equation gives the product

$$q_1q_2 = -\frac{r^2F_a}{k} = -\frac{(0.500\,\text{m})^2(0.108\,\text{N})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = -3.00\times10^{-12}\,\text{C}^2,$$

and the second gives the sum

$$q_1+q_2 = 2r\sqrt{\frac{F_b}{k}} = 2(0.500\,\text{m})\sqrt{\frac{0.0360\,\text{N}}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}} = 2.00\times10^{-6}\,\text{C}$$

where we have taken the positive root (which amounts to assuming $q_1 + q_2 \geq 0$). Thus, the product result provides the relation

$$q_2 = \frac{-(3.00\times10^{-12}\,\text{C}^2)}{q_1}$$

which we substitute into the sum result, producing

$$q_1 - \frac{3.00\times10^{-12}\,\text{C}^2}{q_1} = 2.00\times10^{-6}\,\text{C}.$$

Multiplying by q_1 and rearranging, we obtain a quadratic equation

$$q_1^2 - (2.00\times10^{-6}\,\text{C})q_1 - 3.00\times10^{-12}\,\text{C}^2 = 0.$$

The solutions are

$$q_1 = \frac{2.00\times10^{-6}\,\text{C} \pm\sqrt{(-2.00\times10^{-6}\,\text{C})^2 - 4(-3.00\times10^{-12}\,\text{C}^2)}}{2}.$$

If the positive sign is used, $q_1 = 3.00 \times 10^{-6}$ C, and if the negative sign is used, $q_1 = -1.00\times10^{-6}$ C.

(a) Using $q_2 = (-3.00 \times 10^{-12})/q_1$ with $q_1 = 3.00 \times 10^{-6}$ C, we get $q_2 = -1.00\times10^{-6}$ C.

(b) If we instead work with the $q_1 = -1.00 \times 10^{-6}$ C root, then we find $q_2 = 3.00\times10^{-6}$ C.

LEARN Note that since the spheres are identical, the solutions are essentially the same: one sphere originally had charge -1.00×10^{-6} C and the other had charge $+3.00 \times 10^{-6}$ C. What happens if we had not made the assumption, above, that $q_1 + q_2 \geq 0$? If the signs of the charges were reversed (so $q_1 + q_2 < 0$), then the forces remain the same, so a charge of $+1.00 \times 10^{-6}$ C on one sphere and a charge of -3.00×10^{-6} C on the other also satisfies the conditions of the problem.

10. For ease of presentation (of the computations below) we assume $Q > 0$ and $q < 0$ (although the final result does not depend on this particular choice).

(a) The x-component of the force experienced by $q_1 = Q$ is

$$F_{1x} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{(Q)(Q)}{\left(\sqrt{2}a\right)^2}\cos 45° + \frac{(|q|)\,(Q)}{a^2}\right) = \frac{Q|q|}{4\pi\varepsilon_0 a^2}\left(-\frac{Q/|q|}{2\sqrt{2}} + 1\right)$$

which (upon requiring $F_{1x} = 0$) leads to $Q/|q| = 2\sqrt{2}$, or $Q/q = -2\sqrt{2} = -2.83$.

(b) The y-component of the net force on $q_2 = q$ is

$$F_{2y} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q|^2}{\left(\sqrt{2}a\right)^2}\sin 45° - \frac{(|q|)\,(Q)}{a^2}\right) = \frac{|q|^2}{4\pi\varepsilon_0 a^2}\left(\frac{1}{2\sqrt{2}} - \frac{Q}{|q|}\right)$$

which (if we demand $F_{2y} = 0$) leads to $Q/q = -1/2\sqrt{2}$. The result is inconsistent with that obtained in part (a). Thus, we are unable to construct an equilibrium configuration with this geometry, where the only forces present are given by Eq. 21-1.

11. The force experienced by q_3 is

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{|q_3||q_1|}{a^2}\hat{\text{j}} + \frac{|q_3||q_2|}{(\sqrt{2}a)^2}(\cos 45°\hat{\text{i}} + \sin 45°\hat{\text{j}}) + \frac{|q_3||q_4|}{a^2}\hat{\text{i}}\right)$$

(a) Therefore, the x-component of the resultant force on q_3 is

$$F_{3x} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(\frac{|q_2|}{2\sqrt{2}} + |q_4|\right) = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{2\left(1.0\times10^{-7}\ \text{C}\right)^2}{(0.050\ \text{m})^2}\left(\frac{1}{2\sqrt{2}} + 2\right) = 0.17\ \text{N}.$$

(b) Similarly, the y-component of the net force on q_3 is

$$F_{3y} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(-|q_1| + \frac{|q_2|}{2\sqrt{2}}\right) = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{2\left(1.0\times10^{-7}\ \text{C}\right)^2}{(0.050\ \text{m})^2}\left(-1 + \frac{1}{2\sqrt{2}}\right)$$
$$= -0.046\ \text{N}.$$

12. (a) For the net force to be in the $+x$ direction, the y components of the individual forces must cancel. The angle of the force exerted by the q_1 = 40 μC charge on $q_3 = 20\mu C$ is 45°, and the angle of force exerted on q_3 by Q is at $-\theta$ where

$$\theta = \tan^{-1}\left(\frac{2.0 \text{ cm}}{3.0 \text{ cm}}\right) = 33.7°.$$

Therefore, cancellation of y components requires

$$k\frac{q_1 q_3}{\left(0.02\sqrt{2}\text{ m}\right)^2}\sin 45° = k\frac{|Q|\,q_3}{\left(\sqrt{(0.030\text{ m})^2 + (0.020\text{ m})^2}\right)^2}\sin\theta$$

from which we obtain $|Q|$ = 83 μC. Charge Q is "pulling" on q_3, so (since $q_3 > 0$) we conclude Q = –83 μC.

(b) Now, we require that the x components cancel, and we note that in this case, the angle of force on q_3 exerted by Q is $+\theta$ (it is repulsive, and Q is positive-valued). Therefore,

$$k\frac{q_1 q_3}{\left(0.02\sqrt{2}\text{ m}\right)^2}\cos 45° = k\frac{Q q_3}{\left(\sqrt{(0.030\text{ m})^2 + (0.020\text{ m})^2}\right)^2}\cos\theta$$

from which we obtain Q = 55.2 μC $\approx 55\ \mu$C.

13. (a) There is no equilibrium position for q_3 *between* the two fixed charges, because it is being pulled by one and pushed by the other (since q_1 and q_2 have different signs); in this region this means the two force arrows on q_3 are in the same direction and cannot cancel. It should also be clear that off-axis (with the axis defined as that which passes through the two fixed charges) there are no equilibrium positions. On the semi-infinite region of the axis that is nearest q_2 and furthest from q_1 an equilibrium position for q_3 cannot be found because $|q_1| < |q_2|$ and the magnitude of force exerted by q_2 is everywhere (in that region) stronger than that exerted by q_1 on q_3. Thus, we must look in the semi-infinite region of the axis which is nearest q_1 and furthest from q_2, where the net force on q_3 has magnitude

$$\left| k\frac{|q_1 q_3|}{L_0^2} - k\frac{|q_2 q_3|}{\left(L + L_0\right)^2} \right|$$

with L = 10 cm and L_0 is assumed to be *positive*. We set this equal to zero, as required by the problem, and cancel k and q_3. Thus, we obtain

$$\frac{|q_1|}{L_0^2}-\frac{|q_2|}{\left(L+L_0\right)^2}=0 \;\Rightarrow\; \left(\frac{L+L_0}{L_0}\right)^2=\left|\frac{q_2}{q_1}\right|=\left|\frac{-3.0\ \mu\text{C}}{+1.0\ \mu\text{C}}\right|=3.0$$

which yields (after taking the square root)

$$\frac{L+L_0}{L_0}=\sqrt{3} \;\Rightarrow\; L_0=\frac{L}{\sqrt{3}-1}=\frac{10\text{ cm}}{\sqrt{3}-1}\approx 14\,\text{cm}$$

for the distance between q_3 and q_1. That is, q_3 should be placed at $x=-14$ cm along the x-axis.

(b) As stated above, $y = 0$.

14. (a) The individual force magnitudes (acting on Q) are, by Eq. 21-1,

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1|Q}{\left(-a-a/2\right)^2}=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|Q}{\left(a-a/2\right)^2}$$

which leads to $|q_1| = 9.0\ |q_2|$. Since Q is located between q_1 and q_2, we conclude q_1 and q_2 are like-sign. Consequently, $q_1/q_2 = 9.0$.

(b) Now we have

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1|Q}{\left(-a-3a/2\right)^2}=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|Q}{\left(a-3a/2\right)^2}$$

which yields $|q_1| = 25\ |q_2|$. Now, Q is not located between q_1 and q_2; one of them must push and the other must pull. Thus, they are unlike-sign, so $q_1/q_2 = -25$.

15. (a) The distance between q_1 and q_2 is

$$r_{12}=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}=\sqrt{\left(-0.020\text{ m}-0.035\text{ m}\right)^2+\left(0.015\text{ m}-0.005\text{ m}\right)^2}=0.056\text{ m.}$$

The magnitude of the force exerted by q_1 on q_2 is

$$F_{21}=k\frac{|q_1q_2|}{r_{12}^2}=\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.0\times10^{-6}\text{C}\right)\left(4.0\times10^{-6}\text{C}\right)}{(0.056\text{ m})^2}=35\text{ N.}$$

(b) The vector $\vec{F}_{21}$ is directed toward q_1 and makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \tan^{-1}\left(\frac{1.5\text{ cm} - 0.5\text{ cm}}{-2.0\text{ cm} - 3.5\text{ cm}}\right) = -10.3° \approx -10°.$$

(c) Let the third charge be located at (x_3, y_3), a distance r from q_2. We note that q_1, q_2, and q_3 must be collinear; otherwise, an equilibrium position for any one of them would be impossible to find. Furthermore, we cannot place q_3 on the same side of q_2 where we also find q_1, since in that region both forces (exerted on q_2 by q_3 and q_1) would be in the same direction (since q_2 is attracted to both of them). Thus, in terms of the angle found in part (a), we have $x_3 = x_2 - r\cos\theta$ and $y_3 = y_2 - r\sin\theta$ (which means $y_3 > y_2$ since θ is negative). The magnitude of force exerted on q_2 by q_3 is $F_{23} = k|q_2q_3|/r^2$, which must equal that of the force exerted on it by q_1 (found in part (a)). Therefore,

$$k\frac{|q_2q_3|}{r^2} = k\frac{|q_1q_2|}{r_{12}^2} \;\Rightarrow\; r = r_{12}\sqrt{\frac{q_3}{q_1}} = 0.0645\,\text{m} = 6.45\text{ cm}.$$

Consequently, $x_3 = x_2 - r\cos\theta = -2.0\text{ cm} - (6.45\text{ cm})\cos(-10°) = -8.4\text{ cm}$,

(d) and $y_3 = y_2 - r\sin\theta = 1.5\text{ cm} - (6.45\text{ cm})\sin(-10°) = 2.7\text{ cm}$.

16. (a) According to the graph, when q_3 is very close to q_1 (at which point we can consider the force exerted by particle 1 on 3 to dominate) there is a (large) force in the positive x direction. This is a repulsive force, then, so we conclude q_1 has the same sign as q_3. Thus, q_3 is a positive-valued charge.

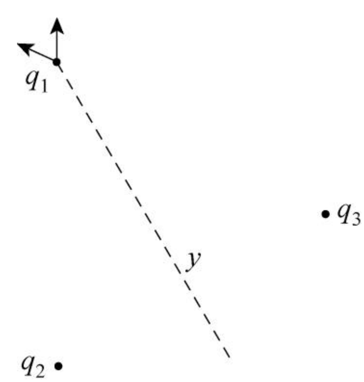
(b) Since the graph crosses zero and particle 3 is *between* the others, q_1 must have the same sign as q_2, which means it is also positive-valued. We note that it crosses zero at r = 0.020 m (which is a distance d = 0.060 m from q_2). Using Coulomb's law at that point, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_3q_2}{d^2} \;\Rightarrow\; q_2 = \left(\frac{d}{r}\right)^2 q_1 = \left(\frac{0.060\text{ m}}{0.020\text{ m}}\right)^2 q_1 = 9.0\,q_1,$$

or $q_2/q_1 = 9.0$.

17. (a) Equation 21-1 gives

$$F_{12} = k\frac{q_1q_2}{d^2} = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(20.0\times10^{-6}\text{C}\right)^2}{(1.50\text{m})^2} = 1.60\text{ N}.$$



(b) On the right, a force diagram is shown as well as our choice of y axis (the dashed line).

The y axis is meant to bisect the line between q_2 and q_3 in order to make use of the symmetry in the problem (equilateral triangle of side length d, equal-magnitude charges $q_1 = q_2 = q_3 = q$). We see that the resultant force is along this symmetry axis, and we obtain

$$|F_y| = 2\left(k\frac{q^2}{d^2}\right)\cos 30° = 2\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(20.0\times10^{-6}\,\text{C}\right)^2}{\left(1.50\text{m}\right)^2}\cos 30° = 2.77\,\text{N}.$$

18. Since the forces involved are proportional to q, we see that the essential difference between the two situations is $F_a \propto q_B + q_C$ (when those two charges are on the same side) versus $F_b \propto -q_B + q_C$ (when they are on opposite sides). Setting up ratios, we have

$$\frac{F_a}{F_b} = \frac{q_B + q_C}{-q_B + q_C} \quad \Rightarrow \quad \frac{2.014\times10^{-23}\text{ N}}{-2.877\times10^{-24}\text{ N}} = \frac{1 + q_C/q_B}{-1 + q_C/q_B}.$$

After noting that the ratio on the left hand side is very close to – 7, then, after a couple of algebra steps, we are led to

$$\frac{q_C}{q_B} = \frac{7+1}{7-1} = \frac{8}{6} = 1.333.$$

19. **THINK** Our system consists of two charges in a straight line. We'd like to place a third charge so that all three charges are in equilibrium.

EXPRESS If the system of three charges is to be in equilibrium, the force on each charge must be zero. The third charge q_3 must lie between the other two or else the forces acting on it due to the other charges would be in the same direction and q_3 could not be in equilibrium. Suppose q_3 is at a distance x from q, and $L - x$ from $4.00q$. The force acting on it is then given by

$$F_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} - \frac{4qq_3}{(L-x)^2}\right)$$

where the positive direction is rightward. We require $F_3 = 0$ and solve for x.

ANALYZE (a) Canceling common factors yields $1/x^2 = 4/(L - x)^2$ and taking the square root yields $1/x = 2/(L - x)$. The solution is $x = L/3$. With $L = 9.00$ cm, we have $x = 3.00$ cm.

(b) Similarly, the y coordinate of q_3 is $y = 0$.

(c) The force on q is

$$F_q = \frac{-1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} + \frac{4.00q^2}{L^2}\right).$$

The signs are chosen so that a negative force value would cause q to move leftward. We require $F_q = 0$ and solve for q_3:

$$q_3 = -\frac{4qx^2}{L^2} = -\frac{4}{9}q \quad \Rightarrow \quad \frac{q_3}{q} = -\frac{4}{9} = -\,0.444$$

where $x = L/3$ is used.

LEARN We may also verify that the force on $4.00q$ also vanishes:

$$F_{4q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4qq_0}{(L-x)^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2}\right) = 0\,.$$

20. We note that the problem is examining the force on charge A, so that the respective distances (involved in the Coulomb force expressions) between B and A, and between C and A, do not change as particle B is moved along its circular path. We focus on the endpoints ($\theta = 0°$ and 180°) of each graph, since they represent cases where the forces (on A) due to B and C are either parallel or anti-parallel (yielding maximum or minimum force magnitudes, respectively). We note, too, that since Coulomb's law is inversely proportional to r^2 then (if, say, the charges were all the same) the force due to C would be one-fourth as big as that due to B (since C is twice as far away from A). The charges, it turns out, are not the same, so there is also a factor of the charge ratio ξ (the charge of C divided by the charge of B), as well as the aforementioned ¼ factor. That is, the force exerted by C is, by Coulomb's law, equal to $\pm\frac{1}{4}\xi$ multiplied by the force exerted by B.

(a) The maximum force is $2F_0$ and occurs when $\theta = 180°$ (B is to the left of A, while C is the right of A). We choose the minus sign and write

$$2\,F_0 = (1 - \tfrac{1}{4}\xi)\,F_0 \quad \Rightarrow \quad \xi = -\,4\,.$$

One way to think of the minus sign choice is $\cos(180°) = -1$. This is certainly consistent with the minimum force ratio (zero) at $\theta = 0°$ since that would also imply

$$0 = 1 + \tfrac{1}{4}\xi \quad \Rightarrow \quad \xi = -\,4\,.$$

(b) The ratio of maximum to minimum forces is 1.25/0.75 = 5/3 in this case, which implies

$$\frac{5}{3} = \frac{1 + \frac{1}{4}\xi}{1 - \frac{1}{4}\xi} \quad \Rightarrow \quad \xi = 16\,.$$

Of course, this could also be figured as illustrated in part (a), looking at the maximum force ratio by itself and solving, or looking at the minimum force ratio (¾) at $\theta = 180°$ and solving for ξ.

21. The charge dq within a thin shell of thickness dr is $dq = \rho dV = \rho A dr$ where $A = 4\pi r^2$. Thus, with $\rho = b/r$, we have

$$q = \int dq = 4\pi b \int_{r_1}^{r_2} r\,dr = 2\pi b\left(r_2^2 - r_1^2\right).$$

With $b = 3.0\ \mu C/m^2$, $r_2 = 0.06$ m, and $r_1 = 0.04$ m, we obtain $q = 0.038\ \mu C = 3.8 \times 10^{-8}$ C.

22. (a) We note that $\cos(30°) = \frac{1}{2}\sqrt{3}$, so that the dashed line distance in the figure is $r = 2d/\sqrt{3}$. The net force on q_1 due to the two charges q_3 and q_4 (with $|q_3| = |q_4| = 1.60 \times 10^{-19}$ C) on the y axis has magnitude

$$2\frac{|q_1 q_3|}{4\pi\varepsilon_0 r^2}\cos(30°) = \frac{3\sqrt{3}\,|q_1 q_3|}{16\pi\varepsilon_0 d^2}.$$

This must be set equal to the magnitude of the force exerted on q_1 by $q_2 = 8.00 \times 10^{-19}$ C $= 5.00\,|q_3|$ in order that its net force be zero:

$$\frac{3\sqrt{3}\,|q_1 q_3|}{16\pi\varepsilon_0 d^2} = \frac{|q_1 q_2|}{4\pi\varepsilon_0 (D+d)^2} \quad \Rightarrow \quad D = d\left(2\sqrt{\frac{5}{3\sqrt{3}}} - 1\right) = 0.9245\ d.$$

Given $d = 2.00$ cm, this then leads to $D = 1.92$ cm.

(b) As the angle decreases, its cosine increases, resulting in a larger contribution from the charges on the y axis. To offset this, the force exerted by q_2 must be made stronger, so that it must be brought closer to q_1 (keep in mind that Coulomb's law is *inversely* proportional to distance-squared). Thus, D must be decreased.

23. If θ is the angle between the force and the x-axis, then

$$\cos\theta = \frac{x}{\sqrt{x^2 + d^2}}.$$

We note that, due to the symmetry in the problem, there is no y component to the net force on the third particle. Thus, F represents the magnitude of force exerted by q_1 or q_2 on q_3. Let $e = +1.60 \times 10^{-19}$ C, then $q_1 = q_2 = +2e$ and $q_3 = 4.0e$ and we have

$$F_{\text{net}} = 2F\cos\theta = \frac{2(2e)(4e)}{4\pi\varepsilon_o (x^2 + d^2)}\frac{x}{\sqrt{x^2 + d^2}} = \frac{4e^2 x}{\pi\varepsilon_o (x^2 + d^2)^{3/2}}.$$

(a) To find where the force is at an extremum, we can set the derivative of this expression equal to zero and solve for x, but it is good in any case to graph the function for a fuller understanding of its behavior, and as a quick way to see whether an extremum point is a maximum or a miminum. In this way, we find that the value coming from the derivative procedure is a maximum (and will be presented in part (b)) and that the minimum is found at the lower limit of the interval. Thus, the net force is found to be zero at $x = 0$, which is the smallest value of the net force in the interval $5.0\text{ m} \geq x \geq 0$.

(b) The maximum is found to be at $x = d/\sqrt{2}$ or roughly 12 cm.

(c) The value of the net force at $x = 0$ is $F_{\text{net}} = 0$.

(d) The value of the net force at $x = d/\sqrt{2}$ is $F_{\text{net}} = 4.9 \times 10^{-26}$ N.

24. (a) Equation 21-1 gives

$$F = \frac{\left(8.99 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\left(1.00 \times 10^{-16}\ \text{C}\right)^2}{\left(1.00 \times 10^{-2}\ \text{m}\right)^2} = 8.99 \times 10^{-19}\ \text{N}.$$

(b) If n is the number of excess electrons (of charge $-e$ each) on each drop then

$$n = -\frac{q}{e} = -\frac{-1.00 \times 10^{-16}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = 625.$$

25. Equation 21-11 (in absolute value) gives $n = \dfrac{|q|}{e} = \dfrac{1.0 \times 10^{-7}\ \text{C}}{1.6 \times 10^{-19}\ \text{C}} = 6.3 \times 10^{11}$.

26. The magnitude of the force is

$$F = k\frac{e^2}{r^2} = \left(8.99 \times 10^{9}\ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(2.82 \times 10^{-10}\ \text{m}\right)^2} = 2.89 \times 10^{-9}\ \text{N}.$$

27. **THINK** The magnitude of the electrostatic force between two charges q_1 and q_2 separated by a distance r is given by Coulomb's law.

EXPRESS Let the charge of the ions be q. With $q_1 = q_2 = +q$, the magnitude of the force between the (positive) ions is given by

$$F = \frac{(q)(q)}{4\pi\varepsilon_0 r^2} = k\frac{q^2}{r^2},$$

where $k = 1/4\pi\varepsilon_0 = 8.99 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2$.

ANALYZE (a) We solve for the charge:

$$q = r\sqrt{\frac{F}{k}} = \left(5.0 \times 10^{-10}\ \text{m}\right)\sqrt{\frac{3.7 \times 10^{-9}\ \text{N}}{8.99 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2}} = 3.2 \times 10^{-19}\ \text{C}.$$

(b) Let n be the number of electrons missing from each ion. Then, $ne = q$, or

$$n = \frac{q}{e} = \frac{3.2\times10^{-9}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 2.$$

LEARN Electric charge is quantized. This means that any charge can be written as $q = ne$, where n is an integer (positive or negative), and $e = 1.6\times10^{-19}$ C is the elementary charge.

28. Keeping in mind that an ampere is a coulomb per second (1 A = 1 C/s), and that a minute is 60 seconds, the charge (in absolute value) that passes through the chest is

$$|\,q\,| = (\ 0.300\ \text{C/s}\ )\ (\ 120\ \text{s}\ ) = 36.0\ \text{C}\ .$$

This charge consists of n electrons (each of which has an absolute value of charge equal to e). Thus,

$$n = \frac{|\,q\,|}{e} = \frac{36.0\ \text{C}}{1.60\ \text{x}\ 10^{-19}\ \text{C}} = 2.25\times10^{20}\ .$$

29. (a) We note that tan(30°) = $1/\sqrt{3}$. In the initial (highly symmetrical) configuration, the net force on the central bead is in the $-y$ direction and has magnitude $3F$ where F is the Coulomb's law force of one bead on another at distance d = 10 cm. This is due to the fact that the forces exerted on the central bead (in the initial situation) by the beads on the x axis cancel each other; also, the force exerted "downward" by bead 4 on the central bead is four times larger than the "upward" force exerted by bead 2. This net force along the y axis does not change as bead 1 is now moved, though there is now a nonzero x-component F_x. The components are now related by

$$\tan(30°) = \frac{F_x}{F_y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{F_x}{3F}$$

which implies $F_x = \sqrt{3}\,F$. Now, bead 3 exerts a "leftward" force of magnitude F on the central bead, while bead 1 exerts a "rightward" force of magnitude F'. Therefore,

$$F' - F = \sqrt{3}\,F. \quad \Rightarrow \quad F' = (\sqrt{3} + 1)\,F\ .$$

The fact that Coulomb's law depends inversely on distance-squared then implies

$$r^2 = \frac{d^2}{\sqrt{3}+1} \Rightarrow r = \frac{d}{\sqrt{\sqrt{3}+1}} = \frac{10\ \text{cm}}{\sqrt{\sqrt{3}+1}} = \frac{10\ \text{cm}}{1.65} = 6.05\ \text{cm}$$

where r is the distance between bead 1 and the central bead. This corresponds to $x = -6.05$ cm.

(b) To regain the condition of high symmetry (in particular, the cancellation of x-components) bead 3 must be moved closer to the central bead so that it, too, is the distance r (as calculated in part (a)) away from it.

30. (a) Let x be the distance between particle 1 and particle 3. Thus, the distance between particle 3 and particle 2 is $L - x$. Both particles exert leftward forces on q_3 (so long as it is on the line between them), so the magnitude of the net force on q_3 is

$$F_{\text{net}} = |\vec{F}_{13}| + |\vec{F}_{23}| = \frac{|q_1 q_3|}{4\pi\varepsilon_0 x^2} + \frac{|q_2 q_3|}{4\pi\varepsilon_0 (L-x)^2} = \frac{e^2}{\pi\varepsilon_0}\left(\frac{1}{x^2} + \frac{27}{(L-x)^2}\right)$$

with the values of the charges (stated in the problem) plugged in. Finding the value of x that minimizes this expression leads to $x = \frac{1}{4}L$. Thus, $x = 2.00$ cm.

(b) Substituting $x = \frac{1}{4}L$ back into the expression for the net force magnitude and using the standard value for e leads to $F_{\text{net}} = 9.21 \times 10^{-24}$ N.

31. The unit ampere is discussed in Section 21-4. The proton flux is given as 1500 protons per square meter per second, where each proton provides a charge of $q = +e$. The current through the spherical area $4\pi R^2 = 4\pi(6.37 \times 10^6 \text{ m})^2 = 5.1 \times 10^{14} \text{ m}^2$ would be

$$i = \left(5.1\times10^{14}\,\text{m}^2\right)\left(1500\,\frac{\text{protons}}{\text{s}\cdot\text{m}^2}\right)\left(1.6\times10^{-19}\,\text{C/proton}\right) = 0.122\,\text{A}.$$

32. Since the graph crosses zero, q_1 must be positive-valued: $q_1 = +8.00e$. We note that it crosses zero at $r = 0.40$ m. Now the asymptotic value of the force yields the magnitude and sign of q_2:

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = F \quad \Rightarrow \quad q_2 = \left(\frac{1.5\times10^{-25}}{kq_1}\right) r^2 = 2.086 \times 10^{-18}\,\text{C} = 13e.$$

33. The volume of 250 cm^3 corresponds to a mass of 250 g since the density of water is 1.0 g/cm^3. This mass corresponds to 250/18 = 14 moles since the molar mass of water is 18. There are ten protons (each with charge $q = +e$) in each molecule of H_2O, so

$$Q = 14N_A q = 14\left(6.02\times10^{23}\right)(10)\left(1.60\times10^{-19}\,\text{C}\right) = 1.3\times10^7\,\text{C}.$$

34. Let d be the vertical distance from the coordinate origin to $q_3 = -q$ and $q_4 = -q$ on the $+y$ axis, where the symbol q is assumed to be a positive value. Similarly, d is the (positive) distance from the origin $q_4 = -$ on the $-y$ axis. If we take each angle θ in the figure to be positive, then we have

$$\tan\theta = d/R \text{ and } \cos\theta = R/r,$$

where r is the dashed line distance shown in the figure. The problem asks us to consider θ to be a variable in the sense that, once the charges on the x axis are fixed in place (which determines R), d can then be arranged to some multiple of R, since $d = R \tan\theta$. The aim of this exploration is to show that if q is bounded then θ (and thus d) is also bounded.

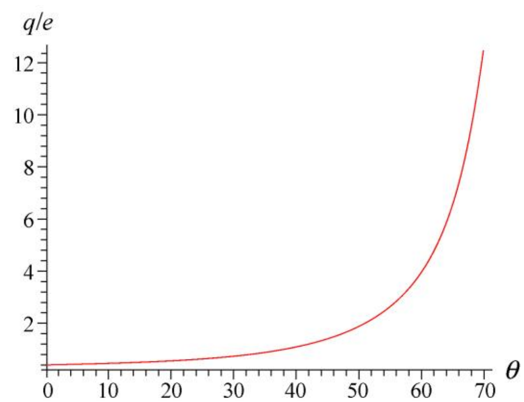
From symmetry, we see that there is no net force in the vertical direction on $q_2 = -e$ sitting at a distance R to the left of the coordinate origin. We note that the net x force caused by q_3 and q_4 on the y-axis will have a magnitude equal to

$$2\frac{qe}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{2qe\cos\theta}{4\pi\varepsilon_0 (R/\cos\theta)^2} = \frac{2qe\cos^3\theta}{4\pi\varepsilon_0 R^2} \ .$$

Consequently, to achieve a zero net force along the x axis, the above expression must equal the magnitude of the repulsive force exerted on q_2 by $q_1 = -e$. Thus,

$$\frac{2qe\cos^3\theta}{4\pi\varepsilon_0 R^2} = \frac{e^2}{4\pi\varepsilon_0 R^2} \quad \Rightarrow \quad q = \frac{e}{2\cos^3\theta} \, .$$

Below we plot q/e as a function of the angle (in degrees):



The graph suggests that $q/e < 5$ for $\theta < 60^\circ$, roughly. We can be more precise by solving the above equation. The requirement that $q \le 5e$ leads to

$$\frac{e}{2\cos^3\theta} \le 5e \quad \Rightarrow \quad \frac{1}{(10)^{1/3}} \le \cos\theta$$

which yields $\theta \le 62.34^\circ$. The problem asks for "physically possible values," and it is reasonable to suppose that only positive-integer-multiple values of e are allowed for q. If we let $q = ne$, for $n = 1 \ldots 5$, then θ_N will be found by taking the inverse cosine of the cube root of $(1/2n)$.

(a) The smallest value of angle is θ_1 = 37.5º (or 0.654 rad).

(b) The second smallest value of angle is θ_2 = 50.95º (or 0.889 rad).

(c) The third smallest value of angle is θ_3 = 56.6º (or 0.988 rad).

35. **THINK** Our system consists of 8 Cs^+ ions at the corners of a cube and a Cl^- ion at the cube's center. To calculate the electrostatic force on the Cl^- ion, we apply the superposition principle and make use of the symmetry property of the configuration.

EXPRESS In (a) where all 8 Cs^+ ions are present, every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is attractive and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube.

In (b) where one Cs^+ ion is missing at the corner, rather than remove a cesium ion, we superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

ANALYZE (a) Since the two Cs^+ ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) The length of a body diagonal of a cube is $\sqrt{3}a$, where a is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = \left(\sqrt{3}/2\right)a$. The force has magnitude

$$F = k\frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(3/4)\left(0.40\times10^{-9}\ \text{m}\right)^2} = 1.9\times10^{-9}\,\text{N}.$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

LEARN When solving electrostatic problems involving a discrete number of charges, symmetry argument can often be applied to simplify the problem.

36. (a) Since the proton is positively charged, the emitted particle must be a positron (as opposed to the negatively charged electron) in accordance with the law of charge conservation.

(b) In this case, the initial state had zero charge (the neutron is neutral), so the sum of charges in the final state must be zero. Since there is a proton in the final state, there should also be an electron (as opposed to a positron) so that $\Sigma q = 0$.

37. **THINK** Charges are conserved in nuclear reactions.

EXPRESS We note that none of the reactions given include a beta decay (see Chapter 42), so the number of protons (Z), the number of neutrons (N), and the number of electrons are each conserved. The mass number (total number of nucleons) is defined as $A = N + Z$. Atomic numbers (number of protons) and molar masses can be found in Appendix F of the text.

ANALYZE (a) ^{1}H has 1 proton, 1 electron, and 0 neutrons and ^{9}Be has 4 protons, 4 electrons, and 9 – 4 = 5 neutrons, so X has 1 + 4 = 5 protons, 1 + 4 = 5 electrons, and 0 + 5 – 1 = 4 neutrons. One of the neutrons is freed in the reaction. X must be boron with a molar mass of 5 + 4 = 9 g/mol: ^{9}B.

(b) ^{12}C has 6 protons, 6 electrons, and 12 – 6 = 6 neutrons and ^{1}H has 1 proton, 1 electron, and 0 neutrons, so X has 6 + 1 = 7 protons, 6 + 1 = 7 electrons, and 6 + 0 = 6 neutrons. It must be nitrogen with a molar mass of 7 + 6 = 13 g/mol: ^{13}N.

(c) ^{15}N has 7 protons, 7 electrons, and 15 – 7 = 8 neutrons; ^{1}H has 1 proton, 1 electron, and 0 neutrons; and ^{4}He has 2 protons, 2 electrons, and 4 – 2 = 2 neutrons; so X has 7 + 1 – 2 = 6 protons, 6 electrons, and 8 + 0 – 2 = 6 neutrons. It must be carbon with a molar mass of 6 + 6 = 12 g/mol: ^{12}C.

LEARN A general expression for the reaction can be expressed as

$$^{A_1}_{Z_1}X1_{N_1} + {}^{A_2}_{Z_2}X2_{N_2} \rightarrow {}^{A_3}_{Z_3}X3_{N_3} + {}^{A_4}_{Z_4}X4_{N_4}$$

where $A_i = Z_i + N_i$. Since the number of protons (Z), the number of neutrons (N), and the number of nucleons (A) are each conserved, we have $A_1 + A_2 = A_3 + A_4$, $Z_1 + Z_2 = Z_3 + Z_4$ and $N_1 + N_2 = N_3 + N_4$.

38. As a result of the first action, both sphere W and sphere A possess charge $\frac{1}{2}q_A$, where q_A is the initial charge of sphere A. As a result of the second action, sphere W has charge

$$\frac{1}{2}\left(\frac{q_A}{2} - 32e\right).$$

As a result of the final action, sphere W now has charge equal to

$$\frac{1}{2}\left[\frac{1}{2}\left(\frac{q_A}{2}-32e\right)+48e\right].$$

Setting this final expression equal to +18e as required by the problem leads (after a couple of algebra steps) to the answer: $q_A = +16e$.

39. **THINK** We have two discrete charges in the xy-plane. The electrostatic force on particle 2 due to particle 1 has both x and y components.

EXPRESS Using Coulomb's law, the magnitude of the force of particle 1 on particle 2 is $F_{21} = k\frac{q_1 q_2}{r^2}$, where $r=\sqrt{d_1^2+d_2^2}$ and $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. Since both q_1 and q_2 are positively charged, particle 2 is repelled by particle 1, so the direction of $\vec{F}_{21}$ is away from particle 1 and toward 2. In unit-vector notation, $\vec{F}_{21} = F_{21}\hat{\text{r}}$, where

$$\hat{\text{r}} = \frac{\vec{r}}{r} = \frac{d_2\hat{\text{i}} - d_1\hat{\text{j}}}{\sqrt{d_1^2+d_2^2}}.$$

The x component of $\vec{F}_{21}$ is $F_{21,x} = F_{21}d_2/\sqrt{d_1^2+d_2^2}$.

ANALYZE Combining the expressions above, we obtain

$$\begin{aligned}F_{21,x} &= k\frac{q_1q_2d_2}{r^3} = k\frac{q_1q_2d_2}{(d_1^2+d_2^2)^{3/2}}\\ &= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4\cdot1.60\times10^{-19}\ \text{C})(6\cdot1.60\times10^{-19}\ \text{C})(6.00\times10^{-3}\ \text{m})}{\left[(2.00\times10^{-3}\ \text{m})^2+(6.00\times10^{-3}\ \text{m})^2\right]^{3/2}}\\ &= 1.31\times10^{-22}\ \text{N}\end{aligned}$$

LEARN In a similar manner, we find the y component of $\vec{F}_{21}$ to be

$$\begin{aligned}F_{21,y} &= -k\frac{q_1q_2d_1}{r^3} = -k\frac{q_1q_2d_1}{(d_1^2+d_2^2)^{3/2}}\\ &= -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4\cdot1.60\times10^{-19}\ \text{C})(6\cdot1.60\times10^{-19}\ \text{C})(2.00\times10^{-3}\ \text{m})}{\left[(2.00\times10^{-3}\ \text{m})^2+(6.00\times10^{-3}\ \text{m})^2\right]^{3/2}}\\ &= -0.437\times10^{-22}\ \text{N}.\end{aligned}$$

Thus, $\vec{F}_{21} = (1.31\times10^{-22}\ \text{N})\hat{\text{i}} - (0.437\times10^{-22}\ \text{N})\hat{\text{j}}$.

40. Regarding the forces on q_3 exerted by q_1 and q_2, one must "push" and the other must "pull" in order that the net force is zero; hence, q_1 and q_2 have opposite signs. For individual forces to cancel, their magnitudes must be equal:

$$k\frac{|q_1||q_3|}{(L_{12}+L_{23})^2}=k\frac{|q_2||q_3|}{(L_{23})^2}.$$

With $L_{23}=2.00L_{12}$, the above expression simplifies to $\frac{|q_1|}{9}=\frac{|q_2|}{4}$. Therefore, $q_1=-9q_2/4$, or $q_1/q_2=-2.25$.

41. (a) The magnitudes of the gravitational and electrical forces must be the same:

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}=G\frac{mM}{r^2}$$

where q is the charge on either body, r is the center-to-center separation of Earth and Moon, G is the universal gravitational constant, M is the mass of Earth, and m is the mass of the Moon. We solve for q:

$$q=\sqrt{4\pi\varepsilon_0 GmM}.$$

According to Appendix C of the text, $M = 5.98 \times 10^{24}$ kg, and $m = 7.36 \times 10^{22}$ kg, so (using $4\pi\varepsilon_0 = 1/k$) the charge is

$$q=\sqrt{\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(7.36\times10^{22}\ \mathrm{kg})(5.98\times10^{24}\ \mathrm{kg})}{8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}}}=5.7\times10^{13}\ \mathrm{C}.$$

(b) The distance r cancels because both the electric and gravitational forces are proportional to $1/r^2$.

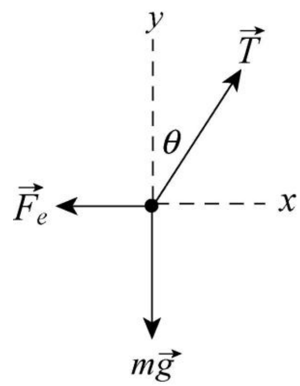
(c) The charge on a hydrogen ion is $e = 1.60 \times 10^{-19}$ C, so there must be

$$n=\frac{q}{e}=\frac{5.7\times10^{13}\ \mathrm{C}}{1.6\times10^{-19}\ \mathrm{C}}=3.6\times10^{32}\ \text{ions}.$$

Each ion has a mass of $m_i = 1.67 \times 10^{-27}$ kg, so the total mass needed is

$$m=nm_i=(3.6\times10^{32})(1.67\times10^{-27}\ \mathrm{kg})=6.0\times10^{5}\ \mathrm{kg}.$$

42. (a) A force diagram for one of the balls is shown below. The force of gravity $m\vec{g}$ acts downward, the electrical force $\vec{F}_e$ of the other ball acts to the left, and the tension in the thread acts along the thread, at the angle θ to the vertical. The ball is in equilibrium, so its acceleration is zero. The y component of Newton's second law yields $T\cos\theta - mg = 0$ and the x component yields $T\sin\theta - F_e = 0$. We solve the first equation for T and obtain $T = mg/\cos\theta$. We substitute the result into the second to obtain $mg\tan\theta - F_e = 0$.



Examination of the geometry of the figure shown leads to $\tan\theta = \dfrac{x/2}{\sqrt{L^2 - (x/2)^2}}$.

If L is much larger than x (which is the case if θ is very small), we may neglect $x/2$ in the denominator and write $\tan\theta \approx x/2L$. This is equivalent to approximating $\tan\theta$ by $\sin\theta$. The magnitude of the electrical force of one ball on the other is

$$F_e = \frac{q^2}{4\pi\varepsilon_0 x^2}$$

by Eq. 21-4. When these two expressions are used in the equation $mg\tan\theta = F_e$, we obtain

$$\frac{mgx}{2L} \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} \;\Rightarrow\; x \approx \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}.$$

(b) We solve $x^3 = 2kq^2L/mg$ for the charge (using Eq. 21-5):

$$q = \sqrt{\frac{mgx^3}{2kL}} = \sqrt{\frac{(0.010\,\text{kg})(9.8\,\text{m/s}^2)(0.050\,\text{m})^3}{2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.20\,\text{m})}} = \pm 2.4\times10^{-8}\,\text{C}.$$

Thus, the magnitude is $|q| = 2.4\times10^{-8}\,\text{C}$.

43. (a) If one of them is discharged, there would no electrostatic repulsion between the two balls and they would both come to the position $\theta = 0$, making contact with each other.

(b) A redistribution of the remaining charge would then occur, with each of the balls getting $q/2$. Then they would again be separated due to electrostatic repulsion, which results in the new equilibrium separation

$$x' = \left[\frac{(q/2)^2 L}{2\pi\varepsilon_0 mg}\right]^{1/3} = \left(\frac{1}{4}\right)^{1/3} x = \left(\frac{1}{4}\right)^{1/3}(5.0\text{ cm}) = 3.1\text{ cm}.$$

44. **THINK** The problem compares the electrostatic force between two protons and the gravitational force by Earth on a proton.

EXPRESS The magnitude of the gravitational force on a proton near the surface of the Earth is $F_g = mg$, where $m = 1.67\times10^{-27}$ kg is the mass of the proton. On the other hand, the electrostatic force between two protons separated by a distance r is $F_e = kq^2/r$. When the two forces are equal, we have $kq^2/r^2 = mg$.

ANALYZE Solving for r, we obtain

$$r = q\sqrt{\frac{k}{mg}} = (1.60\times10^{-19}\text{ C})\sqrt{\frac{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}{(1.67\times10^{-27}\text{ kg})(9.8\text{ m/s}^2)}} = 0.119\text{ m}.$$

LEARN The electrostatic force at this distance is $F_e = F_g = 1.64\times10^{-26}$ N.

45. There are two protons (each with charge $q = +e$) in each molecule, so

$$Q = N_A q = (6.02\times10^{23})(2)(1.60\times10^{-19}\text{ C}) = 1.9\times10^5\text{ C} = 0.19\text{ MC}.$$

46. Let $\vec{F}_{12}$ denotes the force on q_1 exerted by q_2 and F_{12} be its magnitude.

(a) We consider the net force on q_1. $\vec{F}_{12}$ points in the $+x$ direction since q_1 is attracted to q_2. $\vec{F}_{13}$ and $\vec{F}_{14}$ both point in the $-x$ direction since q_1 is repelled by q_3 and q_4. Thus, using $d = 0.0200$ m, the net force is

$$F_1 = F_{12} - F_{13} - F_{14} = \frac{2e|-e|}{4\pi\varepsilon_0 d^2} - \frac{(2e)(e)}{4\pi\varepsilon_0 (2d)^2} - \frac{(2e)(4e)}{4\pi\varepsilon_0 (3d)^2} = \frac{11}{18}\frac{e^2}{4\pi\varepsilon_0 d^2}$$

$$= \frac{11}{18}\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(2.00\times10^{-2}\text{ m})^2} = 3.52\times10^{-25}\text{ N}$$

or $\vec{F}_1 = (3.52\times10^{-25}\text{ N})\hat{\text{i}}$.

(b) We now consider the net force on q_2. We note that $\vec{F}_{21} = -\vec{F}_{12}$ points in the $-x$ direction, and $\vec{F}_{23}$ and $\vec{F}_{24}$ both point in the $+x$ direction. The net force is

$$F_{23} + F_{24} - F_{21} = \frac{4e|-e|}{4\pi\varepsilon_0 (2d)^2} + \frac{e|-e|}{4\pi\varepsilon_0 d^2} - \frac{2e|-e|}{4\pi\varepsilon_0 d^2} = 0 .$$

47. We are looking for a charge q that, when placed at the origin, experiences $\vec{F}_{\text{net}} = 0$, where

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 .$$

The magnitude of these individual forces are given by Coulomb's law, Eq. 21-1, and without loss of generality we assume $q > 0$. The charges q_1 (+6 μC), q_2 (–4 μC), and q_3 (unknown), are located on the $+x$ axis, so that we know $\vec{F}_1$ points toward $-x$, $\vec{F}_2$ points toward $+x$, and $\vec{F}_3$ points toward $-x$ if $q_3 > 0$ and points toward $+x$ if $q_3 < 0$. Therefore, with r_1 = 8 m, r_2 = 16 m and r_3 = 24 m, we have

$$0 = -k\frac{q_1 q}{r_1^2} + k\frac{|q_2| q}{r_2^2} - k\frac{q_3 q}{r_3^2} .$$

Simplifying, this becomes

$$0 = -\frac{6}{8^2} + \frac{4}{16^2} - \frac{q_3}{24^2}$$

where q_3 is now understood to be in μC. Thus, we obtain q_3 = –45 μC.

48. (a) Since q_A = –2.00 nC and q_C = +8.00 nC, Eq. 21-4 leads to

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-2.00\times10^{-9}\ \text{C})(8.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 3.60\times10^{-6}\ \text{N}.$$

(b) After making contact with each other, both A and B have a charge of

$$\frac{q_A + q_B}{2} = \left(\frac{-2.00 + (-4.00)}{2}\right)\ \text{nC} = -3.00\ \text{nC}.$$

When B is grounded its charge is zero. After making contact with C, which has a charge of +8.00 nC, B acquires a charge of [0 + (–8.00 nC)]/2 = –4.00 nC, which charge C has as well. Finally, we have Q_A = –3.00 nC and $Q_B = Q_C$ = –4.00 nC. Therefore,

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-3.00\times10^{-9}\ \text{C})(-4.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 2.70\times10^{-6}\ \text{N}.$$

(c) We also obtain

$$|\vec{F}_{BC}| = \frac{|q_B q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(-4.00\times10^{-9}\ \mathrm{C})(-4.00\times10^{-9}\ \mathrm{C})|}{(0.200\ \mathrm{m})^2} = 3.60\times10^{-6}\ \mathrm{N}.$$

49. Coulomb's law gives

$$F = \frac{|q|^2}{4\pi\varepsilon_0 r^2} = \frac{k(e/3)^2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(1.60\times10^{-19}\ \mathrm{C})^2}{9(2.6\times10^{-15}\ \mathrm{m})^2} = 3.8\,\mathrm{N}.$$

50. (a) Since the rod is in equilibrium, the net force acting on it is zero, and the net torque about any point is also zero. We write an expression for the net torque about the bearing, equate it to zero, and solve for x. The charge Q on the left exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\ (qQ/h^2)$, at a distance $L/2$ from the bearing. We take the torque to be negative. The attached weight exerts a downward force of magnitude W, at a distance $x - L/2$ from the bearing. This torque is also negative. The charge Q on the right exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\ (2qQ/h^2)$, at a distance $L/2$ from the bearing. This torque is positive. The equation for rotational equilibrium is

$$\frac{-1}{4\pi\varepsilon_0}\frac{qQ}{h^2}\frac{L}{2} - W\left(x - \frac{L}{2}\right) + \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2}\frac{L}{2} = 0.$$

The solution for x is

$$x = \frac{L}{2}\left(1 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2 W}\right).$$

(b) If F_N is the magnitude of the upward force exerted by the bearing, then Newton's second law (with zero acceleration) gives

$$W - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2} - \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2} - F_N = 0.$$

We solve for h so that $F_N = 0$. The result is

$$h = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{3qQ}{W}}.$$

51. The charge dq within a thin section of the rod (of thickness dx) is $\rho A dx$ where $A = 4.00\times10^{-4}\,\mathrm{m}^2$ and ρ is the charge per unit volume. The number of (excess) electrons in the rod (of length L = 2.00 m) is $n = q/(-e)$ where e is given in Eq. 21-12.

(a) In the case where $\rho = -4.00\times10^{-6}\ \mathrm{C/m^3}$, we have

$$n = \frac{q}{-e} = \frac{\rho A}{-e}\int_0^L dx = \frac{|\rho|AL}{e} = 2.00\times10^{10}.$$

(b) With $\rho = bx^2$ ($b = -2.00 \times 10^{-6}$ C/m^5) we obtain

$$n = \frac{bA}{-e}\int_0^L x^2\,dx = \frac{|b|AL^3}{3e} = 1.33\times10^{10}.$$

52. For the Coulomb force to be sufficient for circular motion at that distance (where r = 0.200 m and the acceleration needed for circular motion is $a = v^2/r$) the following equality is required:

$$\frac{Qq}{4\pi\varepsilon_0 r^2} = -\frac{mv^2}{r}.$$

With $q = 4.00 \times 10^{-6}$ C, m = 0.000800 kg, v = 50.0 m/s, this leads to

$$Q = -\frac{4\pi\varepsilon_0 rmv^2}{q} = -\frac{(0.200\text{ m})(8.00\times10^{-4}\text{ kg})(50.0\text{ m/s})^2}{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(4.00\times10^{-6}\text{C})} = -1.11\times10^{-5}\text{C}.$$

53. (a) Using Coulomb's law, we obtain

$$F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.00\text{C}\right)^2}{\left(1.00\text{m}\right)^2} = 8.99\times10^9\text{ N}.$$

(b) If r = 1000 m, then

$$F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.00\text{C}\right)^2}{\left(1.00\times10^3\text{m}\right)^2} = 8.99\times10^3\text{ N}.$$

54. Let q_1 be the charge of one part and q_2 that of the other part; thus, $q_1 + q_2 = Q = 6.0$ μC. The repulsive force between them is given by Coulomb's law:

$$F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} = \frac{q_1(Q-q_1)}{4\pi\varepsilon_0 r^2}.$$

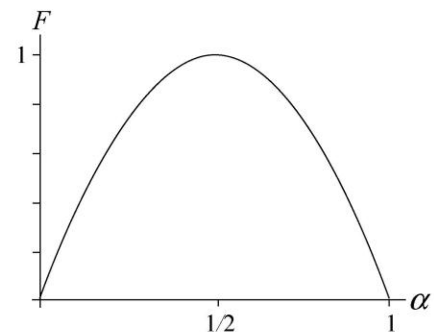
If we maximize this expression by taking the derivative with respect to q_1 and setting equal to zero, we find $q_1 = Q/2$, which might have been anticipated (based on symmetry arguments). This implies $q_2 = Q/2$ also. With r = 0.0030 m and $Q = 6.0 \times 10^{-6}$ C, we find

$$F = \frac{(Q/2)(Q/2)}{4\pi\varepsilon_0 r^2} = \frac{1}{4}\frac{Q^2}{4\pi\varepsilon_0 r^2} = \frac{1}{4}\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\,\text{C}\right)^2}{\left(3.00\times10^{-3}\,\text{m}\right)^2} \approx 9.0\times10^3\ \text{N}.$$

55. The two charges are $q = \alpha Q$ (where α is a pure number presumably less than 1 and greater than zero) and $Q - q = (1 - \alpha)Q$. Thus, Eq. 21-4 gives

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(\alpha Q)((1-\alpha)Q)}{d^2} = \frac{Q^2\alpha(1-\alpha)}{4\pi\varepsilon_0 d^2}.$$

The graph below, of F versus α, has been scaled so that the maximum is 1. In actuality, the maximum value of the force is $F_{max} = Q^2/16\pi\varepsilon_0 d^2$.



(a) It is clear that $\alpha = 1/2 = 0.5$ gives the maximum value of F.

(b) Seeking the half-height points on the graph is difficult without grid lines or some of the special tracing features found in a variety of modern calculators. It is not difficult to algebraically solve for the half-height points (this involves the use of the quadratic formula). The results are

$$\alpha_1 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right) \approx 0.15 \quad \text{and} \quad \alpha_2 = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) \approx 0.85.$$

Thus, the smaller value of α is $\alpha_1 = 0.15$,

(c) and the larger value of α is $\alpha_2 = 0.85$.

56. (a) Equation 21-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{2.00\times10^{-6}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 1.25\times10^{13}\ \text{electrons}.$$

(b) Since you have the excess electrons (and electrons are lighter and more mobile than protons) then the electrons "leap" from you to the faucet instead of protons moving from the faucet to you (in the process of neutralizing your body).

(c) Unlike charges attract, and the faucet (which is grounded and is able to gain or lose any number of electrons due to its contact with Earth's large reservoir of mobile charges) becomes positively charged, especially in the region closest to your (negatively charged) hand, just before the spark.

(d) The cat is positively charged (before the spark), and by the reasoning given in part (b) the flow of charge (electrons) is from the faucet to the cat.

(e) If we think of the nose as a conducting sphere, then the side of the sphere closest to the fur is of one sign (of charge) and the side furthest from the fur is of the opposite sign (which, additionally, is oppositely charged from your bare hand, which had stroked the cat's fur). The charges in your hand and those of the furthest side of the "sphere" therefore attract each other, and when close enough, manage to neutralize (due to the "jump" made by the electrons) in a painful spark.

57. If the relative difference between the proton and electron charges (in absolute value) were

$$\frac{q_p - |q_e|}{e} = 0.0000010$$

then the actual difference would be $q_p - |q_e| = 1.6 \times 10^{-25}$ C. Amplified by a factor of 29 × 3 × 10^{22} as indicated in the problem, this amounts to a deviation from perfect neutrality of

$$\Delta q = (29 \times 3 \times 10^{22})(1.6 \times 10^{-25}\ \text{C}) = 0.14\ \text{C}$$

in a copper penny. Two such pennies, at r = 1.0 m, would therefore experience a very large force. Equation 21-1 gives

$$F = k\frac{(\Delta q)^2}{r^2} = 1.7 \times 10^8\ \text{N}.$$

58. Charge $q_1 = -80 \times 10^{-6}$ C is at the origin, and charge $q_2 = +40 \times 10^{-6}$ C is at x = 0.20 m. The force on $q_3 = +20 \times 10^{-6}$ C is due to the attractive and repulsive forces from q_1 and q_2, respectively. In symbols, $\vec{F}_{3\,\text{net}} = \vec{F}_{31} + \vec{F}_{32}$, where

$$\left|\vec{F}_{31}\right| = k\frac{q_3\,|q_1|}{r_{31}^2}, \quad |\vec{F}_{32}| = k\frac{q_3 q_2}{r_{32}^2}.$$

(a) In this case r_{31} = 0.40 m and r_{32} = 0.20 m, with $\vec{F}_{31}$ directed toward $-x$ and $\vec{F}_{32}$ directed in the $+x$ direction. Using the value of k in Eq. 21-5, we obtain

$$\vec{F}_{3\,\text{net}} = -\left|\vec{F}_{31}\right|\hat{\text{i}} + |\vec{F}_{32}|\,\hat{\text{i}} = \left(-k\frac{q_3|q_1|}{r_{31}^2} + k\frac{q_3q_2}{r_{32}^2}\right)\hat{\text{i}} = kq_3\left(-\frac{|q_1|}{r_{31}^2} + \frac{q_2}{r_{32}^2}\right)\hat{\text{i}}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(20\times10^{-6}\,\text{C})\left(\frac{-80\times10^{-6}\,\text{C}}{(0.40\text{m})^2} + \frac{+40\times10^{-6}\,\text{C}}{(0.20\text{m})^2}\right)\hat{\text{i}}$$

$$= (89.9\ \text{N})\hat{\text{i}}\ .$$

(b) In this case r_{31} = 0.80 m and r_{32} = 0.60 m, with $\vec{F}_{31}$ directed toward $-x$ and $\vec{F}_{32}$ toward $+x$. Now we obtain

$$\vec{F}_{3\,\text{net}} = -\left|\vec{F}_{31}\right|\hat{\text{i}} + |\vec{F}_{32}|\,\hat{\text{i}} = \left(-k\frac{q_3|q_1|}{r_{31}^2} + k\frac{q_3q_2}{r_{32}^2}\right)\hat{\text{i}} = kq_3\left(-\frac{|q_1|}{r_{31}^2} + \frac{q_2}{r_{32}^2}\right)\hat{\text{i}}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(20\times10^{-6}\,\text{C})\left(\frac{-80\times10^{-6}\,\text{C}}{(0.80\text{m})^2} + \frac{+40\times10^{-6}\,\text{C}}{(0.60\text{m})^2}\right)\hat{\text{i}}$$

$$= -(2.50\ \text{N})\hat{\text{i}}\ .$$

(c) Between the locations treated in parts (a) and (b), there must be one where $\vec{F}_{3\,\text{net}} = 0$. Writing r_{31} = x and r_{32} = x – 0.20 m, we equate $\left|\vec{F}_{31}\right|$ and $\left|\vec{F}_{32}\right|$, and after canceling common factors, arrive at

$$\frac{|q_1|}{x^2} = \frac{q_2}{(x-0.20\text{ m})^2}.$$

This can be further simplified to

$$\frac{(x-0.20\text{ m})^2}{x^2} = \frac{q_2}{|q_1|} = \frac{1}{2}.$$

Taking the (positive) square root and solving, we obtain x = 0.683 m. If one takes the negative root and 'solves', one finds the location where the net force *would* be zero *if* q_1 and q_2 were of like sign (which is not the case here).

(d) From the above, we see that y = 0.

59. The mass of an electron is $m = 9.11 \times 10^{-31}$ kg, so the number of electrons in a collection with total mass M = 75.0 kg is

$$n = \frac{M}{m} = \frac{75.0\,\text{kg}}{9.11\times10^{-31}\,\text{kg}} = 8.23\times10^{31}\ \text{electrons}.$$

The total charge of the collection is

$$q = -ne = -\left(8.23\times10^{31}\right)\left(1.60\times10^{-19}\text{ C}\right) = -1.32\times10^{13}\text{ C}.$$

60. We note that, as result of the fact that the Coulomb force is inversely proportional to r^2, a particle of charge Q that is distance d from the origin will exert a force on some charge q_o at the origin of equal strength as a particle of charge $4Q$ at distance $2d$ would exert on q_o. Therefore, $q_6 = +8e$ on the $-y$ axis could be replaced with a $+2e$ closer to the origin (at half the distance); this would add to the $q_5 = +2e$ already there and produce $+4e$ below the origin, which exactly cancels the force due to $q_2 = +4e$ above the origin.

Similarly, $q_4 = +4e$ to the far right could be replaced by a $+e$ at half the distance, which would add to $q_3 = +e$ already there to produce a $+2e$ at distance d to the right of the central charge q_7. The horizontal force due to this $+2e$ is cancelled exactly by that of $q_1 = +2e$ on the $-x$ axis, so that the net force on q_7 is zero.

61. (a) Charge $Q_1 = +80 \times 10^{-9}$ C is on the y axis at $y = 0.003$ m, and charge $Q_2 = +80\times10^{-9}\text{C}$ is on the y axis at $y = -0.003$ m. The force on particle 3 (which has a charge of $q = +18 \times 10^{-9}$ C) is due to the vector sum of the repulsive forces from Q_1 and Q_2. In symbols, $\vec{F}_{31} + \vec{F}_{32} = \vec{F}_3$, where

$$|\vec{F}_{31}| = k\frac{q_3|q_1|}{r_{31}^2}, \quad |\vec{F}_{32}| = k\frac{q_3 q_2}{r_{32}^2}.$$

Using the Pythagorean theorem, we have $r_{31} = r_{32} = 0.005$ m. In magnitude-angle notation (particularly convenient if one uses a vector-capable calculator in polar mode), the indicated vector addition becomes

$$\vec{F}_3 = \left(0.518\angle -37°\right) + \left(0.518\angle 37°\right) = \left(0.829\angle 0°\right).$$

Therefore, the net force is $\vec{F}_3 = (0.829\text{ N})\hat{\text{i}}$.

(b) Switching the sign of Q_2 amounts to reversing the direction of its force on q. Consequently, we have

$$\vec{F}_3 = \left(0.518\angle -37°\right) + \left(0.518\angle -143°\right) = \left(0.621\angle -90°\right).$$

Therefore, the net force is $\vec{F}_3 = -(0.621\text{ N})\hat{\text{j}}$.

62. **THINK** We have four discrete charges in the xy-plane. We use superposition principle to calculate the net electrostatic force on particle 4 due to the other three particles.

EXPRESS Using Coulomb's law, the magnitude of the force on particle 4 by particle *i* is $F_{4i} = k\frac{q_4 q_i}{r_{4i}^2}$. For example, the magnitude of $\vec{F}_{41}$ is

$$F_{41} = k\frac{|q_4||q_1|}{r_{41}^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\ \text{C})(3.20\times10^{-19}\ \text{C})}{(0.0300\ \text{m})^2} = 1.02\times10^{-24}\ \text{N}.$$

Since the force is attractive, $\hat{r}_{41} = -\cos\theta_1\hat{i} - \sin\theta_1\hat{j} = -\cos35°\hat{i} - \sin35°\hat{j} = -0.82\hat{i} - 0.57\hat{j}$. In unit-vector notation, we have

$$\vec{F}_{41} = F_{41}\hat{r}_{41} = (1.02\times10^{-24}\ \text{N})(-0.82\hat{i} - 0.57\hat{j}) = -(8.36\times10^{-25}\ \text{N})\hat{i} - (5.85\times10^{-24}\ \text{N})\hat{j}.$$

Similarly,

$$\vec{F}_{42} = -k\frac{|q_4||q_2|}{r_{42}^2}\hat{j} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\ \text{C})(3.20\times10^{-19}\ \text{C})}{(0.0200\ \text{m})^2}\hat{j} = -(2.30\times10^{-24}\ \text{N})\hat{j}$$

and

$$\vec{F}_{43} = -k\frac{|q_4||q_3|}{r_{43}^2}\hat{i} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.40\times10^{-19}\ \text{C})(3.20\times10^{-19}\ \text{C})}{(0.0200\ \text{m})^2}\hat{i} = -(4.60\times10^{-24}\ \text{N})\hat{j}.$$

ANALYZE (a) The net force on particle 4 is

$$\vec{F}_{4,\text{net}} = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43} = -(5.44\times10^{-24}\ \text{N})\hat{i} - (2.89\times10^{-24}\ \text{N})\hat{j}.$$

The magnitude of the force is

$$F_{4,\text{net}} = \sqrt{(-5.44\times10^{-24}\ \text{N})^2 + (-2.89\times10^{-24}\ \text{N})^2} = 6.16\times10^{-24}\ \text{N}.$$

(b) The direction of the net force is at an angle of

$$\varphi = \tan^{-1}\left(\frac{F_{4y,\text{net}}}{F_{4x,\text{net}}}\right) = \tan^{-1}\left(\frac{-2.89\times10^{-24}\ \text{N}}{-5.44\times10^{-24}\ \text{N}}\right) = 208°,$$

measured counterclockwise from the +*x* axis.

LEARN A nonzero net force indicates that particle 4 will be accelerated in the direction of the force.

63. The magnitude of the net force on the $q = 42 \times 10^{-6}$ C charge is

$$k\frac{q_1 q}{0.28^2} + k\frac{|q_2|q}{0.44^2}$$

where $q_1 = 30 \times 10^{-9}$ C and $|q_2| = 40 \times 10^{-9}$ C. This yields 0.22 N. Using Newton's second law, we obtain

$$m = \frac{F}{a} = \frac{0.22\,\text{N}}{100\times10^3\ \text{m/s}^2} = 2.2\times10^{-6}\,\text{kg}.$$

64. Let the two charges be q_1 and q_2. Then $q_1 + q_2 = Q = 5.0 \times 10^{-5}$ C. We use Eq. 21-1:

$$1.0\ \text{N} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)q_1 q_2}{\left(2.0\ \text{m}\right)^2}.$$

We substitute $q_2 = Q - q_1$ and solve for q_1 using the quadratic formula. The two roots obtained are the values of q_1 and q_2, since it does not matter which is which. We get 1.2×10^{-5} C and 3.8×10^{-5} C. Thus, the charge on the sphere with the smaller charge is 1.2×10^{-5} C.

65. When sphere C touches sphere A, they divide up their total charge (Q/2 plus Q) equally between them. Thus, sphere A now has charge 3Q/4, and the magnitude of the force of attraction between A and B becomes

$$F = k\frac{(3Q/4)(Q/4)}{d^2} = 4.68\times10^{-19}\,\text{N}.$$

66. With $F = m_e g$, Eq. 21-1 leads to

$$y^2 = \frac{ke^2}{m_e g} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(9.8\text{m/s}^2\right)}$$

which leads to $y = \pm 5.1$ m. We choose $y = -5.1$ m since the second electron must be below the first one, so that the repulsive force (acting on the first) is in the direction opposite to the pull of Earth's gravity.

67. **THINK** Our system consists of two charges along a straight line. We'd like to place a third charge so that the net force on it due to charges 1 and 2 vanishes.

EXPRESS The net force on particle 3 is the vector sum of the forces due to particles 1 and 2: $\vec{F}_{3,\text{net}} = \vec{F}_{31} + \vec{F}_{32}$. In order that $\vec{F}_{3,\text{net}} = 0$, particle 3 must be on the x axis and be attracted by one and repelled by another. As the result, it cannot be between particles 1 and 2, but instead either to the left of particle 1 or to the right of particle 2. Let q_3 be placed a distance x to the right of $q_1 = -5.00q$. Then its attraction to q_1 particle will be exactly balanced by its repulsion from $q_2 = +2.00q$:

$$F_{3x,\text{net}} = k\left[\frac{q_1 q_3}{x^2} + \frac{q_2 q_3}{(x-L)^2}\right] = kq_3 q\left[\frac{-5}{x^2} + \frac{2}{(x-L)^2}\right] = 0 .$$

ANALYZE (a) Cross-multiplying and taking the square root, we obtain

$$\frac{x}{x-L} = \sqrt{\frac{5}{2}}$$

which can be rearranged to produce

$$x = \frac{L}{1-\sqrt{2/5}} \approx 2.72\,L .$$

(b) The y coordinate of particle 3 is $y = 0$.

LEARN We can use the result obtained above for consistency check. We find the force on particle 3 due to particle 1 to be

$$F_{31} = k\frac{q_1 q_3}{x^2} = k\frac{(-5.00q)(q_3)}{(2.72L)^2} = -0.675\frac{kqq_3}{L^2} .$$

Similarly, the force on particle 3 due to particle 2 is

$$F_{32} = k\frac{q_2 q_3}{x^2} = k\frac{(+2.00q)(q_3)}{(2.72L-L)^2} = +0.675\frac{kqq_3}{L^2} .$$

Indeed, the sum of the two forces is zero.

68. The net charge carried by John whose mass is m is roughly

$$\begin{aligned} q &= (0.0001)\frac{mN_A Ze}{M} \\ &= (0.0001)\frac{(90\,\text{kg})(6.02\times10^{23}\text{ molecules/mol})(18\text{ electron proton pairs/molecule})(1.6\times10^{-19}\,\text{C})}{0.018\,\text{kg/mol}} \\ &= 8.7\times10^{5}\,\text{C}, \end{aligned}$$

and the net charge carried by Mary is half of that. So the electrostatic force between them is estimated to be

$$F \approx k\frac{q(q/2)}{d^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(8.7\times10^5\ \text{C})^2}{2(30\text{m})^2} \approx 4\times10^{18}\ \text{N}.$$

Thus, the order of magnitude of the electrostatic force is 10^{18} N .

69. We are concerned with the charges in the nucleus (not the "orbiting" electrons, if there are any). The nucleus of Helium has 2 protons and that of thorium has 90.

(a) Equation 21-1 gives

$$F = k\frac{q^2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2(1.60\times10^{-19}\ \text{C}))(90(1.60\times10^{-19}\ \text{C}))}{(9.0\times10^{-15}\ \text{m})^2} = 5.1\times10^2\ \text{N}.$$

(b) Estimating the helium nucleus mass as that of 4 protons (actually, that of 2 protons and 2 neutrons, but the neutrons have approximately the same mass), Newton's second law leads to

$$a = \frac{F}{m} = \frac{5.1\times10^2\ \text{N}}{4(1.67\times10^{-27}\ \text{kg})} = 7.7\times10^{28}\ \text{m/s}^2 .$$

70. For the net force on $q_1 = +Q$ to vanish, the x force component due to $q_2 = q$ must exactly cancel the force of attraction caused by $q_4 = -2Q$. Consequently,

$$\frac{Qq}{4\pi\varepsilon_0 a^2} = \frac{Q|2Q|}{4\pi\varepsilon_0(\sqrt{2}a)^2}\cos 45° = \frac{Q^2}{4\pi\varepsilon_0\sqrt{2}a^2}$$

or $q = Q/\sqrt{2}$. This implies that $q/Q = 1/\sqrt{2} = 0.707$.

71. (a) The second shell theorem states that a charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell. Thus, inside the spherical metal shell at $r = 0.500R < R$, the net force on the electron is zero, and therefore, $a = 0$.

(b) The first shell theorem states that a charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center. Thus, the magnitude of the Coulomb force on the electron at $r = 2.00R$ is

$$\begin{aligned} F &= k\frac{Q|e|}{r^2} = k\frac{(4\pi R^2\sigma)|e|}{(2.0R)^2} = k\pi\sigma|e| \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\pi(6.90\times10^{-13}\ \text{C/m}^2)(1.60\times10^{-19}\ \text{C}) \\ &= 3.12\times10^{-21}\ \text{N}, \end{aligned}$$

and the corresponding acceleration is

$$a = \frac{F}{m} = \frac{3.12\times10^{-21}\ \text{N}}{9.11\times10^{-31}\ \text{kg}} = 3.43\times10^{9}\ \text{m/s}^2 .$$

72. Since the total energy is conserved,

$$\frac{1}{2}m_e v_i^2 = \frac{1}{2}m_e v_f^2 - \frac{ke^2}{r_f}$$

where r_f is the distance between the electron and the proton. For $v_f = 2v_i$, we solve for r_f and obtain

$$r_f = \frac{2ke^2}{m_e(v_f^2 - v_i^2)} = \frac{2ke^2}{3m_e v_i^2} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{3(9.11\times10^{-31}\ \text{kg})(3.2\times10^5\ \text{m/s})^2}$$
$$= 1.64\times10^{-9}\ \text{m}$$

or about 1.6 nm.

73. (a) The Coulomb force between the electron and the proton provides the centripetal force that keeps the electron in circular orbit about the proton:

$$\frac{k|e|^2}{r^2} = \frac{m_e v^2}{r}$$

The smallest orbital radius is $r_1 = a_0 = 52.9\times10^{-12}$ m. The corresponding speed of the electron is

$$v_1 = \sqrt{\frac{k|e|^2}{m_e r_1}} = \sqrt{\frac{k|e|^2}{m_e a_0}} = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(52.9\times10^{-12}\ \text{m})}}$$
$$= 2.19\times10^6\ \text{m/s}.$$

(b) The radius of the second smallest orbit is $r_2 = (2)^2 a_0 = 4a_0$. Thus, the speed of the electron is

$$v_2 = \sqrt{\frac{k|e|^2}{m_e r_2}} = \sqrt{\frac{k|e|^2}{m_e(4a_0)}} = \frac{1}{2}v_1 = \frac{1}{2}(2.19\times10^6\ \text{m/s})$$
$$= 1.09\times10^6\ \text{m/s}.$$

(c) Since the speed is inversely proportional to $r^{1/2}$, the speed of the electron will decrease if it moves to larger orbits.

74. Electric current i is the rate dq/dt at which charge passes a point. With i = 0.83A, the time it takes for one mole of electron to pass through the lamp is

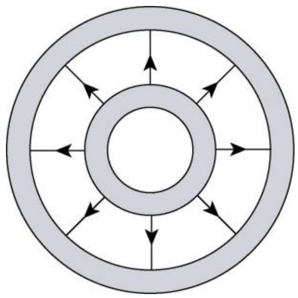
$$\Delta t = \frac{\Delta q}{i} = \frac{N_A e}{i} = \frac{(6.02\times10^{23})(1.6\times10^{-19}\text{ C})}{0.83\text{ A}} = 1.16\times10^5\text{ s} \approx 1.3\text{ days.}$$

75. The electrical force between an electron and a positron separated by a distance r is $F_e = ke^2/r^2$. On the other hand, the gravitational force between the two charges is $F_g = Gm_e^2/r^2$. Thus, the ratio of the two forces is

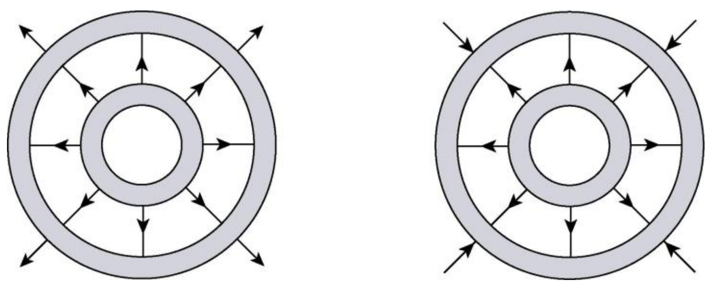
$$\frac{F_e}{F_g} = \frac{ke^2/r^2}{Gm_e^2/r^2} = \frac{ke^2}{Gm_e^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{ C})^2}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\text{ kg})^2} = 4.16\times10^{42}.$$

Chapter 22

1. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge that resides on the larger shell. The following sketch is for $q_1 = q_2$.



The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).



2. (a) We note that the electric field points leftward at both points. Using $\vec{F} = q_0\vec{E}$, and orienting our x axis rightward (so $\hat{\text{i}}$ points right in the figure), we find

$$\vec{F} = \left(+1.6\times10^{-19}\,\text{C}\right)\left(-40\frac{\text{N}}{\text{C}}\,\hat{\text{i}}\right) = (-6.4\times10^{-18}\,\text{N})\,\hat{\text{i}}$$

which means the magnitude of the force on the proton is 6.4×10^{-18} N and its direction $(-\hat{\text{i}})$ is leftward.

(b) As the discussion in Section 22-2 makes clear, the field strength is proportional to the "crowdedness" of the field lines. It is seen that the lines are twice as crowded at A than at B, so we conclude that $E_A = 2E_B$. Thus, $E_B = 20$ N/C.

3. **THINK** Since the nucleus is treated as a sphere with uniform surface charge distribution, the electric field at the surface is exactly the same as it would be if the charge were all at the center.

EXPRESS The nucleus has a radius R = 6.64 fm and a total charge $q = Ze$, where $Z = 94$ for Pu. Thus, the magnitude of the electric field at the nucleus surface is

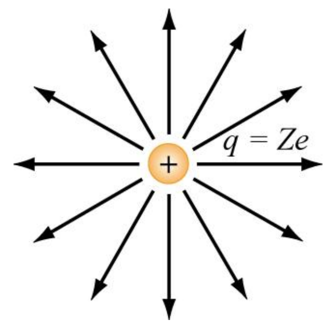
$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{Ze}{4\pi\varepsilon_0 R^2}.$$

ANALYZE (a) Substituting the values given, we find the field to be

$$E = \frac{Ze}{4\pi\varepsilon_0 R^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(94\right)\left(1.60\times10^{-19}\ \text{C}\right)}{\left(6.64\times10^{-15}\ \text{m}\right)^2} = 3.07\times10^{21}\ \text{N/C}.$$

(b) The field is normal to the surface. In addition, since the charge is positive, it points outward from the surface.

LEARN The direction of electric field lines is radially outward for a positive charge, and radially inward for a negative charge. The field lines of our nucleus are shown on the right.



4. With x_1 = 6.00 cm and x_2 = 21.00 cm, the point midway between the two charges is located at x = 13.5 cm. The values of the charge are

$$q_1 = -q_2 = -2.00\times10^{-7}\ \text{C},$$

and the magnitudes and directions of the individual fields are given by:

$$\vec{E}_1 = -\frac{|q_1|}{4\pi\varepsilon_0(x-x_1)^2}\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)|-2.00\times10^{-7}\ \text{C}|}{\left(0.135\ \text{m}-0.060\ \text{m}\right)^2}\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

$$\vec{E}_2 = -\frac{q_2}{4\pi\varepsilon_0(x-x_2)^2}\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-7}\ \text{C})}{\left(0.135\ \text{m}-0.210\ \text{m}\right)^2}\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

Thus, the net electric field is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = -(6.39\times10^5\ \text{N/C})\hat{\text{i}}$.

5. **THINK** The magnitude of the electric field produced by a point charge q is given by $E = |q|/4\pi\varepsilon_0 r^2$, where r is the distance from the charge to the point where the field has magnitude E.

EXPRESS From $E = |q|/4\pi\varepsilon_0 r^2$, the magnitude of the charge is $|q| = 4\pi\varepsilon_0 r^2 E$.

ANALYZE With $E = 2.0$ N/C at $r = 50$ cm $= 0.50$ m, we obtain

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.50\,\text{m})^2(2.0\,\text{N/C})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 5.6\times10^{-11}\,\text{C}.$$

LEARN To determine the sign of the charge, we would need to know the direction of the field. The field lines extend away from a positive charge and toward a negative charge.

6. We find the charge magnitude $|q|$ from $E = |q|/4\pi\varepsilon_0 r^2$:

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(1.00\,\text{N/C})(1.00\,\text{m})^2}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 1.11\times10^{-10}\,\text{C}.$$

7. **THINK** Our system consists of four point charges that are placed at the corner of a square. The total electric field at a point is the vector sum of the electric fields of individual charges.

EXPRESS Applying the superposition principle, the net electric field at the center of the square is

$$\vec{E} = \sum_{i=1}^{4}\vec{E}_i = \sum_{i=1}^{4}\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\hat{\text{r}}_i.$$

With $q_1 = +10$ nC, $q_2 = -20$ nC, $q_3 = +20$ nC, and $q_4 = -10$ nC, the x component of the electric field at the center of the square is given by, taking the signs of the charges into consideration,

$$\begin{aligned}E_x &= \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} - \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45^\circ \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(|q_1| + |q_2| - |q_3| - |q_4|\right)\frac{1}{\sqrt{2}}.\end{aligned}$$

Similarly, the y component of the electric field is

$$\begin{aligned}E_y &= \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} + \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45^\circ \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(-|q_1| + |q_2| + |q_3| - |q_4|\right)\frac{1}{\sqrt{2}}.\end{aligned}$$

The magnitude of the net electric field is $E = \sqrt{E_x^2 + E_y^2}$.

ANALYZE Substituting the values given, we obtain

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(|q_1|+|q_2|-|q_3|-|q_4|\right) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(10\text{ nC}+20\text{ nC}-20\text{ nC}-10\text{ nC}\right)=0$$

and

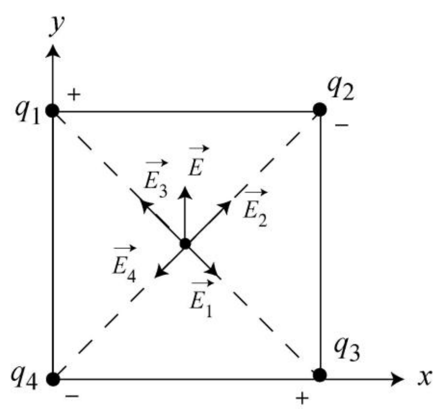
$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(-|q_1|+|q_2|+|q_3|-|q_4|\right) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(-10\text{ nC}+20\text{ nC}+20\text{ nC}-10\text{ nC}\right)$$

$$= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.0\times10^{-8}\ \text{C})\sqrt{2}}{(0.050\ \text{m})^2}$$

$$= 1.02\times10^5\ \text{N/C}.$$

Thus, the electric field at the center of the square is $\vec{E} = E_y\hat{\text{j}} = (1.02\times10^5\ \text{N/C})\hat{\text{j}}$.

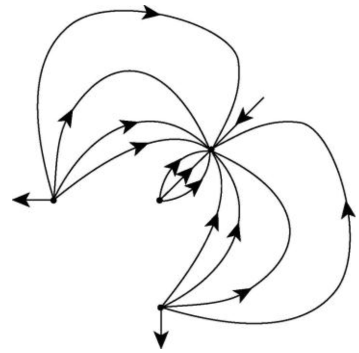
LEARN The net electric field at the center of the square is depicted in the figure below (not to scale). The field, pointing to the $+y$ direction, is the vector sum of the electric fields of individual charges.



8. We place the origin of our coordinate system at point P and orient our y axis in the direction of the $q_4 = -12q$ charge (passing through the $q_3 = +3q$ charge). The x axis is perpendicular to the y axis, and thus passes through the identical $q_1 = q_2 = +5q$ charges. The individual magnitudes $|\vec{E}_1|, |\vec{E}_2|, |\vec{E}_3|$, and $|\vec{E}_4|$ are figured from Eq. 22-3, where the absolute value signs for q_1, q_2, and q_3 are unnecessary since those charges are positive (assuming $q > 0$). We note that the contribution from q_1 cancels that of q_2 (that is, $|\vec{E}_1| = |\vec{E}_2|$), and the net field (if there is any) should be along the y axis, with magnitude equal to

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_4|}{(2d)^2} - \frac{q_3}{d^2}\right)\hat{\text{j}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{12q}{4d^2} - \frac{3q}{d^2}\right)\hat{\text{j}}$$

which is seen to be zero. A rough sketch of the field lines is shown next:



9. (a) The vertical components of the individual fields (due to the two charges) cancel, by symmetry. Using d = 3.00 m and y = 4.00 m, the horizontal components (both pointing to the $-x$ direction) add to give a magnitude of

$$E_{x,\text{net}} = \frac{2|q|d}{4\pi\varepsilon_0(d^2+y^2)^{3/2}} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\ \text{C})(3.00\ \text{m})}{[(3.00\ \text{m})^2+(4.00\ \text{m})^2]^{3/2}} \\ = 1.38\times10^{-10}\ \text{N/C}\ .$$

(b) The net electric field points in the $-x$ direction, or 180° counterclockwise from the $+x$ axis.

10. For it to be possible for the net field to vanish at some $x > 0$, the two individual fields (caused by q_1 and q_2) must point in opposite directions for $x > 0$. Given their locations in the figure, we conclude they are therefore oppositely charged. Further, since the net field points more strongly leftward for the small positive x (where it is very close to q_2) then we conclude that q_2 is the negative-valued charge. Thus, q_1 is a positive-valued charge. We write each charge as a multiple of some positive number ξ (not determined at this point). Since the problem states the absolute value of their ratio, and we have already inferred their signs, we have $q_1 = 4\,\xi$ and $q_2 = -\xi$. Using Eq. 22-3 for the individual fields, we find

$$E_{\text{net}} = E_1 + E_2 = \frac{4\,\xi}{4\pi\varepsilon_0\,(L+x)^2} - \frac{\xi}{4\pi\varepsilon_0\,x^2}$$

for points along the positive x axis. Setting $E_{\text{net}} = 0$ at x = 20 cm (see graph) immediately leads to L = 20 cm.

(a) If we differentiate E_{net} with respect to x and set equal to zero (in order to find where it is maximum), we obtain (after some simplification) that location:

$$x = \left(\frac{2}{3}\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{4} + \frac{1}{3}\right)L = 1.70(20\ \text{cm}) = 34\ \text{cm}.$$

We note that the result for part (a) does not depend on the particular value of ξ.

(b) Now we are asked to set $\xi = 3e$, where $e = 1.60 \times 10^{-19}$ C, and evaluate E_{net} at the value of x (converted to meters) found in part (a). The result is 2.2×10^{-8} N/C .

11. **THINK** Our system consists of two point charges of opposite signs fixed to the x axis. Since the net electric field at a point is the vector sum of the electric fields of individual charges, there exists a location where the net field is zero.

EXPRESS At points between the charges, the individual electric fields are in the same direction and do not cancel. Since charge $q_2 = -4.00\, q_1$ located at $x_2 = 70$ cm has a greater magnitude than $q_1 = 2.1 \times 10^{-8}$ C located at $x_1 = 20$ cm, a point of zero field must be closer to q_1 than to q_2. It must be to the left of q_1.

Let x be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_2|}{(x-x_2)^2} - \frac{|q_1|}{(x-x_1)^2}\right).$$

ANALYZE If the field is to vanish, then

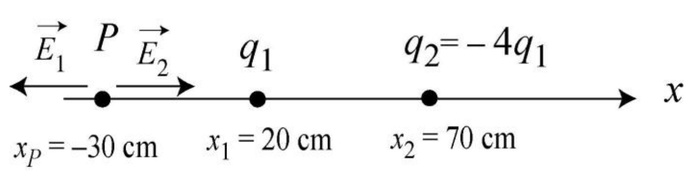
$$\frac{|q_2|}{(x-x_2)^2} = \frac{|q_1|}{(x-x_1)^2} \quad\Rightarrow\quad \frac{|q_2|}{|q_1|} = \frac{(x-x_2)^2}{(x-x_1)^2}.$$

Taking the square root of both sides, noting that $|q_2|/|q_1| = 4$, we obtain

$$\frac{x - 70\text{ cm}}{x - 20\text{ cm}} = \pm 2.0\,.$$

Choosing –2.0 for consistency, the value of x is found to be $x = -30$ cm.

LEARN The results are depicted in the figure below. At P, the field $\vec{E}_1$ due to q_1 points to the left, while the field $\vec{E}_2$ due to q_2 points to the right. Since $|\vec{E}_1| = |\vec{E}_2|$, the net field at P is zero.



12. The field of each charge has magnitude

$$E = \frac{kq}{r^2} = k\frac{e}{(0.020\,\text{m})^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{1.60\times10^{-19}\,\text{C}}{(0.020\ \text{m})^2} = 3.6\times10^{-6}\ \text{N/C}.$$

The directions are indicated in standard format below. We use the magnitude-angle notation (convenient if one is using a vector-capable calculator in polar mode) and write (starting with the proton on the left and moving around clockwise) the contributions to $\vec{E}_{\text{net}}$ as follows:

$$\left(E\angle -20°\right)+\left(E\angle 130°\right)+\left(E\angle -100°\right)+\left(E\angle -150°\right)+\left(E\angle 0°\right).$$

This yields $\left(3.93\times10^{-6}\angle -76.4°\right)$, with the N/C unit understood.

(a) The result above shows that the magnitude of the net electric field is $|\vec{E}_{\text{net}}| = 3.93\times10^{-6}$ N/C.

(b) Similarly, the direction of $\vec{E}_{\text{net}}$ is –76.4° from the x-axis.

13. (a) The electron e_c is a distance $r = z = 0.020$ m away. Thus,

$$E_C = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})}{(0.020\ \text{m})^2} = 3.60\times10^{-6}\ \text{N/C}.$$

(b) The horizontal components of the individual fields (due to the two e_s charges) cancel, and the vertical components add to give

$$E_{\text{s,net}} = \frac{2ez}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})(0.020\ \text{m})}{[(0.020\ \text{m})^2+(0.020\ \text{m})^2]^{3/2}}$$
$$= 2.55\times10^{-6}\ \text{N/C}.$$

(c) Calculation similar to that shown in part (a) now leads to a stronger field $E_c = 3.60\times10^{-4}$ N/C from the central charge.

(d) The field due to the side charges may be obtained from calculation similar to that shown in part (b). The result is $E_{\text{s, net}} = 7.09\times10^{-7}$ N/C.

(e) Since E_c is inversely proportional to z^2, this is a simple result of the fact that z is now much smaller than in part (a). For the net effect due to the side charges, it is the "trigonometric factor" for the y component (here expressed as $z/\sqrt{r}$) that shrinks almost linearly (as z decreases) for very small z, plus the fact that the x components cancel, which leads to the decreasing value of $E_{\text{s, net}}$.

14. (a) The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 22-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on whether $\vec{E}_1$ is in the same, or opposite,

direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $\left|\vec{E}_1\right|$ is everywhere bigger than $\left|\vec{E}_2\right|$ in this region. In the region between the charges ($0 < x < L$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > L$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\text{net}} = \left(|\vec{E}_2| - |\vec{E}_1|\right)\hat{\text{i}}.$$

Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\text{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > L$ region:

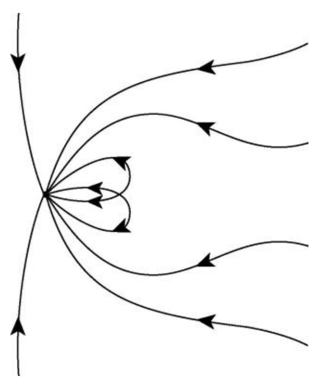
$$|\vec{E}_1| = |\vec{E}_2| \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-L)^2}$$

which leads to

$$\frac{x-L}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{2}{5}}.$$

Thus, we obtain $x = \dfrac{L}{1-\sqrt{2/5}} \approx 2.72L$.

(b) A sketch of the field lines is shown in the figure below:



15. By symmetry we see that the contributions from the two charges $q_1 = q_2 = +e$ cancel each other, and we simply use Eq. 22-3 to compute magnitude of the field due to $q_3 = +2e$.

(a) The magnitude of the net electric field is

$$|\vec{E}_{\text{net}}| = \frac{1}{4\pi\varepsilon_0}\frac{2e}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2e}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{a^2}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.60\times10^{-19}\ \text{C})}{(6.00\times10^{-6}\ \text{m})^2} = 160\ \text{N/C}.$$

(b) This field points at 45.0°, counterclockwise from the *x* axis.

16. The net field components along the *x* and *y* axes are

$$E_{\text{net},x} = \frac{q_1}{4\pi\varepsilon_0 R^2} - \frac{q_2\cos\theta}{4\pi\varepsilon_0 R^2}, \quad E_{\text{net},y} = -\frac{q_2\sin\theta}{4\pi\varepsilon_0 R^2}.$$

The magnitude is the square root of the sum of the components squared. Setting the magnitude equal to $E = 2.00 \times 10^5$ N/C, squaring and simplifying, we obtain

$$E^2 = \frac{q_1^2 + q_1^2 - 2q_1q_2\cos\theta}{(4\pi\varepsilon_0 R^2)^2}.$$

With R = 0.500 m, q_1 = 2.00 × 10^{-6} C, and q_2 = 6.00 × 10^{-6} C, we can solve this expression for cos θ and then take the inverse cosine to find the angle:

$$\theta = \cos^{-1}\left(\frac{q_1^2 + q_1^2 - (4\pi\varepsilon_0 R^2)^2 E^2}{2q_1q_2}\right).$$

There are two answers.

(a) The positive value of angle is θ = 67.8°.

(b) The positive value of angle is θ = – 67.8°.

17. We make the assumption that bead 2 is in the lower half of the circle, partly because it would be awkward for bead 1 to "slide through" bead 2 if it were in the path of bead 1 (which is the upper half of the circle) and partly to eliminate a second solution to the problem (which would have opposite angle and charge for bead 2). We note that the net *y* component of the electric field evaluated at the origin is negative (points *down*) for all positions of bead 1, which implies (with our assumption in the previous sentence) that bead 2 is a negative charge.

(a) When bead 1 is on the +*y* axis, there is no *x* component of the net electric field, which implies bead 2 is on the –*y* axis, so its angle is –90°.

(b) Since the downward component of the net field, when bead 1 is on the +*y* axis, is of largest magnitude, then bead 1 must be a positive charge (so that its field is in the same direction as that of bead 2, in that situation). Comparing the values of E_y at 0° and at 90° we see that the absolute values of the charges on beads 1 and 2 must be in the ratio of 5 to 4. This checks with the 180° value from the E_x graph, which further confirms our belief that bead 1 is positively charged. In fact, the 180° value from the E_x graph allows us to solve for its charge (using Eq. 22-3):

$$q_1 = 4\pi\varepsilon_0 r^2 E = 4\pi\left(8.854\times10^{-12}\,\frac{\text{C}^2}{\text{N m}^2}\right)(0.60\text{ m})^2\left(5.0\times10^4\,\frac{\text{N}}{\text{C}}\right) = 2.0\times10^{-6}\text{ C}.$$

(c) Similarly, the 0° value from the E_y graph allows us to solve for the charge of bead 2:

$$q_2 = 4\pi\varepsilon_0 r^2 E = 4\pi\left(8.854\times10^{-12}\,\frac{\text{C}^2}{\text{N m}^2}\right)(0.60\text{ m})^2\left(-4.0\times10^4\,\frac{\text{N}}{\text{C}}\right) = -1.6\times10^{-6}\text{ C}.$$

18. Referring to Eq. 22-6, we use the binomial expansion (see Appendix E) but keeping higher order terms than are shown in Eq. 22-7:

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\left(1+\frac{d}{z}+\frac{3}{4}\frac{d^2}{z^2}+\frac{1}{2}\frac{d^3}{z^3}+\ldots\right)-\left(1-\frac{d}{z}+\frac{3}{4}\frac{d^2}{z^2}-\frac{1}{2}\frac{d^3}{z^3}+\ldots\right)\right)$$

$$= \frac{q\,d}{2\pi\varepsilon_0 z^3}+\frac{q\,d^3}{4\pi\varepsilon_0 z^5}+\ldots$$

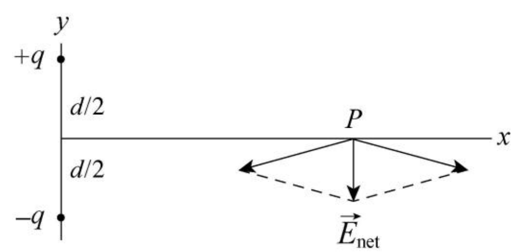
Therefore, in the terminology of the problem, $E_{\text{next}} = q\,d^3/4\pi\varepsilon_0 z^5$.

19. (a) Consider the figure below. The magnitude of the net electric field at point P is

$$\left|\vec{E}_{\text{net}}\right| = 2E_1\sin\theta = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(d/2)^2+r^2}\right]\frac{d/2}{\sqrt{(d/2)^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[(d/2)^2+r^2\right]^{3/2}}$$

For $r \gg d$, we write $[(d/2)^2+r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$|\vec{E}_{\text{net}}| \approx \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}.$$



(b) From the figure, it is clear that the net electric field at point P points in the $-\hat{j}$ direction, or $-90°$ from the $+x$ axis.

20. According to the problem statement, E_{act} is Eq. 22-5 (with $z = 5d$)

$$E_{\text{act}} = \frac{q}{4\pi\varepsilon_0(4.5d)^2}-\frac{q}{4\pi\varepsilon_0(5.5d)^2} = \frac{160}{9801}\cdot\frac{q}{4\pi\varepsilon_0 d^2}$$

and E_{approx} is

$$E_{\text{approx}} = \frac{2qd}{4\pi\varepsilon_0(5d)^3} = \frac{2}{125}\cdot\frac{q}{4\pi\varepsilon_0 d^2}.$$

The ratio is $\dfrac{E_{\text{approx}}}{E_{\text{act}}} = 0.9801 \approx 0.98$.

21. **THINK** The electric quadrupole is composed of two dipoles, each with a dipole moment of magnitude $p = qd$. The dipole moments point in the opposite directions and produce fields in the opposite directions at points on the quadrupole axis.

EXPRESS Consider the point *P* on the axis, a distance *z* to the right of the quadrupole center and take a rightward pointing field to be positive. Then the field produced by the right dipole of the pair is given by $qd/2\pi\varepsilon_0(z - d/2)^3$ while the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$.

ANALYZE Use the binomial expansions

$$(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$$

$$(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$$

we obtain

$$E = \frac{qd}{2\pi\varepsilon_0(z-d/2)^3} - \frac{qd}{2\pi\varepsilon_0(z+d/2)^3} \approx \frac{qd}{2\pi\varepsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4}.$$

Since the quadrupole moment is $Q = 2qd^2$, we have $E = \dfrac{3Q}{4\pi\varepsilon_0 z^4}$.

LEARN For a quadrupole moment *Q*, the electric field varies with *z* as $E \sim Q/z^4$. For a point charge *q*, the dependence is $E \sim q/z^2$, and for a dipole *p*, we have $E \sim p/z^3$.

22. (a) We use the usual notation for the linear charge density: $\lambda = q/L$. The arc length is $L = r\theta$ with θ is expressed in radians. Thus,

$$L = (0.0400 \text{ m})(0.698 \text{ rad}) = 0.0279 \text{ m}.$$

With $q = -300(1.602 \times 10^{-19}\text{ C})$, we obtain $\lambda = -1.72 \times 10^{-15}$ C/m.

(b) We consider the same charge distributed over an area $A = \pi r^2 = \pi(0.0200 \text{ m})^2$ and obtain

$$\sigma = q/A = -3.82 \times 10^{-14} \text{ C/m}^2.$$

(c) Now the area is four times larger than in the previous part ($A_{\text{sphere}} = 4\pi r^2$) and thus obtain an answer that is one-fourth as big:

$$\sigma = q/A_{\text{sphere}} = -9.56 \times 10^{-15} \text{ C/m}^2.$$

(d) Finally, we consider that same charge spread throughout a volume of $V = 4\pi r^3/3$ and obtain the charge density $\rho = q/V = -1.43 \times 10^{-12}$ C/m^3.

23. We use Eq. 22-3, assuming both charges are positive. At *P*, we have

$$E_{\text{left ring}} = E_{\text{right ring}} \quad \Rightarrow \quad \frac{q_1 R}{4\pi\varepsilon_0\left(R^2+R^2\right)^{3/2}} = \frac{q_2(2R)}{4\pi\varepsilon_0[(2R)^2+R^2]^{3/2}}$$

Simplifying, we obtain

$$\frac{q_1}{q_2} = 2\left(\frac{2}{5}\right)^{3/2} \approx 0.506.$$

24. (a) It is clear from symmetry (also from Eq. 22-16) that the field vanishes at the center.

(b) The result (E = 0) for points infinitely far away can be reasoned directly from Eq. 22-16 (it goes as $1/z^2$ as $z \to \infty$) or by recalling the starting point of its derivation (Eq. 22-11, which makes it clearer that the field strength decreases as $1/r^2$ at distant points).

(c) Differentiating Eq. 22-16 and setting equal to zero (to obtain the location where it is maximum) leads to

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}} = 0 \quad \Rightarrow \quad z = +\frac{R}{\sqrt{2}} = 0.707R\,.$$

(d) Plugging this value back into Eq. 22-16 with the values stated in the problem, we find $E_{\max} = 3.46 \times 10^7$ N/C.

25. The smallest arc is of length $L_1 = \pi r_1/2 = \pi R/2$; the middle-sized arc has length $L_2 = \pi r_2/2 = \pi(2R)/2 = \pi R$; and, the largest arc has $L_3 = \pi(3R)/2$. The charge per unit length for each arc is $\lambda = q/L$ where each charge q is specified in the figure. Thus, we find the net electric field to be

$$E_{\text{net}} = \frac{\lambda_1(2\sin 45°)}{4\pi\varepsilon_0 r_1} + \frac{\lambda_2(2\sin 45°)}{4\pi\varepsilon_0 r_2} + \frac{\lambda_3(2\sin 45°)}{4\pi\varepsilon_0 r_3} = \frac{Q}{\sqrt{2}\pi^2\varepsilon_0 R^2}$$

which yields $E_{\text{net}} = 1.62 \times 10^6$ N/C .

(b) The direction is – 45º, measured counterclockwise from the $+x$ axis.

26. Studying Sample Problem 22.03 — "Electric field of a charged circular rod," we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\bigg|_{-\theta}^{\theta}$$

along the symmetry axis, with $\lambda = q/r\theta$ with θ in radians. In this problem, each charged quarter-circle produces a field of magnitude

$$|\vec{E}| = \frac{|q|}{r\pi/2}\frac{1}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\pi/4}^{\pi/4} = \frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}\,|q|}{\pi r^2}.$$

That produced by the positive quarter-circle points at –45°, and that of the negative quarter-circle points at +45°.

(a) The magnitude of the net field is

$$E_{\text{net},x} = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}\,|q|}{\pi r^2}\right)\cos 45° = \frac{1}{4\pi\varepsilon_0}\frac{4|q|}{\pi r^2}$$
$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)4(4.50\times10^{-12}\,\text{C})}{\pi(5.00\times10^{-2}\ \text{m})^2} = 20.6\ \text{N/C}.$$

(b) By symmetry, the net field points vertically downward in the $-\hat{\text{j}}$ direction, or $-90°$ counterclockwise from the $+x$ axis.

27. From symmetry, we see that the net field at P is twice the field caused by the upper semicircular charge $+q = \lambda(\pi R)$ (and that it points downward). Adapting the steps leading to Eq. 22-21, we find

$$\vec{E}_{\text{net}} = 2\left(-\hat{\text{j}}\right)\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_{-90°}^{90°} = -\left(\frac{q}{\varepsilon_0\pi^2 R^2}\right)\hat{\text{j}}.$$

(a) With $R = 8.50\times10^{-2}$ m and $q = 1.50\times10^{-8}$ C, $|\vec{E}_{\text{net}}| = 23.8$ N/C.

(b) The net electric field $\vec{E}_{\text{net}}$ points in the $-\hat{\text{j}}$ direction, or $-90°$ counterclockwise from the $+x$ axis.

28. We find the maximum by differentiating Eq. 22-16 and setting the result equal to zero.

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}} = 0$$

which leads to $z = R/\sqrt{2}$. With R = 2.40 cm, we have z = 1.70 cm.

29. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22.03 — "Electric field of a charged circular

rod" illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ if θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}.$$

The problem asks for the ratio $E_{\text{particle}} / E_{\text{arc}}$, where E_{particle} is given by Eq. 22-3:

$$\frac{E_{\text{particle}}}{E_{\text{arc}}} = \frac{Q/4\pi\varepsilon_0 R^2}{2Q\sin(\theta/2)/4\pi\varepsilon_0 R^2\theta} = \frac{\theta}{2\sin(\theta/2)}.$$

With $\theta = \pi$, we have

$$\frac{E_{\text{particle}}}{E_{\text{arc}}} = \frac{\pi}{2} \approx 1.57.$$

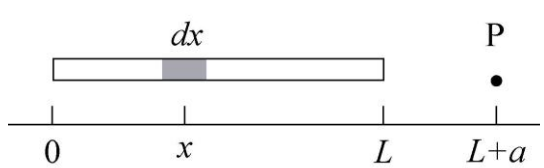
30. We use Eq. 22-16, with "q" denoting the charge on the larger ring:

$$\frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} + \frac{qz}{4\pi\varepsilon_0[z^2+(3R)^2]^{3/2}} = 0 \;\Rightarrow\; q = -Q\left(\frac{13}{5}\right)^{3/2} = -4.19Q.$$

Note: We set $z = 2R$ in the above calculation.

31. **THINK** Our system is a non-conducting rod with uniform charge density. Since the rod is an extended object and not a point charge, the calculation of electric field requires an integration.

EXPRESS The linear charge density λ is the charge per unit length of rod. Since the total charge $-q$ is uniformly distributed on the rod of length L, we have $\lambda = -q/L$. To calculate the electric at the point P shown in the figure, we position the x-axis along the rod with the origin at the left end of the rod, as shown in the diagram below.



Let dx be an infinitesimal length of rod at x. The charge in this segment is $dq = \lambda\, dx$. The charge dq may be considered to be a point charge. The electric field it produces at point P has only an x component and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(L+a-x)^2}.$$

The total electric field produced at P by the whole rod is the integral

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(L+a-x)^2} = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{L+a-x}\bigg|_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{L+a}\right)$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{a(L+a)} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)},$$

upon substituting $-q = \lambda L$.

ANALYZE (a) With q = 4.23 × 10^{-15} C, L = 0.0815 m, and a = 0.120 m, the linear charge density of the rod is

$$\lambda = \frac{-q}{L} = \frac{-4.23\times10^{-15}\text{ C}}{0.0815\text{ m}} = -5.19\times10^{-14}\text{ C/m}.$$

(b) Similarly, we obtain

$$E_x = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)} = -\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(4.23\times10^{-15}\text{ C})}{(0.120\text{ m})(0.0815\text{ m}+0.120\text{ m})} = -1.57\times10^{-3}\text{ N/C},$$

or $|E_x| = 1.57\times10^{-3}$ N/C.

(c) The negative sign in E_x indicates that the field points in the –x direction, or −180° counterclockwise from the +x axis.

(d) If a is much larger than L, the quantity L + a in the denominator can be approximated by a, and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2}.$$

Since $a = 50\text{ m} \gg L = 0.0815\text{ m}$, the above approximation applies and we have $E_x = -1.52\times10^{-8}$ N/C, or $|E_x| = 1.52\times10^{-8}$ N/C.

(e) For a particle of charge $-q = -4.23\times10^{-15}$ C, the electric field at a distance a = 50 m away has a magnitude $|E_x| = 1.52\times10^{-8}$ N/C.

LEARN At a distance much greater than the length of the rod ($a \gg L$), the rod can be effectively regarded as a point charge $-q$, and the electric field can be approximated as $E_x \approx \dfrac{-q}{4\pi\varepsilon_0 a^2}$.

32. We assume $q > 0$. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element dx of the rod contains charge $dq = \lambda\, dx$. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin\theta = R/r$ where $r = \sqrt{x^2 + R^2}$.

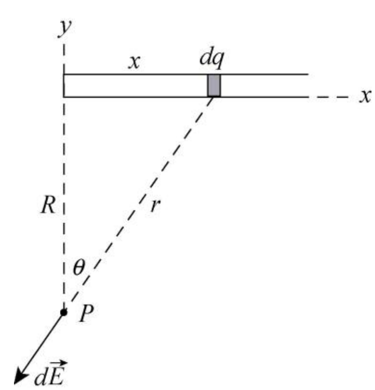
(a) Using Eq. 22-3 (with the 2 and $\sin\theta$ factors just discussed) the magnitude is

$$\begin{aligned}|\vec{E}| &= 2\int_0^{L/2}\left(\frac{dq}{4\pi\varepsilon_0 r^2}\right)\sin\theta = \frac{2}{4\pi\varepsilon_0}\int_0^{L/2}\left(\frac{\lambda\, dx}{x^2+R^2}\right)\left(\frac{y}{\sqrt{x^2+R^2}}\right)\\ &= \frac{\lambda R}{2\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{(q/L)R}{2\pi\varepsilon_0}\cdot\left.\frac{x}{R^2\sqrt{x^2+R^2}}\right|_0^{L/2}\\ &= \frac{q}{2\pi\varepsilon_0 LR}\frac{L/2}{\sqrt{(L/2)^2+R^2}} = \frac{q}{2\pi\varepsilon_0 R}\frac{1}{\sqrt{L^2+4R^2}}\end{aligned}$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals). With $q = 7.81\times10^{-12}$ C, $L = 0.145$ m, and $R = 0.0600$ m, we have $|\vec{E}| = 12.4$ N/C.

(b) As noted above, the electric field $\vec{E}$ points in the $+y$ direction, or $+90°$ counterclockwise from the $+x$ axis.

33. Consider an infinitesimal section of the rod of length dx, a distance x from the left end, as shown in the following diagram. It contains charge $dq = \lambda dx$ and is a distance r from P. The magnitude of the field it produces at P is given by



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}.$$

The x and the y components are

$$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{r^2}\sin\theta$$

and

$$dE_y = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{r^2}\cos\theta\,,$$

respectively. We use θ as the variable of integration and substitute $r = R/\cos\,\theta$, $x = R\tan\theta$ and $dx = (R/\cos^2\,\theta)\,d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus,

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\sin\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\cos\,\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

and

$$E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\sin\,\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}.$$

We notice that $E_x = E_y$ no matter what the value of R. Thus, $\vec{E}$ makes an angle of 45° with the rod for all values of R.

34. From Eq. 22-26, we obtain

$$E = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right) = \frac{5.3\times10^{-6}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right)}\left[1-\frac{12\text{cm}}{\sqrt{(12\text{cm})^2+(2.5\text{cm})^2}}\right] = 6.3\times10^3\ \text{N/C}.$$

35. **THINK** Our system is a uniformly charged disk of radius R. We compare the field strengths at different points on its axis of symmetry.

EXPRESS At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is given by Eq. 22-26:

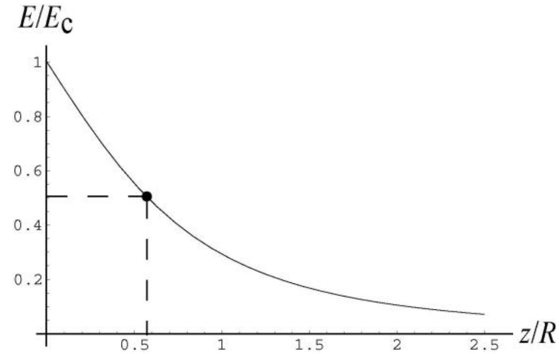
$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]$$

where R is the radius of the disk and σ is the surface charge density on the disk. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of z such that $E/E_c = 1/2$. This means

$$1-\frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2} \Rightarrow \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2}.$$

ANALYZE Squaring both sides, then multiplying them by $z^2 + R^2$, we obtain $z^2 = (z^2/4) + (R^2/4)$. Thus, $z^2 = R^2/3$, or $z = R/\sqrt{3}$. With $R = 0.600$ m, we have $z = 0.346$ m.

LEARN The ratio of the electric field strengths, $E/E_c = 1-(z/R)/\sqrt{(z/R)^2+1}$, as a function of z/R, is plotted below. From the plot, we readily see that at $z/R = (0.346\text{ m})/(0.600\text{ m}) = 0.577$, the ratio indeed is 1/2.



36. From $dA = 2\pi r\, dr$ (which can be thought of as the differential of $A = \pi r^2$) and $dq = \sigma\, dA$ (from the definition of the surface charge density σ), we have

$$dq = \left(\frac{Q}{\pi R^2}\right) 2\pi r\, dr$$

where we have used the fact that the disk is uniformly charged to set the surface charge density equal to the total charge (Q) divided by the total area (πR^2). We next set r = 0.0050 m and make the approximation $dr \approx 30 \times 10^{-6}$ m. Thus we get $dq \approx 2.4 \times 10^{-16}$ C.

37. We use Eq. 22-26, noting that the disk in Figure 22-57(b) is effectively equivalent to the disk in Figure 22-57(a) plus a concentric smaller disk (of radius R/2) with the opposite value of σ. That is,

$$E_{(b)} = E_{(a)} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{2R}{\sqrt{(2R)^2 + (R/2)^2}}\right)$$

where

$$E_{(a)} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{2R}{\sqrt{(2R)^2 + R^2}}\right).$$

We find the relative difference and simplify:

$$\frac{E_{(a)} - E_{(b)}}{E_{(a)}} = \frac{1-2/\sqrt{4+1/4}}{1-2/\sqrt{4+1}} = \frac{1-2/\sqrt{17/4}}{1-2/\sqrt{5}} = \frac{0.0299}{0.1056} = 0.283$$

or approximately 28%.

38. We write Eq. 22-26 as

$$\frac{E}{E_{\max}} = 1 - \frac{z}{(z^2+R^2)^{1/2}}$$

and note that this ratio is $\frac{1}{2}$ (according to the graph shown in the figure) when z = 4.0 cm. Solving this for R we obtain $R = z\sqrt{3}$ = 6.9 cm.

39. When the drop is in equilibrium, the force of gravity is balanced by the force of the electric field: $mg = -qE$, where m is the mass of the drop, q is the charge on the drop, and E is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where r is its radius and ρ is its mass density. Thus,

$$q = -\frac{mg}{E} = -\frac{4\pi r^3 \rho g}{3E} = -\frac{4\pi\left(1.64\times10^{-6}\,\text{m}\right)^3\left(851\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)}{3\left(1.92\times10^5\,\text{N/C}\right)} = -8.0\times10^{-19}\,\text{C}$$

and $q/e = (-8.0 \times 10^{-19}\text{ C})/(1.60 \times 10^{-19}\text{ C}) = -5$, or $q = -5e$.

40. (a) The initial direction of motion is taken to be the $+x$ direction (this is also the direction of $\vec{E}$). We use $v_f^2 - v_i^2 = 2a\Delta x$ with v_f = 0 and $\vec{a} = \vec{F}/m = -e\vec{E}/m_e$ to solve for distance Δx:

$$\Delta x = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-\left(9.11\times10^{-31}\text{ kg}\right)\left(5.00\times10^6\text{ m/s}\right)^2}{-2\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^3\text{ N/C}\right)} = 7.12\times10^{-2}\text{ m}.$$

(b) Equation 2-17 leads to

$$t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_i} = \frac{2\left(7.12\times10^{-2}\text{ m}\right)}{5.00\times10^6\text{ m/s}} = 2.85\times10^{-8}\text{ s}.$$

(c) Using $\Delta v^2 = 2a\Delta x$ with the new value of Δx, we find

$$\frac{\Delta K}{K_i} = \frac{\Delta\left(\frac{1}{2}m_e v^2\right)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta x}{v_i^2} = \frac{-2eE\Delta x}{m_e v_i^2}$$
$$= \frac{-2\left(1.60\times10^{-19}\text{C}\right)\left(1.00\times10^3\text{ N/C}\right)\left(8.00\times10^{-3}\text{m}\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(5.00\times10^6\text{ m/s}\right)^2} = -0.112.$$

Thus, the fraction of the initial kinetic energy lost in the region is 0.112 or 11.2%.

41. **THINK** In this problem we compare the strengths between the electrostatic force and the gravitational force.

EXPRESS The magnitude of the electrostatic force on a point charge of magnitude q is given by $F = qE$, where E is the magnitude of the electric field at the location of the particle. On the other hand, the force of gravity on a particle of mass m is $F_g = mg$.

ANALYZE (a) With $q = -2.0\times10^{-9}$ C and $F = 3.0\times10^{-6}$ N, the magnitude of the electric field strength is

$$E = \frac{F}{q} = \frac{3.0\times10^{-6}\ \text{N}}{2.0\times10^{-9}\ \text{C}} = 1.5\times10^{3}\ \text{N/C}.$$

In vector notation, $\vec{F} = q\vec{E}$. Since the force points downward and the charge is negative, the field $\vec{E}$ must points upward (in the opposite direction of $\vec{F}$).

(b) The magnitude of the electrostatic force on a proton is

$$F_{el} = eE = \left(1.60\times10^{-19}\ \text{C}\right)\left(1.5\times10^{3}\ \text{N/C}\right) = 2.4\times10^{-16}\text{N}.$$

(c) A proton is positively charged, so the force is in the same direction as the field, upward.

(d) The magnitude of the gravitational force on the proton is

$$F_g = mg = \left(1.67\times10^{-27}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right) = 1.6\times10^{-26}\text{N}.$$

The force is downward.

(e) The ratio of the forces is

$$\frac{F_{el}}{F_g} = \frac{2.4\times10^{-16}\ \text{N}}{1.64\times10^{-26}\ \text{N}} = 1.5\times10^{10}.$$

LEARN The force of gravity on the proton is much smaller than the electrostatic force on the proton due to the field of strength $E = 1.5\times10^{3}$ N/C. For the two forces to have equal strength, the electric field would have to be very small:

$$E = \frac{mg}{q} = \frac{(1.67\times10^{-27}\ \text{kg})(9.8\ \text{m/s}^2)}{1.6\times10^{-19}\ \text{C}} = 1.02\times10^{-7}\ \text{N/C}.$$

42. (a) $F_e = Ee = (3.0 \times 10^6$ N/C$)(1.6 \times 10^{-19}$ C$) = 4.8 \times 10^{-13}$ N.

(b) $F_i = Eq_{\text{ion}} = Ee = (3.0 \times 10^6$ N/C$)(1.6 \times 10^{-19}$ C$) = 4.8 \times 10^{-13}$ N.

43. **THINK** The acceleration of the electron is given by Newton's second law: $F = ma$, where F is the electrostatic force.

EXPRESS The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. Using Newton's second law, the acceleration of the electron is

$$a = \frac{F}{m} = \frac{eE}{m}.$$

ANALYZE With $e = 1.6\times10^{-19}$ C, $E = 2.00\times10^{4}$ N/C, and $m = 9.11\times10^{-31}$ kg, we find the acceleration to be

$$a = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.00\times10^{4}\ \text{N/C}\right)}{9.11\times10^{-31}\ \text{kg}} = 3.51\times10^{15}\ \text{m/s}^2.$$

LEARN In vector notation, $\vec{a} = \vec{F}/m = -e\vec{E}/m$, so $\vec{a}$ is in the opposite direction of $\vec{E}$. The magnitude of electron's acceleration is proportional to the field strength E: the greater the value of E, the greater the acceleration.

44. (a) Vertical equilibrium of forces leads to the equality

$$q\left|\vec{E}\right| = mg \ \Rightarrow\ \left|\vec{E}\right| = \frac{mg}{2e}.$$

Substituting the values given in the problem, we obtain

$$\left|\vec{E}\right| = \frac{mg}{2e} = \frac{(6.64\times10^{-27}\ \text{kg})(9.8\ \text{m/s}^2)}{2(1.6\times10^{-19}\text{C})} = 2.03\times10^{-7}\ \text{N/C}.$$

(b) Since the force of gravity is downward, then $q\vec{E}$ must point upward. Since $q > 0$ in this situation, this implies $\vec{E}$ must itself point upward.

45. We combine Eq. 22-9 and Eq. 22-28 (in absolute values).

$$F = |q|E = |q|\left(\frac{p}{2\pi\varepsilon_0 z^3}\right) = \frac{2kep}{z^3}$$

where we have used Eq. 21-5 for the constant k in the last step. Thus, we obtain

$$F = \frac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(3.6\times10^{-29}\text{C}\cdot\text{m}\right)}{\left(25\times10^{-9}\text{m}\right)^3} = 6.6\times10^{-15}\ \text{N}.$$

If the dipole is oriented such that $\vec{p}$ is in the $+z$ direction, then $\vec{F}$ points in the $-z$ direction.

46. Equation 22-28 gives

$$\vec{E}=\frac{\vec{F}}{q}=\frac{m\vec{a}}{(-e)}=-\left(\frac{m}{e}\right)\vec{a}$$

using Newton's second law.

(a) With *east* being the $\hat{\text{i}}$ direction, we have

$$\vec{E}=-\left(\frac{9.11\times10^{-31}\text{ kg}}{1.60\times10^{-19}\text{ C}}\right)\left(1.80\times10^{9}\text{ m/s}^2\,\hat{\text{i}}\right)=(-0.0102\text{ N/C})\,\hat{\text{i}}$$

which means the field has a magnitude of 0.0102 N/C .

(b) The result shows that the field $\vec{E}$ is directed in the $-x$ direction, or westward.

47. **THINK** The acceleration of the proton is given by Newton's second law: $F=ma$, where F is the electrostatic force.

EXPRESS The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus,

$$a=\frac{F}{m}=\frac{eE}{m}.$$

We assume that the proton starts from rest ($v_0=0$) and apply the kinematic equation $v^2=v_0^2+2ax$ (or else $x=\frac{1}{2}at^2$ and $v = at$). Thus, the speed of the proton after having traveling a distance x is $v=\sqrt{2ax}$.

ANALYZE (a) With $e=1.6\times10^{-19}$ C, $E=2.00\times10^{4}$ N/C, and $m=1.67\times10^{-27}$ kg, we find the acceleration to be

$$a=\frac{eE}{m}=\frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{4}\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}}=1.92\times10^{12}\text{ m/s}^2.$$

(b) With $x=1.00\text{ cm}=1.0\times10^{-2}\text{ m}$, the speed of the proton is

$$v = \sqrt{2ax} = \sqrt{2\left(1.92 \times 10^{12}\ \text{m/s}^2\right)\left(0.0100\,\text{m}\right)} = 1.96 \times 10^5\ \text{m/s}.$$

LEARN The time it takes for the proton to attain the final speed is

$$t = \frac{v}{a} = \frac{1.96 \times 10^5\ \text{m/s}}{1.92 \times 10^{12}\ \text{m/s}^2} = 1.02 \times 10^{-7}\ \text{s}.$$

The distance the proton travels can be written as

$$x = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{eE}{m}\right)t^2.$$

48. We are given $\sigma = 4.00 \times 10^{-6}$ C/m^2 and various values of z (in the notation of Eq. 22-26, which specifies the field E of the charged disk). Using this with $F = eE$ (the magnitude of Eq. 22-28 applied to the electron) and $F = ma$, we obtain $a = F/m = eE/m$.

(a) The magnitude of the acceleration at a distance R is

$$a = \frac{e\,\sigma\,(2 - \sqrt{2}\,)}{4\,m\,\varepsilon_0} = 1.16 \times 10^{16}\ \text{m/s}^2\ .$$

(b) At a distance R/100, $a = \dfrac{e\,\sigma\,(10001 - \sqrt{10001}\,)}{20002\,m\,\varepsilon_0} = 3.94 \times 10^{16}\ \text{m/s}^2$.

(c) At a distance R/1000, $a = \dfrac{e\,\sigma\,(1000001 - \sqrt{1000001}\,)}{2000002\,m\,\varepsilon_0} = 3.97 \times 10^{16}\ \text{m/s}^2$.

(d) The field due to the disk becomes more uniform as the electron nears the center point. One way to view this is to consider the forces exerted on the electron by the charges near the edge of the disk; the net force on the electron caused by those charges will decrease due to the fact that their contributions come closer to canceling out as the electron approaches the middle of the disk.

49. (a) Using Eq. 22-28, we find

$$\begin{aligned}\vec{F} &= \left(8.00\times10^{-5}\text{C}\right)\left(3.00\times10^3\ \text{N/C}\right)\hat{\text{i}} + \left(8.00\times10^{-5}\text{C}\right)\left(-600\,\text{N/C}\right)\hat{\text{j}} \\ &= \left(0.240\ \text{N}\right)\hat{\text{i}} - \left(0.0480\ \text{N}\right)\hat{\text{j}}.\end{aligned}$$

Therefore, the force has magnitude equal to

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(0.240\ \text{N}\right)^2 + \left(-0.0480\ \text{N}\right)^2} = 0.245\ \text{N}.$$

(b) The angle the force $\vec{F}$ makes with the +*x* axis is

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.0480\ \text{N}}{0.240\ \text{N}}\right) = -11.3°$$

measured counterclockwise from the +*x* axis.

(c) With m = 0.0100 kg, the (x, y) coordinates at t = 3.00 s can be found by combining Newton's second law with the kinematics equations of Chapters 2–4. The *x* coordinate is

$$x = \frac{1}{2}a_x t^2 = \frac{F_x t^2}{2m} = \frac{(0.240\ \text{N})(3.00\ \text{s})^2}{2(0.0100\ \text{kg})} = 108\ \text{m}.$$

(d) Similarly, the *y* coordinate is

$$y = \frac{1}{2}a_y t^2 = \frac{F_y t^2}{2m} = \frac{(-0.0480\ \text{N})(3.00\ \text{s})^2}{2(0.0100\ \text{kg})} = -21.6\ \text{m}.$$

50. We assume there are no forces or force-components along the *x* direction. We combine Eq. 22-28 with Newton's second law, then use Eq. 4-21 to determine time *t* followed by Eq. 4-23 to determine the final velocity (with $-g$ replaced by the a_y of this problem); for these purposes, the velocity components *given* in the problem statement are re-labeled as v_{0x} and v_{0y}, respectively.

(a) We have $\vec{a} = q\vec{E}/m = -(e/m)\vec{E}$, which leads to

$$\vec{a} = -\left(\frac{1.60\times10^{-19}\text{C}}{9.11\times10^{-31}\text{kg}}\right)\left(120\frac{\text{N}}{\text{C}}\right)\hat{\text{j}} = -(2.1\times10^{13}\ \text{m/s}^2)\,\hat{\text{j}}.$$

(b) Since $v_x = v_{0x}$ in this problem (that is, $a_x = 0$), we obtain

$$t = \frac{\Delta x}{v_{0x}} = \frac{0.020\,\text{m}}{1.5\times10^5\ \text{m/s}} = 1.3\times10^{-7}\,\text{s}$$

$$v_y = v_{0y} + a_y t = 3.0\times10^3\ \text{m/s} + \left(-2.1\times10^{13}\ \text{m/s}^2\right)\left(1.3\times10^{-7}\,\text{s}\right)$$

which leads to $v_y = -2.8 \times 10^6$ m/s. Therefore, the final velocity is

$$\vec{v} = (1.5\times10^5\ \text{m/s})\,\hat{\text{i}} - (2.8\times10^6\ \text{m/s})\,\hat{\text{j}}.$$

51. We take the charge $Q = 45.0\text{ pC}$ of the bee to be concentrated as a particle at the center of the sphere. The magnitude of the induced charges on the sides of the grain is $|q| = 1.000\text{ pC}$.

(a) The electrostatic force on the grain by the bee is

$$F = \frac{kQq}{(d+D/2)^2} + \frac{kQ(-q)}{(D/2)^2} = -kQ|q|\left[\frac{1}{(D/2)^2} - \frac{1}{(d+D/2)^2}\right]$$

where $D = 1.000\text{ cm}$ is the diameter of the sphere representing the honeybee, and $d = 40.0\mu\text{m}$ is the diameter of the grain. Substituting the values, we obtain

$$F = -\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(45.0\times10^{-12}\text{C})(1.000\times10^{-12}\text{C})\left[\frac{1}{(5.00\times10^{-3}\text{ m})^2} - \frac{1}{(5.04\times10^{-3}\text{ m})^2}\right]$$
$$= -2.56\times10^{-10}\text{ N}.$$

The negative sign implies that the force between the bee and the grain is attractive. The magnitude of the force is $|F| = 2.56\times10^{-10}\text{ N}$.

(b) Let $|Q'| = 45.0\text{ pC}$ be the magnitude of the charge on the tip of the stigma. The force on the grain due to the stigma is

$$F' = \frac{k|Q'|q}{(d+D')^2} + \frac{k|Q'|(-q)}{(D')^2} = -k|Q'||q|\left[\frac{1}{(D')^2} - \frac{1}{(d+D')^2}\right]$$

where $D' = 1.000\text{ mm}$ is the distance between the grain and the tip of the stigma. Substituting the values given, we have

$$F' = -\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(45.0\times10^{-12}\text{C})(1.000\times10^{-12}\text{C})\left[\frac{1}{(1.000\times10^{-3}\text{ m})^2} - \frac{1}{(1.040\times10^{-3}\text{ m})^2}\right]$$
$$= -3.06\times10^{-8}\text{ N}.$$

The negative sign implies that the force between the grain and the stigma is attractive. The magnitude of the force is $|F'| = 3.06\times10^{-8}\text{ N}$.

(c) Since $|F'| > |F|$, the grain will move to the stigma.

52. (a) Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the same direction as the velocity leads to deceleration. Thus, with $t = 1.5\times10^{-9}$ s, we find

$$v = v_0 - |a|t = v_0 - \frac{eE}{m}t = 4.0\times10^4 \text{ m/s} - \frac{(1.6\times10^{-19}\text{ C})(50\text{ N/C})}{9.11\times10^{-31}\text{ kg}}(1.5\times10^{-9}\text{ s})$$
$$= 2.7\times10^4 \text{ m/s}.$$

(b) The displacement is equal to the distance since the electron does not change its direction of motion. The field is uniform, which implies the acceleration is constant. Thus,

$$d = \frac{v+v_0}{2}t = 5.0\times10^{-5}\text{m}.$$

53. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where E is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time t is $x = \frac{1}{2}a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_e t^2$. They pass each other when their coordinates are the same, or

$$\frac{1}{2}a_p t^2 = L + \frac{1}{2}a_e t^2.$$

This means $t^2 = 2L/(a_p - a_e)$ and

$$x = \frac{a_p}{a_p - a_e}L = \frac{eE/m_p}{(eE/m_p)+(eE/m_e)}L = \left(\frac{m_e}{m_e+m_p}\right)L$$
$$= \left(\frac{9.11\times10^{-31}\text{kg}}{9.11\times10^{-31}\text{kg}+1.67\times10^{-27}\text{kg}}\right)(0.050\text{ m})$$
$$= 2.7\times10^{-5}\text{m}.$$

54. Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the $+y$ direction (which we will call "upward") leads to a downward acceleration. This is exactly like a projectile motion problem as treated in Chapter 4 (but with g replaced with $a = eE/m = 8.78 \times 10^{11}$ m/s^2). Thus, Eq. 4-21 gives

$$t = \frac{x}{v_0\cos\theta_0} = \frac{3.00\text{ m}}{(2.00\times10^6\text{ m/s})\cos 40.0°} = 1.96\times10^{-6}\text{ s}.$$

This leads (using Eq. 4-23) to

$$v_y = v_0\sin\theta_0 - at = (2.00\times10^6\text{ m/s})\sin 40.0° - (8.78\times10^{11}\text{ m/s}^2)(1.96\times10^{-6}\text{ s})$$
$$= -4.34\times10^5\text{ m/s}.$$

Since the x component of velocity does not change, then the final velocity is

$$\vec{v} = (1.53 \times 10^6 \text{ m/s})\,\hat{\text{i}} - (4.34 \times 10^5 \text{ m/s})\,\hat{\text{j}}\,.$$

55. (a) We use $\Delta x = v_{avg}t = vt/2$:

$$v = \frac{2\Delta x}{t} = \frac{2\left(2.0\times10^{-2}\text{ m}\right)}{1.5\times10^{-8}\text{ s}} = 2.7\times10^6 \text{ m/s}.$$

(b) We use $\Delta x = \frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E = \frac{ma}{e} = \frac{2\Delta xm}{et^2} = \frac{2\left(2.0\times10^{-2}\text{ m}\right)\left(9.11\times10^{-31}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.5\times10^{-8}\text{ s}\right)^2} = 1.0\times10^3 \text{ N/C}.$$

56. (a) Equation 22-33 leads to $\tau = pE\sin 0° = 0.$

(b) With $\theta = 90°$, the equation gives

$$\tau = pE = \left[2\left(1.6\times10^{-19}\text{ C}\right)\left(0.78\times10^{-9}\text{ m}\right)\right]\left(3.4\times10^6\text{ N/C}\right) = 8.5\times10^{-22}\text{ N}\cdot\text{m}.$$

(c) Now the equation gives $\tau = pE\sin 180° = 0.$

57. **THINK** The potential energy of the electric dipole placed in an electric field depends on its orientation relative to the electric field.

EXPRESS The magnitude of the electric dipole moment is $p = qd$, where q is the magnitude of the charge, and d is the separation between the two charges. When placed in an electric field, the potential energy of the dipole is given by Eq. 22-38:

$$U(\theta) = -\vec{p}\cdot\vec{E} = -pE\cos\theta\,.$$

Therefore, if the initial angle between $\vec{p}$ and $\vec{E}$ is θ_0 and the final angle is θ, then the change in potential energy would be

$$\Delta U = U(\theta) - U_0(\theta) = -pE\left(\cos\theta - \cos\theta_0\right).$$

ANALYZE (a) With $q = 1.50\times10^{-9}$ C and $d = 6.20\times10^{-6}$ m, we find the magnitude of the dipole moment to be

$$p = qd = \left(1.50\times10^{-9}\text{ C}\right)\left(6.20\times10^{-6}\text{ m}\right) = 9.30\times10^{-15}\text{ C}\cdot\text{m}.$$

(b) The initial and the final angles are $\theta_0 = 0$ (parallel) and $\theta = 180°$ (anti-parallel), so we find ΔU to be

$$\Delta U = U(180°) - U(0) = 2pE = 2\left(9.30\times10^{-15}\,\text{C}\cdot\text{m}\right)\left(1100\text{ N/C}\right) = 2.05\times10^{-11}\,\text{J}.$$

LEARN The potential energy is a maximum ($U_{max} = +pE$) when the dipole is oriented antiparallel to $\vec{E}$, and is a minimum ($U_{min} = -pE$) when it is parallel to $\vec{E}$.

58. Examining the lowest value on the graph, we have (using Eq. 22-38)

$$U = -\vec{p}\cdot\vec{E} = -1.00\times10^{-28}\,\text{J}.$$

If E = 20 N/C, we find $p = 5.0 \times 10^{-28}$ C·m.

59. Following the solution to part (c) of Sample Problem 22.05 — "Torque and energy of an electric dipole in an electric field," we find

$$\begin{aligned} W &= U(\theta_0+\pi) - U(\theta_0) = -pE\left(\cos(\theta_0+\pi) - \cos(\theta_0)\right) = 2pE\cos\theta_0 \\ &= 2(3.02\times10^{-25}\,\text{C}\cdot\text{m})(46.0\text{ N/C})\cos64.0° \\ &= 1.22\times10^{-23}\text{ J}. \end{aligned}$$

60. Using Eq. 22-35, considering θ as a variable, we note that it reaches its maximum value when $\theta = -90°$: $\tau_{max} = pE$. Thus, with E = 40 N/C and $\tau_{max} = 100 \times 10^{-28}$ N·m (determined from the graph), we obtain the dipole moment: $p = 2.5 \times 10^{-28}$ C·m.

61. Equation 22-35 $\left(\tau = -pE\sin\theta\right)$ captures the sense as well as the magnitude of the effect. That is, this is a restoring torque, trying to bring the tilted dipole back to its aligned equilibrium position. If the amplitude of the motion is small, we may replace sin θ with θ in radians. Thus, $\tau \approx -pE\theta$. Since this exhibits a simple negative proportionality to the angle of rotation, the dipole oscillates in simple harmonic motion, like a torsional pendulum with torsion constant $\kappa = pE$. The angular frequency ω is given by

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I}$$

where I is the rotational inertia of the dipole. The frequency of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}.$$

62. (a) We combine Eq. 22-28 (in absolute value) with Newton's second law:

$$a = \frac{|q|E}{m} = \left(\frac{1.60\times10^{-19}\,\text{C}}{9.11\times10^{-31}\,\text{kg}}\right)\left(1.40\times10^{6}\,\frac{\text{N}}{\text{C}}\right) = 2.46\times10^{17}\ \text{m/s}^2 .$$

(b) With $v = \frac{c}{10} = 3.00\times10^{7}$ m/s, we use Eq. 2-11 to find

$$t = \frac{v - v_o}{a} = \frac{3.00\times10^{7}\ \text{m/s}}{2.46\times10^{17}\ \text{m/s}^2} = 1.22\times10^{-10}\ \text{s}.$$

(c) Equation 2-16 gives

$$\Delta x = \frac{v^2 - v_o^2}{2a} = \frac{\left(3.00\times10^{7}\ \text{m/s}\right)^2}{2\left(2.46\times10^{17}\ \text{m/s}^2\right)} = 1.83\times10^{-3}\ \text{m}.$$

63. (a) Using the density of water (ρ = 1000 kg/m^3), the weight mg of the spherical drop (of radius $r = 6.0\times10^{-7}$ m) is

$$W = \rho V g = \left(1000\,\text{kg/m}^3\right)\left(\frac{4\pi}{3}\left(6.0\times10^{-7}\ \text{m}\right)^3\right)\left(9.8\,\text{m/s}^2\right) = 8.87\times10^{-15}\ \text{N}.$$

(b) Vertical equilibrium of forces leads to $mg = qE = neE$, which we solve for n, the number of excess electrons:

$$n = \frac{mg}{eE} = \frac{8.87\times10^{-15}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(462\,\text{N/C}\right)} = 120.$$

64. The two closest charges produce fields at the midpoint that cancel each other out. Thus, the only significant contribution is from the furthest charge, which is a distance $r = \sqrt{3}d/2$ away from that midpoint. Plugging this into Eq. 22-3 immediately gives the result:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0(\sqrt{3}d/2)^2} = \frac{4}{3}\frac{Q}{4\pi\varepsilon_0 d^2} .$$

65. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22.03 — "Electric field of a charged circular rod," illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r} .$$

Now, the arc length is $L = r\theta$ with θ expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} .$$

Thus, the problem requires $E_{\text{arc}} = \frac{1}{2}\, E_{\text{particle}}$, where E_{particle} is given by Eq. 22-3. Hence,

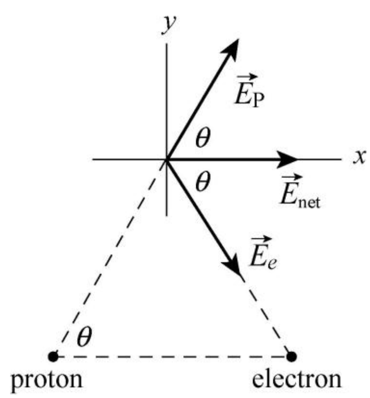
$$\frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2} \;\Rightarrow\; \sin\frac{\theta}{2} = \frac{\theta}{4}$$

where we note, again, that the angle is in radians. The approximate solution to this equation is $\theta = 3.791$ rad $\approx 217°$.

66. We denote the electron with subscript *e* and the proton with *p*. From the figure below we see that

$$\left|\vec{E}_e\right| = \left|\vec{E}_p\right| = \frac{e}{4\pi\varepsilon_0 d^2}$$

where $d = 2.0 \times 10^{-6}$ m. We note that the components along the *y* axis cancel during the vector summation. With $k = 1/4\pi\varepsilon_0$ and $\theta = 60°$, the magnitude of the net electric field is obtained as follows:



$$\left|\vec{E}_{\text{net}}\right| = E_x = 2E_e\cos\theta = 2\left(\frac{e}{4\pi\varepsilon_0 d^2}\right)\cos\theta = 2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.6\times10^{-19}\ \text{C}\right)}{\left(2.0\times10^{-6}\ \text{m}\right)^2}\cos 60°$$

$$= 3.6\times10^2\ \text{N/C}.$$

67. A small section of the distribution that has charge *dq* is $\lambda\, dx$, where $\lambda = 9.0 \times 10^{-9}$ C/m. Its contribution to the field at $x_P = 4.0$ m is

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 \left(x - x_P\right)^2}$$

pointing in the $+x$ direction. Thus, we have

$$\vec{E} = \int_0^{3.0\,\text{m}} \frac{\lambda\, dx}{4\pi\varepsilon_0 \left(x - x_P\right)^2}\,\hat{\text{i}}$$

which becomes, using the substitution $u = x - x_P$,

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int_{-4.0\,\text{m}}^{-1.0\,\text{m}} \frac{du}{u^2}\,\hat{\text{i}} = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{-1.0\,\text{m}} - \frac{-1}{-4.0\,\text{m}}\right)\hat{\text{i}}$$

which yields 61 N/C in the $+x$ direction.

68. Most of the individual fields, caused by diametrically opposite charges, will cancel, except for the pair that lie on the x axis passing through the center. This pair of charges produces a field pointing to the right

$$\vec{E} = \frac{3q}{4\pi\varepsilon_0 d^2}\,\hat{\text{i}} = \frac{3e}{4\pi\varepsilon_0 d^2}\,\hat{\text{i}} = \frac{3\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{\left(0.020\text{m}\right)^2}\,\hat{\text{i}} = (1.08\times10^{-5}\ \text{N/C})\hat{\text{i}}\,.$$

69. (a) From symmetry, we see the net field component along the x axis is zero; the net field component along the y axis points upward. With $\theta = 60°$,

$$E_{\text{net},y} = 2\frac{Q\sin\theta}{4\pi\varepsilon_0 a^2}\ .$$

Since $\sin(60°) = \sqrt{3}/2$, we can write this as $E_{\text{net}} = kQ\sqrt{3}/a^2$ (using the notation of the constant k defined in Eq. 21-5). Numerically, this gives roughly 47 N/C.

(b) From symmetry, we see in this case that the net field component along the y axis is zero; the net field component along the x axis points rightward. With $\theta = 60°$,

$$E_{\text{net},x} = 2\frac{Q\cos\theta}{4\pi\varepsilon_0 a^2}\,.$$

Since $\cos(60°) = 1/2$, we can write this as $E_{\text{net}} = kQ/a^2$ (using the notation of Eq. 21-5). Thus, $E_{\text{net}} \approx 27$ N/C.

70. Our approach (based on Eq. 22-29) consists of several steps. The first is to find an *approximate* value of e by taking differences between all the given data. The smallest difference is between the fifth and sixth values:

$$18.08 \times 10^{-19}\ \text{C} - 16.48 \times 10^{-19}\ \text{C} = 1.60 \times 10^{-19}\ \text{C}$$

which we denote e_{approx}. The goal at this point is to assign integers n using this approximate value of e:

datum1	$\dfrac{6.563\times10^{-19}\text{C}}{e_{\text{approx}}}=4.10\Rightarrow n_1=4$	datum6	$\dfrac{18.08\times10^{-19}\text{C}}{e_{\text{appeox}}}=11.30\Rightarrow n_6=11$
datum2	$\dfrac{8.204\times10^{-19}\text{C}}{e_{\text{approx}}}=5.13\Rightarrow n_2=5$	datum7	$\dfrac{19.71\times10^{-19}\text{C}}{e_{\text{approx}}}=12.32\Rightarrow n_7=12$
datum3	$\dfrac{11.50\times10^{-19}\text{C}}{e_{\text{approx}}}=7.19\Rightarrow n_3=7$	datum8	$\dfrac{22.89\times10^{-19}\text{C}}{e_{\text{approx}}}=14.31\Rightarrow n_8=14$
datum4	$\dfrac{13.13\times10^{-19}\text{C}}{e_{\text{approx}}}=8.21\Rightarrow n_4=8$	datum9	$\dfrac{26.13\times10^{-19}\text{C}}{e_{\text{approx}}}=16.33\Rightarrow n_9=16$
datum5	$\dfrac{16.48\times10^{-19}\text{C}}{e_{\text{approx}}}=10.30\Rightarrow n_5=10$		

Next, we construct a new data set (e_1, e_2, e_3, …) by dividing the given data by the respective exact integers n_i (for i = 1, 2, 3, …):

$$(e_1,e_2,e_3,\ldots)=\left(\frac{6.563\times10^{-19}\text{C}}{n_1},\frac{8.204\times10^{-19}\text{C}}{n_2},\frac{11.50\times10^{-19}\text{C}}{n_3},\ldots\right)$$

which gives (carrying a few more figures than are significant)

$$\left(1.64075\times10^{-19}\text{C},\ 1.6408\times10^{-19}\text{C},\ 1.64286\times10^{-19}\text{C},\ldots\right)$$

as the new data set (our experimental values for e). We compute the average and standard deviation of this set, obtaining

$$e_{\text{exptal}}=e_{\text{avg}}\pm\Delta e=(1.641\pm0.004)\times10^{-19}\,\text{C}$$

which does not agree (to within one standard deviation) with the modern accepted value for e. The lower bound on this spread is $e_{\text{avg}}-\Delta e = 1.637\times10^{-19}$ C, which is still about 2% too high.

71. Studying Sample Problem 22.03 — "Electric field of a charged circular rod," we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E}=\frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\theta}^{\theta}$$

along the symmetry axis, where $\lambda = q/\ell = q/r\theta$ with θ in radians. Here ℓ is the length of the arc, given as $\ell = 4.0\,\text{m}$. Therefore, the angle is $\theta = \ell/r = 4.0/2.0 = 2.0\,\text{rad}$. Thus, with $q = 20 \times 10^{-9}$ C, we obtain

$$\left|\vec{E}\right| = \frac{(q/\ell)}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-1.0\text{ rad}}^{1.0\text{ rad}} = 38\text{ N/C}.$$

72. The electric field at a point on the axis of a uniformly charged ring, a distance z from the ring center, is given by

$$E = \frac{qz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}$$

where q is the charge on the ring and R is the radius of the ring (see Eq. 22-16). For q positive, the field points upward at points above the ring and downward at points below the ring. We take the positive direction to be upward. Then, the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}.$$

For small amplitude oscillations $z \ll R$ and z can be neglected in the denominator. Thus,

$$F = -\frac{eqz}{4\pi\varepsilon_0 R^3}.$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to z, just as if the electron were attached to a spring with spring constant $k = eq/4\pi\varepsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\varepsilon_0 mR^3}}$$

where m is the mass of the electron.

73. **THINK** We have a positive charge in the xy plane. From the electric fields it produces at two different locations, we can determine the position and the magnitude of the charge.

EXPRESS Let the charge be placed at (x_0, y_0). In Cartesian coordinates, the electric field at a point (x, y) can be written as

$$\vec{E} = E_x\hat{\mathrm{i}} + E_y\hat{\mathrm{j}} = \frac{q}{4\pi\varepsilon_0}\frac{(x-x_0)\hat{\mathrm{i}} + (y-y_0)\hat{\mathrm{j}}}{\left[(x-x_0)^2 + (y-y_0)^2\right]^{3/2}}.$$

The ratio of the field components is

$$\frac{E_y}{E_x} = \frac{y-y_0}{x-x_0}.$$

ANALYZE (a) The fact that the second measurement at the location (2.0 cm, 0) gives $\vec{E} = (100\ \mathrm{N/C})\hat{\mathrm{i}}$ indicates that $y_0 = 0,$ that is, the charge must be somewhere on the x axis. Thus, the above expression can be simplified to

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{(x-x_0)\hat{\mathrm{i}} + y\hat{\mathrm{j}}}{\left[(x-x_0)^2 + y^2\right]^{3/2}}.$$

On the other hand, the field at (3.0 cm, 3.0 cm) is $\vec{E} = (7.2\ \mathrm{N/C})(4.0\hat{\mathrm{i}} + 3.0\hat{\mathrm{j}}),$ which gives $E_y / E_x = 3/4.$ Thus, we have

$$\frac{3}{4} = \frac{3.0\ \mathrm{cm}}{3.0\ \mathrm{cm} - x_0}$$

which implies $x_0 = -1.0$ cm.

(b) As shown above, the y coordinate is $y_0 = 0$.

(c) To calculate the magnitude of the charge, we note that the field magnitude measured at (2.0 cm, 0) (which is $r = 0.030$ m from the charge) is

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 100\ \mathrm{N/C}.$$

Therefore,

$$q = 4\pi\varepsilon_0\left|\vec{E}\right|r^2 = \frac{(100\ \mathrm{N/C})(0.030\ \mathrm{m})^2}{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}} = 1.0\times10^{-11}\mathrm{C}.$$

LEARN Alternatively, we may calculate q by noting that at (3.0 cm, 3.00 cm)

$$E_x = 28.8\ \mathrm{N/C} = \frac{q}{4\pi\varepsilon_0}\frac{0.040\ \mathrm{m}}{\left[(0.040\ \mathrm{m})^2 + (0.030\ \mathrm{m})^2\right]^{3/2}} = \frac{q}{4\pi\varepsilon_0}\left(320/\mathrm{m}^2\right).$$

This gives

$$q = \frac{28.8\ \mathrm{N/C}}{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(320/\mathrm{m^2})} = 1.0\times10^{-11}\ \mathrm{C},$$

in agreement with that calculated above.

74. (a) Let $E = \sigma/2\varepsilon_0 = 3 \times 10^6$ N/C. With $\sigma = |q|/A$, this leads to

$$|q| = \pi R^2\sigma = 2\pi\varepsilon_0 R^2 E = \frac{R^2E}{2k} = \frac{\left(2.5\times10^{-2}\,\text{m}\right)^2\left(3.0\times10^6\,\text{N/C}\right)}{2\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = 1.0\times10^{-7}\,\text{C},$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$.

(b) Setting up a simple proportionality (with the areas), the number of atoms is estimated to be

$$n = \frac{\pi\left(2.5\times10^{-2}\,\text{m}\right)^2}{0.015\times10^{-18}\,\text{m}^2} = 1.3\times10^{17}.$$

(c) The fraction is

$$\frac{q}{Ne} = \frac{1.0\times10^{-7}\,\text{C}}{\left(1.3\times10^{17}\right)\left(1.6\times10^{-19}\,\text{C}\right)} \approx 5.0\times10^{-6}.$$

75. On the one hand, the conclusion (that $Q = +1.00\ \mu$C) is clear from symmetry. If a more in-depth justification is desired, one should use Eq. 22-3 for the electric field magnitudes of the three charges (each at the same distance $r = a/\sqrt{3}$ from C) and then find field components along suitably chosen axes, requiring each component-sum to be zero. If the y axis is vertical, then (assuming $Q > 0$) the component-sum along that axis leads to $2kq\sin 30°/r^2 = kQ/r^2$ where q refers to either of the charges at the bottom corners. This yields $Q = 2q \sin 30° = q$ and thus to the conclusion mentioned above.

76. Equation 22-38 gives $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$. We note that $\theta_i = 110°$ and $\theta_f = 70.0°$. Therefore,

$$\Delta U = -pE\left(\cos 70.0° - \cos 110°\right) = -3.28\times10^{-21}\,\text{J}.$$

77. (a) Since the two charges in question are of the same sign, the point $x = 2.0$ mm should be located in between them (so that the field vectors point in the opposite direction). Let the coordinate of the second particle be x' ($x' > 0$). Then, the magnitude of the field due to the charge $-q_1$ evaluated at x is given by $E = q_1/4\pi\varepsilon_0 x^2$, while that due to the second charge $-4q_1$ is $E' = 4q_1/4\pi\varepsilon_0(x'-x)^2$. We set the net field equal to zero:

$$\vec{E}_{\text{net}} = 0 \Rightarrow E = E'$$

so that

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{4q_1}{4\pi\varepsilon_0\left(x'-x\right)^2}.$$

Thus, we obtain $x' = 3x = 3(2.0\text{ mm}) = 6.0$ mm.

(b) In this case, with the second charge now positive, the electric field vectors produced by both charges are in the negative x direction, when evaluated at x = 2.0 mm. Therefore, the net field points in the negative x direction, or 180°, measured counterclockwise from the $+x$ axis.

78. Let q_1 denote the charge at $y = d$ and q_2 denote the charge at $y = -d$. The individual magnitudes $|\vec{E}_1|$ and $|\vec{E}_2|$ are figured from Eq. 22-3, where the absolute value signs for q are unnecessary since these charges are both positive. The distance from q_1 to a point on the x axis is the same as the distance from q_2 to a point on the x axis: $r = \sqrt{x^2 + d^2}$. By symmetry, the y component of the net field along the x axis is zero. The x component of the net field, evaluated at points on the positive x axis, is

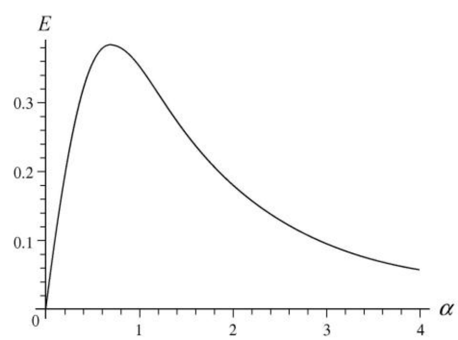
$$E_x = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{x^2 + d^2}\right)\left(\frac{x}{\sqrt{x^2 + d^2}}\right)$$

where the last factor is $\cos\theta = x/r$ with θ being the angle for each individual field as measured from the x axis.

(a) If we simplify the above expression, and plug in $x = \alpha d$, we obtain

$$E_x = \frac{q}{2\pi\varepsilon_0 d^2} \frac{\alpha}{(\alpha^2 + 1)^{3/2}}.$$

(b) The graph of $E = E_x$ versus α is shown below. For the purposes of graphing, we set d = 1 m and $q = 5.56 \times 10^{-11}$ C.



(c) From the graph, we estimate E_{max} occurs at about α = 0.71. More accurate computation shows that the maximum occurs at $\alpha = 1/\sqrt{2}$.

(d) The graph suggests that "half-height" points occur at $\alpha \approx 0.2$ and $\alpha \approx 2.0$. Further numerical exploration leads to the values: α = 0.2047 and α = 1.9864.

79. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o'clock (–q) and seven o'clock (–7q) positions is clearly equivalent to that of a single –6q charge sitting at the seven o'clock position. Similarly, the net field due to just the charges in the six o'clock (–6q) and twelve o'clock (–12q) positions is the same as that due to a single –6q charge sitting at the twelve o'clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o'clock, eight o'clock, … twelve o'clock positions. Thus, the resultant field of all of these points, by symmetry, is directed toward the position midway between seven and twelve o'clock. Therefore, $\vec{E}_{\text{resultant}}$ points toward the nine-thirty position.

80. The magnitude of the dipole moment is given by $p = qd$, where q is the positive charge in the dipole and d is the separation of the charges. For the dipole described in the problem,

$$p = (1.60\times10^{-19}\text{ C})(4.30\times10^{-9}\text{ m}) = 6.88\times10^{-28}\text{ C}\cdot\text{m}.$$

The dipole moment is a vector that points from the negative toward the positive charge.

81. (a) Since $\vec{E}$ points down and we need an upward electric force (to cancel the downward pull of gravity), then we require the charge of the sphere to be negative. The magnitude of the charge is found by working with the absolute value of Eq. 22-28:

$$|q| = \frac{F}{E} = \frac{mg}{E} = \frac{4.4\text{N}}{150\text{ N/C}} = 0.029\text{C},$$

or $q = -0.029$ C.

(b) The feasibility of this experiment may be studied by using Eq. 22-3 (using k for $1/4\pi\varepsilon_0$). We have $E = k|q|/r^2$ with

$$\rho_{\text{sulfur}}\left(\frac{4}{3}\pi r^3\right) = m_{\text{sphere}}$$

Since the mass of the sphere is 4.4/9.8 ≈ 0.45 kg and the density of sulfur is about 2.1×10^3 kg/m^3 (see Appendix F), then we obtain

$$r = \left(\frac{3m_{\text{sphere}}}{4\pi\rho_{\text{sulfur}}}\right)^{1/3} = 0.037\,\text{m} \;\Rightarrow\; E = k\frac{|q|}{r^2} \approx 2\times10^{11}\text{ N/C}$$

which is much too large a field to maintain in air.

82. We interpret the linear charge density, $\lambda = |Q|/L$, to indicate a positive quantity (so we can relate it to the magnitude of the field). Sample Problem 22.03 — "Electric field of a charged circular rod" illustrates the simplest approach to circular arc field problems.

Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ with θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(|Q|/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(|Q|/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2|Q|\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} .$$

With $|Q| = 6.25\times10^{-12}$ C, $\theta = 2.40$ rad $= 137.5°$, and $R = 9.00\times10^{-2}$ m, the magnitude of the electric field is $E = 5.39$ N/C.

83. **THINK** The potential energy of the electric dipole placed in an electric field depends on its orientation relative to the electric field. The field causes a torque that tends to align the dipole with the field.

EXPRESS When placed in an electric field $\vec{E}$, the potential energy of the dipole $\vec{p}$ is given by Eq. 22-38:

$$U(\theta) = -\vec{p}\cdot\vec{E} = -pE\cos\theta .$$

The torque caused by the electric field is (see Eq. 22-34) $\vec{\tau} = \vec{p}\times\vec{E}$.

ANALYZE (a) From Eq. 22-38 (and the facts that $\hat{\text{i}}\cdot\hat{\text{i}} = 1$ and $\hat{\text{j}}\cdot\hat{\text{i}} = 0$), the potential energy is

$$\begin{aligned} U &= -\vec{p}\cdot\vec{E} = -\left[\left(3.00\hat{\text{i}} + 4.00\hat{\text{j}}\right)\left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right)\right]\cdot\left[\left(4000\,\text{N/C}\right)\hat{\text{i}}\right] \\ &= -1.49\times10^{-26}\ \text{J}. \end{aligned}$$

(b) From Eq. 22-34 (and the facts that $\hat{\text{i}}\times\hat{\text{i}} = 0$ and $\hat{\text{j}}\times\hat{\text{i}} = -\hat{\text{k}}$), the torque is

$$\vec{\tau} = \vec{p}\times\vec{E} = \left[\left(3.00\hat{\text{i}} + 4.00\hat{\text{j}}\right)\left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right)\right]\times\left[\left(4000\,\text{N/C}\right)\hat{\text{i}}\right] = \left(-1.98\times10^{-26}\,\text{N}\cdot\text{m}\right)\hat{\text{k}}.$$

(c) The work done is

$$\begin{aligned} W &= \Delta U = \Delta\left(-\vec{p}\cdot\vec{E}\right) = \left(\vec{p}_i - \vec{p}_f\right)\cdot\vec{E} \\ &= \left[\left(3.00\hat{\text{i}} + 4.00\hat{\text{j}}\right) - \left(-4.00\hat{\text{i}} + 3.00\hat{\text{j}}\right)\right]\left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right)\cdot\left(4000\,\text{N/C}\right)\hat{\text{i}} \\ &= 3.47\times10^{-26}\ \text{J}. \end{aligned}$$

LEARN The work done by the agent is equal to the change in the potential energy of the dipole.

84. (a) The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is *a* = *eE*/*m*, where *E* is the magnitude of the field and *m* is the mass of the electron. Its numerical value is

$$a = \frac{(1.60 \times 10^{-19}\,\text{C})(2.00 \times 10^{3}\,\text{N/C})}{9.11 \times 10^{-31}\,\text{kg}} = 3.51 \times 10^{14}\ \text{m/s}^2 .$$

We put the origin of a coordinate system at the initial position of the electron. We take the *x* axis to be horizontal and positive to the right; take the *y* axis to be vertical and positive toward the top of the page. The kinematic equations are

$$x = v_0 t\cos\theta, \quad y = v_0 t\sin\theta - \frac{1}{2}at^2, \quad \text{and} \quad v_y = v_0\sin\theta - at.$$

First, we find the greatest *y* coordinate attained by the electron. If it is less than *d*, the electron does not hit the upper plate. If it is greater than *d*, it will hit the upper plate if the corresponding *x* coordinate is less than *L*. The greatest *y* coordinate occurs when $v_y = 0$. This means $v_0 \sin\theta - at = 0$ or $t = (v_0/a)\sin\theta$ and

$$y_{\max} = \frac{v_0^2\sin^2\theta}{a} - \frac{1}{2}a\frac{v_0^2\sin^2\theta}{a^2} = \frac{1}{2}\frac{v_0^2\sin^2\theta}{a} = \frac{\left(6.00\times10^6\ \text{m/s}\right)^2\sin^2 45°}{2\left(3.51\times10^{14}\ \text{m/s}^2\right)}$$
$$= 2.56\times10^{-2}\,\text{m}.$$

Since this is greater than *d* = 2.00 cm, the electron might hit the upper plate.

(b) Now, we find the *x* coordinate of the position of the electron when *y* = *d*. Since

$$v_0\sin\theta = (6.00\times10^6\ \text{m/s})\sin 45° = 4.24\times10^6\ \text{m/s}$$

and

$$2ad = 2(3.51\times10^{14}\ \text{m/s}^2)(0.0200\,\text{m}) = 1.40\times10^{13}\ \text{m}^2/\text{s}^2$$

the solution to $d = v_0 t\sin\theta - \frac{1}{2}at^2$ is

$$t = \frac{v_0\sin\theta - \sqrt{v_0^2\sin^2\theta - 2ad}}{a} = \frac{(4.24\times10^6\ \text{m/s}) - \sqrt{\left(4.24\times10^6\ \text{m/s}\right)^2 - 1.40\times10^{13}\ \text{m}^2/\text{s}^2}}{3.51\times10^{14}\ \text{m/s}^2}$$
$$= 6.43\times10^{-9}\,\text{s}.$$

The negative root was used because we want the *earliest* time for which $y = d$. The x coordinate is

$$x = v_0 t\cos\theta = \left(6.00\times10^{6}\ \text{m/s}\right)\left(6.43\times10^{-9}\ \text{s}\right)\cos 45° = 2.72\times10^{-2}\ \text{m}.$$

This is less than L so the electron hits the upper plate at x = 2.72 cm.

85. (a) If we subtract each value from the next larger value in the table, we find a set of numbers that are suggestive of a basic unit of charge: 1.64×10^{-19}, 3.3×10^{-19}, 1.63×10^{-19}, 3.35×10^{-19}, 1.6×10^{-19}, 1.63×10^{-19}, 3.18×10^{-19}, 3.24×10^{-19}, where the SI unit Coulomb is understood. These values are either close to a common $e \approx 1.6\times10^{-19}\text{C}$ value or are double that. Taking this, then, as a crude approximation to our experimental e we divide it into all the values in the original data set and round to the nearest integer, obtaining n = 4, 5, 7, 8,10, 11, 12, 14, and 16.

(b) When we perform a least squares fit of the original data set versus these values for n we obtain the linear equation:

$$q = 7.18 \times 10^{-21} + 1.633 \times 10^{-19} n\ .$$

If we dismiss the constant term as unphysical (representing, say, systematic errors in our measurements) then we obtain $e = 1.63 \times 10^{-19}$ when we set n = 1 in this equation.

86. (a) From symmetry, we see the net force component along the y axis is zero.

(b) The net force component along the x axis points rightward. With θ = 60°,

$$F_3 = 2\frac{q_3 q_1 \cos\theta}{4\pi\varepsilon_0 a^2}\ .$$

Since cos(60°) =1/2, we can write this as

$$F_3 = \frac{kq_3q_1}{a^2} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-12}\ \text{C})(2.00\times10^{-12}\ \text{C})}{(0.0950\ \text{m})^2} = 9.96\times10^{-12}\ \text{N}.$$

87. (a) For point A, we have (in SI units)

$$\vec{E}_A = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{q_2}{4\pi\varepsilon_0 r_2^2}\right](-\hat{\text{i}}) = \frac{(8.99\times10^{9})(1.00\times10^{-12}\ \text{C})}{\left(5.00\times10^{-2}\right)^2}(-\hat{\text{i}}) + \frac{(8.99\times10^{9})\,|-2.00\times10^{-12}\ \text{C}|}{\left(2\times5.00\times10^{-2}\right)^2}(+\hat{\text{i}})$$

$$= (-1.80\ \text{N/C})\hat{\text{i}}.$$

(b) Similar considerations leads to

$$\vec{E}_B = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\mathrm{i}} = \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\mathrm{C}\right)}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}} + \frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\mathrm{C}|}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}}$$

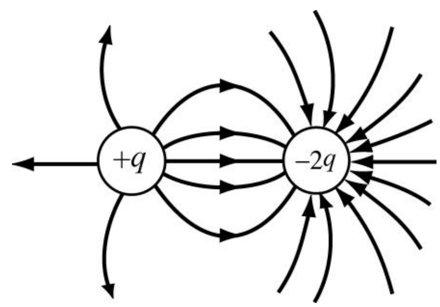
$$= (43.2\,\mathrm{N/C})\hat{\mathrm{i}}.$$

(c) For point C, we have

$$\vec{E}_C = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} - \frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\mathrm{i}} = \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\mathrm{C}\right)}{\left(2.00\times5.00\times10^{-2}\right)^2}\hat{\mathrm{i}} - \frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\mathrm{C}|}{\left(5.00\times10^{-2}\right)^2}\hat{\mathrm{i}}$$

$$= -(6.29\,\mathrm{N/C})\hat{\mathrm{i}}.$$

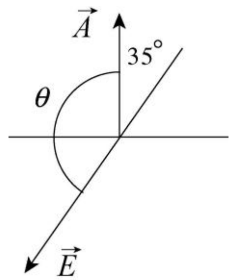
(d) The field lines are shown to the right. Note that there are twice as many field lines "going into" the negative charge $-2q$ as compared to that flowing out from the positive charge $+q$.



Chapter 23

1. **THINK** This exercise deals with electric flux through a square surface.

EXPRESS The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram below.



The electric flux through the surface is given by $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$.

EXPRESS The angle θ between $\vec{A}$ and $\vec{E}$ is 180° – 35° = 145°, so the electric flux through the area is

$$\Phi = EA\cos\theta = (1800\ \text{N/C})(3.2\times10^{-3}\ \text{m})^2\cos 145° = -1.5\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}.$$

LEARN The flux is a maximum when $\vec{A}$ and $\vec{E}$ points in the same direction ($\theta = 0$), and is zero when the two vectors are perpendicular to each other ($\theta = 90$).

2. We use $\Phi = \int \vec{E}\cdot d\vec{A}$ and note that the side length of the cube is (3.0 m–1.0 m) = 2.0 m.

(a) On the top face of the cube y = 2.0 m and $d\vec{A} = (dA)\hat{\text{j}}$. Therefore, we have $\vec{E} = 4\hat{\text{i}} - 3\left((2.0)^2 + 2\right)\hat{\text{j}} = 4\hat{\text{i}} - 18\hat{\text{j}}$. Thus the flux is

$$\Phi = \int_{\text{top}} \vec{E}\cdot d\vec{A} = \int_{\text{top}} \left(4\hat{\text{i}} - 18\hat{\text{j}}\right)\cdot(dA)\hat{\text{j}} = -18\int_{\text{top}} dA = (-18)(2.0)^2\ \text{N}\cdot\text{m}^2/\text{C} = -72\,\text{N}\cdot\text{m}^2/\text{C}.$$

(b) On the bottom face of the cube y = 0 and $d\vec{A} = (dA)(-\hat{\text{j}})$. Therefore, we have $E = 4\hat{\text{i}} - 3(0^2 + 2)\hat{\text{j}} = 4\hat{\text{i}} - 6\hat{\text{j}}$. Thus, the flux is

$$\Phi = \int_{\text{bottom}} \vec{E}\cdot d\vec{A} = \int_{\text{bottom}} \left(4\hat{\text{i}} - 6\hat{\text{j}}\right)\cdot(dA)\left(-\hat{\text{j}}\right) = 6\int_{\text{bottom}} dA = 6(2.0)^2\ \text{N}\cdot\text{m}^2/\text{C} = +24\ \text{N}\cdot\text{m}^2/\text{C}.$$

(c) On the left face of the cube $d\vec{A} = (dA)(-\hat{\text{i}})$. So

$$\Phi = \int_{\text{left}} \hat{E} \cdot d\vec{A} = \int_{\text{left}} \left(4\hat{\text{i}} + E_y\hat{\text{j}}\right) \cdot (dA)\left(-\hat{\text{i}}\right) = -4\int_{\text{bottom}} dA = -4(2.0)^2 \text{ N}\cdot\text{m}^2/\text{C} = -16 \text{ N}\cdot\text{m}^2/\text{C}.$$

(d) On the back face of the cube $d\vec{A} = (dA)(-\hat{\text{k}})$. But since $\vec{E}$ has no z component $\vec{E} \cdot d\vec{A} = 0$. Thus, Φ = 0.

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or +16 N·m^2/C. Thus the net flux through the cube is

$$\Phi = (-72 + 24 - 16 + 0 + 0 + 16)\ \text{N}\cdot\text{m}^2/\text{C} = -48\ \text{N}\cdot\text{m}^2/\text{C}.$$

3. We use $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\hat{\text{j}} = (1.40\text{m})^2\,\hat{\text{j}}$.

(a) $\Phi = (6.00\ \text{N/C})\hat{\text{i}} \cdot (1.40\ \text{m})^2\,\hat{\text{j}} = 0.$

(b) $\Phi = (-2.00\ \text{N/C})\hat{\text{j}} \cdot (1.40\ \text{m})^2\,\hat{\text{j}} = -3.92\ \text{N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi = \left[(-3.00\ \text{N/C})\hat{\text{i}} + (400\,\text{N/C})\hat{\text{k}}\right] \cdot (1.40\ \text{m})^2\,\hat{\text{j}} = 0$.

(d) The total flux of a uniform field through a closed surface is always zero.

4. The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E.$ Thus, the flux through the netting is

$$\Phi' = -\Phi = -\pi a^2 E = -\pi(0.11\ \text{m})^2(3.0\times10^{-3}\ \text{N/C}) = -1.1\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C}.$$

5. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length d, with a proton of charge $q = +1.6\times10^{-19}$ C situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{\text{net}} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus,

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{1.6\times10^{-19}\ \text{C}}{6(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.01\times10^{-9}\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

6. There is no flux through the sides, so we have two “inward” contributions to the flux, one from the top (of magnitude (34)(3.0)^2) and one from the bottom (of magnitude

(20)(3.0)^2). With "inward" flux being negative, the result is Φ = – 486 N·m^2/C. Gauss' law then leads to

$$q_{\text{enc}} = \varepsilon_0 \Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-486\ \ \text{N}\cdot\text{m}^2/\text{C}) = -4.3\times10^{-9}\,\text{C}.$$

7. We use Gauss' law: $\varepsilon_0\Phi = q$, where Φ is the total flux through the cube surface and q is the net charge inside the cube. Thus,

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.8\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \ \text{C}^2/\text{N}\cdot\text{m}^2} = 2.0\times10^5\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

8. (a) The total surface area bounding the bathroom is

$$A = 2(2.5\times3.0) + 2(3.0\times2.0) + 2(2.0\times2.5) = 37\ \text{m}^2.$$

The absolute value of the total electric flux, with the assumptions stated in the problem, is

$$|\Phi| = |\sum \vec{E}\cdot\vec{A}| = |\vec{E}|\,A = (600\ \text{N/C})(37\ \text{m}^2) = 22\times10^3\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

By Gauss' law, we conclude that the enclosed charge (in absolute value) is $|q_{\text{enc}}| = \varepsilon_0|\Phi| = 2.0\times10^{-7}$ C. Therefore, with volume V = 15 m^3, and recognizing that we are dealing with negative charges, the charge density is

$$\rho = \frac{q_{\text{enc}}}{V} = \frac{-2.0\times10^{-7}\ \text{C}}{15\ \text{m}^3} = -1.3\times10^{-8}\ \text{C/m}^3.$$

(b) We find $(|q_{\text{enc}}|/e)/V$ = (2.0 × 10^{-7} C/1.6 × 10^{-19} C)/15 m^3 = 8.2 × 10^{10} excess electrons per cubic meter.

9. (a) Let A = (1.40 m)^2. Then

$$\Phi = \left(3.00y\,\hat{\text{j}}\right)\cdot\left(-A\,\hat{\text{j}}\right)\Big|_{y=0} + \left(3.00y\,\hat{\text{j}}\right)\cdot\left(\text{A}\,\hat{\text{j}}\right)\Big|_{y=1.40} = (3.00)(1.40)(1.40)^2 = 8.23\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

(b) The charge is given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

(c) The electric field can be re-written as $\vec{E} = 3.00y\,\hat{\text{j}} + \vec{E}_0$, where $\vec{E}_0 = -4.00\hat{\text{i}} + 6.00\hat{\text{j}}$ is a constant field which does not contribute to the net flux through the cube. Thus Φ is still 8.23 N·m^2/C.

(d) The charge is again given by

$$q_{\text{enc}} = \varepsilon_0 \Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

10. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only. In Si units, we have

$$E_{\text{nonconstant}} = 3x\,\hat{\text{i}}\ .$$

The face of the cube located at $x = 0$ (in the yz plane) has area $A = 4\text{ m}^2$ (and it "faces" the $+\hat{\text{i}}$ direction) and has a "contribution" to the flux equal to $E_{\text{nonconstant}}A = (3)(0)(4) = 0$. The face of the cube located at $x = -2$ m has the same area A (and this one "faces" the $-\hat{\text{i}}$ direction) and a contribution to the flux:

$$-E_{\text{nonconstant}}A = -(3)(-2)(4) = 24\ \text{N}\cdot\text{m/C}^2.$$

Thus, the net flux is $\Phi = 0 + 24 = 24\ \text{N}\cdot\text{m/C}^2$. According to Gauss' law, we therefore have $q_{\text{enc}} = \varepsilon_0\Phi = 2.13\times10^{-10}$ C.

11. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only:

$$E_{\text{nonconstant}} = (-4.00y^2)\ \hat{\text{i}}\ \text{(in SI units)}\ .$$

The face of the cube located at $y = 4.00$ has area $A = 4.00\text{ m}^2$ (and it "faces" the $+\hat{\text{j}}$ direction) and has a "contribution" to the flux equal to

$$E_{\text{nonconstant}}A = (-4)(4^2)(4) = -256\ \text{N}\cdot\text{m/C}^2.$$

The face of the cube located at $y = 2.00$ m has the same area A (however, this one "faces" the $-\hat{\text{j}}$ direction) and a contribution to the flux:

$$-E_{\text{nonconstant}}A = -(-4)(2^2)(4) = 64\ \text{N}\cdot\text{m/C}^2.$$

Thus, the net flux is $\Phi = (-256 + 64)\ \text{N}\cdot\text{m/C}^2 = -192\ \text{N}\cdot\text{m/C}^2$. According to Gauss's law, we therefore have

$$q_{\text{enc}} = \varepsilon_0\Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-192\ \text{N}\cdot\text{m}^2/\text{C}) = -1.70\times10^{-9}\text{C}.$$

12. We note that only the smaller shell contributes a (nonzero) field at the designated point, since the point is inside the radius of the large sphere (and $E = 0$ inside of a spherical charge), and the field points toward the $-x$ direction. Thus, with $R = 0.020$ m (the radius of the smaller shell), $L = 0.10$ m and $x = 0.020$ m, we obtain

$$\vec{E}=E(-\hat{\mathrm{j}})=-\frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathrm{j}}=-\frac{4\pi R^2\sigma_2}{4\pi\varepsilon_0(L-x)^2}\hat{\mathrm{j}}=-\frac{R^2\sigma_2}{\varepsilon_0(L-x)^2}\hat{\mathrm{j}}$$

$$=-\frac{(0.020\text{ m})^2(4.0\times10^{-6}\text{C/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.10\text{ m}-0.020\text{ m})^2}\hat{\mathrm{j}}=\left(-2.8\times10^4\text{ N/C}\right)\hat{\mathrm{j}}.$$

13. **THINK** A cube has six surfaces. The total flux through the cube is the sum of fluxes through each individual surface. We use Gauss' law to find the net charge inside the cube.

EXPRESS Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_l be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero (because their area vectors are parallel to the field), so the total flux through the cube surface is

$$\Phi = A(E_\ell - E_u).$$

The net charge inside the cube is given by Gauss' law: $q=\varepsilon_0\Phi$.

ANALYZE Substituting the values given, we find the net charge to be

$$q=\varepsilon_0\Phi=\varepsilon_0 A(E_\ell-E_u)=(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(100\text{ m})^2(100\text{ N/C}-60.0\text{ N/C})$$
$$=3.54\times10^{-6}\text{ C}=3.54\,\mu\text{C}.$$

LEARN Since $\Phi>0$, we conclude that the cube encloses a net positive charge.

14. Equation 23-6 (Gauss' law) gives $\varepsilon_0\Phi = q_{\text{enc}}$.

(a) Thus, the value $\Phi=2.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$ for small r leads to

$$q_{\text{central}}=\varepsilon_0\Phi=(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.0\times10^5\text{ N}\cdot\text{m}^2/\text{C})=1.77\times10^{-6}\text{C}\approx1.8\times10^{-6}\text{C}.$$

(b) The next value that Φ takes is $\Phi=-4.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$, which implies that $q_{\text{enc}}=-3.54\times10^{-6}\text{C}$. But we have already accounted for some of that charge in part (a), so the result for part (b) is

$$q_A = q_{\text{enc}} - q_{\text{central}} = -5.3\times10^{-6}\text{ C}.$$

(c) Finally, the large r value for Φ is $\Phi=6.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$, which implies that $q_{\text{total enc}}=5.31\times10^{-6}\text{C}$. Considering what we have already found, then the result is $q_{\text{total enc}}-q_A-q_{\text{central}}=+8.9\mu C$.

15. The total flux through any surface that completely surrounds the point charge is q/ε_0.

(a) If we stack identical cubes side by side and directly on top of each other, we will find that eight cubes meet at any corner. Thus, one-eighth of the field lines emanating from the point charge pass through a cube with a corner at the charge, and the total flux through the surface of such a cube is $q/8\varepsilon_0$. Now the field lines are radial, so at each of the three cube faces that meet at the charge, the lines are parallel to the face and the flux through the face is zero.

(b) The fluxes through each of the other three faces are the same, so the flux through each of them is one-third of the total. That is, the flux through each of these faces is $(1/3)(q/8\varepsilon_0) = q/24\varepsilon_0$. Thus, the multiple is 1/24 = 0.0417.

16. The total electric flux through the cube is $\Phi = \oint \vec{E} \cdot d\vec{A}$. The net flux through the two faces parallel to the *yz* plane is

$$\Phi_{yz} = \iint \left[E_x(x = x_2) - E_x(x = x_1) \right] dydz = \int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz \left[10 + 2(4) - 10 - 2(1) \right]$$
$$= 6\int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz = 6(1)(2) = 12.$$

Similarly, the net flux through the two faces parallel to the *xz* plane is

$$\Phi_{xz} = \iint \left[E_y(y = y_2) - E_y(y = y_1) \right] dxdz = \int_{x_1=1}^{x_2=4} dy \int_{z_1=1}^{z_2=3} dz [-3 - (-3)] = 0,$$

and the net flux through the two faces parallel to the *xy* plane is

$$\Phi_{xy} = \iint \left[E_z(z = z_2) - E_z(z = z_1) \right] dxdy = \int_{x_1=1}^{x_2=4} dx \int_{y_1=0}^{y_2=1} dy \left(3b - b \right) = 2b(3)(1) = 6b.$$

Applying Gauss' law, we obtain

$$q_{\text{enc}} = \varepsilon_0 \Phi = \varepsilon_0 (\Phi_{xy} + \Phi_{xz} + \Phi_{yz}) = \varepsilon_0 (6.00b + 0 + 12.0) = 24.0\varepsilon_0$$

which implies that $b = 2.00 \text{ N/C} \cdot \text{m}$.

17. **THINK** The system has spherical symmetry, so our Gaussian surface is a sphere of radius *R* with a surface area $A = 4\pi R^2$.

EXPRESS The charge on the surface of the sphere is the product of the surface charge density σ and the surface area of the sphere: $q = \sigma A = \sigma(4\pi R^2)$. We calculate the total electric flux leaving the surface of the sphere using Gauss' law: $q = \varepsilon_0 \Phi$.

ANALYZE (a) With $R = (1.20 \text{ m})/2 = 0.60 \text{ m}$ and $\sigma = 8.1 \times 10^{-6} \text{ C/m}^2$, the charge on the surface is

$$q = 4\pi R^2\sigma = 4\pi\left(0.60\text{ m}\right)^2(8.1\times10^{-6}\text{ C/m}^2) = 3.7\times10^{-5}\text{ C}.$$

(b) We choose a Gaussian surface in the form of a sphere, concentric with the conducting sphere and with a slightly larger radius. By Gauss's law, the flux is

$$\Phi = \frac{q}{\varepsilon_0} = \frac{3.66\times10^{-5}\text{ C}}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 4.1\times10^{6}\text{ N}\cdot\text{m}^2/\text{C}.$$

LEARN Since there is no charge inside the conducting sphere, the total electric flux through the surface of the sphere only depends on the charge residing on the surface of the sphere.

18. Using Eq. 23-11, the surface charge density is

$$\sigma = E\varepsilon_0 = \left(2.3\times10^{5}\text{ N/C}\right)\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right) = 2.0\times10^{-6}\text{ C/m}^2.$$

19. (a) The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

$$\sigma = \frac{q}{\pi D^2} = \frac{2.4\times10^{-6}\text{ C}}{\pi\left(1.3\text{ m}\right)^2} = 4.5\times10^{-7}\text{ C/m}^2.$$

(b) Equation 23-11 gives

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.5\times10^{-7}\text{ C/m}^2}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 5.1\times10^{4}\text{ N/C}.$$

20. Equation 23-6 (Gauss' law) gives $\varepsilon_o\Phi = q_{\text{enc}}$.

(a) The value $\Phi = -9.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$ for small r leads to $q_{\text{central}} = -7.97\times10^{-6}$ C or roughly $-8.0\ \mu$C.

(b) The next (nonzero) value that Φ takes is $\Phi = +4.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$, which implies $q_{\text{enc}} = 3.54\times10^{-6}$C. But we have already accounted for some of that charge in part (a), so the result is

$$q_A = q_{\text{enc}} - q_{\text{central}} = 11.5\times10^{-6}\text{ C} \approx 12\ \mu\text{C}.$$

(c) Finally, the large r value for Φ is $\Phi = -2.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$, which implies $q_{\text{total enc}} = -1.77\times10^{-6}$C. Considering what we have already found, then the result is

$$q_{\text{total enc}} - q_A - q_{\text{central}} = -5.3\ \mu\text{C}.$$

21. (a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it

encloses is zero. The net charge is the sum of the charge q in the cavity and the charge q_w on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0 \times 10^{-6}$C.

(b) The net charge Q of the conductor is the sum of the charge on the cavity wall and the charge q_s on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_\omega = \left(10\times10^{-6}\ \text{C}\right) - \left(-3.0\times10^{-6}\ \text{C}\right) = +1.3\times10^{-5}\ \text{C}.$$

22. We combine Newton's second law ($F = ma$) with the definition of electric field ($F = qE$) and with Eq. 23-12 (for the field due to a line of charge). In terms of magnitudes, we have (if r = 0.080 m and $\lambda = 6.0\times10^{-6}$C/m)

$$ma = eE = \frac{e\lambda}{2\pi\varepsilon_0 r} \quad \Rightarrow \quad a = \frac{e\lambda}{2\pi\varepsilon_0 r\, m} = 2.1 \times 10^{17}\ \text{m/s}^2\ .$$

23. (a) The side surface area A for the drum of diameter D and length h is given by $A = \pi D h$. Thus,

$$\begin{aligned} q &= \sigma A = \sigma\pi D h = \pi\varepsilon_0 E D h \\ &= \pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.3\times10^5\ \text{N/C}\right)\left(0.12\ \text{m}\right)\left(0.42\ \text{m}\right) \\ &= 3.2\times10^{-7}\ \text{C}. \end{aligned}$$

(b) The new charge is

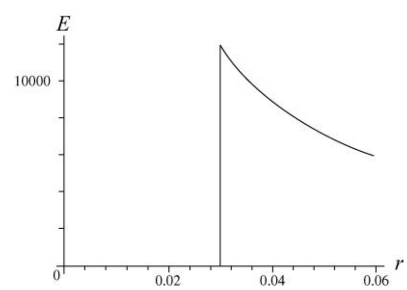
$$q' = q\left(\frac{A'}{A}\right) = q\left(\frac{\pi D'h'}{\pi D h}\right) = \left(3.2\times10^{-7}\text{C}\right)\left[\frac{(8.0\ \text{cm})(28\ \text{cm})}{(12\ \text{cm})(42\ \text{cm})}\right] = 1.4\times10^{-7}\ \text{C}.$$

24. We imagine a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry $\oint_A \vec{E}\cdot d\vec{A} = 2\pi r E = \frac{q_{\text{enc}}}{\varepsilon_0}$.

(a) For $r < R$, $q_{\text{enc}} = 0$, so $E = 0$.

(b) For $r > R$, $q_{\text{enc}} = \lambda$, so $E(r) = \lambda / 2\pi r\varepsilon_0$. With $\lambda = 2.00\times10^{-8}$ C/m and $r = 2.00R$ = 0.0600 m, we obtain

$$E = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.0600\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 5.99\times10^3\ \text{N/C}.$$



(c) The plot of E vs. r is shown to the right. Here, the maximum value is

$$E_{\max} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{\left(2.0\times10^{-8}\text{ C/m}\right)}{2\pi\left(0.030\text{ m}\right)\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)} = 1.2\times10^4\text{ N/C.}$$

25. **THINK** Our system is an infinitely long line of charge. Since the system possesses cylindrical symmetry, we may apply Gauss' law and take the Gaussian surface to be in the form of a closed cylinder.

EXPRESS We imagine a cylindrical Gaussian surface *A* of radius *r* and length *h* concentric with the metal tube. Then by symmetry,

$$\oint_A \vec{E}\cdot d\vec{A} = 2\pi rhE = \frac{q}{\varepsilon_0},$$

where *q* is the amount of charge enclosed by the Gaussian cylinder. Thus, the magnitude of the electric field produced by a uniformly charged infinite line is

$$E = \frac{q/h}{2\pi\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where λ is the linear charge density and *r* is the distance from the line to the point where the field is measured.

ANALYZE Substituting the values given, we have

$$\lambda = 2\pi\varepsilon_0 Er = 2\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(4.5\times10^4\text{ N/C}\right)\left(2.0\text{ m}\right)$$
$$= 5.0\times10^{-6}\text{ C/m.}$$

LEARN Since $\lambda > 0$, the direction of $\vec{E}$ is radially outward from the line of charge. Note that the field varies with *r* as $E \sim 1/r$, in contrast to the $1/r^2$ dependence due to a point charge.

26. As we approach *r* = 3.5 cm from the inside, we have

$$E_{\text{internal}} = \frac{2\lambda}{4\pi\varepsilon_0 r} = 1000\text{ N/C}.$$

And as we approach *r* = 3.5 cm from the outside, we have

$$E_{\text{external}} = \frac{2\lambda}{4\pi\varepsilon_0 r} + \frac{2\lambda'}{4\pi\varepsilon_0 r} = -3000\text{ N/C}.$$

Considering the difference ($E_{\text{external}} - E_{\text{internal}}$) allows us to find λ' (the charge per unit length on the larger cylinder). Using r = 0.035 m, we obtain $\lambda' = -5.8 \times 10^{-9}$ C/m.

27. We denote the radius of the thin cylinder as R = 0.015 m. Using Eq. 23-12, the net electric field for $r > R$ is given by

$$E_{\text{net}} = E_{\text{wire}} + E_{\text{cylinder}} = \frac{-\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda'}{2\pi\varepsilon_0 r}$$

where $-\lambda = -3.6$ nC/m is the linear charge density of the wire and λ' is the linear charge density of the thin cylinder. We note that the surface and linear charge densities of the thin cylinder are related by

$$q_{\text{cylinder}} = \lambda' L = \sigma(2\pi RL) \Rightarrow \lambda' = \sigma(2\pi R).$$

Now, E_{net} outside the cylinder will equal zero, provided that $2\pi R\sigma = \lambda$, or

$$\sigma = \frac{\lambda}{2\pi R} = \frac{3.6\times10^{-6}\text{ C/m}}{(2\pi)(0.015\text{ m})} = 3.8\times10^{-8}\text{ C/m}^2.$$

28. (a) In Eq. 23-12, $\lambda = q/L$ where q is the net charge enclosed by a cylindrical Gaussian surface of radius r. The field is being measured outside the system (the charged rod coaxial with the neutral cylinder) so that the net enclosed charge is only that which is on the rod. Consequently,

$$\left|\vec{E}\right| = \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{2(2.0\times10^{-9}\text{C/m})}{4\pi\varepsilon_0(0.15\text{ m})} = 2.4\times10^{2}\text{ N/C}.$$

(b) Since the field is zero inside the conductor (in an electrostatic configuration), then there resides on the inner surface charge $-q$, and on the outer surface, charge $+q$ (where q is the charge on the rod at the center). Therefore, with r_i = 0.05 m, the surface density of charge is

$$\sigma_{\text{inner}} = \frac{-q}{2\pi r_i L} = -\frac{\lambda}{2\pi r_i} = -\frac{2.0\times10^{-9}\text{C/m}}{2\pi(0.050\text{ m})} = -6.4\times10^{-9}\text{ C/m}^2$$

for the inner surface.

(c) With r_o = 0.10 m, the surface charge density of the outer surface is

$$\sigma_{\text{outer}} = \frac{+q}{2\pi r_o L} = \frac{\lambda}{2\pi r_o} = +3.2\times10^{-9}\text{ C/m}^2.$$

29. **THINK** The charge densities of both the conducting cylinder and the shell are uniform, and we neglect fringing effect. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

EXPRESS We take the Gaussian surface to be a cylinder of length L, coaxial with the given cylinders and of radius r. The flux through this surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Gauss' law yields $q_{enc} = \varepsilon_0\Phi = 2\pi r\varepsilon_0 LE$, where q_{enc} is the charge enclosed by the Gaussian surface.

ANALYZE (a) In this case, we take the radius of our Gaussian cylinder to be

$$r = 2.00R_2 = 20.0R_1 = (20.0)(1.3\times10^{-3}\text{ m}) = 2.6\times10^{-2}\text{ m}.$$

The charge enclosed is

$$q_{enc} = Q_1 + Q_2 = -Q_1 = -3.40\times10^{-12}\text{ C}.$$

Consequently, Gauss' law yields

$$E = \frac{q_{enc}}{2\pi\varepsilon_0 Lr} = \frac{-3.40\times10^{-12}\text{ C}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(11.0\text{ m})(2.60\times10^{-2}\text{m})} = -0.214\text{ N/C},$$

or $|E| = 0.214$ N/C.

(b) The negative sign in E indicates that the field points inward.

(c) Next, for $r = 5.00\ R_1$, the charge enclosed by the Gaussian surface is $q_{enc} = Q_1 = 3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{enc}$, or

$$E = \frac{q_{enc}}{2\pi\varepsilon_0 Lr} = \frac{3.40\times10^{-12}\text{ C}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(11.0\text{ m})(5.00\times1.30\times10^{-3}\text{m})} = 0.855\text{ N/C}.$$

(d) The positive sign indicates that the field points outward.

(e) We consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod has charge Q_1, the inner surface of the shell must have charge $Q_{in} = -Q_1 = -3.40\times10^{-12}$ C.

(f) Since the shell is known to have total charge $Q_2 = -2.00Q_1$, it must have charge $Q_{out} = Q_2 - Q_{in} = -Q_1 = -3.40\times10^{-12}$ C on its outer surface.

LEARN Cylindrical symmetry of the system allows us to apply Gauss' law to the problem. Since electric field is zero inside the conducting shell, by Gauss' law, any net charge must be distributed on the surfaces of the shells.

30. We reason that point *P* (the point on the *x* axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that *P* is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for *P*. Using Eq. 23-12, we have

$$E_{\text{net}} = E_1 + E_2 = \frac{2\lambda_1}{4\pi\varepsilon_0(x+L/2)} + \frac{2\lambda_2}{4\pi\varepsilon_0(x-L/2)} \ .$$

Setting this equal to zero and solving for *x* we find

$$x = \left(\frac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2}\right)\frac{L}{2} = \left(\frac{6.0\mu\text{C/m}-(-2.0\mu\text{C/m})}{6.0\mu\text{C/m}+(-2.0\mu\text{C/m})}\right)\frac{8.0\text{ cm}}{2} = 8.0\text{ cm}\,.$$

31. We denote the inner and outer cylinders with subscripts *i* and *o*, respectively.

(a) Since $r_i < r = 4.0$ cm $< r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\text{ C/m}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\text{ m})} = 2.3\times10^6\text{ N/C.}$$

(b) The electric field $\vec{E}(r)$ points radially outward.

(c) Since $r > r_o$,

$$E(r=8.0\text{ cm}) = \frac{\lambda_i+\lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\text{ C/m}-7.0\times10^{-6}\text{ C/m}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\text{ m})} = -4.5\times10^5\text{ N/C,}$$

or $|E(r=8.0\text{ cm})| = 4.5\times10^5$ N/C.

(d) The minus sign indicates that $\vec{E}(r)$ points radially inward.

32. To evaluate the field using Gauss' law, we employ a cylindrical surface of area $2\pi r L$ where *L* is very large (large enough that contributions from the ends of the cylinder become irrelevant to the calculation). The volume within this surface is $V = \pi r^2 L$, or expressed more appropriate to our needs: $dV = 2\pi rLdr$. The charge enclosed is, with $A = 2.5\times10^{-6}\,\text{C/m}^5$,

$$q_{\text{enc}} = \int_0^r A r^2\, 2\pi r L\, dr = \frac{\pi}{2} A L r^4.$$

By Gauss' law, we find $\Phi = |\vec{E}|(2\pi rL) = q_{\text{enc}}/\varepsilon_0$; we thus obtain $\left|\vec{E}\right| = \dfrac{A r^3}{4\varepsilon_0}$.

(a) With r = 0.030 m, we find $|\vec{E}| = 1.9$ N/C.

(b) Once outside the cylinder, Eq. 23-12 is obeyed. To find λ = q/L we must find the total charge q. Therefore,

$$\frac{q}{L} = \frac{1}{L}\int_0^{0.04} A r^2\, 2\pi r L\, dr = 1.0\times10^{-11}\ \text{C/m}.$$

And the result, for r = 0.050 m, is $|\vec{E}| = \lambda/2\pi\varepsilon_0 r = 3.6$ N/C.

33. We use Eq. 23-13.

(a) To the left of the plates:

$$\vec{E} = \left(\sigma/2\varepsilon_0\right)(-\hat{\text{i}})\ \text{(from the right plate)} + (\sigma/2\varepsilon_0)\hat{\text{i}}\ \text{(from the left one)} = 0.$$

(b) To the right of the plates:

$$\vec{E} = \left(\sigma/2\varepsilon_0\right)\hat{\text{i}}\ \text{(from the right plate)} + \left(\sigma/2\varepsilon_0\right)(-\hat{\text{i}})\ \text{(from the left one)} = 0.$$

(c) Between the plates:

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\text{i}}) + \left(\frac{\sigma}{2\varepsilon_0}\right)\left(-\hat{\text{i}}\right) = \left(\frac{\sigma}{\varepsilon_0}\right)(-\hat{\text{i}}) = -\left(\frac{7.00\times10^{-22}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)\hat{\text{i}}$$
$$= \left(-7.91\times10^{-11}\ \text{N/C}\right)\hat{\text{i}}.$$

34. The charge distribution in this problem is equivalent to that of an infinite sheet of charge with surface charge density $\sigma = 4.50\times10^{-12}$ C/m^2 plus a small circular pad of radius R = 1.80 cm located at the middle of the sheet with charge density $-\sigma$. We denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. Using Eq. 22-26 for $\vec{E}_2$, the net electric field $\vec{E}$ at a distance z = 2.56 cm along the central axis is then

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\text{k}} + \frac{(-\sigma)}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{\text{k}} = \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+R^2}}\hat{\text{k}}$$
$$= \frac{(4.50\times10^{-12}\ \text{C/m}^2)(2.56\times10^{-2}\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\sqrt{(2.56\times10^{-2}\ \text{m})^2 + (1.80\times10^{-2}\ \text{m})^2}}\hat{\text{k}} = (0.208\ \text{N/C})\,\hat{\text{k}}.$$

35. In the region between sheets 1 and 2, the net field is $E_1 - E_2 + E_3 = 2.0 \times 10^5$ N/C .

In the region between sheets 2 and 3, the net field is at its greatest value:

$$E_1 + E_2 + E_3 = 6.0 \times 10^5 \text{ N/C} .$$

The net field vanishes in the region to the right of sheet 3, where $E_1 + E_2 = E_3$. We note the implication that σ_3 is negative (and is the largest surface-density, in magnitude). These three conditions are sufficient for finding the fields:

$$E_1 = 1.0 \times 10^5 \text{ N/C} ,\ E_2 = 2.0 \times 10^5 \text{ N/C} ,\ E_3 = 3.0 \times 10^5 \text{ N/C} .$$

From Eq. 23-13, we infer (from these values of E)

$$\frac{|\sigma_3|}{|\sigma_2|} = \frac{3.0 \text{ x } 10^5 \text{ N/C}}{2.0 \text{ x } 10^5 \text{ N/C}} = 1.5.$$

Recalling our observation, above, about σ_3, we conclude that $\frac{\sigma_3}{\sigma_2} = -1.5.$

36. According to Eq. 23-13 the electric field due to either sheet of charge with surface charge density $\sigma = 1.77\times 10^{-22}$ C/m^2 is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0 = (1.77 \times 10^{-22}\text{ C/m}^2)/(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 2.00\times10^{-11}$ N/C, pointing in the upward direction, or $\vec{E} = (2.00\times10^{-11}\text{ N/C})\hat{\text{j}}$;

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down, or $\vec{E} = -(2.00\times10^{-11}\text{ N/C})\hat{\text{j}}$.

37. **THINK** To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we replace the finite plate with an infinite plate having the same charge density. Planar symmetry then allows us to apply Gauss' law to calculate the electric field.

EXPRESS Using Gauss' law, we find the magnitude of the field to be $E = \sigma/\varepsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus, $\sigma = q/2A$.

ANALYZE (a) With $q = 6.0\times10^{-6}$ C and $A = (0.080\text{ m})^2$, we obtain

$$\sigma = \frac{q}{2A} = \frac{6.0\times10^{-6}\ \text{C}}{2(0.080\ \text{m})^2} = 4.69\times10^{-4}\ \text{C/m}^2.$$

The magnitude of the field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.69\times10^{-4}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.3\times10^{7}\ \text{N/C}.$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\varepsilon_0 r^2 = kq/r^2$, where r is the distance from the plate. Thus,

$$E = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\ \text{C}\right)}{\left(30\ \text{m}\right)^2} = 60\ \text{N/C}.$$

LEARN In summary, the electric field is nearly uniform ($E = \sigma/\varepsilon_0$) close to the plate, but resembles that of a point charge far away from the plate.

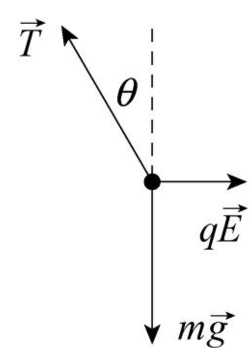
38. The field due to the sheet is $E = \frac{\sigma}{2\varepsilon_o}$. The force (in magnitude) on the electron (due to that field) is $F = eE$, and assuming it's the *only* force then the acceleration is

$$a = \frac{e\sigma}{2\varepsilon_o m} = \text{slope of the graph}\ \ (= 2.0\times10^{5}\ \text{m/s divided by}\ 7.0\times10^{-12}\ \text{s})\ .$$

Thus we obtain $\sigma = 2.9\times10^{-6}\ \text{C/m}^2$.

39. **THINK** Since the non-conducting charged ball is in equilibrium with the non-conducting charged sheet (see Fig. 23-49), both the vertical and horizontal components of the net force on the ball must be zero.

EXPRESS The forces acting on the ball are shown in the diagram to the right. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the magnitude of the electric field at the position of the ball; and the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical. Since the ball is in



equilibrium the net force on it vanishes. The sum of the horizontal components yields

$$qE - T\sin\theta = 0$$

and the sum of the vertical components yields

$$T\cos\theta - mg = 0\,.$$

We solve for the electric field E and deduce σ, the charge density of the sheet, from $E = \sigma/2\varepsilon_0$ (see Eq. 23-13).

ANALYZE The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform sheet of charge is given by $E = \sigma/2\varepsilon_0$, so

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and we have

$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0\times10^{-6}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\tan 30^\circ}{2.0\times10^{-8}\ \text{C}}$$
$$= 5.0\times10^{-9}\ \text{C/m}^2.$$

LEARN Since both the sheet and the ball are positively charged, the force between them is repulsive. This is balanced by the horizontal component of the tension in the thread. The angle the thread makes with the vertical direction increases with the charge density of the sheet.

40. The point where the individual fields cancel cannot be in the region between the sheet and the particle ($-d < x < 0$) since the sheet and the particle have opposite-signed charges. The point(s) could be in the region to the right of the particle ($x > 0$) and in the region to the left of the sheet ($x < d$); this is where the condition

$$\frac{|\sigma|}{2\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

must hold. Solving this with the given values, we find $r = x = \pm\sqrt{3/2\pi} \approx \pm\ 0.691$ m.

If $d = 0.20$ m (which is less than the magnitude of r found above), then neither of the points ($x \approx \pm\ 0.691$ m) is in the "forbidden region" between the particle and the sheet. Thus, both values are allowed. Thus, we have

(a) $x = 0.691$ m on the positive axis, and

(b) $x = -\,0.691$ m on the negative axis.

(c) If, however, $d = 0.80$ m (greater than the magnitude of r found above), then one of the points ($x \approx -0.691$ m) is in the "forbidden region" between the particle and the sheet and is disallowed. In this part, the fields cancel only at the point $x \approx +0.691$ m.

41. The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. We take the initial direction of motion of the electron to be positive. Then, the electric field is given by $E = \sigma/\varepsilon_0$, where σ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\varepsilon_0$ and the acceleration is

$$a = \frac{F}{m} = -\frac{e\sigma}{\varepsilon_0 m}$$

where m is the mass of the electron. The force is constant, so we use constant acceleration kinematics. If v_0 is the initial velocity of the electron, v is the final velocity, and x is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax$. Set $v = 0$ and replace a with $-e\sigma/\varepsilon_0 m$, then solve for x. We find

$$x = -\frac{v_0^2}{2a} = \frac{\varepsilon_0 m v_0^2}{2e\sigma}.$$

Now $\frac{1}{2}mv_0^2$ is the initial kinetic energy K_0, so

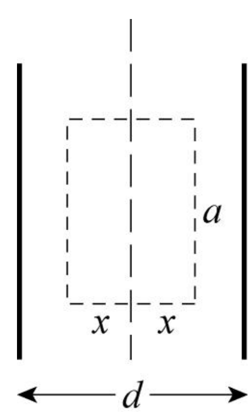
$$x = \frac{\varepsilon_0 K_0}{e\sigma} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.60\times10^{-17}\ \text{J}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.0\times10^{-6}\ \text{C/m}^2\right)} = 4.4\times10^{-4}\ \text{m}.$$

42. The surface charge density is given by

$$E = \sigma/\varepsilon_0 \Rightarrow \sigma = \varepsilon_0 E = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(55\ \text{N/C}) = 4.9\times10^{-10}\ \text{C/m}^2.$$

Since the area of the plates is $A = 1.0\ \text{m}^2$, the magnitude of the charge on the plate is $Q = \sigma A = 4.9\times10^{-10}$ C.

43. We use a Gaussian surface in the form of a box with rectangular sides. The cross section is shown with dashed lines in the diagram to the right. It is centered at the central plane of the slab, so the left and right faces are each a distance x from the central plane. We take the thickness of the rectangular solid to be a, the same as its length, so the left and right faces are squares.



The electric field is normal to the left and right faces and is uniform over them. Since ρ = 5.80 fC/m^3 is positive, it points outward at both faces: toward the left at the left face and toward the right at the right face. Furthermore, the magnitude is the same at both faces. The electric flux through each of these faces is Ea^2. The field is parallel to the other faces of the Gaussian surface and the flux through them is zero. The total flux through the Gaussian surface is $\Phi = 2Ea^2$. The volume enclosed by the Gaussian surface is $2a^2x$ and the charge contained within it is $q = 2a^2x\rho$. Gauss' law yields

$$2\varepsilon_0 Ea^2 = 2a^2x\rho.$$

We solve for the magnitude of the electric field: $E = \rho x / \varepsilon_0$.

(a) For $x = 0$, $E = 0$.

(b) For $x = 2.00$ mm $= 2.00 \times 10^{-3}$ m,

$$E = \frac{\rho x}{\varepsilon_0} = \frac{(5.80\times10^{-15}\ \text{C/m}^3)(2.00\times10^{-3}\ \text{m})}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.31\times10^{-6}\ \text{N/C}.$$

(c) For $x = d/2 = 4.70$ mm $= 4.70 \times 10^{-3}$ m,

$$E = \frac{\rho x}{\varepsilon_0} = \frac{(5.80\times10^{-15}\ \text{C/m}^3)(4.70\times10^{-3}\ \text{m})}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 3.08\times10^{-6}\ \text{N/C}.$$

(d) For $x = 26.0$ mm $= 2.60 \times 10^{-2}$ m, we take a Gaussian surface of the same shape and orientation, but with $x > d/2$, so the left and right faces are outside the slab. The total flux through the surface is again $\Phi = 2Ea^2$ but the charge enclosed is now $q = a^2d\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = a^2d\rho$, so

$$E = \frac{\rho d}{2\varepsilon_0} = \frac{(5.80\times10^{-15}\ \text{C/m}^3)(9.40\times10^{-3}\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.08\times10^{-6}\ \text{N/C}.$$

44. We determine the (total) charge on the ball by examining the maximum value (E = 5.0 × 10^7 N/C) shown in the graph (which occurs at r = 0.020 m). Thus, from $E = q/4\pi\varepsilon_0 r^2$, we obtain

$$q = 4\pi\varepsilon_0 r^2 E = \frac{(0.020\ \text{m})^2(5.0\times10^7\ \text{N/C})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.2\times10^{-6}\text{C} .$$

45. (a) Since $r_1 = 10.0$ cm $< r = 12.0$ cm $< r_2 = 15.0$ cm,

$$E(r)=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00\times10^{-8}\ \text{C}\right)}{\left(0.120\ \text{m}\right)^2}=2.50\times10^4\ \text{N/C}.$$

(b) Since $r_1 < r_2 < r = 20.0$ cm,

$$E(r)=\frac{1}{4\pi\varepsilon_0}\frac{q_1+q_2}{r^2}=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00+2.00\right)\left(1\times10^{-8}\ \text{C}\right)}{\left(0.200\ \text{m}^2\right)}=1.35\times10^4\ \text{N/C}.$$

46. The field at the proton's location (but not caused by the proton) has magnitude E. The proton's charge is e. The ball's charge has magnitude q. Thus, as long as the proton is at $r \geq R$ then the force on the proton (caused by the ball) has magnitude

$$F = eE = e\left(\frac{q}{4\pi\varepsilon_o r^2}\right) = \frac{e\,q}{4\pi\varepsilon_o r^2}$$

where r is measured from the center of the ball (to the proton). This agrees with Coulomb's law from Chapter 22. We note that if $r = R$ then this expression becomes

$$F_R = \frac{e\,q}{4\pi\varepsilon_o R^2}.$$

(a) If we require $F = \frac{1}{2}F_R$, and solve for r, we obtain $r = \sqrt{2}\,R$. Since the problem asks for the measurement from the surface then the answer is $\sqrt{2}\,R - R = 0.41R$.

(b) Now we require $F_{\text{inside}} = \frac{1}{2}F_R$ where $F_{\text{inside}} = eE_{\text{inside}}$ and E_{inside} is given by Eq. 23-20. Thus,

$$e\left(\frac{q}{4\pi\varepsilon_o R^2}\right)r = \frac{1}{2}\,\frac{e\,q}{4\pi\varepsilon_o R^2} \quad\Rightarrow\quad r = \frac{1}{2}R = 0.50\,R.$$

47. **THINK** The unknown charge is distributed uniformly over the surface of the conducting solid sphere.

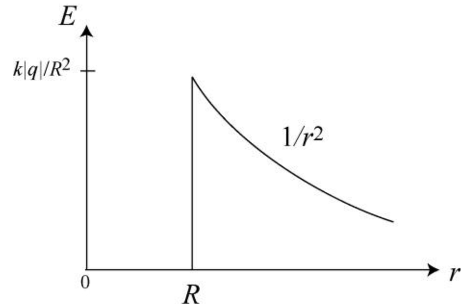
EXPRESS The electric field produced by the unknown charge at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = |q|/4\pi\varepsilon_0 r^2$, where $|q|$ is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured.

ANALYZE Thus, we have

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{\left(0.15\ \text{m}\right)^2\left(3.0\times10^3\ \text{N/C}\right)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}=7.5\times10^{-9}\ \text{C}.$$

The field points inward, toward the sphere center, so the charge is negative, i.e., $q = -7.5\times10^{-9}$C.

LEARN The electric field strength as a function of r is shown to the right. Inside the metal sphere, E = 0; outside the sphere, $E = k\,|\,q\,|\,/\,r^2$, where $k = 1/4\pi\varepsilon_0$.



48. Let E_A designate the magnitude of the field at r = 2.4 cm. Thus E_A = 2.0 × 10^7 N/C, and is totally due to the particle. Since $E_{\text{particle}} = q/4\pi\varepsilon_0 r^2$, then the field due to the particle at any other point will relate to E_A by a ratio of distances squared. Now, we note that at r = 3.0 cm the total contribution (from particle and shell) is 8.0 × 10^7 N/C. Therefore,

$$E_{\text{shell}} + E_{\text{particle}} = E_{\text{shell}} + (2.4/3)^2 E_A = 8.0 \times 10^7 \text{ N/C} .$$

Using the value for E_A noted above, we find E_{shell} = 6.6 × 10^7 N/C. Thus, with r = 0.030 m, we find the charge Q using $E_{\text{shell}} = Q/4\pi\varepsilon_0 r^2$:

$$Q = 4\pi\varepsilon_0 r^2 E_{\text{shell}} = \frac{r^2 E_{\text{shell}}}{k} = \frac{(0.030\text{ m})^2(6.6\times10^7\text{ N/C})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 6.6\times10^{-6}\text{C}$$

49. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E$, where r is the radius of the Gaussian surface.

For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \;\Rightarrow\; E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For r = 0, the above equation implies E = 0.

(b) For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{2(2.00\times10^{-2}\,\text{m})^2} = 5.62\times10^{-2}\text{ N/C}.$$

(c) For $r = a$, we have

$$E=\frac{q_1}{4\pi\varepsilon_0 a^2}=\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{(2.00\times10^{-2}\,\text{m})^2}=0.112\text{ N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

$$4\pi r^2 E=\frac{q_1}{\varepsilon_0} \quad\Rightarrow\quad E=\frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E=\frac{q_1}{4\pi\varepsilon_0 r^2}=\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{(1.50\times2.00\times10^{-2}\,\text{m})^2}=0.0499\text{ N/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E=0 \Rightarrow E=0.$ Thus, $E = 0$ at $r = 3.50a$.

(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E}\cdot d\vec{A}=0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -5.00$ fC.

(h) Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means

$$Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0.$$

50. The point where the individual fields cancel cannot be in the region between the shells since the shells have opposite-signed charges. It cannot be inside the radius R of one of the shells since there is only one field contribution there (which would not be canceled by another field contribution and thus would not lead to zero net field). We note shell 2 has greater magnitude of charge ($|\sigma_2|A_2$) than shell 1, which implies the point is not to the right of shell 2 (any such point would always be closer to the larger charge and thus no possibility for cancellation of equal-magnitude fields could occur). Consequently, the point should be in the region to the left of shell 1 (at a distance $r > R_1$ from its center); this is where the condition

$$E_1=E_2 \quad\Rightarrow\quad \frac{|q_1|}{4\pi\varepsilon_0 r^2}=\frac{|q_2|}{4\pi\varepsilon_0 (r+L)^2}$$

or

$$\frac{\sigma_1 A_1}{4\pi\varepsilon_0 r^2}=\frac{|\sigma_2|A_2}{4\pi\varepsilon_0 (r+L)^2}\ .$$

Using the fact that the area of a sphere is $A = 4\pi R^2$, this condition simplifies to

$$r = \frac{L}{(R_2/R_1)\sqrt{|\sigma_2|/\sigma_1} - 1} = 3.3 \text{ cm} .$$

We note that this value satisfies the requirement $r > R_1$. The answer, then, is that the net field vanishes at $x = -r = -3.3$ cm.

51. **THINK** Since our system possesses spherical symmetry, to calculate the electric field strength, we may apply Gauss' law and take the Gaussian surface to be in the form of a sphere of radius r.

EXPRESS To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, choose A so the field does not depend on the distance. We use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center. The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho \, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2 \, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 dr = 4\pi \; A \int_a^{r_g} r \, dr = 2\pi \; A \left(r_g^2 - a^2 \right).$$

The total charge inside the Gaussian surface is

$$q_{\text{enc}} = q + q_s = q + 2\pi A (r_g^2 - a^2).$$

The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E,$ where E is the magnitude of the field. Gauss' law yields

$$\Phi = q_{\text{enc}} / \varepsilon_0 \quad \Rightarrow \quad 4\pi\varepsilon_0 E r_g^2 = q + 2\pi A (r_g^2 - a^2).$$

We solve for E:

$$E = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{r_g^2} + 2\pi \, A - \frac{2\pi A a^2}{r_g^2} \right].$$

ANALYZE For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$. With $a = 2.00 \times 10^{-2}$ m and $q = 45.0 \times 10^{-15}$ C, we have $A = 1.79 \times 10^{-11} \text{C/m}^2$.

LEARN The value we have found for A ensures the uniformity of the field strength inside the shell. Using the result found above, we can readily show that the electric field in the region $a \le r \le b$ is

$$E = \frac{2\pi A}{4\pi\varepsilon_0} = \frac{A}{2\varepsilon_0} = \frac{1.79\times10^{-11}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.01\ \text{N/C}.$$

52. The field is zero for $0 \le r \le a$ as a result of Eq. 23-16. Thus,

(a) $E = 0$ at $r = 0$,

(b) $E = 0$ at $r = a/2.00$, and

(c) $E = 0$ at $r = a$.

For $a \le r \le b$ the enclosed charge q_{enc} (for $a \le r \le b$) is related to the volume by

$$q_{enc} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right).$$

Therefore, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right) = \frac{\rho}{3\varepsilon_0}\frac{r^3 - a^3}{r^2}$$

for $a \le r \le b$.

(d) For $r = 1.50a$, we have

$$E = \frac{\rho}{3\varepsilon_0}\frac{(1.50a)^3 - a^3}{(1.50a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{2.375}{2.25}\right) = \frac{(1.84\times10^{-9}\ \text{C/m}^3)(0.100\ \text{m})}{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{2.375}{2.25}\right) = 7.32\ \text{N/C}.$$

(e) For $r = b = 2.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(2.00a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{4}\right) = \frac{(1.84\times10^{-9}\ \text{C/m}^3)(0.100\ \text{m})}{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{7}{4}\right) = 12.1\ \text{N/C}.$$

(f) For $r \ge b$ we have $E = q_{total}/4\pi\varepsilon_0 r^2$ or

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2}.$$

Thus, for $r = 3.00b = 6.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(6.00a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{36}\right) = \frac{(1.84\times10^{-9}\ \text{C/m}^3)(0.100\ \text{m})}{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{7}{36}\right) = 1.35\ \text{N/C}.$$

53. (a) We integrate the volume charge density over the volume and require the result be equal to the total charge:

$$\int dx \int dy \int dz\, \rho = 4\pi \int_0^R dr\ r^2\ \rho = Q.$$

Substituting the expression $\rho = \rho_s r/R$, with ρ_s= 14.1 pC/m^3, and performing the integration leads to

$$4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{R^4}{4}\right) = Q$$

or

$$Q = \pi\rho_s R^3 = \pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})^3 = 7.78\times10^{-15}\ \text{C}.$$

(b) At r = 0, the electric field is zero (E = 0) since the enclosed charge is zero.

At a certain point within the sphere, at some distance r from the center, the field (see Eq. 23-8 through Eq. 23-10) is given by Gauss' law:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2}$$

where q_{enc} is given by an integral similar to that worked in part (a):

$$q_{\text{enc}} = 4\pi\int_0^r dr\, r^2 \rho = 4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{r^4}{4}\right).$$

Therefore,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^4}{Rr^2} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^2}{R}.$$

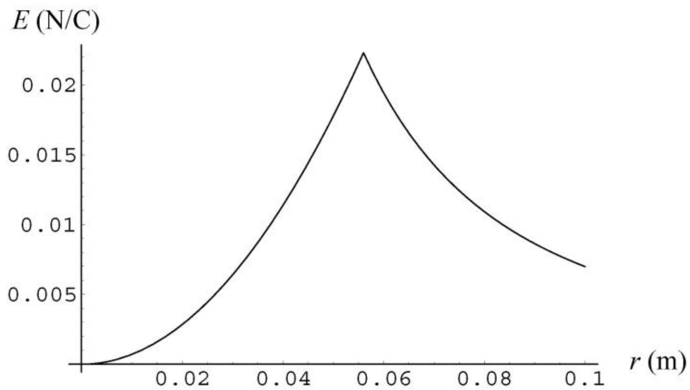
(c) For $r = R/2.00$, where R = 5.60 cm, the electric field is

$$\begin{aligned} E &= \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s (R/2.00)^2}{R} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R}{4.00} \\ &= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})}{4.00} \\ &= 5.58\times10^{-3}\,\text{N/C}. \end{aligned}$$

(d) For $r = R$, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R^2}{R} = \frac{\pi\rho_s R}{4\pi\varepsilon_0} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\pi(14.1\times10^{-12}\ \text{C/m}^3)(0.0560\ \text{m})$$
$$= 2.23\times10^{-2}\ \text{N/C}.$$

(e) The electric field strength as a function of *r* is depicted below:



54. Applying Eq. 23-20, we have

$$E_1 = \frac{|q_1|}{4\pi\varepsilon_0 R^3} r_1 = \frac{|q_1|}{4\pi\varepsilon_0 R^3}\left(\frac{R}{2}\right) = \frac{1}{2}\frac{|q_1|}{4\pi\varepsilon_0 R^2} .$$

Also, outside sphere 2 we have

$$E_2 = \frac{|q_2|}{4\pi\varepsilon_0 r^2} = \frac{|q_2|}{4\pi\varepsilon_0 (1.50R)^2} .$$

Equating these and solving for the ratio of charges, we arrive at $\frac{q_2}{q_1} = \frac{9}{8} = 1.125$.

55. We use

$$E(r) = \frac{q_{\text{enc}}}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\int_0^r \rho(r)4\pi r^2 dr$$

to solve for $\rho(r)$ and obtain

$$\rho(r) = \frac{\varepsilon_0}{r^2}\frac{d}{dr}\ r^2E(r)\ = \frac{\varepsilon_0}{r^2}\frac{d}{dr}\left(Kr^6\right) = 6K\varepsilon_0 r^3.$$

56. (a) There is no flux through the sides, so we have two contributions to the flux, one from the $x = 2$ end (with $\Phi_2 = +(2 + 2)(\pi(0.20)^2) = 0.50\ \text{N}\cdot\text{m}^2/\text{C}$) and one from the $x = 0$ end (with $\Phi_0 = -(2)(\pi(0.20)^2)$).

(b) By Gauss' law we have $q_{\text{enc}} = \varepsilon_0(\Phi_2 + \Phi_0) = 2.2\times10^{-12}$ C.

57. (a) For $r < R$, $E = 0$ (see Eq. 23-16).

(b) For r slightly greater than R,

$$E_R = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \approx \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-7}\ \text{C}\right)}{\left(0.250\ \text{m}\right)^2} = 2.88\times10^4\ \text{N/C}.$$

(c) For $r > R$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = E_R\left(\frac{R}{r}\right)^2 = \left(2.88\times10^4\ \text{N/C}\right)\left(\frac{0.250\ \text{m}}{3.00\ \text{m}}\right)^2 = 200\ \text{N/C}.$

58. From Gauss's law, we have

$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{\sigma\pi r^2}{\varepsilon_0} = \frac{(8.0\times10^{-9}\ \text{C/m}^2)\pi(0.050\ \text{m})^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 7.1\ \text{N}\cdot\text{m}^2/\text{C}\ .$$

59. (a) At $x = 0.040$ m, the net field has a rightward ($+x$) contribution (computed using Eq. 23-13) from the charge lying between $x = -0.050$ m and $x = 0.040$ m, and a leftward ($-x$) contribution (again computed using Eq. 23-13) from the charge in the region from $x = 0.040$ m to $x = 0.050$ m. Thus, since $\sigma = q/A = \rho V/A = \rho\Delta x$ in this situation, we have

$$\left|\vec{E}\right| = \frac{\rho(0.090\,\text{m})}{2\varepsilon_0} - \frac{\rho(0.010\,\text{m})}{2\varepsilon_0} = \frac{(1.2\times10^{-9}\ \text{C/m}^3)(0.090\,\text{m} - 0.010\,\text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 5.4\ \text{N/C}.$$

(b) In this case, the field contributions from all layers of charge point rightward, and we obtain

$$\left|\vec{E}\right| = \frac{\rho(0.100\,\text{m})}{2\varepsilon_0} = \frac{(1.2\times10^{-9}\ \text{C/m}^3)(0.100\,\text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 6.8\ \text{N/C}.$$

60. (a) We consider the radial field produced at points within a uniform cylindrical distribution of charge. The volume enclosed by a Gaussian surface in this case is $L\pi r^2$. Thus, Gauss' law leads to

$$E = \frac{|q_{\text{enc}}|}{\varepsilon_0 A_{\text{cylinder}}} = \frac{|\rho|\left(L\pi r^2\right)}{\varepsilon_0(2\pi rL)} = \frac{|\rho|r}{2\varepsilon_0}.$$

(b) We note from the above expression that the magnitude of the radial field grows with r.

(c) Since the charged powder is negative, the field points radially inward.

(d) The largest value of r that encloses charged material is $r_{\max} = R$. Therefore, with $|\rho| = 0.0011\ \text{C/m}^3$ and $R = 0.050$ m, we obtain

$$E_{\max} = \frac{|\rho| R}{2\varepsilon_0} = \frac{(0.0011\ \text{C/m}^3)(0.050\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.1\times10^6\ \text{N/C}.$$

(e) According to condition 1 mentioned in the problem, the field is high enough to produce an electrical discharge (at $r = R$).

61. **THINK** Our system consists of two concentric metal shells. We apply the superposition principle and Gauss' law to calculate the electric field everywhere.

EXPRESS At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the metal shells of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so

$$\Phi = \oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E = \frac{q_{\text{enc}}}{\varepsilon_0},$$

where r is the radius of the Gaussian surface.

ANALYZE (a) For $r < a$, the charge enclosed is $q_{\text{enc}} = 0$, so $E = 0$ in the region inside the shell.

(b) For $a < r < b$, the charged enclosed by the Gaussian surface is $q_{\text{enc}} = q_a$, so the field strength is $E = q_a/4\pi\varepsilon_0 r^2$.

(c) For $r > b$, the charged enclosed by the Gaussian surface is $q_{\text{enc}} = q_a + q_b$, so the field strength is $E = (q_a + q_b)/4\pi\varepsilon_0 r^2$.

(d) Since $E = 0$ for $r < a$ the charge on the inner surface of the inner shell is always zero. The charge on the outer surface of the inner shell is therefore q_a. Since $E = 0$ inside the metallic outer shell the net charge enclosed in a Gaussian surface that lies in between the inner and outer surfaces of the outer shell is zero. Thus the inner surface of the outer shell must carry a charge $-q_a$, leaving the charge on the outer surface of the outer shell to be $q_b + q_a$.

LEARN The concepts involved in this problem are discussed in Section 23-9 of the text. In the case of a single shell of radius R and charge q, the field strength is $E = 0$ for $r < R$, and $E = q/4\pi\varepsilon_0 r^2$ for $r > R$ (see Eqs. 23-15 and 23-16).

62. (a) The direction of the electric field at P_1 is away from q_1 and its magnitude is

$$\left|\vec{E}\right| = \frac{q}{4\pi\varepsilon_0 r_1^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-7}\ \text{C})}{(0.015\,\text{m})^2} = 4.0\times10^6\ \text{N/C}.$$

(b) $\vec{E} = 0$, since P_2 is inside the metal.

63. The proton is in uniform circular motion, with the electrical force of the sphere on the proton providing the centripetal force. According to Newton's second law, $F = mv^2/r$, where F is the magnitude of the force, v is the speed of the proton, and r is the radius of its orbit, essentially the same as the radius of the sphere. The magnitude of the force on the proton is $F = e|q|/4\pi\varepsilon_0 r^2$, where $|q|$ is the magnitude of the charge on the sphere. Thus,

$$\frac{1}{4\pi\varepsilon_0}\frac{e\,|\,q\,|}{r^2} = \frac{mv^2}{r}$$

so

$$|\,q\,| = \frac{4\pi\varepsilon_0 mv^2 r}{e} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.00\times10^5\ \text{m/s}\right)^2\left(0.0100\ \text{m}\right)}{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-9}\ \text{C}\right)} = 1.04\times10^{-9}\ \text{C}.$$

The force must be inward, toward the center of the sphere, and since the proton is positively charged, the electric field must also be inward. The charge on the sphere is negative: $q = -1.04 \times 10^{-9}$ C.

64. We interpret the question as referring to the field *just* outside the sphere (that is, at locations roughly equal to the radius r of the sphere). Since the area of a sphere is $A = 4\pi r^2$ and the surface charge density is $\sigma = q/A$ (where we assume q is positive for brevity), then

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{q}{4\pi r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

which we recognize as the field of a point charge (see Eq. 22-3).

65. (a) Since the volume contained within a radius of $\frac{1}{2}R$ is one-eighth the volume contained within a radius of R, the charge at $0 < r < R/2$ is $Q/8$. The fraction is 1/8 = 0.125.

(b) At $r = R/2$, the magnitude of the field is

$$E = \frac{Q/8}{4\pi\varepsilon_0 (R/2)^2} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2}$$

and is equivalent to *half* the field at the surface. Thus, the ratio is 0.500.

66. (a) The flux is still $-750\ \text{N}\cdot\text{m}^2/\text{C}$, since it depends only on the amount of charge enclosed.

(b) We use $\Phi = q/\varepsilon_0$ to obtain the charge q:

$$q = \varepsilon_0\Phi = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(-750\ \text{N}\cdot\text{m}^2/\text{C}\right) = -6.64\times10^{-9}\ \text{C}.$$

67. **THINK** The electric field at P is due to the charge on the surface of the metallic conductor and the point charge Q.

EXPRESS The initial field (evaluated "just outside the outer surface" which means it is evaluated at R_2 = 0.20 m, the outer radius of the conductor) is related to the charge q on the hollow conductor by Eq. 23-15: $E_{\text{initial}} = q/4\pi\varepsilon_0 R_2^2$. After the point charge Q is placed at the geometric center of the hollow conductor, the final field at that point is a combination of the initial and that due to Q (determined by Eq. 22-3):

$$E_{\text{final}} = E_{\text{initial}} + \frac{Q}{4\pi\varepsilon_0 R_2^2}.$$

ANALYZE (a) The charge on the spherical shell is

$$q = 4\pi\varepsilon_0 R_2^2 E_{\text{initial}} = \frac{(0.20\ \text{m})^2(450\ \text{N/C})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.0\times10^{-9}\ \text{C}.$$

(b) Similarly, using the equation above, we find the point charge to be

$$Q = 4\pi\varepsilon_0 R_2^2\left(E_{\text{final}} - E_{\text{initial}}\right) = \frac{(0.20\ \text{m})^2(180\ \text{N/C} - 450\ \text{N/C})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = -1.2\times10^{-9}\ \text{C}.$$

(c) In order to cancel the field (due to Q) within the conducting material, there must be an amount of charge equal to $-Q$ distributed uniformly on the inner surface (of radius R_1). Thus, the answer is $+1.2\times10^{-9}$ C.

(d) Since the total excess charge on the conductor is q and is located on the surfaces, then the outer surface charge must equal the total minus the inner surface charge. Thus, the answer is $2.0\times10^{-9}\ \text{C} - 1.2\times10^{-9}\ \text{C} = +0.80\times10^{-9}$ C.

LEARN The key idea here is to realize that the electric field inside the conducting shell ($R_1 < r < R_2$) must be zero, so the charge must be distributed in such a way that the charge enclosed by a Gaussian sphere of radius r ($R_1 < r < R_2$) is zero.

68. Let $\Phi_0 = 10^3\ \text{N}\cdot\text{m}^2/\text{C}$. The net flux through the entire surface of the dice is given by

$$\Phi = \sum_{n=1}^{6} \Phi_n = \sum_{n=1}^{6} (-1)^n n \Phi_0 = \Phi_0 (-1+2-3+4-5+6) = 3\Phi_0 .$$

Thus, the net charge enclosed is

$$q = \varepsilon_0 \Phi = 3\varepsilon_0 \Phi_0 = 3\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(10^3\,\text{N}\cdot\text{m}^2/\text{C}\right) = 2.66\times10^{-8}\text{C}.$$

69. Since the fields involved are uniform, the precise location of *P* is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward $(+\hat{\text{j}})$, and (from Eq. 23-13) its magnitude is

$$|\vec{E}| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0\times10^{-6}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 5.65\times10^4\ \text{N/C}.$$

In unit-vector notation, we have $\vec{E} = (5.65\times10^4\ \text{N/C})\hat{\text{j}}$.

70. Since the charge distribution is uniform, we can find the total charge *q* by multiplying ρ by the spherical volume ($\frac{4}{3}\pi r^3$) with $r = R =$ 0.050 m. This gives q = 1.68 nC.

(a) Applying Eq. 23-20 with r = 0.035 m, we have $E_{\text{internal}} = \dfrac{|q|\,r}{4\pi\varepsilon_0 R^3} = 4.2\times10^3\ \text{N/C}$.

(b) Outside the sphere we have (with r = 0.080 m)

$$E_{\text{external}} = \frac{|q|}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.68\times10^{-9}\,\text{C})}{(0.080\ \text{m})^2} = 2.4\times10^3\ \text{N/C}\ .$$

71. We choose a coordinate system whose origin is at the center of the flat base, such that the base is in the *xy* plane and the rest of the hemisphere is in the $z > 0$ half space.

(a) $\Phi = \pi R^2\left(-\hat{\text{k}}\right)\cdot E\hat{\text{k}} = -\pi R^2 E = -\pi(0.0568\ \text{m})^2(2.50\ \text{N/C}) = -0.0253\ \text{N}\cdot\text{m}^2/\text{C}.$

(b) Since the flux through the entire hemisphere is zero, the flux through the curved surface is $\vec{\Phi}_c = -\Phi_{\text{base}} = \pi R^2 E = 0.0253\ \text{N}\cdot\text{m}^2/\text{C}.$

72. The net enclosed charge *q* is given by

$$q = \varepsilon_0 \Phi = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(-48\ \text{N}\cdot\text{m}^2/\text{C}\right) = -4.2\times10^{-10}\text{C}.$$

73. (a) From Gauss' law, we get $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^3}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{(4\pi\rho r^3/3)\vec{r}}{r^3} = \frac{\rho\vec{r}}{3\varepsilon_0}$.

(b) The charge distribution in this case is equivalent to that of a whole sphere of charge density ρ plus a smaller sphere of charge density $-\rho$ that fills the void. By superposition

$$\vec{E}(\vec{r}) = \frac{\rho\vec{r}}{3\varepsilon_0} + \frac{(-\rho)(\vec{r}-\vec{a})}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0}.$$

74. (a) The cube is totally within the spherical volume, so the charge enclosed is

$$q_{enc} = \rho V_{cube} = (500 \times 10^{-9}\ \text{C/m}^3)(0.0400\ \text{m})^3 = 3.20 \times 10^{-11}\ \text{C}.$$

By Gauss' law, we find $\Phi = q_{enc}/\varepsilon_0 = 3.62\ \text{N}\cdot\text{m}^2/\text{C}$.

(b) Now the sphere is totally contained within the cube (note that the radius of the sphere is less than half the side-length of the cube). Thus, the total charge is

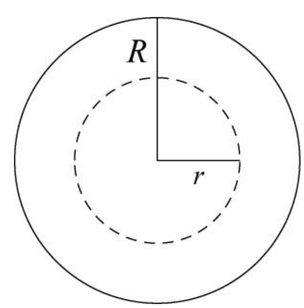
$$q_{enc} = \rho V_{sphere} = 4.5 \times 10^{-10}\ \text{C}.$$

By Gauss' law, we find $\Phi = q_{enc}/\varepsilon_0 = 51.1\ \text{N}\cdot\text{m}^2/\text{C}$.

75. The electric field is radially outward from the central wire. We want to find its magnitude in the region between the wire and the cylinder as a function of the distance r from the wire. Since the magnitude of the field at the cylinder wall is known, we take the Gaussian surface to coincide with the wall. Thus, the Gaussian surface is a cylinder with radius R and length L, coaxial with the wire. Only the charge on the wire is actually enclosed by the Gaussian surface; we denote it by q. The area of the Gaussian surface is $2\pi RL$, and the flux through it is $\Phi = 2\pi RLE$. We assume there is no flux through the ends of the cylinder, so this Φ is the total flux. Gauss' law yields $q = 2\pi\varepsilon_0 RLE$. Thus,

$$q = 2\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.014\ \text{m})(0.16\ \text{m})\ (2.9\times10^4\ \text{N/C}) = 3.6\times10^{-9}\ \text{C}.$$

76. (a) The diagram shows a cross section (or, perhaps more appropriately, "end view") of the charged cylinder (solid circle).



Consider a Gaussian surface in the form of a cylinder with radius r and length ℓ, coaxial with the charged cylinder. An "end view" of the Gaussian surface is shown as a dashed circle. The charge enclosed by it is $q = \rho V = \pi r^2 \ell\rho$, where $V = \pi r^2\ell$ is the volume of the cylinder. If ρ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it. Thus, the total flux through the Gaussian cylinder is $\Phi = EA_{\text{cylinder}} = E(2\pi r\ell)$. Now, Gauss' law leads to

$$2\pi\varepsilon_0 r\ell E = \pi r^2 \ell\rho \quad\Rightarrow\quad E = \frac{\rho r}{2\varepsilon_0}.$$

(b) Next, we consider a cylindrical Gaussian surface of radius $r > R$. If the external field E_{ext} then the flux is $\Phi = 2\pi r\ell E_{\text{ext}}$. The charge enclosed is the total charge in a section of the charged cylinder with length ℓ. That is, $q = \pi R^2 \ell\rho$. In this case, Gauss' law yields

$$2\pi\varepsilon_0 r\ell E_{\text{ext}} = \pi R^2 \ell\rho \quad\Rightarrow\quad E_{\text{ext}} = \frac{R^2\rho}{2\varepsilon_0 r}.$$

77. **THINK** The total charge on the conducting shell is equal to the sum of the charges on the shell's inner surface and the outer surface.

EXPRESS Let q_{in} be the charge on the inner surface and q_{out} the charge on the outer surface. The net charge on the shell is $Q = q_{\text{in}} + q_{\text{out}}$.

ANALYZE (a) In order to have net charge $Q = -10\ \mu$C when the charge on the outer surface is $q_{\text{out}} = -14\ \mu\text{C}$, then there must be

$$q_{\text{in}} = Q - q_{\text{out}} = -10\ \mu\text{C} - (-14\ \mu\text{C}) = +4\ \mu\text{C}$$

on the inner surface (since charges reside on the surfaces of a conductor in electrostatic situations).

(b) Let q be the charge of the particle. In order to cancel the electric field inside the conducting material, the contribution from the $q_{\text{in}} = +4\ \mu\text{C}$ on the inner surface must be canceled by that of the charged particle in the hollow, that is, $q_{\text{enc}} = q + q_{\text{in}} = 0$. Thus, the particle's charge is $q = -q_{\text{in}} = -4\ \mu\text{C}$.

LEARN The key idea here is to realize that the electric field inside the conducting shell must be zero. Thus, in the presence of a point charge in the hollow, the charge on the shell must be redistributed between its inner and outer surfaces in such a way that the net charge enclosed by a Gaussian sphere of radius r ($R_1 < r < R_2$, where R_1 is the inner radius and R_2 is the outer radius) remains zero.

78. (a) Outside the sphere, we use Eq. 23-15 and obtain

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-12}\ \text{C})}{(0.0600\ \text{m})^2} = 15.0\ \text{N/C}.$$

(b) With $q = +6.00 \times 10^{-12}$ C, Eq. 23-20 leads to $E = 25.3\,\text{N/C}$.

79. (a) The mass flux is $wd\rho v$ = (3.22 m) (1.04 m) (1000 kg/m^3) (0.207 m/s) = 693 kg/s.

(b) Since water flows only through area wd, the flux through the larger area is still 693 kg/s.

(c) Now the mass flux is $(wd/2)\rho v$ = (693 kg/s)/2 = 347 kg/s.

(d) Since the water flows through an area $(wd/2)$, the flux is 347 kg/s.

(e) Now the flux is $\left(wd\cos\theta\right)\rho v = \left(693\,\text{kg/s}\right)\left(\cos 34°\right) = 575\ \text{kg/s}$.

80. The field due to a sheet of charge is given by Eq. 23-13. Both sheets are horizontal (parallel to the xy plane), producing vertical fields (parallel to the z axis). At points above the z = 0 sheet (sheet A), its field points upward (toward $+z$); at points above the z = 2.0 sheet (sheet B), its field does likewise. However, below the z = 2.0 sheet, its field is oriented downward.

(a) The magnitude of the net field in the region between the sheets is

$$|\vec{E}| = \frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0} = \frac{8.00\times10^{-9}\,\text{C/m}^2 - 3.00\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.82\times10^2\ \text{N/C}.$$

(b) The magnitude of the net field at points above both sheets is

$$|\vec{E}| = \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0} = \frac{8.00\times10^{-9}\,\text{C/m}^2 + 3.00\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 6.21\times10^2\ \text{N/C}.$$

81. (a) The field maximum occurs at the outer surface:

$$E_{\text{max}} = \left(\frac{|q|}{4\pi\varepsilon_o r^2}\right)_{\text{at } r = R} = \frac{|q|}{4\pi\varepsilon_o R^2}$$

Applying Eq. 23-20, we have

$$E_{\text{internal}} = \frac{|q|}{4\pi\varepsilon_o R^3}\, r = \frac{1}{4} E_{\text{max}} \quad \Rightarrow \quad r = \frac{R}{4} = 0.25\,R.$$

(b) Outside sphere 2 we have

$$E_{\text{external}} = \frac{|q|}{4\pi\varepsilon_o r^2} = \frac{1}{4} E_{\text{max}} \quad \Rightarrow \quad r = 2.0R.$$

Chapter 24

1. **THINK** Ampere is the SI unit for current. An ampere is one coulomb per second.

EXPRESS To calculate the total charge through the circuit, we note that $1\text{ A} = 1\text{ C/s}$ and $1\text{ h} = 3600\text{ s}$.

ANALYZE (a) Thus,

$$84\text{ A}\cdot\text{h} = \left(84\frac{\text{C}\cdot\text{h}}{\text{s}}\right)\left(3600\frac{\text{s}}{\text{h}}\right) = 3.0\times10^{5}\text{ C}.$$

(b) The change in potential energy is $\Delta U = q\,\Delta V = (3.0\times10^{5}\text{ C})(12\text{ V}) = 3.6\times10^{6}\text{ J}$.

LEARN Potential difference is the change of potential energy per unit charge. Unlike electric field, potential difference is a scalar quantity.

2. The magnitude is $\Delta U = e\Delta V = 1.2\times10^{9}\text{ eV} = 1.2\text{ GeV}$.

3. (a) The change in energy of the transferred charge is

$$\Delta U = q\,\Delta V = (30\text{ C})(1.0\times10^{9}\text{ V}) = 3.0\times10^{10}\text{ J}.$$

(b) If all this energy is used to accelerate a 1000-kg car from rest, then $\Delta U = K = \frac{1}{2}mv^2$, and we find the car's final speed to be

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\Delta U}{m}} = \sqrt{\frac{2(3.0\times10^{10}\text{ J})}{1000\text{ kg}}} = 7.7\times10^{3}\text{ m/s}.$$

4. (a) $E = F/e = \left(3.9\times10^{-15}\text{ N}\right)/\left(1.60\times10^{-19}\text{ C}\right) = 2.4\times10^{4}\text{ N/C} = 2.4\times10^{4}\text{ V/m}.$

(b) $\Delta V = E\Delta s = \left(2.4\times10^{4}\text{ N/C}\right)\left(0.12\text{ m}\right) = 2.9\times10^{3}\text{ V}.$

5. **THINK** The electric field produced by an infinite sheet of charge is normal to the sheet and is uniform.

EXPRESS The magnitude of the electric field produced by the infinite sheet of charge is $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by

$$\Delta V = E\Delta x = (\sigma/2\varepsilon_0)\Delta x.$$

ANALYZE Thus, for $\sigma = 0.10\times10^{-6}\ \text{C/m}^2$ and $\Delta V = 50\ \text{V}$, we have

$$\Delta x = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(50\ \text{V}\right)}{0.10\times10^{-6}\ \text{C/m}^2} = 8.8\times10^{-3}\ \text{m}.$$

LEARN Equipotential surfaces are always perpendicular to the electric field lines. Figure 24-5(a) depicts the electric field lines and equipotential surfaces for a uniform electric field.

6. (a) $V_B - V_A = \Delta U/q = -W/(-e) = -(3.94\times10^{-19}\ \text{J})/(-1.60\times10^{-19}\ \text{C}) = 2.46\ \text{V}$.

(b) $V_C - V_A = V_B - V_A = 2.46\ \text{V}$.

(c) $V_C - V_B = 0$ (since C and B are on the same equipotential line).

7. We connect A to the origin with a line along the y axis, along which there is no change of potential (Eq. 24-18: $\int \vec{E}\cdot d\vec{s} = 0$). Then, we connect the origin to B with a line along the x axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4} \vec{E}\cdot d\vec{s} = -4.00\int_0^4 x\,dx = -4.00\left(\frac{4^2}{2}\right)$$

which yields $V_B - V_A = -32.0\ \text{V}$.

8. (a) By Eq. 24-18, the change in potential is the negative of the "area" under the curve. Thus, using the area-of-a-triangle formula, we have

$$V - 10 = -\int_0^{x=2} \vec{E}\cdot d\vec{s} = \frac{1}{2}(2)(20)$$

which yields $V = 30\ \text{V}$.

(b) For any region within $0 < x < 3\,\text{m}$, $-\int \vec{E}\cdot d\vec{s}$ is positive, but for any region for which $x > 3$ m it is negative. Therefore, $V = V_{max}$ occurs at $x = 3$ m.

$$V - 10 = -\int_0^{x=3} \vec{E}\cdot d\vec{s} = \frac{1}{2}(3)(20)$$

which yields $V_{max} = 40$ V.

(c) In view of our result in part (b), we see that now (to find $V = 0$) we are looking for some $X > 3$ m such that the "area" from $x = 3$ m to $x = X$ is 40 V. Using the formula for a triangle ($3 < x < 4$) and a rectangle ($4 < x < X$), we require

$$\frac{1}{2}(1)(20) + (X - 4)(20) = 40.$$

Therefore, $X = 5.5$ m.

9. (a) The work done by the electric field is

$$W = \int_i^f q_0\vec{E}\cdot d\vec{s} = \frac{q_0\sigma}{2\varepsilon_0}\int_0^d dz = \frac{q_0\sigma d}{2\varepsilon_0} = \frac{(1.60\times10^{-19}\text{C})(5.80\times10^{-12}\text{ C/m}^2)(0.0356\text{ m})}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}$$
$$= 1.87\times10^{-21}\text{J}.$$

(b) Since

$$V - V_0 = -W/q_0 = -\sigma z/2\varepsilon_0,$$

with V_0 set to be zero on the sheet, the electric potential at P is

$$V = -\frac{\sigma z}{2\varepsilon_0} = -\frac{(5.80\times10^{-12}\text{ C/m}^2)(0.0356\text{ m})}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)} = -1.17\times10^{-2}\text{ V}.$$

10. In the "inside" region between the plates, the individual fields (given by Eq. 24-13) are in the same direction ($-\hat{\text{i}}$):

$$\vec{E}_{\text{in}} = -\left(\frac{50\times10^{-9}\text{ C/m}^2}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)} + \frac{25\times10^{-9}\text{ C/m}^2}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\right)\hat{\text{i}} = -(4.2\times10^3\text{ N/C})\hat{\text{i}}.$$

In the "outside" region where $x > 0.5$ m, the individual fields point in opposite directions:

$$\vec{E}_{\text{out}} = -\frac{50\times10^{-9}\text{C/m}^2}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\hat{\text{i}} + \frac{25\times10^{-9}\text{ C/m}^2}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\hat{\text{i}} = -(1.4\times10^3\text{ N/C})\hat{\text{i}}.$$

Therefore, by Eq. 24-18, we have

$$\Delta V = -\int_0^{0.8}\vec{E}\cdot d\vec{s} = -\int_0^{0.5}\left|\vec{E}_{\text{in}}\right|dx - \int_{0.5}^{0.8}\left|\vec{E}_{\text{out}}\right|dx = -\left(4.2\times10^3\right)(0.5) - \left(1.4\times10^3\right)(0.3)$$
$$= 2.5\times10^3\text{ V}.$$

11. (a) The potential as a function of *r* is

$$V(r) = V(0) - \int_0^r E(r)dr = 0 - \int_0^r \frac{qr}{4\pi\varepsilon_0 R^3}dr = -\frac{qr^2}{8\pi\varepsilon_0 R^3}$$
$$= -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\,\text{C})(0.0145\ \text{m})^2}{2(0.0231\ \text{m})^3} = -2.68\times10^{-4}\ \text{V}.$$

(b) Since $\Delta V = V(0) - V(R) = q/8\pi\varepsilon_0 R$, we have

$$V(R) = -\frac{q}{8\pi\varepsilon_0 R} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\,\text{C})}{2(0.0231\ \text{m})} = -6.81\times10^{-4}\ \text{V}.$$

12. The charge is

$$q = 4\pi\varepsilon_0 RV = \frac{(10\text{m})(-1.0\text{V})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = -1.1\times10^{-9}\,\text{C}.$$

13. (a) The charge on the sphere is

$$q = 4\pi\varepsilon_0 VR = \frac{(200\ \text{V})(0.15\ \text{m})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 3.3\times10^{-9}\ \text{C}.$$

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma = \frac{q}{4\pi R^2} = \frac{3.3\times10^{-9}\ \text{C}}{4\pi(0.15\ \text{m})^2} = 1.2\times10^{-8}\ \text{C/m}^2.$$

14. (a) The potential difference is

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0 r_A} - \frac{q}{4\pi\varepsilon_0 r_B} = \left(1.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{2.0\ \text{m}} - \frac{1}{1.0\ \text{m}}\right)$$
$$= -4.5\times10^3\ \text{V}.$$

(b) Since $V(r)$ depends only on the magnitude of $\vec{r}$, the result is unchanged.

15. **THINK** The electric potential for a spherically symmetric charge distribution falls off as $1/r$, where *r* is the radial distance from the center of the charge distribution.

EXPRESS The electric potential *V* at the surface of a drop of charge *q* and radius *R* is given by $V = q/4\pi\varepsilon_0 R$.

ANALYZE (a) With V = 500 V and $q = 30\times10^{-12}$ C, we find the radius to be

$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(30\times10^{-12}\ \text{C}\right)}{500\ \text{V}} = 5.4\times10^{-4}\ \text{m}.$$

(b) After the two drops combine to form one big drop, the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of the original drop: $q' = 2q$. Thus,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500\ \text{V}) \approx 790\ \text{V}.$$

LEARN A positively charged configuration produces a positive electric potential, and a negatively charged configuration produces a negative electric potential. Adding more charge increases the electric potential.

16. In applying Eq. 24-27, we are assuming $V \to 0$ as $r \to \infty$. All corner particles are equidistant from the center, and since their total charge is

$$2q_1 - 3q_1 + 2\,q_1 - q_1 = 0,$$

then their contribution to Eq. 24-27 vanishes. The net potential is due, then, to the two $+4q_2$ particles, each of which is a distance of $a/2$ from the center:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} + \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} = \frac{16q_2}{4\pi\varepsilon_0 a} = \frac{16(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-12}\text{C})}{0.39\ \text{m}}$$
$$= 2.21\ \text{V}.$$

17. A charge $-5q$ is a distance $2d$ from P, a charge $-5q$ is a distance d from P, and two charges $+5q$ are each a distance d from P, so the electric potential at P is

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{2d} - \frac{1}{d} + \frac{1}{d} + \frac{1}{d}\right] = \frac{q}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\text{C})}{2(4.00\times10^{-2}\ \text{m})}$$
$$= 5.62\times10^{-4}\,\text{V}.$$

The zero of the electric potential was taken to be at infinity.

18. When the charge q_2 is infinitely far away, the potential at the origin is due only to the charge q_1 :

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 d} = 5.76\times10^{-7}\,\text{V}.$$

Thus, $q_1/d = 6.41 \times 10^{-17}$ C/m. Next, we note that when q_2 is located at $x = 0.080$ m, the net potential vanishes ($V_1 + V_2 = 0$). Therefore,

$$0 = \frac{kq_2}{0.08\text{ m}} + \frac{kq_1}{d}$$

Thus, we find $q_2 = -(q_1/d)(0.08\text{ m}) = -5.13 \times 10^{-18}\text{ C} = -32\,e$.

19. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$. With $d = 24.0$ cm, we have $x = 6.00$ cm.

(b) Similarly, for $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. With $d = 24.0$ cm, we have $x = -12.0$ cm.

20. Since according to the problem statement there is a point in between the two charges on the x axis where the net electric field is zero, the fields at that point due to q_1 and q_2 must be directed opposite to each other. This means that q_1 and q_2 must have the same sign (i.e., either both are positive or both negative). Thus, the potentials due to either of them must be of the same sign. Therefore, the net electric potential cannot possibly be zero anywhere except at infinity.

21. We use Eq. 24-20:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.47\times3.34\times10^{-30}\text{ C}\cdot\text{m}\right)}{\left(52.0\times10^{-9}\text{ m}\right)^2} = 1.63\times10^{-5}\text{ V}.$$

22. From Eq. 24-30 and Eq. 24-14, we have (for $\theta_i = 0°$)

$$W_a = q\Delta V = e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2} - \frac{p\cos\theta_i}{4\pi\varepsilon_0 r^2}\right) = \frac{ep\cos\theta}{4\pi\varepsilon_0 r^2}(\cos\theta - 1)$$

with $r = 20 \times 10^{-9}$ m. For $\theta = 180°$ the graph indicates $W_a = -4.0 \times 10^{-30}$ J, from which we can determine p. The magnitude of the dipole moment is therefore $5.6 \times 10^{-37}\ \text{C}\cdot\text{m}$.

23. (a) From Eq. 24-35, we find the potential to be

$$\begin{aligned} V &= 2\frac{\lambda}{4\pi\varepsilon_0}\ \ln\left[\frac{L/2 + \sqrt{(L^2/4) + d^2}}{d}\right] \\ &= 2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.68\times10^{-12}\ \text{C/m})\ln\left[\frac{(0.06\ \text{m}/2) + \sqrt{(0.06\ \text{m})^2/4 + (0.08\ \text{m})^2}}{0.08\ \text{m}}\right] \\ &= 2.43\times10^{-2}\ \text{V}. \end{aligned}$$

(b) The potential at P is $V = 0$ due to superposition.

24. The potential is

$$\begin{aligned} V_P &= \frac{1}{4\pi\varepsilon_0}\int_{\text{rod}}\frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\varepsilon_0 R} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(25.6\times10^{-12}\ \text{C})}{3.71\times10^{-2}\ \text{m}} \\ &= -6.20\ \text{V}. \end{aligned}$$

We note that the result is exactly what one would expect for a point-charge $-Q$ at a distance R. This "coincidence" is due, in part, to the fact that V is a scalar quantity.

25. (a) All the charge is the same distance R from C, so the electric potential at C is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} - \frac{6Q_1}{R}\right) = -\frac{5Q_1}{4\pi\varepsilon_0 R} = -\frac{5(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.20\times10^{-12}\ \text{C})}{8.20\times10^{-2}\ \text{m}} = -2.30\ \text{V},$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from P. That distance is $\sqrt{R^2 + D^2}$, so the electric potential at P is

$$\begin{aligned} V &= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{R^2 + D^2}} - \frac{6Q_1}{\sqrt{R^2 + D^2}}\right] = -\frac{5Q_1}{4\pi\varepsilon_0\sqrt{R^2 + D^2}} \\ &= -\frac{5(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.20\times10^{-12}\ \text{C})}{\sqrt{(8.20\times10^{-2}\ \text{m})^2 + (6.71\times10^{-2}\ \text{m})^2}} \\ &= -1.78\ \text{V}. \end{aligned}$$

26. The derivation is shown in the book (Eq. 24-33 through Eq. 24-35) except for the change in the lower limit of integration (which is now $x = D$ instead of $x = 0$). The result is therefore (cf. Eq. 24-35)

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{L+\sqrt{L^2+d^2}}{D+\sqrt{D^2+d^2}}\right) = \frac{2.0\times10^{-6}}{4\pi\varepsilon_0}\ln\left(\frac{4+\sqrt{17}}{1+\sqrt{2}}\right) = 2.18\times10^4\ \text{V}.$$

27. Letting d denote 0.010 m, we have

$$V = \frac{Q_1}{4\pi\varepsilon_0 d} + \frac{3Q_1}{8\pi\varepsilon_0 d} - \frac{3Q_1}{16\pi\varepsilon_0 d} = \frac{Q_1}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(30\times10^{-9}\ \text{C})}{2(0.01\ \text{m})}$$
$$= 1.3\times10^4\ \text{V}.$$

28. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right)$$
$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(56.1\times10^{-15}\ \text{C})}{0.12\ \text{m}}\ln\left(1+\frac{0.12\ \text{m}}{0.025\ \text{m}}\right)$$
$$= 7.39\times10^{-3}\ \text{V}.$$

29. Since the charge distribution on the arc is equidistant from the point where V is evaluated, its contribution is identical to that of a point charge at that distance. We assume $V \to 0$ as $r \to \infty$ and apply Eq. 24-27:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+4Q_1}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R}$$
$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.21\times10^{-12}\ \text{C})}{2.00\ \text{m}}$$
$$= 3.24\times10^{-2}\ \text{V}.$$

30. The dipole potential is given by Eq. 24-30 (with $\theta = 90^\circ$ in this case)

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos 90°}{4\pi\varepsilon_0 r^2} = 0$$

since cos(90º) = 0 . The potential due to the short arc is $q_1 / 4\pi\varepsilon_0 r_1$ and that caused by the long arc is $q_2 / 4\pi\varepsilon_0 r_2$. Since q_1 = +2 μC, r_1 = 4.0 cm, q_2 = –3 μC, and r_2 = 6.0 cm, the potentials of the arcs cancel. The result is zero.

31. **THINK** Since the disk is uniformly charged, when the full disk is present each quadrant contributes equally to the electric potential at *P*.

EXPRESS Electrical potential is a scalar quantity. The potential at *P* due to a single quadrant is one-fourth the potential due to the entire disk. We first find an expression for the potential at *P* due to the entire disk. To do so, consider a ring of charge with radius *r* and (infinitesimal) width *dr*. Its area is 2π*r dr* and it contains charge *dq* = 2πσ*r dr*. All the charge in it is at a distance $\sqrt{r^2 + D^2}$ from *P*, so the potential it produces at *P* is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma r dr}{\sqrt{r^2+D^2}} = \frac{\sigma r dr}{2\varepsilon_0\sqrt{r^2+D^2}}.$$

ANALYZE Integrating over *r*, the total potential at *P* is

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{rdr}{\sqrt{r^2+D^2}} = \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+D^2}\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+D^2} - D\right].$$

Therefore, the potential V_{sq} at *P* due to a single quadrant is

$$V_{sq} = \frac{V}{4} = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2} - D\right] = \frac{(7.73\times10^{-15}\,\text{C/m}^2)}{8(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left[\sqrt{(0.640\ \text{m})^2 + (0.259\ \text{m})^2} - 0.259\ \text{m}\right]$$
$$= 4.71\times10^{-5}\ \text{V}.$$

LEARN Consider the limit $D \gg R$. The potential becomes

$$V_{sq} = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2} - D\right] \approx \frac{\sigma}{8\varepsilon_0}\left[D\left(1 + \frac{1}{2}\frac{R^2}{D^2} + \cdots\right) - D\right]$$
$$= \frac{\sigma}{8\varepsilon_0}\frac{R^2}{2D} = \frac{\pi R^2\sigma/4}{4\pi\varepsilon_0 D} = \frac{q_{sq}}{4\pi\varepsilon_0 D}$$

where $q_{sq} = \pi R^2\sigma/4$ is the charge on the quadrant. In this limit, we see that the potential resembles that due to a point charge q_{sq}.

32. Equation 24-32 applies with $dq = \lambda\, dx = bx\, dx$ (along $0 \le x \le 0.20$ m).

(a) Here $r = x > 0$, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bx\,dx}{x} = \frac{b(0.20)}{4\pi\varepsilon_0} = 36 \text{ V}.$$

(b) Now $r = \sqrt{x^2 + d^2}$ where $d = 0.15$ m, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bxdx}{\sqrt{x^2+d^2}} = \frac{b}{4\pi\varepsilon_0}\left(\sqrt{x^2+d^2}\right)\Bigg|_0^{0.20} = 18 \text{ V}.$$

33. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx = cx\, dx$. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain

$$\begin{aligned} V &= \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{xdx}{d+x} = \frac{c}{4\pi\varepsilon_0}[x - d\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\varepsilon_0}\left[L - d\ln\left(1+\frac{L}{d}\right)\right] \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(28.9\times10^{-12}\,\text{C/m}^2)\left[0.120\text{ m} - (0.030\text{ m})\ln\left(1+\frac{0.120\text{ m}}{0.030\text{ m}}\right)\right] \\ &= 1.86\times10^{-2}\ \text{V}. \end{aligned}$$

34. The magnitude of the electric field is given by

$$|E| = \left|-\frac{\Delta V}{\Delta x}\right| = \frac{2(5.0\text{V})}{0.015\text{m}} = 6.7\times10^2\ \text{V/m}.$$

At any point in the region between the plates, $\vec{E}$ points away from the positively charged plate, directly toward the negatively charged one.

35. We use Eq. 24-41:

$$E_x(x,y) = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = -2(2.0\text{V}/\text{m}^2)x;$$
$$E_y(x,y) = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = 2(3.0\text{V}/\text{m}^2)y.$$

We evaluate at $x = 3.0$ m and $y = 2.0$ m to obtain

$$\vec{E} = (-12\ \text{V/m})\hat{\text{i}} + (12\ \text{V/m})\hat{\text{j}}.$$

36. We use Eq. 24-41. This is an ordinary derivative since the potential is a function of only one variable.

$$\vec{E} = -\left(\frac{dV}{dx}\right)\hat{\text{i}} = -\frac{d}{dx}(1500x^2)\hat{\text{i}} = (-3000x)\hat{\text{i}} = (-3000\,\text{V/m}^2)(0.0130\,\text{m})\hat{\text{i}}$$
$$= (-39\,\text{V/m})\hat{\text{i}}.$$

(a) Thus, the magnitude of the electric field is E = 39 V/m.

(b) The direction of $\vec{E}$ is $-\hat{\text{i}}$, or toward plate 1.

37. **THINK** The component of the electric field $\vec{E}$ in a given direction is the negative of the rate at which potential changes with distance in that direction.

EXPRESS With $V = 2.00xyz^2$, we apply Eq. 24-41 to calculate the x, y, and z components of the electric field:

$$E_x = -\frac{\partial V}{\partial x} = -2.00yz^2$$
$$E_y = -\frac{\partial V}{\partial y} = -2.00xz^2$$
$$E_z = -\frac{\partial V}{\partial z} = -4.00xyz$$

which, at (x, y, z) = (3.00 m, –2.00 m, 4.00 m), gives

$$(E_x, E_y, E_z) = (64.0\text{ V/m}, -96.0\text{ V/m}, 96.0\text{ V/m}).$$

ANALYZE The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(64.0\text{ V/m})^2 + (-96.0\text{ V/m})^2 + (96.0\text{ V/m})^2}$$
$$= 150\,\text{V/m} = 150\,\text{N/C}.$$

LEARN If the electric potential increases along some direction, say x, with $\partial V/\partial x > 0$, then there is a corresponding nonvanishing component of $\vec{E}$ in the opposite direction $(-E_x \neq 0)$.

38. (a) From the result of Problem 24-28, the electric potential at a point with coordinate x is given by

$$V=\frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x-L}{x}\right).$$

At $x = d$ we obtain

$$V=\frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{d+L}{d}\right)=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(43.6\times10^{-15}\,\text{C})}{0.135\ \text{m}}\ln\left(1+\frac{0.135\ \text{m}}{d}\right)$$
$$=(2.90\times10^{-3}\ \text{V})\ln\left(1+\frac{0.135\ \text{m}}{d}\right).$$

(b) We differentiate the potential with respect to x to find the x component of the electric field:

$$E_x=-\frac{\partial V}{\partial x}=-\frac{Q}{4\pi\varepsilon_0 L}\frac{\partial}{\partial x}\ln\left(\frac{x-L}{x}\right)=-\frac{Q}{4\pi\varepsilon_0 L}\frac{x}{x-L}\left(\frac{1}{x}-\frac{x-L}{x^2}\right)=-\frac{Q}{4\pi\varepsilon_0 x(x-L)}$$
$$=-\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(43.6\times10^{-15}\,\text{C})}{x(x+0.135\ \text{m})}=-\frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{x(x+0.135\ \text{m})},$$

or

$$|E_x|=\frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{x(x+0.135\ \text{m})}.$$

(c) Since $E_x<0$, its direction relative to the positive x axis is $180°$.

(d) At $x = d = 6.20$ cm, we obtain

$$|E_x|=\frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{(0.0620\ \text{m})(0.0620\ \text{m}+0.135\ \text{m})}=0.0321\ \text{N/C}.$$

(e) Consider two points an equal infinitesimal distance on either side of P_1, along a line that is perpendicular to the x axis. The difference in the electric potential divided by their separation gives the transverse component of the electric field. Since the two points are situated symmetrically with respect to the rod, their potentials are the same and the potential difference is zero. Thus, the transverse component of the electric field E_y is zero.

39. The electric field (along some axis) is the (negative of the) derivative of the potential V with respect to the corresponding coordinate. In this case, the derivatives can be read off of the graphs as slopes (since the graphs are of straight lines). Thus,

$$E_x=-\frac{\partial V}{\partial x}=-\left(\frac{-500\ \text{V}}{0.20\ \text{m}}\right)=2500\ \text{V/m}=2500\ \text{N/C}$$
$$E_y=-\frac{\partial V}{\partial y}=-\left(\frac{300\ \text{V}}{0.30\ \text{m}}\right)=-1000\ \text{V/m}=-1000\ \text{N/C}.$$

These components imply the electric field has a magnitude of 2693 N/C and a direction of –21.8º (with respect to the positive x axis). The force on the electron is given by $\vec{F} = q\vec{E}$ where $q = -e$. The minus sign associated with the value of q has the implication that $\vec{F}$ points in the opposite direction from $\vec{E}$ (which is to say that its angle is found by adding 180º to that of $\vec{E}$). With $e = 1.60 \times 10^{-19}$ C, we obtain

$$\vec{F} = (-1.60\times10^{-19}\,\text{C})[(2500\text{ N/C})\hat{\text{i}} - (1000\text{ N/C})\hat{\text{j}}] = (-4.0\times10^{-16}\text{ N})\hat{\text{i}} + (1.60\times10^{-16}\text{ N})\hat{\text{j}}.$$

40. (a) Consider an infinitesimal segment of the rod from x to $x + dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}dx.$$

Thus,

$$\begin{aligned} V &= \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}dx = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2}-y\right) \\ &= (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\text{C/m}^2)\left(\sqrt{(0.100\text{ m})^2+(0.0356\text{ m})^2}-0.0356\text{ m}\right) \\ &= 3.16\times10^{-2}\text{ V}. \end{aligned}$$

(b) The y component of the field there is

$$\begin{aligned} E_y &= -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}\left(\sqrt{L^2+y^2}-y\right) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right) \\ &= (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\text{C/m}^2)\left(1-\frac{0.0356\text{ m}}{\sqrt{(0.100\text{ m})^2+(0.0356\text{ m})^2}}\right) \\ &= 0.298\text{ N/C}. \end{aligned}$$

(c) We obtained above the value of the potential at any point P strictly on the y-axis. In order to obtain $E_x(x, y)$ we need to first calculate $V(x, y)$. That is, we must find the potential for an arbitrary point located at (x, y). Then $E_x(x, y)$ can be obtained from $E_x(x,y) = -\partial V(x,y)/\partial x$.

41. We apply conservation of energy for the particle with $q = 7.5 \times 10^{-6}$ C (which has zero initial kinetic energy):

$$U_0 = K_f + U_f,$$

where $U = \dfrac{qQ}{4\pi\varepsilon_0 r}$.

(a) The initial value of r is 0.60 m and the final value is (0.6 + 0.4) m = 1.0 m (since the particles repel each other). Conservation of energy, then, leads to K_f = 0.90 J.

(b) Now the particles attract each other so that the final value of r is 0.60 − 0.40 = 0.20 m. Use of energy conservation yields K_f = 4.5 J in this case.

42. (a) We use Eq. 24-43 with $q_1 = q_2 = -e$ and r = 2.00 nm:

$$U = k\frac{q_1 q_2}{r} = k\frac{e^2}{r} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(1.60\times10^{-19}\,\text{C})^2}{2.00\times10^{-9}\,\text{m}} = 1.15\times10^{-19}\,\text{J}.$$

(b) Since $U > 0$ and $U \propto r^{-1}$ the potential energy U decreases as r increases.

43. **THINK** The work required to set up the arrangement is equal to the potential energy of the system.

EXPRESS We choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$U_f = \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right) = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right).$$

Thus the amount of work required to set up the system is given by

$$W = \Delta U = U_f - U_i = U_f = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right) = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.30\times10^{-12}\,\text{C})^2}{0.640\ \text{m}}\left(\frac{1}{\sqrt{2}}-2\right)$$
$$= -1.92\times10^{-13}\,\text{J}.$$

LEARN The work done in assembling the system is negative. This means that an external agent would have to supply $W_{\text{ext}} = +1.92\times10^{-13}\,\text{J}$ in order to take apart the arrangement completely.

44. The work done must equal the change in the electric potential energy. From Eq. 24-14 and Eq. 24-26, we find (with r = 0.020 m)

$$W = \frac{(3e-2e+2e)(6e)}{4\pi\varepsilon_0 r} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(18)(1.60\times10^{-19}\,\text{C})^2}{0.020\ \text{m}} = 2.1\times10^{-25}\ \text{J}.$$

45. We use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\varepsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\varepsilon_0 r_2$,

and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the final speed of the particle. Conservation of energy yields

$$\frac{q^2}{4\pi\varepsilon_0 r_1} = \frac{q^2}{4\pi\varepsilon_0 r_2} + \frac{1}{2}mv^2 .$$

The solution for v is

$$v = \sqrt{\frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2)(3.1\times10^{-6}\ \text{C})^2}{20\times10^{-6}\text{kg}}\left(\frac{1}{0.90\times10^{-3}\ \text{m}} - \frac{1}{2.5\times10^{-3}\ \text{m}}\right)}$$
$$= 2.5\times10^3\ \text{m/s}.$$

46. Let r = 1.5 m, x = 3.0 m, q_1 = –9.0 nC, and q_2 = –6.0 pC. The work done by an external agent is given by

$$W = \Delta U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + x^2}}\right)$$
$$= \left(-9.0\times10^{-9}\ \text{C}\right)\left(-6.0\times10^{-12}\ \text{C}\right)\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\cdot\left(\frac{1}{1.5\ \text{m}} - \frac{1}{\sqrt{(1.5\ \text{m})^2 + (3.0\ \text{m})^2}}\right)$$
$$= 1.8\times10^{-10}\ \text{J}.$$

47. The *escape speed* may be calculated from the requirement that the initial kinetic energy (of *launch*) be equal to the absolute value of the initial potential energy (compare with the gravitational case in Chapter 14). Thus,

$$\frac{1}{2}mv^2 = \frac{eq}{4\pi\varepsilon_0 r}$$

where $m = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C, $q = 10000e$, and r = 0.010 m. This yields $v = 22490\ \text{m/s} \approx 2.2\times10^4\ \text{m/s}$.

48. The change in electric potential energy of the electron-shell system as the electron starts from its initial position and just reaches the shell is $\Delta U = (-e)(-V) = eV$. Thus from $\Delta U = K = \frac{1}{2}m_e v_i^2$ we find the initial electron speed to be

$$v_i = \sqrt{\frac{2\Delta U}{m_e}} = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(125\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 6.63\times10^6\ \text{m/s}.$$

49. We use conservation of energy, taking the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then $U_f = 2e^2/4\pi\varepsilon_0 d$, where d is half the distance between the fixed electrons. The initial

kinetic energy is $K_i = \frac{1}{2}mv^2$, where *m* is the mass of an electron and *v* is the initial speed of the moving electron. The final kinetic energy is zero. Thus,

$$K_i = U_f \quad \Rightarrow \quad \frac{1}{2}mv^2 = 2e^2/4\pi\varepsilon_0 d.$$

Hence,

$$v = \sqrt{\frac{4e^2}{4\pi\varepsilon_0 dm}} = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4)(1.60\times10^{-19}\ \text{C})^2}{(0.010\ \text{m})(9.11\times10^{-31}\ \text{kg})}} = 3.2\times10^2\ \text{m/s}.$$

50. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1Q}{2d} + \frac{q_2Q}{d}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1Q}{2d} + \frac{(-q_1/2)Q}{d}\right) = 0.$$

51. (a) Let $\ell = 0.15\,\text{m}$ be the length of the rectangle and $w = 0.050$ m be its width. Charge q_1 is a distance ℓ from point *A* and charge q_2 is a distance *w*, so the electric potential at *A* is

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\ell} + \frac{q_2}{w}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-5.0\times10^{-6}\,\text{C}}{0.15\,\text{m}} + \frac{2.0\times10^{-6}\,\text{C}}{0.050\ \text{m}}\right)$$
$$= 6.0\times10^4\,\text{V}.$$

(b) Charge q_1 is a distance *w* from point *b* and charge q_2 is a distance ℓ, so the electric potential at *B* is

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{w} + \frac{q_2}{\ell}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-5.0\times10^{-6}\,\text{C}}{0.050\ \text{m}} + \frac{2.0\times10^{-6}\,\text{C}}{0.15\,\text{m}}\right)$$
$$= -7.8\times10^5\,\text{V}.$$

(c) Since the kinetic energy is zero at the beginning and end of the trip, the work done by an external agent equals the change in the potential energy of the system. The potential energy is the product of the charge q_3 and the electric potential. If U_A is the potential energy when q_3 is at *A* and U_B is the potential energy when q_3 is at *B*, then the work done in moving the charge from *B* to *A* is

$$W = U_A - U_B = q_3(V_A - V_B) = (3.0\times10^{-6}\ \text{C})(6.0\times10^4\ \text{V} + 7.8\times10^5\ \text{V}) = 2.5\ \text{J}.$$

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter which path is used.

52. From Eq. 24-30 and Eq. 24-7, we have (for $\theta = 180^\circ$)

$$U = qV = -e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right) = \frac{ep}{4\pi\varepsilon_0 r^2}$$

where r = 0.020 m. Using energy conservation, we set this expression equal to 100 eV and solve for p. The magnitude of the dipole moment is therefore $p = 4.5 \times 10^{-12}\ \text{C}\cdot\text{m}$.

53. (a) The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-6}\ \text{C}\right)^2}{1.00\ \text{m}} = 0.225\ \text{J}$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-6}\ \text{C}\right)^2}{\left(1.00\ \text{m}\right)^2} = 0.225\ \text{N}.$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let m_A and m_B be the masses of the spheres. The acceleration of sphere A is

$$a_A = \frac{F}{m_A} = \frac{0.225\ \text{N}}{5.0\times10^{-3}\ \text{kg}} = 45.0\ \text{m/s}^2$$

and the acceleration of sphere B is

$$a_B = \frac{F}{m_B} = \frac{0.225\ \text{N}}{10\times10^{-3}\ \text{kg}} = 22.5\ \text{m/s}^2\ .$$

(c) Energy is conserved. The initial potential energy is U = 0.225 J, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, where v_A and v_B are the final velocities. Thus,

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2.$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B.$$

These equations may be solved simultaneously for v_A and v_B. Substituting $v_B = -(m_A/m_B)v_A$, from the momentum equation into the energy equation, and collecting terms, we obtain

$$U = \tfrac{1}{2}(m_A/m_B)(m_A+m_B)v_A^2.$$

Thus,

$$v_A = \sqrt{\frac{2Um_B}{m_A(m_A+m_B)}} = \sqrt{\frac{2(0.225\text{ J})(10\times10^{-3}\text{ kg})}{(5.0\times10^{-3}\text{ kg})(5.0\times10^{-3}\text{ kg}+10\times10^{-3}\text{ kg})}} = 7.75\text{ m/s}.$$

We thus obtain

$$v_B = -\frac{m_A}{m_B}v_A = -\left(\frac{5.0\times10^{-3}\text{ kg}}{10\times10^{-3}\text{ kg}}\right)(7.75\text{ m/s}) = -3.87\text{ m/s},$$

or $|v_B| = 3.87$ m/s.

54. (a) Using $U = qV$ we can "translate" the graph of voltage into a potential energy graph (in eV units). From the information in the problem, we can calculate its kinetic energy (which is its total energy at $x = 0$) in those units: $K_i = 284$ eV. This is less than the "height" of the potential energy "barrier" (500 eV high once we've translated the graph as indicated above). Thus, it must reach a turning point and then reverse its motion.

(b) Its final velocity, then, is in the negative x direction with a magnitude equal to that of its initial velocity. That is, its speed (upon leaving this region) is 1.0×10^7 m/s.

55. Let the distance in question be r. The initial kinetic energy of the electron is $K_i = \tfrac{1}{2}m_e v_i^2$, where $v_i = 3.2\times10^5$ m/s. As the speed doubles, K becomes $4K_i$. Thus

$$\Delta U = \frac{-e^2}{4\pi\varepsilon_0 r} = -\Delta K = -(4K_i - K_i) = -3K_i = -\frac{3}{2}m_e v_i^2,$$

or

$$r = \frac{2e^2}{3(4\pi\varepsilon_0)m_e v_i^2} = \frac{2\left(1.6\times10^{-19}\text{ C}\right)^2\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)}{3\left(9.11\times10^{-19}\text{ kg}\right)\left(3.2\times10^5\text{ m/s}\right)^2} = 1.6\times10^{-9}\text{m}.$$

56. When particle 3 is at $x = 0.10$ m, the total potential energy vanishes. Using Eq. 24-43, we have (with meters understood at the length unit)

$$0 = \frac{q_1q_2}{4\pi\varepsilon_0 d} + \frac{q_1q_3}{4\pi\varepsilon_0(d+0.10\text{ m})} + \frac{q_3q_2}{4\pi\varepsilon_0(0.10\text{ m})}$$

This leads to

$$q_3\left(\frac{q_1}{d+0.10\text{ m}}+\frac{q_2}{0.10\text{ m}}\right)=-\frac{q_1q_2}{d}$$

which yields $q_3 = -5.7\ \mu$C.

57. **THINK** Mechanical energy is conserved in the process.

EXPRESS The electric potential at (0, y) due to the two charges Q held fixed at $(\pm x, 0)$ is

$$V=\frac{2Q}{4\pi\varepsilon_0\sqrt{x^2+y^2}}\ .$$

Thus, the potential energy of the particle of charge q at (0, y) is

$$U=qV=\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y^2}}\ .$$

Conservation of mechanical energy ($K_i + U_i = K_f + U_f$) gives

$$K_f=K_i+U_i-U_f=K_i+\frac{2Qq}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y_i^2}}-\frac{1}{\sqrt{x^2+y_f^2}}\right),$$

where y_i and y_f are the initial and final coordinates of the moving charge along the y axis.

ANALYZE (a) With $q=-15\times10^{-6}$ C, $Q = 50\times10^{-6}$ C, $x = \pm3$ m, $y_i = 4$ m, and $y_f = 0$, we obtain

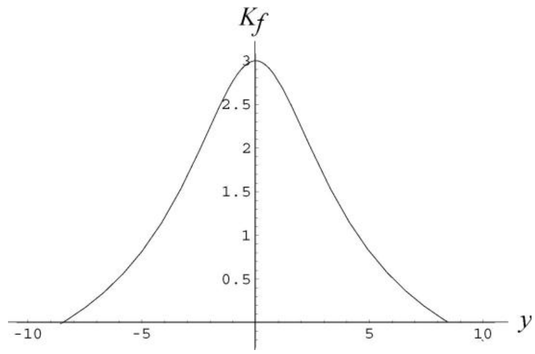
$$\begin{aligned}K_f&=1.2\text{ J}+\frac{2(50\times10^{-6}\text{ C})(-15\times10^{-6}\text{ C})}{4\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{1}{\sqrt{(3.0\text{ m})^2+(4.0\text{ m})^2}}-\frac{1}{\sqrt{(3.0\text{ m})^2}}\right)\\&=3.0\text{ J.}\end{aligned}$$

(b) We set $K_f = 0$ and solve for y_f (choosing the negative root, as indicated in the problem statement):

$$K_i+U_i=U_f\ \Rightarrow\ 1.2\text{ J}+\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y_i^2}}=\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y_f^2}}\ .$$

Substituting the values given, we have $U_i = -2.7$ J, and $y_f = -8.5$ m.

LEARN The dependence of the final kinetic energy of the particle on y is plotted below. From the plot, we see that $K_f = 3.0$ J at $y = 0$, and $K_f = 0$ at $y = -8.5$ m. The particle oscillates between the two end-points $y_f = \pm 8.5$ m.



58. (a) When the proton is released, its energy is $K + U = 4.0$ eV + 3.0 eV (the latter value is inferred from the graph). This implies that if we draw a horizontal line at the 7.0 volt "height" in the graph and find where it intersects the voltage plot, then we can determine the turning point. Interpolating in the region between 1.0 cm and 3.0 cm, we find the turning point is at roughly $x = 1.7$ cm.

(b) There is no turning point toward the right, so the speed there is nonzero, and is given by energy conservation:

$$v = \sqrt{\frac{2(7.0\text{ eV})}{m}} = \sqrt{\frac{2(7.0\text{ eV})(1.6 \times 10^{-19}\text{ J/eV})}{1.67 \times 10^{-27}\text{ kg}}} = 20\text{ km/s}.$$

(c) The electric field at any point P is the (negative of the) slope of the voltage graph evaluated at P. Once we know the electric field, the force on the proton follows immediately from $\vec{F} = q\vec{E}$, where $q = +e$ for the proton. In the region just to the left of $x = 3.0$ cm, the field is $\vec{E} = (+300\text{ V/m})\hat{\text{i}}$ and the force is $F = +4.8 \times 10^{-17}$ N.

(d) The force $\vec{F}$ points in the $+x$ direction, as the electric field $\vec{E}$.

(e) In the region just to the right of $x = 5.0$ cm, the field is $\vec{E} = (-200\text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F = 3.2 \times 10^{-17}$ N.

(f) The force $\vec{F}$ points in the $-x$ direction, as the electric field $\vec{E}$.

59. (a) The electric field between the plates is leftward in Fig, 24-59 since it points toward lower values of potential. The force (associated with the field, by Eq. 23-28) is evidently leftward, from the problem description (indicating deceleration of the rightward moving particle), so that $q > 0$ (ensuring that $\vec{F}$ is parallel to $\vec{E}$); it is a proton.

(b) We use conservation of energy:

$$K_0 + U_0 = K + U \;\;\Rightarrow\;\; \frac{1}{2} m_p v_0^2 + qV_1 = \frac{1}{2} m_p v^2 + qV_2 \;.$$

Using $q = +1.6 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ kg, $v_0 = 90 \times 10^3$ m/s, $V_1 = -70$ V, and $V_2 = -50$ V, we obtain the final speed $v = 6.53 \times 10^4$ m/s. We note that the value of d is not used in the solution.

60. (a) The work done results in a potential energy gain:

$$W = q\,\Delta V = (-e)\left(\frac{Q}{4\pi\varepsilon_0 R}\right) = +2.16 \times 10^{-13}\ \text{J}\,.$$

With R = 0.0800 m, we find $Q = -1.20 \times 10^{-5}$ C.

(b) The work is the same, so the increase in the potential energy is $\Delta U = +2.16 \times 10^{-13}$ J.

61. We note that for two points on a circle, separated by angle θ (in radians), the direct-line distance between them is $r = 2R\sin(\theta/2)$. Using this fact, distinguishing between the cases where N = odd and N = even, and counting the pair-wise interactions very carefully, we arrive at the following results for the total potential energies. We use $k = 1/4\pi\varepsilon_0$. For configuration 1 (where all N electrons are on the circle), we have

$$U_{1,N=\text{even}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta/2)} + \frac{1}{2}\right), \quad U_{1,N=\text{odd}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N-1}{2}}\frac{1}{\sin(j\theta/2)}\right)$$

where $\theta = \dfrac{2\pi}{N}$. For configuration 2, we find

$$U_{2,N=\text{even}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta'/2)} + 2\right), \; U_{2,N=\text{odd}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N-3}{2}}\frac{1}{\sin(j\theta'/2)} + \frac{5}{2}\right)$$

where $\theta' = \dfrac{2\pi}{N-1}$. The results are all of the form

$$U_{1\text{or}2}\,\frac{ke^2}{2R} \times \text{a pure number.}$$

In our table below we have the results for those "pure numbers" as they depend on N and on which configuration we are considering. The values listed in the U rows are the potential energies divided by $ke^2/2R$.

N	4	5	6	7	8	9	10	11	12	13	14	15
U_1	3.83	6.88	10.96	16.13	22.44	29.92	38.62	48.58	59.81	72.35	86.22	101.5
U_2	4.73	7.83	11.88	16.96	23.13	30.44	39.92	48.62	59.58	71.81	85.35	100.2

We see that the potential energy for configuration 2 is greater than that for configuration 1 for $N < 12$, but for $N \geq 12$ it is configuration 1 that has the greatest potential energy.

(a) $N = 12$ is the smallest value such that $U_2 < U_1$.

(b) For $N = 12$, configuration 2 consists of 11 electrons distributed at equal distances around the circle, and one electron at the center. A specific electron e_0 on the circle is R distance from the one in the center, and is

$$r = 2R\sin\left(\frac{\pi}{11}\right) \approx 0.56R$$

distance away from its nearest neighbors on the circle (of which there are two — one on each side). Beyond the nearest neighbors, the next nearest electron on the circle is

$$r = 2R\sin\left(\frac{2\pi}{11}\right) \approx 1.1R$$

distance away from e_0. Thus, we see that there are only two electrons closer to e_0 than the one in the center.

62. (a) Since the two conductors are connected V_1 and V_2 must be equal to each other.

Let $V_1 = q_1/4\pi\varepsilon_0 R_1 = V_2 = q_2/4\pi\varepsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. We solve for q_1 and q_2: $q_1 = q/3$, $q_2 = 2q/3$, or

(b) $q_1/q = 1/3 = 0.333$.

(c) Similarly, $q_2/q = 2/3 = 0.667$.

(d) The ratio of surface charge densities is $\dfrac{\sigma_1}{\sigma_2} = \dfrac{q_1/4\text{p}R_1^2}{q_2/4\text{p}R_2^2} = \left(\dfrac{q_1}{q_2}\right)\left(\dfrac{R_2}{R_1}\right)^2 = 2.00.$

63. **THINK** The electric potential is the sum of the contributions of the individual spheres.

EXPRESS Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ (= 1.0 m) from the center of each sphere.

For parts (b) and (c), we note that the distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere as well as the charge on the other sphere.

ANALYZE (a) The potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\varepsilon_0 \, d/2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.0\times10^{-8}\,\text{C} - 3.0\times10^{-8}\,\text{C}\right)}{1.0\ \text{m}} = -1.8\times10^2\,\text{V}.$$

(b) The potential at the surface of sphere 1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{1.0\times10^{-8}\,\text{C}}{0.030\,\text{m}} - \frac{3.0\times10^{-8}\,\text{C}}{2.0\,\text{m} - 0.030\,\text{m}}\right] = 2.9\times10^3\,\text{V}.$$

(c) Similarly, the potential at the surface of sphere 2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{1.0\times10^{-8}\,\text{C}}{2.0\,\text{m} - 0.030\,\text{m}} - \frac{3.0\times10^{-8}\,\text{C}}{0.030\,\text{m}}\right] = -8.9\times10^3\,\text{V}.$$

LEARN In the limit where $d \to \infty$, the spheres are isolated from each other and the electric potentials at the surface of each individual sphere become

$$V_{10} = \frac{q_1}{4\pi\varepsilon_0 R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-8}\,\text{C})}{0.030\,\text{m}} = 3.0\times10^3\,\text{V},$$

and

$$V_{20} = \frac{q_2}{4\pi\varepsilon_0 R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-3.0\times10^{-8}\,\text{C})}{0.030\,\text{m}} = -8.99\times10^3\,\text{V}.$$

64. Since the electric potential throughout the entire conductor is a constant, the electric potential at its center is also +400 V.

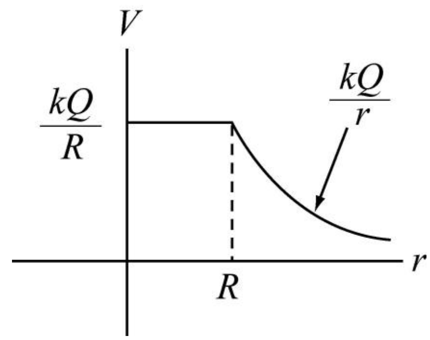
65. **THINK** If the electric potential is zero at infinity, then the potential at the surface of the sphere is given by $V = Q/4\pi\varepsilon_0 R$, where Q is the charge on the sphere and R is its radius.

EXPRESS From $V = Q/4\pi\varepsilon_0 R$, we find the charge to be $Q = 4\pi\varepsilon_0 RV$.

ANALYZE With $R = 0.15$ m and $V = 1500$ V, we have

$$Q = 4\pi\varepsilon_0 RV = \frac{(0.15\ \text{m})(1500\ \text{V})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.5\times10^{-8}\,\text{C}.$$

LEARN A plot of the electric potential as a function of r is shown to the right with $k = 1/4\pi\varepsilon_0$. Note that the potential is constant inside the conducting sphere.



66. Since the charge distribution is spherically symmetric we may write

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r},$$

where q_{enc} is the charge enclosed in a sphere of radius r centered at the origin.

(a) For $r = 4.00$ m, $R_2 = 1.00$ m, and $R_1 = 0.500$ m, with $r > R_2 > R_1$ we have

$$E(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C} + 1.00\times10^{-6}\,\text{C})}{(4.00\ \text{m})^2} = 1.69\times10^3\ \text{V/m}.$$

(b) For $R_2 > r = 0.700$ m $> R_2$,

$$E(r) = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C})}{(0.700\ \text{m})^2} = 3.67\times10^4\ \text{V/m}.$$

(c) For $R_2 > R_1 > r$, the enclosed charge is zero. Thus, $E = 0$.

The electric potential may be obtained using Eq. 24-18:

$$V(r) - V(r') = \int_r^{r'} E(r)\,dr.$$

(d) For $r = 4.00$ m $> R_2 > R_1$, we have

$$V(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C} + 1.00\times10^{-6}\,\text{C})}{(4.00\ \text{m})} = 6.74\times10^3\ \text{V}.$$

(e) For $r = 1.00$ m $= R_2 > R_1$, we have

$$V(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C} + 1.00\times10^{-6}\,\text{C})}{(1.00\ \text{m})} = 2.70\times10^4\ \text{V}.$$

(f) For $R_2 > r = 0.700$ m $> R_2$,

$$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})\left(\frac{2.00\times10^{-6}\,\mathrm{C}}{0.700\ \mathrm{m}}+\frac{1.00\times10^{-6}\,\mathrm{C}}{1.00\ \mathrm{m}}\right)$$
$$=3.47\times10^4\ \mathrm{V}.$$

(g) For $R_2 > r = 0.500$ m $= R_2$,

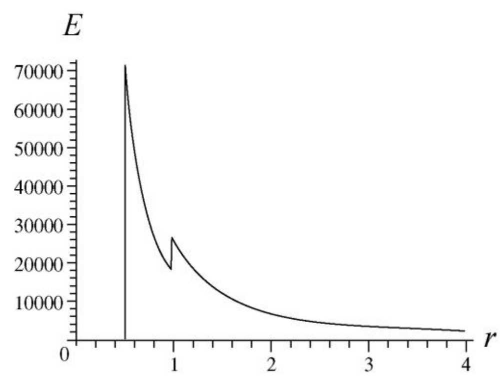
$$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})\left(\frac{2.00\times10^{-6}\,\mathrm{C}}{0.500\ \mathrm{m}}+\frac{1.00\times10^{-6}\,\mathrm{C}}{1.00\ \mathrm{m}}\right)$$
$$=4.50\times10^4\ \mathrm{V}.$$

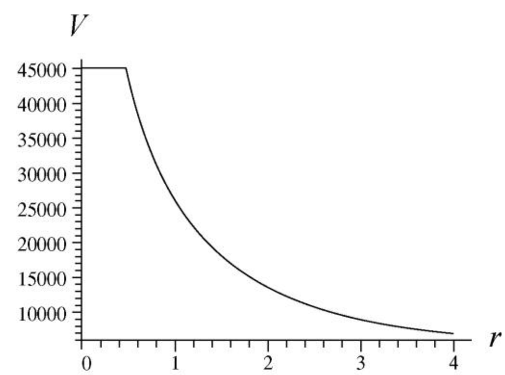
(h) For $R_2 > R_1 > r$,

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})\left(\frac{2.00\times10^{-6}\,\mathrm{C}}{0.500\ \mathrm{m}}+\frac{1.00\times10^{-6}\,\mathrm{C}}{1.00\ \mathrm{m}}\right)$$
$$=4.50\times10^4\ \mathrm{V}.$$

(i) At $r = 0$, the potential remains constant, $V = 4.50\times10^4\ \mathrm{V}$.

(j) The electric field and the potential as a function of r are depicted below:





67. (a) The magnitude of the electric field is

$$E=\frac{\sigma}{\varepsilon_0}=\frac{q}{4\pi\varepsilon_0 R^2}=\frac{(3.0\times10^{-8}\,\mathrm{C})(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})}{(0.15\,\mathrm{m})^2}=1.2\times10^4\ \mathrm{N/C}.$$

(b) $V = RE = (0.15\ \mathrm{m})(1.2\times10^4\ \mathrm{N/C}) = 1.8\times10^3\ \mathrm{V}$.

(c) Let the distance be x. Then

$$\Delta V = V(x) - V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R+x}-\frac{1}{R}\right] = -500\,\mathrm{V},$$

which gives

$$x = \frac{R\Delta V}{-V-\Delta V} = \frac{(0.15\,\text{m})(-500\,\text{V})}{-1800\,\text{V}+500\,\text{V}} = 5.8\times10^{-2}\,\text{m}.$$

68. The potential energy of the two-charge system is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\,\text{C})(-4.00\times10^{-6}\,\text{C})}{\sqrt{(3.50+2.00)^2+(0.500-1.50)^2}\,\text{cm}}$$
$$= -1.93\ \text{J}.$$

Thus, –1.93 J of work is needed.

69. **THINK** To calculate the potential, we first apply Gauss' law to calculate the electric field of the charged cylinder of radius *R*. The Gaussian surface is a cylindrical surface that is concentric with the cylinder.

EXPRESS We imagine a cylindrical Gaussian surface *A* of radius *r* and length *h* concentric with the cylinder. Then, by Gauss' law,

$$\oint_A \vec{E}\cdot d\vec{A} = 2\pi rhE = \frac{q_{\text{enc}}}{\varepsilon_0},$$

where q_{enc} is the amount of charge enclosed by the Gaussian cylinder. Inside the charged cylinder ($r < R$), $q_{\text{enc}} = 0$, so the electric field is zero. On the other hand, outside the cylinder ($r > R$), $q_{\text{enc}} = \lambda h$ so the magnitude of the electric field is

$$E = \frac{q/h}{2\pi\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where λ is the linear charge density and *r* is the distance from the line to the point where the field is measured. The potential difference between two points 1 and 2 is

$$V(r_2) - V(r_1) = -\int_{r_1}^{r_2} E(r)\,dr.$$

ANALYZE (a) The radius of the cylinder (0.020 m, the same as R_B) is denoted *R*, and the field magnitude there (160 N/C) is denoted E_B. From the equation above, we see that the electric field beyond the surface of the cylinder is inversely proportional with *r*:

$$E = E_B\frac{R_B}{r}, \quad r \ge R_B.$$

Thus, if $r = R_C = 0.050$ m, we obtain

$$E_C = E_B \frac{R_B}{R_C} = (160\text{ N/C})\left(\frac{0.020\text{ m}}{0.050\text{ m}}\right) = 64\text{ N/C}.$$

(b) The potential difference between V_B and V_C is

$$V_B - V_C = -\int_{R_C}^{R_B} \frac{E_B R_B}{r}\,dr = E_B R_B \ln\left(\frac{R_C}{R_B}\right) = (160\text{ N/C})(0.020\text{ m})\ln\left(\frac{0.050\text{ m}}{0.020\text{ m}}\right)$$
$$= 2.9\text{ V}.$$

(c) The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged cylinder: $V_A - V_B = 0$.

LEARN The electric potential at a distance $r > R_B$ can be written as

$$V(r) = V_B - E_B R_B \ln\left(\frac{r}{R_B}\right).$$

We see that $V(r)$ decreases logarithmically with r.

70. (a) We use Eq. 24-18 to find the potential: $V_{\text{wall}} - V = -\int_r^R E\,dr$, or

$$0 - V = -\int_r^R \left(\frac{\rho r}{2\varepsilon_0}\right) \quad \Rightarrow \quad -V = -\frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right).$$

Consequently, $V = \rho(R^2 - r^2)/4\varepsilon_0$.

(b) The value at $r = 0$ is

$$V_{\text{center}} = \frac{-1.1\times10^{-3}\text{ C/m}^3}{4\left(8.85\times10^{-12}\text{ C/V}\cdot\text{m}\right)}\left((0.05\text{ m})^2 - 0\right) = -7.8\times10^4\text{ V}.$$

Thus, the difference is $|V_{\text{center}}| = 7.8\times10^4$ V.

71. **THINK** The component of the electric field $\vec{E}$ in any direction is the negative of the rate at which potential changes with distance in that direction.

EXPRESS From Eq. 24-30, the electric potential of a dipole at a point a distance r away is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

where p is the magnitude of the dipole moment $\vec{p}$ and θ is the angle between $\vec{p}$ and the position vector of the point. The potential at infinity is taken to be zero.

ANALYZE On the dipole axis $\theta = 0$ or π, so |cos θ| = 1. Therefore, magnitude of the electric field is

$$|E(r)| = \left|-\frac{\partial V}{\partial r}\right| = \frac{p}{4\pi\varepsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\varepsilon_0 r^3}\ .$$

LEARN Take the z axis to be the dipole axis. For $r = z > 0\,(\theta = 0)$, $E = p/2\pi\varepsilon_0 z^3$. On the other hand, for $r = -z < 0\ (\theta = \pi)$, $E = -p/2\pi\varepsilon_0 z^3$.

72. Using Eq. 24-18, we have

$$\Delta V = -\int_2^3 \frac{A}{r^4}\,dr = \frac{A}{3}\left(\frac{1}{2^3} - \frac{1}{3^3}\right) = A(0.029/\text{m}^3).$$

73. (a) The potential on the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(4.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.10\,\text{m}} = 3.6\times10^5\ \text{V}\ .$$

(b) The field just outside the sphere would be

$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{V}{R} = \frac{3.6\times10^5\ \text{V}}{0.10\,\text{m}} = 3.6\times10^6\ \text{V/m}\ ,$$

which would have exceeded 3.0 MV/m. So this situation cannot occur.

74. The work done is equal to the change in the (total) electric potential energy U of the system, where

$$U = \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_3q_2}{4\pi\varepsilon_0 r_{23}} + \frac{q_1q_3}{4\pi\varepsilon_0 r_{13}}$$

and the notation r_{13} indicates the distance between q_1 and q_3 (similar definitions apply to r_{12} and r_{23}).

(a) We consider the difference in U where initially $r_{12} = b$ and $r_{23} = a$, and finally $r_{12} = a$ and $r_{23} = b$ (r_{13} doesn't change). Converting the values given in the problem to SI units (μC to C, cm to m), we obtain $\Delta U = -24$ J.

(b) Now we consider the difference in U where initially $r_{23} = a$ and $r_{13} = a$, and finally r_{23} is again equal to a and r_{13} is also again equal to a (and of course, r_{12} doesn't change in this case). Thus, we obtain $\Delta U = 0$.

75. Assume the charge on Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of Earth it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on Earth and $R = 6.37 \times 10^6$ m is the radius of Earth. The magnitude of the electric field at the surface is $E = q/4\pi\varepsilon_0 R^2$, so

$$V = ER = (100 \text{ V/m})\,(6.37 \times 10^6 \text{ m}) = 6.4 \times 10^8 \text{ V}.$$

76. Using Gauss' law, $q = \varepsilon_0 \Phi = +495.8$ nC. Consequently,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.958\times10^{-7}\text{ C})}{0.120 \text{ m}} = 3.71\times10^4 \text{ V}.$$

77. The potential difference is

$$\Delta V = E\Delta s = (1.92 \times 10^5 \text{ N/C})(0.0150 \text{ m}) = 2.90 \times 10^3 \text{ V}.$$

78. The charges are equidistant from the point where we are evaluating the potential — which is computed using Eq. 24-27 (or its integral equivalent). Equation 24-27 implicitly assumes $V \to 0$ as $r \to \infty$. Thus, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+3Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2Q_1}{R}$$
$$= \frac{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.52\times10^{-12}\text{ C})}{0.0850 \text{ m}} = 0.956 \text{ V}.$$

79. The electric potential energy in the presence of the dipole is

$$U = qV_{\text{dipole}} = \frac{qp\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{(-e)(ed)\cos\theta}{4\pi\varepsilon_0 r^2}\ .$$

Noting that $\theta_i = \theta_f = 0°$, conservation of energy leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow \quad v = \sqrt{\frac{2e^2}{4\pi\varepsilon_0 m d}\left(\frac{1}{25} - \frac{1}{49}\right)} = 7.0 \times 10^5 \text{ m/s}\ .$$

80. We treat the system as a superposition of a disk of surface charge density σ and radius R and a smaller, oppositely charged, disk of surface charge density $-\sigma$ and radius r. For each of these, Eq 24-37 applies (for $z > 0$)

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right) + \frac{-\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + r^2} - z\right).$$

This expression does vanish as $r \to \infty$, as the problem requires. Substituting $r = 0.200R$ and $z = 2.00R$ and simplifying, we obtain

$$V = \frac{\sigma R}{\varepsilon_0}\left(\frac{5\sqrt{5} - \sqrt{101}}{10}\right) = \frac{(6.20\times10^{-12}\,\text{C/m}^2)(0.130\text{ m})}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2}\left(\frac{5\sqrt{5} - \sqrt{101}}{10}\right)$$
$$= 1.03\times10^{-2}\text{ V}.$$

81. (a) When the electron is released, its energy is

$$K + U = 3.0\text{ eV} - 6.0\text{ eV}$$

(the latter value is inferred from the graph along with the fact that $U = qV$ and $q = -e$). Because of the minus sign (of the charge) it is convenient to imagine the graph multiplied by a minus sign so that it represents potential energy in eV. Thus, the 2 V value shown at $x = 0$ would become –2 eV, and the 6 V value at $x = 4.5$ cm becomes –6 eV, and so on. The total energy (– 3.0 eV) is constant and can then be represented on our (imagined) graph as a horizontal line at – 3.0 V. This intersects the potential energy plot at a point we recognize as the turning point. Interpolating in the region between 1.0 cm and 4.0 cm, we find the turning point is at $x = 1.75$ cm $\approx$ 1.8 cm.

(b) There is no turning point toward the right, so the speed there is nonzero. Noting that the kinetic energy at $x = 7.0$ cm is

$$K = -3.0\text{ eV} - (-5.0\text{ eV}) = 2.0\text{ eV},$$

we find the speed using energy conservation:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.0\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 8.4\times10^5\text{ m/s}.$$

(c) The electric field at any point P is the (negative of the) slope of the voltage graph evaluated at P. Once we know the electric field, the force on the electron follows immediately from $\vec{F} = q\vec{E}$, where $q = -e$ for the electron. In the region just to the left of $x = 4.0$ cm, the electric field is $\vec{E} = (-133\text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F = 2.1\times10^{-17}$ N.

(d) The force points in the +*x* direction.

(e) In the region just to the right of *x* = 5.0 cm, the field is $\vec{E}$ = +100 V/m $\hat{\text{i}}$ and the force is $\vec{F}$ = (–1.6 x 10^{-17} N) $\hat{\text{i}}$. Thus, the magnitude of the force is $F = 1.6\times10^{-17}$ N.

(f) The minus sign indicates that $\vec{F}$ points in the –*x* direction.

82. (a) The potential would be

$$\begin{aligned} V_e &= \frac{Q_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e^2\sigma_e}{4\pi\varepsilon_0 R_e} = 4\pi R_e\sigma_e k \\ &= 4\pi\left(6.37\times10^6\text{ m}\right)\left(1.0\,\text{electron/m}^2\right)\left(-1.6\times10^{-9}\text{ C/electron}\right)\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right) \\ &= -0.12\,\text{V}. \end{aligned}$$

(b) The electric field is

$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\,\text{V}}{6.37\times10^6\,\text{m}} = -1.8\times10^{-8}\text{ N/C},$$

or $|E| = 1.8\times10^{-8}$ N/C.

(c) The minus sign in *E* indicates that $\vec{E}$ is radially inward.

83. (a) Using *d* = 2 m, we find the potential at *P*:

$$\begin{aligned} V_P &= \frac{2e}{4\pi\varepsilon_0 d} + \frac{-2e}{4\pi\varepsilon_0(2d)} = \frac{e}{4\pi\varepsilon_0 d} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{C})}{2.00\text{ m}} \\ &= 7.192\times10^{-10}\text{ V}. \end{aligned}$$

Note that we are implicitly assuming that $V \to 0$ as $r \to \infty$.

(b) Since $U = qV$, then the movable particle's contribution of the potential energy when it is at *r* = ∞ is zero, and its contribution to U_{system} when it is at *P* is

$$U = qV_P = 2(1.6\times10^{-19}\text{C})(7.192\times10^{-10}\text{ V}) = 2.30\times10^{-28}\text{ J}.$$

Thus, the work done is approximately equal to $W_{\text{app}} = 2.30 \times 10^{-28}$ J.

(c) Now, combining the contribution to U_{system} from part (b) and from the original pair of fixed charges

$$U_{\text{fixed}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(-2e)}{\sqrt{(4.00\text{ m})^2+(2.00\text{ m})^2}} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(4)(1.60\times10^{-19}\text{C})^2}{\sqrt{20.0}\text{ m}}$$
$$= -2.058\times10^{-28}\text{ J}$$

we obtain

$$U_{\text{system}} = W_{\text{app}} + U_{\text{fixed}} = 2.43\times10^{-29}\text{ J}.$$

84. The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged sphere:

$$V_A = V_S = \frac{q}{4\pi\varepsilon_0 R}$$

where $q = 30\times10^{-9}$ C and $R = 0.030$ m. For points beyond the surface of the sphere, the potential follows Eq. 24-26:

$$V_B = \frac{q}{4\pi\varepsilon_0 r}$$

where $r = 0.050$ m.

(a) We see that

$$V_S - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right) = 3.6\times10^3\text{ V}.$$

(b) Similarly,

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right) = 3.6\times10^3\text{ V}.$$

85. We note that the net potential (due to the "fixed" charges) is zero at the first location ("at ∞") being considered for the movable charge q (where $q = +2e$). Thus, with $D = 4.00$ m and $e = 1.60\times10^{-19}$ C, we obtain

$$V = \frac{+2e}{4\pi\varepsilon_0(2D)} + \frac{+e}{4\pi\varepsilon_0 D} = \frac{2e}{4\pi\varepsilon_0 D} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(1.60\times10^{-19}\text{C})}{4.00\text{ m}}$$
$$= 7.192\times10^{-10}\text{ V} .$$

The work required is equal to the potential energy in the final configuration:

$$W_{\text{app}} = qV = (2e)(7.192\times10^{-10}\text{ V}) = 2.30\times10^{-28}\text{ J}.$$

86. Since the electric potential is a scalar quantity, this calculation is far simpler than it would be for the electric field. We are able to simply take half the contribution that

would be obtained from a complete (whole) sphere. If it were a whole sphere (of the same density) then its charge would be $q_{\text{whole}} = 8.00\ \mu\text{C}$. Then

$$V = \frac{1}{2}V_{\text{whole}} = \frac{1}{2}\,\frac{q_{\text{whole}}}{4\pi\varepsilon_0 r} = \frac{1}{2}\,\frac{8.00 \times 10^{-6}\ \text{C}}{4\pi\varepsilon_0(0.15\ \text{m})} = 2.40 \times 10^5\ \text{V}\,.$$

87. **THINK** The work done is equal to the change in potential energy.

EXPRESS The initial potential energy of the system is

$$U_i = \frac{2q^2}{4\pi\varepsilon_0 L} + U_0$$

where q is the charge on each particle, L is the length of the triangle side, and U_0 is the potential energy associated with the interaction of the two fixed charges. After moving to the midpoint of the line joining the two fixed charges, the final energy of the configuration is

$$U_f = \frac{2q^2}{4\pi\varepsilon_0 (L/2)} + U_0\,.$$

Thus, the work done by the external agent is

$$W = \Delta U = U_f - U_i = \frac{2q^2}{4\pi\varepsilon_0}\left(\frac{2}{L} - \frac{1}{L}\right) = \frac{2q^2}{4\pi\varepsilon_0 L}.$$

ANALYZE Substituting the values given, we have

$$W = \frac{2q^2}{4\pi\varepsilon_0 L} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.12\ \text{C})^2}{1.7\ \text{m}} = 1.5\times10^8\ \text{J}.$$

At a rate of $P = 0.83 \times 10^3$ joules per second, it would take $W/P = 1.8 \times 10^5$ seconds or about 2.1 days to do this amount of work.

LEARN Since all three particles are positively charged, positive work is required by the external agent in order to bring them closer.

88. (a) The charges are equal and are the same distance from C. We use the Pythagorean theorem to find the distance

$$r = \sqrt{(d/2)^2 + (d/2)^2} = d/\sqrt{2}\,.$$

The electric potential at C is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$V = \frac{2q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{d} = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(2)\sqrt{2}\left(2.0\times10^{-6}\,\text{C}\right)}{0.020\text{ m}}$$
$$= 2.5\times10^6\,\text{V}.$$

(b) As you move the charge into position from far away the potential energy changes from zero to qV, where V is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in:

$$W = qV = \left(2.0\times10^{-6}\,\text{C}\right)\left(2.54\times10^6\,\text{V}\right) = 5.1\text{ J}.$$

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is d so this potential energy is $q^2/4\pi\varepsilon_0 d$. The total potential energy is

$$U = W + \frac{q^2}{4\pi\varepsilon_0 d} = 5.1\text{ J} + \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.0\times10^{-6}\,\text{C}\right)^2}{0.020\,\text{m}} = 6.9\text{ J}.$$

89. The net potential at point P (the place where we are to place the third electron) due to the fixed charges is computed using Eq. 24-27 (which assumes $V \to 0$ as $r \to \infty$):

$$V_P = \frac{-e}{4\pi\varepsilon_0 d} + \frac{-e}{4\pi\varepsilon_0 d} = -\frac{2e}{4\pi\varepsilon_0 d}.$$

Thus, with $d = 2.00 \times 10^{-6}$ m and $e = 1.60 \times 10^{-19}$ C, we find

$$V_P = -\frac{2e}{4\pi\varepsilon_0 d} = -\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(1.60\times10^{-19}\text{C})}{2.00\times10^{-6}\text{ m}} = -1.438\times10^{-3}\text{ V}.$$

Then the required "applied" work is, by Eq. 24-14,

$$W_{\text{app}} = (-e)\,V_P = 2.30 \times 10^{-22}\text{ J}.$$

90. The particle with charge $-q$ has both potential and kinetic energy, and both of these change when the radius of the orbit is changed. We first find an expression for the total energy in terms of the orbit radius r. The charge Q provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\varepsilon_0 r^2$. The acceleration of $-q$ is v^2/r, where v is its speed. Newton's second law yields

$$\frac{Q_q}{4\pi\varepsilon_0 r^2}=\frac{mv^2}{r} \;\Rightarrow\; mv^2=\frac{Qq}{4\pi\varepsilon_0 r},$$

and the kinetic energy is

$$K=\frac{1}{2}mv^2=\frac{Qq}{8\pi\varepsilon_0 r}.$$

The potential energy is $U = -Qq/4\pi\varepsilon_0 r$, and the total energy is

$$E=K+U=\frac{Qq}{8\pi\varepsilon_0 r}-\frac{Qq}{4\pi\varepsilon_0 r}=-\frac{Qq}{8\pi\varepsilon_0 r}.$$

When the orbit radius is r_1 the energy is $E_1 = -Qq/8\pi\varepsilon_0 r_1$ and when it is r_2 the energy is $E_2 = -Qq/8\pi\varepsilon_0 r_2$. The difference $E_2 - E_1$ is the work W done by an external agent to change the radius:

$$W=E_2-E_1=-\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)=\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

91. The initial speed v_i of the electron satisfies

$$K_i=\tfrac{1}{2}m_e v_i^2=e\Delta V,$$

which gives

$$v_i=\sqrt{\frac{2e\Delta V}{m_e}}=\sqrt{\frac{2(1.60\times10^{-19}\text{ J})(625\text{ V})}{9.11\times10^{-31}\text{ kg}}}=1.48\times10^{7}\text{ m/s}.$$

92. The net electric potential at point P is the sum of those due to the six charges:

$$V_P=\sum_{i=1}^{6}V_{Pi}=\sum_{i=1}^{6}\frac{q_i}{4\pi\varepsilon_0 r_i}=\frac{10^{-15}}{4\pi\varepsilon_0}\left[\frac{5.00}{\sqrt{d^2+(d/2)^2}}+\frac{-2.00}{d/2}+\frac{-3.00}{\sqrt{d^2+(d/2)^2}}\right.$$
$$\left.+\frac{3.00}{\sqrt{d^2+(d/2)^2}}+\frac{-2.00}{d/2}+\frac{+5.00}{\sqrt{d^2+(d/2)^2}}\right]=\frac{9.4\times10^{-16}}{4\pi\varepsilon_0(2.54\times10^{-2})}$$
$$=3.34\times10^{-4}\text{ V}.$$

93. **THINK** To calculate the potential at point B due to the charged ring, we note that all points on the ring are at the same distance from B.

EXPRESS Let point B be at $(0, 0, z)$. The electric potential at B is given by

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

where q is the charge on the ring. The potential at infinity is taken to be zero.

ANALYZE With $q = 16 \times 10^{-6}$ C, $z = 0.040$ m, and $R = 0.0300$ m, we find the potential difference between points A (located at the origin) and B to be

$$\begin{aligned} V_B - V_A &= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{R}\right) \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(16.0\times10^{-6}\ \text{C})\left(\frac{1}{\sqrt{(0.030\ \text{m})^2+(0.040\ \text{m})^2}} - \frac{1}{0.030\ \text{m}}\right) \\ &= -1.92\times10^6\ \text{V}. \end{aligned}$$

LEARN In the limit $z \gg R$, the potential approaches its "point-charge" limit:

$$V \approx \frac{q}{4\pi\varepsilon_0 z}.$$

94. (a) Using Eq. 24-26, we calculate the radius r of the sphere representing the 30 V equipotential surface:

$$r = \frac{q}{4\pi\varepsilon_0 V} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.50\times10^{-8}\text{C})}{30\ \text{V}} = 4.5\,\text{m}.$$

(b) If the potential were a linear function of r then it would have equally spaced equipotentials, but since $V \propto 1/r$ they are spaced more and more widely apart as r increases.

95. **THINK** To calculate the electric potential, we first apply Gauss' law to calculate the electric field of the spherical shell. The Gaussian surface is a sphere that is concentric with the shell.

EXPRESS At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so the flux through the surface is given by $\Phi = \oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E = q_{\text{enc}}/\varepsilon_0$, where r is the radius of the Gaussian surface and q_{enc} is the charge enclosed. (i) In the region $r < r_1$, the enclosed charge is $q_{\text{enc}} = 0$ and therefore,

$E = 0$. (ii) In the region $r_1 < r < r_2$, the volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)},$$

where Q is the total charge on the spherical shell. Thus, the charge enclosed by the Gaussian surface is

$$q_{\text{enc}} = \left(\frac{4\pi}{3}\right)\left(r^3 - r_1^3\right)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right).$$

Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2\left(r_2^3 - r_1^3\right)}.$$

(iii) In the region $r > r_2$, the charge enclosed is $q_{\text{enc}} = Q$, and the electric field is like that of a point charge:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$$

ANALYZE (a) For $r > r_2$ the field is like that of a point charge, and so is the potential:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r},$$

where the potential was taken to be zero at infinity.

(b) In the region $r_1 < r < r_2$, we have

$$E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2\left(r_2^3 - r_1^3\right)}.$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$V = V_s - \int_{r_2}^{r} E\,dr = V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr$$

$$= V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right).$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\varepsilon_0 r_2$. We make this substitution and collect terms to find

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right).$$

Since $\rho = 3Q/4\pi\left(r_2^3 - r_1^3\right)$ this can also be written as

$$V(r) = \frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right).$$

(c) For $r < r_1$, the electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. We put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{3\left(r_2^2 - r_1^2\right)}{2\left(r_2^3 - r_1^3\right)},$$

or in terms of the charge density $V = \dfrac{\rho}{2\varepsilon_0}\left(r_2^2 - r_1^2\right)$.

(d) Using the expression for $V(r)$ found in (b), we have

$$V(r_1) = \frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2} - \frac{r_1^2}{2} - \frac{r_1^3}{r_1}\right) = \frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2} - \frac{3r_1^2}{2}\right) = \frac{\rho}{2\varepsilon_0}\left(r_2^2 - r_1^2\right)$$

and

$$V(r_2) = \frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2} - \frac{r_2^2}{2} - \frac{r_1^3}{r_2}\right) = \frac{\rho}{3\varepsilon_0}\left(r_2^2 - \frac{r_1^3}{r_2}\right) = \frac{\rho}{3\varepsilon_0 r_2}\left(r_2^3 - r_1^3\right) = \frac{3Q/4\pi}{3\varepsilon_0 r_2} = \frac{Q}{4\pi\varepsilon_0 r_2}.$$

So the solutions agree at $r = r_1$ and at $r = r_2$.

LEARN Electric potential must be continuous at the boundaries at $r = r_1$ and $r = r_2$. In the region where the electric field is zero, no work is required to move the charge around. Thus, there's no change in potential energy and the electric potential is constant.

96. (a) We use Gauss' law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside.

Outside the charge distribution the magnitude of the field is $E = q/4\pi\varepsilon_0 r^2$ and the potential is $V = q/4\pi\varepsilon_0 r$, where r is the distance from the center of the distribution. This is the same as the field and potential of a point charge at the center of the spherical distribution. To find an expression for the magnitude of the field inside the charge distribution, we use a Gaussian surface in the form of a sphere with radius r, concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is $4\pi r^2 E$. The charge enclosed is qr^3/R^3. Gauss' law becomes

$$4\pi\varepsilon_0 r^2 E = \frac{qr^3}{R^3} \quad \Rightarrow \quad E = \frac{qr}{4\pi\varepsilon_0 R^3}.$$

If V_s is the potential at the surface of the distribution ($r = R$) then the potential at a point inside, a distance r from the center, is

$$V = V_s - \int_R^r E\,dr = V_s - \frac{q}{4\pi\varepsilon_0 R^3}\int_R^r r\,dr = V_s - \frac{qr^2}{8\pi\varepsilon_0 R^3} + \frac{q}{8\pi\varepsilon_0 R}.$$

The potential at the surface can be found by replacing r with R in the expression for the potential at points outside the distribution. It is $V_s = q/4\pi\varepsilon_0 R$. Thus,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right] = \frac{q}{8\pi\varepsilon_0 R^3}\left(3R^2 - r^2\right).$$

(b) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\varepsilon_0 R} - \frac{3q}{8\pi\varepsilon_0 R} = -\frac{q}{8\pi\varepsilon_0 R},$$

or $|\Delta V| = q/8\pi\varepsilon_0 R$.

97. **THINK** The increase in electric potential at the surface of the copper sphere is proportional to the increase in electric charge.

EXPRESS The electric potential at the surface of a sphere of radius R is given by $V = q/4\pi\varepsilon_0 R,$ where q is the charge on the sphere. Thus, $q = 4\pi\varepsilon_0 RV.$ The number of electrons entering the copper sphere is $N = q/e,$ but this must be equal to $(\lambda/2)t,$ where λ is the decay rate of the nickel.

ANALYZE (a) With $R = 0.010$ m, when $V = 1000$ V, the net charge on the sphere is

$$q = 4\pi\varepsilon_0 RV = \frac{(0.010\text{ m})(1000\text{ V})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 1.11\times10^{-9}\text{ C}.$$

Dividing q by e yields

$$N = (1.11\times10^{-9}\ \text{C})/(1.6\times10^{-19}\ \text{C}) = 6.95\times10^{9}$$

electrons that entered the copper sphere. So the time required is

$$t = \frac{N}{\lambda/2} = \frac{6.95\times10^{9}}{(3.7\times10^{8}/\text{s})/2} = 38\ \text{s}.$$

(b) The energy deposited by each electron that enters the sphere is E_0 = 100 keV = 1.6 × 10^{-14} J. Using the given heat capacity, we note that a temperature increase of ΔT = 5.0 K = 5.0 ºC required

$$E = C\Delta T = (14\ \text{J/K})(5.0\ \text{K}) = 70\ \text{J}$$

of energy. Dividing this by E_0 gives the number of electrons needed to enter the sphere (in order to achieve that temperature change):

$$N' = \frac{E}{E_0} = \frac{70\ \text{J}}{1.6\times10^{-14}\ \text{J}} = 4.375\times10^{15}$$

Thus, the time needed is

$$t' = \frac{N'}{\lambda/2} = \frac{4.375\times10^{15}}{(3.7\times10^{8}/\text{s})/2} = 2.36\times10^{7}\ \text{s}$$

or roughly 270 days.

LEARN As more electrons get into copper, more energy is deposited, and the copper sample gets hotter.

98. (a) The potential difference is

$$\Delta V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{15\times10^{-6}\ \text{C}}{0.060\ \text{m}} - \frac{5.0\times10^{-6}\ \text{C}}{0.030\ \text{m}}\right)$$
$$= 7.49\times10^{5}\ \text{V}.$$

(b) By connecting the two metal spheres with a wire, we now have one conductor, and any excess charge must reside on the surface of the conductor. Therefore, the charge on the small sphere is zero.

(c) Since all the charges reside on the surface of the large sphere, we have

$$Q' = Q + q = 15.0\ \mu\text{C} + 5.00\ \mu\text{C} = 20.0\ \mu\text{C}.$$

99. (a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r = \sqrt{z^2 + R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2+R^2}}.$$

(b) The electric field is along the axis and its component is given by

$$E = -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z)$$
$$= \frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}.$$

This agrees with Eq. 23-16.

100. The distance r being looked for is that where the alpha particle has (momentarily) zero kinetic energy. Thus, energy conservation leads to

$$K_0 + U_0 = K + U \;\Rightarrow\; (0.48\times10^{-12}\text{ J}) + \frac{(2e)(92e)}{4\pi\varepsilon_0 r_0} = 0 + \frac{(2e)(92e)}{4\pi\varepsilon_0 r}\;.$$

If we set $r_0 = \infty$ (so $U_0 = 0$) then we obtain $r = 8.8\times10^{-14}$ m.

101. (a) Let the quark-quark separation be r. To "naturally" obtain the eV unit, we only plug in for one of the e values involved in the computation:

$$U_{\text{up}-\text{up}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e/3)(2e/3)}{r} = \frac{4ke}{9r}e = \frac{4\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{C}\right)}{9\left(1.32\times10^{-15}\text{m}\right)}e$$
$$= 4.84\times10^5\text{eV} = 0.484\,\text{MeV}.$$

(b) The total consists of all pair-wise terms:

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(2e/3)(2e/3)}{r} + \frac{(-e/3)(2e/3)}{r} + \frac{(-e/3)(2e/3)}{r}\right] = 0.$$

102. We imagine moving all the charges on the surface of the sphere to the center of the the sphere. Using Gauss' law, we see that this would not change the electric field *outside* the sphere.

The magnitude of the electric field E of the uniformly charged sphere as a function of r, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\varepsilon_0 r^2)$ for $r > R$.

Here R is the radius of the sphere. Thus, the potential V at the surface of the sphere (where $r = R$) is given by

$$V(R) = V\big|_{r=\infty} + \int_R^\infty E(r)\,dr = \int_\infty^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.50\times10^8\ \text{C}\right)}{0.160\text{m}}$$

$$= 8.43\times10^2\ \text{V}.$$

103. Since the electric potential energy is not changed by the introduction of the third particle, we conclude that the net electric potential evaluated at P caused by the original two particles must be zero:

$$\frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} = 0\,.$$

Setting $r_1 = 5d/2$ and $r_2 = 3d/2$ we obtain $q_1 = -\,5q_2/3$, or $q_1/q_2 = -5/3 \approx -1.7$.

Chapter 25

1. (a) The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70\,\text{pC}}{20\,\text{V}} = 3.5\,\text{pF}.$$

(b) The capacitance is independent of q; it is still 3.5 pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200\,\text{pC}}{3.5\,\text{pF}} = 57\,\text{V}.$$

2. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus,

$$q = (25 \times 10^{-6}\ \text{F})(120\ \text{V}) = 3.0 \times 10^{-3}\ \text{C}.$$

3. **THINK** The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where A is the area of each plate and d is the plate separation.

EXPRESS Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates.

ANALYZE (a) Substituting the values given, the capacitance is

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{\left(8.85\times10^{-12}\ \text{F/m}\right)\pi\left(8.2\times10^{-2}\ \text{m}\right)^2}{1.3\times10^{-3}\,\text{m}} = 1.44\times10^{-10}\,\text{F} = 144\ \text{pF}.$$

(b) Similarly, the charge on the plate when $V = 120$ V is

$$q = (1.44 \times 10^{-10}\ \text{F})(120\ \text{V}) = 1.73 \times 10^{-8}\ \text{C} = 17.3\ \text{nC}.$$

LEARN Capacitance depends only on geometric factors, namely, the plate area and plate separation.

4. (a) We use Eq. 25-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\text{mm})(38.0\,\text{mm})}{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0\,\text{mm}-38.0\,\text{mm})} = 84.5\ \text{pF}.$$

(b) Let the area required be A. Then $C = \varepsilon_0 A/(b-a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\text{pF})(40.0\,\text{mm}-38.0\,\text{mm})}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 191\,\text{cm}^2.$$

5. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \quad \Rightarrow \quad R' = 2^{1/3}R.$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 R.$$

With R = 2.00 mm, we obtain $C = 5.04\pi\left(8.85\times10^{-12}\ \text{F/m}\right)\left(2.00\times10^{-3}\,\text{m}\right) = 2.80\times10^{-13}\,\text{F}$.

6. (a) We use $C = A\varepsilon_0/d$. The distance between the plates is

$$d = \frac{A\varepsilon_0}{C} = \frac{\left(1.00\,\text{m}^2\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{1.00\,\text{F}} = 8.85\times10^{-12}\ \text{m}.$$

(b) Since d is much less than the size of an atom (~ 10^{-10} m), this capacitor cannot be constructed.

7. For a given potential difference V, the charge on the surface of the plate is

$$q = Ne = (nAd)e$$

where d is the depth from which the electrons come in the plate, and n is the density of conduction electrons. The charge collected on the plate is related to the capacitance and the potential difference by $q = CV$ (Eq. 25-1). Combining the two expressions leads to

$$\frac{C}{A} = ne\frac{d}{V}.$$

With $d/V = d_s/V_s = 5.0\times10^{-14}$ m/V and $n = 8.49\times10^{28}/\text{m}^3$ (see, for example, Sample Problem 25.01 — "Charging the plates in a parallel-plate capacitor"), we obtain

$$\frac{C}{A}=(8.49\times10^{28}/\text{m}^3)(1.6\times10^{-19}\,\text{C})(5.0\times10-14\text{ m/V})=6.79\times10^{-4}\,\text{F/m}^2.$$

8. The equivalent capacitance is given by $C_{eq} = q/V$, where q is the total charge on all the capacitors and V is the potential difference across any one of them. For N identical capacitors in parallel, $C_{eq} = NC$, where C is the capacitance of one of them. Thus, $NC = q/V$ and

$$N=\frac{q}{VC}=\frac{1.00\,\text{C}}{(110\,\text{V})\left(1.00\times10^{-6}\,\text{F}\right)}=9.09\times10^3.$$

9. The charge that passes through meter A is

$$q = C_{eq}V = 3CV = 3(25.0\,\mu\text{F})(4200\,\text{V}) = 0.315\,\text{C}.$$

10. The equivalent capacitance is

$$C_{eq}=C_3+\frac{C_1C_2}{C_1+C_2}=4.00\,\mu\text{F}+\frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}}=7.33\,\mu\text{F}.$$

11. The equivalent capacitance is

$$C_{eq}=\frac{(C_1+C_2)C_3}{C_1+C_2+C_3}=\frac{(10.0\,\mu\text{F}+5.00\,\mu\text{F})(4.00\,\mu\text{F})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}+4.00\,\mu\text{F}}=3.16\,\mu\text{F}.$$

12. The two 6.0 μF capacitors are in parallel and are consequently equivalent to $C_{eq} = 12\ \mu\text{F}$. Thus, the total charge stored (before the squeezing) is

$$q_{\text{total}}=C_{eq}V=(12\,\mu\text{F})\,(10.0\,\text{V})=120\ \mu\text{C}.$$

(a) and (b) As a result of the squeezing, one of the capacitors is now 12 μF (due to the inverse proportionality between C and d in Eq. 25-9), which represents an increase of $6.0\ \mu\text{F}$ and thus a charge increase of

$$\Delta q_{\text{total}}=\Delta C_{eq}V=(6.0\,\mu\text{F})\,(10.0\,\text{V})=60\ \mu\text{C}.$$

13. **THINK** Charge remains conserved when a fully charged capacitor is connected to an uncharged capacitor.

EXPRESS The charge initially on the charged capacitor is given by $q = C_1V_0$, where C_1 = 100 pF is the capacitance and V_0 = 50 V is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1V$, where $V = 35$ V is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where C_2 is the capacitance of the second capacitor.

ANALYZE Substituting C_1V_0 for q and C_1V for q_1, we obtain $q_2 = C_1(V_0 - V)$. The potential difference across the second capacitor is also V, so the capacitance of the second capacitor is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V}C_1 = \frac{50\,\text{V} - 35\,\text{V}}{35\,\text{V}}(100\,\text{pF}) = 42.86\text{ pF} \approx 43\,\text{pF}.$$

LEARN Capacitors in parallel have the same potential difference. To verify charge conservation explicitly, we note that the initial charge on the first capacitor is $q = C_1V_0 = (100\text{ pF})(50\text{ V}) = 5000\text{ pC}.$ After the connection, the charges on each capacitor are

$$q_1 = C_1V = (100\text{ pF})(35\text{ V}) = 3500\text{ pC}$$
$$q_2 = C_2V = (42.86\text{ pF})(35\text{ V}) = 1500\text{ pC}.$$

Indeed, $q = q_1 + q_2$.

14. (a) The potential difference across C_1 is $V_1 = 10.0$ V. Thus,

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\text{ V}) = 1.00 \times 10^{-4}\text{ C}.$$

(b) Let $C = 10.0\ \mu$F. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

$$C_{\text{eq}} = C + \frac{C_2C}{C + C_2} = 1.50\ C.$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C + C_{\text{eq}}} = \frac{CV_1}{C + 1.50\ C} = 0.40V_1.$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10.0\,\mu\text{F})\left(\frac{10.0\text{V}}{5}\right) = 2.00\times10^{-5}\,\text{C}.$$

15. (a) First, the equivalent capacitance of the two 4.00 μF capacitors connected in series is given by 4.00 μF/2 = 2.00 μF. This combination is then connected in parallel with two other 2.00-μF capacitors (one on each side), resulting in an equivalent capacitance C = 3(2.00 μF) = 6.00 μF. This is now seen to be in series with another combination, which

consists of the two 3.0-μF capacitors connected in parallel (which are themselves equivalent to $C' = 2(3.00\ \mu\text{F}) = 6.00\ \mu\text{F}$). Thus, the equivalent capacitance of the circuit is

$$C_{\text{eq}} = \frac{CC'}{C+C'} = \frac{(6.00\mu\text{F})(6.00\mu\text{F})}{6.00\mu\text{F}+6.00\mu\text{F}} = 3.00\mu\text{F}.$$

(b) Let $V = 20.0$ V be the potential difference supplied by the battery. Then

$$q = C_{\text{eq}}V = (3.00\ \mu\text{F})(20.0\ \text{V}) = 6.00 \times 10^{-5}\ \text{C}.$$

(c) The potential difference across C_1 is given by

$$V_1 = \frac{CV}{C+C'} = \frac{(6.00\mu\text{F})(20.0\text{V})}{6.00\mu\text{F}+6.00\mu\text{F}} = 10.0\text{V}.$$

(d) The charge carried by C_1 is $q_1 = C_1V_1 = (3.00\ \mu\text{F})(10.0\ \text{V}) = 3.00 \times 10^{-5}$ C.

(e) The potential difference across C_2 is given by $V_2 = V - V_1 = 20.0\ \text{V} - 10.0\ \text{V} = 10.0$ V.

(f) The charge carried by C_2 is $q_2 = C_2V_2 = (2.00\ \mu\text{F})(10.0\ \text{V}) = 2.00 \times 10^{-5}$ C.

(g) Since this voltage difference V_2 is divided equally between C_3 and the other 4.00-μF capacitors connected in series with it, the voltage difference across C_3 is given by $V_3 = V_2/2 = 10.0\ \text{V}/2 = 5.00$ V.

(h) Thus, $q_3 = C_3V_3 = (4.00\ \mu\text{F})(5.00\ \text{V}) = 2.00 \times 10^{-5}$ C.

16. We determine each capacitance from the slope of the appropriate line in the graph. Thus, $C_1 = (12\ \mu\text{C})/(2.0\ \text{V}) = 6.0\ \mu\text{F}$. Similarly, $C_2 = 4.0\ \mu\text{F}$ and $C_3 = 2.0\ \mu\text{F}$. The total equivalent capacitance is given by

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2+C_3} = \frac{C_1+C_2+C_3}{C_1(C_2+C_3)},$$

or

$$C_{123} = \frac{C_1(C_2+C_3)}{C_1+C_2+C_3} = \frac{(6.0\ \mu\text{F})(4.0\ \mu\text{F}+2.0\ \mu\text{F})}{6.0\ \mu\text{F}+4.0\ \mu\text{F}+2.0\ \mu\text{F}} = \frac{36}{12}\ \mu\text{F} = 3.0\ \mu\text{F}.$$

This implies that the charge on capacitor 1 is $q_1 = (3.0\ \mu\text{F})(6.0\ \text{V}) = 18\ \mu\text{C}$. The voltage across capacitor 1 is therefore $V_1 = (18\ \mu\text{C})/(6.0\ \mu\text{F}) = 3.0$ V. From the discussion in section 25-4, we conclude that the voltage across capacitor 2 must be 6.0 V – 3.0 V = 3.0 V. Consequently, the charge on capacitor 2 is $(4.0\ \mu\text{F})(3.0\ \text{V}) = 12\ \mu\text{C}$.

17. (a) and (b) The original potential difference V_1 across C_1 is

$$V_1 = \frac{C_{eq}V}{C_1 + C_2} = \frac{(3.16\,\mu\text{F})(100.0\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 21.1\,\text{V}.$$

Thus ΔV_1 = 100.0 V – 21.1 V = 78.9 V and

$$\Delta q_1 = C_1 \Delta V_1 = (10.0\ \mu\text{F})(78.9\ \text{V}) = 7.89 \times 10^{-4}\ \text{C}.$$

18. We note that the voltage across C_3 is V_3 = (12 V – 2 V – 5 V) = 5 V. Thus, its charge is $q_3 = C_3 V_3 = 4\ \mu$C.

(a) Therefore, since C_1, C_2 and C_3 are in series (so they have the same charge), then

$$C_1 = \frac{4\ \mu\text{C}}{2\ \text{V}} = 2.0\ \mu\text{F}\ .$$

(b) Similarly, $C_2 = 4/5 = 0.80\ \mu$F.

19. (a) and (b) We note that the charge on C_3 is $q_3 = 12\ \mu\text{C} - 8.0\ \mu\text{C} = 4.0\ \mu$C. Since the charge on C_4 is $q_4 = 8.0\ \mu$C, then the voltage across it is $q_4/C_4 = 2.0$ V. Consequently, the voltage V_3 across C_3 is 2.0 V $\Rightarrow C_3 = q_3/V_3 = 2.0\ \mu$F.

Now C_3 and C_4 are in parallel and are thus equivalent to 6 μF capacitor which would then be in series with C_2; thus, Eq 25-20 leads to an equivalence of 2.0 μF which is to be thought of as being in series with the unknown C_1. We know that the total effective capacitance of the circuit (in the sense of what the battery "sees" when it is hooked up) is $(12\ \mu\text{C})/V_{\text{battery}} = 4\ \mu\text{F}/3$. Using Eq 25-20 again, we find

$$\frac{1}{2\ \mu\text{F}} + \frac{1}{C_1} = \frac{3}{4\ \mu\text{F}} \quad \Rightarrow \quad C_1 = 4.0\ \mu\text{F}\ .$$

20. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of $(n - 1)$ identical single capacitors connected in parallel. Each capacitor has surface area A and plate separation d so its capacitance is given by $C_0 = \varepsilon_0 A/d$. Thus, the total capacitance of the combination is

$$C = (n-1)C_0 = \frac{(n-1)\varepsilon_0 A}{d} = \frac{(8-1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.25\times10^{-4}\ \text{m}^2)}{3.40\times10^{-3}\ \text{m}} = 2.28\times10^{-12}\,\text{F}.$$

21. **THINK** After the switches are closed, the potential differences across the capacitors are the same and they are connected in parallel.

EXPRESS The potential difference from a to b is given by $V_{ab} = Q/C_{eq}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance.

ANALYZE (a) The equivalent capacitance is $C_{eq} = C_1 + C_2 = 4.0 \times 10^{-6}$ F. The total charge on the combination is the net charge on either pair of connected plates. The initial charge on capacitor 1 is

$$q_1 = C_1 V = \left(1.0\times10^{-6}\text{ F}\right)\left(100\text{ V}\right) = 1.0\times10^{-4}\text{ C}$$

and the initial charge on capacitor 2 is

$$q_2 = C_2 V = \left(3.0\times10^{-6}\text{F}\right)\left(100\text{ V}\right) = 3.0\times10^{-4}\text{ C}.$$

With opposite polarities, the net charge on the combination is

$$Q = 3.0 \times 10^{-4}\text{ C} - 1.0 \times 10^{-4}\text{ C} = 2.0 \times 10^{-4}\text{ C}.$$

The potential difference is

$$V_{ab} = \frac{Q}{C_{eq}} = \frac{2.0\times10^{-4}\text{C}}{4.0\times10^{-6}\text{F}} = 50\text{ V}.$$

(b) The charge on capacitor 1 is now $q_1' = C_1 V_{ab} = (1.0 \times 10^{-6}\text{ F})(50\text{ V}) = 5.0 \times 10^{-5}$ C.

(c) The charge on capacitor 2 is now $q_2' = C_2 V_{ab} = (3.0 \times 10^{-6}\text{ F})(50\text{ V}) = 1.5 \times 10^{-4}$ C.

LEARN The potential difference $V_{ab} = 50$ V is half of the original V (= 100 V), so the final charges on the capacitors are also halved.

22. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{bat} = 100\ \mu$C, and q_1, q_2 and q_3 are the charges on C_1, C_2 and C_3 after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2 + q_3.$$

Since the parallel pair C_2 and C_3 are identical, it is clear that $q_2 = q_3$. They are in parallel with C_1 so that $V_1 = V_3$, or

$$\frac{q_1}{C_1} = \frac{q_3}{C_3}$$

which leads to $q_1 = q_3/2$. Therefore,

$$Q = (q_3/2) + q_3 + q_3 = 5q_3/2$$

which yields $q_3 = 2Q/5 = 2(100\ \mu\text{C})/5 = 40\ \mu\text{C}$ and consequently $q_1 = q_3/2 = 20\ \mu$C.

23. We note that the total equivalent capacitance is $C_{123} = [(C_3)^{-1} + (C_1 + C_2)^{-1}]^{-1} = 6\ \mu$F.

(a) Thus, the charge that passed point *a* is $C_{123}\, V_{\text{batt}} = (6\ \mu\text{F})(12\ \text{V}) = 72\ \mu\text{C}$. Dividing this by the value $e = 1.60 \times 10^{-19}$ C gives the number of electrons: 4.5×10^{14}, which travel to the left, toward the positive terminal of the battery.

(b) The equivalent capacitance of the parallel pair is $C_{12} = C_1 + C_2 = 12\ \mu$F. Thus, the voltage across the pair (which is the same as the voltage across C_1 and C_2 individually) is

$$\frac{72\ \mu\text{C}}{12\ \mu\text{F}} = 6\ \text{V}.$$

Thus, the charge on C_1 is

$$q_1 = (4\ \mu\text{F})(6\ \text{V}) = 24\ \mu\text{C},$$

and dividing this by *e* gives $N_1 = q_1 / e = 1.5 \times 10^{14}$, the number of electrons that have passed (upward) through point *b*.

(c) Similarly, the charge on C_2 is $q_2 = (8\ \mu\text{F})(6\ \text{V}) = 48\ \mu\text{C}$, and dividing this by *e* gives $N_2 = q_2 / e = 3.0 \times 10^{14}$, the number of electrons which have passed (upward) through point *c*.

(d) Finally, since C_3 is in series with the battery, its charge is the same charge that passed through the battery (the same as passed through the switch). Thus, 4.5×10^{14} electrons passed rightward though point *d*. By leaving the rightmost plate of C_3, that plate is then the positive plate of the fully charged capacitor, making its leftmost plate (the one closest to the negative terminal of the battery) the negative plate, as it should be.

(e) As stated in (b), the electrons travel up through point *b*.

(f) As stated in (c), the electrons travel up through point *c*.

24. Using Equation 25-14, the capacitances are

$$C_1 = \frac{2\pi\varepsilon_0 L_1}{\ln(b_1/a_1)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.050\ \text{m})}{\ln(15\ \text{mm}/5.0\ \text{mm})} = 2.53\ \text{pF}$$

$$C_2 = \frac{2\pi\varepsilon_0 L_2}{\ln(b_2/a_2)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.090\ \text{m})}{\ln(10\ \text{mm}/2.5\ \text{mm})} = 3.61\ \text{pF}\ .$$

Initially, the total equivalent capacitance is

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2} \Rightarrow C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2.53\ \text{pF})(3.61\ \text{pF})}{2.53\ \text{pF} + 3.61\ \text{pF}} = 1.49\ \text{pF}\ ,$$

and the charge on the positive plate of each one is (1.49 pF)(10 V) = 14.9 pC. Next, capacitor 2 is modified as described in the problem, with the effect that

$$C_2' = \frac{2\pi\varepsilon_0 L_2}{\ln(b_2'/a_2)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.090\ \text{m})}{\ln(25\ \text{mm}/2.5\ \text{mm})} = 2.17\ \text{pF}\ .$$

The new total equivalent capacitance is

$$C_{12}' = \frac{C_1 C_2'}{C_1 + C_2'} = \frac{(2.53\ \text{pF})(2.17\ \text{pF})}{2.53\ \text{pF} + 2.17\ \text{pF}} = 1.17\ \text{pF}$$

and the new charge on the positive plate of each one is (1.17 pF)(10 V) = 11.7 pC. Thus we see that the charge transferred from the battery (considered in absolute value) as a result of the modification is 14.9 pC – 11.7 pC = 3.2 pC.

(a) This charge, divided by e gives the number of electrons that pass point P. Thus,

$$N = \frac{3.2\times10^{-12}\,\text{C}}{1.6\times10^{-19}\,\text{C}} = 2.0\times10^7\ .$$

(b) These electrons move rightward in the figure (that is, away from the battery) since the positive plates (the ones closest to point P) of the capacitors have suffered a *decrease* in their positive charges. The usual reason for a metal plate to be positive is that it has more protons than electrons. Thus, in this problem some electrons have "returned" to the positive plates (making them less positive).

25. Equation 23-14 applies to each of these capacitors. Bearing in mind that $\sigma = q/A$, we find the total charge to be

$$q_{\text{total}} = q_1 + q_2 = \sigma_1 A_1 + \sigma_2 A_2 = \varepsilon_0 E_1 A_1 + \varepsilon_0 E_2 A_2 = 3.6\ \text{pC}$$

where we have been careful to convert cm^2 to m^2 by dividing by 10^4.

26. Initially the capacitors C_1, C_2, and C_3 form a combination equivalent to a single capacitor which we denote C_{123}. This obeys the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)}\ .$$

Hence, using $q = C_{123}V$ and the fact that $q = q_1 = C_1 V_1$, we arrive at

$$V_1 = \frac{q_1}{C_1} = \frac{q}{C_1} = \frac{C_{123}}{C_1}V = \frac{C_2 + C_3}{C_1 + C_2 + C_3}V\ .$$

(a) As $C_3 \to \infty$ this expression becomes $V_1 = V$. Since the problem states that V_1 approaches 10 volts in this limit, so we conclude $V = 10$ V.

(b) and (c) At $C_3 = 0$, the graph indicates $V_1 = 2.0$ V. The above expression consequently implies $C_1 = 4C_2$. Next we note that the graph shows that, at $C_3 = 6.0\ \mu$F, the voltage across C_1 is exactly half of the battery voltage. Thus,

$$\frac{1}{2} = \frac{C_2 + 6.0\ \mu\text{F}}{C_1 + C_2 + 6.0\ \mu\text{F}} = \frac{C_2 + 6.0\ \mu\text{F}}{4C_2 + C_2 + 6.0\ \mu\text{F}}$$

which leads to $C_2 = 2.0\ \mu$F. We conclude, too, that $C_1 = 8.0\ \mu$F.

27. (a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1 C_3 V}{C_1 + C_3} = \frac{(1.00\,\mu\text{F})\,(3.00\,\mu\text{F})\,(12.0\text{V})}{1.00\,\mu\text{F}+3.00\,\mu\text{F}} = 9.00\,\mu\text{C}.$$

(b) Capacitors 2 and 4 are also in series:

$$q_2 = q_4 = \frac{C_2 C_4 V}{C_2 + C_4} = \frac{(2.00\,\mu\text{F})\,(4.00\,\mu\text{F})\,(12.0\text{V})}{2.00\,\mu\text{F}+4.00\,\mu\text{F}} = 16.0\,\mu\text{C}.$$

(c) $q_3 = q_1 = 9.00\,\mu\text{C}.$

(d) $q_4 = q_2 = 16.0\,\mu\text{C}.$

(e) With switch 2 also closed, the potential difference V_1 across C_1 must equal the potential difference across C_2 and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4}\,V = \frac{(3.00\,\mu\text{F}+4.00\,\mu\text{F})(12.0\text{V})}{1.00\,\mu\text{F}+2.00\,\mu\text{F}+3.00\,\mu\text{F}+4.00\,\mu\text{F}} = 8.40\text{ V}.$$

Thus, $q_1 = C_1V_1 = (1.00\ \mu\text{F})(8.40\text{ V}) = 8.40\ \mu\text{C}$.

(f) Similarly, $q_2 = C_2V_1 = (2.00\ \mu\text{F})(8.40\text{ V}) = 16.8\ \mu\text{C}$.

(g) $q_3 = C_3(V - V_1) = (3.00\ \mu\text{F})(12.0\text{ V} - 8.40\text{ V}) = 10.8\ \mu\text{C}$.

(h) $q_4 = C_4(V - V_1) = (4.00\ \mu\text{F})(12.0\text{ V} - 8.40\text{ V}) = 14.4\ \mu\text{C}$.

28. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{eq}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}.$$

Thus, $C_{eq} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination, and the potential difference across the equivalent capacitor is given by q_2/C_{eq}. The potential difference across capacitor 1 is q_1/C_1, where q_1 is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{eq}$.

Now, some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where V_0 is the original potential difference across capacitor 1.

(a) Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{eq}}$$

$$q_1 + q_2 = C_1 V_0$$

for q_1 and q_2, we obtain

$$q_1 = \frac{C_1^2 V_0}{C_{eq} + C_1} = \frac{C_1^2 V_0}{\frac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1C_2 + C_1C_3 + C_2C_3}.$$

With $V_0 = 12.0$ V, $C_1 = 4.00\ \mu$F, $C_2 = 6.00\ \mu$F and $C_3 = 3.00\ \mu$F, we find $C_{eq} = 2.00\ \mu$F and $q_1 = 32.0\ \mu$C.

(b) The charge on capacitors 2 is

$$q_2 = C_1 V_0 - q_1 = (4.00 \mu\text{F})(12.0\text{ V}) - 32.0\ \mu\text{C} = 16.0\ \mu\text{C}.$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3 = C_1 V_0 - q_1 = (4.00\ \mu\text{F})(12.0\text{ V}) - 32.0\ \mu\text{C} = 16.0\ \mu\text{C}.$$

29. The energy stored by a capacitor is given by $U = \frac{1}{2}CV^2$, where V is the potential difference across its plates. We convert the given value of the energy to Joules. Since $1\text{ J} = 1\text{ W}\cdot\text{s}$, we multiply by $(10^3\text{ W/kW})(3600\text{ s/h})$ to obtain $10\text{ kW}\cdot\text{h} = 3.6\times10^7\text{ J}$. Thus,

$$C = \frac{2U}{V^2} = \frac{2(3.6\times10^7\text{ J})}{(1000\text{ V})^2} = 72\text{ F}.$$

30. Let $\mathcal{V} = 1.00\ \text{m}^3$. Using Eq. 25-25, the energy stored is

$$U = u\mathcal{V} = \frac{1}{2}\varepsilon_0 E^2 \mathcal{V} = \frac{1}{2}\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(150\ \text{V/m})^2\left(1.00\ \text{m}^3\right) = 9.96\times10^{-8}\ \text{J}.$$

31. **THINK** The total electrical energy is the sum of the energies stored in the individual capacitors.

EXPRESS The energy stored in a charged capacitor is

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2.$$

Since we have two capacitors that are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

$$U_{\text{tot}} = U_1 + U_2 = \frac{1}{2}\left(C_1 + C_2\right)V^2.$$

ANALYZE Substituting the values given, we have

$$U = \frac{1}{2}\left(C_1 + C_2\right)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\ \text{F} + 4.0\times10^{-6}\ \text{F}\right)(300\ \text{V})^2 = 0.27\ \text{J}.$$

LEARN The energy stored in a capacitor is equal to the amount of work required to charge the capacitor.

32. (a) The capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(40\times10^{-4}\ \text{m}^2\right)}{1.0\times10^{-3}\text{m}} = 3.5\times10^{-11}\ \text{F} = 35\ \text{pF}.$$

(b) $q = CV = (35\ \text{pF})(600\ \text{V}) = 2.1\times10^{-8}\ \text{C} = 21\ \text{nC}$.

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}\left(35\ \text{pF}\right)\left(21\ \text{nC}\right)^2 = 6.3\times10^{-6}\ \text{J} = 6.3\ \mu\text{J}$.

(d) $E = V/d = 600\ \text{V}/1.0\times10^{-3}\ \text{m} = 6.0\times10^5\ \text{V/m}$.

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3\times10^{-6}\,\text{J}}{\left(40\times10^{-4}\text{m}^2\right)\left(1.0\times10^{-3}\,\text{m}\right)} = 1.6\ \text{J/m}^3.$$

33. We use $E = q/4\pi\varepsilon_0 R^2 = V/R$. Thus

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{8000\,\text{V}}{0.050\ \text{m}}\right)^2 = 0.11\ \text{J/m}^3.$$

34. (a) The charge q_3 in the figure is $q_3 = C_3V = (4.00\,\mu\text{F})(100\ \text{V}) = 4.00\times10^{-4}\ \text{C}$.

(b) $V_3 = V = 100$ V.

(c) Using $U_i = \frac{1}{2}C_iV_i^2$, we have $U_3 = \frac{1}{2}C_3V_3^2 = 2.00\times10^{-2}\,\text{J}$.

(d) From the figure,

$$q_1 = q_2 = \frac{C_1C_2V}{C_1+C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(100\ \text{V})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}} = 3.33\times10^{-4}\text{C}.$$

(e) $V_1 = q_1/C_1 = 3.33\times10^{-4}\ \text{C}/10.0\ \mu\text{F} = 33.3$ V.

(f) $U_1 = \frac{1}{2}C_1V_1^2 = 5.55\times10^{-3}\,\text{J}$.

(g) From part (d), we have $q_2 = q_1 = 3.33\times10^{-4}\text{C}$.

(h) $V_2 = V - V_1 = 100\ \text{V} - 33.3\ \text{V} = 66.7$ V.

(i) $U_2 = \frac{1}{2}C_2V_2^2 = 1.11\times10^{-2}\,\text{J}$.

35. The energy per unit volume is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{32\pi^2\varepsilon_0 r^4}\ .$$

(a) At $r = 1.00\times10^{-3}\,\text{m}$, with $e = 1.60\times10^{-19}\,\text{C}$ and $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$, we have $u = 9.16\times10^{-18}\ \text{J/m}^3$.

(b) Similarly, at $r = 1.00\times10^{-6}\,\text{m}$, $u = 9.16\times10^{-6}\ \text{J/m}^3$.

(c) At $r = 1.00\times10^{-9}\,\text{m}$, $u = 9.16\times10^{6}\ \text{J/m}^3$.

(d) At $r = 1.00\times10^{-12}\,\text{m}$, $u = 9.16\times10^{18}\ \text{J/m}^3$.

(e) From the expression above, $u \propto r^{-4}$. Thus, for $r \to 0$, the energy density $u \to \infty$.

36. (a) We calculate the charged surface area of the cylindrical volume as follows:

$$A = 2\pi rh + \pi r^2 = 2\pi(0.20\ \text{m})(0.10\ \text{m}) + \pi(0.20\ \text{m})^2 = 0.25\ \text{m}^2$$

where we note from the figure that although the bottom is charged, the top is not. Therefore, the charge is $q = \sigma A = -0.50\ \mu\text{C}$ on the exterior surface, and consequently (according to the assumptions in the problem) that same charge q is induced in the interior of the fluid.

(b) By Eq. 25-21, the energy stored is

$$U = \frac{q^2}{2C} = \frac{(5.0\times10^{-7}\ \text{C})^2}{2(35\times10^{-12}\ \text{F})} = 3.6\times10^{-3}\ \text{J}.$$

(c) Our result is within a factor of three of that needed to cause a spark. Our conclusion is that it will probably not cause a spark; however, there is not enough of a safety factor to be sure.

37. **THINK** The potential difference between the plates of a parallel-plate capacitor depends on their distance of separation.

EPXRESS Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $C_i = \varepsilon_0 A/d_i$, the charge is $q_i = C_i V_i = \varepsilon_0 A V_i/d_i$. After the plates are pulled apart, their separation is d_f and the final potential difference is V_f. Thus, the final charge is $q_f = \varepsilon_0 A V_f/2d_f$. Since charge remains unchanged, $q_i = q_f$, we have

$$V_f = \frac{q_f}{C_f} = \frac{d_f}{\varepsilon_0 A} q_f = \frac{d_f}{\varepsilon_0 A}\frac{\varepsilon_0 A}{d_i} V_i = \frac{d_f}{d_i} V_i .$$

ANALYZE (a) With $d_i = 3.00\times10^{-3}\,\text{m}$, $V_i = 6.00\ \text{V}$ and $d_f = 8.00\times10^{-3}\,\text{m}$, the final potential difference is $V_f = 16.0\ \text{V}$.

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV_i^2 = \frac{\varepsilon_0 A V_i^2}{2d_i} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.50\times10^{-4}\ \text{m}^2)(6.00\ \text{V})^2}{2(3.00\times10^{-3}\ \text{m})}$$
$$= 4.51\times10^{-11}\ \text{J}.$$

(c) The final energy stored is

$$U_f = \frac{1}{2}C_f V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}\left(\frac{d_f}{d_i}V_i\right)^2 = \frac{d_f}{d_i}\left(\frac{\varepsilon_0 A V_i^2}{d_i}\right) = \frac{d_f}{d_i}U_i.$$

With $d_f / d_i = 8.00/3.00$, we have $U_f = 1.20\times10^{-10}$ J.

(d) The work done to pull the plates apart is the difference in the energy:

$$W = U_f - U_i = 7.52\times10^{-11}\text{ J}.$$

LEARN In a parallel-plate capacitor, the energy density (energy per unit volume) is given by $u = \varepsilon_0 E^2/2$ (see Eq. 25-25), where E is constant at all points between the plates. Thus, increasing the plate separation increases the volume (= Ad), and hence the total energy of the system.

38. (a) The potential difference across C_1 (the same as across C_2) is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(15.0\,\mu\text{F})(100\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 15.0\,\mu\text{F}} = 50.0\text{V}.$$

Also, $V_3 = V - V_1 = V - V_2$ = 100 V – 50.0 V = 50.0 V. Thus,

$$q_1 = C_1 V_1 = (10.0\,\mu\text{F})(50.0\text{V}) = 5.00\times10^{-4}\,\text{C}$$
$$q_2 = C_2 V_2 = (5.00\,\mu\text{F})(50.0\text{V}) = 2.50\times10^{-4}\,\text{C}$$
$$q_3 = q_1 + q_2 = 5.00\times10^{-4}\,\text{C} + 2.50\times10^{-4}\,\text{C} = 7.50\times10^{-4}\,\text{C}.$$

(b) The potential difference V_3 was found in the course of solving for the charges in part (a). Its value is V_3 = 50.0 V.

(c) The energy stored in C_3 is $U_3 = C_3 V_3^2/2 = (15.0\,\mu\text{F})(50.0\text{V})^2/2 = 1.88\times10^{-2}$ J.

(d) From part (a), we have $q_1 = 5.00\times10^{-4}\,\text{C}$, and

(e) V_1 = 50.0 V, as shown in (a).

(f) The energy stored in C_1 is $U_1 = \frac{1}{2}C_1 V_1^2 = \frac{1}{2}(10.0\,\mu\text{F})(50.0\text{V})^2 = 1.25\times10^{-2}$ J.

(g) Again, from part (a), $q_2 = 2.50\times10^{-4}\,\text{C}$.

(h) V_2 = 50.0 V, as shown in (a).

(i) The energy stored in C_2 is $U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(5.00\,\mu\text{F})(50.0\,\text{V})^2 = 6.25\times10^{-3}$ J.

39. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 μF, then the voltage across the 20 μF capacitor is 50 V and the voltage across the 25 μF capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 25-21 or Eq. 25-22, we sum the energies on the capacitors and obtain U_{total} = 0.095 J.

40. If the original capacitance is given by $C = \varepsilon_0 A/d$, then the new capacitance is $C' = \varepsilon_0\kappa A/2d$. Thus $C'/C = \kappa/2$ or

$$\kappa = 2C'/C = 2(2.6\text{ pF}/1.3\text{ pF}) = 4.0.$$

41. **THINK** Our system, a coaxial cable, is a cylindrical capacitor filled with polystyrene, a dielectric.

EXPRESS Using Eqs. 25-17 and 25-27, the capacitance of a cylindrical capacitor can be written as

$$C = \kappa C_0 = \frac{2\pi\kappa\varepsilon_0 L}{\ln(b/a)},$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius.

ANALYZE With κ = 2.6 for polystyrene, the capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\varepsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85\times10^{-12}\text{ F/m})}{\ln[(0.60\text{ mm})/(0.10\text{ mm})]} = 8.1\times10^{-11}\text{F/m} = 81\text{ pF/m}.$$

LEARN When the space between the plates of a capacitor is completely filled with a dielectric material, the capacitor increases by a factor κ, the dielectric constant characteristic of the material.

42. (a) We use $C = \varepsilon_0 A/d$ to solve for d:

$$d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(0.35\text{ m}^2)}{50\times10^{-12}\text{F}} = 6.2\times10^{-2}\text{ m}.$$

(b) We use $C \propto \kappa$. The new capacitance is

$$C' = C(\kappa/\kappa_{\text{air}}) = (50 \text{ pf})(5.6/1.0) = 2.8\times10^2 \text{ pF}.$$

43. The capacitance with the dielectric in place is given by $C = \kappa C_0$, where C_0 is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4\times10^{-6} \text{ J})}{(7.4\times10^{-12} \text{ F})(652 \text{ V})^2} = 4.7.$$

According to Table 25-1, you should use Pyrex.

44. (a) We use Eq. 25-14:

$$C = 2\pi\varepsilon_0\kappa\frac{L}{\ln(b/a)} = \frac{(4.7)(0.15 \text{ m})}{2\left(8.99\times10^9 \,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\ln(3.8 \text{ cm}/3.6 \text{ cm})} = 0.73 \text{ nF}.$$

(b) The breakdown potential is (14 kV/mm) (3.8 cm – 3.6 cm) = 28 kV.

45. Using Eq. 25-29, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200\times10^3 \text{ N/C}$$

which yields $q = 3.3 \times 10^{-7}$ C. Eq. 25-21 and Eq. 25-27 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6\times10^{-5} \text{ J} .$$

46. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know C_1 and C_2. From Eq. 25-9,

$$C_2 = \frac{\varepsilon_0 A}{d} = 2.21 \times 10^{-11} \text{ F} ,$$

and from Eq. 25-27,

$$C_1 = \frac{\kappa\varepsilon_0 A}{d} = 6.64 \times 10^{-11} \text{ F} .$$

This leads to

$$q_1 = C_1V_1 = 8.00 \times 10^{-10} \text{ C},\; q_2 = C_2V_2 = 2.66 \times 10^{-10} \text{ C}.$$

The addition of these gives the desired result: $q_{\text{tot}} = 1.06 \times 10^{-9}$ C. Alternatively, the circuit could be reduced to find the q_{tot}.

47. **THINK** Dielectric strength is the maximum value of the electric field a dielectric material can tolerate without breakdown.

EXPRESS The capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation. The electric field between the plates is given by $E = V/d$, where V is the potential difference between the plates. Thus, $d = V/E$ and $C = \kappa\varepsilon_0 AE/V$. Therefore, we find the plate area to be

$$A = \frac{CV}{\kappa\varepsilon_0 E}.$$

ANALYZE For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0\times10^{-8}\,\text{F})(4.0\times10^{3}\,\text{V})}{2.8(8.85\times10^{-12}\,\text{F/m})(18\times10^{6}\ \text{V/m})} = 0.63\ \text{m}^2.$$

LEARN If the area is smaller than the minimum value found above, then electric breakdown occurs and the dielectric is no longer insulating and will start to conduct.

48. The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constants κ_1 and κ_2, respectively. Thus, (in SI units),

$$C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1+\kappa_2}{2}\right)$$
$$= \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.56\times10^{-4}\ \text{m}^2)}{5.56\times10^{-3}\ \text{m}}\left(\frac{7.00+12.00}{2}\right) = 8.41\times10^{-12}\ \text{F}.$$

49. We assume there is charge q on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1 \varepsilon_0 A},$$

where A is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2 \varepsilon_0 A}.$$

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\varepsilon_0 A}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right) = \frac{qd}{2\varepsilon_0 A}\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

so

$$C = \frac{q}{V} = \frac{2\varepsilon_0 A}{d}\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$

This expression is exactly the same as that for C_{eq} of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2. Each has plate area A and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1 \varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area A, plate separation d, and dielectric constant κ_1.

With $A = 7.89\times10^{-4}\,\text{m}^2$, $d = 4.62\times10^{-3}\,\text{m}$, $\kappa_1 = 11.0$, and $\kappa_2 = 12.0$, the capacitance is

$$C = \frac{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(7.89\times10^{-4}\ \text{m}^2)}{4.62\times10^{-3}\ \text{m}}\frac{(11.0)(12.0)}{11.0+12.0} = 1.73\times10^{-11}\,\text{F}.$$

50. Let

$$C_1 = \varepsilon_0(A/2)\kappa_1/2d = \varepsilon_0 A\kappa_1/4d,$$
$$C_2 = \varepsilon_0(A/2)\kappa_2/d = \varepsilon_0 A\kappa_2/2d,$$
$$C_3 = \varepsilon_0 A\kappa_3/2d.$$

Note that C_2 and C_3 are effectively connected in series, while C_1 is effectively connected in parallel with the C_2-C_3 combination. Thus,

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A\kappa_1}{4d} + \frac{(\varepsilon_0\, A/d)(\kappa_2/2)(\kappa_3/2)}{\kappa_2/2 + \kappa_3/2} = \frac{\varepsilon_0 A}{4d}\left(\kappa_1 + \frac{2\kappa_2\kappa_3}{\kappa_2 + \kappa_3}\right).$$

With $A = 1.05\times10^{-3}\,\text{m}^2$, $d = 3.56\times10^{-3}\,\text{m}$, $\kappa_1 = 21.0$, $\kappa_2 = 42.0$ and $\kappa_3 = 58.0$, we find the capacitance to be

$$C = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.05\times10^{-3}\ \text{m}^2)}{4(3.56\times10^{-3}\ \text{m})}\left(21.0 + \frac{2(42.0)(58.0)}{42.0+58.0}\right) = 4.55\times10^{-11}\,\text{F}.$$

51. **THINK** We have a parallel-plate capacitor, so the capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation.

EXPRESS The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. Since the separation can be written as $d = \kappa\varepsilon_0 A / C$, we have $E = VC / \kappa\varepsilon_0 A$. The free charge on the plates is $q_f = CV$.

ANALYZE (a) Substituting the values given, we find the magnitude of the field strength to be

$$E = \frac{VC}{\kappa\varepsilon_0 A} = \frac{(50\ \text{V})(100\times10^{-12}\ \text{F})}{5.4(8.85\times10^{-12}\ \text{F/m})(100\times10^{-4}\ \text{m}^2)} = 1.0\times10^4\ \text{V/m}.$$

(b) Similarly, we have $q_f = CV = (100 \times 10^{-12}\ \text{F})(50\ \text{V}) = 5.0 \times 10^{-9}$ C.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 AE = 5.0\times10^{-9}\,\text{C} - (8.85\times10^{-12}\ \text{F/m})(100\times10^{-4}\,\text{m}^2)(1.0\times10^4\ \text{V/m}) \\ &= 4.1\times10^{-9}\,\text{C} = 4.1\,\text{nC}. \end{aligned}$$

LEARN An alternative way to calculate the induced charge is to apply Eq. 25-35:

$$q_i = q_f\left(1 - \frac{1}{\kappa}\right) = (5.0\,\text{nC})\left(1 - \frac{1}{5.4}\right) = 4.1\,\text{nC}.$$

Note that there's no induced charge ($q_i = 0$) in the absence of dielectric ($\kappa = 1$).

52. (a) The electric field E_1 in the free space between the two plates is $E_1 = q/\varepsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\kappa\varepsilon_0 A$. Thus,

$$V_0 = E_1(d - b) + E_2 b = \left(\frac{q}{\varepsilon_0 A}\right)\left(d - b + \frac{b}{\kappa}\right),$$

and the capacitance is

$$C = \frac{q}{V_0} = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(115\times10^{-4}\,\text{m}^2)(2.61)}{(2.61)(0.0124\,\text{m} - 0.00780\,\text{m}) + (0.00780\,\text{m})} = 13.4\,\text{pF}.$$

(b) $q = CV = (13.4 \times 10^{-12}\ \text{F})(85.5\ \text{V}) = 1.15\ \text{nC}$.

(c) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15\times10^{-9}\ \text{C}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(115\times10^{-4}\ \text{m}^2\right)} = 1.13\times10^4\ \text{N/C}.$$

(d) Using Eq. 25-34, we obtain

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13\times10^4\ \text{N/C}}{2.61} = 4.33\times10^3\ \text{N/C}.$$

53. (a) Initially, the capacitance is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.12\ \text{m}^2)}{1.2\times10^{-2}\ \text{m}} = 89\ \text{pF}.$$

(b) Working through Sample Problem 25.06 — "Dielectric partially filling the gap in a capacitor" algebraically, we find:

$$C = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.12\,\text{m}^2)(4.8)}{(4.8)(1.2-0.40)(10^{-2}\ \text{m})+(4.0\times10^{-3}\ \text{m})} = 1.2\times10^2\ \text{pF}.$$

(c) Before the insertion, $q = C_0V$ (89 pF)(120 V) = 11 nC.

(d) Since the battery is disconnected, q will remain the same after the insertion of the slab, with q = 11 nC.

(e) $E = q/\varepsilon_0 A = 11\times10^{-9}\ \text{C}/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.12\ \text{m}^2) = 10\ \text{kV/m}.$

(f) $E' = E/\kappa$ = (10 kV/m)/4.8 = 2.1 kV/m.

(g) The potential difference across the plates is

$V = E(d-b) + E'b$ = (10 kV/m)(0.012 m – 0.0040 m)+ (2.1 kV/m)(0.40 × 10^{-3} m) = 88 V.

(h) The work done is

$$W_{\text{ext}} = \Delta U = \frac{q^2}{2}\left(\frac{1}{C}-\frac{1}{C_0}\right) = \frac{(11\times10^{-9}\ \text{C})^2}{2}\left(\frac{1}{89\times10^{-12}\ \text{F}}-\frac{1}{120\times10^{-12}\ \text{F}}\right) = -1.7\times10^{-7}\ \text{J}.$$

54. (a) We apply Gauss's law with dielectric: $q/\varepsilon_0 = \kappa EA$, and solve for κ:

$$\kappa = \frac{q}{\varepsilon_0 EA} = \frac{8.9\times10^{-7}\ \text{C}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.4\times10^{-6}\ \text{V/m}\right)\left(100\times10^{-4}\,\text{m}^2\right)} = 7.2.$$

(b) The charge induced is $q' = q\left(1-\frac{1}{\kappa}\right) = \left(8.9\times10^{-7}\ \text{C}\right)\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\ \text{C}.$

55. (a) According to Eq. 25-17 the capacitance of an air-filled spherical capacitor is given by

$$C_0 = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right).$$

When the dielectric is inserted between the plates the capacitance is greater by a factor of the dielectric constant κ. Consequently, the new capacitance is

$$C = 4\pi\kappa\varepsilon_0\left(\frac{ab}{b-a}\right) = \frac{23.5}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\cdot\frac{(0.0120\ \text{m})(0.0170\ \text{m})}{0.0170\ \text{m}-0.0120\ \text{m}} = 0.107\ \text{nF}.$$

(b) The charge on the positive plate is $q = CV = (0.107\ \text{nF})(73.0\ \text{V}) = 7.79\ \text{nC}.$

(c) Let the charge on the inner conductor be $-q$. Immediately adjacent to it is the induced charge q'. Since the electric field is less by a factor $1/\kappa$ than the field when no dielectric is present, then $-q + q' = -q/\kappa$. Thus,

$$q' = \frac{\kappa-1}{\kappa}q = 4\pi(\kappa-1)\varepsilon_0\frac{ab}{b-a}V = \left(\frac{23.5-1.00}{23.5}\right)(7.79\ \text{nC}) = 7.45\ \text{nC}.$$

56. (a) The potential across C_1 is 10 V, so the charge on it is

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 100\ \mu\text{C}.$$

(b) Reducing the right portion of the circuit produces an equivalence equal to 6.00 μF, with 10.0 V across it. Thus, a charge of 60.0 μC is on it, and consequently also on the bottom right capacitor. The bottom right capacitor has, as a result, a potential across it equal to

$$V = \frac{q}{C} = \frac{60\ \mu\text{C}}{10\ \mu\text{F}} = 6.00\ \text{V}$$

which leaves 10.0 V – 6.00 V = 4.00 V across the group of capacitors in the upper right portion of the circuit. Inspection of the arrangement (and capacitance values) of that group reveals that this 4.00 V must be equally divided by C_2 and the capacitor directly below it (in series with it). Therefore, with 2.00 V across C_2 we find

$$q_2 = C_2V_2 = (10.0\ \mu\text{F})(2.00\ \text{V}) = 20.0\ \mu\text{C}.$$

57. **THINK** Figure 25-51 depicts a system of capacitors. The pair C_3 and C_4 are in parallel.

EXPRESS Since C_3 and C_4 are in parallel, we replace them with an equivalent capacitance $C_{34} = C_3 + C_4 = 30\ \mu\text{F}$. Now, C_1, C_2, and C_{34} are in series, and all are numerically 30 μF, we observe that each has one-third the battery voltage across it. Hence, 3.0 V is across C_4.

ANALYZE The charge on capacitor 4 is $q_4 = C_4V_4 = (15\ \mu\text{F})(3.0\ \text{V}) = 45\ \mu\text{C}$.

LEARN Alternatively, one may show that the equivalent capacitance of the arrangement is given by

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{34}} = \frac{1}{30\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} = \frac{1}{10\ \mu\text{F}}$$

or $C_{1234} = 10\ \mu\text{F}$. Thus, the charge across C_1, C_2, and C_{34} are

$$q_1 = q_2 = q_{34} = q_{1234} = C_{1234}V = (10\ \mu\text{F})(9.0\ \text{V}) = 90\ \text{nC}.$$

Now, since C_3 and C_4 are in parallel, and $C_3 = C_4$, the charge on C_4 (as well as on C_3) is $q_3 = q_4 = q_{34}/2 = (90\ \mu\text{F})/2 = 45\ \mu\text{F}$.

58. (a) Here D is not attached to anything, so that the 6C and 4C capacitors are in series (equivalent to 2.4C). This is then in parallel with the 2C capacitor, which produces an equivalence of 4.4C. Finally the 4.4C is in series with C and we obtain

$$C_{\text{eq}} = \frac{(C)(4.4C)}{C + 4.4C} = 0.82C = 0.82(50\ \mu\text{F}) = 41\ \mu\text{F}$$

where we have used the fact that $C = 50\ \mu\text{F}$.

(b) Now, B is the point that is not attached to anything, so that the 6C and 2C capacitors are now in series (equivalent to 1.5C), which is then in parallel with the 4C capacitor (and thus equivalent to 5.5C). The 5.5C is then in series with the C capacitor; consequently,

$$C_{\text{eq}} = \frac{(C)(5.5C)}{C + 5.5C} = 0.85C = 42\ \mu\text{F}\ .$$

59. The pair C_1 and C_2 are in parallel, as are the pair C_3 and C_4; they reduce to equivalent values 6.0 μF and 3.0 μF, respectively. These are now in series and reduce to 2.0 μF,

across which we have the battery voltage. Consequently, the charge on the 2.0 μF equivalence is (2.0 μF)(12 V) = 24 μC. This charge on the 3.0 μF equivalence (of C_3 and C_4) has a voltage of

$$V = \frac{q}{C} = \frac{24\ \mu\text{C}}{3\ \mu\text{F}} = 8.0\ \text{V}.$$

Finally, this voltage on capacitor C_4 produces a charge (2.0 μF)(8.0 V) = 16 μC.

60. (a) Equation 25-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(200\times10^{-12}\ \text{F}\right)\left(7.0\times10^3\ \text{V}\right)^2 = 4.9\times10^{-3}\ \text{J}.$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.

61. Initially the capacitors C_1, C_2, and C_3 form a series combination equivalent to a single capacitor, which we denote C_{123}. Solving the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_1C_2 + C_2C_3 + C_1C_3}{C_1C_2C_3},$$

we obtain C_{123} = 2.40 μF. With V = 12.0 V, we then obtain $q = C_{123}V$ = 28.8 μC. In the final situation, C_2 and C_4 are in parallel and are thus effectively equivalent to $C_{24} = 12.0\ \mu\text{F}$. Similar to the previous computation, we use

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_{24}} + \frac{1}{C_3} = \frac{C_1C_{24} + C_{24}C_3 + C_1C_3}{C_1C_{24}C_3}$$

and find C_{1234} = 3.00 μF. Therefore, the final charge is $q = C_{1234}V$ = 36.0 μC.

(a) This represents a change (relative to the initial charge) of Δq = 7.20 μC.

(b) The capacitor C_{24} which we imagined to replace the parallel pair C_2 and C_4, is in series with C_1 and C_3 and thus also has the final charge q =36.0 μC found above. The voltage across C_{24} would be

$$V_{24} = \frac{q}{C_{24}} = \frac{36.0\ \mu\text{C}}{12.0\ \mu\text{F}} = 3.00\ \text{V}.$$

This is the same voltage across each of the parallel pairs. In particular, V_4 = 3.00 V implies that $q_4 = C_4 V_4$ = 18.0 μC.

(c) The battery supplies charges only to the plates where it is connected. The charges on the rest of the plates are due to electron transfers between them, in accord with the new distribution of voltages across the capacitors. So, the battery does not directly supply the charge on capacitor 4.

62. In series, their equivalent capacitance (and thus their total energy stored) is smaller than either one individually (by Eq. 25-20). In parallel, their equivalent capacitance (and thus their total energy stored) is larger than either one individually (by Eq. 25-19). Thus, the middle two values quoted in the problem must correspond to the individual capacitors. We use Eq. 25-22 and find

(a) $100\ \mu\text{J} = \frac{1}{2} C_1 (10\text{ V})^2 \Rightarrow C_1 = 2.0\ \mu\text{F}$;

(b) $300\ \mu\text{J} = \frac{1}{2} C_2 (10\text{ V})^2 \Rightarrow C_2 = 6.0\ \mu\text{F}$.

63. Initially, the total equivalent capacitance is $C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 3.0\ \mu\text{F}$, and the charge on the positive plate of each one is $(3.0\ \mu\text{F})(10\text{ V}) = 30\ \mu\text{C}$. Next, the capacitor (call is C_1) is squeezed as described in the problem, with the effect that the new value of C_1 is 12 μF (see Eq. 25-9). The new total equivalent capacitance then becomes

$$C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 4.0\ \mu\text{F},$$

and the new charge on the positive plate of each one is $(4.0\ \mu\text{F})(10\text{ V}) = 40\ \mu\text{C}$.

(a) Thus we see that the charge transferred from the battery as a result of the squeezing is $40\ \mu\text{C} - 30\ \mu\text{C} = 10\ \mu\text{C}$.

(b) The total increase in positive charge (on the respective positive plates) stored on the capacitors is twice the value found in part (a) (since we are dealing with two capacitors in series): 20 μC.

64. (a) We reduce the parallel group C_2, C_3 and C_4, and the parallel pair C_5 and C_6, obtaining equivalent values $C' = 12\ \mu\text{F}$ and $C'' = 12\ \mu\text{F}$, respectively. We then reduce the series group C_1, C' and C'' to obtain an equivalent capacitance of $C_{eq} = 3\ \mu\text{F}$ hooked to the battery. Thus, the charge stored in the system is $q_{sys} = C_{eq}V_{bat} = 36\ \mu\text{C}$.

(b) Since $q_{sys} = q_1$, then the voltage across C_1 is

$$V_1 = \frac{q_1}{C_1} = \frac{36\ \mu\text{C}}{6.0\ \mu\text{F}} = 6.0\text{ V}.$$

The voltage across the series-pair C' and C'' is consequently $V_{bat} - V_1 = 6.0$ V. Since $C' = C''$, we infer $V' = V'' = 6.0/2 = 3.0$ V, which, in turn, is equal to V_4, the potential across C_4. Therefore,

$$q_4 = C_4V_4 = (4.0\ \mu\text{F})(3.0\text{ V}) = 12\ \mu\text{C}.$$

65. **THINK** We may think of the arrangement as two capacitors connected in series.

EXPRESS Let the capacitances be C_1 and C_2, with the former filled with the $\kappa_1 = 3.00$ material and the latter with the $\kappa_2 = 4.00$ material. Upon using Eq. 25-9, Eq. 25-27, and reducing C_1 and C_2 to an equivalent capacitance, we have

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\kappa_1 \varepsilon_0 A/d} + \frac{1}{\kappa_2 \varepsilon_0 A/d} = \left(\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2}\right)\frac{d}{\varepsilon_0 A}$$

or $C_{eq} = \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\frac{\varepsilon_0 A}{d}$. The charge stored on the capacitor is $q = C_{eq}V$.

ANALYZE Substituting the values given, we find

$$C_{eq} = \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\frac{\varepsilon_0 A}{d} = 1.52 \times 10^{-10} \text{ F},$$

Therefore, $q = C_{eq}V = 1.06 \times 10^{-9}$ C.

LEARN In the limit where $\kappa_1 = \kappa_2 = \kappa$, our expression for C_{eq} becomes $C_{eq} = \frac{\kappa \varepsilon_0 A}{2d}$, where $2d$ is the plate separation.

66. We first need to find an expression for the energy stored in a cylinder of radius R and length L, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where E is the magnitude of the electric field at that point. If q is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance r from the cylinder axis is given by (see Eq. 25-12)

$$E = \frac{q}{2\pi\varepsilon_0 L r},$$

and the energy density at that point is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2 \varepsilon_0 L^2 r^2}.$$

The corresponding energy in the cylinder is the volume integral $U_R = \int u\, d\mathcal{V}$. Now, $d\mathcal{V} = 2\pi r L\, dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2 \varepsilon_0 L^2 r^2} 2\pi r L\, dr = \frac{q^2}{4\pi\varepsilon_0 L}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L}\ln\left(\frac{R}{a}\right).$$

To find an expression for the total energy stored in the capacitor, we replace R with b:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right).$$

We want the ratio U_R/U_b to be 1/2, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln(\sqrt{b/a})$, $\ln(R/a) = \ln(\sqrt{b/a})$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

67. (a) The equivalent capacitance is $C_{\text{eq}} = \dfrac{C_1C_2}{C_1+C_2} = \dfrac{(6.00\ \mu\text{F})(4.00\mu\text{F})}{6.00\ \mu\text{F} + 4.00\ \mu\text{F}} = 2.40\ \mu\text{F}$.

(b) $q_1 = C_{\text{eq}}V = (2.40\ \mu\text{F})(200\ \text{V}) = 4.80 \times 10^{-4}$ C.

(c) $V_1 = q_1/C_1 = 4.80 \times 10^{-4}\ \text{C}/6.00\ \mu\text{F} = 80.0$ V.

(d) $q_2 = q_1 = 4.80 \times 10^{-4}$ C.

(e) $V_2 = V - V_1 = 200\ \text{V} - 80.0\ \text{V} = 120$ V.

68. (a) Now $C_{\text{eq}} = C_1 + C_2 = 6.00\ \mu\text{F} + 4.00\ \mu\text{F} = 10.0\ \mu\text{F}$.

(b) $q_1 = C_1V = (6.00\ \mu\text{F})(200\ \text{V}) = 1.20 \times 10^{-3}$ C.

(c) $V_1 = 200$ V.

(d) $q_2 = C_2V = (4.00\ \mu\text{F})(200\ \text{V}) = 8.00 \times 10^{-4}$ C.

(e) $V_2 = V_1 = 200$ V.

69. We use $U = \frac{1}{2}CV^2$. As V is increased by ΔV, the energy stored in the capacitor increases correspondingly from U to $U + \Delta U$: $U + \Delta U = \frac{1}{2}C(V + \Delta V)^2$. Thus, $(1 + \Delta V/V)^2 = 1 + \Delta U/U$, or

$$\frac{\Delta V}{V} = \sqrt{1 + \frac{\Delta U}{U}} - 1 = \sqrt{1 + 10\%} - 1 = 4.9\%\ .$$

70. (a) The length d is effectively shortened by b so $C' = \varepsilon_0 A/(d - b) = 0.708$ pF.

(b) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'}=\frac{q^2/2C}{q^2/2C'}=\frac{C'}{C}=\frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d}=\frac{d}{d-b}=\frac{5.00}{5.00-2.00}=1.67.$$

(c) The work done is

$$W=\Delta U=U'-U=\frac{q^2}{2}\left(\frac{1}{C'}-\frac{1}{C}\right)=\frac{q^2}{2\varepsilon_0 A}(d-b-d)=-\frac{q^2 b}{2\varepsilon_0 A}=-5.44\text{ J}.$$

(d) Since $W < 0$, the slab is sucked in.

71. (a) $C' = \varepsilon_0 A/(d - b) = 0.708$ pF, the same as part (a) in Problem 25-70.

(b) The ratio of the stored energy is now

$$\frac{U}{U'}=\frac{\frac{1}{2}CV^2}{\frac{1}{2}C'V^2}=\frac{C}{C'}=\frac{\varepsilon_0 A/d}{\varepsilon_0 A/(d-b)}=\frac{d-b}{d}=\frac{5.00-2.00}{5.00}=0.600.$$

(c) The work done is

$$W=\Delta U=U'-U=\frac{1}{2}(C'-C)V^2=\frac{\varepsilon_0 A}{2}\left(\frac{1}{d-b}-\frac{1}{d}\right)V^2=\frac{\varepsilon_0 AbV^2}{2d(d-b)}=1.02\times10^{-9}\text{ J}.$$

(d) In Problem 25-70 where the capacitor is disconnected from the battery and the slab is sucked in, F is certainly given by $-dU/dx$. However, that relation does not hold when the battery is left attached because the force on the slab is not conservative. The charge distribution in the slab causes the slab to be sucked into the gap by the charge distribution on the plates. This action causes an increase in the potential energy stored by the battery in the capacitor.

72. (a) The equivalent capacitance is $C_{eq} = C_1C_2/(C_1 + C_2)$. Thus the charge q on each capacitor is

$$q=q_1=q_2=C_{eq}V=\frac{C_1C_2V}{C_1+C_2}=\frac{(2.00\mu\text{F})(8.00\mu\text{F})(300\text{V})}{2.00\mu\text{F}+8.00\mu\text{F}}=4.80\times10^{-4}\text{C}.$$

(b) The potential difference is $V_1 = q/C_1 = 4.80 \times 10^{-4}$ C/2.0 μF = 240 V.

(c) As noted in part (a), $q_2=q_1=4.80\times10^{-4}\text{C}.$

(d) $V_2 = V - V_1 = 300$ V – 240 V = 60.0 V.

Now we have $q'_1/C_1 = q'_2/C_2 = V'$ (V' being the new potential difference across each capacitor) and $q'_1 + q'_2 = 2q$. We solve for q'_1, q'_2 and V:

(e) $q_1' = \dfrac{2C_1 q}{C_1+C_2} = \dfrac{2(2.00\,\mu\text{F})(4.80\times10^{-4}C)}{2.00\,\mu\text{F}+8.00\,\mu\text{F}} = 1.92\times10^{-4}\,\text{C}.$

(f) $V_1' = \dfrac{q_1'}{C_1} = \dfrac{1.92\times10^{-4}\,C}{2.00\,\mu\text{F}} = 96.0\,\text{V}.$

(g) $q_2' = 2q - q_1 = 7.68\times10^{-4}\,C.$

(h) $V_2' = V_1' = 96.0\,\text{V}.$

(i) In this circumstance, the capacitors will simply discharge themselves, leaving $q_1 = 0$,

(j) $V_1 = 0$,

(k) $q_2 = 0$,

(l) and $V_2 = V_1 = 0$.

73. The voltage across capacitor 1 is

$$V_1 = \frac{q_1}{C_1} = \frac{30\,\mu\text{C}}{10\,\mu\text{F}} = 3.0\ \text{V} .$$

Since $V_1 = V_2$, the total charge on capacitor 2 is

$$q_2 = C_2V_2 = (20\,\mu\text{F})(2\ \text{V}) = 60\,\mu\text{C} ,$$

which means a total of 90 μC of charge is on the pair of capacitors C_1 and C_2. This implies there is a total of 90 μC of charge also on the C_3 and C_4 pair. Since $C_3 = C_4$, the charge divides equally between them, so $q_3 = q_4 = 45\ \mu$C. Thus, the voltage across capacitor 3 is

$$V_3 = \frac{q_3}{C_3} = \frac{45\,\mu\text{C}}{20\,\mu\text{F}} = 2.3\,\text{V} .$$

Therefore, $|V_A - V_B| = V_1 + V_3 = 5.3$ V.

74. We use $C = \varepsilon_0 \kappa A/d \propto \kappa/d$. To maximize C we need to choose the material with the greatest value of κ/d. It follows that the mica sheet should be chosen.

75. We cannot expect simple energy conservation to hold since energy is presumably dissipated either as heat in the hookup wires or as radio waves while the charge oscillates in the course of the system "settling down" to its final state (of having 40 V across the parallel pair of capacitors C and 60 μF). We do expect charge to be conserved. Thus, if Q is the charge originally stored on C and q_1, q_2 are the charges on the parallel pair after "settling down," then

$$Q = q_1 + q_2 \quad \Rightarrow \quad C(100\,\text{V}) = C(40\,\text{V}) + (60\,\mu\text{F})(40\,\text{V})$$

which leads to the solution $C = 40\ \mu$F.

76. One way to approach this is to note that since they are identical, the voltage is evenly divided between them. That is, the voltage across each capacitor is $V = (10/n)$ volt. With $C = 2.0 \times 10^{-6}$ F, the electric energy stored by each capacitor is $\frac{1}{2}CV^2$. The total energy stored by the capacitors is n times that value, and the problem requires the total be equal to 25×10^{-6} J. Thus,

$$\frac{n}{2}(2.0 \times 10^{-6})\left(\frac{10}{n}\right)^2 = 25 \times 10^{-6},$$

which leads to $n = 4$.

77. **THINK** We have two parallel-plate capacitors that are connected in parallel. They both have the same plate separation and same potential difference across their plates.

EXPRESS The magnitude of the electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The surface charge density on the plate is $\sigma = q/A$.

ANALYZE (a) With $d = 0.00300$ m and $V = 600$ V, we have

$$E_A = \frac{V}{d} = \frac{600\ \text{V}}{3.00\times10^{-3}\ \text{m}} = 2.00\times10^5\ \text{V/m}.$$

(b) Since $d = 0.00300$ m and $V = 600$ V in capacitor B as well, $E_B = 2.00\times10^5$ V/m.

(c) For the air-filled capacitor, Eq. 25-4 leads to

$$\sigma_A = \frac{q_A}{A} = \frac{C_A V}{A} = \frac{(\varepsilon_0 A/d)V}{A} = \frac{\varepsilon_0 V}{d} = \varepsilon_0 E_A = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00\times10^5\ \text{V/m})$$
$$= 1.77\times10^{-6}\ \text{C/m}^2.$$

(d) For the dielectric-filled capacitor, we use Eq. 25-29:

$$\sigma_B = \kappa\varepsilon_0 E_B = (2.60)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00\times10^5\ \text{V/m}) = 4.60\times10^{-6}\ \text{C/m}^2.$$

(e) Although the discussion in Section 25-8 of the textbook is in terms of the charge being held fixed (while a dielectric is inserted), it is readily adapted to this situation (where comparison is made of two capacitors that have the same *voltage* and are identical except for the fact that one has a dielectric). The fact that capacitor *B* has a relatively large charge but only produces the field that *A* produces (with its smaller charge) is in line with the point being made (in the text) with Eq. 25-34 and in the material that follows. Adapting Eq. 25-35 to this problem, we see that the difference in charge densities between parts (c) and (d) is due, in part, to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma_{\text{ind}} = \sigma_A - \sigma_B = (1.77\times10^{-6}\ \text{C/m}^2) - (4.60\times10^{-6}\ \text{C/m}^2) = -2.83\times10^{-6}\ \text{C/m}^2\ .$$

LEARN We note that the electric field in capacitor *B* is produced by both the charge on the plates ($\sigma_B A$) and the induced charges ($\sigma_{\text{ind}} A$), while the field in capacitor A is produced by the charge on the plates alone ($\sigma_A A$). Since $E_A = E_B$, we have $\sigma_A = \sigma_B + \sigma_{\text{ind}}$, or $\sigma_{\text{ind}} = \sigma_A - \sigma_B$.

78. (a) Put five such capacitors in series. Then, the equivalent capacitance is 2.0 μF/5 = 0.40 μF. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now C_{eq} = 3(0.40 μF) = 1.2 μF. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

79. (a) For a capacitor with surface area *A* and plate separation *x* its capacitance is given by $C_0 = \varepsilon_0 A/x$. The energy stored in the capacitor can be written as

$$U = \frac{q^2}{2C} = \frac{q^2}{2(\varepsilon_0 A/x)} = \frac{q^2 x}{2\varepsilon_0 A}.$$

The change in energy if the separation between plates increases to $x + dx$ is

$$dU = \frac{q^2}{2\varepsilon_0 A}dx.$$

Thus, the force between the plates is

$$F = -\frac{dU}{dx} = -\frac{q^2}{2\varepsilon_0 A}.$$

The negative sign means that the force between the plates is attractive.

(b) The magnitude of the electrostatic stress is

$$\frac{|F|}{A}=\frac{q^2}{2\varepsilon_0 A^2}=\frac{\sigma^2}{2\varepsilon_0}=\frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2=\frac{1}{2}\varepsilon_0 E^2$$

where $E=\sigma/\varepsilon_0$ is the magnitude of the electric field in the region between the plates.

80. The energy initially stored in one capacitor is $U_0=q_0^2/2C=4.00$ J. After a second capacitor is connected to it in parallel, with $q_1=q_2=q_0/2$, the energy stored in the first capacitor becomes

$$U_1=\frac{q_1^2}{2C}=\frac{(q_0/2)^2}{2C}=\frac{U_0}{4}=1.00\text{ J}$$

which is the same as that stored in the second capacitor. Thus, the total energy is

$$U=U_1+U_2=\frac{U_0}{2}=2.00\text{ J}.$$

(b) The wires connecting the capacitors have resistance, so some energy is converted to thermal energy in the wires, as well as electromagnetic radiation.

Chapter 26

1. (a) The charge that passes through any cross section is the product of the current and time. Since t = 4.0 min = (4.0 min)(60 s/min) = 240 s,

$$q = it = (5.0\text{ A})(240\text{ s}) = 1.2\times 10^3\text{ C}.$$

(b) The number of electrons N is given by $q = Ne$, where e is the magnitude of the charge on an electron. Thus,

$$N = q/e = (1200\text{ C})/(1.60\times 10^{-19}\text{ C}) = 7.5\times 10^{21}.$$

2. Suppose the charge on the sphere increases by Δq in time Δt. Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\varepsilon_0 r},$$

where r is the radius of the sphere. This means $\Delta q = 4\pi\varepsilon_0 r\,\Delta V$. Now, $\Delta q = (i_{in} - i_{out})\,\Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus,

$$\Delta t = \frac{\Delta q}{i_{in}-i_{out}} = \frac{4\pi\varepsilon_0 r\,\Delta V}{i_{in}-i_{out}} = \frac{(0.10\text{ m})(1000\text{ V})}{(8.99\times10^9\text{ F/m})(1.0000020\text{ A}-1.0000000\text{ A})}$$
$$= 5.6\times10^{-3}\text{ s}.$$

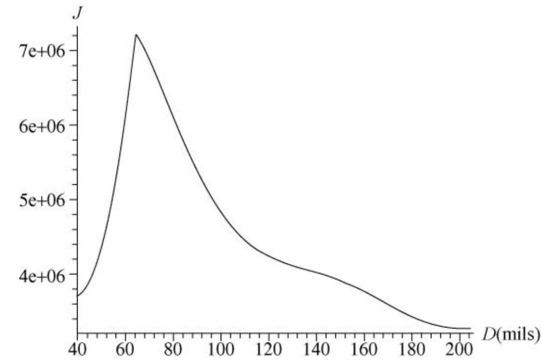
3. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using σ for the charge per unit area and w for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma v w$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100\times10^{-6}\text{ A}}{(30\text{ m/s})(50\times10^{-2}\text{ m})} = 6.7\times10^{-6}\text{ C/m}^2.$$

4. We express the magnitude of the current density vector in SI units by converting the diameter values in mils to inches (by dividing by 1000) and then converting to meters (by multiplying by 0.0254) and finally using

$$J = \frac{i}{A} = \frac{i}{\pi R^2} = \frac{4i}{\pi D^2}.$$

For example, the gauge 14 wire with D = 64 mil = 0.0016 m is found to have a (maximum safe) current density of $J = 7.2 \times 10^6$ A/m^2. In fact, this is the wire with the largest value of J allowed by the given data. The values of J in SI units are plotted below as a function of their diameters in mils.



5. **THINK** The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles.

EXPRESS In vector form, we have (see Eq. 26-7) $\vec{J} = nq\vec{v}_d$. Current density $\vec{J}$ is related to the current i by (see Eq. 26-4): $i = \int \vec{J} \cdot d\vec{A}$.

ANALYZE (a) The particle concentration is $n = 2.0 \times 10^8/\text{cm}^3 = 2.0 \times 10^{14}\ \text{m}^{-3}$, the charge is

$$q = 2e = 2(1.60 \times 10^{-19}\ \text{C}) = 3.20 \times 10^{-19}\ \text{C},$$

and the drift speed is 1.0×10^5 m/s. Thus, we find the current density to be

$$J = \left(2\times10^{14}\ /\ \text{m}\right)\left(3.2\times10^{-19}\ \text{C}\right)\left(1.0\times10^{5}\ \text{m/s}\right) = 6.4\ \text{A/m}^2.$$

(b) Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

LEARN That the current density is in the direction of the motion of the *positive* charge carriers means that it is in the opposite direction of the motion of the negatively charged electrons.

6. (a) Circular area depends, of course, on r^2, so the horizontal axis of the graph in Fig. 26-24(b) is effectively the same as the area (enclosed at variable radius values), except for a factor of π. The fact that the current increases linearly in the graph means that $i/A = J$ = constant. Thus, the answer is "yes, the current density is uniform."

(b) We find $i/(\pi r^2) = (0.005 \text{ A})/(\pi \times 4 \times 10^{-6} \text{ m}^2) = 398 \approx 4.0 \times 10^2 \text{ A/m}^2$.

7. The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius (half its thickness). The magnitude of the current density vector is

$$J = i / A = i / \pi r^2,$$

so

$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50 \text{ A}}{\pi\left(440\times10^4 \text{ A/m}^2\right)}} = 1.9\times10^{-4} \text{ m}.$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4} \text{ m}) = 3.8 \times 10^{-4}$ m.

8. (a) The magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{i}{\pi d^2/4} = \frac{4\left(1.2\times10^{-10} \text{ A}\right)}{\pi\left(2.5\times10^{-3} \text{ m}\right)^2} = 2.4\times10^{-5} \text{ A/m}^2.$$

(b) The drift speed of the current-carrying electrons is

$$v_d = \frac{J}{ne} = \frac{2.4\times10^{-5} \text{ A/m}^2}{\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19} \text{ C}\right)} = 1.8\times10^{-15} \text{ m/s}.$$

9. We note that the radial width $\Delta r = 10$ μm is small enough (compared to $r = 1.20$ mm) that we can make the approximation

$$\int Br2\pi r dr \approx Br2\pi r\Delta r$$

Thus, the enclosed current is $2\pi Br^2\Delta r = 18.1\ \mu\text{A}$. Performing the integral gives the same answer.

10. Assuming $\vec{J}$ is directed along the wire (with no radial flow) we integrate, starting with Eq. 26-4,

$$i = \int |\vec{J}|\, dA = \int_{9R/10}^{R} (kr^2) 2\pi r dr = \frac{1}{2}k\pi\left(R^4 - 0.656R^4\right)$$

where $k = 3.0 \times 10^8$ and SI units are understood. Therefore, if $R = 0.00200$ m, we obtain $i = 2.59\times10^{-3}$ A.

11. (a) The current resulting from this non-uniform current density is

$$i=\int_{\text{cylinder}} J_a\,dA=\frac{J_0}{R}\int_0^R r\cdot 2\pi r dr=\frac{2}{3}\pi R^2 J_0=\frac{2}{3}\pi(3.40\times10^{-3}\,\text{m})^2(5.50\times10^4\,\text{A/m}^2)$$
$$=1.33\ \text{A}.$$

(b) In this case,

$$i=\int_{\text{cylinder}} J_b\,dA=\int_0^R J_0\left(1-\frac{r}{R}\right)2\pi r dr=\frac{1}{3}\pi R^2 J_0=\frac{1}{3}\pi(3.40\times10^{-3}\,\text{m})^2(5.50\times10^4\,\text{A/m}^2)$$
$$=0.666\ \text{A}.$$

(c) The result is different from that in part (a) because J_b is higher near the center of the cylinder (where the area is smaller for the same radial interval) and lower outward, resulting in a lower average current density over the cross section and consequently a lower current than that in part (a). So, J_a has its maximum value near the surface of the wire.

12. (a) Since 1 cm^3 = 10^{-6} m^3, the magnitude of the current density vector is

$$J=nev=\left(\frac{8.70}{10^{-6}\,\text{m}^3}\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(470\times10^3\,\text{m/s}\right)=6.54\times10^{-7}\,\text{A/m}^2.$$

(b) Although the total surface area of Earth is $4\pi R_E^2$ (that of a sphere), the area to be used in a computation of how many protons in an approximately unidirectional beam (the solar wind) will be captured by Earth is its projected area. In other words, for the beam, the encounter is with a "target" of circular area πR_E^2. The rate of charge transport implied by the influx of protons is

$$i=AJ=\pi R_E^2 J=\pi\left(6.37\times10^6\,\text{m}\right)^2\left(6.54\times10^{-7}\,\text{A/m}^2\right)=8.34\times10^7\,\text{A}.$$

13. We use $v_d = J/ne = i/Ane$. Thus,

$$t=\frac{L}{v_d}=\frac{L}{i/Ane}=\frac{LAne}{i}=\frac{(0.85\text{m})\left(0.21\times10^{-14}\,\text{m}^2\right)\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\,\text{C}\right)}{300\,\text{A}}$$
$$=8.1\times10^2\,\text{s}=13\,\text{min}.$$

14. Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50\times10^{-3}\ \text{A})(2000\ \Omega) = 100$ V.

15. **THINK** The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire.

EXPRESS Since each turn of wire has length $2\pi r$, where r is the radius of the coil, then

$$L = (250)2\pi r = (250)(2\pi)(0.12 \text{ m}) = 188.5 \text{ m}.$$

If r_w is the radius of the wire itself, then its cross-sectional area is

$$A = \pi r_w^2 = \pi(0.65 \times 10^{-3} \text{ m})^2 = 1.33 \times 10^{-6} \text{ m}^2.$$

According to Table 26-1, the resistivity of copper is $\rho = 1.69\times10^{-8}\,\Omega\cdot\text{m}$.

ANALYZE Thus, the resistance of the copper coil is

$$R = \frac{\rho L}{A} = \frac{\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(188.5 \text{ m}\right)}{1.33\times10^{-6}\text{ m}^2} = 2.4\,\Omega.$$

LEARN Resistance R is the property of an object (depending on quantities such as L and A), while resistivity is a property of the material.

16. We use $R/L = \rho/A = 0.150\ \Omega/\text{km}$.

(a) For copper $J = i/A = (60.0 \text{ A})(0.150\ \Omega/\text{km})/(1.69 \times 10^{-8}\ \Omega\cdot\text{m}) = 5.32 \times 10^5 \text{ A/m}^2$.

(b) We denote the mass densities as ρ_m. For copper,

$$(m/L)_c = (\rho_m A)_c = (8960 \text{ kg/m}^3)\,(1.69 \times 10^{-8}\ \Omega\cdot\text{m})/(0.150\ \Omega/\text{km}) = 1.01 \text{ kg/m}.$$

(c) For aluminum $J = (60.0 \text{ A})(0.150\ \Omega/\text{km})/(2.75 \times 10^{-8}\ \Omega\cdot\text{m}) = 3.27 \times 10^5 \text{ A/m}^2$.

(d) The mass density of aluminum is

$$(m/L)_a = (\rho_m A)_a = (2700 \text{ kg/m}^3)(2.75 \times 10^{-8}\ \Omega\cdot\text{m})/(0.150\ \Omega/\text{km}) = 0.495 \text{ kg/m}.$$

17. We find the conductivity of Nichrome (the reciprocal of its resistivity) as follows:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0 \text{ m})(4.0 \text{ A})}{(2.0 \text{ V})\left(1.0\times10^{-6}\text{ m}^2\right)} = 2.0\times10^6\,/\,\Omega\cdot\text{m}.$$

18. (a) $i = V/R = 23.0 \text{ V}/15.0 \times 10^{-3}\ \Omega = 1.53 \times 10^3 \text{ A}$.

(b) The cross-sectional area is $A = \pi r^2 = \frac{1}{4}\pi D^2$. Thus, the magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4\left(1.53\times10^{-3}\text{ A}\right)}{\pi\left(6.00\times10^{-3}\text{ m}\right)^2} = 5.41\times10^7 \text{ A/m}^2.$$

(c) The resistivity is

$$\rho = \frac{RA}{L} = \frac{(15.0\times10^{-3}\,\Omega)\pi(6.00\times10^{-3}\text{ m})^2}{4(4.00\text{ m})} = 10.6\times10^{-8}\ \Omega\cdot\text{m}.$$

(d) The material is platinum.

19. **THINK** The resistance of the wire is given by $R = \rho L/A$, where ρ is the resistivity of the material, L is the length of the wire, and A is its cross-sectional area.

EXPRESS In this case, the cross-sectional area is

$$A = \pi r^2 = \pi\left(0.50\times10^{-3}\text{ m}\right)^2 = 7.85\times10^{-7}\text{ m}^2.$$

ANALYZE Thus, the resistivity of the wire is

$$\rho = \frac{RA}{L} = \frac{\left(50\times10^{-3}\,\Omega\right)\left(7.85\times10^{-7}\text{ m}^2\right)}{2.0\text{m}} = 2.0\times10^{-8}\,\Omega\cdot\text{m}.$$

LEARN Resistance R is the property of an object (depending on quantities such as L and A), while resistivity is a property of the material itself. The equation $R = \rho L/A$ implies that the larger the cross-sectional area A, the smaller the resistance R.

20. The thickness (diameter) of the wire is denoted by D. We use $R \propto L/A$ (Eq. 26-16) and note that $A = \frac{1}{4}\pi D^2 \propto D^2$. The resistance of the second wire is given by

$$R_2 = R\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right) = R\left(\frac{D_1}{D_2}\right)^2\left(\frac{L_2}{L_1}\right) = R(2)^2\left(\frac{1}{2}\right) = 2R.$$

21. The resistance at operating temperature T is $R = V/i$ = 2.9 V/0.30 A = 9.67 Ω. Thus, from $R - R_0 = R_0\alpha(T - T_0)$, we find

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20°\text{C} + \left(\frac{1}{4.5\times10^{-3}/\text{K}}\right)\left(\frac{9.67\,\Omega}{1.1\,\Omega} - 1\right) = 1.8\times10^3\ °\text{C}.$$

Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. Table 26-1 has been used.

22. Let $r = 2.00$ mm be the radius of the kite string and $t = 0.50$ mm be the thickness of the water layer. The cross-sectional area of the layer of water is

$$A = \pi\left[(r+t)^2 - r^2\right] = \pi[(2.50\times10^{-3}\text{ m})^2 - (2.00\times10^{-3}\text{ m})^2] = 7.07\times10^{-6}\text{ m}^2.$$

Using Eq. 26-16, the resistance of the wet string is

$$R = \frac{\rho L}{A} = \frac{(150\,\Omega\cdot\text{m})(800\text{ m})}{7.07\times10^{-6}\text{ m}^2} = 1.698\times10^{10}\,\Omega.$$

The current through the water layer is

$$i = \frac{V}{R} = \frac{1.60\times10^{8}\,\text{V}}{1.698\times10^{10}\,\Omega} = 9.42\times10^{-3}\,\text{A}.$$

23. We use $J = E/\rho$, where E is the magnitude of the (uniform) electric field in the wire, J is the magnitude of the current density, and ρ is the resistivity of the material. The electric field is given by $E = V/L$, where V is the potential difference along the wire and L is the length of the wire. Thus $J = V/L\rho$ and

$$\rho = \frac{V}{LJ} = \frac{115\text{ V}}{(10\text{ m})(1.4\times10^{8}\text{ A/m}^2)} = 8.2\times10^{-8}\,\Omega\cdot\text{m}.$$

24. (a) Since the material is the same, the resistivity ρ is the same, which implies (by Eq. 26-11) that the electric fields (in the various rods) are directly proportional to their current-densities. Thus, J_1: J_2: J_3 are in the ratio 2.5/4/1.5 (see Fig. 26-25). Now the currents in the rods must be the same (they are "in series") so

$$J_1 A_1 = J_3 A_3\,, \qquad J_2 A_2 = J_3 A_3\,.$$

Since $A = \pi r^2$, this leads (in view of the aforementioned ratios) to

$$4r_2^2 = 1.5r_3^2\,, \qquad 2.5r_1^2 = 1.5r_3^2\,.$$

Thus, with $r_3 = 2$ mm, the latter relation leads to $r_1 = 1.55$ mm.

(b) The $4r_2^2 = 1.5r_3^2$ relation leads to $r_2 = 1.22$ mm.

25. **THINK** The resistance of an object depends on its length and the cross-sectional area.

EXPRESS Since the mass and density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0A_0 = LA$ and

$$A = L_0A_0/L = L_0A_0/3L_0 = A_0/3.$$

ANALYZE The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0,$$

where R_0 is the original resistance. Thus, R = 9(6.0 Ω) = 54 Ω.

LEARN In general, the resistances of two objects made of the same material but different cross-sectional areas and lengths may be related by

$$R_2 = R_1\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right).$$

26. The absolute values of the slopes (for the straight-line segments shown in the graph of Fig. 26-25(b)) are equal to the respective electric field magnitudes. Thus, applying Eq. 26-5 and Eq. 26-13 to the three sections of the resistive strip, we have

$$J_1 = \frac{i}{A} = \sigma_1 E_1 = \sigma_1 (0.50 \times 10^3 \text{ V/m})$$

$$J_2 = \frac{i}{A} = \sigma_2 E_2 = \sigma_2 (4.0 \times 10^3 \text{ V/m})$$

$$J_3 = \frac{i}{A} = \sigma_3 E_3 = \sigma_3 (1.0 \times 10^3 \text{ V/m}) \ .$$

We note that the current densities are the same since the values of i and A are the same (see the problem statement) in the three sections, so $J_1 = J_2 = J_3$.

(a) Thus we see that $\sigma_1 = 2\sigma_3 = 2\,(3.00 \times 10^7 (\Omega\cdot\text{m})^{-1}) = 6.00 \times 10^7\ (\Omega\cdot\text{m})^{-1}$.

(b) Similarly, $\sigma_2 = \sigma_3/4 = (3.00 \times 10^7 (\Omega\cdot\text{m})^{-1})/4 = 7.50 \times 10^6\ (\Omega\cdot\text{m})^{-1}$.

27. **THINK** In this problem we compare the resistances of two conductors that are made of the same materials.

EXPRESS The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2},$$

where r_A is the radius of the conductor. If r_o is the outside diameter of conductor B and r_i is its inside diameter, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$, and its resistance is

$$R_B = \frac{\rho L}{\pi\left(r_o^2 - r_i^2\right)}.$$

ANALYZE The ratio of the resistances is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0.$$

LEARN The resistance R of an object depends on how the electric potential is applied to the object. Also, R depends on the ratio L/A, according to $R = \rho L / A$.

28. The cross-sectional area is $A = \pi r^2 = \pi(0.002\text{ m})^2$. The resistivity from Table 26-1 is $\rho = 1.69 \times 10^{-8}\,\Omega \cdot \text{m}$. Thus, with $L = 3$ m, Ohm's Law leads to $V = iR = i\rho L/A$, or

$$12 \times 10^{-6}\text{ V} = i\,(1.69 \times 10^{-8}\,\Omega \cdot \text{m})(3.0\text{ m})/\,\pi(0.002\text{ m})^2$$

which yields $i = 0.00297$ A or roughly 3.0 mA.

29. First we find the resistance of the copper wire to be

$$R = \frac{\rho L}{A} = \frac{(1.69\times10^{-8}\,\Omega\cdot\text{m})(0.020\text{ m})}{\pi(2.0\times10^{-3}\text{ m})^2} = 2.69\times10^{-5}\,\Omega.$$

With potential difference $V = 3.00\text{ nV}$, the current flowing through the wire is

$$i = \frac{V}{R} = \frac{3.00\times10^{-9}\text{ V}}{2.69\times10^{-5}\,\Omega} = 1.115\times10^{-4}\text{ A}.$$

Therefore, in 3.00 ms, the amount of charge drifting through a cross section is

$$\Delta Q = i\Delta t = (1.115\times10^{-4}\text{ A})(3.00\times10^{-3}\text{ s}) = 3.35\times10^{-7}\text{ C}.$$

30. We use $R \propto L/A$. The diameter of a 22-gauge wire is 1/4 that of a 10-gauge wire. Thus from $R = \rho L/A$ we find the resistance of 25 ft of 22-gauge copper wire to be

$$R = (1.00\,\Omega)(25\text{ ft}/1000\text{ ft})(4)^2 = 0.40\,\Omega.$$

31. (a) The current in each strand is $i = 0.750\text{ A}/125 = 6.00 \times 10^{-3}$ A.

(b) The potential difference is $V = iR = (6.00 \times 10^{-3}\text{ A})\,(2.65 \times 10^{-6}\,\Omega) = 1.59 \times 10^{-8}$ V.

(c) The resistance is $R_{\text{total}} = 2.65 \times 10^{-6}\,\Omega/125 = 2.12 \times 10^{-8}\,\Omega$.

32. We use $J = \sigma E = (n_+ + n_-)ev_d$, which combines Eq. 26-13 and Eq. 26-7.

(a) The magnitude of the current density is

$$J = \sigma E = (2.70 \times 10^{-14} / \Omega \cdot \text{m})\,(120\ \text{V/m}) = 3.24 \times 10^{-12}\ \text{A/m}^2.$$

(b) The drift velocity is

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{\left(2.70\times10^{-14}/\Omega\cdot\text{m}\right)\left(120\ \text{V/m}\right)}{\left[\left(620+550\right)/\text{cm}^3\right]\left(1.60\times10^{-19}\ \text{C}\right)} = 1.73\ \text{cm/s}.$$

33. (a) The current in the block is $i = V/R = 35.8\ \text{V}/935\ \Omega = 3.83 \times 10^{-2}$ A.

(b) The magnitude of current density is

$$J = i/A = (3.83 \times 10^{-2}\ \text{A})/(3.50 \times 10^{-4}\ \text{m}^2) = 109\ \text{A/m}^2.$$

(c) $v_d = J/ne = (109\ \text{A/m}^2)/[(5.33 \times 10^{22}/\text{m}^3)\,(1.60 \times 10^{-19}\ \text{C})] = 1.28 \times 10^{-2}$ m/s.

(d) $E = V/L = 35.8\ \text{V}/0.158\ \text{m} = 227$ V/m.

34. The number density of conduction electrons in copper is $n = 8.49 \times 10^{28}\ /\text{m}^3$. The electric field in section 2 is $(10.0\ \mu\text{V})/(2.00\ \text{m}) = 5.00\ \mu\text{V/m}$. Since $\rho = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$ for copper (see Table 26-1) then Eq. 26-10 leads to a current density vector of magnitude

$$J_2 = (5.00\ \mu\text{V/m})/(1.69 \times 10^{-8}\ \Omega \cdot \text{m}) = 296\ \text{A/m}^2$$

in section 2. Conservation of electric current from section 1 into section 2 implies

$$J_1 A_1 = J_2 A_2 \quad \Rightarrow \quad J_1(4\pi R^2) = J_2(\pi R^2)$$

(see Eq. 26-5). This leads to $J_1 = 74\ \text{A/m}^2$. Now, for the drift speed of conduction-electrons in section 1, Eq. 26-7 immediately yields

$$v_d = \frac{J_1}{ne} = 5.44\times10^{-9}\ \text{m/s}$$

35. (a) The current i is shown in Fig. 26-30 entering the truncated cone at the left end and leaving at the right. This is our choice of positive x direction. We make the assumption that the current density J at each value of x may be found by taking the ratio i/A where $A = \pi r^2$ is the cone's cross-section area at that particular value of x.

The direction of $\vec{J}$ is identical to that shown in the figure for i (our $+x$ direction). Using Eq. 26-11, we then find an expression for the electric field at each value of x, and next find the potential difference V by integrating the field along the x axis, in accordance with the ideas of Chapter 25. Finally, the resistance of the cone is given by $R = V/i$. Thus,

$$J = \frac{i}{\pi r^2} = \frac{E}{\rho}$$

where we must deduce how r depends on x in order to proceed. We note that the radius increases linearly with x, so (with c_1 and c_2 to be determined later) we may write $r = c_1 + c_2 x$.

Choosing the origin at the left end of the truncated cone, the coefficient c_1 is chosen so that $r = a$ (when $x = 0$); therefore, $c_1 = a$. Also, the coefficient c_2 must be chosen so that (at the right end of the truncated cone) we have $r = b$ (when $x = L$); therefore, $c_2 = (b-a)/L$. Our expression, then, becomes

$$r = a + \left(\frac{b-a}{L}\right)x.$$

Substituting this into our previous statement and solving for the field, we find

$$E = \frac{i\rho}{\pi}\left(a + \frac{b-a}{L}x\right)^{-2}.$$

Consequently, the potential difference between the faces of the cone is

$$V = -\int_0^L E\,dx = -\frac{i\rho}{\pi}\int_0^L \left(a + \frac{b-a}{L}x\right)^{-2} dx = \frac{i\rho}{\pi}\frac{L}{b-a}\left(a + \frac{b-a}{L}x\right)^{-1}\Bigg|_0^L$$
$$= \frac{i\rho}{\pi}\frac{L}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{i\rho}{\pi}\frac{L}{b-a}\frac{b-a}{ab} = \frac{i\rho L}{\pi ab}.$$

The resistance is therefore

$$R = \frac{V}{i} = \frac{\rho L}{\pi ab} = \frac{(731\ \Omega\cdot\text{m})(1.94\times10^{-2}\ \text{m})}{\pi(2.00\times10^{-3}\ \text{m})(2.30\times10^{-3}\ \text{m})} = 9.81\times10^{5}\ \Omega$$

Note that if $b = a$, then $R = \rho L/\pi a^2 = \rho L/A$, where $A = \pi a^2$ is the cross-sectional area of the cylinder.

36. Since the current spreads uniformly over the hemisphere, the current density at any given radius r from the striking point is $J = I/2\pi r^2$. From Eq. 26-10, the magnitude of the electric field at a radial distance r is

$$E = \rho_w J = \frac{\rho_w I}{2\pi r^2},$$

where $\rho_w = 30\,\Omega\cdot\text{m}$ is the resistivity of water. The potential difference between a point at radial distance D and a point at $D+\Delta r$ is

$$\Delta V = -\int_D^{D+\Delta r} E dr = -\int_D^{D+\Delta r} \frac{\rho_w I}{2\pi r^2} dr = \frac{\rho_w I}{2\pi}\left(\frac{1}{D+\Delta r} - \frac{1}{D}\right) = -\frac{\rho_w I}{2\pi}\frac{\Delta r}{D(D+\Delta r)},$$

which implies that the current across the swimmer is

$$i = \frac{|\Delta V|}{R} = \frac{\rho_w I}{2\pi R}\frac{\Delta r}{D(D+\Delta r)}.$$

Substituting the values given, we obtain

$$i = \frac{(30.0\,\Omega\cdot\text{m})(7.80\times10^4\,\text{A})}{2\pi(4.00\times10^3\,\Omega)}\frac{0.70\text{ m}}{(35.0\text{ m})(35.0\text{ m}+0.70\text{ m})} = 5.22\times10^{-2}\,\text{A}.$$

37. From Eq. 26-25, $\rho \propto \tau^{-1} \propto v_{\text{eff}}$. The connection with v_{eff} is indicated in part (b) of Sample Problem 26.05 —"Mean free time and mean free distance," which contains useful insight regarding the problem we are working now. According to Chapter 20, $v_{\text{eff}} \propto \sqrt{T}$. Thus, we may conclude that $\rho \propto \sqrt{T}$.

38. The slope of the graph is $P = 5.0 \times 10^{-4}$ W. Using this in the $P = V^2/R$ relation leads to $V = 0.10$ Vs.

39. Eq. 26-26 gives the rate of thermal energy production:

$$P = iV = (10.0\,\text{A})(120\,\text{V}) = 1.20\text{ kW}.$$

Dividing this into the 180 kJ necessary to cook the three hotdogs leads to the result $t = 150$ s.

40. The resistance is $R = P/i^2 = (100\text{ W})/(3.00\text{ A})^2 = 11.1\ \Omega$.

41. **THINK** In an electrical circuit, the electrical energy is dissipated through the resistor as heat.

EXPRESS Electrical energy is converted to heat at a rate given by $P = V^2/R$, where V is the potential difference across the heater and R is the resistance of the heater.

ANALYZE With $V = 120$ V and $R = 14\ \Omega$, we have

$$P = \frac{(120\text{ V})^2}{14\ \Omega} = 1.0\times10^3\text{ W} = 1.0\text{ kW}.$$

(b) The cost is given by $(1.0\,\text{kW})(5.0\,\text{h})(5.0\,\text{cents/kW}\cdot\text{h}) = \text{US\$}0.25$.

LEARN The energy transferred is lost because the process is irreversible. The thermal energy causes the temperature of the resistor to rise.

42. (a) Referring to Fig. 26-33, the electric field would point down (toward the bottom of the page) in the strip, which means the current density vector would point down, too (by Eq. 26-11). This implies (since electrons are negatively charged) that the conduction electrons would be "drifting" upward in the strip.

(b) Equation 24-6 immediately gives 12 eV, or (using $e = 1.60 \times 10^{-19}$ C) 1.9×10^{-18} J for the work done by the field (which equals, in magnitude, the potential energy change of the electron).

(c) Since the electrons don't (on average) gain kinetic energy as a result of this work done, it is generally dissipated as heat. The answer is as in part (b): 12 eV or 1.9×10^{-18} J.

43. The relation $P = V^2/R$ implies $P \propto V^2$. Consequently, the power dissipated in the second case is

$$P = \left(\frac{1.50\text{ V}}{3.00\text{ V}}\right)^2 (0.540\text{ W}) = 0.135\text{ W}.$$

44. Since $P = iV$, the charge is

$$q = it = Pt/V = (7.0\text{ W})\,(5.0\text{ h})\,(3600\text{ s/h})/9.0\text{ V} = 1.4 \times 10^4\text{ C}.$$

45. **THINK** Let P be the power dissipated, i be the current in the heater, and V be the potential difference across the heater. The three quantities are related by $P = iV$.

EXPRESS The current is given by $i = P/V$. Using Ohm's law $V = iR$, the resistance of the heater can be written as

$$R = \frac{V}{i} = \frac{V}{P/V} = \frac{V^2}{P}.$$

ANALYZE (a) Substituting the values given, we have $i = \dfrac{P}{V} = \dfrac{1250\text{ W}}{115\text{ V}} = 10.9\text{ A}.$

(b) Similarly, the resistance is

$$R = \frac{V^2}{P} = \frac{(115\text{ V})^2}{1250\text{ W}} = 10.6\ \Omega.$$

(c) The thermal energy E generated by the heater in time $t = 1.0$ h = 3600 s is

$$E = Pt = (1250\,\text{W})(3600\,\text{s}) = 4.50\times10^{6}\ \text{J}.$$

LEARN Current in the heater produces a transfer of mechanical energy to thermal energy, with a rate of the transfer equal to $P = iV = V^2/R$.

46. (a) Using Table 26-1 and Eq. 26-10 (or Eq. 26-11), we have

$$|\vec{E}| = \rho|\vec{J}| = \left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(\frac{2.00\,\text{A}}{2.00\times10^{-6}\,\text{m}^2}\right) = 1.69\times10^{-2}\ \text{V/m}.$$

(b) Using L = 4.0 m, the resistance is found from Eq. 26-16:

$$R = \rho L/A = 0.0338\ \Omega.$$

The rate of thermal energy generation is found from Eq. 26-27:

$$P = i^2 R = (2.00\ \text{A})^2(0.0338\ \Omega) = 0.135\ \text{W}.$$

Assuming a steady rate, the amount of thermal energy generated in 30 minutes is found to be (0.135 J/s)(30 × 60 s) = 2.43×10^2 J.

47. (a) From $P = V^2/R = AV^2/\rho L$, we solve for the length:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60\times10^{-6}\ \text{m}^2)(75.0\ \text{V})^2}{(5.00\times10^{-7}\ \Omega\cdot\text{m})(500\ \text{W})} = 5.85\ \text{m}.$$

(b) Since $L \propto V^2$ the new length should be $L' = L\left(\frac{V'}{V}\right)^2 = (5.85\ \text{m})\left(\frac{100\ \text{V}}{75.0\ \text{V}}\right)^2 = 10.4\ \text{m}.$

48. The mass of the water over the length is

$$m = \rho AL = (1000\ \text{kg/m}^3)(15\times10^{-5}\ \text{m}^2)(0.12\ \text{m}) = 0.018\ \text{kg},$$

and the energy required to vaporize the water is

$$Q = Lm = (2256\ \text{kJ/kg})(0.018\ \text{kg}) = 4.06\times10^{4}\ \text{J}.$$

The thermal energy is supplied by joule heating of the resistor:

$$Q = P\Delta t = I^2 R\Delta t.$$

Since the resistance over the length of water is

$$R = \frac{\rho_w L}{A} = \frac{(150\,\Omega\cdot\text{m})(0.120\text{ m})}{15\times10^{-5}\text{ m}^2} = 1.2\times10^5\,\Omega,$$

the average current required to vaporize water is

$$I = \sqrt{\frac{Q}{R\Delta t}} = \sqrt{\frac{4.06\times10^4\text{ J}}{(1.2\times10^5\,\Omega)(2.0\times10^{-3}\text{ s})}} = 13.0\text{ A}.$$

49. (a) Assuming a 31-day month, the monthly cost is

$$(100\text{ W})(24\text{ h/day})(31\text{days/month})\,(6\text{ cents/kW}\cdot\text{h}) = 446\text{ cents} = \text{US\$4.46}.$$

(b) $R = V^2/P = (120\text{ V})^2/100\text{ W} = 144\ \Omega$.

(c) $i = P/V = 100\text{ W}/120\text{ V} = 0.833\text{ A}$.

50. The slopes of the lines yield $P_1 = 8$ mW and $P_2 = 4$ mW. Their sum (by energy conservation) must be equal to that supplied by the battery: $P_{\text{batt}} = (8 + 4)$ mW = 12 mW.

51. **THINK** Our system is made up of two wires that are joined together. To calculate the electrical potential difference between two points, we first calculate their resistances.

EXPRESS The potential difference between points 1 and 2 is $\Delta V_{12} = iR_C$, where R_C is the resistance of wire *C*. Similarly, the potential difference between points 2 and 3 is $\Delta V_{23} = iR_D$, where R_D is the resistance of wire *D*. The corresponding rates of energy dissipation are $P_{12} = i^2 R_C$ and $P_{23} = i^2 R_D$, respectively.

ANALYZE (a) Using Eq. 26-16, we find the resistance of wire *C* to be

$$R_C = \rho_C \frac{L_C}{\pi r_C^2} = (2.0\times10^{-6}\,\Omega\cdot\text{m})\frac{1.0\text{ m}}{\pi(0.00050\text{ m})^2} = 2.55\,\Omega.$$

Thus, $\Delta V_{12} = iR_C = (2.0\text{ A})(2.55\,\Omega) = 5.1\text{V}$.

(b) Similarly, the resistance for wire *D* is

$$R_D = \rho_D \frac{L_D}{\pi r_D^2} = (1.0\times10^{-6}\,\Omega\cdot\text{m})\frac{1.0\text{ m}}{\pi(0.00025\text{ m})^2} = 5.09\,\Omega$$

and the potential difference is $\Delta V_{23} = iR_D = (2.0\text{ A})(5.09\,\Omega) = 10.2\text{V} \approx 10\text{V}$.

(c) The power dissipated between points 1 and 2 is $P_{12} = i^2 R_C = 10\text{W}$.

(d) Similarly, the power dissipated between points 2 and 3 is $P_{23} = i^2 R_D = 20\,\text{W}$.

LEARN The results may be summarized in terms of the following ratios:

$$\frac{P_{23}}{P_{12}} = \frac{\Delta V_{23}}{\Delta V_{12}} = \frac{R_D}{R_C} = \frac{\rho_D}{\rho_C} \cdot \frac{L_D}{L_C} \cdot \left(\frac{r_C}{r_D}\right)^2 = \frac{1}{2} \cdot 1 \cdot (2)^2 = 2.$$

52. Assuming the current is along the wire (not radial) we find the current from Eq. 26-4:

$$i = \int |\vec{J}|\, dA = \int_0^R kr^2 2\pi r dr = \frac{1}{2} k\pi R^4 = 3.50 \text{ A}$$

where $k = 2.75 \times 10^{10}$ A/m^4 and $R = 0.00300$ m. The rate of thermal energy generation is found from Eq. 26-26: $P = iV = 210$ W. Assuming a steady rate, the thermal energy generated in 40 s is $Q = P\Delta t = (210 \text{ J/s})(3600 \text{ s}) = 7.56 \times 10^5$ J.

53. (a) From $P = V^2/R$ we find $R = V^2/P = (120 \text{ V})^2/500 \text{ W} = 28.8\ \Omega$.

(b) Since $i = P/V$, the rate of electron transport is

$$\frac{i}{e} = \frac{P}{eV} = \frac{500 \text{ W}}{(1.60 \times 10^{-19} \text{ C})(120 \text{ V})} = 2.60 \times 10^{19} \text{ / s}.$$

54. From $P = V^2/R$, we have

$$R = (5.0 \text{ V})^2/(200 \text{ W}) = 0.125\ \Omega.$$

To meet the conditions of the problem statement, we must therefore set

$$\int_0^L 5.00x\, dx = 0.125\ \Omega$$

Thus,

$$\frac{5}{2} L^2 = 0.125 \quad \Rightarrow \quad L = 0.224 \text{ m}.$$

55. **THINK** Since the resistivity of Nichrome varies with temperature, the power dissipated through the Nichrome wire will also depend on temperature.

EXPRESS Let R_H be the resistance at the higher temperature (800°C) and let R_L be the resistance at the lower temperature (200°C). Since the potential difference is the same for the two temperatures, the power dissipated at the lower temperature is $P_L = V^2/R_L$, and the power dissipated at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now,

$$R_H = \frac{\rho_H L}{A} = \frac{\rho_0 L}{A}\left[1+\alpha(T_H - T_0)\right]$$
$$R_L = \frac{\rho_L L}{A} = \frac{\rho_0 L}{A}\left[1+\alpha(T_L - T_0)\right]$$

so that

$$R_L = R_H + \alpha R_H \Delta T,$$

where ΔT is the temperature difference: $T_L - T_H$ = –600 C° = –600 K.

ANALYZE Thus, the dissipation rate at 200°C is

$$P_L = \frac{R_H}{R_H + \alpha R_H \Delta T} P_H = \frac{P_H}{1+\alpha\Delta T} = \frac{500\text{ W}}{1+(4.0\times10^{-4}/\text{K})(-600\text{ K})} = 660\text{ W}.$$

LEARN Since the power dissipated is inversely proportional to R, at lower temperature where $R_L < R_H$, we expect a higher rate of energy dissipation: $P_L > P_H$.

56. (a) The current is

$$i = \frac{V}{R} = \frac{V}{\rho L/A} = \frac{\pi V d^2}{4\rho L} = \frac{\pi(1.20\text{ V})[(0.0400\text{ in.})(2.54\times10^{-2}\text{m/in.})]^2}{4(1.69\times10^{-8}\Omega\cdot\text{m})(33.0\text{ m})} = 1.74\text{ A}.$$

(b) The magnitude of the current density vector is

$$|\vec{J}| = \frac{i}{A} = \frac{4i}{\pi d^2} = \frac{4(1.74\text{ A})}{\pi[(0.0400\text{ in.})(2.54\times10^{-2}\text{ m/in.})]^2} = 2.15\times10^6\text{ A/m}^2.$$

(c) $E = V/L$ = 1.20 V/33.0 m = 3.63×10^{-2} V/m.

(d) $P = Vi$ = (1.20 V)(1.74 A) = 2.09 W.

57. We find the current from Eq. 26-26: $i = P/V$ = 2.00 A. Then, from Eq. 26-1 (with constant current), we obtain

$$\Delta q = i\Delta t = 2.88 \times 10^4\text{ C}.$$

58. We denote the copper rod with subscript c and the aluminum rod with subscript a.

(a) The resistance of the aluminum rod is

$$R = \rho_a \frac{L}{A} = \frac{(2.75\times10^{-8}\,\Omega\cdot\text{m})(1.3\text{ m})}{(5.2\times10^{-3}\text{ m})^2} = 1.3\times10^{-3}\,\Omega.$$

(b) Let $R = \rho_c L/(\pi d^2/4)$ and solve for the diameter d of the copper rod:

$$d=\sqrt{\frac{4\rho_c L}{\pi R}}=\sqrt{\frac{4\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(1.3\text{ m}\right)}{\pi\left(1.3\times10^{-3}\,\Omega\right)}}=4.6\times10^{-3}\text{ m}.$$

59. (a) Since

$$\rho=\frac{RA}{L}=\frac{R(\pi d^2/4)}{L}=\frac{(1.09\times10^{-3}\,\Omega)\pi(5.50\times10^{-3}\text{ m})^2/4}{1.60\text{ m}}=1.62\times10^{-8}\;\Omega\cdot\text{m},$$

the material is silver.

(b) The resistance of the round disk is

$$R=\rho\frac{L}{A}=\frac{4\rho L}{\pi d^2}=\frac{4(1.62\times10^{-8}\,\Omega\cdot\text{m})(1.00\times10^{-3}\text{m})}{\pi(2.00\times10^{-2}\text{m})^2}=5.16\times10^{-8}\,\Omega.$$

60. (a) Current is the transport of charge; here it is being transported "in bulk" due to the volume rate of flow of the powder. From Chapter 14, we recall that the volume rate of flow is the product of the cross-sectional area (of the stream) and the (average) stream velocity. Thus, $i = \rho A v$ where ρ is the charge per unit volume. If the cross section is that of a circle, then $i = \rho\pi R^2 v$.

(b) Recalling that a coulomb per second is an ampere, we obtain

$$i=\left(1.1\times10^{-3}\text{ C/m}^3\right)\pi\left(0.050\,\text{m}\right)^2\left(2.0\,\text{m/s}\right)=1.7\times10^{-5}\text{ A}.$$

(c) The motion of charge is not in the same direction as the potential difference computed in problem 70 of Chapter 24. It might be useful to think of (by analogy) Eq. 7-48; there, the scalar (dot) product in $P=\vec{F}\cdot\vec{v}$ makes it clear that $P = 0$ if $\vec{F}\perp\vec{v}$. This suggests that a radial potential difference and an axial flow of charge will not together produce the needed transfer of energy (into the form of a spark).

(d) With the assumption that there is (at least) a voltage equal to that computed in problem 70 of Chapter 24, in the proper direction to enable the transference of energy (into a spark), then we use our result from that problem in Eq. 26-26:

$$P=iV=\left(1.7\times10^{-5}\text{ A}\right)\left(7.8\times10^{4}\text{ V}\right)=1.3\,\text{W}.$$

(e) Recalling that a joule per second is a watt, we obtain (1.3 W)(0.20 s) = 0.27 J for the energy that can be transferred at the exit of the pipe.

(f) This result is greater than the 0.15 J needed for a spark, so we conclude that the spark was likely to have occurred at the exit of the pipe, going into the silo.

61. **THINK** The amount of charge that strikes the surface in time Δt is given by $\Delta q = i\,\Delta t$, where i is the current.

EXPRESS Since each alpha particle carries charge $q = +2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i\Delta t}{2e}.$$

For part (b), let N' be the number of particles in a length L of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where v is the particle speed. The current is the charge that moves through the cross section per unit time. That is,

$$i = \frac{2eN'}{t} = \frac{2eN'v}{L}.$$

Thus $N' = iL/2ev$.

ANALYZE (a) Substituting the values given, we have

$$N = \frac{\Delta q}{2e} = \frac{i\Delta t}{2e} = \frac{\left(0.25\times10^{-6}\,\text{A}\right)\left(3.0\,\text{s}\right)}{2\left(1.6\times10^{-19}\,\text{C}\right)} = 2.34\times10^{12}.$$

(b) To find the particle speed, we note the kinetic energy of a particle is

$$K = 20\,\text{MeV} = \left(20\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right) = 3.2\times10^{-12}\,\text{J}.$$

Since $K = \frac{1}{2}mv^2$, the speed is $v = \sqrt{2K/m}$. The mass of an alpha particle is (very nearly) 4 times the mass of a proton, or $m = 4(1.67 \times 10^{-27}\text{ kg}) = 6.68 \times 10^{-27}$ kg, so

$$v = \sqrt{\frac{2\left(3.2\times10^{-12}\,\text{J}\right)}{6.68\times10^{-27}\,\text{kg}}} = 3.1\times10^{7}\text{ m/s}.$$

Therefore, the number of particles in a length $L = 20$ cm of the beam is

$$N' = \frac{iL}{2ev} = \frac{\left(0.25\times10^{-6}\right)\left(20\times10^{-2}\text{ m}\right)}{2\left(1.60\times10^{-19}\text{ C}\right)\left(3.1\times10^{7}\text{ m/s}\right)} = 5.0\times10^{3}.$$

(c) We use conservation of energy, where the initial kinetic energy is zero and the final kinetic energy is 20 MeV = 3.2×10^{-12} J. We note too, that the initial potential energy is

$$U_i = qV = 2eV,$$

and the final potential energy is zero. Here V is the electric potential through which the particles are accelerated. Consequently, $K_f = U_i = 2eV$, which gives

$$V = \frac{K_f}{2e} = \frac{3.2\times10^{-12}\text{ J}}{2\left(1.60\times10^{-19}\text{ C}\right)} = 1.0\times10^{7}\text{ V}.$$

LEARN By the work-kinetic energy theorem, the work done on 2.34×10^{12} such alpha particles is

$$W = (2.34\times10^{12})(20\text{ MeV}) = (2.34\times10^{12})(3.2\times10^{-12}\text{ J}) = 7.5\text{ J}.$$

The same result can also be obtained from

$$W = q\Delta V = (i\Delta t)\Delta V = (0.25\times10^{-6}\text{ A})(3.0\text{ s})(1.0\times10^{7}\text{ V}) = 7.5\text{ J}.$$

62. We use Eq. 26-28: $R = \dfrac{V^2}{P} = \dfrac{(200\text{ V})^2}{3000\text{ W}} = 13.3\,\Omega$.

63. Combining Eq. 26-28 with Eq. 26-16 demonstrates that the power is inversely proportional to the length (when the voltage is held constant, as in this case). Thus, a new length equal to 7/8 of its original value leads to

$$P = \frac{8}{7}(2.0\text{ kW}) = 2.4\text{ kW}.$$

64. (a) Since $P = i^2 R = J^2 A^2 R$, the current density is

$$J = \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{P}{\rho L/A}} = \sqrt{\frac{P}{\rho LA}} = \sqrt{\frac{1.0\text{ W}}{\pi\left(3.5\times10^{-5}\Omega\cdot\text{m}\right)\left(2.0\times10^{-2}\text{ m}\right)\left(5.0\times10^{-3}\text{ m}\right)^2}}$$

$$= 1.3\times10^{5}\text{ A/m}^2.$$

(b) From $P = iV = JAV$ we get

$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0\text{ W}}{\pi\left(5.0\times10^{-3}\text{ m}\right)^2\left(1.3\times10^{5}\text{ A/m}^2\right)} = 9.4\times10^{-2}\text{ V}.$$

65. We use $P = i^2 R = i^2\rho L/A$, or $L/A = P/i^2\rho$.

(a) The new values of L and A satisfy

$$\left(\frac{L}{A}\right)_{\text{new}} = \left(\frac{P}{i^2\rho}\right)_{\text{new}} = \frac{30}{4^2}\left(\frac{P}{i^2\rho}\right)_{\text{old}} = \frac{30}{16}\left(\frac{L}{A}\right)_{\text{old}}.$$

Consequently, $(L/A)_{\text{new}} = 1.875(L/A)_{\text{old}}$, and

$$L_{\text{new}} = \sqrt{1.875}L_{\text{old}} = 1.37L_{\text{old}} \quad \Rightarrow \quad \frac{L_{\text{new}}}{L_{\text{old}}} = 1.37.$$

(b) Similarly, we note that $(LA)_{\text{new}} = (LA)_{\text{old}}$, and

$$A_{\text{new}} = \sqrt{1/1.875}A_{\text{old}} = 0.730A_{\text{old}} \quad \Rightarrow \quad \frac{A_{\text{new}}}{A_{\text{old}}} = 0.730.$$

66. The horsepower required is $P = \dfrac{iV}{0.80} = \dfrac{(10\text{A})(12\text{ V})}{(0.80)(746\text{ W/hp})} = 0.20$ hp.

67. (a) We use $P = V^2/R \propto V^2$, which gives $\Delta P \propto \Delta V^2 \approx 2V\,\Delta V$. The percentage change is roughly

$$\Delta P/P = 2\Delta V/V = 2(110 - 115)/115 = -8.6\%.$$

(b) A drop in V causes a drop in P, which in turn lowers the temperature of the resistor in the coil. At a lower temperature R is also decreased. Since $P \propto R^{-1}$ a decrease in R will result in an increase in P, which partially offsets the decrease in P due to the drop in V. Thus, the actual drop in P will be smaller when the temperature dependency of the resistance is taken into consideration.

68. We use Eq. 26-17: $\rho - \rho_0 = \rho\alpha(T - T_0)$, and solve for T:

$$T = T_0 + \frac{1}{\alpha}\left(\frac{\rho}{\rho_0} - 1\right) = 20°\text{C} + \frac{1}{4.3\times10^{-3}/\text{K}}\left(\frac{58\,\Omega}{50\,\Omega} - 1\right) = 57°\text{C}.$$

We are assuming that $\rho/\rho_0 = R/R_0$.

69. We find the rate of energy consumption from Eq. 26-28:

$$P = \frac{V^2}{R} = \frac{(90\text{ V})^2}{400\,\Omega} = 20.3\text{ W}$$

Assuming a steady rate, the energy consumed is $(20.3\text{ J/s})(2.00 \times 3600\text{ s}) = 1.46 \times 10^5$ J.

70. (a) The potential difference between the two ends of the caterpillar is

$$V = iR = i\rho\frac{L}{A} = \frac{(12\text{ A})(1.69\times10^{-8}\ \Omega\cdot\text{m})(4.0\times10^{-2}\text{ m})}{\pi(5.2\times10^{-3}\text{ m}/2)^2} = 3.8\times10^{-4}\text{ V}.$$

(b) Since it moves in the direction of the electron drift, which is against the direction of the current, its tail is negative compared to its head.

(c) The time of travel relates to the drift speed:

$$t = \frac{L}{v_d} = \frac{lAne}{i} = \frac{\pi L d^2 ne}{4i} = \frac{\pi(1.0\times10^{-2}\text{ m})(5.2\times10^{-3}\text{ m})^2(8.47\times10^{28}/\text{m}^3)(1.60\times10^{-19}\text{ C})}{4(12\text{ A})}$$
$$= 238\text{ s} = 3\text{ min } 58\text{ s}.$$

71. **THINK** The resistance of copper increases with temperature.

EXPRESS According to Eq. 26-17, the resistance of copper at temperature T can be written as

$$R = \frac{\rho L}{A} = \frac{\rho_0 L}{A}[1+\alpha(T-T_0)]$$

where $T_0 = 20\ ^\circ\text{C}$ is the reference temperature. Thus, the resistance is $R_0 = \rho_0 L/A$ at $T_0 = 20\ ^\circ\text{C}$. The temperature at which $R = 2R_0$ (or equivalently, $\rho = 2\rho_0$) can be found by solving

$$2 = \frac{R}{R_0} = 1 + \alpha(T-T_0) \quad\Rightarrow\quad \alpha(T-T_0) = 1.$$

ANALYZE (a) From the above equation, we find the temperature to be

$$T = T_0 + \frac{1}{\alpha} = 20°\text{C} + \frac{1}{4.3\times10^{-3}/\text{K}} \approx 250°\text{C}.$$

(b) Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved.

LEARN It is worth noting that our result agrees well with Fig. 26-10.

72. Since 100 cm = 1 m, then $10^4\text{ cm}^2 = 1\text{ m}^2$. Thus,

$$R = \frac{\rho L}{A} = \frac{(3.00\times10^{-7}\ \Omega\cdot\text{m})(10.0\times10^3\text{ m})}{56.0\times10^{-4}\text{ m}^2} = 0.536\ \Omega.$$

73. The rate at which heat is being supplied is

$$P = iV = (5.2 \text{ A})(12 \text{ V}) = 62.4 \text{ W}.$$

Considered on a one-second time frame, this means 62.4 J of heat are absorbed by the liquid each second. Using Eq. 18-16, we find the heat of transformation to be

$$L = \frac{Q}{m} = \frac{62.4 \text{ J}}{21\times10^{-6} \text{ kg}} = 3.0\times10^{6} \text{ J/kg}.$$

74. We find the drift speed from Eq. 26-7:

$$v_d = \frac{|\vec{J}|}{ne} = \frac{2.0\times10^{6} \text{ A/m}^2}{(8.49\times10^{28}/\text{m}^3)(1.6\times10^{-19}\text{C})} = 1.47\times10^{-4} \text{ m/s}.$$

At this (average) rate, the time required to travel L = 5.0 m is

$$t = \frac{L}{v_d} = \frac{5.0 \text{ m}}{1.47\times10^{-4} \text{ m/s}} = 3.4\times10^{4} \text{ s}.$$

75. The power dissipated is given by the product of the current and the potential difference:

$$P = iV = (7.0\times10^{-3} \text{ A})(80\times10^{3} \text{ V}) = 560 \text{ W}.$$

76. (a) The current is 4.2×10^{18} e divided by 1 second. Using $e = 1.60 \times 10^{-19}$ C we obtain 0.67 A for the current.

(b) Since the electric field points away from the positive terminal (high potential) and toward the negative terminal (low potential), then the current density vector (by Eq. 26-11) must also point toward the negative terminal.

77. For the temperature of the gas to remain unchanged, the rate of the thermal energy dissipated through the resistor, $P_R = i^2R$, must be equal to the rate of increase of mechanical energy of the piston, $P_m = mg(dh/dt) = mgv$. Thus,

$$i^2R = mgv \quad \Rightarrow \quad v = \frac{i^2R}{mg} = \frac{(0.240 \text{ A})^2(550\ \Omega)}{(12 \text{ kg})(9.8 \text{ m/s}^2)} = 0.27 \text{ m/s}.$$

78. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using σ for the charge per unit area and w for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma vw$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100\times10^{-6} \text{ A}}{(30 \text{ m/s})(50\times10^{-2} \text{ m})} = 6.7\times10^{-6} \text{ C/m}^2.$$

79. (a) The total current density is equal to the sum of the contributions from the alpha particles and the electron. Using the general expression $J = nqv,$ and noting that $n_e = 2n_\alpha$ (two electrons for each α particle), we have

$$\begin{aligned} J_{\text{total}} &= n_\alpha q_\alpha v_\alpha + n_e q_e v_e = n_\alpha (2e) v_\alpha + (2n_\alpha)(e) v_e = 2n_\alpha e (v_\alpha + v_e) \\ &= 2(2.80\times10^{21}/\text{m}^3)(1.6\times10^{-19}\text{ C})(88 \text{ m/s} + 25 \text{ m/s}) \\ &= 1.01\times10^5 \text{ A/m}^2 = 10.1 \text{ A/cm}^2 \end{aligned}$$

(b) The direction of the current is eastward (same as the motion of the alpha particles).

80. (a) Let ΔT be the change in temperature and κ be the coefficient of linear expansion for copper. Then $\Delta L = \kappa L \, \Delta T$ and

$$\frac{\Delta L}{L} = \kappa \Delta T = (1.7\times10^{-5}/\text{K})(1.0°\text{C}) = 1.7\times10^{-5}.$$

This is equivalent to 0.0017%. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of κ used in this calculation is not inconsistent with the other units involved.

(b) Incorporating a factor of 2 for the two-dimensional nature of A, the fractional change in area is

$$\frac{\Delta A}{A} = 2\kappa\Delta T = 2(1.7\times10^{-5}/\text{K})(1.0°\text{C}) = 3.4\times10^{-5}$$

which is 0.0034%.

(c) For small changes in the resistivity ρ, length L, and area A of a wire, the change in the resistance is given by

$$\Delta R = \frac{\partial R}{\partial \rho}\Delta\rho + \frac{\partial R}{\partial L}\Delta L + \frac{\partial R}{\partial A}\Delta A.$$

Since $R = \rho L/A$, $\partial R/\partial\rho = L/A = R/\rho$, $\partial R/\partial L = \rho/A = R/L$, and $\partial R/\partial A = -\rho L/A^2 = -R/A$. Furthermore, $\Delta\rho/\rho = \alpha\Delta T$, where α is the temperature coefficient of resistivity for copper (4.3×10^{-3}/K = 4.3×10^{-3}/C°, according to Table 27-1). Thus,

$$\begin{aligned} \frac{\Delta R}{R} &= \frac{\Delta\rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} = (\alpha + \kappa - 2\kappa)\Delta T = (\alpha - \kappa)\Delta T \\ &= (4.3\times10^{-3}/\text{C°} - 1.7\times10^{-5}/\text{C°})(1.0 \text{ C°}) = 4.3\times10^{-3}. \end{aligned}$$

This is 0.43%, which we note (for the purposes of the next part) is primarily determined by the $\Delta\rho/\rho$ term in the above calculation.

(d) The fractional change in resistivity is much larger than the fractional change in length and area. Changes in length and area affect the resistance much less than changes in resistivity.

81. (a) Using $i = dq/dt = e(dN/dt),$ we obtain

$$\frac{dN}{dt} = \frac{i}{e} = \frac{15\times10^{-6}\,\text{A}}{1.6\times10^{-19}\,\text{C}} = 9.4\times10^{13}\,/\,\text{s}.$$

(b) The rate of thermal energy production is

$$P = \frac{dU}{dt} = \left(\frac{dN}{dt}\right)U_1 = (9.4\times10^{13}\,/\,\text{s})(16\text{ MeV})\left(\frac{1.6\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = 240\text{ W}.$$

82. (a) The charge q that flows past any cross section of the beam in time Δt is given by $q = i\Delta t$, and the number of electrons is $N = q/e = (i/e)\,\Delta t$. This is the number of electrons that are accelerated. Thus,

$$N = \frac{(0.50\text{ A})(0.10\times10^{-6}\text{ s})}{1.60\times10^{-19}\text{ C}} = 3.1\times10^{11}.$$

(b) Over a long time t the total charge is $Q = nqt$, where n is the number of pulses per unit time and q is the charge in one pulse. The average current is given by $i_{\text{avg}} = Q/t = nq$. Now $q = i\Delta t = (0.50\text{ A})\,(0.10\times10^{-6}\text{ s}) = 5.0\times10^{-8}$ C, so

$$i_{\text{avg}} = (500/\text{s})(5.0\times10^{-8}\text{ C}) = 2.5\times10^{-5}\text{ A}.$$

(c) The accelerating potential difference is $V = K/e$, where K is the final kinetic energy of an electron. Since $K = 50$ MeV, the accelerating potential is $V = 50$ kV $= 5.0\times10^{7}$ V. During a pulse the power output is

$$P = iV = (0.50\text{ A})(5.0\times10^{7}\text{ V}) = 2.5\times10^{7}\text{ W}.$$

This is the peak power. The average power is

$$P_{\text{avg}} = i_{\text{avg}}V = (2.5\times10^{-5}\text{ A})(5.0\times10^{7}\text{ V}) = 1.3\times10^{3}\text{ W}.$$

83. With the voltage reduced by 6.00% while resistance remains unchanged, the current through the heating element also decreases by 6.00% ($i' = 0.94i$). The power delivered is now

$$P' = i'^2R = (0.94i)^2R = 0.884i^2R = 0.884P,$$

where $P = i^2 R$ is the power delivered to the heating element under normal circumstance. Since the energy required to heat the water remains the same in both cases, $P\Delta t = P'\Delta t'$, the time required becomes

$$\Delta t' = \left(\frac{P}{P'}\right)\Delta t = \frac{100\text{ min}}{0.884} = 113\text{ min.}$$

84. (a) The mass of the water is $m = \rho V = (1000\text{ kg/m}^3)(2.0\text{ L})(10^{-3}\text{m}^3/\text{L}) = 2.00\text{ kg}$. The energy required to raise the water temperature to the boiling point is

$$Q_1 = mc\Delta T = (2.00\text{ kg})(4187\text{ J/kg}\cdot\text{C°})(100\text{ °C} - 20\text{ °C}) = 6.70\times10^5\text{ J.}$$

With P = 400 W at 80% efficiency, we find the time needed to be

$$\Delta t_1 = \frac{Q_1}{P_{\text{eff}}} = \frac{6.70\times10^5\text{ J}}{(0.80)(400\text{ W})} = 2.09\times10^3\text{s} \approx 35\text{ min.}$$

(b) The energy required to vaporize half of the water is

$$Q_2 = L_V(m/2) = (2.256\times10^6\text{ J/kg})(2.00\text{ kg/2}) = 2.256\times10^6\text{ J.}$$

Thus, the additional time elapsed is

$$\Delta t_2 = \frac{Q_2}{P_{\text{eff}}} = \frac{2.256\times10^6\text{ J}}{(0.80)(400\text{ W})} = 7.05\times10^3\text{s} \approx 118\text{ min,}$$

or about 1.96 h.

85. (a) At t = 0.500 s, the charge on the capacitor is

$$q = CV = C(6.00 + 4.00t - 2.00t^2) = (30\times10^{-6}\text{F})\left[6.00 + 4.00(0.500) - 2.00(0.500)^2\right]$$
$$= 225\times10^{-6}\text{C} = 225\ \mu\text{C.}$$

(b) The current flowing into the capacitor is

$$i = \frac{dq}{dt} = C\frac{dV}{dt} = C\frac{d}{dt}\left(6.00 + 4.00t - 2.00t^2\right) = C(4.00 - 4.00t)$$
$$= (30\times10^{-6}\text{F})\left[4.00 - 4.00(0.500)\right] = 60.0\times10^{-6}\text{A} = 60.0\ \mu\text{A.}$$

(c) The corresponding power output is

$$P = iV = (60.0\times10^{-6}\text{A})\left[6.00 + 4.00(0.500) - 2.00(0.500)^2\right] = 4.50\times10^{-4}\text{W.}$$

Chapter 27

1. **THINK** The circuit consists of two batteries and two resistors. We apply Kirchhoff's loop rule to solve for the current.

EXPRESS Let i be the current in the circuit and take it to be positive if it is to the left in R_1. Kirchhoff's loop rule gives

$$\varepsilon_1 - iR_2 - iR_1 - \varepsilon_2 = 0.$$

For parts (b) and (c), we note that if i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2 R$.

ANALYZE (a) We solve for i:

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{12\ \text{V} - 6.0\ \text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\ \text{A}.$$

A positive value is obtained, so the current is counterclockwise around the circuit.

(b) For R_1, the dissipation rate is $P_1 = i^2 R_1 = (0.50\ \text{A})^2(4.0\ \Omega) = 1.0\ \text{W}$.

(c) For R_2, the rate is $P_2 = i^2 R_2 = (0.50\ \text{A})^2\,(8.0\ \Omega) = 2.0\ \text{W}$.

If i is the current in a battery with emf ε, then the battery supplies energy at the rate $P = i\varepsilon$ provided the current and emf are in the same direction. On the other hand, the battery absorbs energy at the rate $P = i\varepsilon$ if the current and emf are in opposite directions.

(d) For ε_1, $P_1 = i\varepsilon_1 = (0.50\ \text{A})(12\ \text{V}) = 6.0\ \text{W}$.

(e) For ε_2, $P_2 = i\varepsilon_2 = (0.50\ \text{A})(6.0\ \text{V}) = 3.0\ \text{W}$.

(f) In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging.

(g) The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

LEARN Multiplying the equation obtained from Kirchhoff's loop rule by idt leads to the "energy-method" equation discussed in Section 27-4:

$$i\varepsilon_1 dt - i^2 R_1 dt - i^2 R_2 dt - i\varepsilon_2 dt = 0.$$

The first term represents the rate of work done by battery 1, the second and third terms the thermal energies that appear in resistors R_1 and R_2, and the last term the work done *on* battery 2.

2. The current in the circuit is

$$i = (150 \text{ V} - 50 \text{ V})/(3.0 \ \Omega + 2.0 \ \Omega) = 20 \text{ A}.$$

So from $V_Q + 150 \text{ V} - (2.0 \ \Omega)i = V_P$, we get

$$V_Q = 100 \text{ V} + (2.0 \ \Omega)(20 \text{ A}) - 150 \text{ V} = -10 \text{ V}.$$

3. (a) The potential difference is $V = \varepsilon + ir = 12 \text{ V} + (50 \text{ A})(0.040 \ \Omega) = 14 \text{ V}$.

(b) $P = i^2 r = (50 \text{ A})^2(0.040 \ \Omega) = 1.0\times10^2 \text{ W}$.

(c) $P' = iV = (50 \text{ A})(12 \text{ V}) = 6.0\times10^2 \text{ W}$.

(d) In this case $V = \varepsilon - ir = 12 \text{ V} - (50 \text{ A})(0.040 \ \Omega) = 10 \text{ V}$.

(e) $P_r = i^2 r = (50 \text{ A})^2(0.040 \ \Omega) = 1.0\times10^2 \text{ W}$.

4. (a) The loop rule leads to a voltage-drop across resistor 3 equal to 5.0 V (since the total drop along the upper branch must be 12 V). The current there is consequently $i = (5.0 \text{ V})/(200 \ \Omega) = 25 \text{ mA}$. Then the resistance of resistor 1 must be $(2.0 \text{ V})/i = 80 \ \Omega$.

(b) Resistor 2 has the same voltage-drop as resistor 3; its resistance is 200 Ω.

5. The chemical energy of the battery is reduced by $\Delta E = q\varepsilon$, where q is the charge that passes through in time $\Delta t = 6.0$ min, and ε is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and

$$\Delta E = i\varepsilon\,\Delta t = (5.0 \text{ A})(6.0 \text{ V})\,(6.0 \text{ min})\,(60 \text{ s/min}) = 1.1 \times 10^4 \text{ J}.$$

We note the conversion of time from minutes to seconds.

6. (a) The cost is $(100 \text{ W} \cdot 8.0 \text{ h}/2.0 \text{ W} \cdot \text{h})\,(\$0.80) = \$3.2 \times 10^2$.

(b) The cost is $(100 \text{ W} \cdot 8.0 \text{ h}/10^3 \text{ W} \cdot \text{h})\,(\$0.06) = \$0.048 = 4.8$ cents.

7. (a) The energy transferred is

$$U = Pt = \frac{\varepsilon^2 t}{r+R} = \frac{(2.0\text{ V})^2(2.0\,\text{min})(60\,\text{s/min})}{1.0\,\Omega + 5.0\,\Omega} = 80\text{ J}.$$

(b) The amount of thermal energy generated is

$$U' = i^2 Rt = \left(\frac{\varepsilon}{r+R}\right)^2 Rt = \left(\frac{2.0\text{ V}}{1.0\,\Omega+5.0\,\Omega}\right)^2 (5.0\,\Omega)(2.0\text{ min})(60\text{ s/min}) = 67\text{ J}.$$

(c) The difference between U and U', which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

8. If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and ε is the emf of the battery, then $\Delta E = q\varepsilon$. Equating the two expressions for ΔE and solving for Δt, we obtain

$$\Delta t = \frac{q\varepsilon}{P} = \frac{(120\,\text{A}\cdot\text{h})(12.0\,\text{V})}{100\,\text{W}} = 14.4\text{ h}.$$

9. (a) The work done by the battery relates to the potential energy change:

$$q\Delta V = eV = e\left(12.0\,\text{V}\right) = 12.0\text{ eV}.$$

(b) $P = iV = neV = (3.40 \times 10^{18}/\text{s})(1.60 \times 10^{-19}\text{ C})(12.0\text{ V}) = 6.53\text{ W}$.

10. (a) We solve $i = (\varepsilon_2 - \varepsilon_1)/(r_1 + r_2 + R)$ for R:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{i} - r_1 - r_2 = \frac{3.0\text{ V} - 2.0\text{ V}}{1.0\times 10^{-3}\text{ A}} - 3.0\,\Omega - 3.0\,\Omega = 9.9\times 10^2\,\Omega.$$

(b) $P = i^2R = (1.0 \times 10^{-3}\text{ A})^2(9.9 \times 10^2\,\Omega) = 9.9 \times 10^{-4}\text{ W}$.

11. **THINK** As shown in Fig. 27-29, the circuit contains an emf device X. How it is connected to the rest of the circuit can be deduced from the power dissipated and the potential drop across it.

EXPRESS The power absorbed by a circuit element is given by $P = i\Delta V$, where i is the current and ΔV is the potential difference across the element. The end-to-end potential difference is given by

$$V_A - V_B = +iR + \varepsilon,$$

where ε is the emf of device X and is taken to be positive if it is to the left in the diagram.

ANALYZE (a) The potential difference between A and B is

$$\Delta V = \frac{P}{i} = \frac{50 \text{ W}}{1.0 \text{ A}} = 50 \text{ V}.$$

Since the energy of the charge decreases, point *A* is at a higher potential than point *B*; that is, $V_A - V_B = 50$ V.

(b) From the equation above, we find the emf of device *X* to be

$$\varepsilon = V_A - V_B - iR = 50 \text{ V} - (1.0 \text{ A})(2.0 \ \Omega) = 48 \text{ V}.$$

(c) A positive value was obtained for ε, so it is toward the left. The negative terminal is at *B*.

LEARN Writing the potential difference as $V_A - iR - \varepsilon = V_B$, we see that our result is consistent with the resistance and emf rules. Namely, starting at point *A*, the change in potential is $-iR$ for a move through a resistance *R* in the direction of the current, and the change in potential is $-\varepsilon$ for a move through an emf device in the opposite direction of the emf arrow (which points from negative to positive terminals).

12. (a) For each wire, $R_{\text{wire}} = \rho L/A$ where $A = \pi r^2$. Consequently, we have

$$R_{\text{wire}} = (1.69 \times 10^{-8} \ \Omega \cdot \text{m})(0.200 \text{ m})/\pi(0.00100 \text{ m})^2 = 0.0011 \ \Omega.$$

The total resistive load on the battery is therefore

$$R_{\text{tot}} = 2R_{\text{wire}} + R = 2(0.0011 \ \Omega) + 6.00 \ \Omega = 6.0022 \ \Omega.$$

Dividing this into the battery emf gives the current

$$i = \frac{\varepsilon}{R_{\text{tot}}} = \frac{12.0 \text{ V}}{6.0022 \Omega} = 1.9993 \text{ A}.$$

The voltage across the $R = 6.00 \ \Omega$ resistor is therefore

$$V = iR = (1.9993 \text{ A})(6.00 \ \Omega) = 11.996 \text{ V} \approx 12.0 \text{ V}.$$

(b) Similarly, we find the voltage-drop across each wire to be

$$V_{\text{wire}} = iR_{\text{wire}} = (1.9993 \text{ A})(0.0011 \ \Omega) = 2.15 \text{ mV}.$$

(c) $P = i^2R = (1.9993 \text{ A})(6.00 \ \Omega)^2 = 23.98 \text{ W} \approx 24.0 \text{ W}$.

(d) Similarly, we find the power dissipated in each wire to be 4.30 mW.

13. (a) We denote L = 10 km and α = 13 Ω/km. Measured from the east end we have

$$R_1 = 100\ \Omega = 2\alpha(L - x) + R,$$

and measured from the west end $R_2 = 200\ \Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\Omega - 100\Omega}{4(13\Omega/\text{km})} + \frac{10\,\text{km}}{2} = 6.9\,\text{km}.$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\Omega + 200\Omega}{2} - (13\Omega/\text{km})(10\,\text{km}) = 20\Omega\,.$$

14. (a) Here we denote the battery emf's as V_1 and V_2. The loop rule gives

$$V_2 - ir_2 + V_1 - ir_1 - iR = 0 \;\Rightarrow\; i = \frac{V_2 + V_1}{r_1 + r_2 + R}\;.$$

The terminal voltage of battery 1 is V_{1T} and (see Fig. 27-4(a)) is easily seen to be equal to $V_1 - ir_1$; similarly for battery 2. Thus,

$$V_{1T} = V_1 - \frac{r_1(V_2 + V_1)}{r_1 + r_2 + R},\; V_{2T} = V_2 - \frac{r_1(V_2 + V_1)}{r_1 + r_2 + R}\;.$$

The problem tells us that V_1 and V_2 each equal 1.20 V. From the graph in Fig. 27-32(b) we see that $V_{2T} = 0$ and $V_{1T} = 0.40$ V for R = 0.10 Ω. This supplies us (in view of the above relations for terminal voltages) with simultaneous equations, which, when solved, lead to r_1 = 0.20 Ω.

(b) The simultaneous solution also gives r_2 = 0.30 Ω.

15. Let the emf be V. Then $V = iR = i'(R + R')$, where i = 5.0 A, i' = 4.0 A, and R' = 2.0 Ω. We solve for R:

$$R = \frac{i'R'}{i - i'} = \frac{(4.0\text{ A})(2.0\Omega)}{5.0\text{ A} - 4.0\text{ A}} = 8.0\ \Omega.$$

16. (a) Let the emf of the solar cell be ε and the output voltage be V. Thus,

$$V = \varepsilon - ir = \varepsilon - \left(\frac{V}{R}\right) r$$

for both cases. Numerically, we get

$$0.10 \text{ V} = \varepsilon - (0.10 \text{ V}/500\ \Omega)r$$
$$0.15 \text{ V} = \varepsilon - (0.15 \text{ V}/1000\ \Omega)r.$$

We solve for ε and r.

(a) $r = 1.0\times10^3\ \Omega$.

(b) $\varepsilon = 0.30$ V.

(c) The efficiency is

$$\frac{V^2/R}{P_{\text{received}}} = \frac{0.15\text{V}}{(1000\Omega)\left(5.0\,\text{cm}^2\right)\left(2.0\times10^{-3}\,\text{W/cm}^2\right)} = 2.3\times10^{-3} = 0.23\%.$$

17. **THINK** A zero terminal-to-terminal potential difference implies that the emf of the battery is equal to the voltage drop across its internal resistance, that is, $\varepsilon = ir$.

EXPRESS To be as general as possible, we refer to the individual emf's as ε_1 and ε_2 and wait until the latter steps to equate them ($\varepsilon_1 = \varepsilon_2 = \varepsilon$). The batteries are placed in series in such a way that their voltages add; that is, they do not "oppose" each other. The total resistance in the circuit is therefore $R_{\text{total}} = R + r_1 + r_2$ (where the problem tells us $r_1 > r_2$), and the "net emf" in the circuit is $\varepsilon_1 + \varepsilon_2$. Since battery 1 has the higher internal resistance, it is the one capable of having a zero terminal voltage, as the computation in part (a) shows.

ANALYZE (a) The current in the circuit is

$$i = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R},$$

and the requirement of zero terminal voltage leads to $\varepsilon_1 = ir_1$, or

$$R = \frac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{\varepsilon_1} = \frac{(12.0\text{ V})(0.016\,\Omega) - (12.0\text{ V})(0.012\,\Omega)}{12.0\text{ V}} = 0.0040\ \Omega.$$

Note that $R = r_1 - r_2$ when we set $\varepsilon_1 = \varepsilon_2$.

(b) As mentioned above, this occurs in battery 1.

LEARN If we assume the potential difference across battery 2 to be zero and repeat the calculation above, we would find $R = r_2 - r_1 < 0$, which is physically impossible. Thus, only the potential difference across the battery with the larger internal resistance can be made zero with suitable choice of R.

18. The currents i_1, i_2 and i_3 are obtained from Eqs. 27-18 through 27-20:

$$i_1 = \frac{\varepsilon_1(R_2+R_3)-\varepsilon_2R_3}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\text{V})(10\,\Omega+5.0\,\Omega)-(1.0\,\text{V})(5.0\ \Omega)}{(10\,\Omega)(10\,\Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\ \Omega)} = 0.275\ \text{A},$$

$$i_2 = \frac{\varepsilon_1R_3-\varepsilon_2(R_1+R_2)}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\ \text{V})(5.0\,\Omega)-(1.0\ \text{V})(10\,\Omega+5.0\ \Omega)}{(10\,\Omega)(10\ \Omega)+(10\,\Omega)(5.0\,\Omega)+(10\,\Omega)(5.0\ \Omega)} = 0.025\ \text{A},$$

$$i_3 = i_2 - i_1 = 0.025\text{A} - 0.275\text{A} = -0.250\text{A}\ .$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get

$$V_d - V_c = i_2R_2 = (0.0250\text{ A})\ (10\ \Omega) = +0.25\text{ V};$$

from $V_d + i_3R_3 + \varepsilon_2 = V_c$ we get

$$V_d - V_c = i_3R_3 - \varepsilon_2 = -\ (-\ 0.250\text{ A})\ (5.0\ \Omega) - 1.0\text{ V} = +0.25\text{ V}.$$

19. (a) Since $R_{\text{eq}} < R$, the two resistors (R = 12.0 Ω and R_x) must be connected in parallel:

$$R_{\text{eq}} = 3.00\,\Omega = \frac{R_xR}{R+R_x} = \frac{R_x\left(12.0\,\Omega\right)}{12.0\,\Omega + R_x}.$$

We solve for R_x: $R_x = R_{\text{eq}}R/(R - R_{\text{eq}})$ = (3.00 Ω)(12.0 Ω)/(12.0 Ω – 3.00 Ω) = 4.00 Ω.

(b) As stated above, the resistors must be connected in parallel.

20. Let the resistances of the two resistors be R_1 and R_2, with $R_1 < R_2$. From the statements of the problem, we have

$$R_1R_2/(R_1 + R_2) = 3.0\ \Omega \text{ and } R_1 + R_2 = 16\ \Omega.$$

So R_1 and R_2 must be 4.0 Ω and 12 Ω, respectively.

(a) The smaller resistance is R_1 = 4.0 Ω.

(b) The larger resistance is R_2 = 12 Ω.

21. The potential difference across each resistor is V = 25.0 V. Since the resistors are identical, the current in each one is

$$i = V/R = (25.0\text{ V})/(18.0\ \Omega) = 1.39\text{ A}.$$

The total current through the battery is then i_{total} = 4(1.39 A) = 5.56 A. One might alternatively use the idea of equivalent resistance; for four identical resistors in parallel the equivalent resistance is given by

$$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R} = \frac{4}{R}.$$

When a potential difference of 25.0 V is applied to the equivalent resistor, the current through it is the same as the total current through the four resistors in parallel. Thus

$$i_{\text{total}} = V/R_{\text{eq}} = 4V/R = 4(25.0\text{ V})/(18.0\ \Omega) = 5.56\text{ A}.$$

22. (a) R_{eq} (*FH*) = (10.0 Ω)(10.0 Ω)(5.00 Ω)/[(10.0 Ω)(10.0 Ω) + 2(10.0 Ω)(5.00 Ω)] = 2.50 Ω.

(b) R_{eq} (*FG*) = (5.00 Ω) $R/(R$ + 5.00 Ω), where

$$R = 5.00\ \Omega + (5.00\ \Omega)(10.0\ \Omega)/(5.00\ \Omega + 10.0\ \Omega) = 8.33\ \Omega.$$

So R_{eq} (*FG*) = (5.00 Ω)(8.33 Ω)/(5.00 Ω + 8.33 Ω) = 3.13 Ω.

23. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward.

(a) When the loop rule is applied to the lower loop, the result is

$$\varepsilon_2 - i_1 R_1 = 0 .$$

The equation yields

$$i_1 = \frac{\varepsilon_2}{R_1} = \frac{5.0\text{ V}}{100\Omega} = 0.050\text{ A}.$$

(b) When it is applied to the upper loop, the result is

$$\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - i_2 R_2 = 0 .$$

The equation gives

$$i_2 = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3}{R_2} = \frac{6.0\text{ V} - 5.0\text{ V} - 4.0\text{ V}}{50\Omega} = -0.060\text{ A},$$

or $|i_2| = 0.060$ A. The negative sign indicates that the current in R_2 is actually downward.

(c) If V_b is the potential at point *b*, then the potential at point *a* is $V_a = V_b + \varepsilon_3 + \varepsilon_2$, so

$$V_a - V_b = \varepsilon_3 + \varepsilon_2 = 4.0\text{ V} + 5.0\text{ V} = 9.0\text{ V}.$$

24. We note that two resistors in parallel, R_1 and R_2, are equivalent to

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \; R_{12} = \frac{R_1 R_2}{R_1 + R_2}.$$

This situation consists of a parallel pair that are then in series with a single R_3 = 2.50 Ω resistor. Thus, the situation has an equivalent resistance of

$$R_{eq} = R_3 + R_{12} = 2.50\Omega + \frac{(4.00\Omega)(4.00\Omega)}{4.00\Omega + 4.00\Omega} = 4.50\Omega.$$

25. **THINK** The resistance of a copper wire varies with its cross-sectional area, or its diameter.

EXPRESS Let r be the resistance of each of the narrow wires. Since they are in parallel the equivalent resistance R_{eq} of the composite is given by

$$\frac{1}{R_{eq}} = \frac{9}{r},$$

or $R_{eq} = r/9$. Now each thin wire has a resistance $r = 4\rho\ell/\pi d^2$, where ρ is the resistivity of copper, and $A = \pi d^2/4$ is the cross-sectional area of a single thin wire. On the other hand, the resistance of the thick wire of diameter D is $R = 4\rho\ell/\pi D^2$, where the cross-sectional area is $\pi D^2/4$.

ANALYZE If the single thick wire is to have the same resistance as the composite of 9 thin wires, $R = R_{eq}$, then

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2}.$$

Solving for D, we obtain $D = 3d$.

LEARN The equivalent resistance R_{eq} is smaller than r by a factor of 9. Since $r \sim 1/A \sim 1/d^2$, increasing the diameter of the wire threefold will also reduce the resistance by a factor of 9.

26. The part of R_0 connected in parallel with R is given by $R_1 = R_0 x/L$, where L = 10 cm. The voltage difference across R is then $V_R = \varepsilon R'/R_{eq}$, where $R' = RR_1/(R + R_1)$ and

$$R_{eq} = R_0(1 - x/L) + R'.$$

Thus,

$$P_R = \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\varepsilon RR_1/(R+R_1)}{R_0(1-x/L) + RR_1/(R+R_1)}\right)^2 = \frac{100R(\varepsilon x/R_0)^2}{\left(100R/R_0 + 10x - x^2\right)^2},$$

where x is measured in cm.

27. Since the potential differences across the two paths are the same, $V_1 = V_2$ (V_1 for the left path, and V_2 for the right path), we have $i_1 R_1 = i_2 R_2$, where $i = i_1 + i_2 = 5000$ A. With $R = \rho L / A$ (see Eq. 26-16), the above equation can be rewritten as

$$i_1 d = i_2 h \quad \Rightarrow \quad i_2 = i_1 (d/h).$$

With $d/h = 0.400$, we get $i_1 = 3571$ A and $i_2 = 1429$ A. Thus, the current through the person is $i_1 = 3571$ A, or approximately 3.6 kA.

28. Line 1 has slope R_1 = 6.0 kΩ. Line 2 has slope R_2 = 4.0 kΩ. Line 3 has slope R_3 = 2.0 kΩ. The parallel pair equivalence is $R_{12} = R_1R_2/(R_1+R_2)$ = 2.4 kΩ. That in series with R_3 gives an equivalence of

$$R_{123} = R_{12} + R_3 = 2.4 \text{ k}\Omega + 2.0 \text{ k}\Omega = 4.4 \text{ k}\Omega.$$

The current through the battery is therefore $i = \varepsilon / R_{123}$ = (6 V)/(4.4 kΩ) and the voltage drop across R_3 is (6 V)(2 kΩ)/(4.4 kΩ) = 2.73 V. Subtracting this (because of the loop rule) from the battery voltage leaves us with the voltage across R_2. Then Ohm's law gives the current through R_2: (6 V – 2.73 V)/(4 kΩ) = 0.82 mA.

29. (a) The parallel set of three identical R_2 = 18 Ω resistors reduce to R = 6.0 Ω, which is now in series with the R_1 = 6.0 Ω resistor at the top right, so that the total resistive load across the battery is $R' = R_1 + R$ = 12 Ω. Thus, the current through R' is (12V)/R' = 1.0 A, which is the current through R. By symmetry, we see one-third of that passes through any one of those 18 Ω resistors; therefore, i_1 = 0.333 A.

(b) The direction of i_1 is clearly rightward.

(c) We use Eq. 26-27: $P = i^2R'$ = (1.0 A)^2(12 Ω) = 12 W. Thus, in 60 s, the energy dissipated is (12 J/s)(60 s) = 720 J.

30. Using the junction rule ($i_3 = i_1 + i_2$) we write two loop rule equations:

$$10.0 \text{ V} - i_1R_1 - (i_1 + i_2)\, R_3 = 0$$

$$5.00 \text{ V} - i_2R_2 - (i_1 + i_2)\, R_3 = 0.$$

(a) Solving, we find $i_2 = 0$, and

(b) $i_3 = i_1 + i_2$ = 1.25 A (downward, as was assumed in writing the equations as we did).

31. **THINK** This problem involves a multi-loop circuit. We first simplify the circuit by finding the equivalent resistance. We then apply Kirchhoff's loop rule to calculate the current in the loop, and the potentials at various points in the circuit.

EXPRESS We first reduce the parallel pair of identical 2.0-Ω resistors (on the right side) to $R' = 1.0\ \Omega$, and we reduce the series pair of identical 2.0-Ω resistors (on the upper left side) to $R'' = 4.0\ \Omega$. With R denoting the 2.0-Ω resistor at the bottom (between V_2 and V_1), we now have three resistors in series which are equivalent to

$$R_{\text{eq}} = R + R' + R'' = 7.0\ \Omega$$

across which the voltage is $\varepsilon_2 - \varepsilon_1$ = 7.0 V (by the loop rule, this is 12 V – 5.0 V), implying that the current is

$$i = \frac{\varepsilon_2 - \varepsilon_1}{R_{\text{eq}}} = \frac{7.0\ \text{V}}{7.0\ \Omega} = 1.0\ \text{A}.$$

The direction of i is upward in the right-hand emf device. Knowing i allows us to solve for V_1 and V_2.

ANALYZE (a) The voltage across R' is (1.0 A)(1.0 Ω) = 1.0 V, which means that (examining the right side of the circuit) the voltage difference between *ground* and V_1 is 12 V – 1.0 V = 11 V. Noting the orientation of the battery, we conclude that $V_1 = -11\ \text{V}$.

(b) The voltage across R'' is (1.0 A)(4.0 Ω) = 4.0 V, which means that (examining the left side of the circuit) the voltage difference between *ground* and V_2 is 5.0 V + 4.0 V = 9.0 V. Noting the orientation of the battery, we conclude V_2 = –9.0 V.

LEARN The potential difference between points 1 and 2 is

$$V_2 - V_1 = -9.0\ \text{V} - (-11.0\ \text{V}) = 2.0\ \text{V},$$

which is equal to $iR = (1.0\ \text{A})(2.0\ \Omega) = 2.0\ \text{V}.$

32. (a) For typing convenience, we denote the emf of battery 2 as V_2 and the emf of battery 1 as V_1. The loop rule (examining the left-hand loop) gives $V_2 + i_1 R_1 - V_1 = 0$. Since V_1 is held constant while V_2 and i_1 vary, we see that this expression (for large enough V_2) will result in a negative value for i_1, so the downward sloping line (the line that is dashed in Fig. 27-43(b)) must represent i_1. It appears to be zero when V_2 = 6 V. With i_1 = 0, our loop rule gives $V_1 = V_2$, which implies that V_1 = 6.0 V.

(b) At V_2 = 2 V (in the graph) it appears that i_1 = 0.2 A. Now our loop rule equation (with the conclusion about V_1 found in part (a)) gives R_1 = 20 Ω.

(c) Looking at the point where the upward-sloping i_2 line crosses the axis (at $V_2 = 4$ V), we note that $i_1 = 0.1$ A there and that the loop rule around the right-hand loop should give

$$V_1 - i_1 R_1 = i_1 R_2$$

when $i_1 = 0.1$ A and $i_2 = 0$. This leads directly to $R_2 = 40\ \Omega$.

33. First, we note in V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5 + R_6) = (1.40\text{ A})(8.00\ \Omega + 4.00\ \Omega) = 16.8\text{ V}.$$

The current through R_4 is then equal to $i_4 = V_4/R_4 = 16.8\text{ V}/(16.0\ \Omega) = 1.05$ A.

By the junction rule, the current in R_2 is

$$i_2 = i_4 + i_6 = 1.05\text{ A} + 1.40\text{ A} = 2.45\text{ A},$$

so its voltage is $V_2 = (2.00\ \Omega)(2.45\text{ A}) = 4.90$ V.

The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4 = 21.7$ V (implying that the current through it is $i_3 = V_3/(2.00\ \Omega) = 10.85$ A).

The junction rule now gives the current in R_1 as

$$i_1 = i_2 + i_3 = 2.45\text{ A} + 10.85\text{ A} = 13.3\text{ A},$$

implying that the voltage across it is $V_1 = (13.3\text{ A})(2.00\ \Omega) = 26.6$ V. Therefore, by the loop rule,

$$\varepsilon = V_1 + V_3 = 26.6\text{ V} + 21.7\text{ V} = 48.3\text{ V}.$$

34. (a) By the loop rule, it remains the same. This question is aimed at student conceptualization of voltage; many students apparently confuse the concepts of voltage and current and speak of "voltage going through" a resistor – which would be difficult to rectify with the conclusion of this problem.

(b) The loop rule still applies, of course, but (by the junction rule and Ohm's law) the voltages across R_1 and R_3 (which were the same when the switch was open) are no longer equal. More current is now being supplied by the battery, which means more current is in R_3, implying its voltage drop has increased (in magnitude). Thus, by the loop rule (since the battery voltage has not changed) the voltage across R_1 has decreased a corresponding amount. When the switch was open, the voltage across R_1 was 6.0 V (easily seen from symmetry considerations). With the switch closed, R_1 and R_2 are equivalent (by Eq. 27-24) to 3.0 Ω, which means the total load on the battery is 9.0 Ω. The current therefore is 1.33 A, which implies that the voltage drop across R_3 is 8.0 V. The loop rule then tells us that the voltage drop across R_1 is 12 V – 8.0 V = 4.0 V. This is a decrease of 2.0 volts from the value it had when the switch was open.

35. (a) The symmetry of the problem allows us to use i_2 as the current in *both* of the R_2 resistors and i_1 for the R_1 resistors. We see from the junction rule that $i_3 = i_1 - i_2$. There are only two independent loop rule equations:

$$\varepsilon - i_2 R_2 - i_1 R_1 = 0$$
$$\varepsilon - 2 i_1 R_1 - (i_1 - i_2) R_3 = 0$$

where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find $i_1 = 0.002625$ A, $i_2 = 0.00225$ A and $i_3 = i_1 - i_2 = 0.000375$ A. Therefore,

$$V_A - V_B = i_1 R_1 = 5.25 \text{ V}.$$

(b) It follows also that $V_B - V_C = i_3 R_3 = 1.50$ V.

(c) We find $V_C - V_D = i_1 R_1 = 5.25$ V.

(d) Finally, $V_A - V_C = i_2 R_2 = 6.75$ V.

36. (a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\varepsilon_1 - i_2 R_2 - (i_2 + i_3) R_1 = 0$$
$$\varepsilon_2 - i_3 R_3 - (i_2 + i_3) R_1 = 0.$$

Solving, we find $i_2 = 0.0109$ A (rightward, as was assumed in writing the equations as we did), $i_3 = 0.0273$ A (leftward), and $i_1 = i_2 + i_3 = 0.0382$ A (downward).

(b) The direction is downward. See the results in part (a).

(c) $i_2 = 0.0109$ A . See the results in part (a).

(d) The direction is rightward. See the results in part (a).

(e) $i_3 = 0.0273$ A. See the results in part (a).

(f) The direction is leftward. See the results in part (a).

(g) The voltage across R_1 equals V_A: (0.0382 A)(100 Ω) = +3.82 V.

37. The voltage difference across R_3 is $V_3 = \varepsilon R' /(R' + 2.00\ \Omega)$, where

$$R' = (5.00\ \Omega R)/(5.00\ \Omega + R_3).$$

Thus,

$$P_3 = \frac{V_3^2}{R_3} = \frac{1}{R_3}\left(\frac{\varepsilon R'}{R' + 2.00\,\Omega}\right)^2 = \frac{1}{R_3}\left(\frac{\varepsilon}{1 + 2.00\,\Omega/R'}\right)^2 = \frac{\varepsilon^2}{R_3}\left[1 + \frac{(2.00\,\Omega)(5.00\,\Omega + R)}{(5.00\,\Omega)R_3}\right]^{-2}$$
$$\equiv \frac{\varepsilon^2}{f(R_3)}$$

where we use the equivalence symbol ≡ to define the expression $f(R_3)$. To maximize P_3 we need to minimize the expression $f(R_3)$. We set

$$\frac{df(R_3)}{dR_3} = -\frac{4.00\,\Omega^2}{R_3^{\ 2}} + \frac{49}{25} = 0$$

to obtain $R_3 = \sqrt{(4.00\,\Omega^2)(25)/49} = 1.43\,\Omega$.

38. (a) The voltage across R_3 = 6.0 Ω is $V_3 = iR_3$= (6.0 A)(6.0 Ω) = 36 V. Now, the voltage across R_1 = 2.0 Ω is

$$(V_A - V_B) - V_3 = 78 - 36 = 42 \text{ V},$$

which implies the current is i_1 = (42 V)/(2.0 Ω) = 21 A. By the junction rule, then, the current in R_2 = 4.0 Ω is

$$i_2 = i_1 - i = 21 \text{ A} - 6.0 \text{ A} = 15 \text{ A}.$$

The total power dissipated by the resistors is (using Eq. 26-27)

$$i_1^2\,(2.0\ \Omega) + i_2^2\,(4.0\ \Omega) + i^2\,(6.0\ \Omega) = 1998 \text{ W} \approx 2.0 \text{ kW}.$$

By contrast, the power supplied (externally) to this section is $P_A = i_A\,(V_A - V_B)$ where $i_A = i_1$ = 21 A. Thus, P_A = 1638 W. Therefore, the "Box" must be providing energy.

(b) The rate of supplying energy is (1998 − 1638)W = 3.6×10^2 W.

39. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let i be the current in either battery and take it to be positive to the left. According to the junction rule the current in R is $2i$ and it is positive to the right. The loop rule applied to either loop containing a battery and R yields

$$\varepsilon - ir - 2iR = 0 \quad\Rightarrow\quad i = \frac{\varepsilon}{r + 2R}.$$

The power dissipated in R is

$$P = (2i)^2 R = \frac{4\varepsilon^2 R}{(r+2R)^2}.$$

We find the maximum by setting the derivative with respect to R equal to zero. The derivative is

$$\frac{dP}{dR} = \frac{4\varepsilon^2}{(r+2R)^3} - \frac{16\varepsilon^2 R}{(r+2R)^3} = \frac{4\varepsilon^2 (r-2R)}{(r+2R)^3}.$$

The derivative vanishes (and P is a maximum) if $R = r/2$. With $r = 0.300\ \Omega$, we have $R = 0.150\ \Omega$.

(b) We substitute $R = r/2$ into $P = 4\varepsilon^2 R/(r + 2R)^2$ to obtain

$$P_{\max} = \frac{4\varepsilon^2 (r/2)}{[r+2(r/2)]^2} = \frac{\varepsilon^2}{2r} = \frac{(12.0\ \text{V})^2}{2(0.300\ \Omega)} = 240\ \text{W}.$$

40. (a) By symmetry, when the two batteries are connected in parallel the current i going through either one is the same. So from $\varepsilon = ir + (2i)R$ with $r = 0.200\ \Omega$ and $R = 2.00r$, we get

$$i_R = 2i = \frac{2\varepsilon}{r+2R} = \frac{2(12.0\text{V})}{0.200\Omega + 2(0.400\Omega)} = 24.0\ \text{A}.$$

(b) When connected in series $2\varepsilon - i_R r - i_R r - i_R R = 0$, or $i_R = 2\varepsilon/(2r + R)$. The result is

$$i_R = 2i = \frac{2\varepsilon}{2r+R} = \frac{2(12.0\text{V})}{2(0.200\Omega) + 0.400\Omega} = 30.0\ \text{A}.$$

(c) They are in series arrangement, since $R > r$.

(d) If $R = r/2.00$, then for parallel connection,

$$i_R = 2i = \frac{2\varepsilon}{r+2R} = \frac{2(12.0\text{V})}{0.200\Omega + 2(0.100\Omega)} = 60.0\ \text{A}.$$

(e) For series connection, we have

$$i_R = 2i = \frac{2\varepsilon}{2r+R} = \frac{2(12.0\text{V})}{2(0.200\Omega) + 0.100\Omega} = 48.0\ \text{A}.$$

(f) They are in parallel arrangement, since $R < r$.

41. We first find the currents. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is upward. The junction rule produces

$$i_1 + i_2 + i_3 = 0\ .$$

The loop rule applied to the left-hand loop produces

$$\varepsilon_1 - i_1 R_1 + i_3 R_3 = 0$$

and applied to the right-hand loop produces

$$\varepsilon_2 - i_2 R_2 + i_3 R_3 = 0.$$

We substitute $i_3 = -i_2 - i_1$, from the first equation, into the other two to obtain

$$\varepsilon_1 - i_1 R_1 - i_2 R_3 - i_1 R_3 = 0$$

and

$$\varepsilon_2 - i_2 R_2 - i_2 R_3 - i_1 R_3 = 0.$$

Solving the above equations yield

$$i_1 = \frac{\varepsilon_1(R_2 + R_3) - \varepsilon_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(3.00\text{ V})(2.00\,\Omega + 5.00\,\Omega) - (1.00\text{ V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = 0.421\text{ A}.$$

$$i_2 = \frac{\varepsilon_2(R_1 + R_3) - \varepsilon_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(1.00\text{ V})(4.00\,\Omega + 5.00\,\Omega) - (3.00\text{ V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = -0.158\text{ A}.$$

$$i_3 = -\frac{\varepsilon_2 R_1 + \varepsilon_1 R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3} = -\frac{(1.00\text{ V})(4.00\,\Omega) + (3.00\text{ V})(2.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = -0.263\text{ A}.$$

Note that the current i_3 in R_3 is actually downward and the current i_2 in R_2 is to the right. The current i_1 in R_1 is to the right.

(a) The power dissipated in R_1 is $P_1 = i_1^2 R_1 = (0.421\text{ A})^2 (4.00\,\Omega) = 0.709\text{ W}.$

(b) The power dissipated in R_2 is $P_2 = i_2^2 R_2 = (-0.158\text{ A})^2 (2.00\,\Omega) = 0.0499\text{ W} \approx 0.050\text{ W}.$

(c) The power dissipated in R_3 is $P_3 = i_3^2 R_3 = (-0.263\text{ A})^2 (5.00\,\Omega) = 0.346\text{ W}.$

(d) The power supplied by ε_1 is $i_3\varepsilon_1$ = (0.421 A)(3.00 V) = 1.26 W.

(e) The power "supplied" by ε_2 is $i_2\varepsilon_2$ = (–0.158 A)(1.00 V) = –0.158 W. The negative sign indicates that ε_2 is actually absorbing energy from the circuit.

42. The equivalent resistance in Fig. 27-52 (with n parallel resistors) is

$$R_{\text{eq}} = R + \frac{R}{n} = \left(\frac{n+1}{n}\right)R \ .$$

The current in the battery in this case should be

$$i_n = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n}{n+1}\frac{V_{\text{battery}}}{R} \ .$$

If there were n +1 parallel resistors, then

$$i_{n+1} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n+1}{n+2}\frac{V_{\text{battery}}}{R} \ .$$

For the relative increase to be 0.0125 (= 1/80), we require

$$\frac{i_{n+1} - i_n}{i_n} = \frac{i_{n+1}}{i_n} - 1 = \frac{(n+1)/(n+2)}{n/(n+1)} - 1 = \frac{1}{80} \ .$$

This leads to the second-degree equation $n^2 + 2n - 80 = (n + 10)(n - 8) = 0$.

Clearly the only physically interesting solution to this is n = 8. Thus, there are eight resistors in parallel (as well as that resistor in series shown toward the bottom) in Fig. 27-52.

43. Let the resistors be divided into groups of n resistors each, with all the resistors in the same group connected in series. Suppose there are m such groups that are connected in parallel with each other. Let R be the resistance of any one of the resistors. Then the equivalent resistance of any group is nR, and R_{eq}, the equivalent resistance of the whole array, satisfies

$$\frac{1}{R_{\text{eq}}} = \sum_{1}^{m} \frac{1}{nR} = \frac{m}{nR} .$$

Since the problem requires R_{eq} = 10 Ω = R, we must select $n = m$. Next we make use of Eq. 27-16. We note that the current is the same in every resistor and there are $n \cdot m = n^2$ resistors, so the maximum total power that can be dissipated is $P_{\text{total}} = n^2 P$, where $P = 1.0$ W is the maximum power that can be dissipated by any one of the resistors. The problem demands $P_{total} \geq 5.0P$, so n^2 must be at least as large as 5.0. Since n must be an integer, the smallest it can be is 3. The least number of resistors is $n^2 = 9$.

44. (a) Resistors R_2, R_3, and R_4 are in parallel. By finding a common denominator and simplifying, the equation $1/R = 1/R_2 + 1/R_3 + 1/R_4$ gives an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50.0\Omega)(50.0\Omega)(75.0\Omega)}{(50.0\Omega)(50.0\Omega)+(50.0\Omega)(75.0\Omega)+(50.0\Omega)(75.0\Omega)}$$
$$= 18.8\Omega.$$

Thus, considering the series contribution of resistor R_1, the equivalent resistance for the network is $R_{eq} = R_1 + R = 100\ \Omega + 18.8\ \Omega = 118.8\ \Omega \approx 119\ \Omega$.

(b) $i_1 = \varepsilon/R_{eq} = 6.0\ V/(118.8\ \Omega) = 5.05 \times 10^{-2}$ A.

(c) $i_2 = (\varepsilon - V_1)/R_2 = (\varepsilon - i_1R_1)/R_2 = [6.0V - (5.05 \times 10^{-2}\ A)(100\Omega)]/50\ \Omega = 1.90 \times 10^{-2}$ A.

(d) $i_3 = (\varepsilon - V_1)/R_3 = i_2R_2/R_3 = (1.90 \times 10^{-2}\ A)(50.0\ \Omega/50.0\ \Omega) = 1.90 \times 10^{-2}$ A.

(e) $i_4 = i_1 - i_2 - i_3 = 5.05 \times 10^{-2}\ A - 2(1.90 \times 10^{-2}\ A) = 1.25 \times 10^{-2}$ A.

45. (a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\varepsilon_2 = \varepsilon_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through ε_2 and ε_3 are the same: $i_2 = i_3 = i$. Therefore, the current through ε_1 is $i_1 = 2i$. Then from $V_b - V_a = \varepsilon_2 - iR_2 = \varepsilon_1 + (2R_1)(2i)$ we get

$$i = \frac{\varepsilon_2 - \varepsilon_1}{4R_1 + R_2} = \frac{4.0\,V - 2.0\,V}{4(1.0\,\Omega) + 2.0\,\Omega} = 0.33\,A.$$

Therefore, the current through ε_1 is $i_1 = 2i = 0.67$ A.

(b) The direction of i_1 is downward.

(c) The current through ε_2 is $i_2 = 0.33$ A.

(d) The direction of i_2 is upward.

(e) From part (a), we have $i_3 = i_2 = 0.33$ A.

(f) The direction of i_3 is also upward.

(g) $V_a - V_b = -iR_2 + \varepsilon_2 = -(0.333\ A)(2.0\ \Omega) + 4.0\ V = 3.3$ V.

46. (a) When $R_3 = 0$ all the current passes through R_1 and R_3 and avoids R_2 altogether. Since that value of the current (through the battery) is 0.006 A (see Fig. 27-55(b)) for $R_3 = 0$ then (using Ohm's law)

$$R_1 = (12 \text{ V})/(0.006 \text{ A}) = 2.0\times10^3\,\Omega.$$

(b) When $R_3 = \infty$ all the current passes through R_1 and R_2 and avoids R_3 altogether. Since that value of the current (through the battery) is 0.002 A (stated in problem) for $R_3 = \infty$ then (using Ohm's law)

$$R_2 = (12 \text{ V})/(0.002 \text{ A}) - R_1 = 4.0\times10^3\,\Omega.$$

47. **THINK** The copper wire and the aluminum sheath are connected in parallel, so the potential difference is the same for them.

EXPRESS Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, i_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is $R_C = \rho_C L/\pi a^2$, and the resistance of the aluminum sheath is $R_A = \rho_A L/\pi(b^2 - a^2)$. We substitute these expressions into $i_C R_C = i_A R_A$, and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2}.$$

We solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. We find

$$i_C = \frac{r_C^2 \rho_C i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2 \rho_A}$$

and

$$i_A = \frac{\left(r_A^2 - r_C^2\right)\rho_C\, i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2 \rho_A}.$$

ANALYZE (a) The denominators are the same and each has the value

$$\begin{aligned}\left(b^2 - a^2\right)\rho_C + a^2 \rho_A &= \left[\left(0.380\times10^{-3}\text{ m}\right)^2 - \left(0.250\times10^{-3}\text{ m}\right)^2\right]\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right) \\ &\quad + \left(0.250\times10^{-3}\text{ m}\right)^2\left(2.75\times10^{-8}\,\Omega\cdot\text{m}\right) \\ &= 3.10\times10^{-15}\,\Omega\cdot\text{m}^3.\end{aligned}$$

Thus,

$$i_C = \frac{(0.250\times10^{-3}\text{ m})^2(2.75\times10^{-8}\,\Omega\cdot\text{m})(2.00\text{ A})}{3.10\times10^{-15}\,\Omega\cdot\text{m}^3} = 1.11\text{ A}.$$

(b) Similarly,

$$i_A = \frac{\left[(0.380\times10^{-3}\text{ m})^2-(0.250\times10^{-3}\text{ m})^2\right](1.69\times10^{-8}\,\Omega\cdot\text{m})(2.00\text{A})}{3.10\times10^{-15}\,\Omega\cdot\text{m}^3} = 0.893\text{A}.$$

(c) Consider the copper wire. If V is the potential difference, then the current is given by $V = i_C R_C = i_C\rho_C L/\pi a^2$, so the length of the composite wire is

$$L = \frac{\pi a^2 V}{i_C\rho_C} = \frac{(\pi)(0.250\times10^{-3}\text{ m})^2(12.0\text{ V})}{(1.11\text{ A})(1.69\times10^{-8}\,\Omega\cdot\text{m})} = 126\text{ m}.$$

LEARN The potential difference can also be written as $V = i_A R_A = i_A\rho_A L/\pi(b^2 - a^2)$. Thus,

$$L = \frac{\pi(b^2-a^2)V}{i_A\rho_A} = \frac{\pi\left[(0.380\times10^{-3}\text{ m})^2-(0.250\times10^{-3}\text{ m})^2\right](12.0\text{ V})}{(0.893\text{ A})(2.75\times10^{-8}\,\Omega\cdot\text{m})} = 126\text{ m},$$

in agreement with the result found in (c).

48. (a) We use $P = \varepsilon^2/R_{eq}$, where

$$R_{eq} = 7.00\,\Omega + \frac{(12.0\,\Omega)(4.00\,\Omega)R}{(12.0\,\Omega)(4.0\,\Omega)+(12.0\,\Omega)R+(4.00\,\Omega)R}.$$

Put $P = 60.0$ W and $\varepsilon = 24.0$ V and solve for R: $R = 19.5\,\Omega$.

(b) Since $P \propto R_{eq}$, we must minimize R_{eq}, which means $R = 0$.

(c) Now we must maximize R_{eq}, or set $R = \infty$.

(d) Since $R_{eq,\,min} = 7.00\,\Omega$, $P_{max} = \varepsilon^2/R_{eq,\,min} = (24.0\text{ V})^2/7.00\,\Omega = 82.3$ W.

(e) Since $R_{eq,\,max} = 7.00\,\Omega + (12.0\,\Omega)(4.00\,\Omega)/(12.0\,\Omega + 4.00\,\Omega) = 10.0\,\Omega$,

$$P_{min} = \varepsilon^2/R_{eq,\,max} = (24.0\text{ V})^2/10.0\,\Omega = 57.6\text{ W}.$$

49. (a) The current in R_1 is given by

$$i_1 = \frac{\varepsilon}{R_1 + R_2 R_3 / (R_2 + R_3)} = \frac{5.0\text{ V}}{2.0\Omega + (4.0\Omega)(6.0\Omega)/(4.0\Omega + 6.0\Omega)} = 1.14\text{ A}.$$

Thus,

$$i_3 = \frac{\varepsilon - V_1}{R_3} = \frac{\varepsilon - i_1 R_1}{R_3} = \frac{5.0\text{ V} - (1.14\text{ A})(2.0\Omega)}{6.0\Omega} = 0.45\text{ A}.$$

(b) We simply interchange subscripts 1 and 3 in the equation above. Now

$$i_3 = \frac{\varepsilon}{R_3 + \left(R_2 R_1 / (R_2 + R_1)\right)} = \frac{5.0\text{V}}{6.0\Omega + \left((2.0\Omega)(4.0\Omega)/(2.0\Omega + 4.0\Omega)\right)} = 0.6818\text{A}$$

and

$$i_1 = \frac{5.0\text{ V} - (0.6818\text{ A})(6.0\Omega)}{2.0\Omega} = 0.45\text{ A},$$

the same as before.

50. Note that there is no voltage drop across the ammeter. Thus, the currents in the bottom resistors are the same, which we call i (so the current through the battery is $2i$ and the voltage drop across each of the bottom resistors is iR). The resistor network can be reduced to an equivalence of

$$R_{\text{eq}} = \frac{(2R)(R)}{2R + R} + \frac{(R)(R)}{R + R} = \frac{7}{6}R$$

which means that we can determine the current through the battery (and also through each of the bottom resistors):

$$2i = \frac{\varepsilon}{R_{\text{eq}}} \Rightarrow i = \frac{\varepsilon}{2R_{\text{eq}}} = \frac{\varepsilon}{2(7R/6)} = \frac{3\varepsilon}{7R}.$$

By the loop rule (going around the left loop, which includes the battery, resistor $2R$, and one of the bottom resistors), we have

$$\varepsilon - i_{2R}(2R) - iR = 0 \Rightarrow i_{2R} = \frac{\varepsilon - iR}{2R}.$$

Substituting $i = 3\varepsilon/7R$, this gives $i_{2R} = 2\varepsilon/7R$. The difference between i_{2R} and i is the current through the ammeter. Thus,

$$i_{\text{ammeter}} = i - i_{2R} = \frac{3\varepsilon}{7R} - \frac{2\varepsilon}{7R} = \frac{\varepsilon}{7R} \quad \Rightarrow \quad \frac{i_{\text{ammeter}}}{\varepsilon / R} = \frac{1}{7} = 0.143.$$

51. Since the current in the ammeter is i, the voltmeter reading is

$$V' = V + i\,R_A = i\,(R + R_A),$$

or $R = V'/i - R_A = R' - R_A$, where $R' = V'/i$ is the apparent reading of the resistance. Now, from the lower loop of the circuit diagram, the current through the voltmeter is $i_V = \varepsilon/(R_{eq} + R_0)$, where

$$\frac{1}{R_{eq}} = \frac{1}{R_V} + \frac{1}{R_A + R} \quad \Rightarrow \quad R_{eq} = \frac{R_V(R + R_A)}{R_V + R + R_A} = \frac{(300\,\Omega)(85.0\,\Omega + 3.00\,\Omega)}{300\,\Omega + 85.0\,\Omega + 3.00\,\Omega} = 68.0\,\Omega.$$

The voltmeter reading is then

$$V' = i_V R_{eq} = \frac{\varepsilon R_{eq}}{R_{eq} + R_0} = \frac{(12.0\text{ V})(68.0\ \Omega)}{68.0\ \Omega + 100\Omega} = 4.86\text{ V}.$$

(a) The ammeter reading is

$$i = \frac{V'}{R + R_A} = \frac{4.86\text{ V}}{85.0\ \Omega + 3.00\Omega} = 0.0552\text{ A}.$$

(b) As shown above, the voltmeter reading is $V' = 4.86$ V.

(c) $R' = V'/i = 4.86\text{ V}/(5.52 \times 10^{-2}\text{ A}) = 88.0\ \Omega$.

(d) Since $R = R' - R_A$, if R_A is decreased, the difference between R' and R decreases. In fact, when $R_A = 0$, $R' = R$.

52. (a) Since $i = \varepsilon/(r + R_{ext})$ and $i_{max} = \varepsilon/r$, we have $R_{ext} = R(i_{max}/i - 1)$ where $r = 1.50\text{ V}/1.00\text{ mA} = 1.50 \times 10^3\ \Omega$. Thus,

$$R_{ext} = (1.5 \times 10^3\ \Omega)(1/0.100 - 1) = 1.35 \times 10^4\ \Omega.$$

(b) $R_{ext} = (1.5 \times 10^3\ \Omega)(1/0.500 - 1) = 1.5 \times 10^3\ \Omega$.

(c) $R_{ext} = (1.5 \times 10^3\ \Omega)(1/0.900 - 1) = 167\ \Omega$.

(d) Since $r = 20.0\ \Omega + R$, $R = 1.50 \times 10^3\ \Omega - 20.0\ \Omega = 1.48 \times 10^3\ \Omega$.

53. The current in R_2 is i. Let i_1 be the current in R_1 and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. We apply the loop rule to the left-hand loop:

$$\varepsilon - iR_2 - i_1 R_1 - ir = 0.$$

Similarly, applying the loop rule to the right-hand loop gives

$$i_1R_1 - (i - i_1)R_V = 0.$$

The second equation yields

$$i = \frac{R_1 + R_V}{R_V} i_1.$$

We substitute this into the first equation to obtain

$$\varepsilon - \frac{(R_2 + r)(R_1 + R_V)}{R_V} i_1 + R_1 i_1 = 0.$$

This has the solution

$$i_1 = \frac{\varepsilon R_V}{(R_2 + r)(R_1 + R_V) + R_1 R_V}.$$

The reading on the voltmeter is

$$i_1R_1 = \frac{\varepsilon R_V R_1}{(R_2 + r)\,(R_1 + R_V) + R_1 R_V} = \frac{(3.0\text{V})\,(5.0\times10^3\,\Omega)\,(250\Omega)}{(300\Omega + 100\Omega)(250\Omega + 5.0\times10^3\,\Omega) + (250\Omega)(5.0\times10^3\,\Omega)}$$
$$= 1.12 \text{ V}.$$

The current in the absence of the voltmeter can be obtained by taking the limit as R_V becomes infinitely large. Then

$$i_1R_1 = \frac{\varepsilon R_1}{R_1 + R_2 + r} = \frac{(3.0\,\text{V})(250\,\Omega)}{250\ \Omega + 300\ \Omega + 100\,\Omega} = 1.15\,\text{V}.$$

The fractional error is (1.12 – 1.15)/(1.15) = –0.030, or –3.0%.

54. (a) $\varepsilon = V + ir$ = 12 V + (10.0 A) (0.0500 Ω) = 12.5 V.

(b) Now $\varepsilon = V' + (i_{\text{motor}} + 8.00\text{ A})r$, where

$$V' = i'_A R_{\text{light}} = (8.00\text{ A})\,(12.0\text{ V}/10\text{ A}) = 9.60\text{ V}.$$

Therefore,

$$i_{\text{motor}} = \frac{\varepsilon - V'}{r} - 8.00\,\text{A} = \frac{12.5\,\text{V} - 9.60\,\text{V}}{0.0500\,\Omega} - 8.00\,\text{A} = 50.0\,\text{A}.$$

55. Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$(R_1+R_2)i_1=(R_x+R_s)\frac{R_1}{R_s}i_1 \;\Rightarrow\; R_x=\frac{R_2R_s}{R_1}.$$

56. The currents in R and R_V are i and $i'-i$, respectively. Since $V = iR = (i'-i)R_V$ we have, by dividing both sides by V, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$. Thus,

$$\frac{1}{R}=\frac{1}{R'}-\frac{1}{R_V} \;\Rightarrow\; R'=\frac{RR_V}{R+R_V}.$$

The equivalent resistance of the circuit is $R_{\text{eq}}=R_A+R_0+R'=R_A+R_0+\frac{RR_V}{R+R_V}$.

(a) The ammeter reading is

$$i'=\frac{\varepsilon}{R_{\text{eq}}}=\frac{\varepsilon}{R_A+R_0+R_V\,R/(R+R_V)}=\frac{12.0\text{V}}{3.00\Omega+100\Omega+(300\Omega)\,(85.0\Omega)/(300\Omega+85.0\Omega)}$$
$$=7.09\times10^{-2}\text{ A}.$$

(b) The voltmeter reading is

$$V=\varepsilon-i'(R_A+R_0)=12.0\text{ V}-(0.0709\text{ A})(103.00\ \Omega)=4.70\text{ V}.$$

(c) The apparent resistance is $R' = V/i' = 4.70\text{ V}/(7.09\times10^{-2}\text{ A}) = 66.3\ \Omega$.

(d) If R_V is increased, the difference between R and R' decreases. In fact, $R'\to R$ as $R_V\to\infty$.

57. Here we denote the battery emf as V. Then the requirement stated in the problem that the resistor voltage be equal to the capacitor voltage becomes $iR = V_{\text{cap}}$, or

$$Ve^{-t/RC}=V(1-e^{-t/RC})$$

where Eqs. 27-34 and 27-35 have been used. This leads to $t = RC\ln 2$, or $t =$ 0.208 ms.

58. (a) $\tau = RC = (1.40\times10^{6}\ \Omega)(1.80\times10^{-6}\text{ F}) = 2.52$ s.

(b) $q_o = \varepsilon C = (12.0\text{ V})(1.80\ \mu\text{F}) = 21.6\ \mu\text{C}$.

(c) The time t satisfies $q = q_0(1-e^{-t/RC})$, or

$$t=RC\ln\left(\frac{q_0}{q_0-q}\right)=(2.52\text{ s})\ln\left(\frac{21.6\,\mu\text{C}}{21.6\,\mu\text{C}-16.0\,\mu\text{C}}\right)=3.40\text{ s}.$$

59. **THINK** We have an *RC* circuit that is being charged. When fully charged, the charge on the capacitor is equal to $C\varepsilon$.

EXPRESS During charging, the charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where C is the capacitance, ε is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{eq} = C\varepsilon$, so we require $q = 0.99q_{eq} = 0.99C\varepsilon$.

ANALYZE The time required to reach 99% of its final charge is given by

$$0.99 = 1 - e^{-t/\tau}.$$

Thus, $e^{-t/\tau} = 0.01$. Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.61$ or $t = 4.61\tau$.

LEARN The corresponding current in a charging capacitor is given by

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R}e^{-t/\tau}.$$

The current has an initial value ε/R but decays exponentially to zero as the capacitor becomes fully charged. The plots of $q(t)$ and $i(t)$ are shown in Fig. 27-16 of the text.

60. (a) We use $q = q_0 e^{-t/\tau}$, or $t = \tau \ln(q_0/q)$, where $\tau = RC$ is the capacitive time constant. Thus,

$$t_{1/3} = \tau \ln\left(\frac{q_0}{2q_0/3}\right) = \tau \ln\left(\frac{3}{2}\right) = 0.41\tau \;\Rightarrow\; \frac{t_{1/3}}{\tau} = 0.41.$$

(b) $t_{2/3} = \tau \ln\left(\frac{q_0}{q_0/3}\right) = \tau \ln 3 = 1.1\tau \;\Rightarrow\; \frac{t_{2/3}}{\tau} = 1.1.$

61. (a) The voltage difference V across the capacitor is $V(t) = \varepsilon(1 - e^{-t/RC})$. At $t = 1.30\ \mu s$ we have $V(t) = 5.00$ V, so $5.00\text{ V} = (12.0\text{ V})(1 - e^{-1.30\ \mu s/RC})$, which gives

$$\tau = (1.30\ \mu\text{s})/\ln(12/7) = 2.41\ \mu\text{s}.$$

(b) The capacitance is $C = \tau/R = (2.41\ \mu\text{s})/(15.0\text{ k}\Omega) = 161$ pF.

62. The time it takes for the voltage difference across the capacitor to reach V_L is given by $V_L = \varepsilon\left(1 - e^{-t/RC}\right)$. We solve for R:

$$R = \frac{t}{C \ln \varepsilon / (\varepsilon - V_L)} = \frac{0.500\,\text{s}}{(0.150 \times 10^{-6}\,\text{F}) \ln 95.0\,\text{V} / (95.0\,\text{V} - 72.0\,\text{V})} = 2.35 \times 10^{6}\,\Omega$$

where we used t = 0.500 s given (implicitly) in the problem.

63. **THINK** We have a multi-loop circuit with a capacitor that's being charged. Since at t = 0 the capacitor is completely uncharged, the current in the capacitor branch is as it would be if the capacitor were replaced by a wire.

EXPRESS Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0.$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R.

ANALYZE (a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2(1.2 \times 10^{3}\,\text{V})}{3(0.73 \times 10^{6}\,\Omega)} = 1.1 \times 10^{-3}\,\text{A},$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2\times10^{3}\,\text{V}}{3\left(0.73\times10^{6}\,\Omega\right)} = 5.5\times10^{-4}\,\text{A},$

(c) and $i_3 = i_2 = 5.5\times10^{-4}\,\text{A}.$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields $\varepsilon - i_1 R_1 - i_1 R_2 = 0$.

(d) The solution is $i_1 = \dfrac{\varepsilon}{2R} = \dfrac{1.2\times10^{3}\,\text{V}}{2\left(0.73\times10^{6}\,\Omega\right)} = 8.2\times10^{-4}\,\text{A}$

(e) and $i_2 = i_1 = 8.2\times10^{-4}\,\text{A}.$

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1 R - i_2 R = 0$$

$$-\frac{q}{C} - i_3 R + i_2 R = 0.$$

We use the first equation to substitute for i_1 in the second and obtain

$$\varepsilon - 2i_2 R - i_3 R = 0.$$

Thus $i_2 = (\varepsilon - i_3 R)/2R$. We substitute this expression into the third equation above to obtain

$$-(q/C) - (i_3 R) + (\varepsilon/2) - (i_3 R/2) = 0.$$

Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2}.$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{C\varepsilon}{2}\left(1 - e^{-2t/3RC}\right).$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R} e^{-2t/3RC}.$$

The current in the center branch is

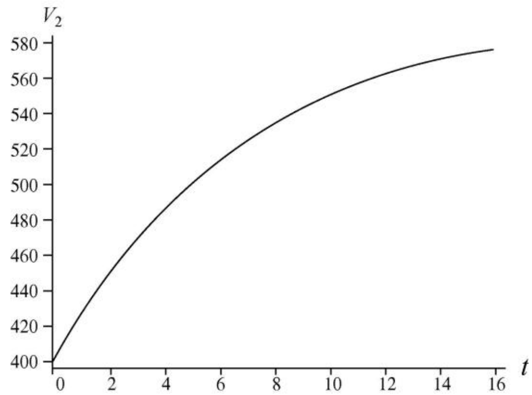
$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R} e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

and the potential difference across R_2 is $V_2(t) = i_2 R = \frac{\varepsilon}{6}\left(3 - e^{-2t/3RC}\right)$.

(g) For $t = 0, e^{-2t/3RC} = 1$ and $V_2 = \varepsilon/3 = \left(1.2\times10^3\,\text{V}\right)/3 = 4.0\times10^2\,\text{V}$.

(h) For $t = \infty, e^{-2t/3RC} \to 0$ and $V_2 = \varepsilon/2 = \left(1.2\times20^3\,\text{V}\right)/2 = 6.0\times10^2\,\text{V}$.

(i) A plot of V_2 as a function of time is shown in the following graph.



LEARN A capacitor that is being charged initially behaves like an ordinary connecting wire relative to the charging current. However, a long time later after it's fully charged, it acts like a broken wire.

64. (a) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0\, e^{-t/\tau}$, this means $V = V_0\, e^{-t/\tau}$ where V_0 is the initial potential difference. We solve for the time constant τ by dividing by V_0 and taking the natural logarithm:

$$\tau = -\frac{t}{\ln\left(V/V_0\right)} = -\frac{10.0\,\text{s}}{\ln\left[\left(1.00\,\text{V}\right)/\left(100\,\text{V}\right)\right]} = 2.17\,\text{s}.$$

(b) At $t = 17.0$ s, $t/\tau = (17.0\text{ s})/(2.17\text{ s}) = 7.83$, so

$$V = V_0 e^{-t/\tau} = \left(100\,\text{V}\right)e^{-7.83} = 3.96\times10^{-2}\,\text{V}\,.$$

65. In the steady state situation, the capacitor voltage will equal the voltage across R_2 = 15 kΩ:

$$V_0 = R_2\,\frac{\varepsilon}{R_1+R_2} = \left(15.0\text{ k}\Omega\right)\left(\frac{20.0\text{V}}{10.0\text{ k}\Omega+15.0\text{ k}\Omega}\right) = 12.0\text{V}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across R_2 = 15.0 kΩ) after the switch is opened (at t = 0). Thus, with t = 0.00400 s, we obtain

$$V = \left(12\right)e^{-0.004/\left(15000\right)\left(0.4\times10^{-6}\right)} = 6.16\,\text{V}.$$

Therefore, using Ohm's law, the current through R_2 is 6.16/15000 = 4.11×10^{-4} A.

66. We apply Eq. 27-39 to each capacitor, demand their initial charges are in a ratio of 3:2 as described in the problem, and solve for the time. With

$$\tau_1 = R_1C_1 = (20.0\ \Omega)(5.00\times10^{-6}\ \text{F}) = 1.00\times10^{-4}\ \text{s}$$
$$\tau_2 = R_2C_2 = (10.0\ \Omega)(8.00\times10^{-6}\ \text{F}) = 8.00\times10^{-5}\ \text{s},$$

we obtain

$$t = \frac{\ln(3/2)}{\tau_2^{-1} - \tau_1^{-1}} = \frac{\ln(3/2)}{1.25\times10^4\ \text{s}^{-1} - 1.00\times10^4\ \text{s}^{-1}} = 1.62\times10^{-4}\,\text{s}.$$

67. The potential difference across the capacitor varies as a function of time t as $V(t) = V_0 e^{-t/RC}$. Using $V = V_0/4$ at $t = 2.0$ s, we find

$$R = \frac{t}{C\ln(V_0/V)} = \frac{2.0\,\text{s}}{(2.0\times10^{-6}\ \text{F})\ln 4} = 7.2\times10^5\ \Omega.$$

68. (a) The initial energy stored in a capacitor is given by $U_C = q_0^2/2C$, where C is the capacitance and q_0 is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2(1.0\times10^{-6}\ \text{F})(0.50\,\text{J})} = 1.0\times10^{-3}\ \text{C}.$$

(b) The charge as a function of time is given by $q = q_0 e^{-t/\tau}$, where τ is the capacitive time constant. The current is the derivative of the charge

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau} e^{-t/\tau},$$

and the initial current is $i_0 = q_0/\tau$. The time constant is

$$\tau = RC = \left(1.0\times10^{-6}\ \text{F}\right)\left(1.0\times10^{6}\ \Omega\right) = 1.0\ \text{s}.$$

Thus $i_0 = (1.0\times10^{-3}\ \text{C})/(1.0\,\text{s}) = 1.0\times10^{-3}\ \text{A}$.

(c) We substitute $q = q_0 e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C} e^{-t/\tau} = \left(\frac{1.0\times10^{-3}\ \text{C}}{1.0\times10^{-6}\ \text{F}}\right) e^{-t/1.0\,\text{s}} = \left(1.0\times10^{3}\ \text{V}\right) e^{-1.0t},$$

where t is measured in seconds.

(d) We substitute $i = (q_0/\tau)e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0 R}{\tau} e^{-t/\tau} = \frac{(1.0\times10^{-3}\text{ C})(1.0\times10^{6}\ \Omega)}{1.0\text{s}} e^{-t/1.0\text{ s}} = (1.0\times10^{3}\text{ V})e^{-1.0t},$$

where t is measured in seconds.

(e) We substitute $i = (q_0/\tau)e^{-t/\tau}$ into $P = i^2 R$ to obtain

$$P = \frac{q_0^2 R}{\tau^2} e^{-2t/\tau} = \frac{(1.0\times10^{-3}\text{ C})^2(1.0\times10^{6}\ \Omega)}{(1.0\text{s})^2} e^{-2t/1.0\text{ s}} = (1.0\text{ W})e^{-2.0t},$$

where t is again measured in seconds.

69. (a) The charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where ε is the emf of the battery, C is the capacitance, and τ is the time constant. The value of τ is

$$\tau = RC = (3.00 \times 10^{6}\ \Omega)(1.00 \times 10^{-6}\text{ F}) = 3.00\text{ s}.$$

At $t = 1.00$ s, $t/\tau = (1.00\text{ s})/(3.00\text{ s}) = 0.333$ and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\varepsilon}{\tau} e^{-t/\tau} = \frac{(1.00\times10^{-6}\text{ F})(4.00\text{ V})}{3.00\text{s}} e^{-0.333} = 9.55\times10^{-7}\text{ C/s}.$$

(b) The energy stored in the capacitor is given by $U_C = \dfrac{q^2}{2C}$, and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt}.$$

Now

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right) = \left(1.00\times10^{-6}\right)(4.00\text{ V})\left(1 - e^{-0.333}\right) = 1.13\times10^{-6}\text{ C},$$

so

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt} = \left(\frac{1.13\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ F}}\right)(9.55\times10^{-7}\text{ C/s}) = 1.08\times10^{-6}\text{ W}.$$

(c) The rate at which energy is being dissipated in the resistor is given by $P = i^2R$. The current is 9.55×10^{-7} A, so

$$P = (9.55\times10^{-7}\text{ A})^2(3.00\times10^{6}\ \Omega) = 2.74\times10^{-6}\text{ W}.$$

(d) The rate at which energy is delivered by the battery is

$$i\varepsilon = (9.55\times10^{-7}\text{ A})(4.00\text{ V}) = 3.82\times10^{-6}\text{ W}.$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\varepsilon = (q/C)(dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.

70. (a) From symmetry we see that the current through the top set of batteries (i) is the same as the current through the second set. This implies that the current through the $R = 4.0\ \Omega$ resistor at the bottom is $i_R = 2i$. Thus, with r denoting the internal resistance of each battery (equal to 4.0 Ω) and ε denoting the 20 V emf, we consider one loop equation (the outer loop), proceeding counterclockwise:

$$3(\varepsilon - ir) - (2i)R = 0.$$

This yields $i = 3.0$ A. Consequently, $i_R = 6.0$ A.

(b) The terminal voltage of each battery is $\varepsilon - ir = 8.0$ V.

(c) Using Eq. 27-17, we obtain $P = i\varepsilon = (3)(20) = 60$ W.

(d) Using Eq. 26-27, we have $P = i^2r = 36$ W.

71. (a) If S_1 is closed, and S_2 and S_3 are open, then $i_a = \varepsilon/2R_1 = 120\text{ V}/40.0\ \Omega = 3.00$ A.

(b) If S_3 is open while S_1 and S_2 remain closed, then

$$R_{eq} = R_1 + R_1(R_1 + R_2)/(2R_1 + R_2) = 20.0\ \Omega + (20.0\ \Omega)\times(30.0\ \Omega)/(50.0\ \Omega) = 32.0\ \Omega,$$

so $i_a = \varepsilon/R_{eq} = 120\text{ V}/32.0\ \Omega = 3.75$ A.

(c) If all three switches S_1, S_2, and S_3 are closed, then $R_{eq} = R_1 + R_1 R'/(R_1 + R')$ where

$$R' = R_2 + R_1(R_1 + R_2)/(2R_1 + R_2) = 22.0\ \Omega,$$

that is,

$$R_{eq} = 20.0\ \Omega + (20.0\ \Omega)(22.0\ \Omega)/(20.0\ \Omega + 22.0\ \Omega) = 30.5\ \Omega,$$

so $i_a = \varepsilon/R_{eq} = 120\text{ V}/30.5\ \Omega = 3.94$ A.

72. (a) The four resistors R_1, R_2, R_3, and R_4 on the left reduce to

$$R_{eq} = R_{12} + R_{34} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = 7.0\,\Omega + 3.0\,\Omega = 10\,\Omega .$$

With $\varepsilon = 30$ V across R_{eq} the current there is $i_2 = 3.0$ A.

(b) The three resistors on the right reduce to

$$R'_{eq} = R_{56} + R_7 = \frac{R_5 R_6}{R_5 + R_6} + R_7 = \frac{(6.0\,\Omega)(2.0\,\Omega)}{6.0\,\Omega + 2.0\,\Omega} + 1.5\,\Omega = 3.0\,\Omega .$$

With $\varepsilon = 30$ V across R'_{eq} the current there is $i_4 = 10$ A.

(c) By the junction rule, $i_1 = i_2 + i_4 = 13$ A.

(d) By symmetry, $i_3 = \frac{1}{2} i_2 = 1.5$ A.

(e) By the loop rule (proceeding clockwise),

$$30V - i_4(1.5\,\Omega) - i_5(2.0\,\Omega) = 0$$

readily yields $i_5 = 7.5$ A.

73. **THINK** Since the wires are connected in series, the current is the same in both wires.

EXPRESS Let i be the current in the wires and V be the applied potential difference. Using Kirchhoff's loop rule, we have $V - iR_A - iR_B = 0$. Thus, the current is $i = V/(R_A + R_B)$, and the corresponding current density is

$$J = \frac{i}{A} = \frac{V}{R_A + R_B}.$$

ANALYZE (a) For wire A, the magnitude of the current density vector is

$$J_A = \frac{i}{A} = \frac{V}{(R_A + R_B)A} = \frac{4V}{(R_1 + R_2)\pi D^2} = \frac{4(60.0\text{V})}{\pi(0.127\Omega + 0.729\Omega)(2.60\times10^{-3}\text{m})^2}$$
$$= 1.32\times10^7 \text{ A/m}^2 .$$

(b) The potential difference across wire A is

$$V_A = iR_A = V R_A/(R_A + R_B) = (60.0\text{ V})(0.127\ \Omega)/(0.127\ \Omega + 0.729\ \Omega) = 8.90\text{ V}.$$

(c) The resistivity of wire A is

$$\rho_A = \frac{R_A A}{L_A} = \frac{\pi R_A D^2}{4L_A} = \frac{\pi(0.127\,\Omega)(2.60\times10^{-3}\text{ m})^2}{4(40.0\,\text{m})} = 1.69\times10^{-8}\,\Omega\cdot\text{m}.$$

So wire A is made of copper.

(d) Since wire B has the same length and diameter as wire A, and the currents are the same, we have $J_B = J_A = 1.32\times10^7\ \text{A/m}^2$.

(e) The potential difference across wire B is $V_B = V - V_A = 60.0\text{ V} - 8.9\text{ V} = 51.1\text{ V}$.

(f) The resistivity of wire B is

$$\rho_B = \frac{R_B A}{L_B} = \frac{\pi R_B D^2}{4L_B} = \frac{\pi(0.729\ \Omega)(2.60\times10^{-3}\text{ m})^2}{4(40.0\,\text{m})} = 9.68\times10^{-8}\,\Omega\cdot\text{m},$$

so wire B is made of iron.

LEARN Resistance R is the property of an object (depending on quantities such as L and A), while resistivity is a property of the material itself. Knowing the value of ρ allows us to deduce what material the wire is made of.

74. The resistor by the letter i is above three other resistors; together, these four resistors are equivalent to a resistor $R = 10\ \Omega$ (with current i). As if we were presented with a maze, we find a path through R that passes through any number of batteries (10, it turns out) but no other resistors, which — as in any good maze — winds "all over the place." Some of the ten batteries are opposing each other (particularly the ones along the outside), so that their net emf is only $\varepsilon = 40$ V.

(a) The current through R is then $i = \varepsilon/R = 4.0$ A.

(b) The direction is upward in the figure.

75. (a) In the process described in the problem, no charge is gained or lost. Thus, q = constant. Hence,

$$q = C_1V_1 = C_2V_2 \quad \Rightarrow \quad V_2 = V_1\frac{C_1}{C_2} = (200)\left(\frac{150}{10}\right) = 3.0\times10^3\,\text{V}.$$

(b) Equation 27-39, with $\tau = RC$, describes not only the discharging of q but also of V. Thus,

$$V = V_0 e^{-t/\tau} \;\Rightarrow\; t = RC\ln\left(\frac{V_0}{V}\right) = \left(300\times10^{9}\ \Omega\right)\left(10\times10^{-12}\ \text{F}\right)\ln\left(\frac{3000}{100}\right)$$

which yields t = 10 s. This is a longer time than most people are inclined to wait before going on to their next task (such as handling the sensitive electronic equipment).

(c) We solve $V = V_0 e^{-t/RC}$ for R with the new values V_0 = 1400 V and t = 0.30 s. Thus,

$$R = \frac{t}{C\ln(V_0/V)} = \frac{0.30\ \text{s}}{(10\times10^{-12}\ \text{F})\ln(1400/100)} = 1.1\times10^{10}\ \Omega\ .$$

76. (a) We reduce the parallel pair of resistors (at the bottom of the figure) to a single R' =1.00 Ω resistor and then reduce it with its series 'partner' (at the lower left of the figure) to obtain an equivalence of R'' = 2.00 Ω +1.00Ω =3.00 Ω. It is clear that the current through R'' is the i_1 we are solving for. Now, we employ the loop rule, choose a path that includes R'' and all the batteries (proceeding clockwise). Thus, assuming i_1 goes leftward through R'', we have

$$5.00\ \text{V} + 20.0\ \text{V} - 10.0\ \text{V} - i_1 R'' = 0$$

which yields i_1 = 5.00 A.

(b) Since i_1 is positive, our assumption regarding its direction (leftward) was correct.

(c) Since the current through the ε_1 = 20.0 V battery is "forward", battery 1 is supplying energy.

(d) The rate is P_1 = (5.00 A)(20.0 V) = 100 W.

(e) Reducing the parallel pair (which are in parallel to the ε_2 = 10.0 V battery) to a single R' = 1.00 Ω resistor (and thus with current i' = (10.0 V)/(1.00 Ω) = 10.0 A downward through it), we see that the current through the battery (by the junction rule) must be $i = i' - i_1$ = 5.00 A *upward* (which is the "forward" direction for that battery). Thus, battery 2 is supplying energy.

(f) Using Eq. 27-17, we obtain P_2 = 50.0 W.

(g) The set of resistors that are in parallel with the ε_3 = 5 V battery is reduced to R''' = 0.800 Ω (accounting for the fact that two of those resistors are actually reduced in series, first, before the parallel reduction is made), which has current i''' = (5.00 V)/(0.800 Ω) = 6.25 A downward through it. Thus, the current through the battery (by the junction rule) must be $i = i''' + i_1$ = 11.25 A *upward* (which is the "forward" direction for that battery). Thus, battery 3 is supplying energy.

(h) Equation 27-17 leads to P_3 = 56.3 W.

77. **THINK** The silicon resistor and the iron resistor are connected in series. Both resistors are temperature-dependent, but we want the combination to be independent of temperature.

EXPRESS We denote silicon with subscript *s* and iron with *i*. Let T_0 = 20°. The resistances of the two resistors can be written as

$$R_s(T) = R_s(T_0)\left[1+\alpha_s(T-T_0)\right], \qquad R_i(T) = R_i(T_0)\left[1+\alpha_i(T-T_0)\right].$$

The resistors are in series connection so

$$\begin{aligned} R(T) = R_s(T)+R_i(T) &= R_s(T_0)\left[1+\alpha_s(T-T_0)\right]+R_i(T_0)\left[1+\alpha_i(T-T_0)\right] \\ &= R_s(T_0)+R_i(T_0)+\left[R_s(T_0)\alpha_s+R_i(T_0)\alpha_i\right](T-T_0). \end{aligned}$$

Now, if $R(T)$ is to be temperature-independent, we must require that $R_s(T_0)\alpha_s + R_i(T_0)\alpha_i = 0$. Also note that $R_s(T_0) + R_i(T_0) = R$ = 1000 Ω.

ANALYZE (a) We solve for $R_s(T_0)$ and $R_i(T_0)$ to obtain

$$R_s(T_0) = \frac{R\alpha_i}{\alpha_i-\alpha_s} = \frac{(1000\,\Omega)\left(6.5\times10^{-3}/\text{K}\right)}{(6.5\times10^{-3}/\text{K})-(-70\times10^{-3}/\text{K})} = 85.0\,\Omega.$$

(b) Similarly, $R_i(T_0)$ = 1000 Ω – 85.0 Ω = 915 Ω.

LEARN The temperature independence of the combined resistor was possible because α_i and α_s, the temperature coefficients of resistivity of the two materials have opposite signs, so their temperature dependences can cancel.

78. The current in the ammeter is given by

$$i_A = \varepsilon/(r + R_1 + R_2 + R_A).$$

The current in R_1 and R_2 without the ammeter is $i = \varepsilon/(r + R_1 + R_2)$. The percent error is then

$$\begin{aligned} \frac{\Delta i}{i} = \frac{i-i_A}{i} = 1-\frac{r+R_1+R_2}{r+R_1+R_2+R_A} = \frac{R_A}{r+R_1+R_2+R_A} = \frac{0.10\,\Omega}{2.0\,\Omega+5.0\,\Omega+4.0\,\Omega+0.10\,\Omega} \\ = 0.90\%. \end{aligned}$$

79. **THINK** As the capacitor in an *RC* circuit is being charged, some energy supplied by the emf device also goes to the resistor as thermal energy.

EXPRESS The charge q on the capacitor as a function of time is $q(t) = (\varepsilon C)(1 - e^{-t/RC})$, so the charging current is $i(t) = dq/dt = (\varepsilon/R)e^{-t/RC}$. The rate at which the emf device supplies energy is $P_\varepsilon = i\varepsilon dt$.

ANALYZE (a) The energy supplied by the emf is then

$$U = \int_0^\infty P_\varepsilon dt = \int_0^\infty \varepsilon i\, dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-t/RC} dt = C\varepsilon^2 = 2U_C$$

where $U_C = \frac{1}{2}C\varepsilon^2$ is the energy stored in the capacitor.

(b) By directly integrating i^2R we obtain

$$U_R = \int_0^\infty i^2 R dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC} dt = \frac{1}{2}C\varepsilon^2.$$

LEARN Half of the energy supplied by the emf device is stored in the capacitor as electrical energy, while the other half is dissipated in the resistor as thermal energy.

80. In the steady state situation, there is no current going to the capacitors, so the resistors all have the same current. By the loop rule,

$$20.0 \text{ V} = (5.00\ \Omega)i + (10.0\ \Omega)i + (15.0\ \Omega)i$$

which yields $i = \frac{2}{3}$ A. Consequently, the voltage across the R_1 = 5.00 Ω resistor is (5.00 Ω)(2/3 A) = 10/3 V, and is equal to the voltage V_1 across the C_1 = 5.00 μF capacitor. Using Eq. 26-22, we find the stored energy on that capacitor:

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(5.00\times10^{-6}\text{ F})\left(\frac{10}{3}\text{V}\right)^2 = 2.78\times10^{-5}\text{ J}.$$

Similarly, the voltage across the R_2 = 10.0 Ω resistor is (10.0 Ω)(2/3 A) = 20/3 V and is equal to the voltage V_2 across the C_2 = 10.0 μF capacitor. Hence,

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(10.0\times10^{-6}\text{ F})\left(\frac{20}{3}\text{V}\right)^2 = 2.22\times10^{-5}\text{ J}$$

Therefore, the total capacitor energy is $U_1 + U_2 = 2.50 \times 10^{-4}$ J.

81. The potential difference across R_2 is

$$V_2 = iR_2 = \frac{\varepsilon R_2}{R_1 + R_2 + R_3} = \frac{(12\,\text{V})(4.0\,\Omega)}{3.0\,\Omega + 4.0\,\Omega + 5.0\,\Omega} = 4.0\,\text{V}.$$

82. From $V_a - \varepsilon_1 = V_c - ir_1 - iR$ and $i = (\varepsilon_1 - \varepsilon_2)/(R + r_1 + r_2)$, we get

$$\begin{aligned} V_a - V_c &= \varepsilon_1 - i(r_1 + R) = \varepsilon_1 - \left(\frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2}\right)(r_1 + R) \\ &= 4.4\,\text{V} - \left(\frac{4.4\,\text{V} - 2.1\,\text{V}}{5.5\,\Omega + 1.8\,\Omega + 2.3\,\Omega}\right)(2.3\,\Omega + 5.5\,\Omega) \\ &= 2.5\,\text{V}. \end{aligned}$$

83. **THINK** The time constant in an RC circuit is $\tau = RC$, where R is the resistance and C is the capacitance. A greater value of τ means a longer discharging time.

EXPRESS The potential difference across the capacitor varies as a function of time t as $V(t) = V_0 e^{-t/\tau}$, where $\tau = RC$. Thus, $R = \dfrac{t}{C\ln(V_0/V)}$.

ANALYZE (a) Then, for the smaller time interval $t_{\min} = 10.0\ \mu\text{s}$

$$R_{\min} = \frac{10.0\,\mu\text{s}}{(0.220\,\mu\text{F})\ln(5.00/0.800)} = 24.8\,\Omega.$$

(b) Similarly, for the larger time interval $t_{\max} = 6.00$ ms,

$$R_{\max} = \frac{6.00\times10^{-3}\,\text{s}}{(0.220\,\mu\text{F})\ln(5.00\,\text{V}/0.800\,\text{V})} = 1.49\times10^{4}\,\Omega.$$

LEARN The two extrema of the resistances are related by

$$\frac{R_{\max}}{R_{\min}} = \frac{t_{\max}}{t_{\min}}.$$

The larger the value of R for a given capacitance, the longer the discharging time.

84. (a) Since $R_{\text{tank}} = 140\,\Omega, i = 12\,\text{V}/(10\,\Omega + 140\,\Omega) = 8.0\times10^{-2}\,\text{A}$.

(b) Now, $R_{\text{tank}} = (140\ \Omega + 20\ \Omega)/2 = 80\ \Omega$, so $i = 12\ \text{V}/(10\ \Omega + 80\ \Omega) = 0.13$ A.

(c) When full, $R_{\text{tank}} = 20\ \Omega$ so $i = 12\ \text{V}/(10\ \Omega + 20\ \Omega) = 0.40$ A.

85. **THINK** One of the three parts could be defective: the battery, the motor, or the cable.

EXPRESS All three circuit elements are connected in series, so the current is the same in all of them. The battery is discharging, so the potential drop across the terminals is $V_{\text{battery}} = \varepsilon - ir$, where ε is the emf and r is the internal resistance. On the other hand, the resistances in the cable and the motor are $R_{\text{cable}} = V_{\text{cable}} / i$ and $R_{\text{motor}} = V_{\text{motor}} / i$, respectively.

ANALYZE The internal resistance of the battery is

$$r = \frac{\varepsilon - V_{\text{battery}}}{i} = \frac{12\text{ V} - 11.4\text{ V}}{50\text{ A}} = 0.012\,\Omega$$

which is less than 0.020 Ω. So the battery is OK. For the motor, we have

$$R_{\text{motor}} = \frac{V_{\text{motor}}}{i} = \frac{11.4\text{ V} - 3.0\text{ V}}{50\text{ A}} = 0.17\,\Omega$$

which is less than 0.20 Ω. So the motor is OK. Now, the resistance of the cable is

$$R_{\text{cable}} = \frac{V_{\text{cable}}}{i} = \frac{3.0\text{ V}}{50\text{ A}} = 0.060\,\Omega$$

which is greater than 0.040 Ω. So the cable is defective.

LEARN In this exercise, we see that a defective component has a resistance outside its the range of acceptance.

86. When connected in series, the rate at which electric energy dissipates is $P_s = \varepsilon^2/(R_1 + R_2)$. When connected in parallel, the corresponding rate is $P_p = \varepsilon^2(R_1 + R_2)/R_1R_2$. Letting $P_p/P_s = 5$, we get $(R_1 + R_2)^2/R_1R_2 = 5$, where $R_1 = 100\ \Omega$. We solve for R_2: $R_2 = 38\ \Omega$ or 260 Ω.

(a) Thus, the smaller value of R_2 is 38 Ω.

(b) The larger value of R_2 is 260 Ω.

87. When S is open for a long time, the charge on C is $q_i = \varepsilon_2 C$. When S is closed for a long time, the current i in R_1 and R_2 is

$$i = (\varepsilon_2 - \varepsilon_1)/(R_1 + R_2) = (3.0\text{ V} - 1.0\text{ V})/(0.20\ \Omega + 0.40\ \Omega) = 3.33\text{ A}.$$

The voltage difference V across the capacitor is then

$$V = \varepsilon_2 - iR_2 = 3.0\text{ V} - (3.33\text{ A})(0.40\ \Omega) = 1.67\text{ V}.$$

Thus the final charge on C is $q_f = VC$. So the change in the charge on the capacitor is

$$\Delta q = q_f - q_i = (V - \varepsilon_2)C = (1.67\text{ V} - 3.0\text{ V})(10\ \mu\text{F}) = -13\ \mu\text{C}.$$

88. Using the junction and the loop rules, we have

$$\begin{aligned} 20.0 - i_1R_1 - i_3R_3 &= 0 \\ 20.0 - i_1R_1 - i_2R_2 - 50 &= 0 \\ i_2 + i_3 &= i_1 \end{aligned}$$

Requiring no current through the battery 1 means that $i_1 = 0$, or $i_2 = i_3$. Solving the above equations with $R_1 = 10.0\Omega$ and $R_2 = 20.0\Omega$, we obtain

$$i_1 = \frac{40 - 3R_3}{20 + 3R_3} = 0 \quad \Rightarrow \quad R_3 = \frac{40}{3} = 13.3\ \Omega\,.$$

89. The bottom two resistors are in parallel, equivalent to a 2.0R resistance. This, then, is in series with resistor R on the right, so that their equivalence is $R' = 3.0R$. Now, near the top left are two resistors (2.0R and 4.0R) that are in series, equivalent to $R'' = 6.0R$. Finally, R' and R'' are in parallel, so the net equivalence is

$$R_{\text{eq}} = \frac{(R')\,(R'')}{R' + R''} = 2.0R = 20\ \Omega$$

where in the final step we use the fact that $R = 10\ \Omega$.

90. (a) Using Eq. 27-4, we take the derivative of the power $P = i^2R$ with respect to R and set the result equal to zero:

$$\frac{dP}{dR} = \frac{d}{dR}\left(\frac{\varepsilon^2 R}{(R+r)^2}\right) = \frac{\varepsilon^2(r-R)}{(R+r)^3} = 0$$

which clearly has the solution $R = r$.

(b) When $R = r$, the power dissipated in the external resistor equals

$$P_{\max} = \left.\frac{\varepsilon^2 R}{(R+r)^2}\right|_{R=r} = \frac{\varepsilon^2}{4r}.$$

91. (a) We analyze the lower left loop and find

$$i_1 = \varepsilon_1/R = (12.0\text{ V})/(4.00\ \Omega) = 3.00\text{ A}.$$

(b) The direction of i_1 is downward.

(c) Letting R = 4.00 Ω, we apply the loop rule to the tall rectangular loop in the center of the figure (proceeding clockwise):

$$\varepsilon_2 + \left(+i_1 R\right) + \left(-i_2 R\right) + \left(-\frac{i_2}{2} R\right) + \left(-i_2 R\right) = 0\,.$$

Using the result from part (a), we find i_2 = 1.60 A.

(d) The direction of i_2 is downward (as was assumed in writing the equation as we did).

(e) Battery 1 is supplying this power since the current is in the "forward" direction through the battery.

(f) We apply Eq. 27-17: The current through the ε_1 = 12.0 V battery is, by the junction rule, 3.00 A + 1.60 A = 4.60 A and

$$P = (4.60\text{ A})(12.0\text{ V}) = 55.2\text{ W}.$$

(g) Battery 2 is supplying this power since the current is in the "forward" direction through the battery.

(h) $P = i_2(4.00\text{ V}) = 6.40$ W.

92. The equivalent resistance of the series pair of $R_3 = R_4 = 2.0\ \Omega$ is R_{34}= 4.0 Ω, and the equivalent resistance of the parallel pair of $R_1 = R_2 = 4.0\ \Omega$ is R_{12}= 2.0 Ω. Since the voltage across R_{34} must equal that across R_{12}:

$$V_{34} = V_{12} \quad \Rightarrow \quad i_{34}R_{34} = i_{12}R_{12} \quad \Rightarrow \quad i_{34} = \frac{1}{2} i_{12}$$

This relation, plus the junction rule condition $I = i_{12} + i_{34} = 6.00$ A, leads to the solution $i_{12} = 4.0$ A. It is clear by symmetry that $i_1 = i_{12}/2 = 2.00$ A.

93. (a) From $P = V^2/R$ we find $V = \sqrt{PR} = \sqrt{(10\text{ W})(0.10\,\Omega)} = 1.0\text{ V}$.

(b) From $i = V/R = (\varepsilon - V)/r$ we find

$$r = R\left(\frac{\varepsilon - V}{V}\right) = \left(0.10\,\Omega\right)\left(\frac{1.5\text{ V} - 1.0\text{ V}}{1.0\text{ V}}\right) = 0.050\,\Omega.$$

94. (a) $R_{eq}(AB)$ = 20.0 Ω/3 = 6.67 Ω (three 20.0 Ω resistors in parallel).

(b) $R_{eq}(AC)$ = 20.0 Ω/3 = 6.67 Ω (three 20.0 Ω resistors in parallel).

(c) $R_{eq}(BC)$ = 0 (as *B* and *C* are connected by a conducting wire).

95. The maximum power output is (120 V)(15 A) = 1800 W. Since 1800 W/500 W = 3.6, the maximum number of 500 W lamps allowed is 3.

96. Here we denote the battery emf as *V*. Eq. 27-30 leads to

$$i = \frac{\varepsilon}{R} - \frac{q}{RC} = \frac{12\text{ V}}{4.0\,\Omega} - \frac{8.0\times10^{-6}\text{ C}}{(4.0\,\Omega)(4.0\times10^{-6}\text{F})} = 2.50\text{ A}.$$

97. **THINK** To calculate the current in the resistor *R*, we first find the equivalent resistance of the *N* batteries.

EXPRESS When all the batteries are connected in parallel, the emf is ε and the equivalent resistance is $R_{\text{parallel}} = R + r/N$, so the current is

$$i_{\text{parallel}} = \frac{\varepsilon}{R_{\text{parallel}}} = \frac{\varepsilon}{R + r/N} = \frac{N\varepsilon}{NR + r}.$$

Similarly, when all the batteries are connected in series, the total emf is $N\varepsilon$ and the equivalent resistance is $R_{\text{series}} = R + Nr$. Therefore,

$$i_{\text{series}} = \frac{N\varepsilon}{R_{\text{series}}} = \frac{N\varepsilon}{R + Nr}.$$

ANALYZE Comparing the two expressions, we see that the two currents i_{parallel} and i_{series} are equal if $R = r$, with

$$i_{\text{parallel}} = i_{\text{series}} = \frac{N\varepsilon}{(N+1)r}.$$

LEARN In general, the current difference is

$$i_{\text{parallel}} - i_{\text{series}} = \frac{N\varepsilon}{NR + r} - \frac{N\varepsilon}{R + Nr} = \frac{N\varepsilon(N-1)(r-R)}{(NR+r)(R+Nr)}.$$

If $R > r$, then $i_{\text{parallel}} < i_{\text{series}}$.

98. **THINK** The rate of energy supplied by the battery is $i\varepsilon$. So we first calculate the current in the circuit.

EXPRESS With R_2 and R_3 in parallel, and the combination in series with R_1, the equivalent resistance for the circuit is

$$R_{eq} = R_1 + \frac{R_2R_3}{R_2+R_3} = \frac{R_1R_2+R_1R_3+R_2R_3}{R_2+R_3}$$

and the current is

$$i = \frac{\varepsilon}{R_{eq}} = \frac{(R_2+R_3)\varepsilon}{R_1R_2+R_1R_3+R_2R_3}.$$

The rate at which the battery supplies energy is

$$P = i\varepsilon = \frac{(R_2+R_3)\varepsilon^2}{R_1R_2+R_1R_3+R_2R_3}.$$

To find the value of R_3 that maximizes P, we differentiate P with respect to R_3.

ANALYZE (a) With a little algebra, we find

$$\frac{dP}{dR_3} = -\frac{R_2^2\varepsilon^2}{(R_1R_2+R_1R_3+R_2R_3)^2}.$$

The derivative is negative for all positive value of R_3. Thus, we see that P is maximized when $R_3 = 0$.

(b) With the value of R_3 set to zero, we obtain $P = \frac{\varepsilon^2}{R_1} = \frac{(12.0\text{ V})^2}{10.0\,\Omega} = 14.4\text{ W}.$

LEARN Mathematically speaking, the function P is a monotonically decreasing function of R_3 (as well as R_2 and R_1), so P is a maximum at $R_3 = 0$.

99. **THINK** A capacitor that is being charged initially behaves like an ordinary connecting wire relative to the charging current.

EXPRESS The capacitor is *initially* uncharged. So immediately after the switch is closed, by the Kirchhoff's loop rule, there is zero voltage (at $t = 0$) across the $R_2 = 10\text{ k}\Omega$ resistor, and that $\varepsilon = 30$ V is across the $R_1 = 20\text{ k}\Omega$ resistor.

ANALYZE (a) By Ohm's law, the initial current in R_1 is

$$i_{10} = \varepsilon / R_1 = (30\text{ V})/(20\text{ k}\Omega) = 1.5 \times 10^{-3}\text{ A}.$$

(b) Similarly, the initial current in R_2 is $i_{20} = 0$.

(c) As $t \to \infty$ the current to the capacitor reduces to zero and the R_1 = 20 kΩ and R_2 = 10 kΩ resistors behave more like a series pair (having the same current), equivalent to

$$R_{eq} = R_1 + R_2 = 30 \text{ k}\Omega.$$

The current through them, then, at long times, is

$$i = \varepsilon / R_{eq} = (30 \text{ V})/(30 \text{ k}\Omega) = 1.0 \times 10^{-3} \text{ A}.$$

LEARN A long time later after a capacitor is being fully charged, it acts like a broken wire.

100. (a) Reducing the bottom two series resistors to a single R' = 4.00 Ω (with current i_1 through it), we see we can make a path (for use with the loop rule) that passes through R, the ε_4 = 5.00 V battery, the ε_1 = 20.0 V battery, and the ε_3 = 5.00 V. This leads to

$$i_1 = \frac{\varepsilon_1 + \varepsilon_3 + \varepsilon_4}{R'} = \frac{20.0 \text{ V} + 5.00 \text{ V} + 5.00 \text{ V}}{40.0\ \Omega} = \frac{30.0 \text{ V}}{4.0\ \Omega} = 7.50 \text{ A}.$$

(b) The direction of i_1 is leftward.

(c) The voltage across the bottom series pair is i_1R' = 30.0 V. This must be the same as the voltage across the two resistors directly above them, one of which has current i_2 through it and the other (by symmetry) has current $\frac{1}{2} i_2$ through it. Therefore,

$$30.0 \text{ V} = i_2 (2.00\ \Omega) + \tfrac{1}{2} i_2 (2.00\ \Omega)$$

which leads to i_2 = (30.0 V)/(3.00 Ω) = 10.0 A.

(d) The direction of i_2 is also leftward.

(e) We use Eq. 27-17: $P_4 = (i_1 + i_2)\varepsilon_4 = (7.50 \text{ A} + 10.0 \text{ A})(5.00 \text{ V}) = 87.5 \text{ W}.$

(f) The energy is being supplied to the circuit since the current is in the "forward" direction through the battery.

101. Consider the lowest branch with the two resistors R_4 = 3.00 Ω and R_5 = 5.00 Ω. The voltage difference across R_5 is

$$V = i_5 R_5 = \frac{\varepsilon R_5}{R_4 + R_5} = \frac{(120\text{V})(5.00\Omega)}{3.00\Omega + 5.00\Omega} = 7.50\text{V}.$$

102. (a) Here we denote the battery emf as V. See Fig. 27-4(a): $V_T = V - ir$.

(b) Doing a least squares fit for the V_T versus i values listed, we obtain

$$V_T = 13.61 - 0.0599i$$

which implies $V = 13.6$ V.

(c) It also implies the internal resistance is 0.060 Ω.

103. (a) The loop rule (proceeding counterclockwise around the right loop) leads to $\varepsilon_2 - i_1R_1 = 0$ (where i_1 was assumed downward). This yields $i_1 = 0.0600$ A.

(b) The direction of i_1 is downward.

(c) The loop rule (counterclockwise around the left loop) gives

$$(+\varepsilon_1) + (+i_1R_1) + (-i_2R_2) = 0$$

where i_2 has been assumed leftward. This yields $i_3 = 0.180$ A.

(d) A positive value of i_3 implies that our assumption on the direction is correct, i.e., it flows leftward.

(e) The junction rule tells us that the current through the 12 V battery is 0.180 + 0.0600 = 0.240 A.

(f) The direction is upward.

104. (a) Since $P = \varepsilon^2/R_{eq}$, the higher the power rating the smaller the value of R_{eq}. To achieve this, we can let the low position connect to the larger resistance (R_1), middle position connect to the smaller resistance (R_2), and the high position connect to both of them in parallel.

(b) For $P = 300$ W, $R_{eq} = R_1R_2/(R_1 + R_2) = (144\ \Omega)R_2/(144\ \Omega + R_2) = (120\ \text{V})^2/(300\ \text{W})$. We obtain $R_2 = 72\ \Omega$.

(c) For $P = 100$ W, $R_{eq} = R_1 = \varepsilon^2/P = (120\ \text{V})^2/100\ \text{W} = 144\ \Omega$;

105. (a) The six resistors to the left of $\varepsilon_1 = 16$ V battery can be reduced to a single resistor $R = 8.0\ \Omega$, through which the current must be $i_R = \varepsilon_1/R = 2.0$ A. Now, by the loop rule, the current through the 3.0 Ω and 1.0 Ω resistors at the upper right corner is

$$i' = \frac{16.0\,\text{V} - 8.0\,\text{V}}{3.0\,\Omega + 1.0\,\Omega} = 2.0\,\text{A}$$

in a direction that is "backward" relative to the ε_2 = 8.0 V battery. Thus, by the junction rule, $i_1 = i_R + i' = 4.0\,\text{A}$.

(b) The direction of i_1 is upward (that is, in the "forward" direction relative to ε_1).

(c) The current i_2 derives from a succession of symmetric splittings of i_R (reversing the procedure of reducing those six resistors to find R in part (a)). We find

$$i_2 = \frac{1}{2}\left(\frac{1}{2}i_R\right) = 0.50\,\text{A}.$$

(d) The direction of i_2 is clearly downward.

(e) Using our conclusion from part (a) in Eq. 27-17, we have

$$P = i_1\varepsilon_1 = (4.0\text{ A})(16\text{ V}) = 64\text{ W}.$$

(f) Using results from part (a) in Eq. 27-17, we obtain $P = i'\varepsilon_2$ = (2.0 A)(8.0 V) = 16 W.

(g) Energy is being supplied in battery 1.

(h) Energy is being absorbed in battery 2.

Chapter 28

1. **THINK** The magnetic force on a charged parti014cle is given by $\vec{F}_B = q\vec{v}\times\vec{B}$, where $\vec{v}$ is the velocity of the charged particle and $\vec{B}$ is the magnetic field.

EXPRESS The magnitude of the magnetic force on the proton (of charge +*e*) is $F_B = evB\sin\phi$, where ϕ is the angle between $\vec{v}$ and $\vec{B}$.

ANALYZE (a) The speed of the proton is

$$v = \frac{F_B}{eB\sin\phi} = \frac{6.50\times10^{-17}\,\text{N}}{(1.60\times10^{-19}\,\text{C})(2.60\times10^{-3}\,\text{T})\sin 23.0^\circ} = 4.00\times10^5\ \text{m/s}.$$

(b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\,\text{kg})(4.00\times10^5\ \text{m/s})^2 = 1.34\times10^{-16}\ \text{J},$$

which is equivalent to

$$K = (1.34\times10^{-16}\ \text{J}) / (1.60\times10^{-19}\ \text{J/eV}) = 835\ \text{eV}.$$

LEARN from the definition of $\vec{B}$ given by the expression $\vec{F}_B = q\vec{v}\times\vec{B}$, we see that the magnetic force $\vec{F}_B$ is always perpendicular to $\vec{v}$ and $\vec{B}$.

2. The force associated with the magnetic field must point in the $\hat{\text{j}}$ direction in order to cancel the force of gravity in the $-\hat{\text{j}}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{\text{k}}$ direction (since $\hat{\text{i}}\times(-\hat{\text{k}}) = \hat{\text{j}}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by Eq. 28-3 (with $\phi = 90°$). Therefore, with $m = 1.0\times10^{-2}$ kg, $v = 2.0\times10^4$ m/s, and $q = 8.0\times10^{-5}$ C, we find

$$\vec{B} = B_z\hat{\text{k}} = -\left(\frac{mg}{qv}\right)\hat{\text{k}} = (-0.061\ \text{T})\hat{\text{k}}.$$

3. (a) The force on the electron is

$$\vec{F}_B = q\vec{v}\times\vec{B} = q\left(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}}\right)\times\left(B_x\hat{\mathrm{i}} + B_y\vec{j}\right) = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}}$$
$$= \left(-1.6\times10^{-19}\,\mathrm{C}\right)\left[\left(2.0\times10^{6}\,\mathrm{m/s}\right)\left(-0.15\,\mathrm{T}\right) - \left(3.0\times10^{6}\,\mathrm{m/s}\right)\left(0.030\,\mathrm{T}\right)\right]$$
$$= \left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.$$

Thus, the magnitude of $\vec{F}_B$ is 6.2×10^{14} N, and $\vec{F}_B$ points in the positive z direction.

(b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative z direction, namely, $\vec{F}_B = -\left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.$

4. (a) We use Eq. 28-3:

$$F_B = |q|\, vB \sin\phi = (+\,3.2 \times 10^{-19}\ \mathrm{C})\ (550\ \mathrm{m/s})\ (0.045\ \mathrm{T})\ (\sin 52°) = 6.2 \times 10^{-18}\ \mathrm{N}.$$

(b) The acceleration is

$$a = F_B/m = (6.2 \times 10^{-18}\ \mathrm{N}) / (6.6 \times 10^{-27}\ \mathrm{kg}) = 9.5 \times 10^{8}\ \mathrm{m/s^2}.$$

(c) Since it is perpendicular to $\vec{v}, \vec{F}_B$ does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

5. Using Eq. 28-2 and Eq. 3-30, we obtain

$$\vec{F} = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}} = q\left(v_x\left(3B_x\right) - v_yB_x\right)\hat{\mathrm{k}}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\,\hat{\mathrm{k}}$ where $F_z = 6.4 \times 10^{-19}$ N, then we are led to the condition

$$q\left(3v_x - v_y\right)B_x = F_z \quad\Rightarrow\quad B_x = \frac{F_z}{q\left(3v_x - v_y\right)}.$$

Substituting $v_x = 2.0$ m/s, $v_y = 4.0$ m/s, and $q = -1.6 \times 10^{-19}$ C, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)} = \frac{6.4\times10^{-19}\ \mathrm{N}}{(-1.6\times10^{-19}\mathrm{C})[3(2.0\ \mathrm{m/s}) - 4.0\ \mathrm{m}]} = -2.0\ \mathrm{T}.$$

6. The magnetic force on the proton is given by $\vec{F} = q\vec{v}\times\vec{B}$, where $q = +e$. Using Eq. 3-30 this becomes

$(4 \times 10^{-17})\hat{i} + (2 \times 10^{-17})\hat{j} = e[(0.03v_y + 40)\hat{i} + (20 - 0.03v_x)\hat{j} - (0.02v_x + 0.01v_y)\hat{k}]$

with SI units understood. Equating corresponding components, we find

(a) $v_x = -3.5\times10^3$ m/s, and

(b) $v_y = 7.0\times10^3$ m/s.

7. We apply $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) = m_e\vec{a}$ to solve for $\vec{E}$:

$$\begin{aligned}\vec{E} &= \frac{m_e\vec{a}}{q} + \vec{B}\times\vec{v}\\ &= \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.00\times10^{12}\ \text{m/s}^2\right)\hat{i}}{-1.60\times10^{-19}\,\text{C}} + \left(400\,\mu\text{T}\right)\hat{i}\times\left[\left(12.0\,\text{km/s}\right)\hat{j} + \left(15.0\,\text{km/s}\right)\hat{k}\right]\\ &= \left(-11.4\hat{i} - 6.00\hat{j} + 4.80\hat{k}\right)\text{V/m}.\end{aligned}$$

8. Letting $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) = 0$, we get $vB\sin\phi = E$. We note that (for given values of the fields) this gives a minimum value for speed whenever the sin ϕ factor is at its maximum value (which is 1, corresponding to $\phi = 90°$). So

$$v_{\min} = \frac{E}{B} = \frac{1.50\times10^3\ \text{V/m}}{0.400\ \text{T}} = 3.75\times10^3\ \text{m/s}.$$

9. Straight-line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) = 0$. Note that $\vec{v}\perp\vec{B}$ so $\left|\vec{v}\times\vec{B}\right| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$B = \frac{E}{v} = \frac{E}{\sqrt{2K/m_e}} = \frac{100\ \text{V}/(20\times10^{-3}\text{m})}{\sqrt{2\left(1.0\times10^3\text{V}\right)\left(1.60\times10^{-19}\text{C}\right)/\left(9.11\times10^{-31}\text{kg}\right)}} = 2.67\times10^{-4}\,\text{T}.$$

In unit-vector notation, $\vec{B} = -(2.67\times10^{-4}\ \text{T})\hat{k}$.

10. (a) The net force on the proton is given by

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B} = \left(1.60\times10^{-19}\text{C}\right)\left[\left(4.00\,\text{V/m}\right)\hat{k} + \left(2000\,\text{m/s}\right)\hat{j}\times\left(-2.50\times10^{-3}\,\text{T}\right)\hat{i}\right]\\ &= \left(1.44\times10^{-18}\,\text{N}\right)\hat{k}.\end{aligned}$$

(b) In this case, we have

$$\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B}$$
$$= \left(1.60\times10^{-19}\,\text{C}\right)\left[\left(-4.00\,\text{V/m}\right)\hat{\text{k}} + \left(2000\,\text{m/s}\right)\hat{\text{j}}\times\left(-2.50\,\text{mT}\right)\hat{\text{i}}\right]$$
$$= \left(1.60\times10^{-19}\,\text{N}\right)\hat{\text{k}}.$$

(c) In the final case, we have

$$\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B}$$
$$= \left(1.60\times10^{-19}\,\text{C}\right)\left[\left(4.00\,\text{V/m}\right)\hat{\text{i}} + \left(2000\,\text{m/s}\right)\hat{\text{j}}\times\left(-2.50\,\text{mT}\right)\hat{\text{i}}\right]$$
$$= \left(6.41\times10^{-19}\,\text{N}\right)\hat{\text{i}} + \left(8.01\times10^{-19}\,\text{N}\right)\hat{\text{k}}.$$

11. Since the total force given by $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v} \times \vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $mv^2/2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B\sqrt{\frac{2eV}{m}} = \left(1.2\,\text{T}\right)\sqrt{\frac{2\left(1.60\times10^{-19}\,\text{C}\right)\left(10\times10^{3}\,\text{V}\right)}{\left(9.99\times10^{-27}\,\text{kg}\right)}} = 6.8\times10^{5}\,\text{V/m}.$$

12. (a) The force due to the electric field ($\vec{F} = q\vec{E}$) is distinguished from that associated with the magnetic field ($\vec{F} = q\vec{v} \times \vec{B}$) in that the latter vanishes when the speed is zero and the former is independent of speed. The graph shows that the force (y-component) is negative at $v = 0$ (specifically, its value is -2.0×10^{-19} N there), which (because $q = -e$) implies that the electric field points in the $+y$ direction. Its magnitude is

$$E = \frac{F_{\text{net},y}}{|q|} = \frac{2.0\times10^{-19}\,\text{N}}{1.6\times10^{-19}\,\text{C}} = 1.25\,\text{N/C} = 1.25\,\text{V/m}.$$

(b) We are told that the x and z components of the force remain zero throughout the motion, implying that the electron continues to move along the x axis, even though magnetic forces generally cause the paths of charged particles to curve (Fig. 28-11). The exception to this is discussed in Section 28-3, where the forces due to the electric and magnetic fields cancel. This implies (Eq. 28-7) $B = E/v = 2.50 \times 10^{-2}$ T.

For $\vec{F} = q\vec{v} \times \vec{B}$ to be in the opposite direction of $\vec{F} = q\vec{E}$ we must have $\vec{v} \times \vec{B}$ in the <u>opposite</u> direction from $\vec{E}$, which points in the $+y$ direction, as discussed in part (a). Since the velocity is in the $+x$ direction, then (using the right-hand rule) we conclude that the magnetic field must point in the +*z* direction ($\hat{\text{i}} \times \hat{\text{k}} = -\hat{\text{j}}$). In unit-vector notation, we have $\vec{B} = (2.50 \times 10^{-2}\ \text{T})\hat{\text{k}}$.

13. We use Eq. 28-12 to solve for *V*:

$$V = \frac{iB}{nle} = \frac{(23\text{A})(0.65\ \text{T})}{(8.47 \times 10^{28}/\text{m}^3)(150\mu\text{m})(1.6 \times 10^{-19}\text{C})} = 7.4 \times 10^{-6}\,\text{V}.$$

14. For a free charge *q* inside the metal strip with velocity $\vec{v}$ we have $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. We set this force equal to zero and use the relation between (uniform) electric field and potential difference. Thus,

$$v = \frac{E}{B} = \frac{|V_x - V_y|/d_{xy}}{B} = \frac{(3.90 \times 10^{-9}\ \text{V})}{(1.20 \times 10^{-3}\,\text{T})(0.850 \times 10^{-2}\ \text{m})} = 0.382\ \text{m/s}.$$

15. (a) We seek the electrostatic field established by the separation of charges (brought on by the magnetic force). With Eq. 28-10, we define the magnitude of the electric field as

$$|\vec{E}| = v|\vec{B}| = (20.0\ \text{m/s})(0.030\ \text{T}) = 0.600\ \text{V/m}.$$

Its direction may be inferred from Figure 28-8; its direction is opposite to that defined by $\vec{v} \times \vec{B}$. In summary,

$$\vec{E} = -(0.600\,\text{V/m})\hat{\text{k}}$$

which insures that $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ vanishes.

(b) Equation 28-9 yields $V = Ed = (0.600\ \text{V/m})(2.00\ \text{m}) = 1.20\ \text{V}$.

16. We note that $\vec{B}$ must be along the *x* axis because when the velocity is along that axis there is no induced voltage. Combining Eq. 28-7 and Eq. 28-9 leads to

$$d = \frac{V}{E} = \frac{V}{vB}$$

where one must interpret the symbols carefully to ensure that $\vec{d}, \vec{v}$, and $\vec{B}$ are mutually perpendicular. Thus, when the velocity if parallel to the *y* axis the absolute value of the voltage (which is considered in the same "direction" as $\vec{d}$) is 0.012 V, and

$$d = d_z = \frac{0.012\ \text{V}}{(3.0\ \text{m/s})(0.020\ \text{T})} = 0.20\ \text{m}.$$

On the other hand, when the velocity is parallel to the z axis the absolute value of the appropriate voltage is 0.018 V, and

$$d = d_y = \frac{0.018\text{ V}}{(3.0\text{ m/s})(0.020\text{ T})} = 0.30\text{ m}.$$

Thus, our answers are

(a) d_x = 25 cm (which we arrive at "by elimination," since we already have figured out d_y and d_z),

(b) d_y = 30 cm, and

(c) d_z = 20 cm.

17. (a) Using Eq. 28-16, we obtain

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\,\text{u}} = \frac{2\left(4.50\times10^{-2}\text{ m}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(1.20\text{ T}\right)}{\left(4.00\,\text{u}\right)\left(1.66\times10^{-27}\text{ kg/u}\right)} = 2.60\times10^{6}\text{ m/s}.$$

(b) $T = 2\pi r/v = 2\pi(4.50\times10^{-2}\text{ m})/(2.60\times10^{6}\text{ m/s}) = 1.09\times10^{-7}$ s.

(c) The kinetic energy of the alpha particle is

$$K = \frac{1}{2}m_\alpha v^2 = \frac{\left(4.00\,\text{u}\right)\left(1.66\times10^{-27}\text{ kg/u}\right)\left(2.60\times10^{6}\text{ m/s}\right)^2}{2\left(1.60\times10^{-19}\text{ J/eV}\right)} = 1.40\times10^{5}\text{ eV}.$$

(d) $\Delta V = K/q = 1.40\times10^{5}\text{ eV}/2e = 7.00\times10^{4}$ V.

18. With the $\vec{B}$ pointing "out of the page," we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle's path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent toward the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 28-3 (with angle ϕ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e|\vec{B}|} = 4.99\times10^{6}\text{ m/s}.$$

(b) Using either Eq. 28-14 or Eq. 28-16, we find r = 0.00710 m.

(c) Using Eq. 28-17 (in either its first or last form) readily yields $T = 8.93\times10^{-9}$ s.

19. Let ξ stand for the ratio ($m/|q|$) we wish to solve for. Then Eq. 28-17 can be written as $T = 2\pi\xi/B$. Noting that the horizontal axis of the graph (Fig. 28-37) is inverse-field ($1/B$) then we conclude (from our previous expression) that the slope of the line in the graph must be equal to $2\pi\xi$. We estimate that slope is 7.5×10^{-9} T·s, which implies

$$\xi = m/|q| = 1.2\times10^{-9}\ \text{kg/C}.$$

20. Combining Eq. 28-16 with energy conservation ($eV = \frac{1}{2}m_e v^2$ in this particular application) leads to the expression

$$r = \frac{m_e}{eB}\sqrt{\frac{2eV}{m_e}}$$

which suggests that the slope of the r versus $\sqrt{V}$ graph should be $\sqrt{2m_e/eB^2}$. From Fig. 28-38, we estimate the slope to be 5×10^{-5} in SI units. Setting this equal to $\sqrt{2m_e/eB^2}$ and solving, we find $B = 6.7 \times 10^{-2}$ T.

21. **THINK** The electron is in circular motion because the magnetic force acting on it points toward the center of the circle.

EXPRESS The kinetic energy of the electron is given by $K = \frac{1}{2}m_e v^2$, where m_e is the mass of electron and v is its speed. The magnitude of the magnetic force on the electron is $F_B = evB$ which is equal to the centripetal force:

$$evB = \frac{m_e v^2}{r}.$$

ANALYZE (a) From $K = \frac{1}{2}m_e v^2$ we get

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.20\times10^3\,\text{eV})(1.60\times10^{-19}\ \text{eV/J})}{9.11\times10^{-31}\,\text{kg}}} = 2.05\times10^7\ \text{m/s}.$$

(b) Since $evB = m_e v^2/r$, we find the magnitude of the magnetic field to be

$$B = \frac{m_e v}{er} = \frac{(9.11\times10^{-31}\text{kg})(2.05\times10^7\ \text{m/s})}{(1.60\times10^{-19}\text{C})(25.0\times10^{-2}\,\text{m})} = 4.67\times10^{-4}\ \text{T}.$$

(c) The "orbital" frequency is

$$f = \frac{v}{2\pi r} = \frac{2.07\times10^{7}\ \text{m/s}}{2\pi\left(25.0\times10^{-2}\text{m}\right)} = 1.31\times10^{7}\ \text{Hz}.$$

(d) The period is simply equal to the reciprocal of frequency:

$$T = 1/f = (1.31 \times 10^{7}\ \text{Hz})^{-1} = 7.63 \times 10^{-8}\ \text{s}.$$

LEARN The period of the electron's circular motion can be written as

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|e|B} = \frac{2\pi m}{|e|B}.$$

The period is inversely proportional to B.

22. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $K = (rqB)^2/2m \propto q^2m^{-1}$.

(a) $K_\alpha = \left(q_\alpha/q_p\right)^2\left(m_p/m_\alpha\right)K_p = \left(2\right)^2\left(1/4\right)K_p = K_p = 1.0\text{MeV};$

(b) $K_d = \left(q_d/q_p\right)^2\left(m_p/m_d\right)K_p = \left(1\right)^2\left(1/2\right)K_p = 1.0\ \text{MeV}/2 = 0.50\text{MeV}.$

23. From Eq. 28-16, we find

$$B = \frac{m_e v}{er} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(1.30\times10^{6}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(0.350\ \text{m}\right)} = 2.11\times10^{-5}\ \text{T}.$$

24. (a) The accelerating process may be seen as a conversion of potential energy eV into kinetic energy. Since it starts from rest, $\frac{1}{2}m_e v^2 = eV$ and

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2\left(1.60\times10^{-19}\text{C}\right)\left(350\ \text{V}\right)}{9.11\times10^{-31}\text{kg}}} = 1.11\times10^{7}\ \text{m/s}.$$

(b) Equation 28-16 gives

$$r=\frac{m_e v}{eB}=\frac{(9.11\times10^{-31}\,\text{kg})(1.11\times10^{7}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(200\times10^{-3}\,\text{T})}=3.16\times10^{-4}\,\text{m}.$$

25. (a) The frequency of revolution is

$$f=\frac{Bq}{2\pi m_e}=\frac{(35.0\times10^{-6}\,\text{T})(1.60\times10^{-19}\,\text{C})}{2\pi(9.11\times10^{-31}\,\text{kg})}=9.78\times10^{5}\,\text{Hz}.$$

(b) Using Eq. 28-16, we obtain

$$r=\frac{m_e v}{qB}=\frac{\sqrt{2m_eK}}{qB}=\frac{\sqrt{2(9.11\times10^{-31}\,\text{kg})(100\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}}{(1.60\times10^{-19}\,\text{C})(35.0\times10^{-6}\,\text{T})}=0.964\,\text{m}\,.$$

26. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v}\times\vec{B}$ points leftward, which indeed seems to be the direction it is "pushed''; therefore, $q > 0$ (it is a proton).

(a) Equation 28-17 becomes $T=2\pi m_p/e|\vec{B}|$, or

$$2\left(130\times10^{-9}\right)=\frac{2\pi\left(1.67\times10^{-27}\right)}{\left(1.60\times10^{-19}\right)|\vec{B}|}$$

which yields $|\vec{B}|=0.252\,\text{T}$.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130$ ns.

27. (a) We solve for B from $m = B^2qx^2/8V$ (see Sample Problem 28.04 — "Uniform circular motion of a charged particle in a magnetic field"):

$$B=\sqrt{\frac{8Vm}{qx^2}}\,.$$

We evaluate this expression using $x = 2.00$ m:

$$B=\sqrt{\frac{8(100\times10^{3}\,\text{V})(3.92\times10^{-25}\,\text{kg})}{(3.20\times10^{-19}\,\text{C})(2.00\,\text{m})^2}}=0.495\,\text{T}\,.$$

(b) Let N be the number of ions that are separated by the machine per unit time. The current is $i = qN$ and the mass that is separated per unit time is $M = mN$, where m is the mass of a single ion. M has the value

$$M = \frac{100\times10^{-6}\text{ kg}}{3600\text{ s}} = 2.78\times10^{-8}\text{ kg/s}.$$

Since $N = M/m$ we have

$$i = \frac{qM}{m} = \frac{\left(3.20\times10^{-19}\text{ C}\right)\left(2.78\times10^{-8}\text{ kg/s}\right)}{3.92\times10^{-25}\text{ kg}} = 2.27\times10^{-2}\text{ A}.$$

(c) Each ion deposits energy qV in the cup, so the energy deposited in time Δt is given by

$$E = NqV\,\Delta t = \frac{iqV}{q}\Delta t = iV\,\Delta t.$$

For $\Delta t = 1.0$ h,

$$E = \left(2.27\times10^{-2}\text{ A}\right)\left(100\times10^{3}\text{ V}\right)\left(3600\text{ s}\right) = 8.17\times10^{6}\text{ J}.$$

To obtain the second expression, i/q is substituted for N.

28. Using $F = mv^2/r$ (for the centripetal force) and $K = mv^2/2$, we can easily derive the relation

$$K = \frac{1}{2}Fr.$$

With the values given in the problem, we thus obtain $K = 2.09\times10^{-22}$ J.

29. Reference to Fig. 28-11 is very useful for interpreting this problem. The distance traveled parallel to $\vec{B}$ is $d_{\|} = v_{\|}T = v_{\|}(2\pi m_e/|q|B)$ using Eq. 28-17. Thus,

$$v_{\|} = \frac{d_{\|}eB}{2\pi m_e} = 50.3\text{ km/s}$$

using the values given in this problem. Also, since the magnetic force is $|q|Bv_{\perp}$, then we find $v_{\perp} = 41.7$ km/s. The speed is therefore $v = \sqrt{v_{\perp}^2 + v_{\|}^2} = 65.3$ km/s.

30. Eq. 28-17 gives $T = 2\pi m_e/eB$. Thus, the total time is

$$\left(\frac{T}{2}\right)_1 + t_{\text{gap}} + \left(\frac{T}{2}\right)_2 = \frac{\pi m_e}{e}\left(\frac{1}{B_1} + \frac{1}{B_2}\right) + t_{\text{gap}}.$$

The time spent in the gap (which is where the electron is accelerating in accordance with Eq. 2-15) requires a few steps to figure out: letting $t = t_{gap}$ then we want to solve

$$d = v_0 t + \frac{1}{2}at^2 \;\Rightarrow\; 0.25\text{ m} = \sqrt{\frac{2K_0}{m_e}}\,t + \frac{1}{2}\left(\frac{e\Delta V}{m_e d}\right)t^2$$

for t. We find in this way that the time spent in the gap is $t \approx 6$ ns. Thus, the total time is 8.7 ns.

31. Each of the two particles will move in the same circular path, initially going in the opposite direction. After traveling half of the circular path they will collide. Therefore, using Eq. 28-17, the time is given by

$$t = \frac{T}{2} = \frac{\pi m}{Bq} = \frac{\pi\left(9.11\times10^{-31}\text{kg}\right)}{(3.53\times10^{-3}\text{T})\left(1.60\times10^{-19}\text{ C}\right)} = 5.07\times10^{-9}\text{ s}.$$

32. Let $v_{\|} = v\cos\theta$. The electron will proceed with a uniform speed $v_{\|}$ in the direction of $\vec{B}$ while undergoing uniform circular motion with frequency f in the direction perpendicular to B: $f = eB/2\pi m_e$. The distance d is then

$$d = v_{\|}T = \frac{v_{\|}}{f} = \frac{(v\cos\theta)2\pi m_e}{eB} = \frac{2\pi\left(1.5\times10^{7}\text{ m/s}\right)\left(9.11\times10^{-31}\text{kg}\right)\left(\cos10°\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.0\times10^{-3}\text{ T}\right)} = 0.53\text{m}.$$

33. **THINK** The path of the positron is helical because its velocity $\vec{v}$ has components parallel and perpendicular to the magnetic field $\vec{B}$.

EXPRESS If v is the speed of the positron then $v \sin\phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here $\phi = 89°$ is the angle between the velocity and the field. Newton's second law yields $eBv \sin\phi = m_e(v \sin\phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v/eB)\sin\phi$. The period is given by

$$T = \frac{2\pi r}{v\sin\phi} = \frac{2\pi m_e}{eB}.$$

The equation for r is substituted to obtain the second expression for T. For part (b), the pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT\cos\phi$.

ANALYZE (a) Substituting the values given, we find the period to be

$$T=\frac{2\pi m_e}{eB}=\frac{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.100\,\text{T}\right)}=3.58\times10^{-10}\,\text{s}.$$

(b) We use the kinetic energy, $K=\frac{1}{2}m_e v^2$, to find the speed:

$$v=\sqrt{\frac{2K}{m_e}}=\sqrt{\frac{2\left(2.00\times10^{3}\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{9.11\times10^{-31}\,\text{kg}}}=2.65\times10^{7}\ \text{m/s}\,.$$

Thus, the pitch is $p=\left(2.65\times10^{7}\ \text{m/s}\right)\left(3.58\times10^{-10}\ \text{s}\right)\cos 89°=1.66\times10^{-4}\ \text{m}$.

(c) The orbit radius is

$$R=\frac{m_e v\sin\phi}{eB}=\frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.65\times10^{7}\ \text{m/s}\right)\sin 89°}{\left(1.60\times10^{-19}\ \text{C}\right)\left(0.100\ \text{T}\right)}=1.51\times10^{-3}\ \text{m}\,.$$

LEARN The parallel component of the velocity, $v_{\parallel}=v\cos\phi$, is what determines the pitch of the helix. On the other hand, the perpendicular component, $v_{\perp}=v\sin\phi$, determines the radius of the helix.

34. (a) Equation 3-20 gives $\phi = \cos^{-1}(2/19) = 84°$.

(b) No, the magnetic field can only change the direction of motion of a free (unconstrained) particle, not its speed or its kinetic energy.

(c) No, as reference to Fig. 28-11 should make clear.

(d) We find $v_{\perp} = v \sin \phi = 61.3$ m/s, so $r = mv_{\perp}/eB =$ 5.7 nm.

35. (a) By conservation of energy (using qV for the potential energy, which is converted into kinetic form) the kinetic energy gained in each pass is 200 eV.

(b) Multiplying the part (a) result by n = 100 gives $\Delta K = n(200\text{ eV}) = 20.0$ keV.

(c) Combining Eq. 28-16 with the kinetic energy relation (n(200 eV) = $m_p v^2/2$ in this particular application) leads to the expression

$$r=\frac{m_p}{e\,B}\sqrt{\frac{2n(200\text{ eV})}{m_p}}$$

which shows that r is proportional to $\sqrt{n}$. Thus, the percent increase defined in the problem in going from n = 100 to n = 101 is $\sqrt{101/100}\, - 1$ = 0.00499 or 0.499%.

36. (a) The magnitude of the field required to achieve resonance is

$$B=\frac{2\pi fm_p}{q}=\frac{2\pi(12.0\times10^6\ \text{Hz})\left(1.67\times10^{-27}\,\text{kg}\right)}{1.60\times10^{-19}\,\text{C}}=0.787\,\text{T}.$$

(b) The kinetic energy is given by

$$K=\tfrac{1}{2}mv^2=\frac{1}{2}m\left(2\pi Rf\right)^2=\frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)4\pi^2(0.530\text{ m})^2(12.0\times10^6\ \text{Hz})^2$$
$$=1.33\times10^{-12}\,\text{J}=8.34\times10^6\,\text{eV}.$$

(c) The required frequency is

$$f=\frac{qB}{2\pi m_p}=\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.57\,\text{T}\right)}{2\pi\left(1.67\times10^{-27}\,\text{kg}\right)}=2.39\times10^7\,\text{Hz}.$$

(d) The kinetic energy is given by

$$K=\tfrac{1}{2}mv^2=\frac{1}{2}m\left(2\pi Rf\right)^2=\frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)4\pi^2(0.530\text{ m})^2(2.39\times10^7\ \text{Hz})^2$$
$$=5.3069\times10^{-12}\,\text{J}=3.32\times10^7\,\text{eV}.$$

37. We approximate the total distance by the number of revolutions times the circumference of the orbit corresponding to the average energy. This should be a good approximation since the deuteron receives the same energy each revolution and its period does not depend on its energy. The deuteron accelerates twice in each cycle, and each time it receives an energy of $qV = 80 \times 10^3$ eV. Since its final energy is 16.6 MeV, the number of revolutions it makes is

$$n=\frac{16.6\times10^6\,\text{eV}}{2\left(80\times10^3\,\text{eV}\right)}=104\ .$$

Its average energy during the accelerating process is 8.3 MeV. The radius of the orbit is given by $r = mv/qB$, where v is the deuteron's speed. Since this is given by $v=\sqrt{2K/m}$, the radius is

$$r=\frac{m}{qB}\sqrt{\frac{2K}{m}}=\frac{1}{qB}\sqrt{2Km}\ .$$

For the average energy

$$r=\frac{\sqrt{2\left(8.3\times10^6\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)\left(3.34\times10^{-27}\,\text{kg}\right)}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.57\,\text{T}\right)}=0.375\,\text{m}\ .$$

The total distance traveled is about

$$n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2 \text{ m}.$$

38. (a) Using Eq. 28-23 and Eq. 28-18, we find

$$f_{\text{osc}} = \frac{qB}{2\pi m_p} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.20\,\text{T}\right)}{2\pi\left(1.67\times10^{-27}\,\text{kg}\right)} = 1.83\times10^{7}\,\text{Hz}.$$

(b) From $r = m_p v/qB = \sqrt{2m_P k}/qB$ we have

$$K = \frac{\left(rqB\right)^2}{2m_p} = \frac{\left[\left(0.500\text{m}\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(1.20\,\text{T}\right)\right]^2}{2\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 1.72\times10^{7}\,\text{eV}.$$

39. **THINK** The magnetic force on a wire that carries a current i is given by $\vec{F}_B = i\vec{L}\times\vec{B}$, where $\vec{L}$ is the length vector of the wire and $\vec{B}$ is the magnetic field.

EXPRESS The magnitude of the magnetic force on the wire is given by $F_B = iLB \sin \phi$, where ϕ is the angle between the current and the field.

ANALYZE (a) With $\phi = 70°$, we have

$$F_B = \left(5000\,\text{A}\right)\left(100\,\text{m}\right)\left(60.0\times10^{-6}\ \text{T}\right)\sin 70° = 28.2\,\text{N}.$$

(b) We apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L}\times\vec{B}$ to show that the force is to the west.

LEARN From the expression $\vec{F}_B = i\vec{L}\times\vec{B}$, we see that the magnetic force acting on a current-carrying wire is a maximum when $\vec{L}$ is perpendicular to $\vec{B}$ ($\phi = 90°$), and is zero when $\vec{L}$ is parallel to $\vec{B}$ ($\phi = 0°$).

40. The magnetic force on the (straight) wire is

$$F_B = iBL\sin\theta = \left(13.0\text{A}\right)\left(1.50\text{T}\right)\left(1.80\text{m}\right)\left(\sin 35.0°\right) = 20.1\text{N}.$$

41. (a) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \;\Rightarrow\; i = \frac{mg}{LB} = \frac{(0.0130\,\text{kg})(9.8\,\text{m/s}^2)}{(0.620\,\text{m})(0.440\,\text{T})} = 0.467\,\text{A}.$$

(b) Applying the right-hand rule reveals that the current must be from left to right.

42. (a) From symmetry, we conclude that any x-component of force will vanish (evaluated over the entirety of the bent wire as shown). By the right-hand rule, a field in the $\hat{k}$ direction produces on each part of the bent wire a y-component of force pointing in the $-\hat{j}$ direction; each of these components has magnitude

$$|F_y| = i\ell\,|\vec{B}|\sin 30° = (2.0\text{ A})(2.0\text{ m})(4.0\text{ T})\sin 30° = 8\,\text{N}.$$

Therefore, the force on the wire shown in the figure is $(-16\hat{j})\,\text{N}$.

(b) The force exerted on the left half of the bent wire points in the $-\hat{k}$ direction, by the right-hand rule, and the force exerted on the right half of the wire points in the $+\hat{k}$ direction. It is clear that the magnitude of each force is equal, so that the force (evaluated over the entirety of the bent wire as shown) must necessarily vanish.

43. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $\ell_y = 50$ cm side runs along the $+y$ axis, while the $\ell_x = 120$ cm side runs along the $+x$ axis. The angle made by the hypotenuse (of length 130 cm) is

$$\theta = \tan^{-1}(50/120) = 22.6°,$$

relative to the 120 cm side. If one measures the angle counterclockwise from the $+x$ direction, then the angle for the hypotenuse is 180° – 22.6° = +157°. Since we are only asked to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the $+z$ axis. We take $\vec{B}$ to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\vec{B}| = 0.0750\,\text{T}$,

$$B_x = -B\cos\theta = -0.0692\,\text{T}\;, \quad B_y = B\sin\theta = 0.0288\,\text{T}.$$

(a) Equation 28-26 produces zero force when $\vec{L}\,||\,\vec{B}$ so there is no force exerted on the hypotenuse of length 130 cm.

(b) On the 50 cm side, the B_x component produces a force $i\ell_y B_x\hat{k}$, and there is no contribution from the B_y component. Using SI units, the magnitude of the force on the ℓ_y side is therefore

$$(4.00\,\text{A})(0.500\,\text{m})(0.0692\,\text{T}) = 0.138\,\text{N}.$$

(c) On the 120 cm side, the B_y component produces a force $i\ell_x B_y \hat{k}$, and there is no contribution from the B_x component. The magnitude of the force on the ℓ_x side is also

$$(4.00\,\text{A})(1.20\,\text{m})(0.0288\,\text{T}) = 0.138\,\text{N}.$$

(d) The net force is

$$i\ell_y B_x \hat{k} + i\ell_x B_y \hat{k} = 0,$$

keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed $\vec{B}$ went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$, but $B_y < 0$ and a zero net force would still be the result.

44. Consider an infinitesimal segment of the loop, of length ds. The magnetic field is perpendicular to the segment, so the magnetic force on it has magnitude $dF = iB\,ds$. The horizontal component of the force has magnitude

$$dF_h = (iB\cos\theta)ds$$

and points inward toward the center of the loop. The vertical component has magnitude

$$dF_y = (iB\sin\theta)ds$$

and points upward. Now, we sum the forces on all the segments of the loop. The horizontal component of the total force vanishes, since each segment of wire can be paired with another, diametrically opposite, segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$F_v = iB\sin\theta\int ds = 2\pi a iB\sin\theta = 2\pi(0.018\text{ m})(4.6\times10^{-3}\text{ A})(3.4\times10^{-3}\text{ T})\sin 20°$$
$$= 6.0\times10^{-7}\text{ N}.$$

We note that i, B, and θ have the same value for every segment and so can be factored from the integral.

45. The magnetic force on the wire is

$$\begin{aligned}\vec{F}_B &= i\vec{L}\times\vec{B} = iL\hat{i}\times\left(B_y\hat{j}+B_z\hat{k}\right) = iL\left(-B_z\hat{j}+B_y\hat{k}\right)\\ &= (0.500\text{A})\,(0.500\text{m})\left[-(0.0100\text{T})\,\hat{j}+(0.00300\text{T})\,\hat{k}\right]\\ &= \left(-2.50\times10^{-3}\hat{j}+0.750\times10^{-3}\hat{k}\right)\text{N}.\end{aligned}$$

46. (a) The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus

$$v = at = \frac{F_B t}{m} = \frac{idBt}{m} = \frac{(9.13\times10^{-3}\,\text{A})(2.56\times10^{-2}\,\text{m})(5.63\times10^{-2}\,\text{T})(0.0611\,\text{s})}{2.41\times10^{-5}\,\text{kg}}$$
$$= 3.34\times10^{-2}\,\text{m/s}.$$

(b) The direction is to the left (away from the generator).

47. (a) The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; mg, the magnitude of the (downward) force of gravity; $\vec{F}_N$, the normal force exerted by the stationary rails upward on the rod; and $\vec{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that $\vec{f}$ points westward (and is equal to its maximum possible value $\mu_s F_N$). Thus, $\vec{F}$ has an eastward component F_x and an upward component F_y, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the right-hand rule, a downward component (B_d) of $\vec{B}$ will produce the eastward F_x, and a westward component (B_w) will produce the upward F_y. Specifically,

$$F_x = iLB_d, \quad F_y = iLB_w.$$

Considering forces along a vertical axis, we find

$$F_N = mg - F_y = mg - iLB_w$$

so that

$$f = f_{s,\max} = \mu_s(mg - iLB_w).$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x - f = 0 \quad \Rightarrow \quad iLB_d = \mu_s\left(mg - iLB_w\right).$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB\cos\theta = \mu_s\left(mg - iLB\sin\theta\right) \quad \Rightarrow \quad B = \frac{\mu_s mg}{iL\left(\cos\theta + \mu_s\sin\theta\right)}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°.$$

Consequently,

$$B_{\min} = \frac{0.60(1.0\,\text{kg})(9.8\,\text{m/s}^2)}{(50\,\text{A})(1.0\,\text{m})(\cos 31° + 0.60 \sin 31°)} = 0.10\,\text{T}.$$

(b) As shown above, the angle is $\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°$.

48. We use $d\vec{F}_B = id\vec{L} \times \vec{B}$, where $d\vec{L} = dx\hat{\text{i}}$ and $\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}}$. Thus,

$$\vec{F}_B = \int id\vec{L} \times \vec{B} = \int_{x_i}^{x_f} idx\hat{\text{i}} \times (B_x\hat{\text{i}} + B_y\hat{\text{j}}) = i\int_{x_i}^{x_f} B_y dx\hat{\text{k}}$$
$$= (-5.0\,\text{A})\left(\int_{1.0}^{3.0} (8.0x^2 dx)\ (\text{m}\cdot\text{mT})\right)\hat{\text{k}} = (-0.35\text{N})\hat{\text{k}}.$$

49. **THINK** Magnetic forces on the loop produce a torque that rotates it about the hinge line. Our applied field has two components: $B_x > 0$ and $B_z > 0$.

EXPRESS Considering each straight segment of the rectangular coil, we note that Eq. 28-26 produces a nonzero force only for the component of $\vec{B}$ which is perpendicular to that segment; we also note that the equation is effectively multiplied by $N = 20$ due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight segment of the coil that lies along the *y* axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight segments experience forces due to Eq. 28-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the ±*y* directions and are thus unable to produce a torque about the *y* axis. Consequently, the torque derives completely from the force exerted on the straight segment located at $x = 0.050$ m, which has length $L = 0.10$ m and is shown in Fig. 28-45 carrying current in the –*y* direction.

Now, the B_z component will produce a force on this straight segment which points in the –*x* direction (back toward the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B\cos\theta$ where $B = 0.50$ T and $\theta = 30°$) produces a force equal to $F = NiLB_x$ which points (by the right-hand rule) in the +*z* direction.

ANALYZE Since the action of the force *F* is perpendicular to the plane of the coil, and is located a distance *x* away from the hinge, then the torque has magnitude

$$\tau = (NiLB_x)(x) = NiLxB\cos\theta = (20)(0.10\text{ A})(0.10\text{ m})(0.050\text{ m})(0.50\text{ T})\cos 30°$$
$$= 0.0043\text{ N}\cdot\text{m}.$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is $\vec{\tau} = (-4.3\times10^{-3}\ \text{N}\cdot\text{m})\hat{\text{j}}$

LEARN An alternative way to do this problem is through the use of Eq. 28-37: $\vec{\tau} = \vec{\mu}\times\vec{B}$. The magnetic moment vector is

$$\vec{\mu} = -(NiA)\,\hat{\text{k}} = -(20)(0.10\,\text{A})(0.0050\,\text{m}^2)\hat{\text{k}} = -(0.01\,\text{A}\cdot\text{m}^2)\hat{\text{k}}.$$

The torque on the loop is

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = (-\mu\,\hat{\text{k}})\times(B\cos\theta\,\hat{\text{i}} + B\sin\theta\,\hat{\text{k}}) = -(\mu B\cos\theta)\hat{\text{j}} \\ &= -(0.01\,\text{A}\cdot\text{m}^2)(0.50\ \text{T})\cos30°\hat{\text{j}} \\ &= (-4.3\times10^{-3}\ \text{N}\cdot\text{m})\hat{\text{j}}.\end{aligned}$$

50. We use $\tau_{\max} = |\vec{\mu}\times\vec{B}|_{\max} = \mu B = i\pi r^2 B$, and note that $i = qf = qv/2\pi r$. So

$$\tau_{\max} = \left(\frac{qv}{2\pi r}\right)\pi r^2 B = \frac{1}{2}qvrB = \frac{1}{2}(1.60\times10^{-19}\,\text{C})(2.19\times10^{6}\,\text{m/s})(5.29\times10^{-11}\,\text{m})(7.10\times10^{-3}\,\text{T})$$
$$= 6.58\times10^{-26}\,\text{N}\cdot\text{m}.$$

51. We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline F_N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B\sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B\sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = Ni(2rL)$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\ \text{kg})(9.8\ \text{m/s}^2)}{2(10.0)(0.100\ \text{m})(0.500\ \text{T})} = 2.45\ \text{A}.$$

52. The insight central to this problem is that for a given length of wire (formed into a rectangle of various possible aspect ratios), the maximum possible area is enclosed when the ratio of height to width is 1 (that is, when it is a square). The maximum possible value for the width, the problem says, is $x = 4$ cm (this is when the height is very close to zero, so the total length of wire is effectively 8 cm). Thus, when it takes the shape of a square the value of x must be ¼ of 8 cm; that is, $x = 2$ cm when it encloses maximum area (which leads to a maximum torque by Eq. 28-35 and Eq. 28-37) of $A = (0.020\ \text{m})^2 = 0.00040\ \text{m}^2$. Since $N = 1$ and the torque in this case is given as $4.8 \times 10^{-4}\ \text{N}\cdot\text{m}$, then the aforementioned equations lead immediately to $i = 0.0030$ A.

53. We replace the current loop of arbitrary shape with an assembly of small adjacent rectangular loops filling the same area that was enclosed by the original loop (as nearly as possible). Each rectangular loop carries a current i flowing in the same sense as the original loop. As the sizes of these rectangles shrink to infinitesimally small values, the assembly gives a current distribution equivalent to that of the original loop. The magnitude of the torque $\Delta\vec{\tau}$ exerted by $\vec{B}$ on the nth rectangular loop of area ΔA_n is given by $\Delta\tau_n = NiB\sin\theta\Delta A_n$. Thus, for the whole assembly

$$\tau = \sum_n \Delta\tau_n = NiB\sum_n \Delta A_n = NiAB\sin\theta.$$

54. (a) The kinetic energy gained is due to the potential energy decrease as the dipole swings from a position specified by angle θ to that of being aligned (zero angle) with the field. Thus,

$$K = U_i - U_f = -\mu B\cos\theta - (-\mu B\cos 0°).$$

Therefore, using SI units, the angle is

$$\theta = \cos^{-1}\left(1 - \frac{K}{\mu B}\right) = \cos^{-1}\left(1 - \frac{0.00080}{(0.020)(0.052)}\right) = 77°.$$

(b) Since we are making the assumption that no energy is dissipated in this process, then the dipole will continue its rotation (similar to a pendulum) until it reaches an angle $\theta = 77°$ on the other side of the alignment axis.

55. **THINK** Our system consists of two concentric current-carrying loops. The net magnetic dipole moment is the vector sum of the individual contributions.

EXPRESS The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. Each of the loops is a circle, so the area is $A = \pi r^2$, where r is the radius of the loop.

ANALYZE (a) Since the currents are in the same direction, the magnitude of the magnetic moment vector is

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi(7.00\,\text{A})\left[(0.200\,\text{m})^2 + (0.300\,\text{m})^2\right] = 2.86\,\text{A}\cdot\text{m}^2.$$

(b) Now, the two currents flow in the opposite directions, so the magnitude of the magnetic moment vector is

$$\mu = \pi r_2^2 i_2 - \pi r_1^2 i_1 = \pi(7.00\,\text{A})\left[(0.300\,\text{m})^2 - (0.200\,\text{m})^2\right] = 1.10\,\text{A}\cdot\text{m}^2.$$

LEARN In both cases, the directions of the dipole moments are into the page. The direction of $\vec{\mu}$ is that of the normal vector $\vec{n}$ to the plane of the coil, in accordance with the right-hand rule shown in Fig. 28-19(b).

56. (a) $\mu = NAi = \pi r^2 i = \pi(0.150\,\text{m})^2(2.60\,\text{A}) = 0.184\,\text{A}\cdot\text{m}^2$.

(b) The torque is

$$\tau = \left|\vec{\mu} \times \vec{B}\right| = \mu B \sin\theta = (0.184\,\text{A}\cdot\text{m}^2)(12.0\,\text{T})\sin 41.0° = 1.45\,\text{N}\cdot\text{m}.$$

57. **THINK** Magnetic forces on a current-carrying loop produce a torque that tends to align the magnetic dipole moment with the magnetic field.

EXPRESS The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where r is the radius of a turn.

ANALYZE (a) Thus, the current is

$$i = \frac{\mu}{N\pi r^2} = \frac{2.30\,\text{A}\cdot\text{m}^2}{(160)(\pi)(0.0190\,\text{m})^2} = 12.7\,\text{A}.$$

(b) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by

$$\tau_{max} = \mu B = (2.30\,\text{A}\cdot\text{m}^2)(35.0\times10^{-3}\,\text{T}) = 8.05\times10^{-2}\,\text{N}\cdot\text{m}.$$

LEARN The torque on the coil can be written as $\vec{\tau} = \vec{\mu} \times \vec{B}$, with $\tau = |\vec{\tau}| = \mu B \sin\theta$, where θ is the angle between $\vec{\mu}$ and $\vec{B}$. Thus, τ is a maximum when $\theta = 90°$, and zero when $\theta = 0°$.

58. From $\mu = NiA = i\pi r^2$ we get

$$i = \frac{\mu}{\pi r^2} = \frac{8.00\times 10^{22}\ \text{J/T}}{\pi\left(3500\times 10^3\ \text{m}\right)^2} = 2.08\times 10^9\ \text{A}.$$

59. (a) The area of the loop is $A = \frac{1}{2}(30\,\text{cm})(40\,\text{cm}) = 6.0\times 10^2\,\text{cm}^2$, so

$$\mu = iA = (5.0\,\text{A})\left(6.0\times 10^{-2}\,\text{m}^2\right) = 0.30\,\text{A}\cdot\text{m}^2.$$

(b) The torque on the loop is

$$\tau = \mu B\sin\theta = \left(0.30\,\text{A}\cdot\text{m}^2\right)\left(80\times 10^3\,\text{T}\right)\sin 90° = 2.4\times 10^{-2}\,\text{N}\cdot\text{m}.$$

60. Let a = 30.0 cm, b = 20.0 cm, and c = 10.0 cm. From the given hint, we write

$$\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2 = iab\left(-\hat{k}\right) + iac\left(\hat{j}\right) = ia\left(c\hat{j} - b\hat{k}\right) = (5.00\,\text{A})(0.300\,\text{m})\left[(0.100\,\text{m})\hat{j} - (0.200\,\text{m})\hat{k}\right]$$
$$= \left(0.150\hat{j} - 0.300\hat{k}\right)\text{A}\cdot\text{m}^2.$$

61. **THINK** Magnetic forces on a current-carrying coil produce a torque that tends to align the magnetic dipole moment with the field. The magnetic energy of the dipole depends on its orientation relative to the field.

EXPRESS The magnetic potential energy of the dipole is given by $U = -\vec{\mu}\cdot\vec{B}$, where $\vec{\mu}$ is the magnetic dipole moment of the coil and $\vec{B}$ is the magnetic field. The magnitude of $\vec{\mu}$ is $\mu = NiA$, where i is the current in the coil, N is the number of turns, A is the area of the coil. On the other hand, the torque on the coil is given by the vector product $\vec{\tau} = \vec{\mu}\times\vec{B}$.

ANALYZE (a) By using the right-hand rule, we see that $\vec{\mu}$ is in the $-y$ direction. Thus, we have

$$\vec{\mu} = (NiA)(-\hat{j}) = -(3)(2.00\ \text{A})(4.00\times 10^{-3}\ \text{m}^2)\hat{j} = -(0.0240\ \text{A}\cdot\text{m}^2)\hat{j}.$$

The corresponding magnetic energy is

$$U = -\vec{\mu}\cdot\vec{B} = -\mu_y B_y = -(-0.0240\ \text{A}\cdot\text{m}^2)(-3.00\times 10^{-3}\ \text{T}) = -7.20\times 10^{-5}\ \text{J}.$$

(b) Using the fact that $\hat{\mathrm{j}}\cdot\hat{\mathrm{i}}=0$, $\hat{\mathrm{j}}\times\hat{\mathrm{j}}=0$, and $\hat{\mathrm{j}}\times\hat{\mathrm{k}}=\hat{\mathrm{i}}$, the torque on the coil is

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = \mu_y B_z \hat{\mathrm{i}} - \mu_y B_x \hat{\mathrm{k}} \\ &= (-0.0240\ \mathrm{A\cdot m^2})(-4.00\times10^{-3}\ \mathrm{T})\hat{\mathrm{i}} - (-0.0240\ \mathrm{A\cdot m^2})(2.00\times10^{-3}\ \mathrm{T})\hat{\mathrm{k}} \\ &= (9.60\times10^{-5}\ \mathrm{N\cdot m})\hat{\mathrm{i}} + (4.80\times10^{-5}\ \mathrm{N\cdot m})\hat{\mathrm{k}}.\end{aligned}$$

LEARN The magnetic energy is highest when $\vec{\mu}$ is in the opposite direction of $\vec{B}$, and lowest when $\vec{\mu}$ lines up with $\vec{B}$.

62. Looking at the point in the graph (Fig. 28-51(b)) corresponding to $i_2 = 0$ (which means that coil 2 has no magnetic moment) we are led to conclude that the magnetic moment of coil 1 must be $\mu_1 = 2.0\times10^{-5}\ \mathrm{A\cdot m^2}$. Looking at the point where the line crosses the axis (at $i_2 = 5.0$ mA) we conclude (since the magnetic moments cancel there) that the magnitude of coil 2's moment must also be $\mu_2 = 2.0\times10^{-5}\ \mathrm{A\cdot m^2}$ when $i_2 = 0.0050$ A, which means (Eq. 28-35)

$$N_2A_2 = \frac{\mu_2}{i_2} = \frac{2.0\times10^{-5}\ \mathrm{A\cdot m^2}}{0.0050\ \mathrm{A}} = 4.0\times10^{-3}\ \mathrm{m^2}.$$

Now the problem has us consider the direction of coil 2's current changed so that the net moment is the sum of two (positive) contributions, from coil 1 and coil 2, specifically for the case that $i_2 = 0.007$ A. We find that total moment is

$$\mu = (2.0\times10^{-5}\ \mathrm{A\cdot m^2}) + (N_2A_2\, i_2) = 4.8\times10^{-5}\ \mathrm{A\cdot m^2}.$$

63. The magnetic dipole moment is $\vec{\mu} = \mu\left(0.60\,\hat{\mathrm{i}} - 0.80\,\hat{\mathrm{j}}\right)$, where

$$\mu = NiA = Ni\pi r^2 = 1(0.20\ \mathrm{A})\pi(0.080\ \mathrm{m})^2 = 4.02\times10^{-4}\ \mathrm{A\cdot m^2}.$$

Here i is the current in the loop, N is the number of turns, A is the area of the loop, and r is its radius.

(a) The torque is

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = \mu\left(0.60\hat{\mathrm{i}} - 0.80\hat{\mathrm{j}}\right)\times\left(0.25\hat{\mathrm{i}} + 0.30\hat{\mathrm{k}}\right) \\ &= \mu\left[(0.60)(0.30)\left(\hat{\mathrm{i}}\times\hat{\mathrm{k}}\right) - (0.80)(0.25)\left(\hat{\mathrm{j}}\times\hat{\mathrm{i}}\right) - (0.80)(0.30)\left(\hat{\mathrm{j}}\times\hat{\mathrm{k}}\right)\right] \\ &= \mu\left(-0.18\hat{\mathrm{j}} + 0.20\hat{\mathrm{k}} - 0.24\hat{\mathrm{i}}\right).\end{aligned}$$

Here $\hat{\mathrm{i}}\times\hat{\mathrm{k}}=-\hat{\mathrm{j}}, \hat{\mathrm{j}}\times\hat{\mathrm{i}}=-\hat{\mathrm{k}},$ and $\hat{\mathrm{j}}\times\hat{\mathrm{k}}=\hat{\mathrm{i}}$ are used. We also use $\hat{\mathrm{i}}\times\hat{\mathrm{i}}=0$. Now, we substitute the value for μ to obtain

$$\vec{\tau}=\left(-9.7\times10^{-4}\,\hat{\mathrm{i}}-7.2\times10^{-4}\,\hat{\mathrm{j}}+8.0\times10^{-4}\,\hat{\mathrm{k}}\right)\mathrm{N\cdot m}.$$

(b) The orientation energy of the dipole is given by

$$U=-\vec{\mu}\cdot\vec{B}=-\mu\left(0.60\hat{\mathrm{i}}-0.80\hat{\mathrm{j}}\right)\cdot\left(0.25\hat{\mathrm{i}}+0.30\hat{\mathrm{k}}\right)=-\mu(0.60)(0.25)=-0.15\mu=-6.0\times10^{-4}\,\mathrm{J}.$$

Here $\hat{\mathrm{i}}\cdot\hat{\mathrm{i}}=1, \hat{\mathrm{i}}\cdot\hat{\mathrm{k}}=0, \hat{\mathrm{j}}\cdot\hat{\mathrm{i}}=0,$ and $\hat{\mathrm{j}}\cdot\hat{\mathrm{k}}=0$ are used.

64. Eq. 28-39 gives $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi$, so at $\phi = 0$ (corresponding to the lowest point on the graph in Fig. 28-52) the mechanical energy is

$$K + U = K_o + (-\mu B) = 6.7 \times 10^{-4}\ \mathrm{J} + (-5 \times 10^{-4}\ \mathrm{J}) = 1.7 \times 10^{-4}\ \mathrm{J}.$$

The turning point occurs where $K = 0$, which implies $U_{\text{turn}} = 1.7 \times 10^{-4}$ J. So the angle where this takes place is given by

$$\phi=-\cos^{-1}\left(\frac{1.7\times10^{-4}\ \mathrm{J}}{\mu B}\right)=110°$$

where we have used the fact (see above) that $\mu B = 5 \times 10^{-4}$ J.

65. **THINK** The torque on a current-carrying coil is a maximum when its dipole moment is perpendicular to the magnetic field.

EXPRESS The magnitude of the torque on the coil is given by $\tau=|\vec{\tau}|=\mu B\sin\theta,$ where θ is the angle between $\vec{\mu}$ and $\vec{B}$. The magnitude of $\vec{\mu}$ is $\mu=NiA,$ where i is the current in the coil, N is the number of turns, A is the area of the coil. Thus, if N closed loops are formed from the wire of length L, the circumference of each loop is L/N, the radius of each loop is $R = L/2\pi N$, and the area of each loop is

$$A=\pi R^2=\pi\left(L/2\pi N\right)^2=L^2/4\pi N^2.$$

ANALYZE (a) For maximum torque, we orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular (i.e., at a $90°$ angle) to the field.

(b) The magnitude of the torque is then

$$\tau=NiAB=\left(Ni\right)\left(\frac{L^2}{4\pi N^2}\right)B=\frac{iL^2B}{4\pi N}.$$

To maximize the torque, we take the number of turns N to have the smallest possible value, 1. Then $\tau = iL^2B/4\pi$.

(c) The magnitude of the maximum torque is

$$\tau = \frac{iL^2B}{4\pi} = \frac{(4.51\times10^{-3}\ \text{A})(0.250\ \text{m})^2(5.71\times10^{-3}\,\text{T})}{4\pi} = 1.28\times10^{-7}\ \text{N}\cdot\text{m}.$$

LEARN The torque tends to align $\vec{\mu}$ with $\vec{B}$. The magnitude of the torque is a maximum when the angle between $\vec{\mu}$ and $\vec{B}$ is $\theta = 90°$, and is zero when $\theta = 0°$.

66. The equation of motion for the proton is

$$\vec{F} = q\vec{v}\times\vec{B} = q\left(v_x\hat{\text{i}} + v_y\hat{\text{j}} + v_z\hat{\text{k}}\right)\times B\hat{\text{i}} = qB\left(v_z\hat{\text{j}} - v_y\hat{\text{k}}\right)$$
$$= m_p\vec{a} = m_p\left[\left(\frac{dv_x}{dt}\right)\hat{\text{i}} + \left(\frac{dv_y}{dt}\right)\hat{\text{j}} + \left(\frac{dv_z}{dt}\right)\hat{\text{k}}\right].$$

Thus,

$$\frac{dv_x}{dt} = 0, \quad \frac{dv_y}{dt} = \omega v_z, \quad \frac{dv_z}{dt} = -\omega v_y,$$

where $\omega = eB/m$. The solution is $v_x = v_{0x}$, $v_y = v_{0y}\cos\omega t$, and $v_z = -v_{0y}\sin\omega t$. In summary, we have

$$\vec{v}(t) = v_{0x}\hat{\text{i}} + v_{0y}\cos(\omega t)\hat{\text{j}} - v_{0y}(\sin\omega t)\hat{\text{k}}.$$

67. (a) We use $\vec{\tau} = \vec{\mu}\times\vec{B}$, where $\vec{\mu}$ points into the wall (since the current goes clockwise around the clock). Since $\vec{B}$ points toward the one-hour (or "5-minute'') mark, and (by the properties of vector cross products) $\vec{\tau}$ must be perpendicular to it, then (using the right-hand rule) we find $\vec{\tau}$ points at the 20-minute mark. So the time interval is 20 min.

(b) The torque is given by

$$\tau = |\vec{\mu}\times\vec{B}| = \mu B\sin 90° = NiAB = \pi Nir^2B = 6\pi\left(2.0\text{A}\right)\left(0.15\text{m}\right)^2\left(70\times10^{-3}\text{T}\right)$$
$$= 5.9\times10^{-2}\,\text{N}\cdot\text{m}.$$

68. The unit vector associated with the current element (of magnitude $d\ell$) is $-\hat{\text{j}}$. The (infinitesimal) force on this element is

$$d\vec{F} = i\,d\ell\left(-\hat{\text{j}}\right)\times\left(0.3y\hat{\text{i}} + 0.4y\hat{\text{j}}\right)$$

with SI units (and 3 significant figures) understood. Since $\hat{\mathrm{j}}\times\hat{\mathrm{i}} = -\hat{\mathrm{k}}$ and $\hat{\mathrm{j}}\times\hat{\mathrm{j}} = 0$, we obtain

$$d\vec{F} = 0.3iy\,d\ell\ \hat{\mathrm{k}} = \left(6.00\times10^{-4}\ \mathrm{N/m^2}\right) y\,d\ell\ \hat{\mathrm{k}}.$$

We integrate the force element found above, using the symbol ξ to stand for the coefficient 6.00×10^{-4} N/m^2, and obtain

$$\vec{F} = \int d\vec{F} = \xi\ \hat{\mathrm{k}}\int_0^{0.25} y dy = \xi\ \hat{\mathrm{k}}\left(\frac{0.25^2}{2}\right) = (1.88\times10^{-5}\ \mathrm{N})\hat{\mathrm{k}}\ .$$

69. From $m = B^2qx^2/8V$ we have $\Delta m = (B^2q/8V)(2x\Delta x)$. Here $x = \sqrt{8Vm/B^2q}$, which we substitute into the expression for Δm to obtain

$$\Delta m = \left(\frac{B^2q}{8V}\right) 2\sqrt{\frac{8mV}{B^2q}}\Delta x = B\sqrt{\frac{mq}{2V}}\Delta x\,.$$

Thus, the distance between the spots made on the photographic plate is

$$\Delta x = \frac{\Delta m}{B}\sqrt{\frac{2V}{mq}} = \frac{\left(37\,\mathrm{u} - 35\,\mathrm{u}\right)\left(1.66\times10^{-27}\ \mathrm{kg/u}\right)}{0.50\,\mathrm{T}}\sqrt{\frac{2\left(7.3\times10^3\ \mathrm{V}\right)}{\left(36\,\mathrm{u}\right)\left(1.66\times10^{-27}\ \mathrm{kg/u}\right)\left(1.60\times10^{-19}\mathrm{C}\right)}}$$
$$= 8.2\times10^{-3}\,\mathrm{m}.$$

70. (a) Equating the magnitude of the electric force ($F_e = eE$) with that of the magnetic force (Eq. 28-3), we obtain $B = E / v \sin\phi$. The field is smallest when the $\sin\phi$ factor is at its largest value; that is, when $\phi = 90°$. Now, we use $K = \frac{1}{2}mv^2$ to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(2.5\times10^3\mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)}{9.11\times10^{-31}\mathrm{kg}}} = 2.96\times10^7\ \mathrm{m/s}.$$

Thus,

$$B = \frac{E}{v} = \frac{10\times10^3\ \mathrm{V/m}}{2.96\times10^7\ \mathrm{m/s}} = 3.4\times10^{-4}\mathrm{T}.$$

The direction of the magnetic field must be perpendicular to both the electric field ($-\hat{\mathrm{j}}$) and the velocity of the electron ($+\hat{\mathrm{i}}$). Since the electric force $\vec{F}_e = (-e)\vec{E}$ points in the $+\hat{\mathrm{j}}$ direction, the magnetic force $\vec{F}_B = (-e)\vec{v}\times\vec{B}$ points in the $-\hat{\mathrm{j}}$ direction. Hence, the direction of the magnetic field is $-\hat{\mathrm{k}}$. In unit-vector notation, $\vec{B} = (-3.4\times10^{-4}\mathrm{T})\hat{\mathrm{k}}$.

71. The period of revolution for the iodine ion is

$$T = 2\pi r/v = 2\pi m/Bq,$$

which gives

$$m = \frac{BqT}{2\pi} = \frac{\left(45.0\times10^{-3}\,\text{T}\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(1.29\times10^{-3}\,\text{s}\right)}{(7)(2\pi)\left(1.66\times10^{-27}\,\text{kg/u}\right)} = 127\,\text{u}.$$

72. (a) For the magnetic field to have an effect on the moving electrons, we need a non-negligible component of $\vec{B}$ to be perpendicular to $\vec{v}$ (the electron velocity). It is most efficient, therefore, to orient the magnetic field so it is perpendicular to the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, and the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_e v^2/r$, so the radius of the circle is given by $r = m_e v/eB$ in agreement with Eq. 28-16. The kinetic energy of the electron is $K = \frac{1}{2}m_e v^2$, so $v = \sqrt{2K/m_e}$. Thus,

$$r = \frac{m_e}{eB}\sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_e K}{e^2B^2}}\ .$$

This must be less than d, so $\sqrt{\frac{2m_e K}{e^2B^2}} \le d$, or $B \ge \sqrt{\frac{2m_e K}{e^2 d^2}}$.

(b) If the electrons are to travel as shown in Fig. 28-53, the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path, as it must be (in order to make the circular motion possible).

73. **THINK** The electron moving in the Earth's magnetic field is being accelerated by the magnetic force acting on it.

EXPRESS Since the electron is moving in a line that is parallel to the horizontal component of the Earth's magnetic field, the magnetic force on the electron is due to the vertical component of the field only. The magnitude of the force acting on the electron is given by $F = evB$, where B represents the downward component of Earth's field. With $F = m_e a$, the acceleration of the electron is $a = evB/m_e$.

ANALYZE (a) The electron speed can be found from its kinetic energy $K = \frac{1}{2}m_e v^2$:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(12.0\times10^3\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}{9.11\times10^{-31}\,\text{kg}}} = 6.49\times10^7\ \text{m/s}.$$

Therefore,

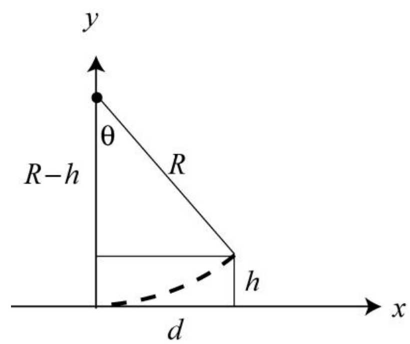
$$a = \frac{evB}{m_e} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(6.49\times10^{7}\ \text{m/s}\right)\left(55.0\times10^{-6}\,\text{T}\right)}{9.11\times10^{-31}\,\text{kg}}$$
$$= 6.27\times10^{14}\ \text{m/s}^2 \approx 6.3\times10^{14}\ \text{m/s}^2 .$$

(b) We ignore any vertical deflection of the beam that might arise due to the horizontal component of Earth's field. Then, the path of the electron is a circular arc. The radius of the path is given by $a = v^2/R$, or

$$R = \frac{v^2}{a} = \frac{(6.49\times10^{7}\ \text{m/s})^2}{6.27\times10^{14}\ \text{m/s}^2} = 6.72\ \text{m}.$$

The dashed curve shown represents the path. Let the deflection be *h* after the electron has traveled a distance *d* along the *x* axis. With $d = R\sin\theta$, we have

$$h = R(1-\cos\theta) = R\left(1-\sqrt{1-\sin^2\theta}\right)$$
$$= R\left(1-\sqrt{1-(d/R)^2}\right).$$



Substituting *R* = 6.72 m and *d* = 0.20 m into the expression, we obtain *h* = 0.0030 m.

LEARN The deflection is so small that many of the technicalities of circular geometry may be ignored, and a calculation along the lines of projectile motion analysis (see Chapter 4) provides an adequate approximation:

$$d = vt \;\Rightarrow\; t = \frac{d}{v} = \frac{0.200\text{m}}{6.49\times10^{7}\ \text{m/s}} = 3.08\times10^{-9}\ \text{s}.$$

Then, with our *y* axis oriented eastward,

$$h = \frac{1}{2}at^2 = \frac{1}{2}\left(6.27\times10^{14}\right)\left(3.08\times10^{-9}\right)^2 = 0.00298\text{m} \approx 0.0030\ \text{m}.$$

74. Letting $B_x = B_y = B_1$ and $B_z = B_2$ and using Eq. 28-2 ($\vec{F} = q\vec{v}\times\vec{B}$) and Eq. 3-30, we obtain (with SI units understood)

$$4\hat{\text{i}} - 20\hat{\text{j}} + 12\hat{\text{k}} = 2\left(\left(4B_2 - 6B_1\right)\hat{\text{i}} + \left(6B_1 - 2B_2\right)\hat{\text{j}} + \left(2B_1 - 4B_1\right)\hat{\text{k}}\right).$$

Equating like components, we find $B_1 = -3$ and $B_2 = -4$. In summary,

$$\vec{B} = \left(-3.0\hat{\text{i}} - 3.0\hat{\text{j}} - 4.0\hat{\text{k}}\right)\text{T}.$$

75. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $r \propto \sqrt{mK}\big/qB$.

(a) $\frac{r_d}{r_p} = \sqrt{\frac{m_d K_d}{m_p K_p}}\frac{q_p}{q_d} = \sqrt{\frac{2.0\text{u}}{1.0\text{u}}}\frac{e}{e} = \sqrt{2} \approx 1.4$, and

(b) $\frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}}\frac{q_p}{q_\alpha} = \sqrt{\frac{4.0\text{u}}{1.0\text{u}}}\frac{e}{2e} = 1.0.$

76. Using Eq. 28-16, the charge-to-mass ratio is $\frac{q}{m} = \frac{v}{B'r}$. With the speed of the ion given by $v = E/B$ (using Eq. 28-7), the expression becomes

$$\frac{q}{m} = \frac{E/B}{B'r} = \frac{E}{BB'r}.$$

77. **THINK** Since both electric and magnetic fields are present, the net force on the electron is the vector sum of the electric force and the magnetic force.

EXPRESS The force on the electron is given by $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, and $\vec{v}$ is the velocity of the electron. The fact that the fields are uniform with the feature that the charge moves in a straight line, implies that the speed is constant. Thus, the net force must vanish.

ANALYZE The condition $\vec{F} = 0$ implies that

$$E = vB = 500\,\text{V/m}.$$

Its direction (so that $\vec{F} = 0$) is downward, or $-\hat{\text{j}}$, in the "page" coordinates. In unit-vector notation, $\vec{E} = (-500\text{ V/m})\hat{\text{j}}$

LEARN Electron moves in a straight line only when the condition $E = vB$ is met. In many experiments, a velocity selector can be set up so that only electrons with a speed given by $v = E/B$ can pass through.

78. (a) In Chapter 27, the electric field (called E_C in this problem) that "drives" the current through the resistive material is given by Eq. 27-11, which (in magnitude) reads $E_C = \rho J$. Combining this with Eq. 27-7, we obtain

$$E_C = \rho n e v_d .$$

Now, regarding the Hall effect, we use Eq. 28-10 to write $E = v_d B$. Dividing one equation by the other, we get $E/E_c = B/ne\rho$.

(b) Using the value of copper's resistivity given in Chapter 26, we obtain

$$\frac{E}{E_c} = \frac{B}{ne\rho} = \frac{0.65\ \text{T}}{\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(1.69\times10^{-8}\ \Omega\cdot\text{m}\right)} = 2.84\times10^{-3}.$$

79. **THINK** We have charged particles that are accelerated through an electric potential difference, and then moved through a region of uniform magnetic field. Energy is conserved in the process.

EXPRESS The kinetic energy of a particle is given by $K = qV$, where q is the particle's charge and V is the potential difference. With $K = mv^2/2$, the speed of the particle is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2qV}{m}}.$$

In the region with uniform magnetic field, the magnetic force on a particle of charge q is qvB, which according to Newton's second law, is equal to mv^2/r, where r is the radius of the orbit. Thus, we have

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2mK}}{qB}.$$

ANALYZE (a) Since $K = qV$ we have $K_p = \frac{1}{2}K_\alpha \left(\text{as } q_\alpha = 2K_p\right)$, or $K_p/K_\alpha = 0.50$.

(b) Similarly, $q_\alpha = 2K_d$, $K_d/K_\alpha = 0.50$.

(c) Since $r \propto \sqrt{mK}/q$, we have

$$r_d = \sqrt{\frac{m_d K_d}{m_p K_p}}\frac{q_p}{q_d} r_p = \sqrt{\frac{(2.00\ \text{u})K_p}{(1.00\ \text{u})K_p}}\, r_p = 10\sqrt{2}\ \text{cm} = 14\ \text{cm}.$$

(d) Similarly, for the alpha particle, we have

$$r_\alpha = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}}\frac{q_p}{q_\alpha}r_p = \sqrt{\frac{(4.00\text{u})K_\alpha}{(1.00\text{u})(K_\alpha/2)}}\frac{e}{2e}r_p = 10\sqrt{2}\text{ cm} = 14\text{ cm}.$$

LEARN The radius of the particle's path, given by $r = \sqrt{2mK}/qB$, depends on its mass. kinetic energy, and charge, in addition to the field strength.

80. (a) The largest value of force occurs if the velocity vector is perpendicular to the field. Using Eq. 28-3,

$$F_{B,\max} = |q|\, vB \sin(90°) = ev\,B = (1.60 \times 10^{-19}\text{ C})(7.20 \times 10^6\text{ m/s})(83.0 \times 10^{-3}\text{ T}) = 9.56 \times 10^{-14}\text{ N}.$$

(b) The smallest value occurs if they are parallel: $F_{B,\min} = |q|\, vB \sin(0) = 0$.

(c) By Newton's second law, $a = F_B/m_e = |q|\, vB \sin\theta/m_e$, so the angle θ between $\vec{v}$ and $\vec{B}$ is

$$\theta = \sin^{-1}\left(\frac{m_e a}{|q|vB}\right) = \sin^{-1}\left[\frac{(9.11\times10^{-31}\text{ kg})(4.90\times10^{14}\text{ m/s}^2)}{(1.60\times10^{-16}\text{C})(7.20\times10^6\text{ m/s})(83.0\times10^{-3}\text{T})}\right] = 0.267°.$$

81. The contribution to the force by the magnetic field $\left(\vec{B} = B_x\hat{\text{i}} = (-0.020\text{ T})\hat{\text{i}}\right)$ is given by Eq. 28-2:

$$\vec{F}_B = q\vec{v}\times\vec{B} = q\left(\left(17000\hat{\text{i}}\times B_x\hat{\text{i}}\right) + \left(-11000\hat{\text{j}}\times B_x\hat{\text{i}}\right) + \left(7000\hat{\text{k}}\times B_x\hat{\text{i}}\right)\right)$$
$$= q\left(-220\hat{\text{k}} - 140\hat{\text{j}}\right)$$

in SI units. And the contribution to the force by the electric field $\left(\vec{E} = E_y\hat{\text{j}} = 300\hat{\text{j}}\text{ V/m}\right)$ is given by Eq. 23-1: $\vec{F}_E = qE_y\hat{\text{j}}$. Using $q = 5.0 \times 10^{-6}$ C, the net force on the particle is

$$\vec{F} = (0.00080\hat{\text{j}} - 0.0011\hat{\text{k}})\text{ N}.$$

82. (a) We use Eq. 28-10: $v_d = E/B = (10 \times 10^{-6}\text{V}/1.0 \times 10^{-2}\text{ m})/(1.5\text{ T}) = 6.7 \times 10^{-4}$ m/s.

(b) We rewrite Eq. 28-12 in terms of the electric field:

$$n = \frac{Bi}{V\ell e} = \frac{Bi}{(Ed)\ell e} = \frac{Bi}{EAe}$$

where we use $A = \ell d$. In this experiment, $A = (0.010\text{ m})(10 \times 10^{-6}\text{ m}) = 1.0 \times 10^{-7}\text{ m}^2$. By Eq. 28-10, v_d equals the ratio of the fields (as noted in part (a)), so we are led to

$$n = \frac{Bi}{E\,Ae} = \frac{i}{v_d Ae} = \frac{3.0\text{ A}}{\left(6.7\times10^{-4}\text{ m/s}\right)\left(1.0\times10^{-7}\text{ m}^2\right)\left(1.6\times10^{-19}\text{ C}\right)} = 2.8\times10^{29}/\text{m}^3 .$$

(c) Since a drawing of an inherently 3-D situation can be misleading, we describe it in terms of horizontal *north, south, east, west* and vertical *up* and *down* directions. We assume $\vec{B}$ points up and the conductor's width of 0.010 m is along an east-west line. We take the current going northward. The conduction electrons experience a westward magnetic force (by the right-hand rule), which results in the west side of the conductor being negative and the east side being positive (with reference to the Hall voltage that becomes established).

83. **THINK** The force on the charged particle is given by $\vec{F} = q\vec{v}\times\vec{B},$ where q is the charge, $\vec{B}$ is the magnetic field, and $\vec{v}$ is the velocity of the electron.

EXPRESS We write $\vec{B} = B\hat{\text{i}}$ and take the velocity of the particle to be $\vec{v} = v_x\hat{\text{i}} + v_y\hat{\text{j}}$. Thus,

$$\vec{F} = q\vec{v}\times\vec{B} = q(v_x\hat{\text{i}} + v_y\hat{\text{j}})\times(B\hat{\text{i}}) = -qv_yB\hat{\text{k}}.$$

For the force to point along $+\hat{\text{k}}$, we must have $q < 0$.

ANALYZE The charge of the particle is

$$q = -\frac{F}{v_yB} = -\frac{0.48\text{ N}}{(4.0\times10^3\text{ m/s})(\sin 37°)(0.0050\text{ T})} = -4.0\times10^{-2}\text{ C}.$$

LEARN The component of the velocity, v_x, being parallel to the magnetic field, does not contribute to the magnetic force $\vec{F}$; only v_y, the component of $\vec{v}$ that is perpendicular to $\vec{B}$, contributes to $\vec{F}$.

84. The current is in the $+\hat{\text{i}}$ direction. Thus, the $\hat{\text{i}}$ component of $\vec{B}$ has no effect, and (with x in meters) we evaluate

$$\vec{F} = (3.00\text{A})\int_0^1 \left(-0.600\text{ T/m}^2\right)x^2\,dx\left(\hat{\text{i}}\times\hat{\text{j}}\right) = \left(-1.80\frac{1^3}{3}\text{ A}\cdot\text{T}\cdot\text{m}\right)\hat{\text{k}} = (-0.600\text{N})\hat{\text{k}}.$$

85. (a) We use Eq. 28-2 and Eq. 3-30:

$$\vec{F} = q\vec{v}\times\vec{B} = (+e)\left((v_yB_z - v_zB_y)\hat{\mathrm{i}} + (v_zB_x - v_xB_z)\hat{\mathrm{j}} + (v_xB_y - v_yB_x)\hat{\mathrm{k}}\right)$$
$$= (1.60\times10^{-19})\Big(\big((4)(0.008) - (-6)(-0.004)\big)\hat{\mathrm{i}} +$$
$$\big((-6)(0.002) - (-2)(0.008)\big)\hat{\mathrm{j}} + \big((-2)(-0.004) - (4)(0.002)\big)\hat{\mathrm{k}}\Big)$$
$$= (1.28\times10^{-21})\hat{\mathrm{i}} + (6.41\times10^{-22})\hat{\mathrm{j}}$$

with SI units understood.

(b) By definition of the cross product, $\vec{v}\perp\vec{F}$. This is easily verified by taking the dot (scalar) product of $\vec{v}$ with the result of part (a), yielding zero, provided care is taken not to introduce any round-off error.

(c) There are several ways to proceed. It may be worthwhile to note, first, that if B_z were 6.00 mT instead of 8.00 mT then the two vectors would be exactly antiparallel. Hence, the angle θ between $\vec{B}$ and $\vec{v}$ is presumably "close" to 180°. Here, we use Eq. 3-20:

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{B}}{|\vec{v}||\vec{B}|}\right) = \cos^{-1}\left(\frac{-68}{\sqrt{56}\sqrt{84}}\right) = 173° .$$

86. (a) We are given $\vec{B} = B_x\hat{\mathrm{i}} = (6\times10^{-5}\,\mathrm{T})\hat{\mathrm{i}}$, so that $\vec{v}\times\vec{B} = -v_yB_x\hat{\mathrm{k}}$ where $v_y = 4\times10^4$ m/s. We note that the magnetic force on the electron is $(-e)(-v_yB_x\hat{\mathrm{k}})$ and therefore points in the $+\hat{\mathrm{k}}$ direction, at the instant the electron enters the field-filled region. In these terms, Eq. 28-16 becomes

$$r = \frac{m_e v_y}{e B_x} = 0.0038\,\mathrm{m}.$$

(b) One revolution takes $T = 2\pi r/v_y = 0.60$ μs, and during that time the "drift" of the electron in the x direction (which is the *pitch* of the helix) is $\Delta x = v_xT = 0.019$ m where $v_x = 32\times10^3$ m/s.

(c) Returning to our observation of force direction made in part (a), we consider how this is perceived by an observer at some point on the $-x$ axis. As the electron moves away from him, he sees it enter the region with positive v_y (which he might call "upward'') but "pushed" in the $+z$ direction (to his right). Hence, he describes the electron's spiral as clockwise.

87. (a) The magnetic force on the electrons is given by $\vec{F} = q\vec{v}\times\vec{B}$. Since the field $\vec{B}$ points to the left, and an electron (with $q = -e$) is forced to rotate clockwise (out of the page at the top of the rotor), using the right-hand-rule, the direction of the magnetic force is up the figure.

(b) The magnitude of the magnetic force can be written as $F = evB = e\omega rB$, where ω is the angular velocity and r is the distance from the axis. Since $F \sim r$, the force is greater near the rim.

(c) The work per unit charge done by the force in moving the charge along the radial line from the center to the rim, or the voltage, is

$$V = \frac{W}{e} = \frac{1}{e}\int_0^R e\omega Br dr = \frac{1}{2}\omega BR^2 = \frac{1}{2}(2\pi f)BR^2 = \pi fBR^2$$
$$= \pi(4000\ /\text{s})(60\times10^{-3}\ \text{T})(0.250\ \text{m})^2 = 47.1\ \text{V}.$$

(d) The emf of the device is simply equal to the voltage calculated in part (c): $\varepsilon = 47.1$ V.

(e) The power produced is $P = iV = (50.0\ \text{A})(47.1\ \text{V}) = 2.36\times10^3$ W.

88. The magnetic force exerted on the U-shaped wire is given by $F = iLB$. Using the impulse-momentum theorem, we have

$$\Delta p = m\Delta v = \int F dt = \int iLB dt = LB\int i dt = LBq,$$

where q is the charge in the pulse. Since the wire is initially at rest, the speed at which the wire jumps is $v = LBq/m$. On the other hand, energy conservation gives $\frac{1}{2}mv^2 = mgh$. Combining the above expressions leads to

$$h = \frac{v^2}{2g} = \frac{1}{2g}\left(\frac{LBq}{m}\right)^2$$

Solving for q, we find

$$q = \frac{m\sqrt{2gh}}{LB} = \frac{(0.0100\ \text{kg})\sqrt{2(9.80\ \text{m/s}^2)(3.00\ \text{m})}}{(0.200\ \text{m})(0.100\ \text{T})} = 3.83\ \text{C}.$$

89. Just before striking the plate, the electric force on the electron is $F_E = eE = eV/d$, in the upward direction. Since the kinetic energy of the electron is $K = \frac{1}{2}mv^2 = eV$, $v = \sqrt{2eV/m}$. On the other hand, the magnetic force is

$$F_B = evB = eB\sqrt{\frac{2eV}{m}}$$

in the downward direction. To prevent the electron from striking the plate, we require $F_B > F_E$, or

$$eB\sqrt{\frac{2eV}{m}} > \frac{eV}{d} \quad \Rightarrow \quad B > \frac{V}{d}\sqrt{\frac{m}{2eV}} = \sqrt{\frac{mV}{2ed^2}}$$

90. The average current in the loop is $i = \frac{q}{T} = \frac{q}{2\pi r / v} = \frac{qv}{2\pi r}$ and its magnetic dipole moment is

$$\mu = iA = \left(\frac{qv}{2\pi r}\right)(\pi r^2) = \frac{1}{2}qvr.$$

With $\vec{\tau} = \vec{\mu} \times \vec{B}$, we find the maximum torque exerted on the loop by a uniform magnetic field to be

$$\tau_{\max} = \mu B = \frac{1}{2}qvrB.$$

91. When the electric and magnetic forces are in balance, $eE = ev_d B$, where v_d is the drift speed of the electrons. In addition, since the current density is $J = nev_d$, we solve for n and find

$$n = \frac{J}{ev_d} = \frac{J}{e(E/B)} = \frac{JB}{eE}.$$

92. With $F_z = v_z = B_x = 0$, Eq. 28-2 (and Eq. 3-30) gives

$$F_x \hat{\text{i}} + F_y \hat{\text{j}} = q\,(v_y B_z \hat{\text{i}} - v_x B_z \hat{\text{j}} + v_x B_y \hat{\text{k}})$$

where $q = -e$ for the electron. The last term immediately implies $B_y = 0$, and either of the other two terms (along with the values stated in the problem, bearing in mind that "fN" means femto-newtons or 10^{-15} N) can be used to solve for B_z:

$$B_z = \frac{F_x}{-ev_y} = \frac{-4.2\times10^{-15}\,\text{N}}{-(1.6\times10^{-19}\,\text{C})(35{,}000\,\text{m/s})} = 0.75\,\text{T}.$$

We therefore find that the magnetic field is given by $\vec{B} = (0.75\,\text{T})\hat{\text{k}}.$

Chapter 29

1. (a) The magnitude of the magnetic field due to the current in the wire, at a point a distance r from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r}.$$

With r = 20 ft = 6.10 m, we have

$$B = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(100\ \text{A}\right)}{2\pi\left(6.10\ \text{m}\right)} = 3.3\times 10^{-6}\ \text{T} = 3.3\ \mu\text{T}.$$

(b) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

2. Equation 29-1 is maximized (with respect to angle) by setting θ = 90º (= π/2 rad). Its value in this case is

$$dB_{\max} = \frac{\mu_0 i}{4\pi}\frac{ds}{R^2}.$$

From Fig. 29-35(b), we have $B_{\max} = 60\times 10^{-12}$ T. We can relate this $B_{\max}$ to our $dB_{\max}$ by setting "ds" equal to 1 × 10^{-6} m and R = 0.025 m. This allows us to solve for the current: i = 0.375 A. Plugging this into Eq. 29-4 (for the infinite wire) gives B_∞ = 3.0 μT.

3. **THINK** The magnetic field produced by a current-carrying wire can be calculated using the Biot-Savart law.

EXPRESS The magnitude of the magnetic field at a distance r from a long straight wire carrying current i is, using the Biot-Savart law, $B = \mu_0 i/2\pi r$.

ANALYZE (a) The field due to the wire, at a point 8.0 cm from the wire, must be 39 μT and must be directed due south. Therefore,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(0.080\ \text{m}\right)\left(39\times 10^{-6}\ \text{T}\right)}{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}} = 16\ \text{A}.$$

(b) The current must be from west to east to produce a field that is directed southward at points below it.

LEARN The direction of the current is given by the right-hand rule: grasp the element in your right hand with your thumb pointing in the direction of the current. The direction of the field due to the current-carrying element corresponds to the direction your fingers naturally curl.

4. The straight segment of the wire produces no magnetic field at C (see the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current"). Also, the fields from the two semicircular loops cancel at C (by symmetry). Therefore, $B_C = 0$.

5. (a) We find the field by superposing the results of two semi-infinite wires (Eq. 29-7) and a semicircular arc (Eq. 29-9 with $\phi = \pi$ rad). The direction of $\vec{B}$ is out of the page, as can be checked by referring to Fig. 29-7(c). The magnitude of $\vec{B}$ at point a is therefore

$$B_a = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i\pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi}+\frac{1}{2}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{A})}{2(0.0050\ \text{m})}\left(\frac{1}{\pi}+\frac{1}{2}\right) = 1.0\times10^{-3}\ \text{T}$$

upon substituting $i = 10$ A and $R = 0.0050$ m.

(b) The direction of this field is out of the page, as Fig. 29-7(c) makes clear.

(c) The last remark in the problem statement implies that treating b as a point midway between two infinite wires is a good approximation. Thus, using Eq. 29-4,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{A})}{\pi(0.0050\ \text{m})} = 8.0\times10^{-4}\ \text{T}.$$

(d) This field, too, points out of the page.

6. With the "usual" x and y coordinates used in Fig. 29-38, then the vector $\vec{r}$ pointing from a current element to P is $\vec{r} = -s\,\hat{\text{i}} + R\,\hat{\text{j}}$. Since $d\vec{s} = ds\,\hat{\text{i}}$, then $|d\vec{s}\times\vec{r}| = Rds$. Therefore, with $r = \sqrt{s^2+R^2}$, Eq. 29-3 gives

$$dB = \frac{\mu_0}{4\pi}\frac{iR\,ds}{(s^2+R^2)^{3/2}}.$$

(a) Clearly, considered as a function of s (but thinking of "ds" as some finite-sized constant value), the above expression is maximum for $s = 0$. Its value in this case is $dB_{\max} = \mu_0 i\,ds/4\pi R^2$.

(b) We want to find the s value such that $dB = dB_{\max}/10$. This is a nontrivial algebra exercise, but is nonetheless straightforward. The result is $s = \sqrt{10^{2/3}-1}\,R$. If we set $R = 2.00$ cm, then we obtain $s = 3.82$ cm.

7. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with *P* do not contribute to the field at that point. Using Eq. 29-9 (with $\phi = \theta$) and the right-hand rule, we find that the current in the semicircular arc of radius *b* contributes $\mu_0 i\theta/4\pi b$ (out of the page) to the field at *P*. Also, the current in the large radius arc contributes $\mu_0 i\theta/4\pi a$ (into the page) to the field there. Thus, the net field at *P* is

$$B = \frac{\mu_0 i\theta}{4}\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.411\,\text{A})(74°\cdot\pi/180°)}{4\pi}\left(\frac{1}{0.107\,\text{m}} - \frac{1}{0.135\,\text{m}}\right)$$
$$= 1.02\times10^{-7}\,\text{T}.$$

(b) The direction is out of the page.

8. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in segments *AH* and *JD* do not contribute to the field at point *C*. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc *H J* contributes $\mu_0 i/4R_1$ (into the page) to the field at *C*. Also, arc *D A* contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at *C* is

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.281\,\text{A})}{4}\left(\frac{1}{0.0315\,\text{m}} - \frac{1}{0.0780\,\text{m}}\right) = 1.67\times10^{-6}\,\text{T}.$$

(b) The direction of the field is into the page.

9. **THINK** The net magnetic field at a point half way between the two long straight wires is the vector sum of the magnetic fields due to the currents in the two wires.

EXPRESS Since the magnitude of the magnetic field at a distance *r* from a long straight wire carrying current *i* is given by $B = \mu_0 i/2\pi r$, at a point half way between the two sires, the magnetic field is $\vec{B} = \vec{B}_1 + \vec{B}_2$, where $B_1 = B_2 = \mu_0 i/2\pi r$ (assuming the two wires to be 2*r* apart). The directions of $\vec{B}_1$ and $\vec{B}_2$ are determined by the right-hand rule.

ANALYZE (a) The currents must be opposite or anti-parallel, so that the resulting fields are in the same direction in the region between the wires. If the currents are parallel, then the two fields are in opposite directions in the region between the wires. Since the currents are the same, the total field is zero along the line that runs halfway between the wires.

(b) The total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi(0.040\text{ m})(300\times10^{-6}\text{T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 30\text{ A}.$$

LEARN For two parallel wires carrying currents in the opposite directions, a point that is a distance d from one wire and $2r - d$ from the other, the magnitude of the magnetic field is

$$B = B_1 + B_2 = \frac{\mu_0 i}{2\pi d} + \frac{\mu_0 i}{2\pi(2r-d)} = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d} + \frac{1}{2r-d}\right).$$

10. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with C do not contribute to the field at that point.

Equation 29-9 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at C. Thus, the magnitude of the magnetic field is

$$B = \frac{\mu_0 i}{4R} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(0.0348\text{A})}{4(0.0926\text{m})} = 1.18\times10^{-7}\text{ T}.$$

(b) The right-hand rule shows that this field is into the page.

11. (a) $B_{P_1} = \mu_0 i_1 / 2\pi r_1$ where i_1 = 6.5 A and $r_1 = d_1 + d_2$ = 0.75 cm + 1.5 cm = 2.25 cm, and $B_{P_2} = \mu_0 i_2 / 2\pi r_2$ where $r_2 = d_2$ = 1.5 cm. From $B_{P1} = B_{P2}$ we get

$$i_2 = i_1\left(\frac{r_2}{r_1}\right) = (6.5\text{ A})\left(\frac{1.5\text{ cm}}{2.25\text{ cm}}\right) = 4.3\text{ A}.$$

(b) Using the right-hand rule, we see that the current i_2 carried by wire 2 must be out of the page.

12. (a) Since they carry current in the same direction, then (by the right-hand rule) the only region in which their fields might cancel is between them. Thus, if the point at which we are evaluating their field is r away from the wire carrying current i and is $d - r$ away from the wire carrying current $3.00i$, then the canceling of their fields leads to

$$\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 (3i)}{2\pi(d-r)} \quad \Rightarrow \quad r = \frac{d}{4} = \frac{16.0\text{ cm}}{4} = 4.0\text{ cm}.$$

(b) Doubling the currents does not change the location where the magnetic field is zero.

13. Our x axis is along the wire with the origin at the midpoint. The current flows in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are out of

the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where θ (the angle between the segment and a line drawn from the segment to P_1) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and sin θ with $R/r = R\big/\sqrt{x^2+R^2}$, we integrate from x = –L/2 to x = L/2. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}$$

$$= \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.0582\ \text{A}\right)}{2\pi\left(0.131\,\text{m}\right)}\frac{0.180\text{m}}{\sqrt{(0.180\text{m})^2+4(0.131\text{m})^2}} = 5.03\times 10^{-8}\ \text{T}.$$

14. We consider Eq. 29-6 but with a finite upper limit (L/2 instead of ∞). This leads to

$$B = \frac{\mu_0 i}{2\pi R}\frac{L/2}{\sqrt{(L/2)^2+R^2}}\,.$$

In terms of this expression, the problem asks us to see how large L must be (compared with R) such that the infinite wire expression B_∞ (Eq. 29-4) can be used with no more than a 1% error. Thus we must solve

$$\frac{B_\infty - B}{B} = 0.01.$$

This is a nontrivial algebra exercise, but is nonetheless straightforward. The result is

$$L = \frac{200R}{\sqrt{201}} \approx 14.1R \quad \Rightarrow \quad \frac{L}{R} \approx 14.1\,.$$

15. (a) As discussed in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," the radial segments do not contribute to $\vec{B}_P$ and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\text{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0\left(0.40\,\text{A}\right)\left(\pi\,\text{rad}\right)}{4\pi\left(0.050\,\text{m}\right)}\hat{\text{k}} - \frac{\mu_0\left(0.80\,\text{A}\right)\left(2\pi/3\,\text{rad}\right)}{4\pi\left(0.040\,\text{m}\right)}\hat{\text{k}} = -(1.7\times 10^{-6}\ \text{T})\hat{\text{k}}$$

or $|\vec{B}| = 1.7\times 10^{-6}\,\text{T}$.

(b) The direction is $-\hat{\text{k}}$, or into the page.

(c) If the direction of i_1 is reversed, we then have

$$\vec{B} = -\frac{\mu_0 (0.40\,\text{A})(\pi\,\text{rad})}{4\pi(0.050\,\text{m})}\hat{\text{k}} - \frac{\mu_0 (0.80\,\text{A})(2\pi/3\,\text{rad})}{4\pi(0.040\,\text{m})}\hat{\text{k}} = -(6.7\times10^{-6}\ \text{T})\hat{\text{k}}$$

or $|\vec{B}| = 6.7\times10^{-6}\,\text{T}$.

(d) The direction is $-\hat{\text{k}}$, or into the page.

16. Using the law of cosines and the requirement that B = 100 nT, we have

$$\theta = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{-2B_1B_2}\right) = 144°,$$

where Eq. 29-10 has been used to determine B_1 (168 nT) and B_2 (151 nT).

17. **THINK** We apply the Biot-Savart law to calculate the magnetic field at point P_2. An integral is required since the length of the wire is finite.

EXPRESS We take the x axis to be along the wire with the origin at the right endpoint. The current is in the $+x$ direction. All segments of the wire produce magnetic fields at P_2 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_2 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where θ (the angle between the segment and a line drawn from the segment to P_2) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and sin θ with $R/r = R\big/\sqrt{x^2+R^2}$, we integrate from $x = -L$ to $x = 0$.

ANALYZE The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{(x^2+R^2)^{1/2}}\Bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}$$
$$= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.693\,\text{A})}{4\pi(0.251\,\text{m})}\frac{0.136\,\text{m}}{\sqrt{(0.136\,\text{m})^2+(0.251\,\text{m})^2}} = 1.32\times10^{-7}\,\text{T}.$$

LEARN In calculating B at P_2, we could have chosen the origin to be at the left endpoint. This only changes the integration limit, but the result remains the same:

$$B = \frac{\mu_0 iR}{4\pi}\int_0^L \frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_0^L = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}} .$$

18. In the one case we have $B_{\text{small}} + B_{\text{big}}$ = 47.25 μT, and the other case gives $B_{\text{small}} - B_{\text{big}}$ = 15.75 μT (cautionary note about our notation: B_{small} refers to the field at the center of the small-radius arc, which is actually a bigger field than B_{big}!). Dividing one of these equations by the other and canceling out common factors (see Eq. 29-9) we obtain

$$\frac{(1/r_{\text{small}}) + (1/r_{\text{big}})}{(1/r_{\text{small}}) - (1/r_{\text{big}})} = \frac{1+(r_{\text{small}}/r_{\text{big}})}{1-(r_{\text{small}}/r_{\text{big}})} = 3 .$$

The solution of this is straightforward: $r_{\text{small}} = r_{\text{big}}/2$. Using the given fact that the $r_{\text{big}} = 4.00$ cm, then we conclude that the small radius is $r_{\text{small}} = 2.00$ cm.

19. The contribution to $\vec{B}_{\text{net}}$ from the first wire is (using Eq. 29-4)

$$\vec{B}_1 = \frac{\mu_0 i_1}{2\pi r_1}\hat{\text{k}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(30\ \text{A})}{2\pi(2.0\ \text{m})}\hat{\text{k}} = (3.0\times10^{-6}\ \text{T})\hat{\text{k}}.$$

The distance from the second wire to the point where we are evaluating $\vec{B}_{\text{net}}$ is r_2 = 4 m − 2 m = 2 m. Thus,

$$\vec{B}_2 = \frac{\mu_0 i_2}{2\pi r_2}\hat{\text{i}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(40\ \text{A})}{2\pi(2.0\ \text{m})}\hat{\text{i}} = (4.0\times10^{-6}\ \text{T})\hat{\text{i}}.$$

and consequently is perpendicular to $\vec{B}_1$. The magnitude of $\vec{B}_{\text{net}}$ is therefore

$$|\vec{B}_{\text{net}}| = \sqrt{(3.0\times10^{-6}\ \text{T})^2 + (4.0\times10^{-6}\ \text{T})^2} = 5.0\times10^{-6}\ \text{T} .$$

20. (a) The contribution to B_C from the (infinite) straight segment of the wire is

$$B_{C1} = \frac{\mu_0 i}{2\pi R}.$$

The contribution from the circular loop is $B_{C2} = \frac{\mu_0 i}{2R}$. Thus,

$$B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R}\left(1+\frac{1}{\pi}\right) = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.78\times10^{-3}\ \text{A}\right)}{2\left(0.0189\ \text{m}\right)}\left(1+\frac{1}{\pi}\right) = 2.53\times10^{-7}\,\text{T}.$$

$\vec{B}_C$ points out of the page, or in the $+z$ direction. In unit-vector notation, $\vec{B}_C = (2.53\times10^{-7}\,\text{T})\hat{\text{k}}$

(b) Now, $\vec{B}_{C1} \perp \vec{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1+\frac{1}{\pi^2}} = \frac{\left(4\text{p}\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(5.78\times10^{-3}\,\text{A}\right)}{2\left(0.0189\,\text{m}\right)}\sqrt{1+\frac{1}{\pi^2}} = 2.02\times10^{-7}\,\text{T}.$$

and $\vec{B}_C$ points at an angle (relative to the plane of the paper) equal to

$$\tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 17.66°.$$

In unit-vector notation,

$$\vec{B}_C = 2.02\times10^{-7}\,\text{T}(\cos17.66°\hat{\text{i}} + \sin17.66°\hat{\text{k}}) = (1.92\times10^{-7}\,\text{T})\hat{\text{i}} + (6.12\times10^{-8}\,\text{T})\hat{\text{k}}.$$

21. Using the right-hand rule (and symmetry), we see that $\vec{B}_{\text{net}}$ points along what we will refer to as the y axis (passing through P), consisting of two equal magnetic field y-components. Using Eq. 29-17,

$$|\vec{B}_{\text{net}}| = 2\frac{\mu_0 i}{2\pi r}\sin\theta$$

where i = 4.00 A, r = $r = \sqrt{d_2^2 + d_1^2/4} = 5.00$ m, and

$$\theta = \tan^{-1}\left(\frac{d_2}{d_1/2}\right) = \tan^{-1}\left(\frac{4.00\text{ m}}{6.00\text{ m}/2}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°.$$

Therefore,

$$|\vec{B}_{\text{net}}| = \frac{\mu_0 i}{\pi r}\sin\theta = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.00\text{ A})}{\pi(5.00\text{ m})}\sin53.1° = 2.56\times10^{-7}\text{ T}.$$

22. The fact that $B_y = 0$ at x = 10 cm implies the currents are in opposite directions. Thus,

$$B_y = \frac{\mu_0 i_1}{2\pi(L+x)} - \frac{\mu_0 i_2}{2\pi x} = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+x} - \frac{1}{x}\right)$$

using Eq. 29-4 and the fact that $i_1 = 4i_2$. To get the maximum, we take the derivative with respect to x and set equal to zero. This leads to $3x^2 - 2Lx - L^2 = 0$, which factors and becomes $(3x + L)(x - L) = 0$, which has the physically acceptable solution: $x = L$. This produces the maximum B_y: $\mu_0 i_2/2\pi L$. To proceed further, we must determine L.

Examination of the datum at x = 10 cm in Fig. 29-50(b) leads (using our expression above for B_y and setting that to zero) to L = 30 cm.

(a) The maximum value of B_y occurs at $x = L$ = 30 cm.

(b) With i_2 = 0.003 A we find $\mu_0 i_2 / 2\pi L$ = 2.0 nT.

(c) and (d) Figure 29-50(b) shows that as we get very close to wire 2 (where its field strongly dominates over that of the more distant wire 1) B_y points along the $-y$ direction. The right-hand rule leads us to conclude that wire 2's current is consequently is *into the page*. We previously observed that the currents were in opposite directions, so wire 1's current is *out of the page*.

23. We assume the current flows in the $+x$ direction and the particle is at some distance d in the $+y$ direction (away from the wire). Then, the magnetic field at the location of a proton with charge q is $\vec{B} = (\mu_0 i / 2\pi d)\,\hat{\mathrm{k}}$. Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{\mu_0 iq}{2\pi d}\left(\vec{v} \times \hat{\mathrm{k}}\right).$$

In this situation, $\vec{v} = v\left(-\hat{\mathrm{j}}\right)$ (where v is the speed and is a positive value), and $q > 0$. Thus,

$$\vec{F} = \frac{\mu_0 iqv}{2\pi d}\left(\left(-\hat{\mathrm{j}}\right) \times \hat{\mathrm{k}}\right) = -\frac{\mu_0 iqv}{2\pi d}\,\hat{\mathrm{i}} = -\frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.350\mathrm{A})(1.60\times10^{-19}\,\mathrm{C})(200\mathrm{m/s})}{2\pi(0.0289\text{ m})}\,\hat{\mathrm{i}}$$
$$= (-7.75\times10^{-23}\,\mathrm{N})\hat{\mathrm{i}}.$$

24. Initially, we have $B_{\mathrm{net},y} = 0$ and $B_{\mathrm{net},x} = B_2 + B_4 = 2(\mu_0 i / 2\pi d)$ using Eq. 29-4, where $d = 0.15$ m. To obtain the 30º condition described in the problem, we must have

$$B_{\mathrm{net},y} = B_{\mathrm{net},x}\tan(30°) \quad \Rightarrow \quad B_1' - B_3 = 2\left(\frac{\mu_0 i}{2\pi d}\right)\tan(30°)$$

where $B_3 = \mu_0 i / 2\pi d$ and $B_1' = \mu_0 i / 2\pi d'$. Since tan(30º) = $1/\sqrt{3}$, this leads to

$$d' = \frac{\sqrt{3}}{\sqrt{3}+2}\,d = 0.464d\,.$$

(a) With d = 15.0 cm, this gives d' = 7.0 cm. Being very careful about the geometry of the situation, then we conclude that we must move wire 1 to x = –7.0 cm.

(b) To restore the initial symmetry, we would have to move wire 3 to x = +7.0 cm.

25. **THINK** The magnetic field at the center of the circle is the vector sum of the fields of the two straight wires and the arc.

EXPRESS Each of the semi-infinite straight wires contributes $B_{\text{straight}} = \mu_0 i/4\pi R$ (Eq. 29-7) to the field at the center of the circle (both contributions pointing "out of the page"). The current in the arc contributes a term given by Eq. 29-9: $B_{\text{arc}} = \dfrac{\mu_0 i\phi}{4\pi R}$, pointing into the page.

ANALYZE The total magnetic field is

$$B = 2B_{\text{straight}} - B_{\text{arc}} = 2\left(\frac{\mu_0 i}{4\pi R}\right) - \frac{\mu_0 i\phi}{4\pi R} = \frac{\mu_0 i}{4\pi R}(2-\phi).$$

Therefore, $\phi = 2.00$ rad would produce zero total field at the center of the circle.

LEARN The total contribution of the two semi-infinite wires is the same as that of an infinite wire. Note that the angle ϕ is in radians rather than degrees.

26. Using the Pythagorean theorem, we have

$$B^2 = B_1^2 + B_2^2 = \left(\frac{\mu_0 i_1\phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i_2}{2\pi R}\right)^2$$

which, when thought of as the equation for a line in a B^2 versus i_2^2 graph, allows us to identify the first term as the "y-intercept" (1×10^{-10}) and the part of the second term that multiplies i_2^2 as the "slope" (5×10^{-10}). The latter observation leads to

$$5.00\times10^{-10}\ \text{T}^2/\text{A}^2 = \left(\frac{\mu_0}{2\pi R}\right)^2 = \left(\frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi R}\right)^2$$

or

$$R^2 = \frac{4.00\times10^{-14}\ \text{T}^2\cdot\text{m}^2/\text{A}^2}{5.00\times10^{-10}\ \text{T}^2/\text{A}^2} = 8.00\times10^{-5}\,\text{m}^2 \;\Rightarrow\; R = 8.94\times10^{-3}\ \text{m} \approx 8.9\ \text{mm}.$$

The other observation about the "y-intercept" determines the angle subtended by the arc:

$$1.00\times10^{-10}\ \text{T}^2 = \left(\frac{\mu_0 i_1\phi}{4\pi R}\right)^2 = \left(\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.50\ \text{A})}{4\pi(8.94\times10^{-3}\text{m})}\right)^2\phi^2 = (3.13\times10^{-11}\phi^2)\ \text{T}^2$$

or

$$\phi^2 = \frac{1.00\times10^{-10}\ \text{T}^2}{3.13\times10^{-11}\ \text{T}^2} = 3.19 \;\Rightarrow\; \phi = 1.79\ \text{rad} \approx 1.8\ \text{rad}.$$

27. We use Eq. 29-4 to relate the magnitudes of the magnetic fields B_1 and B_2 to the currents (i_1 and i_2, respectively) in the two long wires. The angle of their net field is

$$\theta = \tan^{-1}(B_2/B_1) = \tan^{-1}(i_2/i_1) = 53.13°.$$

The accomplish the net field rotation described in the problem, we must achieve a final angle $\theta' = 53.13° - 20° = 33.13°$. Thus, the final value for the current i_1 must be $i_2/\tan\theta'$ = 61.3 mA.

28. Letting "out of the page" in Fig. 29-56(a) be the positive direction, the net field is

$$B = \frac{\mu_0 i_1 \phi}{4\pi R} - \frac{\mu_0 i_2}{2\pi(R/2)}$$

from Eqs. 29-9 and 29-4. Referring to Fig. 29-56, we see that $B = 0$ when $i_2 = 0.5$ A, so (solving the above expression with B set equal to zero) we must have

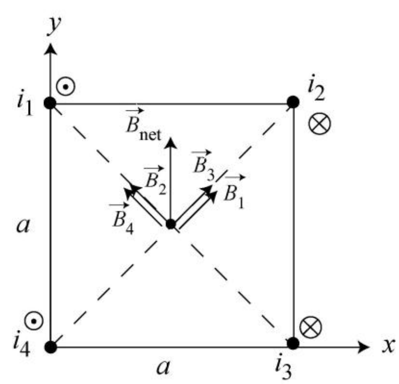
$$\phi = 4(i_2/i_1) = 4(0.5/2) = 1.00 \text{ rad (or } 57.3°).$$

29. **THINK** Our system consists of four long straight wires whose cross section form a square of length a. The magnetic field at the center of the square is the vector sum of the fields of the four wires.

EXPRESS Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left (wire 1) and lower right (wire 3) corners both point toward the upper right corner of the square. The fields due to the wires at the upper right (wire 2) and lower left (wire 4) corners both point toward the upper left corner.

ANALYZE The horizontal components of the fields cancel and the vertical components sum to

$$B_{\text{net}} = 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45° = \frac{2\mu_0 i}{\pi a} = \frac{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20\ \text{A}\right)}{\pi\left(0.20\ \text{m}\right)}$$
$$= 8.0\times10^{-5}\ \text{T}.$$



In the calculation, cos 45° was replaced with $1/\sqrt{2}$. The total field points upward, or in the $+y$ direction. Thus, $\vec{B}_{\text{net}} = (8.0\times10^{-5}\ \text{T})\hat{\text{j}}$.

LEARN In the figure to the right, we show the contributions from the individual wires. The directions of the fields are deduced using the right-hand rule.

30. We note that when there is no y-component of magnetic field from wire 1 (which, by the right-hand rule, relates to when wire 1 is at 90º = $\pi/2$ rad), the total y-component of magnetic field is zero (see Fig. 29-58(c)). This means wire #2 is either at $+\pi/2$ rad or $-\pi/2$ rad.

(a) We now make the assumption that wire #2 must be at $-\pi/2$ rad (–90º, the bottom of the cylinder) since it would pose an obstacle for the motion of wire #1 (which is needed to make these graphs) if it were anywhere in the top semicircle.

(b) Looking at the θ_1 = 90º datum in Fig. 29-58(b)), where there is a *maximum* in $B_{\text{net }x}$ (equal to +6 μT), we are led to conclude that $B_{1x} = 6.0\ \mu\text{T} - 2.0\ \mu\text{T} = 4.0\ \mu\text{T}$ in that situation. Using Eq. 29-4, we obtain

$$i_1 = \frac{2\pi R B_{1x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(4.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 4.0\text{ A}.$$

(c) The fact that Fig. 29-58(b) increases as θ_1 progresses from 0 to 90º implies that wire 1's current is *out of the page*, and this is consistent with the cancellation of $B_{\text{net }y}$ at $\theta_1 = 90°$, noted earlier (with regard to Fig. 29-58(c)).

(d) Referring now to Fig. 29-58(b) we note that there is no x-component of magnetic field from wire 1 when θ_1 = 0, so that plot tells us that B_{2x} = +2.0 μT. Using Eq. 29-4, we find the magnitudes of the current to be

$$i_2 = \frac{2\pi R B_{2x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 2.0\text{ A}.$$

(e) We can conclude (by the right-hand rule) that wire 2's current is *into the page*.

31. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with P do not contribute to the field at that point. We use the result of Problem 29-21 to evaluate the contributions to the field at P, noting that the nearest wire segments (each of length a) produce magnetism into the page at P and the further wire segments (each of length $2a$) produce magnetism pointing out of the page at P. Thus, we find (into the page)

$$B_P = 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi a}\right) - 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi(2a)}\right) = \frac{\sqrt{2}\mu_0 i}{8\pi a} = \frac{\sqrt{2}\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(13\text{ A})}{8\pi\left(0.047\text{ m}\right)}$$
$$= 1.96\times10^{-5}\text{T} \approx 2.0\times10^{-5}\text{T}.$$

(b) The direction of the field is into the page.

32. Initially we have

$$B_i = \frac{\mu_0 i\phi}{4\pi R} + \frac{\mu_0 i\phi}{4\pi r}$$

using Eq. 29-9. In the final situation we use Pythagorean theorem and write

$$B_f^2 = B_z^2 + B_y^2 = \left(\frac{\mu_0 i\phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i\phi}{4\pi r}\right)^2.$$

If we square B_i and divide by B_f^2, we obtain

$$\left(\frac{B_i}{B_f}\right)^2 = \frac{[(1/R)+(1/r)]^2}{(1/R)^2+(1/r)^2} .$$

From the graph (see Fig. 29-60(c), note the maximum and minimum values) we estimate $B_i/B_f = 12/10 = 1.2$, and this allows us to solve for r in terms of R:

$$r = R\,\frac{1 \pm 1.2\sqrt{2 - 1.2^2}}{1.2^2 - 1} = 2.3 \text{ cm} \quad \text{or} \quad 43.1 \text{ cm}.$$

Since we require $r < R$, then the acceptable answer is $r = 2.3$ cm.

33. **THINK** The magnetic field at point P produced by the current-carrying ribbon (shown in Fig. 29-61) can be calculated using the Biot-Savart law.

EXPRESS Consider a section of the ribbon of thickness dx located a distance x away from point P. The current it carries is $di = i\,dx/w$, and its contribution to B_P is

$$dB_P = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 i dx}{2\pi x w}.$$

ANALYZE Integrating over the length of the ribbon, we obtain

$$B_P = \int dB_P = \frac{\mu_0 i}{2\pi w}\int_d^{d+w}\frac{dx}{x} = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.61\times10^{-6}\ \text{A})}{2\pi(0.0491\,\text{m})}\ln\left(1+\frac{0.0491}{0.0216}\right)$$
$$= 2.23\times10^{-11}\,\text{T}.$$

and $\vec{B}_P$ points upward. In unit-vector notation, $\vec{B}_P = (2.23\times10^{-11}\,\text{T})\hat{\text{j}}$.

LEARN In the limit where $d \gg w$, using

$$\ln(1+x) = x - x^2/2 + \cdots,$$

the magnetic field becomes

$$B_P = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) \approx \frac{\mu_0 i}{2\pi w}\cdot\frac{w}{d} = \frac{\mu_0 i}{2\pi d}$$

which is the same as that due to a thin wire.

34. By the right-hand rule (which is "built-into" Eq. 29-3) the field caused by wire 1's current, evaluated at the coordinate origin, is along the $+y$ axis. Its magnitude B_1 is given by Eq. 29-4. The field caused by wire 2's current will generally have both an x and a y component, which are related to its magnitude B_2 (given by Eq. 29-4), and sines and cosines of some angle. A little trig (and the use of the right-hand rule) leads us to conclude that when wire 2 is at angle θ_2 (shown in Fig. 29-62) then its components are

$$B_{2x} = B_2 \sin\theta_2, \quad B_{2y} = -B_2\cos\theta_2 .$$

The magnitude-squared of their net field is then (by Pythagoras' theorem) the sum of the square of their net x-component and the square of their net y-component:

$$B^2 = (B_2\sin\theta_2)^2 + (B_1 - B_2\cos\theta_2)^2 = B_1^2 + B_2^2 - 2B_1B_2\cos\theta_2 .$$

(since $\sin^2\theta + \cos^2\theta = 1$), which we could also have gotten directly by using the law of cosines. We have

$$B_1 = \frac{\mu_0 i_1}{2\pi R} = 60 \text{ nT}, \quad B_2 = \frac{\mu_0 i_2}{2\pi R} = 40 \text{ nT}.$$

With the requirement that the net field have magnitude B = 80 nT, we find

$$\theta_2 = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{2B_1B_2}\right) = \cos^{-1}(-1/4) = 104°,$$

where the positive value has been chosen.

35. **THINK** The magnitude of the force of wire 1 on wire 2 is given by $F_{21} = \mu_0 i_1 i_2 L / 2\pi r$, where i_1 is the current in wire 1, i_2 is the current in wire 2, and r is the distance between the wires.

EXPRESS The distance between the wires is $r = \sqrt{d_1^2 + d_2^2}$. The x component of the force is $F_{21,x} = F_{21}\cos\phi$, where $\cos\phi = d_2/\sqrt{d_1^2 + d_2^2}$.

ANALYZE Substituting the values given, the x component of the force per unit length is

$$\frac{F_{21,x}}{L}=\frac{\mu_0 i_1 i_2 d_2}{2\pi(d_1^2+d_2^2)}=\frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(4.00\times10^{-3}\ \mathrm{A})(6.80\times10^{-3}\ \mathrm{A})(0.050\ \mathrm{m})}{2\pi[(0.0240\ \mathrm{m})^2+(0.050\ \mathrm{m})^2]}$$
$$=8.84\times10^{-11}\ \mathrm{N/m}.$$

LEARN Since the two currents flow in the opposite directions, the force between the wires is repulsive. Thus, the direction of $\vec{F}_{21}$ is along the line that joins the wire and is away from wire 1.

36. We label these wires 1 through 5, left to right, and use Eq. 29-13. Then,

(a) The magnetic force on wire 1 is

$$\vec{F}_1=\frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{4d}\right)\hat{\mathrm{j}}=\frac{25\mu_0 i^2 l}{24\pi d}\,\hat{\mathrm{j}}=\frac{25\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(3.00\mathrm{A}\right)^2(10.0\mathrm{m})}{24\pi\left(8.00\times10^{-2}\,\mathrm{m}\right)}\,\hat{\mathrm{j}}$$
$$=(4.69\times10^{-4}\,\mathrm{N})\,\hat{\mathrm{j}}.$$

(b) Similarly, for wire 2, we have

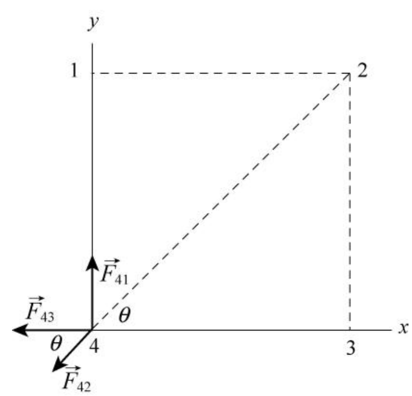
$$\vec{F}_2=\frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{2d}+\frac{1}{3d}\right)\hat{\mathrm{j}}=\frac{5\mu_0 i^2 l}{12\pi d}\,\hat{\mathrm{j}}=(1.88\times10^{-4}\mathrm{N})\,\hat{\mathrm{j}}.$$

(c) F_3 = 0 (because of symmetry).

(d) $\vec{F}_4=-\vec{F}_2=(-1.88\times10^{-4}\mathrm{N})\hat{\mathrm{j}}$, and

(e) $\vec{F}_5=-\vec{F}_1=-(4.69\times10^{-4}\mathrm{N})\hat{\mathrm{j}}$.

37. We use Eq. 29-13 and the superposition of forces: $\vec{F}_4=\vec{F}_{14}+\vec{F}_{24}+\vec{F}_{34}$. With $\theta = 45°$, the situation is as shown on the right.



The components of $\vec{F}_4$ are given by

$$F_{4x}=-F_{43}-F_{42}\cos\theta=-\frac{\mu_0 i^2}{2\mathrm{p}a}-\frac{\mu_0 i^2\cos45°}{2\sqrt{2}\mathrm{p}a}=-\frac{3\mu_0 i^2}{4\mathrm{p}a}$$

and

$$F_{4y}=F_{41}-F_{42}\sin\theta=\frac{\mu_0 i^2}{2\mathrm{p}a}-\frac{\mu_0 i^2\sin45°}{2\sqrt{2}\mathrm{p}a}=\frac{\mu_0 i^2}{4\mathrm{p}a}.$$

Thus,

$$F_4 = \left(F_{4x}^2 + F_{4y}^2\right)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a} = \frac{\sqrt{10}\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(7.50\mathrm{A}\right)^2}{4\pi\left(0.135\mathrm{m}\right)}$$
$$= 1.32\times10^{-4}\ \mathrm{N/m}$$

and $\vec{F}_4$ makes an angle ϕ with the positive x axis, where

$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162°.$$

In unit-vector notation, we have

$$\vec{F}_1 = (1.32\times10^{-4}\ \mathrm{N/m})[\cos162°\hat{\mathrm{i}} + \sin162°\hat{\mathrm{j}}] = (-1.25\times10^{-4}\ \mathrm{N/m})\hat{\mathrm{i}} + (4.17\times10^{-5}\ \mathrm{N/m})\hat{\mathrm{j}}$$

38. (a) The fact that the curve in Fig. 29-65(b) passes through zero implies that the currents in wires 1 and 3 exert forces in opposite directions on wire 2. Thus, current i_1 points *out of the page*. When wire 3 is a great distance from wire 2, the only field that affects wire 2 is that caused by the current in wire 1; in this case the force is negative according to Fig. 29-65(b). This means wire 2 is attracted to wire 1, which implies (by the discussion in Section 29-2) that wire 2's current is in the same direction as wire 1's current: *out of the page*. With wire 3 infinitely far away, the force per unit length is given (in magnitude) as 6.27×10^{-7} N/m. We set this equal to $F_{12} = \mu_0 i_1 i_2 / 2\pi d$. When wire 3 is at x = 0.04 m the curve passes through the zero point previously mentioned, so the force between 2 and 3 must equal F_{12} there. This allows us to solve for the distance between wire 1 and wire 2:

$$d = (0.04\ \mathrm{m})(0.750\ \mathrm{A})/(0.250\ \mathrm{A}) = 0.12\ \mathrm{m}.$$

Then we solve 6.27×10^{-7} N/m= $\mu_0 i_1 i_2 / 2\pi d$ and obtain i_2 = 0.50 A.

(b) The direction of i_2 is out of the page.

39. Using a magnifying glass, we see that all but i_2 are directed into the page. Wire 3 is therefore attracted to all but wire 2. Letting d = 0.500 m, we find the net force (per meter length) using Eq. 29-13, with positive indicated a rightward force:

$$\frac{|\vec{F}|}{\ell} = \frac{\mu_0 i_3}{2\pi}\left(-\frac{i_1}{2d} + \frac{i_2}{d} + \frac{i_4}{d} + \frac{i_5}{2d}\right)$$

which yields $|\vec{F}|/\ell = 8.00\times10^{-7}\ \mathrm{N/m}$.

40. Using Eq. 29-13, the force on, say, wire 1 (the wire at the upper left of the figure) is along the diagonal (pointing toward wire 3, which is at the lower right). Only the forces

(or their components) along the diagonal direction contribute. With $\theta = 45°$, we find the force per unit meter on wire 1 to be

$$F_1 = |\vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}| = 2F_{12}\cos\theta + F_{13} = 2\left(\frac{\mu_0 i^2}{2\pi a}\right)\cos 45° + \frac{\mu_0 i^2}{2\sqrt{2}\pi a} = \frac{3}{2\sqrt{2}\pi}\left(\frac{\mu_0 i^2}{a}\right)$$

$$= \frac{3}{2\sqrt{2}\pi}\frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(15.0\text{A}\right)^2}{\left(8.50\times10^{-2}\text{m}\right)} = 1.12\times10^{-3}\text{ N/m}.$$

The direction of $\vec{F}_1$ is along $\hat{r} = (\hat{\text{i}} - \hat{\text{j}})/\sqrt{2}$. In unit-vector notation, we have

$$\vec{F}_1 = \frac{(1.12\times10^{-3}\text{ N/m})}{\sqrt{2}}(\hat{\text{i}} - \hat{\text{j}}) = (7.94\times10^{-4}\text{ N/m})\hat{\text{i}} + (-7.94\times10^{-4}\text{ N/m})\hat{\text{j}}$$

41. The magnitudes of the forces on the sides of the rectangle that are parallel to the long straight wire (with i_1 = 30.0 A) are computed using Eq. 29-13, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp\text{ sides}} = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y}\, dy.$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a+b)}$$

$$= \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(30.0\text{A}\right)\left(20.0\text{A}\right)\left(8.00\text{cm}\right)\left(300\times10^{-2}\text{ m}\right)}{2\pi\left(1.00\text{cm} + 8.00\text{cm}\right)} = 3.20\times10^{-3}\text{ N},$$

and $\vec{F}$ points toward the wire, or $+\hat{\text{j}}$. That is, $\vec{F} = (3.20\times10^{-3}\text{ N})\hat{\text{j}}$ in unit-vector notation.

42. The area enclosed by the loop L is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\oint_c \vec{B}\cdot d\vec{s} = \mu_0 i = \mu_0 jA = \left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(15\text{ A/m}^2\right)\left(6\right)\left(0.20\text{m}\right)^2 = 4.5\times10^{-6}\text{ T}\cdot\text{m}.$$

43. We use Eq. 29-20 $B = \mu_0 i r / 2\pi a^2$ for the B-field inside the wire ($r < a$) and Eq. 29-17 $B = \mu_0 i / 2\pi r$ for that outside the wire ($r > a$).

(a) At $r = 0$, $B = 0$.

(b) At $r = 0.0100\,\text{m}$, $B = \dfrac{\mu_0 i r}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0100\text{m})}{2\pi(0.0200\text{m})^2} = 8.50\times10^{-4}\,\text{T}.$

(c) At $r = a = 0.0200\,\text{m}$, $B = \dfrac{\mu_0 i r}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0200\text{m})}{2\pi(0.0200\text{m})^2} = 1.70\times10^{-3}\,\text{T}.$

(d) At $r = 0.0400\,\text{m}$, $B = \dfrac{\mu_0 i}{2\pi r} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})}{2\pi(0.0400\text{m})} = 8.50\times10^{-4}\,\text{T}.$

44. We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop.

(a) For path 1, the result is

$$\oint_1 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\text{A} + 3.0\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-2.0\text{A}\right) = -2.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

(b) For path 2, we find

$$\oint_2 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\text{A} - 5.0\text{A} - 3.0\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-13.0\text{A}\right) = -1.6\times10^{-5}\,\text{T}\cdot\text{m}.$$

45. **THINK** The value of the line integral $\oint \vec{B}\cdot d\vec{s}$ is proportional to the net current enclosed.

EXPRESS By Ampere's law, we have $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where i_{enc} is the current enclosed by the closed path.

ANALYZE (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path, or "Amperian loop" is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B}\cdot d\vec{s} = -\mu_0 i = -(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(2.0\,\text{A}\right) = -2.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}} = 0$.

LEARN The value of $\oint \vec{B}\cdot d\vec{s}$ depends only on the current enclosed, and not the shape of the Amperian loop.

46. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\left(7i - 6i + 3i + i\right) = 5\mu_0 i = 5\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(4.50\times10^{-3}\ \text{A}\right) = 2.83\times10^{-8}\ \text{T}\cdot\text{m}.$$

47. For $r \le a$,

$$B(r) = \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0}{2\pi r}\int_0^r J(r)\,2\pi r\,dr = \frac{\mu_0}{2\pi}\int_0^r J_0\left(\frac{r}{a}\right)2\pi r\,dr = \frac{\mu_0 J_0 r^2}{3a}.$$

(a) At $r = 0$, $B = 0$.

(b) At $r = a/2$, we have

$$B(r) = \frac{\mu_0 J_0 r^2}{3a} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(310\ \text{A/m}^2)(3.1\times10^{-3}\ \text{m}/2)^2}{3(3.1\times10^{-3}\ \text{m})} = 1.0\times10^{-7}\ \text{T}.$$

(c) At $r = a$,

$$B(r=a) = \frac{\mu_0 J_0 a}{3} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(310\ \text{A/m}^2)(3.1\times10^{-3}\ \text{m})}{3} = 4.0\times10^{-7}\ \text{T}.$$

48. (a) The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi(3R)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R}.$$

For the wire we have $B_{P,\,\text{wire}} > B_{C,\,\text{wire}}$. Thus, for $B_P = B_C = B_{C,\,\text{wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\text{wire}} - B_{P,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)}.$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8 = 3(8.00\times10^{-3}\ \text{A})/8 = 3.00\times10^{-3}\ \text{A}$.

(b) The direction is into the page.

49. (a) We use Eq. 29-24. The inner radius is r = 15.0 cm, so the field there is

$$B = \frac{\mu_0 i N}{2\pi r} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.800\ \text{A}\right)\left(500\right)}{2\pi\left(0.150\ \text{m}\right)} = 5.33\times10^{-4}\ \text{T}.$$

(b) The outer radius is r = 20.0 cm. The field there is

$$B=\frac{\mu_0 iN}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.800\ \text{A}\right)\left(500\right)}{2\pi\left(0.200\text{m}\right)}=4.00\times10^{-4}\ \text{T}.$$

50. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil radius) is the preferred method:

$$B=\mu_0 in=\mu_0 i\left(\frac{N}{\ell}\right)$$

where i = 3.60 A, $\ell=0.950$ m, and N = 1200. This yields B = 0.00571 T.

51. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil diameter) is the preferred method:

$$B=\mu_0 in=\mu_0 i\left(\frac{N}{\ell}\right)$$

where i = 0.30 A, $\ell=0.25$ m, and N = 200. This yields $B=3.0\times10^{-4}$ T.

52. We find N, the number of turns of the solenoid, from the magnetic field $B=\mu_0 in=\mu_o iN/\ell$: $N=B\ell/\mu_0 i$. Thus, the total length of wire used in making the solenoid is

$$2\pi rN=\frac{2\pi rB\ell}{\mu_0 i}=\frac{2\pi\left(2.60\times10^{-2}\ \text{m}\right)\left(23.0\times10^{-3}\ \text{T}\right)\left(1.30\ \text{m}\right)}{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(18.0\ \text{A}\right)}=108\ \text{m}.$$

53. The orbital radius for the electron is

$$r=\frac{mv}{eB}=\frac{mv}{e\mu_0 ni}$$

which we solve for i:

$$i=\frac{mv}{e\mu_0 nr}=\frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.0460\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(100/0.0100\text{m}\right)\left(2.30\times10^{-2}\ \text{m}\right)}$$
$$=0.272\ \text{A}.$$

54. As the problem states near the end, some idealizations are being made here to keep the calculation straightforward (but are slightly unrealistic). For circular motion (with speed, $v_\perp$, which represents the magnitude of the component of the velocity perpendicular to the magnetic field [the field is shown in Fig. 29-20]), the period is (see Eq. 28-17)

$$T = 2\pi r/v_\perp = 2\pi m/eB.$$

Now, the time to travel the length of the solenoid is $t = L/v_\parallel$ where $v_\parallel$ is the component of the velocity in the direction of the field (along the coil axis) and is equal to $v\cos\theta$ where θ = 30º. Using Eq. 29-23 ($B = \mu_0 in$) with $n = N/L$, we find the number of revolutions made is $t/T = 1.6 \times 10^6$.

55. **THINK** The net field at a point inside the solenoid is the vector sum of the fields of the solenoid and that of the long straight wire along the central axis of the solenoid.

EXPRESS The magnetic field at a point P is given by $\vec{B} = \vec{B}_s + \vec{B}_w$, where $\vec{B}_s$ and $\vec{B}_w$ are the fields due to the solenoid and the wire, respectively. The direction of $\vec{B}_s$ is along the axis of the solenoid, and the direction of $\vec{B}_w$ is perpendicular to it, so the two fields are perpendicular to each other, $\vec{B}_s \perp \vec{B}_w$. For the net field $\vec{B}$ to be at 45° with the axis, we must have $B_s = B_w$.

ANALYZE (a) Thus,

$$B_s = B_w \Rightarrow \quad \mu_0 i_s n = \frac{\mu_0 i_w}{2\pi d},$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\text{A}}{2\pi\left(20.0\times10^{-3}\,\text{A}\right)\left(10\,\text{turns/cm}\right)} = 4.77\,\text{cm}.$$

(b) The magnetic field strength is

$$B = \sqrt{2}B_s = \sqrt{2}\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20.0\times10^{-3}\,\text{A}\right)\left(10\ \text{turns}/0.0100\ \text{m}\right) = 3.55\times10^{-5}\,\text{T}.$$

LEARN In general, the angle $\vec{B}$ makes with the solenoid axis is give by

$$\phi = \tan^{-1}\left(\frac{B_w}{B_s}\right) = \tan^{-1}\left(\frac{\mu_0 i_w/2\pi d}{\mu_0 i_s n}\right) = \tan^{-1}\left(\frac{i_w}{2\pi d\; n i_s}\right).$$

56. We use Eq. 29-26 and note that the contributions to $\vec{B}_P$ from the two coils are the same. Thus,

$$B_P = \frac{2\mu_0 i R^2 N}{2\left[R^2 + (R/2)^2\right]^{3/2}} = \frac{8\mu_0 N i}{5\sqrt{5}R} = \frac{8\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(200)\left(0.0122\text{A}\right)}{5\sqrt{5}\left(0.25\text{m}\right)} = 8.78\times10^{-6}\ \text{T}.$$

$\vec{B}_P$ is in the positive x direction.

57. **THINK** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area.

EXPRESS The cross-sectional area is a circle, so $A = \pi R^2$, where R is the radius. The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 29-27: $B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}$.

ANALYZE (a) Substituting the values given, we find the magnitude of the dipole moment to be

$$\mu = Ni\pi R^2 = (300)(4.0\,\text{A})\pi(0.025\,\text{m})^2 = 2.4\,\text{A}\cdot\text{m}^2.$$

(b) Solving for z, we obtain

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(2.36\,\text{A}\cdot\text{m}^2\right)}{2\pi\left(5.0\times10^{-6}\ \text{T}\right)}\right)^{1/3} = 46\,\text{cm}.$$

LEARN Note the similarity between $B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}$, the magnetic field of a magnetic dipole μ and $E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$, the electric field of an electric dipole p (see Eq. 22-9).

58. (a) We set $z = 0$ in Eq. 29-26 (which is equivalent using to Eq. 29-10 multiplied by the number of loops). Thus, $B(0) \propto i/R$. Since case b has two loops,

$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4.0\,.$$

(b) The ratio of their magnetic dipole moments is

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} = 0.50.$$

59. **THINK** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area.

EXPRESS The cross-sectional area is a circle, so $A = \pi R^2$, where R is the radius.

ANALYZE With $N = 200$, $i = 0.30$ A, and $R = 0.050$ m, the magnitude of the dipole moment is

$$\mu = (200)(0.30\,\text{A})\pi(0.050\,\text{m})^2 = 0.47\,\text{A}\cdot\text{m}^2.$$

LEARN The direction of $\vec{\mu}$ is that of the normal vector $\vec{n}$ to the plane of the coil, in accordance with the right-hand rule shown in Fig. 28-19.

60. Using Eq. 29-26, we find that the net y-component field is

$$B_y = \frac{\mu_0 i_1 R^2}{2(R^2 + z_1^2)^{3/2}} - \frac{\mu_0 i_2 R^2}{2(R^2 + z_2^2)^{3/2}},$$

where $z_1^2 = L^2$ (see Fig. 29-74(a)) and $z_2^2 = y^2$ (because the central axis here is denoted y instead of z). The fact that there is a minus sign between the two terms, above, is due to the observation that the datum in Fig. 29-74(b) corresponding to $B_y = 0$ would be impossible without it (physically, this means that one of the currents is clockwise and the other is counterclockwise).

(a) As $y \to \infty$, only the first term contributes and (with $B_y = 7.2 \times 10^{-6}$ T given in this case) we can solve for i_1:

$$i_1 = \frac{2(R^2 + z_1^2)^{3/2} B_y}{\mu_0 R^2} = \frac{2R[1 + (L/R)^2]^{3/2} B_y}{\mu_0}$$
$$= \frac{2(0.040\text{ m})[1 + (0.030\text{ m}/0.040\text{ m})^2]^{3/2}(7.2\times 10^{-6}\text{ T})}{4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}} = 0.895\text{ A} \approx 0.90\text{ A}.$$

(b) With loop 2 at $y = 0.06$ m (see Fig. 29-74(b)) we are able to determine i_2 from

$$\frac{\mu_0 i_1 R^2}{2(R^2 + L^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2 + y^2)^{3/2}}.$$

We obtain $i_2 = (117\sqrt{13}/50\pi)$ A ≈ 2.7 A.

61. (a) We denote the large loop and small coil with subscripts 1 and 2, respectively.

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A})(15\,\text{A})}{2(0.12\,\text{m})} = 7.9\times 10^{-5}\,\text{T}.$$

(b) The torque has magnitude equal to

$$\tau = |\vec{\mu}_2 \times \vec{B}_1| = \mu_2 B_1 \sin 90° = N_2 i_2 A_2 B_1 = \pi N_2 i_2 r_2^2 B_1 = \pi(50)(1.3\text{A})(0.82\times10^{-2}\text{m})^2(7.9\times10^{-5}\text{T})$$
$$= 1.1\times10^{-6}\,\text{N}\cdot\text{m}.$$

62. (a) To find the magnitude of the field, we use Eq. 29-9 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$B = \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.0562\text{A})}{4}\left(\frac{1}{0.0572\text{m}}+\frac{1}{0.0936\text{m}}\right)$$
$$= 4.97\times10^{-7}\,\text{T}.$$

(b) By the right-hand rule, $\vec{B}$ points into the paper at P (see Fig. 29-7(c)).

(c) The enclosed area is $A = (\pi a^2 + \pi b^2)/2$, which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2}(a^2+b^2) = \frac{\pi(0.0562\text{A})}{2}[(0.0572\text{m})^2 + (0.0936\text{m})^2] = 1.06\times10^{-3}\,\text{A}\cdot\text{m}^2.$$

(d) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper.

63. By imagining that each of the segments *bg* and *cf* (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude (i) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.

(a) The dipole moment of path *abcdefgha* is

$$\vec{\mu} = \vec{\mu}_{bcfgb} + \vec{\mu}_{abgha} + \vec{\mu}_{cdefc} = (ia^2)(\hat{\mathrm{j}} - \hat{\mathrm{i}} + \hat{\mathrm{i}}) = ia^2\hat{\mathrm{j}}$$
$$= (6.0\,\text{A})(0.10\,\text{m})^2\,\hat{\mathrm{j}} = (6.0\times10^{-2}\,\text{A}\cdot\text{m}^2)\,\hat{\mathrm{j}}\,.$$

(b) Since both points are far from the cube we can use the dipole approximation. For (x, y, z) = (0, 5.0 m, 0),

$$\vec{B}(0,\,5.0\text{ m},\,0) \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{y^3} = \frac{(1.26\times10^{-6}\ \text{T}\cdot\text{m/A})(6.0\times10^{-2}\text{m}^2\cdot\text{A})\,\hat{\mathrm{j}}}{2\pi(5.0\text{ m})^3} = (9.6\times10^{-11}\ \text{T})\,\hat{\mathrm{j}}\,.$$

64. (a) The radial segments do not contribute to $\vec{B}_P$, and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\mathrm{k}}$ designates the direction "out of the page" then

$$\vec{B}_P = \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(4.00\text{ m})}\hat{\text{k}} - \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(2.00\text{ m})}\hat{\text{k}}$$

where $i = 0.200$ A. This yields $\vec{B} = -2.75 \times 10^{-8}\,\hat{\text{k}}$ T, or $|\vec{B}| = 2.75 \times 10^{-8}$ T.

(b) The direction is $-\hat{\text{k}}$, or into the page.

65. Using Eq. 29-20,

$$|\vec{B}| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r,$$

we find that $r = 0.00128$ m gives the desired field value.

66. (a) We designate the wire along $y = r_A = 0.100$ m wire A and the wire along $y = r_B =$ 0.050 m wire B. Using Eq. 29-4, we have

$$\vec{B}_{\text{net}} = \vec{B}_A + \vec{B}_B = -\frac{\mu_0 i_A}{2\pi r_A}\hat{\text{k}} - \frac{\mu_0 i_B}{2\pi r_B}\hat{\text{k}} = (-52.0\times10^{-6}\text{ T})\hat{\text{k}}.$$

(b) This will occur for some value $r_B < y < r_A$ such that

$$\frac{\mu_0 i_A}{2\pi(r_A - y)} = \frac{\mu_0 i_B}{2\pi(y - r_B)}.$$

Solving, we find $y = 13/160 \approx 0.0813$ m.

(c) We eliminate the $y < r_B$ possibility due to wire B carrying the larger current. We expect a solution in the region $y > r_A$ where

$$\frac{\mu_0 i_A}{2\pi(y - r_A)} = \frac{\mu_0 i_B}{2\pi(y - r_B)}.$$

Solving, we find $y = 7/40 \approx 0.0175$ m.

67. Let the length of each side of the square be a. The center of a square is a distance $a/2$ from the nearest side. There are four sides contributing to the field at the center. The result is

$$B_{\text{center}} = 4\left(\frac{\mu_0 i}{2\pi(a/2)}\right)\left(\frac{a}{\sqrt{a^2 + 4(a/2)^2}}\right) = \frac{2\sqrt{2}\mu_0 i}{\pi a}.$$

On the other hand, the magnetic field at the center of a circular wire of radius R is $\mu_0 i/2R$ (e.g., Eq. 29-10). Thus, the problem is equivalent to showing that

$$\frac{2\sqrt{2}\mu_0 i}{\pi a} > \frac{\mu_0 i}{2R} \quad \Rightarrow \quad \frac{4\sqrt{2}}{\pi a} > \frac{1}{R} \; .$$

To do this we must relate the parameters a and R. If both wires have the same length L then the geometrical relationships $4a = L$ and $2\pi R = L$ provide the necessary connection:

$$4a = 2\pi R \quad \Rightarrow \quad a = \frac{\pi R}{2}.$$

Thus, our proof consists of the observation that

$$\frac{4\sqrt{2}}{\pi a} = \frac{8\sqrt{2}}{\pi^2 R} > \frac{1}{R},$$

as one can check numerically (that $8\sqrt{2}/\pi^2 > 1$).

68. We take the current (i = 50 A) to flow in the $+x$ direction, and the electron to be at a point P, which is r = 0.050 m above the wire (where “up” is the $+y$ direction). Thus, the field produced by the current points in the $+z$ direction at P. Then, combining Eq. 29-4 with Eq. 28-2, we obtain

$$\vec{F}_e = \left(-e\mu_0 i/2\pi r\right)\left(\vec{v} \times \hat{\mathrm{k}}\right).$$

(a) The electron is moving down: $\vec{v} = -v\hat{\mathrm{j}}$ (where $v = 1.0 \times 10^7$ m/s is the speed) so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r}\left(-\hat{\mathrm{i}}\right) = (3.2\times10^{-16}\ \mathrm{N})\,\hat{\mathrm{i}}\,,$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N.

(b) In this case, the electron is in the same direction as the current: $\vec{v} = v\hat{\mathrm{i}}$ so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r}\left(-\hat{\mathrm{j}}\right) = (3.2\times10^{-16}\ \mathrm{N})\,\hat{\mathrm{j}},$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N.

(c) Now, $\vec{v} = \pm v\hat{\mathrm{k}}$ so $\vec{F}_e \propto \hat{\mathrm{k}} \times \hat{\mathrm{k}} = 0.$

69. (a) By the right-hand rule, the magnetic field $\vec{B}_1$ (evaluated at a) produced by wire 1 (the wire at bottom left) is at ϕ = 150° (measured counterclockwise from the $+x$ axis, in the xy plane), and the field produced by wire 2 (the wire at bottom right) is at ϕ = 210°. By symmetry $\left(\vec{B}_1 = \vec{B}_2\right)$ we observe that only the x-components survive, yielding

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \left(2\frac{\mu_0 i}{2\pi\ell}\cos 150°\right)\hat{\text{i}} = (-3.46\times10^{-5}\,\text{T})\hat{\text{i}}$$

where i = 10 A, ℓ = 0.10 m, and Eq. 29-4 has been used. To cancel this, wire b must carry current into the page (that is, the $-\hat{\text{k}}$ direction) of value

$$i_b = B\frac{2\pi r}{\mu_0} = \left(3.46\times10^{-5}\ \text{T}\right)\frac{2\pi(0.087\ \text{m})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 15\,\text{A}$$

where $r = \sqrt{3}\,\ell/2 = 0.087$ m and Eq. 29-4 has again been used.

(b) As stated above, to cancel this, wire b must carry current into the page (that is, the $-z$ direction).

70. The radial segments do not contribute to $\vec{B}$ (at the center), and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\text{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0 i(\pi\ \text{rad})}{4\pi(4.00\ \text{m})}\hat{\text{k}} + \frac{\mu_0 i(\pi/2\ \text{rad})}{4\pi(2.00\ \text{m})}\hat{\text{k}} - \frac{\mu_0 i(\pi/2\ \text{rad})}{4\pi(4.00\ \text{m})}\hat{\text{k}}$$

where i = 2.00 A. This yields $\vec{B}$ = (1.57 × 10^{-7} T) $\hat{\text{k}}$, or $|\vec{B}| = 1.57\times10^{-7}\,\text{T}$.

71. Since the radius is R = 0.0013 m, then the i = 50 A produces

$$B = \frac{\mu_0 i}{2\pi R} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(50\ \text{A})}{2\pi(0.0013\ \text{m})} = 7.7\times10^{-3}\,\text{T}$$

at the edge of the wire. The three equations, Eq. 29-4, Eq. 29-17, and Eq. 29-20, agree at this point.

72. (a) With cylindrical symmetry, we have, external to the conductors,

$$\left|\vec{B}\right| = \frac{\mu_0\, i_{\text{enc}}}{2\pi r}$$

which produces i_{enc} = 25 mA from the given information. Therefore, the thin wire must carry 5.0 mA.

(b) The direction is downward, opposite to the 30 mA carried by the thin conducting surface.

73. (a) The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and

has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current i, which is uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

at a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi\left(a^2 - b^2\right)},$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}.$$

The current in the cylinder that fills the hole is

$$I_2 = \pi J b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}.$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi\left(a^2 - b^2\right)} = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})\left(5.25\ \text{A}\right)(0.0200\text{m})}{2\pi[(0.0400\text{m})^2 - (0.0150\text{m})^2]} = 1.53\times 10^{-5}\ \text{T}$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$ the formula for the field becomes $B = \frac{\mu_0 i d}{2\pi a^2}$. This correctly gives the field of a solid cylinder carrying a uniform current *i*, at a point inside the cylinder a distance *d* from the axis. If $d = 0$ the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

Note: One may apply Ampere's law to show that the magnetic field in the hole is uniform. Consider a rectangular path with two long sides (side 1 and 2, each with length *L*) and two short sides (each of length less than *b*). If side 1 is directly along the axis of the hole, then side 2 would also be parallel to it and in the hole. To ensure that the short sides do not contribute significantly to the integral in Ampere's law, we might wish to make *L* *very* long (perhaps longer than the length of the cylinder), or we might appeal to an argument regarding the angle between $\vec{B}$ and the short sides (which is 90° at the axis of the hole). In any case, the integral in Ampere's law reduces to

$$\oint_{\text{rectangle}} \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$

$$\int_{side 1} \vec{B} \cdot d\vec{s} + \int_{\text{side 2}} \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{in hole}}$$

$$\left(B_{\text{side 1}} - B_{\text{side 2}}\right) L = 0$$

where $B_{\text{side 1}}$ is the field along the axis found in part (a). This shows that the field at off-axis points (where $B_{\text{side 2}}$ is evaluated) is the same as the field at the center of the hole; therefore, the field in the hole is uniform.

74. Equation 29-4 gives

$$i = \frac{2\pi RB}{\mu_0} = \frac{2\pi(0.880\,\text{m})(7.30\times10^{-6}\,\text{T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 32.1\,\text{A}.$$

75. **THINK** In this problem, we apply the Biot-Savart law to calculate the magnetic field due to a current-carrying segment at various locations.

EXPRESS The Biot-Savart law can be written as

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi}\frac{i\Delta\vec{s}\times\hat{\text{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{i\Delta\vec{s}\times\vec{r}}{r^3}.$$

With $\Delta\vec{s} = \Delta s\hat{\text{j}}$ and $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, their cross product is

$$\Delta\vec{s}\times\vec{r} = (\Delta s\hat{\text{j}})\times(x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}) = \Delta s(z\hat{\text{i}} - x\hat{\text{k}})$$

where we have used $\hat{\text{j}}\times\hat{\text{i}}=-\hat{\text{k}}$, $\hat{\text{j}}\times\hat{\text{j}}=0$, and $\hat{\text{j}}\times\hat{\text{k}}=\hat{\text{i}}$. Thus, the Biot-Savart equation becomes

$$\vec{B}(x,y,z)=\frac{\mu_0 i\,\Delta s\left(z\hat{\text{i}}-x\hat{\text{k}}\right)}{4\pi\left(x^2+y^2+z^2\right)^{3/2}}.$$

ANALYZE (a) The field on the z axis (at $x = 0$, $y = 0$, and $z = 5.0$ m) is

$$\vec{B}(0,0,5.0\text{ m})=\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(2.0\,\text{A})\left(3.0\times10^{-2}\,\text{m}\right)(5.0\text{ m})\hat{\text{i}}}{4\pi\left(0^2+0^2+(5.0\text{ m})^2\right)^{3/2}}=(2.4\times10^{-10}\,\text{T})\hat{\text{i}}.$$

(b) Similarly, $\vec{B}$ (0, 6.0 m, 0) = 0, since $x = z = 0$.

(c) The field in the xy plane, at (x, y, z) = (7 m, 7 m, 0), is

$$\vec{B}(7.0\text{ m},7.0\text{ m},0)=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times10^{-2}\,\text{m})(-7.0\,\text{m})\hat{\text{k}}}{4\pi\left((7.0\text{ m})^2+(7.0\text{ m})^2+0^2\right)^{3/2}}=(-4.3\times10^{-11}\,\text{T})\hat{\text{k}}.$$

(d) The field in the xy plane, at (x, y, z) = (–3, –4, 0), is

$$\vec{B}(-3.0\,\text{m},-4.0\text{m},0)=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times10^{-2}\,\text{m})(3.0\,\text{m})\hat{\text{k}}}{4\pi\left((-3.0\,\text{m})^2+(-4.0\,\text{m})^2+0^2\right)^{3/2}}=(1.4\times10^{-10}\,\text{T})\hat{\text{k}}.$$

LEARN Along the x and z axes, the expressions for $\vec{B}$ simplify to

$$\vec{B}(x,0,0)=-\frac{\mu_0}{4\pi}\frac{i\,\Delta s}{x^2}\hat{\text{k}},\quad \vec{B}(0,0,z)=\frac{\mu_0}{4\pi}\frac{i\,\Delta s}{z^2}\hat{\text{i}}.$$

The magnetic field at any point on the y axis vanishes because the current flows in the $+y$ direction, so $d\vec{s}\times\hat{\text{r}}=0$.

76. We note that the distance from each wire to P is $r=d/\sqrt{2}=0.071\,\text{m}$. In both parts, the current is i = 100 A.

(a) With the currents parallel, application of the right-hand rule (to determine each of their contributions to the field at P) reveals that the vertical components cancel and the horizontal components add, yielding the result:

$$B=2\left(\frac{\mu_0 i}{2\text{p}r}\right)\cos 45.0^\circ=4.00\times10^{-4}\,\text{T}$$

and directed in the –*x* direction. In unit-vector notation, we have $\vec{B} = (-4.00\times10^{-4}\ \text{T})\hat{\text{i}}$.

(b) Now, with the currents anti-parallel, application of the right-hand rule shows that the horizontal components cancel and the vertical components add. Thus,

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\sin 45.0° = 4.00\times10^{-4}\ \text{T}$$

and directed in the +*y* direction. In unit-vector notation, we have $\vec{B} = (4.00\times10^{-4}\ \text{T})\hat{\text{j}}$.

77. We refer to the center of the circle (where we are evaluating $\vec{B}$) as *C*. Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments that are collinear with *C* do not contribute to the field there. Eq. 29-9 (with $\phi = \pi/2$ rad) and the right-hand rule indicates that the currents in the two arcs contribute

$$\frac{\mu_0 i(\pi/2)}{4\pi R} - \frac{\mu_0 i(\pi/2)}{4\pi R} = 0$$

to the field at *C*. Thus, the nonzero contributions come from those straight segments that are not collinear with *C*. There are two of these "semi-infinite" segments, one a vertical distance *R* above *C* and the other a horizontal distance *R* to the left of *C*. Both contribute fields pointing out of the page (see Fig. 29-7(c)). Since the magnitudes of the two contributions (governed by Eq. 29-7) add, then the result is

$$B = 2\left(\frac{\mu_0 i}{4\pi R}\right) = \frac{\mu_0 i}{2\pi R}$$

exactly what one would expect from a single infinite straight wire (see Eq. 29-4). For such a wire to produce such a field (out of the page) with a leftward current requires that the point of evaluating the field be below the wire (again, see Fig. 29-7(c)).

78. The points must be along a line parallel to the wire and a distance *r* from it, where *r* satisfies $B_{\text{wire}} = \frac{\mu_0 i}{2\pi r} = B_{\text{ext}}$, or

$$r = \frac{\mu_0 i}{2\pi B_{\text{ext}}} = \frac{(1.26\times10^{-6}\ \text{T}\cdot\text{m/A})(100\ \text{A})}{2\pi(5.0\times10^{-3}\ \text{T})} = 4.0\times10^{-3}\ \text{m}.$$

79. (a) The field in this region is entirely due to the long wire (with, presumably, negligible thickness). Using Eq. 29-17,

$$\left|\vec{B}\right| = \frac{\mu_0\, i_w}{2\pi r} = 4.8\times 10^{-3}\ \text{T}$$

where i_w = 24 A and r = 0.0010 m.

(b) Now the field consists of two contributions (which are anti-parallel) — from the wire (Eq. 29-17) and from a portion of the conductor (Eq. 29-20 modified for annular area):

$$|\vec{B}| = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_c}{2\pi r}\left(\frac{\pi r^2 - \pi R_i^2}{\pi R_0^2 - \pi R_i^2}\right)$$

where r = 0.0030 m, R_i = 0.0020 m, R_o = 0.0040 m, and i_c = 24 A. Thus, we find $|\vec{B}| = 9.3\times 10^{-4}$ T.

(c) Now, in the external region, the individual fields from the two conductors cancel completely (since $i_c = i_w$): $\vec{B} = 0$.

80. Using Eq. 29-20 and Eq. 29-17, we have

$$|\vec{B}_1| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r_1 \qquad |\vec{B}_2| = \frac{\mu_0 i}{2\pi r_2}$$

where $r_1 = 0.0040\,\text{m}$, $\left|\vec{B}_1\right| = 2.8\times 10^{-4}$ T, $r_2 = 0.010\,\text{m}$, and $|\vec{B}_2| = 2.0\times 10^{-4}$ T. Point 2 is known to be external to the wire since $|\vec{B}_2| < |\vec{B}_1|$. From the second equation, we find i = 10 A. Plugging this into the first equation yields $R = 5.3\times 10^{-3}$ m.

81. **THINK** The objective of this problem is to calculate the magnetic field due to an infinite current sheet by applying Ampere's law.

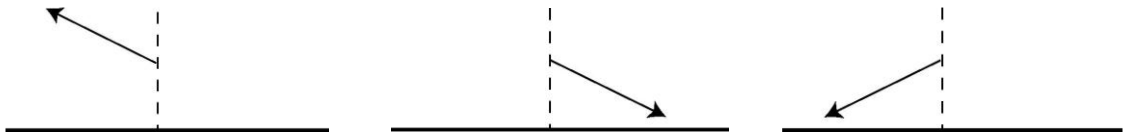
EXPRESS The "current per unit x-length" may be viewed as current density multiplied by the thickness Δy of the sheet; thus, $\lambda = J\Delta y$. Ampere's law may be (and often is) expressed in terms of the current density vector as follows:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \int \vec{J}\cdot d\vec{A}$$

where the area integral is over the region enclosed by the path relevant to the line integral (and $\vec{J}$ is in the $+z$ direction, out of the paper). With J uniform throughout the sheet, then it is clear that the right-hand side of this version of Ampere's law should reduce, in this problem, to

$$\mu_0 JA = \mu_0 J\,\Delta y\Delta x = \mu_0 \lambda\,\Delta x.$$

ANALYZE (a) Figure 29-84 certainly has the horizontal components of $\vec{B}$ drawn correctly at points P and P', so the question becomes: is it possible for $\vec{B}$ to have vertical components in the figure?



Our focus is on point *P*. Suppose the magnetic field is not parallel to the sheet, as shown in the upper left diagram. If we reverse the direction of the current, then the direction of the field will also be reversed (as shown in the upper middle diagram). Now, if we rotate the sheet by 180° about a line that is perpendicular to the sheet, the field will rotate and point in the direction shown in the diagram on the upper right. The current distribution now is exactly the same as the original; however, comparing the upper left and upper right diagrams, we see that the fields are not the same, unless the original field is parallel to the sheet and only has a horizontal component. That is, the field at *P* must be purely horizontal, as drawn in Fig. 29-84.

(b) The path used in evaluating $\oint \vec{B}\cdot d\vec{s}$ is rectangular, of horizontal length Δx (the horizontal sides passing through points *P* and *P′*, respectively) and vertical size $\delta y > \Delta y$. The vertical sides have no contribution to the integral since $\vec{B}$ is purely horizontal (so the scalar dot product produces zero for those sides), and the horizontal sides contribute two equal terms, as shown next. Ampere's law yields

$$2B\Delta x = \mu_0 \lambda \Delta x \;\Rightarrow\; B = \frac{1}{2}\mu_0 \lambda .$$

LEARN In order to apply Ampere's law, the system must possess certain symmetry. In the case of an infinite current sheet, the symmetry is planar.

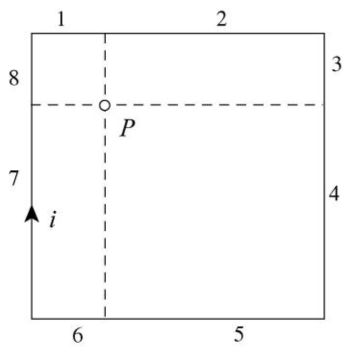
82. Equation 29-17 applies for each wire, with $r = \sqrt{R^2 + (d/2)^2}$ (by the Pythagorean theorem). The vertical components of the fields cancel, and the two (identical) horizontal components add to yield the final result

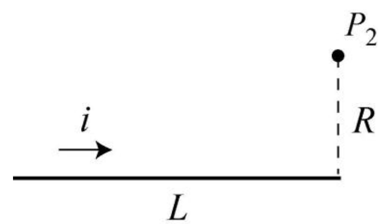
$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{d/2}{r}\right) = \frac{\mu_0 i d}{2\pi\left(R^2 + (d/2)^2\right)} = 1.25\times10^{-6}\ \text{T},$$

where $(d/2)/r$ is a trigonometric factor to select the horizontal component. It is clear that this is equivalent to the expression in the problem statement. Using the right-hand rule, we find both horizontal components point in the $+x$ direction. Thus, in unit-vector notation, we have $\vec{B} = (1.25\times10^{-6}\ \text{T})\hat{\text{i}}$.

83. **THINK** The magnetic field at *P* is the vector sum of the fields of the individual wire segments.

EXPRESS The two small wire segments, each of length $a/4$, shown in Fig. 29-86 nearest to point P, are labeled 1 and 8 in the figure (below left). Let $-\hat{k}$ be a unit vector pointing into the page.





We use the result of Problem 29-17: namely, the magnetic field at P_2 (shown in Fig. 29-44 and upper right) is

$$B_{P_2} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}.$$

Therefore, the magnetic fields due to the 8 segments are

$$B_{P1} = B_{P8} = \frac{\sqrt{2}\mu_0 i}{8\pi(a/4)} = \frac{\sqrt{2}\mu_0 i}{2\pi a},$$

$$B_{P4} = B_{P5} = \frac{\sqrt{2}\mu_0 i}{8\pi(3a/4)} = \frac{\sqrt{2}\mu_0 i}{6\pi a},$$

$$B_{P2} = B_{P7} = \frac{\mu_0 i}{4\pi(a/4)}\cdot\frac{3a/4}{\left[(3a/4)^2+(a/4)^2\right]^{1/2}} = \frac{3\mu_0 i}{\sqrt{10}\pi a},$$

and

$$B_{P3} = B_{P6} = \frac{\mu_0 i}{4\pi(3a/4)}\cdot\frac{a/4}{\left[(a/4)^2+(3a/4)^2\right]^{1/2}} = \frac{\mu_0 i}{3\sqrt{10}\pi a}.$$

ANALYZE Adding up all the contributions, the total magnetic field at P is

$$\begin{aligned}\vec{B}_P &= \sum_{n=1}^{8} B_{Pn}(-\hat{k}) = 2\frac{\mu_0 i}{\pi a}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)(-\hat{k})\\ &= \frac{2(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(10\text{A})}{\pi(8.0\times10^{-2}\,\text{m})}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)(-\hat{k})\\ &= (2.0\times10^{-4}\,\text{T})(-\hat{k}).\end{aligned}$$

LEARN If point P is located at the center of the square, then each segment would contribute

$$B_{P1} = B_{P2} = \cdots B_{P8} = \frac{\sqrt{2}\mu_0 i}{4\pi a},$$

making the total field

$$B_{\text{center}} = 8B_{P1} = \frac{8\sqrt{2}\mu_0 i}{4\pi a}.$$

84. (a) All wires carry parallel currents and attract each other; thus, the "top" wire is pulled downward by the other two:

$$|\vec{F}| = \frac{\mu_0 L(5.0\,\text{A})(3.2\,\text{A})}{2\pi(0.10\,\text{m})} + \frac{\mu_0 L(5.0\,\text{A})(5.0\,\text{A})}{2\pi(0.20\,\text{m})}$$

where L = 3.0 m. Thus, $|\vec{F}| = 1.7 \times 10^{-4}$ N.

(b) Now, the "top" wire is pushed upward by the center wire and pulled downward by the bottom wire:

$$|\vec{F}| = \frac{\mu_0 L(5.0\text{A})(3.2\text{A})}{2\pi(0.10\text{m})} - \frac{\mu_0 L(5.0\text{A})(5.0\text{A})}{2\pi(0.20\text{m})} = 2.1\times10^{-5}\,\text{N}.$$

85. **THINK** The hollow conductor has cylindrical symmetry, so Ampere's law can be applied to calculate the magnetic field due to the current distribution.

EXPRESS Ampere's law states that $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where i_{enc} is the current enclosed by the closed path, or Amperian loop. We choose the Amperian loop to be a circle of radius r and concentric with the cylindrical shell. Since the current is uniformly distributed throughout the cross section of the shell, the enclosed current is

$$i_{\text{enc}} = i\,\frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)} = i\left(\frac{r^2 - b^2}{a^2 - b^2}\right).$$

ANALYZE (a) Thus, in the region $b < r < a$, we have

$$\oint \vec{B}\cdot d\vec{s} = 2\pi r B = \mu_0 i_{\text{enc}} = \mu_0 i\left(\frac{r^2 - b^2}{a^2 - b^2}\right)$$

which gives $B = \dfrac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\dfrac{r^2 - b^2}{r}\right)$.

(b) At $r = a$, the magnetic field strength is

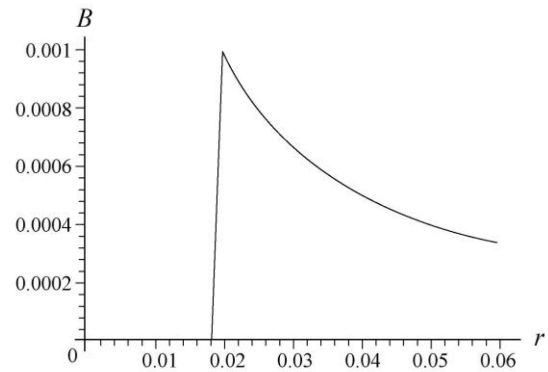
$$\frac{\mu_0 i}{2\pi(a^2-b^2)}\left(\frac{a^2-b^2}{a}\right)=\frac{\mu_0 i}{2\pi a}.$$

At $r=b, B\propto r^2-b^2=0$. Finally, for $b = 0$

$$B=\frac{\mu_0 i}{2\pi a^2}\frac{r^2}{r}=\frac{\mu_0 i r}{2\pi a^2}$$

which agrees with Eq. 29-20.

(c) The field is zero for $r < b$ and is equal to Eq. 29-17 for $r > a$, so this along with the result of part (a) provides a determination of B over the full range of values. The graph (with SI units understood) is shown below.



LEARN For $r < b$, the field is zero, and for $r > a$, the field decreases as $1/r$. In the region $b < r < a$, the field increases with r as $r-b^2/r$.

86. We refer to the side of length L as the long side and that of length W as the short side. The center is a distance $W/2$ from the midpoint of each long side, and is a distance $L/2$ from the midpoint of each short side. There are two of each type of side, so the result of Problem 29-17 leads to

$$B=2\frac{\mu_0 i}{2\pi(W/2)}\frac{L}{\sqrt{L^2+4(W/2)^2}}+2\frac{\mu_0 i}{2\pi(L/2)}\frac{W}{\sqrt{W^2+4(L/2)^2}}.$$

The final form of this expression, shown in the problem statement, derives from finding the common denominator of the above result and adding them, while noting that

$$\frac{L^2+W^2}{\sqrt{W^2+L^2}}=\sqrt{W^2+L^2}.$$

87. (a) Equation 29-20 applies for $r < c$. Our sign choice is such that i is positive in the smaller cylinder and negative in the larger one.

$$B = \frac{\mu_0 i r}{2\pi c^2},\ r \le c.$$

(b) Equation 29-17 applies in the region between the conductors:

$$B = \frac{\mu_0 i}{2\pi r},\ c \le r \le b.$$

(c) Within the larger conductor we have a superposition of the field due to the current in the inner conductor (still obeying Eq. 29-17) plus the field due to the (negative) current in that part of the outer conductor at radius less than r. The result is

$$B = \frac{\mu_0 i}{2\pi r} - \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - b^2}{a^2 - b^2}\right),\ \ b < r \le a.$$

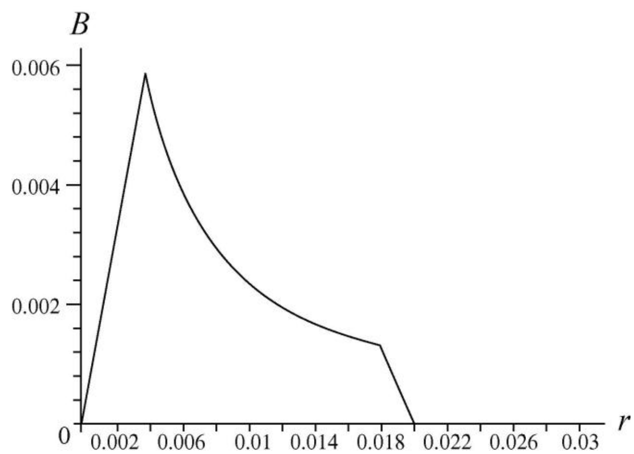
If desired, this expression can be simplified to read

$$B = \frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right).$$

(d) Outside the coaxial cable, the net current enclosed is zero. So $B = 0$ for $r \ge a$.

(e) We test these expressions for one case. If $a \to \infty$ and $b \to \infty$ (such that $a > b$) then we have the situation described on page 696 of the textbook.

(f) Using SI units, the graph of the field is shown to the right.



88. (a) Consider a segment of the projectile between y and $y + dy$. We use Eq. 29-12 to find the magnetic force on the segment, and Eq. 29-7 for the magnetic field of each semi-infinite wire (the top rail referred to as wire 1 and the bottom as wire 2). The current in rail 1 is in the $+\hat{\mathrm{i}}$ direction, and the current in rail 2 is in the $-\hat{\mathrm{i}}$ direction. The field (in the region between the wires) set up by wire 1 is into the paper (the $-\hat{\mathrm{k}}$ direction) and that set up by wire 2 is also into the paper. The force element (a function of y) acting on the segment of the projectile (in which the current flows in the $-\hat{\mathrm{j}}$ direction) is given below. The coordinate origin is at the bottom of the projectile.

$$d\vec{F} = d\vec{F}_1 + d\vec{F}_2 = i\,dy\left(-\hat{\mathrm{j}}\right)\times\vec{B}_1 + dy\left(-\hat{\mathrm{j}}\right)\times\vec{B}_2 = i\left[B_1 + B_2\right]\hat{\mathrm{i}}\,dy$$

$$= i\left[\frac{\mu_0 i}{4\pi\left(2R+w-y\right)} + \frac{\mu_0 i}{4\pi y}\right]\hat{\mathrm{i}}\,dy.$$

Thus, the force on the projectile is

$$\vec{F} = \int d\vec{F} = \frac{i^2\mu_0}{4\pi}\int_R^{R+w}\left(\frac{1}{2R+w-y} + \frac{1}{y}\right)dy\,\hat{\mathrm{i}} = \frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)\hat{\mathrm{i}}.$$

(b) Using the work-energy theorem, we have

$$\Delta K = \tfrac{1}{2}mv_f^2 = W_{\text{ext}} = \int \vec{F}\cdot d\vec{s} = FL.$$

Thus, the final speed of the projectile is

$$v_f = \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\,\frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)L\right]^{1/2}$$

$$= \left[\frac{2\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(450\times10^{3}\ \mathrm{A}\right)^2\ \ln\left(1+1.2\ \mathrm{cm}/6.7\ \mathrm{cm}\right)\left(4.0\ \mathrm{m}\right)}{2\pi\left(10\times10^{-3}\ \mathrm{kg}\right)}\right]^{1/2}$$

$$= 2.3\times10^{3}\ \mathrm{m/s}.$$

Chapter 30

1. The flux $\Phi_B = BA\cos\theta$ does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is zero.

2. Using Faraday's law, the induced emf is

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d\left(BA\right)}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(\pi r^2\right)}{dt} = -2\pi r B\frac{dr}{dt} \\ &= -2\pi\left(0.12\,\text{m}\right)\left(0.800\,\text{T}\right)\left(-0.750\,\text{m/s}\right) \\ &= 0.452\,\text{V}.\end{aligned}$$

3. **THINK** Changing the current in the solenoid changes the flux, and therefore, induces a current in the coil.

EXPRESS Using Faraday's law, the total induced emf is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 n i) = -N\mu_0 n A\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt}$$

By Ohm's law, the induced current in the coil is $i_{\text{ind}} = |\varepsilon|/R,$ where R is the resistance of the coil.

ANALYZE Substituting the values given, we obtain

$$\begin{aligned}\varepsilon &= -N\mu_0 n(\pi r^2)\frac{di}{dt} = -(120)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(22000/\text{m})\,\pi\left(0.016\,\text{m}\right)^2\left(\frac{1.5\,\text{A}}{0.025\,\text{s}}\right) \\ &= 0.16\,\text{V}.\end{aligned}$$

Ohm's law then yields $i_{\text{ind}} = \dfrac{|\varepsilon|}{R} = \dfrac{0.016\,\text{V}}{5.3\,\Omega} = 0.030\,\text{A}.$

LEARN The direction of the induced current can be deduced from Lenz's law, which states that the direction of the induced current is such that the magnetic field which it produces opposes the change in flux that induces the current.

4. (a) We use $\varepsilon = -d\Phi_B/dt = -\pi r^2 dB/dt$. For $0 < t < 2.0$ s:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi\left(0.12\text{m}\right)^2\left(\frac{0.5\,\text{T}}{2.0\,\text{s}}\right) = -1.1\times 10^{-2}\ \text{V}.$$

(b) For 2.0 s < t < 4.0 s: $\varepsilon \propto dB/dt = 0$.

(c) For 4.0 s < t < 6.0 s:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi\left(0.12\text{m}\right)^2\left(\frac{-0.5\text{T}}{6.0\text{s} - 4.0\text{s}}\right) = 1.1\times 10^{-2}\ \text{V}.$$

5. The field (due to the current in the straight wire) is out of the page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

6. From the datum at $t = 0$ in Fig. 30-37(b) we see 0.0015 A = V_{battery}/R, which implies that the resistance is

$$R = (6.00\ \mu\text{V})/(0.0015\ \text{A}) = 0.0040\ \Omega.$$

Now, the value of the current during 10 s < t < 20 s leads us to equate

$$(V_{\text{battery}} + \varepsilon_{\text{induced}})/R = 0.00050\ \text{A}.$$

This shows that the induced emf is $\varepsilon_{\text{induced}} = -4.0\ \mu\text{V}$. Now we use Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -A\ a\ .$$

Plugging in $\varepsilon = -4.0\times 10^{-6}$ V and $A = 5.0\times 10^{-4}\ \text{m}^2$, we obtain $a = 0.0080$ T/s.

7. (a) The magnitude of the emf is

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}\left(6.0t^2 + 7.0t\right) = 12t + 7.0 = 12\left(2.0\right) + 7.0 = 31\,\text{mV}.$$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to the left through R.

8. The resistance of the loop is

$$R = \rho\frac{L}{A} = \left(1.69\times 10^{-8}\Omega\cdot\text{m}\right)\frac{\pi\left(0.10\,\text{m}\right)}{\pi\left(2.5\times 10^{-3}\ \text{m}\right)^2/4} = 1.1\times 10^{-3}\Omega.$$

We use $i = |\varepsilon|/R = |d\Phi_B/dt|/R = (\pi r^2/R)|dB/dt|$. Thus

$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{(10\,\text{A})(1.1\times10^{-3}\,\Omega)}{\pi(0.05\,\text{m})^2} = 1.4\ \text{T/s}.$$

9. The amplitude of the induced emf in the loop is

$$\varepsilon_m = A\mu_0 n i_0 \omega = (6.8\times10^{-6}\,\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(85400/\text{m})(1.28\ \text{A})(212\ \text{rad/s})$$
$$= 1.98\times10^{-4}\,\text{V}.$$

10. (a) The magnetic flux Φ_B through the loop is given by

$$\Phi_B = 2B\left(\pi r^2/2\right)\left(\cos 45°\right) = \pi r^2 B/\sqrt{2}\,.$$

Thus,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) = -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right) = -\frac{\pi\left(3.7\times10^{-2}\,\text{m}\right)^2}{\sqrt{2}}\left(\frac{0-76\times10^{-3}\,\text{T}}{4.5\times10^{-3}\,\text{s}}\right)$$
$$= 5.1\times10^{-2}\,\text{V}.$$

(a) The direction of the induced current is clockwise when viewed along the direction of $\vec{B}$.

11. (a) It should be emphasized that the result, given in terms of sin($2\pi ft$), could as easily be given in terms of cos($2\pi ft$) or even cos($2\pi ft + \phi$) where ϕ is a phase constant as discussed in Chapter 15. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon = -N\frac{d\left(BA\cos\theta\right)}{dt} = -NBA\frac{d\cos\left(2\pi ft\right)}{dt} = N\,Bab2\pi f\sin\left(2\pi ft\right)$$

which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0 \sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0 = 2\pi f NabB$.

(b) We solve

$$\varepsilon_0 = 150\ \text{V} = 2\pi f NabB$$

when $f = 60.0$ rev/s and $B = 0.500$ T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $Nab = 0.796\ \text{m}^2$.

12. To have an induced emf, the magnetic field must be perpendicular (or have a nonzero component perpendicular) to the coil, and must be changing with time.

(a) For $\vec{B} = (4.00\times10^{-2}\,\text{T/m})y\hat{\text{k}}$, $dB/dt = 0$ and hence $\varepsilon = 0$.

(b) None.

(c) For $\vec{B} = (6.00\times10^{-2}\,\text{T/s})t\hat{\text{k}}$,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -(0.400\text{ m}\times0.250\text{ m})(0.0600\text{ T/s}) = -6.00\text{ mV},$$

or $|\varepsilon| = 6.00$ mV.

(d) Clockwise.

(e) For $\vec{B} = (8.00\times10^{-2}\,\text{T/m}\cdot\text{s})yt\,\hat{\text{k}}$, $\Phi_B = (0.400)(0.0800t)\int y\,dy = 1.00\times10^{-3}t$,

in SI units. The induced emf is $\varepsilon = -d\Phi B/dt = -1.00$ mV, or $|\varepsilon| = 1.00$ mV.

(f) Clockwise.

(g) $\Phi_B = 0 \;\Rightarrow\; \varepsilon = 0$.

(h) None.

(i) $\Phi_B = 0 \;\Rightarrow\; \varepsilon = 0$.

(j) None.

13. The amount of charge is

$$q(t) = \frac{1}{R}[\Phi_B(0) - \Phi_B(t)] = \frac{A}{R}[B(0) - B(t)] = \frac{1.20\times10^{-3}\,\text{m}^2}{13.0\,\Omega}[1.60\,\text{T} - (-1.60\,\text{T})]$$
$$= 2.95\times10^{-2}\,\text{C}.$$

14. Figure 30-42(b) demonstrates that dB/dt (the slope of that line) is 0.003 T/s. Thus, in absolute value, Faraday's law becomes

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt}$$

where $A = 8 \times 10^{-4}$ m^2. We related the induced emf to resistance and current using Ohm's law. The current is estimated from Fig. 30-42(c) to be $i = dq/dt = 0.002$ A (the slope of that line). Therefore, the resistance of the loop is

$$R = \frac{|\varepsilon|}{i} = \frac{A\,|dB/dt|}{i} = \frac{(8.0\times 10^{-4}\ \text{m}^2)(0.0030\ \text{T/s})}{0.0020\ \text{A}} = 0.0012\,\Omega.$$

15. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2 B/2$, and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870$ T/s. Thus,

$$\varepsilon_i = \frac{(2.00\ \text{m})^2}{2}(0.870\ \text{T/s}) = 1.74\ \text{V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0\ \text{V} + 1.74\ \text{V} = 21.7\ \text{V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

16. (a) Since the flux arises from a dot product of vectors, the result of one sign for B_1 and B_2 and of the opposite sign for B_3 (we choose the minus sign for the flux from B_1 and B_2, and therefore a plus sign for the flux from B_3). The induced emf is

$$\begin{aligned}\varepsilon &= -\sum \frac{d\Phi_B}{dt} = A\left(\frac{dB_1}{dt} + \frac{dB_2}{dt} - \frac{dB_3}{dt}\right)\\ &= (0.10\ \text{m})(0.20\ \text{m})\left(2.0\times 10^{-6}\ \text{T/s} + 1.0\times 10^{-6}\ \text{T/s} - 5.0\times 10^{-6}\ \text{T/s}\right)\\ &= -4.0\times 10^{-8}\ \text{V}.\end{aligned}$$

The minus sign means that the effect is dominated by the changes in B_3. Its magnitude (using Ohm's law) is $|\varepsilon|/R = 8.0\ \mu$A.

(b) Consideration of Lenz's law leads to the conclusion that the induced current is therefore counterclockwise.

17. Equation 29-10 gives the field at the center of the large loop with $R = 1.00$ m and current $i(t)$. This is approximately the field throughout the area ($A = 2.00 \times 10^{-4}$ m^2) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$, where $i_0 = 200$ A and

$$k = (-200\text{ A} - 200\text{ A})/1.00\text{ s} = -400\text{ A/s},$$

we find

(a) $B(t=0) = \dfrac{\mu_0 i_0}{2R} = \dfrac{(4\pi\times10^{-7}\text{ H/m})(200\text{A})}{2(1.00\text{m})} = 1.26\times10^{-4}\text{ T},$

(b) $B(t=0.500\text{s}) = \dfrac{(4\pi\times10^{-7}\text{ H/m})[200\text{A} - (400\text{A/s})(0.500\text{s})]}{2(1.00\text{m})} = 0,$ and

(c) $B(t=1.00\text{s}) = \dfrac{(4\pi\times10^{-7}\text{ H/m})[200\text{A} - (400\text{A/s})(1.00\text{s})]}{2(1.00\text{m})} = -1.26\times10^{-4}\text{ T},$

or $|B(t=1.00\text{s})| = 1.26\times10^{-4}\text{ T}.$

(d) Yes, as indicated by the flip of sign of $B(t)$ in (c).

(e) Let the area of the small loop be a. Then $\Phi_B = Ba$, and Faraday's law yields

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a\frac{dB}{dt} = -a\left(\frac{\Delta B}{\Delta t}\right)\\ &= -(2.00\times10^{-4}\text{m}^2)\left(\frac{-1.26\times10^{-4}\text{ T} - 1.26\times10^{-4}\text{ T}}{1.00\text{ s}}\right)\\ &= 5.04\times10^{-8}\text{ V}.\end{aligned}$$

18. (a) The "height" of the triangular area enclosed by the rails and bar is the same as the distance traveled in time v: $d = vt$, where $v = 5.20$ m/s. We also note that the "base" of that triangle (the distance between the intersection points of the bar with the rails) is $2d$. Thus, the area of the triangle is

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2vt)(vt) = v^2t^2 \ .$$

Since the field is a uniform $B = 0.350$ T, then the magnitude of the flux (in SI units) is

$$\Phi_B = BA = (0.350)(5.20)^2t^2 = 9.46t^2.$$

At $t = 3.00$ s, we obtain $\Phi_B = 85.2$ Wb.

(b) The magnitude of the emf is the (absolute value of) Faraday's law:

$$\varepsilon = \frac{d\Phi_B}{dt} = 9.46\frac{dt^2}{dt} = 18.9t$$

in SI units. At t = 3.00 s, this yields ε = 56.8 V.

(c) Our calculation in part (b) shows that n = 1.

19. First we write $\Phi_B = BA\cos\theta$. We note that the angular position θ of the rotating coil is measured from some reference line or plane, and we are implicitly making such a choice by writing the magnetic flux as $BA\cos\theta$ (as opposed to, say, $BA\sin\theta$). Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ if θ is understood to be in radians (here, $\omega = 2\pi f$ is the angular velocity of the coil in radians per second, and f = 1000 rev/min ≈ 16.7 rev/s is the frequency). Since the area of the rectangular coil is A = (0.500 m) × (0.300 m) = 0.150 m^2, Faraday's law leads to

$$\varepsilon = -N\frac{d\left(BA\cos\theta\right)}{dt} = -NBA\frac{d\cos\left(2\pi ft\right)}{dt} = NBA2\pi f\sin\left(2\pi ft\right)$$

which means it has a voltage amplitude of

$$\varepsilon_{\max} = 2\pi fNAB = 2\pi\left(16.7\,\text{rev/s}\right)\left(100\,\text{turns}\right)\left(0.15\,\text{m}^2\right)\left(3.5\,\text{T}\right) = 5.50\times10^3\ \text{V}.$$

20. We note that 1 gauss = 10^{-4} T. The amount of charge is

$$q(t) = \frac{N}{R}[BA\cos 20° - (-BA\cos 20°)] = \frac{2NBA\cos 20°}{R}$$
$$= \frac{2(1000)(0.590\times10^{-4}\ \text{T})\pi(0.100\ \text{m})^2(\cos 20°)}{85.0\ \Omega + 140\ \Omega} = 1.55\times10^{-5}\,\text{C}.$$

Note that the axis of the coil is at 20°, not 70°, from the magnetic field of the Earth.

21. (a) The frequency is

$$f = \frac{\omega}{2\pi} = \frac{(40\ \text{rev/s})(2\pi\ \text{rad/rev})}{2\pi} = 40\ \text{Hz}.$$

(b) First, we define angle relative to the plane of Fig. 30-46, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0, which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position).

Since the area of the semicircle is $\pi a^2/2$, then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ, is

$$A = A_0 + \frac{\pi a^2}{2}\cos\theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi ft$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(A_0 + (\pi a^2/2)\cos\theta\right)}{dt} = -B\frac{\pi a^2}{2}\frac{d\cos\left(2\pi ft\right)}{dt}$$

which yields $\varepsilon = B\pi^2 a^2 f\sin(2\pi ft)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude:

$$\varepsilon_m = B\pi^2 a^2 f = (0.020\text{ T})\pi^2(0.020\text{ m})^2(40/\text{s}) = 3.2\times10^{-3}\text{ V}.$$

22. Since $\dfrac{d\cos\phi}{dt} = -\sin\phi\dfrac{d\phi}{dt}$, Faraday's law (with $N = 1$) becomes

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d(BA\cos\phi)}{dt} = BA\sin\phi\frac{d\phi}{dt}.$$

Substituting the values given yields $|\varepsilon| = 0.018$ V.

23. **THINK** Increasing the separation between the two loops changes the flux through the smaller loop and, therefore, induces a current in the smaller loop.

EXPRESS The magnetic flux through a surface is given by $\Phi_B = \int \vec{B}\cdot d\vec{A}$, where $\vec{B}$ is the magnetic field and $d\vec{A}$ is a vector of magnitude dA that is normal to a differential area dA. In the case where $\vec{B}$ is uniform and perpendicular to the plane of the loop, $\Phi_B = BA$.

In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Equation 29-27, with $z = x$ (taken to be much greater than R), gives $\vec{B} = \dfrac{\mu_0 i R^2}{2x^3}\hat{\text{i}}$, where the $+x$ direction is upward in Fig. 30-47. The area of the smaller loop is $A = \pi r^2$.

ANALYZE (a) The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area of the smaller loop:

$$\Phi_B = BA = \frac{\pi\mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) As the smaller loop moves upward, the flux through it decreases. The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

LEARN The situation in this problem is like that shown in Fig. 30-5(d). The induced magnetic field is in the same direction as the initial magnetic field.

24. (a) Since $\vec{B} = B\,\hat{\text{i}}$ uniformly, then only the area "projected" onto the *yz* plane will contribute to the flux (due to the scalar [dot] product). This "projected" area corresponds to one-fourth of a circle. Thus, the magnetic flux Φ_B through the loop is

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \frac{1}{4}\pi r^2 B .$$

Thus,

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}\left(\frac{1}{4}\pi r^2 B\right)\right| = \frac{\pi r^2}{4}\left|\frac{dB}{dt}\right| = \frac{1}{4}\pi(0.10\,\text{m})^2(3.0\times10^{-3}\,\text{T/s}) = 2.4\times10^{-5}\,\text{V} .$$

(b) We have a situation analogous to that shown in Fig. 30-5(a). Thus, the current in segment *bc* flows from *c* to *b* (following Lenz's law).

25. (a) We refer to the (very large) wire length as *L* and seek to compute the flux per meter: Φ_B/L. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of anti-parallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at what we will call $x = \ell/2$, where $\ell = 20\,\text{mm} = 0.020\,\text{m}$); the net field at any point $0 < x < \ell/2$ is the same at its "mirror image" point $\ell - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = \ell$. We make use of the symmetry by integrating over $0 < x < \ell/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{\ell/2} B\,dA = 2\int_0^{d/2} B\left(L\,dx\right) + 2\int_{d/2}^{\ell/2} B\left(L\,dx\right)$$

where d = 0.0025 m is the diameter of each wire. We will use $R = d/2$, and r instead of x in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{L} &= 2\int_0^R\left(\frac{\mu_0 i}{2\pi R^2}r+\frac{\mu_0 i}{2\pi(\ell-r)}\right)dr+2\int_R^{\ell/2}\left(\frac{\mu_0 i}{2\pi r}+\frac{\mu_0 i}{2\pi(\ell-r)}\right)dr\\ &=\frac{\mu_0 i}{2\pi}\left(1-2\ln\left(\frac{\ell-R}{\ell}\right)\right)+\frac{\mu_0 i}{\pi}\ln\left(\frac{\ell-R}{R}\right)\\ &=0.23\times10^{-5}\,\mathrm{T\cdot m}+1.08\times10^{-5}\,\mathrm{T\cdot m}\end{aligned}$$

which yields $\Phi_B/L = 1.3\times10^{-5}$ T·m or 1.3×10^{-5} Wb/m.

(b) The flux (per meter) existing within the regions of space occupied by one or the other wire was computed above to be 0.23×10^{-5} T·m. Thus,

$$\frac{0.23\times10^{-5}\,\mathrm{T\cdot m}}{1.3\times10^{-5}\,\mathrm{T\cdot m}}=0.17=17\%\ .$$

(c) What was described in part (a) as a symmetry plane at $x=\ell/2$ is now (in the case of parallel currents) a plane of vanishing field (the fields subtract from each other in the region between them, as the right-hand rule shows). The flux in the $0<x<\ell/2$ region is now of opposite sign of the flux in the $\ell/2<x<\ell$ region, which causes the total flux (or, in this case, flux per meter) to be zero.

26. (a) First, we observe that a large portion of the figure contributes flux that "cancels out." The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is $b-a$, then a strip below the wire (where the strip borders the long wire, and extends a distance $b-a$ away from it) has exactly the equal but opposite flux that cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B=\int B\,dA=\int_{b-a}^{a}\left(\frac{\mu_0 i}{2\pi r}\right)(b\,dr)=\frac{\mu_0 ib}{2\pi}\ln\left(\frac{a}{b-a}\right).$$

Faraday's law, then, (with SI units and 3 significant figures understood) leads to

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt}=-\frac{d}{dt}\left[\frac{\mu_0 ib}{2\pi}\ln\left(\frac{a}{b-a}\right)\right]=-\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{di}{dt}\\ &=-\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{d}{dt}\left(\frac{9}{2}t^2-10t\right)\\ &=\frac{-\mu_0 b(9t-10)}{2\pi}\ln\left(\frac{a}{b-a}\right).\end{aligned}$$

With a = 0.120 m and b = 0.160 m, then, at t = 3.00 s, the magnitude of the emf induced in the rectangular loop is

$$|\varepsilon| = \frac{\left(4\pi\times10^{-7}\right)\left(0.16\right)\left(9\left(3\right)-10\right)}{2\pi}\ln\left(\frac{0.12}{0.16-0.12}\right) = 5.98\times10^{-7}\text{ V}.$$

(b) We note that $di/dt > 0$ at t = 3 s. The situation is roughly analogous to that shown in Fig. 30-5(c). From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

27. (a) Consider a (thin) strip of area of height dy and width $\ell = 0.020\,\text{m}$. The strip is located at some $0 < y < \ell$. The element of flux through the strip is

$$d\Phi_B = B\,dA = \left(4t^2 y\right)\left(\ell\,dy\right)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^{\ell}\left(4t^2 y\ell\right)dy = 2t^2\ell^3 .$$

Thus, Faraday's law yields

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = 4t\ell^3 .$$

At t = 2.5 s, the magnitude of the induced emf is 8.0×10^{-5} V.

(b) Its "direction" (or "sense'') is clockwise, by Lenz's law.

28. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 29-17). We integrate according to Eq. 30-1, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2}\left(\frac{\mu_0 i}{2\pi r}\right)(a\,dr) = \frac{\mu_0 i a}{2\pi}\ \ln\left(\frac{r+b/2}{r-b/2}\right).$$

When $r = 1.5b$, we have

$$|\Phi_B| = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.7\text{A})(0.022\text{m})}{2\pi}\ln(2.0) = 1.4\times10^{-8}\,\text{Wb}.$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $dr/dt = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$i_{\text{loop}} = \left|\frac{\varepsilon}{R}\right| = -\frac{\mu_0 ia}{2\pi R}\left|\frac{d}{dt}\ln\left(\frac{r+b/2}{r-b/2}\right)\right| = \frac{\mu_0 iabv}{2\pi R[r^2-(b/2)^2]}$$

$$= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.7\text{A})(0.022\text{m})(0.0080\text{m})(3.2\times10^{-3}\,\text{m/s})}{2\pi(4.0\times10^{-4}\,\Omega)[2(0.0080\text{m})^2]}$$

$$= 1.0\times10^{-5}\,\text{A}.$$

29. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (0.350\ \text{T})(0.250\ \text{m})(0.55\ \text{m/s}) = 0.0481\ \text{V}.$$

(b) By Ohm's law, the induced current is

$$i = 0.0481\ \text{V}/18.0\ \Omega = 0.00267\ \text{A}.$$

By Lenz's law, the current is clockwise in Fig. 30-52.

(c) Equation 26-27 leads to $P = i^2R = 0.000129$ W.

30. Equation 26-28 gives ε^2/R as the rate of energy transfer into thermal forms (dE_{th}/dt, which, from Fig. 30-53(c), is roughly 40 nJ/s). Interpreting ε as the induced emf (in absolute value) in the single-turn loop ($N = 1$) from Faraday's law, we have

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}.$$

Equation 29-23 gives $B = \mu_0 ni$ for the solenoid (and note that the field is zero outside of the solenoid, which implies that $A = A_{\text{coil}}$), so our expression for the magnitude of the induced emf becomes

$$\varepsilon = A\frac{dB}{dt} = A_{\text{coil}}\frac{d}{dt}\left(\mu_0 n i_{\text{coil}}\right) = \mu_0 n A_{\text{coil}}\frac{di_{\text{coil}}}{dt}.$$

where Fig. 30-53(b) suggests that $di_{\text{coil}}/dt = 0.5$ A/s. With $n = 8000$ (in SI units) and $A_{\text{coil}} = \pi(0.02)^2$ (note that the loop radius does not come into the computations of this problem, just the coil's), we find V = 6.3 μV. Returning to our earlier observations, we can now solve for the resistance:

$$R = \varepsilon^2/(dE_{\text{th}}/dt) = 1.0\ \text{m}\Omega.$$

31. **THINK** Thermal energy is generated at the rate given by $P = \varepsilon^2/R$ (see Eq. 27-23), where ε is the emf in the wire and R is the resistance of the wire.

EXPRESS Using Eq. 27-16, the resistance is given by $R = \rho L/A$, where the resistivity is $1.69 \times 10^{-8}\ \Omega\cdot$m (by Table 27-1) and $A = \pi d^2/4$ is the cross-sectional area of the wire (d = 0.00100 m is the wire thickness). The area *enclosed* by the loop is

$$A_{\text{loop}} = \pi r_{\text{loop}}^2 = \pi\left(\frac{L}{2\pi}\right)^2$$

since the length of the wire (L = 0.500 m) is the circumference of the loop. This enclosed area is used in Faraday's law to give the induced emf:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A_{\text{loop}}\frac{dB}{dt} = -\frac{L^2}{4\pi}\frac{dB}{dt}.$$

ANALYZE The rate of change of the field is dB/dt = 0.0100 T/s. Thus, we obtain

$$P = \frac{|\varepsilon|^2}{R} = \frac{(L^2/4\pi)^2(dB/dt)^2}{\rho L/(\pi d^2/4)} = \frac{d^2L^3}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = \frac{(1.00\times10^{-3}\text{ m})^2(0.500\text{ m})^3}{64\pi(1.69\times10^{-8}\,\Omega\cdot\text{m})}\left(0.0100\text{ T/s}\right)^2$$
$$= 3.68\times10^{-6}\,\text{W}.$$

LEARN The rate of thermal energy generated is proportional to $(dB/dt)^2$.

32. Noting that $|\Delta B| = B$, we find the thermal energy is

$$P_{\text{thermal}}\Delta t = \frac{\varepsilon^2\Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2\Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2\Delta t = \frac{A^2B^2}{R\Delta t}$$
$$= \frac{(2.00\times10^{-4}\,\text{m}^2)^2(17.0\times10^{-6}\,\text{T})^2}{(5.21\times10^{-6}\,\Omega)(2.96\times10^{-3}\,\text{s})} = 7.50\times10^{-10}\,\text{J}.$$

33. (a) Letting x be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 29-17, the field is $B = \mu_0 i/2\pi r$, where r is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length x and width dr, parallel to the wire and a distance r from it; it has area $A = x\,dr$ and the flux is

$$d\Phi_B = B\,dA = \frac{\mu_0 i}{2\pi r}x\,dr.$$

By Eq. 30-1, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 i x}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i x}{2\pi}\ln\left(\frac{a+L}{a}\right).$$

According to Faraday's law the emf induced in the loop is

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 i v}{2\pi}\ln\left(\frac{a+L}{a}\right)$$
$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(100\,\text{A}\right)\left(5.00\,\text{m/s}\right)}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right) = 2.40\times10^{-4}\,\text{V}.$$

(b) By Ohm's law, the induced current is

$$i_\ell = \varepsilon / R = \left(2.40\times10^{-4}\,\text{V}\right)/\left(0.400\,\Omega\right) = 6.00\times10^{-4}\,\text{A}.$$

Since the flux is increasing, the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate

$$P = i_\ell^2 R = \left(6.00\times10^{-4}\,\text{A}\right)^2\left(0.400\,\Omega\right) = 1.44\times10^{-7}\,\text{W}.$$

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length *dr* at a distance *r* from the long straight wire, is

$$dF_B = i_\ell B\,dr = \left(\mu_0 i_\ell i / 2\pi r\right)dr.$$

We integrate to find the magnitude of the total magnetic force on the rod:

$$F_B = \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right)$$
$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(6.00\times10^{-4}\,\text{A}\right)\left(100\,\text{A}\right)}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right)$$
$$= 2.87\times10^{-8}\,\text{N}.$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of 2.87×10^{-8} N, to the left.

(e) By Eq. 7-48, the external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8}\text{ N})(5.00\text{ m/s}) = 1.44 \times 10^{-7}\text{ W}.$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

34. Noting that $F_{\text{net}} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_t L}{R},$$

which yields $v_t = mgR/B^2L^2$.

35. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (1.2\text{ T})(0.10\text{ m})(5.0\text{ m/s}) = 0.60\text{ V}.$$

(b) By Lenz's law, the induced emf is clockwise. In the rod itself, we would say the emf is directed up the page.

(c) By Ohm's law, the induced current is $i = 0.60\text{ V}/0.40\ \Omega = 1.5\text{ A}$.

(d) The direction is clockwise.

(e) Equation 26-28 leads to $P = i^2R = 0.90$ W.

(f) From Eq. 29-2, we find that the force on the rod associated with the uniform magnetic field is directed rightward and has magnitude

$$F = iLB = (1.5\text{ A})(0.10\text{ m})(1.2\text{ T}) = 0.18\text{ N}.$$

To keep the rod moving at constant velocity, therefore, a leftward force (due to some external agent) having that same magnitude must be continuously supplied to the rod.

(g) Using Eq. 7-48, we find the power associated with the force being exerted by the external agent:

$$P = Fv = (0.18\text{ N})(5.0\text{ m/s}) = 0.90\text{ W},$$

which is the same as our result from part (e).

36. (a) For path 1, we have

$$\oint_1 \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{B1}}{dt} = \frac{d}{dt}(B_1A_1) = A_1\frac{dB_1}{dt} = \pi r_1^2\frac{dB_1}{dt} = \pi(0.200\text{m})^2(-8.50\times10^{-3}\text{ T/s})$$
$$= -1.07\times10^{-3}\text{ V}.$$

(b) For path 2, the result is

$$\oint_2 \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{B2}}{dt} = \pi r_2^2 \frac{dB_2}{dt} = \pi\left(0.300\,\text{m}\right)^2\left(-8.50\times10^{-3}\,\text{T/s}\right) = -2.40\times10^{-3}\,\text{V}.$$

(c) For path 3, we have

$$\oint_3 \vec{E}\cdot d\vec{s} = \oint_1 \vec{E}\cdot d\vec{s} - \oint_2 \vec{E}\cdot d\vec{s} = -1.07\times10^{-3}\,\text{V} - \left(-2.4\times10^{-3}\,\text{V}\right) = 1.33\times10^{-3}\,\text{V}.$$

37. **THINK** Changing magnetic field induces an electric field.

EXPRESS The induced electric field is given by Eq. 30-20: $\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$.

ANALYZE (a) The point at which we are evaluating the field is inside the solenoid, so

$$E(2\pi r) = -(\pi r^2)\frac{dB}{dt} \quad\Rightarrow\quad E = -\frac{1}{2}\frac{dB}{dt}r.$$

The magnitude of the induced electric field is

$$|E| = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}\left(6.5\times10^{-3}\,\text{T/s}\right)\left(0.0220\,\text{m}\right) = 7.15\times10^{-5}\,\text{V/m}.$$

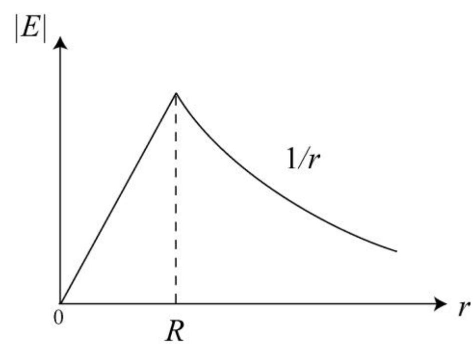
(b) Now the point at which we are evaluating the field is outside the solenoid, so

$$E(2\pi r) = -(\pi R^2)\frac{dB}{dt} \quad\Rightarrow\quad E = -\frac{1}{2}\frac{dB}{dt}\frac{R^2}{r}.$$

The magnitude of the induced field is

$$|E| = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}\left(6.5\times10^{-3}\,\text{T/s}\right)\frac{\left(0.0600\,\text{m}\right)^2}{0.0820\,\text{m}} = 1.43\times10^{-4}\,\text{V/m}.$$

LEARN The magnitude of the induced electric field as a function of r is shown to the right. Inside the solenoid, $r < R$, the field $|E|$ is linear in r. However, outside the solenoid, $r > R$, $|E| \sim 1/r$.



38. From the "kink" in the graph of Fig. 30-57, we conclude that the radius of the circular region is 2.0 cm. For values of r less than that, we have (from the absolute value of Eq. 30-20)

$$E(2\pi r) = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \pi r^2 a$$

which means that $E/r = a/2$. This corresponds to the slope of that graph (the linear portion for small values of r) which we estimate to be 0.015 (in SI units). Thus, $a = 0.030$ T/s.

39. The magnetic field B can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0),$$

where B_0 = (30.0 T + 29.6 T)/2 = 29.8 T and B_1 = (30.0 T – 29.6 T)/2 = 0.200 T. Then from Eq. 30-25

$$E = \frac{1}{2}\left(\frac{dB}{dt}\right)r = \frac{r}{2}\frac{d}{dt}\left[B_0 + B_1\sin(\omega t + \phi_0)\right] = \frac{1}{2}B_1\omega r\cos(\omega t + \phi_0).$$

We note that $\omega = 2\pi f$ and that the factor in front of the cosine is the maximum value of the field. Consequently,

$$E_{\max} = \frac{1}{2}B_1(2\pi f)r = \frac{1}{2}(0.200\,\text{T})(2\pi)(15\,\text{Hz})(1.6\times10^{-2}\text{ m}) = 0.15\text{ V/m}.$$

40. Since $N\Phi_B = Li$, we obtain

$$\Phi_B = \frac{Li}{N} = \frac{(8.0\times10^{-3}\text{ H})(5.0\times10^{-3}\text{ A})}{400} = 1.0\times10^{-7}\,\text{Wb}.$$

41. (a) We interpret the question as asking for N multiplied by the flux through one turn:

$$\Phi_{\text{turns}} = N\Phi_B = NBA = NB(\pi r^2) = (30.0)(2.60\times10^{-3}\text{ T})(\pi)(0.100\text{ m})^2 = 2.45\times10^{-3}\text{ Wb}.$$

(b) Equation 30-33 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45\times10^{-3}\text{ Wb}}{3.80\,\text{A}} = 6.45\times10^{-4}\text{ H}.$$

42. (a) We imagine dividing the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is

$$B = \mu_0 n\Delta i = \mu_0\left(\frac{N}{W}\right)\left(\frac{i}{N}\right) = \frac{\mu_0 i}{W} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.035\text{A})}{0.16\text{m}} = 2.7\times10^{-7}\,\text{T}.$$

(b) Equation 30-33 leads to

$$L=\frac{\Phi_B}{i}=\frac{\pi R^2 B}{i}=\frac{\pi R^2\left(\mu_0 i/W\right)}{i}=\frac{\pi\mu_0 R^2}{W}=\frac{\pi(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.018\mathrm{m})^2}{0.16\mathrm{m}}=8.0\times10^{-9}\,\mathrm{H}.$$

43. We refer to the (very large) wire length as ℓ and seek to compute the flux per meter: Φ_B/ℓ. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at $x = d/2$); the net field at any point $0 < x < d/2$ is the same at its "mirror image" point $d - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = d$. We make use of the symmetry by integrating over $0 < x < d/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{d/2} B\,dA = 2\int_0^a B\left(\ell\,dx\right)+2\int_a^{d/2} B\left(\ell\,dx\right)$$

where d = 0.0025 m is the diameter of each wire. We will use r instead of x in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{\ell} &= 2\int_0^a \left(\frac{\mu_0 i}{2\pi a^2}r+\frac{\mu_0 i}{2\pi\left(d-r\right)}\right)dr+2\int_a^{d/2}\left(\frac{\mu_0 i}{2\pi r}+\frac{\mu_0 i}{2\pi\left(d-r\right)}\right)dr\\ &=\frac{\mu_0 i}{2\pi}\left(1-2\ln\left(\frac{d-a}{d}\right)\right)+\frac{\mu_0 i}{\pi}\ln\left(\frac{d-a}{a}\right)\end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately $\Phi_B \approx \mu_0 i\ell/\pi\,\ln\left(\left(d-a\right)/a\right)$. Now, we use Eq. 30-33 (with $N = 1$) to obtain the inductance per unit length:

$$\frac{L}{\ell}=\frac{\Phi_B}{\ell i}=\frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)=\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})}{\pi}\ln\left(\frac{142-1.53}{1.53}\right)=1.81\times10^{-6}\,\mathrm{H/m}.$$

44. Since $\varepsilon = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt}=-\frac{\varepsilon}{L}=-\frac{60\mathrm{V}}{12\mathrm{H}}=-5.0\,\mathrm{A/s},$$

or $|di/dt| = 5.0\,\mathrm{A/s}$. We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

45. (a) Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then i must be in the process of decreasing.

(b) From Eq. 30-35 (in absolute value) we get

$$L = \left|\frac{\varepsilon}{di/dt}\right| = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8\times 10^{-4}\ \text{H}.$$

46. During periods of time when the current is varying linearly with time, Eq. 30-35 (in absolute values) becomes $|\varepsilon| = L\,|\Delta i/\Delta t|$. For simplicity, we omit the absolute value signs in the following.

(a) For $0 < t < 2$ ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\ \text{H})(7.0\ \text{A} - 0)}{2.0\times 10^{-3}\ \text{s}} = 1.6\times 10^{4}\ \text{V}.$$

(b) For 2 ms $< t <$ 5 ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\ \text{H})(5.0\ \text{A} - 7.0\ \text{A})}{(5.0 - 2.0)(10^{-3}\ \text{s})} = 3.1\times 10^{3}\ \text{V}.$$

(c) For 5 ms $< t <$ 6 ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\ \text{H})(0 - 5.0\ \text{A})}{(6.0 - 5.0)(10^{-3}\ \text{s})} = 2.3\times 10^{4}\ \text{V}.$$

47. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add (V_1 + V_2), then inductances in series must add, $L_{\text{eq}} = L_1 + L_2$, just as was the case for resistances. Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that magnetic field lines from one inductor should not have significant presence in any other.

(b) Just as with resistors, $L_{\text{eq}} = \sum_{n=1}^{N} L_n$.

48. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the Eq. 27-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt}.$$

Thus, although the inductance equation Eq. 30-35 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also apply to inductors. Therefore,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that the field of one inductor not to have significant influence (or "coupling'') in the next.

(b) Just as with resistors, $\frac{1}{L_{eq}} = \sum_{n=1}^{N} \frac{1}{L_n}$.

49. Using the results from Problems 30-47 and 30-48, the equivalent resistance is

$$L_{eq} = L_1 + L_4 + L_{23} = L_1 + L_4 + \frac{L_2 L_3}{L_2 + L_3} = 30.0 \text{ mH} + 15.0 \text{ mH} + \frac{(50.0 \text{ mH})(20.0 \text{ mH})}{50.0 \text{ mH} + 20.0 \text{ mH}}$$
$$= 59.3 \text{ mH}.$$

50. The steady state value of the current is also its maximum value, ε/R, which we denote as i_m. We are told that $i = i_m/3$ at $t_0 = 5.00$ s. Equation 30-41 becomes $i = i_m\left(1 - e^{-t_0/\tau_L}\right)$, which leads to

$$\tau_L = -\frac{t_0}{\ln(1 - i/i_m)} = -\frac{5.00 \text{ s}}{\ln(1 - 1/3)} = 12.3 \text{ s}.$$

51. The current in the circuit is given by $i = i_0 e^{-t/\tau_L}$, where i_0 is the current at time $t = 0$ and τ_L is the inductive time constant (L/R). We solve for τ_L. Dividing by i_0 and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L}.$$

This yields

$$\tau_L = -\frac{t}{\ln(i/i_0)} = -\frac{1.0 \text{ s}}{\ln\left[\left(10 \times 10^{-3} \text{ A}\right)/(1.0 \text{ A})\right]} = 0.217 \text{ s}.$$

Therefore, $R = L/\tau_L = 10 \text{ H}/0.217 \text{ s} = 46\ \Omega$.

52. (a) Immediately after the switch is closed, $\varepsilon - \varepsilon_L = iR$. But $i = 0$ at this instant, so $\varepsilon_L = \varepsilon$, or $\varepsilon_L/\varepsilon = 1.00$.

(b) $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L} = \varepsilon e^{-2.0\tau_L/\tau_L} = \varepsilon e^{-2.0} = 0.135\varepsilon$, or $\varepsilon_L/\varepsilon = 0.135$.

(c) From $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L}$ we obtain

$$\frac{t}{\tau_L} = \ln\left(\frac{\varepsilon}{\varepsilon_L}\right) = \ln 2 \quad \Rightarrow \quad t = \tau_L \ln 2 = 0.693\tau_L \quad \Rightarrow \quad t/\tau_L = 0.693.$$

53. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it.

EXPRESS If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$.

(a) We want to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \;\Rightarrow\; e^{-t/\tau_L} = 0.200.$$

Taking the natural logarithm of both sides, we obtain

$$-(t/\tau_L) = \ln(0.200) = -1.609.$$

Thus,

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\text{H})}{1.20\times10^3\,\Omega} = 8.45\times10^{-9}\,\text{s}.$$

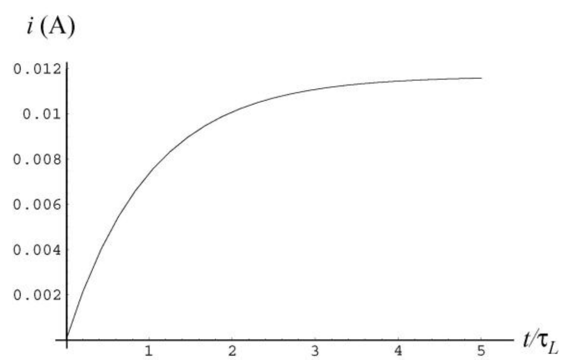
(b) At $t = 1.0\,\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\text{V}}{1.20\times10^3\,\Omega}\right)(1 - e^{-1.0}) = 7.37\times10^{-3}\ \text{A}.$$

LEARN At $t = 0$, the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is

$$i_0 = \frac{\varepsilon}{R} = \frac{14.0\,\text{V}}{1.20\times10^3\,\Omega} = 0.0117\ \text{A}.$$

The current as a function of t/τ_L is plotted to the right.



54. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{R_1 + R_2} = \frac{100\,\text{V}}{10.0\,\Omega + 20.0\,\Omega} = 3.33\,\text{A}.$$

(b) $i_2 = i_1 = 3.33\,\text{A}.$

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0$$

$$\varepsilon - i_1 R_1 - (i_1 - i_2) R_3 = 0.$$

We solve these simultaneously for i_1 and i_2, and find

$$i_1 = \frac{\varepsilon (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(100\,\text{V})(20.0\,\Omega + 30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 4.55\,\text{A},$$

(d) and

$$i_2 = \frac{\varepsilon R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(100\,\text{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 2.73\,\text{A}.$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$).

(f) The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is 4.55 A – 2.73 A = 1.82 A. The current in R_2 is the same but in the opposite direction as that in R_3, that is, $i_2 = -1.82$ A.

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g) $i_1 = 0$, and

(h) $i_2 = 0$.

55. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it.

EXPRESS Starting with zero current at t = 0 (the moment the switch is closed) the current in the circuit increases according to

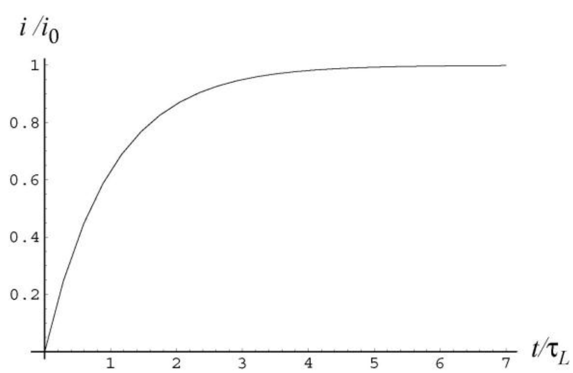
$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$ is the inductive time constant and ε is the battery emf.

ANALYZE To calculate the time at which $i = 0.9990\varepsilon/R$, we solve for t:

$$0.990\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) \;\Rightarrow\; \ln(0.0010) = -\frac{t}{\tau_L} \;\Rightarrow\; \frac{t}{\tau_L} = 6.91.$$

LEARN At t = 0, the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is $i_0 = \varepsilon / R$. The current (in terms of i/i_0) as a function of t/τ_L is plotted below.



56. From the graph we get $\Phi/i = 2 \times 10^{-4}$ in SI units. Therefore, with N = 25, we find the self-inductance is $L = N\Phi/i = 5 \times 10^{-3}$ H. From the derivative of Eq. 30-41 (or a combination of that equation and Eq. 30-39) we find (using the symbol V to stand for the battery emf)

$$\frac{di}{dt} = \frac{V}{R}\frac{R}{L}\,e^{-t/\tau_L} = \frac{V}{L}\,e^{-t/\tau_L} = 7.1 \times 10^2 \text{ A/s}.$$

57. (a) Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\varepsilon - L\,di/dt = 0$. So $i = \varepsilon t/L$. As the fuse blows at $t = t_0$, $i = i_0$ = 3.0 A. Thus,

$$t_0 = \frac{i_0 L}{\varepsilon} = \frac{(3.0\,\text{A})(5.0\,\text{H})}{10\,\text{V}} = 1.5\ \text{s}.$$

(b) We do not show the graph here; qualitatively, it would be similar to Fig. 30-15.

58. Applying the loop theorem,

$$\varepsilon - L\left(\frac{di}{dt}\right) = iR \ ,$$

we solve for the (time-dependent) emf, with SI units understood:

$$\varepsilon = L\frac{di}{dt} + iR = L\frac{d}{dt}(3.0+5.0t) + (3.0+5.0t)R = (6.0)(5.0) + (3.0+5.0t)(4.0)$$
$$= (42+20t).$$

59. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it. We are interested in the currents in the resistor and the current in the inductor as a function of time.

EXPRESS We assume i to be from left to right through the closed switch. We let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor, also assumed downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1R - L(di_2/dt) = 0$. According to the junction rule, $(di_1/dt) = -(di_2/dt)$. We substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1R = 0.$$

This equation is similar to Eq. 30-46, and its solution is the function given as Eq. 30-47: $i_1 = i_0e^{-Rt/L}$, where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that moment $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$.

ANALYZE (a) The currents in the resistor and the inductor as a function of time are:

$$i_1 = ie^{-Rt/L}, \quad i_2 = i - i_1 = i\left(1-e^{-Rt/L}\right).$$

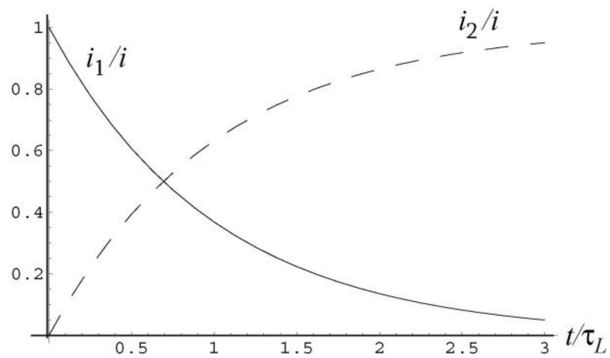
(b) When $i_2 = i_1$, we have

$$e^{-Rt/L} = 1 - e^{-Rt/L} \ \Rightarrow \ e^{-Rt/L} = \frac{1}{2}.$$

Taking the natural logarithm of both sides and using $\ln(1/2) = -\ln 2$, we obtain

$$\left(\frac{Rt}{L}\right) = \ln 2 \ \Rightarrow \ t = \frac{L}{R}\ln 2.$$

LEARN A plot of i_1/i (solid line, for resistor) and i_2/i (dashed line, for inductor) as a function of t/τ_L is shown next.



60. (a) Our notation is as follows: *h* is the height of the toroid, *a* its inner radius, and *b* its outer radius. Since it has a square cross section, $h = b - a = 0.12$ m – 0.10 m = 0.02 m. We derive the flux using Eq. 29-24 and the self-inductance using Eq. 30-33:

$$\Phi_B = \int_a^b B\,dA = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right) h\,dr = \frac{\mu_0 Nih}{2\pi}\ln\left(\frac{b}{a}\right)$$

and

$$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right).$$

Now, since the inner circumference of the toroid is $l = 2\pi a = 2\pi(10 \text{ cm}) \approx 62.8$ cm, the number of turns of the toroid is roughly $N \approx 62.8$ cm/1.0 mm = 628. Thus

$$L = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right) \approx \frac{\left(4\pi\times10^{-7}\text{ H/m}\right)(628)^2(0.02\text{m})}{2\pi}\ln\left(\frac{12}{10}\right) = 2.9\times10^{-4}\text{H}.$$

(b) Noting that the perimeter of a square is four times its sides, the total length ℓ of the wire is $\ell = (628)(4)(2.0\,\text{cm}) = 50\,\text{m}$, and the resistance of the wire is

$$R = (50\text{ m})(0.02\ \Omega/\text{m}) = 1.0\ \Omega.$$

Thus,

$$\tau_L = \frac{L}{R} = \frac{2.9\times10^{-4}\text{ H}}{1.0\Omega} = 2.9\times10^{-4}\text{s}.$$

61. **THINK** Inductance *L* is related to the inductive time constant of an *RL* circuit by $L = \tau_L R$, where *R* is the resistance in the circuit. The energy stored by an inductor carrying current *i* is given by $U_B = Li^2/2$.

EXPRESS If the battery is applied at time $t = 0$ the current is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where ε is the emf of the battery, R is the resistance, and τ_L is the inductive time constant (L/R). This leads to

$$e^{-t/\tau_L} = 1 - \frac{iR}{\varepsilon} \Rightarrow -\frac{t}{\tau_L} = \ln\left(1 - \frac{iR}{\varepsilon}\right).$$

Since

$$\ln\left(1 - \frac{iR}{\varepsilon}\right) = \ln\left[1 - \frac{(2.00\times10^{-3}\,\text{A})(10.0\times10^{3}\,\Omega)}{50.0\,\text{V}}\right] = -0.5108,$$

the inductive time constant is $\tau_L = t/0.5108 = (5.00 \times 10^{-3}\ \text{s})/0.5108 = 9.79 \times 10^{-3}$ s.

ANALYZE (a) The inductance is

$$L = \tau_L R = (9.79\times10^{-3}\,\text{s})(10.0\times10^{3}\,\Omega) = 97.9\,\text{H}.$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\text{H})(2.00\times10^{-3}\ \text{A})^2 = 1.96\times10^{-4}\ \text{J}.$$

LEARN Note the similarity between $U_B = \frac{1}{2}Li^2$ and $U_C = \frac{q^2}{2C}$, the electric energy stored in a capacitor.

62. (a) From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = Li\frac{di}{dt} = L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right) = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}.$$

Now,

$$\tau_L = L/R = 2.0\ \text{H}/10\ \Omega = 0.20\ \text{s}$$

and $\varepsilon = 100$ V, so the above expression yields $dU_B/dt = 2.4 \times 10^2$ W when $t = 0.10$ s.

(b) From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\varepsilon^2}{R^2}\left(1 - e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1 - e^{-t/\tau_L}\right)^2.$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.5 \times 10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9 \times 10^2 \text{ W}.$$

We note that this result could alternatively have been found from Eq. 28-14 (with Eq. 30-41).

63. From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(Li^2/2\right)}{dt} = Li\frac{di}{dt} = L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right) = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}$$

where $\tau_L = L/R$ has been used. From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

We equate this to dU_B/dt, and solve for the time:

$$\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2 = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L} \quad \Rightarrow \quad t = \tau_L \ln 2 = (37.0\,\text{ms})\ln 2 = 25.6\,\text{ms}.$$

64. Let $U_B(t) = \frac{1}{2}Li^2(t)$. We require the energy at time t to be half of its final value: $U(t) = \frac{1}{2}U_B(t \to \infty) = \frac{1}{4}Li_f^2$. This gives $i(t) = i_f/\sqrt{2}$. But $i(t) = i_f(1-e^{-t/\tau_L})$, so

$$1-e^{-t/\tau_L} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{t}{\tau_L} = -\ln\left(1-\frac{1}{\sqrt{2}}\right) = 1.23.$$

65. (a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 30-41 for the current):

$$\begin{aligned}\int_0^t P_{\text{battery}}\,dt &= \int_0^t \frac{\varepsilon^2}{R}\left(1-e^{-Rt/L}\right)dt = \frac{\varepsilon^2}{R}\left[t + \frac{L}{R}\left(e^{-Rt/L}-1\right)\right] \\ &= \frac{(10.0\,\text{V})^2}{6.70\ \Omega}\left[2.00\ \text{s} + \frac{(5.50\,\text{H})\left(e^{-(6.70\ \Omega)(2.00\ \text{s})/5.50\ \text{H}}-1\right)}{6.70\ \Omega}\right] \\ &= 18.7\ \text{J}.\end{aligned}$$

(b) The energy stored in the magnetic field is given by Eq. 30-49:

$$U_B = \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2\left(1-e^{-Rt/L}\right)^2 = \frac{1}{2}(5.50\,\text{H})\left(\frac{10.0\,\text{V}}{6.70\,\Omega}\right)^2\left[1-e^{-(6.70\,\Omega)(2.00\,\text{s})/5.50\,\text{H}}\right]^2$$
$$= 5.10\text{ J}.$$

(c) The difference of the previous two results gives the amount "lost" in the resistor: 18.7 J – 5.10 J = 13.6 J.

66. (a) The magnitude of the magnetic field at the center of the loop, using Eq. 29-9, is

$$B = \frac{\mu_0 i}{2R} = \frac{(4\pi\times10^{-7}\text{ H/m})(100\text{ A})}{2(50\times10^{-3}\text{ m})} = 1.3\times10^{-3}\text{ T}.$$

(b) The energy per unit volume in the immediate vicinity of the center of the loop is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(1.3\times10^{-3}\text{ T})^2}{2(4\pi\times10^{-7}\text{ H/m})} = 0.63\text{ J/m}^3.$$

67. **THINK** The magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point.

EXPRESS Inside a solenoid, the magnitude of the magnetic field is $B = \mu_0 ni$, where

$$n = (950\text{ turns})/(0.850\text{ m}) = 1.118\times10^3\text{ m}^{-1}.$$

Thus, the energy density is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(\mu_0 ni)^2}{2\mu_0} = \frac{1}{2}\mu_0 n^2 i^2.$$

Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B\mathcal{V}$, where $\mathcal{V}$ is the volume of the solenoid.

ANALYZE (a) Substituting the values given, we find the magnetic energy density to be

$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1.118\times10^3\text{ m}^{-1}\right)^2\left(6.60\text{ A}\right)^2 = 34.2\text{ J/m}^3.$$

(b) The volume $\mathcal{V}$ is calculated as the product of the cross-sectional area and the length. Thus,

$$U_B = \left(34.2\text{ J/m}^3\right)\left(17.0\times10^{-4}\text{ m}^2\right)\left(0.850\text{ m}\right) = 4.94\times10^{-2}\text{ J}.$$

LEARN Note the similarity between $u_B = \dfrac{B^2}{2\mu_0}$, the energy density at a point in a magnetic field, and $u_E = \dfrac{1}{2}\varepsilon_0 E^2$, the energy density at a point in an electric field. Both quantities are proportional to the square of the fields.

68. The magnetic energy stored in the toroid is given by $U_B = \frac{1}{2}Li^2$, where L is its inductance and i is the current. By Eq. 30-54, the energy is also given by $U_B = u_B\mathcal{V}$, where u_B is the average energy density and $\mathcal{V}$ is the volume. Thus

$$i = \sqrt{\frac{2u_B\mathcal{V}}{L}} = \sqrt{\frac{2\left(70.0\,\text{J/m}^3\right)\left(0.0200\,\text{m}^3\right)}{90.0\times10^{-3}\,\text{H}}} = 5.58\,\text{A}\ .$$

69. We set $u_E = \frac{1}{2}\varepsilon_0 E^2 = u_B = \frac{1}{2}B^2/\mu_0$ and solve for the magnitude of the electric field:

$$E = \frac{B}{\sqrt{\varepsilon_0\mu_0}} = \frac{0.50\,\text{T}}{\sqrt{\left(8.85\times10^{-12}\,\text{F/m}\right)\left(4\pi\times10^{-7}\,\text{H/m}\right)}} = 1.5\times10^8\,\text{V/m}\ .$$

70. It is important to note that the x that is used in the graph of Fig. 30-67(b) is not the x at which the energy density is being evaluated. The x in Fig. 30-67(b) is the location of wire 2. The energy density (Eq. 30-54) is being evaluated at the coordinate origin throughout this problem. We note the curve in Fig. 30-67(b) has a zero; this implies that the magnetic fields (caused by the individual currents) are in opposite directions (at the origin), which further implies that the currents have the same direction. Since the magnitudes of the fields must be equal (for them to cancel) when the x of Fig. 30-67(b) is equal to 0.20 m, then we have (using Eq. 29-4) $B_1 = B_2$, or

$$\frac{\mu_0 i_1}{2\pi d} = \frac{\mu_0 i_2}{2\pi(0.20\text{ m})}$$

which leads to $d = (0.20\text{ m})/3$ once we substitute $i_1 = i_2/3$ and simplify. We can also use the given fact that when the energy density is completely caused by B_1 (this occurs when x becomes infinitely large because then $B_2 = 0$) its value is $u_B = 1.96 \times 10^{-9}$ (in SI units) in order to solve for B_1:

$$B_1 = \sqrt{2\mu_0 u_B}\ .$$

(a) This combined with $B_1 = \mu_0 i_1/2\pi d$ allows us to find wire 1's current: $i_1 \approx 23$ mA.

(b) Since $i_2 = 3i_1$ then $i_2 = 70$ mA (approximately).

71. (a) The energy per unit volume associated with the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi R}\right)^2 = \frac{\mu_0 i^2}{8\pi^2 R^2} = \frac{\left(4\pi\times10^{-7}\text{ H/m}\right)\left(10\text{ A}\right)^2}{8\pi^2\left(2.5\times10^{-3}\text{ m/2}\right)^2} = 1.0\text{ J/m}^3 .$$

(b) The electric energy density is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}(\rho J)^2 = \frac{\varepsilon_0}{2}\left(\frac{iR}{\ell}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\text{ F/m}\right)\left[(10\text{A})\left(3.3\Omega/10^3\text{ m}\right)\right]^2$$
$$= 4.8\times10^{-15}\text{ J/m}^3 .$$

Here we used $J = i/A$ and $R = \rho\ell/A$ to obtain $\rho J = iR/\ell$.

72. (a) The flux in coil 1 is

$$\frac{L_1 i_1}{N_1} = \frac{(25\text{mH})(6.0\text{mA})}{100} = 1.5\,\mu\text{Wb}.$$

(b) The magnitude of the self-induced emf is

$$L_1\frac{di_1}{dt} = (25\text{mH})(4.0\text{ A/s}) = 1.0\times10^2\text{ mV}.$$

(c) In coil 2, we find

$$\Phi_{21} = \frac{Mi_1}{N_2} = \frac{(3.0\text{mH})(6.0\text{mA})}{200} = 90\text{nWb} .$$

(d) The mutually induced emf is

$$\varepsilon_{21} = M\frac{di_1}{dt} = (3.0\text{mH})(4.0\text{ A/s}) = 12\text{mV}.$$

73. **THINK** If two coils are near each other, mutual induction can take place whereby a changing current in one coil can induce an emf in the other.

EXPRESS The mutual inductance is given by

$$\varepsilon_1 = -M\frac{di_2}{dt}$$

where ε_1 is the induced emf in coil 1 due to the changing current in coil 2. The flux linkage in coil 2 is $N_2\Phi_{21} = Mi_1$.

ANALYZE (a) From the equation above, we find the mutual inductance to be

$$M = \frac{|\varepsilon_1|}{di_2/dt} = \frac{25.0\,\text{mV}}{15.0\,\text{A/s}} = 1.67\,\text{mH}.$$

(b) Similarly, the flux linkage in coil 2 is

$$N_2\Phi_{21} = Mi_1 = (1.67\,\text{mH})(3.60\,\text{A}) = 6.00\,\text{mWb}.$$

LEARN The emf induced in one coil is proportional to the rate at which current in the other coil is changing:

$$\varepsilon_1 = -M_{12}\frac{di_2}{dt}, \quad \varepsilon_2 = -M_{21}\frac{di_1}{dt}.$$

The proportionality constants, M_{12} and M_{21}, are the same, $M_{12} = M_{21} = M$, so we simply write

$$\varepsilon_1 = -M\frac{di_2}{dt}, \quad \varepsilon_2 = -M\frac{di_1}{dt}.$$

74. We use $\varepsilon_2 = -M\, di_1/dt \approx M|\Delta i/\Delta t|$ to find M:

$$M = \left|\frac{\varepsilon}{\Delta i_1/\Delta t}\right| = \frac{30\times 10^3\ \text{V}}{6.0\,\text{A}/(2.5\times 10^{-3}\ \text{s})} = 13\,\text{H}\ .$$

75. The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}}\, l dr = \int_a^{a+b} \frac{\mu_0 il}{2\pi r}\, dr = \frac{\mu_0 il}{2\pi}\ln\left(1+\frac{b}{a}\right).$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi}\ln\left(1+\frac{b}{a}\right).$$

From the formula for M obtained above, we have

$$M = \frac{(100)(4\pi\times 10^{-7}\ \text{H/m})(0.30\,\text{m})}{2\pi}\ln\left(1+\frac{8.0}{1.0}\right) = 1.3\times 10^{-5}\ \text{H}\ .$$

76. (a) The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N\left(\mu_0 i_s n\pi R^2\right)}{i_s} = \mu_0 \pi R^2 nN \ .$$

(b) As long as the magnetic field of the solenoid is entirely contained within the cross section of the coil we have $\Phi_{sc} = B_s A_s = B_s \pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

77. **THINK** To find the equivalent inductance, we calculate the total emf across both coils.

EXPRESS We assume the current to be changing at (nonzero) a rate di/dt. The induced emf's can take on the following form:

$$\varepsilon_1 = -\left(L_1 \pm M\right)\frac{di}{dt}, \quad \varepsilon_2 = -\left(L_2 \pm M\right)\frac{di}{dt}$$

The relative sign between L and M depends on how the coils are connected, as we shall see below.

ANALYZE (a) The connection is shown in Fig. 30-70. First consider coil 1. The magnetic field due to the current in that coil points to the right. The magnetic field due to the current in coil 2 also points to the right. When the current increases, both fields increase and both changes in flux contribute emfs in the same direction. Thus, the induced emfs are

$$\varepsilon_1 = -\left(L_1 + M\right)\frac{di}{dt}, \quad \varepsilon_2 = -\left(L_2 + M\right)\frac{di}{dt} \ .$$

Therefore, the total emf across both coils is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = -\left(L_1 + L_2 + 2M\right)\frac{di}{dt}$$

which is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{eq} = L_1 + L_2 + 2M$.

(b) We imagine reversing the leads of coil 2 so the current enters at the back of the coil rather than the front (as pictured in Fig. 30-70). Then the field produced by coil 2 at the site of coil 1 is opposite to the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil, but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\varepsilon_1 = -\left(L_1 - M\right)\frac{di}{dt} \ .$$

Similarly, the emf across coil 2 is

$$\varepsilon_2 = -\left(L_2 - M\right)\frac{di}{dt}.$$

The total emf across both coils is

$$\varepsilon = -\left(L_1 + L_2 - 2M\right)\frac{di}{dt}.$$

This is the same as the emf that would be produced by a single coil with inductance

$$L_{eq} = L_1 + L_2 - 2M.$$

LEARN The sign of the mutual inductance term is determined by the senses of the coil winding. The induced emfs can either reinforce one another ($L + M$), or oppose one another ($L - M$).

78. Taking the derivative of Eq. 30-41, we have

$$\frac{di}{dt} = \frac{d}{dt}\left[\frac{\varepsilon}{R}(1-e^{-t/\tau_L})\right] = \frac{\varepsilon}{R\tau_L}e^{-t/\tau_L} = \frac{\varepsilon}{L}e^{-t/\tau_L}.$$

With $\tau_L = L/R$ (Eq. 30-42), L = 0.023 H and ε = 12 V, t = 0.00015 s, and di/dt = 280 A/s, we obtain $e^{-t/\tau_L} = 0.537$. Taking the natural log and rearranging leads to R = 95.4 Ω.

79. **THINK** The inductor in the RL circuit initially acts to oppose changes in current through it.

EXPRESS When the switch S is just closed, $V_1 = \varepsilon$ and no current flows through the inductor. A long time later, the currents have reached their equilibrium values and the inductor acts as an ordinary connecting wire; we can solve the multi-loop circuit problem by applying Kirchhoff's junction and loop rules.

ANALYZE (a) Applying the loop rule to the left loop gives $\varepsilon - i_1 R_1 = 0$, so

$$i_1 = \varepsilon/R_1 = 10 \text{ V}/5.0\ \Omega = 2.0 \text{ A}.$$

(b) Since now $\varepsilon_L = \varepsilon$, we have $i_2 = 0$.

(c) The junction rule gives $i_s = i_1 + i_2 = 2.0 \text{ A} + 0 = 2.0 \text{ A}$.

(d) Since $V_L = \varepsilon$, the potential difference across resistor 2 is $V_2 = \varepsilon - \varepsilon_L = 0$.

(e) The potential difference across the inductor is $V_L = \varepsilon = 10$ V.

(f) The rate of change of current is $\frac{di_2}{dt} = \frac{V_L}{L} = \frac{\varepsilon}{L} = \frac{10 \text{ V}}{5.0 \text{ H}} = 2.0 \text{ A/s}$.

(g) After a long time, we still have $V_1 = \varepsilon$, so $i_1 = 2.0$ A.

(h) Since now $V_L = 0$, $i_2 = \varepsilon/R_2 = 10 \text{ V}/10\ \Omega = 1.0$ A.

(i) The current through the switch is now $i_s = i_1 + i_2 = 2.0 \text{ A} + 1.0 \text{ A} = 3.0$ A.

(j) Since $V_L = 0$, $V_2 = \varepsilon - V_L = \varepsilon = 10$ V.

(k) With the inductor acting as an ordinary connecting wire, we have $V_L = 0$.

(l) The rate of change of current in resistor 2 is $\dfrac{di_2}{dt} = \dfrac{V_L}{L} = 0$.

LEARN In analyzing an *RL* circuit immediately after closing the switch and a very long time after that, there is no need to solve any differential equation.

80. Using Eq. 30-41: $i = \dfrac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$, where $\tau_L = 2.0$ ns, we find

$$t = \tau_L \ln\left(\frac{1}{1 - iR/\varepsilon}\right) \approx 1.0 \text{ ns}.$$

81. Using Ohm's law, we relate the induced current to the emf and (the absolute value of) Faraday's law:

$$i = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right|.$$

As the loop is crossing the boundary between regions 1 and 2 (so that "x" amount of its length is in region 2 while "$D - x$" amount of its length remains in region 1) the flux is

$$\Phi_B = xHB_2 + (D - x)HB_1 = DHB_1 + xH(B_2 - B_1)$$

which means

$$\frac{d\Phi_B}{dt} = \frac{dx}{dt}H(B_2 - B_1) = vH(B_2 - B_1) \Rightarrow \quad i = vH(B_2 - B_1)/R.$$

Similar considerations hold (replacing "B_1" with 0 and "B_2" with B_1) for the loop crossing initially from the zero-field region (to the left of Fig. 30-72(a)) into region 1.

(a) In this latter case, appeal to Fig. 30-72(b) leads to

$$3.0 \times 10^{-6} \text{ A} = (0.40 \text{ m/s})(0.015 \text{ m})\, B_1/(0.020\ \Omega)$$

which yields $B_1 = 10\ \mu$T.

(b) Lenz's law considerations lead us to conclude that the direction of the region 1 field is *out of the page*.

(c) Similarly, $i = vH(B_2 - B_1)/R$ leads to $B_2 = 3.3\mu\text{T}$.

(d) The direction of $\vec{B}_2$ is out of the page.

82. Faraday's law (for a single turn, with B changing in time) gives

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt} .$$

In this problem, we find $\frac{dB}{dt} = -\frac{B_0}{\tau} e^{-t/\tau}$. Thus, $\varepsilon = \pi r^2 \frac{B_0}{\tau} e^{-t/\tau}$.

83. Equation 30-41 applies, and the problem requires

$$iR = L\frac{di}{dt} = \varepsilon - iR$$

at some time t (where Eq. 30-39 has been used in that last step). Thus, we have $2iR = \varepsilon$, or

$$\varepsilon = 2iR = 2\left[\frac{\varepsilon}{R}(1 - e^{-t/\tau_L})\right]R = 2\varepsilon\left(1 - e^{-t/\tau_L}\right)$$

where Eq. 30-42 gives the inductive time constant as $\tau_L = L/R$. We note that the emf ε cancels out of that final equation, and we are able to rearrange (and take the natural log) and solve. We obtain $t = 0.520$ ms.

84. In absolute value, Faraday's law (for a single turn, with B changing in time) gives

$$\frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \pi R^2 \frac{dB}{dt}$$

for the magnitude of the induced emf. Dividing it by R^2 then allows us to relate this to the slope of the graph in Fig. 30-73(b) [particularly the first part of the graph], which we estimate to be 80 $\mu\text{V/m}^2$.

(a) Thus, $\frac{dB_1}{dt} = (80\ \mu\text{V/m}^2)/\pi \approx 25\ \mu\text{T/s}$.

(b) Similar reasoning for region 2 (corresponding to the slope of the second part of the graph in Fig. 30-73(b)) leads to an emf equal to

$$\pi r_1^2\left(\frac{dB_1}{dt}-\frac{dB_2}{dt}\right)+\pi R^2\frac{dB_2}{dt}$$

which means the second slope (which we estimate to be 40 μV/m^2) is equal to $\pi\dfrac{dB_2}{dt}$. Therefore, $\dfrac{dB_2}{dt}$ = (40 μV/m^2)/π ≈ 13 μT/s.

(c) Considerations of Lenz's law leads to the conclusion that B_2 is increasing.

85. **THINK** Changing magnetic field induces an electric field.

EXPRESS The induced electric field is given by Eq. 30-20:

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}.$$

The electric field lines are circles that are concentric with the cylindrical region. Thus,

$$E(2\pi r) = -(\pi r^2)\frac{dB}{dt} \quad \Rightarrow \quad E = -\frac{1}{2}\frac{dB}{dt}r.$$

The force on the electron is $\vec{F} = -e\vec{E},$ so by Newton's second law, the acceleration is $\vec{a} = -e\vec{E}/m.$

ANALYZE (a) At point a,

$$E = -\frac{r}{2}\left(\frac{dB}{dt}\right) = -\frac{1}{2}(5.0\times10^{-2}\,\text{m})(-10\times10^{-3}\,\text{T/s}) = 2.5\times10^{-4}\ \text{V/m}.$$

With the normal taken to be into the page, in the direction of the magnetic field, the positive direction for $\vec{E}$ is clockwise. Thus, the direction of the electric field at point a is to the left, that is $\vec{E} = -(2.5\times10^{-4}\ \text{V/m})\hat{\text{i}}.$ The resulting acceleration is

$$\vec{a}_a = \frac{-e\vec{E}}{m} = \frac{(-1.60\times10^{-19}\,\text{C})(-2.5\times10^{-4}\ \text{V/m})}{9.11\times10^{-31}\text{kg}}\hat{\text{i}} = (4.4\times10^{7}\ \text{m/s}^2)\hat{\text{i}}.$$

The acceleration is to the right.

(b) At point b we have r_b = 0, so the acceleration is zero.

(c) The electric field at point *c* has the same magnitude as the field in *a*, but with its direction reversed. Thus, the acceleration of the electron released at point *c* is

$$\vec{a}_c = -\vec{a}_a = -(4.4\times10^7 \text{ m/s}^2)\hat{\text{i}}.$$

LEARN Inside the cylindrical region, the induced electric field increases with *r*. Therefore, the greater the value of *r*, the greater the magnitude of acceleration.

86. Because of the decay of current (Eq. 30-45) that occurs after the switches are closed on *B*, the flux will decay according to

$$\Phi_1 = \Phi_{10} e^{-t/\tau_{L1}}, \quad \Phi_2 = \Phi_{20} e^{-t/\tau_{L2}}$$

where each time constant is given by Eq. 30-42. Setting the fluxes equal to each other and solving for time leads to

$$t = \frac{\ln(\Phi_{20}/\Phi_{10})}{(R_2/L_2)-(R_1/L_1)} = \frac{\ln(1.50)}{(30.0\,\Omega/0.0030\text{ H})-(25\,\Omega/0.0050\text{ H})} = 81.1\mu\text{s} \ .$$

87. **THINK** Changing the area of the loop changes the flux through it. An induced emf is produced to oppose this change.

EXPRESS The magnetic flux through the loop is $\Phi_B = BA$, where *B* is the magnitude of the magnetic field and *A* is the area of the loop. According to Faraday's law, the magnitude of the average induced emf is

$$\varepsilon_{\text{avg}} = \left|\frac{-d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{B|\Delta A|}{\Delta t}.$$

ANALYZE (a) substituting the values given, we obtain

$$\varepsilon_{\text{avg}} = \frac{B|\Delta A|}{\Delta t} = \frac{(2.0\text{ T})(0.20\text{ m})^2}{0.20\text{ s}} = 0.40\text{V}.$$

(b) The average induced current is $i_{\text{avg}} = \dfrac{\varepsilon_{\text{avg}}}{R} = \dfrac{0.40\text{ V}}{20\times10^{-3}\Omega} = 20\text{ A}$.

LEARN By Lenz's law, the more rapidly the area is changing, the greater the induced current in

88. (a) From Eq. 30-28, we have

$$L = \frac{N\Phi}{i} = \frac{(150)(50\times10^{-9}\ \text{T}\cdot\text{m}^2)}{2.00\times10^{-3}\ \text{A}} = 3.75\ \text{mH}\,.$$

(b) The answer for L (which should be considered the *constant* of proportionality in Eq. 30-35) does not change; it is still 3.75 mH.

(c) The equations of Chapter 28 display a simple proportionality between magnetic field and the current that creates it. Thus, if the current has doubled, so has the field (and consequently the flux). The answer is 2(50) = 100 nWb.

(d) The magnitude of the induced emf is (from Eq. 30-35)

$$L\left.\frac{di}{dt}\right|_{\max} = (0.00375\ \text{H})(0.0030\ \text{A})(377\ \text{rad/s}) = 4.24\times10^{-3}\ \text{V}\,.$$

89. (a) $i_0 = \varepsilon/R$ = 100 V/10 Ω = 10 A.

(b) $U_B = \frac{1}{2}Li_0^2 = \frac{1}{2}(2.0\text{H})(10\text{A})^2 = 1.0\times10^2\,\text{J}$.

90. We write $i = i_0 e^{-t/\tau_L}$ and note that i = 10% i_0. We solve for t:

$$t = \tau_L \ln\left(\frac{i_0}{i}\right) = \frac{L}{R}\ln\left(\frac{i_0}{i}\right) = \frac{2.00\,\text{H}}{3.00\,\Omega}\ln\left(\frac{i_0}{0.100 i_0}\right) = 1.54\,\text{s}\,.$$

91. **THINK** We have an RL circuit in which the inductor is in series with the battery.

EXPRESS As the switch closes at t = 0, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit.

ANALYZE (a) At t = 0, the current through the battery is also zero.

(b) With no current anywhere in the circuit at t = 0, the loop rule requires the emf of the inductor ε_L to cancel that of the battery (ε = 40 V). Thus, the absolute value of Eq. 30-35 yields

$$\frac{di_{\text{bat}}}{dt} = \frac{|\varepsilon_L|}{L} = \frac{40\ \text{V}}{0.050\ \text{H}} = 8.0\times10^2\ \text{A/s}\,.$$

(c) This circuit becomes equivalent to that analyzed in Section 30-9 when we replace the parallel set of 20000 Ω resistors with R = 10000 Ω. Now, with $\tau_L = L/R = 5\times10^{-6}$ s, we have t/τ_L = 3/5, and we apply Eq. 30-41:

$$i_{\text{bat}} = \frac{\varepsilon}{R}\left(1-e^{-3/5}\right) \approx 1.8\times10^{-3}\,\text{A}.$$

(d) The rate of change of the current is figured from the loop rule (and Eq. 30-35):

$$\varepsilon - i_{\text{bat}} R - |\varepsilon_L| = 0.$$

Using the values from part (c), we obtain $|\varepsilon_L| \approx 22$ V. Then,

$$\frac{di_{\text{bat}}}{dt} = \frac{|\varepsilon_L|}{L} = \frac{22\text{ V}}{0.050\text{ H}} \approx 4.4\times10^2 \text{ A/s}.$$

(e) As $t \to \infty$, the circuit reaches a steady-state condition, so that $di_{\text{bat}}/dt = 0$ and $\varepsilon_L = 0$. The loop rule then leads to

$$\varepsilon - i_{\text{bat}} R - |\varepsilon_L| = 0 \;\Rightarrow\; i_{\text{bat}} = \frac{40\text{ V}}{10000\,\Omega} = 4.0\times10^{-3}\text{ A}.$$

(f) As $t \to \infty$, the circuit reaches a steady-state condition, $di_{\text{bat}}/dt = 0$.

LEARN In summary, at $t = 0$ immediately after the switch is closed, the inductor opposes any change in current, and with the inductor and the battery being connected in series, the induced emf in the inductor is equal to the emf of the battery, $\varepsilon_L = \varepsilon$. A long time later after all the currents have reached their steady-state values, $\varepsilon_L = 0$, and the inductor can be treated as an ordinary connecting wire. In this limit, the circuit can be analyzed as if L were not present.

92. (a) $L = \Phi/i = 26 \times 10^{-3}$ Wb/5.5 A $= 4.7 \times 10^{-3}$ H.

(b) We use Eq. 30-41 to solve for t:

$$t = -\tau_L \ln\left(1 - \frac{iR}{\varepsilon}\right) = -\frac{L}{R}\ln\left(1-\frac{iR}{\varepsilon}\right) = -\frac{4.7\times10^{-3}\text{ H}}{0.75\Omega}\ln\left[1 - \frac{(2.5\text{A})(0.75\Omega)}{6.0\text{V}}\right]$$
$$= 2.4\times10^{-3}\text{s}.$$

93. The energy stored when the current is i is $U_B = \frac{1}{2}Li^2$, where L is the self-inductance. The rate at which this is developed is

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

where i is given by Eq. 30-41 and di/dt is obtained by taking the derivative of that equation (or by using Eq. 30-37). Thus, using the symbol V to stand for the battery voltage (12.0 volts) and R for the resistance (20.0 Ω), we have, at $t = 1.61\tau_L$,

$$\frac{dU_B}{dt} = \frac{V^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L} = \frac{(12.0\text{ V})^2}{20.0\Omega}\left(1-e^{-1.61}\right)e^{-1.61} = 1.15\text{ W}.$$

94. (a) The self-inductance per meter is

$$\frac{L}{\ell} = \mu_0 n^2 A = \left(4\pi\times10^{-7}\text{ H/m}\right)\left(100\text{ turns/cm}\right)^2(\pi)\left(1.6\text{ cm}\right)^2 = 0.10\text{ H/m}.$$

(b) The induced emf per meter is

$$\frac{\varepsilon}{\ell} = \frac{L}{\ell}\frac{di}{dt} = (0.10\text{ H/m})(13\text{ A/s}) = 1.3\text{ V/m}.$$

95. (a) As the switch closes at $t = 0$, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf (ε_{L1}) of the $L_1 = 0.30$ H inductor to cancel that of the battery. We now apply (the absolute value of) Eq. 30-35

$$\frac{di}{dt} = \frac{|\varepsilon_{L1}|}{L_1} = \frac{6.0}{0.30} = 20\text{ A/s}.$$

(b) What is being asked for is essentially the current in the battery when the emfs of the inductors vanish (as $t \to \infty$). Applying the loop rule to the outer loop, with $R_1 = 8.0\ \Omega$, we have

$$\varepsilon - iR_1 - |\varepsilon_{L1}| - |\varepsilon_{L2}| = 0 \;\Rightarrow\; i = \frac{6.0\text{ V}}{R_1} = 0.75\text{ A}.$$

96. Since $A = \ell^2$, we have $dA/dt = 2\ell\, d\ell/dt$. Thus, Faraday's law, with $N = 1$, becomes

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -2\ell B\frac{d\ell}{dt}$$

which yields $\varepsilon = 0.0029$ V.

97. The self-inductance and resistance of the coil may be treated as a "pure" inductor in series with a "pure" resistor, in which case the situation described in the problem may be addressed by using Eq. 30-41. The derivative of that solution is

$$\frac{di}{dt} = \frac{d}{dt}\left[\frac{\varepsilon}{R}(1-e^{-t/\tau_L})\right] = \frac{\varepsilon}{R\tau_L}e^{-t/\tau_L} = \frac{\varepsilon}{L}e^{-t/\tau_L}$$

With τ_L = 0.28 ms (by Eq. 30-42), L = 0.050 H, and ε = 45 V, we obtain di/dt = 12 A/s when t = 1.2 ms.

98. (a) From Eq. 30-35, we find L = (3.00 mV)/(5.00 A/s) = 0.600 mH.

(b) Since $N\Phi = iL$ (where Φ = 40.0 μWb and i = 8.00 A), we obtain N = 120.

99. We use 1 ly = 9.46×10^{15} m, and use the symbol $\mathcal{V}$ for volume.

$$U_B = \mathcal{V}u_B = \frac{\mathcal{V}B^2}{2\mu_0} = \frac{\left(9.46\times10^{15}\text{ m}\right)^3\left(1\times10^{-10}\text{ T}\right)^2}{2\left(4\pi\times10^{-7}\text{ H/m}\right)} = 3\times10^{36}\text{ J}.$$

100. (a) The total length of the closed loop formed by the two radii plus the arc is

$$L = 2r + r\theta = r(2+\theta),$$

where r is the radius. The total resistance is

$$R = \frac{\rho L}{A} = \frac{\rho r(2+\theta)}{A} = \frac{(1.7\times10^{-8}\,\Omega\cdot\text{m})(0.24\text{ m})(2+\theta)}{1.20\times10^{-6}\,\text{m}^2}$$
$$= (3.4\times10^{-3})(2+\theta)\,\Omega.$$

(b) The area of the loop is $A = \frac{1}{2}r^2\theta$. Thus, the magnetic flux through the loop is

$$\Phi_B = BA = \frac{1}{2}Br^2\theta = \frac{1}{2}(0.150\text{ T})(0.240\text{ m})^2\theta = (4.32\times10^{-3}\theta)\text{ Wb}.$$

(c) The induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{1}{2}Br^2\theta\right) = -\frac{1}{2}Br^2\frac{d\theta}{dt} = -\frac{1}{2}Br^2\omega$$

which gives

$$i = \frac{|\varepsilon|}{R} = \frac{Br^2\omega}{2R} = \frac{Br^2\omega}{2(3.4\times10^{-3})(2+\theta)} = \frac{Br^2\alpha t}{2(3.4\times10^{-3})(2+\alpha t^2/2)}$$

as the magnitude of the induced current. Note that in the last step, we have substituted $\omega = \alpha t$ and $\theta = \frac{1}{2}\alpha t^2$, for constant angular acceleration α. Differentiating i with respect to t gives

$$\frac{di}{dt} = \frac{Br^2\alpha(4-\alpha t^2)}{(3.4\times10^{-3})(4+\alpha t^2)^2}.$$

The induced current is at a maximum when $4-\alpha t^2=0$, or $t=\sqrt{4/\alpha}$. At this instant, the angle is

$$\theta=\frac{1}{2}\alpha t^2=\frac{1}{2}\alpha\left(\frac{4}{\alpha}\right)=2.0\text{ rad}.$$

(d) When current is at a maximum, $\omega=\alpha t=\alpha\sqrt{4/\alpha}=\sqrt{4\alpha}$. Thus,

$$i_{\max}=\frac{Br^2\omega}{2R}=\frac{Br^2\sqrt{4\alpha}}{2R}=\frac{Br^2\sqrt{4\alpha}}{2(3.4\times10^{-3})(2+\theta)}=\frac{(0.150\text{ T})(0.24\text{ m})^2\sqrt{4(12\text{ rad/s}^2)}}{2(3.4\times10^{-3})(2+2.0)}=2.20\text{ A}.$$

101. (a) We use $U_B=\frac{1}{2}Li^2$ to solve for the self-inductance:

$$L=\frac{2U_B}{i^2}=\frac{2\left(25.0\times10^{-3}\text{ J}\right)}{\left(60.0\times10^{-3}\text{ A}\right)^2}=13.9\text{ H}.$$

(b) Since $U_B \propto i^2$, for U_B to increase by a factor of 4, i must increase by a factor of 2. Therefore, i should be increased to 2(60.0 mA) = 120 mA.

Chapter 31

1. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If Q is the maximum charge on the capacitor, then the total energy is

$$U = \frac{Q^2}{2C} = \frac{\left(2.90\times10^{-6}\,\text{C}\right)^2}{2\left(3.60\times10^{-6}\,\text{F}\right)} = 1.17\times10^{-6}\,\text{J}.$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If I is the maximum current, then $U = LI^2/2$ leads to

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.168\times10^{-6}\,\text{J}\right)}{75\times10^{-3}\,\text{H}}} = 5.58\times10^{-3}\,\text{A}.$$

2. (a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 31-1. The values of t when plate A will again have maximum positive charge are multiples of the period:

$$t_A = nT = \frac{n}{f} = \frac{n}{2.00\times10^3\,\text{Hz}} = n\left(5.00\,\mu\text{s}\right),$$

where n = 1, 2, 3, 4, …. The earliest time is (n = 1) $t_A = 5.00\,\mu\text{s}$.

(b) We note that it takes $t = \frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps a and e in Fig. 31-1). This is when plate A acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t = \frac{1}{2}T + (n-1)T = \frac{1}{2}(2n-1)T = \frac{(2n-1)}{2f} = \frac{(2n-1)}{2\left(2\times10^3\,\text{Hz}\right)} = (2n-1)(2.50\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n = 1) $t = 2.50\,\mu\text{s}$.

(c) At $t = \frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps a and c in Fig. 31-1). Later this will repeat every half-period (compare steps c and g in Fig. 31-1). Therefore,

$$t_L = \frac{T}{4} + \frac{(n-1)T}{2} = (2n-1)\frac{T}{4} = (2n-1)(1.25\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n = 1) $t = 1.25\,\mu$s.

3. (a) The period is $T = 4(1.50\ \mu\text{s}) = 6.00\ \mu\text{s}$.

(b) The frequency is the reciprocal of the period: $f = \dfrac{1}{T} = \dfrac{1}{6.00\,\mu\text{s}} = 1.67\times 10^{5}\ \text{Hz}$.

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or 3.00 μs.

4. We find the capacitance from $U = \frac{1}{2}Q^2/C$:

$$C = \frac{Q^2}{2U} = \frac{\left(1.60\times 10^{-6}\ \text{C}\right)^2}{2\left(140\times 10^{-6}\ \text{J}\right)} = 9.14\times 10^{-9}\ \text{F}.$$

5. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00\times 10^{-6}\ \text{C}}{\sqrt{\left(1.10\times 10^{-3}\ \text{H}\right)\left(4.00\times 10^{-6}\ \text{F}\right)}} = 4.52\times 10^{-2}\ \text{A}.$$

6. (a) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0\ \text{N}}{\left(2.0\times 10^{-13}\ \text{m}\right)\left(0.50\ \text{kg}\right)}} = 89\ \text{rad/s}.$$

(b) The period is $1/f$ and $f = \omega/2\pi$. Therefore, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{89\ \text{rad/s}} = 7.0\times 10^{-2}\ \text{s}$.

(c) From $\omega = (LC)^{-1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{\left(89\ \text{rad/s}\right)^2\left(5.0\ \text{H}\right)} = 2.5\times 10^{-5}\ \text{F}.$$

7. **THINK** This problem explores the analogy between an oscillating LC system and an oscillating mass–spring system.

EXPRESS Table 31-1 provides a comparison of energies in the two systems. From the table, we see the following correspondences:

$$x \leftrightarrow q, \quad k \leftrightarrow \frac{1}{C}, \; m \leftrightarrow L, \quad v = \frac{dx}{dt} \leftrightarrow \frac{dq}{dt} = i,$$

$$\frac{1}{2}kx^2 \leftrightarrow \frac{q^2}{2C}, \quad \frac{1}{2}mv^2 \leftrightarrow \frac{1}{2}Li^2.$$

ANALYZE (a) The mass m corresponds to the inductance, so m = 1.25 kg.

(b) The spring constant k corresponds to the reciprocal of the capacitance, $1/C$. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance, we have

$$C = \frac{Q^2}{2U} = \frac{\left(175\times 10^{-6}\text{ C}\right)^2}{2\left(5.70\times 10^{-6}\text{ J}\right)} = 2.69\times 10^{-3}\text{ F}$$

and

$$k = \frac{1}{2.69\times 10^{-3}\text{ m/N}} = 372 \text{ N/m}.$$

(c) The maximum displacement corresponds to the maximum charge, so $x_{\max} = 1.75\times 10^{-4}$ m.

(d) The maximum speed $v_{\max}$ corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175\times 10^{-6}\text{ C}}{\sqrt{(1.25\text{ H})(2.69\times 10^{-3}\text{ F})}} = 3.02\times 10^{-3}\text{ A}.$$

Consequently, $v_{\max} = 3.02 \times 10^{-3}$ m/s.

LEARN The correspondences suggest that an oscillating LC system is mathematically equivalent to an oscillating mass–spring system. The electrical mechanical analogy can also be seen by comparing their angular frequencies of oscillation:

$$\omega = \sqrt{\frac{k}{m}} \text{ (mass-spring system)}, \quad \omega = \frac{1}{\sqrt{LC}} \text{ (LC circuit)}$$

8. We apply the loop rule to the entire circuit:

$$\varepsilon_{\text{total}} = \varepsilon_{L_1} + \varepsilon_{C_1} + \varepsilon_{R_1} + \cdots = \sum_j \left(\varepsilon_{L_j} + \varepsilon_{C_j} + \varepsilon_{R_j}\right) = \sum_j \left(L_j \frac{di}{dt} + \frac{q}{C_j} + iR_j\right) = L\frac{di}{dt} + \frac{q}{C} + iR$$

with

$$L = \sum_j L_j, \quad \frac{1}{C} = \sum_j \frac{1}{C_j}, \quad R = \sum_j R_j$$

and we require $\varepsilon_{total} = 0$. This is equivalent to the simple *LRC* circuit shown in Fig. 31-27(b).

9. The time required is $t = T/4$, where the period is given by $T = 2\pi/\omega = 2\pi\sqrt{LC}$. Consequently,

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.050\,\text{H})(4.0\times10^{-6}\,\text{F})}}{4} = 7.0\times10^{-4}\,\text{s}.$$

10. We find the inductance from $f = \omega/2\pi = \left(2\pi\sqrt{LC}\right)^{-1}$.

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2\left(10\times10^3\,\text{Hz}\right)^2\left(6.7\times10^{-6}\,\text{F}\right)} = 3.8\times10^{-5}\,\text{H}.$$

11. **THINK** The frequency of oscillation f in an *LC* circuit is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$.

EXPRESS Since $f \sim 1/\sqrt{C}$, the smaller value of C gives the larger value of f, while the larger value of C gives the smaller value of f. Consequently, $f_{max} = 1/2\pi\sqrt{LC_{min}}$, and $f_{min} = 1/2\pi\sqrt{LC_{max}}$.

ANALYZE (a) The ratio of the maximum frequency to the minimum frequency is

$$\frac{f_{max}}{f_{min}} = \frac{\sqrt{C_{max}}}{\sqrt{C_{min}}} = \frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}} = 6.0.$$

(b) An additional capacitance C is chosen so the desired ratio of the frequencies is

$$r = \frac{1.60\,\text{MHz}}{0.54\,\text{MHz}} = 2.96.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads (pF), then

$$\frac{\sqrt{C + 365\,\text{pF}}}{\sqrt{C + 10\,\text{pF}}} = 2.96.$$

The solution for C is

$$C = \frac{(365\,\text{pF}) - (2.96)^2(10\,\text{pF})}{(2.96)^2 - 1} = 36\,\text{pF}.$$

(c) We solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency, C = 365 pF + 36 pF = 401 pF and f = 0.54 MHz. Thus, the inductance is

$$L = \frac{1}{(2\pi)^2 Cf^2} = \frac{1}{(2\pi)^2(401\times10^{-12}\,\text{F})(0.54\times10^6\,\text{Hz})^2} = 2.2\times10^{-4}\,\text{H}.$$

LEARN One could also use the maximum frequency condition to solve for the inductance of the coil in (d). The capacitance is C = 10 pF + 36 pF = 46 pF and f = 1.60 MHz, so

$$L = \frac{1}{(2\pi)^2 Cf^2} = \frac{1}{(2\pi)^2(46\times10^{-12}\,\text{F})(1.60\times10^6\,\text{Hz})^2} = 2.2\times10^{-4}\,\text{H}.$$

12. (a) Since the percentage of energy stored in the electric field of the capacitor is $(1-75.0\%) = 25.0\%$, then

$$\frac{U_E}{U} = \frac{q^2/2C}{Q^2/2C} = 25.0\%$$

which leads to $q/Q = \sqrt{0.250} = 0.500.$

(b) From

$$\frac{U_B}{U} = \frac{Li^2/2}{LI^2/2} = 75.0\%,$$

we find $i/I = \sqrt{0.750} = 0.866.$

13. (a) The charge (as a function of time) is given by $q = Q\sin\omega t$, where Q is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that q = 0 at time t = 0. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q\cos\omega t,$$

and at t = 0, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC},$

$$Q = I\sqrt{LC} = (2.00\,\text{A})\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 1.80\times10^{-4}\,\text{C}.$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2 \sin^2 \omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2\omega \sin\omega t \cos\omega t}{C}$$

We use the trigonometric identity $\cos\omega t \sin\omega t = \frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C}\sin(2\omega t).$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2$ rad. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} = \frac{\pi}{4}\sqrt{(3.00\times10^{-3}\text{ H})(2.70\times10^{-6}\text{ F})} = 7.07\times10^{-5}\text{s}.$$

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}.$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times10^{-3}\text{ H})(2.70\times10^{-6}\text{ F})} = 5.655\times10^{-4}\text{s}$, so

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{\pi(1.80\times10^{-4}\text{ C})^2}{(5.655\times10^{-4}\text{ s})(2.70\times10^{-6}\text{ F})} = 66.7\text{ W}.$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

14. The capacitors C_1 and C_2 can be used in four different ways: (1) C_1 only; (2) C_2 only; (3) C_1 and C_2 in parallel; and (4) C_1 and C_2 in series.

(a) The smallest oscillation frequency is

$$f_3 = \frac{1}{2\pi\sqrt{L(C_1+C_2)}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\text{ H})(2.0\times10^{-6}\text{ F}+5.0\times10^{-6}\text{ F})}}.$$
$$= 6.0\times10^2\text{ Hz}$$

(b) The second smallest oscillation frequency is

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\,\text{H})(5.0\times10^{-6}\,\text{F})}} = 7.1\times10^2\,\text{Hz}.$$

(c) The second largest oscillation frequency is

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\,\text{H})(2.0\times10^{-6}\,\text{F})}} = 1.1\times10^3\,\text{Hz}.$$

(d) The largest oscillation frequency is

$$f_4 = \frac{1}{2\pi\sqrt{LC_1C_2/(C_1+C_2)}} = \frac{1}{2\pi}\sqrt{\frac{2.0\times10^{-6}\,\text{F}+5.0\times10^{-6}\,\text{F}}{(1.0\times10^{-2}\,\text{H})(2.0\times10^{-6}\,\text{F})(5.0\times10^{-6}\,\text{F})}} = 1.3\times10^3\,\text{Hz}.$$

15. (a) The maximum charge is

$$Q = CV_{\text{max}} = (1.0 \times 10^{-9}\ \text{F})(3.0\ \text{V}) = 3.0 \times 10^{-9}\ \text{C}.$$

(b) From $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ we get

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.0\times10^{-9}\ \text{C}}{\sqrt{(3.0\times10^{-3}\ \text{H})(1.0\times10^{-9}\ \text{F})}} = 1.7\times10^{-3}\ \text{A}.$$

(c) When the current is at a maximum, the magnetic energy is at a maximum also:

$$U_{B,\text{max}} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0\times10^{-3}\ \text{H})(1.7\times10^{-3}\ \text{A})^2 = 4.5\times10^{-9}\ \text{J}.$$

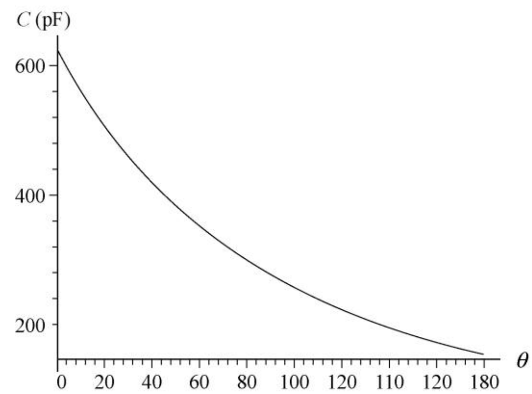
16. The linear relationship between θ (the knob angle in degrees) and frequency f is

$$f = f_0\left(1+\frac{\theta}{180^\circ}\right) \Rightarrow \theta = 180^\circ\left(\frac{f}{f_0}-1\right)$$

where $f_0 = 2 \times 10^5$ Hz. Since $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, we are able to solve for C in terms of θ:

$$C = \frac{1}{4\pi^2 L f_0^2 (1+\theta/180^\circ)^2} = \frac{81}{400000\pi^2(180^\circ+\theta)^2}$$

with SI units understood. After multiplying by 10^{12} (to convert to picofarads), this is plotted next:



17. (a) After the switch is thrown to position *b* the circuit is an *LC* circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(54.0\times10^{-3}\text{ H})(6.20\times10^{-6}\text{ F})}} = 275\text{ Hz}.$$

(b) When the switch is thrown, the capacitor is charged to V = 34.0 V and the current is zero. Thus, the maximum charge on the capacitor is

$$Q = VC = (34.0\text{ V})(6.20\times10^{-6}\text{ F}) = 2.11\times10^{-4}\text{ C}.$$

The current amplitude is

$$I = \omega Q = 2\pi f Q = 2\pi(275\text{ Hz})(2.11\times10^{-4}\text{ C}) = 0.365\text{ A}.$$

18. (a) From $V = IX_C$ we find $\omega = I/CV$. The period is then $T = 2\pi/\omega = 2\pi CV/I$ = 46.1 μs.

(b) The maximum energy stored in the capacitor is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}(2.20\times10^{-7}\text{ F})(0.250\text{ V})^2 = 6.88\times10^{-9}\text{ J}.$$

(c) The maximum energy stored in the inductor is also $U_B = LI^2/2 = 6.88\text{ nJ}$.

(d) We apply Eq. 30-35 as $V = L(di/dt)_{max}$. We can substitute $L = CV^2/I^2$ (combining what we found in part (a) with Eq. 31-4) into Eq. 30-35 (as written above) and solve for $(di/dt)_{max}$. Our result is

$$\left(\frac{di}{dt}\right)_{max} = \frac{V}{L} = \frac{V}{CV^2/I^2} = \frac{I^2}{CV} = \frac{(7.50\times10^{-3}\text{ A})^2}{(2.20\times10^{-7}\text{ F})(0.250\text{ V})} = 1.02\times10^3\text{ A/s}.$$

(e) The derivative of $U_B = \frac{1}{2}Li^2$ leads to

$$\frac{dU_B}{dt} = LI^2\omega \sin\omega t \cos\omega t = \frac{1}{2}LI^2\omega \sin 2\omega t .$$

Therefore, $\left(\frac{dU_B}{dt}\right)_{\max} = \frac{1}{2}LI^2\omega = \frac{1}{2}IV = \frac{1}{2}(7.50\times10^{-3}\text{ A})(0.250\text{ V}) = 0.938\text{ mW}.$

19. The loop rule, for just two devices in the loop, reduces to the statement that the magnitude of the voltage across one of them must equal the magnitude of the voltage across the other. Consider that the capacitor has charge q and a voltage (which we'll consider positive in this discussion) $V = q/C$. Consider at this moment that the current in the inductor at this moment is directed in such a way that the capacitor charge is increasing (so $i = +dq/dt$). Equation 30-35 then produces a positive result equal to the V across the capacitor: $V = -L(di/dt)$, and we interpret the fact that $-di/dt > 0$ in this discussion to mean that $d(dq/dt)/dt = d^2q/dt^2 < 0$ represents a "deceleration" of the charge-buildup process on the capacitor (since it is approaching its maximum value of charge). In this way we can "check" the signs in Eq. 31-11 (which states $q/C = -L\, d^2q/dt^2$) to make sure we have implemented the loop rule correctly.

20. (a) We use $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ to solve for L:

$$L = \frac{1}{C}\left(\frac{Q}{I}\right)^2 = \frac{1}{C}\left(\frac{CV_{\max}}{I}\right)^2 = C\left(\frac{V_{\max}}{I}\right)^2 = \left(4.00\times10^{-6}\text{ F}\right)\left(\frac{1.50\text{ V}}{50.0\times10^{-3}\text{ A}}\right)^2 = 3.60\times10^{-3}\text{ H}.$$

(b) Since $f = \omega/2\pi$, the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(3.60\times10^{-3}\text{ H}\right)\left(4.00\times10^{-6}\text{ F}\right)}} = 1.33\times10^3\text{ Hz}.$$

(c) Referring to Fig. 31-1, we see that the required time is one-fourth of a period (where the period is the reciprocal of the frequency). Consequently,

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{4\left(1.33\times10^3\text{ Hz}\right)} = 1.88\times10^{-4}\text{ s}.$$

21. (a) We compare this expression for the current with $i = I\sin(\omega t+\phi_0)$. Setting $(\omega t+\phi) = 2500t + 0.680 = \pi/2$, we obtain $t = 3.56\times10^{-4}$ s.

(b) Since $\omega = 2500$ rad/s $= (LC)^{-1/2}$,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500\,\text{rad/s})^2(64.0\times10^{-6}\,\text{F})} = 2.50\times10^{-3}\ \text{H}.$$

(c) The energy is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(2.50\times10^{-3}\,\text{H})(1.60\,\text{A})^2 = 3.20\times10^{-3}\ \text{J}.$$

22. For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\omega' = \frac{1}{\sqrt{L_{\text{eq}}C_{\text{eq}}}} = \frac{1}{\sqrt{(L_1+L_2)C_1C_2/(C_1+C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2+L_2C_2C_1)/(C_1+C_2)}}$$
$$= \frac{1}{\sqrt{L_1C_1}}\frac{1}{\sqrt{(C_1+C_2)/(C_1+C_2)}} = \omega,$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

23. (a) The total energy U is the sum of the energies in the inductor and capacitor:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{i^2L}{2} = \frac{(3.80\times10^{-6}\,\text{C})^2}{2(7.80\times10^{-6}\,\text{F})} + \frac{(9.20\times10^{-3}\,\text{A})^2(25.0\times10^{-3}\,\text{H})}{2} = 1.98\times10^{-6}\ \text{J}.$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2(7.80\times10^{-6}\,\text{F})(1.98\times10^{-6}\,\text{J})} = 5.56\times10^{-6}\ \text{C}.$$

(c) From $U = I^2L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.98\times10^{-6}\,\text{J})}{25.0\times10^{-3}\,\text{H}}} = 1.26\times10^{-2}\ \text{A}.$$

(d) If q_0 is the charge on the capacitor at time $t = 0$, then $q_0 = Q\cos\phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80\times10^{-6}\,\text{C}}{5.56\times10^{-6}\,\text{C}}\right) = \pm 46.9°.$$

For $\phi = +46.9°$ the charge on the capacitor is decreasing, for $\phi = -46.9°$ it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for $t = 0$.

We obtain $-\omega Q \sin\phi$, which we wish to be positive. Since sin(+46.9°) is positive and sin(–46.9°) is negative, the correct value for increasing charge is $\phi = -46.9°$.

(e) Now we want the derivative to be negative and $\sin\phi$ to be positive. Thus, we take $\phi = +46.9°$.

24. The charge q after N cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 31-25:

$$\begin{aligned} q &= Qe^{-Rt/2L}\cos\left(\omega' t+\phi\right) = Qe^{-RNT/2L}\cos\left[\omega'\left(2\pi N/\omega'\right)+\phi\right] \\ &= Qe^{-RN\left(2\pi\sqrt{L/C}\right)/2L}\cos\left(2\pi N+\phi\right) \\ &= Qe^{-N\pi R\sqrt{C/L}}\cos\phi. \end{aligned}$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q\cos\phi$, where $q_0 = 6.2\ \mu$C is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0\exp\left(-N\pi R\sqrt{C/L}\right)$.

(a) For $N = 5$, $q_5 = (6.2\,\mu\text{C})\exp\left(-5\pi(7.2\,\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.85\,\mu\text{C}$.

(b) For $N = 10$, $q_{10} = (6.2\,\mu\text{C})\exp\left(-10\pi(7.2\,\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.52\,\mu\text{C}$.

(c) For $N = 100$, $q_{100} = (6.2\,\mu\text{C})\exp\left(-100\pi(7.2\,\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 1.93\,\mu\text{C}$.

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$\begin{aligned} t &= 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) = 50\left(2\pi\sqrt{\left(220\times10^{-3}\,\text{H}\right)\left(12.0\times10^{-6}\,\text{F}\right)}\right) \\ &= 0.5104\,\text{s}. \end{aligned}$$

The maximum charge on the capacitor decays according to $q_{\max} = Qe^{-Rt/2L}$ (this is called the *exponentially decaying amplitude* in Section 31-5), where Q is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 31-25). Dividing by Q and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R=-\frac{2L}{t}\ln\left(\frac{q_{max}}{Q}\right)=-\frac{2\left(220\times10^{-3}\text{ H}\right)}{0.5104\text{ s}}\ln\left(0.99\right)=8.66\times10^{-3}\ \Omega.$$

26. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 31-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{max}^2/2C$, where q_{max} is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{max}^2}{2C}=\frac{1}{2}\frac{Q^2}{2C}\Rightarrow q_{max}=\frac{Q}{\sqrt{2}}.$$

Now q_{max} (referred to as the *exponentially decaying amplitude* in Section 31-5) is related to Q (and the other parameters of the circuit) by

$$q_{max}=Qe^{-Rt/2L}\Rightarrow\ln\left(\frac{q_{max}}{Q}\right)=-\frac{Rt}{2L}.$$

Setting $q_{max}=Q/\sqrt{2}$, we solve for t:

$$t=-\frac{2L}{R}\ln\left(\frac{q_{max}}{Q}\right)=-\frac{2L}{R}\ln\left(\frac{1}{\sqrt{2}}\right)=\frac{L}{R}\ln 2\ .$$

The identities $\ln(1/\sqrt{2})=-\ln\sqrt{2}=-\frac{1}{2}\ln 2$ were used to obtain the final form of the result.

27. **THINK** With the presence of a resistor in the *RLC* circuit, oscillation is damped, and the total electromagnetic energy of the system is no longer conserved, as some energy is transferred to thermal energy in the resistor.

EXPRESS Let t be a time at which the capacitor is fully charged in some cycle and let $q_{max\,1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t)=\frac{q_{max1}^2}{2C}=\frac{Q^2}{2C}e^{-Rt/L}$$

where

$$q_{max1}=Qe^{-Rt/2L}$$

(see the discussion of the *exponentially decaying amplitude* in Section 31-5). One period later the charge on the fully charged capacitor is

$$q_{max2}=Qe^{-R(t+T)2/L}$$

where $T = \dfrac{2\pi}{\omega'}$, and the energy is

$$U(t+T) = \frac{q_{\max 2}^2}{2C} = \frac{Q^2}{2C} e^{-R(t+T)/L}.$$

ANALYZE The fractional loss in energy is

$$\frac{|\Delta U|}{U} = \frac{U(t) - U(t+T)}{U(t)} = \frac{e^{-Rt/L} - e^{-R(t+T)/L}}{e^{-Rt/L}} = 1 - e^{-RT/L}.$$

Assuming that RT/L is very small compared to 1 (which would be the case if the resistance is small), we expand the exponential (see Appendix E). The first few terms are:

$$e^{-RT/L} \approx 1 - \frac{RT}{L} + \frac{R^2T^2}{2L^2} + \cdots.$$

If we approximate $\omega \approx \omega'$, then we can write T as $2\pi/\omega$. As a result, we obtain

$$\frac{|\Delta U|}{U} \approx 1 - \left(1 - \frac{RT}{L} + \cdots\right) \approx \frac{RT}{L} = \frac{2\pi R}{\omega L}.$$

LEARN The ratio $|\Delta U|/U$ can be rewritten as

$$\frac{|\Delta U|}{U} = \frac{2\pi}{Q}$$

where $Q = \omega L/R$ (not to confuse Q with charge) is called the "quality factor" of the oscillating circuit. A high-Q circuit has low resistance and hence, low fractional energy loss.

28. (a) We use $I = \varepsilon/X_c = \omega_d C\varepsilon$:

$$I = \omega_d C\varepsilon_m = 2\pi f_d C\varepsilon_m = 2\pi(1.00\times10^3\,\text{Hz})(1.50\times10^{-6}\,\text{F})(30.0\text{ V}) = 0.283\text{ A}.$$

(b) $I = 2\pi(8.00 \times 10^3\text{ Hz})(1.50 \times 10^{-6}\text{ F})(30.0\text{ V}) = 2.26\text{ A}$.

29. (a) The current amplitude I is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \varepsilon_m$. Therefore,

$$I = \frac{V_L}{X_L} = \frac{\varepsilon_m}{2\pi f_d L} = \frac{30.0\text{V}}{2\pi(1.00\times10^3\,\text{Hz})(50.0\times10^{-3}\text{H})} = 0.0955\,\text{A} = 95.5\text{ mA}.$$

(b) The frequency is now eight times larger than in part (a), so the inductive reactance X_L is eight times larger and the current is one-eighth as much. The current is now

$$I = (0.0955 \text{ A})/8 = 0.0119 \text{ A} = 11.9 \text{ mA}.$$

30. (a) The current through the resistor is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0\,\text{V}}{50.0\,\Omega} = 0.600 \text{ A} .$$

(b) Regardless of the frequency of the generator, the current is the same, $I = 0.600$ A .

31. (a) The inductive reactance for angular frequency ω_d is given by $X_L = \omega_d L$, and the capacitive reactance is given by $X_C = 1/\omega_d C$. The two reactances are equal if $\omega_d L = 1/\omega_d C$, or $\omega_d = 1/\sqrt{LC}$. The frequency is

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.0\times10^{-3}\text{H})(10\times10^{-6}\text{F})}} = 6.5\times10^{2} \text{ Hz}.$$

(b) The inductive reactance is

$$X_L = \omega_d L = 2\pi f_d L = 2\pi(650 \text{ Hz})(6.0 \times 10^{-3} \text{ H}) = 24\ \Omega.$$

The capacitive reactance has the same value at this frequency.

(c) The natural frequency for free LC oscillations is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, the same as we found in part (a).

32. (a) The circuit consists of one generator across one inductor; therefore, $\varepsilon_m = V_L$. The current amplitude is

$$I = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{\omega_d L} = \frac{25.0 \text{ V}}{(377 \text{ rad/s})(12.7 \text{ H})} = 5.22\times10^{-3} \text{ A} .$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 30-35 gives $\varepsilon_L = 0$ at that instant. Stated another way, since $\varepsilon(t)$ and $i(t)$ have a 90° phase difference, then $\varepsilon(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 31-28 with $\varepsilon = -\varepsilon_m/2$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 31-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I\sin\left(2n\pi - \frac{5\pi}{6} - \frac{\pi}{2}\right) = (5.22\times10^{-3}\text{ A})\left(\frac{\sqrt{3}}{2}\right) = 4.51\times10^{-3}\text{ A}.$$

33. **THINK** Our circuit consists of an ac generator that produces an alternating current, as well as a load that could be purely resistive, capacitive, or inductive. The nature of the load can be determined by the phase angle between the current and the emf.

EXPRESS The generator emf and the current are given by

$$\varepsilon = \varepsilon_m\sin(\omega_d - \pi/4), \quad i(t) = I\sin(\omega_d - 3\pi/4).$$

The expressions show that the emf is maximum when $\sin(\omega_d t - \pi/4) = 1$ or

$$\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi \quad [n = \text{integer}].$$

Similarly, the current is maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or

$$\omega_d t - 3\pi/4 = (\pi/2) \pm 2n\pi \quad [n = \text{integer}].$$

ANALYZE (a) The first time the emf reaches its maximum after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ (that is, $n = 0$). Therefore,

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350\text{ rad/s})} = 6.73\times10^{-3}\text{ s}.$$

(b) The first time the current reaches its maximum after $t = 0$ is when $\omega_d t - 3\pi/4 = \pi/2$, as in part (a) with $n = 0$. Therefore,

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350\text{ rad/s})} = 1.12\times10^{-2}\text{ s}.$$

(c) The current lags the emf by $+\pi/2$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \varepsilon_m$. Thus, $\varepsilon_m = I\omega_d L$ and

$$L = \frac{\varepsilon_m}{I\omega_d} = \frac{30.0\text{ V}}{(620\times10^{-3}\text{A})(350\text{ rad/s})} = 0.138\text{ H}.$$

LEARN The current in the circuit can be rewritten as

$$i(t) = I\sin\left(\omega_d - \frac{3\pi}{4}\right) = I\sin\left(\omega_d - \frac{\pi}{4} - \phi\right)$$

where $\phi = +\pi/2$. In a purely inductive circuit, the current lags the voltage by $90°$.

34. (a) The circuit consists of one generator across one capacitor; therefore, $\varepsilon_m = V_C$. Consequently, the current amplitude is

$$I = \frac{\varepsilon_m}{X_C} = \omega C\varepsilon_m = (377\ \text{rad/s})(4.15\times10^{-6}\,\text{F})(25.0\ \text{V}) = 3.91\times10^{-2}\,\text{A}\ .$$

(b) When the current is at a maximum, the charge on the capacitor is changing at its largest rate. This happens not when it is fully charged ($\pm q_{\max}$), but rather as it passes through the (momentary) states of being uncharged ($q = 0$). Since $q = CV$, then the voltage across the capacitor (and at the generator, by the loop rule) is zero when the current is at a maximum. Stated more precisely, the time-dependent emf $\varepsilon(t)$ and current $i(t)$ have a $\phi = -90°$ phase relation, implying $\varepsilon(t) = 0$ when $i(t) = I$. The fact that $\phi = -90° = -\pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\varepsilon = -\frac{1}{2}\varepsilon_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I\sin\left(2n\pi - \frac{5\pi}{6} + \frac{\pi}{2}\right) = (3.91\times10^{-3}\,\text{A})\left(-\frac{\sqrt{3}}{2}\right) = -3.38\times10^{-2}\,\text{A},$$

or $|i| = 3.38\times10^{-2}\,\text{A}$.

35. The resistance of the coil is related to the reactances and the phase constant by Eq. 31-65. Thus,

$$\frac{X_L - X_C}{R} = \frac{\omega_d L - 1/\omega_d C}{R} = \tan\phi\ ,$$

which we solve for R:

$$R = \frac{1}{\tan\phi}\left(\omega_d L - \frac{1}{\omega_d C}\right) = \frac{1}{\tan 75°}\left[(2\pi)(930\,\text{Hz}(8.8\times10^{-2}\,\text{H}) - \frac{1}{(2\pi)(930\,\text{Hz})(0.94\times10^{-6}\,\text{F}}\right]$$
$$= 89\,\Omega.$$

36. (a) The circuit has a resistor and a capacitor (but no inductor). Since the capacitive reactance decreases with frequency, then the asymptotic value of Z must be the resistance: $R = 500\ \Omega$.

(b) We describe three methods here (each using information from different points on the graph):

method 1: At $\omega_d = 50$ rad/s, we have $Z \approx 700\ \Omega$, which gives $C = (\omega_d\sqrt{Z^2 - R^2}\,)^{-1} = 41\ \mu$F.

method 2: At $\omega_d = 50$ rad/s, we have $X_C \approx 500\ \Omega$, which gives $C = (\omega_d X_C)^{-1} = 40\ \mu$F.

method 3: At $\omega_d = 250$ rad/s, we have $X_C \approx 100\ \Omega$, which gives $C = (\omega_d X_C)^{-1} = 40\ \mu$F.

37. The rms current in the motor is

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + X_L^2}} = \frac{420\,\text{V}}{\sqrt{(45.0\,\Omega)^2 + (32.0\,\Omega)^2}} = 7.61\,\text{A}.$$

38. (a) The graph shows that the resonance angular frequency is 25000 rad/s, which means (using Eq. 31-4)

$$C = (\omega^2 L)^{-1} = [(25000)^2 \times 200 \times 10^{-6}]^{-1} = 8.0\ \mu\text{F}.$$

(b) The graph also shows that the current amplitude at resonance is 4.0 A, but at resonance the impedance Z becomes purely resistive ($Z = R$) so that we can divide the emf amplitude by the current amplitude at resonance to find R: 8.0/4.0 = 2.0 Ω.

39. (a) Now $X_L = 0$, while $R = 200\ \Omega$ and $X_C = 1/2\pi f_d C = 177\ \Omega$. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200\,\Omega)^2 + (177\,\Omega)^2} = 267\,\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{200\,\Omega}\right) = -41.5°$$

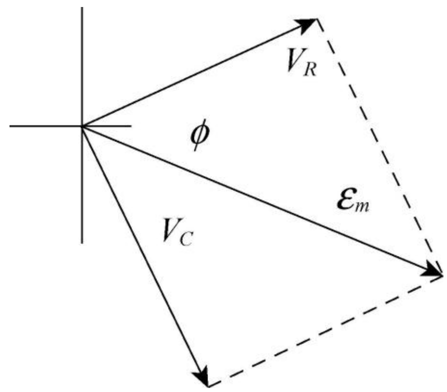
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{267\,\Omega} = 0.135\ \text{A}\ .$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.135\,\text{A})(200\,\Omega) \approx 27.0\,\text{V}$$
$$V_C = IX_C = (0.135\,\text{A})(177\,\Omega) \approx 23.9\,\text{V}$$

The circuit is capacitive, so *I* leads ε_m. The phasor diagram is drawn to scale next.



40. A phasor diagram very much like Fig. 31-14(d) leads to the condition:

$$V_L - V_C = (6.00\text{ V})\sin(30°) = 3.00\text{ V}.$$

With the magnitude of the capacitor voltage at 5.00 V, this gives a inductor voltage magnitude equal to 8.00 V. Since the capacitor and inductor voltage phasors are 180° out of phase, the potential difference across the inductor is -8.00 V.

41. **THINK** We have a series *RLC* circuit. Since *R*, *L*, and *C* are in series, the same current is driven in all three of them.

EXPRESS The capacitive and the inductive reactances can be written as

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C}, \quad X_L = \omega_d L = 2\pi f_d L.$$

The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the current amplitude is given by $I = \varepsilon_m / Z$.

ANALYZE (a) Substituting the values given, we find the capacitive reactance to be

$$X_C = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\text{ Hz})(70.0\times10^{-6}\text{ F})} = 37.9\ \Omega.$$

Similarly, the inductive reactance is

$$X_L = 2\pi f_d L = 2\pi(60.0\text{ Hz})(230\times10^{-3}\text{ H}) = 86.7\ \Omega.$$

Thus, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (37.9\ \Omega - 86.7\ \Omega)^2} = 206\ \Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\ \Omega - 37.9\ \Omega}{200\ \Omega}\right) = 13.7°.$$

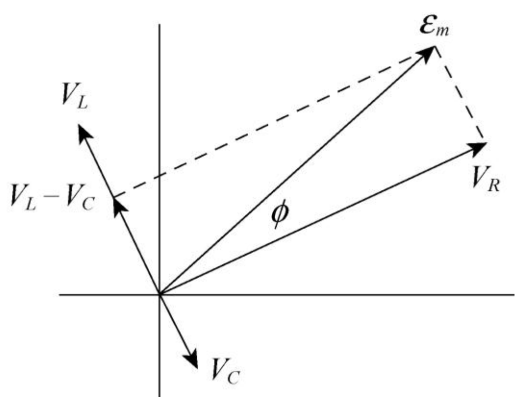
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{206\Omega} = 0.175\,\text{A}.$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.175\ \text{A})(200\ \Omega) = 35.0\ \text{V}$$
$$V_L = IX_L = (0.175\ \text{A})(86.7\ \Omega) = 15.2\ \text{V}$$
$$V_C = IX_C = (0.175\ \text{A})(37.9\ \Omega) = 6.62\,\text{V}$$

Note that $X_L > X_C$, so that ε_m leads *I*. The phasor diagram is drawn to scale below.



LEARN The circuit in this problem is more inductive since $X_L > X_C$. The phase angle is positive, so the current lags behind the applied emf.

42. (a) Since $Z = \sqrt{R^2 + X_L^2}$ and $X_L = \omega_d L$, then as $\omega_d \to 0$ we find $Z \to R = 40\ \Omega$.

(b) $L = X_L/\omega_d = slope = 60$ mH.

43. (a) Now $X_C = 0$, while $R = 200\ \Omega$ and

$$X_L = \omega L = 2\pi f_d L = 86.7\ \Omega$$

both remain unchanged. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(200\ \Omega)^2 + (86.7\ \Omega)^2} = 218\ \Omega\ .$$

(b) The phase angle is, from Eq. 31-65,

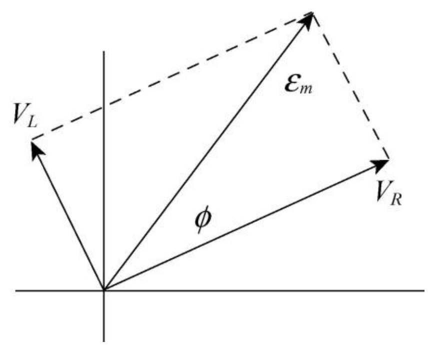
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 0}{200\ \Omega}\right) = 23.4°\ .$$

(c) The current amplitude is now found to be $I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{218\ \Omega} = 0.165\ \text{A}$.

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.165\ \text{A})(200\Omega) \approx 33\,\text{V}$$
$$V_L = IX_L = (0.165\,\text{A})(86.7\Omega) \approx 14.3\,\text{V}.$$

This is an inductive circuit, so ε_m leads I. The phasor diagram is drawn to scale next.



44. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\ \text{Hz})(24.0\times10^{-6}\text{F})} = 16.6\ \Omega\ .$$

(b) The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi fL - X_C)^2}$$
$$= \sqrt{(220\,\Omega)^2 + [2\pi(400\ \text{Hz})(150\times10^{-3}\ \text{H}) - 16.6\ \Omega]^2} = 422\ \Omega\ .$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{220\,\text{V}}{422\,\Omega} = 0.521\ \text{A}\ .$$

(d) Now $X_C \propto C_{\text{eq}}^{-1}$. Thus, X_C increases as C_{eq} decreases.

(e) Now $C_{\text{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\ \Omega)^2 + [2\pi(400\text{ Hz})(150\times10^{-3}\text{ H}) - 2(16.6\ \Omega)]^2} = 408\ \Omega < 422\ \Omega .$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

45. (a) Yes, the voltage amplitude across the inductor can be much larger than the amplitude of the generator emf.

(b) The amplitude of the voltage across the inductor in an *RLC* series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\text{ H}}{\sqrt{(1.0\text{ H})(1.0\times10^{-6}\text{F})}} = 1000\ \Omega .$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

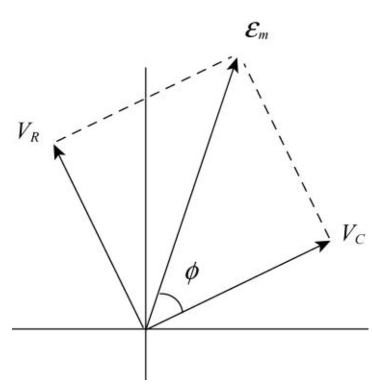
$$I = \left.\frac{\varepsilon_m}{Z}\right|_{\text{resonance}} = \frac{\varepsilon_m}{R} = \frac{10\text{ V}}{10\ \Omega} = 1.0\text{ A} .$$

The voltage amplitude across the inductor is therefore

$$V_L = IX_L = (1.0\text{ A})(1000\ \Omega) = 1.0\times10^3\text{ V}$$

which is much larger than the amplitude of the generator emf.

46. (a) A sketch of the phasor diagram is shown to the right.



(b) We have $I R = I X_C$, or

$$I R = I X_C \rightarrow R = \frac{1}{\omega_d C}$$

which yields

$$f = \frac{\omega_d}{2\pi} = \frac{1}{2\pi RC} = \frac{1}{2\pi(50.0\ \Omega)(2.00\times10^{-5}\text{ F})} = 159\text{ Hz}.$$

(c) $\phi = \tan^{-1}(-V_C/V_R) = -45°$.

(d) $\omega_d = 1/RC = 1.00 \times 10^3$ rad/s.

(e) $I = (12\text{ V})/\sqrt{R^2 + X_C^2} = 6/(25\sqrt{2}) \approx 170$ mA.

47. **THINK** In a driven *RLC* circuit, the current amplitude is maximum at resonance, where the driven angular frequency is equal to the natural angular frequency.

EXPRESS For a given amplitude ε_m of the generator emf, the current amplitude is given by

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}.$$

To explicitly show that I is maximum when $\omega_d = \omega = 1/\sqrt{LC}$, we differentiate I with respect to ω_d and set the derivative to zero:

$$\frac{dI}{d\omega_d} = -(E)_m[R^2 + (\omega_d L - 1/\omega_d C)^2]^{-3/2}\left(\omega_d L - \frac{1}{\omega_d C}\right)\left(L + \frac{1}{\omega_d^2 C}\right).$$

The only factor that can equal zero is when $\omega_d L - (1/\omega_d C)$, or $\omega_d = 1/\sqrt{LC} = \omega$.

ANALYZE (a) For this circuit, the driving angular frequency is

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\text{ H})(20.0\times 10^{-6}\text{ F})}} = 224\text{ rad/s}.$$

(b) When $\omega_d = \omega$, the impedance is $Z = R$, and the current amplitude is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0\text{ V}}{5.00\ \Omega} = 6.00\text{ A}.$$

(c) We want to find the (positive) values of ω_d for which $I = \varepsilon_m/2R$:

$$\frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{\varepsilon_m}{2R}.$$

This may be rearranged to give

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2.$$

Taking the square root of both sides (acknowledging the two ± roots) and multiplying by $\omega_d C$, we obtain

$$\omega_d^2(LC) \pm \omega_d\left(\sqrt{3}CR\right) - 1 = 0.$$

Using the quadratic formula, we find the smallest positive solution

$$\omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{-\sqrt{3}(20.0\times10^{-6}\text{ F})(5.00\ \Omega)}{2(1.00\text{ H})(20.0\times10^{-6}\text{ F})}$$
$$+\frac{\sqrt{3(20.0\times10^{-6}\text{ F})^2(5.00\ \Omega)^2 + 4(1.00\text{ H})(20.0\times10^{-6}\text{ F})}}{2(1.00\text{ H})(20.0\times10^{-6}\text{ F})}$$
$$= 219\text{ rad/s}.$$

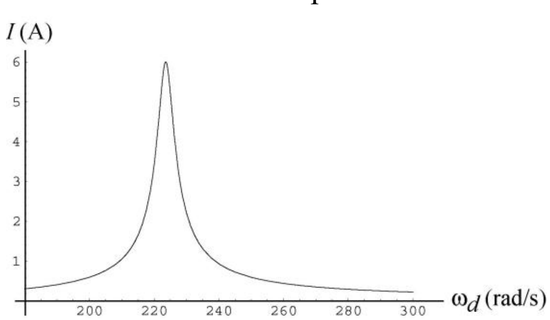
(d) The largest positive solution

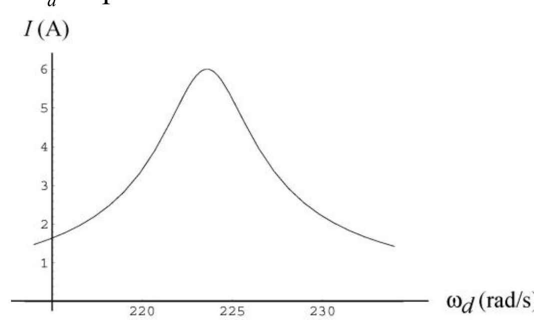
$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{+\sqrt{3}(20.0\times10^{-6}\text{ F})(5.00\Omega)}{2(1.00\text{ H})(20.0\times10^{-6}\text{ F})}$$
$$+\frac{\sqrt{3(20.0\times10^{-6}\text{ F})^2(5.00\ \Omega)^2 + 4(1.00\text{ H})(20.0\times10^{-6}\text{F})}}{2(1.00\text{ H})(20.0\times10^{-6}\text{ F})}$$
$$= 228\text{ rad/s}.$$

(e) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega} = \frac{228\text{ rad/s} - 219\text{ rad/s}}{224\text{ rad/s}} = 0.040.$$

LEARN The current amplitude as a function of ω_d is plotted below.





We see that I is a maximum at $\omega_d = \omega = 224$ rad/s, and is at half maximum (3 A) at 219 rad/s and 228 rad/s.

48. (a) With both switches closed (which effectively removes the resistor from the circuit), the impedance is just equal to the (net) reactance and is equal to

$$X_{net} = (12\ \text{V})/(0.447\ \text{A}) = 26.85\ \Omega.$$

With switch 1 closed but switch 2 open, we have the same (net) reactance as just discussed, but now the resistor is part of the circuit; using Eq. 31-65 we find

$$R = \frac{X_{net}}{\tan\phi} = \frac{26.85\,\Omega}{\tan 15°} = 100\,\Omega\,.$$

(b) For the first situation described in the problem (both switches open) we can reverse our reasoning of part (a) and find

$$X_{\text{net first}} = R\tan\phi' = (100\ \Omega)\tan(-30.9°) = -59.96\ \Omega.$$

We observe that the effect of switch 1 implies

$$X_C = X_{net} - X_{\text{net first}} = 26.85\ \Omega - (-59.96\ \Omega) = 86.81\ \Omega.$$

Then Eq. 31-39 leads to $C = 1/\omega X_C = 30.6\ \mu\text{F}$.

(c) Since $X_{net} = X_L - X_C$, then we find $L = X_L/\omega = 301$ mH .

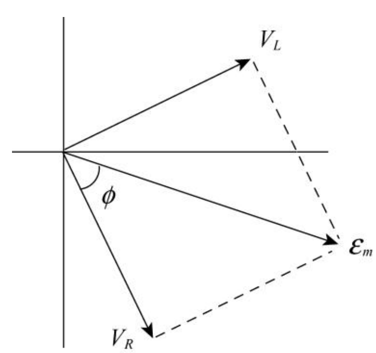
49. (a) Since $L_{eq} = L_1 + L_2$ and $C_{eq} = C_1 + C_2 + C_3$ for the circuit, the resonant frequency is

$$\omega = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_1+C_2+C_3)}}$$
$$= \frac{1}{2\pi\sqrt{(1.70\times10^{-3}\ \text{H}+2.30\times10^{-3}\ \text{H})(4.00\times10^{-6}\ \text{F}+2.50\times10^{-6}\ \text{F}+3.50\times10^{-6}\ \text{F})}}$$
$$= 796\,\text{Hz}.$$

(b) The resonant frequency does not depend on R so it will not change as R increases.

(c) Since $\omega \propto (L_1 + L_2)^{-1/2}$, it will decrease as L_1 increases.

(d) Since $\omega \propto C_{eq}^{-1/2}$ and C_{eq} decreases as C_3 is removed, ω will increase.



50. (a) A sketch of the phasor diagram is shown to the right.

(b) We have $V_R = V_L$, which implies

$$IR = IX_L \;\rightarrow\; R = \omega_d L$$

which yields $f = \omega_d/2\pi = R/2\pi L = 318$ Hz.

(c) $\phi = \tan^{-1}(V_L/V_R) = +45°$.

(d) $\omega_d = R/L = 2.00\times10^3$ rad/s.

(e) $I = (6\text{ V})/\sqrt{R^2 + X_L^2} = 3/(40\sqrt{2}) \approx 53.0$ mA.

51. **THINK** In a driven *RLC* circuit, the current amplitude is maximum at resonance, where the driven angular frequency is equal to the natural angular frequency. It then falls off rapidly away from resonance.

EXPRESS We use the expressions found in Problem 31-47:

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}, \quad \omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} .$$

The resonance angular frequency is $\omega = 1/\sqrt{LC}$.

ANALYZE Thus, the fractional half width is

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}}.$$

LEARN Note that the value of $\Delta\omega_d/\omega$ increases linearly with R; that is, the larger the resistance, the broader the peak. As an example, the data of Problem 31-47 gives

$$\frac{\Delta\omega_d}{\omega} = (5.00\ \Omega)\sqrt{\frac{3\left(20.0\times10^{-6}\text{ F}\right)}{1.00\text{ H}}} = 3.87\times10^{-2}.$$

This is in agreement with the result of Problem 31-47. The method used there, however, gives only one significant figure since two numbers close in value are subtracted ($\omega_1 - \omega_2$). Here the subtraction is done algebraically, and three significant figures are obtained.

52. Since the impedance of the voltmeter is large, it will not affect the impedance of the circuit when connected in parallel with the circuit. So the reading will be 100 V in all three cases.

53. **THINK** Energy is supplied by the 120 V rms ac line to keep the air conditioner running.

EXPRESS The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the average rate of energy delivery is

$$P_{\text{avg}} = I_{\text{rms}}^2 R = \left(\frac{\varepsilon_{\text{rms}}}{Z}\right)^2 R = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2}.$$

ANALYZE (a) Substituting the values given, the impedance is

$$Z = \sqrt{(12.0\ \Omega)^2 + (1.30\ \Omega - 0)^2} = 12.1\ \Omega.$$

(b) The average rate at which energy has been supplied is

$$P_{\text{avg}} = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2} = \frac{(120\ \text{V})^2 (12.0\ \Omega)}{(12.07\ \Omega)^2} = 1.186 \times 10^3\ \text{W} \approx 1.19 \times 10^3\ \text{W}.$$

LEARN In a steady-state operation, the total energy stored in the capacitor and the inductor stays constant. Thus, the net energy transfer is from the generator to the resistor, where electromagnetic energy is dissipated in the form of thermal energy.

54. The amplitude (peak) value is

$$V_{\max} = \sqrt{2} V_{\text{rms}} = \sqrt{2}(100\ \text{V}) = 141\ \text{V}.$$

55. The average power dissipated in resistance R when the current is alternating is given by $P_{\text{avg}} = I_{\text{rms}}^2 R$, where I_{rms} is the root-mean-square current. Since $I_{\text{rms}} = I/\sqrt{2}$, where I is the current amplitude, this can be written $P_{\text{avg}} = I^2R/2$. The power dissipated in the same resistor when the current i_d is direct is given by $P = i_d^2 R$. Setting the two powers equal to each other and solving, we obtain

$$i_d = \frac{I}{\sqrt{2}} = \frac{2.60\ \text{A}}{\sqrt{2}} = 1.84\ \text{A}.$$

56. (a) The power consumed by the light bulb is $P = I^2R/2$. So we must let $P_{\max}/P_{\min} = (I/I_{\min})^2 = 5$, or

$$\left(\frac{I}{I_{\min}}\right)^2 = \left(\frac{\varepsilon_m / Z_{\min}}{\varepsilon_m / Z_{\max}}\right)^2 = \left(\frac{Z_{\max}}{Z_{\min}}\right)^2 = \left(\frac{\sqrt{R^2 + (\omega L_{\max})^2}}{R}\right)^2 = 5.$$

We solve for $L_{\max}$:

$$L_{\max} = \frac{2R}{\omega} = \frac{2(120\ \text{V})^2 / 1000\ \text{W}}{2\pi(60.0\ \text{Hz})} = 7.64 \times 10^{-2}\ \text{H}.$$

(b) Yes, one could use a variable resistor.

(c) Now we must let

$$\left(\frac{R_{\max}+R_{\text{bulb}}}{R_{\text{bulb}}}\right)^2=5,$$

or

$$R_{\max}=\left(\sqrt{5}-1\right)R_{\text{bulb}}=\left(\sqrt{5}-1\right)\frac{(120\text{ V})^2}{1000\text{ W}}=17.8\ \Omega.$$

(d) This is not done because the resistors would consume, rather than temporarily store, electromagnetic energy.

57. We shall use

$$P_{\text{avg}}=\frac{\varepsilon_m^2 R}{2Z^2}=\frac{\varepsilon_m^2 R}{2\left[R^2+\left(\omega_d L-1/\omega_d C\right)^2\right]}.$$

where $Z=\sqrt{R^2+\left(\omega_d L-1/\omega_d C\right)^2}$ is the impedance.

(a) Considered as a function of *C*, P_{avg} has its largest value when the factor $R^2+\left(\omega_d L-1/\omega_d C\right)^2$ has the smallest possible value. This occurs for $\omega_d L=1/\omega_d C$, or

$$C=\frac{1}{\omega_d^2 L}=\frac{1}{(2\pi)^2(60.0\text{ Hz})^2\left(60.0\times10^{-3}\text{ H}\right)}=1.17\times10^{-4}\text{ F}.$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as *C* becomes very small. Thus, the smallest average power occurs for *C* = 0 (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}}=\frac{\varepsilon_m^2}{2R},$$

so the maximum average power is in the resonant case and is equal to

$$P_{\text{avg}}=\frac{(30.0\text{ V})^2}{2(5.00\ \Omega)}=90.0\text{ W}.$$

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0,$$

which implies $\phi = 0°$.

(e) At maximum power, the power factor is cos ϕ = cos 0° = 1.

(f) The minimum average power is P_{avg} = 0 (as it would be for an open switch).

(g) On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that ϕ = –90°.

(h) At minimum power, the power factor is cos ϕ = cos(–90°) = 0.

58. This circuit contains no reactances, so $\varepsilon_{rms} = I_{rms}R_{total}$. Using Eq. 31-71, we find the average dissipated power in resistor R is

$$P_R = I_{rms}^2 R = \left(\frac{\varepsilon_m}{r+R}\right)^2 R.$$

In order to maximize P_R we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\varepsilon_m^2\left[(r+R)^2 - 2(r+R)R\right]}{(r+R)^4} = \frac{\varepsilon_m^2(r-R)}{(r+R)^3} = 0 \quad\Rightarrow\quad R = r$$

59. (a) The rms current is

$$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{\varepsilon_{rms}}{\sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}}$$

$$= \frac{75.0\text{V}}{\sqrt{(15.0\Omega)^2 + \left\{2\pi(550\text{Hz})(25.0\text{mH}) - 1/\left[2\pi(550\text{Hz})(4.70\mu\text{F})\right]\right\}^2}}$$

$$= 2.59\text{A}.$$

(b) The rms voltage across R is $V_{ab} = I_{rms}R = (2.59\,\text{A})(15.0\Omega) = 38.8\,\text{V}$.

(c) The rms voltage across C is

$$V_{bc} = I_{rms}X_C = \frac{I_{rms}}{2\pi fC} = \frac{2.59\text{A}}{2\pi(550\,\text{Hz})(4.70\mu\text{F})} = 159\,\text{V}.$$

(d) The rms voltage across L is

$$V_{cd} = I_{\text{rms}} X_L = 2\pi I_{\text{rms}} fL = 2\pi(2.59\,\text{A})(550\,\text{Hz})(25.0\,\text{mH}) = 224\,\text{V}.$$

(e) The rms voltage across *C* and *L* together is

$$V_{bd} = |V_{bc} - V_{cd}| = |159.5\,\text{V} - 223.7\,\text{V}| = 64.2\,\text{V}.$$

(f) The rms voltage across *R*, *C*, and *L* together is

$$V_{ad} = \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{(38.8\,\text{V})^2 + (64.2\,\text{V})^2} = 75.0\,\text{V}.$$

(g) For the resistor *R*, the power dissipated is $P_R = \dfrac{V_{ab}^2}{R} = \dfrac{(38.8\,\text{V})^2}{15.0\,\Omega} = 100\,\text{W}.$

(h) No energy dissipation in *C*.

(i) No energy dissipation in *L*.

60. The current in the circuit satisfies $i(t) = I\sin(\omega_d t - \phi)$, where

$$\begin{aligned} I &= \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \\ &= \frac{45.0\,\text{V}}{\sqrt{(16.0\,\Omega)^2 + \{(3000\,\text{rad/s})(9.20\,\text{mH}) - 1/[(3000\,\text{rad/s})(31.2\,\mu\text{F})]\}^2}} \\ &= 1.93\,\text{A} \end{aligned}$$

and

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - 1/\omega_d C}{R}\right) \\ &= \tan^{-1}\left[\frac{(3000\,\text{rad/s})(9.20\,\text{mH})}{16.0\,\Omega} - \frac{1}{(3000\,\text{rad/s})(16.0\,\Omega)(31.2\,\mu\text{F})}\right] \\ &= 46.5^\circ. \end{aligned}$$

(a) The power supplied by the generator is

$$\begin{aligned} P_g &= i(t)\varepsilon(t) = I\sin(\omega_d t - \phi)\,\varepsilon_m \sin\omega_d t \\ &= (1.93\,\text{A})(45.0\,\text{V})\sin[(3000\,\text{rad/s})(0.442\,\text{ms})]\sin[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5^\circ] \\ &= 41.4\,\text{W}. \end{aligned}$$

(b) With

$$v_c(t) = V_c \sin(\omega_d t - \phi - \pi/2) = -V_c \cos(\omega_d t - \phi)$$

where $V_c = I/\omega_d C$, the rate at which the energy in the capacitor changes is

$$\begin{aligned} P_c &= \frac{d}{dt}\left(\frac{q^2}{2C}\right) = i\frac{q}{C} = iv_c \\ &= -I\sin(\omega_d t - \phi)\left(\frac{I}{\omega_d C}\right)\cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C}\sin\left[2(\omega_d t - \phi)\right] \\ &= -\frac{(1.93\,\text{A})^2}{2(3000\,\text{rad/s})(31.2\times10^{-6}\,\text{F})}\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)\right] \\ &= -17.0\ \text{W}. \end{aligned}$$

(c) The rate at which the energy in the inductor changes is

$$\begin{aligned} P_L &= \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = LI\sin(\omega_d t - \phi)\frac{d}{dt}\left[I\sin(\omega_d t - \phi)\right] = \frac{1}{2}\omega_d LI^2\sin\left[2(\omega_d t - \phi)\right] \\ &= \frac{1}{2}(3000\,\text{rad/s})(1.93\,\text{A})^2(9.20\,\text{mH})\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)\right] \\ &= 44.1\ \text{W}. \end{aligned}$$

(d) The rate at which energy is being dissipated by the resistor is

$$\begin{aligned} P_R &= i^2R = I^2R\sin^2(\omega_d t - \phi) = (1.93\,\text{A})^2(16.0\,\Omega)\sin^2\left[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5°\right] \\ &= 14.4\ \text{W}. \end{aligned}$$

(e) Equal. $P_L + P_R + P_c = 44.1\,\text{W} - 17.0\ \text{W} + 14.4\,\text{W} = 41.5\ \text{W} = P_g$.

61. **THINK** We have an ac generator connected to a "black box," whose load is of the form of an *RLC* circuit. Given the functional forms of the emf and the current in the circuit, we can deduce the nature of the load.

EXPRESS In general, the driving emf and the current can be written as

$$\varepsilon(t) = \varepsilon_m \sin\omega_d t, \quad i(t) = I\sin(\omega_d t - \phi).$$

Thus, we have $\varepsilon_m = 75$ V, I = 1.20 A, and $\phi = -42°$ for this circuit. The power factor of the circuit is simply given by $\cos\phi$.

ANALYZE (a) With $\phi = -42.0°$, we obtain $\cos\phi = \cos(-42.0°) = 0.743$.

(b) Since the phase constant is negative, $\phi < 0$, $\omega t - \phi > \omega t$. The current leads the emf.

(c) The phase constant is related to the reactance difference by $\tan\phi = (X_L - X_C)/R$. We have

$$\tan\phi = \tan(-42.0°) = -0.900,$$

a negative number. Therefore, $X_L - X_C$ is negative, which implies that $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit were in resonance, X_L would be the same as X_C, then $\tan\phi$ would be zero, and ϕ would be zero as well. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e) Since $\tan\phi$ is negative and finite, neither the capacitive reactance nor the resistance is zero. This means the box must contain a capacitor and a resistor.

(f) The inductive reactance may be zero, so there need not be an inductor.

(g) Yes, there is a resistor.

(h) The average power is

$$P_{\text{avg}} = \frac{1}{2}\varepsilon_m I\cos\phi = \frac{1}{2}(75.0\text{ V})(1.20\text{ A})(0.743) = 33.4\text{ W}.$$

(i) The answers above depend on the frequency only through the phase constant ϕ, which is given. If values were given for R, L, and C, then the value of the frequency would also be needed to compute the power factor.

LEARN The phase constant ϕ allows us to calculate the power factor and deduce the nature of the load in the circuit. In (f) we stated that the inductance may be set to zero. If there is an inductor, then its reactance must be smaller than the capacitive reactance, $X_L < X_C$.

62. We use Eq. 31-79 to find

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (100\text{ V})\left(\frac{500}{50}\right) = 1.00\times10^3\text{ V}.$$

63. **THINK** The transformer in this problem is a step-down transformer.

EXPRESS If N_p is the number of primary turns, and N_s is the number of secondary turns, then the step-down voltage in the secondary circuit is

$$V_s = V_p\left(\frac{N_s}{N_p}\right).$$

By Ohm's law, the current in the secondary circuit is given by $I_s = V_s / R_s$.

ANALYZE (a) The step-down voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (120\,\text{V})\left(\frac{10}{500}\right) = 2.4\,\text{V}.$$

(b) The current in the secondary is $I_s = \dfrac{V_s}{R_s} = \dfrac{2.4\,\text{V}}{15\,\Omega} = 0.16\,\text{A}.$

We find the primary current from Eq. 31-80:

$$I_p = I_s\left(\frac{N_s}{N_p}\right) = (0.16\,\text{A})\left(\frac{10}{500}\right) = 3.2\times10^{-3}\,\text{A}.$$

(c) As shown above, the current in the secondary is $I_s = 0.16\,\text{A}$.

LEARN In a transformer, the voltages and currents in the secondary circuit are related to that in the primary circuit by

$$V_s = V_p\left(\frac{N_s}{N_p}\right), \qquad I_s = I_p\left(\frac{N_p}{N_s}\right).$$

64. For step-up transformer:

(a) The smallest value of the ratio V_s / V_p is achieved by using T_2T_3 as primary and T_1T_3 as secondary coil: V_{13}/V_{23} = (800 + 200)/800 = 1.25.

(b) The second smallest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_2T_3 as secondary coil: V_{23}/V_{13} = 800/200 = 4.00.

(c) The largest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_1T_3 as secondary coil: V_{13}/V_{12} = (800 + 200)/200 = 5.00.

For the step-down transformer, we simply exchange the primary and secondary coils in each of the three cases above.

(d) The smallest value of the ratio V_s / V_p is 1/5.00 = 0.200.

(e) The second smallest value of the ratio V_s / V_p is 1/4.00 = 0.250.

(f) The largest value of the ratio V_s / V_p is 1/1.25 = 0.800.

65. (a) The rms current in the cable is $I_{\text{rms}} = P/V_t = 250\times10^3\,\text{W}/\left(80\times10^3\,\text{V}\right) = 3.125\,\text{A}$. Therefore, the rms voltage drop is $\Delta V = I_{\text{rms}}R = (3.125\,\text{A})(2)(0.30\,\Omega) = 1.9\,\text{V}$.

(b) The rate of energy dissipation is $P_d = I_{\text{rms}}^2 R = (3.125\,\text{A})(2)(0.60\,\Omega) = 5.9\,\text{W}$.

(c) Now $I_{\text{rms}} = 250\times10^3\,\text{W}/\left(8.0\times10^3\,\text{V}\right) = 31.25\,\text{A}$, so $\Delta V = (31.25\,\text{A})(0.60\,\Omega) = 19\,\text{V}$.

(d) $P_d = (3.125\,\text{A})^2(0.60\,\Omega) = 5.9\times10^2\,\text{W}$.

(e) $I_{\text{rms}} = 250\times10^3\,\text{W}/\left(0.80\times10^3\,\text{V}\right) = 312.5\,\text{A}$, so $\Delta V = (312.5\,\text{A})(0.60\,\Omega) = 1.9\times10^2\,\text{V}$.

(f) $P_d = (312.5\,\text{A})^2(0.60\,\Omega) = 5.9\times10^4\,\text{W}$.

66. (a) The amplifier is connected across the primary windings of a transformer and the resistor R is connected across the secondary windings.

(b) If I_s is the rms current in the secondary coil then the average power delivered to R is $P_{\text{avg}} = I_s^2 R$. Using $I_s = (N_p/N_s)I_p$, we obtain

$$P_{\text{avg}} = \left(\frac{I_p N_p}{N_s}\right)^2 R.$$

Next, we find the current in the primary circuit. This is effectively a circuit consisting of a generator and two resistors in series. One resistance is that of the amplifier (r), and the other is the equivalent resistance R_{eq} of the secondary circuit. Therefore,

$$I_p = \frac{\varepsilon_{\text{rms}}}{r + R_{\text{eq}}} = \frac{\varepsilon_{\text{rms}}}{r + (N_p/N_s)^2 R}$$

where Eq. 31-82 is used for R_{eq}. Consequently,

$$P_{\text{avg}} = \frac{\varepsilon^2 (N_p/N_s)^2 R}{\left[r + (N_p/N_s)^2 R\right]^2}.$$

Now, we wish to find the value of N_p/N_s such that P_{avg} is a maximum. For brevity, let $x = (N_p/N_s)^2$. Then

$$P_{\text{avg}} = \frac{\varepsilon^2 Rx}{(r + xR)^2},$$

and the derivative with respect to x is

$$\frac{dP_{\text{avg}}}{dx} = \frac{\varepsilon^2 R(r - xR)}{(r + xR)^3}.$$

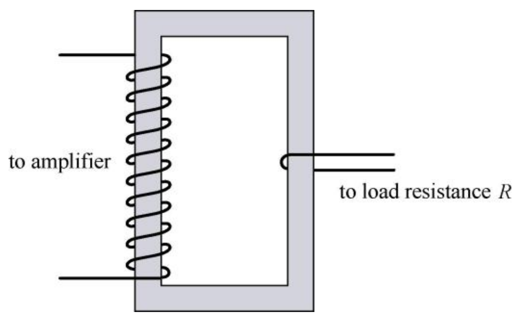
This is zero for

$$x = r/R = (1000\,\Omega)/(10\,\Omega) = 100.$$

We note that for small x, P_{avg} increases linearly with x, and for large x it decreases in proportion to $1/x$. Thus $x = r/R$ is indeed a maximum, not a minimum. Recalling $x = (N_p/N_s)^2$, we conclude that the maximum power is achieved for

$$N_p / N_s = \sqrt{x} = 10.$$

The diagram that follows is a schematic of a transformer with a ten to one turns ratio. An actual transformer would have many more turns in both the primary and secondary coils.



67. (a) Let $\omega t - \pi/4 = \pi/2$ to obtain $t = 3\pi/4\omega = 3\pi/[4(350\,\text{rad/s})] = 6.73\times10^{-3}$ s.

(b) Let $\omega t + \pi/4 = \pi/2$ to obtain $t = \pi/4\omega = \pi/[4(350\,\text{rad/s})] = 2.24\times10^{-3}$ s.

(c) Since i leads ε in phase by $\pi/2$, the element must be a capacitor.

(d) We solve C from $X_C = (\omega C)^{-1} = \varepsilon_m / I$:

$$C = \frac{I}{\varepsilon_m \omega} = \frac{6.20\times10^{-3}\ \text{A}}{(30.0\ \text{V})(350\,\text{rad/s})} = 5.90\times10^{-5}\ \text{F}.$$

68. (a) We observe that ω_d = 12566 rad/s. Consequently, X_L = 754 Ω and X_C = 199 Ω. Hence, Eq. 31-65 gives

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 1.22 \text{ rad} .$$

(b) We find the current amplitude from Eq. 31-60:

$$I = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}} = 0.288 \text{ A} .$$

69. (a) Using $\omega = 2\pi f$, $X_L = \omega L$, $X_C = 1/\omega C$ and $\tan(\phi) = (X_L - X_C)/R$, we find

$$\phi = \tan^{-1}[(16.022 - 33.157)/40.0] = -0.40473 \approx -0.405 \text{ rad}.$$

(b) Equation 31-63 gives $I = 120/\sqrt{40^2 + (16\text{-}33)^2} = 2.7576 \approx 2.76$ A.

(c) $X_C > X_L \Rightarrow$ capacitive.

70. (a) We find L from $X_L = \omega L = 2\pi f L$:

$$f = \frac{X_L}{2\pi L} = \frac{1.30\times10^3\,\Omega}{2\pi\left(45.0\times10^{-3}\,\text{H}\right)} = 4.60\times10^3\,\text{Hz}.$$

(b) The capacitance is found from $X_C = (\omega C)^{-1} = (2\pi f C)^{-1}$:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\left(4.60\times10^3\,\text{Hz}\right)\left(1.30\times10^3\,\Omega\right)} = 2.66\times10^{-8}\,\text{F}.$$

(c) Noting that $X_L \propto f$ and $X_C \propto f^{-1}$, we conclude that when f is doubled, X_L doubles and X_C reduces by half. Thus,

$$X_L = 2(1.30 \times 10^3 \ \Omega) = 2.60 \times 10^3 \ \Omega .$$

(d) $X_C = 1.30 \times 10^3 \ \Omega/2 = 6.50 \times 10^2 \ \Omega$.

71. (a) The impedance is Z = (80.0 V)/(1.25 A) = 64.0 Ω.

(b) We can write $\cos \phi = R/Z$. Therefore,

$$R = (64.0 \ \Omega)\cos(0.650 \text{ rad}) = 50.9 \ \Omega.$$

(c) Since the current leads the emf, the circuit is capacitive.

72. (a) From Eq. 31-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L/1.50)}{(V_L/2.00)}\right)$$

which becomes $\tan^{-1}(2/3) = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 31-60, we have

$$\varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(9.98\text{ V})^2 + (20.0\text{ V} - 13.3\text{ V})^2} = 12.0\text{ V}.$$

73. (a) From Eq. 31-4, we have $L = (\omega^2 C)^{-1} = ((2\pi f)^2 C)^{-1} = 2.41\ \mu$H.

(b) The total energy is the maximum energy on either device (see Fig. 31-4). Thus, we have $U_{max} = \frac{1}{2}LI^2 = 21.4$ pJ.

(c) Of several methods available to do this part, probably the one most "in the spirit" of this problem (considering the energy that was calculated in part (b)) is to appeal to $U_{max} = \frac{1}{2}Q^2/C$ (from Chapter 26) to find the maximum charge: $Q = \sqrt{2CU_{max}} = 82.2$ nC.

74. (a) Equation 31-4 directly gives $1/\sqrt{LC} \approx 5.77\times10^3$ rad/s.

(b) Equation 16-5 then yields $T = 2\pi/\omega = 1.09$ ms.

(c) Although we do not show the graph here, we describe it: it is a cosine curve with amplitude 200 μC and period given in part (b).

75. (a) The impedance is $Z = \dfrac{\varepsilon_m}{I} = \dfrac{125\text{ V}}{3.20\text{ A}} = 39.1\ \Omega.$

(b) From $V_R = IR = \varepsilon_m \cos\phi$, we get

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(125\text{ V})\cos(0.982\text{ rad})}{3.20\text{ A}} = 21.7\ \Omega.$$

(c) Since $X_L - X_C \propto \sin\phi = \sin(-0.982\text{ rad})$, we conclude that $X_L < X_C$. The circuit is predominantly capacitive.

76. (a) Equation 31-39 gives $f = \omega/2\pi = (2\pi C X_C)^{-1} = 8.84$ kHz.

(b) Because of its inverse relationship with frequency, the reactance will go down by a factor of 2 when f increases by a factor of 2. The answer is $X_C = 6.00\ \Omega$.

77. **THINK** The three-phase generator has three ac voltages that are 120° out of phase with each other.

EXPRESS To calculate the potential difference between any two wires, we use the following trigonometric identity:

$$\sin\alpha - \sin\beta = 2\sin\left[(\alpha-\beta)/2\right]\cos\left[(\alpha+\beta)/2\right],$$

where α and β are any two angles.

ANALYZE (a) We consider the following combinations: $\Delta V_{12} = V_1 - V_2$, $\Delta V_{13} = V_1 - V_3$, and $\Delta V_{23} = V_2 - V_3$. For ΔV_{12},

$$\Delta V_{12} = A\sin(\omega_d t) - A\sin(\omega_d t - 120°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 120°}{2}\right) = \sqrt{3}A\cos\left(\omega_d t - 60°\right)$$

where $\sin 60° = \sqrt{3}/2$. Similarly,

$$\Delta V_{13} = A\sin(\omega_d t) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{240°}{2}\right)\cos\left(\frac{2\omega_d t - 240°}{2}\right)$$
$$= \sqrt{3}A\cos\left(\omega_d t - 120°\right)$$

and

$$\Delta V_{23} = A\sin(\omega_d t - 120°) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 360°}{2}\right)$$
$$= \sqrt{3}A\cos\left(\omega_d t - 180°\right).$$

All three expressions are sinusoidal functions of t with angular frequency ω_d.

(b) We note that each of the above expressions has an amplitude of $\sqrt{3}A$.

LEARN A three-phase generator provides a smoother flow of power than a single-phase generator.

78. (a) The effective resistance R_{eff} satisfies $I_{\text{rms}}^2 R_{\text{eff}} = P_{\text{mechanical}}$, or

$$R_{\text{eff}} = \frac{P_{\text{mechanical}}}{I_{\text{rms}}^2} = \frac{(0.100\ \text{hp})(746\ \text{W/hp})}{(0.650\ \text{A})^2} = 177\ \Omega.$$

(b) This is not the same as the resistance R of its coils, but just the effective resistance for power transfer from electrical to mechanical form. In fact $I_{\rm rms}^2 R$ would not give $P_{\rm mechanical}$ but rather the rate of energy loss due to thermal dissipation.

79. **THINK** The total energy in the LC circuit is the sum of electrical energy stored in the capacitor, and the magnetic energy stored in the inductor. Energy is conserved.

EXPRESS Let U_E be the electrical energy in the capacitor and U_B be the magnetic energy in the inductor. The total energy is $U = U_E + U_B$. When $U_E = 0.500U_B$ (at time t), then $U_B = 2.00U_E$ and $U = U_E + U_B = 3.00U_E$. Now, U_E is given by $q^2/2C$, where q is the charge on the capacitor at time t. The total energy U is given by $Q^2/2C$, where Q is the maximum charge on the capacitor.

ANALYZE (a) Thus,

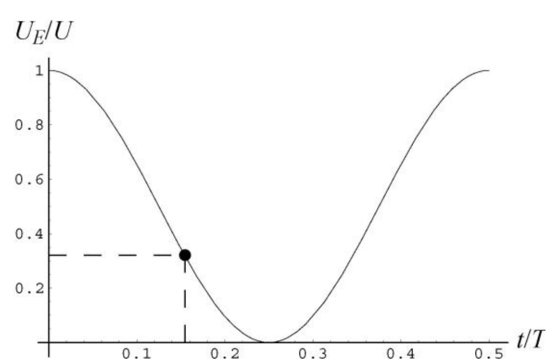
$$\frac{Q^2}{2C} = \frac{3.00q^2}{2C} \;\Rightarrow\; q = \frac{Q}{\sqrt{3.00}} = 0.577Q\,.$$

(b) If the capacitor is fully charged at time $t = 0$, then the time-dependent charge on the capacitor is given by $q = Q\cos\omega t$. This implies that the condition $q = 0.577Q$ is satisfied when $\cos\omega t$ = 0.557, or ωt = 0.955 rad. Since $\omega = 2\pi/T$ (where T is the period of oscillation), $t = 0.955T/2\pi = 0.152T$, or $t/T = 0.152$.

LEARN The fraction of total energy that is of electrical nature at a given time t is given by

$$\frac{U_E}{U} = \frac{(Q^2/2C)\cos^2\omega t}{Q^2/2C} = \cos^2\omega t = \cos^2\left(\frac{2\pi t}{T}\right).$$

A plot of U_E/U as a function of t/T is given below.



From the plot, we see that $U_E/U = 1/3$ at $t/T = 0.152$.

80. (a) The reactances are as follows:

$$X_L = 2\pi f_d L = 2\pi(400\text{ Hz})(0.0242\text{ H}) = 60.82\ \Omega$$
$$X_C = (2\pi f_d C)^{-1} = [2\pi(400\text{ Hz})(1.21\times10^{-5}\text{ F})]^{-1} = 32.88\ \Omega$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20.0\ \Omega)^2 + (60.82\ \Omega - 32.88\ \Omega)^2} = 34.36\ \Omega\,.$$

With $\varepsilon = 90.0$ V, we have

$$I = \frac{\varepsilon}{Z} = \frac{90.0\text{ V}}{34.36\ \Omega} = 2.62\text{ A} \quad\Rightarrow\quad I_{\text{rms}} = \frac{I}{\sqrt{2}} = \frac{2.62\text{ A}}{\sqrt{2}} = 1.85\text{ A}\,.$$

Therefore, the rms potential difference across the resistor is $V_{R\,\text{rms}} = I_{\text{rms}}R = 37.0$ V.

(b) Across the capacitor, the rms potential difference is $V_{C\,\text{rms}} = I_{\text{rms}}X_C = 60.9$ V.

(c) Similarly, across the inductor, the rms potential difference is $V_{L\,\text{rms}} = I_{\text{rms}}X_L = 113$ V.

(d) The average rate of energy dissipation is $P_{\text{avg}} = (I_{\text{rms}})^2R = 68.6$ W.

81. **THINK** Since the current lags the generator emf, the phase angle is positive and the circuit is more inductive than capacitive.

EXPRESS Let V_L be the maximum potential difference across the inductor, V_C be the maximum potential difference across the capacitor, and V_R be the maximum potential difference across the resistor. The phase constant is given by

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right).$$

The maximum emf is related to the current amplitude by $\varepsilon_m = IZ$, where Z is the impedance.

ANALYZE (a) With $V_C = V_L/2.00$ and $V_R = V_L/2.00$, we find the phase constant to be

$$\phi = \tan^{-1}\left(\frac{V_L - V_L/2.00}{V_L/2.00}\right) = \tan^{-1}(1.00) = 45.0°.$$

(b) The resistance is related to the impedance by $R = Z\cos\phi$. Thus,

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(30.0\text{ V})(\cos 45°)}{300\times10^{-3}\text{ A}} = 70.7\ \Omega.$$

LEARN With R and I known, the inductive and capacitive reactances are, respectively, $X_L = 2.00R = 141\ \Omega$, and $X_C = R = 70.7\ \Omega$. Similarly, the impedance of the circuit is

$$Z = \frac{\mathcal{E}_m}{I} = (30.0 \text{ V})/(300 \times 10^{-3} \text{ A}) = 100\ \Omega .$$

82. From $U_{\max} = \frac{1}{2}LI^2$ we get I = 0.115 A.

83. From Eq. 31-4 we get $f = 1/2\pi\sqrt{LC}$ = 1.84 kHz.

84. (a) With a phase constant of 45º the (net) reactance must equal the resistance in the circuit, which means the circuit impedance becomes

$$Z = R\sqrt{2} \Rightarrow R = Z/\sqrt{2} = 707\ \Omega.$$

(b) Since f = 8000 Hz, then $\omega_d = 2\pi(8000)$ rad/s. The net reactance (which, as observed, must equal the resistance) is therefore

$$X_L - X_C = \omega_d L - (\omega_d C)^{-1} = 707\ \Omega.$$

We are also told that the resonance frequency is 6000 Hz, which (by Eq. 31-4) means

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (6000 \text{ Hz})^2 L} .$$

Substituting this for C in our previous expression (for the net reactance) we obtain an equation that can be solved for the self-inductance. Our result is L = 32.2 mH.

(c) $C = ((2\pi(6000))^2 L)^{-1}$ = 21.9 nF.

85. **THINK** The current and the charge undergo sinusoidal oscillations in the LC circuit. Energy is conserved.

EXPRESS The angular frequency oscillation is related to the capacitance C and inductance L by $\omega = 1/\sqrt{LC}$. The electrical energy and magnetic energy in the circuit as a function of time are given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}L\omega^2 Q^2 \sin^2(\omega t + \phi) = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

The maximum value of U_E is $Q^2/2C$, which is the total energy in the circuit, U. Similarly, the maximum value of U_B is also $Q^2/2C$, which can also be written as $LI^2/2$ using $I = \omega Q$.

ANALYZE (a) Using the fact that $\omega = 2\pi f$, the inductance is

$$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \left(10.4\times10^3\ \text{Hz}\right)^2 \left(340\times10^{-6}\ \text{F}\right)} = 6.89\times10^{-7}\,\text{H}.$$

(b) The total energy may be calculated from the inductor (when the current is at maximum):

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(6.89\times10^{-7}\ \text{H}\right)\left(7.20\times10^{-3}\ \text{A}\right)^2 = 1.79\times10^{-11}\ \text{J}.$$

(c) We solve for Q from $U = \frac{1}{2}Q^2/C$:

$$Q = \sqrt{2CU} = \sqrt{2\left(340\times10^{-6}\ \text{F}\right)\left(1.79\times10^{-11}\ \text{J}\right)} = 1.10\times10^{-7}\ \text{C}.$$

LEARN Figure 31-4 of the textbook illustrates the oscillations of electrical and magnetic energies. The total energy $U = U_E + U_B = Q^2/2C$ remains constant. When U_E is maximum, U_B is zero, and vice versa.

86. From Eq. 31-60, we have $(220\ \text{V}/3.00\ \text{A})^2 = R^2 + X_L^2 \Rightarrow X_L = 69.3\,\Omega$.

87. When the switch is open, we have a series LRC circuit involving just the one capacitor near the upper right corner. Equation 31-65 leads to

$$\frac{\omega_d L - \dfrac{1}{\omega_d C}}{R} = \tan\phi_o = \tan(-20°) = -\tan 20°.$$

Now, when the switch is in position 1, the equivalent capacitance in the circuit is $2C$. In this case, we have

$$\frac{\omega_d L - \dfrac{1}{2\omega_d C}}{R} = \tan\phi_1 = \tan 10.0°.$$

Finally, with the switch in position 2, the circuit is simply an LC circuit with current amplitude

$$I_2 = \frac{\varepsilon_m}{Z_{LC}} = \frac{\varepsilon_m}{\sqrt{\left(\omega_d L - \dfrac{1}{\omega_d C}\right)^2}} = \frac{\varepsilon_m}{\dfrac{1}{\omega_d C} - \omega_d L}$$

where we use the fact that $(\omega_d C)^{-1} > \omega_d L$ in simplifying the square root (this fact is evident from the description of the first situation, when the switch was open). We solve for *L, R* and *C* from the three equations above, and the results are as follows:

(a) $R = \dfrac{-\varepsilon_m}{I_2 \tan\phi_o} = \dfrac{-120\text{V}}{(2.00\,\text{A})\tan(-20.0°)} = 165\,\Omega$,

(b) $L = \dfrac{\varepsilon_m}{\omega_d I_2}\left(1 - 2\dfrac{\tan\phi_1}{\tan\phi_o}\right) = \dfrac{120\,\text{V}}{2\pi(60.0\text{ Hz})(2.00\text{ A})}\left(1 - 2\dfrac{\tan 10.0°}{\tan(-20.0°)}\right) = 0.313\text{ H}$,

(c) and

$$C = \frac{I_2}{2\omega_d \varepsilon_m \left(1 - \tan\phi_1/\tan\phi_0\right)} = \frac{2.00\text{ A}}{2(2\pi)(60.0\text{ Hz})(120\text{ V})\left(1 - \tan 10.0°/\tan(-20.0°)\right)}$$
$$= 1.49\times10^{-5}\text{ F}.$$

88. (a) Eqs. 31-4 and 31-14 lead to

$$Q = \frac{1}{\omega} = I\sqrt{LC} = 1.27\times10^{-6}\,\text{C}.$$

(b) We choose the phase constant in Eq. 31-12 to be $\phi = -\pi/2$, so that $i_0 = I$ in Eq. 31-15). Thus, the energy in the capacitor is

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}(\sin\omega t)^2 .$$

Differentiating and using the fact that 2 sin θ cos θ = sin 2θ, we obtain

$$\frac{dU_E}{dt} = \frac{Q^2}{2C}\omega\sin 2\omega t .$$

We find the maximum value occurs whenever sin $2\omega t = 1$, which leads (with n = *odd* integer) to

$$t = \frac{1}{2\omega}\frac{n\pi}{2} = \frac{n\pi}{4\omega} = \frac{n\pi}{4}\sqrt{LC} = 8.31\times10^{-5}\text{s},\ 2.49\times10^{-4}\text{s},\ \ldots .$$

The earliest time is $t = 8.31\times10^{-5}$s.

(c) Returning to the above expression for dU_E/dt with the requirement that $\sin 2\omega t = 1$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{Q^2}{2C}\omega = \frac{\left(I\sqrt{LC}\right)^2}{2C}\frac{I}{\sqrt{LC}} = \frac{I^2}{2}\sqrt{\frac{L}{C}} = 5.44\times10^{-3}\ \text{J/s}\ .$$

89. **THINK** In this problem, we demonstrate that in a driven *RLC* circuit, the energies stored in the capacitor and the inductor stay constant; however, energy is transferred from the driving emf device to the resistor.

EXPRESS The energy stored in the capacitor is given by $U_E = q^2/2C$. Similarly, the energy stored in the inductor is $U_B = \frac{1}{2}Li^2$. The rate of energy supply by the driving emf device is $P_\varepsilon = i\varepsilon$, where $i = I\sin(\omega_d - \phi)$ and $\varepsilon = \varepsilon_m \sin\omega_d t$. The rate with which energy dissipates in the resistor is $P_R = i^2 R$.

ANALYZE (a) Since the charge q is a periodic function of t with period T, so must be U_E. Consequently, U_E will not be changed over one complete cycle. Actually, U_E has period $T/2$, which does not alter our conclusion.

(b) Since the current i is a periodic function of t with period T, so must be U_B.

(c) The energy supplied by the emf device over one cycle is

$$U_\varepsilon = \int_0^T P_\varepsilon dt = I\varepsilon_m \int_0^T \sin(\omega_d t - \phi)\sin(\omega_d t)dt = I\varepsilon_m \int_0^T [\sin\omega_d t\cos\phi - \cos\omega_d t\sin\phi]\sin(\omega_d t)dt$$
$$= \frac{T}{2}I\varepsilon_m\cos\phi,$$

where we have used

$$\int_0^T \sin^2(\omega_d t)dt = \frac{T}{2}, \qquad \int_0^T \sin(\omega_d t)\cos(\omega_d t)dt = 0.$$

(d) Over one cycle, the energy dissipated in the resistor is

$$U_R = \int_0^T P_R dt = I^2 R\int_0^T \sin^2(\omega_d t - \phi)dt = \frac{T}{2}I^2 R.$$

(e) Since $\varepsilon_m I\cos\phi = \varepsilon_m I\left(V_R/\varepsilon_m\right) = \varepsilon_m I\left(IR/\varepsilon_m\right) = I^2 R$, the two quantities are indeed the same.

LEARN In solving for (c) and (d), we could have used Eqs. 31-74 and 31-71: By doing so, we find the energy supplied by the generator to be

$$P_{\text{avg}}T = \left(I_{\text{rms}}\varepsilon_{\text{rms}}\cos\phi\right)T = \left(\frac{1}{2}T\right)\varepsilon_m I\cos\phi$$

where we substitute $I_{\text{rms}} = I/\sqrt{2}$ and $\varepsilon_{\text{rms}} = \varepsilon_m/\sqrt{2}$. Similarly, the energy dissipated by the resistor is

$$P_{\text{avg,resistor}}\,T = \left(I_{\text{rms}}V_R\right)T = I_{\text{rms}}\left(I_{\text{rms}}R\right)T = \left(\frac{1}{2}T\right)I^2R.$$

The same results are obtained without any integration.

90. From Eq. 31-4, we have $C = (\omega^2L)^{-1} = ((2\pi f)^2L)^{-1} = 1.59\ \mu\text{F}$.

91. Resonance occurs when the inductive reactance equals the capacitive reactance. Reactances of a certain type add (in series) just like resistances. Thus, since the resonance ω values are the same for both circuits, we have for each circuit:

$$\omega L_1 = \frac{1}{\omega C_1}, \quad \omega L_2 = \frac{1}{\omega C_2}$$

and adding these equations we find

$$\omega\left(L_1 + L_2\right) = \frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right).$$

Since $L_{\text{eq}} = L_1 + L_2$ and $C_{\text{eq}}^{-1} = (C_1^{-1} + C_2^{-1})$,

$$\omega L_{\text{eq}} = \frac{1}{\omega C_{\text{eq}}} \quad \Rightarrow \quad \text{resonance in the combined circuit.}$$

92. When switch S_1 is closed and the others are open, the inductor is essentially out of the circuit and what remains is an RC circuit. The time constant is $\tau_C = RC$. When switch S_2 is closed and the others are open, the capacitor is essentially out of the circuit. In this case, what we have is an LR circuit with time constant $\tau_L = L/R$. Finally, when switch S_3 is closed and the others are open, the resistor is essentially out of the circuit and what remains is an LC circuit that oscillates with period $T = 2\pi\sqrt{LC}$. Substituting $L = R\tau_L$ and $C = \tau_C/R$, we obtain $T = 2\pi\sqrt{\tau_C\tau_L}$.

93. (a) We note that we obtain the maximum value in Eq. 31-28 when we set

$$t = \frac{\pi}{2\omega_d} = \frac{1}{4f} = \frac{1}{4(60)} = 0.00417\ \text{s}$$

or 4.17 ms. The result is $\varepsilon_m \sin(\pi/2) = \varepsilon_m \sin(90°) = 36.0\ \text{V}$.

(b) At $t = 4.17$ ms, the current is

$$i = I\sin(\omega_d t - \phi) = I\sin(90° - (-24.3°)) = (0.164\,\text{A})\cos(24.3°)$$
$$= 0.1495\,\text{A} \approx 0.150\ \text{A}.$$

Ohm's law directly gives

$$v_R = iR = (0.1495\,\text{A})(200\,\Omega) = 29.9\,\text{V}.$$

(c) The capacitor voltage phasor is 90° less than that of the current. Thus, at t = 4.17 ms, we obtain

$$v_C = I\sin(90° - (-24.3°) - 90°)X_C = IX_C\sin(24.3°) = (0.164\,\text{A})(177\,\Omega)\sin(24.3°)$$
$$= 11.9\,\text{V}.$$

(d) The inductor voltage phasor is 90° more than that of the current. Therefore, at t = 4.17 ms, we find

$$v_L = I\sin(90° - (-24.3°) + 90°)X_L = -IX_L\sin(24.3°) = -(0.164\,\text{A})(86.7\,\Omega)\sin(24.3°)$$
$$= -5.85\,\text{V}.$$

(e) Our results for parts (b), (c) and (d) add to give 36.0 V, the same as the answer for part (a).

Chapter 32

1. We use $\sum_{n=1}^{6} \Phi_{Bn} = 0$ to obtain

$$\Phi_{B6} = -\sum_{n=1}^{5} \Phi_{Bn} = -\left(-1\,\text{Wb} + 2\,\text{Wb} - 3\,\text{Wb} + 4\,\text{Wb} - 5\,\text{Wb}\right) = +3\,\text{Wb}\ .$$

2. (a) The flux through the top is $+(0.30\ \text{T})\pi r^2$ where $r = 0.020$ m. The flux through the bottom is +0.70 mWb as given in the problem statement. Since the *net* flux must be zero then the flux through the sides must be negative and exactly cancel the total of the previously mentioned fluxes. Thus (in magnitude) the flux though the sides is 1.1 mWb.

(b) The fact that it is negative means it is inward.

3. **THINK** Gauss' law for magnetism states that the net magnetic flux through any closed surface is zero.

EXPRESS Mathematically, Gauss' law for magnetism is expressed as $\oint \vec{B}\cdot d\vec{A} = 0$. Now, our Gaussian surface has the shape of a right circular cylinder with two end caps and a curved surface. Thus,

$$\oint \vec{B}\cdot d\vec{A} = \Phi_1 + \Phi_2 + \Phi_C,$$

where Φ_1 is the magnetic flux through the first end cap, Φ_2 is the magnetic flux through the second end cap, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is $\Phi_1 = -25.0\ \mu$Wb. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder.

ANALYZE (a) Substituting the values given, the flux through the second end is

$$\Phi_2 = \pi\left(0.120\,\text{m}\right)^2\left(1.60\times10^{-3}\,\text{T}\right) = +7.24\times10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb}.$$

Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}.$$

Thus, the magnitude is $|\Phi_C| = 47.4\,\mu\text{Wb}$.

(b) The minus sign in Φ_C indicates that the flux is inward through the curved surface.

LEARN Gauss' law for magnetism implies that magnetic monopoles do not exist; the simplest magnetic structure is a magnetic dipole (having a north pole and a south pole).

4. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2, the portion of the *xz* plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\Phi_B(S_1) = \Phi_B(S_2) = \int_{-r}^{r} B(x)L\,dx = 2\int_{-r}^{r} B_{\text{left}}(x)L\,dx = 2\int_{-r}^{r} \frac{\mu_0 i}{2\pi}\frac{1}{2r-x}L\,dx = \frac{\mu_0 iL}{\pi}\ln 3\ .$$

5. **THINK** Changing electric flux induces a magnetic field.

EXPRESS Consider a circle of radius r between the plates, with its center on the axis of the capacitor. Since there is no current between the capacitor plates, the Ampere-Maxwell's law reduces to

$$\oint \vec{B}\cdot d\vec{A} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}\ ,$$

where $\vec{B}$ is the magnetic field at points on the circle, and Φ_E is the electric flux through the circle. Since the $\vec{B}$ field on the circle is in the tangential direction, and $\Phi_E = AE = \pi R^2 E,$ where R is the radius of the capacitor, we have

$$2\pi rB = \mu_0\varepsilon_0\pi R^2 \frac{dE}{dt}$$

or

$$B = \frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt} \qquad (r \ge R).$$

ANALYZE Solving for dE/dt, we obtain

$$\frac{dE}{dt} = \frac{2Br}{\mu_0\varepsilon_0 R^2} = \frac{2\left(2.0\times10^{-7}\ \text{T}\right)\left(6.0\times10^{-3}\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3.0\times10^{-3}\text{m}\right)^2} = 2.4\times10^{13}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

LEARN Outside the capacitor, the induced magnetic field decreases with increased radial distance r, from a maximum value at the plate edge $r = R$.

6. The integral of the field along the indicated path is, by Eq. 32-18 and Eq. 32-19, equal to

$$\mu_0 i_d\left(\frac{\text{enclosed area}}{\text{total area}}\right) = \mu_0(0.75\ \text{A})\frac{(4.0\ \text{cm})(2.0\ \text{cm})}{12\ \text{cm}^2} = 52\ \text{nT}\cdot\text{m}\ .$$

7. (a) Inside we have (by Eq. 32-16) $B=\mu_0 i_d r_1/2\pi R^2$, where $r_1 = 0.0200$ m, $R = 0.0300$ m, and the displacement current is given by Eq. 32-38 (in SI units):

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{-3}\ \text{V/m}\cdot\text{s}) = 2.66\times10^{-14}\ \text{A}\,.$$

Thus, we find

$$B=\frac{\mu_0 i_d r_1}{2\pi R^2}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})(0.0200\ \text{m})}{2\pi(0.0300\ \text{m})^2}=1.18\times10^{-19}\ \text{T}\,.$$

(b) Outside we have (by Eq. 32-17) $B=\mu_0 i_d/2\pi r_2$ where r_2 = 0.0500 cm. Here we obtain

$$B=\frac{\mu_0 i_d}{2\pi r_2}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})}{2\pi(0.0500\ \text{m})}=1.06\times10^{-19}\ \text{T}$$

8. (a) Application of Eq. 32-3 along the circle referred to in the second sentence of the problem statement (and taking the derivative of the flux expression given in that sentence) leads to

$$B(2\pi r)=\varepsilon_0\mu_0\left(0.60\ \text{V}\cdot\text{m/s}\right)\frac{r}{R}\,.$$

Using r = 0.0200 m (which, in any case, cancels out) and R = 0.0300 m, we obtain

$$B=\frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi R}=\frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0300\ \text{m})}$$
$$=3.54\times10^{-17}\ \text{T}\,.$$

(b) For a value of r larger than R, we must note that the flux enclosed has already reached its full amount (when $r = R$ in the given flux expression). Referring to the equation we wrote in our solution of part (a), this means that the final fraction (r/R) should be replaced with unity. On the left hand side of that equation, we set r = 0.0500 m and solve. We now find

$$B=\frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi r}=\frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0500\ \text{m})}$$
$$=2.13\times10^{-17}\ \text{T}\,.$$

9. (a) Application of Eq. 32-7 with $A = \pi r^2$ (and taking the derivative of the field expression given in the problem) leads to

$$B(2\pi r)=\varepsilon_0\mu_0\pi r^2\left(0.00450\ \text{V/m}\cdot\text{s}\right).$$

For r = 0.0200 m, this gives

$$\begin{aligned}B &= \frac{1}{2}\varepsilon_0\mu_0 r(0.00450\ \text{V/m}\cdot\text{s})\\ &= \frac{1}{2}(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.0200\ \text{m})(0.00450\ \text{V/m}\cdot\text{s})\\ &= 5.01\times10^{-22}\ \text{T}.\end{aligned}$$

(b) With $r > R$, the expression above must replaced by

$$B(2\pi r) = \varepsilon_0\mu_0\pi R^2\left(0.00450\ \text{V/m}\cdot\text{s}\right).$$

Substituting r = 0.050 m and R = 0.030 m, we obtain $B = 4.51 \times 10^{-22}$ T.

10. (a) Here, the enclosed electric flux is found by integrating

$$\Phi_E = \int_0^r E\ 2\pi r dr = t(0.500\ \text{V/m}\cdot\text{s})(2\pi)\int_0^r\left(1-\frac{r}{R}\right)r dr = t\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$

with SI units understood. Then (after taking the derivative with respect to time) Eq. 32-3 leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right).$$

For r = 0.0200 m and R = 0.0300 m, this gives $B = 3.09 \times 10^{-20}$ T.

(b) The integral shown above will no longer (since now $r > R$) have r as the upper limit; the upper limit is now R. Thus,

$$\Phi_E = t\pi\left(\frac{1}{2}R^2 - \frac{R^3}{3R}\right) = \frac{1}{6}t\pi R^2.$$

Consequently, Eq. 32-3 becomes

$$B(2\pi r) = \frac{1}{6}\varepsilon_0\mu_0\pi R^2$$

which for r = 0.0500 m, yields

$$B = \frac{\varepsilon_0\mu_0 R^2}{12r} = \frac{(8.85\times10^{-12}\ )(4\pi\times10^{-7})(0.030)^2}{12(0.0500)} = 1.67\times10^{-20}\ \text{T}.$$

11. (a) Noting that the magnitude of the electric field (assumed uniform) is given by $E = V/d$ (where d = 5.0 mm), we use the result of part (a) in Sample Problem 32.01 – "Magnetic field induced by changing electric field:"

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 r}{2d}\frac{dV}{dt} \qquad (r \le R).$$

We also use the fact that the time derivative of sin (ωt) (where $\omega = 2\pi f = 2\pi(60) \approx 377$/s in this problem) is $\omega\cos(\omega t)$. Thus, we find the magnetic field as a function of r (for $r \le R$; note that this neglects "fringing" and related effects at the edges):

$$B = \frac{\mu_0 \varepsilon_0 r}{2d} V_{\max}\,\omega\cos(\omega t) \quad \Rightarrow \quad B_{\max} = \frac{\mu_0 \varepsilon_0 r V_{\max}\,\omega}{2d}$$

where $V_{\max}$ = 150 V. This grows with r until reaching its highest value at $r = R$ = 30 mm:

$$B_{\max}\Big|_{r=R} = \frac{\mu_0 \varepsilon_0 R V_{\max}\,\omega}{2d} = \frac{(4\pi\times10^{-7}\text{ H/m})(8.85\times10^{-12}\text{ F/m})(30\times10^{-3}\text{m})(150\text{ V})(377/\text{s})}{2(5.0\times10^{-3}\text{m})}$$

$$= 1.9\times10^{-12}\text{ T}.$$

(b) For $r \le 0.03$ m, we use the expression

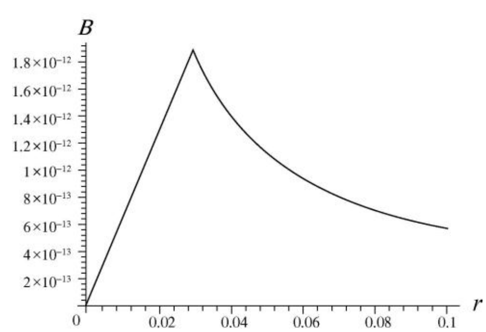
$$B_{\max} = \mu_0 \varepsilon_0 r V_{\max}\,\omega / 2d$$

found in part (a) (note the $B \propto r$ dependence), and for $r \ge 0.03$ m we perform a similar calculation starting with the result of part (b) in Sample Problem 32.01 — "Magnetic field induced by changing electric field:"

$$B_{\max} = \left(\frac{\mu_0 \varepsilon_0 R^2}{2r}\frac{dE}{dt}\right)_{\max} = \left(\frac{\mu_0 \varepsilon_0 R^2}{2rd}\frac{dV}{dt}\right)_{\max} = \left(\frac{\mu_0 \varepsilon_0 R^2}{2rd}V_{\max}\,\omega\cos(\omega t)\right)_{\max}$$

$$= \frac{\mu_0 \varepsilon_0 R^2 V_{\max}\,\omega}{2rd} \qquad (\text{for } r \ge R)$$

(note the $B \propto r^{-1}$ dependence — see also Eqs. 32-16 and 32-17). The plot, with SI units understood, is shown to the right.



12. From Sample Problem 32.01 — "Magnetic field induced by changing electric field," we know that $B \propto r$ for $r \le R$ and $B \propto r^{-1}$ for $r \ge R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of $B_{\max}$. We denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$.

(a) Inside the capacitor, $0.75\, B_{\max}/B_{\max} = r_1/R$, or $r_1 = 0.75\,R = 0.75$ (40 mm) =30 mm.

(b) Outside the capacitor, $0.75\, B_{\max}/B_{\max} = (r_2/R)^{-1}$, or

$$r_2 = R/0.75 = 4R/3 = (4/3)(40\text{ mm}) = 53\text{ mm}.$$

(c) From Eqs. 32-15 and 32-17,

$$B_{\max} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(6.0\text{ A}\right)}{2\pi\left(0.040\text{ m}\right)} = 3.0\times10^{-5}\text{ T}.$$

13. Let the area plate be A and the plate separation be d. We use Eq. 32-10:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(AE\right) = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right),$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\varepsilon_0 A} = \frac{i_d}{C} = \frac{1.5\text{ A}}{2.0\times10^{-6}\text{ F}} = 7.5\times10^5\text{ V/s}.$$

Therefore, we need to change the voltage difference across the capacitor at the rate of $7.5\times10^5\text{ V/s}$.

14. Consider an area A, normal to a uniform electric field $\vec{E}$. The displacement current density is uniform and normal to the area. Its magnitude is given by $J_d = i_d/A$. For this situation , $i_d = \varepsilon_0 A(dE/dt)$, so

$$J_d = \frac{1}{A}\varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{dE}{dt}.$$

15. **THINK** The displacement current is related to the changing electric flux by $i_d = \varepsilon_0 (d\Phi_E / dt)$.

EXPRESS Let A be the area of a plate and E be the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation.

ANALYZE Thus, the displacement current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}.$$

Now, $\varepsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor (not filled with a dielectric), so

$$i_d = C\frac{dV}{dt}.$$

LEARN The real current charging the capacitor is

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}.$$

Thus, we see that $i = i_d$.

16. We use Eq. 32-14: $i_d = \varepsilon_0 A(dE/dt)$. Note that, in this situation, A is the area over which a changing electric field is present. In this case $r > R$, so $A = \pi R^2$. Thus,

$$\frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{i_d}{\varepsilon_0 \pi R^2} = \frac{2.0\,\text{A}}{\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.10\,\text{m}\right)^2} = 7.2\times10^{12}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

17. (a) Using Eq. 27-10, we find $E = \rho J = \dfrac{\rho i}{A} = \dfrac{\left(1.62\times10^{-8}\,\Omega\cdot\text{m}\right)\left(100\,\text{A}\right)}{5.00\times10^{-6}\,\text{m}^2} = 0.324\ \text{V/m}.$

(b) The displacement current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0 A\frac{d}{dt}\left(\frac{\rho i}{A}\right) = \varepsilon_0 \rho\frac{di}{dt} = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(1.62\times10^{-8}\,\Omega\right)\left(2000\,\text{A/s}\right)$$
$$= 2.87\times10^{-16}\,\text{A}.$$

(c) The ratio of fields is $\dfrac{B(\text{due to } i_d)}{B(\text{due to } i)} = \dfrac{\mu_0 i_d/2\pi r}{\mu_0 i/2\pi r} = \dfrac{i_d}{i} = \dfrac{2.87\times10^{-16}\ \text{A}}{100\,\text{A}} = 2.87\times10^{-18}.$

18. From Eq. 28-11, we have $i = (\varepsilon / R)\, e^{-t/\tau}$ since we are ignoring the self-inductance of the capacitor. Equation 32-16 gives

$$B = \frac{\mu_0 i_d r}{2\pi R^2}.$$

Furthermore, Eq. 25-9 yields the capacitance

$$C = \frac{\varepsilon_0 \pi (0.05\ \text{m})^2}{0.003\ \text{m}} = 2.318\times10^{-11}\,\text{F},$$

so that the capacitive time constant is

$$\tau = (20.0\times10^6\,\Omega)(2.318\times10^{-11}\,\text{F}) = 4.636\times10^{-4}\,\text{s}.$$

At $t = 250\times10^{-6}$ s, the current is

$$i = \frac{12.0\ \text{V}}{20.0\ \text{x}\ 10^6\,\Omega}\ e^{-t/\tau} = 3.50\ \times10^{-7}\,\text{A}.$$

Since $i = i_d$ (see Eq. 32-15) and r = 0.0300 m, then (with plate radius R = 0.0500 m) we find

$$B = \frac{\mu_0 i_d r}{2\pi R^2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.50\times10^{-7}\ \text{A})(0.030\ \text{m})}{2\pi(0.050\ \text{m})^2} = 8.40\times10^{-13}\ \text{T}.$$

19. (a) Equation 32-16 (with Eq. 26-5) gives, with $A = \pi R^2$,

$$B = \frac{\mu_0 i_d r}{2\pi R^2} = \frac{\mu_0 J_d A r}{2\pi R^2} = \frac{\mu_0 J_d (\pi R^2) r}{2\pi R^2} = \frac{1}{2}\mu_0 J_d r$$
$$= \frac{1}{2}(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(6.00\ \text{A/m}^2)(0.0200\ \text{m}) = 75.4\ \text{nT}.$$

(b) Similarly, Eq. 32-17 gives $B = \frac{\mu_0 i_d}{2\pi r} = \frac{\mu_0 J_d \pi R^2}{2\pi r} = 67.9\ \text{nT}$.

20. (a) Equation 32-16 gives $B = \frac{\mu_0 i_d r}{2\pi R^2} = 2.22\ \mu\text{T}$.

(b) Equation 32-17 gives $B = \frac{\mu_0 i_d}{2\pi r} = 2.00\ \mu\text{T}$.

21. (a) Equation 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\text{enc}}$. It is the enclosed portion of the displacement current, and if we related this to the displacement current density J_d, then

$$i_{d\,\text{enc}} = \int_0^r J_d 2\pi r\,dr = (4.00\ \text{A/m}^2)(2\pi)\int_0^r \left(1 - r/R\right) r\,dr = 8\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$

with SI units understood. Now, we apply Eq. 32-17 (with i_d replaced with $i_{d,\text{enc}}$) or start from scratch with Eq. 32-11, to get $B = \frac{\mu_0 i_{d\,\text{enc}}}{2\pi r} = 27.9\ \text{nT}$.

(b) The integral shown above will no longer (since now $r > R$) have r as the upper limit; the upper limit is now R. Thus,

$$i_{d\,\text{enc}} = i_d = 8\pi\left(\frac{1}{2}R^2 - \frac{R^3}{3R}\right) = \frac{4}{3}\pi R^2.$$

Now Eq. 32-17 gives $B = \frac{\mu_0 i_d}{2\pi r} = 15.1\ \text{nT}$.

22. (a) Eq. 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\text{enc}}$. It is the enclosed portion of the displacement current. Thus Eq. 32-17 (which derives from Eq. 32-11) becomes, with i_d replaced with $i_{d,\text{enc}}$,

$$B = \frac{\mu_0 i_{d\,\text{enc}}}{2\pi r} = \frac{\mu_0 (3.00\text{ A})(r/R)}{2\pi r}$$

which yields (after canceling r, and setting R = 0.0300 m) B = 20.0 μT.

(b) Here i_d = 3.00 A, and we get $B = \frac{\mu_0 i_d}{2\pi r} = 12.0\ \mu\text{T}$.

23. **THINK** The electric field between the plates in a parallel-plate capacitor is changing, so there is a nonzero displacement current $i_d = \varepsilon_0 (d\Phi_E / dt)$ between the plates.

EXPRESS Let A be the area of a plate and E be the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. The current into the positive plate of the capacitor is

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{d(Ed)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt},$$

which is the same as the displacement current.

ANALYZE (a) Thus, at any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires: $i_d = i$ = 2.0 A.

(b) The rate of change of the electric field is

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right) = \frac{i_d}{\varepsilon_0 A} = \frac{2.0\text{ A}}{\left(8.85\times 10^{-12}\text{ F/m}\right)\left(1.0\text{ m}\right)^2} = 2.3\times 10^{11}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

(c) The displacement current through the indicated path is

$$i'_d = i_d\left(\frac{d^2}{L^2}\right) = (2.0\text{ A})\left(\frac{0.50\text{m}}{1.0\text{m}}\right)^2 = 0.50\text{ A}.$$

(d) The integral of the field around the indicated path is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i'_d = \left(1.26\times 10^{-16}\text{ H/m}\right)\left(0.50\text{ A}\right) = 6.3\times 10^{-7}\text{ T}\cdot\text{m}.$$

LEARN the displacement through the dashed path is proportional to the area encircled by the path since the displacement current is uniformly distributed over the full plate area.

24. (a) From Eq. 32-10,

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} \varepsilon_0 A \frac{d}{dt}\left[\left(4.0\times10^5\right) - \left(6.0\times10^4 t\right)\right] = -\varepsilon_0 A\left(6.0\times10^4 \text{ V/m}\cdot\text{s}\right)$$
$$= -\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(4.0\times10^{-2}\text{m}^2\right)\left(6.0\times10^4\text{ V/m}\cdot\text{s}\right)$$
$$= -2.1\times10^{-8}\text{ A}.$$

Thus, the magnitude of the displacement current is $|i_d| = 2.1\times10^{-8}\text{ A}$.

(b) The negative sign in i_d implies that the direction is downward.

(c) If one draws a counterclockwise circular loop s around the plates, then according to Eq. 32-18,

$$\oint_s \vec{B}\cdot d\vec{s} = \mu_0 i_d < 0,$$

which means that $\vec{B}\cdot d\vec{s} < 0$. Thus $\vec{B}$ must be clockwise.

25. (a) We use $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$ to find

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r} = \frac{\mu_0\left(J_d \pi r^2\right)}{2\pi r} = \frac{1}{2}\mu_0 J_d r = \frac{1}{2}\left(1.26\times10^{-6}\text{ H/m}\right)\left(20\text{ A/m}^2\right)\left(50\times10^{-3}\text{ m}\right)$$
$$= 6.3\times10^{-7}\text{ T}.$$

(b) From $i_d = J_d \pi r^2 = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \pi r^2 \frac{dE}{dt}$, we get

$$\frac{dE}{dt} = \frac{J_d}{\varepsilon_0} = \frac{20\text{ A/m}^2}{8.85\times10^{-12}\text{ F/m}} = 2.3\times10^{12}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

26. (a) Since $i = i_d$ (Eq. 32-15) then the portion of displacement current enclosed is

$$i_{d,\text{enc}} = i\frac{\pi\left(R/3\right)^2}{\pi R^2} = \frac{i}{9} = 1.33\text{ A}.$$

(b) We see from Sample Problem 32.01 — "Magnetic field induced by changing electric field" that the maximum field is at $r = R$ and that (in the interior) the field is simply proportional to r. Therefore,

$$\frac{B}{B_{\max}} = \frac{3.00\text{ mT}}{12.0\text{ mT}} = \frac{r}{R}$$

which yields $r = R/4 = (1.20\text{ cm})/4 = 0.300\text{ cm}$.

(c) We now look for a solution in the exterior region, where the field is inversely proportional to r (by Eq. 32-17):

$$\frac{B}{B_{\max}} = \frac{3.00\,\text{mT}}{12.0\,\text{mT}} = \frac{R}{r}$$

which yields $r = 4R = 4(1.20 \text{ cm}) = 4.80$ cm.

27. (a) In region a of the graph,

$$|i_d| = \varepsilon_0 \left|\frac{d\Phi_E}{dt}\right| = \varepsilon_0 A \left|\frac{dE}{dt}\right| = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(1.6\,\text{m}^2\right)\left|\frac{4.5\times10^5\,\text{N/C} - 6.0\times10^5\,\text{N/C}}{4.0\times10^{-6}\,\text{s}}\right| = 0.71\,\text{A}.$$

(b) $i_d \propto dE/dt = 0$.

(c) In region c of the graph,

$$|i_d| = \varepsilon_0 A \left|\frac{dE}{dt}\right| = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(1.6\,\text{m}^2\right)\left|\frac{-4.0\times10^5\,\text{N/C}}{2.0\times10^{-6}\,\text{s}}\right| = 2.8\,\text{A}.$$

28. (a) Figure 32-35 indicates that $i = 4.0$ A when $t = 20$ ms. Thus,

$$B_i = \mu_0 i/2\pi r = 0.089 \text{ mT}.$$

(b) Figure 32-35 indicates that $i = 8.0$ A when $t = 40$ ms. Thus, $B_i \approx 0.18$ mT.

(c) Figure 32-35 indicates that $i = 10$ A when $t > 50$ ms. Thus, $B_i \approx 0.220$ mT.

(d) Equation 32-4 gives the displacement current in terms of the time-derivative of the electric field: $i_d = \varepsilon_0 A(dE/dt)$, but using Eq. 26-5 and Eq. 26-10 we have $E = \rho i/A$ (in terms of the real current); therefore, $i_d = \varepsilon_0\rho(di/dt)$. For $0 < t < 50$ ms, Fig. 32-35 indicates that $di/dt = 200$ A/s. Thus,

$$B_{id} = \mu_0 i_d/2\pi r = 6.4 \times 10^{-22} \text{ T}.$$

(e) As in (d), $B_{id} = \mu_0 i_d/2\pi r = 6.4 \times 10^{-22}$ T.

(f) Here $di/dt = 0$, so (by the reasoning in the previous step) $B = 0$.

(g) By the right-hand rule, the direction of $\vec{B}_i$ at $t = 20$ s is out of the page.

(h) By the right-hand rule, the direction of $\vec{B}_{id}$ at $t = 20$ s is out of the page.

29. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_{\max} = i_{d\,\max} = 7.60\ \mu$A.

(b) Since $i_d = \varepsilon_0 (d\Phi_E/dt)$, we have

$$\left(\frac{d\Phi_E}{dt}\right)_{\max} = \frac{i_{d\max}}{\varepsilon_0} = \frac{7.60\times10^{-6}\,\text{A}}{8.85\times10^{-12}\,\text{F/m}} = 8.59\times10^{5}\,\text{V}\cdot\text{m/s}.$$

(c) Let the area plate be A and the plate separation be d. The displacement current is

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0\frac{d}{dt}(AE) = \varepsilon_0 A\frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right).$$

Now the potential difference across the capacitor is the same in magnitude as the emf of the generator, so $V = \varepsilon_m \sin \omega t$ and $dV/dt = \omega\varepsilon_m \cos \omega t$. Thus, $i_d = (\varepsilon_0 A\omega\varepsilon_m / d)\cos\omega t$ and $i_{d\max} = \varepsilon_0 A\omega\varepsilon_m / d$. This means

$$d = \frac{\varepsilon_0 A\omega\varepsilon_m}{i_{d\max}} = \frac{(8.85\times10^{-12}\,\text{F/m})\pi(0.180\,\text{m})^2(130\,\text{rad/s})(220\,\text{V})}{7.60\times10^{-6}\,\text{A}} = 3.39\times10^{-3}\,\text{m},$$

where $A = \pi R^2$ was used.

(d) We use the Ampere-Maxwell law in the form $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_d$, where the path of integration is a circle of radius r between the plates and parallel to them. I_d is the displacement current through the area bounded by the path of integration. Since the displacement current density is uniform between the plates, $I_d = (r^2/R^2)i_d$, where i_d is the total displacement current between the plates and R is the plate radius. The field lines are circles centered on the axis of the plates, so $\vec{B}$ is parallel to $d\vec{s}$. The field has constant magnitude around the circular path, so $\oint \vec{B}\cdot d\vec{s} = 2\pi rB$. Thus,

$$2\pi rB = \mu_0\left(\frac{r^2}{R^2}\right)i_d \quad\Rightarrow\quad B = \frac{\mu_0 i_d r}{2\pi R^2}.$$

The maximum magnetic field is given by

$$B_{\max} = \frac{\mu_0 i_{d\max} r}{2\pi R^2} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(7.6\times10^{-6}\,\text{A})(0.110\text{m})}{2\pi(0.180\,\text{m})^2} = 5.16\times10^{-12}\,\text{T}.$$

30. (a) The flux through Arizona is

$$\Phi = -B_r A = -(43\times10^{-6}\,\text{T})(295{,}000\,\text{km}^2)(10^3\,\text{m/km})^2 = -1.3\times10^{7}\,\text{Wb},$$

inward. By Gauss' law this is equal to the negative value of the flux Φ' through the rest of the surface of the Earth. So $\Phi' = 1.3 \times 10^7$ Wb.

(b) The direction is outward.

31. The horizontal component of the Earth's magnetic field is given by $B_h = B\cos\phi_i$, where B is the magnitude of the field and ϕ_i is the inclination angle. Thus

$$B = \frac{B_h}{\cos\phi_i} = \frac{16\,\mu\text{T}}{\cos 73°} = 55\,\mu\text{T}\ .$$

32. (a) The potential energy of the atom in association with the presence of an external magnetic field $\vec{B}_{\text{ext}}$ is given by Eqs. 32-31 and 32-32:

$$U = -\mu_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}} = -m_\ell \mu_B B_{\text{ext}}\,.$$

For level E_1 there is no change in energy as a result of the introduction of $\vec{B}_{\text{ext}}$, so $U \propto m_\ell$ = 0, meaning that $m_\ell = 0$ for this level.

(b) For level E_2 the single level splits into a triplet (i.e., three separate ones) in the presence of $\vec{B}_{\text{ext}}$, meaning that there are three different values of m_ℓ. The middle one in the triplet is unshifted from the original value of E_2 so its m_ℓ must be equal to 0. The other two in the triplet then correspond to $m_\ell = -1$ and $m_\ell = +1$, respectively.

(c) For any pair of adjacent levels in the triplet, $|\Delta m_\ell| = 1$. Thus, the spacing is given by

$$\Delta U = |\,\Delta(-m_\ell \mu_B B)\,| = |\,\Delta m_\ell\,|\,\mu_B B = \mu_B B = (9.27\times10^{-24}\ \text{J/T})(0.50\text{T}) = 4.64\times10^{-24}\ \text{J}.$$

33. **THINK** An electron in an atom has both orbital angular momentum and spin angular momentum; the *z* components of the angular momenta are quantized.

EXPRESS The *z* component of the orbital angular momentum is give by

$$L_{\text{orb},z} = \frac{m_\ell h}{2\pi}$$

where h is the Planck constant and m_ℓ is the orbital magnetic quantum number. The corresponding *z* component of the orbital magnetic dipole moment is

$$\mu_{\text{orb},z} = -m_\ell \mu_{\text{B}}$$

where $\mu_B = eh/4\pi m$ is the Bohr magneton. When placed in an external field $\vec{B}_{ext}$, the energy associated with the orientation of $\vec{\mu}_{orb}$ is given by

$$U = -\vec{\mu}_{orb} \cdot \vec{B}_{ext}.$$

ANALYZE (a) Since $m_\ell = 0$, $L_{orb,z} = m_\ell h/2\pi = 0$.

(b) Since $m_\ell = 0$, $\mu_{orb,z} = -m_\ell \mu_B = 0$.

(c) Since $m_\ell = 0$, then from Eq. 32-32, $U = -\mu_{orb,z}B_{ext} = -m_\ell \mu_B B_{ext} = 0$.

(d) Regardless of the value of m_ℓ, we find for the spin part

$$U = -\mu_{s,z}B = \pm\mu_B B = \pm\left(9.27\times10^{-24}\ \text{J/T}\right)\left(35\ \text{mT}\right) = \pm3.2\times10^{-25}\ \text{J}\ .$$

(e) Now $m_\ell = -3$, so

$$L_{orb,z} = \frac{m_\ell h}{2\pi} = \frac{(-3)\left(6.63\times10^{-27}\ \text{J}\cdot\text{s}\right)}{2\pi} = -3.16\times10^{-34}\ \text{J}\cdot\text{s} \approx -3.2\times10^{-34}\ \text{J}\cdot\text{s}$$

(f) and $\mu_{orb,z} = -m_\ell\mu_B = -(-3)\left(9.27\times10^{-24}\ \text{J/T}\right) = 2.78\times10^{-23}\ \text{J/T} \approx 2.8\times10^{-23}\ \text{J/T}$.

(g) The potential energy associated with the electron's orbital magnetic moment is now

$$U = -\mu_{orb,z}B_{ext} = -\left(2.78\times10^{-23}\ \text{J/T}\right)\left(35\times10^{-3}\ \text{T}\right) = -9.7\times10^{-25}\ \text{J}.$$

(h) On the other hand, the potential energy associated with the electron spin, being independent of m_ℓ, remains the same: $\pm3.2 \times 10^{-25}$ J.

LEARN Spin is an intrinsic angular momentum that is not associated with the motion of the electron. Its z component is quantized, and can be written as

$$S_z = \frac{m_s h}{2\pi}$$

where $m_s = \pm1/2$ is the spin magnetic quantum number.

34. We use Eq. 32-27 to obtain

$$\Delta U = -\Delta(\mu_{s,z}B) = -B\Delta\mu_{s,z},$$

where $\mu_{s,z} = \pm eh/4\pi m_e = \pm \mu_B$ (see Eqs. 32-24 and 32-25). Thus,

$$\Delta U = -B\left(\mu_B - (-\mu_B)\right) = 2\mu_B B = 2\left(9.27\times10^{-24}\ \text{J/T}\right)\left(0.25\ \text{T}\right) = 4.6\times10^{-24}\ \text{J}\ .$$

35. We use Eq. 32-31: $\mu_{\text{orb},\,z} = -m_\ell \mu_B$.

(a) For $m_\ell = 1$, $\mu_{\text{orb},z} = -(1)\,(9.3\times10^{-24}\ \text{J/T}) = -9.3\times10^{-24}$ J/T.

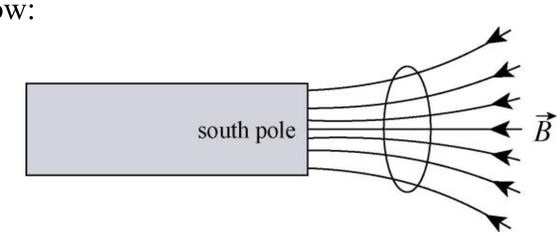
(b) For $m_\ell = -2$, $\mu_{\text{orb},z} = -(-2)\,(9.3\times10^{-24}\ \text{J/T}) = 1.9\times10^{-23}$ J/T.

36. Combining Eq. 32-27 with Eqs. 32-22 and 32-23, we see that the energy difference is

$$\Delta U = 2\mu_B B$$

where μ_B is the Bohr magneton (given in Eq. 32-25). With $\Delta U = 6.00\times10^{-25}$ J, we obtain $B = 32.3$ mT.

37. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The primary conclusion of Section 32-9 is two-fold: $\vec{u}$ is opposite to $\vec{B}$, and the effect of $\vec{F}$ is to move the material toward regions of smaller $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the right in our sketch, or in the $+x$ direction.

(c) The direction of the current is clockwise (from the perspective of the bar magnet).

(d) Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is toward the right in our sketch, or in the $+x$ direction.

38. An electric field with circular field lines is induced as the magnetic field is turned on. Suppose the magnetic field increases linearly from zero to B in time t. According to Eq. 31-27, the magnitude of the electric field at the orbit is given by

$$E = \left(\frac{r}{2}\right)\frac{dB}{dt} = \left(\frac{r}{2}\right)\frac{B}{t}\ ,$$

where r is the radius of the orbit. The induced electric field is tangent to the orbit and changes the speed of the electron, the change in speed being given by

$$\Delta v = at = \frac{eE}{m_e}t = \left(\frac{e}{m_e}\right)\left(\frac{r}{2}\right)\left(\frac{B}{t}\right)t = \frac{erB}{2m_e} .$$

The average current associated with the circulating electron is $i = ev/2\pi r$ and the dipole moment is

$$\mu = Ai = \left(\pi r^2\right)\left(\frac{ev}{2\pi r}\right) = \frac{1}{2}evr .$$

The change in the dipole moment is

$$\Delta\mu = \frac{1}{2}er\Delta v = \frac{1}{2}er\left(\frac{erB}{2m_e}\right) = \frac{e^2r^2B}{4m_e} .$$

39. For the measurements carried out, the largest ratio of the magnetic field to the temperature is (0.50 T)/(10 K) = 0.050 T/K. Look at Fig. 32-14 to see if this is in the region where the magnetization is a linear function of the ratio. It is quite close to the origin, so we conclude that the magnetization obeys Curie's law.

40. (a) From Fig. 32-14 we estimate a slope of B/T = 0.50 T/K when $M/M_{\max}$ = 50%. So

$$B = 0.50 \text{ T} = (0.50 \text{ T/K})(300 \text{ K}) = 1.5\times10^2 \text{ T}.$$

(b) Similarly, now $B/T \approx 2$ so $B = (2)(300) = 6.0\times10^2$ T.

(c) Except for very short times and in very small volumes, these values are not attainable in the lab.

41. **THINK** As defined in Eq. 32-38, magnetization is the dipole moment per unit volume.

EXPRESS Let M be the magnetization and $\mathcal{V}$ be the volume of the cylinder ($\mathcal{V} = \pi r^2 L$, where r is the radius of the cylinder and L is its length). The dipole moment is given by $\mu = M\mathcal{V}$.

ANALYZE Substituting the values given, we obtain

$$\mu = M\pi r^2 L = \left(5.30\times10^3 \text{ A/m}\right)\pi\left(0.500\times10^{-2} \text{ m}\right)^2\left(5.00\times10^{-2} \text{ m}\right) = 2.08\times10^{-2} \text{ J/T} .$$

LEARN In a sample with N atoms, the magnetization reaches maximum, or saturation, when all the dipoles are completely aligned, leading to $M_{\max} = N\mu/\mathcal{V}$.

42. Let

$$K = \frac{3}{2}kT = \left|\vec{\mu}\cdot\vec{B} - \left(-\vec{\mu}\cdot\vec{B}\right)\right| = 2\mu B$$

which leads to

$$T = \frac{4\mu B}{3k} = \frac{4\left(1.0\times10^{-23}\ \text{J/T}\right)\left(0.50\ \text{T}\right)}{3\left(1.38\times10^{-23}\ \text{J/K}\right)} = 0.48\ \text{K}.$$

43. (a) A charge e traveling with uniform speed v around a circular path of radius r takes time $T = 2\pi r/v$ to complete one orbit, so the average current is

$$i = \frac{e}{T} = \frac{ev}{2\pi r}.$$

The magnitude of the dipole moment is this multiplied by the area of the orbit:

$$\mu = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}.$$

Since the magnetic force with magnitude evB is centripetal, Newton's law yields $evB = m_e v^2/r$, so $r = m_e v/eB$. Thus,

$$\mu = \frac{1}{2}(ev)\left(\frac{m_e v}{eB}\right) = \left(\frac{1}{B}\right)\left(\frac{1}{2}m_e v^2\right) = \frac{K_e}{B}.$$

The magnetic force $-e\vec{v}\times\vec{B}$ must point toward the center of the circular path. If the magnetic field is directed out of the page (defined to be $+z$ direction), the electron will travel counterclockwise around the circle. Since the electron is negative, the current is in the opposite direction, clockwise and, by the right-hand rule for dipole moments, the dipole moment is into the page, or in the $-z$ direction. That is, the dipole moment is directed opposite to the magnetic field vector.

(b) We note that the charge canceled in the derivation of $\mu = K_e/B$. Thus, the relation $\mu = K_i/B$ holds for a positive ion.

(c) The direction of the dipole moment is $-z$, opposite to the magnetic field.

(d) The magnetization is given by $M = \mu_e n_e + \mu_i n_i$, where μ_e is the dipole moment of an electron, n_e is the electron concentration, μ_i is the dipole moment of an ion, and n_i is the ion concentration. Since $n_e = n_i$, we may write n for both concentrations. We substitute $\mu_e = K_e/B$ and $\mu_i = K_i/B$ to obtain

$$M = \frac{n}{B}\left(K_e + K_i\right) = \frac{5.3\times10^{21}\text{m}^{-3}}{1.2\text{T}}\left(6.2\times10^{-20}\text{J} + 7.6\times10^{-21}\text{J}\right) = 3.1\times10^{2}\ \text{A/m}.$$

44. Section 32-10 explains the terms used in this problem and the connection between *M* and μ. The graph in Fig. 32-39 gives a slope of

$$\frac{M/M_{\text{max}}}{B_{\text{ext}}/T} = \frac{0.15}{0.20\ \text{T/K}} = 0.75\ \text{K/T}\ .$$

Thus we can write

$$\frac{\mu}{\mu_{\text{max}}} = (0.75\ \text{K/T})\frac{0.800\ \text{T}}{2.00\ \text{K}} = 0.30\ .$$

45. **THINK** According to statistical mechanics, the probability of a magnetic dipole moment placed in an external magnetic field having energy *U* is $P = e^{-U/kT}$, where *k* is the Boltzmann's constant.

EXPRESS The orientation energy of a dipole in a magnetic field is given by $U = -\vec{\mu}\cdot\vec{B}$. So if a dipole is parallel with $\vec{B}$, then $U = -\mu B$; however, $U = +\mu B$ if the alignment is anti-parallel. We use the notation $P(\mu) = e^{\mu B/kT}$ for the probability of a dipole that is parallel to $\vec{B}$, and $P(-\mu) = e^{-\mu B/kT}$ for the probability of a dipole that is anti-parallel to the field. The magnetization may be thought of as a "weighted average" in terms of these probabilities.

ANALYZE (a) With *N* atoms per unit volume, we find the magnetization to be

$$M = \frac{N\mu P(\mu) - N\mu P(-\mu)}{P(\mu)+P(-\mu)} = \frac{N\mu\left(e^{\mu B/kT} - e^{-\mu B/kT}\right)}{e^{\mu B/kT} + e^{-\mu B/kT}} = N\mu\tanh\left(\frac{\mu B}{kT}\right).$$

(b) For $\mu B \ll kT$ (that is, $\mu B/kT \ll 1$) we have $e^{\pm\mu B/kT} \approx 1 \pm \mu B/kT$, so

$$M = N\mu\tanh\left(\frac{\mu B}{kT}\right) \approx \frac{N\mu\left[(1+\mu B/kT) - (1-\mu B/kT)\right]}{(1+\mu B/kT)+(1-\mu B/kT)} = \frac{N\mu^2 B}{kT}\ .$$

(c) For $\mu B \gg kT$ we have $\tanh(\mu B/kT) \approx 1$, so $M = N\mu\tanh\left(\frac{\mu B}{kT}\right) \approx N\mu$.

(d) One can easily plot the tanh function using, for instance, a graphical calculator. One can then note the resemblance between such a plot and Fig. 32-14. By adjusting the parameters used in one's plot, the curve in Fig. 32-14 can reliably be fit with a tanh function.

LEARN As can be seen from Fig. 32-14, the magnetization *M* is linear in B/kT in the regime $B/T \ll 1$. On the other hand, when $B \gg T$, *M* approaches a constant.

46. From Eq. 29-37 (see also Eq. 29-36) we write the torque as $\tau = -\mu B_h \sin\theta$ where the minus indicates that the torque opposes the angular displacement θ (which we will assume is small and in radians). The small angle approximation leads to $\tau \approx -\mu B_h \theta$, which is an indicator for simple harmonic motion (see section 16-5, especially Eq. 16-22). Comparing with Eq. 16-23, we then find the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{\mu B_h}}$$

where I is the rotational inertial that we asked to solve for. Since the frequency is given as 0.312 Hz, then the period is $T = 1/f = 1/(0.312\text{ Hz}) = 3.21$ s. Similarly, $B_h = 18.0 \times 10^{-6}$ T and $\mu = 6.80 \times 10^{-4}$ J/T. The above relation then yields $I = 3.19 \times 10^{-9}\ \text{kg}\cdot\text{m}^2$.

47. **THINK** In this problem, we model the Earth's magnetic dipole moment with a magnetized iron sphere.

EXPRESS If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\text{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}.$$

We substitute this into $\mu_{\text{total}} = N\mu$ to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3\mu}{3m} \quad\Rightarrow\quad R = \left(\frac{3m\mu_{\text{total}}}{4\pi\rho\mu}\right)^{1/3}.$$

ANALYZE (a) The mass of an iron atom is

$$m = 56\,\text{u} = (56\,\text{u})(1.66\times10^{-27}\ \text{kg/u}) = 9.30\times10^{-26}\ \text{kg}.$$

Therefore, the radius of the iron sphere is

$$R = \left[\frac{3(9.30\times10^{-26}\ \text{kg})(8.0\times10^{22}\ \text{J/T})}{4\pi(14\times10^{3}\ \text{kg/m}^3)(2.1\times10^{-23}\ \text{J/T})}\right]^{1/3} = 1.8\times10^{5}\,\text{m}.$$

(b) The volume of the sphere is $V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(1.82\times10^5\,\text{m}\right)^3 = 2.53\times10^{16}\,\text{m}^3$ and the volume of the Earth is

$$V_E = \frac{4\pi}{3}R_E^3 = \frac{4\pi}{3}\left(6.37\times10^6\,\text{m}\right)^3 = 1.08\times10^{21}\,\text{m}^3,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{V_s}{V_E} = \frac{2.53\times10^{16}\,\text{m}^3}{1.08\times10^{21}\,\text{m}^3} = 2.3\times10^{-5}.$$

LEARN The finding that $V_s \ll V_E$ makes it unlikely that our simple model of a magnetized iron sphere could explain the origin of Earth's magnetization.

48. (a) The number of iron atoms in the iron bar is

$$N = \frac{\left(7.9\,\text{g/cm}^3\right)\left(5.0\,\text{cm}\right)\left(1.0\,\text{cm}^2\right)}{\left(55.847\,\text{g/mol}\right)/\left(6.022\times10^{23}/\text{mol}\right)} = 4.3\times10^{23}.$$

Thus the dipole moment of the iron bar is

$$\mu = \left(2.1\times10^{-23}\,\text{J/T}\right)\left(4.3\times10^{23}\right) = 8.9\,\text{A}\cdot\text{m}^2.$$

(b) $\tau = \mu B \sin 90° = (8.9\ \text{A}\cdot\text{m}^2)(1.57\ \text{T}) = 13\ \text{N}\cdot\text{m}$.

49. **THINK** Exchange coupling is a quantum phenomenon in which electron spins of one atom interact with those of neighboring atoms.

EXPRESS The field of a dipole along its axis is given by Eq. 30-29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3},$$

where μ is the dipole moment and z is the distance from the dipole. The energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi,$$

where ϕ is the angle between the dipole moment and the field.

ANALYZE (a) Thus, the magnitude of the magnitude field at a distance 10 nm away from the atom is

$$B=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.5\times10^{-23}\ \text{J/T}\right)}{2\pi\left(10\times10^{-9}\ \text{m}\right)}=3.0\times10^{-6}\ \text{T}.$$

(b) The energy required to turn it end-for-end (from $\phi = 0°$ to $\phi = 180°$) is

$$\Delta U = 2\mu B = 2\left(1.5\times10^{-23}\ \text{J/T}\right)\left(3.0\times10^{-6}\ \text{T}\right) = 9.0\times10^{-29}\ \text{J} = 5.6\times10^{-10}\ \text{eV}.$$

(c) The mean kinetic energy of translation at room temperature is about 0.04 eV. Thus, if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

LEARN The persistent alignment of magnetic dipole moments despite the randomizing tendency due to thermal agitation is what gives the ferromagnetic materials their permanent magnetism.

50. (a) Equation 29-36 gives

$$\tau = \mu_{\text{rod}} B \sin\theta = (2700\ \text{A/m})(0.06\ \text{m})\pi(0.003\ \text{m})^2(0.035\ \text{T})\sin(68°) = 1.49\times10^{-4}\ \text{N}\cdot\text{m}.$$

We have used the fact that the volume of a cylinder is its length times its (circular) cross sectional area.

(b) Using Eq. 29-38, we have

$$\begin{aligned}\Delta U &= -\mu_{\text{rod}} B(\cos\theta_f - \cos\theta_i)\\ &= -(2700\ \text{A/m})(0.06\ \text{m})\pi(0.003\text{m})^2(0.035\text{T})[\cos(34°) - \cos(68°)]\\ &= -72.9\ \mu\text{J}.\end{aligned}$$

51. The saturation magnetization corresponds to complete alignment of all atomic dipoles and is given by $M_{\text{sat}} = \mu n$, where n is the number of atoms per unit volume and μ is the magnetic dipole moment of an atom. The number of nickel atoms per unit volume is $n = \rho/m$, where ρ is the density of nickel. The mass of a single nickel atom is calculated using $m = M/N_A$, where M is the atomic mass of nickel and N_A is Avogadro's constant. Thus,

$$\begin{aligned}n &= \frac{\rho N_A}{M} = \frac{\left(8.90\ \text{g/cm}^3\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)}{58.71\ \text{g/mol}} = 9.126\times10^{22}\ \text{atoms/cm}^3\\ &= 9.126\times10^{28}\ \text{atoms/m}^3.\end{aligned}$$

The dipole moment of a single atom of nickel is

$$\mu = \frac{M_{\text{sat}}}{n} = \frac{4.70\times10^5\ \text{A/m}}{9.126\times10^{28}\ \text{m}^3} = 5.15\times10^{-24}\ \text{A}\cdot\text{m}^2.$$

52. The Curie temperature for iron is 770°C. If x is the depth at which the temperature has this value, then 10°C + (30°C/km)x = 770°C. Therefore,

$$x = \frac{770°\,\mathrm{C} - 10°\,\mathrm{C}}{30°\,\mathrm{C/km}} = 25\,\mathrm{km}.$$

53. (a) The magnitude of the toroidal field is given by $B_0 = \mu_0 n i_p$, where n is the number of turns per unit length of toroid and i_p is the current required to produce the field (in the absence of the ferromagnetic material). We use the average radius (r_{avg} = 5.5 cm) to calculate n:

$$n = \frac{N}{2\pi r_{\text{avg}}} = \frac{400 \text{ turns}}{2\pi(5.5\times10^{-2}\,\mathrm{m})} = 1.16\times10^{3} \text{ turns/m}.$$

Thus,

$$i_p = \frac{B_0}{\mu_0 n} = \frac{0.20\times10^{-3}\ \mathrm{T}}{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(1.16\times10^{3}\,/\,\mathrm{m})} = 0.14\ \mathrm{A}.$$

(b) If Φ is the magnetic flux through the secondary coil, then the magnitude of the emf induced in that coil is $\varepsilon = N(d\Phi/dt)$ and the current in the secondary is $i_s = \varepsilon/R$, where R is the resistance of the coil. Thus,

$$i_s = \left(\frac{N}{R}\right)\frac{d\Phi}{dt}.$$

The charge that passes through the secondary when the primary current is turned on is

$$q = \int i_s\, dt = \frac{N}{R}\int \frac{d\Phi}{dt}\,dt = \frac{N}{R}\int_0^{\Phi} d\Phi = \frac{N\Phi}{R}.$$

The magnetic field through the secondary coil has magnitude $B = B_0 + B_M = 801B_0$, where B_M is the field of the magnetic dipoles in the magnetic material. The total field is perpendicular to the plane of the secondary coil, so the magnetic flux is $\Phi = AB$, where A is the area of the Rowland ring (the field is inside the ring, not in the region between the ring and coil). If r is the radius of the ring's cross section, then $A = \pi r^2$. Thus,

$$\Phi = 801\pi r^2 B_0.$$

The radius r is (6.0 cm – 5.0 cm)/2 = 0.50 cm and

$$\Phi = 801\pi(0.50\times10^{-2}\ \mathrm{m})^2(0.20\times10^{-3}\,\mathrm{T}) = 1.26\times10^{-5}\ \mathrm{Wb}.$$

Consequently, $q = \dfrac{50(1.26\times10^{-5}\ \mathrm{Wb})}{8.0\ \Omega} = 7.9\times10^{-5}\ \mathrm{C}$.

54. (a) At a distance r from the center of the Earth, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1+3\sin^2 \lambda_m} \ ,$$

where μ is the Earth's dipole moment and λ_m is the magnetic latitude. The ratio of the field magnitudes for two different distances at the same latitude is

$$\frac{B_2}{B_1} = \frac{r_1^3}{r_2^3} \ .$$

With B_1 being the value at the surface and B_2 being half of B_1, we set r_1 equal to the radius R_e of the Earth and r_2 equal to $R_e + h$, where h is altitude at which B is half its value at the surface. Thus,

$$\frac{1}{2} = \frac{R_e^3}{(R_e + h)^3} \ .$$

Taking the cube root of both sides and solving for h, we get

$$h = \left(2^{1/3} - 1\right) R_e = \left(2^{1/3} - 1\right)\left(6370 \text{km}\right) = 1.66\times 10^3 \text{ km}.$$

(b) For maximum B, we set $\sin \lambda_m = 1.00$. Also, r = 6370 km – 2900 km = 3470 km. Thus,

$$B_{\max} = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1+3\sin^2 \lambda_m} = \frac{\left(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(8.00\times 10^{22} \text{ A}\cdot\text{m}^2\right)}{4\pi\left(3.47\times 10^6 \text{ m}\right)^3}\sqrt{1+3\left(1.00\right)^2}$$
$$= 3.83\times 10^{-4} \text{ T}.$$

(c) The angle between the magnetic axis and the rotational axis of the Earth is 11.5°, so λ_m = 90.0° – 11.5° = 78.5° at Earth's geographic north pole. Also $r = R_e$ = 6370 km. Thus,

$$B = \frac{\mu_0 \mu}{4\pi R_E^3}\sqrt{1+3\sin^2 \lambda_m} = \frac{\left(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(8.0\times 10^{22} \text{ J/T}\right)\sqrt{1+3\sin^2 78.5^\circ}}{4\pi\left(6.37\times 10^6 \text{ m}\right)^3}$$
$$= 6.11\times 10^{-5} \text{ T}.$$

(d) $\phi_i = \tan^{-1}(2\tan 78.5^\circ) = 84.2^\circ$.

(e) A plausible explanation to the discrepancy between the calculated and measured values of the Earth's magnetic field is that the formulas we used are based on dipole approximation, which does not accurately represent the Earth's actual magnetic field distribution on or near its surface. (Incidentally, the dipole approximation becomes more reliable when we calculate the Earth's magnetic field far from its center.)

55. (a) From $\mu = iA = i\pi R_e^2$ we get

$$i = \frac{\mu}{\pi R_e^2} = \frac{8.0\times10^{22}\ \text{J/T}}{\pi(6.37\times10^6\ \text{m})^2} = 6.3\times10^8\ \text{A}\ .$$

(b) Yes, because far away from the Earth the fields of both the Earth itself and the current loop are dipole fields. If these two dipoles cancel each other out, then the net field will be zero.

(c) No, because the field of the current loop is not that of a magnetic dipole in the region close to the loop.

56. (a) The period of rotation is $T = 2\pi/\omega$, and in this time all the charge passes any fixed point near the ring. The average current is $i = q/T = q\omega/2\pi$ and the magnitude of the magnetic dipole moment is

$$\mu = iA = \frac{q\omega}{2\pi}\pi r^2 = \frac{1}{2}q\omega r^2\ .$$

(b) We curl the fingers of our right hand in the direction of rotation. Since the charge is positive, the thumb points in the direction of the dipole moment. It is the same as the direction of the angular momentum vector of the ring.

57. The interacting potential energy between the magnetic dipole of the compass and the Earth's magnetic field is

$$U = -\vec{\mu}\cdot\vec{B}_e = -\mu B_e\cos\theta,$$

where θ is the angle between $\vec{\mu}$ and $\vec{B}_e$. For small angle θ,

$$U(\theta) = -\mu B_e\cos\theta \approx -\mu B_e\left(1-\frac{\theta^2}{2}\right) = \frac{1}{2}\kappa\theta^2 - \mu B_e$$

where $\kappa = \mu B_e$. Conservation of energy for the compass then gives

$$\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\kappa\theta^2 = \text{const.}$$

This is to be compared with the following expression for the mechanical energy of a spring-mass system:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 = \text{const.}\ ,$$

which yields $\omega = \sqrt{k/m}$. So by analogy, in our case

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{\mu B_e}{I}} = \sqrt{\frac{\mu B_e}{ml^2/12}} ,$$

which leads to

$$\mu = \frac{ml^2\omega^2}{12B_e} = \frac{(0.050\,\text{kg})(4.0\times10^{-2}\ \text{m})^2(45\,\text{rad/s})^2}{12(16\times10^{-6}\ \text{T})} = 8.4\times10^2\ \text{J/T} .$$

58. (a) Equation 30-22 gives $B = \dfrac{\mu_0 i r}{2\pi R^2} = 222\ \mu\text{T}$.

(b) Equation 30-19 (or Eq. 30-6) gives $B = \dfrac{\mu_0 i}{2\pi r} = 167\ \mu\text{T}$.

(c) As in part (b), we obtain a field of $B = \dfrac{\mu_0 i}{2\pi r} = 22.7\ \mu\text{T}$.

(d) Equation 32-16 (with Eq. 32-15) gives $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 1.25\ \mu\text{T}$.

(e) As in part (d), we get $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 3.75\ \mu\text{T}$.

(f) Equation 32-17 yields B = 22.7 μT.

(g) Because the displacement current in the gap is spread over a larger cross-sectional area, values of B within that area are relatively small. Outside that cross-sectional area, the two values of B are identical.

59. (a) We use the result of part (a) in Sample Problem 32.01 — "Magnetic field induced by changing electric field:"

$$B = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt} \quad (\text{for } r \le R) ,$$

where $r = 0.80R$, and

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{1}{d}\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = -\frac{V_0}{\tau d}e^{-t/\tau} .$$

Here V_0 = 100 V. Thus,

$$B(t) = \left(\frac{\mu_0 \varepsilon_0 r}{2}\right)\left(-\frac{V_0}{\tau d} e^{-t/\tau}\right) = -\frac{\mu_0 \varepsilon_0 V_0 r}{2\tau d} e^{-t/\tau}$$

$$= -\frac{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(8.85 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2})(100\ \mathrm{V})(0.80)(16\ \mathrm{mm})}{2(12 \times 10^{-3}\ \mathrm{s})(5.0\ \mathrm{mm})} e^{-t/12\ \mathrm{ms}}$$

$$= -(1.2 \times 10^{-13}\ \mathrm{T})e^{-t/12\ \mathrm{ms}}\ .$$

The magnitude is $|B(t)| = (1.2\times10^{-13}\ \mathrm{T})e^{-t/12\ \mathrm{ms}}$.

(b) At time $t = 3\tau$, $B(t) = -(1.2 \times 10^{-13}\ \mathrm{T})e^{-3\tau/\tau} = -5.9 \times 10^{-15}$ T, with a magnitude $|B(t)|= 5.9 \times 10^{-15}$ T.

60. (a) From Eq. 32-1, we have

$$(\Phi_B)_{\mathrm{in}} = (\Phi_B)_{\mathrm{out}} = 0.0070\,\mathrm{Wb} + (0.40\,\mathrm{T})(\pi r^2) = 9.2\times10^{-3}\,\mathrm{Wb}.$$

Thus, the magnetic of the magnetic flux is 9.2 mWb.

(b) The flux is inward.

61. **THINK** The Earth's magnetic field at a given latitude has both horizontal and vertical components.

EXPRESS Let B_h and B_v be the horizontal and vertical components of the Earth's magnetic field, respectively. Since B_h and B_v are perpendicular to each other, the Pythagorean theorem leads to $B = \sqrt{B_h^2 + B_v^2}$. The tangent of the inclination angle is given by $\tan\phi_i = B_v / B_h$.

ANALYZE (a) Substituting the expression given in the problem statement, we have

$$B = \sqrt{B_h^2 + B_v^2} = \sqrt{\left(\frac{\mu_0\mu}{4\pi r^3}\cos\lambda_m\right)^2 + \left(\frac{\mu_0\mu}{2\pi r^3}\sin\lambda_m\right)^2} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m + 4\sin^2\lambda_m}$$

$$= \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m},$$

where $\cos^2\lambda_m + \sin^2\lambda_m = 1$ was used.

(b) The inclination ϕ_i is related to λ_m by $\tan\phi_i = \frac{B_v}{B_h} = \frac{(\mu_0\mu/2\pi r^3)\sin\lambda_m}{(\mu_0\mu/4\pi r^3)\cos\lambda_m} = 2\tan\lambda_m$.

LEARN At the magnetic equator ($\lambda_m = 0$), $\phi_i = 0°$, and the field is

$$B = \frac{\mu_0 \mu}{4\pi r^3} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.00\times10^{22}\ \text{A}\cdot\text{m}^2\right)}{4\pi\left(6.37\times10^{6}\ \text{m}\right)^3} = 3.10\times10^{-5}\ \text{T}.$$

62. (a) At the magnetic equator ($\lambda_m = 0$), the field is

$$B = \frac{\mu_0 \mu}{4\pi r^3} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.00\times10^{22}\ \text{A}\cdot\text{m}^2\right)}{4\pi\left(6.37\times10^{6}\ \text{m}\right)^3} = 3.10\times10^{-5}\ \text{T}.$$

(b) $\phi_i = \tan^{-1}(2\tan\lambda_m) = \tan^{-1}(0) = 0°$.

(c) At $\lambda_m = 60.0°$, we find

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = \left(3.10\times10^{-5}\right)\sqrt{1+3\sin^2 60.0°} = 5.59\times10^{-5}\ \text{T}.$$

(d) $\phi_i = \tan^{-1}(2\tan 60.0°) = 73.9°$.

(e) At the north magnetic pole ($\lambda_m = 90.0°$), we obtain

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = \left(3.10\times10^{-5}\right)\sqrt{1+3\left(1.00\right)^2} = 6.20\times10^{-5}\ \text{T}.$$

(f) $\phi_i = \tan^{-1}(2\tan 90.0°) = 90.0°$.

63. Let R be the radius of a capacitor plate and r be the distance from axis of the capacitor. For points with $r \le R$, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt},$$

and for $r \ge R$, it is

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r}\frac{dE}{dt}.$$

The maximum magnetic field occurs at points for which $r = R$, and its value is given by either of the formulas above:

$$B_{\max} = \frac{\mu_0 \varepsilon_0 R}{2}\frac{dE}{dt}.$$

There are two values of r for which $B = B_{\max}/2$: one less than R and one greater.

(a) To find the one that is less than R, we solve

$$\frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}=\frac{\mu_0\varepsilon_0 R}{4}\frac{dE}{dt}$$

for r. The result is $r = R/2 = (55.0 \text{ mm})/2 = 27.5$ mm.

(b) To find the one that is greater than R, we solve

$$\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}=\frac{\mu_0\varepsilon_0 R}{4}\frac{dE}{dt}$$

for r. The result is $r = 2R = 2(55.0 \text{ mm}) = 110$ mm.

64. (a) Again from Fig. 32-14, for $M/M_{\max} = 50\%$ we have $B/T = 0.50$. So $T = B/0.50 = 2/0.50 = 4$ K.

(b) Now $B/T = 2.0$, so $T = 2/2.0 = 1$ K.

65. Let the area of each circular plate be A and that of the central circular section be a. Then

$$\frac{A}{a}=\frac{\pi R^2}{\pi\left(R/2\right)^2}=4\ .$$

Thus, from Eqs. 32-14 and 32-15 the total discharge current is given by $i = i_d = 4(2.0 \text{ A}) = 8.0$ A.

66. Ignoring points where the determination of the slope is problematic, we find the interval of largest $|\Delta\vec{E}|/\Delta t$ is 6 μs < t < 7 μs. During that time, we have, from Eq. 32-14,

$$i_d=\varepsilon_0 A\frac{|\Delta\vec{E}|}{\Delta t}=(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m}^2)(2.0\times10^6\ \text{V/m})=3.5\times10^{-5}\,\text{A}.$$

67. (a) Using Eq. 32-13 but noting that the capacitor is being *discharged*, we have

$$\frac{d\,|\vec{E}\,|}{dt}=-\frac{i}{\varepsilon_0 A}=-\frac{5.0\ \text{A}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0080\ \text{m})^2}=-8.8\times10^{15}\ \text{V/m}\cdot\text{s}\ .$$

(b) Assuming a perfectly uniform field, even so near to an edge (which is consistent with the fact that fringing is neglected in Section 32-4), we follow part (a) of Sample Problem 32.02 — "Treating a changing electric field as a displacement current" and relate the (absolute value of the) line integral to the portion of displacement current enclosed:

$$\left|\oint \vec{B}\cdot d\vec{s}\right| = \mu_0 i_{d,\text{enc}} = \mu_0\left(\frac{WH}{L^2}i\right) = 5.9\times10^{-7}\,\text{Wb/m}.$$

68. (a) Using Eq. 32-31, we find

$$\mu_{\text{orb},z} = -3\mu_B = -2.78\times10^{-23}\ \text{J/T}.$$

That these are acceptable units for magnetic moment is seen from Eq. 32-32 or Eq. 32-27; they are equivalent to $\text{A}\cdot\text{m}^2$.

(b) Similarly, for $m_\ell = -4$ we obtain $\mu_{\text{orb},z} = 3.71\times10^{-23}$ J/T.

69. (a) Since the field lines of a bar magnet point toward its South pole, then the $\vec{B}$ arrows in one's sketch should point generally toward the left and also towards the central axis.

(b) The sign of $\vec{B}\cdot d\vec{A}$ for every $d\vec{A}$ on the side of the paper cylinder is negative.

(c) No, because Gauss' law for magnetism applies to an *enclosed* surface only. In fact, if we include the top and bottom of the cylinder to form an enclosed surface S then $\oint_s \vec{B}\cdot d\vec{A} = 0$ will be valid, as the flux through the open end of the cylinder near the magnet is positive.

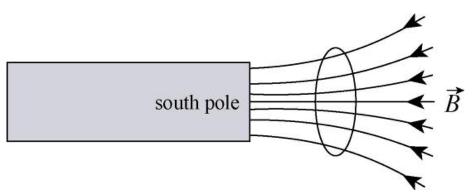
70. (a) From Eq. 21-3,

$$E = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{(1.60\times10^{-19}\ \text{C})(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}{(5.2\times10^{-11}\ \text{m})^2} = 5.3\times10^{11}\ \text{N/C}.$$

(b) We use Eq. 29-28: $B = \dfrac{\mu_0}{2\pi}\dfrac{\mu_p}{r^3} = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.4\times10^{-26}\ \text{J/T})}{2\pi(5.2\times10^{-11}\ \text{m})^3} = 2.0\times10^{-2}\ \text{T}$.

(c) From Eq. 32-30, $\dfrac{\mu_{\text{orb}}}{\mu_p} = \dfrac{eh/4\pi m_e}{\mu_p} = \dfrac{\mu_B}{\mu_p} = \dfrac{9.27\times10^{-24}\ \text{J/T}}{1.4\times10^{-26}\ \text{J/T}} = 6.6\times10^2$.

71. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) For paramagnetic materials, the dipole moment $\vec{\mu}$ is in the same direction as $\vec{B}$. From the above figure, $\vec{\mu}$ points in the $-x$ direction.

(c) Form the right-hand rule, since $\vec{\mu}$ points in the $-x$ direction, the current flows counterclockwise, from the perspective of the bar magnet.

(d) The effect of $\vec{F}$ is to move the material toward regions of larger $|\vec{B}|$ values. Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is toward the left, or $-x$.

72. (a) Inside the gap of the capacitor, $B_1 = \mu_0 i_d r_1/2\pi R^2$ (Eq. 32-16); outside the gap the magnetic field is $B_2 = \mu_0 i_d/2\pi r_2$ (Eq. 32-17). Consequently, $B_2 = B_1 R^2/r_1 r_2 = 16.7$ nT.

(b) The displacement current is $i_d = 2\pi B_1 R^2/\mu_0 r_1 = 5.00$ mA.

73. **THINK** The z component of the orbital angular momentum is give by $L_{\text{orb},z} = m_\ell h/2\pi$, where h is the Planck constant and m_ℓ is the orbital magnetic quantum number.

EXPRESS The "limit" for m_ℓ is 3. This means that the allowed values of m_ℓ are: $0, \pm 1, \pm 2,$ and ± 3.

ANALYZE (a) The number of different m_ℓ's is 2(3) + 1 = 7. Since $L_{\text{orb},z} \propto m_\ell$, there are a total of seven different values of $L_{\text{orb},z}$.

(b) Similarly, since $\mu_{\text{orb},z} \propto m_\ell$, there are also a total of seven different values of $\mu_{\text{orb},z}$.

(c) The greatest allowed value of $L_{\text{orb},z}$ is given by $|m_\ell|_{\max} h/2\pi = 3h/2\pi$.

(d) Similar to part (c), since $\mu_{\text{orb},z} = -m_\ell \mu_B$, the greatest allowed value of $\mu_{\text{orb},z}$ is given by $|m_\ell|_{\max}\mu_B = 3eh/4\pi m_e$.

(e) From Eqs. 32-23 and 32-29 the z component of the net angular momentum of the electron is given by

$$L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = \frac{m_\ell h}{2\pi} + \frac{m_s h}{2\pi}.$$

For the maximum value of $L_{\text{net},z}$ let $m_\ell = [m_\ell]_{\max} = 3$ and $m_s = \frac{1}{2}$. Thus

$$\left(L_{\text{net},z}\right)_{\max} = \left(3 + \frac{1}{2}\right)\frac{h}{2\pi} = \frac{3.5h}{2\pi}.$$

(f) Since the maximum value of $L_{net,z}$ is given by $[m_J]_{max}h/2\pi$ with $[m_J]_{max} = 3.5$ (see the last part above), the number of allowed values for the z component of $L_{net,z}$ is given by $2[m_J]_{max} + 1 = 2(3.5) + 1 = 8$.

LEARN As we shall see in Chapter 40, the allowed values of m_ℓ range from $-\ell$ to $+\ell$, where ℓ is called the orbital quantum number.

74. The definition of displacement current is Eq. 32-10, and the formula of greatest convenience here is Eq. 32-17:

$$i_d = \frac{2\pi r\, B}{\mu_0} = \frac{2\pi(0.0300\,\text{m})(2.00\times10^{-6}\,\text{T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 0.300\ \text{A}\,.$$

75. (a) The complete set of values are

$$\{-4, -3, -2, -1, 0, +1, +2, +3, +4\} \quad\Rightarrow\quad \text{nine values in all.}$$

(b) The maximum value is $4\mu_B = 3.71 \times 10^{-23}$ J/T.

(c) Multiplying our result for part (b) by 0.250 T gives $U = +9.27 \times 10^{-24}$ J.

(d) Similarly, for the lower limit, $U = -9.27 \times 10^{-24}$ J.

76. (a) The z component of the orbital magnetic dipole moment is

$$\mu_{\text{orb},z} = -m_\ell \mu_B$$

where $\mu_B = eh/4\pi m = 9.27\times10^{-24}\,\text{J/T}$ is the Bohr magneton. For $m_l = 3,$ we have

$$\mu_{\text{orb},z} = -m_\ell \mu_B - (3)(9.27\times10^{-24}\ \text{J/T}) = -2.78\times10^{-23}\ \text{J/T}.$$

(b) Similarly, for $m_l = -4,$ the result is

$$\mu_{\text{orb},z} = -m_\ell \mu_B - (-4)(9.27\times10^{-24}\ \text{J/T}) = 3.71\times10^{-23}\ \text{J/T}.$$

Chapter 33

1. Since $\Delta\lambda \ll \lambda$, we find Δf is equal to

$$\left|\Delta\left(\frac{c}{\lambda}\right)\right| \approx \frac{c\Delta\lambda}{\lambda^2} = \frac{(3.0\times10^8\text{ m/s})(0.0100\times10^{-9}\text{ m})}{(632.8\times10^{-9}\text{ m})^2} = 7.49\times10^9\text{ Hz.}$$

2. (a) The frequency of the radiation is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^8\text{ m/s}}{(1.0\times10^5)(6.4\times10^6\text{ m})} = 4.7\times10^{-3}\text{ Hz.}$$

(b) The period of the radiation is

$$T = \frac{1}{f} = \frac{1}{4.7\times10^{-3}\text{ Hz}} = 212\text{ s} = 3\min 32\text{ s.}$$

3. (a) From Fig. 33-2 we find the smaller wavelength in question to be about 515 nm.

(b) Similarly, the larger wavelength is approximately 610 nm.

(c) From Fig. 33-2 the wavelength at which the eye is most sensitive is about 555 nm.

(d) Using the result in (c), we have

$$f = \frac{c}{\lambda} = \frac{3.00\times10^8\text{ m/s}}{555\text{ nm}} = 5.41\times10^{14}\text{ Hz}.$$

(e) The period is $T = 1/f = (5.41\times10^{14}\text{ Hz})^{-1} = 1.85\times10^{-15}$ s.

4. In air, light travels at roughly $c = 3.0\times10^8$ m/s. Therefore, for $t = 1.0$ ns, we have a distance of

$$d = ct = (3.0\times10^8\text{ m/s})(1.0\times10^{-9}\text{ s}) = 0.30\text{ m.}$$

5. **THINK** The frequency of oscillation of the current in the LC circuit of the generator is $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. This frequency is the same as the frequency of an electromagnetic wave.

EXPRESS If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. Thus,

$$\frac{\lambda}{2\pi\sqrt{LC}} = c.$$

ANALYZE The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 Cc^2} = \frac{(550\times10^{-9}\text{ m})^2}{4\pi^2(17\times10^{-12}\text{ F})(2.998\times10^8\text{ m/s})^2} = 5.00\times10^{-21}\text{ H}.$$

This is exceedingly small.

LEARN The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^8\text{ m/s}}{550\times10^{-9}\text{ m}} = 5.45\times10^{14}\text{ Hz}.$$

The EM wave is in the visible spectrum.

6. The emitted wavelength is

$$\lambda = \frac{c}{f} = 2\pi c\sqrt{LC} = 2\pi\left(2.998\times10^8\text{ m/s}\right)\sqrt{\left(0.253\times10^{-6}\text{ H}\right)\left(25.0\times10^{-12}\text{ F}\right)} = 4.74\text{ m}.$$

7. The intensity is the average of the Poynting vector:

$$I = S_{\text{avg}} = \frac{cB_m^2}{2\mu_0} = \frac{(3.0\times10^8\text{ m/s})(1.0\times10^{-4}\text{ T})^2}{2(1.26\times10^{-6}\text{ H/m})} = 1.2\times10^6\text{ W/m}^2.$$

8. The intensity of the signal at Proxima Centauri is

$$I = \frac{P}{4\pi r^2} = \frac{1.0\times10^6\text{ W}}{4\pi\left[(4.3\text{ ly})(9.46\times10^{15}\text{ m/ly})\right]^2} = 4.8\times10^{-29}\text{ W/m}^2.$$

9. If P is the power and Δt is the time interval of one pulse, then the energy in a pulse is

$$E = P\Delta t = (100\times10^{12}\text{ W})(1.0\times10^{-9}\text{ s}) = 1.0\times10^5\text{ J}.$$

10. (a) Setting $v = c$ in the wave relation $kv = \omega = 2\pi f$, we find $f = 1.91\times10^8$ Hz.

(b) $E_{\text{rms}} = E_m/\sqrt{2} = B_m/c\sqrt{2} = 18.2$ V/m.

(c) $I = (E_{\text{rms}})^2/c\mu_o = 0.878$ W/m^2.

11. (a) The amplitude of the magnetic field is

$$B_m = \frac{E_m}{c} = \frac{2.0\text{V/m}}{2.998\times10^8\,\text{m/s}} = 6.67\times10^{-9}\,\text{T} \approx 6.7\times10^{-9}\,\text{T}.$$

(b) Since the $\vec{E}$-wave oscillates in the z direction and travels in the x direction, we have $B_x = B_z = 0$. So, the oscillation of the magnetic field is parallel to the y axis.

(c) The direction ($+x$) of the electromagnetic wave propagation is determined by $\vec{E}\times\vec{B}$. If the electric field points in $+z$, then the magnetic field must point in the $-y$ direction.

With SI units understood, we may write

$$B_y = B_m \cos\left[\pi\times10^{15}\left(t-\frac{x}{c}\right)\right] = \frac{2.0\cos\left[10^{15}\pi\left(t-x/c\right)\right]}{3.0\times10^8}$$
$$= \left(6.7\times10^{-9}\right)\cos\left[10^{15}\pi\left(t-\frac{x}{c}\right)\right]$$

12. (a) The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{5.00\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 1.67\times10^{-8}\,\text{T}.$$

(b) The intensity is the average of the Poynting vector:

$$I = S_{\text{avg}} = \frac{E_m^2}{2\mu_0 c} = \frac{\left(5.00\,\text{V/m}\right)^2}{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(2.998\times10^8\,\text{m/s}\right)} = 3.31\times10^{-2}\,\text{W/m}^2.$$

13. (a) We use $I = E_m^2/2\mu_0 c$ to calculate E_m:

$$E_m = \sqrt{2\mu_0 I_c} = \sqrt{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(1.40\times10^3\,\text{W/m}^2\right)\left(2.998\times10^8\,\text{m/s}\right)}$$
$$= 1.03\times10^3\,\text{V/m}.$$

(b) The magnetic field amplitude is therefore

$$B_m = \frac{E_m}{c} = \frac{1.03\times10^4\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 3.43\times10^{-6}\,\text{T}.$$

14. From the equation immediately preceding Eq. 33-12, we see that the maximum value of $\partial B/\partial t$ is ωB_m. We can relate B_m to the intensity:

$$B_m = \frac{E_m}{c} = \frac{\sqrt{2c\mu_0 I}}{c},$$

and relate the intensity to the power P (and distance r) using Eq. 33-27. Finally, we relate ω to wavelength λ using $\omega = kc = 2\pi c/\lambda$. Putting all this together, we obtain

$$\left(\frac{\partial B}{\partial t}\right)_{\max} = \sqrt{\frac{2\mu_0 P}{4\pi c}}\,\frac{2\pi c}{\lambda r} = 3.44\times10^{6}\ \text{T/s}.$$

15. (a) The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2/2\mu_0 c$, so

$$E_m = \sqrt{2\mu_0 cI} = \sqrt{2\left(4\pi\times10^{-7}\ \text{H/m}\right)\left(2.998\times10^{8}\ \text{m/s}\right)\left(10\times10^{-6}\ \text{W/m}^2\right)}$$
$$= 8.7\times10^{-2}\ \text{V/m}.$$

(b) The amplitude of the magnetic field is given by

$$B_m = \frac{E_m}{c} = \frac{8.7\times10^{-2}\ \text{V/m}}{2.998\times10^{8}\ \text{m/s}} = 2.9\times10^{-10}\ \text{T}.$$

(c) At a distance r from the transmitter, the intensity is $I = P/2\pi r^2$, where P is the power of the transmitter over the hemisphere having a surface area $2\pi r^2$. Thus

$$P = 2\pi r^2 I = 2\pi\left(10\times10^{3}\ \text{m}\right)^2\left(10\times10^{-6}\ \text{W/m}^2\right) = 6.3\times10^{3}\ \text{W}.$$

16. (a) The power received is

$$P_r = \left(1.0\times10^{-12}\ \text{W}\right)\frac{\pi(300\ \text{m})^2/4}{4\pi\left(6.37\times10^{6}\ \text{m}\right)^2} = 1.4\times10^{-22}\ \text{W}.$$

(b) The power of the source would be

$$P = 4\pi r^2 I = 4\pi\left[\left(2.2\times10^{4}\ \text{ly}\right)\left(9.46\times10^{15}\ \text{m/ly}\right)\right]^2\left[\frac{1.0\times10^{-12}\ \text{W}}{4\pi\left(6.37\times10^{6}\ \text{m}\right)^2}\right] = 1.1\times10^{15}\ \text{W}.$$

17. (a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 6.7\times10^{-9}\,\text{T}.$$

(b) The intensity is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(2.0\,\text{V/m})^2}{2(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.998\times10^8\,\text{m/s})} = 5.3\times10^{-3}\,\text{W/m}^2.$$

(c) The power of the source is

$$P = 4\pi r^2 I_{\text{avg}} = 4\pi(10\,\text{m})^2\left(5.3\times10^{-3}\,\text{W/m}^2\right) = 6.7\,\text{W}.$$

18. Equation 33-27 suggests that the slope in an intensity versus inverse-square-distance graph (I plotted versus r^{-2}) is $P/4\pi$. We estimate the slope to be about 20 (in SI units), which means the power is $P = 4\pi(30) \approx 2.5\times10^2$ W.

19. **THINK** The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity.

EXPRESS The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation.

ANALYZE Thus, the radiation pressure is

$$p_r = \frac{2I}{c} = \frac{2P}{Ac} = \frac{2\left(1.5\times10^9\,\text{W}\right)}{\left(1.00\times10^{-6}\,\text{m}^2\right)\left(2.998\times10^8\,\text{m/s}\right)} = 1.0\times10^7\,\text{Pa}.$$

LEARN In the case of total absorption, the radiation pressure would be $p_r = I/c$, a factor of 2 smaller than the case of total reflection.

20. (a) The radiation pressure produces a force equal to

$$F_r = p_r\left(\pi R_e^2\right) = \left(\frac{I}{c}\right)\left(\pi R_e^2\right) = \frac{\pi\left(1.4\times10^3\,\text{W/m}^2\right)\left(6.37\times10^6\,\text{m}\right)^2}{2.998\times10^8\,\text{m/s}} = 6.0\times10^8\,\text{N}.$$

(b) The gravitational pull of the Sun on the Earth is

$$F_{\text{grav}} = \frac{GM_sM_e}{d_{es}^2} = \frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.0\times10^{30}\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(1.5\times10^{11}\,\text{m}\right)^2}$$

$$= 3.6\times10^{22}\,\text{N},$$

which is much greater than F_r.

21. Since the surface is perfectly absorbing, the radiation pressure is given by $p_r = I/c$, where I is the intensity. Since the bulb radiates uniformly in all directions, the intensity a distance r from it is given by $I = P/4\pi r^2$, where P is the power of the bulb. Thus

$$p_r = \frac{P}{4\pi r^2 c} = \frac{500\,\text{W}}{4\pi(1.5\text{ m})^2(2.998\times10^8\text{ m/s})} = 5.9\times10^{-8}\text{ Pa}.$$

22. The radiation pressure is

$$p_r = \frac{I}{c} = \frac{10\,\text{W/m}^2}{2.998\times10^8\text{ m/s}} = 3.3\times10^{-8}\text{ Pa}.$$

23. (a) The upward force supplied by radiation pressure in this case (Eq. 33-32) must be equal to the magnitude of the pull of gravity (mg). For a sphere, the "projected" area (which is a factor in Eq. 33-32) is that of a circle $A = \pi r^2$ (not the entire surface area of the sphere) and the volume (needed because the mass is given by the density multiplied by the volume: $m = \rho V$) is $V = 4\pi r^3/3$. Finally, the intensity is related to the power P of the light source and another area factor $4\pi R^2$, given by Eq. 33-27. In this way, with $\rho = 1.9\times10^4$ kg/m^3, equating the forces leads to

$$P = 4\pi R^2 c\left(\rho\frac{4\pi r^3 g}{3}\right)\frac{1}{\pi r^2} = 4.68\times10^{11}\text{ W}.$$

(b) Any chance disturbance could move the sphere from being directly above the source, and then the two force vectors would no longer be along the same axis.

24. We require $F_{\text{grav}} = F_r$ or

$$G\frac{mM_s}{d_{es}^2} = \frac{2IA}{c},$$

and solve for the area A:

$$A = \frac{cGmM_s}{2Id_{es}^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1500\text{ kg})(1.99\times10^{30}\text{ kg})(2.998\times10^8\text{ m/s})}{2(1.40\times10^3\text{ W/m}^2)(1.50\times10^{11}\text{ m})^2}$$
$$= 9.5\times10^5\text{ m}^2 = 0.95\text{ km}^2.$$

25. **THINK** In this problem we relate radiation pressure to energy density in the incident beam.

EXPRESS Let f be the fraction of the incident beam intensity that is reflected. The fraction absorbed is $1 - f$. The reflected portion exerts a radiation pressure of

$$p_r = \frac{2fI_0}{c}$$

and the absorbed portion exerts a radiation pressure of

$$p_a = \frac{(1-f)I_0}{c},$$

where I_0 is the incident intensity. The factor 2 enters the first expression because the momentum of the reflected portion is reversed. The total radiation pressure is the sum of the two contributions:

$$p_{\text{total}} = p_r + p_a = \frac{2fI_0 + (1-f)I_0}{c} = \frac{(1+f)I_0}{c}.$$

ANALYZE To relate the intensity and energy density, we consider a tube with length ℓ and cross-sectional area A, lying with its axis along the propagation direction of an electromagnetic wave. The electromagnetic energy inside is $U = uA\ell$, where u is the energy density. All this energy passes through the end in time $t = \ell / c$, so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc.$$

Thus $u = I/c$. The intensity and energy density are positive, regardless of the propagation direction. For the partially reflected and partially absorbed wave, the intensity just outside the surface is

$$I = I_0 + fI_0 = (1 + f)I_0,$$

where the first term is associated with the incident beam and the second is associated with the reflected beam. Consequently, the energy density is

$$u = \frac{I}{c} = \frac{(1+f)I_0}{c},$$

the same as radiation pressure.

LEARN In the case of total reflection, $f = 1$, and $p_{\text{total}} = p_r = 2I_0 / c$. On the other hand, the energy density is $u = I / c = 2I_0 / c$, which is the same as p_{total}. Similarly, for total absorption, $f = 0$, $p_{\text{total}} = p_a = I_0 / c$, and since $I = I_0$, we have $u = I / c = I_0 / c$, which again is the same as p_{total}.

26. The mass of the cylinder is $m = \rho(\pi D^2 / 4)H$, where D is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \frac{\pi H D^2 g\rho}{4} - \left(\frac{\pi D^2}{4}\right)\left(\frac{2I}{c}\right) = 0.$$

We solve for H:

$$H = \frac{2I}{gc\rho} = \left(\frac{2P}{\pi D^2/4}\right)\frac{1}{gc\rho}$$
$$= \frac{2(4.60\,\text{W})}{[\pi(2.60\times10^{-3}\,\text{m})^2/4](9.8\,\text{m/s}^2)(3.0\times10^8\,\text{m/s})(1.20\times10^3\,\text{kg/m}^3)}$$
$$= 4.91\times10^{-7}\,\text{m}.$$

27. **THINK** Electromagnetic waves travel at speed of light, and carry both linear momentum and energy.

EXPRESS The speed of the electromagnetic wave is $c = \lambda f$, where λ is the wavelength and f is the frequency of the wave. The angular frequency is $\omega = 2\pi f$, and the angular wave number is $k = 2\pi/\lambda$. The magnetic field amplitude is related to the electric field amplitude by $B_m = E_m/c$. The intensity of the wave is given by Eq. 33-26:

$$I = \frac{1}{c\mu_0}E_{\text{rms}}^2 = \frac{1}{2c\mu_0}E_m^2.$$

ANALYZE (a) With $\lambda = 3.0$ m, the frequency of the wave is

$$f = \frac{c}{\lambda} = \frac{2.998\times10^8\ \text{m/s}}{3.0\ \text{m}} = 1.0\times10^8\ \text{Hz}.$$

(b) From the value of f obtained in (a), we find the angular frequency to be

$$\omega = 2\pi f = 2\pi(1.0\times10^8\ \text{Hz}) = 6.3\times10^8\ \text{rad/s}.$$

(c) The corresponding angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\ \text{m}} = 2.1\ \text{rad/m}.$$

(d) With $E_m = 300$ V/m, the magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 1.0\times10^{-6}\,\text{T}.$$

(e) Since $\vec{E}$ is in the positive *y* direction, $\vec{B}$ must be in the positive *z* direction so that their cross product $\vec{E}\times\vec{B}$ points in the positive *x* direction (the direction of propagation).

(f) The intensity of the wave is

$$I=\frac{E_m^2}{2\mu_0 c}=\frac{(300\,\text{V/m})^2}{2(4\pi\times10^{-7}\,\text{H/m})(2.998\times10^8\,\text{m/s})}=119\,\text{W/m}^2\approx1.2\times10^2\,\text{W/m}^2.$$

(g) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is *I*/*c*, so

$$\frac{dp}{dt}=\frac{IA}{c}=\frac{(119\ \text{W/m}^2)(2.0\ \text{m}^2)}{2.998\times10^8\ \text{m/s}}=8.0\times10^{-7}\ \text{N}.$$

(h) The radiation pressure is

$$p_r=\frac{dp/dt}{A}=\frac{8.0\times10^{-7}\ \text{N}}{2.0\ \text{m}^2}=4.0\times10^{-7}\ \text{Pa}.$$

LEARN The energy density is given by

$$u=\frac{I}{c}=\frac{119\ \text{W/m}^2}{2.998\times10^8\ \text{m/s}}=4.0\times10^{-7}\ \text{J/m}^3$$

which is the same as the radiation pressure p_r.

28. (a) Assuming complete absorption, the radiation pressure is

$$p_r=\frac{I}{c}=\frac{1.4\times10^3\ \text{W/m}^2}{3.0\times10^8\ \text{m/s}}=4.7\times10^{-6}\ \text{N/m}^2.$$

(b) We compare values by setting up a ratio:

$$\frac{p_r}{p_0}=\frac{4.7\times10^{-6}\ \text{N/m}^2}{1.0\times10^5\ \text{N/m}^2}=4.7\times10^{-11}.$$

29. **THINK** The laser beam carries both energy and momentum. The total momentum of the spaceship and light is conserved.

EXPRESS If the beam carries energy *U* away from the spaceship, then it also carries momentum *p* = *U*/*c* away. By momentum conservation, this is the magnitude of the momentum acquired by the spaceship. If *P* is the power of the laser, then the energy carried away in time *t* is *U* = *Pt*.

ANALYZE We note that there are 86400 seconds in a day. Thus, $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10\times10^3\ \text{W})(86400\ \text{s})}{(1.5\times10^3\ \text{kg})(2.998\times10^8\ \text{m/s})} = 1.9\times10^{-3}\ \text{m/s}.$$

LEARN As expected, the speed of the spaceship is proportional to the power of the laser beam.

30. (a) We note that the cross-section area of the beam is $\pi d^2/4$, where d is the diameter of the spot ($d = 2.00\lambda$). The beam intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{5.00\times10^{-3}\ \text{W}}{\pi\left[(2.00)(633\times10^{-9}\ \text{m})\right]^2/4} = 3.97\times10^9\ \text{W/m}^2.$$

(b) The radiation pressure is

$$p_r = \frac{I}{c} = \frac{3.97\times10^9\ \text{W/m}^2}{2.998\times10^8\ \text{m/s}} = 13.2\ \text{Pa}.$$

(c) In computing the corresponding force, we can use the power and intensity to eliminate the area (mentioned in part (a)). We obtain

$$F_r = \left(\frac{\pi d^2}{4}\right)p_r = \left(\frac{P}{I}\right)p_r = \frac{(5.00\times10^{-3}\ \text{W})(13.2\ \text{Pa})}{3.97\times10^9\ \text{W/m}^2} = 1.67\times10^{-11}\ \text{N}.$$

(d) The acceleration of the sphere is

$$a = \frac{F_r}{m} = \frac{F_r}{\rho(\pi d^3/6)} = \frac{6(1.67\times10^{-11}\ \text{N})}{\pi(5.00\times10^3\ \text{kg/m}^3)[(2.00)(633\times10^{-9}\ \text{m})]^3}$$
$$= 3.14\times10^3\ \text{m/s}^2.$$

31. We shall assume that the Sun is far enough from the particle to act as an isotropic point source of light.

(a) The forces that act on the dust particle are the radially outward radiation force $\vec{F}_r$ and the radially inward (toward the Sun) gravitational force $\vec{F}_g$. Using Eqs. 33-32 and 33-27, the radiation force can be written as

$$F_r = \frac{IA}{c} = \frac{P_S}{4\pi r^2}\frac{\pi R^2}{c} = \frac{P_S R^2}{4r^2 c},$$

where R is the radius of the particle, and $A = \pi R^2$ is the cross-sectional area. On the other hand, the gravitational force on the particle is given by Newton's law of gravitation (Eq. 13-1):

$$F_g = \frac{GM_S m}{r^2} = \frac{GM_S \rho(4\pi R^3/3)}{r^2} = \frac{4\pi GM_S \rho R^3}{3r^2},$$

where $m = \rho(4\pi R^3/3)$ is the mass of the particle. When the two forces balance, the particle travels in a straight path. The condition that $F_r = F_g$ implies

$$\frac{P_S R^2}{4r^2 c} = \frac{4\pi GM_S \rho R^3}{3r^2},$$

which can be solved to give

$$R = \frac{3P_S}{16\pi c\rho GM_S} = \frac{3(3.9\times10^{26}\ \text{W})}{16\pi(3\times10^8\ \text{m/s})(3.5\times10^3\ \text{kg/m}^3)(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99\times10^{30}\ \text{kg})}$$
$$= 1.7\times10^{-7}\ \text{m}.$$

(b) Since F_g varies with R^3 and F_r varies with R^2, if the radius R is larger, then $F_g > F_r$, and the path will be curved toward the Sun (like path 3).

32. After passing through the first polarizer the initial intensity I_0 reduces by a factor of 1/2. After passing through the second one it is further reduced by a factor of $\cos^2(\pi - \theta_1 - \theta_2) = \cos^2(\theta_1 + \theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\pi - \theta_2 - \theta_3) = \cos^2(\theta_2 + \theta_3)$. Therefore,

$$\frac{I_f}{I_0} = \frac{1}{2}\cos^2(\theta_1+\theta_2)\cos^2(\theta_2+\theta_3) = \frac{1}{2}\cos^2(50°+50°)\cos^2(50°+50°)$$
$$= 4.5\times10^{-4}.$$

Thus, 0.045% of the light's initial intensity is transmitted.

33. **THINK** Unpolarized light becomes polarized when it is sent through a polarizing sheet. In this problem, three polarizing sheets are involved, we work through the system sheet by sheet, applying either the one-half rule or the cosine-squared rule.

EXPRESS Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is, by one-half rule, $I_1 = \frac{1}{2}I_0$, and the direction of polarization of the transmitted light is $\theta_1 = 40°$ *counterclockwise* from the y axis in the diagram. For the second sheet (and the third one as well), we apply the cosine-squared rule:

$$I_2 = I_1 \cos^2\theta_2'$$

where θ_2' is the angle between the direction of polarization that is incident on that sheet and the polarizing direction of the sheet.

ANALYZE The polarizing direction of the second sheet is θ_2 = 20° *clockwise* from the *y* axis, so $\theta_2' = 40° + 20° = 60°$. The transmitted intensity is

$$I_2 = I_1 \cos^2 60° = \frac{1}{2} I_0 \cos^2 60°,$$

and the direction of polarization of the transmitted light is 20° clockwise from the *y* axis. The polarizing direction of the third sheet is θ_3 = 40° *counterclockwise* from the *y* axis. Consequently, the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is 20° + 40° = 60°. The transmitted intensity is

$$I_3 = I_2 \cos^2 60° = \frac{1}{2} I_0 \cos^4 60° = 3.1\times10^{-2} I_0.$$

Thus, 3.1% of the light's initial intensity is transmitted.

LEARN When two polarizing sheets are crossed ($\theta = 90°$), no light passes through and the transmitted intensity is zero.

34. In this case, we replace $I_0 \cos^2 70°$ by $\frac{1}{2} I_0$ as the intensity of the light after passing through the first polarizer. Therefore,

$$I_f = \frac{1}{2} I_0 \cos^2(90°-70°) = \frac{1}{2}(43 \text{ W/m}^2)(\cos^2 20°) = 19 \text{ W/m}^2.$$

35. The angle between the direction of polarization of the light incident on the first polarizing sheet and the polarizing direction of that sheet is θ_1 = 70°. If I_0 is the intensity of the incident light, then the intensity of the light transmitted through the first sheet is

$$I_1 = I_0 \cos^2\theta_1 = (43 \text{ W/m}^2)\cos^2 70° = 5.03 \text{ W/m}^2.$$

The direction of polarization of the transmitted light makes an angle of 70° with the vertical and an angle of θ_2 = 20° with the horizontal. θ_2 is the angle it makes with the polarizing direction of the second polarizing sheet. Consequently, the transmitted intensity is

$$I_2 = I_1 \cos^2\theta_2 = (5.03 \text{ W/m}^2)\cos^2 20° = 4.4 \text{ W/m}^2.$$

36. (a) The fraction of light that is transmitted by the glasses is

$$\frac{I_f}{I_0}=\frac{E_f^2}{E_0^2}=\frac{E_v^2}{E_v^2+E_h^2}=\frac{E_v^2}{E_v^2+(2.3E_v)^2}=0.16.$$

(b) Since now the horizontal component of $\vec{E}$ will pass through the glasses,

$$\frac{I_f}{I_0}=\frac{E_h^2}{E_v^2+E_h^2}=\frac{(2.3E_v)^2}{E_v^2+(2.3E_v)^2}=0.84.$$

37. **THINK** A polarizing sheet can change the direction of polarization of the incident beam since it allows only the component that is parallel to its polarization direction to pass.

EXPRESS The 90° rotation of the polarization direction cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted.

ANALYZE (a) The 90° rotation of the polarization direction can be done with two sheets. We place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is

$$I=I_0\cos^2\theta\cos^2(90°-\theta)=I_0\cos^2\theta\sin^2\theta,$$

where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

(b) Consider *n* sheets, with the polarizing direction of the first sheet making an angle of θ = 90°/*n* relative to the direction of polarization of the incident radiation. The polarizing direction of each successive sheet is rotated 90°/*n* in the same sense from the polarizing direction of the previous sheet. The transmitted radiation is polarized, with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is

$$I=I_0\cos^{2n}(90°/n).$$

We want the smallest integer value of *n* for which this is greater than $0.60I_0$. We start with *n* = 2 and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, we have obtained the solution. If it is less, increase *n* by 1 and try again. We repeat this process, increasing *n* by 1 each time, until we have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be *n* = 5.

LEARN The intensities associated with *n* = 1 to 5 are:

$$
\begin{aligned}
I_{n=1} &= I_0 \cos^2(90°) = 0 \\
I_{n=2} &= I_0 \cos^4(45°) = I_0/4 = 0.25I_0 \\
I_{n=3} &= I_0 \cos^6(30°) = 0.422I_0 \\
I_{n=4} &= I_0 \cos^8(22.5°) = 0.531I_0 \\
I_{n=5} &= I_0 \cos^{10}(18°) = 0.605I_0
\end{aligned}
$$

38. We note the points at which the curve is zero (θ_2 = 0° and 90°) in Fig. 33-43. We infer that sheet 2 is perpendicular to one of the other sheets at θ_2 = 0°, and that it is perpendicular to the *other* of the other sheets when θ_2 = 90°. Without loss of generality, we choose θ_1 = 0°, θ_3 = 90°. Now, when θ_2 = 30°, it will be $\Delta\theta$ = 30° relative to sheet 1 and $\Delta\theta'$ = 60° relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 9.4\% .$$

39. (a) Since the incident light is unpolarized, half the intensity is transmitted and half is absorbed. Thus the transmitted intensity is I = 5.0 mW/m^2. The intensity and the electric field amplitude are related by $I = E_m^2/2\mu_0 c$, so

$$
\begin{aligned}
E_m &= \sqrt{2\mu_0 cI} = \sqrt{2(4\pi\times10^{-7}\ \text{H/m})(3.00\times10^8\ \text{m/s})(5.0\times10^{-3}\ \text{W/m}^2)} \\
&= 1.9\ \text{V/m}.
\end{aligned}
$$

(b) The radiation pressure is $p_r = I_a/c$, where I_a is the absorbed intensity. Thus

$$p_r = \frac{5.0\times10^{-3}\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}} = 1.7\times10^{-11}\ \text{Pa}.$$

40. We note the points at which the curve is zero (θ_2 = 60° and 140°) in Fig. 33-44. We infer that sheet 2 is perpendicular to one of the other sheets at θ_2 = 60°, and that it is perpendicular to the *other* of the other sheets when θ_2 = 140°. Without loss of generality, we choose θ_1 = 150°, θ_3 = 50°. Now, when θ_2 = 90°, it will be $|\Delta\theta|$ = 60° relative to sheet 1 and $|\Delta\theta'|$ = 40° relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 7.3\% .$$

41. As the polarized beam of intensity I_0 passes the first polarizer, its intensity is reduced to $I_0\cos^2\theta$. After passing through the second polarizer, which makes a 90° angle with the first filter, the intensity is

$$I = (I_0\cos^2\theta)\sin^2\theta = I_0/10$$

which implies $\sin^2\theta\cos^2\theta = 1/10$, or $\sin\theta\cos\theta = \sin 2\theta/2 = 1/\sqrt{10}$. This leads to $\theta = 70°$ or 20°.

42. We examine the point where the graph reaches zero: $\theta_2 = 160º$. Since the polarizers must be "crossed" for the intensity to vanish, then $\theta_1 = 160º - 90º = 70º$. Now we consider the case $\theta_2 = 90º$ (which is hard to judge from the graph). Since θ_1 is still equal to 70º, then the angle between the polarizers is now $\Delta\theta = 20º$. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is

$$\tfrac{1}{2}\cos^2(\Delta\theta) = 0.442 \approx 44\%.$$

43. Let I_0 be the intensity of the incident beam and f be the fraction that is polarized. Thus, the intensity of the polarized portion is fI_0. After transmission, this portion contributes $fI_0\cos^2\theta$ to the intensity of the transmitted beam. Here θ is the angle between the direction of polarization of the radiation and the polarizing direction of the filter. The intensity of the unpolarized portion of the incident beam is $(1-f)I_0$ and after transmission, this portion contributes $(1-f)I_0/2$ to the transmitted intensity. Consequently, the transmitted intensity is

$$I = fI_0\cos^2\theta + \frac{1}{2}(1-f)I_0.$$

As the filter is rotated, $\cos^2\theta$ varies from a minimum of 0 to a maximum of 1, so the transmitted intensity varies from a minimum of

$$I_{\text{min}} = \frac{1}{2}(1-f)I_0$$

to a maximum of

$$I_{\text{max}} = fI_0 + \frac{1}{2}(1-f)I_0 = \frac{1}{2}(1+f)I_0.$$

The ratio of I_{max} to I_{min} is

$$\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{1+f}{1-f}.$$

Setting the ratio equal to 5.0 and solving for f, we get $f = 0.67$.

44. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2}I_0\cos^2\theta_2\cos^2(90° - \theta_2).$$

Using trig identities, we rewrite this as $\frac{I}{I_0} = \frac{1}{8}\sin^2(2\theta_2)$.

(a) Therefore we find $\theta_2 = \frac{1}{2}\sin^{-1}\sqrt{0.40} = 19.6°$.

(b) Since the first expression we wrote is symmetric under the exchange $\theta_2 \leftrightarrow 90° - \theta_2$, we see that the angle's complement, 70.4°, is also a solution.

45. Note that the normal to the refracting surface is vertical in the diagram. The angle of refraction is $\theta_2 = 90°$ and the angle of incidence is given by $\tan\theta_1 = L/D$, where D is the height of the tank and L is its width. Thus

$$\theta_1 = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{1.10\text{ m}}{0.850\text{ m}}\right) = 52.31°.$$

The law of refraction yields

$$n_1 = n_2\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\left(\frac{\sin 90°}{\sin 52.31°}\right) = 1.26,$$

where the index of refraction of air was taken to be unity.

46. (a) For the angles of incidence and refraction to be equal, the graph in Fig. 33-47(b) would consist of a "$y = x$" line at 45° in the plot. Instead, the curve for material 1 falls under such a "$y = x$" line, which tells us that all refraction angles are less than incident ones. With $\theta_2 < \theta_1$ Snell's law implies $n_2 > n_1$.

(b) Using the same argument as in (a), the value of n_2 for material 2 is also greater than that of water (n_1).

(c) It's easiest to examine the topmost point of each curve. With $\theta_2 = 90°$ and $\theta_1 = ½(90°)$, and with $n_2 = 1.33$ (Table 33-1), we find $n_1 = 1.9$ from Snell's law.

(d) Similarly, with $\theta_2 = 90°$ and $\theta_1 = ¾(90°)$, we obtain $n_1 = 1.4$.

47. The law of refraction states

$$n_1\sin\theta_1 = n_2\sin\theta_2.$$

We take medium 1 to be the vacuum, with $n_1 = 1$ and $\theta_1 = 32.0°$. Medium 2 is the glass, with $\theta_2 = 21.0°$. We solve for n_2:

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2} = (1.00)\left(\frac{\sin 32.0°}{\sin 21.0°}\right) = 1.48.$$

48. (a) For the angles of incidence and refraction to be equal, the graph in Fig. 33-48(b) would consist of a "$y = x$" line at 45º in the plot. Instead, the curve for material 1 falls under such a "$y = x$" line, which tells us that all refraction angles are less than incident ones. With $\theta_2 < \theta_1$ Snell's law implies $n_2 > n_1$.

(b) Using the same argument as in (a), the value of n_2 for material 2 is also greater than that of water (n_1).

(c) It's easiest to examine the right end-point of each curve. With θ_1 = 90º and θ_2 = ¾(90º), and with n_1 = 1.33 (Table 33-1) we find, from Snell's law, n_2 = 1.4 for material 1.

(d) Similarly, with θ_1 = 90º and θ_2 = ½(90º), we obtain n_2 = 1.9.

49. The angle of incidence for the light ray on mirror B is $90° - \theta$. So the outgoing ray r' makes an angle $90° - (90° - \theta) = \theta$ with the vertical direction, and is antiparallel to the incoming one. The angle between i and r' is therefore 180°.

50. (a) From $n_1\sin\theta_1 = n_2\sin\theta_2$ and $n_2\sin\theta_2 = n_3\sin\theta_3$, we find $n_1\sin\theta_1 = n_3\sin\theta_3$. This has a simple implication: that $\theta_1 = \theta_3$ when $n_1 = n_3$. Since we are given θ_1 = 40º in Fig. 33-50(a), then we look for a point in Fig. 33-50(b) where θ_3 = 40º. This seems to occur at n_3 = 1.6, so we infer that n_1 = 1.6.

(b) Our first step in our solution to part (a) shows that information concerning n_2 disappears (cancels) in the manipulation. Thus, we cannot tell; we need more information.

(c) From $1.6\sin 70° = 2.4\sin\theta_3$ we obtain $\theta_3 = 39°$.

51. (a) Approximating n = 1 for air, we have

$$n_1 \sin\theta_1 = (1)\sin\theta_5 \quad \Rightarrow \quad 56.9° = \theta_5$$

and with the more accurate value for n_{air} in Table 33-1, we obtain 56.8°.

(b) Equation 33-44 leads to

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_4 \sin\theta_4$$

so that

$$\theta_4 = \sin^{-1}\left(\frac{n_1}{n_4}\sin\theta_1\right) = 35.3°.$$

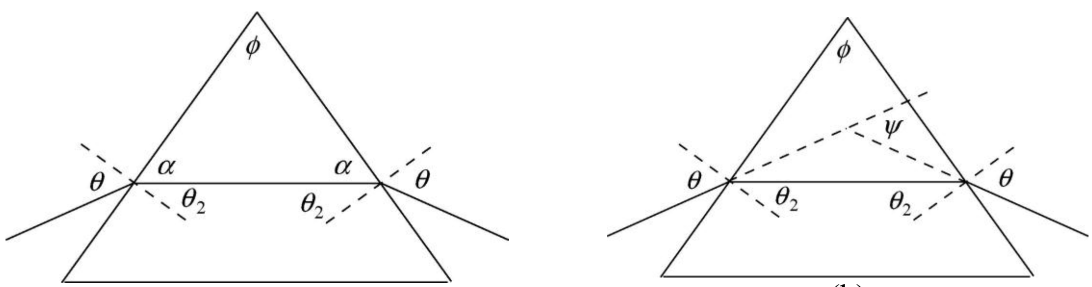
52. (a) A simple implication of Snell's law is that $\theta_2 = \theta_1$ when $n_1 = n_2$. Since the angle of incidence is shown in Fig. 33-52(a) to be 30º, we look for a point in Fig. 33-52(b) where $\theta_2 = 30º$. This seems to occur when $n_2 = 1.7$. By inference, then, $n_1 = 1.7$.

(b) From $1.7\sin(60º) = 2.4\sin(\theta_2)$ we get $\theta_2 = 38°$.

53. **THINK** The angle with which the light beam emerges from the triangular prism depends on the index of refraction of the prism.

EXPRESS Consider diagram (a) shown next. The incident angle is θ and the angle of refraction is θ_2. Since $\theta_2 + \alpha = 90°$ and $\phi + 2\alpha = 180°$, we have

$$\theta_2 = 90° - \alpha = 90° - \frac{1}{2}\left(180° - \phi\right) = \frac{\phi}{2}.$$



(a) (b)

ANALYZE Next, examine diagram (b) and consider the triangle formed by the two normals and the ray in the interior. One can show that ψ is given by

$$\psi = 2(\theta - \theta_2).$$

Upon substituting $\phi/2$ for θ_2, we obtain $\psi = 2(\theta - \phi/2)$ which yields $\theta = (\phi + \psi)/2$. Thus, using the law of refraction, we find the index of refraction of the prism to be

$$n = \frac{\sin\theta}{\sin\theta_2} = \frac{\sin\frac{1}{2}(\phi+\psi)}{\sin\frac{1}{2}\phi}.$$

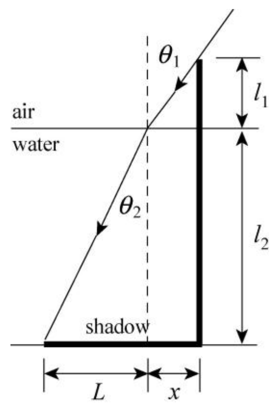
LEARN The angle ψ is called the deviation angle. Physically, it represents the total angle through which the beam has turned while passing through the prism. This angle is minimum when the beam passes through the prism "symmetrically," as it does in this case. Knowing the value of ϕ and ψ allows us to determine the value of n for the prism material.

54. (a) Snell's law gives $n_{air}\sin(50º) = n_{2b}\sin\theta_{2b}$ and $n_{air}\sin(50º) = n_{2r}\sin\theta_{2r}$ where we use subscripts b and r for the blue and red light rays. Using the common approximation for air's index ($n_{air} = 1.0$) we find the two angles of refraction to be 30.176° and 30.507°. Therefore, $\Delta\theta = 0.33°$.

(b) Both of the refracted rays emerge from the other side with the same angle (50°) with which they were incident on the first side (generally speaking, light comes into a block at the same angle that it emerges with from the opposite parallel side). There is thus no difference (the difference is 0°) and thus there is no dispersion in this case.

55. **THINK** Light is refracted at the air–water interface. To calculate the length of the shadow of the pole, we first calculate the angle of refraction using the Snell's law.

EXPRESS Consider a ray that grazes the top of the pole, as shown in the diagram below.



Here θ_1 = 90° − θ = 90° −55° = 35°, $\ell_1 = 0.50$ m, and $\ell_2 = 1.50$ m. The length of the shadow is $d = x + L$.

ANALYZE The distance x is given by

$$x = \ell_1 \tan\theta_1 = (0.50 \text{ m})\tan 35° = 0.35 \text{ m}.$$

According to the law of refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. We take $n_1 = 1$ and $n_2 = 1.33$ (from Table 33-1). Then,

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 35.0°}{1.33}\right) = 25.55°.$$

L is given by

$$L = \ell_2 \tan\theta_2 = (1.50 \text{ m})\tan 25.55° = 0.72 \text{ m}.$$

Thus, the length of the shadow is d = 0.35 m + 0.72 m = 1.07 m.

LEARN If the pole were empty with no water, then $\theta_1 = \theta_2$ and the length of the shadow would be

$$d' = \ell_1 \tan\theta_1 + \ell_2 \tan\theta_1 = (\ell_1 + \ell_2)\tan\theta_1$$

by simple geometric consideration.

56. (a) We use subscripts *b* and *r* for the blue and red light rays. Snell's law gives

$$\theta_{2b} = \sin^{-1}\left(\frac{1}{1.343}\sin(70°)\right) = 44.403°$$
$$\theta_{2r} = \sin^{-1}\left(\frac{1}{1.331}\sin(70°)\right) = 44.911°$$

for the refraction angles at the first surface (where the normal axis is vertical). These rays strike the second surface (where *A* is) at complementary angles to those just calculated (since the normal axis is horizontal for the second surface). Taking this into consideration, we again use Snell's law to calculate the second refractions (with which the light re-enters the air):

$$\theta_{3b} = \sin^{-1}[1.343\sin(90° - \theta_{2b})] = 73.636°$$
$$\theta_{3r} = \sin^{-1}[1.331\sin(90° - \theta_{2r})] = 70.497°$$

which differ by 3.1° (thus giving a rainbow of angular width 3.1°).

(b) Both of the refracted rays emerge from the bottom side with the same angle (70°) with which they were incident on the topside (the occurrence of an intermediate reflection [from side 2] does not alter this overall fact: light comes into the block at the same angle that it emerges with from the opposite parallel side). There is thus no difference (the difference is 0°) and thus there is no rainbow in this case.

57. Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. Thus, the diameter *D* of the circle in question is

$$D = 2h\tan\theta_c = 2h\tan\left[\sin^{-1}\left(\frac{1}{n_w}\right)\right] = 2(80.0\,\text{cm})\tan\left[\sin^{-1}\left(\frac{1}{1.33}\right)\right] = 182\text{ cm.}$$

58. The critical angle is $\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.8}\right) = 34°.$

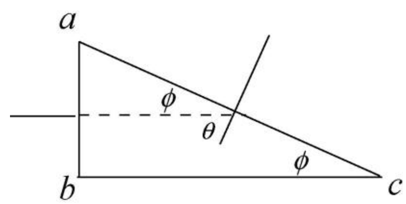
59. **THINK** Total internal reflection happens when the angle of incidence exceeds a critical angle such that Snell's law gives $\sin\theta_2 > 1$.

EXPRESS When light reaches the interfaces between two materials with indices of refraction n_1 and n_2, if $n_1 > n_2$, and the incident angle exceeds a critical value given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

then total internal reflection will occur.

In our case, the incident light ray is perpendicular to the face *ab*. Thus, no refraction occurs at the surface *ab*, so the angle of incidence at surface *ac* is $\theta = 90° - \phi$, as shown in the figure below.



ANALYZE (a) For total internal reflection at the second surface, $n_g \sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, we want the largest value of ϕ for which $n_g \cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g \cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9°.$$

The index of refraction for air is taken to be unity.

(b) We now replace the air with water. If $n_w = 1.33$ is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0°.$$

LEARN Total internal reflection cannot occur if the incident light is in the medium with lower index of refraction. With $\theta_c = \sin^{-1}(n_2/n_1)$, we see that the larger the ratio n_2/n_1, the larger the value of θ_c.

60. (a) The condition (in Eq. 33-44) required in the critical angle calculation is $\theta_3 = 90°$. Thus (with $\theta_2 = \theta_c$, which we don't compute here),

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$

leads to $\theta_1 = \theta = \sin^{-1} n_3/n_1 = 54.3°$.

(b) Yes. Reducing θ leads to a reduction of θ_2 so that it becomes less than the critical angle; therefore, there will be some transmission of light into material 3.

(c) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leading to $\theta = 51.1°$.

(d) No. Reducing θ leads to an increase of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle. Therefore, there will be no transmission of light into material 3.

61. (a) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leading to $\theta = 26.8°$.

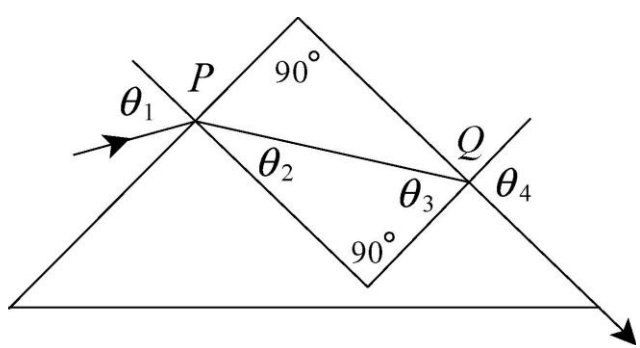
(b) Increasing θ leads to a decrease of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle; therefore, there will be some transmission of light into material 3.

62. (a) Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by $d = 2h \tan\theta_c$. For water $n = 1.33$, so Eq. 33-47 gives $\sin\theta_c = 1/1.33$, or $\theta_c = 48.75°$. Thus,

$$d = 2h\tan\theta_c = 2(2.00 \text{ m})(\tan 48.75°) = 4.56 \text{ m}.$$

(b) The diameter *d* of the circle will increase if the fish descends (increasing *h*).

63. (a) A ray diagram is shown below.



Let θ_1 be the angle of incidence and θ_2 be the angle of refraction at the first surface. Let θ_3 be the angle of incidence at the second surface. The angle of refraction there is $\theta_4 = 90°$. The law of refraction, applied to the second surface, yields $n \sin \theta_3 = \sin \theta_4 = 1$. As shown in the diagram, the normals to the surfaces at P and Q are perpendicular to each other. The interior angles of the triangle formed by the ray and the two normals must sum to 180°, so $\theta_3 = 90° - \theta_2$ and

$$\sin\theta_3 = \sin(90° - \theta_2) = \cos\theta_2 = \sqrt{1-\sin^2\theta_2}.$$

According to the law of refraction, applied at Q, $n\sqrt{1-\sin^2\theta_2} = 1.$ The law of refraction, applied to point P, yields $\sin \theta_1 = n \sin \theta_2$, so $\sin \theta_2 = (\sin \theta_1)/n$ and

$$n\sqrt{1-\frac{\sin^2\theta_1}{n^2}} = 1.$$

Squaring both sides and solving for n, we get

$$n = \sqrt{1+\sin^2\theta_1}.$$

(b) The greatest possible value of $\sin^2 \theta_1$ is 1, so the greatest possible value of n is $n_{\max} = \sqrt{2} = 1.41.$

(c) For a given value of n, if the angle of incidence at the first surface is greater than θ_1, the angle of refraction there is greater than θ_2 and the angle of incidence at the second face is less than θ_3 (= $90° - \theta_2$). That is, it is less than the critical angle for total internal reflection, so light leaves the second surface and emerges into the air.

(d) If the angle of incidence at the first surface is less than θ_1, the angle of refraction there is less than θ_2 and the angle of incidence at the second surface is greater than θ_3. This is greater than the critical angle for total internal reflection, so all the light is reflected at Q.

64. (a) We refer to the entry point for the original incident ray as point A (which we take to be on the left side of the prism, as in Fig. 33-53), the prism vertex as point B, and the point where the interior ray strikes the right surface of the prism as point C. The angle between line AB and the interior ray is β (the complement of the angle of refraction at the first surface), and the angle between the line BC and the interior ray is α (the complement of its angle of incidence when it strikes the second surface). When the incident ray is at the minimum angle for which light is able to exit the prism, the light exits along the second face. That is, the angle of refraction at the second face is 90°, and the angle of incidence there for the interior ray is the critical angle for total internal reflection. Let θ_1 be the angle of incidence for the original incident ray and θ_2 be the angle of refraction at the first face, and let θ_3 be the angle of incidence at the second face. The law of refraction, applied to point C, yields $n \sin \theta_3 = 1$, so

$$\sin\theta_3 = 1/n = 1/1.60 = 0.625 \Rightarrow \theta_3 = 38.68°.$$

The interior angles of the triangle *ABC* must sum to 180°, so $\alpha + \beta = 120°$. Now, $\alpha = 90° - \theta_3 = 51.32°$, so $\beta = 120° - 51.32° = 69.68°$. Thus, $\theta_2 = 90° - \beta = 21.32°$. The law of refraction, applied to point *A*, yields

$$\sin\theta_1 = n\sin\theta_2 = 1.60\sin 21.32° = 0.5817.$$

Thus $\theta_1 = 35.6°$.

(b) We apply the law of refraction to point *C*. Since the angle of refraction there is the same as the angle of incidence at *A*, $n\sin\theta_3 = \sin\theta_1$. Now, $\alpha + \beta = 120°$, $\alpha = 90° - \theta_3$, and $\beta = 90° - \theta_2$, as before. This means $\theta_2 + \theta_3 = 60°$. Thus, the law of refraction leads to

$$\sin\theta_1 = n\sin\left(60° - \theta_2\right) \Rightarrow \sin\theta_1 = n\sin 60°\cos\theta_2 - n\cos 60°\sin\theta_2$$

where the trigonometric identity

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

is used. Next, we apply the law of refraction to point *A*:

$$\sin\theta_1 = n\sin\theta_2 \Rightarrow \sin\theta_2 = \left(1/n\right)\sin\theta_1$$

which yields $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\left(1/n^2\right)\sin^2\theta_1}$. Thus,

$$\sin\theta_1 = n\sin 60°\sqrt{1-\left(1/n\right)^2\sin^2\theta_1} - \cos 60°\sin\theta_1$$

or

$$\left(1+\cos 60°\right)\sin\theta_1 = \sin 60°\sqrt{n^2-\sin^2\theta_1}.$$

Squaring both sides and solving for $\sin\theta_1$, we obtain

$$\sin\theta_1 = \frac{n\sin 60°}{\sqrt{\left(1+\cos 60°\right)^2+\sin^2 60°}} = \frac{1.60\sin 60°}{\sqrt{\left(1+\cos 60°\right)^2+\sin^2 60°}} = 0.80$$

and $\theta_1 = 53.1°$.

65. When examining Fig. 33-61, it is important to note that the angle (measured from the central axis) for the light ray in air, θ, is not the angle for the ray in the glass core, which we denote θ'. The law of refraction leads to

$$\sin\theta' = \frac{1}{n_1}\sin\theta$$

assuming $n_{\text{air}} = 1$. The angle of incidence for the light ray striking the coating is the complement of θ', which we denote as θ'_{comp}, and recall that

$$\sin\theta'_{\text{comp}} = \cos\theta' = \sqrt{1-\sin^2\theta'}.$$

In the critical case, θ'_{comp} must equal θ_c specified by Eq. 33-47. Therefore,

$$\frac{n_2}{n_1} = \sin\theta'_{\text{comp}} = \sqrt{1-\sin^2\theta'} = \sqrt{1-\left(\frac{1}{n_1}\sin\theta\right)^2}$$

which leads to the result: $\sin\theta = \sqrt{n_1^2 - n_2^2}$. With n_1 = 1.58 and n_2 = 1.53, we obtain

$$\theta = \sin^{-1}\left(\sqrt{1.58^2 - 1.53^2}\right) = 23.2°.$$

66. (a) We note that the upper-right corner is at an angle (measured from the point where the light enters, and measured relative to a normal axis established at that point the normal at that point would be horizontal in Fig. 33-62) is at $\tan^{-1}(2/3)$ = 33.7º. The angle of refraction is given by

$$n_{\text{air}} \sin 40° = 1.56 \sin \theta_2$$

which yields θ_2 = 24.33º if we use the common approximation n_{air} = 1.0, and yields θ_2 = 24.34º if we use the more accurate value for n_{air} found in Table 33-1. The value is less than 33.7º, which means that the light goes to side 3.

(b) The ray strikes a point on side 3, which is 0.643 cm below that upper-right corner, and then (using the fact that the angle is symmetrical upon reflection) strikes the top surface (side 2) at a point 1.42 cm to the left of that corner. Since 1.42 cm is certainly less than 3 cm we have a self-consistency check to the effect that the ray does indeed strike side 2 as its second reflection (if we had gotten 3.42 cm instead of 1.42 cm, then the situation would be quite different).

(c) The normal axes for sides 1 and 3 are both horizontal, so the angle of incidence (in the plastic) at side 3 is the same as the angle of refraction was at side 1. Thus,

$$1.56 \sin 24.3° = n_{\text{air}} \sin \theta_{\text{air}} \quad \Rightarrow \quad \theta_{\text{air}} = 40° .$$

(d) It strikes the top surface (side 2) at an angle (measured from the normal axis there, which in this case would be a vertical axis) of 90º − θ_2 = 66º, which is much greater than the critical angle for total internal reflection ($\sin^{-1}(n_{air}/1.56) = 39.9°$). Therefore, no refraction occurs when the light strikes side 2.

(e) In this case, we have

$$n_{air} \sin 70° = 1.56 \sin \theta_2$$

which yields $\theta_2 = 37.04°$ if we use the common approximation $n_{air} = 1.0$, and yields $\theta_2 = 37.05°$ if we use the more accurate value for n_{air} found in Table 33-1. This is greater than the 33.7° mentioned above (regarding the upper-right corner), so the ray strikes side 2 instead of side 3.

(f) After bouncing from side 2 (at a point fairly close to that corner) it goes to side 3.

(g) When it bounced from side 2, its angle of incidence (because the normal axis for side 2 is orthogonal to that for side 1) is $90° - \theta_2 = 53°$, which is much greater than the critical angle for total internal reflection (which, again, is $\sin^{-1}(n_{air}/1.56) = 39.9°$). Therefore, no refraction occurs when the light strikes side 2.

(h) For the same reasons implicit in the calculation of part (c), the refracted ray emerges from side 3 with the same angle (70°) that it entered side 1. We see that the occurrence of an intermediate reflection (from side 2) does not alter this overall fact: light comes into the block at the same angle that it emerges with from the opposite parallel side.

67. (a) In the notation of this problem, Eq. 33-47 becomes

$$\theta_c = \sin^{-1}\frac{n_3}{n_2}$$

which yields $n_3 = 1.39$ for $\theta_c = \phi = 60°$.

(b) Applying Eq. 33-44 to the interface between material 1 and material 2, we have

$$n_2 \sin 30° = n_1 \sin \theta$$

which yields $\theta = 28.1°$.

(c) Decreasing θ will increase ϕ and thus cause the ray to strike the interface (between materials 2 and 3) at an angle larger than θ_c. Therefore, no transmission of light into material 3 can occur.

68. (a) We use Eq. 33-49: $\theta_B = \tan^{-1} n_w = \tan^{-1}(1.33) = 53.1°$.

(b) Yes, since n_w depends on the wavelength of the light.

69. **THINK** A reflected wave will be fully polarized if it strikes the boundary at the Brewster angle.

EXPRESS The angle of incidence for which reflected light is fully polarized is given by Eq. 33-48:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

where n_1 is the index of refraction for the medium of incidence and n_2 is the index of refraction for the second medium. The angle θ_B is called the Brewster angle.

ANALYZE With $n_1 = 1.33$ and $n_2 = 1.53$, we obtain

$$\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.53/1.33) = 49.0°.$$

LEARN In general, reflected light is partially polarized, having components both parallel and perpendicular to the plane of incidence. However, it can be completely polarized when incident at the Brewster angle.

70. Since the layers are parallel, the angle of refraction regarding the first surface is the same as the angle of incidence regarding the second surface (as is suggested by the notation in Fig. 33-64). We recall that as part of the derivation of Eq. 33-49 (Brewster's angle), the refracted angle is the complement of the incident angle:

$$\theta_2 = (\theta_1)_c = 90° - \theta_1.$$

We apply Eq. 33-49 to both refractions, setting up a product:

$$\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right) = (\tan\theta_{B1\to2})(\tan\theta_{B2\to3}) \quad \Rightarrow \quad \frac{n_3}{n_1} = (\tan\theta_1)(\tan\theta_2).$$

Now, since θ_2 is the complement of θ_1 we have

$$\tan\theta_2 = \tan(\theta_1)_c = \frac{1}{\tan\theta_1}.$$

Therefore, the product of tangents cancel and we obtain $n_3/n_1 = 1$. Consequently, the third medium is air: $n_3 = 1.0$.

71. **THINK** All electromagnetic waves, including visible light, travel at the same speed c in vacuum.

EXPRESS The time for light to travel a distance d in free space is $t = d/c$, where c is the speed of light (3.00×10^8 m/s).

ANALYZE (a) We take d to be 150 km = 150 × 10^3 m. Then,

$$t = \frac{d}{c} = \frac{150\times 10^3\ \text{m}}{3.00\times 10^8\ \text{m/s}} = 5.00\times 10^{-4}\ \text{s}.$$

(b) At full moon, the Moon and Sun are on opposite sides of Earth, so the distance traveled by the light is

$$d = (1.5\times 10^8\ \text{km}) + 2\,(3.8\times 10^5\ \text{km}) = 1.51\times 10^8\ \text{km} = 1.51\times 10^{11}\ \text{m}.$$

The time taken by light to travel this distance is

$$t = \frac{d}{c} = \frac{1.51\times 10^{11}\ \text{m}}{3.00\times 10^8\ \text{m/s}} = 500\ \text{s} = 8.4\ \text{min}.$$

(c) We take d to be 2(1.3 × 10^9 km) = 2.6 × 10^{12} m. Then,

$$t = \frac{d}{c} = \frac{2.6\times 10^{12}\ \text{m}}{3.00\times 10^8\ \text{m/s}} = 8.7\times 10^3\ \text{s} = 2.4\ \text{h}.$$

(d) We take d to be 6500 ly and the speed of light to be 1.00 ly/y. Then,

$$t = \frac{d}{c} = \frac{6500\ \text{ly}}{1.00\ \text{ly/y}} = 6500\ \text{y}.$$

The explosion took place in the year 1054 – 6500 = –5446 or 5446 B.C.

LEARN Since the speed c is constant, the travel time is proportional to the distance. The radio signals at 150 km away reach you almost instantly.

72. (a) The expression $E_y = E_m \sin(kx - \omega t)$ fits the requirement "at point P … [it] is decreasing with time" if we imagine P is just to the right ($x > 0$) of the coordinate origin (but at a value of x less than $\pi/2k = \lambda/4$ which is where there would be a maximum, at t = 0). It is important to bear in mind, in this description, that the wave is moving to the right. Specifically, $x_P = (1/k)\sin^{-1}(1/4)$ so that E_y = (1/4) E_m at t = 0, there. Also, E_y = 0 with our choice of expression for E_y . Therefore, part (a) is answered simply by solving for x_P. Since $k = 2\pi f/c$ we find

$$x_P = \frac{c}{2\pi f}\sin^{-1}\left(\frac{1}{4}\right) = 30.1\ \text{nm}.$$

(b) If we proceed to the right on the x axis (still studying this "snapshot" of the wave at t = 0) we find another point where E_y = 0 at a distance of one-half wavelength from the previous point where $E_y = 0$. Thus (since $\lambda = c/f$) the next point is at $x = \frac{1}{2}\lambda = \frac{1}{2}c/f$ and is consequently a distance $c/2f - x_P = 345$ nm to the right of *P*.

73. **THINK** The electric and magnetic components of the electromagnetic waves are always in phase, perpendicular to each other, and perpendicular to the direction of propagation of the wave.

EXPRESS The electric and magnetic fields can be written as sinusoidal functions of position and time as:

$$E = E_m \sin(kx + \omega t), \quad B = B_m \sin(kx + \omega t)$$

where E_m and B_m are the amplitudes of the fields, and ω and k, are the angular frequency and angular wave number of the wave, respectively. The two amplitudes are related by Eq. 33-4: $E_m / B_m = c$, where c is the speed of the wave.

ANALYZE (a) From $kc = \omega$ where $k = 1.00 \times 10^6$ m^{-1}, we obtain $\omega = 3.00 \times 10^{14}$ rad/s. The magnetic field amplitude is, from Eq. 33-5,

$$B_m = E_m/c = (5.00 \text{ V/m})/c = 1.67 \times 10^{-8} \text{ T}.$$

From the argument of the sinusoidal fucntion for E, we see that the direction of propagation is in the $-z$ direction. Since $\vec{E} = E_y\hat{\text{j}}$, and that $\vec{B}$ is perpendicular to $\vec{E}$ and $\vec{E}\times\vec{B}$,, we conclude that the only non-zero component of $\vec{B}$ is B_x, so that we have

$$B_x = (1.67\times10^{-8}\text{ T})\sin[(1.00\times10^6/\text{m})z + (3.00\times10^{14}/\text{s})t].$$

(b) The wavelength is $\lambda = 2\pi/k = 6.28 \times 10^{-6}$ m.

(c) The period is $T = 2\pi/\omega = 2.09 \times 10^{-14}$ s.

(d) The intensity is

$$I = \frac{1}{c\mu_0}\left(\frac{5.00\text{ V/m}}{\sqrt{2}}\right)^2 = 0.0332\text{ W/m}^2.$$

(e) As noted in part (a), the only nonzero component of $\vec{B}$ is B_x. The magnetic field oscillates along the x axis.

(f) The wavelength found in part (b) places this in the infrared portion of the spectrum.

LEARN Electromagnetic wave is a transverse wave. Knowing the functional form of the electric field allows us to determine the corresponding magnetic field, and vice versa.

74. (a) Let r be the radius and ρ be the density of the particle. Since its volume is $(4\pi/3)r^3$, its mass is $m = (4\pi/3)\rho r^3$. Let R be the distance from the Sun to the particle and let M be the mass of the Sun. Then, the gravitational force of attraction of the Sun on the particle has magnitude

$$F_g = \frac{GMm}{R^2} = \frac{4\pi GM\rho r^3}{3R^2}.$$

If P is the power output of the Sun, then at the position of the particle, the radiation intensity is $I = P/4\pi R^2$, and since the particle is perfectly absorbing, the radiation pressure on it is

$$p_r = \frac{I}{c} = \frac{P}{4\pi R^2 c}.$$

All of the radiation that passes through a circle of radius r and area $A = \pi r^2$, perpendicular to the direction of propagation, is absorbed by the particle, so the force of the radiation on the particle has magnitude

$$F_r = p_r A = \frac{\pi P r^2}{4\pi R^2 c} = \frac{Pr^2}{4R^2 c}.$$

The force is radially outward from the Sun. Notice that both the force of gravity and the force of the radiation are inversely proportional to R^2. If one of these forces is larger than the other at some distance from the Sun, then that force is larger at all distances. The two forces depend on the particle radius r differently: F_g is proportional to r^3 and F_r is proportional to r^2. We expect a small radius particle to be blown away by the radiation pressure and a large radius particle with the same density to be pulled inward toward the Sun. The critical value for the radius is the value for which the two forces are equal. Equating the expressions for F_g and F_r, we solve for r:

$$r = \frac{3P}{16\pi GM\rho c}.$$

(b) According to Appendix C, $M = 1.99 \times 10^{30}$ kg and $P = 3.90 \times 10^{26}$ W. Thus,

$$r = \frac{3(3.90\times10^{26}\text{ W})}{16\pi(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})(1.0\times10^3\text{ kg/m}^3)(3.00\times10^8\text{ m/s})}$$
$$= 5.8\times10^{-7}\text{ m}.$$

75. **THINK** Total internal reflection happens when the angle of incidence exceeds a critical angle such that Snell's law gives $\sin\theta_2 > 1$.

EXPRESS When light reaches the interfaces between two materials with indices of refraction n_1 and n_2, if $n_1 > n_2$, and the incident angle exceeds a critical value given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

then total internal reflection will occur.

Referring to Fig. 33-65, let $\theta_1 = 45°$ be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is

$$n \sin \theta_3 \geq 1.$$

We want to find the smallest value of the index of refraction n for which this inequality holds. The law of refraction, applied to the first surface, yields

$$n \sin \theta_2 = \sin \theta_1.$$

Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus, the condition for total internal reflection becomes

$$1 \leq n \sin(90° - \theta_2) = n \cos \theta_2.$$

Squaring this equation and using $\sin^2 \theta_2 + \cos^2 \theta_2 = 1$, we obtain $1 \leq n^2 (1 - \sin^2 \theta_2)$. Substituting $\sin \theta_2 = (1/n) \sin \theta_1$ now leads to

$$1 \leq n^2\left(1 - \frac{\sin^2\theta_1}{n^2}\right) = n^2 - \sin^2\theta_1.$$

The smallest value of n for which this equation is true is given by $1 = n^2 - \sin^2 \theta_1$. We solve for n:

$$n = \sqrt{1+\sin^2\theta_1} = \sqrt{1+\sin^2 45°} = 1.22.$$

LEARN With $n = 1.22$, we have $\theta_2 = \sin^{-1}[(1/1.22)\sin 45°] = 35°$, which gives $\theta_3 = 90° - 35° = 55°$ as the angle of incidence at the second surface. We can readily verify that $n \sin \theta_3 = (1.22) \sin 55° = 1$, meeting the threshold condition for total internal reflection.

76. Since some of the angles in Fig. 33-66 are measured from vertical axes and some are measured from horizontal axes, we must be very careful in taking differences. For instance, the angle difference between the first polarizer struck by the light and the second is 110º (or 70º depending on how we measure it; it does not matter in the final result whether we put $\Delta\theta_1 = 70º$ or put $\Delta\theta_1 = 110º$). Similarly, the angle difference between the second and the third is $\Delta\theta_2 = 40º$, and between the third and the fourth is $\Delta\theta_3$

= 40º, also. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is the incident intensity multiplied by

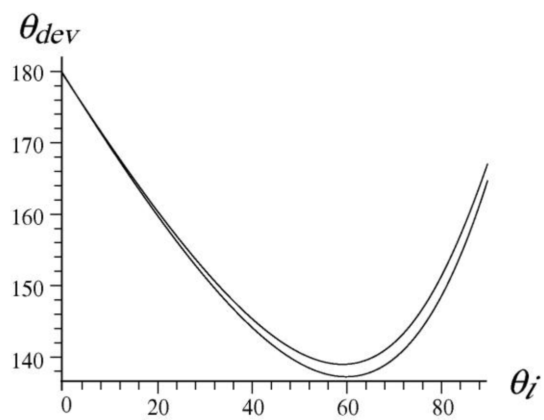
$$\frac{1}{2}\cos^2(\Delta\theta_1)\cos^2(\Delta\theta_2)\cos^2(\Delta\theta_3)\,.$$

Thus, the light that emerges from the system has intensity equal to 0.50 W/m^2.

77. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution to the overall deviation is the reflection. Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after the reflection) is $\delta\theta_2 = 180° - 2\theta_r$. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 180° + 2\theta_i - 4\theta_r.$$

(b) We substitute $\theta_r = \sin^{-1}(\frac{1}{n}\sin\theta_i)$ into the expression derived in part (a), using the two given values for n. The higher curve is for the blue light.



(c) We can expand the graph and try to estimate the minimum, or search for it with a more sophisticated numerical procedure. We find that the θ_{dev} minimum for red light is $137.63° \approx 137.6°$, and this occurs at $\theta_i = 59.52°$.

(d) For blue light, we find that the θ_{dev} minimum is $139.35° \approx 139.4°$, and this occurs at $\theta_i = 59.52°$.

(e) The difference in θ_{dev} in the previous two parts is 1.72°.

78. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution(s) to the overall deviation is (are) the reflection(s).

Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after [each] reflection) is $\delta\theta_r = 180° - 2\theta_r$. Thus, for k reflections, we have $\delta\theta_2 = k\theta_r$ to account for these contributions. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 2(\theta_i - \theta_r) + k(180° - 2\theta_r) = k(180°) + 2\theta_i - 2(k+1)\theta_r.$$

(b) For k = 2 and n = 1.331 (given in Problem 33-77), we search for the second-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is $230.37° \approx 230.4°$, and this occurs at θ_i = 71.90°.

(c) Similarly, we find that the second-order θ_{dev} minimum for blue light (for which n = 1.343) is $233.48° \approx 233.5°$, and this occurs at θ_i = 71.52°.

(d) The difference in θ_{dev} in the previous two parts is approximately 3.1°.

(e) Setting k = 3, we search for the third-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 317.5°, and this occurs at θ_i = 76.88°.

(f) Similarly, we find that the third-order θ_{dev} minimum for blue light is 321.9°, and this occurs at θ_i = 76.62°.

(g) The difference in θ_{dev} in the previous two parts is 4.4°.

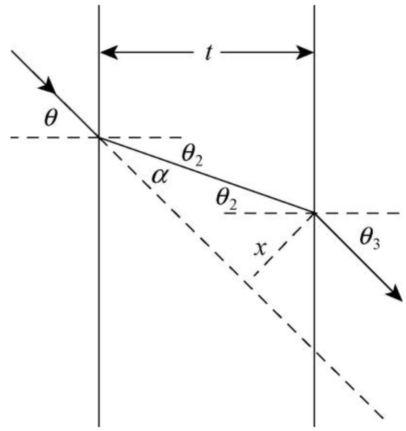
79. **THINK** We apply law of refraction to both interfaces to calculate the sideway displacement.

EXPRESS Let θ be the angle of incidence and θ_2 be the angle of refraction at the left face of the plate. Let n be the index of refraction of the glass. Then, the law of refraction yields

$$\sin\theta = n\sin\theta_2.$$

The angle of incidence at the right face is also θ_2. If θ_3 is the angle of emergence there, then

$$n\sin\theta_2 = \sin\theta_3.$$



ANALYZE (a) Combining the two expressions gives sin θ_3 = sin θ, which implies that $\theta_3 = \theta$. Thus, the emerging ray is parallel to the incident ray.

(b) We wish to derive an expression for x in terms of θ. If D is the length of the ray in the glass, then $D \cos\theta_2 = t$ and $D = t/\cos\theta_2$. The angle α in the diagram equals $\theta - \theta_2$ and

$$x = D\sin\alpha = D\sin(\theta - \theta_2).$$

Thus,

$$x = \frac{t\sin(\theta-\theta_2)}{\cos\theta_2}.$$

If all the angles θ, θ_2, θ_3, and $\theta - \theta_2$ are small and measured in radians, then $\sin\theta \approx \theta$, $\sin\theta_2 \approx \theta_2$, $\sin(\theta - \theta_2) \approx \theta - \theta_2$, and $\cos\theta_2 \approx 1$. Thus $x \approx t(\theta - \theta_2)$. The law of refraction applied to the point of incidence at the left face of the plate is now $\theta \approx n\theta_2$, so $\theta_2 \approx \theta/n$ and

$$x \approx t\left(\theta - \frac{\theta}{n}\right) = \frac{(n-1)t\theta}{n}.$$

LEARN The thicker the glass, the greater the displacement x. Note in the limit $n = 1$ (no glass), $x = 0$, as expected.

80. (a) The magnitude of the magnetic field is

$$B = \frac{E}{c} = \frac{100\ \text{V/m}}{3.0\times10^8\ \text{m/s}} = 3.3\times10^{-7}\ \text{T}.$$

(b) With $\vec{E}\times\vec{B} = \mu_0\vec{S}$, where $\vec{E} = E\hat{\text{k}}$ and $\vec{S} = S(-\hat{\text{j}})$, one can verify easily that since $\hat{\text{k}}\times(-\hat{\text{i}}) = -\hat{\text{j}}$, $\vec{B}$ has to be in the $-x$ direction.

81. (a) The polarization direction is defined by the electric field (which is perpendicular to the magnetic field in the wave, and also perpendicular to the direction of wave travel). The given function indicates the magnetic field is along the x axis (by the subscript on B)

and the wave motion is along $-y$ axis (see the argument of the sine function). Thus, the electric field direction must be parallel to the z axis.

(b) Since k is given as 1.57×10^7/m, then $\lambda = 2\pi/k = 4.0 \times 10^{-7}$ m, which means $f = c/\lambda = 7.5 \times 10^{14}$ Hz.

(c) The magnetic field amplitude is given as $B_m = 4.0 \times 10^{-6}$ T. The electric field amplitude E_m is equal to B_m divided by the speed of light c. The rms value of the electric field is then E_m divided by $\sqrt{2}$. Equation 33-26 then gives $I = 1.9$ kW/m^2.

82. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2 \theta_1' \cos^2 \theta_2'$$

where $\theta_1' = 90° - \theta_1 = 60°$ and $\theta_2' = 90° - \theta_2 = 60°$. This yields $I/I_0 = 0.031$.

83. **THINK** The index of refraction encountered by light generally depends on the wavelength of the light.

EXPRESS The critical angle for total internal reflection is given by sin $\theta_c = 1/n$. With an index of refraction $n = 1.456$ at the red end, the critical angle is $\theta_c = 43.38°$ for red. Similarly, with $n = 1.470$ at the blue end, the critical angle is $\theta_c = 42.86°$ for blue.

ANALYZE (a) An angle of incidence of $\theta_1 = 42.00°$ is less than the critical angles for both red and blue light, so the refracted light is white.

(b) An angle of incidence of $\theta_1 = 43.10°$ is slightly less than the critical angle for red light but greater than the critical angle for blue light, so the refracted light is dominated by red end.

(c) An angle of incidence of $\theta_1 = 44.00°$ is greater than the critical angles for both red and blue light, so there is no refracted light.

LEARN The dependence of the index of refraction of fused quartz on wavelength is shown in Fig. 33-18. From the figure, we see that the index of refraction is greater for a shorter wavelength. Such dependence results in the spreading of light as it enters or leaves quartz, a phenomenon called "chromatic dispersion."

84. Using Eqs. 33-40 and 33-42, we obtain

$$\frac{I_{\text{final}}}{I_0} = \frac{(I_0/2)(\cos^2 45°)(\cos^2 45°)}{I_0} = \frac{1}{8} = 0.125.$$

85. We write $m = \rho\mathcal{V}$ where $\mathcal{V} = 4\pi R^3/3$ is the volume. Plugging this into $F = ma$ and then into Eq. 33-32 (with $A = \pi R^2$, assuming the light is in the form of plane waves), we find

$$\rho\frac{4\pi R^3}{3}a = \frac{I\pi R^2}{c}.$$

This simplifies to

$$a = \frac{3I}{4\rho cR}$$

which yields $a = 1.5 \times 10^{-9}$ m/s^2.

86. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is

$$\tfrac{1}{2}(\cos^2(30°))^3 = 0.21.$$

87. **THINK** Since the radar beam is emitted uniformly over a hemisphere, the source power is also the same everywhere within the hemisphere.

EXPRESS The intensity of the beam is given by

$$I = \frac{P}{A} = \frac{P}{2\pi r^2}$$

where $A = 2\pi r^2$ is the area of a hemisphere. The power of the aircraft's reflection is equal to the product of the intensity at the aircraft's location and its cross-sectional area: $P_r = IA_r$. The intensity is related to the amplitude of the electric field by Eq. 33-26: $I = E_{\rm rms}^2/c\mu_0 = E_m^2/2c\mu_0$.

ANALYZE (a) Substituting the values given we get

$$I = \frac{P}{2\pi r^2} = \frac{180\times10^3\ \text{W}}{2\pi(90\times10^3\ \text{m})^2} = 3.5\times10^{-6}\ \text{W/m}^2.$$

(b) The power of the aircraft's reflection is

$$P_r = IA_r = (3.5\times10^{-6}\ \text{W/m}^2)(0.22\ \text{m}^2) = 7.8\times10^{-7}\ \text{W}.$$

(c) Back at the radar site, the intensity is

$$I_r = \frac{P_r}{2\pi r^2} = \frac{7.8\times10^{-7}\ \text{W}}{2\pi(90\times10^3\ \text{m})^2} = 1.5\times10^{-17}\ \text{W/m}^2.$$

(d) From $I_r = E_m^2/2c\mu_0$, we find the amplitude of the electric field to be

$$E_m = \sqrt{2c\mu_0 I_r} = \sqrt{2(3.0\times10^8 \text{ m/s})(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(1.5\times10^{-17} \text{ W/m}^2)}$$
$$= 1.1\times10^{-7} \text{ V/m}.$$

(e) The rms value of the magnetic field is

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{E_m}{\sqrt{2}c} = \frac{1.1\times10^{-7} \text{ V/m}}{\sqrt{2}(3.0\times10^8 \text{ m/s})} = 2.5\times10^{-16} \text{T}.$$

LEARN The intensity due to a power source decreases with the square of the distance. Also, as emphasized in Sample Problem — "Light wave: rms values of the electric and magnetic fields," one cannot compare the values of the two fields because they are measured in different units. Both components are on the same basis from the perspective of wave propagation, and they have the same average energy.

88. The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{3.20\times10^{-4}\text{ V/m}}{2.998\times10^8 \text{ m/s}} = 1.07\times10^{-12} \text{ T}.$$

89. From Fig. 33-19 we find $n_{\max} = 1.470$ for $\lambda = 400$ nm and $n_{\min} = 1.456$ for $\lambda = 700$ nm.
(a) The corresponding Brewster's angles are

$$\theta_{\text{B,max}} = \tan^{-1} n_{\max} = \tan^{-1}(1.470) = 55.8°,$$

(b) and $\theta_{\text{B,min}} = \tan^{-1}(1.456) = 55.5°$.

90. (a) Suppose there are a total of N transparent layers ($N = 5$ in our case). We label these layers from left to right with indices 1, 2, …, N. Let the index of refraction of the air be n_0. We denote the initial angle of incidence of the light ray upon the air-layer boundary as θ_i and the angle of the emerging light ray as θ_f. We note that, since all the boundaries are parallel to each other, the angle of incidence θ_j at the boundary between the j-th and the $(j + 1)$-th layers is the same as the angle between the transmitted light ray and the normal in the j-th layer. Thus, for the first boundary (the one between the air and the first layer)

$$\frac{n_1}{n_0} = \frac{\sin\theta_i}{\sin\theta_1},$$

for the second boundary

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2},$$

and so on. Finally, for the last boundary

$$\frac{n_0}{n_N} = \frac{\sin\theta_N}{\sin\theta_f},$$

Multiplying these equations, we obtain

$$\left(\frac{n_1}{n_0}\right)\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right)\cdots\left(\frac{n_0}{n_N}\right) = \left(\frac{\sin\theta_i}{\sin\theta_1}\right)\left(\frac{\sin\theta_1}{\sin\theta_2}\right)\left(\frac{\sin\theta_2}{\sin\theta_3}\right)\cdots\left(\frac{\sin\theta_N}{\sin\theta_f}\right).$$

We see that the L.H.S. of the equation above can be reduced to n_0/n_0 while the R.H.S. is equal to $\sin\theta_i/\sin\theta_f$. Equating these two expressions, we find

$$\sin\theta_f = \left(\frac{n_0}{n_0}\right)\sin\theta_i = \sin\theta_i,$$

which gives $\theta_i = \theta_f$. So for the two light rays in the problem statement, the angle of the emerging light rays are both the same as their respective incident angles. Thus, $\theta_f = 0$ for ray *a*,

(b) and $\theta_f = 20°$ for ray *b*.

(c) In this case, all we need to do is to change the value of n_0 from 1.0 (for air) to 1.5 (for glass). This does not change the result above. That is, we still have $\theta_f = 0$ for ray *a*,

(d) and $\theta_f = 20°$ for ray *b*.

Note that the result of this problem is fairly general. It is independent of the number of layers and the thickness and index of refraction of each layer.

91. (a) At $r = 40$ m, the intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{P}{\pi(\theta r)^2/4} = \frac{4(3.0\times10^{-3}\,\text{W})}{\pi\left[\left(0.17\times10^{-3}\,\text{rad}\right)\left(40\,\text{m}\right)\right]^2} = 83\,\text{W/m}^2.$$

(b) $P' = 4\pi r^2 I = 4\pi(40\text{m})^2(83\,\text{W/m}^2) = 1.7\times10^6\,\text{W}.$

92. The law of refraction requires that

$$\sin\theta_1/\sin\theta_2 = n_{\text{water}} = \text{const.}$$

We can check that this is indeed valid for any given pair of θ_1 and θ_2. For example, sin 10° / sin 8° = 1.3, and sin 20° / sin 15°30′ = 1.3, etc. Therefore, the index of refraction of water is $n_{\text{water}} = 1.3$.

93. We remind ourselves that when the unpolarized light passes through the first sheet, its intensity is reduced by a factor of 2. Thus, to end up with an overall reduction of one-third, the second sheet must cause a further decrease by a factor of two-thirds (since (1/2)(2/3) = 1/3). Thus, $\cos^2\theta = 2/3 \quad \Rightarrow \quad \theta = 35°$.

94. (a) The magnitude of the electric field at point P is

$$E = \frac{V}{l} = \frac{iR}{l} = (25.0\text{ A})\left(\frac{1.00\,\Omega}{300\text{ m}}\right) = 0.0833\text{ V/m}.$$

The direction of $\vec{E}$ at point P is in the $+x$ direction, same as the current.

(b) We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop. The magnitude of the magnetic field is

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(25.0\text{ A}\right)}{2\pi\left(1.25\times10^{-3}\text{m}\right)} = 4.00\times10^{-3}\text{ T}.$$

The direction of $\vec{B}$ at point P is in the $+z$ direction (out of the page).

(c) From $\vec{S} = \vec{E}\times\vec{B}/\mu_0$, we find the magnitude of the Poynting vector to be

$$S = \frac{EB}{\mu_0} = \frac{(0.0833\text{ V/m})(4.0\times10^{-3}\text{ T})}{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)} = 265\text{ W/m}^2.$$

(d) Since $\vec{S}$ points in the direction of $\vec{E}\times\vec{B}$, using the right-hand-rule, the direction of $\vec{S}$ at point P is in the $-y$ direction.

95. (a) For the cylindrical resistor shown in Figure 33-74, the magnetic field is in the $-\hat{\theta}$, or clockwise direction. On the other hand, the electric field is in the same direction as the current, $-\hat{z}$. Since $\vec{S} = \vec{E}\times\vec{B}/\mu_0$, $\vec{S}$ is in the direction of $(-\hat{z})\times(-\hat{\theta}) = -\hat{r}$, or radially inward.

(b) The magnitudes of the electric and magnetic fields are $E = V/l = iR/l$ and $B = \mu_0 i/2\pi a$, respectively. Thus,

$$S = \frac{EB}{\mu_0} = \frac{1}{\mu_0}\left(\frac{iR}{l}\right)\left(\frac{\mu_0 i}{2\pi a}\right) = \frac{i^2 R}{2\pi a l}.$$

Noting that the magnitude of the Poynting vector S is constant, we have

$$\int \vec{S}\cdot d\vec{A} = SA = \left(\frac{i^2 R}{2\pi a l}\right)(2\pi a l) = i^2 R.$$

96. The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2/2\mu_0 c$, implying that the rate of energy absorbed is $P_{\text{abs}} = IA = E_m^2 A/2\mu_0 c$. If all the energy is used to heat up the sheet (converting to its internal energy), then

$$P_{\text{abs}} = \frac{dE_{\text{int}}}{dt} = mc_s \frac{dT}{dt},$$

where c_s is the specific heat of the material. Solving for dT/dt, we find

$$mc_s \frac{dT}{dt} = \frac{E_m^2 A}{2\mu_0 c} \quad \Rightarrow \quad \frac{dT}{dt} = \frac{E_m^2 A}{2mc_s \mu_0 c}.$$

97. Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is, by one-half rule, $I_1 = \frac{1}{2} I_0$. For the second sheet, we apply the cosine-squared rule:

$$I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2\theta$$

where θ is the angle between the direction of polarization of the two sheets. With $I_2/I_0 = p/100$, we solve for θ and obtain

$$\frac{I_2}{I_0} = \frac{p}{100} = \frac{1}{2}\cos^2\theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\frac{p}{50}}\right).$$

98. The cross-sectional area of the beam on the surface is $A\cos\theta$. In a time interval Δt, the volume of the beam that's been reflected is $\Delta V = (A\cos\theta)c\Delta t$, and the momentum carried by this volume is $p = (I/c^2)(A\cos\theta)c\Delta t$. Upon being reflected, the change in momentum is

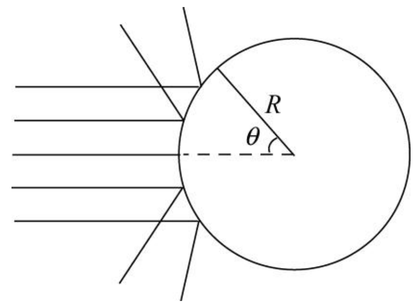
$$\Delta p = 2p\cos\theta = 2IA\cos^2\theta \Delta t / c$$

Thus, the radiation pressure is

$$p_r = \frac{F_r}{A} = \frac{\Delta p}{A\Delta t} = \frac{2I}{c}\cos^2\theta = p_{r\perp}\cos^2\theta$$

where $p_{r\perp} = 2I/c$ is the radiation pressure when $\theta = 0$.

99. Consider the figure shown to the right. The y-component of the force cancels out, and we're left with the x-component:



$$dF_x = 2dF\cos\theta = 2(p_r dA)\cos\theta .$$

Using the result from Problem 98: $p_r = (2I/c)\cos^2\theta$, and $dA = RLd\theta$, where L is the length of the cylinder, we obtain

$$\frac{F_x}{L} = \int 2(2I\cos\theta/c)\cos\theta\, Rd\theta = \frac{4IR}{c}\int_0^{\pi/2}\cos^3\theta d\theta = \frac{8IR}{3c}.$$

100. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2\theta_1' \cos^2\theta_2'$$

where $\theta_1' = (90° - \theta_1) + \theta_2 = 110°$ is the relative angle between the first and the second polarizing sheets, and $\theta_2' = 90° - \theta_2 = 50°$ is the relative angle between the second and the third polarizing sheets. Thus, we have $I/I_0 = 0.024$.

101. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2\theta' \cos^2\theta''.$$

With $\theta' = \theta_2 - \theta_1$ = 60° – 20° = 40° and $\theta'' = \theta_3 + (\pi/2 - \theta_2)$ = 40° + 30° = 70°, we get $I/I_0 = 0.034$.

102. We use Eq. 33-33 for the force, where A is the area of the reflecting surface (4.0 m^2). The intensity is gotten from Eq. 33-27 where $P = P_S$ is in Appendix C (see also Sample Problem 33-2) and $r = 3.0 \times 10^{11}$ m (given in the problem statement). Our result for the force is 9.2 μN.

103. Eq. 33-5 gives $B = E/c$, which relates the field values at any instant — and so relates rms values to rms values, and amplitude values to amplitude values, as the case may be. Thus, the rms value of the magnetic field is

$$B_{\text{rms}} = (0.200 \text{ V/m})/(3 \times 10^8 \text{ m/s}) = 6.67 \times 10^{-10} \text{ T},$$

which (upon multiplication by $\sqrt{2}$) yields an amplitude value of magnetic field equal to 9.43×10^{-10} T.

104. (a) The Sun is far enough away that we approximate its rays as "parallel" in this Figure. That is, if the sunray makes angle θ from horizontal when the bird is in one position, then it makes the same angle θ when the bird is any other position. Therefore, its shadow on the ground moves as the bird moves: at 15 m/s.

(b) If the bird is in a position, a distance $x > 0$ from the wall, such that its shadow is on the wall at a distance $0 \geq y \geq h$ from the top of the wall, then it is clear from the Figure that $\tan\theta = y/x$. Thus,

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta = (-15\ \text{m/s})\tan 30° = -8.7\ \text{m/s},$$

which means that the distance y (which was measured as a positive number downward from the top of the wall) is shrinking at the rate of 8.7 m/s.

(c) Since $\tan\theta$ grows as $0 \leq \theta < 90°$ increases, then a larger value of $|dy/dt|$ implies a larger value of θ. The Sun is higher in the sky when the hawk glides by.

(d) With $|dy/dt| = 45$ m/s, we find

$$v_{\text{hawk}} = \left|\frac{dx}{dt}\right| = \frac{|dy/dt|}{\tan\theta}$$

so that we obtain $\theta = 72°$ if we assume $v_{\text{hawk}} = 15$ m/s.

105. (a) The wave is traveling in the $-y$ direction (see §16-5 for the significance of the relative sign between the spatial and temporal arguments of the wave function).

(b) Figure 33-5 may help in visualizing this. The direction of propagation (along the y axis) is perpendicular to $\vec{B}$ (presumably along the x axis, since the problem gives B_x and no other component) and both are perpendicular to $\vec{E}$ (which determines the axis of polarization). Thus, the wave is z-polarized.

(c) Since the magnetic field amplitude is $B_m = 4.00\ \mu$T, then (by Eq. 33-5) $E_m = 1199$ V/m $\approx 1.20 \times 10^3$ V/m. Dividing by $\sqrt{2}$ yields $E_{\text{rms}} = 848$ V/m. Then, Eq. 33-26 gives

$$I = \frac{I}{c\mu_0}E_{\text{rms}}^2 = 1.91 \times 10^3\ \text{W/m}^2.$$

(d) Since $kc = \omega$ (equivalent to $c = f\lambda$), we have

$$k = \frac{2.00\times 10^{15}}{c} = 6.67\times 10^{6}\ \text{m}^{-1}.$$

Summarizing the information gathered so far, we have (with SI units understood)

$$E_z = (1.2\times 10^3\ \text{V/m})\sin[(6.67\times 10^6\ /\ \text{m})y + (2.00\times 10^{15}\ /\ \text{s})t].$$

(e) $\lambda = 2\pi/k = 942$ nm.

(f) This is an infrared light.

106. (a) The angle of incidence $\theta_{B,1}$ at B is the complement of the critical angle at A; its sine is

$$\sin\theta_{B,1} = \cos\theta_c = \sqrt{1-\left(\frac{n_3}{n_2}\right)^2}$$

so that the angle of refraction $\theta_{B,2}$ at B becomes

$$\theta_{B,2} = \sin^{-1}\left(\frac{n_2}{n_3}\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}\right) = \sin^{-1}\sqrt{\left(\frac{n_2}{n_3}\right)^2 - 1} = 35.1^\circ .$$

(b) From $n_1 \sin\theta = n_2 \sin\theta_c = n_2(n_3/n_2)$, we find

$$\theta = \sin^{-1}\left(\frac{n_3}{n_1}\right) = 49.9^\circ .$$

(c) The angle of incidence $\theta_{A,1}$ at A is the complement of the critical angle at B; its sine is

$$\sin\theta_{A,1} = \cos\theta_c = \sqrt{1-\left(\frac{n_3}{n_2}\right)^2} .$$

so that the angle of refraction $\theta_{A,2}$ at A becomes

$$\theta_{A,2} = \sin^{-1}\left(\frac{n_2}{n_3}\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}\right) = \sin^{-1}\sqrt{\left(\frac{n_2}{n_3}\right)^2 - 1} = 35.1^\circ .$$

(d) From

$$n_1 \sin\theta = n_2 \sin\theta_{A,1} = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2} ,$$

we find

$$\theta = \sin^{-1}\left(\frac{\sqrt{n_2^2 - n_3^2}}{n_1}\right) = 26.1°$$

(e) The angle of incidence $\theta_{B,1}$ at B is the complement of the Brewster angle at A; its sine is

$$\sin\theta_{B,1} = \frac{n_2}{\sqrt{n_2^2 + n_3^2}}$$

so that the angle of refraction $\theta_{B,2}$ at B becomes

$$\theta_{B,2} = \sin^{-1}\left(\frac{n_2^2}{n_3\sqrt{n_2^2 + n_3^2}}\right) = 60.7°.$$

(f) From

$$n_1 \sin\theta = n_2 \sin\theta_{\text{Brewster}} = n_2 \frac{n_3}{\sqrt{n_2^2 + n_3^2}} ,$$

we find

$$\theta = \sin^{-1}\left(\frac{n_2 n_3}{n_1\sqrt{n_2^2 + n_3^2}}\right) = 35.3° .$$

107. (a) and (b) At the Brewster angle, $\theta_{\text{incident}} + \theta_{\text{refracted}} = \theta_B + 32.0° = 90.0°$, so $\theta_B = 58.0°$ and

$$n_{\text{glass}} = \tan\theta_B = \tan 58.0° = 1.60.$$

108. We take the derivative with respect to x of both sides of Eq. 33-11:

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right) = \frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial^2 B}{\partial x \partial t}.$$

Now we differentiate both sides of Eq. 33-18 with respect to t:

$$\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial x}\right) = -\frac{\partial^2 B}{\partial x \partial t} = \frac{\partial}{\partial t}\left(\varepsilon_0\mu_0 \frac{\partial E}{\partial t}\right) = \varepsilon_0\mu_0 \frac{\partial^2 E}{\partial t^2}.$$

Substituting $\partial^2 E/\partial x^2 = -\partial^2 B/\partial x \partial t$ from the first equation above into the second one, we get

$$\varepsilon_0\mu_0 \frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} \quad \Rightarrow \quad \frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2 E}{\partial x^2}.$$

Similarly, we differentiate both sides of Eq. 33-11 with respect to t

$$\frac{\partial^2 E}{\partial x \partial t} = -\frac{\partial^2 B}{\partial t^2},$$

and differentiate both sides of Eq. 33-18 with respect to x

$$-\frac{\partial^2 B}{\partial x^2} = \varepsilon_0 \mu_0 - \frac{\partial^2 E}{\partial x \partial t}.$$

Combining these two equations, we get

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

109. (a) From Eq. 33-1,

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2}{\partial t^2} \; E_m \sin(kx - \omega t) \; = -\omega^2 E_m \sin(kx - \omega t),$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2}{\partial x^2} \; E_m \sin(kx - \omega t) \; = -k^2 c^2 \sin(kx - \omega t) = -\omega^2 E_m \sin(kx - \omega t).$$

Consequently,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}$$

is satisfied. Analogously, one can show that Eq. 33-2 satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

(b) From $E = E_m f(kx \pm \omega t)$,

$$\frac{\partial^2 E}{\partial t^2} = E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = \omega^2 E_m \left.\frac{d^2 f}{du^2}\right|_{u = kx \pm \omega t}$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = c^2 E_m k^2 \left.\frac{d^2 f}{du^2}\right|_{u = kx \pm \omega t}$$

Since $\omega = ck$ the right-hand sides of these two equations are equal. Therefore,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}.$$

Changing E to B and repeating the derivation above shows that $B = B_m f(kx \pm \omega t)$ satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

110. Since intensity is power divided by area (and the area is spherical in the isotropic case), then the intensity at a distance of r = 20 m from the source is

$$I = \frac{P}{4\pi r^2} = 0.040\,\mathrm{W/m^2}.$$

as illustrated in Sample Problem 33-2. Now, in Eq. 33-32 for a totally absorbing area A, we note that the exposed area of the small sphere is that on a flat circle $A = \pi(0.020\text{ m})^2 = 0.0013\text{ m}^2$. Therefore,

$$F = \frac{IA}{c} = \frac{(0.040)(0.0013)}{3\times 10^8} = 1.7\times 10^{-13}\,\mathrm{N}.$$

Chapter 34

1. The bird is a distance d_2 in front of the mirror; the plane of its image is that same distance d_2 behind the mirror. The lateral distance between you and the bird is d_3 = 5.00 m. We denote the distance from the camera to the mirror as d_1, and we construct a right triangle out of d_3 and the distance between the camera and the image plane ($d_1 + d_2$). Thus, the focus distance is

$$d=\sqrt{(d_1+d_2)^2+d_3^2}=\sqrt{(4.30\text{ m}+3.30\text{ m})^2+(5.00\text{ m})^2}=9.10\text{ m}.$$

2. The image is 10 cm behind the mirror and you are 30 cm in front of the mirror. You must focus your eyes for a distance of 10 cm + 30 cm = 40 cm.

3. The intensity of light from a point source varies as the inverse of the square of the distance from the source. Before the mirror is in place, the intensity at the center of the screen is given by $I_P = A/d^2$, where A is a constant of proportionality. After the mirror is in place, the light that goes directly to the screen contributes intensity I_P, as before. Reflected light also reaches the screen. This light appears to come from the image of the source, a distance d behind the mirror and a distance $3d$ from the screen. Its contribution to the intensity at the center of the screen is

$$I_r=\frac{A}{(3d)^2}=\frac{A}{9d^2}=\frac{I_P}{9}.$$

The total intensity at the center of the screen is

$$I=I_P+I_r=I_P+\frac{I_P}{9}=\frac{10}{9}I_P.$$

The ratio of the new intensity to the original intensity is $I/I_P = 10/9 = 1.11$.

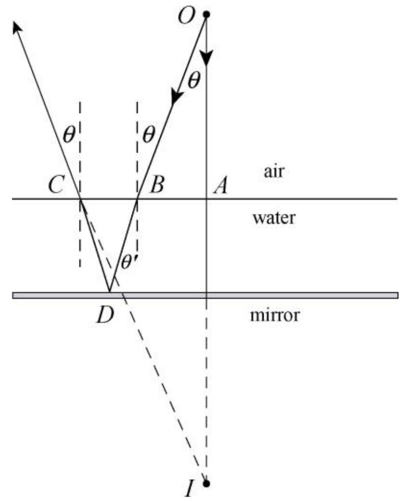
4. When S is barely able to see B, the light rays from B must reflect to S off the edge of the mirror. The angle of reflection in this case is 45°, since a line drawn from S to the mirror's edge makes a 45° angle relative to the wall. By the law of reflection, we find

$$\frac{x}{d/2}=\tan 45°=1 \quad\Rightarrow\quad x=\frac{d}{2}=\frac{3.0\,\text{m}}{2}=1.5\,\text{m}.$$

5. **THINK** This problem involves refraction at air–water interface and reflection from a plane mirror at the bottom of the pool.

EXPRESS We apply the law of refraction, assuming all angles are in radians:

$$\frac{\sin\theta}{\sin\theta'} = \frac{n_w}{n_{\text{air}}},$$



which in our case reduces to $\theta' \approx \theta/n_w$ (since both θ and θ' are small, and $n_{\text{air}} \approx 1$). We refer to our figure on the right.

The object O is a vertical distance d_1 above the water, and the water surface is a vertical distance d_2 above the mirror. We are looking for a distance d (treated as a positive number) below the mirror where the image I of the object is formed. In the triangle $O\,AB$

$$|AB| = d_1 \tan\theta \approx d_1\theta,$$

and in the triangle CBD

$$|BC| = 2d_2 \tan\theta' \approx 2d_2\theta' \approx \frac{2d_2\theta}{n_w}.$$

Finally, in the triangle ACI, we have $|AI| = d + d_2$.

ANALYZE Therefore,

$$d = |AI| - d_2 = \frac{|AC|}{\tan\theta} - d_2 \approx \frac{|AB| + |BC|}{\theta} - d_2 = \left(d_1\theta + \frac{2d_2\theta}{n_w}\right)\frac{1}{\theta} - d_2 = d_1 + \frac{2d_2}{n_w} - d_2$$
$$= 250\,\text{cm} + \frac{2(200\,\text{cm})}{1.33} - 200\,\text{cm} = 351\,\text{cm}.$$

LEARN If the pool were empty without water, then $\theta = \theta'$, and the distance would be $d = d_1 + 2d_2 - d_2 = d_1 + d_2$. This is precisely what we expect from a plane mirror.

6. We note from Fig. 34-34 that $m = \frac{1}{2}$ when p = 5 cm. Thus Eq. 34-7 (the magnification equation) gives us i = –10 cm in that case. Then, by Eq. 34-9 (which applies to mirrors and thin lenses) we find the focal length of the mirror is f = 10 cm. Next, the problem asks us to consider p = 14 cm. With the focal length value already determined, then Eq. 34-9 yields i = 35 cm for this new value of object distance. Then, using Eq. 34-7 again, we find $m = i/p = -2.5$.

7. We use Eqs. 34-3 and 34-4, and note that $m = -i/p$. Thus,

$$\frac{1}{p}-\frac{1}{pm}=\frac{1}{f}=\frac{2}{r}.$$

We solve for p: $p=\frac{r}{2}\left(1-\frac{1}{m}\right)=\frac{35.0\,\text{cm}}{2}\left(1-\frac{1}{2.50}\right)=10.5\text{ cm}.$

8. The graph in Fig. 34-35 implies that f = 20 cm, which we can plug into Eq. 34-9 (with p = 70 cm) to obtain i = +28 cm.

9. **THINK** A concave mirror has a positive value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f=r/2$. The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is $m=-i/p$. The value of m is positive for upright (not inverted) images, and negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With f = +12 cm and p = +18 cm, the radius of curvature is r = 2f = 2(12 cm) = + 24 cm.

(b) The image distance is $i=\frac{pf}{p-f}=\frac{(18\text{ cm})(12\text{ cm})}{18\text{ cm}-12\text{ cm}}=36\text{ cm}.$

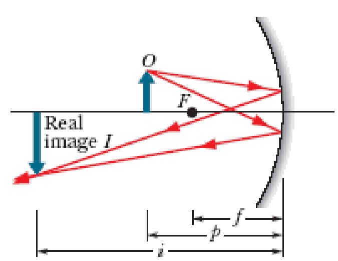
(c) The lateral magnification is $m = -i/p = -$ (36 cm)/(18 cm) = -2.0.

(d) Since the image distance i is positive, the image is real (R).

(e) Since the magnification m is negative, the image is inverted (I).

(f) A real image is formed on the same side as the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-10(c). The object is outside the focal point, and its image is real and inverted.



10. A concave mirror has a positive value of focal length.

(a) Then (with $f = +10$ cm and $p = +15$ cm), the radius of curvature is $r = 2f = +20\text{ cm}$.

(b) Equation 34-9 yields $i = pf/(p - f) = +30$ cm.

(c)Then, by Eq. 34-7, $m = -i/p = -2.0$.

(d) Since the image distance computation produced a positive value, the image is real (R).

(e) The magnification computation produced a negative value, so it is inverted (I).

(f) A real image is formed on the <u>same</u> side as the object.

11. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length *f* is related to the radius of curvature *r* by $f = r/2$. The object distance *p*, the image distance *i*, and the focal length *f* are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of *i* is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is

$$m = -\frac{i}{p}.$$

The value of *m* is positive for upright (not inverted) images, and negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With $f = -10$ cm and $p = +8$ cm, the radius of curvature is $r = 2f = -20$ cm.

(b) The image distance is $i = \dfrac{pf}{p-f} = \dfrac{(8\text{ cm})(-10\text{ cm})}{8\text{ cm} - (-10)\text{ cm}} = -4.44$ cm.

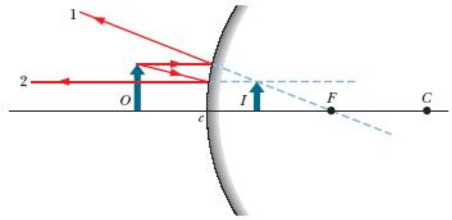
(c) The lateral magnification is $m = -i/p = -(-4.44\text{ cm})/(8.0\text{ cm}) = +0.56$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification *m* is positive, so the image is upright [not inverted] (NI).

(f) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



12. A concave mirror has a positive value of focal length.

(a) Then (with f = +36 cm and p = +24 cm), the radius of curvature is $r = 2f$ = + 72 cm.

(b) Equation 34-9 yields $i = pf/(p-f)$ = –72 cm.

(c) Then, by Eq. 34-7, $m = -i/p$ = +3.0.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

13. **THINK** A concave mirror has a positive value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$.

The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of i is positive for real images and negative for virtual images.

The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE With f = +18 cm and p = +12 cm, the radius of curvature is $r = 2f$ = + 36 cm.

(b) Equation 34-9 yields $i = pf/(p-f)$ = –36 cm.

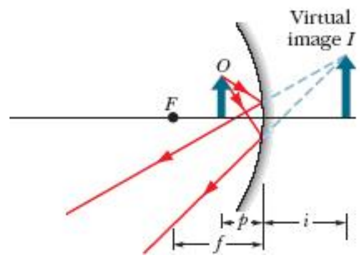
(c) Then, by Eq. 34-7, $m = -i/p$ = +3.0.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(a). The mirror is concave, and its image is virtual, enlarged, and upright.



14. A convex mirror has a negative value of focal length.

(a) Then (with $f = –35$ cm and $p = +22$ cm), the radius of curvature is $r = 2f = –70$ cm.

(b) Equation 34-9 yields $i = pf/(p-f) = -14$ cm.

(c) Then, by Eq. 34-7, $m = –i/p = +0.61$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) The side where a virtual image forms is opposite from the side where the object is.

15. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length *f* is related to the radius of curvature *r* by $f = r/2$.

The object distance *p*, the image distance *i*, and the focal length *f* are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of *i* is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is $m = -i/p$. The value of *m* is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With f = –8 cm and p = +10 cm, the radius of curvature is $r = 2f = 2(-8$ cm) = –16 cm.

(b) The image distance is $i = \frac{pf}{p-f} = \frac{(10\text{ cm})(-8\text{ cm})}{10\text{ cm}-(-8)\text{ cm}} = -4.44$ cm.

(c) The lateral magnification is $m = -i/p = -(-4.44$ cm)/(10 cm) = +0.44.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification m is positive, so the image is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.

16. A convex mirror has a negative value of focal length.

(a) Then (with f = –14 cm and p = +17 cm), the radius of curvature is $r = 2f$ = –28 cm.

(b) Equation 34-9 yields $i = pf/(p-f) = -7.7$ cm.

(c) Then, by Eq. 34-7, $m = -i/p$ = +0.45.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

17. (a) The mirror is concave.

(b) f = +20 cm (positive, because the mirror is concave).

(c) $r = 2f$ = 2(+20 cm) = +40 cm.

(d) The object distance p = +10 cm, as given in the table.

(e) The image distance is $i = (1/f - 1/p)^{-1} = (1/20\text{ cm} - 1/10\text{ cm})^{-1} = -20$ cm.

(f) $m = -i/p = -(-20$ cm/10 cm) = +2.0.

(g) The image is virtual (V).

(h) The image is upright or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

18. (a) Since the image is inverted, we can scan Figs. 34-8, 34-10, and 34-11 in the textbook and find that the mirror must be concave.

(b) This also implies that we must put a minus sign in front of the "0.50" value given for m. To solve for f, we first find $i = -pm = +12$ cm from Eq. 34-6 and plug into Eq. 34-4; the result is $f = +8$ cm.

(c) Thus, $r = 2f = +16$ cm.

(d) $p = +24$ cm, as given in the table.

(e) As shown above, $i = -pm = +12$ cm.

(f) $m = -0.50$, with a minus sign.

(g) The image is real (R), since $i > 0$.

(h) The image is inverted (I), as noted above.

(i) A real image is formed on the same side as the object.

19. (a) Since $r < 0$ then (by Eq. 34-3) $f < 0$, which means the mirror is convex.

(b) The focal length is $f = r/2 = -20$ cm.

(c) $r = -40$ cm, as given in the table.

(d) Equation 34-4 leads to $p = +20$ cm.

(e) $i = -10$ cm, as given in the table.

(f) Equation 34-6 gives $m = +0.50$.

(g) The image is virtual (V).

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

20. (a) From Eq. 34-7, we get $i = -mp = +28$ cm, which implies the image is real (R) and on the same side as the object. Since $m < 0$, we know it was inverted (I). From Eq. 34-9,

we obtain $f = ip/(i + p) = +16$ cm, which tells us (among other things) that the mirror is concave.

(b) $f = ip/(i + p) = +16$ cm.

(c) $r = 2f = +32$ cm.

(d) $p = +40$ cm, as given in the table.

(e) $i = -mp = +28$ cm.

(f) $m = -0.70$, as given in the table.

(g) The image is real (R).

(h) The image is inverted (I).

(i) A real image is formed on the same side as the object.

21. (a) Since $f > 0$, the mirror is concave.

(b) $f = + 20$ cm, as given in the table.

(c) Using Eq. 34-3, we obtain $r = 2f = +40$ cm.

(d) $p = + 10$ cm, as given in the table.

(e) Equation 34-4 readily yields $i = pf/(p - f) = +60$ cm.

(f) Equation 34-6 gives $m = -i/p = -2.0$.

(g) Since $i > 0$, the image is real (R).

(h) Since $m < 0$, the image is inverted (I).

(i) A real image is formed on the same side as the object.

22. (a) Since $0 < m < 1$, the image is upright but smaller than the object. With that in mind, we examine the various possibilities in Figs. 34-8, 34-10, and 34-11, and note that such an image (for reflections from a single mirror) can only occur if the mirror is convex.

(b) Thus, we must put a minus sign in front of the "20" value given for f, that is, $f = -20$ cm.

(c) Equation 34-3 then gives $r = 2f = -40$ cm.

(d) To solve for i and p we must set up Eq. 34-4 and Eq. 34-6 as a simultaneous set and solve for the two unknowns. The results are p = +180 cm = +1.8 m, and

(e) $i = -18$ cm.

(f) m = 0.10, as given in the table.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI), as already noted.

(i) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

23. **THINK** A positive value for the magnification means that the image is upright (not inverted).

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) The magnification is given by $m = -i/p$. Since $p > 0$, a positive value for m means that the image distance (i) is negative, implying a virtual image. A positive magnification of magnitude less than unity is only possible for <u>convex</u> mirrors.

(b) With $i = -mp$, we may write $p = f(1-1/m)$. For $0 < m < 1$, a positive value for p can be obtained only if $f < 0$. Thus, with a minus sign, we have $f = -30$ cm.

(c) The radius of curvature is $r = 2f = -60$ cm.

(d) The object distance is $p = f(1 - 1/m) = (-30 \text{ cm})(1 - 1/0.20) = +120 \text{ cm} = 1.2$ m.

(e) The image distance is $i = -mp = -(0.20)(120 \text{ cm}) = -24$ cm.

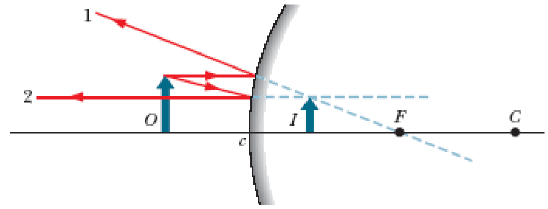
(f) The magnification is $m = +0.20$, as given in the Table.

(g) As discussed in (a), the image is virtual (V).

(h) As discussed in (a), the image is upright, or not inverted (NI).

(i) A virtual image is formed on the <u>opposite</u> side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



24. (a) Since $m = -1/2 < 0$, the image is inverted. With that in mind, we examine the various possibilities in Figs. 34-8, 34-10, and 34-11, and note that an inverted image (for reflections from a single mirror) can only occur if the mirror is concave (and if $p > f$).

(b) Next, we find i from Eq. 34-6 (which yields $i = mp = 30$ cm) and then use this value (and Eq. 34-4) to compute the focal length; we obtain $f = +20$ cm.

(c) Then, Eq. 34-3 gives $r = 2f = +40$ cm.

(d) $p = 60$ cm, as given in the table.

(e) As already noted, $i = +30$ cm.

(f) $m = -1/2$, as given.

(g) Since $i > 0$, the image is real (R).

(h) As already noted, the image is inverted (I).

(i) A real image is formed on the <u>same</u> side as the object.

25. (a) As stated in the problem, the image is inverted (I), which implies that it is real (R). It also (more directly) tells us that the magnification is equal to a negative value: $m = -0.40$. By Eq. 34-7, the image distance is consequently found to be $i = +12$ cm. Real images don't arise (under normal circumstances) from convex mirrors, so we conclude that this mirror is concave.

(b) The focal length is $f = +8.6$ cm, using Eq. 34-9, $f = +8.6$ cm.

(c) The radius of curvature is $r = 2f = +17.2$ cm ≈ 17 cm.

(d) $p = +30$ cm, as given in the table.

(e) As noted above, $i = +12$ cm.

(f) Similarly, $m = -0.40$, with a minus sign.

(g) The image is real (R).

(h) The image is inverted (I).

(i) A real image is formed on the same side as the object.

26. (a) We are told that the image is on the same side as the object; this means the image is real (R) and further implies that the mirror is concave.

(b) The focal distance is $f = +20$ cm.

(c) The radius of curvature is $r = 2f = +40$ cm.

(d) $p = +60$ cm, as given in the table.

(e) Equation 34-9 gives $i = pf/(p - f) = +30$ cm.

(f) Equation 34-7 gives $m = -i/p = -0.50$.

(g) As noted above, the image is real (R).

(h) The image is inverted (I) since $m < 0$.

(i) A real image is formed on the same side as the object.

27. (a) The fact that the focal length is given as a negative value means the mirror is convex.

(b) $f = -30$ cm, as given in the Table.

(c) The radius of curvature is $r = 2f = -60$ cm.

(d) Equation 34-9 gives $p = if/(i - f) = +30$ cm.

(e) $i = -15$, as given in the table.

(f) From Eq. 34-7, we get $m = +1/2 = 0.50$.

(g) The image distance is given as a negative value (as it would have to be, since the mirror is convex), which means the image is virtual (V).

(h) Since $m > 0$, the image is upright (not inverted: NI).

(i) The image is on the opposite side of the mirror as the object.

28. (a) The fact that the magnification is 1 means that the mirror is flat (plane).

(b) Flat mirrors (and flat "lenses" such as a window pane) have $f = \infty$ (or $f = -\infty$ since the sign does not matter in this extreme case).

(c) The radius of curvature is $r = 2f = \infty$ (or $r = -\infty$) by Eq. 34-3.

(d) $p = +\,10$ cm, as given in the table.

(e) Equation 34-4 readily yields $i = pf/(p-f) = -10$ cm.

(f) The magnification is $m = -i/p = +1.0$.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

29. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p, the image distance i, and the focal length f are related by Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) The mirror is convex, as given.

(b) Since the mirror is convex, the radius of curvature is negative, so $r = -\,40$ cm. Then, the focal length is $f = r/2 = (-40 \text{ cm})/2 = -20$ cm.

(c) The radius of curvature is $r = -\,40$ cm.

(d) The fact that the mirror is convex also means that we need to insert a minus sign in front of the "4.0" value given for i, since the image in this case must be virtual. Eq. 34-4 leads to

$$p = \frac{if}{i-f} = \frac{(-4.0 \text{ cm})(-20 \text{ cm})}{-4.0 \text{ cm} - (-20 \text{ cm})} = 5.0 \text{ cm}$$

(e) As noted above, $i = -\,4.0$ cm.

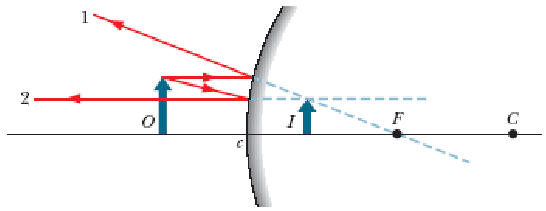
(f) The magnification is $m = -i/p = -(-4.0\text{ cm})/(5.0\text{ cm}) = +0.80$.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



30. We note that there is "singularity" in this graph (Fig. 34-36) like there was in Fig. 34-35), which tells us that there is no point where $p = f$ (which causes Eq. 34-9 to "blow up"). Since $p > 0$, as usual, then this means that the focal length is not positive. We know it is not a flat mirror since the curve shown does decrease with p, so we conclude it is a convex mirror. We examine the point where $m = 0.50$ and $p = 10$ cm. Combining Eq. 34-7 and Eq. 34-9 we obtain

$$m = -\frac{i}{p} = -\frac{f}{p-f}.$$

This yields $f = -10$ cm (verifying our expectation that the mirror is convex). Now, for $p = 21$ cm, we find $m = -f/(p-f) = +0.32$.

31. (a) From Eqs. 34-3 and 34-4, we obtain

$$i = \frac{pf}{p-f} = \frac{pr}{2p-r}.$$

Differentiating both sides with respect to time and using $v_O = -dp/dt$, we find

$$v_I = \frac{di}{dt} = \frac{d}{dt}\left(\frac{pr}{2p-r}\right) = \frac{-rv_O(2p-r)+2v_O pr}{(2p-r)^2} = \left(\frac{r}{2p-r}\right)^2 v_O.$$

(b) If $p = 30$ cm, we obtain $v_I = \left[\frac{15\text{ cm}}{2(30\text{ cm}) - 15\text{ cm}}\right]^2 (5.0\text{ cm/s}) = 0.56\text{ cm/s}.$

(c) If $p = 8.0$ cm, we obtain $v_I = \left[\frac{15\text{ cm}}{2(8.0\text{ cm}) - 15\text{ cm}}\right]^2 (5.0\text{ cm/s}) = 1.1\times 10^3\text{ cm/s}.$

(d) If p = 1.0 cm, we obtain $v_I = \left[\frac{15\,\text{cm}}{2(1.0\,\text{cm}) - 15\,\text{cm}}\right]^2 (5.0\,\text{cm/s}) = 6.7\,\text{cm/s}.$

32. In addition to n_1 =1.0, we are given (a) n_2 = 1.5, (b) p = +10 cm, and (c) r = +30 cm.

(d) Equation 34-8 yields $i = n_2\left(\frac{n_2 - n_1}{r} - \frac{n_1}{p}\right)^{-1} = 1.5\left(\frac{1.5-1.0}{30\text{ cm}} - \frac{1.0}{10\text{ cm}}\right) = -18\text{ cm}.$

(e) The image is virtual (V) and upright since $i < 0$.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(c) in the textbook.

33. **THINK** An image is formed by refraction through a spherical surface. A negative value for the image distance implies that the image is virtual.

EXPRESS Let n_1 be the index of refraction of the material where the object is located, n_2 be the index of refraction of the material on the other side of the refracting surface, and r be the radius of curvature of the surface. The image distance i is related to the object distance p by Eq. 34-8:

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}.$$

The value of i is positive for a real images, and negative for virtual images.

ANALYZE In addition to n_1 =1.0, we are given (a) n_2 = 1.5, (b) p = +10 cm, and (d) $i = -13$ cm.

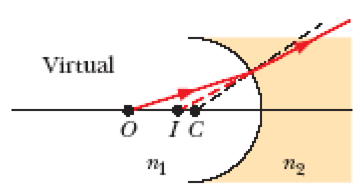
(c) Eq. 34-8 yields

$$r = (n_2 - n_1)\left(\frac{n_1}{p} + \frac{n_2}{i}\right)^{-1} = (1.5 - 1.0)\left(\frac{1.0}{10\text{ cm}} + \frac{1.5}{-13\text{ cm}}\right)^{-1} = -32.5\,\text{cm} \approx -33\text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-12(e). Here refraction always directs the ray away from the central axis; the images are always virtual, regardless of the object distance.



34. In addition to n_1 =1.5, we are given (b) p = +100, (c) r = –30 cm, and (d) $i = +600$ cm.

(a) We manipulate Eq. 34-8 to separate the indices:

$$n_2\left(\frac{1}{r}-\frac{1}{i}\right)=\left(\frac{n_1}{p}+\frac{n_1}{r}\right) \Rightarrow n_2\left(\frac{1}{-30}-\frac{1}{600}\right)=\left(\frac{1.5}{100}+\frac{1.5}{-30}\right) \Rightarrow n_2(-0.035)=-0.035$$

which implies n_2 = 1.0.

(e) The image is real (R) and inverted.

(f) The object and its image are on the opposite side. The ray diagram would be similar to Fig. 34-12(b) in the textbook.

35. **THINK** An image is formed by refraction through a spherical surface. Whether the image is real or virtual depends on the relative values of n_1 and n_2, and on the geometry.

EXPRESS Let n_1 be the index of refraction of the material where the object is located, n_2 be the index of refraction of the material on the other side of the refracting surface, and r be the radius of curvature of the surface. The image distance i is related to the object distance p by Eq. 34-8:

$$\frac{n_1}{p}+\frac{n_2}{i}=\frac{n_2-n_1}{r}.$$

The value of i is positive for a real images, and negative for virtual images.

ANALYZE In addition to n_1 =1.5, we are also given (a) n_2 = 1.0, (b) p = +70 cm, and (c) r = +30 cm. Notice that $n_2 < n_1$.

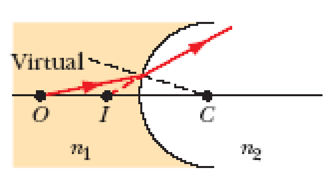
(d) We manipulate Eq. 34-8 to find the image distance:

$$i = n_2\left[\frac{n_2-n_1}{r}-\frac{n_1}{p}\right]^{-1} = 1.0\left[\frac{1.0-1.5}{30\text{ cm}}-\frac{1.5}{70\text{ cm}}\right]^{-1} = -26\text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-12(f). Here refraction always directs the ray away from the central axis; the images are always virtual, regardless of the object distance.



36. In addition to $n_1 = 1.5$, we are given (a) $n_2 = 1.0$, (c) $r = -30$ cm and (d) $i = -7.5$ cm.

(b)We manipulate Eq. 34-8 to find p:

$$p = \frac{n_1}{\frac{n_2 - n_1}{r} - \frac{n_2}{i}} = \frac{1.5}{\frac{1.0 - 1.5}{-30\text{ cm}} - \frac{1.0}{-7.5\text{ cm}}} = 10\text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(d) in the textbook.

37. In addition to $n_1 = 1.5$, we are given (a) $n_2 = 1.0$, (b) $p = +10$ cm, and (d) $i = -6.0$ cm.

(c) We manipulate Eq. 34-8 to find r:

$$r = (n_2 - n_1)\left(\frac{n_1}{p} + \frac{n_2}{i}\right)^{-1} = (1.0 - 1.5)\left(\frac{1.5}{10\text{ cm}} + \frac{1.0}{-6.0\text{ cm}}\right)^{-1} = 30\text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(f) in the textbook, but with the object and the image located closer to the surface.

38. In addition to $n_1 = 1.0$, we are given (a) $n_2 = 1.5$, (c) $r = +30$ cm, and (d) $i = +600$.

(b) Equation 34-8 gives $p = \frac{n_1}{\frac{n_2 - n_1}{r} - \frac{n_2}{i}} = \frac{1.0}{\frac{1.5 - 1.0}{30\text{ cm}} - \frac{1.5}{600\text{ cm}}} = 71\text{cm}.$

(e) With $i > 0$, the image is real (R) and inverted.

(f) The object and its image are on the opposite side. The ray diagram would be similar to Fig. 34-12(a) in the textbook.

39. (a) We use Eq. 34-8 and note that $n_1 = n_{air} = 1.00$, $n_2 = n$, $p = \infty$, and $i = 2r$:

$$\frac{1.00}{\infty} + \frac{n}{2r} = \frac{n-1}{r}.$$

We solve for the unknown index: $n = 2.00$.

(b) Now $i = r$ so Eq. 34-8 becomes

$$\frac{n}{r}=\frac{n-1}{r},$$

which is not valid unless $n\to\infty$ or $r\to\infty$. It is impossible to focus at the center of the sphere.

40. We use Eq. 34-8 (and Fig. 34-11(d) is useful), with n_1 = 1.6 and n_2 = 1 (using the rounded-off value for air):

$$\frac{1.6}{p}+\frac{1}{i}=\frac{1-1.6}{r}.$$

Using the sign convention for r stated in the paragraph following Eq. 34-8 (so that $r=-5.0$ cm), we obtain i = –2.4 cm for objects at p = 3.0 cm. Returning to Fig. 34-38 (and noting the location of the observer), we conclude that the tabletop seems 7.4 cm away.

41. (a) We use Eq. 34-10:

$$f=\left[(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\right]^{-1}=\left[(1.5-1)\left(\frac{1}{\infty}-\frac{1}{-20\text{ cm}}\right)\right]^{-1}=+40\text{ cm}.$$

(b) From Eq. 34-9,

$$i=\left(\frac{1}{f}-\frac{1}{p}\right)^{-1}=\left(\frac{1}{40\text{ cm}}-\frac{1}{40\text{ cm}}\right)^{-1}=\infty.$$

42. Combining Eq. 34-7 and Eq. 34-9, we have $m(p-f)=-f$. The graph in Fig. 34-39 indicates that m = 0.5 where p = 15 cm, so our expression yields f = –15 cm. Plugging this back into our expression and evaluating at p = 35 cm yields m = +0.30.

43. We solve Eq. 34-9 for the image distance:

$$i=\left(\frac{1}{f}-\frac{1}{p}\right)^{-1}=\frac{fp}{p-f}.$$

The height of the image is

$$h_i=mh_p=\left(\frac{i}{p}\right)h_p=\frac{fh_p}{p-f}=\frac{(75\text{ mm})(1.80\text{ m})}{27\text{ m}-0.075\text{ m}}=5.0\text{ mm}.$$

44. The singularity the graph (where the curve goes to $\pm\infty$) is at p = 30 cm, which implies (by Eq. 34-9) that f = 30 cm > 0 (converging type lens). For p = 100 cm, Eq. 34-9 leads to i = +43 cm.

45. Let the diameter of the Sun be d_s and that of the image be d_i. Then, Eq. 34-5 leads to

$$d_i = |m| d_s = \left(\frac{i}{p}\right) d_s \approx \left(\frac{f}{p}\right) d_s = \frac{(20.0\times10^{-2}\,\text{m})(2)(6.96\times10^{8}\,\text{m})}{1.50\times10^{11}\,\text{m}} = 1.86\times10^{-3}\,\text{m} = 1.86\text{ mm}.$$

46. Since the focal length is a constant for the whole graph, then $1/p + 1/i$ = constant. Consider the value of the graph at p = 20 cm; we estimate its value there to be –10 cm. Therefore, 1/20 + 1/(–10) = 1/70 + 1/i_{new}. Thus, i_{new} = –16 cm.

47. **THINK** Our lens is of double-convex type. We apply lens maker's equation to analyze the problem.

EXPRESS The lens maker's equation is given by Eq. 34-10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{2r_1}\right) = \frac{3(n-1)}{2r_1}.$$

ANALYZE (a) We solve for the smaller radius r_1:

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60\text{ mm})}{2} = 45\text{ mm}.$$

(b) The magnitude of the larger radius is $|r_2| = 2r_1 = 90$ mm.

LEARN An image of an object can be formed with a lens because it can bend the light rays, but the bending is possible only if the index of refraction of the lens is different from that of its surrounding medium.

48. Combining Eq. 34-7 and Eq. 34-9, we have $m(p - f) = -f$. The graph in Fig. 34-42 indicates that m = 2 where p = 5 cm, so our expression yields f = 10 cm. Plugging this back into our expression and evaluating at p = 14 cm yields m = –2.5.

49. **THINK** The image is formed on the screen, so the sum of the object distance and the image distance is equal to the distance between the slide and the screen.

EXPRESS Using Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$$

and noting that $p + i = d = 44$ cm, we obtain $p^2 - dp + df = 0$.

ANALYZE The focal length is $f = 11$ cm. Solving the quadratic equation, we find the solution to p to be

$$p = \frac{1}{2}\left(d \pm \sqrt{d^2 - 4df}\right) = 22\text{ cm} \pm \frac{1}{2}\sqrt{(44\text{ cm})^2 - 4(44\text{ cm})(11\text{ cm})} = 22\text{ cm}.$$

LEARN Since $p > f$, the object is outside the focal length. The image distance is $i = d - p = 44 - 22 = 22$ cm.

50. We recall that for a converging (C) lens, the focal length value should be positive ($f = +4$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = +5.3$ cm.

(b) Equation 34-7 gives $m = -i/p = -0.33$.

(c) The fact that the image distance i is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the opposite side of the object (see Fig. 34-16(a)).

51. We recall that for a converging (C) lens, the focal length value should be positive ($f = +16$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = -48$ cm.

(b) Equation 34-7 gives $m = -i/p = +4.0$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(b)).

52. We recall that for a converging (C) lens, the focal length value should be positive ($f = +35$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = -88$ cm.

(b) Equation 34-7 give $m = -i/p = +3.5$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(b)).

53. **THINK** For a diverging (D) lens, the focal length value is negative.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have $f = -12$ cm and $p = +8.0$ cm.

(a) The image distance is $i = \dfrac{pf}{p-f} = \dfrac{(8.0\text{ cm})(-12\text{ cm})}{8.0\text{ cm} - (-12)\text{ cm}} = -4.8$ cm.

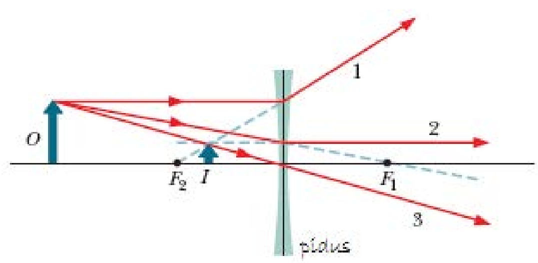
(b) The magnification is $m = -i/p = -(-4.8\text{ cm})/(8.0\text{ cm}) = +0.60$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.



54. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -6$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = -3.8$ cm.

(b) Equation 34-7 gives $m = -i/p = +0.38$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(c)).

55. **THINK** For a diverging (D) lens, the value of the focal length is negative.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m=-i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have f = –14 cm and p = +22.0 cm.

(a) The image distance is $i=\frac{pf}{p-f}=\frac{(22\text{ cm})(-14\text{ cm})}{22\text{ cm}-(-14)\text{ cm}}=-8.6\text{ cm}.$

(b) The magnification is $m=-i/p=-(-8.6\text{ cm})/(22\text{ cm})=+0.39$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.

56. We recall that for a diverging (D) lens, the focal length value should be negative (f = –31 cm).

(a) Equation 34-9 gives $i = pf/(p-f) = -8.7$ cm.

(b) Equation 34-7 gives $m = -i/p = +0.72$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(c)).

57. **THINK** For a converging (C) lens, the focal length value is positive.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m=-i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have $f = +20$ cm and $p = +45.0$ cm.

(a) The image distance is $i=\dfrac{pf}{p-f}=\dfrac{(45\text{ cm})(20\text{ cm})}{45\text{ cm}-20\text{ cm}}=+36\text{ cm}.$

(b) The magnification is $m = -i/p=-(+36\text{ cm})/(45\text{ cm})=-0.80.$

(c) The fact that the image distance is a positive value means the image is real (R).

(d) A negative value of magnification means the image is inverted (I).

(e) The image is on the opposite side of the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(a). The lens is converging, forming a real, inverted image on the opposite side of the object.

58. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -63$ cm.

(b) Equation 34-7 gives $m = -i/p = +2.2$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

59. **THINK** Since r_1 is positive and r_2 is negative, our lens is of double-convex type. We apply lens maker's equation to analyze the problem.

EXPRESS The lens maker's equation is given by Eq. 34-10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

ANALYZE For this lens, we have $r_1 = +30$ cm, $r_2 = -42$ cm, n = 1.55 and $p = +75$ cm.

(a) The focal length is

$$f = \frac{r_1 r_2}{(n-1)(r_2 - r_1)} = \frac{(+30\text{ cm})(-42\text{ cm})}{(1.55-1)(-42\text{ cm} - 30\text{ cm})} = +31.8\text{ cm}.$$

Thus, the image distance is $i = \dfrac{pf}{p-f} = \dfrac{(75\text{ cm})(31.8\text{ cm})}{75\text{ cm} - 31.8\text{ cm}} = +55\text{ cm}.$

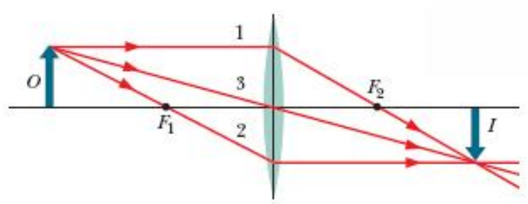
(b) Eq. 34-7 give $m = -i/p = -(55\text{ cm})/(75\text{ cm}) = -0.74$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(a). The lens is converging, forming a real, inverted image on the opposite side of the object.



60. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -26$ cm.

(b) Equation 34-7 gives $m = -i/p = +4.3$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

61. (a) Combining Eq. 34-9 and Eq. 34-10 gives i = –18 cm.

(b) Equation 34-7 gives $m = -i/p = +0.76$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

62. (a) Equation 34-10 yields

$$f = \frac{r_1 r_2}{(n-1)(r_2 - r_1)} = +30 \text{ cm}$$

Since $f > 0$, this must be a converging ("C") lens. From Eq. 34-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{30 \text{ cm}} - \frac{1}{10 \text{ cm}}} = -15 \text{cm}.$$

(b) Equation 34-6 yields $m = -i/p$ = –(–15 cm)/(10 cm) = +1.5.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(b) of the textbook.

63. (a) Combining Eq. 34-9 and Eq. 34-10 gives i = –30 cm.

(b) Equation 34-7 gives $m = -i/p = +0.86$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

64. (a) Equation 34-10 yields

$$f = \frac{1}{n-1}\left(1/r_1 - 1/r_2\right)^{-1} = -120 \text{ cm}.$$

Since $f < 0$, this must be a diverging ("D") lens. From Eq. 34-9, we obtain

$$i=\frac{1}{\frac{1}{f}-\frac{1}{p}}=\frac{1}{\frac{1}{-120\text{ cm}}-\frac{1}{10\text{ cm}}}=-9.2\text{ cm}.$$

(b) Equation 34-6 yields $m = -i/p = -(-9.2\text{ cm})/(10\text{ cm}) = +0.92$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(c) of the textbook.

65. (a) Equation 34-10 yields $f=\frac{1}{n-1}(1/r_1-1/r_2)^{-1}=-30$ cm. Since $f < 0$, this must be a diverging ("D") lens. From Eq. 34-9, we obtain

$$i=\frac{1}{\frac{1}{f}-\frac{1}{p}}=\frac{1}{\frac{1}{-30\text{ cm}}-\frac{1}{10\text{ cm}}}=-7.5\text{ cm}.$$

(b) Equation 34-6 yields $m = -i/p = -(-7.5\text{ cm})/(10\text{ cm}) = +0.75$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(c) of the textbook.

66. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -9.7$ cm.

(b) Equation 34-7 gives $m = -i/p = +0.54$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

67. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = +84$ cm.

(b) Equation 34-7 gives $m = -i/p = -1.4$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object.

68. (a) A convex (converging) lens, since a real image is formed.

(b) Since $i = d - p$ and $i/p = 1/2$,

$$p = \frac{2d}{3} = \frac{2(40.0\text{ cm})}{3} = 26.7\text{ cm}.$$

(c) The focal length is

$$f = \left(\frac{1}{i} + \frac{1}{p}\right)^{-1} = \left(\frac{1}{d/3} + \frac{1}{2d/3}\right)^{-1} = \frac{2d}{9} = \frac{2(40.0\text{ cm})}{9} = 8.89\text{ cm}.$$

69. (a) Since $f > 0$, this is a converging lens ("C").

(d) Equation 34-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10\text{ cm}} - \frac{1}{5.0\text{ cm}}} = -10\text{ cm}.$$

(e) From Eq. 34-6, $m = -(-10\text{ cm})/(5.0\text{ cm}) = +2.0$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

70. (a) The fact that $m < 1$ and that the image is upright (not inverted: NI) means the lens is of the diverging type (D) (it may help to look at Fig. 34-16 to illustrate this).

(b) A diverging lens implies that $f = -20$ cm, with a minus sign.

(d) Equation 34-9 gives $i = -5.7$ cm.

(e) Equation 34-7 gives $m = -i/p = +0.71$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(h) The image is on the same side as the object.

71. (a) Eq. 34-7 yields $i = –mp = –(0.25)(16 \text{ cm}) = –4.0$ cm. Equation 34-9 gives $f = –5.3$ cm, which implies the lens is of the diverging type (D).

(b) From (a), we have $f = –5.3$ cm.

(d) Similarly, $i = –4.0$ cm.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

72. (a) Equation 34-7 readily yields $i = +4.0$ cm. Then Eq. 34-9 gives $f = +3.2$ cm, which implies the lens is of the converging type (C).

(b) From (a), we have $f = +3.2$ cm.

(d) Similarly, $i = +4.0$ cm.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the opposite side of the object.

73. (a) Using Eq. 34-6 (which implies the image is inverted) and the given value of p, we find $i = –mp = +5.0$ cm; it is a real image. Equation 34-9 then yields the focal length: $f = +3.3$ cm. Therefore, the lens is of the converging ("C") type.

(b) From (a), we have $f = +3.3$ cm.

(d) Similarly, $i = –mp = +5.0$ cm.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object. The ray diagram is similar to Fig. 34-16(a) of the textbook.

74. (b) Since this is a converging lens ("C") then $f > 0$, so we should put a plus sign in front of the "10" value given for the focal length.

(d) Equation 34-9 gives

$$i=\frac{1}{\frac{1}{f}-\frac{1}{p}}=\frac{1}{\frac{1}{10\text{ cm}}-\frac{1}{20\text{ cm}}}=+20\,\text{cm}.$$

(e) From Eq. 34-6, $m = -20/20 = -1.0$.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object.

75. **THINK** Since the image is on the same side as the object, it must be a virtual image.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m=-i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE (a) Since the image is virtual (on the same side as the object), the image distance i is negative. By substituting $i=fp/(p-f)$ into $m=-i/p$, we obtain

$$m=-\frac{i}{p}=-\frac{f}{p-f}.$$

The fact that the magnification is less than 1.0 implies that f must be negative. This means that the lens is of the diverging ("D") type.

(b) Thus, the focal length is $f=-10$ cm.

(d) The image distance is $i=\frac{pf}{p-f}=\frac{(5.0\text{ cm})(-10\text{ cm})}{5.0\text{ cm}-(-10\text{ cm})}=-3.3$ cm.

(e) The magnification is $m=-i/p=-(-3.3\text{ cm})/(5.0\text{ cm})=+0.67$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.

76. (a) We are told the magnification is positive and greater than 1. Scanning the single-lens-image figures in the textbook (Figs. 34-16, 34-17, and 34-19), we see that such a magnification (which implies an upright image larger than the object) is only possible if the lens is of the converging ("C") type (and if $p < f$).

(b) We should put a plus sign in front of the "10" value given for the focal length.

(d) Equation 34-9 gives $i = \dfrac{1}{\frac{1}{f} - \frac{1}{p}} = \dfrac{1}{\frac{1}{10\text{ cm}} - \frac{1}{5.0\text{ cm}}} = -10\,\text{cm}.$

(e) $m = -i/p = +2.0$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

77. **THINK** A positive value for the magnification m means that the image is upright (not inverted). In addition, $m > 1$ indicates that the image is enlarged.

EXPRESS The object distance p, the image distance i, and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE (a) Combining Eqs. 34-7 and 34-9, we find the focal length to be

$$f = \frac{p}{1 - 1/m} = \frac{16\text{ cm}}{1 - 1/1.25} = 80\text{ cm}.$$

Since the value of f is positive, the lens is of the converging type (C).

(b) From (a), we have $f = +80$ cm.

(d) The image distance is $i = -mp = -(1.25)(16\text{ cm}) = -20$ cm.

(e) The magnification is $m = +1.25$, as given.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image it is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(b). The lens is converging. With the object placed inside the focal point ($p < f$), we have a virtual image with the same orientation as the object, and on the same side as the object.

78. (a) We are told the absolute value of the magnification is 0.5 and that the image was upright (NI). Thus, $m = +0.5$. Using Eq. 34-6 and the given value of p, we find $i = -5.0$ cm; it is a virtual image. Equation 34-9 then yields the focal length: $f = -10$ cm. Therefore, the lens is of the diverging ("D") type.

(b) From (a), we have $f = -10$ cm.

(d) Similarly, $i = -5.0$ cm.

(e) $m = +0.5$, with a plus sign.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(h) The image is on the same side as the object.

79. (a) The fact that $m > 1$ means the lens is of the converging type (C) (it may help to look at Fig. 34-16 to illustrate this).

(b) A converging lens implies $f = +20$ cm, with a plus sign.

(d) Equation 34-9 then gives $i = -13$ cm.

(e) Equation 34-7 gives $m = -i / p = +1.7$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

80. (a) The image from lens 1 (which has $f_1 = +15$ cm) is at $i_1 = -30$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = +8$ cm) with $p_2 = d - i_1 = 40$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +10$ cm.

(b) Equation 34-11 yields $M = m_1 m_2 = (-i_1/p_1)(-i_2/p_2) = i_1 i_2/p_1 p_2 = -0.75$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (relative to lens 2).

81. (a) The image from lens 1 (which has f_1 = +8 cm) is at i_1 = 24 cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has f_2 = +6 cm) with $p_2 = d - i_1$ = 8 cm. Then Eq. 34-9 (applied to lens 2) yields i_2 = +24 cm.

(b) Equation 34-11 yields $M = m_1 m_2 = (-i_1/p_1)(-i_2/p_2) = i_1 i_2/p_1 p_2 = +6.0$.

(c)The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the side opposite from the object (relative to lens 2).

82. (a) The image from lens 1 (which has f_1 = –6 cm) is at i_1 = –3.4 cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has f_2 = +6 cm) with $p_2 = d - i_1$ = 15.4 cm. Then Eq. 34-9 (applied to lens 2) yields i_2 = +9.8 cm.

(b) Equation 34-11 yields $M = -0.27$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (relative to lens 2).

83. **THINK** In a system with two lenses, the image formed by lens 1 serves the "object" for lens 2.

EXPRESS To analyze two-lens systems, we first ignore lens 2, and apply the standard procedure used for a single-lens system. The object distance p_1, the image distance i_1, and the focal length f_1 are related by:

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{i_1}.$$

Next, we ignore the lens 1 but treat the image formed by lens 1 as the object for lens 2. The object distance p_2 is the distance between lens 2 and the location of the first image. The location of the final image, i_2, is obtained by solving

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{i_2}$$

where f_2 is the focal length of lens 2.

ANALYZE (a) Since lens 1 is converging, f_1 = +9 cm, and we find the image distance to be

$$i_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(20\text{ cm})(9\text{ cm})}{20\text{ cm} - 9\text{ cm}} = 16.4\text{ cm}.$$

This serves as an "object" for lens 2 (which has f_2 = +5 cm) with an object distance given by $p_2 = d - i_1 = -8.4$ cm. The negative sign means that the "object" is behind lens 2. Solving the lens equation, we obtain

$$i_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-8.4\text{ cm})(5.0\text{ cm})}{-8.4\text{ cm} - 5.0\text{ cm}} = 3.13\text{ cm}.$$

(b) Te overall magnification is $M = m_1 m_2 = (-i_1/p_1)(-i_2/p_2) = i_1 i_2 / p_1 p_2 = -0.31$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image it is on the side opposite from the object (relative to lens 2).

LEARN Since this result involves a negative value for p_2 (and perhaps other "non-intuitive" features), we offer a few words of explanation: lens 1 is converging the rays towards an image (that never gets a chance to form due to the intervening presence of lens 2) that would be real and inverted (and 8.4 cm beyond lens 2's location). Lens 2, in a sense, just causes these rays to converge a little more rapidly, and causes the image to form a little closer (to the lens system) than if lens 2 were not present.

84. (a) The image from lens 1 (which has f_1 = +12 cm) is at i_1 = +60 cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has f_2 = +10 cm) with $p_2 = d - i_1 = 7$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -23$ cm.

(b) Equation 34-11 yields $M = m_1 m_2 = (-i_1/p_1)(-i_2/p_2) = i_1 i_2 / p_1 p_2 = -13$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the same side as the object (relative to lens 2).

85. (a) The image from lens 1 (which has f_1 = +6 cm) is at i_1 = –12 cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has f_2 = –6 cm) with $p_2 = d - i_1$ = 20 cm. Then Eq. 34-9 (applied to lens 2) yields i_2 = –4.6 cm.

(b) Equation 34-11 yields M = +0.69.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the same side as the object (relative to lens 2).

86. (a) The image from lens 1 (which has f_1 = +8 cm) is at i_1 = +24 cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has f_2 = –8 cm) with $p_2 = d - i_1$ = 6 cm. Then Eq. 34-9 (applied to lens 2) yields i_2 = –3.4 cm.

(b) Equation 34-11 yields M = –1.1.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the same side as the object (relative to lens 2).

87. (a) The image from lens 1 (which has f_1 = –12 cm) is at i_1 = –7.5 cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has f_2 = –8 cm) with

$$p_2 = d - i_1 = 17.5 \text{ cm}.$$

Then Eq. 34-9 (applied to lens 2) yields i_2 = –5.5 cm.

(b) Equation 34-11 yields M = +0.12.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the same side as the object (relative to lens 2).

88. The minimum diameter of the eyepiece is given by

$$d_{\text{ey}} = \frac{d_{\text{ob}}}{m_\theta} = \frac{75 \text{ mm}}{36} = 2.1 \text{ mm}.$$

89. **THINK** The compound microscope shown in Fig. 34-20 consists of an objective and an eyepiece. It's used for viewing small objects that are very close to the objective.

EXPRESS Let f_{ob} be the focal length of the objective, and f_{ey} be the focal length of the eyepiece. The distance between the two lenses is

$$L = s + f_{ob} + f_{ey},$$

where s is the tube length. The magnification of the objective is

$$m = -\frac{i}{p} = -\frac{s}{f_{ob}}$$

and the angular magnification produced by the eyepiece is $m_\theta = (25 \text{ cm}) / f_{ey}$.

ANALYZE (a) The tube length is

$$s = L - f_{ob} - f_{ey} = 25.0 \text{ cm} - 4.00 \text{ cm} - 8.00 \text{ cm} = 13.0 \text{ cm}.$$

(b) We solve $(1/p) + (1/i) = (1/f_{ob})$ for p. The image distance is

$$i = f_{ob} + s = 4.00 \text{ cm} + 13.0 \text{ cm} = 17.0 \text{ cm},$$

so

$$p = \frac{i f_{ob}}{i - f_{ob}} = \frac{(17.0 \text{ cm})(4.00 \text{ cm})}{17.0 \text{ cm} - 4.00 \text{ cm}} = 5.23 \text{ cm}.$$

(c) The magnification of the objective is $m = -\frac{i}{p} = -\frac{17.0 \text{ cm}}{5.23 \text{ cm}} = -3.25.$

(d) The angular magnification of the eyepiece is $m_\theta = \frac{25 \text{ cm}}{f_{ey}} = \frac{25 \text{ cm}}{8.00 \text{ cm}} = 3.13.$

(e) The overall magnification of the microscope is

$$M = m m_\theta = (-3.25)(3.13) = -10.2.$$

LEARN The objective produces a real image I of the object inside the focal point of the eyepiece ($i > f_{ey}$). Image I then serves as the object for the eyepiece, which produces a virtual image I' seen by the observer.

90. (a) Now, the lens-film distance is $i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{5.0 \text{ cm}} - \frac{1}{100 \text{ cm}}\right)^{-1} = 5.3 \text{ cm}.$

(b) The change in the lens-film distance is 5.3 cm – 5.0 cm = 0.30 cm.

91. **THINK** This problem is about human eyes. We model the cornea and eye lens as a single effective thin lens, with image formed at the retina.

EXPRESS When the eye is relaxed, its lens focuses far-away objects on the retina, a distance i behind the lens. We set $p = \infty$ in the thin lens equation to obtain $1/i = 1/f$, where f is the focal length of the relaxed effective lens. Thus, $i = f = 2.50$ cm. When the eye focuses on closer objects, the image distance i remains the same but the object distance and focal length change.

ANALYZE (a) If p is the new object distance and f' is the new focal length, then

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f'}.$$

We substitute $i = f$ and solve for f': $f' = \dfrac{pf}{f+p} = \dfrac{(40.0\text{ cm})(2.50\text{ cm})}{40.0\text{ cm}+2.50\text{ cm}} = 2.35\text{ cm}.$

(b) Consider the lens maker's equation

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where r_1 and r_2 are the radii of curvature of the two surfaces of the lens and n is the index of refraction of the lens material. For the lens pictured in Fig. 34-46, r_1 and r_2 have about the same magnitude, r_1 is positive, and r_2 is negative. Since the focal length decreases, the combination $(1/r_1) - (1/r_2)$ must increase. This can be accomplished by decreasing the magnitudes of both radii.

LEARN When focusing on an object near the eye, the lens bulges a bit (smaller radius of curvature), and its focal length decreases.

92. We refer to Fig. 34-20. For the intermediate image, $p = 10$ mm and

$$i = (f_{ob} + s + f_{ey}) - f_{ey} = 300\text{ m} - 50\text{ mm} = 250\text{ mm},$$

so

$$\frac{1}{f_{ob}}=\frac{1}{i}+\frac{1}{p}=\frac{1}{250\text{ mm}}+\frac{1}{10\text{ mm}} \Rightarrow f_{ob} = 9.62\text{ mm},$$

and

$$s = (f_{ob} + s + f_{ey}) - f_{ob} - f_{ey} = 300\text{ mm} - 9.62\text{ mm} - 50\text{ mm} = 240\text{ mm}.$$

Then from Eq. 34-14,

$$M = -\frac{s}{f_{ob}}\frac{25\text{ cm}}{f_{ey}} = -\left(\frac{240\text{ mm}}{9.62\text{ mm}}\right)\left(\frac{150\text{ mm}}{50\text{ mm}}\right) = -125.$$

93. (a) Without the magnifier, $\theta = h/P_n$ (see Fig. 34-19). With the magnifier, letting

$$i = -|i| = -P_n,$$

we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n}.$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25\,\text{cm}}{f}.$$

With f = 10 cm, $m_\theta = 1 + \dfrac{25\text{ cm}}{10\text{ cm}} = 3.5$.

(b) In the case where the image appears at infinity, let $i = -|i| \to -\infty$, so that $1/p + 1/i = 1/p = 1/f$, we have

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25\,\text{cm}}{f}.$$

With f = 10 cm, $m_\theta = \dfrac{25\text{ cm}}{10\text{ cm}} = 2.5.$

94. By Eq. 34-9, $1/i + 1/p$ is equal to constant ($1/f$). Thus,

$$1/(-10) + 1/(15) = 1/i_{\text{new}} + 1/(70).$$

This leads to $i_{\text{new}} = -21$ cm.

95. A converging lens has a positive-valued focal length, so $f_1 = +8$ cm, $f_2 = +6$ cm, and $f_3 = +6$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = 24$ cm and $i_2 = -12$ cm. Our final results are as follows:

(a) $i_3 = +8.6$ cm.

(b) $m = +2.6$.

(c) The image is real (R).

(d) The image is not inverted (NI).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

96. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = -6.0$ cm, $f_2 = +6.0$ cm, and $f_3 = +4.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -2.4$ cm and $i_2 = 12$ cm. Our final results are as follows:

(a) $i_3 = -4.0$ cm.

(b) $m = -1.2$.

(c) The image is virtual (V).

(d) The image is inverted (I).

(e) It is on the same side as the object (relative to lens 3) as expected for a virtual image.

97. A converging lens has a positive-valued focal length, so $f_1 = +6.0$ cm, $f_2 = +3.0$ cm, and $f_3 = +3.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = 9.0$ cm and $i_2 = 6.0$ cm. Our final results are as follows:

(a) $i_3 = +7.5$ cm.

(b) $m = -0.75$.

(c) The image is real (R).

(d) The image is inverted (I).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

98. A converging lens has a positive-valued focal length, so $f_1 = +6.0$ cm, $f_2 = +6.0$ cm, and $f_3 = +5.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -3.0$ cm and $i_2 = 9.0$ cm. Our final results are as follows:

(a) $i_3 = +10$ cm.

(b) $m = +0.75$.

(c) The image is real (R).

(d) The image is not inverted (NI).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

99. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = -8.0$ cm, $f_2 = -16$ cm, and $f_3 = +8.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -4.0$ cm and $i_2 = -6.86$ cm. Our final results are as follows:

(a) $i_3 = +24.2$ cm.

(b) $m = -0.58$.

(c) The image is real (R).

(d) The image is inverted (I).

(e) It is on the opposite side of lens 3 from the object (as expected for a real image).

100. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = +6.0$ cm, $f_2 = -4.0$ cm, and $f_3 = -12$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1, etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -12$ cm and $i_2 = -3.33$ cm. Our final results are as follows:

(a) $i_3 = -5.15 \text{ cm} \approx -5.2 \text{ cm}$.

(b) $m = +0.285 \approx +0.29$.

(c) The image is virtual (V).

(d) The image is not inverted (NI).

(e) It is on the same side as the object (relative to lens 3) as expected for a virtual image.

101. **THINK** In this problem we convert the Gaussian form of the thin-lens formula to the Newtonian form.

EXPRESS For a thin lens, the Gaussian form of the thin-lens formula gives (1/*p*) + (1/*i*) = (1/*f*), where *p* is the object distance, *i* is the image distance, and *f* is the focal length. To convert the formula to the Newtonian form, let $p = f + x$, where *x* is positive if the object is outside the focal point and negative if it is inside. In addition, let $i = f + x'$, where x' is positive if the image is outside the focal point and negative if it is inside.

ANALYZE From the Gaussian form, we solve for *I* and obtain:

$$i = \frac{fp}{p-f}.$$

Substituting $p = f + x$ gives

$$i = \frac{f(f+x)}{x}.$$

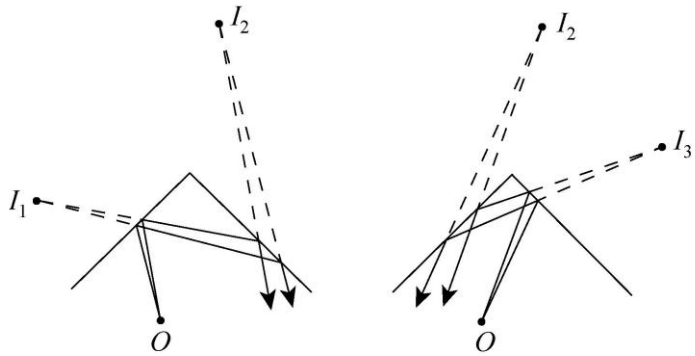
With $i = f + x'$, we have

$$x' = i - f = \frac{f(f+x)}{x} - f = \frac{f^2}{x}$$

which leads to $xx' = f^2$.

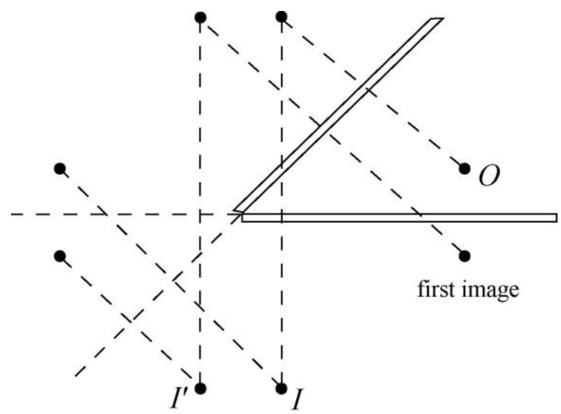
LEARN The Newtonain form is equivalent to the Gaussian form, and it provides another convenient way to analyze problems involving thin lenses.

102. (a) There are three images. Two are formed by single reflections from each of the mirrors and the third is formed by successive reflections from both mirrors. The positions of the images are shown on the two diagrams that follow. The diagram on the left shows the image I_1, formed by reflections from the left-hand mirror. It is the same distance behind the mirror as the object *O* is in front, and lies on the line perpendicular to the mirror and through the object. Image I_2 is formed by light that is reflected from both mirrors.



We may consider I_2 to be the image of I_1 formed by the right-hand mirror, extended. I_2 is the same distance behind the line of the right-hand mirror as I_1 is in front, and it is on the line that is perpendicular to the line of the mirror. The diagram on the right shows image I_3, formed by reflections from the right-hand mirror. It is the same distance behind the mirror as the object is in front, and lies on the line perpendicular to the mirror and through the object. As the diagram shows, light that is first reflected from the right-hand mirror and then from the left-hand mirror forms an image at I_2.

(b) For $\theta = 45°$, we have two images in the second mirror caused by the object and its "first" image, and from these one can construct two new images I and I' behind the first mirror plane. Extending the second mirror plane, we can find two further images of I and I' that are on equal sides of the extension of the first mirror plane. This circumstance implies there are no further images, since these final images are each other's "twins." We show this construction in the figure below. Summarizing, we find 1 + 2 + 2 + 2 = 7 images in this case.



(c) For $\theta = 60°$, we have two images in the second mirror caused by the object and its "first" image, and from these one can construct two new images I and I' behind the first mirror plane. The images I and I' are each other's "twins" in the sense that they are each other's reflections about the extension of the second mirror plane; there are no further images. Summarizing, we find 1 + 2 + 2 = 5 images in this case.

For $\theta = 120°$, we have two images I'_1 and I_2 behind the extension of the second mirror plane, caused by the object and its "first" image (which we refer to here as I_1). No further images can be constructed from I'_1 and I_2, since the method indicated above would place any further possibilities in front of the mirrors. This construction has the disadvantage of deemphasizing the actual ray-tracing, and thus any dependence on where the observer of these images is actually placing his or her eyes. It turns out in this case that the number of images that can be seen ranges from 1 to 3, depending on the locations of both the object and the observer.

(d) Thus, the smallest number of images that can be seen is 1. For example, if the observer's eye is collinear with I_1 and I'_1, then the observer can only see one image (I_1 and not the one behind it). Note that an observer who stands close to the second mirror would probably be able to see two images, I_1 and I_2.

(e) Similarly, the largest number would be 3. This happens if the observer moves further back from the vertex of the two mirrors. He or she should also be able to see the third image, I'_1, which is essentially the "twin" image formed from I_1 relative to the extension of the second mirror plane.

103. **THINK** Two lenses in contact can be treated as one single lens with an effective focal length.

EXPRESS We place an object far away from the composite lens and find the image distance i. Since the image is at a focal point, $i = f$, where f equals the effective focal length of the composite. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens, $(1/p_1) + (1/i_1) = (1/f_1)$, where f_1 is the focal length of this lens and i_1 is the image distance for the image it forms. Since $p_1 = \infty$, $i_1 = f_1$. The thin lens equation, applied to the second lens, is $(1/p_2) + (1/i_2) = (1/f_2)$, where p_2 is the object distance, i_2 is the image distance, and f_2 is the focal length. If the thickness of the lenses can be ignored, the object distance for the second lens is $p_2 = -i_1$. The negative sign must be used since the image formed by the first lens is beyond the second lens if i_1 is positive. This means the object for the second lens is virtual and the object distance is negative. If i_1 is negative, the image formed by the first lens is in front of the second lens and p_2 is positive.

ANALYZE In the thin lens equation, we replace p_2 with $-f_1$ and i_2 with f to obtain

$$-\frac{1}{f_1}+\frac{1}{f}=\frac{1}{f_2}$$

or

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{f_1+f_2}{f_1 f_2}.$$

Thus, the effective focal length of the system is $f=\dfrac{f_1 f_2}{f_1+f_2}$.

LEARN The reciprocal of the focal length, $1/f$, is known as the power of the lens, a quantity used by the optometrists to specify the strength of eyeglasses. From the derivation above, we see that when two lenses are in contact, the power of the effective lens is the sum of the two powers.

104. (a) In the closest mirror M_1, the "first" image I_1 is 10 cm behind M_1 and therefore 20 cm from the object O. This is the smallest distance between the object and an image of the object.

(b) There are images from both O and I_1 in the more distant mirror, M_2: an image I_2 located at 30 cm behind M_2. Since O is 30 cm in front of it, I_2 is 60 cm from O. This is the second smallest distance between the object and an image of the object.

(c) There is also an image I_3 that is 50 cm behind M_2 (since I_1 is 50 cm in front of it). Thus, I_3 is 80 cm from O. In addition, we have another image I_4 that is 70 cm behind M_1 (since I_2 is 70 cm in front of it). The distance from I_4 to O for is 80 cm.

(d) Returning to the closer mirror M_1, there is an image I_5 that is 90 cm behind the mirror (since I_3 is 90 cm in front of it). The distances (measured from O) for I_5 is 100 cm = 1.0 m.

105. (a) The "object" for the mirror that results in that box image is equally in front of the mirror (4 cm). This object is actually the first image formed by the system (produced by the first transmission through the lens); in those terms, it corresponds to $i_1 = 10 - 4 = 6$ cm. Thus, with $f_1 = 2$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow p_1 = 3.00\text{ cm}.$$

(b) The previously mentioned box image (4 cm behind the mirror) serves as an "object" (at $p_3 = 14$ cm) for the return trip of light through the lens ($f_3 = f_1 = 2$ cm). This time, Eq. 34-9 leads to

$$\frac{1}{p_3} + \frac{1}{i_3} = \frac{1}{f_3} \Rightarrow i_3 = 2.33\text{ cm}.$$

106. (a) First, the lens forms a real image of the object located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1$$

to the right of the lens, or at

$$p_2 = 2(f_1 + f_2) - 2f_1 = 2f_2$$

in front of the mirror. The subsequent image formed by the mirror is located at a distance

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{f_2} - \frac{1}{2f_2}\right)^{-1} = 2f_2$$

to the left of the mirror, or at

$$p'_1 = 2(f_1 + f_2) - 2f_2 = 2f_1$$

to the right of the lens. The final image formed by the lens is at a distance i'_1 to the left of the lens, where

$$i'_1 = \left(\frac{1}{f_1} - \frac{1}{p'_1}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1.$$

This turns out to be the same as the location of the original object.

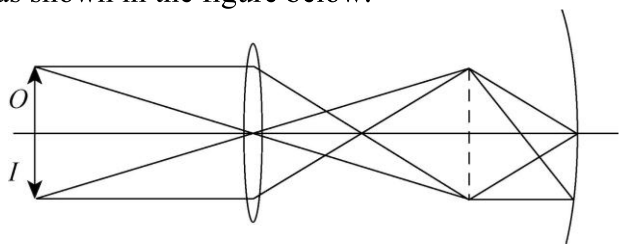
(b) The lateral magnification is

$$m = \left(-\frac{i_1}{p_1}\right)\left(-\frac{i_2}{p_2}\right)\left(-\frac{i'_1}{p'_1}\right) = \left(-\frac{2f_1}{2f_1}\right)\left(-\frac{2f_2}{2f_2}\right)\left(-\frac{2f_1}{2f_1}\right) = -1.0.$$

(c) The final image is real (R).

(d) It is at a distance i'_1 to the left of the lens,

(e) and inverted (I), as shown in the figure below.



107. **THINK** The nature of the lenses, whether converging or diverging, can be determined from the magnification and orientation of the images they produce.

EXPRESS By examining the ray diagrams shown in Fig. 34-16(a) – (c), we see that only a converging lens can produce an enlarged, upright image, while the image produced by a diverging lens is always virtual, reduced in size, and not inverted.

ANALYZE (a) In this case $m > +1$ and we know that lens 1 is converging (producing a virtual image), so that our result for focal length should be positive. Since $|P + i_1| = 20$ cm and $i_1 = -2p_1$, we find $p_1 = 20$ cm and $i_1 = -40$ cm. Substituting these into Eq. 34-9,

$$\frac{1}{p_1}+\frac{1}{i_1}=\frac{1}{f_1}$$

leads to

$$f_1=\frac{p_1 i_1}{p_1+i_1}=\frac{(20\text{ cm})(-40\text{ cm})}{20\text{ cm}+(-40\text{ cm})}=+40\text{ cm},$$

which is positive as we expected.

(b) The object distance is $p_1 = 20$ cm, as shown in part (a).

(c) In this case $0 < m < 1$ and we know that lens 2 is diverging (producing a virtual image), so that our result for focal length should be negative. Since $|p + i_2| = 20$ cm and $i_2 = -p_2/2$, we find $p_2 = 40$ cm and $i_2 = -20$ cm. Substituting these into Eq. 34-9 leads to

$$f_2=\frac{p_2 i_2}{p_2+i_2}=\frac{(40\text{ cm})(-20\text{ cm})}{40\text{ cm}+(-20\text{ cm})}=-40\text{ cm},$$

which is negative as we expected.

(d) The object distance is $p_2 = 40$ cm, as shown in part (c).

LEARN The ray diagram for lens 1 is similar to the one shown in Fig. 34-16(b). The lens is converging. With the fly inside the focal point ($p_1 < f_1$), we have a virtual image with the same orientation, and on the same side as the object. On the other hand, the ray diagram for lens 2 is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation but smaller in size as the object, and on the same side as the object.

108. We use Eq. 34-10, with the conventions for signs discussed in the text.

(a) For lens 1, the biconvex (or double convex) case, we have

$$f = \left[(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]^{-1} = \left[(1.5-1)\left(\frac{1}{40\text{ cm}} - \frac{1}{-40\text{ cm}}\right)\right]^{-1} = 40\text{ cm}.$$

(b) Since $f > 0$ the lens forms a real image of the Sun.

(c) For lens 2, of the planar convex type, we find

$$f = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{-40\text{cm}}\right)\right]^{-1} = 80\text{cm}.$$

(d) The image formed is real (since $f > 0$).

(e) Now for lens 3, of the meniscus convex type, we have

$$f = \left[(1.5-1)\left(\frac{1}{40\text{cm}} - \frac{1}{60\text{cm}}\right)\right]^{-1} = 240\text{cm} = 2.4\text{ m}.$$

(f) The image formed is real (since $f > 0$).

(g) For lens 4, of the biconcave type, the focal length is

$$f = \left[(1.5-1)\left(\frac{1}{-40\text{cm}} - \frac{1}{40\text{cm}}\right)\right]^{-1} = -40\text{cm}.$$

(h) The image formed is virtual (since $f < 0$).

(i) For lens 5 (plane-concave), we have $f = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{40\text{cm}}\right)\right]^{-1} = -80\text{cm}.$

(j) The image formed is virtual (since $f < 0$).

(k) For lens 6 (meniscus concave), $f = \left[(1.5-1)\left(\frac{1}{60\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)\right]^{-1} = -240\,\text{cm} = -2.4$ m.

(l) The image formed is virtual (since $f < 0$).

109. (a) The first image is figured using Eq. 34-8, with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 8/5$.

$$\frac{1}{p} + \frac{8}{5i} = \frac{1.6-1}{r}$$

For a "flat lens" $r = \infty$, so we obtain

$$i = -\,8p/5 = -\,64/5$$

(with the unit cm understood) for that object at $p = 10$ cm. Relative to the second surface, this image is at a distance of 3 + 64/5 = 79/5. This serves as an object in order to find the final image, using Eq. 34-8 again (and $r = \infty$) but with $n_1 = 8/5$ and $n_2 = 4/3$.

$$\frac{8}{5p'} + \frac{4}{3i'} = 0$$

which produces (for $p' = 79/5$)

$$i' = -\,5p/6 = -\,79/6 \approx -\,13.2.$$

This means the observer appears 13.2 + 6.8 = 20 cm from the fish.

(b) It is straightforward to "reverse" the above reasoning, the result being that the final fish image is 7.0 cm to the right of the air-wall interface, and thus 15 cm from the observer.

110. Setting $n_{\text{air}} = 1$, $n_{\text{water}} = n$, and $p = r/2$ in Eq. 34-8 (and being careful with the sign convention for r in that equation), we obtain $i = -r/(1+n)$, or $|i| = r/(1+n)$. Then we use similar triangles (where h is the size of the fish and h' is that of the "virtual fish") to set up the ratio

$$\frac{h'}{r-|i|} = \frac{h}{r/2}\ .$$

Using our previous result for $|i|$, this gives $h'/h = 2(1 - 1/(1+n)) = 1.14$.

111. (a) Parallel rays are bent by positive-f lenses to their focal points F_1, and rays that come from the focal point positions F_2 in front of positive-f lenses are made to emerge parallel. The key, then, to this type of beam expander is to have the rear focal point F_1 of the first lens coincide with the front focal point F_2 of the second lens. Since the triangles that meet at the coincident focal point are similar (they share the same angle; they are

vertex angles), then $W_f/f_2 = W_i/f_1$ follows immediately. Substituting the values given, we have

$$W_f = \frac{f_2}{f_1}W_i = \frac{30.0\text{ cm}}{12.5\text{ cm}}(2.5\text{ mm}) = 6.0\text{ mm}.$$

(b) The area is proportional to W^2. Since intensity is defined as power P divided by area, we have

$$\frac{I_f}{I_i} = \frac{P/W_f^2}{P/W_i^2} = \frac{W_i^2}{W_f^2} = \frac{f_1^2}{f_2^2} \Rightarrow I_f = \left(\frac{f_1}{f_2}\right)^2 I_i = 1.6\text{ kW/m}^2.$$

(c) The previous argument can be adapted to the first lens in the expanding pair being of the diverging type, by ensuring that the front focal point of the first lens coincides with the front focal point of the second lens. The distance between the lenses in this case is

$$f_2 - |f_1| = 30.0\text{ cm} - 26.0\text{ cm} = 4.0\text{ cm}.$$

112. The water is medium 1, so $n_1 = n_w$, which we simply write as n. The air is medium 2, for which $n_2 \approx 1$. We refer to points where the light rays strike the water surface as A (on the left side of Fig. 34-56) and B (on the right side of the picture). The point midway between A and B (the center point in the picture) is C. The penny P is directly below C, and the location of the "apparent" or virtual penny is V. We note that the angle $\angle CVB$ (the same as $\angle CVA$) is equal to θ_2, and the angle $\angle CPB$ (the same as $\angle CPA$) is equal to θ_1. The triangles CVB and CPB share a common side, the horizontal distance from C to B (which we refer to as x). Therefore,

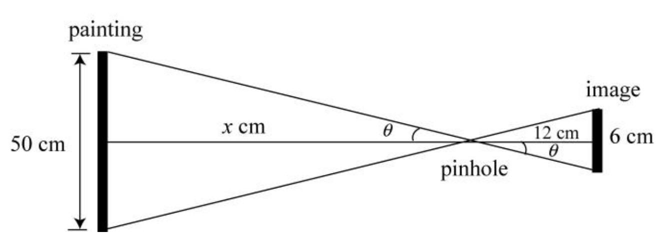
$$\tan\theta_2 = \frac{x}{d_a} \qquad \text{and} \qquad \tan\theta_1 = \frac{x}{d}.$$

Using the small angle approximation (so a ratio of tangents is nearly equal to a ratio of sines) and the law of refraction, we obtain

$$\frac{\tan\theta_2}{\tan\theta_1} \approx \frac{\sin\theta_2}{\sin\theta_1} \Rightarrow \frac{\frac{x}{d_a}}{\frac{x}{d}} \approx \frac{n_1}{n_2} \Rightarrow \frac{d}{d_a} \approx n$$

which yields the desired relation: $d_a = d/n$.

113. The top view of the arrangement is depicted in the figure below.

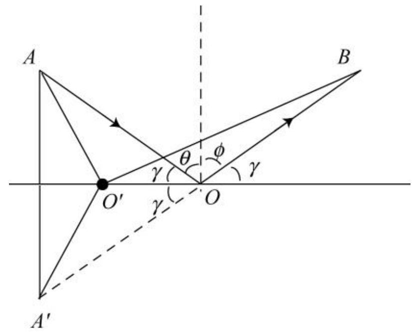


From the figure, we obtain

$$\tan\theta = \frac{25}{x} = \frac{3}{12}$$

which gives $x = 100$ cm.

114. Consider the ray diagram below.



Since $\theta + \gamma = \phi + \gamma = \pi/2$, we readily see that $\theta = \phi$, i.e., the angle of incidence is equal to the angle of reflection. To show that AOB is the shortest path, consider an incident ray AO' with a reflected ray $O'B$, where the angle of incidence is not equal to the angle of reflection. From the figure, we have

$$AO'B = AO' + O'B = A'O' + O'B > A'B = A'O + OB = AO + OB = AOB$$

The inequality comes from the fact that the sum of the two sides of a triangle is always greater than the hypotenuse.

115. We refer to Fig. 34-2 in the textbook. Consider the two light rays, r and r', which are closest to and on either side of the normal ray (the ray that reverses when it reflects). Each of these rays has an angle of incidence equal to θ when they reach the mirror. Consider that these two rays reach the top and bottom edges of the pupil after they have reflected. If ray r strikes the mirror at point A and ray r' strikes the mirror at B, the distance between A and B (call it x) is

$$x = 2d_o \tan\theta$$

where d_o is the distance from the mirror to the object. We can construct a right triangle starting with the image point of the object (a distance d_o behind the mirror; see I in Fig. 34-2). One side of the triangle follows the extended normal axis (which would reach from I to the middle of the pupil), and the hypotenuse is along the extension of ray r (after reflection). The distance from the pupil to I is $d_{ey} + d_o$, and the small angle in this triangle is again θ. Thus,

$$\tan\theta = \frac{R}{d_{ey} + d_o}$$

where R is the pupil radius (2.5 mm). Combining these relations, we find

$$x = 2d_o \frac{R}{d_{ey} + d_o} = 2(100\,\text{mm}) \frac{2.5\,\text{mm}}{300\,\text{mm} + 100\,\text{mm}}$$

which yields x = 1.67 mm. Now, x serves as the diameter of a circular area A on the mirror, in which all rays that reflect will reach the eye. Therefore,

$$A = \frac{1}{4}\pi x^2 = \frac{\pi}{4}(1.67\,\text{mm})^2 = 2.2\,\text{mm}^2\ .$$

116. For an object in front of a thin lens, the object distance p and the image distance i are related by $(1/p) + (1/i) = (1/f)$, where f is the focal length of the lens. For the situation described by the problem, all quantities are positive, so the distance x between the object and image is $x = p + i$. We substitute $i = x - p$ into the thin lens equation and solve for x:

$$x = \frac{p^2}{p - f}.$$

To find the minimum value of x, we set $dx/dp = 0$ and solve for p. Since

$$\frac{dx}{dp} = \frac{p(p - 2f)}{(p - f)^2},$$

the result is $p = 2f$. The minimum distance is

$$x_{\min} = \frac{p^2}{p - f} = \frac{(2f)^2}{2f - f} = 4f.$$

This is a minimum, rather than a maximum, since the image distance i becomes large without bound as the object approaches the focal point.

117. (a) If the object distance is x, then the image distance is $D - x$ and the thin lens equation becomes

$$\frac{1}{x} + \frac{1}{D - x} = \frac{1}{f}.$$

We multiply each term in the equation by $fx(D - x)$ and obtain $x^2 - Dx + Df = 0$. Solving for x, we find that the two object distances for which images are formed on the screen are

$$x_1 = \frac{D - \sqrt{D(D - 4f)}}{2} \quad \text{and} \quad x_2 = \frac{D + \sqrt{D(D - 4f)}}{2}.$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)}.$$

(b) The ratio of the image sizes is the same as the ratio of the lateral magnifications. If the object is at $p = x_1$, the magnitude of the lateral magnification is

$$|m_1| = \frac{i_1}{p_1} = \frac{D - x_1}{x_1}.$$

Now $x_1 = \frac{1}{2}(D-d)$, where $d = \sqrt{D(D-f)}$, so

$$|m_1| = \frac{D - (D-d)/2}{(D-d)/2} = \frac{D+d}{D-d}.$$

Similarly, when the object is at x_2, the magnitude of the lateral magnification is

$$|m_2| = \frac{I_2}{p_2} = \frac{D - x_2}{x_2} = \frac{D - (D+d)/2}{(D+d)/2} = \frac{D-d}{D+d}.$$

The ratio of the magnifications is

$$\frac{m_2}{m_1} = \frac{(D-d)/(D+d)}{(D+d)/(D-d)} = \left(\frac{D-d}{D+d}\right)^2.$$

118. (a) Our first step is to form the image from the first lens. With p_1 = 10 cm and $f_1 = -15$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \quad \Rightarrow \quad i_1 = -6.0\,\text{cm}.$$

The corresponding magnification is $m_1 = -i_1/p_1 = 0.60$. This image serves the role of "object" for the second lens, with p_2 = 12 + 6.0 = 18 cm, and f_2 = 12 cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \quad \Rightarrow \quad i_2 = 36 \text{ cm}.$$

(b) The corresponding magnification is $m_2 = -i_2/p_2 = -2.0$, which results in a net magnification of $m = m_1 m_2 = -1.2$. The height of the final image is (in absolute value) (1.2)(1.0 cm) = 1.2 cm.

(c) The fact that i_2 is positive means that the final image is real.

(d) The fact that m is negative means that the orientation of the final image is inverted with respect to the (original) object.

119. (a) Without the diverging lens (lens 2), the real image formed by the converging lens (lens 1) is located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{20\text{ cm}} - \frac{1}{40\text{ cm}}\right)^{-1} = 40\text{ cm}$$

to the right of lens 1. This image now serves as an object for lens 2, with $p_2 = -(40\text{ cm} - 10\text{ cm}) = -30$ cm. So

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{-15\text{cm}} - \frac{1}{-30\text{cm}}\right)^{-1} = -30\text{cm}.$$

Thus, the image formed by lens 2 is located 30 cm to the left of lens 2.

(b) The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1.0 > 0$, so the image is not inverted.

(c) The image is virtual since $i_2 < 0$.

(d) The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1.0$, so the image has the same size as the object.

120. (a) For the image formed by the first lens

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{10\text{cm}} - \frac{1}{20\text{cm}}\right)^{-1} = 20\text{cm}.$$

For the subsequent image formed by the second lens $p_2 = 30\text{ cm} - 20\text{ cm} = 10$ cm, so

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{12.5\text{cm}} - \frac{1}{10\text{cm}}\right)^{-1} = -50\text{cm}.$$

Thus, the final image is 50 cm to the left of the second lens, which means that it coincides with the object.

(b) The magnification is

$$m = \left(\frac{i_1}{p_1}\right)\left(\frac{i_2}{p_2}\right) = \left(\frac{20\text{cm}}{20\text{cm}}\right)\left(\frac{-50\text{cm}}{10\text{cm}}\right) = -5.0,$$

which means that the final image is five times larger than the original object.

(c) The image is virtual since $i_2 < 0$.

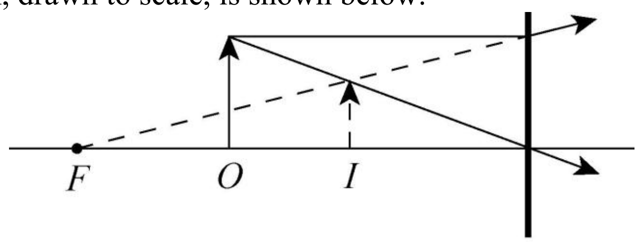
(d) The image is inverted since $m < 0$.

121. (a) We solve Eq. 34-9 for the image distance i: $i = pf/(p - f)$. The lens is diverging, so its focal length is $f = -30$ cm. The object distance is $p = 20$ cm. Thus,

$$i = \frac{(20\text{ cm})(-30\text{ cm})}{(20\text{ cm}) - (-30\text{ cm})} = -12\text{ cm}.$$

The negative sign indicates that the image is virtual and is on the same side of the lens as the object.

(b) The ray diagram, drawn to scale, is shown below.



122. (a) Suppose that the lens is placed to the left of the mirror. The image formed by the converging lens is located at a distance

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{0.50\text{ m}} - \frac{1}{1.0\text{ m}}\right)^{-1} = 1.0\text{ m}$$

to the right of the lens, or 2.0 m – 1.0 m = 1.0 m in front of the mirror. The image formed by the mirror for this real image is then at 1.0 m to the right of the mirror, or 2.0 m + 1.0 m = 3.0 m to the right of the lens. This image then results in another image formed by the lens, located at a distance

$$i' = \left(\frac{1}{f} - \frac{1}{p'}\right)^{-1} = \left(\frac{1}{0.50\text{m}} - \frac{1}{3.0\text{m}}\right)^{-1} = 0.60\text{m}$$

to the left of the lens (that is, 2.6 cm from the mirror).

(b) The lateral magnification is

$$m = \left(-\frac{i}{p}\right)\left(-\frac{i'}{p'}\right) = \left(-\frac{1.0\text{ m}}{1.0\text{ m}}\right)\left(-\frac{0.60\text{ m}}{3.0\text{ m}}\right) = +0.20\ .$$

(c) The final image is real since $i' > 0$.

(d) The image is to the left of the lens.

(e) It also has the same orientation as the object since $m > 0$. Therefore, the image is not inverted.

123. (a) We use Eq. 34-8 (and Fig. 34-12(b) is useful), with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 1.5$.

$$\frac{1}{p}+\frac{1.5}{i}=\frac{1.5-1}{r}$$

Using the sign convention for r stated in the paragraph following Eq. 34-8 (so that $r = +6.0$ cm), we obtain $i = -90$ cm for objects at $p = 10$ cm. Thus, the object and image are 80 cm apart.

(b) The image distance i is negative with increasing magnitude as p increases from very small values to some value p_0 at which point $i \to -\infty$. Since $1/(-\infty) = 0$, the above equation yields

$$\frac{1}{p_0}=\frac{1.5-1}{r} \quad \Rightarrow \quad p_0 = 2r.$$

Thus, the range for producing virtual images is $0 < p \le 12$ cm.

124. (a) Suppose one end of the object is a distance p from the mirror and the other end is a distance $p + L$. The position i_1 of the image of the first end is given by

$$\frac{1}{p}+\frac{1}{i_1}=\frac{1}{f}$$

where f is the focal length of the mirror. Thus, $i_1 = \frac{f_p}{p-f}$. The image of the other end is located at

$$i_2 = \frac{f(p+L)}{p+L-f},$$

so the length of the image is

$$L' = i_1 - i_2 = \frac{fp}{p-f} - \frac{f(p+L)}{p+L-f} = \frac{f^2 L}{(p-f)(p+L-f)}.$$

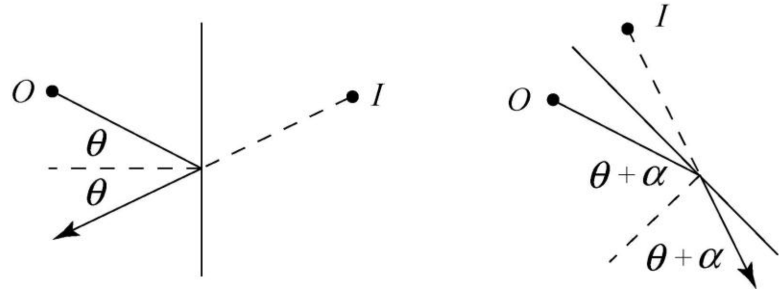
Since the object is short compared to $p - f$, we may neglect the L in the denominator and write

$$L' = L\left(\frac{f}{p-f}\right)^2.$$

(b) The lateral magnification is $m = -i/p$ and since $i = fp/(p - f)$, this can be written $m = -f/(p - f)$. The longitudinal magnification is

$$m' = \frac{L'}{L} = \left(\frac{f}{p-f}\right)^2 = m^2.$$

125. Consider a single ray from the source to the mirror and let θ be the angle of incidence. The angle of reflection is also θ and the reflected ray makes an angle of 2θ with the incident ray.



Now we rotate the mirror through the angle α so that the angle of incidence increases to $\theta + \alpha$. The reflected ray now makes an angle of $2(\theta + \alpha)$ with the incident ray. The reflected ray has been rotated through an angle of 2α. If the mirror is rotated so the angle of incidence is decreased by α, then the reflected ray makes an angle of $2(\theta - \alpha)$ with the incident ray. Again it has been rotated through 2α. The diagrams below show the situation for $\alpha = 45°$. The ray from the object to the mirror is the same in both cases and the reflected rays are 90° apart.

126. The fact that it is inverted implies $m < 0$. Therefore, with $m = -1/2$, we have $i = p/2$, which we substitute into Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{p}+\frac{2}{p}=\frac{1}{f}$$

or

$$\frac{3}{30.0 \text{ cm}}=\frac{1}{f}.$$

Consequently, we find $f = (30.0 \text{ cm})/3 = 10.0$ cm. The fact that $f > 0$ implies the mirror is concave.

127. (a) The mirror has focal length $f = 12.0$ cm. With $m = +3$, we have $i = -3p$. We substitute this into Eq. 34-4:

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{p}+\frac{1}{-3p}=\frac{1}{12 \text{ cm}}$$

or

$$\frac{2}{3p} = \frac{1}{12 \text{ cm}}.$$

Consequently, we find p = 2(12 cm)/3 = 8.0 cm.

(b) With m = –3, we have i = +3p, which we substitute into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{p} + \frac{1}{3p} = \frac{1}{12}$$

or

$$\frac{4}{3p} = \frac{1}{12 \text{ cm}}.$$

Consequently, we find p = 4(12 cm)/3 = 16 cm.

(c) With m = –1/3, we have i = p/3. Thus, Eq. 34-4 leads to

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{p} + \frac{3}{p} = \frac{1}{12 \text{ cm}}$$

or

$$\frac{4}{p} = \frac{1}{12 \text{ cm}}.$$

Consequently, we find p = 4(12 cm) = 48 cm.

128. Since 0 < m < 1, we conclude the lens is of the diverging type (so f = –40 cm). Thus, substituting i = –3p/10 into Eq. 34-9 produces

$$\frac{1}{p} - \frac{10}{3p} = -\frac{7}{3p} = \frac{1}{f}.$$

Therefore, we find p = 93.3 cm and i = –28.0 cm, or | i | = 28.0 cm.

129. (a) We show the α = 0.500 rad, r =12 cm, p = 20 cm calculation in detail. The understood length unit is the centimeter:

The distance from the object to point x:

$$d = p - r + x = 8 + x$$
$$y = d \tan\alpha = 4.3704 + 0.54630x$$

From the solution of $x^2 + y^2 = r^2$ we get x = 8.1398.

$$\beta = \tan^{-1}(y/x) = 0.8253 \text{ rad}$$

$$\gamma = 2\beta - \alpha = 1.151 \text{ rad}$$

From the solution of $\tan(\gamma) = y/(x + i - r)$ we get $i = 7.799$. The other results are shown without the intermediate steps:

For $\alpha = 0.100$ rad, we get $i = 8.544$ cm; for $\alpha = 0.0100$ rad, we get $i = 8.571$ cm. Eq. 34-3 and Eq. 34-4 (the mirror equation) yield $i = 8.571$ cm.

(b) Here the results are: ($\alpha = 0.500$ rad, $i = -13.56$ cm), ($\alpha = 0.100$ rad, $i = -12.05$ cm), ($\alpha = 0.0100$ rad, $i = -12.00$ cm). The mirror equation gives $i = -12.00$ cm.

130. (a) Since $m = +0.250$, we have $i = -0.25p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex and that $f < 0$; in fact, $f = -2.00$ cm. Substituting $i = -p/4$ into Eq. 34-4 produces

$$\frac{1}{p} - \frac{4}{p} = -\frac{3}{p} = \frac{1}{f}$$

Therefore, we find $p = 6.00$ cm and $i = -1.50$ cm, or $|i| = 1.50\text{ cm}$.

(b) The focal length is negative.

(c) As shown in (a), the image is virtual.

131. First, we note that — *relative to the water* — the index of refraction of the carbon tetrachloride should be thought of as $n = 1.46/1.33 = 1.1$ (this notation is chosen to be consistent with Problem 34-122). Now, if the observer were in the water, directly above the 40 mm deep carbon tetrachloride layer, then the apparent depth of the penny as measured below the surface of the carbon tetrachloride is $d_a = 40$ mm/1.1 = 36.4 mm. This "apparent penny" serves as an "object" for the rays propagating upward through the 20 mm layer of water, where this "object" should be thought of as being 20 mm + 36.4 mm = 56.4 mm from the top surface. Using the result of Problem 34-122 again, we find the perceived location of the penny, for a person at the normal viewing position above the water, to be 56.4 mm/1.33 = 42 mm below the water surface.

132. The sphere (of radius 0.35 m) is a convex mirror with focal length $f = -0.175$ m. We adopt the approximation that the rays are close enough to the central axis for Eq. 34-4 to be applicable.

(a) With $p = 1.0$ m, the equation $1/p + 1/i = 1/f$ yields $i = -0.15$ m, which means the image is 0.15 m from the front surface, appearing to be *inside* the sphere.

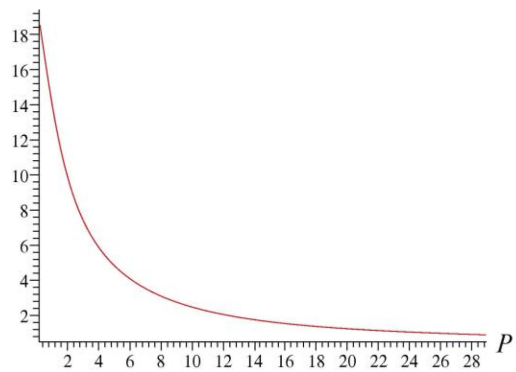
(b) The lateral magnification is $m = -i/p$ which yields $m = 0.15$. Therefore, the image distance is (0.15)(2.0 m) = 0.30 m.

(c) Since $m > 0$, the image is upright, or not inverted (NI).

133. (a) In this case $i < 0$ so $i = -|i|$, and Eq. 34-9 becomes $1/f = 1/p - 1/|i|$. We differentiate this with respect to time (t) to obtain

$$\frac{d|i|}{dt} = \left(\frac{|i|}{p}\right)^2 \frac{dp}{dt} \quad .$$

As the object is moved toward the lens, p is decreasing, so $dp/dt < 0$. Consequently, the above expression shows that $d|i|/dt < 0$; that is, the image moves in from infinity. The angular magnification $m_\theta = \theta'/\theta$ also increases as the following graph shows ("read" the graph from left to right since we are considering decreasing p from near the focal length to near 0). To obtain this graph of m_θ, we chose $f = 30$ cm and $h = 2$ cm.



(b) When the image appears to be at the near point (that is, $|i| = P_n$), m_θ is at its maximum usable value. Since one generally takes P_n to be equal to 25 cm (this value, too, was used in making the above graph).

(c) In this case,

$$p = \frac{if}{i-f} = \frac{|i|f}{|i|+f} = \frac{P_n f}{P_n + f} \quad .$$

If we use the small angle approximation, we have $\theta' \approx h'/|i|$ and $\theta \approx h/P_n$ (note: this approximation was not used in obtaining the graph, above). We therefore find

$$m_\theta \approx (h'/|i|)/(h/P_n)$$

which (using Eq. 34-7 relating the ratio of heights to the ratio of distances) becomes

$$m_\theta \approx \frac{h'}{h} \cdot \frac{P_n}{|i|} = \frac{|i|}{p} \cdot \frac{P_n}{|i|} = \frac{P_n}{p} = \frac{P_n}{P_n f/(P_n + f)} = \frac{P_n + f}{f}$$

which readily simplifies to the desired result.

(d) The linear magnification (Eq. 34-7) is given by $(h'/h) \approx m_\theta\,(|i|/P_n)$ (see the first in the chain of equalities, above). Once we set $|i| = P_n$ (see part (b)) then this shows the equality in the magnifications.

134. (a) The discussion in the textbook of the refracting telescope applies to the Newtonian arrangement if we replace the objective lens of Fig. 34-21 with an objective mirror (with the light incident on it from the right). This might suggest that the incident light would be blocked by the person's head in Fig. 34-21, which is why Newton added the mirror M' in his design (to move the head and eyepiece out of the way of the incoming light). The beauty of the idea of characterizing both lenses and mirrors by focal lengths is that it is easy, in a case like this, to simply carry over the results of the objective-lens telescope to the objective-mirror telescope, so long as we replace a positive f device with another positive f device. Thus, the converging lens serving as the objective of Fig. 34-21 must be replaced (as Newton has done in Fig. 34-58) with a concave mirror. With this change of language, the discussion in the textbook leading up to Eq. 34-15 applies equally as well to the Newtonian telescope: $m_\theta = -f_{ob}/f_{ey}$.

(b) A meter stick (held perpendicular to the line of sight) at a distance of 2000 m subtends an angle of

$$\theta_{\text{stick}} \approx \frac{1\text{ m}}{2000\text{ m}} = 0.0005\text{ rad.}$$

multiplying this by the mirror focal length gives (16.8 m) (0.0005) = 8.4 mm for the size of the image.

(c) With r = 10 m, Eq. 34-3 gives f_{ob} = 5 m. Plugging this into (the absolute value of) Eq. 34-15 leads to f_{ey} = 5/200 = 2.5 cm.

135. (a) If we let $p \to \infty$ in Eq. 34-8, we get $i = n_2 r/(n_2 - n_1)$. If we set $n_1 = 1$ (for air) and restrict n_2 so that $1 < n_2 < 2$, then this suggests that $i > 2r$ (so this image does form before the rays strike the opposite side of the sphere). We can still consider this as a sort of "virtual" object for the second imaging event, where this "virtual" object distance is

$$2r - i = (n-2)\,r/(n-1),$$

where we have simplified the notation by writing $n_2 = n$. Putting this in for p in Eq. 34-8 and being careful with the sign convention for r in that equation, we arrive at the final image location: $i' = (0.5)(2-n)r/(n-1)$.

(b) The image is to the right of the right side of the sphere.

136. We set up an xyz coordinate system where the individual planes (xy, yz, xz) serve as the mirror surfaces. Suppose an incident ray of light A first strikes the mirror in the xy plane. If the unit vector denoting the direction of A is given by

$$\cos(\alpha)\hat{\text{i}} + \cos(\beta)\hat{\text{j}} + \cos(\gamma)\hat{\text{k}}$$

where α, β, γ are the angles A makes with the axes, then after reflection off the xy plane the unit vector becomes $\cos(\alpha)\hat{\text{i}} + \cos(\beta)\hat{\text{j}} - \cos(\gamma)\hat{\text{k}}$ (one way to rationalize this is to think of the reflection as causing the angle γ to become $\pi - \gamma$). Next suppose it strikes the mirror in the xz plane. The unit vector of the reflected ray is now $\cos(\alpha)\hat{\text{i}} - \cos(\beta)\hat{\text{j}} - \cos(\gamma)\hat{\text{k}}$. Finally as it reflects off the mirror in the yz plane α becomes $\pi - \alpha$, so the unit vector in the direction of the reflected ray is given by $-\cos(\alpha)\hat{\text{i}} - \cos(\beta)\hat{\text{j}} - \cos(\gamma)\hat{\text{k}}$, exactly reversed from A's original direction. A further observation may be made: this argument would fail if the ray could strike any given surface twice and some consideration (perhaps an illustration) should convince the student that such an occurrence is not possible.

137. Since $m = -2$ and $p = 4.00$ cm, then $i = 8.00$ cm (and is real). Eq. 34-9 is

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

and leads to $f = 2.67$ cm (which is positive, as it must be for a converging lens).

138. (a) Since $m = +0.200$, we have $i = -0.2p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex (and that $f = -40.0$ cm).

(b) Substituting $i = -p/5$ into Eq. 34-4 produces

$$\frac{1}{p} - \frac{5}{p} = -\frac{4}{p} = \frac{1}{f}.$$

Therefore, we find $p = -4f = -4(-40.0 \text{ cm}) = 160 \text{ cm}$.

139. (a) Our first step is to form the image from the first lens. With $p_1 = 3.00$ cm and $f_1 = +4.00$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow i_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(4.00 \text{ cm})(3.00 \text{ cm})}{3.00 \text{ cm} - 4.00 \text{ cm}} = -12.0 \text{cm}.$$

The corresponding magnification is $m_1 = -i_1/p_1 = 4$. This image serves the role of "object" for the second lens, with $p_2 = 8.00 + 12.0 = 20.0$ cm, and $f_2 = -4.00$ cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \Rightarrow i_2 = \frac{f_2 p_2}{p_2 - f_2} = \frac{(-4.00 \text{ cm})(20.0 \text{ cm})}{20.0 \text{ cm} - (-4.00 \text{ cm})} = -3.33 \text{ cm},$$

or $|i_2| = 3.33 \text{ cm}$.

(b) The fact that i_2 is negative means that the final image is virtual (and therefore to the left of the second lens).

(c) The image is virtual.

(d) With $m_2 = -i_2/p_2 = 1/6$, the net magnification is $m = m_1 m_2 = 2/3 > 0$. The fact that m is positive means that the orientation of the final image is the same as the (original) object. Therefore, the image is not inverted.

140. The far point of the person is 50 cm = 0.50 m from the eye. The object distance is taken to be at infinity, and the corrected lens will allow the image to be formed at the near point. Thus,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} = \frac{1}{\infty} + \frac{1}{-0.50\text{ m}}$$

and we find the focal length of the lens to $f = -0.50$ m.

(b) Since $f < 0$, the lens is diverging.

(c) The power of the lens is $P = \frac{1}{f} = \frac{1}{-0.50\text{ m}} = -2.0\text{ diopters}$.

141. (a) Without the magnifier, $\theta = h/P_n$. With the magnifier, letting $p = p_n$ and $i = -|i| = -P_n$, we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n}.$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25\text{ cm}}{f}.$$

(b) Now $i = -|i| \to -\infty$, so $1/p + 1/i = 1/p = 1/f$ and

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25\text{ cm}}{f}.$$

(c) For $f = 10$ cm, we find the magnifications to be $m_\theta = 1 + \frac{25\text{ cm}}{10\text{ cm}} = 3.5$ for cases (a), and $m_\theta = \frac{25\text{ cm}}{10\text{ cm}} = 2.5$ for case (b).

Chapter 35

1. The fact that wave W_2 reflects two additional times has no substantive effect on the calculations, since two reflections amount to a $2(\lambda/2) = \lambda$ phase difference, which is effectively not a phase difference at all. The substantive difference between W_2 and W_1 is the extra distance $2L$ traveled by W_2.

(a) For wave W_2 to be a half-wavelength "behind" wave W_1, we require $2L = \lambda/2$, or $L = \lambda/4 = (620 \text{ nm})/4 = 155$ nm using the wavelength value given in the problem.

(b) Destructive interference will again appear if W_2 is $\frac{3}{2}\lambda$ "behind" the other wave. In this case, $2L' = 3\lambda/2$, and the difference is

$$L' - L = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} = \frac{620 \text{ nm}}{2} = 310 \text{ nm} .$$

2. We consider waves W_2 and W_1 with an initial effective phase difference (in wavelengths) equal to $\frac{1}{2}$, and seek positions of the sliver that cause the wave to constructively interfere (which corresponds to an integer-valued phase difference in wavelengths). Thus, the extra distance $2L$ traveled by W_2 must amount to $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, and so on. We may write this requirement succinctly as

$$L = \frac{2m+1}{4}\lambda \quad \text{where } m = 0, 1, 2, \ldots .$$

(a) Thus, the smallest value of L/λ that results in the final waves being exactly in phase is when $m = 0$, which gives $L/\lambda = 1/4 = 0.25$.

(b) The second smallest value of L/λ that results in the final waves being exactly in phase is when $m = 1$, which gives $L/\lambda = 3/4 = 0.75$.

(c) The third smallest value of L/λ that results in the final waves being exactly in phase is when $m = 2$, which gives $L/\lambda = 5/4 = 1.25$.

3. **THINK** The wavelength of light in a medium depends on the index of refraction of the medium. The nature of the interference, whether constructive or destructive, depends on the phase difference of the two waves.

EXPRESS We take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1 L - \omega t$, where k_1 $(= 2\pi/\lambda_1)$ is the angular wave number and λ_1 is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 =$

$k_2L - \omega t$, where k_2 (= $2\pi/\lambda_2$) is the angular wave number and λ_2 is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)L.$$

Now, $\lambda_1 = \lambda_{air}/n_1$, where λ_{air} is the wavelength in air and n_1 is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{air}/n_2$, where n_2 is the index of refraction of the plastic. This means that the phase difference is

$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda_{air}}(n_1 - n_2)L.$$

ANALYZE (a) The value of L that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{air}}{2\pi(n_1 - n_2)} = \frac{5.65(400 \times 10^{-9}\ \text{m})}{2\pi(1.60 - 1.50)} = 3.60 \times 10^{-6}\ \text{m}.$$

(b) A phase difference of 5.65 rad is less than 2π rad = 6.28 rad, the phase difference for completely constructive interference, but greater than π rad (= 3.14 rad), the phase difference for completely destructive interference. The interference is, therefore, intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

LEARN The phase difference of two light waves can change when they travel through different materials having different indices of refraction.

4. Note that Snell's law (the law of refraction) leads to $\theta_1 = \theta_2$ when $n_1 = n_2$. The graph indicates that $\theta_2 = 30°$ (which is what the problem gives as the value of θ_1) occurs at $n_2 =$ 1.5. Thus, $n_1 = 1.5$, and the speed with which light propagates in that medium is

$$v = \frac{c}{n_1} = \frac{2.998 \times 10^8\ \text{m/s}}{1.5} = 2.0 \times 10^8\ \text{m/s}.$$

5. Comparing the light speeds in sapphire and diamond, we obtain

$$\Delta v = v_s - v_d = c\left(\frac{1}{n_s} - \frac{1}{n_d}\right) = (2.998 \times 10^8\ \text{m/s})\left(\frac{1}{1.77} - \frac{1}{2.42}\right) = 4.55 \times 10^7\ \text{m/s}.$$

6. (a) The frequency of yellow sodium light is

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\ \text{m/s}}{589 \times 10^{-9}\ \text{m}} = 5.09 \times 10^{14}\ \text{Hz}.$$

(b) When traveling through the glass, its wavelength is

$$\lambda_n = \frac{\lambda}{n} = \frac{589\,\text{nm}}{1.52} = 388\text{ nm.}$$

(c) The light speed when traveling through the glass is

$$v = f\,\lambda_n = \left(5.09\times10^{14}\,\text{Hz}\right)\left(388\times10^{-9}\,\text{m}\right) = 1.97\times10^{8}\text{ m/s}.$$

7. The index of refraction is found from Eq. 35-3:

$$n = \frac{c}{v} = \frac{2.998\times10^{8}\text{ m/s}}{1.92\times10^{8}\text{ m/s}} = 1.56.$$

8. (a) The time t_2 it takes for pulse 2 to travel through the plastic is

$$t_2 = \frac{L}{c/1.55} + \frac{L}{c/1.70} + \frac{L}{c/1.60} + \frac{L}{c/1.45} = \frac{6.30L}{c}.$$

Similarly for pulse 1:

$$t_1 = \frac{2L}{c/1.59} + \frac{L}{c/1.65} + \frac{L}{c/1.50} = \frac{6.33L}{c}.$$

Thus, pulse 2 travels through the plastic in less time.

(b) The time difference (as a multiple of L/c) is

$$\Delta t = t_2 - t_1 = \frac{6.33L}{c} - \frac{6.30L}{c} = \frac{0.03L}{c}.$$

Thus, the multiple is 0.03.

9. (a) We wish to set Eq. 35-11 equal to $1/2$, since a half-wavelength phase difference is equivalent to a π radians difference. Thus,

$$L_{\min} = \frac{\lambda}{2(n_2 - n_1)} = \frac{620\,\text{nm}}{2(1.65 - 1.45)} = 1550\,\text{nm} = 1.55\ \mu\text{m}.$$

(b) Since a phase difference of $\frac{3}{2}$ (wavelengths) is effectively the same as what we required in part (a), then

$$L = \frac{3\lambda}{2(n_2 - n_1)} = 3L_{\min} = 3(1.55\,\mu\text{m}) = 4.65\,\mu\text{m}.$$

10. (a) The exiting angle is 50º, the same as the incident angle, due to what one might call the "transitive" nature of Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = \ldots$

(b) Due to the fact that the speed (in a certain medium) is c/n (where n is that medium's index of refraction) and that speed is distance divided by time (while it's constant), we find

$$t = nL/c = (1.45)(25 \times 10^{-19}\text{ m})/(3.0 \times 10^{8}\text{ m/s}) = 1.4 \times 10^{-13}\text{ s} = 0.14\text{ ps}.$$

11. (a) Equation 35-11 (in absolute value) yields

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(8.50\times10^{-6}\text{ m})}{500\times10^{-9}\text{ m}}(1.60-1.50) = 1.70.$$

(b) Similarly,

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(8.50\times10^{-6}\text{ m})}{500\times10^{-9}\text{ m}}(1.72-1.62) = 1.70.$$

(c) In this case, we obtain

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(3.25\times10^{-6}\text{ m})}{500\times10^{-9}\text{ m}}(1.79-1.59) = 1.30.$$

(d) Since their phase differences were identical, the brightness should be the same for (a) and (b). Now, the phase difference in (c) differs from an integer by 0.30, which is also true for (a) and (b). Thus, their effective phase differences are equal, and the brightness in case (c) should be the same as that in (a) and (b).

12. (a) We note that ray 1 travels an extra distance $4L$ more than ray 2. To get the least possible L that will result in destructive interference, we set this extra distance equal to half of a wavelength:

$$4L = \frac{\lambda}{2} \quad \Rightarrow \quad L = \frac{\lambda}{8} = \frac{420.0\text{ nm}}{8} = 52.50\text{ nm}.$$

(b) The next case occurs when that extra distance is set equal to $\frac{3}{2}\lambda$. The result is

$$L = \frac{3\lambda}{8} = \frac{3(420.0\text{ nm})}{8} = 157.5\text{ nm}.$$

13. (a) We choose a horizontal x axis with its origin at the left edge of the plastic. Between $x = 0$ and $x = L_2$ the phase difference is that given by Eq. 35-11 (with L in that

equation replaced with L_2). Between $x = L_2$ and $x = L_1$ the phase difference is given by an expression similar to Eq. 35-11 but with L replaced with $L_1 - L_2$ and n_2 replaced with 1 (since the top ray in Fig. 35-36 is now traveling through air, which has index of refraction approximately equal to 1). Thus, combining these phase differences with $\lambda = 0.600$ μm, we have

$$\frac{L_2}{\lambda}(n_2 - n_1) + \frac{L_1 - L_2}{\lambda}(1 - n_1) = \frac{3.50\ \mu\text{m}}{0.600\ \mu\text{m}}(1.60 - 1.40) + \frac{4.00\ \mu\text{m} - 3.50\ \mu\text{m}}{0.600\ \mu\text{m}}(1 - 1.40)$$
$$= 0.833.$$

(b) Since the answer in part (a) is closer to an integer than to a half-integer, the interference is more nearly constructive than destructive.

14. (a) For the maximum adjacent to the central one, we set $m = 1$ in Eq. 35-14 and obtain

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right)\Bigg|_{m=1} = \sin^{-1}\left[\frac{(1)(\lambda)}{100\lambda}\right] = 0.010\,\text{rad}.$$

(b) Since $y_1 = D \tan \theta_1$ (see Fig. 35-10(a)), we obtain

$$y_1 = (500\text{ mm}) \tan (0.010\text{ rad}) = 5.0\text{ mm}.$$

The separation is $\Delta y = y_1 - y_0 = y_1 - 0 = 5.0$ mm.

15. **THINK** The interference at a point depends on the path-length difference of the light rays reaching that point from the two slits.

EXPRESS The angular positions of the maxima of a two-slit interference pattern are given by $\Delta L = d \sin\theta = m\lambda$, where ΔL is the path-length difference, d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$.

ANALYZE Let λ' be the wavelength for which the angular separation is greater by10.0%. Then, $1.10\lambda/d = \lambda'/d$. or

$$\lambda' = 1.10\lambda = 1.10(589\text{ nm}) = 648\text{ nm}.$$

LEARN The angular separation $\Delta\theta$ is proportional to the wavelength of the light. For small θ, we have

$$\Delta\theta' = \left(\frac{\lambda'}{\lambda}\right)\Delta\theta.$$

16. The distance between adjacent maxima is given by $\Delta y = \lambda D/d$ (see Eqs. 35-17 and 35-18). Dividing both sides by D, this becomes $\Delta\theta = \lambda/d$ with θ in radians. In the steps that follow, however, we will end up with an expression where degrees may be directly used. Thus, in the present case,

$$\Delta\theta_n = \frac{\lambda_n}{d} = \frac{\lambda}{nd} = \frac{\Delta\theta}{n} = \frac{0.20°}{1.33} = 0.15°.$$

17. **THINK** Interference maxima occur at angles θ such that $d \sin \theta = m\lambda$, where m is an integer.

EXPRESS Since d = 2.0 m and λ = 0.50 m, this means that $\sin\theta = 0.25m$. We want all values of m (positive and negative) for which $|0.25m| \le 1$. These are –4, –3, –2, –1, 0, +1, +2, +3, and +4.

ANALYZE For each of these except –4 and +4, there are two different values for θ. A single value of θ (–90°) is associated with m = –4 and a single value (+90°) is associated with m = +4. There are sixteen different angles in all and, therefore, sixteen maxima.

LEARN The angles at which the maxima occur are given by

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(0.25m)$$

Similarly, the condition for interference minima (destructive interference) is

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, ...$$

18. (a) The phase difference (in wavelengths) is

$$\phi = d \sin\theta/\lambda = (4.24\ \mu\text{m})\sin(20°)/(0.500\ \mu\text{m}) = 2.90\ .$$

(b) Multiplying this by 2π gives ϕ = 18.2 rad.

(c) The result from part (a) is greater than $\frac{5}{2}$ (which would indicate the third minimum) and is less than 3 (which would correspond to the third side maximum).

19. **THINK** The condition for a maximum in the two-slit interference pattern is $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction.

EXPRESS If θ is small, $\sin \theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer m and the other associated with the integer $m + 1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance D away is given by

$$\Delta y = D\,\Delta\theta = \lambda D/d.$$

ANALYZE Thus,

$$\Delta y = \frac{(500\times10^{-9}\,\text{m})(5.40\,\text{m})}{1.20\times10^{-3}\,\text{m}} = 2.25\times10^{-3}\,\text{m} = 2.25\text{ mm}.$$

LEARN For small θ, the spacing is nearly uniform. However, away from the center of the pattern, θ increases and the spacing gets larger.

20. (a) We use Eq. 35-14 with $m = 3$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{2(550\times10^{-9}\,\text{m})}{7.70\times10^{-6}\,\text{m}}\right] = 0.216\,\text{rad}.$$

(b) $\theta = (0.216)\,(180°/\pi) = 12.4°$.

21. The maxima of a two-slit interference pattern are at angles θ given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be replaced by θ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is

$$\Delta\theta = (m/d)(\lambda_2 - \lambda_1),$$

and their separation on a screen a distance D away is

$$\Delta y = D\tan\Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1)$$
$$= \left[\frac{3(1.0\,\text{m})}{5.0\times10^{-3}\,\text{m}}\right](600\times10^{-9}\,\text{m} - 480\times10^{-9}\,\text{m}) = 7.2\times10^{-5}\,\text{m}.$$

The small angle approximation $\tan\Delta\theta \approx \Delta\theta$ (in radians) is made.

22. Imagine a y axis midway between the two sources in the figure. Thirty points of destructive interference (to be considered in the xy plane of the figure) implies there are $7+1+7=15$ on each side of the y axis. There is no point of destructive interference on the y axis itself since the sources are in phase and any point on the y axis must therefore correspond to a zero phase difference (and corresponds to $\theta = 0$ in Eq. 35-14). In other words, there are 7 "dark" points in the first quadrant, one along the $+x$ axis, and 7 in the fourth quadrant, constituting the 15 dark points on the right-hand side of the y axis. Since the y axis corresponds to a minimum phase difference, we can count (say, in the first quadrant) the m values for the destructive interference (in the sense of Eq. 35-16)

beginning with the one closest to the *y* axis and going clockwise until we reach the *x* axis (at any point beyond S_2). This leads us to assign $m = 7$ (in the sense of Eq. 35-16) to the point on the *x* axis itself (where the path difference for waves coming from the sources is simply equal to the separation of the sources, *d*); this would correspond to $\theta = 90°$ in Eq. 35-16. Thus,

$$d = (\ 7 + \tfrac{1}{2}\ )\lambda = \ 7.5\ \lambda \ \ \Rightarrow \ \ \frac{d}{\lambda} = 7.5\,.$$

23. Initially, source *A* leads source *B* by 90°, which is equivalent to 1/4 wavelength. However, source *A* also lags behind source *B* since r_A is longer than r_B by 100 m, which is $100\,\text{m}/400\,\text{m} = 1/4$ wavelength. So the net phase difference between *A* and *B* at the detector is zero.

24. (a) We note that, just as in the usual discussion of the double slit pattern, the $x = 0$ point on the screen (where that vertical line of length *D* in the picture intersects the screen) is a bright spot with phase difference equal to zero (it would be the middle fringe in the usual double slit pattern). We are not considering $x < 0$ values here, so that negative phase differences are not relevant (and if we did wish to consider $x < 0$ values, we could limit our discussion to absolute values of the phase difference, so that, again, negative phase differences do not enter it). Thus, the $x = 0$ point is the one with the minimum phase difference.

(b) As noted in part (a), the phase difference $\phi = 0$ at $x = 0$.

(c) The path length difference is greatest at the rightmost "edge" of the screen (which is assumed to go on forever), so ϕ is maximum at $x = \infty$.

(d) In considering $x = \infty$, we can treat the rays from the sources as if they are essentially horizontal. In this way, we see that the difference between the path lengths is simply the distance (2*d*) between the sources. The problem specifies $2d = 6.00\ \lambda$, or $2d/\lambda = 6.00$.

(e) Using the Pythagorean theorem, we have

$$\phi = \frac{\sqrt{D^2 + (x+d)^2}}{\lambda} - \frac{\sqrt{D^2 + (x-d)^2}}{\lambda} = 1.71$$

where we have plugged in $D = 20\lambda$, $d = 3\lambda$ and $x = 6\lambda$. Thus, the phase difference at that point is 1.71 wavelengths.

(f) We note that the answer to part (e) is closer to $\frac{3}{2}$ (destructive interference) than to 2 (constructive interference), so that the point is "intermediate" but closer to a minimum than to a maximum.

25. Let the distance in question be x. The path difference (between rays originating from S_1 and S_2 and arriving at points on the $x > 0$ axis) is

$$\sqrt{d^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda,$$

where we are requiring destructive interference (half-integer wavelength phase differences) and $m = 0, 1, 2, \ldots$. After some algebraic steps, we solve for the distance in terms of m:

$$x = \frac{d^2}{(2m+1)\lambda} - \frac{(2m+1)\lambda}{4}.$$

To obtain the largest value of x, we set $m = 0$:

$$x_0 = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(3.00\lambda)^2}{\lambda} - \frac{\lambda}{4} = 8.75\lambda = 8.75(900 \text{ nm}) = 7.88\times10^3 \text{ nm} = 7.88\,\mu\text{m}.$$

26. (a) We use Eq. 35-14 to find d:

$$d\sin\theta = m\lambda \quad \Rightarrow \quad d = (4)(450 \text{ nm})/\sin(90°) = 1800 \text{ nm}.$$

For the third-order spectrum, the wavelength that corresponds to $\theta = 90°$ is

$$\lambda = d\sin(90°)/3 = 600 \text{ nm}.$$

Any wavelength greater than this will not be seen. Thus, $600 \text{ nm} < \theta \le 700 \text{ nm}$ are absent.

(b) The slit separation d needs to be decreased.

(c) In this case, the 400 nm wavelength in the $m = 4$ diffraction is to occur at 90°. Thus

$$d_{\text{new}}\sin\theta = m\lambda \quad \Rightarrow \quad d_{\text{new}} = (4)(400 \text{ nm})/\sin(90°) = 1600 \text{ nm}.$$

This represents a change of

$$|\Delta d| = d - d_{\text{new}} = 200 \text{ nm} = 0.20\,\mu\text{m}.$$

27. Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness x is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x}{\lambda}(n-1)$$

where λ_m is the wavelength in the mica and n is the index of refraction of the mica. The relationship $\lambda_m = \lambda/n$ is used to substitute for λ_m. Since the waves are now in phase at the screen,

$$\frac{2\pi x}{\lambda}(n-1) = 14\pi$$

or

$$x = \frac{7\lambda}{n-1} = \frac{7(550\times10^{-9}\ \text{m})}{1.58-1} = 6.64\times10^{-6}\ \text{m}.$$

28. The problem asks for "the greatest value of x… exactly out of phase," which is to be interpreted as the value of x where the curve shown in the figure passes through a phase value of π radians. This happens as some point P on the x axis, which is, of course, a distance x from the top source and (using Pythagoras' theorem) a distance $\sqrt{d^2+x^2}$ from the bottom source. The difference (in normal length units) is therefore $\sqrt{d^2+x^2} - x$, or (expressed in radians) is

$$\frac{2\pi}{\lambda}\left(\sqrt{d^2+x^2} - x\right).$$

We note (looking at the leftmost point in the graph) that at $x = 0$, this latter quantity equals 6π, which means $d = 3\lambda$. Using this value for d, we now must solve the condition

$$\frac{2\pi}{\lambda}\left(\sqrt{d^2+x^2} - x\right) = \pi .$$

Straightforward algebra then leads to $x = (35/4)\lambda$, and using $\lambda = 400$ nm we find $x = 3500$ nm, or 3.5 μm.

29. **THINK** The intensity is proportional to the square of the resultant field amplitude.

EXPRESS Let the electric field components of the two waves be written as

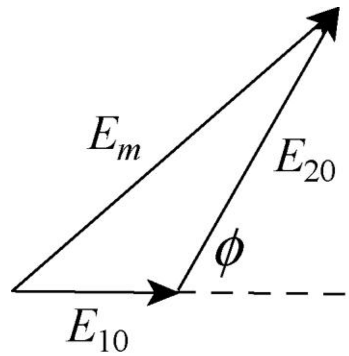
$$E_1 = E_{10}\sin\omega t$$
$$E_2 = E_{20}\sin(\omega t+\phi),$$

where $E_{10} = 1.00$, $E_{20} = 2.00$, and $\phi = 60°$. The resultant field is $E = E_1 + E_2$. We use phasor diagram to calculate the amplitude of E.

ANALYZE The phasor diagram is shown next.

The resultant amplitude E_m is given by the trigonometric law of cosines:

$$E_m^2 = E_{10}^2 + E_{20}^2 - 2E_{10}E_{20}\cos(180° - \phi) .$$



Thus,

$$E_m = \sqrt{(1.00)^2 + (2.00)^2 - 2(1.00)(2.00)\cos 120°} = 2.65 .$$

LEARN Summing over the horizontal components of the two fields gives

$$\sum E_h = E_{10}\cos 0 + E_{20}\cos 60° = 1.00 + (2.00)\cos 60° = 2.00$$

Similarly, the sum over the vertical components is

$$\sum E_v = E_{10}\sin 0 + E_{20}\sin 60° = 1.00\sin 0° + (2.00)\sin 60° = 1.732 .$$

The resultant amplitude is

$$E_m = \sqrt{(2.00)^2 + (1.732)^2} = 2.65 ,$$

which agrees with what we found above. The phase angle relative to the phasor representing E_1 is

$$\beta = \tan^{-1}\left(\frac{1.732}{2.00}\right) = 40.9°$$

Thus, the resultant field can be written as $E = (2.65)\sin(\omega t + 40.9°)$.

30. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ and add them as vectors:

$$y_h = 10\cos 0° + 8.0\cos 30° = 16.9$$
$$y_v = 10\sin 0° + 8.0\sin 30° = 4.0$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 17.4$$
$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 13.3° .$$

Thus,

$$y = y_1 + y_2 = y_R \sin(\omega t + \beta) = 17.4\sin(\omega t + 13.3°) .$$

Quoting the answer to two significant figures, we have $y \approx 17\sin\left(\omega t + 13°\right)$.

31. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ and add them as vectors:

$$y_h = 10\cos 0° + 15\cos 30° + 5.0\cos(-45°) = 26.5$$
$$y_v = 10\sin 0° + 15\sin 30° + 5.0\sin(-45°) = 4.0$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 26.8 \approx 27$$
$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 8.5°.$$

Thus, $y = y_1 + y_2 + y_3 = y_R \sin\left(\omega t + \beta\right) = 27\sin\left(\omega t + 8.5°\right)$.

32. (a) We can use phasor techniques or use trig identities. Here we show the latter approach. Since

$$\sin a + \sin(a + b) = 2\cos(b/2)\sin(a + b/2),$$

we find

$$E_1 + E_2 = 2E_0\cos(\phi/2)\sin(\omega t + \phi/2)$$

where E_0 = 2.00 μV/m, ω = 1.26 × 10^{15} rad/s, and ϕ = 39.6 rad. This shows that the electric field amplitude of the resultant wave is

$$E = 2E_0\cos(\phi/2) = 2(2.00\ \mu\text{V/m})\cos(19.2\text{ rad}) = 2.33\ \mu\text{V/m}.$$

(b) Equation 35-22 leads to

$$I = 4I_0\cos^2(\phi/2) = 1.35\ I_0$$

at point P, and

$$I_{\text{center}} = 4I_0\cos^2(0) = 4\ I_0$$

at the center. Thus, $I/I_{\text{center}} = 1.35/4 = 0.338$.

(c) The phase difference ϕ (in wavelengths) is gotten from ϕ in radians by dividing by 2π. Thus, $\phi = 39.6/2\pi = 6.3$ wavelengths. Thus, point P is between the sixth side maximum (at which ϕ = 6 wavelengths) and the seventh minimum (at which $\phi = 6\frac{1}{2}$ wavelengths).

(d) The rate is given by ω = 1.26 × 10^{15} rad/s.

(e) The angle between the phasors is $\phi = 39.6$ rad = 2270° (which would look like about 110° when drawn in the usual way).

33. With phasor techniques, this amounts to a vector addition problem $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ where (in magnitude-angle notation) $\vec{A} = (10\angle 0°), \vec{B} = (5\angle 45°)$, and $\vec{C} = (5\angle -45°)$, where the magnitudes are understood to be in μV/m. We obtain the resultant (especially efficient on a vector-capable calculator in polar mode):

$$\vec{R} = (10\angle 0°) + (5\angle 45°) + (5\angle -45°) = (17.1\angle 0°)$$

which leads to

$$E_R = (17.1\,\mu\text{V/m})\sin(\omega t)$$

where $\omega = 2.0 \times 10^{14}$ rad/s.

34. (a) Referring to Figure 35-10(a) makes clear that

$$\theta = \tan^{-1}(y/D) = \tan^{-1}(0.205/4) = 2.93°.$$

Thus, the phase difference at point P is $\phi = d\sin\theta/\lambda = 0.397$ wavelengths, which means it is between the central maximum (zero wavelength difference) and the first minimum ($\frac{1}{2}$ wavelength difference). Note that the above computation could have been simplified somewhat by avoiding the explicit use of the tangent and sine functions and making use of the small-angle approximation ($\tan\theta \approx \sin\theta$).

(b) From Eq. 35-22, we get (with $\phi = (0.397)(2\pi) = 2.495$ rad)

$$I = 4I_0\cos^2(\phi/2) = 0.404\, I_0$$

at point P and

$$I_{\text{center}} = 4I_0\cos^2(0) = 4\, I_0$$

at the center. Thus, $I/I_{\text{center}} = 0.404/4 = 0.101$.

35. **THINK** For complete destructive interference, we want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of π rad.

EXPRESS Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of π rad on reflection. If L is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where λ_c is the wavelength in the coating. If n is the index of refraction of the coating, $\lambda_c = \lambda/n$, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi$$

for L. Here m is an integer. The result is $L = \frac{(2m+1)\lambda}{4n}$.

ANALYZE To find the least thickness for which destructive interference occurs, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{600\times10^{-9}\,\text{m}}{4(1.25)} = 1.20\times10^{-7}\,\text{m}.$$

LEARN A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling.

36. (a) On both sides of the soap is a medium with lower index (air) and we are examining the reflected light, so the condition for strong reflection is Eq. 35-36. With lengths in nm,

$$\lambda = \frac{2n_2L}{m+\frac{1}{2}} = \begin{cases} 3360 & \text{for } m = 0 \\ 1120 & \text{for } m = 1 \\ 672 & \text{for } m = 2 \\ 480 & \text{for } m = 3 \\ 373 & \text{for } m = 4 \\ 305 & \text{for } m = 5 \end{cases}$$

from which we see the latter *four* values are in the given range.

(b) We now turn to Eq. 35-37 and obtain

$$\lambda = \frac{2n_2L}{m} = \begin{cases} 1680 & \text{for } m = 1 \\ 840 & \text{for } m = 2 \\ 560 & \text{for } m = 3 \\ 420 & \text{for } m = 4 \\ 336 & \text{for } m = 5 \end{cases}$$

from which we see the latter *three* values are in the given range.

37. Light reflected from the front surface of the coating suffers a phase change of π rad while light reflected from the back surface does not change phase. If L is the thickness of the coating, light reflected from the back surface travels a distance $2L$ farther than light reflected from the front surface. The difference in phase of the two waves is $2L(2\pi/\lambda_c) - \pi$, where λ_c is the wavelength in the coating. If λ is the wavelength in vacuum, then $\lambda_c = \lambda/n$, where n is the index of refraction of the coating. Thus, the phase difference is

$2nL(2\pi/\lambda) - \pi$. For fully constructive interference, this should be a multiple of 2π. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) - \pi = 2m\pi$$

for L. Here m is an integer. The solution is

$$L = \frac{(2m+1)\lambda}{4n} .$$

To find the smallest coating thickness, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{560\times10^{-9}\text{ m}}{4(2.00)} = 7.00\times10^{-8}\text{ m} .$$

38. (a) We are dealing with a thin film (material 2) in a situation where $n_1 > n_2 > n_3$, looking for strong *reflections*; the appropriate condition is the one expressed by Eq. 35-37. Therefore, with lengths in nm and $L = 500$ and $n_2 = 1.7$, we have

$$\lambda = \frac{2n_2L}{m} = \begin{cases} 1700 & \text{for } m = 1 \\ 850 & \text{for } m = 2 \\ 567 & \text{for } m = 3 \\ 425 & \text{for } m = 4 \end{cases}$$

from which we see the latter *two* values are in the given range. The longer wavelength (m=3) is $\lambda = 567$ nm.

(b) The shorter wavelength ($m = 4$) is $\lambda = 425$ nm.

(c) We assume the temperature dependence of the refraction index is negligible. From the proportionality evident in the part (a) equation, longer L means longer λ.

39. For constructive interference, we use Eq. 35-36:

$$2n_2L = (m+1/2)\lambda .$$

For the smallest value of L, let $m = 0$:

$$L_0 = \frac{\lambda/2}{2n_2} = \frac{624\text{nm}}{4(1.33)} = 117\text{nm} = 0.117\,\mu\text{m}.$$

(b) For the second smallest value, we set $m = 1$ and obtain

$$L_1 = \frac{(1+1/2)\lambda}{2n_2} = \frac{3\lambda}{2n_2} = 3L_0 = 3(0.1173\,\mu\text{m}) = 0.352\,\mu\text{m}.$$

40. The incident light is in a low index medium, the thin film of acetone has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, the condition

$$2L = \left(m + \tfrac{1}{2}\right)\frac{l}{n_2} \quad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. This is the same as Eq. 35-36, which was developed for the opposite situation (constructive interference) regarding a thin film surrounded on both sides by air (a very different context from the one in this problem). By analogy, we expect Eq. 35-37 to apply in this problem to reflection *maxima*. Thus, using Eq. 35-37 with $n_2 = 1.25$ and $\lambda = 700$ nm yields

$$L = 0,\ 280\,\text{nm},\ 560\,\text{nm},\ 840\,\text{nm},\ 1120\,\text{nm}, \ldots$$

for the first several m values. And the equation shown above (equivalent to Eq. 35-36) gives, with $\lambda = 600$ nm,

$$L = 120\,\text{nm}, 360\,\text{nm}, 600\,\text{nm}, 840\,\text{nm}, 1080\,\text{nm}, \ldots$$

for the first several m values. The lowest number these lists have in common is $L = 840$ nm.

41. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$, and the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}\ , \quad m = 0, 1, 2, \ldots$$

The second least thickness is ($m = 1$)

$$L = \left(1 + \frac{1}{2}\right)\frac{342\text{ nm}}{2(1.59)} = 161\text{ nm}\,.$$

42. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, \ldots$$

Thus, we get

$$\lambda = \begin{cases} 4Ln_2 = 4(285\text{ nm})(1.60) = 1824\text{ nm} \quad (m=0) \\ 4Ln_2/3 = 4(285\text{ nm})(1.60)/3 = 608\text{ nm} \quad (m=1) \end{cases}.$$

For the wavelength to be in the visible range, we choose $m = 1$ with $\lambda = 608$ nm.

43. When a thin film of thickness L and index of refraction n_2 is placed between materials 1 and 3 such that $n_1 > n_2$ and $n_3 > n_2$ where n_1 and n_3 are the indexes of refraction of the materials, the general condition for destructive interference for a thin film is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \quad \lambda = \frac{2Ln_2}{m}, \qquad m = 0, 1, 2, \ldots$$

where λ is the wavelength of light as measured in air. Thus, we have, for $m = 1$

$$\lambda = 2Ln_2 = 2(200\text{ nm})(1.40) = 560\text{ nm}.$$

44. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \qquad m = 0, 1, 2, \ldots$$

The second least thickness is ($m = 1$)

$$L = \left(1 + \frac{1}{2}\right)\frac{587\text{ nm}}{2(1.34)} = 329\text{ nm}.$$

45. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \qquad m = 0, 1, 2, \ldots$$

The third least thickness is ($m = 2$)

$$L = \left(2 + \frac{1}{2}\right)\frac{612\text{ nm}}{2(1.60)} = 478\text{ nm}.$$

46. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$, and the condition for destructive interference is

$$2L=\left(m+\frac{1}{2}\right)\frac{\lambda}{n_2}\quad\Rightarrow\lambda=\frac{4Ln_2}{2m+1},\quad m=0,1,2,...$$

Therefore,

$$\lambda=\begin{cases}4Ln_2=4(415\text{ nm})(1.59)=2639\text{ nm }\ (m=0)\\4Ln_2/3=4(415\text{ nm})(1.59)/3=880\text{ nm }\ (m=1)\\4Ln_2/5=4(415\text{ nm})(1.59)/5=528\text{ nm }\ (m=2)\end{cases}.$$

For the wavelength to be in the visible range, we choose m = 3 with $\lambda=528$ nm.

47. **THINK** For a complete destructive interference, we want the waves reflected from the front and back of material 2 of refractive index n_2 to differ in phase by an odd multiple of π rad.

EXPRESS In this setup, we have $n_2<n_1$, so there is no phase change from the first surface. On the other hand $n_2<n_3$, so there is a phase change of π rad from the second surface. Since the second wave travels an extra distance of 2L, the phase difference is

$$\phi=\frac{2\pi}{\lambda_2}(2L)+\pi$$

where $\lambda_2=\lambda/n_2$ is the wavelength in medium 2. The condition for destructive interference is

$$\frac{2\pi}{\lambda_2}(2L)+\pi=(2m+1)\pi,$$

or

$$2L=m\frac{\lambda}{n_2}\quad\Rightarrow\lambda=\frac{2Ln_2}{m},\quad m=0,1,2,...$$

ANALYZE Thus, we have

$$\lambda=\begin{cases}2Ln_2=2(380\text{ nm})(1.34)=1018\text{ nm }\ (m=1)\\Ln_2=(380\text{ nm})(1.34)=509\text{ nm }\ (m=2)\end{cases}$$

For the wavelength to be in the visible range, we choose m = 2 with $\lambda=509$ nm.

LEARN In this setup, the condition for *constructive* interference is

$$\frac{2\pi}{\lambda_2}(2L)+\pi=2m\pi,$$

or

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2}, \quad m = 0, 1, 2, ...$$

48. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0,1,2,...$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{632 \text{ nm}}{2(1.40)} = 339 \text{ nm}.$$

49. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0,1,2,...$$

The third least thickness is (m = 2)

$$L = \left(2 + \frac{1}{2}\right)\frac{382 \text{ nm}}{2(1.75)} = 273 \text{ nm}.$$

50. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$, and the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0,1,2,...$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{482 \text{ nm}}{2(1.46)} = 248 \text{ nm}.$$

51. **THINK** For a complete destructive interference, we want the waves reflected from the front and back of material 2 of refractive index n_2 to differ in phase by an odd multiple of π rad.

EXPRESS In this setup, we have $n_1 < n_2$ and $n_2 < n_3$, which means that both waves are incident on a medium of higher refractive index from a medium of lower refractive index.

Thus, in both cases, there is a phase change of π rad from both surfaces. Since the second wave travels an additional distance of $2L$, the phase difference is

$$\phi = \frac{2\pi}{\lambda_2}(2L)$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in medium 2. The condition for destructive interference is

$$\frac{2\pi}{\lambda_2}(2L) = (2m+1)\pi,$$

or

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, ...$$

ANALYZE Thus,

$$\lambda = \begin{cases} 4Ln_2 = 4(210 \text{ nm})(1.46) = 1226 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(210 \text{ nm})(1.46)/3 = 409 \text{ nm} \ \ (m=1) \end{cases}$$

For the wavelength to be in the visible range, we choose m = 1 with $\lambda = 409$ nm.

LEARN In this setup, the condition for *constructive* interference is

$$\frac{2\pi}{\lambda_2}(2L) = 2m\pi,$$

or

$$2L = m\frac{\lambda}{n_2}, \quad m = 0, 1, 2, ...$$

52. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, ...$$

Thus, we have

$$\lambda = \begin{cases} 4Ln_2 = 4(325 \text{ nm})(1.75) = 2275 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(325 \text{ nm})(1.75)/3 = 758 \text{ nm} \ \ (m=1) \\ 4Ln_2/5 = 4(325 \text{ nm})(1.75)/5 = 455 \text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose m = 2 with $\lambda = 455$ nm.

53. We solve Eq. 35-36 with n_2 = 1.33 and λ = 600 nm for m = 1, 2, 3,…:

$$L = 113\text{ nm}, 338\text{ nm}, 564\text{ nm}, 789\text{ nm}, \ldots$$

And, we similarly solve Eq. 35-37 with the same n_2 and λ = 450 nm:

$$L = 0, 169\text{ nm}, 338\text{ nm}, 508\text{ nm}, 677\text{ nm}, \ldots$$

The lowest number these lists have in common is L = 338 nm.

54. The situation is analogous to that treated in Sample Problem — "Thin-film interference of a coating on a glass lens," in the sense that the incident light is in a low index medium, the thin film of oil has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{l}{n_2} \quad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. With λ = 500 nm and n_2 = 1.30, the possible answers for L are

$$L = 96\text{ nm}, 288\text{ nm}, 481\text{ nm}, 673\text{ nm}, 865\text{ nm}, ...$$

And, with λ = 700 nm and the same value of n_2, the possible answers for L are

$$L = 135\text{ nm}, 404\text{ nm}, 673\text{ nm}, 942\text{ nm}, ...$$

The lowest number these lists have in common is L = 673 nm.

55. **THINK** The index of refraction of oil is greater than that of the air, but smaller than that of the water.

EXPRESS Let the indices of refraction of the air, oil and water be n_1, n_2, and n_3, respectively. Since $n_1 < n_2$ and $n_2 < n_3$, there is a phase change of π rad from both surfaces. Since the second wave travels an additional distance of $2L$, the phase difference is

$$\phi = \frac{2\pi}{\lambda_2}(2L)$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in the oil. The condition for constructive interference is

$$\frac{2\pi}{\lambda_2}(2L) = 2m\pi,$$

or

$$2L = m\frac{\lambda}{n_2}, \quad m = 0, 1, 2, ...$$

ANALYZE (a) For $m = 1, 2, ...,$ maximum reflection occurs for wavelengths

$$\lambda = \frac{2n_2L}{m} = \frac{2(1.20)(460\,\text{nm})}{m} = 1104\,\text{nm}\ ,\ 552\,\text{nm},\ 368\,\text{nm}...$$

We note that only the 552 nm wavelength falls within the visible light range.

(b) Maximum transmission into the water occurs for wavelengths for which reflection is a minimum. The condition for such destructive interference is given by

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \Rightarrow \lambda = \frac{4n_2L}{2m+1}$$

which yields λ = 2208 nm, 736 nm, 442 nm … for the different values of m. We note that only the 442 nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

LEARN A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling. Otherwise, there is no phase change. Note that refraction at an interface does not cause a phase shift.

56. For constructive interference (which is obtained for λ = 600 nm) in this circumstance, we require

$$2L = \frac{k}{2}\lambda_n = \frac{k\lambda}{2n}$$

where k = some positive odd integer and n is the index of refraction of the thin film. Rearranging and plugging in L = 272.7 nm and the wavelength value, this gives

$$n = \frac{k\lambda}{4L} = \frac{k(600\text{ nm})}{4(272.7\text{ nm})} = \frac{k}{1.818} = 0.55k\,.$$

Since we expect $n > 1$, then $k = 1$ is ruled out. However, $k = 3$ seems reasonable, since it leads to $n = 1.65$, which is close to the "typical" values found in Table 34-1. Taking this to be the correct index of refraction for the thin film, we now consider the destructive interference part of the question. Now we have $2L = (\text{integer})\lambda_{\text{dest}}/n$. Thus,

$$\lambda_{\text{dest}} = (900\text{ nm})/(\text{integer}).$$

We note that setting the integer equal to 1 yields a λ_{dest} value outside the range of the visible spectrum. A similar remark holds for setting the integer equal to 3. Thus, we set it equal to 2 and obtain $\lambda_{\text{dest}} = 450$ nm.

57. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}\ , \quad m = 0,1,2,\ldots$$

Therefore,

$$\lambda = \begin{cases} 4Ln_2 = 4(285 \text{ nm})(1.60) = 1824 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(415 \text{ nm})(1.59)/3 = 608 \text{ nm} \ \ (m=1) \end{cases}$$

For the wavelength to be in the visible range, we choose m = 1 with $\lambda = 608$ nm.

58. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}\ , \quad m = 0,1,2,\ldots$$

The third least thickness is (m = 2)

$$L = \left(2 + \frac{1}{2}\right)\frac{382 \text{ nm}}{2(1.75)} = 273 \text{ nm}\,.$$

59. **THINK** Maximum transmission means constructive interference.

EXPRESS As shown in Fig. 35-43, one wave travels a distance of 2L further than the other. This wave is reflected twice, once from the back surface (between materials 2 and 3), and once from the front surface (between materials 1 and 2). Since $n_2 > n_3$, there is no phase change at the back-surface reflection. On the other hand, since $n_2 < n_1$, there is a phase change of π rad due to the front-surface reflection. The phase difference of the two waves as they leave material 2 is

$$\phi = \frac{2\pi}{\lambda_2}(2L) + \pi$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in material 2. The condition for constructive interference is

$$\frac{2\pi}{\lambda_2}(2L)+\pi = 2m\pi,$$

or

$$2L = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,...$$

ANALYZE Thus, we have

$$\lambda = \begin{cases} 4Ln_2 = 4(415\text{ nm})(1.59) = 2639\text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(415\text{ nm})(1.59)/3 = 880\text{ nm} \ \ (m=1) \\ 4Ln_2/5 = 4(415\text{ nm})(1.59)/5 = 528\text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose $m = 2$ with $\lambda = 528$ nm.

LEARN similarly, the condition for destructive interference is

$$\frac{2\pi}{\lambda_2}(2L)+\pi = (2m+1)\pi,$$

or

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{2Ln_2}{m}, \quad m = 0, 1, 2,...$$

60. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{2Ln_2}{m}, \quad m = 0,1,2,...$$

Thus, we obtain

$$\lambda = \begin{cases} 2Ln_2 = 2(380\text{ nm})(1.34) = 1018\text{ nm} \ \ (m=1) \\ Ln_2 = (380\text{ nm})(1.34) = 509\text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose $m = 2$ with $\lambda = 509$ nm.

61. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,...$$

Therefore,

$$\lambda = \begin{cases} 4Ln_2 = 4(325 \text{ nm})(1.75) = 2275 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(415 \text{ nm})(1.59)/3 = 758 \text{ nm} \ \ (m=1) \\ 4Ln_2/5 = 4(415 \text{ nm})(1.59)/5 = 455 \text{ nm} \ \ (m=2) \end{cases}$$

For the wavelength to be in the visible range, we choose $m = 2$ with $\lambda = 455$ nm.

62. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m+\frac{1}{2}\right)\frac{\lambda}{2n_2} \ , \quad m = 0,1,2,...$$

The second least thickness is ($m = 1$)

$$L = \left(1+\frac{1}{2}\right)\frac{342 \text{ nm}}{2(1.59)} = 161 \text{ nm} .$$

63. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m+\frac{1}{2}\right)\frac{\lambda}{2n_2} \ , \quad m = 0,1,2,...$$

The second least thickness is ($m = 1$)

$$L = \left(1+\frac{1}{2}\right)\frac{482 \text{ nm}}{2(1.46)} = 248 \text{ nm} .$$

64. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,...$$

Thus, we have

$$\lambda = \begin{cases} 4Ln_2 = 4(210 \text{ nm})(1.46) = 1226 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(210 \text{ nm})(1.46)/3 = 409 \text{ nm} \ \ (m=1) \end{cases}$$

For the wavelength to be in the visible range, we choose $m = 1$ with $\lambda = 409$ nm.

65. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{632 \text{ nm}}{2(1.40)} = 339 \text{ nm}.$$

66. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{2Ln_2}{m}, \quad m = 0, 1, 2, \ldots$$

Thus, we have (with m =1)

$$\lambda = 2Ln_2 = 2(200 \text{ nm})(1.40) = 560 \text{ nm}.$$

67. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The second least thickness is (m = 1)

$$L = \left(1 + \frac{1}{2}\right)\frac{587 \text{ nm}}{2(1.34)} = 329 \text{ nm}.$$

68. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The third least thickness is (m = 2)

$$L = \left(2 + \frac{1}{2}\right)\frac{612 \text{ nm}}{2(1.60)} = 478 \text{ nm}.$$

69. Assume the wedge-shaped film is in air, so the wave reflected from one surface undergoes a phase change of π rad while the wave reflected from the other surface does not. At a place where the film thickness is L, the condition for fully constructive interference is $2nL = \left(m + \frac{1}{2}\right)\lambda$, where n is the index of refraction of the film, λ is the wavelength in vacuum, and m is an integer. The ends of the film are bright. Suppose the end where the film is narrow has thickness L_1 and the bright fringe there corresponds to $m = m_1$. Suppose the end where the film is thick has thickness L_2 and the bright fringe there corresponds to $m = m_2$. Since there are ten bright fringes, $m_2 = m_1 + 9$. Subtract $2nL_1 = \left(m_1 + \frac{1}{2}\right)\lambda$ from $2nL_2 = \left(m_1 + 9 + \frac{1}{2}\right)\lambda$ to obtain $2n\,\Delta L = 9\lambda$, where $\Delta L = L_2 - L_1$ is the change in the film thickness over its length. Thus,

$$\Delta L = \frac{9\lambda}{2n} = \frac{9\left(630 \times 10^{-9}\text{ m}\right)}{2\left(1.50\right)} = 1.89 \times 10^{-6}\text{ m}.$$

70. (a) The third sentence of the problem implies $m_o = 9.5$ in $2\,d_o = m_o\lambda$ initially. Then, $\Delta t = 15$ s later, we have $m' = 9.0$ in $2d' = m'\lambda$. This means

$$|\Delta d| = d_o - d' = \frac{1}{2}(\,m_o\lambda - m'\lambda) = 155\text{ nm}.$$

Thus, $|\Delta d|$ divided by Δt gives 10.3 nm/s.

(b) In this case, $m_f = 6$ so that

$$d_o - d_f = \frac{1}{2}(m_o\lambda - m_f\lambda) = \frac{7}{4}\lambda = 1085\text{ nm} = 1.09\ \mu\text{m}.$$

71. The (vertical) change between the center of one dark band and the next is

$$\Delta y = \frac{\lambda}{2} = \frac{500\text{ nm}}{2} = 250\text{ nm} = 2.50 \times 10^{-4}\text{ mm}.$$

Thus, with the (horizontal) separation of dark bands given by $\Delta x = 1.2$ mm, we have

$$\theta \approx \tan\theta = \frac{\Delta y}{\Delta x} = 2.08 \times 10^{-4}\text{ rad}.$$

Converting this angle into degrees, we arrive at $\theta = 0.012°$.

72. We apply Eq. 35-27 to both scenarios: $m = 4001$ and $n_2 = n_{\text{air}}$, and $m = 4000$ and $n_2 = n_{\text{vacuum}} = 1.00000$:

$$2L = \left(4001\right)\frac{\lambda}{n_{\text{air}}} \quad\text{and}\quad 2L = \left(4000\right)\frac{\lambda}{1.00000}.$$

Since the 2*L* factor is the same in both cases, we set the right-hand sides of these expressions equal to each other and cancel the wavelength. Finally, we obtain

$$n_{\text{air}} = (1.00000)\frac{4001}{4000} = 1.00025.$$

We remark that this same result can be obtained starting with Eq. 35-43 (which is developed in the textbook for a somewhat different situation) and using Eq. 35-42 to eliminate the $2L/\lambda$ term.

73. **THINK** A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling.

EXPRESS Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by π rad. At a place where the thickness of the air film is *L*, the condition for fully constructive interference is $2L = (m + \frac{1}{2})\lambda$ where λ (= 683 nm) is the wavelength and *m* is an integer.

ANALYZE For $L = 48\ \mu$m, we find the value of m to be

$$m = \frac{2L}{\lambda} - \frac{1}{2} = \frac{2(4.80\times10^{-5}\,\text{m})}{683\times10^{-9}\,\text{m}} - \frac{1}{2} = 140.$$

At the thin end of the air film, there is a bright fringe. It is associated with $m = 0$. There are, therefore, 140 bright fringes in all.

LEARN The number of bright fringes increases with *L*, but decreases with λ.

74. By the condition $m\lambda = 2y$ where *y* is the thickness of the air film between the plates directly underneath the middle of a dark band), the edges of the plates (the edges where they are not touching) are $y = 8\lambda/2 = 2400$ nm apart (where we have assumed that the *middle* of the ninth dark band is at the edge). Increasing that to $y' = 3000$ nm would correspond to $m' = 2y'/\lambda = 10$ (counted as the eleventh dark band, since the first one corresponds to $m = 0$). There are thus 11 dark fringes along the top plate.

75. **THINK** The formation of Newton's rings is due to the interference between the rays reflected from the flat glass plate and the curved lens surface.

EXPRESS Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not.

At a place where the thickness of the wedge is d, the condition for a maximum in intensity is $2d = \left(m+\frac{1}{2}\right)\lambda$, where λ is the wavelength in air and m is an integer. Therefore,

$$d = (2m + 1)\lambda/4.$$

ANALYZE As the geometry of Fig. 35-46 shows, $d = R - \sqrt{R^2 - r^2}$, where R is the radius of curvature of the lens and r is the radius of a Newton's ring. Thus, $\left(2m+1\right)\lambda/4 = R - \sqrt{R^2 - r^2}$. First, we rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4}.$$

Next, we square both sides, rearrange to solve for r^2, then take the square root. We get

$$r = \sqrt{\frac{\left(2m+1\right)R\lambda}{2} - \frac{\left(2m+1\right)^2\lambda^2}{16}}.$$

If R is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{\left(2m+1\right)R\lambda}{2}}.$$

LEARN Similarly, the radii of the dark fringes are given by

$$r = \sqrt{mR\lambda}.$$

76. (a) We find m from the last formula obtained in Problem 35-75:

$$m = \frac{r^2}{R\lambda} - \frac{1}{2} = \frac{\left(10\times10^{-3}\,\text{m}\right)^2}{\left(5.0\,\text{m}\right)\left(589\times10^{-9}\,\text{m}\right)} - \frac{1}{2}$$

which (rounding down) yields $m = 33$. Since the first bright fringe corresponds to $m = 0$, $m = 33$ corresponds to the thirty-fourth bright fringe.

(b) We now replace λ by $\lambda_n = \lambda/n_w$. Thus,

$$m_n = \frac{r^2}{R\lambda_n} - \frac{1}{2} = \frac{n_w r^2}{R\lambda} - \frac{1}{2} = \frac{(1.33)\left(10\times10^{-3}\,\text{m}\right)^2}{(5.0\,\text{m})\left(589\times10^{-9}\,\text{m}\right)} - \frac{1}{2} = 45.$$

This corresponds to the forty-sixth bright fringe (see the remark at the end of our solution in part (a)).

77. We solve for m using the formula $r = \sqrt{(2m+1)R\lambda/2}$ obtained in Problem 35-75 and find $m = r^2/R\lambda - 1/2$. Now, when m is changed to $m + 20$, r becomes r', so

$$m + 20 = r'^2/R\lambda - 1/2.$$

Taking the difference between the two equations above, we eliminate m and find

$$R = \frac{r'^2 - r^2}{20\lambda} = \frac{(0.368\,\text{cm})^2 - (0.162\,\text{cm})^2}{20(546\times10^{-7}\,\text{cm})} = 100\,\text{cm}.$$

78. The time to change from one minimum to the next is $\Delta t = 12$ s. This involves a change in thickness $\Delta L = \lambda/2n_2$ (see Eq. 35-37), and thus a change of volume

$$\Delta V = \pi r^2 \Delta L = \frac{\pi r^2 \lambda}{2n_2} \quad \Rightarrow \quad \frac{dV}{dt} = \frac{\pi r^2 \lambda}{2n_2 \Delta t} = \frac{\pi(0.0180)^2\,(550 \times 10^{-9})}{2(1.40)\,(12)}$$

using SI units. Thus, the rate of change of volume is 1.67×10^{-11} m^3/s.

79. A shift of one fringe corresponds to a change in the optical path length of one wavelength. When the mirror moves a distance d, the path length changes by $2d$ since the light traverses the mirror arm twice. Let N be the number of fringes shifted. Then, $2d = N\lambda$ and

$$\lambda = \frac{2d}{N} = \frac{2(0.233\times10^{-3}\,\text{m})}{792} = 5.88\times10^{-7}\,\text{m} = 588\,\text{nm}.$$

80. According to Eq. 35-43, the number of fringes shifted (ΔN) due to the insertion of the film of thickness L is $\Delta N = (2L/\lambda)\,(n-1)$. Therefore,

$$L = \frac{\lambda \Delta N}{2(n-1)} = \frac{(589\,\text{nm})(7.0)}{2(1.40-1)} = 5.2\,\mu\text{m}.$$

81. **THINK** The wavelength in air is different from the wavelength in vacuum.

EXPRESS Let ϕ_1 be the phase difference of the waves in the two arms when the tube has air in it, and let ϕ_2 be the phase difference when the tube is evacuated. If λ is the wavelength in vacuum, then the wavelength in air is λ/n, where n is the index of refraction of air. This means

$$\phi_1 - \phi_2 = 2L\left[\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda}\right] = \frac{4\pi(n-1)L}{\lambda}$$

where L is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror. Each shift by one fringe corresponds to a change in phase of 2π rad, so if the interference pattern shifts by N fringes as the tube is evacuated, then

$$\frac{4\pi(n-1)L}{\lambda} = 2N\pi .$$

ANALYZE Solving for n, we obtain

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500\times 10^{-9}\text{ m})}{2(5.0\times 10^{-2}\text{ m})} = 1.00030 .$$

LEARN The interferometer provides an accurate way to measure the refractive index of the air (and other gases as well).

82. We apply Eq. 35-42 to both wavelengths and take the difference:

$$N_1 - N_2 = \frac{2L}{\lambda_1} - \frac{2L}{\lambda_2} = 2L\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right).$$

We now require $N_1 - N_2 = 1$ and solve for L:

$$L = \frac{1}{2}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)^{-1} = \frac{1}{2}\left(\frac{1}{588.9950\text{ nm}} - \frac{1}{589.5924\text{ nm}}\right)^{-1} = 2.91\times 10^5\text{ nm} = 291\ \mu\text{m}.$$

83. (a) The path length difference between rays 1 and 2 is $7d - 2d = 5d$. For this to correspond to a half-wavelength requires $5d = \lambda/2$, so that $d = 50.0$ nm.

(b) The above requirement becomes $5d = \lambda/2n$ in the presence of the solution, with $n = 1.38$. Therefore, $d = 36.2$ nm.

84. (a) The minimum path length difference occurs when both rays are nearly vertical. This would correspond to a point as far up in the picture as possible. Treating the screen as if it extended forever, then the point is at $y = \infty$.

(b) When both rays are nearly vertical, there is no path length difference between them. Thus at $y = \infty$, the phase difference is $\phi = 0$.

(c) At $y = 0$ (where the screen crosses the x axis) both rays are horizontal, with the ray from S_1 being longer than the one from S_2 by distance d.

(d) Since the problem specifies d = 6.00λ, then the phase difference here is ϕ = 6.00 wavelengths and is at its maximum value.

(e) With $D = 20\lambda$, use of the Pythagorean theorem leads to

$$\phi = \frac{L_1 - L_2}{\lambda} = \frac{\sqrt{d^2 + (d + D)^2} - \sqrt{d^2 + D^2}}{\lambda} = 5.80$$

which means the rays reaching the point $y = d$ have a phase difference of roughly 5.8 wavelengths.

(f) The result of the previous part is "intermediate" — closer to 6 (constructive interference) than to $5\frac{1}{2}$ (destructive interference).

85. **THINK** The angle between adjacent fringes depends the wavelength of the light and the distance between the slits.

EXPRESS The angular positions of the maxima of a two-slit interference pattern are given by $\Delta L = d\sin\theta = m\lambda$, where ΔL is the path-length difference, d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, sin θ may be approximated by θ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$. When the arrangement is immersed in water, the wavelength changes to $\lambda' = \lambda/n$, and the equation above becomes

$$\Delta\theta' = \frac{\lambda'}{d}.$$

ANALYZE Dividing the equation by $\Delta\theta = \lambda/d$, we obtain

$$\frac{\Delta\theta'}{\Delta\theta} = \frac{\lambda'}{\lambda} = \frac{1}{n}.$$

Therefore, with n = 1.33 and $\Delta\theta$ = 0.30°, we find $\Delta\theta'$ = 0.23°.

LEARN The angular separation decreases with increasing index of refraction; the greater the value of n, the smaller the value of $\Delta\theta$.

86. (a) The graph shows part of a periodic pattern of half-cycle "length" Δn = 0.4. Thus if we set $n = 1.0 + 2\Delta n$ = 1.8 then the maximum at n = 1.0 should repeat itself there.

(b) Continuing the reasoning of part (a), adding another half-cycle "length" we get $1.8 + \Delta n = 2.2$ for the answer.

(c) Since Δn = 0.4 represents a half-cycle, then $\Delta n/2$ represents a quarter-cycle. To accumulate a total change of 2.0 – 1.0 = 1.0 (see problem statement), then we need $2\Delta n + \Delta n/2 = 5/4^{\text{th}}$ of a cycle, which corresponds to 1.25 wavelengths.

87. **THINK** For a completely destructive interference, the intensity produced by the two waves is zero.

EXPRESS When the interference between two waves is completely destructive, their phase difference is given by

$$\phi = (2m+1)\pi, \;\; m = 0, 1, 2, ...$$

The equivalent condition is that their path-length difference is an odd multiple of $\lambda/2$, where λ is the wavelength of the light.

ANALYZE (a) Looking at Fig. 35-52, we see that half of the periodic pattern is of length ΔL = 750 nm (judging from the maximum at x = 0 to the minimum at x = 750 nm); this suggests that $\Delta L = \lambda/2$, and the wavelength (the full length of the periodic pattern) is $\lambda = 2\Delta L$ = 1500 nm. Thus, a maximum should be reached again at x = 1500 nm (and at x = 3000nm, x = 4500 nm, …).

(b) From our discussion in part (a), we expect a minimum to be reached at odd multiple of $\lambda/2$, or x = 750 nm + n(1500 nm), where n = 1, 2, 3 … . For instance, for n = 1 we would find the minimum at x = 2250 nm.

(c) With λ = 1500 nm (found in part (a)), we can express x = 1200 nm as x = 1200/1500 = 0.80 wavelength.

LEARN For a completely destructive interference, the phase difference between two light sources is an odd multiple of π; however, for a completely constructive interference, the phase difference is a multiple of 2π.

88. (a) The difference in wavelengths, with and without the n = 1.4 material, is found using Eq. 35-9:

$$\Delta N = (n-1)\frac{L}{\lambda} = 1.143\,.$$

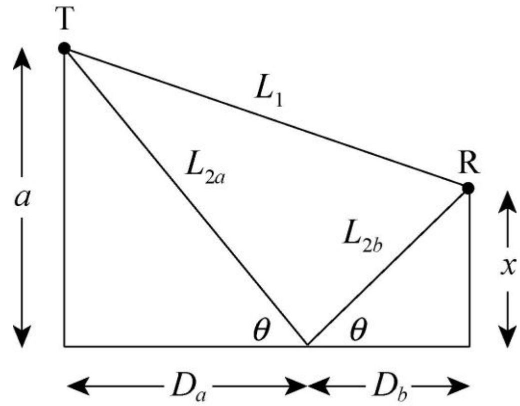
The result is equal to a phase shift of (1.143)(360°) = 411.4°, or

(b) more meaningfully, a shift of 411.4° – 360° = 51.4°.

89. **THINK** Since the index of refraction of water is greater than that of air, the wave that is reflected from the water surface suffers a phase change of π rad on reflection.

EXPRESS Suppose the wave that goes directly to the receiver travels a distance L_1 and the reflected wave travels a distance L_2. The last wave suffers a phase change on reflection of half a wavelength since water has higher refractive index than air. To obtain constructive interference at the receiver, the difference $L_2 - L_1$ must be an odd multiple of a half wavelength.

ANALYZE Consider the diagram below.



The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a/\tan\theta$. The right triangle on the right, formed by the vertical line from the water to the receiver R, the reflected ray, and the water line leads to $D_b = x/\tan\theta$. Since $D_a + D_b = D$,

$$\tan\theta = \frac{a+x}{D} .$$

We use the identity $\sin^2\theta = \tan^2\theta / (1 + \tan^2\theta)$ to show that

$$\sin\theta = (a+x)/\sqrt{D^2+(a+x)^2} .$$

This means

$$L_{2a} = \frac{a}{\sin\theta} = \frac{a\sqrt{D^2+(a+x)^2}}{a+x}$$

and

$$L_{2b} = \frac{x}{\sin\theta} = \frac{x\sqrt{D^2+(a+x)^2}}{a+x}.$$

Therefore,

$$L_2 = L_{2a} + L_{2b} = \frac{(a+x)\sqrt{D^2+(a+x)^2}}{a+x} = \sqrt{D^2+(a+x)^2} .$$

Using the binomial theorem, with D^2 large and $a^2 + x^2$ small, we approximate this expression: $L_2 \approx D + (a+x)^2/2D$. The distance traveled by the direct wave is

$L_1 = \sqrt{D^2 + (a-x)^2}$. Using the binomial theorem, we approximate this expression: $L_1 \approx D + (a-x)^2/2D$. Thus,

$$L_2 - L_1 \approx D + \frac{a^2 + 2ax + x^2}{2D} - D - \frac{a^2 - 2ax + x^2}{2D} = \frac{2ax}{D} .$$

Setting this equal to $(m + \frac{1}{2})\lambda$, where m is zero or a positive integer, we find $x = (m + \frac{1}{2})(\lambda D/2a)$.

LEARN Similarly, the condition for destructive interference is

$$L_2 - L_1 \approx \frac{2ax}{D} = m\lambda,$$

or

$$x = m\frac{\lambda D}{2a}, \quad m = 0, 1, 2, ...$$

90. (a) Since P_1 is equidistant from S_1 and S_2 we conclude the sources are not in phase with each other. Their phase difference is $\Delta\phi_{\text{source}} = 0.60\ \pi$ rad, which may be expressed in terms of "wavelengths" (thinking of the $\lambda \Leftrightarrow 2\pi$ correspondence in discussing a full cycle) as

$$\Delta\phi_{\text{source}} = (0.60\ \pi / 2\pi)\ \lambda = 0.3\ \lambda$$

(with S_2 "leading" as the problem states). Now S_1 is closer to P_2 than S_2 is. Source S_1 is 80 nm ($\Leftrightarrow$ 80/400 $\lambda = 0.2\ \lambda$) from P_2 while source S_2 is 1360 nm ($\Leftrightarrow$ 1360/400 $\lambda = 3.4\ \lambda$) from P_2. Here we find a difference of $\Delta\phi_{\text{path}} = 3.2\ \lambda$ (with S_1 "leading" since it is closer). Thus, the net difference is

$$\Delta\phi_{\text{net}} = \Delta\phi_{\text{path}} - \Delta\phi_{\text{source}} = 2.90\ \lambda,$$

or 2.90 wavelengths.

(b) A whole number (like 3 wavelengths) would mean fully constructive, so our result is of the following nature: intermediate, but close to fully constructive.

91. (a) Applying the law of refraction, we obtain $\sin\theta_2 / \sin\theta_1 = \sin\theta_2 / \sin 30° = v_s/v_d$. Consequently,

$$\theta_2 = \sin^{-1}\left(\frac{v_s \sin 30°}{v_d}\right) = \sin^{-1}\left[\frac{(3.0\,\text{m/s})\sin 30°}{4.0\,\text{m/s}}\right] = 22°.$$

(b) The angle of incidence is gradually reduced due to refraction, such as shown in the calculation above (from 30° to 22°). Eventually after several refractions, θ_2 will be virtually zero. This is why most waves come in normal to a shore.

92. When the depth of the liquid (L_{liq}) is zero, the phase difference ϕ is 60 wavelengths; this must equal the difference between the number of wavelengths in length L = 40 μm (since the liquid initially fills the hole) of the plastic (for ray r_1) and the number in that same length of the air (for ray r_2). That is,

$$\frac{Ln_{\text{plastic}}}{\lambda} - \frac{Ln_{\text{air}}}{\lambda} = 60\,.$$

(a) Since $\lambda = 400 \times 10^{-9}$ m and $n_{\text{air}} = 1$ (to good approximation), we find $n_{\text{plastic}} = 1.6$.

(b) The slope of the graph can be used to determine n_{liq}, but we show an approach more closely based on the above equation:

$$\frac{Ln_{\text{plastic}}}{\lambda} - \frac{Ln_{\text{liq}}}{\lambda} = 20$$

which makes use of the leftmost point of the graph. This readily yields $n_{\text{liq}} = 1.4$.

93. **THINK** Knowing the slit separation and the distance between interference fringes allows us to calculate the wavelength of the light used.

EXPRESS The condition for a minimum in the two-slit interference pattern is $d \sin\theta = (m + ½)\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = (m + ½)\lambda/d$, and the distance from the minimum to the central fringe is

$$y = D\tan\theta \approx D\sin\theta \approx D\theta = \left(m + \frac{1}{2}\right)\frac{D\lambda}{d}\,,$$

where D is the distance from the slits to the screen. For the first minimum m = 0 and for the tenth one, m = 9. The separation is

$$\Delta y = \left(9 + \frac{1}{2}\right)\frac{D\lambda}{d} - \frac{1}{2}\frac{D\lambda}{d} = \frac{9D\lambda}{d}\,.$$

ANALYZE We solve for the wavelength:

$$\lambda = \frac{d\Delta y}{9D} = \frac{(0.15\times10^{-3}\,\text{m})(18\times10^{-3}\,\text{m})}{9(50\times10^{-2}\,\text{m})} = 6.0\times10^{-7}\,\text{m} = 600\text{ nm}.$$

LEARN The distance between two adjacent dark fringes, one associated with the integer *m* and the other associated with the integer *m* + 1, is

$$\Delta y = D\theta = D\lambda/d.$$

94. A light ray traveling directly along the central axis reaches the end in time

$$t_{\text{direct}} = \frac{L}{v_1} = \frac{n_1 L}{c}.$$

For the ray taking the critical zig-zag path, only its velocity component along the core axis direction contributes to reaching the other end of the fiber. That component is $v_1 \cos\theta'$, so the time of travel for this ray is

$$t_{\text{zig zag}} = \frac{L}{v_1 \cos\theta'} = \frac{n_1 L}{c\sqrt{1-(\sin\theta/n_1)^2}}$$

using results from the previous solution. Plugging in $\sin\theta = \sqrt{n_1^2 - n_2^2}$ and simplifying, we obtain

$$t_{\text{zig zag}} = \frac{n_1 L}{c(n_2/n_1)} = \frac{n_1^2 L}{n_2 c}.$$

The difference is

$$\Delta t = t_{\text{zig zag}} - t_{\text{direct}} = \frac{n_1^2 L}{n_2 c} - \frac{n_1 L}{c} = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right).$$

With $n_1 = 1.58$, $n_2 = 1.53$, and $L = 300$ m, we obtain

$$\Delta t = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right) = \frac{(1.58)(300\text{ m})}{3.0\times10^8\text{ m/s}}\left(\frac{1.58}{1.53} - 1\right) = 5.16\times10^{-8}\text{ s} = 51.6\text{ ns}.$$

95. **THINK** The dark band corresponds to a completely destructive interference.

EXPRESS When the interference between two waves is completely destructive, their phase difference is given by

$$\phi = (2m+1)\pi,\ \ m = 0, 1, 2, \ldots$$

The equivalent condition is that their path-length difference is an odd multiple of $\lambda/2$, where λ is the wavelength of the light.

ANALYZE (a) A path length difference of λ/2 produces the first dark band, of 3λ/2 produces the second dark band, and so on. Therefore, the fourth dark band corresponds to a path length difference of 7λ/2 = 1750 nm = 1.75 μm.

(b) In the small angle approximation (which we assume holds here), the fringes are equally spaced, so that if Δ*y* denotes the distance from one maximum to the next, then the distance from the middle of the pattern to the fourth dark band must be 16.8 mm = 3.5 Δ*y*. Therefore, we obtain Δ*y* = (16.8 mm)/3.5 = 4.8 mm.

LEARN The distance from the *m*th maximum to the central fringe is

$$y_{\text{bright}} = D\tan\theta \approx D\sin\theta \approx D\theta = m\frac{D\lambda}{d}.$$

Similarly, the distance from the *m*th minimum to the central fringe is

$$y_{\text{dark}} = \left(m+\frac{1}{2}\right)\frac{D\lambda}{d}.$$

96. We use the formula obtained in Sample Problem — "Thin-film interference of a coating on a glass lens:"

$$L_{\min} = \frac{\lambda}{4n_2} = \frac{\lambda}{4(1.25)} = 0.200\lambda \;\Rightarrow\; \frac{L_{\min}}{\lambda} = 0.200.$$

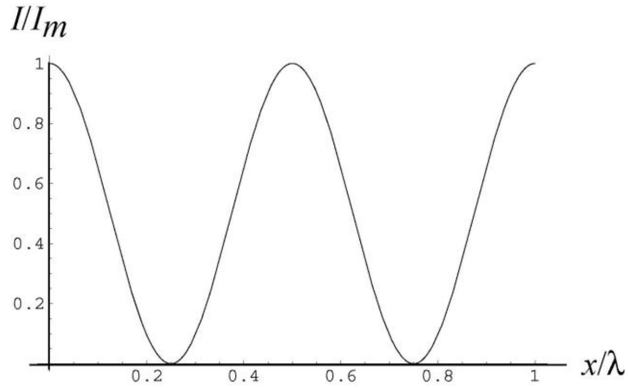
97. **THINK** The intensity of the light observed in the interferometer depends on the phase difference between the two waves.

EXPRESS Let the position of the mirror measured from the point at which $d_1 = d_2$ be *x*. We assume the beam-splitting mechanism is such that the two waves interfere constructively for *x* = 0 (with some beam-splitters, this would not be the case). We can adapt Eq. 35-23 to this situation by incorporating a factor of 2 (since the interferometer utilizes directly reflected light in contrast to the double-slit experiment) and eliminating the $\sin\theta$ factor. Thus, the path difference is 2*x*, and the phase difference between the two light paths is $\Delta\phi = 2(2\pi x/\lambda) = 4\pi x/\lambda$.

ANALYZE From Eq. 35-22, we see that the intensity is proportional to $\cos^2(\Delta\phi/2)$. Thus, writing $4I_0$ as I_m, we find

$$I = I_m\cos^2\left(\frac{\Delta\phi}{2}\right) = I_m\cos^2\left(\frac{2\pi x}{\lambda}\right).$$

LEARN The intensity I/I_m as a function of x/λ is plotted below.



From the figure, we see that the intensity is at a maximum when

$$x = \frac{m}{2}\lambda, \quad m = 0, 1, 2, \ldots$$

Similarly, the condition for minima is

$$x = \frac{1}{4}(2m+1)\lambda, \quad m = 0, 1, 2, \ldots$$

98. We note that ray 1 travels an extra distance $4L$ more than ray 2. For constructive interference (which is obtained for λ = 620 nm) we require

$$4L = m\lambda \qquad \text{where } m = \text{some positive integer.}$$

For destructive interference (which is obtained for λ' = 4196 nm) we require

$$4L = \frac{k}{2}\lambda' \qquad \text{where } k = \text{some positive \underline{odd} integer.}$$

Equating these two equations (since their left-hand sides are equal) and rearranging, we obtain

$$k = 2\,m\,\frac{\lambda}{\lambda'} = 2\,m\,\frac{620}{496} = 2.5\,m\,.$$

We note that this condition is satisfied for $k = 5$ and $m = 2$. It is satisfied for some larger values, too, but recalling that we want the least possible value for L, we choose the solution set $(k, m) = (5, 2)$. Plugging back into either of the equations above, we obtain the distance L:

$$4L = 2\lambda \quad \Rightarrow \quad L = \frac{\lambda}{2} = 310.0 \text{ nm}\,.$$

99. (a) Straightforward application of Eq. 35-3 $n = c/v$ and $v = \Delta x/\Delta t$ yields the result: pistol 1 with a time equal to $\Delta t = n\Delta x/c = 42.0 \times 10^{-12}$ s = 42.0 ps.

(b) For pistol 2, the travel time is equal to 42.3×10^{-12} s.

(c) For pistol 3, the travel time is equal to 43.2×10^{-12} s.

(d) For pistol 4, the travel time is equal to 41.8×10^{-12} s.

(e) We see that the blast from pistol 4 arrives first.

100. We use Eq. 35-36 for constructive interference: $2n_2L = (m + 1/2)\lambda$, or

$$\lambda = \frac{2n_2L}{m+1/2} = \frac{2(1.50)(410\text{ nm})}{m+1/2} = \frac{1230\text{ nm}}{m+1/2},$$

where m = 0, 1, 2, …. The only value of m which, when substituted into the equation above, would yield a wavelength that falls within the visible light range is m = 1. Therefore,

$$\lambda = \frac{1230\text{ nm}}{1+1/2} = 492\text{ nm}.$$

101. In the case of a distant screen the angle θ is close to zero so $\sin\theta \approx \theta$. Thus from Eq. 35-14,

$$\Delta\theta \approx \Delta\sin\theta = \Delta\left(\frac{m\lambda}{d}\right) = \frac{\lambda}{d}\Delta m = \frac{\lambda}{d},$$

or $d \approx \lambda/\Delta\theta = 589 \times 10^{-9}$ m/0.018 rad $= 3.3 \times 10^{-5}$ m = 33 μm.

102. We note that $\Delta\phi = 60° = \frac{\pi}{3}$ rad. The phasors rotate with constant angular velocity

$$\omega = \frac{\Delta\phi}{\Delta t} = \frac{\pi/3\text{ rad}}{2.5\times10^{-16}\text{ s}} = 4.19\times10^{15}\text{ rad/s}.$$

Since we are working with light waves traveling in a medium (presumably air) where the wave speed is approximately c, then $kc = \omega$ (where $k = 2\pi/\lambda$), which leads to

$$\lambda = \frac{2\pi c}{\omega} = 450\text{ nm}.$$

103. (a) Each wave is incident on a medium of higher index of refraction from a medium of lower index (air to oil, and oil to water), so both suffer phase changes of π rad on reflection. If L is the thickness of the oil, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_o)$, where λ_o is the wavelength in oil. If n is the index of refraction of the oil, λ_o =

λ/n, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. The conditions for constructive and destructive interferences are

$$\text{constructive}: 2nL\left(\frac{2\pi}{\lambda}\right) = 2m\pi \quad \Rightarrow \quad 2nL = m\lambda, \quad m = 0,1,2,\ldots$$

$$\text{destructive}: 2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi \quad \Rightarrow 2nL = \left(m+\tfrac{1}{2}\right)\lambda, \quad m = 0,1,2,\ldots$$

Near the rim of the drop, $L < \lambda/4n$, so only the condition for constructive interference with $m = 0$ can be met. So the outer (thinnest) region is bright.

(b) The third band from the rim corresponds to $2nL = 3\lambda/2$. Thus, the film thickness there is

$$L = \frac{3\lambda}{2n} = \frac{3(475\text{ nm})}{2(1.20)} = 594\text{ nm}.$$

(c) The primary reason why colors gradually fade and then disappear as the oil thickness increases is because the colored bands begin to overlap too much to be distinguished. Also, the two reflecting surfaces would be too separated for the light reflecting from them to be coherent.

104. (a) The combination of the direct ray and the reflected ray from the mirror will produce an interference pattern on the screen, like the double-slit experiment. However, in this case, the reflected ray has a phase change of π, causing the locations of the dark and bright fringes to be interchanged. Thus, a zero path difference would correspond to a dark fringe.

(b) The condition for constructive interferences is

$$2h\sin\theta = (m+\tfrac{1}{2})\lambda, \quad m = 0,1,2,\ldots$$

(c) Similarly, the condition for destructive interference is

$$2h\sin\theta = m\lambda, \qquad m = 0,1,2,\ldots$$

105. The *Hint* essentially answers the question, but we put in some algebraic details and arrive at the familiar analytic-geometry expression for a hyperbola. The distance $d/2$ is denoted a and the constant value for the path length difference is denoted ϕ:

$$r_1 - r_2 = \phi$$

$$\sqrt{(a+x)^2 + y^2} - \sqrt{(a-x)^2 + y^2} = \phi$$

Rearranging and squaring, we have

$$\left(\sqrt{(a+x)^2 + y^2}\right)^2 = \left(\sqrt{(a-x)^2 + y^2} + \phi\right)^2$$

$$a^2 + 2ax + x^2 + y^2 = a^2 - 2ax + x^2 + y^2 + \phi^2 + 2\,\phi\,\sqrt{(a-x)^2 + y^2}$$

Many terms on both sides are identical and may be eliminated. This leaves us with

$$-2\,\phi\,\sqrt{(a-x)^2 + y^2} = \phi^2 - 4ax$$

at which point we square both sides again:

$$4\,\phi^2 a^2 - 8\,\phi^2 ax + 4\,\phi^2 x^2 + 4\,\phi^2 y^2 = \phi^4 - 8\,\phi^2 ax + 16a^2x^2$$

We eliminate the $-8\,\phi^2 ax$ term from both sides and plug in $a = 2d$ to get back to the original notation used in the problem statement:

$$\phi^2 d^2 + 4\,\phi^2 x^2 + 4\,\phi^2 y^2 = \phi^4 + 4\,d^2 x^2$$

Then a simple rearrangement puts it in the familiar analytic format for a hyperbola:

$$\phi^2 d^2 - \phi^4 = 4(d^2 - \phi^2)x^2 - 4\,\phi^2 y^2$$

which can be further simplified by dividing through by $\phi^2 d^2 - \phi^4$:

$$1 = \left(\frac{4}{\phi^2}\right)x^2 - \left(\frac{4}{d^2 - \phi^2}\right)y^2 .$$

Chapter 36

1. (a) We use Eq. 36-3 to calculate the separation between the first (m_1 = 1) and fifth $(m_2 = 5)$ minima:

$$\Delta y = D\Delta \sin\theta = D\Delta\left(\frac{m\lambda}{a}\right) = \frac{D\lambda}{a}\Delta m = \frac{D\lambda}{a}\left(m_2 - m_1\right) .$$

Solving for the slit width, we obtain

$$a = \frac{D\lambda(m_2 - m_1)}{\Delta y} = \frac{(400\text{ mm})(550\times10^{-6}\text{ mm})(5-1)}{0.35\text{ mm}} = 2.5\text{ mm} .$$

(b) For m = 1,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(550\times10^{-6}\text{ mm})}{2.5\text{ mm}} = 2.2\times10^{-4} .$$

The angle is $\theta = \sin^{-1}(2.2\times10^{-4}) = 2.2\times10^{-4}$ rad.

2. From Eq. 36-3,

$$\frac{a}{\lambda} = \frac{m}{\sin\theta} = \frac{1}{\sin 45.0°} = 1.41.$$

3. (a) A plane wave is incident on the lens so it is brought to focus in the focal plane of the lens, a distance of 70 cm from the lens.

(b) Waves leaving the lens at an angle θ to the forward direction interfere to produce an intensity minimum if $a \sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. The distance on the screen from the center of the pattern to the minimum is given by $y = D \tan\theta$, where D is the distance from the lens to the screen. For the conditions of this problem,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(590\times10^{-9}\text{ m})}{0.40\times10^{-3}\text{ m}} = 1.475\times10^{-3} .$$

This means $\theta = 1.475\times10^{-3}$ rad and

$$y = (0.70\text{ m})\tan(1.475\times10^{-3}\text{ rad}) = 1.0\times10^{-3}\text{ m}.$$

4. (a) Equations 36-3 and 36-12 imply smaller angles for diffraction for smaller wavelengths. This suggests that diffraction effects in general would decrease.

(b) Using Eq. 36-3 with m = 1 and solving for 2θ (the angular width of the central diffraction maximum), we find

$$2\theta = 2\sin^{-1}\left(\frac{\lambda}{a}\right) = 2\sin^{-1}\left(\frac{0.50\,\text{m}}{5.0\,\text{m}}\right) = 11°.$$

(c) A similar calculation yields 0.23° for λ = 0.010 m.

5. (a) The condition for a minimum in a single-slit diffraction pattern is given by

$$a \sin \theta = m\lambda,$$

where a is the slit width, λ is the wavelength, and m is an integer. For $\lambda = \lambda_a$ and $m = 1$, the angle θ is the same as for $\lambda = \lambda_b$ and $m = 2$. Thus,

$$\lambda_a = 2\lambda_b = 2(350 \text{ nm}) = 700 \text{ nm}.$$

(b) Let m_a be the integer associated with a minimum in the pattern produced by light with wavelength λ_a, and let m_b be the integer associated with a minimum in the pattern produced by light with wavelength λ_b. A minimum in one pattern coincides with a minimum in the other if they occur at the same angle. This means $m_a\lambda_a = m_b\lambda_b$. Since $\lambda_a = 2\lambda_b$, the minima coincide if $2m_a = m_b$. Consequently, every other minimum of the λ_b pattern coincides with a minimum of the λ_a pattern. With m_a =2, we have m_b = 4.

(c) With m_a =3, we have m_b = 6.

6. (a) $\theta = \sin^{-1}$ (1.50 cm/2.00 m) = 0.430°.

(b) For the mth diffraction minimum, $a \sin \theta = m\lambda$. We solve for the slit width:

$$a = \frac{m\lambda}{\sin\theta} = \frac{2(441\,\text{nm})}{\sin 0.430°} = 0.118\,\text{mm} .$$

7. The condition for a minimum of a single-slit diffraction pattern is

$$a\sin\theta = m\lambda$$

where a is the slit width, λ is the wavelength, and m is an integer. The angle θ is measured from the forward direction, so for the situation described in the problem, it is 0.60° for m = 1. Thus,

$$a = \frac{m\lambda}{\sin\theta} = \frac{633\times10^{-9}\text{ m}}{\sin 0.60°} = 6.04\times10^{-5}\text{ m} .$$

8. Let the first minimum be a distance y from the central axis that is perpendicular to the speaker. Then

$$\sin\theta = y\Big/\left(D^2 + y^2\right)^{1/2} = m\lambda/a = \lambda/a \ \text{(for } m = 1\text{)}.$$

Therefore,

$$y = \frac{D}{\sqrt{(a/\lambda)^2 - 1}} = \frac{D}{\sqrt{(af/v_s)^2 - 1}} = \frac{100\,\text{m}}{\sqrt{\left[(0.300\,\text{m})(3000\,\text{Hz})/(343\,\text{m/s})\right]^2 - 1}} = 41.2\,\text{m}\ .$$

9. **THINK** The condition for a minimum of intensity in a single-slit diffraction pattern is given by $a\sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer.

EXPRESS To find the angular position of the first minimum to one side of the central maximum, we set $m = 1$:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{589\times10^{-9}\ \text{m}}{1.00\times10^{-3}\ \text{m}}\right) = 5.89\times10^{-4}\ \text{rad}\ .$$

If D is the distance from the slit to the screen, the distance on the screen from the center of the pattern to the minimum is

$$y_1 = D\tan\theta_1 = (3.00\,\text{m})\tan\left(5.89\times10^{-4}\ \text{rad}\right) = 1.767\times10^{-3}\ \text{m}\ .$$

To find the second minimum, we set $m = 2$:

$$\theta_2 = \sin^{-1}\left(\frac{2\left(589\times10^{-9}\ \text{m}\right)}{1.00\times10^{-3}\ \text{m}}\right) = 1.178\times10^{-3}\ \text{rad}\ .$$

ANALYZE The distance from the center of the pattern to this second minimum is

$$y_2 = D\tan\theta_2 = (3.00\ \text{m})\tan(1.178\times10^{-3}\ \text{rad}) = 3.534\times10^{-3}\ \text{m}.$$

The separation of the two minima is

$$\Delta y = y_2 - y_1 = 3.534\ \text{mm} - 1.767\ \text{mm} = 1.77\ \text{mm}.$$

LEARN The angles θ_1 and θ_2 found above are quite small. In the small-angle approximation, $\sin\theta \approx \tan\theta \approx \theta$, and the separation between two adjacent diffraction minima can be approximated as

$$\Delta y = D(\tan\theta_{m+1} - \tan\theta_m) \approx D(\theta_{m+1} - \theta_m) = \frac{D\lambda}{a}.$$

10. From $y = m\lambda L/a$ we get

$$\Delta y = \Delta\left(\frac{m\lambda L}{a}\right) = \frac{\lambda L}{a}\Delta m = \frac{(632.8\,\text{nm})(2.60)}{1.37\,\text{mm}}[10-(-10)] = 24.0\,\text{mm}\ .$$

11. We note that 1 nm = 1 $\times 10^{-9}$ m = 1 $\times 10^{-6}$ mm. From Eq. 36-4,

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x\sin\theta) = \left(\frac{2\pi}{589\times10^{-6}\ \text{mm}}\right)\left(\frac{0.10\,\text{mm}}{2}\right)\sin 30° = 266.7\ \text{rad}\ .$$

This is equivalent to 266.7 rad – 84π = 2.8 rad = 160°.

12. (a) The slope of the plotted line is 12, and we see from Eq. 36-6 that this slope should correspond to

$$\frac{\pi a}{\lambda} = 12 \ \Rightarrow\ a = \frac{12\lambda}{\pi} = \frac{12(610\text{ nm})}{\pi} = 2330\text{ nm} \approx 2.33\ \mu\text{m}$$

(b) Consider Eq. 36-3 with "continuously variable" m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m_{\max} = \frac{a}{\lambda}(\sin\theta)_{\max} = \frac{a}{\lambda} = \frac{2330\text{ nm}}{610\text{ nm}} \approx 3.82$$

which suggests that, on each side of the central maximum ($\theta_{\text{centr}} = 0$), there are three minima; considering both sides then implies there are six minima in the pattern.

(c) Setting m = 1 in Eq. 36-3 and solving for θ yields 15.2°.

(d) Setting m = 3 in Eq. 36-3 and solving for θ yields 51.8°.

13. (a) $\theta = \sin^{-1}$ (0.011 m/3.5 m) = 0.18°.

(b) We use Eq. 36-6:

$$\alpha = \left(\frac{\pi a}{\lambda}\right)\sin\theta = \frac{\pi(0.025\,\text{mm})\sin 0.18°}{538\times10^{-6}\ \text{mm}} = 0.46\,\text{rad}\ .$$

(c) Making sure our calculator is in radian mode, Eq. 36-5 yields

$$\frac{I(\theta)}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0.93\ .$$

14. We will make use of arctangents and sines in our solution, even though they can be "shortcut" somewhat since the angles are small enough to justify the use of the small angle approximation.

(a) Given y/D = 15/300 (both expressed here in centimeters), then $\theta = \tan^{-1}(y/D) = 2.86°$. Use of Eq. 36-6 (with a = 6000 nm and λ = 500 nm) leads to

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi(6000\,\text{nm})\sin 2.86°}{500\,\text{nm}} = 1.883\,\text{rad}.$$

Thus,

$$\frac{I_p}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0.256\ .$$

(b) Consider Eq. 36-3 with "continuously variable" m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{a\sin\theta}{\lambda} = \frac{(6000\text{ nm})\sin 2.86°}{500\text{ nm}} \approx 0.60,$$

which suggests that the angle takes us to a point between the central maximum ($\theta_{\text{centr}} = 0$) and the first minimum (which corresponds to m = 1 in Eq. 36-3).

15. **THINK** The relative intensity in a single-slit diffraction depends on the ratio a/λ, where a is the slit width and λ is the wavelength.

EXPRESS The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2\alpha}{\alpha^2}$$

where I_m is the maximum intensity and $\alpha = (\pi a/\lambda)\sin\theta$. The angle θ is measured from the forward direction.

ANALYZE (a) We require $I = I_m/2$, so

$$\sin^2\alpha = \frac{1}{2}\alpha^2\ .$$

(b) We evaluate $\sin^2\alpha$ and $\alpha^2/2$ for α = 1.39 rad and compare the results. To be sure that 1.39 rad is closer to the correct value for α than any other value with three significant digits, we could also try 1.385 rad and 1.395 rad.

(c) Since $\alpha = (\pi a/\lambda) \sin \theta$,

$$\theta = \sin^{-1}\left(\frac{\alpha\lambda}{\pi a}\right).$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1}\left(\frac{0.442\lambda}{a}\right).$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{0.442\lambda}{a}\right).$$

(d) For $a/\lambda = 1.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/1.0) = 0.916\,\text{rad} = 52.5°.$$

(e) For $a/\lambda = 5.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/5.0) = 0.177\,\text{rad} = 10.1°.$$

(f) For $a/\lambda = 10$,

$$\Delta\theta = 2\sin^{-1}(0.442/10) = 0.0884\,\text{rad} = 5.06°.$$

LEARN As shown in Fig. 36-8, the wider the slit is (relative to the wavelength), the narrower is the central diffraction maximum.

16. Consider Huygens' explanation of diffraction phenomena. When A is in place only the Huygens' wavelets that pass through the hole get to point P. Suppose they produce a resultant electric field E_A. When B is in place, the light that was blocked by A gets to P and the light that passed through the hole in A is blocked. Suppose the electric field at P is now $\vec{E}_B$. The sum $\vec{E}_A + \vec{E}_B$ is the resultant of all waves that get to P when neither A nor B are present. Since P is in the geometric shadow, this is zero. Thus $\vec{E}_A = -\vec{E}_B$, and since the intensity is proportional to the square of the electric field, the intensity at P is the same when A is present as when B is present.

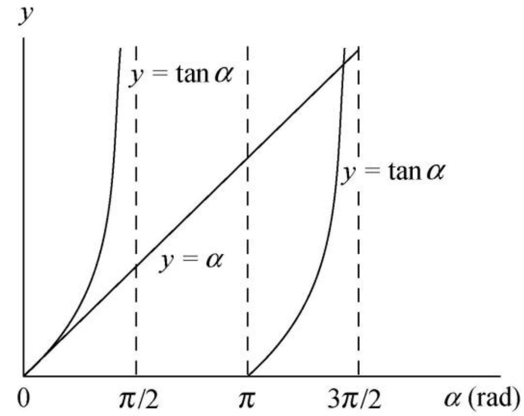
17. (a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2\alpha}{\alpha^2}$$

where α is described in the text (see Eq. 36-6). To locate the extrema, we set the derivative of I with respect to α equal to zero and solve for α. The derivative is

$$\frac{dI}{d\alpha} = 2I_m \frac{\sin\alpha}{\alpha^3}\left(\alpha\cos\alpha - \sin\alpha\right).$$

The derivative vanishes if $\alpha \neq 0$ but sin $\alpha = 0$. This yields $\alpha = m\pi$, where m is a nonzero integer. These are the intensity minima: $I = 0$ for $\alpha = m\pi$. The derivative also vanishes for $\alpha\cos\alpha - \sin\alpha = 0$. This condition can be written tan $\alpha = \alpha$. These implicitly locate the maxima.



(b) The values of α that satisfy tan $\alpha = \alpha$ can be found by trial and error on a pocket calculator or computer. Each of them is slightly less than one of the values $\left(m+\frac{1}{2}\right)\pi$ rad, so we start with these values. They can also be found graphically. As in the diagram that follows, we plot $y = \tan\alpha$ and $y = \alpha$ on the same graph. The intersections of the line with the tan α curves are the solutions. The smallest α is $\alpha = 0$.

(c) We write $\alpha = \left(m+\frac{1}{2}\right)\pi$ for the maxima. For the central maximum, $\alpha = 0$ and $m = -1/2 = -0.500$.

(d) The next one can be found to be $\alpha = 4.493$ rad.

(e) For $\alpha = 4.4934$, $m = 0.930$.

(f) The next one can be found to be $\alpha = 7.725$ rad.

(g) For $\alpha = 7.7252$, $m = 1.96$.

18. Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the maximum distance is

$$L = \frac{D}{\theta_R} = \frac{D}{1.22\lambda/d} = \frac{\left(5.0\times10^{-3}\text{ m}\right)\left(4.0\times10^{-3}\text{ m}\right)}{1.22\left(550\times10^{-9}\text{ m}\right)} = 30\,\text{m}.$$

19. (a) Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye,"

$$L = \frac{D}{1.22\lambda/d} = \frac{2(50\times10^{-6}\text{ m})(1.5\times10^{-3}\text{ m})}{1.22(650\times10^{-9}\text{ m})} = 0.19\text{ m}.$$

(b) The wavelength of the blue light is shorter so $L_{max} \propto \lambda^{-1}$ will be larger.

20. Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \left(6.2\times10^3\text{ m}\right)\frac{(1.22)\left(1.6\times10^{-2}\text{ m}\right)}{2.3\text{m}} = 53\text{m} .$$

21. **THINK** We apply the Rayleigh criterion to estimate the linear separation between the two objects.

EXPRESS If L is the distance from the observer to the objects, then the smallest separation D they can have and still be resolvable is $D = L\theta_R$, where θ_R is measured in radians.

ANALYZE (a) With small angle approximation, $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. Thus,

$$D = \frac{1.22\,L\lambda}{d} = \frac{1.22\left(8.0\times10^{10}\text{ m}\right)\left(550\times10^{-9}\text{ m}\right)}{5.0\times10^{-3}\text{ m}} = 1.1\times10^7\text{ m} = 1.1\times10^4\text{ km} .$$

This distance is greater than the diameter of Mars; therefore, one part of the planet's surface cannot be resolved from another part.

(b) Now d = 5.1 m and $D = \dfrac{1.22\left(8.0\times10^{10}\text{ m}\right)\left(550\times10^{-9}\text{ m}\right)}{5.1\text{m}} = 1.1\times10^4\text{ m} = 11\text{ km} .$

LEARN By the Rayleigh criterion for resolvability, two objects can be resolved only if their angular separation at the observer is greater than $\theta_R = 1.22\lambda/d$.

22. (a) Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \frac{\left(400\times10^3\text{m}\right)(1.22)\left(550\times10^{-9}\text{ m}\right)}{(0.005\text{m})} \approx 50\text{ m}.$$

(b) The Rayleigh criterion suggests that the astronaut will not be able to discern the Great Wall (see the result of part (a)).

(c) The signs of intelligent life would probably be, at most, ambiguous on the sunlit half of the planet. However, while passing over the half of the planet on the opposite side from the Sun, the astronaut would be able to notice the effects of artificial lighting.

23. **THINK** We apply the Rayleigh criterion to determine the conditions that allow the headlights to be resolved.

EXPRESS By the Rayleigh criteria, two point sources can be resolved if the central diffraction maximum of one source is centered on the first minimum of the diffraction pattern of the other. Thus, the angular separation (in radians) of the sources must be at least $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture.

ANALYZE (a) For the headlights of this problem,

$$\theta_R = \frac{1.22\left(550\times10^{-9}\text{ m}\right)}{5.0\times10^{-3}\text{ m}} = 1.34\times10^{-4}\text{ rad},$$

or $1.3\times10^{-4}\text{ rad}$, in two significant figures.

(b) If L is the distance from the headlights to the eye when the headlights are just resolvable and D is the separation of the headlights, then $D = L\theta_R$, where the small angle approximation is made. This is valid for θ_R in radians. Thus,

$$L = \frac{D}{\theta_R} = \frac{1.4\text{ m}}{1.34\times10^{-4}\text{ rad}} = 1.0\times10^{4}\text{ m} = 10\text{ km}.$$

LEARN A distance of 10 km far exceeds what human eyes can resolve. In reality, our visual resolvability depends on other factors such as the relative brightness of the source and their surroundings, turbulence in the air between the lights and the eyes, the health of one's vision.

24. We use Eq. 36-12 with $\theta = 2.5°/2 = 1.25°$. Thus,

$$d = \frac{1.22\lambda}{\sin\theta} = \frac{1.22\left(550\text{ nm}\right)}{\sin 1.25°} = 31\,\mu\text{m}.$$

25. Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the minimum separation is

$$D = L\theta_R = L\left(1.22\frac{\lambda}{d}\right) = \left(3.82\times10^{8}\text{ m}\right)\frac{(1.22)\left(550\times10^{-9}\text{ m}\right)}{5.1\text{ m}} = 50\text{ m}.$$

26. Using the same notation found in Sample Problem — "Pointillistic paintings use the diffraction of your eye,"

$$\frac{D}{L} = \theta_R = 1.22\frac{\lambda}{d}$$

where we will assume a "typical" wavelength for visible light: $\lambda \approx 550 \times 10^{-9}$ m.

(a) With $L = 400 \times 10^3$ m and $D = 0.85$ m, the above relation leads to $d = 0.32$ m.

(b) Now with $D = 0.10$ m, the above relation leads to $d = 2.7$ m.

(c) The military satellites do not use Hubble Telescope-sized apertures. A great deal of very sophisticated optical filtering and digital signal processing techniques go into the final product, for which there is not space for us to describe here.

27. Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye,"

$$L = \frac{D}{\theta_R} = \frac{D}{1.22\lambda / d} = \frac{(5.0\times10^{-2}\text{ m})(4.0\times10^{-3}\text{m})}{1.22(0.10\times10^{-9}\text{ m})} = 1.6\times10^{6}\text{ m} = 1.6\times10^{3}\text{ km}.$$

28. Eq. 36-14 gives $\theta_R = 1.22\lambda/d$, where in our case $\theta_R \approx D/L$, with $D = 60\ \mu$m being the size of the object your eyes must resolve, and L being the maximum viewing distance in question. If $d = 3.00$ mm $= 3000\ \mu$m is the diameter of your pupil, then

$$L = \frac{Dd}{1.22\lambda} = \frac{(60\ \mu\text{m})(3000\ \mu\text{m})}{1.22(0.55\ \mu\text{m})} = 2.7\times10^{5}\ \mu\text{m} = 27\,\text{cm}.$$

29. (a) Using Eq. 36-14, the angular separation is

$$\theta_R = \frac{1.22\lambda}{d} = \frac{(1.22)(550\times10^{-9}\text{ m})}{0.76\,\text{m}} = 8.8\times10^{-7}\text{ rad}.$$

(b) Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the distance between the stars is

$$D = L\theta_R = \frac{(10\,\text{ly})(9.46\times10^{12}\text{ km/ly})(0.18)\pi}{(3600)(180)} = 8.4\times10^{7}\text{ km}.$$

(c) The diameter of the first dark ring is

$$d = 2\theta_R L = \frac{2(0.18)(\pi)(14\,\text{m})}{(3600)(180)} = 2.5\times10^{-5}\text{ m} = 0.025\,\text{mm}.$$

30. From Fig. 36-42(a), we find the diameter D' on the retina to be

$$D' = D\frac{L'}{L} = (2.00\text{ mm})\frac{2.00\text{ cm}}{45.0\text{ cm}} = 0.0889\text{ mm}.$$

Next, using Fig. 36-42(b), the angle from the axis is

$$\theta = \tan^{-1}\left(\frac{D'/2}{x}\right) = \tan^{-1}\left(\frac{0.0889\ \text{mm}/2}{6.00\ \text{mm}}\right) = 0.424° .$$

Since the angle corresponds to the first minimum in the diffraction pattern, we have $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the defect. With $\lambda = 550$ nm, we obtain

$$d = \frac{1.22\lambda}{\sin\theta} = \frac{1.22(550\ \text{nm})}{\sin(0.424°)} = 9.06\times10^{-5}\ \text{m} \approx 91\,\mu\text{m} .$$

31. **THINK** We apply the Rayleigh criterion to calculate the angular width of the central maxima.

EXPRESS The first minimum in the diffraction pattern is at an angular position θ, measured from the center of the pattern, such that $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the antenna. If f is the frequency, then the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{220\times10^{9}\ \text{Hz}} = 1.36\times10^{-3}\ \text{m} .$$

ANALYZE (a) Thus, we have

$$\theta = \sin^{-1}\left(\frac{1.22\,\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22\left(1.36\times10^{-3}\ \text{m}\right)}{55.0\times10^{-2}\ \text{m}}\right) = 3.02\times10^{-3}\ \text{rad} .$$

The angular width of the central maximum is twice this, or 6.04×10^{-3} rad (0.346°).

(b) Now $\lambda = 1.6$ cm and $d = 2.3$ m, so

$$\theta = \sin^{-1}\left(\frac{1.22\left(1.6\times10^{-2}\ \text{m}\right)}{2.3\,\text{m}}\right) = 8.5\times10^{-3}\ \text{rad} .$$

The angular width of the central maximum is 1.7×10^{-2} rad (or 0.97°).

LEARN Using small angle approximation, we can write the angular width as

$$2\theta \approx 2\left(\frac{1.22\,\lambda}{d}\right) = \frac{2.44\,\lambda}{d} .$$

32. (a) We use Eq. 36-12:

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left[\frac{1.22(v_s/f)}{d}\right] = \sin^{-1}\left[\frac{(1.22)(1450\,\text{m/s})}{(25\times10^3\,\text{Hz})(0.60\,\text{m})}\right] = 6.8°.$$

(b) Now $f = 1.0 \times 10^3$ Hz so

$$\frac{1.22\lambda}{d} = \frac{(1.22)(1450\,\text{m/s})}{(1.0\times10^3\,\text{Hz})(0.60\,\text{m})} = 2.9 > 1.$$

Since sin θ cannot exceed 1 there is no minimum.

33. Equation 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — "Pointillistic paintings use the diffraction of your eye."

(a) We are asked to solve for D and are given $\lambda = 1.40 \times 10^{-9}$ m, $d = 0.200 \times 10^{-3}$ m, and $L = 2000\times10^3$ m. Consequently, we obtain $D = 17.1$ m.

(b) Intensity is power over area (with the area assumed spherical in this case, which means it is proportional to radius-squared), so the ratio of intensities is given by the square of a ratio of distances: $(d/D)^2 = 1.37 \times 10^{-10}$.

34. (a) Since $\theta = 1.22\lambda/d$, the larger the wavelength the larger the radius of the first minimum (and second maximum, etc). Therefore, the white pattern is outlined by red lights (with longer wavelength than blue lights).

(b) The diameter of a water drop is

$$d = \frac{1.22\,\lambda}{\theta} \approx \frac{1.22(7\times10^{-7}\,\text{m})}{1.5(0.50°)(\pi/180°)/2} = 1.3\times10^{-4}\,\text{m}.$$

35. Bright interference fringes occur at angles θ given by $d \sin \theta = m\lambda$, where m is an integer. For the slits of this problem, we have $d = 11a/2$, so

$$a \sin \theta = 2m\lambda/11 .$$

The first minimum of the diffraction pattern occurs at the angle θ_1 given by $a \sin \theta_1 = \lambda$, and the second occurs at the angle θ_2 given by $a \sin \theta_2 = 2\lambda$, where a is the slit width. We should count the values of m for which $\theta_1 < \theta < \theta_2$, or, equivalently, the values of m for which $\sin\theta_1 < \sin\theta < \sin\theta_2$. This means $1 < (2m/11) < 2$. The values are m = 6, 7, 8, 9, and 10. There are five bright fringes in all.

36. Following the method of Sample Problem — "Double-slit experiment with diffraction of each slit included," we find

$$\frac{d}{a} = \frac{0.30\times10^{-3}\text{ m}}{46\times10^{-6}\text{ m}} = 6.52$$

which we interpret to mean that the first diffraction minimum occurs slightly farther "out" than the m = 6 interference maximum. This implies that the central diffraction envelope includes the central (m = 0) interference maximum as well as six interference maxima on each side of it. Therefore, there are 6 + 1 + 6 = 13 bright fringes (interference maxima) in the central diffraction envelope.

37. In a manner similar to that discussed in Sample Problem — "Double-slit experiment with diffraction of each slit included," we find the number is $2(d/a) - 1 = 2(2a/a) - 1 = 3$.

38. We note that the central diffraction envelope contains the central bright interference fringe (corresponding to m = 0 in Eq. 36-25) plus ten on either side of it. Since the eleventh order bright interference fringe is not seen in the central envelope, then we conclude the first diffraction minimum (satisfying $\sin\theta = \lambda/a$) coincides with the m = 11 instantiation of Eq. 36-25:

$$d = \frac{m\lambda}{\sin\theta} = \frac{11\,\lambda}{\lambda/a} = 11\,a\,.$$

Thus, the ratio d/a is equal to 11.

39. (a) The first minimum of the diffraction pattern is at 5.00°, so

$$a = \frac{\lambda}{\sin\theta} = \frac{0.440\,\mu\text{m}}{\sin 5.00°} = 5.05\mu\text{m}\,.$$

(b) Since the fourth bright fringe is missing, $d = 4a = 4(5.05\ \mu\text{m}) = 20.2\ \mu\text{m}$.

(c) For the m = 1 bright fringe,

$$\alpha = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(5.05\,\mu\text{m})\sin 1.25°}{0.440\,\mu\text{m}} = 0.787\text{ rad}\,.$$

Consequently, the intensity of the m = 1 fringe is

$$I = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(7.0\text{ mW/cm}^2\right)\left(\frac{\sin 0.787\text{ rad}}{0.787}\right)^2 = 5.7\text{ mW/cm}^2\,,$$

which agrees with Fig. 36-45. Similarly for m = 2, the intensity is I = 2.9 mW/cm^2, also in agreement with Fig. 36-45.

40. (a) We note that the slope of the graph is 80, and that Eq. 36-20 implies that the slope should correspond to

$$\frac{\pi d}{\lambda} = 80 \;\Rightarrow\; d = \frac{80\lambda}{\pi} = \frac{80(435\text{ nm})}{\pi} = 11077\text{ nm} \approx 11.1\,\mu\text{m}.$$

(b) Consider Eq. 36-25 with "continuously variable" m (of course, m should be an integer for interference maxima, but for the moment we will solve for it as if it could be any real number):

$$m_{\max} = \frac{d}{\lambda}(\sin\theta)_{\max} = \frac{d}{\lambda} = \frac{11077\text{ nm}}{435\text{ nm}} \approx 25.5$$

which indicates (on one side of the interference pattern) there are 25 bright fringes. Thus on the other side there are also 25 bright fringes. Including the one in the middle, then, means there are a total of 51 maxima in the interference pattern (assuming, as the problem remarks, that none of the interference maxima have been eliminated by diffraction minima).

(c) Clearly, the maximum closest to the axis is the middle fringe at $\theta = 0°$.

(d) If we set m = 25 in Eq. 36-25, we find

$$m\lambda = d\sin\theta \;\Rightarrow\; \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(25)(435\text{ nm})}{11077\text{ nm}}\right) = 79.0°$$

41. We will make use of arctangents and sines in our solution, even though they can be "shortcut" somewhat since the angles are [almost] small enough to justify the use of the small angle approximation.

(a) Given y/D = (0.700 m)/(4.00 m), then

$$\theta = \tan^{-1}\left(\frac{y}{D}\right) = \tan^{-1}\left(\frac{0.700\text{ m}}{4.00\text{ m}}\right) = 9.93° = 0.173\text{ rad}.$$

Equation 36-20 then gives

$$\beta = \frac{\pi d\sin\theta}{\lambda} = \frac{\pi(24.0\,\mu\text{m})\sin 9.93°}{0.600\,\mu\text{m}} = 21.66\text{ rad}.$$

Thus, use of Eq. 36-21 (with a = 12 μm and λ = 0.60 μm) leads to

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi(12.0\,\mu\text{m})\sin 9.93°}{0.600\,\mu\text{m}} = 10.83 \text{ rad} .$$

Thus,

$$\frac{I}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 (\cos\beta)^2 = \left(\frac{\sin 10.83\,\text{rad}}{10.83}\right)^2 (\cos 21.66 \text{ rad})^2 = 0.00743 .$$

(b) Consider Eq. 36-25 with "continuously variable" m (of course, m should be an integer for interference maxima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{d\sin\theta}{\lambda} = \frac{(24.0\,\mu\text{m})\sin 9.93°}{0.600\,\mu\text{m}} \approx 6.9$$

which suggests that the angle takes us to a point between the sixth minimum (which would have $m = 6.5$) and the seventh maximum (which corresponds to $m = 7$).

(c) Similarly, consider Eq. 36-3 with "continuously variable" m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{a\sin\theta}{\lambda} = \frac{(12.0\,\mu\text{m})\sin 9.93°}{0.600\,\mu\text{m}} \approx 3.4$$

which suggests that the angle takes us to a point between the third diffraction minimum ($m = 3$) and the fourth one ($m = 4$). The maxima (in the smaller peaks of the diffraction pattern) are not exactly midway between the minima; their location would make use of mathematics not covered in the prerequisites of the usual sophomore-level physics course.

42. (a) In a manner similar to that discussed in Sample Problem — "Double-slit experiment with diffraction of each slit included," we find the ratio should be $d/a = 4$. Our reasoning is, briefly, as follows: we let the location of the fourth bright fringe coincide with the first minimum of diffraction pattern, and then set $\sin\theta = 4\lambda/d = \lambda/a$ (so $d = 4a$).

(b) Any bright fringe that happens to be at the same location with a diffraction minimum will vanish. Thus, if we let

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{a} = \frac{m_1\lambda}{4a} ,$$

or $m_1 = 4m_2$ where $m_2 = 1, 2, 3, \ldots$. The fringes missing are the 4th, 8th, 12th, and so on. Hence, every fourth fringe is missing.

43. **THINK** For relatively wide slits, the interference of light from two slits produces bright fringes that do not all have the same intensity; instead, the intensities are modified by diffraction of light passing through each slit.

EXPRESS The angular positions θ of the bright interference fringes are given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a \sin \theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so we should count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or, equivalently, the number of values of m for which

$$-\sin \theta_1 < \sin \theta < +\sin \theta_1.$$

The intensity at the screen is given by

$$I = I_m (\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha} \right)^2$$

where $\alpha = (\pi a/\lambda) \sin \theta$, $\beta = (\pi d/\lambda) \sin \theta$, and I_m is the intensity at the center of the pattern.

ANALYZE (a) The condition above means $-1/a < m/d < 1/a$, or $-d/a < m < +d/a$. Now

$$d/a = (0.150 \times 10^{-3}\text{ m})/(30.0 \times 10^{-6}\text{ m}) = 5.00,$$

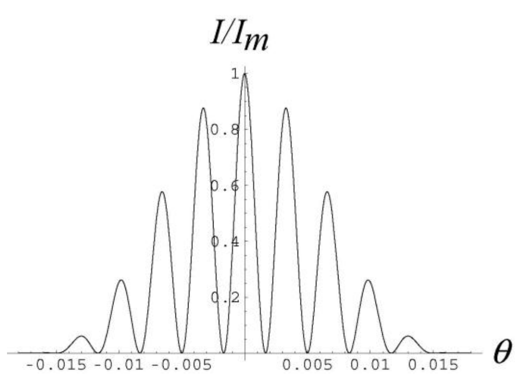
so the values of m are $m = -4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. There are 9 fringes.

(b) For the third bright interference fringe, $d \sin \theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2 \beta = 1$. Similarly, $\alpha = 3\pi a/d = 3\pi/5.00 = 0.600\pi$ rad and

$$\left(\frac{\sin \alpha}{\alpha} \right)^2 = \left(\frac{\sin 0.600\pi}{0.600\pi} \right)^2 = 0.255 .$$

The intensity ratio is $I/I_m = 0.255$.

LEARN The expression for intensity contains two factors: (1) the interference factor $\cos^2 \beta$ due to the interference between two slits with separation d, and (2) the diffraction factor $[(\sin \alpha)/\alpha]^2$ which arises due to diffraction by a single slit of width a. In the limit $a \to 0$, $(\sin \alpha)/\alpha \to 1$, and we recover Eq. 35-22 for the interference between two slits of vanishingly narrow slits separated by d. Similarly, setting $d = 0$ or equivalently, $\beta = 0$, we recover Eq. 36-5 for the diffraction of a single slit of width a. A plot of the relative intensity is shown to the right.



44. We use Eq. 36-25 for diffraction maxima: $d \sin \theta = m\lambda$. In our case, since the angle between the $m = 1$ and $m = -1$ maxima is 26°, the angle θ corresponding to $m = 1$ is $\theta = 26°/2 = 13°$. We solve for the grating spacing:

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(550\,\text{nm})}{\sin 13°} = 2.4\,\mu\text{m} \approx 2\,\mu\text{m}.$$

45. The distance between adjacent rulings is

$$d = 20.0\text{ mm}/6000 = 0.00333\text{ mm} = 3.33\ \mu\text{m}.$$

(a) Let $d\sin\theta = m\lambda\left(m = 0, \pm 1, \pm 2, \ldots\right)$. Since $|m|\lambda/d > 1$ for $|m| \ge 6$, the largest value of θ corresponds to $|m| = 5$, which yields

$$\theta = \sin^{-1}\left(|m|\lambda/d\right) = \sin^{-1}\left(\frac{5(0.589\,\mu\text{m})}{3.33\,\mu\text{m}}\right) = 62.1°.$$

(b) The second largest value of θ corresponds to $|m| = 4$, which yields

$$\theta = \sin^{-1}\left(|m|\lambda/d\right) = \sin^{-1}\left(\frac{4(0.589\,\mu\text{m})}{3.33\,\mu\text{m}}\right) = 45.0°.$$

(c) The third largest value of θ corresponds to $|m| = 3$, which yields

$$\theta = \sin^{-1}\left(|m|\lambda/d\right) = \sin^{-1}\left(\frac{3(0.589\,\mu\text{m})}{3.33\,\mu\text{m}}\right) = 32.0°.$$

46. The angular location of the mth order diffraction maximum is given by $m\lambda = d \sin \theta$. To be able to observe the fifth-order maximum, we must let $\sin \theta|_{m=5} = 5\lambda/d < 1$, or

$$\lambda < \frac{d}{5} = \frac{1.00\,\text{nm}/315}{5} = 635\,\text{nm}.$$

Therefore, the longest wavelength that can be used is $\lambda = 635$ nm.

47. **THINK** Diffraction lines occur at angles θ such that $d \sin \theta = m\lambda$, where d is the grating spacing, λ is the wavelength and m is an integer.

EXPRESS The ruling separation is

$$d = 1/(400\text{ mm}^{-1}) = 2.5 \times 10^{-3}\text{ mm}.$$

Notice that for a given order, the line associated with a long wavelength is produced at a greater angle than the line associated with a shorter wavelength. We take λ to be the longest wavelength in the visible spectrum (700 nm) and find the greatest integer value of m such that θ is less than 90°. That is, find the greatest integer value of m for which $m\lambda < d$.

ANALYZE Since

$$\frac{d}{\lambda} = \frac{2.5\times10^{-6}\,\text{m}}{700\times10^{-9}\,\text{m}} \approx 3.57,$$

that value is $m = 3$. There are three complete orders on each side of the $m = 0$ order. The second and third orders overlap.

LEARN From $\theta = \sin^{-1}(m\lambda/d)$, the condition for maxima or lines, we see that for a given diffraction grating, the angle from the central axis to any line depends on the wavelength of the light being used.

48. (a) For the maximum with the greatest value of $m = M$ we have $M\lambda = a\sin\theta < d$, so $M < d/\lambda = 900\text{ nm}/600\text{ nm} = 1.5$, or $M = 1$. Thus three maxima can be seen, with $m = 0, \pm1$.

(b) From Eq. 36-28, we obtain

$$\Delta\theta_{\text{hw}} = \frac{\lambda}{N d\cos\theta} = \frac{d\sin\theta}{N d\cos\theta} = \frac{\tan\theta}{N} = \frac{1}{N}\tan\left[\sin^{-1}\left(\frac{\lambda}{d}\right)\right]$$
$$= \frac{1}{1000}\tan\left[\sin^{-1}\left(\frac{600\,\text{nm}}{900\,\text{nm}}\right)\right] = 0.051°.$$

49. **THINK** Maxima of a diffraction grating pattern occur at angles θ given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer.

EXPRESS If two lines are adjacent, then their order numbers differ by unity. Let m be the order number for the line with $\sin\theta = 0.2$ and $m + 1$ be the order number for the line with $\sin\theta = 0.3$. Then,

$$0.2d = m\lambda, \quad 0.3d = (m+1)\lambda.$$

ANALYZE (a) We subtract the first equation from the second to obtain $0.1d = \lambda$, or

$$d = \lambda/0.1 = (600\times10^{-9}\text{m})/0.1 = 6.0\times10^{-6}\text{ m}.$$

(b) Minima of the single-slit diffraction pattern occur at angles θ given by $a\sin\theta = m\lambda$, where a is the slit width. Since the fourth-order interference maximum is missing, it must fall at one of these angles. If a is the smallest slit width for which this order is missing, the angle must be given by $a \sin\theta = \lambda$. It is also given by $d \sin\theta = 4\lambda$, so

$$a = d/4 = (6.0 \times 10^{-6}\text{ m})/4 = 1.5 \times 10^{-6}\text{ m}.$$

(c) First, we set $\theta = 90°$ and find the largest value of m for which $m\lambda < d \sin\theta$. This is the highest order that is diffracted toward the screen. The condition is the same as $m < d/\lambda$ and since

$$d/\lambda = (6.0 \times 10^{-6}\text{ m})/(600 \times 10^{-9}\text{ m}) = 10.0,$$

the highest order seen is the $m = 9$ order. The fourth and eighth orders are missing, so the observable orders are $m = 0, 1, 2, 3, 5, 6, 7$, and 9. Thus, the largest value of the order number is $m = 9$.

(d) Using the result obtained in (c), the second largest value of the order number is $m = 7$.

(e) Similarly, the third largest value of the order number is $m = 6$.

LEARN Interference maxima occur when $d \sin\theta = m\lambda$, while the condition for diffraction minima is $a \sin\theta = m'\lambda$. Thus, a particular interference maximum with order m may coincide with the diffraction minimum of order m'. The value of m is given by

$$\frac{d\sin\theta}{a\sin\theta} = \frac{m\lambda}{m'\lambda} \Rightarrow m = \left(\frac{d}{a}\right)m'.$$

Since $m = 4$ when $m' = 1$, we conclude that $d/a = 4$. Thus, $m = 8$ would correspond to the second diffraction minimum ($m' = 2$).

50. We use Eq. 36-25. For $m = \pm 1$

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.73\,\mu\text{m})\sin(\pm 17.6°)}{\pm 1} = 523\text{ nm},$$

and for $m = \pm 2$,

$$\lambda = \frac{(1.73\,\mu\text{m})\sin(\pm 37.3°)}{\pm 2} = 524\text{ nm}.$$

Similarly, we may compute the values of λ corresponding to the angles for $m = \pm 3$. The average value of these λ's is 523 nm.

51. (a) Since $d = (1.00\text{ mm})/180 = 0.0056$ mm, we write Eq. 36-25 as

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(180)(2)\lambda$$

where $\lambda_1 = 4\times10^{-4}$ mm and $\lambda_2 = 5\times10^{-4}$ mm. Thus, $\Delta\theta = \theta_2 - \theta_1 = 2.1°$.

(b) Use of Eq. 36-25 for each wavelength leads to the condition

$$m_1\lambda_1 = m_2\lambda_2$$

for which the smallest possible choices are m_1 = 5 and m_2 = 4. Returning to Eq. 36-25, then, we find

$$\theta = \sin^{-1}\left(\frac{m_1\lambda_1}{d}\right) = \sin^{-1}\left(\frac{5(4.0\times10^{-4}\text{ mm})}{0.0056\text{ mm}}\right) = \sin^{-1}(0.36) = 21°.$$

(c) There are no refraction angles greater than 90°, so we can solve for "$m_{\max}$" (realizing it might not be an integer):

$$m_{\max} = \frac{d\sin 90°}{\lambda_2} = \frac{d}{\lambda_2} = \frac{0.0056\text{ mm}}{5.0\times10^{-4}\text{ mm}} \approx 11$$

where we have rounded down. There are no values of m (for light of wavelength λ_2) greater than m = 11.

52. We are given the "number of lines per millimeter" (which is a common way to express $1/d$ for diffraction gratings); thus,

$$\frac{1}{d} = 160\text{ lines/mm} \quad\Rightarrow\quad d = 6.25\times10^{-6}\text{ m}.$$

(a) We solve Eq. 36-25 for θ with various values of m and λ. We show here the m = 2 and λ = 460 nm calculation:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2(460\times10^{-9}\text{ m})}{6.25\times10^{-6}\text{ m}}\right) = \sin^{-1}(0.1472) = 8.46°.$$

Similarly, we get 11.81° for m = 2 and λ = 640 nm, 12.75° for m = 3 and λ = 460 nm, and 17.89° for m = 3 and λ = 640 nm. The first indication of overlap occurs when we compute the angle for m = 4 and λ = 460 nm; the result is 17.12° which clearly shows overlap with the large-wavelength portion of the m = 3 spectrum.

(b) We solve Eq. 36-25 for m with θ = 90° and λ = 640 nm. In this case, we obtain m = 9.8 which means that the largest order in which the full range (which must include that largest wavelength) is seen is ninth order.

(c) Now with m = 9, Eq. 36-25 gives θ = 41.5° for λ = 460 nm.

(d) It similarly gives $\theta = 67.2°$ for $\lambda = 640$ nm.

(e) We solve Eq. 36-25 for m with $\theta = 90°$ and $\lambda = 460$ nm. In this case, we obtain m = 13.6 which means that the largest order in which the wavelength is seen is the thirteenth order. Now with $m = 13$, Eq. 36-25 gives $\theta = 73.1°$ for $\lambda = 460$ nm.

53. At the point on the screen where we find the inner edge of the hole, we have tan θ = 5.0 cm/30 cm, which gives $\theta = 9.46°$. We note that d for the grating is equal to 1.0 mm/350 = 1.0×10^6 nm/350.

(a) From $m\lambda = d \sin\theta$, we find

$$m = \frac{d\sin\theta}{\lambda} = \frac{(1.0\times10^6 \text{ nm/350})(0.1644)}{\lambda} = \frac{470\,\text{nm}}{\lambda}.$$

Since for white light $\lambda > 400$ nm, the only integer m allowed here is $m = 1$. Thus, at one edge of the hole, $\lambda = 470$ nm. This is the shortest wavelength of the light that passes through the hole.

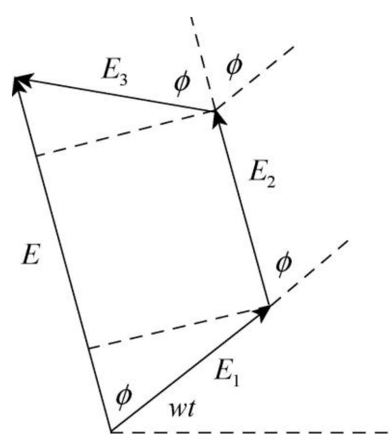
(b) At the other edge, we have tan $\theta' = 6.0$ cm/30 cm, which gives $\theta' = 11.31°$. This leads to

$$\lambda' = d\sin\theta' = \left(\frac{1.0\times10^6\text{ nm}}{350}\right)\sin(11.31°) = 560\text{ nm}.$$

This corresponds to the longest wavelength of the light that passes through the hole.

54. Since the slit width is much less than the wavelength of the light, the central peak of the single-slit diffraction pattern is spread across the screen and the diffraction envelope can be ignored. Consider three waves, one from each slit. Since the slits are evenly spaced, the phase difference for waves from the first and second slits is the same as the phase difference for waves from the second and third slits. The electric fields of the waves at the screen can be written as

$$E_1 = E_0 \sin(\omega t)$$
$$E_2 = E_0 \sin(\omega t + \phi)$$
$$E_3 = E_0 \sin(\omega t + 2\phi)$$



where $\phi = (2\pi d/\lambda)\sin\theta$. Here d is the separation of adjacent slits and λ is the wavelength. The phasor diagram is shown on the right. It yields

$$E = E_0\cos\phi + E_0\cos\phi = E_0(1 + 2\cos\phi).$$

for the amplitude of the resultant wave. Since the intensity of a wave is proportional to the square of the electric field, we may write $I = AE_0^2(1+2\cos\phi)^2$, where A is a constant of proportionality. If I_m is the intensity at the center of the pattern, for which $\phi = 0$, then $I_m = 9AE_0^2$. We take A to be $I_m/9E_0^2$ and obtain

$$I = \frac{I_m}{9}(1+2\cos\phi)^2 = \frac{I_m}{9}\left(1+4\cos\phi+4\cos^2\phi\right).$$

55. **THINK** If a grating just resolves two wavelengths whose average is λ_{avg} and whose separation is $\Delta\lambda$, then its resolving power is defined by $R = \lambda_{avg}/\Delta\lambda$.

EXPRESS As shown in Eq. 36-32, the resolving power can also be written as Nm, where N is the number of rulings in the grating and m is the order of the lines.

ANALYZE Thus $\lambda_{avg}/\Delta\lambda = Nm$ and

$$N = \frac{\lambda_{avg}}{m\Delta\lambda} = \frac{656.3\,\text{nm}}{(1)(0.18\,\text{nm})} = 3.65\times10^3 \text{ rulings.}$$

LEARN A large N (more rulings) means greater resolving power.

56. (a) From $R = \lambda/\Delta\lambda = Nm$ we find

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(415.496\text{ nm}+415.487\text{ nm})/2}{2(415.96\text{ nm}-415.487\text{ nm})} = 23100.$$

(b) We note that $d = (4.0 \times 10^7\text{ nm})/23100 = 1732$ nm. The maxima are found at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(2)(415.5\,\text{nm})}{1732\,\text{nm}}\right] = 28.7°.$$

57. (a) We note that $d = (76 \times 10^6\text{ nm})/40000 = 1900$ nm. For the first order maxima $\lambda = d\sin\theta$, which leads to

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{589\text{ nm}}{1900\text{ nm}}\right) = 18°.$$

Now, substituting $m = d\sin\theta/\lambda$ into Eq. 36-30 leads to

$$D = \tan\theta/\lambda = \tan 18°/589\text{ nm} = 5.5\times10^{-4}\text{ rad/nm} = 0.032°/\text{nm}.$$

(b) For $m = 1$, the resolving power is $R = Nm = 40000\,m = 40000 = 4.0 \times 10^4$.

(c) For $m = 2$ we have $\theta = 38°$, and the corresponding value of dispersion is 0.076°/nm.

(d) For $m = 2$, the resolving power is $R = Nm = 40000\, m = (40000)2 = 8.0 \times 10^4$.

(e) Similarly for $m = 3$, we have $\theta = 68°$, and the corresponding value of dispersion is 0.24°/nm.

(f) For $m = 3$, the resolving power is $R = Nm = 40000\, m = (40000)3 = 1.2 \times 10^5$.

58. (a) We find $\Delta\lambda$ from $R = \lambda/\Delta\lambda = Nm$:

$$\Delta\lambda = \frac{\lambda}{Nm} = \frac{500\,\text{nm}}{(600/\text{mm})(5.0\,\text{mm})(3)} = 0.056\,\text{nm} = 56\,\text{pm}.$$

(b) Since $\sin\theta = m_{max}\lambda/d < 1$,

$$m_{max} < \frac{d}{\lambda} = \frac{1}{(600/\text{mm})(500 \times 10^{-6}\,\text{mm})} = 3.3.$$

Therefore, $m_{max} = 3$. No higher orders of maxima can be seen.

59. Assuming all $N = 2000$ lines are uniformly illuminated, we have

$$\frac{\lambda_{av}}{\Delta\lambda} = Nm$$

from Eq. 36-31 and Eq. 36-32. With $\lambda_{av} = 600$ nm and $m = 2$, we find $\Delta\lambda = 0.15$ nm.

60. Letting $R = \lambda/\Delta\lambda = Nm$, we solve for N:

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(589.6\,\text{nm} + 589.0\,\text{nm})/2}{2(589.6\,\text{nm} - 589.0\,\text{nm})} = 491.$$

61. (a) From $d \sin\theta = m\lambda$ we find

$$d = \frac{m\lambda_{avg}}{\sin\theta} = \frac{3(589.3\,\text{nm})}{\sin 10°} = 1.0 \times 10^4\,\text{nm} = 10\,\mu\text{m}.$$

(b) The total width of the ruling is

$$L = Nd = \left(\frac{R}{m}\right)d = \frac{\lambda_{avg}d}{m\Delta\lambda} = \frac{(589.3\,\text{nm})(10\,\mu\text{m})}{3(589.59\,\text{nm} - 589.00\,\text{nm})} = 3.3 \times 10^3\,\mu\text{m} = 3.3\text{ mm}.$$

62. (a) From the expression for the half-width $\Delta\theta_{hw}$ (given by Eq. 36-28) and that for the resolving power R (given by Eq. 36-32), we find the product of $\Delta\theta_{hw}$ and R to be

$$\Delta\theta_{hw}R = \left(\frac{\lambda}{N d\cos\theta}\right) Nm = \frac{m\lambda}{d\cos\theta} = \frac{d\sin\theta}{d\cos\theta} = \tan\theta,$$

where we used $m\lambda = d\sin\theta$ (see Eq. 36-25).

(b) For first order $m = 1$, so the corresponding angle θ_1 satisfies $d\sin\theta_1 = m\lambda = \lambda$. Thus the product in question is given by

$$\tan\theta_1 = \frac{\sin\theta_1}{\cos\theta_1} = \frac{\sin\theta_1}{\sqrt{1-\sin^2\theta_1}} = \frac{1}{\sqrt{(1/\sin\theta_1)^2-1}} = \frac{1}{\sqrt{(d/\lambda)^2-1}}$$
$$= \frac{1}{\sqrt{(900\text{nm}/600\text{nm})^2-1}} = 0.89.$$

63. The angular positions of the first-order diffraction lines are given by $d\sin\theta = \lambda$. Let λ_1 be the shorter wavelength (430 nm) and θ be the angular position of the line associated with it. Let λ_2 be the longer wavelength (680 nm), and let $\theta + \Delta\theta$ be the angular position of the line associated with it. Here $\Delta\theta = 20°$. Then,

$$\lambda_1 = d\sin\theta, \quad \lambda_2 = d\sin(\theta+\Delta\theta).$$

We write

$$\sin(\theta+\Delta\theta) \text{ as } \sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta,$$

then use the equation for the first line to replace $\sin\theta$ with λ_1/d, and $\cos\theta$ with $\sqrt{1-\lambda_1^2/d^2}$. After multiplying by d, we obtain

$$\lambda_1\cos\Delta\theta + \sqrt{d^2-\lambda_1^2}\sin\Delta\theta = \lambda_2.$$

Solving for d, we find

$$d = \sqrt{\frac{(\lambda_2-\lambda_1\cos\Delta\theta)^2+(\lambda_1\sin\Delta\theta)^2}{\sin^2\Delta\theta}}$$
$$= \sqrt{\frac{[(680\text{ nm})-(430\text{ nm})\cos 20°]^2+[(430\text{ nm})\sin 20°]^2}{\sin^2 20°}}$$
$$= 914\text{ nm} = 9.14\times10^{-4}\text{ mm}.$$

There are $1/d = 1/(9.14\times10^{-4}\text{ mm}) = 1.09\times10^3$ rulings per mm.

64. We use Eq. 36-34. For smallest value of θ, we let $m = 1$. Thus,

$$\theta_{\min} = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left[\frac{(1)(30\text{ pm})}{2(0.30\times 10^3\text{ pm})}\right] = 2.9°.$$

65. (a) For the first beam $2d \sin \theta_1 = \lambda_A$ and for the second one $2d \sin \theta_2 = 3\lambda_B$. The values of d and λ_A can then be determined:

$$d = \frac{3\lambda_B}{2\sin\theta_2} = \frac{3(97\text{ pm})}{2\sin 60°} = 1.7\times 10^2\text{ pm}.$$

(b) $\lambda_A = 2d\sin\theta_1 = 2\left(1.7\times 10^2\text{ pm}\right)\left(\sin 23°\right) = 1.3\times 10^2\text{ pm}.$

66. The x-ray wavelength is $\lambda = 2d \sin \theta = 2(39.8\text{ pm}) \sin 30.0° = 39.8\text{ pm}$.

67. We use Eq. 36-34.

(a) From the peak on the left at angle 0.75° (estimated from Fig. 36-46), we have

$$\lambda_1 = 2d\sin\theta_1 = 2\left(0.94\text{ nm}\right)\sin\left(0.75°\right) = 0.025\text{ nm} = 25\text{ pm}.$$

This is the shorter wavelength of the beam. Notice that the estimation should be viewed as reliable to within ±2 pm.

(b) We now consider the next peak:

$$\lambda_2 = 2d\sin\theta_2 = 2(0.94\text{ nm})\sin 1.15° = 0.038\text{ nm} = 38\text{ pm}.$$

This is the longer wavelength of the beam. One can check that the third peak from the left is the second-order one for λ_1.

68. For x-ray ("Bragg") scattering, we have $2d \sin \theta_m = m\lambda$. This leads to

$$\frac{2d\sin\theta_2}{2d\sin\theta_1} = \frac{2\lambda}{1\lambda} \quad\Rightarrow\quad \sin\theta_2 = 2\sin\theta_1 .$$

Thus, with $\theta_1 = 3.4°$, this yields $\theta_2 = 6.8°$. The fact that θ_2 is very nearly twice the value of θ_1 is due to the small angles involved (when angles are small, $\sin\theta_2 / \sin\theta_1 = \theta_2/\theta_1$).

69. Bragg's law gives the condition for diffraction maximum:

$$2d\sin\theta = m\lambda$$

where d is the spacing of the crystal planes and λ is the wavelength. The angle θ is measured from the surfaces of the planes. For a second-order reflection $m = 2$, so

$$d = \frac{m\lambda}{2\sin\theta} = \frac{2\left(0.12\times10^{-9}\,\text{m}\right)}{2\sin 28°} = 2.56\times10^{-10}\,\text{m} \approx 0.26\,\text{nm}.$$

70. The angle of incidence on the reflection planes is $\theta = 63.8° - 45.0° = 18.8°$, and the plane-plane separation is $d = a_0/\sqrt{2}$. Thus, using $2d \sin\theta = \lambda$, we get

$$a_0 = \sqrt{2}d = \frac{\sqrt{2}\lambda}{2\sin\theta} = \frac{0.260\,\text{nm}}{\sqrt{2}\sin 18.8°} = 0.570\,\text{nm}.$$

71. **THINK** The criterion for diffraction maxima is given by the Bragg's law.

EXPRESS We want the reflections to obey the Bragg condition: $2d \sin\theta = m\lambda$, where θ is the angle between the incoming rays and the reflecting planes, λ is the wavelength, and m is an integer. We solve for θ:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{\left(0.125\times10^{-9}\,\text{m}\right)m}{2\left(0.252\times10^{-9}\,\text{m}\right)}\right) = 0.2480m.$$

ANALYZE (a) For $m = 2$ the above equation gives $\theta = 29.7°$. The crystal should be turned $\phi = 45° - 29.7° = 15.3°$ clockwise.

(b) For $m = 1$ the above equation gives $\theta = 14.4°$. The crystal should be turned $\phi = 45° - 14.4° = 30.6°$ clockwise.

(c) For $m = 3$ the above equation gives $\theta = 48.1°$. The crystal should be turned $\phi = 48.1° - 45° = 3.1°$ counterclockwise.

(d) For $m = 4$ the above equation gives $\theta = 82.8°$. The crystal should be turned $\phi = 82.8° - 45° = 37.8°$ counterclockwise.

LEARN Note that there are no intensity maxima for $m > 4$ as one can verify by noting that $m\lambda/2d$ is greater than 1 for m greater than 4.

72. The wavelengths satisfy

$$m\lambda = 2d \sin\theta = 2(275\text{ pm})(\sin 45°) = 389\text{ pm}.$$

In the range of wavelengths given, the allowed values of m are $m = 3, 4$.

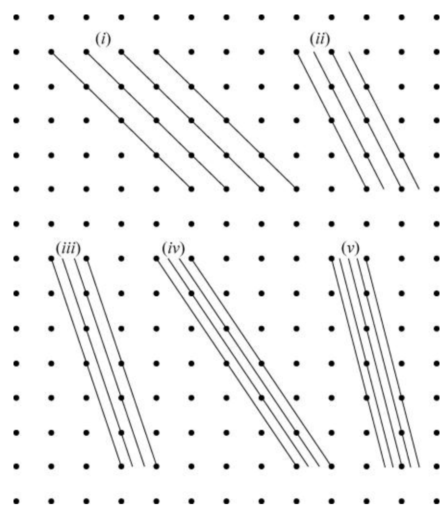
(a) The longest wavelength is 389 pm/3 = 130 pm.

(b) The associated order number is $m = 3$.

(c) The shortest wavelength is 389 pm/4 = 97.2 pm.

(d) The associated order number is $m = 4$.

73. The sets of planes with the next five smaller interplanar spacings (after a_0) are shown in the diagram that follows.



(a) In terms of a_0, the second largest interplanar spacing is $a_0/\sqrt{2} = 0.7071a_0$.

(b) The third largest interplanar spacing is $a_0/\sqrt{5} = 0.4472a_0$.

(c) The fourth largest interplanar spacing is $a_0/\sqrt{10} = 0.3162a_0$.

(d) The fifth largest interplanar spacing is $a_0/\sqrt{13} = 0.2774a_0$.

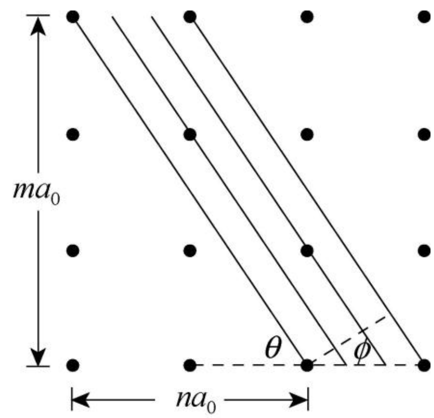
(e) The sixth largest interplanar spacing is $a_0/\sqrt{17} = 0.2425a_0$.

(f) Since a crystal plane passes through lattice points, its slope can be written as the ratio of two integers. Consider a set of planes with slope m/n, as shown in the diagram that follows. The first and last planes shown pass through adjacent lattice points along a horizontal line and there are $m - 1$ planes between. If h is the separation of the first and last planes, then the interplanar spacing is $d = h/m$. If the planes make the angle θ with the horizontal, then the normal to the planes (shown dashed) makes the angle $\phi = 90° - \theta$. The distance h is given by $h = a_0 \cos \phi$ and the interplanar spacing is $d = h/m = (a_0/m) \cos \phi$. Since $\tan \theta = m/n$, $\tan \phi = n/m$ and

$$\cos\phi = 1\Big/\sqrt{1+\tan^2\phi} = m\Big/\sqrt{n^2+m^2}\ .$$

Thus,

$$d = \frac{h}{m} = \frac{a_0\cos\phi}{m} = \frac{a_0}{\sqrt{n^2+m^2}}\ .$$



74. (a) We use Eq. 36-14:

$$\theta_R = 1.22\frac{\lambda}{d} = \frac{(1.22)\left(540\times10^{-6}\text{ mm}\right)}{5.0\text{ mm}} = 1.3\times10^{-4}\text{ rad}\ .$$

(b) The linear separation is $D = L\theta_R = (160 \times 10^3\text{ m})\ (1.3 \times 10^{-4}\text{ rad}) = 21$ m.

75. **THINK** Maxima of a diffraction grating pattern occur at angles θ given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer.

EXPRESS The ruling separation is given by

$$d = \frac{1}{200\text{ mm}^{-1}} = 5.00\times10^{-3}\text{ mm} = 5.00\times10^{-6}\text{ m} = 5000\text{ nm}.$$

Letting $d \sin \theta = m\lambda$, we solve for λ:

$$\lambda = \frac{d\sin\theta}{m} = \frac{(5000\text{ nm})(\sin30°)}{m} = \frac{2500\text{ nm}}{m}$$

where $m = 1, 2, 3 \ldots$. In the visible light range m can assume the following values: $m_1 = 4$, $m_2 = 5$ and $m_3 = 6$.

(a) The longest wavelength corresponds to $m_1 = 4$ with $\lambda_1 = 2500$ nm/4 = 625 nm.

(b) The second longest wavelength corresponds to $m_2 = 5$ with $\lambda_2 = 2500$ nm/5 = 500 nm.

(c) The third longest wavelength corresponds to $m_3 = 6$ with $\lambda_3 = 2500$ nm/6 = 416 nm.

LEARN As shown above, only three values of *m* give wavelengths that are in the visible spectrum. Note that if the light incident on the diffraction grating is not monochromatic, a *spectrum* would be observed since the grating spreads out light into its component wavelength,

76. We combine Eq. 36-31 ($R = \lambda_{avg}/\Delta\lambda$) with Eq. 36-32 ($R = Nm$) and solve for *N*:

$$N = \frac{\lambda_{avg}}{m\,\Delta\lambda} = \frac{590.2 \text{ nm}}{2\,(0.061 \text{ nm})} = 4.84 \times 10^3 .$$

77. **THINK** The condition for a minimum of intensity in a single-slit diffraction pattern is given by $a \sin\theta = m\lambda$, where *a* is the slit width, λ is the wavelength, and *m* is an integer.

EXPRESS As a slit is narrowed, the pattern spreads outward, so the question about "minimum width" suggests that we are looking at the lowest possible values of *m* (the label for the minimum produced by light λ = 600 nm) and *m′* (the label for the minimum produced by light λ′ = 500 nm). Since the angles are the same, then Eq. 36-3 leads to

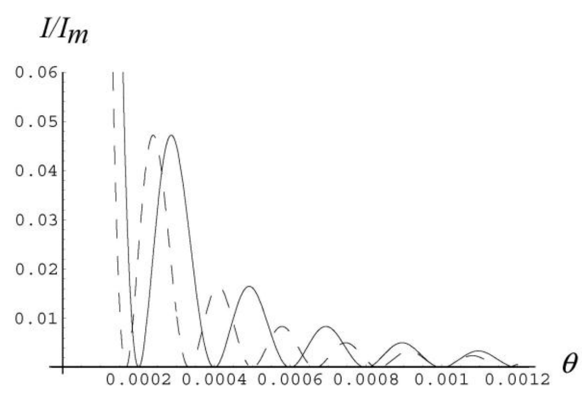
$$m\lambda = m'\lambda'$$

which leads to the choices $m = 5$ and $m' = 6$.

ANALYZE We find the slit width from Eq. 36-3:

$$a = \frac{m\lambda}{\sin\theta} = \frac{5(600\times10^{-9}\text{m})}{\sin(1.00\times10^{-9}\text{rad})} = 3.00\times10^{-3} \text{ m}.$$

LEARN The intensities of the diffraction are shown next (solid line for orange light, and dashed line for blue-green light). The angle $\theta = 0.001$ rad corresponds to $m = 5$ for the orange light, but $m' = 6$ for the blue-green light.



78. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda/a)$. The maxima in the double-slit pattern are located at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$, we find $-6 < m < +6$, or, since m is an integer, $-5 \le m \le +5$. Thus, we find eleven values of m that satisfy this requirement.

79. **THINK** We relate the resolving power of a diffraction grating to the frequency range.

EXPRESS Since the resolving power of a grating is given by $R = \lambda/\Delta\lambda$ and by Nm, the range of wavelengths that can just be resolved in order m is $\Delta\lambda = \lambda/Nm$. Here N is the number of rulings in the grating and λ is the average wavelength. The frequency f is related to the wavelength by $f\lambda = c$, where c is the speed of light. This means $f\Delta\lambda + \lambda\Delta f = 0$, so

$$\Delta\lambda = -\frac{\lambda}{f}\Delta f = -\frac{\lambda^2}{c}\Delta f$$

where $f = c/\lambda$ is used. The negative sign means that an increase in frequency corresponds to a decrease in wavelength.

ANALYZE (a) Equating the two expressions for $\Delta\lambda$, we have

$$\frac{\lambda^2}{c}\Delta f = \frac{\lambda}{Nm}$$

and

$$\Delta f = \frac{c}{Nm\lambda}.$$

(b) The difference in travel time for waves traveling along the two extreme rays is $\Delta t = \Delta L/c$, where ΔL is the difference in path length. The waves originate at slits that are separated by $(N-1)d$, where d is the slit separation and N is the number of slits, so the path difference is $\Delta L = (N-1)d \sin\theta$ and the time difference is

$$\Delta t = \frac{(N-1)d\sin\theta}{c}.$$

If N is large, this may be approximated by $\Delta t = (Nd/c)\sin\theta$. The lens does not affect the travel time.

(c) Substituting the expressions we derived for Δt and Δf, we obtain

$$\Delta f\ \Delta t = \left(\frac{c}{Nm\lambda}\right)\left(\frac{N\,d\sin\theta}{c}\right) = \frac{d\sin\theta}{m\lambda} = 1.$$

The condition $d\sin\theta = m\lambda$ for a diffraction line is used to obtain the last result.

LEARN We take Δf to be positive and interpret it as the range of frequencies that can be resolved.

80. Eq. 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — "Pointillistic paintings use the diffraction of your eye." We are asked to solve for D and are given $\lambda = 500 \times 10^{-9}$ m, $d = 5.00 \times 10^{-3}$ m, and $L = 0.250$ m. Consequently, $D = 3.05 \times 10^{-5}$ m.

81. Consider two of the rays shown in Fig. 36-49, one just above the other. The extra distance traveled by the lower one may be found by drawing perpendiculars from where the top ray changes direction (point P) to the incident and diffracted paths of the lower one. Where these perpendiculars intersect the lower ray's paths are here referred to as points A and C. Where the bottom ray changes direction is point B. We note that angle $\angle APB$ is the same as ψ, and angle BPC is the same as θ (see Fig. 36-49). The difference in path lengths between the two adjacent light rays is

$$\Delta x = |AB| + |BC| = d\sin\psi + d\sin\theta.$$

The condition for bright fringes to occur is therefore

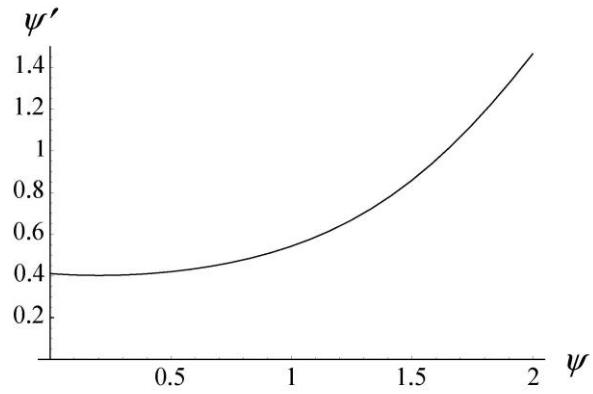
$$\Delta x = d(\sin\psi + \sin\theta) = m\lambda$$

where $m = 0, 1, 2, \ldots$. If we set $\psi = 0$ then this reduces to Eq. 36-25.

82. The angular deviation of a diffracted ray (the angle between the forward extrapolation of the incident ray and its diffracted ray) is $\psi' = \psi + \theta$. For $m = 1$, this becomes

$$\psi' = \psi + \theta = \psi + \sin^{-1}\left(\frac{\lambda}{d} - \sin\psi\right)$$

where the ratio $\lambda/d = 0.40$ using the values given in the problem statement. The graph of this is shown next (with radians used along both axes).



83. **THINK** For relatively wide slits, we consider both the interference of light from two slits, as well as the diffraction of light passing through each slit.

EXPRESS The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda/a)$ is the angle that corresponds to the first diffraction minimum. The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

The equation above sets the range of allowable values of m.

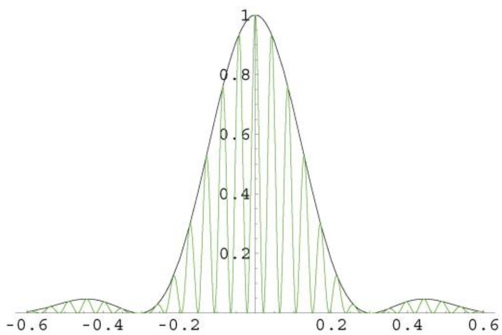
ANALYZE (a) Rewriting the equation as $-d/a < m < +d/a$, noting that $d/a = (14\ \mu\text{m})/(2.0\ \mu\text{m}) = 7$, we arrive at the result $-7 < m < +7$, or (since m must be an integer) $-6 \le m \le +6$, which amounts to 13 distinct values for *m*. Thus, thirteen maxima are within the central envelope.

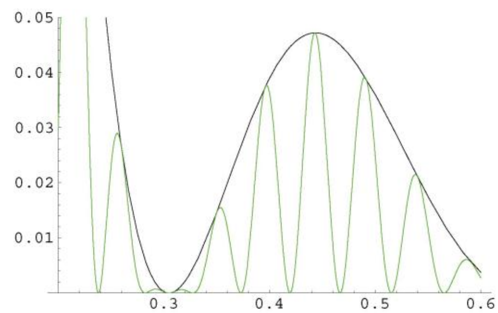
(b) The range (within *one* of the first-order envelopes) is now

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{2\lambda}{a}\right),$$

which leads to $d/a < m < 2d/a$ or $7 < m < 14$. Since *m* is an integer, this means $8 \le m \le 13$ which includes 6 distinct values for *m* in that one envelope. If we were to include the total from both first-order envelopes, the result would be twelve, but the wording of the problem implies six should be the answer (just one envelope).

LEARN The intensity of the double-slit interference experiment is plotted below. The central diffraction envelope contains 13 maxima, and the first-order envelope has 6 on each side.





84. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda / a)$. The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$ we arrive at the result $m_{max} < d/a \le m_{max} + 1$. Due to the symmetry of the pattern, the multiplicity of the m values is $2m_{max} + 1 = 17$ so that $m_{max} = 8$, and the result becomes

$$8 < \frac{d}{a} \le 9$$

where these numbers are as accurate as the experiment allows (that is, "9" means "9.000" if our measurements are that good).

85. We see that the total number of lines on the grating is (1.8 cm)(1400/cm) = 2520 = N. Combining Eq. 36-31 and Eq. 36-32, we find

$$\Delta\lambda = \frac{\lambda_{avg}}{Nm} = \frac{450 \text{ nm}}{(2520)(3)} = 0.0595 \text{ nm} = 59.5 \text{ pm}.$$

86. Use of Eq. 36-21 leads to $D = \frac{1.22\lambda L}{d} = 6.1$ mm.

87. Following the method of Sample Problem — "Pointillistic paintings use the diffraction of your eye," we have

$$\frac{1.22\lambda}{d} = \frac{D}{L}$$

where $\lambda = 550 \times 10^{-9}$ m, $D = 0.60$ m, and $d = 0.0055$ m. Thus we get $L = 4.9 \times 10^3$ m.

88. We use Eq. 36-3 for $m = 2$: $m\lambda = a\sin\theta \Rightarrow \frac{a}{\lambda} = \frac{m}{\sin\theta} = \frac{2}{\sin 37°} = 3.3$.

89. We solve Eq. 36-25 for d:

$$d = \frac{m\lambda}{\sin\theta} = \frac{2(600\times10^{-9} \text{ m})}{\sin 33°} = 2.203\times10^{-6} \text{ m} = 2.203\times10^{-4} \text{ cm}$$

which is typically expressed in reciprocal form as the "number of lines per centimeter" (or per millimeter, or per inch):

$$\frac{1}{d} = 4539 \text{ lines/cm} .$$

The full width is 3.00 cm, so the number of lines is $(4539 \text{ /cm})(3.00 \text{ cm}) = 1.36 \times 10^4$.

90. Although the angles in this problem are not particularly big (so that the small angle approximation could be used with little error), we show the solution appropriate for large as well as small angles (that is, we do not use the small angle approximation here). Equation 36-3 gives

$$m\lambda = a\sin\theta \;\Rightarrow\; \theta = \sin^{-1}(m\lambda/a) = \sin^{-1}[2(0.42\ \mu\text{m})/(5.1\ \mu\text{m})] = 9.48°.$$

The geometry of Figure 35-10(a) is a useful reference (even though it shows a double slit instead of the single slit that we are concerned with here). We see in that figure the relation between y, D, and θ:

$$y = D\tan\theta = (3.2\ \text{m})\tan(9.48°) = 0.534\ \text{m}\ .$$

91. The problem specifies d = 12/8900 using the mm unit, and we note there are no refraction angles greater than 90°. We convert λ = 500 nm to 5×10^{-4} mm and solve Eq. 36-25 for "$m_{\max}$" (realizing it might not be an integer):

$$m_{\max} = \frac{d\sin 90°}{\lambda} = \frac{12}{(8900)(5\times10^{-4})} \approx 2$$

where we have rounded down. There are no values of m (for light of wavelength λ) greater than m = 2.

92. We denote the Earth-Moon separation as L. The energy of the beam of light that is projected onto the Moon is concentrated in a circular spot of diameter d_1, where $d_1/L = 2\theta_R = 2(1.22\lambda/d_0)$, with d_0 the diameter of the mirror on Earth. The fraction of energy picked up by the reflector of diameter d_2 on the Moon is then $\eta' = (d_2/d_1)^2$. This reflected light, upon reaching the Earth, has a circular cross section of diameter d_3 satisfying

$$d_3/L = 2\theta_R = 2(1.22\lambda/d_2).$$

The fraction of the reflected energy that is picked up by the telescope is then $\eta'' = (d_0/d_3)^2$. Consequently, the fraction of the original energy picked up by the detector is

$$\eta = \eta'\eta'' = \left(\frac{d_0}{d_3}\right)^2\left(\frac{d_2}{d_1}\right)^2 = \left[\frac{d_0 d_2}{(2.44\lambda d_{em}/d_0)(2.44\lambda d_{em}/d_2)}\right]^2 = \left(\frac{d_0 d_2}{2.44\lambda d_{em}}\right)^4$$

$$= \left[\frac{(2.6\,\text{m})(0.10\,\text{m})}{2.44(0.69\times10^{-6}\ \text{m})(3.82\times10^{8}\ \text{m})}\right]^4 \approx 4\times10^{-13}\ .$$

93. Since we are considering the *diameter* of the central diffraction maximum, then we are working with *twice* the Rayleigh angle. Using notation similar to that in Sample Problem — "Pointillistic paintings use the diffraction of your eye," we have $2(1.22\lambda/d) = D/L$. Therefore,

$$d = 2\frac{1.22\,\lambda L}{D} = 2\frac{(1.22)(500\times10^{-9}\ \text{m})(3.54\times10^{5}\ \text{m})}{9.1\,\text{m}} = 0.047\,\text{m}\ .$$

94. Letting $d\sin\theta = (L/N)\sin\theta = m\lambda$, we get

$$\lambda = \frac{(L/N)\sin\theta}{m} = \frac{(1.0\times10^{7}\ \text{nm})(\sin 30°)}{(1)(10000)} = 500\ \text{nm}\ .$$

95. **THINK** We use phasors to explore how doubling slit width changes the intensity of the central maximum of diffraction and the energy passing through the slit.

EXPRESS We imagine dividing the original slit into N strips and represent the light from each strip, when it reaches the screen, by a phasor. Then, at the central maximum in the diffraction pattern, we would add the N phasors, all in the same direction and each with the same amplitude. We would find that the intensity there is proportional to N^2.

ANALYZE If we double the slit width, we need $2N$ phasors if they are each to have the amplitude of the phasors we used for the narrow slit. The intensity at the central maximum is proportional to $(2N)^2$ and is, therefore, four times the intensity for the narrow slit. The energy reaching the screen per unit time, however, is only twice the energy reaching it per unit time when the narrow slit is in place. The energy is simply redistributed. For example, the central peak is now half as wide and the integral of the intensity over the peak is only twice the analogous integral for the narrow slit.

LEARN From the discussion above, we see that the intensity of the central maximum increases as N^2. The dependence arises from the following two considerations: (1) The total power reaching the screen is proportional to N, and (2) the width of each maximum (distance between two adjacent minima) is proportional to $1/N$.

96. The condition for a minimum in a single-slit diffraction pattern is given by Eq. 36-3, which we solve for the wavelength:

$$\lambda = \frac{a\sin\theta}{m} = \frac{(0.022\ \text{mm})\sin 1.8°}{1} = 6.91\times10^{-4}\ \text{mm} = 691\ \text{nm}\ .$$

97. Equation 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — "Pointillistic paintings use the diffraction of your eye." We are asked to solve for d and are given $\lambda = 550\times10^{-9}$ m, $D = 30\times10^{-2}$ m, and $L = 160\times10^{3}$ m. Consequently, we obtain $d = 0.358\ \text{m} \approx 36\ \text{cm}$.

98. Following Sample Problem — "Pointillistic paintings use the diffraction of your eye," we use Eq. 36-17 and obtain $L = \dfrac{Dd}{1.22\lambda} = 164\ \text{m}$.

99. (a) Use of Eq. 36-25 for the limit-wavelengths (λ_1 = 700 nm and λ_2 = 550 nm) leads to the condition

$$m_1\lambda_1 \ge m_2\lambda_2$$

for $m_1 + 1 = m_2$ (the low end of a high-order spectrum is what is overlapping with the high end of the next-lower-order spectrum). Assuming equality in the above equation, we can solve for "m_1" (realizing it might not be an integer) and obtain $m_1 \approx 4$ where we have rounded *up*. It is the fourth-order spectrum that is the lowest-order spectrum to overlap with the next higher spectrum.

(b) The problem specifies d = (1/200) mm, and we note there are no refraction angles greater than 90°. We concentrate on the largest wavelength λ = 700 nm = 7×10^{-4} mm and solve Eq. 36-25 for "m_{max}" (realizing it might not be an integer):

$$m_{max} = \frac{d \sin 90°}{\lambda} = \frac{(1/200)\text{ mm}}{7\times 10^{-4}\text{ mm}} \approx 7$$

where we have rounded down. There are no values of m (for the appearance of the full spectrum) greater than m = 7.

100. (a) Maxima of a diffraction grating pattern occur at angles θ given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. With $\theta = 30°$, and $d = (1\text{ mm})/200 = 5.0\times 10^{-6}\text{m}$, the wavelengths for the mth order maxima are given by

$$\lambda = \frac{d\sin\theta}{m} = \frac{(5.0\times 10^{-6}\text{m})\sin 30°}{m} = \frac{2.5\times 10^{-6}\text{m}}{m} = \frac{2500\text{ nm}}{m}$$

For the light to be in the visible spectrum (400 – 750 nm), the values of m are m = 4, 5, and 6. The wavelengths are: $\lambda_4 = (2500\text{ nm})/4 = 625\text{ nm}$, $\lambda_5 = (2500\text{ nm})/5 = 500\text{ nm}$, and $\lambda_6 = (2500\text{ nm})/6 = 417\text{ nm}$.

(c) The three wavelengths correspond to orange, blue-green, and violet, respectively.

101. The dispersion of a grating is given by $D = d\theta/d\lambda$, where θ is the angular position of a line associated with wavelength λ. The angular position and wavelength are related by $\mathbf{d}\sin\theta = m\lambda$, where **d** is the slit separation (which we made boldfaced in order not to confuse it with the d used in the derivative, below) and m is an integer. We differentiate this expression with respect to θ to obtain

$$\frac{d\theta}{d\lambda}\mathbf{d}\cos\theta = m,$$

or

$$D = \frac{d\theta}{d\lambda} = \frac{m}{\mathbf{d}\cos\theta}.$$

Now $m = (\mathbf{d}/\lambda)\sin\theta$, so $D = \frac{\mathbf{d}\sin\theta}{\mathbf{d}\lambda\cos\theta} = \frac{\tan\theta}{\lambda}$.

102. (a) Employing Eq. 36-3 with the small angle approximation ($\sin\theta \approx \tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern), we find (with $m = 1$)

$$D = \frac{ya}{m\lambda} = \frac{(0.90\text{ mm})(0.40\text{ mm})}{4.50\times10^{-4}\text{ mm}} = 800\text{ mm} = 80\text{ cm}$$

which places the screen 80 cm away from the slit.

(b) The above equation gives for the value of y (for $m = 3$)

$$y = \frac{(3)\lambda D}{a} = \frac{(3)(4.50\times10^{-4}\text{ mm})(800\text{ mm})}{(0.40\text{ mm})} = 2.7\text{ mm}.$$

Subtracting this from the first minimum position $y = 0.9$ mm, we find the result $\Delta y = 1.8$ mm.

103. (a) We require that $\sin\theta = m\lambda_{1,2}/d \le \sin 30°$, where $m = 1, 2$ and $\lambda_1 = 500$ nm. This gives

$$d \ge \frac{2\lambda_s}{\sin 30°} = \frac{2(600\text{nm})}{\sin 30°} = 2400\text{nm} = 2.4\,\mu\text{m}.$$

For a grating of given total width L we have $N = L/d \propto d^{-1}$, so we need to minimize d to maximize $R = mN \propto d^{-1}$. Thus we choose $d = 2400$ nm $= 2.4\ \mu$m.

(b) Let the third-order maximum for $\lambda_2 = 600$ nm be the first minimum for the single-slit diffraction profile. This requires that $d\sin\theta = 3\lambda_2 = a\sin\theta$, or

$$a = d/3 = 2400\text{ nm}/3 = 800\text{ nm} = 0.80\ \mu\text{m}.$$

(c) Letting $\sin\theta = m_{\max}\lambda_2/d \le 1$, we obtain

$$m_{\max} \le \frac{d}{\lambda_2} = \frac{2400\text{nm}}{800\text{nm}} = 3.$$

Since the third order is missing the only maxima present are the ones with $m = 0$, 1 and 2. Thus, the largest order of maxima produced by the grating is $m = 2$.

104. For λ = 0.10 nm, we have scattering for order *m*, and for λ′ = 0.075 nm, we have scattering for order *m*′. From Eq. 36-34, we see that we must require $m\lambda = m'\lambda'$, which suggests (looking for the smallest integer solutions) that $m = 3$ and $m' = 4$. Returning with this result and with $d = 0.25$ nm to Eq. 36-34, we obtain

$$\theta = \sin^{-1}\frac{m\lambda}{2d} = 37° .$$

Studying Figure 36-30, we conclude that the angle between incident and scattered beams is $180° - 2\theta = 106°$.

105. The key trigonometric identity used in this proof is $\sin(2\theta) = 2\sin\theta\cos\theta$. Now, we wish to show that Eq. 36-19 becomes (when $d = a$) the pattern for a single slit of width $2a$ (see Eq. 36-5 and Eq. 36-6):

$$I(\theta) = I_m\left(\frac{\sin(2\pi a\sin\theta/\lambda)}{2\pi a\sin\theta/\lambda}\right)^2 .$$

We note from Eq. 36-20 and Eq. 36-21, that the parameters β and α are identical in this case (when $d = a$), so that Eq. 36-19 becomes

$$I(\theta) = I_m\left(\frac{\cos(\pi a\sin\theta/\lambda)\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right)^2 .$$

Multiplying numerator and denominator by 2 and using the trig identity mentioned above, we obtain

$$I(\theta) = I_m\left(\frac{2\cos(\pi a\sin\theta/\lambda)\sin(\pi a\sin\theta/\lambda)}{2\pi a\sin\theta/\lambda}\right)^2 = I_m\left(\frac{\sin(2\pi a\sin\theta/\lambda)}{2\pi a\sin\theta/\lambda}\right)^2$$

which is what we set out to show.

106. Employing Eq. 36-3, we find (with $m = 3$ and all lengths in μm)

$$\theta = \sin^{-1}\frac{m\lambda}{a} = \sin^{-1}\frac{(3)(0.5)}{2}$$

which yields $\theta = 48.6°$. Now, we use the experimental geometry ($\tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern) to find

$$y = D\tan\theta = 2.27 \text{ m}.$$

107. (a) The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where

$$\theta_1 = \sin^{-1}\frac{\lambda}{a},$$

which could be further simplified *if* the small-angle approximation were justified (which it is *not*, since a is so small). The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$ we arrive at the result $m_{\max} < d/a \le m_{\max} + 1$. Due to the symmetry of the pattern, the multiplicity of the m values is $2m_{\max} + 1 = 17$ so that $m_{\max} = 8$, and the result becomes

$$8 < \frac{d}{a} \le 9$$

where these numbers are as accurate as the experiment allows (that is, "9" means "9.000" if our measurements are that good).

108. We refer (somewhat sloppily) to the 400 nm wavelength as "blue" and the 700 nm wavelength as "red." Consider Eq. 36-25 ($m\lambda = d\sin\theta$), for the 3rd order blue, and also for the 2nd order red:

$$(3)\,\lambda_{\text{blue}} = 1200\text{ nm} = d\sin(\theta_{\text{blue}})$$

$$(2)\,\lambda_{\text{red}} = 1400\text{ nm} = d\sin(\theta_{\text{red}})\,.$$

Since sine is an increasing function of angle (in the first quadrant) then the above set of values make clear that $\theta_{\text{red (second order)}} > \theta_{\text{blue (third order)}}$ which shows that the spectrums overlap (regardless of the value of d).

109. One strategy is to divide Eq. 36-25 by Eq. 36-3, assuming the same angle (a point we'll come back to, later) and the same light wavelength for both:

$$\frac{m}{m'} = \frac{m\lambda}{m'\lambda} = \frac{d\sin\theta}{a\sin\theta} = \frac{d}{a}.$$

We recall that d is measured from middle of transparent strip to the middle of the next transparent strip, which in this particular setup means $d = 2a$. Thus, $m/m' = 2$, or $m = 2m'$.

Now we interpret our result. First, the division of the equations is not valid when $m = 0$ (which corresponds to $\theta = 0$), so our remarks do not apply to the $m = 0$ maximum. Second, Eq. 36-25 gives the "bright" interference results, and Eq. 36-3 gives the "dark" diffraction results (where the latter overrules the former in places where they coincide – see Figure 36-17 in the textbook). For m' = any nonzero integer, the relation $m = 2m'$ implies that m = any nonzero *even* integer. As mentioned above, these are occurring at the same angle, so the even integer interference maxima are eliminated by the diffraction minima.

110. The derivation is similar to that used to obtain Eq. 36-27. At the first minimum beyond the mth principal maximum, two waves from adjacent slits have a phase difference of $\Delta\phi = 2\pi m + (2\pi/N)$, where N is the number of slits. This implies a difference in path length of

$$\Delta L = (\Delta\phi/2\pi)\lambda = m\lambda + (\lambda/N).$$

If θ_m is the angular position of the mth maximum, then the difference in path length is also given by $\Delta L = d\sin(\theta_m + \Delta\theta)$. Thus

$$d\sin(\theta_m + \Delta\theta) = m\lambda + (\lambda/N).$$

We use the trigonometric identity

$$\sin(\theta_m + \Delta\theta) = \sin\theta_m \cos\Delta\theta + \cos\theta_m \sin\Delta\theta.$$

Since $\Delta\theta$ is small, we may approximate $\sin\Delta\theta$ by $\Delta\theta$ in radians and $\cos\Delta\theta$ by unity. Thus,

$$d\sin\theta_m + d\,\Delta\theta\cos\theta_m = m\lambda + (\lambda/N).$$

We use the condition $d\sin\theta_m = m\lambda$ to obtain $d\,\Delta\theta\cos\theta_m = \lambda/N$ and

$$\Delta\theta = \frac{\lambda}{N\,d\cos\theta_m}.$$

111. There are two unknowns, the x-ray wavelength λ and the plane separation d, so data for scattering at two angles from the same planes should suffice. The observations obey Bragg's law, so

$$2d\sin\theta_1 = m_1\lambda,\; 2d\sin\theta_2 = m_2\lambda.$$

However, these cannot be solved for the unknowns. For example, we can use the first equation to eliminate λ from the second. We obtain

$$m_2\sin\theta_1 = m_1\sin\theta_2,$$

an equation that does not contain either of the unknowns.

112. The problem specifies d = (1 mm)/500 = 2.00 μm unit, and we note there are no refraction angles greater than 90°. We concentrate on the largest wavelength λ = 700 nm = 0.700 μm and solve Eq. 36-25 for "m_{max}" (realizing it might not be an integer):

$$m_{\max} = \frac{d \sin 90°}{\lambda} = \frac{d}{\lambda} = \frac{2.00\ \mu\text{m}}{0.700\ \mu\text{m}} \approx 2$$

where we have rounded down. There are no values of m (for appearance of the full spectrum) greater than m = 2.

113. When the speaker phase difference is π rad (180°), we expect to see the "reverse" of Fig. 36-15 [translated into the acoustic context, so that "bright" becomes "loud" and "dark" becomes "quiet"]. That is, with 180° phase difference, all the peaks in Fig. 36-15 become valleys and all the valleys become peaks. As the phase changes from zero to 180° (and similarly for the change from 180° back to 360° = original pattern), the peaks should shift (and change height) in a continuous fashion – with the most dramatic feature being a large "dip" in the center diffraction envelope which deepens until it seems to split the central maximum into smaller diffraction maxima which (once the phase difference reaches π rad) will be located at angles given by a sinθ = ± λ. How many interference fringes would actually "be inside" each of these smaller diffraction maxima would, of course, depend on the particular values of a, λ and d.

114. From $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer, we write

$$d \sin(\theta + \Delta\theta) = m(\lambda + \Delta\lambda)$$

Subtracting the first equation from the second gives

$$d\left[\sin(\theta + \Delta\theta) - \sin\theta\right] = m(\lambda + \Delta\lambda) - m\lambda = m\Delta\lambda\,.$$

Noting that

$$\lim_{\Delta\theta \to 0} \frac{\sin(\theta + \Delta\theta) - \sin(\theta + \Delta\theta)}{\Delta\theta} = \cos\theta\,,$$

the above expression simplifies to

$$\cos\theta = \frac{m\Delta\lambda}{d\Delta\theta}\,.$$

Thus,

$$\Delta\theta = \frac{m\Delta\lambda}{d\cos\theta} = \frac{m\Delta\lambda}{d\sqrt{1-\sin^2\theta}} = \frac{m\Delta\lambda}{d\sqrt{1-(m\lambda/d)^2}} = \frac{m\Delta\lambda}{\sqrt{d^2-(m\lambda)^2}} = \frac{\Delta\lambda}{\sqrt{(d/m)^2-\lambda^2}}\,.$$

Chapter 37

1. From the time dilation equation $\Delta t = \gamma\Delta t_0$ (where Δt_0 is the proper time interval, $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = v/c$), we obtain

$$\beta = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2}.$$

The proper time interval is measured by a clock at rest relative to the muon. Specifically, Δt_0 = 2.2000 μs. We are also told that Earth observers (measuring the decays of moving muons) find Δt = 16.000 μs. Therefore,

$$\beta = \sqrt{1-\left(\frac{2.2000\,\mu\text{s}}{16.000\,\mu\text{s}}\right)^2} = 0.99050.$$

2. (a) We find β from $\gamma = 1/\sqrt{1-\beta^2}$:

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(1.0100000)^2}} = 0.14037076.$$

(b) Similarly, $\beta = \sqrt{1-(10.000000)^{-2}} = 0.99498744.$

(c) In this case, $\beta = \sqrt{1-(100.00000)^{-2}} = 0.99995000.$

(d) The result is $\beta = \sqrt{1-(1000.0000)^{-2}} = 0.99999950.$

3. (a) The round-trip (discounting the time needed to "turn around") should be one year according to the clock you are carrying (this is your proper time interval Δt_0) and 1000 years according to the clocks on Earth, which measure Δt. We solve Eq. 37-7 for β:

$$\beta = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1-\left(\frac{1\text{y}}{1000\text{y}}\right)^2} = 0.99999950.$$

(b) The equations do not show a dependence on acceleration (or on the direction of the velocity vector), which suggests that a circular journey (with its constant magnitude centripetal acceleration) would give the same result (if the speed is the same) as the one described in the problem. A more careful argument can be given to support this, but it should be admitted that this is a fairly subtle question that has occasionally precipitated debates among professional physicists.

4. Due to the time-dilation effect, the time between initial and final ages for the daughter is longer than the four years experienced by her father:

$$t_{f\,\text{daughter}} - t_{i\,\text{daughter}} = \gamma(4.000 \text{ y})$$

where γ is the Lorentz factor (Eq. 37-8). Letting T denote the age of the father, then the conditions of the problem require

$$T_i = t_{i\,\text{daughter}} + 20.00 \text{ y}\,,\quad T_f = t_{f\,\text{daughter}} - 20.00 \text{ y}\;.$$

Since $T_f - T_i = 4.000$ y, then these three equations combine to give a single condition from which γ can be determined (and consequently v):

$$44 = 4\gamma \quad\Rightarrow\quad \gamma = 11 \quad\Rightarrow\quad \beta = \frac{2\sqrt{30}}{11} = 0.9959.$$

5. In the laboratory, it travels a distance $d = 0.00105$ m $= vt$, where $v = 0.992c$ and t is the time measured on the laboratory clocks. We can use Eq. 37-7 to relate t to the proper lifetime of the particle t_0:

$$t = \frac{t_0}{\sqrt{1-(v/c)^2}} \quad\Rightarrow\quad t_0 = t\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{d}{0.992c}\sqrt{1-0.992^2}$$

which yields $t_0 = 4.46 \times 10^{-13}$ s $= 0.446$ ps.

6. From the value of Δt in the graph when $\beta = 0$, we infer than Δt_o in Eq. 37-9 is 8.0 s. Thus, that equation (which describes the curve in Fig. 37-22) becomes

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} = \frac{8.0 \text{ s}}{\sqrt{1-\beta^2}}\,.$$

If we set $\beta = 0.98$ in this expression, we obtain approximately 40 s for Δt.

7. We solve the time dilation equation for the time elapsed (as measured by Earth observers):

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(0.9990)^2}}$$

where Δt_0 = 120 y. This yields Δt = 2684 y $\approx 2.68\times10^3$ y.

8. The contracted length of the tube would be

$$L = L_0\sqrt{1-\beta^2} = (3.00\,\text{m})\sqrt{1-(0.999987)^2} = 0.0153\,\text{m}.$$

9. **THINK** The length of the moving spaceship is measured to be shorter by a stationary observer

EXPRESS Let the rest length of the spaceship be L_0. The length measured by the timing station is

$$L = L_0\sqrt{1-(v/c)^2}.$$

ANALYZE (a) The rest length is L_0 = 130 m. With v = 0.740c, we obtain

$$L = L_0\sqrt{1-(v/c)^2} = (130\,\text{m})\sqrt{1-(0.740)^2} = 87.4\,\text{m}.$$

(b) The time interval for the passage of the spaceship is

$$\Delta t = \frac{L}{v} = \frac{87.4\,\text{m}}{(0.740)(3.00\times10^8\,\text{m/s})} = 3.94\times10^{-7}\,\text{s}.$$

LEARN The length of the spaceship appears to be contracted by a factor of

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.740)^2}} = 1.487.$$

10. Only the "component" of the length in the x direction contracts, so its y component stays

$$\ell'_y = \ell_y = \ell\sin 30° = (1.0\text{ m})(0.50) = 0.50\,\text{m}$$

while its x component becomes

$$\ell'_x = \ell_x\sqrt{1-\beta^2} = (1.0\text{ m})(\cos 30°)\sqrt{1-(0.90)^2} = 0.38\,\text{m}.$$

Therefore, using the Pythagorean theorem, the length measured from S' is

$$\ell' = \sqrt{(\ell'_x)^2 + (\ell'_y)^2} = \sqrt{(0.38\text{ m})^2 + (0.50\text{ m})^2} = 0.63\,\text{m}.$$

11. The length L of the rod, as measured in a frame in which it is moving with speed v parallel to its length, is related to its rest length L_0 by $L = L_0/\gamma$, where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Since γ must be greater than 1, L is less than L_0. For this problem, L_0 = 1.70 m and β = 0.630, so

$$L = L_0\sqrt{1-\beta^2} = (1.70\,\text{m})\sqrt{1-(0.630)^2} = 1.32\,\text{m}.$$

12. (a) We solve Eq. 37-13 for v and then plug in:

$$\beta = \sqrt{1-\left(\frac{L}{L_0}\right)^2} = \sqrt{1-\left(\frac{1}{2}\right)^2} = 0.866.$$

(b) The Lorentz factor in this case is $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}} = 2.00$.

13. (a) The speed of the traveler is $v = 0.99c$, which may be equivalently expressed as 0.99 ly/y. Let d be the distance traveled. Then, the time for the trip as measured in the frame of Earth is

$$\Delta t = d/v = (26\text{ ly})/(0.99\text{ ly/y}) = 26.26\text{ y}.$$

(b) The signal, presumed to be a radio wave, travels with speed c and so takes 26.0 y to reach Earth. The total time elapsed, in the frame of Earth, is

$$26.26\text{ y} + 26.0\text{ y} = 52.26\text{ y}.$$

(c) The proper time interval is measured by a clock in the spaceship, so $\Delta t_0 = \Delta t/\gamma$. Now

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.99)^2}} = 7.09.$$

Thus, Δt_0 = (26.26 y)/(7.09) = 3.705 y.

14. From the value of L in the graph when $\beta = 0$, we infer that L_0 in Eq. 37-13 is 0.80 m. Thus, that equation (which describes the curve in Fig. 37-23) with SI units understood becomes

$$L = L_0\sqrt{1-(v/c)^2} = (0.80\,\text{m})\sqrt{1-\beta^2}\ .$$

If we set $\beta = 0.95$ in this expression, we obtain approximately 0.25 m for L.

15. (a) Let d = 23000 ly = 23000 c y, which would give the distance in meters if we included a conversion factor for years $\to$ seconds. With Δt_0 = 30 y and $\Delta t = d/v$ (see Eq. 37-10), we wish to solve for v from Eq. 37-7. Our first step is as follows:

$$\Delta t = \frac{d}{v} = \frac{\Delta t_0}{\sqrt{1-\beta^2}} \quad \Rightarrow \quad \frac{23000 \text{ y}}{\beta} = \frac{30 \text{ y}}{\sqrt{1-\beta^2}},$$

at which point we can cancel the unit year and manipulate the equation to solve for the speed parameter β. This yields

$$\beta = \frac{1}{\sqrt{1+(30/23000)^2}} = 0.99999915.$$

(b) The Lorentz factor is $\gamma = 1/\sqrt{1-\beta^2} = 766.6680752$. Thus, the length of the galaxy measured in the traveler's frame is

$$L = \frac{L_0}{\gamma} = \frac{23000 \text{ ly}}{766.6680752} = 29.99999 \text{ ly} \approx 30 \text{ ly}.$$

16. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 37-21 to apply without additional terms. In part (a), we apply these equations directly with

$$v = +0.400c = 1.199 \times 10^8 \text{ m/s},$$

and in part (c) we simply change $v \to -v$ and recalculate the primed values.

(a) The position coordinate measured in the S' frame is

$$x' = \gamma(x - vt) = \frac{x - vt}{\sqrt{1-\beta^2}} = \frac{3.00\times10^8 \text{ m} - (1.199\times10^8 \text{m/s})(2.50\text{s})}{\sqrt{1-(0.400)^2}} = 2.7\times10^5 \text{ m} \approx 0,$$

where we conclude that the numerical result (2.7×10^5 m or 2.3×10^5 m depending on how precise a value of v is used) is not meaningful (in the significant figures sense) and should be set equal to zero (that is, it is "consistent with zero" in view of the statistical uncertainties involved).

(b) The time coordinate measured in the S' frame is

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t - \beta x/c}{\sqrt{1-\beta^2}} = \frac{2.50\text{s} - (0.400)(3.00\times10^8 \text{ m})/2.998\times10^8 \text{m/s}}{\sqrt{1-(0.400)^2}} = 2.29\text{s}.$$

(c) Now, we obtain

$$x' = \frac{x + vt}{\sqrt{1-\beta^2}} = \frac{3.00\times10^8\ \text{m} + \left(1.199\times10^8\ \text{m/s}\right)\left(2.50\ \text{s}\right)}{\sqrt{1-\left(0.400\right)^2}} = 6.54\times10^8\ \text{m}.$$

(d) Similarly,

$$t' = \gamma\left(t + \frac{vx}{c^2}\right) = \frac{2.50\,\text{s} + \left(0.400\right)\left(3.00\times10^8\ \text{m}\right)/2.998\times10^8\,\text{m/s}}{\sqrt{1-\left(0.400\right)^2}} = 3.16\,\text{s}.$$

17. **THINK** We apply Lorentz transformation to calculate x' and t' according to an observer in S'.

EXPRESS The proper time is not measured by clocks in either frame S or frame S' since a single clock at rest in either frame cannot be present at the origin and at the event. The full Lorentz transformation must be used:

$$x' = \gamma(x - vt), \quad t' = \gamma(t - \beta x / c)$$

where $\beta = v/c = 0.950$ and

$$\gamma = 1\sqrt{1-\beta^2} = 1/\sqrt{1-(0.950)^2} = 3.20256\,.$$

ANALYZE (a) Thus, the spatial coordinate in S' is

$$\begin{aligned} x' &= \gamma(x - vt) = (3.20256)\left(100\times10^3\,\text{m} - (0.950)(2.998\times10^8\ \text{m/s})(200\times10^{-6}\text{s})\right) \\ &= 1.38\times10^5\ \text{m} = 138\ \text{km}. \end{aligned}$$

(b) The temporal coordinate in S' is

$$\begin{aligned} t' &= \gamma(t - \beta x / c) = (3.20256)\left[200\times10^{-6}\,\text{s} - \frac{(0.950)(100\times10^3\ \text{m})}{2.998\times10^8\ \text{m/s}}\right] \\ &= -3.74\times10^{-4}\ \text{s} = -374\,\mu\text{s}\,. \end{aligned}$$

LEARN The time and the location of the collision recorded by an observer S' are different than that by another observer in S.

18. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 37-21 to apply without additional terms. We label the event coordinates with subscripts: $(x_1, t_1) = (0, 0)$ and $(x_2, t_2) = (3000\ \text{m}, 4.0\times10^{-6}\ \text{s})$.

(a) We expect $(x'_1, t'_1) = (0, 0)$, and this may be verified using Eq. 37-21.

(b) We now compute (x'_2, t'_2), assuming $v = +0.60c = +1.799 \times 10^8$ m/s (the sign of v is not made clear in the problem statement, but the figure referred to, Fig. 37-9, shows the motion in the positive x direction).

$$x_2' = \frac{x - vt}{\sqrt{1-\beta^2}} = \frac{3000\text{ m} - (1.799\times10^8\text{ m/s})(4.0\times10^{-6}\text{ s})}{\sqrt{1-(0.60)^2}} = 2.85\times10^3\text{ m}$$

$$t_2' = \frac{t - \beta x/c}{\sqrt{1-\beta^2}} = \frac{4.0\times10^{-6}\text{ s} - (0.60)(3000\text{ m})/(2.998\times10^8\text{ m/s})}{\sqrt{1-(0.60)^2}} = -2.5\times10^{-6}\text{ s}$$

(c) The two events in frame S occur in the order: first 1, then 2. However, in frame S' where $t_2' < 0$, they occur in the reverse order: first 2, then 1. So the two observers see the two events in the reverse sequence.

We note that the distances $x_2 - x_1$ and $x_2' - x_1'$ are larger than how far light can travel during the respective times $(c(t_2 - t_1) = 1.2\text{ km}$ and $c\,|t_2' - t_1'| \approx 750\,\text{m})$, so that no inconsistencies arise as a result of the order reversal (that is, no signal from event 1 could arrive at event 2 or vice versa).

19. (a) We take the flashbulbs to be at rest in frame S, and let frame S' be the rest frame of the second observer. Clocks in neither frame measure the proper time interval between the flashes, so the full Lorentz transformation (Eq. 37-21) must be used. Let t_s be the time and x_s be the coordinate of the small flash, as measured in frame S. Then, the time of the small flash, as measured in frame S', is

$$t_s' = \gamma\left(t_s - \frac{\beta x_s}{c}\right)$$

where $\beta = v/c = 0.250$ and

$$\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.250)^2} = 1.0328\,.$$

Similarly, let t_b be the time and x_b be the coordinate of the big flash, as measured in frame S. Then, the time of the big flash, as measured in frame S', is

$$t_b' = \gamma\left(t_b - \frac{\beta x_b}{c}\right).$$

Subtracting the second Lorentz transformation equation from the first and recognizing that $t_s = t_b$ (since the flashes are simultaneous in S), we find

$$\Delta t' = \frac{\gamma\beta(x_s - x_b)}{c} = \frac{(1.0328)(0.250)(30\times10^3\text{ m})}{3.00\times10^8\text{ m/s}} = 2.58\times10^{-5}\text{ s}$$

where $\Delta t' = t'_b - t'_s$.

(b) Since $\Delta t'$ is negative, t'_b is greater than t'_s . The small flash occurs first in S'.

20. From Eq. 2 in Table 37-2, we have

$$\Delta t = v\,\gamma\,\Delta x'/c^2 + \gamma\,\Delta t'.$$

The coefficient of $\Delta x'$ is the slope (4.0 μs/400 m) of the graph, and the last term involving $\Delta t'$ is the "y-intercept" of the graph. From the first observation, we can solve for $\beta = v/c = 0.949$ and consequently $\gamma = 3.16$. Then, from the second observation, we find

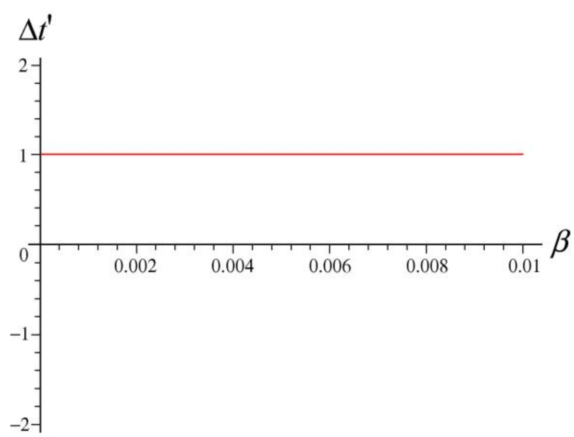
$$\Delta t' = \frac{\Delta t}{\gamma} = \frac{2.00\times10^{-6}\text{s}}{3.16} = 6.3\times10^{-7}\text{s} .$$

21. (a) Using Eq. 2′ of Table 37-2, we have

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\left(\Delta t - \frac{\beta\Delta x}{c}\right) = \gamma\left(1.00\times10^{-6}\text{s} - \frac{\beta(400\,\text{m})}{2.998\times10^8\text{ m/s}}\right)$$

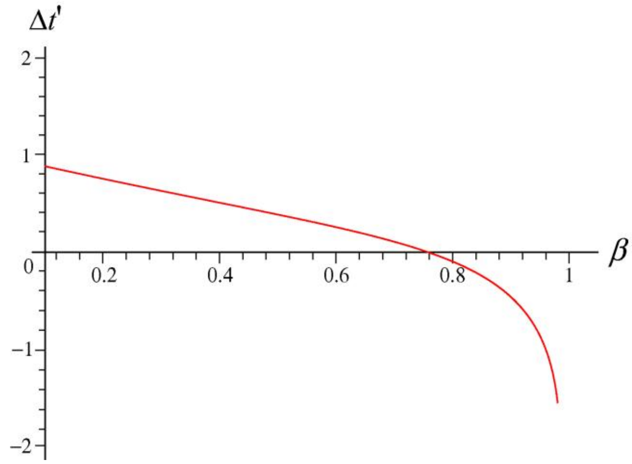
where the Lorentz factor is itself a function of β (see Eq. 37-8).

(b) A plot of $\Delta t'$ as a function of β in the range $0 < \beta < 0.01$ is shown below:



Note the limits of the vertical axis are +2 μs and −2 μs. We note how "flat" the curve is in this graph; the reason is that for low values of β, Bullwinkle's measure of the temporal separation between the two events is approximately our measure, namely +1.0 μs. There are no nonintuitive relativistic effects in this case.

(c) A plot of $\Delta t'$ as a function of β in the range $0.1 < \beta < 1$ is shown below:



(d) Setting

$$\Delta t' = \gamma\left(\Delta t - \frac{\beta \Delta x}{c}\right) = \gamma\left(1.00\times10^{-6}\,\text{s} - \frac{\beta(400\,\text{m})}{2.998\times10^{8}\,\text{m/s}}\right) = 0$$

leads to

$$\beta = \frac{c\Delta t}{\Delta x} = \frac{(2.998\times10^{8}\,\text{m/s})(1.00\times10^{-6}\,\text{s})}{400\,\text{m}} = 0.7495 \approx 0.750\,.$$

(e) For the graph shown in part (c), as we increase the speed, the temporal separation according to Bullwinkle is positive for the lower values and then goes to zero and finally (as the speed approaches that of light) becomes progressively more negative. For the lower speeds with

$$\Delta t' > 0 \Rightarrow t_A' < t_B' \Rightarrow \quad 0 < \beta < 0.750,$$

according to Bullwinkle event *A* occurs before event *B* just as we observe.

(f) For the higher speeds with

$$\Delta t' < 0 \;\Rightarrow\; \quad t_A' > t_B' \Rightarrow 0.750 < \beta < 1,$$

according to Bullwinkle event *B* occurs before event *A* (the opposite of what we observe).

(g) No, event *A* cannot cause event *B* or vice versa. We note that

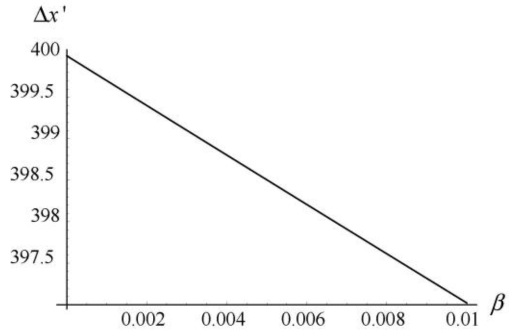
$$\Delta x/\Delta t = (400\text{ m})/(1.00\ \mu\text{s}) = 4.00\times10^{8}\text{ m/s} > c.$$

A signal cannot travel from event *A* to event *B* without exceeding *c*, so causal influences cannot originate at *A* and thus affect what happens at *B*, or vice versa.

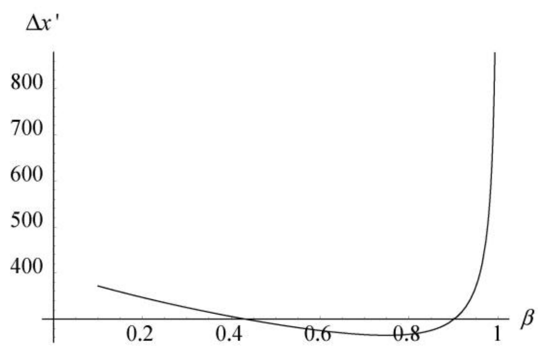
22. (a) From Table 37-2, we find

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right) = \gamma\left(\Delta x - \beta c\Delta t\right) = \gamma[400 \text{ m} - \beta c(1.00\ \mu\text{s})] = \frac{400 \text{ m} - (299.8 \text{ m})\beta}{\sqrt{1-\beta^2}}$$

(b) A plot of $\Delta x'$ as a function of β with $0 < \beta < 0.01$ is shown below:



(c) A plot of $\Delta x'$ as a function of β with $0.1 < \beta < 1$ is shown below:



(d) To find the minimum, we can take a derivative of $\Delta x'$ with respect to β, simplify, and then set equal to zero:

$$\frac{d\Delta x'}{d\beta} = \frac{d}{d\beta}\left(\frac{\Delta x - \beta c\Delta t}{\sqrt{1-\beta^2}}\right) = \frac{\beta\Delta x - c\Delta t}{(1-\beta^2)^{3/2}} = 0$$

This yields

$$\beta = \frac{c\Delta t}{\Delta x} = \frac{(2.998\times10^8 \text{ m/s})(1.00\times10^{-6}\text{s})}{400 \text{ m}} = 0.7495 \approx 0.750$$

(e) Substituting this value of β into the part (a) expression yields $\Delta x'$ = 264.8 m ≈ 265 m for its minimum value.

23. (a) The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.600)^2}} = 1.25 .$$

(b) In the unprimed frame, the time for the clock to travel from the origin to x = 180 m is

$$t = \frac{x}{v} = \frac{180\,\text{m}}{(0.600)(3.00\times10^8\ \text{m/s})} = 1.00\times10^{-6}\ \text{s} .$$

The proper time interval between the two events (at the origin and at x = 180 m) is measured by the clock itself. The reading on the clock at the beginning of the interval is zero, so the reading at the end is

$$t' = \frac{t}{\gamma} = \frac{1.00\times10^{-6}\,\text{s}}{1.25} = 8.00\times10^{-7}\,\text{s} .$$

24. The time-dilation information in the problem (particularly, the 15 s on "his wristwatch… which takes 30.0 s according to you") reveals that the Lorentz factor is γ= 2.00 (see Eq. 37-9), which implies his speed is v = 0.866c.

(a) With γ= 2.00, Eq. 37-13 implies the contracted length is 0.500 m.

(b) There is no contraction along the direction perpendicular to the direction of motion (or "boost" direction), so meter stick 2 still measures 1.00 m long.

(c) As in part (b), the answer is 1.00 m.

(d) Equation 1′ in Table 37-2 gives

$$\begin{aligned}\Delta x' &= x_2' - x_1' = \gamma\left(\Delta x - v\Delta t\right) = (2.00)\left[20.0\ \text{m} - (0.866)(2.998\times10^8\ \text{m/s})(40.0\times10^{-9}\ \text{s})\right]\\ &= 19.2\ \text{m}\end{aligned}$$

(e) Equation 2′ in Table 37-2 gives

$$\begin{aligned}\Delta t' &= t_2' - t_1' = \gamma\left(\Delta t - v\Delta x/c^2\right) = \gamma\left(\Delta t - \beta\Delta x/c\right)\\ &= (2.00)\left[40.0\times10^{-9}\ \text{s} - (0.866)(20.0\ \text{m})/(2.998\times10^8\,\text{m/s})\right]\\ &= -35.5\ \text{ns} .\end{aligned}$$

In absolute value, the two events are separated by 35.5 ns.

(f) The negative sign obtained in part (e) implies event 2 occurred before event 1.

25. (a) In frame *S*, our coordinates are such that x_1 = +1200 m for the big flash, and x_2 = 1200 – 720 = 480 m for the small flash (which occurred later). Thus,

$$\Delta x = x_2 - x_1 = -720 \text{ m}.$$

If we set $\Delta x' = 0$ in Eq. 37-25, we find

$$0 = \gamma(\Delta x - v\Delta t) = \gamma\left(-720\,\text{m} - v(5.00\times10^{-6}\,\text{s})\right)$$

which yields $v = -1.44 \times 10^8$ m/s, or $\beta = v/c = 0.480$.

(b) The negative sign in part (a) implies that frame S' must be moving in the $-x$ direction.

(c) Equation 37-28 leads to

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\left(5.00\times10^{-6}\,\text{s} - \frac{(-1.44\times10^8\,\text{m/s})(-720\,\text{m})}{(2.998\times10^8\,\text{m/s})^2}\right),$$

which turns out to be positive (regardless of the specific value of γ). Thus, the order of the flashes is the same in the S' frame as it is in the *S* frame (where Δt is also positive). Thus, the big flash occurs first, and the small flash occurs later.

(d) Finishing the computation begun in part (c), we obtain

$$\Delta t' = \frac{5.00\times10^{-6}\,\text{s} - (-1.44\times10^8\,\text{m/s})(-720\,\text{m})/(2.998\times10^8\,\text{m/s})^2}{\sqrt{1-0.480^2}} = 4.39\times10^{-6}\,\text{s}.$$

26. We wish to adjust Δt so that

$$0 = \Delta x' = \gamma\left(\Delta x - v\Delta t\right) = \gamma(-720\,\text{m} - v\Delta t)$$

in the limiting case of $|v| \to c$. Thus,

$$\Delta t = \frac{\Delta x}{v} = \frac{\Delta x}{c} = \frac{720\,\text{m}}{2.998\times10^8\,\text{m/s}} = 2.40\times10^{-6}\,\text{s}.$$

27. **THINK** We apply relativistic velocity transformation to calculate the velocity of the particle with respect to frame *S*.

EXPRESS We assume S' is moving in the $+x$ direction. Let u' be the velocity of the particle as measured in S' and v be the velocity of S' relative to *S*, the velocity of the particle as measured in *S* is given by Eq. 37-29:

$$u = \frac{u' + v}{1 + u'v/c^2}.$$

ANALYZE With $u' = +0.40c$ and $v = +0.60c$, we obtain

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.40c + 0.60c}{1 + (0.40c)(+0.60c)/c^2} = 0.81c\ .$$

LEARN The classical Galilean transformation would have given

$$u = u' + v = 0.40c + 0.60c = 1.0c.$$

28. (a) We use Eq. 37-29:

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.47c + 0.62c}{1 + (0.47)(0.62)} = 0.84c\ ,$$

in the direction of increasing x (since $v > 0$). In unit-vector notation, we have $\vec{v} = (0.84c)\hat{\mathrm{i}}$.

(b) The classical theory predicts that $v = 0.47c + 0.62c = 1.1c$, or $\vec{v} = (1.1c)\hat{\mathrm{i}}$.

(c) Now $v' = -0.47c\,\hat{\mathrm{i}}$ so

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{-0.47c + 0.62c}{1 + (-0.47)(0.62)} = 0.21c\ ,$$

or $\vec{v} = (0.21c)\hat{\mathrm{i}}$

(d) By contrast, the classical prediction is $v = 0.62c - 0.47c = 0.15c$, or $\vec{v} = (0.15c)\hat{\mathrm{i}}$.

29. (a) One thing Einstein's relativity has in common with the more familiar (Galilean) relativity is the reciprocity of relative velocity. If Joe sees Fred moving at 20 m/s eastward away from him (Joe), then Fred should see Joe moving at 20 m/s westward away from him (Fred). Similarly, if we see Galaxy A moving away from us at $0.35c$ then an observer in Galaxy A should see our galaxy move away from him at $0.35c$, or 0.35 in multiple of c.

(b) We take the positive axis to be in the direction of motion of Galaxy A, as seen by us. Using the notation of Eq. 37-29, the problem indicates $v = +0.35c$ (velocity of Galaxy A relative to Earth) and $u = -0.35c$ (velocity of Galaxy B relative to Earth). We solve for the velocity of B relative to A:

$$\frac{u'}{c}=\frac{u/c-v/c}{1-uv/c^2}=\frac{(-0.35)-0.35}{1-(-0.35)(0.35)}=-0.62\,,$$

or $|u'/c|=0.62$.

30. Using the notation of Eq. 37-29 and taking "away" (from us) as the positive direction, the problem indicates $v = +0.4c$ and $u = +0.8c$ (with 3 significant figures understood). We solve for the velocity of Q_2 relative to Q_1 (in multiple of c):

$$\frac{u'}{c}=\frac{u/c-v/c}{1-uv/c^2}=\frac{0.8-0.4}{1-(0.8)(0.4)}=0.588$$

in a direction away from Earth.

31. **THINK** Both the spaceship and the micrometeorite are moving relativistically, and we apply relativistic speed transformation to calculate the velocity of the micrometeorite relative to the spaceship.

EXPRESS Let S be the reference frame of the micrometeorite, and S' be the reference frame of the spaceship. We assume S to be moving in the $+x$ direction. Let u be the velocity of the micrometeorite as measured in S and v be the velocity of S' relative to S, the velocity of the micrometeorite as measured in S' can be solved by using Eq. 37-29:

$$u=\frac{u'+v}{1+u'v/c^2}\quad\Rightarrow\; u'=\frac{u-v}{1-uv/c^2}.$$

ANALYZE The problem indicates that $v = -0.82c$ (spaceship velocity) and $u = +0.82c$ (micrometeorite velocity). We solve for the velocity of the micrometeorite relative to the spaceship:

$$u'=\frac{u-v}{1-uv/c^2}=\frac{0.82c-(-0.82c)}{1-(0.82)(-0.82)}=0.98c$$

or 2.94×10^8 m/s. Using Eq. 37-10, we conclude that observers on the ship measure a transit time for the micrometeorite (as it passes along the length of the ship) equal to

$$\Delta t=\frac{d}{u'}=\frac{350\,\text{m}}{2.94\times10^8\,\text{m/s}}=1.2\times10^{-6}\,\text{s}\,.$$

LEARN The classical Galilean transformation would have given

$$u' = u - v = 0.82c -(- 0.82c) = 1.64c,$$

which exceeds c and therefore, is physically impossible.

32. The figure shows that $u' = 0.80c$ when $v = 0$. We therefore infer (using the notation of Eq. 37-29) that $u = 0.80c$. Now, u is a fixed value and v is variable, so u' as a function of v is given by

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.80c - v}{1 - (0.80)v/c}$$

which is Eq. 37-29 rearranged so that u' is isolated on the left-hand side. We use this expression to answer parts (a) and (b).

(a) Substituting $v = 0.90c$ in the expression above leads to $u' = -\,0.357c \approx -\,0.36c$.

(b) Substituting $v = c$ in the expression above leads to $u' = -c$ (regardless of the value of u).

33. (a) In the messenger's rest system (called S_m), the velocity of the armada is

$$v' = \frac{v - v_m}{1 - vv_m/c^2} = \frac{0.80c - 0.95c}{1 - (0.80c)(0.95c)/c^2} = -0.625c\ .$$

The length of the armada as measured in S_m is

$$L_1 = \frac{L_0}{\gamma_{v'}} = (1.0\,\text{ly})\sqrt{1 - (-0.625)^2} = 0.781\ \text{ly}\ .$$

Thus, the length of the trip is

$$t' = \frac{L'}{|v'|} = \frac{0.781\,\text{ly}}{0.625\text{c}} = 1.25\ \text{y}\ .$$

(b) In the armada's rest frame (called S_a), the velocity of the messenger is

$$v' = \frac{v - v_a}{1 - vv_a/c^2} = \frac{0.95c - 0.80c}{1 - (0.95c)(0.80c)/c^2} = 0.625c\ .$$

Now, the length of the trip is

$$t' = \frac{L_0}{v'} = \frac{1.0\,\text{ly}}{0.625c} = 1.60\ \text{y}\ .$$

(c) Measured in system S, the length of the armada is

$$L = \frac{L_0}{\gamma} = 1.0\,\text{ly}\sqrt{1 - (0.80)^2} = 0.60\ \text{ly}\ ,$$

so the length of the trip is

$$t = \frac{L}{v_m - v_a} = \frac{0.60\,\text{ly}}{0.95c - 0.80c} = 4.00 \text{ y} .$$

34. We use the transverse Doppler shift formula, Eq. 37-37: $f = f_0\sqrt{1-\beta^2}$, or

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\sqrt{1-\beta^2}.$$

We solve for $\lambda - \lambda_0$:

$$\lambda - \lambda_0 = \lambda_0\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = (589.00\,\text{mm})\left[\frac{1}{\sqrt{1-(0.100)^2}} - 1\right] = +2.97\,\text{nm} .$$

35. **THINK** This problem deals with the Doppler effect of light. The source is the spaceship that is moving away from the Earth, where the detector is located.

EXPRESS With the source and the detector separating, the frequency received is given directly by Eq. 37-31:

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}}$$

where f_0 is the frequency in the frames of the spaceship, $\beta = v/c$, and v is the speed of the spaceship relative to the Earth.

ANALYZE With $\beta = 0.90$ and $f_0 = 100$ MHz, we obtain

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}} = (100\,\text{MHz})\sqrt{\frac{1-0.9000}{1+0.9000}} = 22.9\,\text{MHz} .$$

LEARN Since the source is moving away from the detector, $f < f_0$. Note that in the low speed limit, $\beta \ll 1$, Eq. 37-31 can be approximated as

$$f \approx f_0\left(1 - \beta + \frac{1}{2}\beta^2\right).$$

36. (a) Equation 37-36 leads to a speed of

$$v = \frac{\Delta\lambda}{\lambda}c = (0.004)(3.0\times10^8\,\text{m/s}) = 1.2\times10^6 \text{ m/s} \approx 1\times10^6 \text{ m/s}.$$

(b) The galaxy is receding.

37. We obtain

$$v=\frac{\Delta\lambda}{\lambda}c=\left(\frac{620\text{ nm}-540\text{ nm}}{620\text{ nm}}\right)c=0.13c.$$

38. (a) Equation 37-36 leads to

$$v=\frac{\Delta\lambda}{\lambda}c=\frac{12.00\text{nm}}{513.0\text{nm}}(2.998\times10^{8}\text{ m/s})=7.000\times10^{6}\text{ m/s}.$$

(b) The line is shifted to a larger wavelength, which means shorter frequency. Recalling Eq. 37-31 and the discussion that follows it, this means galaxy NGC is moving away from Earth.

39. **THINK** This problem deals with the Doppler effect of light. The source is the spaceship that is moving away from the Earth, where the detector is located.

EXPRESS With the source and the detector separating, the frequency received is given directly by Eq. 37-31:

$$f=f_0\sqrt{\frac{1-\beta}{1+\beta}}$$

where f_0 is the frequency in the frames of the spaceship, $\beta = v/c$, and v is the speed of the spaceship relative to the Earth. The frequency and the wavelength are related by $f\lambda=c$. Thus, if λ_0 is the wavelength of the light as seen on the spaceship, using $c=f_0\lambda_0=f\lambda,$ then the wavelength detected on Earth would be

$$\lambda=\lambda_0\left(\frac{f_0}{f}\right)=\lambda_0\sqrt{\frac{1+\beta}{1-\beta}}.$$

ANALYZE (a) With λ_0 = 450 nm and β = 0.20, we obtain

$$\lambda=(450\,\text{nm})\sqrt{\frac{1+0.20}{1-0.20}}=550\text{ nm}.$$

(b) This is in the green-yellow portion of the visible spectrum.

LEARN Since λ_0 = 450 nm, the color of the light as seen on the spaceship is violet-blue. With $\lambda>\lambda_0$, this Doppler shift is red shift.

40. (a) The work-kinetic energy theorem applies as well to relativistic physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use Eq. 37-52

$$W=\Delta K=m_ec^2(\gamma-1)$$

and $m_e c^2$ = 511 keV = 0.511 MeV (Table 37-3), and obtain

$$W = m_e c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = (511\,\text{keV})\left[\frac{1}{\sqrt{1-(0.500)^2}} - 1\right] = 79.1\ \text{keV}\ .$$

(b) $W = (0.511\,\text{MeV})\left[\frac{1}{\sqrt{1-(0.990)^2}} - 1\right] = 3.11\ \text{MeV}.$

(c) $W = (0.511\,\text{MeV})\left[\frac{1}{\sqrt{1-(0.990)^2}} - 1\right] = 10.9\,\text{MeV}.$

41. **THINK** The electron is moving at a relativistic speed since its kinetic energy greatly exceeds its rest energy.

EXPRESS The kinetic energy of the electron is given by Eq. 37-52:

$$K = E - mc^2 = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)\,.$$

Thus, γ = (K/mc^2) + 1. Similarly, by inverting the Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$, we obtain $\beta = \sqrt{1-(1/\gamma)^2}$.

ANALYZE (a) Table 37-3 gives mc^2 = 511 keV = 0.511 MeV for the electron rest energy, so the Lorentz factor is

$$\gamma = \frac{K}{mc^2} + 1 = \frac{100\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 196.695.$$

(b) The speed parameter is

$$\beta = \sqrt{1 - \frac{1}{(196.695)^2}} = 0.999987.$$

Thus, the speed of the electron is 0.999987c, or 99.9987% of the speed of light.

LEARN The classical expression $K = mv^2/2$, for kinetic energy, is adequate only when the speed of the object is well below the speed of light.

42. From Eq. 28-37, we have

$$Q = -\Delta Mc^2 = -\left[3(4.00151\text{u}) - 11.99671\text{u}\right]c^2 = -(0.00782\text{u})(931.5\text{MeV/u})$$
$$= -7.28\text{Mev}.$$

Thus, it takes a minimum of 7.28 MeV supplied to the system to cause this reaction. We note that the masses given in this problem are strictly for the nuclei involved; they are not the "atomic" masses that are quoted in several of the other problems in this chapter.

43. (a) The work-kinetic energy theorem applies as well to relativistic physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K$ where $K = m_e c^2(\gamma - 1)$ (Eq. 37-52), and $m_e c^2 = 511$ keV = 0.511 MeV (Table 37-3). Noting that

$$\Delta K = m_e c^2(\gamma_f - \gamma_i),$$

we obtain

$$W = \Delta K = m_e c^2 \left(\frac{1}{\sqrt{1-\beta_f^2}} - \frac{1}{\sqrt{1-\beta_i^2}} \right) = (511\,\text{keV}) \left(\frac{1}{\sqrt{1-(0.19)^2}} - \frac{1}{\sqrt{1-(0.18)^2}} \right)$$
$$= 0.996 \text{ keV} \approx 1.0 \text{ keV}.$$

(b) Similarly,

$$W = (511\,\text{keV}) \left(\frac{1}{\sqrt{1-(0.99)^2}} - \frac{1}{\sqrt{1-(0.98)^2}} \right) = 1055\,\text{keV} \approx 1.1 \text{ MeV}.$$

We see the dramatic increase in difficulty in trying to accelerate a particle when its initial speed is very close to the speed of light.

44. The mass change is

$$\Delta M = (4.002603\,\text{u} + 15.994915\,\text{u}) - (1.007825\,\text{u} + 18.998405\,\text{u}) = -0.008712\,\text{u}.$$

Using Eq. 37-50 and Eq. 37-46, this leads to

$$Q = -\Delta M\, c^2 = -(-0.008712\,\text{u})(931.5\,\text{MeV/u}) = 8.12\,\text{MeV}.$$

45. The distance traveled by the pion in the frame of Earth is (using Eq. 37-12) $d = v\Delta t$. The proper lifetime Δt_0 is related to Δt by the time-dilation formula: $\Delta t = \gamma\Delta t_0$. To use this equation, we must first find the Lorentz factor γ (using Eq. 37-48). Since the total energy of the pion is given by $E = 1.35 \times 10^5$ MeV and its mc^2 value is 139.6 MeV, then

$$\gamma = \frac{E}{mc^2} = \frac{1.35 \times 10^5\ \text{MeV}}{139.6\,\text{MeV}} = 967.05.$$

Therefore, the lifetime of the moving pion as measured by Earth observers is

$$\Delta t = \gamma\Delta t_0 = (967.1)(35.0 \times 10^{-9}\ \text{s}) = 3.385 \times 10^{-5}\ \text{s},$$

and the distance it travels is

$$d \approx c\Delta t = (2.998\times 10^8 \text{ m/s})(3.385\times 10^{-5} \text{ s}) = 1.015\times 10^4 \text{ m} = 10.15 \text{ km}$$

where we have approximated its speed as *c* (note: its speed can be found by solving Eq. 37-8, which gives v = 0.9999995c; this more precise value for v would not significantly alter our final result). Thus, the altitude at which the pion decays is 120 km – 10.15 km = 110 km.

46. (a) Squaring Eq. 37-47 gives

$$E^2 = (mc^2)^2 + 2mc^2K + K^2$$

which we set equal to Eq. 37-55. Thus,

$$(mc^2)^2 + 2mc^2K + K^2 = (pc)^2 + (mc^2)^2 \quad \Rightarrow \quad m = \frac{(pc)^2 - K^2}{2Kc^2}.$$

(b) At low speeds, the pre-Einsteinian expressions $p = mv$ and $K = \frac{1}{2}mv^2$ apply. We note that $pc \gg K$ at low speeds since $c \gg v$ in this regime. Thus,

$$m \to \frac{(mvc)^2 - (\frac{1}{2}mv^2)^2}{2(\frac{1}{2}mv^2)c^2} \approx \frac{(mvc)^2}{2(\frac{1}{2}mv^2)c^2} = m.$$

(c) Here, pc = 121 MeV, so

$$m = \frac{121^2 - 55^2}{2(55)c^2} = 105.6 \text{ MeV/c}^2.$$

Now, the mass of the electron (see Table 37-3) is m_e = 0.511 MeV/c^2, so our result is roughly 207 times bigger than an electron mass, i.e., $m/m_e \approx 207$. The particle is a muon.

47. **THINK** As a consequence of the theory of relativity, mass can be considered as another form of energy.

EXPRESS The mass of an object and its equivalent energy is given by

$$E_0 = mc^2.$$

ANALYZE The energy equivalent of one tablet is

$$E_0 = mc^2 = (320 \times 10^{-6}\ \text{kg})\,(3.00 \times 10^8\ \text{m/s})^2 = 2.88 \times 10^{13}\ \text{J}.$$

This provides the same energy as

$$(2.88 \times 10^{13}\ \text{J})/(3.65 \times 10^7\ \text{J/L}) = 7.89 \times 10^5\ \text{L}$$

of gasoline. The distance the car can go is

$$d = (7.89 \times 10^5\ \text{L})\,(12.75\ \text{km/L}) = 1.01 \times 10^7\ \text{km}.$$

LEARN The distance is roughly 250 times larger than the circumference of Earth (see Appendix C). However, this is possible only if the mass-energy conversion were perfect.

48. (a) The proper lifetime Δt_0 is 2.20 μs, and the lifetime measured by clocks in the laboratory (through which the muon is moving at high speed) is Δt = 6.90 μs. We use Eq. 37-7 to solve for the speed parameter:

$$\beta = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1 - \left(\frac{2.20\ \mu\text{s}}{6.90\ \mu\text{s}}\right)^2} = 0.948\,.$$

(b) From the answer to part (a), we find γ = 3.136. Thus, with (see Table 37-3)

$$m_\mu c^2 = 207 m_e c^2 = 105.8\ \text{MeV},$$

Eq. 37-52 yields

$$K = m_\mu c^2 (\gamma - 1) = (105.8\,\text{MeV})(3.136 - 1) = 226\,\text{MeV}.$$

(c) We write $m_\mu c$ = 105.8 MeV/c and apply Eq. 37-41:

$$p = \gamma m_\mu v = \gamma m_\mu c\beta = (3.136)(105.8\,\text{MeV}/c)(0.9478) = 314\ \text{MeV}/c$$

which can also be expressed in SI units ($p = 1.7 \times 10^{-19}$ kg·m/s).

49. (a) The strategy is to find the γ factor from $E = 14.24 \times 10^{-9}$ J and $m_p c^2 = 1.5033 \times 10^{-10}$ J and from that find the contracted length. From the energy relation (Eq. 37-48), we obtain

$$\gamma = \frac{E}{m_p c^2} = \frac{14.24 \times 10^{-9}\ \text{J}}{1.5033 \times 10^{-10}\ \text{J}} = 94.73.$$

Consequently, Eq. 37-13 yields

$$L = \frac{L_0}{\gamma} = \frac{21\text{ cm}}{94.73} = 0.222\text{ cm} = 2.22\times10^{-3}\text{ m.}$$

(b) From the γ factor, we find the speed:

$$v = c\sqrt{1-\left(\frac{1}{\gamma}\right)^2} = 0.99994c.$$

Therefore, in our reference frame the time elapsed is

$$\Delta t = \frac{L_0}{v} = \frac{0.21\text{ m}}{(0.99994)(2.998\times10^8\text{ m/s})} = 7.01\times10^{-10}\text{ s}.$$

(c) The time dilation formula (Eq. 37-7) leads to

$$\Delta t = \gamma\Delta t_0 = 7.01\times10^{-10}\text{ s}$$

Therefore, according to the proton, the trip took

$$\Delta t_0 = 2.22\times10^{-3}/0.99994c = 7.40\times10^{-12}\text{ s.}$$

50. From Eq. 37-52, $\gamma = (K/mc^2) + 1$, and from Eq. 37-8, the speed parameter is $\beta = \sqrt{1-(1/\gamma)^2}$.

(a) Table 37-3 gives m_ec^2 = 511 keV = 0.511 MeV, so the Lorentz factor is

$$\gamma = \frac{10.00\text{ MeV}}{0.5110\text{ MeV}} + 1 = 20.57,$$

(b) and the speed parameter is

$$\beta = \sqrt{1-(1/\gamma)^2} = \sqrt{1-\frac{1}{(20.57)^2}} = 0.9988.$$

(c) Using m_pc^2 = 938.272 MeV, the Lorentz factor is

$$\gamma = 1 + 10.00\text{ MeV}/938.272\text{ MeV} = 1.01065 \approx 1.011.$$

(d) The speed parameter is

$$\beta = \sqrt{1-\gamma^{-2}} = 0.144844 \approx 0.1448.$$

(e) With $m_\alpha c^2$ = 3727.40 MeV, we obtain γ = 10.00/3727.4 + 1 = 1.00268 $\approx$ 1.003.

(f) The speed parameter is

$$\beta = \sqrt{1-\gamma^{-2}} = 0.0731037 \approx 0.07310 .$$

51. We set Eq. 37-55 equal to $(3.00mc^2)^2$, as required by the problem, and solve for the speed. Thus,

$$(pc)^2 + (mc^2)^2 = 9.00(mc^2)^2$$

leads to $p = mc\sqrt{8} \approx 2.83mc$.

52. (a) The binomial theorem tells us that, for x small,

$$(1+x)^\nu \approx 1 + \nu x + \tfrac{1}{2}\nu(\nu-1)x^2$$

if we ignore terms involving x^3 and higher powers (this is reasonable since if x is small, say $x = 0.1$, then x^3 is much smaller: $x^3 = 0.001$). The relativistic kinetic energy formula, when the speed v is much smaller than c, has a term that we can apply the binomial theorem to; identifying $-\beta^2$ as x and $-1/2$ as ν, we have

$$\gamma = (1-\beta^2)^{-1/2} \approx 1 + (-\tfrac{1}{2})(-\beta^2) + \tfrac{1}{2}(-\tfrac{1}{2})\big((-\tfrac{1}{2})-1\big)(-\beta^2)^2.$$

Substituting this into Eq. 37-52 leads to

$$K = mc^2(\gamma-1) \approx mc^2\left[(-\tfrac{1}{2})(-\beta^2) + \tfrac{1}{2}(-\tfrac{1}{2})\big((-\tfrac{1}{2})-1\big)(-\beta^2)^2\right]$$

which simplifies to

$$K \approx \frac{1}{2}mc^2\beta^2 + \frac{3}{8}mc^2\beta^4 = \frac{1}{2}mv^2 + \frac{3}{8}mv^4/c^2 .$$

(b) If we use the mc^2 value for the electron found in Table 37-3, then for $\beta = 1/20$, the classical expression for kinetic energy gives

$$K_{\text{classical}} = \frac{1}{2}mv^2 = \frac{1}{2}mc^2\beta^2 = \frac{1}{2}(8.19\times10^{-14}\text{ J})(1/20)^2 = 1.0\times10^{-16}\text{ J} .$$

(c) The first-order correction becomes

$$K_{\text{first-order}} = \frac{3}{8}mv^4/c^2 = \frac{3}{8}mc^2\beta^4 = \frac{3}{8}(8.19\times10^{-14}\text{ J})(1/20)^4 = 1.9\times10^{-19}\text{ J}$$

which we note is much smaller than the classical result.

(d) In this case, $\beta = 0.80 = 4/5$, and the classical expression yields

$$K_{\text{classical}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}mc^2\beta^2 = \tfrac{1}{2}(8.19\times 10^{-14}\text{ J})(4/5)^2 = 2.6\times 10^{-14}\text{ J}.$$

(e) And the first-order correction is

$$K_{\text{first-order}} = \tfrac{3}{8}mv^4/c^2 = \tfrac{3}{8}mc^2\beta^4 = \tfrac{3}{8}(8.19\times 10^{-14}\text{ J})(4/5)^4 = 1.3\times 10^{-14}\text{ J}$$

which is comparable to the classical result. This is a signal that ignoring the higher order terms in the binomial expansion becomes less reliable the closer the speed gets to *c*.

(f) We set the first-order term equal to one-tenth of the classical term and solve for β:

$$\tfrac{3}{8}mc^2\beta^4 = \tfrac{1}{10}\left(\tfrac{1}{2}mc^2\beta^2\right)$$

and obtain $\beta = \sqrt{2/15} \approx 0.37$.

53. Using the classical orbital radius formula $r_0 = mv/|q|B$, the period is

$$T_0 = 2\pi r_0/v = 2\pi m/|q|B.$$

In the relativistic limit, we must use

$$r = \frac{p}{|q|B} = \frac{\gamma mv}{|q|B} = \gamma r_0$$

which yields

$$T = \frac{2\pi r}{v} = \gamma\frac{2\pi m}{|q|B} = \gamma T_0$$

(b) The period *T* is not independent of *v*.

(c) We interpret the given 10.0 MeV to be the kinetic energy of the electron. In order to make use of the mc^2 value for the electron given in Table 37-3 (511 keV = 0.511 MeV) we write the classical kinetic energy formula as

$$K_{\text{classical}} = \frac{1}{2}mv^2 = \frac{1}{2}\left(mc^2\right)\left(\frac{v^2}{c^2}\right) = \frac{1}{2}\left(mc^2\right)\beta^2.$$

If $K_{\text{classical}} = 10.0$ MeV, then

$$\beta = \sqrt{\frac{2K_{\text{classical}}}{mc^2}} = \sqrt{\frac{2(10.0\,\text{MeV})}{0.511\,\text{MeV}}} = 6.256,$$

which, of course, is impossible since it exceeds 1. If we use this value anyway, then the classical orbital radius formula yields

$$r = \frac{mv}{|q|B} = \frac{m\beta c}{eB} = \frac{(9.11\times10^{-31}\,\text{kg})\,(6.256)\,(2.998\times10^{8}\,\text{m/s})}{(1.6\times10^{-19}\,\text{C})\,(2.20\,\text{T})} = 4.85\times10^{-3}\,\text{m}.$$

(d) Before using the relativistically correct orbital radius formula, we must compute β in a relativistically correct way:

$$K = mc^2(\gamma - 1) \;\;\Rightarrow\;\; \gamma = \frac{10.0\text{ MeV}}{0.511\text{ MeV}} + 1 = 20.57$$

which implies (from Eq. 37-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(20.57)^2}} = 0.99882.$$

Therefore,

$$r = \frac{\gamma mv}{|q|B} = \frac{\gamma m\beta c}{eB} = \frac{(20.57)(9.11\times10^{-31}\,\text{kg})(0.99882)(2.998\times10^{8}\,\text{m/s})}{(1.6\times10^{-19}\,\text{C})(2.20\,\text{T})}$$
$$= 1.59\times10^{-2}\,\text{m}.$$

(e) The classical period is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(4.85\times10^{-3}\text{ m})}{(6.256)(2.998\times10^{8}\text{ m/s})} = 1.63\times10^{-11}\text{ s}.$$

(f) The period obtained with relativistic correction is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(0.0159\text{ m})}{(0.99882)(2.998\times10^{8}\text{ m/s})} = 3.34\times10^{-10}\text{ s}.$$

54. (a) We set Eq. 37-52 equal to $2mc^2$, as required by the problem, and solve for the speed. Thus,

$$mc^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = 2mc^2$$

leads to $\beta = 2\sqrt{2}/3 \approx 0.943$.

(b) We now set Eq. 37-48 equal to $2mc^2$ and solve for the speed. In this case,

$$\frac{mc^2}{\sqrt{1-\beta^2}} = 2mc^2$$

leads to $\beta = \sqrt{3}/2 \approx 0.866.$

55. (a) We set Eq. 37-41 equal to *mc*, as required by the problem, and solve for the speed. Thus,

$$\frac{mv}{\sqrt{1-v^2/c^2}} = mc$$

leads to $\beta = 1/\sqrt{2} = 0.707.$

(b) Substituting $\beta = 1/\sqrt{2}$ into the definition of γ, we obtain

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(1/2)}} = \sqrt{2} \approx 1.41.$$

(c) The kinetic energy is

$$K = (\gamma - 1)mc^2 = \left(\sqrt{2}-1\right)mc^2 = 0.414mc^2 = 0.414E_0.$$

which implies $K/E_0 = 0.414$.

56. (a) From the information in the problem, we see that each kilogram of TNT releases $(3.40 \times 10^6$ J/mol)/(0.227 kg/mol) $= 1.50 \times 10^7$ J. Thus,

$$(1.80 \times 10^{14}\text{ J})/(1.50 \times 10^7\text{ J/kg}) = 1.20 \times 10^7\text{ kg}$$

of TNT are needed. This is equivalent to a weight of $\approx 1.2 \times 10^8$ N.

(b) This is certainly more than can be carried in a backpack. Presumably, a train would be required.

(c) We have $0.00080mc^2 = 1.80 \times 10^{14}$ J, and find $m = 2.50$ kg of fissionable material is needed. This is equivalent to a weight of about 25 N, or 5.5 pounds.

(d) This can be carried in a backpack.

57. Since the rest energy E_0 and the mass m of the quasar are related by $E_0 = mc^2$, the rate P of energy radiation and the rate of mass loss are related by

$$P = dE_0/dt = (dm/dt)c^2.$$

Thus,

$$\frac{dm}{dt} = \frac{P}{c^2} = \frac{1\times 10^{41}\ \text{W}}{(2.998\times 10^{8}\ \text{m/s})^2} = 1.11\times 10^{24}\ \text{kg/s}.$$

Since a solar mass is 2.0×10^{30} kg and a year is 3.156×10^{7} s,

$$\frac{dm}{dt} = (1.11\times 10^{24}\ \text{kg/s})\left(\frac{3.156\times 10^{7}\ \text{s/y}}{2.0\times 10^{30}\ \text{kg/smu}}\right) \approx 18\,\text{smu/y}.$$

58. (a) Using $K = m_e c^2 (\gamma - 1)$ (Eq. 37-52) and

$$m_e c^2 = 510.9989\ \text{keV} = 0.5109989\ \text{MeV},$$

we obtain

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1.0000000\,\text{keV}}{510.9989\,\text{keV}} + 1 = 1.00195695 \approx 1.0019570.$$

(b) Therefore, the speed parameter is

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(1.0019570)^2}} = 0.062469542.$$

(c) For $K = 1.0000000$ MeV, we have

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1.0000000\,\text{MeV}}{0.5109989\,\text{MeV}} + 1 = 2.956951375 \approx 2.9569514.$$

(d) The corresponding speed parameter is

$$\beta = \sqrt{1-\gamma^{-2}} = 0.941079236 \approx 0.94107924.$$

(e) For K = 1.0000000 GeV, we have

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1000.0000\,\text{MeV}}{0.5109989\,\text{MeV}} + 1 = 1957.951375 \approx 1957.9514.$$

(f) The corresponding speed parameter is

$$\beta = \sqrt{1-\gamma^{-2}} = 0.99999987\,.$$

59. (a) Before looking at our solution to part (a) (which uses momentum conservation), it might be advisable to look at our solution (and accompanying remarks) for part (b) (where a very different approach is used). Since momentum is a vector, its conservation involves two equations (along the original direction of alpha particle motion, the *x* direction, as well as along the final proton direction of motion, the *y* direction). The problem states that all speeds are much less than the speed of light, which allows us to use the classical formulas for kinetic energy and momentum ($K = \frac{1}{2}mv^2$ and $\vec{p} = m\vec{v}$, respectively). Along the *x* and *y* axes, momentum conservation gives (for the components of $\vec{v}_{\text{oxy}}$):

$$m_\alpha v_\alpha = m_{\text{oxy}} v_{\text{oxy},x} \quad \Rightarrow \quad v_{\text{oxy},x} = \frac{m_\alpha}{m_{\text{oxy}}} v_\alpha \approx \frac{4}{17} v_\alpha$$

$$0 = m_{\text{oxy}} v_{\text{oxy},y} + m_p v_p \quad \Rightarrow \quad v_{\text{oxy},y} = -\frac{m_p}{m_{\text{oxy}}} v_p \approx -\frac{1}{17} v_p.$$

To complete these determinations, we need values (inferred from the kinetic energies given in the problem) for the initial speed of the alpha particle (v_α) and the final speed of the proton (v_p). One way to do this is to rewrite the classical kinetic energy expression as $K = \frac{1}{2}(mc^2)\beta^2$ and solve for β (using Table 37-3 and/or Eq. 37-46). Thus, for the proton, we obtain

$$\beta_p = \sqrt{\frac{2K_p}{m_p c^2}} = \sqrt{\frac{2(4.44 \text{ MeV})}{938 \text{ MeV}}} = 0.0973.$$

This is almost 10% the speed of light, so one might worry that the relativistic expression (Eq. 37-52) should be used. If one does so, one finds β_p = 0.969, which is reasonably close to our previous result based on the classical formula. For the alpha particle, we write

$$m_\alpha c^2 = (4.0026 \text{ u})(931.5 \text{ MeV/u}) = 3728 \text{ MeV}$$

(which is actually an overestimate due to the use of the "atomic mass" value in our calculation, but this does not cause significant error in our result), and obtain

$$\beta_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha c^2}} = \sqrt{\frac{2(7.70 \text{ MeV})}{3728 \text{ MeV}}} = 0.064.$$

Returning to our oxygen nucleus velocity components, we are now able to conclude:

$$v_{\text{oxy},x} \approx \frac{4}{17} v_\alpha \Rightarrow \beta_{\text{oxy},x} \approx \frac{4}{17}\beta_\alpha = \frac{4}{17}(0.064) = 0.015$$

$$|v_{\text{oxy},y}| \approx \frac{1}{17} v_p \Rightarrow \beta_{\text{oxy},y} \approx \frac{1}{17}\beta_p = \frac{1}{17}(0.097) = 0.0057$$

Consequently, with

$$m_{\text{oxy}}c^2 \approx (17\text{ u})(931.5\text{ MeV/u}) = 1.58 \times 10^4\text{ MeV},$$

we obtain

$$K_{\text{oxy}} = \frac{1}{2}(m_{\text{oxy}}c^2)(\beta^2_{\text{oxy},x} + \beta^2_{\text{oxy},y}) = \frac{1}{2}(1.58\times10^4\text{ MeV})(0.015^2 + 0.0057^2)$$
$$\approx 2.08\text{ MeV}.$$

(b) Using Eq. 37-50 and Eq. 37-46,

$$Q = -(1.007825\,\text{u} + 16.99914\,\text{u} - 4.00260\,\text{u} - 14.00307\,\text{u})c^2$$
$$= -(0.001295\,\text{u})(931.5\,\text{MeV/u})$$

which yields $Q = -1.206$ MeV $\approx -1.21\text{ MeV}$. Incidentally, this provides an alternate way to obtain the answer (and a more accurate one at that!) to part (a). Equation 37-49 leads to

$$K_{\text{oxy}} = K_\alpha + Q - K_p = 7.70\,\text{MeV} - 1206\,\text{MeV} - 4.44\,\text{MeV} = 2.05\,\text{MeV}.$$

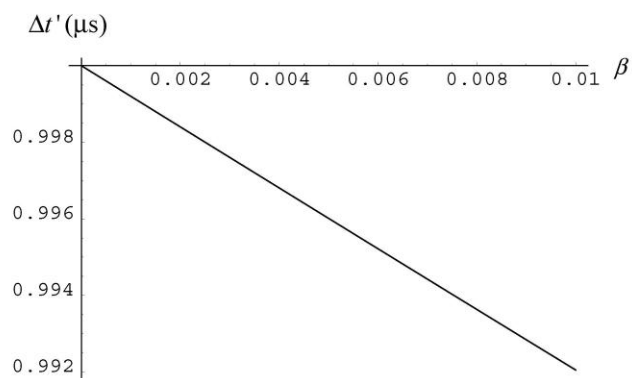
This approach to finding K_{oxy} avoids the many computational steps and approximations made in part (a).

60. (a) Equation 2′ of Table 37-2 becomes

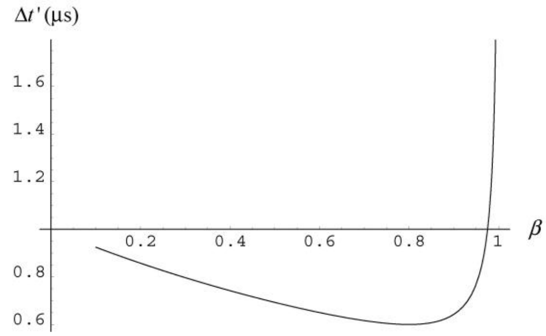
$$\Delta t' = \gamma(\Delta t - \beta\Delta x/c) = \gamma\,[1.00\ \mu\text{s} - \beta(240\text{ m})/(2.998 \times 10^2\text{ m}/\mu\text{s})]$$
$$= \gamma(1.00 - 0.800\beta)\ \mu\text{s}$$

where the Lorentz factor is itself a function of β (see Eq. 37-8).

(b) A plot of $\Delta t'$ is shown for the range $0 < \beta < 0.01$:



(c) A plot of $\Delta t'$ is shown for the range $0.1 < \beta < 1$:



(d) The minimum for the $\Delta t'$ curve can be found by taking the derivative and simplifying and then setting equal to zero:

$$\frac{d\Delta t'}{d\beta} = \gamma^3(\beta\Delta t - \Delta x/c) = 0 .$$

Thus, the value of β for which the curve is minimum is $\beta = \Delta x/c\Delta t = 240/299.8$, or $\beta = 0.801$.

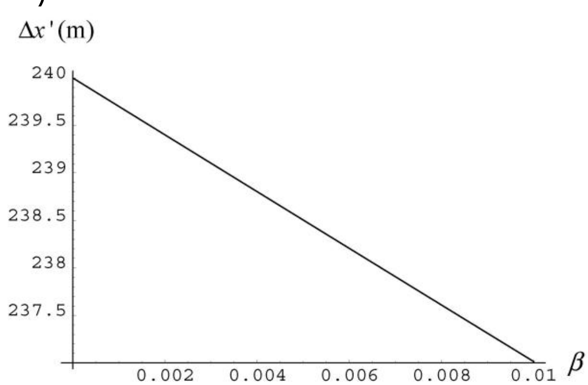
(e) Substituting the value of β from part (d) into the part (a) expression yields the minimum value $\Delta t' = 0.599\ \mu$s.

(f) Yes. We note that $\Delta x/\Delta t = 2.4 \times 10^8$ m/s < c. A signal can indeed travel from event A to event B without exceeding c, so causal influences can originate at A and thus affect what happens at B. Such events are often described as being "time-like separated" – and we see in this problem that it is (always) possible in such a situation for us to find a frame of reference (here with $\beta \approx 0.801$) where the two events will seem to be at the same location (though at different times).

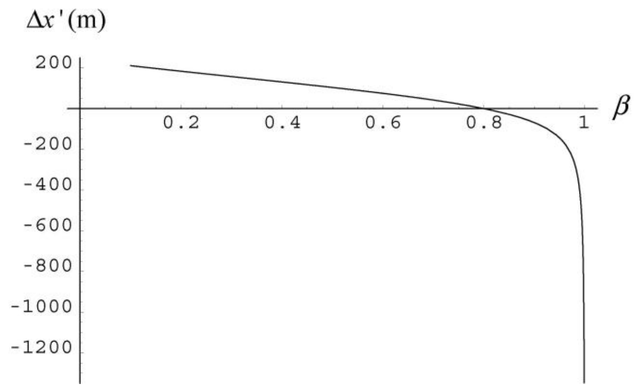
61. (a) Equation 1′ of Table 37-2 becomes

$$\Delta x' = \gamma(\Delta x - \beta c\Delta t) = \gamma[(240 \text{ m}) - \beta(299.8 \text{ m})] .$$

(b) A plot of $\Delta x'$ for $0 < \beta < 0.01$ is shown below:



(c) A plot of $\Delta x'$ for $0.1 < \beta < 1$ is shown below:



We see that $\Delta x'$ decreases from its $\beta = 0$ value (where it is equal to Δx = 240 m) to its zero value (at $\beta \approx 0.8$), and continues (without bound) downward in the graph (where it is negative, implying event B has a *smaller* value of x' than event A!).

(d) The zero value for $\Delta x'$ is easily seen (from the expression in part (b)) to come from the condition $\Delta x - \beta c \Delta t = 0$. Thus $\beta = 0.801$ provides the zero value of $\Delta x'$.

62. By examining the value of u' when $v = 0$ on the graph, we infer $u = -0.20c$. Solving Eq. 37-29 for u' and inserting this value for u, we obtain

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{-0.20c - v}{1 + 0.20v/c}$$

for the equation of the curve shown in the figure.

(a) With $v = 0.80c$, the above expression yields $u' = -0.86c$.

(b) As expected, setting $v = c$ in this expression leads to $u' = -c$.

63. (a) The spatial separation between the two bursts is vt. We project this length onto the direction perpendicular to the light rays headed to Earth and obtain $D_{app} = vt \sin\theta$.

(b) Burst 1 is emitted a time t ahead of burst 2. Also, burst 1 has to travel an extra distance L more than burst 2 before reaching the Earth, where $L = vt \cos\theta$ (see Fig. 37-29); this requires an additional time $t' = L/c$. Thus, the apparent time is given by

$$T_{app} = t - t' = t - \frac{vt\cos\theta}{c} = t\left[1 - \left(\frac{v}{c}\right)\cos\theta\right].$$

(c) We obtain

$$V_{\text{app}} = \frac{D_{\text{app}}}{T_{\text{app}}} = \left(\frac{(v/c)\sin\theta}{1-(v/c)\cos\theta}\right)c = \left(\frac{(0.980)\sin 30.0°}{1-(0.980)\cos 30.0°}\right)c = 3.24\,c.$$

64. The line in the graph is described by Eq. 1 in Table 37-2:

$$\Delta x = v\gamma\Delta t' + \gamma\Delta x' = (\text{"slope"})\Delta t' + \text{"y-intercept"}$$

where the "slope" is 7.0×10^8 m/s. Setting this value equal to $v\gamma$ leads to $v = 2.8 \times 10^8$ m/s and $\gamma = 2.54$. Since the "y-intercept" is 2.0 m, we see that dividing this by γ leads to $\Delta x' = 0.79$ m.

65. Interpreting v_{AB} as the x-component of the velocity of A relative to B, and defining the corresponding speed parameter $\beta_{AB} = v_{AB}/c$, then the result of part (a) is a straightforward rewriting of Eq. 37-29 (after dividing both sides by c). To make the correspondence with Fig. 37-11 clear, the particle in that picture can be labeled A, frame S' (or an observer at rest in that frame) can be labeled B, and frame S (or an observer at rest in it) can be labeled C. The result of part (b) is less obvious, and we show here some of the algebra steps:

$$M_{AC} = M_{AB} \cdot M_{BC} \quad \Rightarrow \quad \frac{1-\beta_{AC}}{1+\beta_{AC}} = \frac{1-\beta_{AB}}{1+\beta_{AB}} \cdot \frac{1-\beta_{BC}}{1+\beta_{BC}}$$

We multiply both sides by factors to get rid of the denominators

$$(1-\beta_{AC})(1+\beta_{AB})(1+\beta_{BC}) = (1-\beta_{AB})(1-\beta_{BC})(1+\beta_{AC})$$

and expand:

$$1 - \beta_{AC} + \beta_{AB} + \beta_{BC} - \beta_{AC}\,\beta_{AB} - \beta_{AC}\,\beta_{BC} + \beta_{AB}\,\beta_{BC} - \beta_{AB}\,\beta_{BC}\,\beta_{AC} =$$
$$1 + \beta_{AC} - \beta_{AB} - \beta_{BC} - \beta_{AC}\,\beta_{AB} - \beta_{AC}\,\beta_{BC} + \beta_{AB}\,\beta_{BC} + \beta_{AB}\,\beta_{BC}\,\beta_{AC}$$

We note that several terms are identical on both sides of the equals sign, and thus cancel, which leaves us with

$$-\beta_{AC} + \beta_{AB} + \beta_{BC} - \beta_{AB}\,\beta_{BC}\,\beta_{AC} = \beta_{AC} - \beta_{AB} - \beta_{BC} + \beta_{AB}\,\beta_{BC}\,\beta_{AC}$$

which can be rearranged to produce

$$2\beta_{AB} + 2\beta_{BC} = 2\beta_{AC} + 2\beta_{AB}\beta_{BC}\beta_{AC} \ .$$

The left-hand side can be written as $2\beta_{AC}\,(1 + \beta_{AB}\,\beta_{BC})$ in which case it becomes clear how to obtain the result from part (a) [just divide both sides by $2(1 + \beta_{AB}\,\beta_{BC})$].

66. We note, because it is a pretty symmetry and because it makes the part (b) computation move along more quickly, that

$$M = \frac{1-\beta}{1+\beta} \;\Rightarrow\; \beta = \frac{1-M}{1+M}.$$

Here, with β_{AB} given as 1/2 (see the problem statement), then M_{AB} is seen to be 1/3 (which is (1 – 1/2) divided by (1 + 1/2)). Similarly for β_{BC} .

(a) Thus,

$$M_{AC} = M_{AB} \cdot M_{BC} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}.$$

(b) Consequently,

$$\beta_{AC} = \frac{1 - M_{AC}}{1 + M_{AC}} = \frac{1-1/9}{1+1/9} = \frac{8}{10} = \frac{4}{5} = 0.80.$$

(c) By the definition of the speed parameter, we finally obtain $v_{AC} = 0.80c$.

67. We note, for use later in the problem, that

$$M = \frac{1-\beta}{1+\beta} \;\Rightarrow\; \beta = \frac{1-M}{1+M}$$

Now, with β_{AB} given as 1/5 (see problem statement), then M_{AB} is seen to be 2/3 (which is (1 – 1/5) divided by (1 + 1/5)). With $\beta_{BC} = -2/5$, we similarly find $M_{BC} = 7/3$, and for $\beta_{CD} = 3/5$ we get $M_{CD} = 1/4$. Thus,

$$M_{AD} = M_{AB} M_{BC} M_{CD} = \frac{2}{3} \cdot \frac{7}{3} \cdot \frac{1}{4} = \frac{7}{18}.$$

Consequently,

$$\beta_{AD} = \frac{1 - M_{AD}}{1 + M_{AD}} = \frac{1-7/18}{1+7/18} = \frac{11}{25} = 0.44.$$

By the definition of the speed parameter, we obtain $v_{AD} = 0.44c$.

68. (a) According to the ship observers, the duration of proton flight is $\Delta t' = (760\ \text{m})/0.980c = 2.59\ \mu\text{s}$ (assuming it travels the entire length of the ship).

(b) To transform to our point of view, we use Eq. 2 in Table 37-2. Thus, with $\Delta x' = -750$ m, we have

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 0.572\mu\text{s}.$$

(c) For the ship observers, firing the proton from back to front makes no difference, and $\Delta t' = 2.59\ \mu\text{s}$ as before.

(d) For us, the fact that now $\Delta x' = +750$ m is a significant change.

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 16.0\mu\text{s}.$$

69. (a) From the length contraction equation, the length L_c' of the car according to Garageman is

$$L_c' = \frac{L_c}{\gamma} = L_c\sqrt{1-\beta^2} = (30.5\text{ m})\sqrt{1-(0.9980)^2} = 1.93\text{ m}.$$

(b) Since the x_g axis is fixed to the garage, $x_{g2} = L_g = 6.00$ m.

(c) As for t_{g2}, note from Fig. 37-32(b) that at $t_g = t_{g1} = 0$ the coordinate of the front bumper of the limo in the x_g frame is L_c', meaning that the front of the limo is still a distance $L_g - L_c'$ from the back door of the garage. Since the limo travels at a speed v, the time it takes for the front of the limo to reach the back door of the garage is given by

$$\Delta t_g = t_{g2} - t_{g1} = \frac{L_g - L_c'}{v} = \frac{6.00\text{ m} - 1.93\text{ m}}{0.9980(2.998\times10^8\text{ m/s})} = 1.36\times10^{-8}\text{ s}.$$

Thus $t_{g2} = t_{g1} + \Delta t_g = 0 + 1.36\times10^{-8}\text{ s} = 1.36\times10^{-8}$ s.

(d) The limo is inside the garage between times t_{g1} and t_{g2}, so the time duration is $t_{g2} - t_{g1} = 1.36\times10^{-8}$ s.

(e) Again from Eq. 37-13, the length L_g' of the garage according to Carman is

$$L_g' = \frac{L_g}{\gamma} = L_g\sqrt{1-\beta^2} = (6.00\text{ m})\sqrt{1-(0.9980)^2} = 0.379\text{ m}.$$

(f) Again, since the x_c axis is fixed to the limo, $x_{c2} = L_c = 30.5$ m.

(g) Now, from the two diagrams described in part (h) below, we know that at $t_c = t_{c2}$ (when event 2 takes place), the distance between the rear bumper of the limo and the back door of the garage is given by $L_c - L_g'$. Since the garage travels at a speed v, the front door of the garage will reach the rear bumper of the limo a time Δt_c later, where Δt_c satisfies

$$\Delta t_c = t_{c1} - t_{c2} = \frac{L_c - L_g'}{v} = \frac{30.5\text{ m} - 0.379\text{ m}}{0.9980(2.998\times10^8\text{ m/s})} = 1.01\times10^{-7}\text{ s}.$$

Thus $t_{c2} = t_{c1} - \Delta t_c = 0 - 1.01\times10^{-7}\text{ s} = -1.01\times10^{-7}$ s.

(h) From Carman's point of view, the answer is clearly no.

(i) Event 2 occurs first according to Carman, since $t_{c2} < t_{c1}$.

(j) We describe the essential features of the two pictures. For event 2, the front of the limo coincides with the back door, and the garage itself seems very short (perhaps failing to reach as far as the front window of the limo). For event 1, the rear of the car coincides with the front door and the front of the limo has traveled a significant distance beyond the back door. In this picture, as in the other, the garage seems very short compared to the limo.

(k) No, the limo cannot be in the garage with both doors shut.

(l) Both Carman and Garageman are correct in their respective reference frames. But, in a sense, Carman should lose the bet since he dropped his physics course before reaching the Theory of Special Relativity!

70. (a) The relative contraction is

$$\frac{|\Delta L|}{L_0} = \frac{L_0(1-\gamma^{-1})}{L_0} = 1-\sqrt{1-\beta^2} \approx 1-\left(1-\frac{1}{2}\beta^2\right) = \frac{1}{2}\beta^2 = \frac{1}{2}\left(\frac{630\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2$$
$$= 2.21\times10^{-12}\,.$$

(b) Letting $|\Delta t - \Delta t_0| = \Delta t_0(\gamma-1) = \tau = 1.00\,\mu\text{s}$, we solve for Δt_0:

$$\Delta t_0 = \frac{\tau}{\gamma-1} = \frac{\tau}{(1-\beta^2)^{-1/2}-1} \approx \frac{\tau}{1+\frac{1}{2}\beta^2-1} = \frac{2\tau}{\beta^2} = \frac{2(1.00\times10^{-6}\,\text{s})(1\,\text{d}/86400\,\text{s})}{[(630\,\text{m/s})/(2.998\times10^8\,\text{m/s})]^2}$$
$$= 5.25\ \text{d}\,.$$

71. **THINK** We calculate the relative speed of the satellites using both the Galilean transformation and the relativistic speed transformation.

EXPRESS Let v be the speed of the satellites relative to Earth. As they pass each other in opposite directions, their relative speed is given by $v_{\text{rel},\,c} = 2v$ according to the classical Galilean transformation. On the other hand, applying relativistic velocity transformation gives

$$v_{\text{rel}} = \frac{2v}{1+v^2/c^2}\,.$$

ANALYZE (a) With v = 27000 km/h, we obtain

$$v_{\text{rel},\,c} = 2v = 2(27000\text{ km/h}) = 5.4\times10^4\ \text{km/h}.$$

(b) We can express c in these units by multiplying by 3.6: $c = 1.08\times10^9$ km/h. The fractional error is

$$\frac{v_{\text{rel},c} - v_{\text{rel}}}{v_{\text{rel},c}} = 1 - \frac{1}{1 + v^2/c^2} = 1 - \frac{1}{1 + [(27000 \text{ km/h})/(1.08 \times 10^9 \text{ km/h})]^2} = 6.3 \times 10^{-10}.$$

LEARN Since the speeds of the satellites are well below the speed of light, calculating their relative speed using the classical Galilean transformation is adequate.

72. Using Eq. 37-10, we obtain $\beta = \frac{v}{c} = \frac{d/c}{t} = \frac{6.0 \text{ y}}{2.0 \text{ y} + 6.0 \text{ y}} = 0.75.$

73. **THINK** The work done to the proton is equal to the change in kinetic energy.

EXPRESS The kinetic energy of the electron is given by Eq. 37-52:

$$K = E - mc^2 = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$$

where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor.

Let v_1 be the initial speed and v_2 be the final speed of the proton. The work required is

$$W = \Delta K = mc^2(\gamma_2 - 1) - mc^2(\gamma_1 - 1) = mc^2(\gamma_2 - \gamma_1) = mc^2 \Delta\gamma \, .$$

ANALYZE When $\beta_2 = 0.9860$, we have $\gamma_2 = 5.9972$, and when $\beta_1 = 0.9850$, we have $\gamma_1 = 5.7953$. Thus, $\Delta\gamma = 0.202$ and the change in kinetic energy (equal to the work) becomes (using Eq. 37-52)

$$W = \Delta K = (mc^2)\Delta\gamma = (938 \text{ MeV})(5.9972 - 5.7953) = 189 \text{ MeV}$$

where $mc^2 = 938$ MeV has been used (see Table 37-3).

LEARN Using the classical expression $K_c = mv^2/2$ for kinetic energy, one would have obtain

$$W_c = \Delta K_c = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}mc^2(\beta_2^2 - \beta_1^2) = \frac{1}{2}(938 \text{ MeV})\left[(0.9860)^2 - (0.9850)^2\right]$$
$$= 0.924 \text{ MeV}$$

which is substantially lowered than that using relativistic formulation.

74. The mean lifetime of a pion measured by observers on the Earth is $\Delta t = \gamma \Delta t_0$, so the distance it can travel (using Eq. 37-12) is

$$d = v\Delta t = \gamma v\Delta t_0 = \frac{(0.99)(2.998\times 10^8 \text{ m/s})(26\times 10^{-9}\text{ s})}{\sqrt{1-(0.99)^2}} = 55\,\text{m}\ .$$

75. **THINK** The electron is moving toward the Earth at a relativistic speed since $E \gg mc^2$, where mc^2 is the rest energy of the electron.

EXPRESS The energy of the electron is given by

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-(v/c)^2}}\ .$$

With E = 1533 MeV and mc^2 = 0.511 MeV (see Table 37-3), we obtain

$$v = c\sqrt{1-\left(\frac{mc^2}{E}\right)^2} = c\sqrt{1-\left(\frac{0.511\text{ MeV}}{1533\text{ MeV}}\right)^2} = 0.99999994c \approx c\ .$$

Thus, in the rest frame of Earth, it took the electron 26 y to reach us. In order to transform to its own "clock" it's useful to compute γ directly from Eq. 37-48:

$$\gamma = \frac{E}{mc^2} = \frac{1533\text{ MeV}}{0.511\text{ MeV}} = 3000$$

though if one is careful one can also get this result from $\gamma = 1/\sqrt{1-(v/c)^2}$.

ANALYZE Then, Eq. 37-7 leads to

$$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{26\text{ y}}{3000} = 0.0087\text{ y}$$

so that the electron "concludes" the distance he traveled is only 0.0087 light-years.

LEARN In the rest frame of the electron, the Earth appears to be rushing toward the electron with a speed $0.99999994c$. Thus, the electron starts its journey from a distance of 0.0087 light-years away.

76. We are asked to solve Eq. 37-48 for the speed v. Algebraically, we find

$$\beta = \sqrt{1-\left(\frac{mc^2}{E}\right)^2}\ .$$

Using $E = 10.611\times10^{-9}$ J and the very accurate values for c and m (in SI units) found in Appendix B, we obtain $\beta = 0.99990$.

77. The speed of the spaceship after the first increment is $v_1 = 0.5c$. After the second one, it becomes

$$v_2 = \frac{v' + v_1}{1+v'v_1/c^2} = \frac{0.50c+0.50c}{1+(0.50c)^2/c^2} = 0.80c,$$

and after the third one, the speed is

$$v_3 = \frac{v'+v_2}{1+v'v_2/c^2} = \frac{0.50c+0.50c}{1+(0.50c)(0.80c)/c^2} = 0.929c.$$

Continuing with this process, we get $v_4 = 0.976c$, $v_5 = 0.992c$, $v_6 = 0.997c$, and $v_7 = 0.999c$. Thus, seven increments are needed.

78. (a) Equation 37-37 yields

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{1-\beta}{1+\beta}} \quad \Rightarrow \quad \beta = \frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2} .$$

With $\lambda_0/\lambda = 434/462$, we obtain $\beta = 0.062439$, or $v = 1.87 \times 10^7$ m/s.

(b) Since it is shifted "toward the red" (toward longer wavelengths) then the galaxy is moving away from us (receding).

79. **THINK** The electron is moving at a relativistic speed since its total energy E is much great than mc^2, the rest energy of the electron.

EXPRESS To calculate the momentum of the electron, we use Eq. 37-54:

$$(pc)^2 = K^2 + 2Kmc^2 .$$

ANALYZE With $K = 2.00$ MeV and $mc^2 = 0.511$ MeV (see Table 37-3), we have

$$pc = \sqrt{K^2+2Kmc^2} = \sqrt{(2.00\text{ MeV})^2 + 2(2.00\text{ MeV})(0.511\text{ MeV})}$$

This readily yields $p = 2.46$ MeV/c.

LEARN Classically, the electron momentum is

$$p_c = \sqrt{2Km} = \frac{\sqrt{2Kmc^2}}{c}\,\frac{\sqrt{2(2.00\text{ MeV})(0.511\text{ MeV})}}{c} = 1.43\text{ MeV}/c$$

which is smaller than that obtained using relativistic formulation.

80. Using Appendix C, we find that the contraction is

$$
\begin{aligned}
|\Delta L| = L_0 - L &= L_0\left(1 - \frac{1}{\gamma}\right) = L_0\left(1 - \sqrt{1-\beta^2}\right) \\
&= 2(6.370\times 10^6\ \text{m})\left(1 - \sqrt{1 - \left(\frac{3.0\times 10^4\ \text{m/s}}{2.998\times 10^8\ \text{m/s}}\right)^2}\right) \\
&= 0.064\ \text{m}.
\end{aligned}
$$

81. We refer to the particle in the first sentence of the problem statement as particle 2. Since the total momentum of the two particles is zero in S', it must be that the velocities of these two particles are equal in magnitude and opposite in direction in S'. Letting the velocity of the S' frame be v relative to S, then the particle that is at rest in S must have a velocity of $u'_1 = -v$ as measured in S', while the velocity of the other particle is given by solving Eq. 37-29 for u':

$$u'_2 = \frac{u_2 - v}{1 - u_2 v/c^2} = \frac{(c/2) - v}{1 - (c/2)(v/c^2)}\ .$$

Letting $u'_2 = -u'_1 = v$, we obtain

$$\frac{(c/2) - v}{1 - (c/2)(v/c^2)} = v \quad \Rightarrow \quad v = c(2 \pm \sqrt{3}) \approx 0.27c$$

where the quadratic formula has been used (with the smaller of the two roots chosen so that $v \le c$).

82. (a) Our lab-based measurement of its lifetime is figured simply from

$$t = L/v = 7.99 \times 10^{-13}\ \text{s}.$$

Use of the time-dilation relation (Eq. 37-7) leads to

$$\Delta t_0 = (7.99\times 10^{-13}\ \text{s})\sqrt{1-(0.960)^2} = 2.24\times 10^{-13}\ \text{s}.$$

(b) The length contraction formula can be used, or we can use the simple speed-distance relation (from the point of view of the particle, who watches the lab and all its meter sticks rushing past him at $0.960c$ until he expires): $L = v\Delta t_0 = 6.44 \times 10^{-5}$ m.

83. (a) For a proton (using Table 37-3), we have

$$E = \gamma m_p c^2 = \frac{938\text{MeV}}{\sqrt{1-(0.990)^2}} = 6.65\,\text{GeV}$$

which gives $K = E - m_p c^2 = 6.65\text{GeV} - 938\text{MeV} = 5.71\text{GeV}$.

(b) From part (a), $E = 6.65\,\text{GeV}$.

(c) Similarly, we have $p = \gamma m_p v = \gamma (m_p c^2)\beta / c = \dfrac{(938\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}} = 6.58\,\text{GeV}/c$.

(d) For an electron, we have

$$E = \gamma m_e c^2 = \frac{0.511\text{MeV}}{\sqrt{1-(0.990)^2}} = 3.62\,\text{MeV}$$

which yields $K = E - m_e c^2 = 3.625\text{MeV} - 0.511\text{MeV} = 3.11\text{MeV}$.

(e) From part (d), $E = 3.62\,\text{MeV}$.

(f) $p = \gamma m_e v = \gamma (m_e c^2)\beta / c = \dfrac{(0.511\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}} = 3.59\,\text{MeV}/c$.

84. (a) Using Eq. 37-7, we expect the dilated time intervals to be

$$\tau = \gamma \tau_0 = \frac{\tau_0}{\sqrt{1-(v/c)^2}} \;.$$

(b) We rewrite Eq. 37-31 using the fact that the period is the reciprocal of frequency ($f_R = \tau_R^{-1}$ and $f_0 = \tau_0^{-1}$):

$$\tau_R = \frac{1}{f_R} = \left(f_0 \sqrt{\frac{1-\beta}{1+\beta}} \right)^{-1} = \tau_0 \sqrt{\frac{1+\beta}{1-\beta}} = \tau_0 \sqrt{\frac{c+v}{c-v}} \;.$$

(c) The Doppler shift combines two physical effects: the time dilation of the moving source *and* the travel-time differences involved in periodic emission (like a sine wave or a series of pulses) from a traveling source to a "stationary" receiver). To isolate the purely time-dilation effect, it's useful to consider "local" measurements (say, comparing the readings on a moving clock to those of two of your clocks, spaced some distance apart, such that the moving clock and each of your clocks can make a close comparison of readings at the moment of passage).

85. Let the reference frame be S in which the particle (approaching the South Pole) is at rest, and let the frame that is fixed on Earth be S'. Then $v = 0.60c$ and $u' = 0.80c$ (calling

"downward" [in the sense of Fig. 37-34] positive). The relative speed is now the speed of the other particle as measured in S:

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.80c + 0.60c}{1 + (0.80c)(0.60c)/c^2} = 0.95c \ .$$

86. (a) $\Delta E = \Delta mc^2 = (3.0 \text{ kg})(0.0010)(2.998 \times 10^8 \text{ m/s})^2 = 2.7 \times 10^{14}$ J.

(b) The mass of TNT is

$$m_{\text{TNT}} = \frac{(2.7 \times 10^{14} \ \text{J})(0.227 \ \text{kg/mol})}{3.4 \times 10^6 \ \text{J}} = 1.8 \times 10^7 \ \text{kg}.$$

(c) The fraction of mass converted in the TNT case is

$$\frac{\Delta m_{\text{TNT}}}{m_{\text{TNT}}} = \frac{(3.0 \text{ kg})(0.0010)}{1.8 \times 10^7 \ \text{kg}} = 1.6 \times 10^{-9},$$

Therefore, the fraction is $0.0010/1.6 \times 10^{-9} = 6.0 \times 10^6$.

87. (a) We assume the electron starts from rest. The classical formula for kinetic energy is Eq. 37-51, so if $v = c$ then this (for an electron) would be $\frac{1}{2}mc^2 = \frac{1}{2}(511 \text{ ke V}) = 255.5 \text{ ke V}$ (using Table 37-3). Setting this equal to the potential energy loss (which is responsible for its acceleration), we find (using Eq. 25-7)

$$V = \frac{255.5 \text{ keV}}{|q|} = \frac{255 \text{ keV}}{e} = 255.5 \text{ kV} \approx 256 \text{ kV}.$$

(b) Setting this amount of potential energy loss ($|\Delta U|$ = 255.5 keV) equal to the correct relativistic kinetic energy, we obtain (using Eq. 37-52)

$$mc^2\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) = |\Delta U| \ \Rightarrow \quad v = c\sqrt{1 + \left(\frac{1}{1 - \Delta U/mc^2}\right)^2}$$

which yields $v = 0.745c = 2.23 \times 10^8$ m/s.

88. We use the relative velocity formula (Eq. 37-29) with the primed measurements being those of the scout ship. We note that $v = -0.900c$ since the velocity of the scout ship relative to the cruiser is opposite to that of the cruiser relative to the scout ship.

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.980c - 0.900c}{1 - (0.980)(0.900)} = 0.678c \ .$$

89. (a) Since both spaceships A and C are approaching B at the same speed (relative to B), with $v_A > v_B > v_C$, using relativistic velocity addition formula, we have $v'_A = -v'_c$, or

$$\frac{v_A - v_B}{1 - v_A v_B / c^2} = \frac{v_B - v_C}{1 - v_B v_C / c^2} \quad \Rightarrow \quad \frac{\beta_A - \beta_B}{1 - \beta_A \beta_B} = \frac{\beta_B - \beta_C}{1 - \beta_B \beta_C}$$

We multiply both sides by factors to get rid of the denominators:

$$(\beta_A - \beta_B)(1 - \beta_B \beta_C) = (\beta_B - \beta_C)(1 - \beta_A \beta_B)$$

Expanding and simplifying gives

$$(\beta_A + \beta_C)\beta_B^2 - 2(1 + \beta_A \beta_C)\beta_B + (\beta_A + \beta_C) = 0$$

Solving the quadratic equation with $\beta_A = 0.90$ and $\beta_C = 0.80$ leads to $\beta_B = 0.858$, or $v_B = 0.858c$.

(b) The relative speed (say, A relative to B) is

$$v'_A = \frac{v_A - v_B}{1 - v_A v_B / c^2} = \frac{0.90c - 0.858c}{1 - (0.90)(0.858)} = 0.185c\,.$$

90. In the rest frame of Cruiser A, Cruiser B is moving at a speed of 0.900c, and has a length of 200 m. The proper length of Cruiser B, according to its pilot, is

$$L_{B0} = \gamma L_B = \frac{200\text{ m}}{\sqrt{1 - (0.900)^2}} = 458.8\text{ m},$$

and the length of Cruiser A is $L_A = L_{A0} / \gamma = \sqrt{1 - (0.900)^2}\,(200\text{ m}) = 87.2\text{ m}$. Therefore, according to pilot in Cruiser B, the time elapsed for the tails to align is

$$\Delta t = \frac{L_{B0} - L_A}{v_A} = \frac{458.8\text{ m} - 87.2\text{ m}}{(0.90)(3.0 \times 10^8\text{ m/s})} = 1.38 \times 10^{-6}\text{ s}\,.$$

91. Let the speed of B relative to the station be v_B. We require the speed of A relative to B to be the same as v_B:

$$v'_A = \frac{v_A - v_B}{1 - v_A v_B / c^2} = v_B\,.$$

The above expression can be rewritten as $v_B^2 - (2c^2/v_A)v_B + c^2 = 0$. Solving the quadratic equation for v_B, with $v_A = 0.80c$, we obtain $v_B = 0.50c$.

92. (a) From the train view, the tunnel appears to be contracted by a factor of

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.900)^2}} = 2.29.$$

Thus, the length is $L_{\text{tunnel}} = L_{\text{tunnel},0}/\gamma = (200\text{ m})/2.29 = 87.2\text{ m}$.

(b) From the train view, since the tunnel appears to be shorter than the train, event FF will occur first.

(c) According to an observer on the train, the time between the two events is

$$\Delta t = \frac{L_{\text{train},0} - L_{\text{tunnel}}}{v} = \frac{200\text{ m} - 87.2\text{ m}}{(0.900)(3.0\times10^8\text{ m/s})} = 0.418\,\mu\text{s}.$$

(d) Since event FF occurs first, the paint will explode.

(e) From the tunnel view, the train appears to be contracted by a factor of

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.900)^2}} = 2.29.$$

Thus, the length is $L_{\text{train}} = L_{\text{train},0}/\gamma = (200\text{ m})/2.29 = 87.2\text{ m}$.

(f) From the tunnel view, since the train appears to be shorter than the tunnel, event RN will occur first.

(g) According to an observer in the rest frame of the tunnel, the time between the two events is

$$\Delta t = \frac{L_{\text{tunnel},0} - L_{\text{train}}}{v} = \frac{200\text{ m} - 87.2\text{ m}}{(0.900)(3.0\times10^8\text{ m/s})} = 0.418\,\mu\text{s}.$$

(h) The bomb will explode also. The reason is that one must take into consideration the time is takes for the deactivation signal to propagate from the rear of the train to the front, which is $\Delta t_{\text{signal}} = \dfrac{L_{\text{train},0}}{v} = \dfrac{200\text{ m}}{(0.900)(3.0\times10^8\text{ m/s})} = 0.741\,\mu\text{s}$. This is longer than the time elapsed between the two events. So the bomb does explode.

93. (a) The condition for energy conservation is $E_A = E_B + E_C$. Similarly, momentum conservation requires $p_B = p_C$ (same magnitude but opposite directions). Using $E = \gamma mc^2$ gives $m_A c^2 = \gamma_B m_B c^2 + \gamma_C m_C c^2$, or

$$200 = 100\gamma_B + 50\gamma_C \;\Rightarrow\; 4 = 2\gamma_B + \gamma_C$$

Now using $p = \gamma mv$, we have

$$\gamma_B m_B v_B = \gamma_C m_C v_C \;\Rightarrow\; \gamma_B m_B \beta_B = \gamma_C m_C \beta_C$$

Noting that $\gamma\beta = \gamma\sqrt{1-1/\gamma^2} = \sqrt{\gamma^2 - 1}$, the above expression can be rewritten as

$$\frac{\sqrt{\gamma_B^2 - 1}}{\sqrt{\gamma_C^2 - 1}} = \frac{m_C}{m_B} = \frac{50\ \text{MeV}/c^2}{100\ \text{MeV}/c^2} = \frac{1}{2}$$

which implies $4\gamma_B^2 = \gamma_C^2 + 3$. Solving the two simultaneous equations gives $\gamma_B = 19/16$ and $\gamma_C = 13/8$. The total energy of *B* is

$$E_B = \gamma_B m_B c^2 = \left(\frac{19}{16}\right)(100\ \text{MeV}) = 119\ \text{MeV}.$$

(b) Using $p = \gamma mv = \sqrt{\gamma^2 - 1}\,\frac{mc^2}{c}$, we find the momentum of *B* to be

$$p_B = \sqrt{\gamma_B^2 - 1}\,\frac{m_B c^2}{c} = \sqrt{(19/16)^2 - 1}\,(100\ \text{MeV}/c) = 64.0\ \text{MeV}/c.$$

(c) The total energy of *C* is $E_C = \gamma_C m_C c^2 = \left(\frac{13}{8}\right)(50\ \text{MeV}) = 81.3\ \text{MeV}.$

(d) The magnitude of momentum of *C* is the same as *B*: $p_C = 64.0\ \text{MeV}/c$.

94. (a) The travel time for trip 1 measured by an Earth observer is $\Delta t_1 = 2D/c$.

(b) For trip 2, we have $\Delta t_2 = 4D/c$,

(c) and $\Delta t_3 = 6D/c$, for trip 3.

(d) In the rest frame of the starship, the distance appears to be shortened by the Lorentz factor γ. Thus, $\Delta t_1' = \frac{2D'}{c} = \frac{2D}{c\gamma_1} = \frac{D}{5c}$.

(e) Similarly, for trip 2, we have $\Delta t_2' = \dfrac{4D'}{c} = \dfrac{4D}{c\gamma_2} = \dfrac{4D}{c(24)} = \dfrac{D}{6c}$.

(f) For trip 3, the time is $\Delta t_3' = \dfrac{6D'}{c} = \dfrac{6D}{c\gamma_3} = \dfrac{6D}{c(30)} = \dfrac{D}{5c}$.

95. The radius *r* of the path is *r* = *γmvqB*. Thus,

$$m = \frac{qBr\sqrt{1-\beta^2}}{v} = \frac{2(1.60\times10^{-19}\ \text{C})(1.00\ \text{T})(6.28\ \text{m})\sqrt{1-(0.710)^2}}{(0.710)(3.00\times10^8\ \text{m/s})} = 6.64\times10^{-27}\ \text{kg}.$$

Since 1.00 u = 1.66 × 10^{-27} kg, the mass is *m* = 4.00 u. The nuclear particle contains four nucleons. Since there must be two protons to provide the charge 2*e*, the nuclear particle is a helium nucleus (usually referred to as an alpha particle) with two protons and two neutrons.

96. We interpret the given 2.50 MeV = 2500 keV to be the kinetic energy of the electron. Using Table 37-3, we find

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{2500\ \text{keV}}{511\ \text{keV}} + 1 = 5.892,$$

and

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9855.$$

Therefore, using the equation *r* = *γmvqB* (with "*q*" interpreted as |*q*|), we obtain

$$B = \frac{\gamma m_e v}{|q|r} = \frac{\gamma m_e \beta c}{er} = \frac{(5.892)(9.11\times10^{-31}\ \text{kg})(0.9855)(2.998\times10^8\ \text{m/s})}{(1.6\times10^{-19}\ \text{C})(0.030\ \text{m})}$$
$$= 0.33\ \text{T}.$$

97. (a) Using Table 37-3 and Eq. 37-58, we find

$$\gamma = \frac{K}{m_p c^2} + 1 = \frac{500\times10^3\ \text{MeV}}{938.3\ \text{MeV}} + 1 = 533.88.$$

(b) From Eq. 37-8, we obtain

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99999825.$$

(c) To make use of the precise $m_p c^2$ value given here, we rewrite the expression introduced in problem 46 (as applied to the proton) as follows:

$$r = \frac{\gamma m v}{qB} = \frac{\gamma (mc^2)\left(\frac{v}{c^2}\right)}{eB} = \frac{\gamma (mc^2)\beta}{ecB}.$$

Therefore, the magnitude of the magnetic field is

$$B = \frac{\gamma (mc^2)\beta}{ecr} = \frac{(533.88)(938.3\text{ MeV})(0.99999825)}{ec(750\text{ m})} = \frac{667.92\times 10^6\text{ V/m}}{c}$$

where we note the cancellation of the "e" in MeV with the e in the denominator. After substituting $c = 2.998 \times 10^8$ m/s, we obtain $B = 2.23$ T.

98. (a) The pulse rate as measured by an observer at the station is

$$R = \frac{\Delta N}{\Delta t} = \frac{\Delta N}{\gamma \Delta t_0} = \frac{R_0}{\gamma} = (150/\text{min})\sqrt{1-(0.900)^2} = 65.4/\text{min}.$$

(b) According to the observer at the station, the stride length appears to be shortened, and the clock runs slower in the spaceship, the speed observed is

$$v = \frac{\Delta L}{\Delta t} = \frac{L_0/\gamma}{\gamma \Delta t_0} = \frac{v_0}{\gamma^2},$$

and the distance the astronaut walked is measured to be

$$d = v\Delta t = \frac{v_0}{\gamma^2}\gamma\Delta t_0 = \frac{v_0 \Delta t_0}{\gamma} = \sqrt{1-(0.900)^2}\,(1.0\text{ m/s})(3600\text{ s}) = 1570\text{ m}.$$

99. The frequency received is given by

$$f = f_0\sqrt{\frac{1+\beta}{1-\beta}} \quad \Rightarrow \quad \frac{c}{\lambda} = \frac{c}{\lambda_0}\sqrt{\frac{1+\beta}{1-\beta}}$$

which implies

$$\lambda = \lambda_0\sqrt{\frac{1-\beta}{1+\beta}} = (650\text{ nm})\sqrt{\frac{1-0.42}{1+0.42}} = 415\text{ nm}.$$

This is in the blue portion of the visible spectrum.

100. (a) Using the classical Doppler equation $f' = \frac{v}{v+v_s} f$, we have

$$v_s = v\left(\frac{f}{f'} - 1\right) = v\left(\frac{\lambda'}{\lambda} - 1\right) = c\left(\frac{3\lambda}{\lambda} - 1\right) = 2c > c.$$

(b) Using $f = f_0\sqrt{\frac{1-\beta}{1+\beta}}$, we solve for β and obtain

$$\beta = \frac{1-(f/f_0)^2}{1+(f/f_0)^2} = \frac{1-(1/3)^2}{1+(1/3)^2} = \frac{8/9}{10/9} = 0.80$$

or $v = 0.80c$.

101. Using $E = mc^2$, we find the required mass to be

$$m = \frac{E}{c^2} = \frac{(2.2\times10^{12}\ \text{kWh})(3.6\times10^{12}\ \text{J/kWh})}{(3\times10^{8}\ \text{m/s})^2} = 88\ \text{kg}.$$

(b) No, the energy consumed is still about 2.2×10^{12} kWh regardless of how it's generated (oil-burning, nuclear, or hydroelectric….).

102. (a) The time an electron with a horizontal component of velocity v takes to travel a horizontal distance L is

$$t = \frac{L}{v} = \frac{20\times10^{-2}\ \text{m}}{(0.992)(2.998\times10^{8}\ \text{m/s})} = 6.72\times10^{-10}\ \text{s}.$$

(b) During this time, it falls a vertical distance

$$y = \frac{1}{2}gt^2 = \frac{1}{2}(9.8\ \text{m/s}^2)(6.72\times10^{-10}\ \text{s})^2 = 2.2\times10^{-18}\ \text{m}.$$

This distance is much less than the radius of a proton.

(c) We can conclude that for particles traveling near the speed of light in a laboratory, Earth may be considered an approximately inertial frame.

103. (a) The speed parameter β is v/c. Thus,

$$\beta = \frac{(3\ \text{cm/y})(0.01\ \text{m/cm})(1\ \text{y}/3.15\times10^{7}\ \text{s})}{3.0\times10^{8}\ \text{m/s}} = 3\times10^{-18}.$$

(b) For the highway speed limit, we find

$$\beta = \frac{(90\,\text{km/h})(1000\,\text{m/km})(1\,\text{h}/3600\,\text{s})}{3.0\times10^{8}\,\text{m/s}} = 8.3\times10^{-8}.$$

(c) Mach 2.5 corresponds to

$$\beta = \frac{(1200\,\text{km/h})(1000\,\text{m/km})(1\,\text{h}/3600\,\text{s})}{3.0\times10^{8}\,\text{m/s}} = 1.1\times10^{-6}.$$

(d) We refer to Table 14-2:

$$\beta = \frac{(11.2\,\text{km/s})(1000\,\text{m/km})}{3.0\times10^{8}\,\text{m/s}} = 3.7\times10^{-5}.$$

(e) For the quasar recession speed, we obtain

$$\beta = \frac{(3.0\times10^{4}\,\text{km/s})(1000\,\text{m/km})}{3.0\times10^{8}\,\text{m/s}} = 0.10.$$

Chapter 38

1. (a) With $E = hc/\lambda_{\min} = 1240\ \text{eV}\cdot\text{nm}/\lambda_{\min} = 0.6$ eV, we obtain $\lambda = 2.1 \times 10^3$ nm = 2.1 μm.

(b) It is in the infrared region.

2. Let

$$\frac{1}{2}m_e v^2 = E_{\text{photon}} = \frac{hc}{\lambda}$$

and solve for v:

$$v = \sqrt{\frac{2hc}{\lambda m_e}} = \sqrt{\frac{2hc}{\lambda m_e c^2}c^2} = c\sqrt{\frac{2hc}{\lambda\left(m_e c^2\right)}}$$
$$= \left(2.998\times10^8\ \text{m/s}\right)\sqrt{\frac{2\left(1240\,\text{eV}\cdot\text{nm}\right)}{\left(590\,\text{nm}\right)\left(511\times10^3\ \text{eV}\right)}} = 8.6\times10^5\ \text{m/s}.$$

Since $v \ll c$, the nonrelativistic formula $K = \frac{1}{2}mv^2$ may be used. The $m_e c^2$ value of Table 37-3 and $hc = 1240\text{eV}\cdot\text{nm}$ are used in our calculation.

3. Let R be the rate of photon emission (number of photons emitted per unit time) of the Sun and let E be the energy of a single photon. Then the power output of the Sun is given by $P = RE$. Now

$$E = hf = hc/\lambda,$$

where $h = 6.626 \times 10^{-34}$ J·s is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and

$$R = \frac{\lambda P}{hc} = \frac{\left(550\,\text{nm}\right)\left(3.9\times10^{26}\ \text{W}\right)}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^8\ \text{m/s}\right)} = 1.0\times10^{45}\ \text{photons/s}.$$

4. We denote the diameter of the laser beam as d. The cross-sectional area of the beam is $A = \pi d^2/4$. From the formula obtained in Problem 38-3, the rate is given by

$$\frac{R}{A} = \frac{\lambda P}{hc\left(\pi d^2/4\right)} = \frac{4\left(633\text{nm}\right)\left(5.0\times10^{-3}\,\text{W}\right)}{\pi\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(2.998\times10^8\,\text{m/s}\right)\left(3.5\times10^{-3}\,\text{m}\right)^2}$$
$$= 1.7\times10^{21}\,\text{photons/m}^2\cdot\text{s}.$$

5. The energy of a photon is given by $E = hf$, where h is the Planck constant and f is the frequency. The wavelength λ is related to the frequency by $\lambda f = c$, so $E = hc/\lambda$. Since $h = 6.626 \times 10^{-34}$ J·s and $c = 2.998 \times 10^8$ m/s,

$$hc = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{(1.602\times10^{-19}\text{ J/eV})(10^{-9}\text{ m/nm})} = 1240\,\text{eV}\cdot\text{nm}.$$

Thus,

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda}.$$

With

$$\lambda = (1,650,763.73)^{-1}\text{ m} = 6.0578021\times10^{-7}\text{ m} = 605.78021\text{ nm},$$

we find the energy to be

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{605.78021\,\text{nm}} = 2.047\,\text{eV}.$$

6. The energy of a photon is given by $E = hf$, where h is the Planck constant and f is the frequency. The wavelength λ is related to the frequency by $\lambda f = c$, so $E = hc/\lambda$. Since $h = 6.626 \times 10^{-34}$ J·s and $c = 2.998 \times 10^8$ m/s,

$$hc = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{(1.602\times10^{-19}\text{ J/eV})(10^{-9}\text{ m/nm})} = 1240\,\text{eV}\cdot\text{nm}.$$

Thus,

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda}.$$

With $\lambda = 589$ nm, we obtain

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{589\,\text{nm}} = 2.11\,\text{eV}.$$

7. The rate at which photons are absorbed by the detector is related to the rate of photon emission by the light source via

$$R_{\text{abs}} = (0.80)\frac{A_{\text{abs}}}{4\pi r^2}R_{\text{emit}}.$$

Given that $A_{\text{abs}} = 2.00\times10^{-6}\text{ m}^2$ and $r = 3.00$ m, with $R_{\text{abs}} = 4.000$ photons/s, we find the rate at which photons are emitted to be

$$R_{\text{emit}} = \frac{4\pi r^2}{(0.80)A_{\text{abs}}}R_{\text{abs}} = \frac{4\pi(3.00\text{ m})^2}{(0.80)(2.00\times10^{-6}\text{ m}^2)}(4.000\text{ photons/s}) = 2.83\times10^{8}\text{ photons/s}.$$

Since the energy of each emitted photon is

$$E_{\text{ph}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{500\,\text{nm}} = 2.48\,\text{eV},$$

the power output of source is

$$P_{\text{emit}} = R_{\text{emit}}E_{\text{ph}} = \left(2.83\times10^{8}\,\text{photons/s}\right)(2.48\,\text{eV}) = 7.0\times10^{8}\ \text{eV/s} = 1.1\times10^{-10}\ \text{W}.$$

8. The rate at which photons are emitted from the argon laser source is given by $R = P/E_{\text{ph}}$, where P = 1.5 W is the power of the laser beam and $E_{\text{ph}} = hc/\lambda$ is the energy of each photon of wavelength λ. Since α = 84% of the energy of the laser beam falls within the central disk, the rate of photon absorption of the central disk is

$$R' = \alpha R = \frac{\alpha P}{hc/\lambda} = \frac{(0.84)(1.5\,\text{W})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})/(515\times10^{-9}\ \text{m})}$$
$$= 3.3\times10^{18}\ \text{photons/s}.$$

9. (a) We assume all the power results in photon production at the wavelength $\lambda = 589\ \text{nm}$. Let R be the rate of photon production and E be the energy of a single photon. Then,

$$P = RE = Rhc/\lambda,$$

where $E = hf$ and $f = c/\lambda$ are used. Here h is the Planck constant, f is the frequency of the emitted light, and λ is its wavelength. Thus,

$$R = \frac{\lambda P}{hc} = \frac{\left(589\times10^{-9}\ \text{m}\right)(100\ \text{W})}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)} = 2.96\times10^{20}\ \text{photon/s}.$$

(b) Let I be the photon flux a distance r from the source. Since photons are emitted uniformly in all directions, $R = 4\pi r^2 I$ and

$$r = \sqrt{\frac{R}{4\pi I}} = \sqrt{\frac{2.96\times10^{20}\ \text{photon/s}}{4\pi\left(1.00\times10^{4}\ \text{photon/m}^2\cdot\text{s}\right)}} = 4.86\times10^{7}\ \text{m}.$$

(c) The photon flux is

$$I = \frac{R}{4\pi r^2} = \frac{2.96\times10^{20}\ \text{photon/s}}{4\pi\left(2.00\ \text{m}\right)^2} = 5.89\times10^{18}\ \frac{\text{photon}}{\text{m}^2\cdot\text{s}}.$$

10. (a) The rate at which solar energy strikes the panel is

$$P = (1.39 \text{ kW/m}^2)(2.60 \text{ m}^2) = 3.61 \text{ kW}.$$

(b) The rate at which solar photons are absorbed by the panel is

$$R = \frac{P}{E_{\text{ph}}} = \frac{3.61\times10^3 \text{ W}}{(6.63\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^8 \text{ m/s})/(550\times10^{-9} \text{ m})}$$
$$= 1.00\times10^{22} \text{ photons/s}.$$

(c) The time in question is given by

$$t = \frac{N_A}{R} = \frac{6.02\times10^{23}}{1.00\times10^{22}\text{ / s}} = 60.2 \text{ s}.$$

11. **THINK** The rate of photon emission is the number of photons emitted per unit time.

EXPRESS Let R be the photon emission rate and E be the energy of a single photon. The power output of a lamp is given by $P = RE$, where we assume that all the power goes into photon production. Now, $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus

$$P = \frac{Rhc}{\lambda} \quad \Rightarrow \quad R = \frac{\lambda P}{hc} \ .$$

ANALYZE (a) The fact that $R \sim \lambda$ means that the lamp that emits light with the longer wavelength (the 700 nm infrared lamp) emits more photons per unit time. The energy of each photon is less, so it must emit photons at a greater rate.

(b) Let R be the rate of photon production for the 700 nm lamp. Then,

$$R = \frac{\lambda P}{hc} = \frac{(700\,\text{nm})(400\,\text{J/s})}{(1.60\times10^{-19}\text{ J/eV})(1240 \text{ eV}\cdot\text{nm})} = 1.41\times10^{21} \text{ photon/s}.$$

LEARN With $P = Rhc/\lambda$, we readily see that when the rate of photon emission is held constant, the shorter the wavelength, the greater the power, or rate of energy emission.

12. Following Sample Problem — "Emission and absorption of light as photons," we have

$$P = \frac{Rhc}{\lambda} = \frac{(100/\text{s})(6.63\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{550\times10^{-9}\text{ m}} = 3.6\times10^{-17} \text{ W}.$$

13. The total energy emitted by the bulb is $E = 0.93Pt$, where $P = 60$ W and

$$t = 730 \text{ h} = (730 \text{ h})(3600 \text{ s/h}) = 2.628 \times 10^6 \text{ s}.$$

The energy of each photon emitted is $E_{ph} = hc/\lambda$. Therefore, the number of photons emitted is

$$N = \frac{E}{E_{ph}} = \frac{0.93Pt}{hc/\lambda} = \frac{(0.93)(60 \text{ W})(2.628 \times 10^6 \text{ s})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})/(630 \times 10^{-9} \text{ m})} = 4.7 \times 10^{26}.$$

14. The average power output of the source is

$$P_{emit} = \frac{\Delta E}{\Delta t} = \frac{7.2 \text{ nJ}}{2 \text{ s}} = 3.6 \text{ nJ/s} = 3.6 \times 10^{-9} \text{ J/s} = 2.25 \times 10^{10} \text{ eV/s}.$$

Since the energy of each photon emitted is

$$E_{ph} = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{600 \text{ nm}} = 2.07 \text{ eV},$$

the rate at which photons are emitted by the source is

$$R_{emit} = \frac{P_{emit}}{E_{ph}} = \frac{2.25 \times 10^{10} \text{ eV/s}}{2.07 \text{ eV}} = 1.09 \times 10^{10} \text{ photons/s}.$$

Given that the source is isotropic, and the detector (located 12.0 m away) has an absorbing area of $A_{abs} = 2.00 \times 10^{-6} \text{ m}^2$ and absorbs 50% of the incident light, the rate of photon absorption is

$$R_{abs} = (0.50)\frac{A_{abs}}{4\pi r^2} R_{emit} = (0.50)\frac{2.00 \times 10^{-6} \text{ m}^2}{4\pi (12.0 \text{ m})^2}\left(1.09 \times 10^{10} \text{ photons/s}\right) = 6.0 \text{ photons/s}.$$

15. **THINK** The energy of an incident photon is $E = hf$, where h is the Planck constant, and f is the frequency of the electromagnetic radiation.

EXPRESS The kinetic energy of the most energetic electron emitted is

$$K_m = E - \Phi = (hc/\lambda) - \Phi,$$

where Φ is the work function for sodium, and $f = c/\lambda$, where λ is the wavelength of the photon.

The stopping potential V_{stop} is related to the maximum kinetic energy by $eV_{stop} = K_m$, so

$$eV_{\text{stop}} = (hc/\lambda) - \Phi$$

and

$$\lambda = \frac{hc}{eV_{\text{stop}} + \Phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{5.0\,\text{eV} + 2.2\,\text{eV}} = 170\,\text{nm}.$$

Here eV_{stop} = 5.0 eV and hc = 1240 eV·nm are used.

LEARN The cutoff frequency for this problem is

$$f_0 = \frac{\Phi}{h} = \frac{(2.2\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{6.626\times10^{-34}\,\text{J}\cdot\text{s}} = 5.3\times10^{14}\text{ Hz}.$$

16. We use Eq. 38-5 to find the maximum kinetic energy of the ejected electrons:

$$K_{\max} = hf - \Phi = \left(4.14\times10^{-15}\text{ eV}\cdot\text{s}\right)\left(3.0\times10^{15}\text{ Hz}\right) - 2.3\,\text{eV} = 10\,\text{eV}.$$

17. The speed v of the electron satisfies

$$K_{\max} = \tfrac{1}{2}m_e v^2 = \tfrac{1}{2}\left(m_e c^2\right)\left(v/c\right)^2 = E_{\text{photon}} - \Phi.$$

Using Table 37-3, we find

$$v = c\sqrt{\frac{2\left(E_{\text{photon}} - \Phi\right)}{m_e c^2}} = \left(2.998\times10^8\text{ m/s}\right)\sqrt{\frac{2\left(5.80\,\text{eV} - 4.50\,\text{eV}\right)}{511\times10^3\text{ eV}}} = 6.76\times10^5\text{ m/s}.$$

18. The energy of the most energetic photon in the visible light range (with wavelength of about 400 nm) is about E = (1240 eV·nm/400 nm) = 3.1 eV (using the value hc = 1240 eV·nm). Consequently, barium and lithium can be used, since their work functions are both lower than 3.1 eV.

19. (a) We use Eq. 38-6:

$$V_{\text{stop}} = \frac{hf - \Phi}{e} = \frac{hc/\lambda - \Phi}{e} = \frac{\left(1240\,\text{eV}\cdot\text{nm}/400\,\text{nm}\right) - 1.8\,\text{eV}}{e} = 1.3\,\text{V}.$$

(b) The speed v of the electron satisfies

$$K_{\max} = \tfrac{1}{2}m_e v^2 = \tfrac{1}{2}\left(m_e c^2\right)\left(v/c\right)^2 = E_{\text{photon}} - \Phi.$$

Using Table 37-3, we find

$$v=\sqrt{\frac{2\left(E_{\text{photon}}-\Phi\right)}{m_e}}=\sqrt{\frac{2eV_{\text{stop}}}{m_e}}=c\sqrt{\frac{2eV_{\text{stop}}}{m_ec^2}}=\left(2.998\times10^8\text{ m/s}\right)\sqrt{\frac{2e\left(1.3\text{V}\right)}{511\times10^3\text{eV}}}$$
$$=6.8\times10^5\text{ m/s.}$$

20. Using the value hc = 1240 eV·nm, the number of photons emitted from the laser per unit time is

$$R=\frac{P}{E_{\text{ph}}}=\frac{2.00\times10^{-3}\text{ W}}{(1240\text{ eV}\cdot\text{nm}/600\text{ nm})(1.60\times10^{-19}\text{ J}/\text{eV})}=6.05\times10^{15}/\text{ s},$$

of which $(1.0\times10^{-16})(6.05\times10^{15}/\text{s})=0.605/\text{s}$ actually cause photoelectric emissions. Thus the current is

$$i=(0.605/\text{s})(1.60\times10^{-19}\text{ C})=9.68\times10^{-20}\text{ A.}$$

21. (a) From $r=m_ev/eB$, the speed of the electron is $v=rBe/m_e$. Thus,

$$K_{\max}=\frac{1}{2}m_ev^2=\frac{1}{2}m_e\left(\frac{rBe}{m_e}\right)^2=\frac{(rB)^2e^2}{2m_e}=\frac{(1.88\times10^{-4}\text{ T}\cdot\text{m})^2(1.60\times10^{-19}\text{ C})^2}{2(9.11\times10^{-31}\text{ kg})(1.60\times10^{-19}\text{ J/eV})}$$
$$=3.1\text{ keV.}$$

(b) Using the value hc = 1240 eV·nm, the work done is

$$W=E_{\text{photon}}-K_{\max}=\frac{1240\text{ eV}\cdot\text{nm}}{71\times10^{-3}\text{ nm}}-3.10\text{ keV}=14\text{ keV.}$$

22. We use Eq. 38-6 and the value hc = 1240 eV·nm:

$$K_{\max}=E_{\text{photon}}-\Phi=\frac{hc}{\lambda}-\frac{hc}{\lambda_{\max}}=\frac{1240\text{eV}\cdot\text{nm}}{254\text{nm}}-\frac{1240\text{eV}\cdot\text{nm}}{325\text{nm}}=1.07\text{eV.}$$

23. **THINK** The kinetic energy K_m of the fastest electron emitted is given by

$$K_m=hf-\Phi,$$

where Φ is the work function of aluminum, and f is the frequency of the incident radiation.

EXPRESS Since $f=c/\lambda$, where λ is the wavelength of the photon, the above expression can be rewritten as

$$K_m=(hc/\lambda)-\Phi.$$

ANALYZE (a) Thus, the kinetic energy of the fastest electron is

$$K_m = \frac{1240\,\text{eV}\cdot\text{nm}}{200\,\text{nm}} - 4.20\,\text{eV} = 2.00\ \text{eV},$$

where we have used hc = 1240 eV·nm.

(b) The slowest electron just breaks free of the surface and so has zero kinetic energy.

(c) The stopping potential V_{stop} is given by $K_m = eV_{\text{stop}}$, so

$$V_{\text{stop}} = K_m/e = (2.00\ \text{eV})/e = 2.00\ \text{V}.$$

(d) The value of the cutoff wavelength is such that $K_m = 0$. Thus, $hc/\lambda_0 = \Phi$, or

$$\lambda_0 = hc/\Phi = (1240\ \text{eV}\cdot\text{nm})/(4.2\ \text{eV}) = 295\ \text{nm}.$$

LEARN If the wavelength is longer than λ_0, the photon energy is less than Φ and a photon does not have sufficient energy to knock even the most energetic electron out of the aluminum sample.

24. (a) For the first and second case (labeled 1 and 2) we have

$$eV_{01} = hc/\lambda_1 - \Phi\ , \quad eV_{02} = hc/\lambda_2 - \Phi,$$

from which h and Φ can be determined. Thus,

$$h = \frac{e(V_1 - V_2)}{c(\lambda_1^{-1} - \lambda_2^{-1})} = \frac{1.85\text{eV} - 0.820\text{eV}}{(3.00\times10^{17}\ \text{nm/s})\left[(300\text{nm})^{-1} - (400\text{nm})^{-1}\right]} = 4.12\times10^{-15}\ \text{eV}\cdot\text{s}.$$

(b) The work function is

$$\Phi = \frac{3(V_2\lambda_2 - V_1\lambda_1)}{\lambda_1 - \lambda_2} = \frac{(0.820\ \text{eV})(400\ \text{nm}) - (1.85\ \text{eV})(300\ \text{nm})}{300\ \text{nm} - 400\ \text{nm}} = 2.27\ \text{eV}.$$

(c) Let $\Phi = hc/\lambda_{\text{max}}$ to obtain

$$\lambda_{\text{max}} = \frac{hc}{\Phi} = \frac{1240\ \text{eV}\cdot\text{nm}}{2.27\ \text{eV}} = 545\ \text{nm}.$$

25. (a) We use the photoelectric effect equation (Eq. 38-5) in the form $hc/\lambda = \Phi + K_m$. The work function depends only on the material and the condition of the surface, and not on the wavelength of the incident light. Let λ_1 be the first wavelength described and λ_2 be the second. Let K_{m1} = 0.710 eV be the maximum kinetic energy of electrons ejected by

light with the first wavelength, and K_{m2} = 1.43 eV be the maximum kinetic energy of electrons ejected by light with the second wavelength. Then,

$$\frac{hc}{\lambda_1} = \Phi + K_{m1}, \qquad \frac{hc}{\lambda_2} = \Phi + K_{m2}.$$

The first equation yields $\Phi = (hc/\lambda_1) - K_{m1}$. When this is used to substitute for Φ in the second equation, the result is

$$(hc/\lambda_2) = (hc/\lambda_1) - K_{m1} + K_{m2}.$$

The solution for λ_2 is

$$\lambda_2 = \frac{hc\lambda_1}{hc + \lambda_1(K_{m2} - K_{m1})} = \frac{(1240\,\text{V}\cdot\text{nm})(491\,\text{nm})}{1240\,\text{eV}\cdot\text{nm} + (491\,\text{nm})(1.43\,\text{eV} - 0.710\,\text{eV})}$$
$$= 382\,\text{nm}.$$

Here hc = 1240 eV·nm has been used.

(b) The first equation displayed above yields

$$\Phi = \frac{hc}{\lambda_1} - K_{m1} = \frac{1240\ \text{eV}\cdot\text{nm}}{491\ \text{nm}} - 0.710\ \text{eV} = 1.82\ \text{eV}.$$

26. To find the longest possible wavelength λ_{max} (corresponding to the lowest possible energy) of a photon that can produce a photoelectric effect in platinum, we set $K_{max} = 0$ in Eq. 38-5 and use $hf = hc/\lambda$. Thus $hc/\lambda_{max} = \Phi$. We solve for λ_{max}:

$$\lambda_{max} = \frac{hc}{\Phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{5.32\,\text{nm}} = 233\,\text{nm}.$$

27. **THINK** The scattering between a photon and an electron initially at rest results in a change or photon's wavelength, or Compton shift.

EXPRESS When a photon scatters off from an electron initially at rest, the change in wavelength is given by

$$\Delta\lambda = (h/mc)(1 - \cos\phi),$$

where m is the mass of an electron and ϕ is the scattering angle.

ANALYZE (a) The Compton wavelength of the electron is $h/mc = 2.43 \times 10^{-12}$ m = 2.43 pm. Therefore, we find the shift to be

$$\Delta\lambda = (h/mc)(1 - \cos\phi) = (2.43\ \text{pm})(1 - \cos 30^\circ) = 0.326\ \text{pm}.$$

The final wavelength is

$$\lambda' = \lambda + \Delta\lambda = 2.4 \text{ pm} + 0.326 \text{ pm} = 2.73 \text{ pm}.$$

(b) With $\phi = 120°$, $\Delta\lambda = (2.43 \text{ pm})(1 - \cos 120°) = 3.645$ pm and

$$\lambda' = 2.4 \text{ pm} + 3.645 \text{ pm} = 6.05 \text{ pm}.$$

LEARN The wavelength shift is greatest when $\phi = 180°$, where $\cos 180° = -1$. At this angle, the photon is scattered back along its initial direction of travel, and $\Delta\lambda = 2h/mc$.

28. (a) The rest energy of an electron is given by $E = m_e c^2$. Thus the momentum of the photon in question is given by

$$p = \frac{E}{c} = \frac{m_e c^2}{c} = m_e c = (9.11\times10^{-31}\text{ kg})(2.998\times10^{8}\text{ m/s}) = 2.73\times10^{-22}\text{ kg}\cdot\text{m/s}$$
$$= 0.511 \text{ MeV}/c.$$

(b) From Eq. 38-7,

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2.73\times10^{-22}\text{ kg}\cdot\text{m/s}} = 2.43\times10^{-12}\text{ m}=2.43\text{ pm}.$$

(c) Using Eq. 38-1,

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\text{ m/s}}{2.43\times10^{-12}\text{ m}} = 1.24\times10^{20}\text{ Hz}.$$

29. (a) The x-ray frequency is

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\text{ m/s}}{35.0\times10^{-12}\text{ m}} = 8.57\times10^{18}\text{ Hz}.$$

(b) The x-ray photon energy is

$$E = hf = (4.14\times10^{-15}\text{ eV}\cdot\text{s})(8.57\times10^{18}\text{ Hz}) = 3.55\times10^{4}\text{ eV}.$$

(c) From Eq. 38-7,

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{35.0\times10^{-12}\text{ m}} = 1.89\times10^{-23}\text{ kg}\cdot\text{m/s} = 35.4\text{ keV}/c.$$

30. The $(1 - \cos\phi)$ factor in Eq. 38-11 is largest when $\phi = 180°$. Thus, using Table 37-3, we obtain

$$\Delta\lambda_{max} = \frac{hc}{m_p c^2}(1-\cos 180°) = \frac{1240\,\text{MeV}\cdot\text{fm}}{938\,\text{MeV}}(1-(-1)) = 2.64\ \text{fm}$$

where we have used the value hc = 1240 eV·nm =1240 MeV·fm.

31. If E is the original energy of the photon and E' is the energy after scattering, then the fractional energy loss is

$$\frac{\Delta E}{E} = \frac{E-E'}{E} = \frac{\Delta\lambda}{\lambda+\Delta\lambda}$$

using the result from Sample Problem – "Compton scattering of light by electrons." Thus

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta E/E}{1-\Delta E/E} = \frac{0.75}{1-0.75} = 3 = 300\ \%.$$

A 300% increase in the wavelength leads to a 75% decrease in the energy of the photon.

32. (a) Equation 38-11 yields

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = (2.43\ \text{pm})(1-\cos 180°) = +4.86\ \text{pm}.$$

(b) Using the value hc = 1240 eV·nm, the change in photon energy is

$$\Delta E = \frac{hc}{\lambda'} - \frac{hc}{\lambda} = (1240\ \text{eV}\cdot\text{nm})\left(\frac{1}{0.01\ \text{nm}+4.86\ \text{pm}} - \frac{1}{0.01\ \text{nm}}\right) = -40.6\ \text{keV}.$$

(c) From conservation of energy, $\Delta K = -\Delta E$ = 40.6 keV.

(d) The electron will move straight ahead after the collision, since it has acquired some of the forward linear momentum from the photon. Thus, the angle between $+x$ and the direction of the electron's motion is zero.

33. (a) The fractional change is

$$\frac{\Delta E}{E} = \frac{\Delta(hc/\lambda)}{hc/\lambda} = \lambda\Delta\left(\frac{1}{\lambda}\right) = \lambda\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) = \frac{\lambda}{\lambda'} - 1 = \frac{\lambda}{\lambda+\Delta\lambda} - 1$$
$$= -\frac{1}{\lambda/\Delta\lambda+1} = -\frac{1}{(\lambda/\lambda_C)(1-\cos\phi)^{-1}+1}.$$

If λ = 3.0 cm = 3.0×10^{10} pm and ϕ = 90°, the result is

$$\frac{\Delta E}{E}=-\frac{1}{(3.0\times10^{10}\text{ pm}/2.43\text{pm})(1-\cos90°)^{-1}+1}=-8.1\times10^{-11}=-8.1\times10^{-9}\ \%.$$

(b) Now λ = 500 nm = 5.00 × 10^5 pm and ϕ = 90°, so

$$\frac{\Delta E}{E}=-\frac{1}{(5.00\times10^{5}\text{ pm}/2.43\text{pm})(1-\cos90°)^{-1}+1}=-4.9\times10^{-6}=-4.9\times10^{-4}\ \%.$$

(c) With λ = 25 pm and ϕ = 90°, we find

$$\frac{\Delta E}{E}=-\frac{1}{(25\text{pm}/2.43\text{pm})(1-\cos90°)^{-1}+1}=-8.9\times10^{-2}=-8.9\ \%.$$

(d) In this case,

$$\lambda = hc/E = 1240 \text{ nm}\cdot\text{eV}/1.0 \text{ MeV} = 1.24 \times 10^{-3} \text{ nm} = 1.24 \text{ pm},$$

so

$$\frac{\Delta E}{E}=-\frac{1}{(1.24\text{pm}/2.43\text{pm})(1-\cos90°)^{-1}+1}=-0.66=-66\ \%.$$

(e) From the calculation above, we see that the shorter the wavelength the greater the fractional energy change for the photon as a result of the Compton scattering. Since $\Delta E/E$ is virtually zero for microwave and visible light, the Compton effect is significant only in the x-ray to gamma ray range of the electromagnetic spectrum.

34. The initial energy of the photon is (using hc = 1240 eV·nm)

$$E=\frac{hc}{\lambda}=\frac{1240\text{eV}\cdot\text{nm}}{0.00300\text{ nm}}=4.13\times10^{5}\text{ eV}.$$

Using Eq. 38-11 (applied to an electron), the Compton shift is given by

$$\Delta\lambda=\frac{h}{m_ec}(1-\cos\phi)=\frac{h}{m_ec}(1-\cos90.0°)=\frac{hc}{m_ec^2}=\frac{1240\text{eV}\cdot\text{nm}}{511\times10^{3}\text{ eV}}=2.43\text{ pm}$$

Therefore, the new photon wavelength is

$$\lambda' = 3.00 \text{ pm} + 2.43 \text{ pm} = 5.43 \text{ pm}.$$

Consequently, the new photon energy is

$$E'=\frac{hc}{\lambda'}=\frac{1240\text{eV}\cdot\text{nm}}{0.00543\text{nm}}=2.28\times10^{5}\text{ eV}$$

By energy conservation, then, the kinetic energy of the electron must be equal to

$$K_e = \Delta E = E - E' = 4.13\times10^5 - 2.28\times10^5 \text{ eV} = 1.85\times10^5 \text{ eV} \approx 3.0\times10^{-14} \text{ J}.$$

35. (a) Since the mass of an electron is $m = 9.109 \times 10^{-31}$ kg, its Compton wavelength is

$$\lambda_C = \frac{h}{mc} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{(9.109\times10^{-31} \text{ kg})(2.998\times10^8 \text{ m/s})} = 2.426\times10^{-12} \text{ m} = 2.43 \text{ pm}.$$

(b) Since the mass of a proton is $m = 1.673 \times 10^{-27}$ kg, its Compton wavelength is

$$\lambda_C = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{(1.673\times10^{-27} \text{ kg})(2.998\times10^8 \text{ m/s})} = 1.321\times10^{-15} \text{ m} = 1.32 \text{ fm}.$$

(c) We note that hc = 1240 eV·nm, which gives E = (1240 eV·nm)/λ, where E is the energy and λ is the wavelength. Thus for the electron,

$$E = (1240 \text{ eV}\cdot\text{nm})/(2.426 \times 10^{-3} \text{ nm}) = 5.11 \times 10^5 \text{ eV} = 0.511 \text{ MeV}.$$

(d) For the proton,

$$E = (1240 \text{ eV}\cdot\text{nm})/(1.321 \times 10^{-6} \text{ nm}) = 9.39 \times 10^8 \text{ eV} = 939 \text{ MeV}.$$

36. (a) Using the value hc = 1240 eV·nm, we find

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ nm}\cdot\text{eV}}{0.511 \text{ MeV}} = 2.43\times10^{-3} \text{ nm} = 2.43 \text{ pm}.$$

(b) Now, Eq. 38-11 leads to

$$\lambda' = \lambda + \Delta\lambda = \lambda + \frac{h}{m_e c}(1-\cos\phi) = 2.43\text{pm} + (2.43\text{pm})(1-\cos 90.0°)$$
$$= 4.86\text{pm}.$$

(c) The scattered photons have energy equal to

$$E' = E\left(\frac{\lambda}{\lambda'}\right) = (0.511 \text{ MeV})\left(\frac{2.43 \text{ pm}}{4.86 \text{ pm}}\right) = 0.255 \text{ MeV}.$$

37. (a) From Eq. 38-11,

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta).$$

In this case ϕ = 180° (so cos ϕ = –1), and the change in wavelength for the photon is given by $\Delta\lambda = 2h/m_e c$. The energy E' of the scattered photon (with initial energy $E = hc/\lambda$) is then

$$E' = \frac{hc}{\lambda+\Delta\lambda} = \frac{E}{1+\Delta\lambda/\lambda} = \frac{E}{1+(2h/m_e c)(E/hc)} = \frac{E}{1+2E/m_e c^2}$$
$$= \frac{50.0\,\text{keV}}{1+2(50.0\,\text{keV})/0.511\,\text{MeV}} = 41.8\,\text{keV}\ .$$

(b) From conservation of energy the kinetic energy K of the electron is given by

$$K = E - E' = 50.0 \text{ keV} - 41.8 \text{ keV} = 8.2 \text{ keV}.$$

38. Referring to Sample Problem — "Compton scattering of light by electrons," we see that the fractional change in photon energy is

$$\frac{E-E_n}{E} = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \frac{(h/mc)(1-\cos\phi)}{(hc/E)+(h/mc)(1-\cos\phi)}\ .$$

Energy conservation demands that $E - E' = K$, the kinetic energy of the electron. In the maximal case, ϕ = 180°, and we find

$$\frac{K}{E} = \frac{(h/mc)(1-\cos 180°)}{(hc/E)+(h/mc)(1-\cos 180°)} = \frac{2h/mc}{(hc/E)+(2h/mc)}\ .$$

Multiplying both sides by E and simplifying the fraction on the right-hand side leads to

$$K = E\left(\frac{2/mc}{c/E+2/mc}\right) = \frac{E^2}{mc^2/2+E}\ .$$

39. The magnitude of the fractional energy change for the photon is given by

$$\left|\frac{\Delta E_{\text{ph}}}{E_{\text{ph}}}\right| = \left|\frac{\Delta(hc/\lambda)}{hc/\lambda}\right| = \left|\lambda\Delta\left(\frac{1}{\lambda}\right)\right| = \lambda\left(\frac{1}{\lambda}-\frac{1}{\lambda+\Delta\lambda}\right) = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \beta$$

where β = 0.10. Thus $\Delta\lambda = \lambda\beta/(1-\beta)$. We substitute this expression for $\Delta\lambda$ in Eq. 38-11 and solve for cos ϕ:

$$\cos\phi = 1-\frac{mc}{h}\Delta\lambda = 1-\frac{mc\lambda\beta}{h(1-\beta)} = 1-\frac{\beta(mc^2)}{(1-\beta)E_{\text{ph}}}$$
$$= 1-\frac{(0.10)(511\text{ keV})}{(1-0.10)(200\,\text{keV})} = 0.716\ .$$

This leads to an angle of $\phi = 44°$.

40. The initial wavelength of the photon is (using hc = 1240 eV·nm)

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{17500\,\text{eV}} = 0.07086 \text{ nm}$$

or 70.86 pm. The maximum Compton shift occurs for $\phi = 180°$, in which case Eq. 38-11 (applied to an electron) yields

$$\Delta\lambda = \left(\frac{hc}{m_e c^2}\right)(1-\cos 180°) = \left(\frac{1240\,\text{eV}\cdot\text{nm}}{511\times 10^3\text{ eV}}\right)(1-(-1)) = 0.00485 \text{ nm}$$

where Table 37-3 is used. Therefore, the new photon wavelength is

$$\lambda' = 0.07086 \text{ nm} + 0.00485 \text{ nm} = 0.0757 \text{ nm}.$$

Consequently, the new photon energy is

$$E' = \frac{hc}{\lambda'} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.0757\,\text{nm}} = 1.64\times 10^4 \text{ eV} = 16.4 \text{ keV} .$$

By energy conservation, then, the kinetic energy of the electron must equal

$$E' - E = 17.5 \text{ keV} - 16.4 \text{ keV} = 1.1 \text{ keV}.$$

41. (a) From Eq. 38-11

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = (2.43\,\text{pm})(1-\cos 90°) = 2.43\,\text{pm} .$$

(b) The fractional shift should be interpreted as $\Delta\lambda$ divided by the original wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{2.425\,\text{pm}}{590\,\text{nm}} = 4.11\times 10^{-6}.$$

(c) The change in energy for a photon with λ = 590 nm is given by

$$\Delta E_{\text{ph}} = \Delta\left(\frac{hc}{\lambda}\right) \approx -\frac{hc\Delta\lambda}{\lambda^2} = -\frac{(4.14\times 10^{-15}\text{ eV}\cdot\text{s})(2.998\times 10^8\text{ m/s})(2.43\,\text{pm})}{(590\,\text{nm})^2}$$
$$= -8.67\times 10^{-6}\text{ eV} .$$

(d) For an x-ray photon of energy E_{ph} = 50 keV, $\Delta\lambda$ remains the same (2.43 pm), since it is independent of E_{ph}.

(e) The fractional change in wavelength is now

$$\frac{\Delta\lambda}{\lambda}=\frac{\Delta\lambda}{hc/E_{ph}}=\frac{(50\times10^{3}\,\text{eV})(2.43\text{pm})}{(4.14\times10^{-15}\,\text{eV}\cdot\text{s})(2.998\times10^{8}\text{m/s})}=9.78\times10^{-2}\,.$$

(f) The change in photon energy is now

$$\Delta E_{ph}=hc\left(\frac{1}{\lambda+\Delta\lambda}-\frac{1}{\lambda}\right)=-\left(\frac{hc}{\lambda}\right)\frac{\Delta\lambda}{\lambda+\Delta\lambda}=-E_{ph}\left(\frac{\alpha}{1+\alpha}\right)$$

where $\alpha = \Delta\lambda/\lambda$. With E_{ph} = 50 keV and $\alpha = 9.78 \times 10^{-2}$, we obtain ΔE_{ph} = –4.45 keV. (Note that in this case $\alpha \approx 0.1$ is not close enough to zero so the approximation $\Delta E_{ph} \approx hc\Delta\lambda/\lambda^2$ is not as accurate as in the first case, in which $\alpha = 4.12 \times 10^{-6}$. In fact if one were to use this approximation here, one would get $\Delta E_{ph} \approx$ –4.89 keV, which does not amount to a satisfactory approximation.)

42. (a) Using Wien's law, $\lambda_{max}T = 2898\ \mu\text{m}\cdot\text{K}$, we obtain

$$\lambda_{max}=\frac{2898\ \mu\text{m}\cdot\text{K}}{T}=\frac{2898\ \mu\text{m}\cdot\text{K}}{5800\ \text{K}}=0.50\ \mu\text{m}=500\ \text{nm}\,.$$

(b) The electromagnetic wave is in the visible spectrum.

(c) If $\lambda_{max}=1.06\ \text{mm}=1060\ \mu\text{m}$, then $T=\dfrac{2898\ \mu\text{m}\cdot\text{K}}{\lambda_{max}}=\dfrac{2898\ \mu\text{m}\cdot\text{K}}{1060\ \mu\text{m}}=2.73\ \text{K}$.

43. (a) Using Wien's law, the wavelength that corresponds to thermal radiation maximum is

$$\lambda_{max}=\frac{2898\ \mu\text{m}\cdot\text{K}}{T}=\frac{2898\ \mu\text{m}\cdot\text{K}}{1.0\times10^{7}\ \text{K}}=2.9\times10^{-4}\ \mu\text{m}=2.9\times10^{-10}\,\text{m}\,.$$

(b) The wave is in the x-ray region of the electromagnetic spectrum.

(c) Using Wien's law, the wavelength that corresponds to thermal radiation maximum is

$$\lambda_{max}=\frac{2898\ \mu\text{m}\cdot\text{K}}{T}=\frac{2898\ \mu\text{m}\cdot\text{K}}{1.0\times10^{5}\ \text{K}}=2.9\times10^{-2}\ \mu\text{m}=2.9\times10^{-8}\,\text{m}$$

(d) The wave is in the ultraviolet region of the electromagnetic spectrum.

44. (a) The intensity per unit length according to the classical radiation law shown in Eq. 38-13 is

$$I_C = \frac{2\pi ckT}{\lambda^4}$$

On the other hand, Planck's radiation law (Eq. 38-14) gives

$$I_P = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}.$$

The ratio of the two expressions can be written as

$$\frac{I_C}{I_P} = \frac{\lambda kT}{hc}\left(e^{hc/\lambda kT}-1\right) = \frac{1}{x}\left(e^x - 1\right)$$

where $x = hc/\lambda kT$. For T = 200 K, and $\lambda = 400$ nm,

$$x = \frac{hc}{\lambda kT} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{(400\times10^{-9}\,\text{m})(1.38\times10^{-23}\ \text{J/K})(2000\ \text{K})} \approx 17.98,$$

and the ratio of the intensities is $\frac{I_C}{I_P} \approx \frac{1}{17.98}\left(e^{17.98}-1\right) \approx 3.6\times10^6$.

(b) For $\lambda = 200\ \mu\text{m}$, we have

$$x = \frac{hc}{\lambda kT} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{(200\times10^{-6}\,\text{m})(1.38\times10^{-23}\ \text{J/K})(2000\ \text{K})} \approx 0.03596,$$

and the ratio of the intensities is

$$\frac{I_C}{I_P} \approx \frac{1}{0.03596}\left(e^{0.03596}-1\right) \approx 1.02.$$

(c) The agreement is better at longer wavelength, with $I_C/I_P \approx 1$.

45. (a) With $T = 98.6\ °\text{F} = 37\ °\text{C} = 310\ \text{K}$, we use Wien's law and find the wavelength that corresponds to spectral radiancy maximum to be

$$\lambda_{\max} = \frac{2898\ \mu\text{m}\cdot\text{K}}{T} = \frac{2898\ \mu\text{m}\cdot\text{K}}{310\ \text{K}} = 9.35\mu\text{m}.$$

(b) With $\lambda = 9.35\ \mu\text{m}$, and T = 310 K, the spectral radiancy is

$$S(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$
$$= \frac{2\pi(2.998\times10^8\ \text{m/s})^2(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(9.35\times10^{-6}\text{m})^5}\left(\exp\left[\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{(9.35\times10^{-6}\text{m})(1.38\times10^{-23}\ \text{J/K})(310\ \text{K})}\right]\right)^{-1}$$
$$= 3.688\times10^7\ \text{W/m}^3$$

For small range of wavelength, the radiated power may be approximated as

$$P = S(\lambda)A\Delta\lambda = (3.688\times10^7\ \text{W/m}^3)(4\times10^{-4}\text{m}^2)(10^{-9}\text{m}) = 1.475\times10^{-5}\ \text{W}.$$

(c) The energy carried by each photon is

$$\varepsilon = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{9.35\times10^{-6}\text{m}} = 2.1246\times10^{-20}\ \text{J}$$

Writing $P = (dN/dt)\varepsilon,$ we find the rate to be

$$\frac{dN}{dt} = \frac{P}{\varepsilon} = \frac{1.475\times10^{-5}\ \text{W}}{2.1246\times10^{-20}\ \text{J}} = 6.94\times10^{14}\ \text{photons/s}\ .$$

(d) If $\lambda = 500$ nm, and T = 310 K, the spectral radiancy is

$$S(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$
$$= \frac{2\pi(2.998\times10^8\ \text{m/s})^2(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(500\times10^{-9}\text{m})^5}\left(\exp\left[\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{(500\times10^{-9}\text{m})(1.38\times10^{-23}\ \text{J/K})(310\ \text{K})}\right]\right)^{-1}$$
$$= 5.95\times10^{-25}\ \text{W/m}^3$$

For small range of wavelength, the radiated power may be approximated as

$$P = S(\lambda)A\Delta\lambda = (5.95\times10^{-25}\ \text{W/m}^3)(4\times10^{-4}\text{m}^2)(10^{-9}\text{m}) = 2.38\times10^{-37}\ \text{W}.$$

(e) The energy carried by each photon is

$$\varepsilon = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{500\times10^{-9}\text{m}} = 3.97\times10^{-19}\ \text{J}$$

The corresponding photon emission rate is

$$\frac{dN}{dt} = \frac{P}{\varepsilon} = \frac{2.38\times10^{-5}\ \text{W}}{3.97\times10^{-19}\ \text{J}} = 5.9\times10^{-19}\ \text{photons/s}$$

46. (a) Using Table 37-3 and the value hc = 1240 eV·nm, we obtain

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{hc}{\sqrt{2m_ec^2K}} = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2(511000\text{eV})(1000\text{eV})}} = 0.0388\text{nm}.$$

(b) A photon's de Broglie wavelength is equal to its familiar wave-relationship value. Using the value hc = 1240 eV·nm,

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{keV}} = 1.24\,\text{nm}\ .$$

(c) The neutron mass may be found in Appendix B. Using the conversion from electron-volts to Joules, we obtain

$$\lambda = \frac{h}{\sqrt{2m_nK}} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(1.675\times10^{-27}\,\text{kg})(1.6\times10^{-16}\,\text{J})}} = 9.06\times10^{-13}\,\text{m}.$$

47. **THINK** The de Broglie wavelength of the electron is given by $\lambda = h/p$, where p is the momentum of the electron.

EXPRESS The momentum of the electron can be written as

$$p = m_ev = \sqrt{2m_eK} = \sqrt{2m_eeV},$$

where V is the accelerating potential and e is the fundamental charge. Thus,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eeV}}.$$

ANALYZE With V = 25.0 kV, we obtain

$$\lambda = \frac{h}{\sqrt{2m_eeV}} = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\ \text{C})(25.0\times10^{3}\ \text{V})}}$$
$$= 7.75\times10^{-12}\ \text{m} = 7.75\,\text{pm}.$$

LEARN The wavelength is of the same order as the Compton wavelength of the electron. Increasing the potential difference V would make the wavelength even smaller.

48. The same resolution requires the same wavelength, and since the wavelength and particle momentum are related by $p = h/\lambda$, we see that the same particle momentum is required. The momentum of a 100 keV photon is

$$p = E/c = (100 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})/(3.00 \times 10^8 \text{ m/s}) = 5.33 \times 10^{-23} \text{ kg·m/s}.$$

This is also the magnitude of the momentum of the electron. The kinetic energy of the electron is

$$K = \frac{p^2}{2m} = \frac{(5.33 \times 10^{-23} \text{ kg} \cdot \text{m/s})^2}{2(9.11 \times 10^{-31} \text{ kg})} = 1.56 \times 10^{-15} \text{ J}.$$

The accelerating potential is

$$V = \frac{K}{e} = \frac{1.56 \times 10^{-15} \text{ J}}{1.60 \times 10^{-19} \text{ C}} = 9.76 \times 10^3 \text{ V}.$$

49. **THINK** The de Broglie wavelength of the sodium ion is given by $\lambda = h/p$, where p is the momentum of the ion.

EXPRESS The kinetic energy acquired is $K = qV$, where q is the charge on an ion and V is the accelerating potential. Thus, the momentum of an ion is $p = \sqrt{2mK}$, and the corresponding de Broglie wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}}$.

ANALYZE (a) The kinetic energy of the ion is

$$K = qV = (1.60 \times 10^{-19} \text{ C})(300 \text{ V}) = 4.80 \times 10^{-17} \text{ J}.$$

The mass of a single sodium atom is, from Appendix F,

$$m = (22.9898 \text{ g/mol})/(6.02 \times 10^{23} \text{ atom/mol}) = 3.819 \times 10^{-23} \text{ g} = 3.819 \times 10^{-26} \text{ kg}.$$

Thus, the momentum of a sodium ion is

$$p = \sqrt{2mK} = \sqrt{2(3.819 \times 10^{-26} \text{ kg})(4.80 \times 10^{-17} \text{ J})} = 1.91 \times 10^{-21} \text{ kg} \cdot \text{m/s}.$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34} \text{ J}\cdot\text{s}}{1.91\times10^{-21} \text{kg}\cdot\text{m/s}} = 3.46\times10^{-13} \text{ m}.$$

LEARN The greater the potential difference, the greater the kinetic energy and momentum, and hence, the smaller the de Broglie wavelength.

50. (a) We need to use the relativistic formula

$$p = \sqrt{(E/c)^2 - m_e^2 c^2} \approx E/c \approx K/c$$

$\left(\text{since } E \gg m_e c^2\right)$. So

$$\lambda = \frac{h}{p} \approx \frac{hc}{K} = \frac{1240\,\text{eV}\cdot\text{nm}}{50\times10^9\,\text{eV}} = 2.5\times10^{-8}\,\text{nm} = 0.025\ \text{fm}.$$

(b) With $R = 5.0$ fm, we obtain $R/\lambda = 2.0\times10^2$.

51. **THINK** The de Broglie wavelength of a particle is given by $\lambda = h/p$, where p is the momentum of the particle.

EXPRESS Let K be the kinetic energy of the electron, in units of electron volts (eV). Since $K = p^2/2m$, the electron momentum is $p = \sqrt{2mK}$. Thus, the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\,\text{J/eV})K}} = \frac{1.226\times10^{-9}\,\text{m}\cdot\text{eV}^{1/2}}{\sqrt{K}}$$
$$= \frac{1.226\ \text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}}.$$

ANALYZE With $\lambda = 590$ nm, the above equation can be inverted to give

$$K = \left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\lambda}\right)^2 = \left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{590\,\text{nm}}\right)^2 = 4.32\times10^{-6}\,\text{eV}.$$

LEARN The analytical expression shows that the kinetic energy is proportional to $1/\lambda^2$. This is so because $K \sim p^2$, while $p \sim 1/\lambda$.

52. Using Eq. 37-8, we find the Lorentz factor to be

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.9900)^2}} = 7.0888\,.$$

With $p = \gamma m v$ (Eq. 37-41), the de Broglie wavelength of the protons is

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(7.0888)(1.67\times10^{-27}\ \text{kg})(0.99\times3.00\times10^{8}\ \text{m/s})} = 1.89\times10^{-16}\ \text{m}.$$

The vertical distance between the second interference minimum and the center point is

$$y_2 = \left(1+\frac{1}{2}\right)\frac{\lambda L}{d} = \frac{3}{2}\frac{\lambda L}{d}$$

where L is the perpendicular distance between the slits and the screen. Therefore, the angle between the center of the pattern and the second minimum is given by

$$\tan\theta = \frac{y_2}{L} = \frac{3\lambda}{2d}.$$

Since $\lambda \ll d$, $\tan\theta \approx \theta$, and we obtain

$$\theta \approx \frac{3\lambda}{2d} = \frac{3(1.89\times10^{-16}\ \text{m})}{2(4.00\times10^{-9}\ \text{m})} = 7.07\times10^{-8}\,\text{rad} = (4.0\times10^{-6})^\circ.$$

53. (a) The momentum of the photon is given by $p = E/c$, where E is its energy. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{eV}} = 1240\ \text{nm}.$$

(b) The momentum of the electron is given by $p = \sqrt{2mK}$, where K is its kinetic energy and m is its mass. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}.$$

If K is given in electron volts, then

$$\lambda = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\,\text{J/eV})K}} = \frac{1.226\times10^{-9}\,\text{m}\cdot\text{eV}^{1/2}}{\sqrt{K}} = \frac{1.226\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}}.$$

For $K = 1.00\ \text{eV}$, we have

$$\lambda = \frac{1.226\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{1.00\,\text{eV}}} = 1.23\ \text{nm}.$$

(c) For the photon,

$$\lambda = \frac{hc}{E} = \frac{1240\text{eV}\cdot\text{nm}}{1.00\times10^{9}\ \text{eV}} = 1.24\times10^{-6}\,\text{nm} = 1.24\ \text{fm}.$$

(d) Relativity theory must be used to calculate the wavelength for the electron. According to Eq. 38-51, the momentum p and kinetic energy K are related by

$$(pc)^2 = K^2 + 2Kmc^2.$$

Thus,

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{\left(1.00\times10^9\ \text{eV}\right)^2 + 2\left(1.00\times10^9\ \text{eV}\right)\left(0.511\times10^6\ \text{eV}\right)}$$
$$= 1.00\times10^9\ \text{eV}.$$

The wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240\ \text{eV}\cdot\text{nm}}{1.00\times10^9\ \text{eV}} = 1.24\times10^{-6}\ \text{nm} = 1.24\ \text{fm}.$$

54. (a) The momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.20\times10^{-9}\ \text{m}} = 3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}.$$

(b) The momentum of the photon is the same as that of the electron: $p = 3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}$.

(c) The kinetic energy of the electron is

$$K_e = \frac{p^2}{2m_e} = \frac{\left(3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)} = 6.0\times10^{-18}\ \text{J} = 38\ \text{eV}.$$

(d) The kinetic energy of the photon is

$$K_{\text{ph}} = pc = \left(3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}\right)\left(2.998\times10^8\ \text{m/s}\right) = 9.9\times10^{-16}\ \text{J} = 6.2\ \text{keV}.$$

55. (a) Setting $\lambda = h/p = h/\sqrt{(E/c)^2 - m_e^2c^2}$, we solve for $K = E - m_ec^2$:

$$K = \sqrt{\left(\frac{hc}{\lambda}\right)^2 + m_e^2c^4} - m_ec^2 = \sqrt{\left(\frac{1240\ \text{eV}\cdot\text{nm}}{10\times10^{-3}\ \text{nm}}\right)^2 + \left(0.511\ \text{MeV}\right)^2} - 0.511\ \text{MeV}$$
$$= 0.015\ \text{MeV} = 15\ \text{keV}.$$

(b) Using the value $hc = 1240\ \text{eV}\cdot\text{nm}$

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{10\times10^{-3}\ \text{nm}} = 1.2\times10^5\ \text{eV} = 120\ \text{keV}.$$

(c) The electron microscope is more suitable, as the required energy of the electrons is much less than that of the photons.

56. (a) Since $K = 7.5\,\text{MeV} \ll m_\alpha c^2 = 4(932\,\text{MeV})$, we may use the nonrelativistic formula $p = \sqrt{2m_\alpha K}$. Using Eq. 38-43 (and noting that 1240 eV·nm = 1240 MeV·fm), we obtain

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2m_\alpha c^2 K}} = \frac{1240\,\text{MeV}\cdot\text{fm}}{\sqrt{2(4\text{u})(931.5\,\text{MeV/u})(7.5\,\text{MeV})}} = 5.2\,\text{fm}.$$

(b) Since $\lambda = 5.2\,\text{fm} \ll 30\,\text{fm}$, to a fairly good approximation, the wave nature of the α particle does not need to be taken into consideration.

57. The wavelength associated with the unknown particle is

$$\lambda_p = \frac{h}{p_p} = \frac{h}{m_p v_p},$$

where p_p is its momentum, m_p is its mass, and v_p is its speed. The classical relationship $p_p = m_p v_p$ was used. Similarly, the wavelength associated with the electron is $\lambda_e = h/(m_e v_e)$, where m_e is its mass and v_e is its speed. The ratio of the wavelengths is

$$\lambda_p/\lambda_e = (m_e v_e)/(m_p v_p),$$

so

$$m_p = \frac{v_e \lambda_e}{v_p \lambda_p} m_e = \frac{9.109\times10^{-31}\,\text{kg}}{3(1.813\times10^{-4})} = 1.675\times10^{-27}\,\text{kg}.$$

According to Appendix B, this is the mass of a neutron.

58. (a) We use the value $hc = 1240\,\text{nm}\cdot\text{eV}$:

$$E_\text{photon} = \frac{hc}{\lambda} = \frac{1240\,\text{nm}\cdot\text{eV}}{1.00\,\text{nm}} = 1.24\,\text{keV}.$$

(b) For the electron, we have

$$K = \frac{p^2}{2m_e} = \frac{(h/\lambda)^2}{2m_e} = \frac{(hc/\lambda)^2}{2m_e c^2} = \frac{1}{2(0.511\,\text{MeV})}\left(\frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{nm}}\right)^2 = 1.50\ \text{eV}.$$

(c) In this case, we find

$$E_{\text{photon}} = \frac{1240\,\text{nm}\cdot\text{eV}}{1.00\times10^{-6}\,\text{nm}} = 1.24\times10^{9}\ \text{eV} = 1.24\ \text{GeV}.$$

(d) For the electron (recognizing that 1240 eV·nm = 1240 MeV·fm)

$$\begin{aligned}K &= \sqrt{p^2c^2 + (m_ec^2)^2} - m_ec^2 = \sqrt{(hc/\lambda)^2 + (m_ec^2)^2} - m_ec^2 \\ &= \sqrt{\left(\frac{1240\,\text{MeV}\cdot\text{fm}}{1.00\,\text{fm}}\right)^2 + (0.511\,\text{MeV})^2} - 0.511\,\text{MeV} \\ &= 1.24\times10^{3}\ \text{MeV} = 1.24\,\text{GeV}.\end{aligned}$$

We note that at short λ (large K) the kinetic energy of the electron, calculated with the relativistic formula, is about the same as that of the photon. This is expected since now $K \approx E \approx pc$ for the electron, which is the same as $E = pc$ for the photon.

59. (a) We solve v from $\lambda = h/p = h/(m_pv)$:

$$v = \frac{h}{m_p\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.6705\times10^{-27}\ \text{kg}\right)\left(0.100\times10^{-12}\ \text{m}\right)} = 3.96\times10^{6}\ \text{m/s}.$$

(b) We set $eV = K = \frac{1}{2}m_pv^2$ and solve for the voltage:

$$V = \frac{m_pv^2}{2e} = \frac{\left(1.6705\times10^{-27}\,\text{kg}\right)\left(3.96\times10^{6}\,\text{m/s}\right)^2}{2\left(1.60\times10^{-19}\,\text{C}\right)} = 8.18\times10^{4}\,\text{V} = 81.8\ \text{kV}.$$

60. The wave function is now given by

$$\Psi(x,t) = \psi_0 e^{-i(kx+\omega t)}.$$

This function describes a plane matter wave traveling in the negative x direction. An example of the actual particles that fit this description is a free electron with linear momentum $\vec{p} = -(hk/2\pi)\hat{\mathrm{i}}$ and kinetic energy

$$K = \frac{p^2}{2m_e} = \frac{h^2k^2}{8\pi^2m_e}\ .$$

61. **THINK** In this problem we solve a special case of the Schrödinger's equation where the potential energy is $U(x) = U_0 = \text{constant}$.

EXPRESS For $U = U_0$, Schrödinger's equation becomes

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}[E-U_0]\psi=0.$$

We substitute $\psi=\psi_0 e^{ikx}$.

ANALYZE The second derivative is $\dfrac{d^2\psi}{dx^2}=-k^2\psi_0 e^{ikx}=-k^2\psi$. The result is

$$-k^2\psi+\frac{8\pi^2 m}{h^2}[E-U_0]\psi=0.$$

Solving for *k*, we obtain

$$k=\sqrt{\frac{8\pi^2 m}{h^2}[E-U_0]}=\frac{2\pi}{h}\sqrt{2m[E-U_0]}.$$

LEARN Another way to realize this is to note that with a constant potential energy $U(x)=U_0$, we can simply redefine the total energy as $E'=E-U_0$, and the Schrödinger's equation looks just like the free-particle case:

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 mE'}{h^2}\psi=0.$$

The solution is $\psi=\psi_0\exp(ik'x)$, where

$$k'^2=\frac{8\pi^2 mE'}{h^2}\;\Rightarrow\; k=\frac{2\pi}{h}\sqrt{2mE'}=\frac{2\pi}{h}\sqrt{2m(E-U_0)}\,.$$

62. We plug Eq. 38-17 into Eq. 38-16, and note that

$$\frac{d\psi}{dx}=\frac{d}{dx}\left(Ae^{ikx}+Be^{-ikx}\right)=ikAe^{ikx}-ikBe^{-ikx}.$$

Also,

$$\frac{d^2\psi}{dx^2}=\frac{d}{dx}\left(ikAe^{ikx}-ikBe^{-ikx}\right)=-k^2\,Ae^{ikx}-k^2\,Be^{ikx}.$$

Thus,

$$\frac{d^2\psi}{dx^2}+k^2\psi=-k^2\,Ae^{ikx}-k^2Be^{ikx}+k^2\left(Ae^{ikx}+Be^{-ikx}\right)=0.$$

63. (a) Using Euler's formula $e^{i\phi}=\cos\phi+i\sin\phi$, we rewrite $\psi(x)$ as

$$\psi(x) = \psi_0 e^{ikx} = \psi_0(\cos kx + i\sin kx) = (\psi_0 \cos kx) + i(\psi_0 \sin kx) = a + ib,$$

where $a = \psi_0 \cos kx$ and $b = \psi_0 \sin kx$ are both real quantities.

(b) The time-dependent wave function is

$$\psi(x,t) = \psi(x)e^{-i\omega t} = \psi_0 e^{ikx} e^{-i\omega t} = \psi_0 e^{i(kx-\omega t)}$$
$$= [\psi_0 \cos(kx - \omega t)] + i[\psi_0 \sin(kx - \omega t)].$$

64. **THINK** The angular wave number k is related to the wavelength λ by $k = 2\pi/\lambda$.

EXPRESS The wavelength is related to the particle momentum p by $\lambda = h/p$, so $k = 2\pi p/h$. Now, the kinetic energy K and the momentum are related by $K = p^2/2m$, where m is the mass of the particle.

ANALYZE Thus, we have $p = \sqrt{2mK}$ and

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{2\pi\sqrt{2mK}}{h}.$$

LEARN The expression obtained above applies to the case of a free particle only. In the presence of interaction, the potential energy is nonzero, and the functional form of k will change. For example, as shown in Problem 38-57, when $U(x) = U_0$, the angular wave number becomes

$$k = \frac{2\pi}{h}\sqrt{2m(E - U_0)}\ .$$

65. (a) The product nn^* can be rewritten as

$$nn^* = (a+ib)(a+ib)^* = (a+ib)(a^* + i^* b^*) = (a+ib)(a-ib)$$
$$= a^2 + iba - iab + (ib)(-ib) = a^2 + b^2,$$

which is always real since both a and b are real.

(b) Straightforward manipulation gives

$$|nm| = |(a+ib)(c+id)| = |ac + iad + ibc + (-i)^2 bd| = |(ac - bd) + i(ad + bc)|$$
$$= \sqrt{(ac-bd)^2 + (ad+bc)^2} = \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}.$$

However, since

$$|n||m| = |a+ib||c+id| = \sqrt{a^2+b^2}\sqrt{c^2+d^2}$$
$$= \sqrt{a^2c^2+b^2d^2+a^2d^2+b^2c^2},$$

we conclude that $|nm| = |n|\,|m|$.

66. (a) The wave function is now given by

$$\Psi(x,t) = \psi_0 \; e^{i(kx-\omega t)} + e^{-i(kx+\omega t)} \; = \psi_0 e^{-i\omega t}(e^{ikx}+e^{-ikx}).$$

Thus,

$$|\Psi(x,t)|^2 = \left|\psi_0 e^{-i\omega t}(e^{ikx}+e^{-ikx})\right|^2 = \left|\psi_0 e^{-i\omega t}\right|^2 \left|e^{ikx}+e^{-ikx}\right|^2 = \psi_0^2\left|e^{ikx}+e^{-ikx}\right|^2$$
$$= \psi_0^2\,|(\cos kx + i\sin kx) + (\cos kx - i\sin kx)|^2 = 4\psi_0^2(\cos kx)^2$$
$$= 2\psi_0^2(1+\cos 2kx).$$

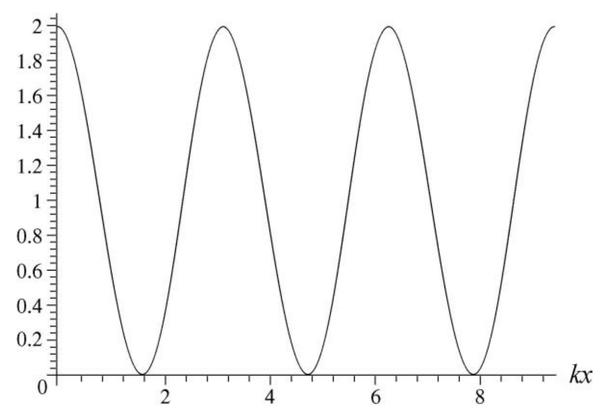
(b) Consider two plane matter waves, each with the same amplitude $\psi_0/\sqrt{2}$ and traveling in opposite directions along the x axis. The combined wave Ψ is a standing wave:

$$\Psi(x,t) = \psi_0 e^{i(kx-\omega t)} + \psi_0 e^{-i(kx+\omega t)} = \psi_0(e^{ikx}+e^{-ikx})e^{-i\omega t} = (2\psi_0\cos kx)e^{-i\omega t}.$$

Thus, the squared amplitude of the matter wave is

$$|\Psi(x,t)|^2 = (2\psi_0\cos kx)^2\left|e^{-i\omega t}\right|^2 = 2\psi_0^2(1+\cos 2kx),$$

which is shown to the right.



(c) We set $\left|\Psi(x,t)\right|^2 = 2\psi_0^2(1+\cos 2kx) = 0$ to obtain $\cos(2kx) = -1$. This gives

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = (2n+1)\pi, \quad (n = 0, 1, 2, 3, \ldots)$$

We solve for x:

$$x = \frac{1}{4}(2n+1)\lambda \;.$$

(d) The most probable positions for finding the particle are where $\left|\Psi(x,t)\right| \propto (1+\cos 2kx)$ reaches its maximum. Thus $\cos 2kx = 1$, or

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = 2n\pi, \quad (n = 0, 1, 2, 3, \ldots)$$

We solve for x and find $x = \frac{1}{2}n\lambda$.

67. If the momentum is measured at the same time as the position, then

$$\Delta p \approx \frac{\hbar}{\Delta x} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\pi(50\ \text{pm})} = 2.1 \times 10^{-24}\ \text{kg}\cdot\text{m/s}\ .$$

68. (a) Using the value $hc = 1240\text{nm}\cdot\text{eV}$, we have

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{nm}\cdot\text{eV}}{10.0\times10^{-3}\ \text{nm}} = 124\,\text{keV}\ .$$

(b) The kinetic energy gained by the electron is equal to the energy decrease of the photon:

$$\begin{aligned}\Delta E &= \Delta\left(\frac{hc}{\lambda}\right) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda + \Delta\lambda}\right) = \left(\frac{hc}{\lambda}\right)\left(\frac{\Delta\lambda}{\lambda + \Delta\lambda}\right) = \frac{E}{1 + \lambda/\Delta\lambda} \\ &= \frac{E}{1 + \frac{\lambda}{\lambda_C(1-\cos\phi)}} = \frac{124\text{keV}}{1 + \frac{10.0\,\text{pm}}{(2.43\,\text{pm})(1-\cos 180°)}} \\ &= 40.5\text{keV}.\end{aligned}$$

(c) It is impossible to "view" an atomic electron with such a high-energy photon, because with the energy imparted to the electron the photon would have knocked the electron out of its orbit.

69. We use the uncertainty relationship $\Delta x \Delta p \geq \hbar$. Letting $\Delta x = \lambda$, the de Broglie wavelength, we solve for the minimum uncertainty in p:

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{h}{2\pi\lambda} = \frac{p}{2\pi}$$

where the de Broglie relationship $p = h/\lambda$ is used. We use $1/2\pi = 0.080$ to obtain $\Delta p = 0.080p$. We would expect the measured value of the momentum to lie between $0.92p$ and $1.08p$. Measured values of zero, $0.5p$, and $2p$ would all be surprising.

70. (a) The potential energy of the electron is $U_b = qV = (-e)(-200\ \text{V}) = 200\,\text{eV}$, so its kinetic energy is

$$K = E - U_b = 500\ \text{eV} - 200\ \text{eV} = 300\ \text{eV}.$$

(b) Using non-relativistic regime approximation, $K = \frac{1}{2}mv^2 = p^2/2m$, we find the momentum of the electron to be

$$p = \sqrt{2mK} = \sqrt{2(9.11\times10^{-31}\text{ kg})(300\text{ eV})(1.6\times10^{-19}\text{ J/eV})} = 9.35\times10^{-24}\text{ kg}\cdot\text{m/s}$$

(c) The speed of the electron is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(300\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 1.03\times10^{7}\text{ m/s}.$$

(d) The corresponding de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{9.35\times10^{-24}\text{ kg}\cdot\text{m/s}} = 7.08\times10^{-11}\text{ m}.$$

(e) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.08\times10^{-11}\text{ m}} = 8.87\times10^{10}\text{ m}^{-1}.$$

71. (a) The angular wave number in region 1 is

$$k = \frac{2\pi}{h}\sqrt{2mE} = \frac{2\pi}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\sqrt{2(9.11\times10^{-31}\text{ kg})(800\text{ eV})(1.6\times10^{-19}\text{ J/eV})}$$
$$= 1.45\times10^{11}\text{ m}^{-1}$$

(b) The angular wave number in region 2 is

$$k_b = \frac{2\pi}{h}\sqrt{2m(E-U_b)} = \frac{2\pi}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\sqrt{2(9.11\times10^{-31}\text{ kg})(800\text{ eV}-200\text{ eV})(1.6\times10^{-19}\text{ J/eV})}$$
$$= \frac{k}{2} = 7.24\times10^{10}\text{ m}^{-1}$$

(c) The wave functions in the two regions can be written as

$$\psi_1(x) = Ae^{ikx} + Be^{-ikx}, \quad \psi_2(x) = Ce^{ik_b x}$$

Matching the boundary conditions leads to

$$A + B = C$$
$$Ak - Bk = Ck_b$$

Since $k_b = k/2$, the above equations can be solved to give $(B/A) = 1/3$ and $(C/A) = 4/3$. The reflection coefficient is

$$R = \frac{|B|^2}{|A|^2} = \frac{1}{9} = 0.111.$$

(d) With $N_0 = 5.00\times10^5$ electrons in the incident beam, the number reflected is

$$N_R = RN_0 = \left(\frac{1}{9}\right)(5.00\times10^5) = 5.56\times10^4.$$

72. (a) The angular wave number in region 1 is given by

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(h/p)} = \frac{2\pi p}{h} = \frac{2\pi mv}{h} = \frac{2\pi(9.11\times10^{-31}\text{ kg})(1.60\times10^7\text{ m/s})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.38\times10^{11}\text{ m}^{-1}$$

(b) The energy of the electron in region 1 is

$$E = K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(1.60\times10^7\text{ m/s})^2 = 1.17\times10^{-16}\text{ J} = 728.8\text{ eV}.$$

In region 2 where $V = -500$ V, the kinetic energy of the electron is

$$K_b = E - U_b = 728.8\text{ eV} - 500\text{ eV} = 228.8\text{ eV}.$$

and the corresponding angular wave number is

$$k_b = \frac{2\pi}{h}\sqrt{2m(E-U_b)} = \frac{2\pi}{h}\sqrt{2mK_b} = \frac{2\pi}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\sqrt{2(9.11\times10^{-31}\text{ kg})(228.8\text{ eV})(1.6\times10^{-19}\text{ J/eV})}$$
$$= 7.74\times10^{10}\text{ m}^{-1}$$

(c) The wave functions in the two regions can be written as

$$\psi_1(x) = Ae^{ikx} + Be^{-ikx}, \quad \psi_2(x) = Ce^{ik_bx}$$

Matching the boundary conditions leads to

$$A + B = C$$
$$Ak - Bk = Ck_b$$

Solving for B and C in terms of A gives

$$\frac{B}{A}=\frac{1-k_b/k}{1+k_b/k}, \qquad \frac{C}{A}=\frac{2}{1+k_b/k}.$$

With $k_b/k=(7.74\times10^{10}\ \text{m}^{-1})/(1.38\times10^{11}\ \text{m}^{-1})=0.56$, we find the reflection coefficient to be

$$R=\frac{|B|^2}{|A|^2}=\left(\frac{1-k_b/k}{1+k_b/k}\right)^2=\left(\frac{1-0.56}{1+0.56}\right)^2=0.0794$$

(d) With $N_0=3.00\times10^9$ electrons in the incident beam, the number reflected is

$$N_R=RN_0=\left(0.0794\right)(3.00\times10^9)=2.38\times10^8.$$

73. The energy of the electron in region 1 is

$$E=K=\frac{1}{2}mv^2=\frac{1}{2}(9.11\times10^{-31}\ \text{kg})(900\text{m/s})^2=3.69\times10^{-25}\ \text{J}=2.306\ \mu\text{eV}.$$

The angular wave number in region 1 is

$$k=\frac{2\pi}{\lambda}=\frac{2\pi}{(h/p)}=\frac{2\pi p}{h}=\frac{2\pi mv}{h}=\frac{2\pi(9.11\times10^{-31}\ \text{kg})(900\text{m/s})}{6.626\times10^{-34}\,\text{J}\cdot\text{s}}=7.77\times10^6\,\text{m}^{-1}$$

In region 2 where $V=-1.25\ \mu\text{V}$, the kinetic energy of the electron is

$$K_b=E-U_b=2.306\ \mu\text{eV}-1.25\ \mu\text{eV}=1.056\ \mu\text{eV}.$$

and the corresponding angular wave number is

$$k_b=\frac{2\pi}{h}\sqrt{2m(E-U_b)}=\frac{2\pi}{h}\sqrt{2mK_b}=\frac{2\pi}{6.626\times10^{-34}\,\text{J}\cdot\text{s}}\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.056\ \mu\text{eV})(1.6\times10^{-25}\ \text{J}/\mu\text{eV})}$$
$$=5.258\times10^6\ \text{m}^{-1}$$

The ratio of the two wave numbers is $k_b/k=(5.258\times10^6\ \text{m}^{-1})/(7.77\times10^6\ \text{m}^{-1})=0.6767$. The reflection coefficient is

$$R=\frac{|B|^2}{|A|^2}=\left(\frac{1-k_b/k}{1+k_b/k}\right)^2=\left(\frac{1-0.6767}{1+0.6767}\right)^2=0.0372,$$

which leads to the following transmission coefficient:

$$T=1-R=1-0.0372=0.9628.$$

Thus, we find the current on the other side of the step boundary to be

$$I_t = TI_0 = (0.9628)(5.00\text{ mA}) = 4.81\text{ mA}.$$

74. With

$$T \approx e^{-2bL} = \exp\left(-2L\sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}\right),$$

we have

$$E = U_b - \frac{1}{2m}\left(\frac{h\ln T}{4\pi L}\right)^2 = 6.0\text{eV} - \frac{1}{2(0.511\text{MeV})}\left[\frac{(1240\text{eV}\cdot\text{nm})(\ln 0.001)}{4\pi(0.70\text{nm})}\right]^2$$
$$= 5.1\text{eV}.$$

75. (a) The transmission coefficient T for a particle of mass m and energy E that is incident on a barrier of height U_b and width L is given by

$$T = e^{-2bL},$$

where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

For the proton, we have

$$b = \sqrt{\frac{8\pi^2(1.6726\times10^{-27}\text{ kg})(10\text{MeV} - 3.0\text{MeV})(1.6022\times10^{-13}\text{ J/MeV})}{(6.6261\times10^{-34}\text{ J}\cdot\text{s})^2}}$$
$$= 5.8082\times10^{14}\text{ m}^{-1}.$$

This gives $bL = (5.8082\times10^{14}\text{ m}^{-1})(10\times10^{-15}\text{ m}) = 5.8082,$ and

$$T = e^{-2(5.8082)} = 9.02\times10^{-6}.$$

The value of b was computed to a greater number of significant digits than usual because an exponential is quite sensitive to the value of the exponent.

(b) Mechanical energy is conserved. Before the proton reaches the barrier, it has a kinetic energy of 3.0 MeV and a potential energy of zero. After passing through the barrier, the proton again has a potential energy of zero, thus a kinetic energy of 3.0 MeV.

(c) Energy is also conserved for the reflection process. After reflection, the proton has a potential energy of zero, and thus a kinetic energy of 3.0 MeV.

(d) The mass of a deuteron is 2.0141 u = 3.3454×10^{-27} kg, so

$$b=\sqrt{\frac{8\pi^2\left(3.3454\times10^{-27}\,\text{kg}\right)\left(10\,\text{MeV}-3.0\,\text{MeV}\right)\left(1.6022\times10^{-13}\ \text{J/MeV}\right)}{\left(6.6261\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}}$$
$$=8.2143\times10^{14}\,\text{m}^{-1}.$$

This gives $bL=\left(8.2143\times10^{14}\,\text{m}^{-1}\right)\left(10\times10^{-15}\,\text{m}\right)=8.2143,$ and $T=e^{-2(8.2143)}=7.33\times10^{-8}$.

(e) As in the case of a proton, mechanical energy is conserved. Before the deuteron reaches the barrier, it has a kinetic energy of 3.0 MeV and a potential energy of zero. After passing through the barrier, the deuteron again has a potential energy of zero, thus a kinetic energy of 3.0 MeV.

(f) Energy is also conserved for the reflection process. After reflection, the deuteron has a potential energy of zero, and thus a kinetic energy of 3.0 MeV.

76. (a) The rate at which incident protons arrive at the barrier is

$$n=1.0\,\text{kA}/1.60\times10^{-19}\,\text{C}=6.25\times10^{21}/\text{s}.$$

Letting $nTt = 1$, we find the waiting time t:

$$t=\left(nT\right)^{-1}=\frac{1}{n}\exp\left(2L\sqrt{\frac{8\pi^2 m_p\left(U_b-E\right)}{h^2}}\right)$$
$$=\left(\frac{1}{6.25\times10^{21}/\text{s}}\right)\exp\left(\frac{2\pi\left(0.70\,\text{nm}\right)}{1240\,\text{eV}\cdot\text{nm}}\sqrt{8\left(938\,\text{MeV}\right)\left(6.0\,\text{eV}-5.0\,\text{eV}\right)}\right)$$
$$=3.37\times10^{111}\,\text{s}\approx10^{104}\,\text{y},$$

which is much longer than the age of the universe.

(b) Replacing the mass of the proton with that of the electron, we obtain the corresponding waiting time for an electron:

$$t=\left(nT\right)^{-1}=\frac{1}{n}\exp\left[2L\sqrt{\frac{8\pi^2 m_e\left(U_b-E\right)}{h^2}}\right]$$
$$=\left(\frac{1}{6.25\times10^{21}/\text{s}}\right)\exp\left[\frac{2\pi\left(0.70\,\text{nm}\right)}{1240\,\text{eV}\cdot\text{nm}}\sqrt{8\left(0.511\,\text{MeV}\right)\left(6.0\,\text{eV}-5.0\,\text{eV}\right)}\right]$$
$$=2.1\times10^{-19}\,\text{s}.$$

The enormous difference between the two waiting times is the result of the difference between the masses of the two kinds of particles.

77. **THINK** Even though $E < U_b$, barrier tunneling can still take place quantum mechanically with finite probability.

EXPRESS If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U_b and width L is given by $T = e^{-2bL}$, where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

If the change ΔU_b in U_b is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dU_b}\Delta U_b = -2LT\frac{db}{dU_b}\Delta U_b.$$

Now,

$$\frac{db}{dU_b} = \frac{1}{2\sqrt{U_b - E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U_b - E)}\sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}} = \frac{b}{2(U_b - E)}.$$

Thus,

$$\Delta T = -LTb\frac{\Delta U_b}{U_b - E}.$$

ANALYZE (a) With

$$b = \sqrt{\frac{8\pi^2\left(9.11\times10^{-31}\text{ kg}\right)\left(6.8\text{ eV} - 5.1\text{ eV}\right)\left(1.6022\times10^{-19}\text{ J/eV}\right)}{\left(6.6261\times10^{-34}\text{ J}\cdot\text{s}\right)^2}} = 6.67\times10^{9}\text{ m}^{-1},$$

we have $bL = (6.67\times10^{9}\text{ m}^{-1})(750\times10^{-12}\text{ m}^{-1}) = 5.0,$ and

$$\frac{\Delta T}{T} = -bL\frac{\Delta U_b}{U_b - E} = -(5.0)\frac{(0.010)(6.8\text{ eV})}{6.8\text{ eV} - 5.1\text{ eV}} = -0.20.$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dL}\Delta L = -2be^{-2bL}\Delta L = -2bT\Delta L$$

and

$$\frac{\Delta T}{T} = -2b\Delta L = -2\left(6.67\times10^{9}\ \text{m}^{-1}\right)\left(0.010\right)\left(750\times10^{-12}\ \text{m}\right) = -0.10\ .$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dE}\Delta E = -2Le^{-2bL}\frac{db}{dE}\Delta E = -2LT\frac{db}{dE}\Delta E.$$

Now, $db/dE = -db/dU_b = -b/2\left(U_b - E\right)$, so

$$\frac{\Delta T}{T} = bL\frac{\Delta E}{U_b - E} = \left(5.0\right)\frac{\left(0.010\right)\left(5.1\,\text{eV}\right)}{6.8\,\text{eV} - 5.1\,\text{eV}} = 0.15\ .$$

There is a 15% increase in the transmission coefficient.

LEARN Increasing the barrier height or the barrier thickness reduces the probability of transmission, while increasing the kinetic energy of the electron increases the probability.

78. The energy of the electron in region 1 is

$$E = K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(1200\ \text{m/s})^2 = 6.56\times10^{-25}\ \text{J} = 4.0995\ \mu\text{eV}.$$

The angular wave number in region 1 is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(h/p)} = \frac{2\pi p}{h} = \frac{2\pi mv}{h} = \frac{2\pi(9.11\times10^{-31}\ \text{kg})(1200\ \text{m/s})}{6.626\times10^{-34}\,\text{J}\cdot\text{s}} = 1.036\times10^{7}\,\text{m}^{-1}$$

The transmission coefficient for a barrier of height U_b and width L is given by

$$T = e^{-2bL},$$

where

$$b = \sqrt{\frac{8\pi^2 m\left(U_b - E\right)}{h^2}} = \sqrt{\frac{8\pi^2\left(9.11\times10^{-31}\ \text{kg}\right)\left(4.719\ \mu\text{eV} - 4.0995\ \mu\text{eV}\right)\left(1.6022\times10^{-25}\ \text{J}/\mu\text{eV}\right)}{\left(6.6261\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}}$$

$$= 4.0298\times10^{6}\,\text{m}^{-1}.$$

Thus,

$$T = \exp(-2bL) = \exp\left[-2(4.0298\times10^{6}\,\text{m}^{-1})(200\times10^{-9}\,\text{m}^{-1})\right] = e^{-1.612} = 0.1995,$$

and the current transmitted is

$$I_t = TI_0 = (0.1995)(9.00\text{ mA}) = 1.795\text{ mA}.$$

79. (a) Since $p_x = p_y = 0,\ \Delta p_x = \Delta p_y = 0$. Thus from Eq. 38-20 both Δx and Δy are infinite. It is therefore impossible to assign a y or z coordinate to the position of an electron.

(b) Since it is independent of y and z the wave function $\Psi(x)$ should describe a plane wave that extends infinitely in both the y and z directions. Also from Fig. 38-12 we see that $|\Psi(x)|^2$ extends infinitely along the x axis. Thus the matter wave described by $\Psi(x)$ extends throughout the entire three-dimensional space.

80. Using the value $hc = 1240\text{eV}\cdot\text{nm}$, we obtain

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{21\times10^{7}\text{ nm}} = 5.9\times10^{-6}\text{ eV} = 5.9\mu\text{eV}.$$

81. We substitute the classical relationship between momentum p and velocity v, $v = p/m$ into the classical definition of kinetic energy, $K = \frac{1}{2}mv^2$ to obtain $K = p^2/2m$. Here m is the mass of an electron. Thus $p = \sqrt{2mK}$. The relationship between the momentum and the de Broglie wavelength λ is $\lambda = h/p$, where h is the Planck constant. Thus,

$$\lambda = \frac{h}{\sqrt{2mK}}.$$

If K is given in electron volts, then

$$\lambda = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\text{ kg})(1.602\times10^{-19}\text{ J/eV})K}} = \frac{1.226\times10^{-9}\text{ m}\cdot\text{eV}^{1/2}}{\sqrt{K}}$$

$$= \frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}}.$$

82. We rewrite Eq. 38-9 as

$$\frac{h}{m\lambda} - \frac{h}{m\lambda'}\cos\phi = \frac{v}{\sqrt{1-(v/c)^2}}\cos\theta,$$

and Eq. 38-10 as

$$\frac{h}{m\lambda'}\sin\phi = \frac{v}{\sqrt{1-(v/c)^2}}\sin\theta.$$

We square both equations and add up the two sides:

$$\left(\frac{h}{m}\right)^2\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\phi\right)^2+\left(\frac{1}{\lambda'}\sin\phi\right)^2\right]=\frac{v^2}{1-(v/c)^2}\ ,$$

where we use $\sin^2\theta+\cos^2\theta=1$ to eliminate θ. Now the right-hand side can be written as

$$\frac{v^2}{1-(v/c)^2}=-c^2\left[1-\frac{1}{1-(v/c)^2}\right],$$

so

$$\frac{1}{1-(v/c)^2}=\left(\frac{h}{mc}\right)^2\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\phi\right)^2+\left(\frac{1}{\lambda'}\sin\phi\right)^2\right]+1\ .$$

Now we rewrite Eq. 38-8 as

$$\frac{h}{mc}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)+1=\frac{1}{\sqrt{1-(v/c)^2}}\ .$$

If we square this, then it can be directly compared with the previous equation we obtained for $[1-(v/c)^2]^{-1}$. This yields

$$\left[\frac{h}{mc}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)+1\right]^2=\left(\frac{h}{mc}\right)^2\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\phi\right)^2+\left(\frac{1}{\lambda'}\sin\phi\right)^2\right]+1\ .$$

We have so far eliminated θ and v. Working out the squares on both sides and noting that $\sin^2\phi+\cos^2\phi=1$, we get

$$\lambda'-\lambda=\Delta\lambda=\frac{h}{mc}(1-\cos\phi)\ .$$

83. (a) The average kinetic energy is

$$K=\frac{3}{2}kT=\frac{3}{2}\left(1.38\times10^{-23}\ \text{J/K}\right)\left(300\,\text{K}\right)=6.21\times10^{-21}\,\text{J}=3.88\times10^{-2}\ \text{eV}.$$

(b) The de Broglie wavelength is

$$\lambda=\frac{h}{\sqrt{2m_nK}}=\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2\left(1.675\times10^{-27}\,\text{kg}\right)\left(6.21\times10^{-21}\,\text{J}\right)}}=1.46\times10^{-10}\,\text{m}.$$

84. (a) The average de Broglie wavelength is

$$\lambda_{avg} = \frac{h}{p_{avg}} = \frac{h}{\sqrt{2mK_{avg}}} = \frac{h}{\sqrt{2m(3kT/2)}} = \frac{hc}{\sqrt{2(mc^2)kT}}$$

$$= \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{3(4)(938\,\text{MeV})(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})}}$$

$$= 7.3\times10^{-11}\,\text{m} = 73\,\text{pm}.$$

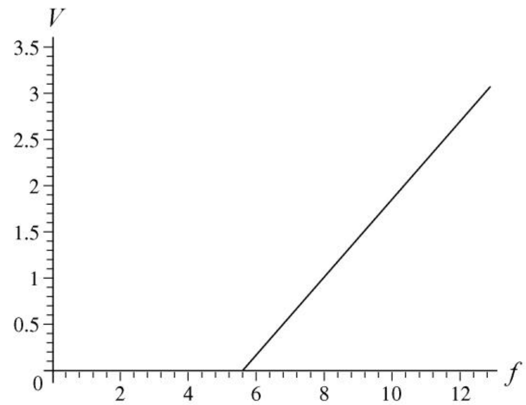
(b) The average separation is

$$d_{avg} = \frac{1}{\sqrt[3]{n}} = \frac{1}{\sqrt[3]{p/kT}} = \sqrt[3]{\frac{(1.38\times10^{-23}\,\text{J/K})(300\text{K})}{1.01\times10^{5}\,\text{Pa}}} = 3.4\,\text{nm}.$$

(c) Yes, since $\lambda_{avg} \ll d_{avg}$.

85. (a) We calculate frequencies from the wavelengths (expressed in SI units) using Eq. 38-1. Our plot of the points and the line that gives the least squares fit to the data is shown below. The vertical axis is in volts and the horizontal axis, when multiplied by 10^{14}, gives the frequencies in Hertz.

From our least squares fit procedure, we determine the slope to be 4.14×10^{-15} V·s, which, upon multiplying by e, gives 4.14×10^{-15} eV·s. The result is in very good agreement with the value given in Eq. 38-3.



(b) Our least squares fit procedure can also determine the y-intercept for that line. The y-intercept is the negative of the photoelectric work function. In this way, we find Φ = 2.31 eV.

86. We note that

$$|e^{ikx}|^2 = (e^{ikx})^*(e^{ikx}) = e^{-ikx}e^{ikx} = 1.$$

Referring to Eq. 38-14, we see therefore that $|\psi|^2 = |\Psi|^2$.

87. From Sample Problem — "Compton scattering of light by electrons," we have

$$\frac{\Delta E}{E}=\frac{\Delta\lambda}{\lambda+\Delta\lambda}=\frac{(h/mc)(1-\cos\phi)}{\lambda'}=\frac{hf'}{mc^2}(1-\cos\phi)$$

where we use the fact that $\lambda + \Delta\lambda = \lambda' = c/f'$.

88. The de Broglie wavelength for the bullet is

$$\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{6.63\times10^{-34}\text{ J.s}}{(40\times10^{-3}\text{ kg})(1000\text{ m/s})}=1.7\times10^{-35}\text{ m}.$$

89. (a) Since

$$E_{ph} = h/\lambda = 1240\text{ eV}\cdot\text{nm}/680\text{ nm} = 1.82\text{ eV} < \Phi = 2.28\text{ eV},$$

there is no photoelectric emission.

(b) The cutoff wavelength is the longest wavelength of photons that will cause photoelectric emission. In sodium, this is given by $E_{ph} = hc/\lambda_{max} = \Phi$, or

$$\lambda_{max} = hc/\Phi = (1240\text{ eV}\cdot\text{nm})/2.28\text{ eV} = 544\text{ nm}.$$

(c) This corresponds to the color green.

90. **THINK** We apply Heisenberg's uncertainty principle to calculate the uncertainty in position.

EXPRESS The uncertainty principle states that $\Delta x\Delta p \geq \hbar$, where Δx and Δp represent the intrinsic uncertainties in measuring the position and momentum, respectively. The uncertainty in the momentum is

$$\Delta p = m\,\Delta v = (0.50\text{ kg})(1.0\text{ m/s}) = 0.50\text{ kg}\cdot\text{m/s},$$

where Δv is the uncertainty in the velocity.

ANALYZE Solving the uncertainty relationship $\Delta x\Delta p \geq \hbar$ for the minimum uncertainty in the coordinate x, we obtain

$$\Delta x=\frac{\hbar}{\Delta p}=\frac{0.60\text{ J}\cdot\text{s}}{2\pi\left(0.50\text{ kg}\cdot\text{m/s}\right)}=0.19\text{ m}.$$

LEARN Heisenberg's uncertainty principle implies that it is impossible to simultaneously measure a particle's position and momentum with infinite accuracy.

Chapter 39

1. According to Eq. 39-4, $E_n \propto L^{-2}$. As a consequence, the new energy level E'_n satisfies

$$\frac{E'_n}{E_n} = \left(\frac{L'}{L}\right)^{-2} = \left(\frac{L}{L'}\right)^{2} = \frac{1}{2},$$

which gives $L' = \sqrt{2}L.$ Thus, the ratio is $L'/L = \sqrt{2} = 1.41.$

2. (a) The ground-state energy is

$$E_1 = \left(\frac{h^2}{8m_e L^2}\right)n^2 = \left(\frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}{8(9.11\times10^{-31}\,\text{kg})\left(200\times10^{-12}\,\text{m}\right)^2}\right)(1)^2 = 1.51\times10^{-18}\,\text{J}$$
$$= 9.42\,\text{eV}.$$

(b) With $m_p = 1.67 \times 10^{-27}$ kg, we obtain

$$E_1 = \left(\frac{h^2}{8m_p L^2}\right)n^2 = \left(\frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}{8(1.67\times10^{-27}\,\text{kg})\left(200\times10^{-12}\,\text{m}\right)^2}\right)(1)^2 = 8.225\times10^{-22}\,\text{J}$$
$$= 5.13\times10^{-3}\,\text{eV}.$$

3. Since $E_n \propto L^{-2}$ in Eq. 39-4, we see that if L is doubled, then E_1 becomes $(2.6\text{ eV})(2)^{-2}$ = 0.65 eV.

4. We first note that since $h = 6.626 \times 10^{-34}$ J·s and $c = 2.998 \times 10^8$ m/s,

$$hc = \frac{\left(6.626\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\,\text{m/s}\right)}{\left(1.602\times10^{-19}\,\text{J/eV}\right)\left(10^{-9}\,\text{m/nm}\right)} = 1240\,\text{eV}\cdot\text{nm}.$$

Using the mc^2 value for an electron from Table 37-3 (511×10^3 eV), Eq. 39-4 can be rewritten as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(mc^2)L^2}.$$

The energy to be absorbed is therefore

$$\Delta E = E_4 - E_1 = \frac{(4^2 - 1^2)h^2}{8m_e L^2} = \frac{15(hc)^2}{8(m_e c^2)L^2} = \frac{15(1240\,\text{eV}\cdot\text{nm})^2}{8(511\times10^3\,\text{eV})(0.250\,\text{nm})^2} = 90.3\,\text{eV}.$$

5. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by writing Eq. 39-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(mc^2)L^2}.$$

For n = 3, we set this expression equal to 4.7 eV and solve for L:

$$L = \frac{n(hc)}{\sqrt{8(mc^2)E_n}} = \frac{3(1240\,\text{eV}\cdot\text{nm})}{\sqrt{8(511\times10^3\,\text{eV})(4.7\,\text{eV})}} = 0.85\,\text{nm}.$$

6. With $m = m_p = 1.67 \times 10^{-27}$ kg, we obtain

$$E_1 = \left(\frac{h^2}{8mL^2}\right)n^2 = \left(\frac{(6.63\times10^{-34}\,\text{J.s})^2}{8(1.67\times10^{-27}\,\text{kg})(100\times10^{12}\,\text{m})^2}\right)(1)^2 = 3.29\times10^{-21}\,\text{J} = 0.0206\,\text{eV}.$$

Alternatively, we can use the mc^2 value for a proton from Table 37-3 (938×10^6 eV) and hc = 1240 eV · nm by writing Eq. 39-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(m_pc^2)L^2}.$$

This alternative approach is perhaps easier to plug into, but it is recommended that both approaches be tried to find which is most convenient.

7. To estimate the energy, we use Eq. 39-4, with n = 1, L equal to the atomic diameter, and m equal to the mass of an electron:

$$E = n^2\frac{h^2}{8mL^2} = \frac{(1)^2(6.63\times10^{-34}\,\text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\,\text{kg})(1.4\times10^{-14}\,\text{m})^2} = 3.07\times10^{-10}\,\text{J} = 1920\,\text{MeV} \approx 1.9\ \text{GeV}.$$

8. The frequency of the light that will excite the electron from the state with quantum number n_i to the state with quantum number n_f is

$$f = \frac{\Delta E}{h} = \frac{h}{8mL^2}\left(n_f^2 - n_i^2\right)$$

and the wavelength of the light is

$$\lambda = \frac{c}{f} = \frac{8mL^2c}{h\left(n_f^2 - n_i^2\right)}.$$

The width of the well is

$$L = \sqrt{\frac{\lambda hc(n_f^2 - n_i^2)}{8mc^2}}.$$

The longest wavelength shown in Figure 39-27 is $\lambda = 80.78$ nm, which corresponds to a jump from $n_i = 2$ to $n_f = 3$. Thus, the width of the well is

$$L = \sqrt{\frac{\lambda hc(n_f^2 - n_i^2)}{8mc^2}} = \sqrt{\frac{(80.78\text{ nm})(1240\,\text{eV}\cdot\text{nm})(3^2 - 2^2)}{8(511\times10^3\,\text{eV})}} = 0.350\,\text{nm} = 350\text{ pm}.$$

9. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by rewriting Eq. 39-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2\left(hc\right)^2}{8\left(mc^2\right)L^2}.$$

(a) The first excited state is characterized by $n = 2$, and the third by $n' = 4$. Thus,

$$\Delta E = \frac{(hc)^2}{8\left(mc^2\right)L^2}\left(n'^2 - n^2\right) = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8\left(511\times10^3\,\text{eV}\right)(0.250\,\text{nm})^2}\left(4^2 - 2^2\right) = (6.02\,\text{eV})(16-4)$$
$$= 72.2\,\text{eV}.$$

Now that the electron is in the $n' = 4$ level, it can "drop" to a lower level (n'') in a variety of ways. Each of these drops is presumed to cause a photon to be emitted of wavelength

$$\lambda = \frac{hc}{E_{n'} - E_{n''}} = \frac{8\left(mc^2\right)L^2}{hc\left(n'^2 - n''^2\right)}.$$

For example, for the transition $n' = 4$ to $n'' = 3$, the photon emitted would have wavelength

$$\lambda = \frac{8\left(511\times10^3\,\text{eV}\right)(0.250\,\text{nm})^2}{(1240\,\text{eV}\cdot\text{nm})\left(4^2 - 3^2\right)} = 29.4\,\text{nm},$$

and once it is then in level $n'' = 3$ it might fall to level $n''' = 2$ emitting another photon. Calculating in this way all the possible photons emitted during the de-excitation of this system, we obtain the following results:

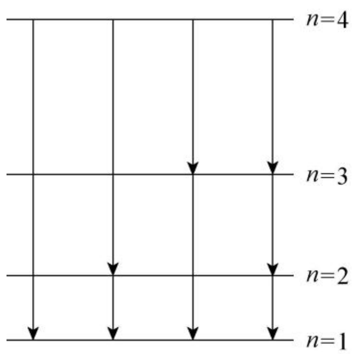
(b) The shortest wavelength that can be emitted is $\lambda_{4\to1} = 13.7\,\text{nm}$.

(c) The second shortest wavelength that can be emitted is $\lambda_{4\to2} = 17.2\,\text{nm}$.

(d) The longest wavelength that can be emitted is $\lambda_{2\to1} = 68.7\,\text{nm}$.

(e) The second longest wavelength that can be emitted is $\lambda_{3\to2} = 41.2\,\text{nm}$.

(f) The possible transitions are shown next. The energy levels are not drawn to scale.



(g) A wavelength of 29.4 nm corresponds to $4 \to 3$ transition. Thus, it could make either the $3 \to 1$ transition or the pair of transitions: $3 \to 2$ and $2 \to 1$. The longest wavelength that can be emitted is $\lambda_{2\to1} = 68.7\,\text{nm}$.

(h) The shortest wavelength that can next be emitted is $\lambda_{3\to1} = 25.8\,\text{nm}$.

10. Let the quantum numbers of the pair in question be n and $n + 1$, respectively. Then

$$E_{n+1} - E_n = E_1\,(n+1)^2 - E_1 n^2 = (2n+1)E_1.$$

Letting

$$E_{n+1} - E_n = (2n+1)E_1 = 3(E_4 - E_3) = 3(4^2 E_1 - 3^2 E_1) = 21E_1,$$

we get $2n + 1 = 21$, or $n = 10$. Thus,

(a) the higher quantum number is $n + 1 = 10 + 1 = 11$, and

(b) the lower quantum number is $n = 10$.

(c) Now letting

$$E_{n+1} - E_n = (2n+1)E_1 = 2(E_4 - E_3) = 2(4^2 E_1 - 3^2 E_1) = 14E_1,$$

we get $2n + 1 = 14$, which does not have an integer-valued solution. So it is impossible to find the pair of energy levels that fits the requirement.

11. Let the quantum numbers of the pair in question be n and $n + 1$, respectively. We note that

$$E_{n+1} - E_n = \frac{(n+1)^2 h^2}{8mL^2} - \frac{n^2 h^2}{8mL^2} = \frac{(2n+1)h^2}{8mL^2}$$

Therefore, $E_{n+1} - E_n = (2n + 1)E_1$. Now

$$E_{n+1} - E_n = E_5 = 5^2 E_1 = 25E_1 = (2n+1)E_1,$$

which leads to $2n + 1 = 25$, or $n = 12$. Thus,

(a) The higher quantum number is $n + 1 = 12 + 1 = 13$.

(b) The lower quantum number is $n = 12$.

(c) Now let

$$E_{n+1} - E_n = E_6 = 6^2 E_1 = 36E_1 = (2n+1)E_1,$$

which gives $2n + 1 = 36$, or $n = 17.5$. This is not an integer, so it is impossible to find the pair that fits the requirement.

12. The energy levels are given by $E_n = n^2h^2/8mL^2$, where h is the Planck constant, m is the mass of an electron, and L is the width of the well. The frequency of the light that will excite the electron from the state with quantum number n_i to the state with quantum number n_f is

$$f = \frac{\Delta E}{h} = \frac{h}{8mL^2}\left(n_f^2 - n_i^2\right)$$

and the wavelength of the light is

$$\lambda = \frac{c}{f} = \frac{8mL^2 c}{h\left(n_f^2 - n_i^2\right)}.$$

We evaluate this expression for $n_i = 1$ and $n_f = 2, 3, 4$, and 5, in turn. We use $h = 6.626 \times 10^{-34}$ J · s, $m = 9.109 \times 10^{-31}$kg, and $L = 250 \times 10^{-12}$ m, and obtain the following results:

(a) 6.87×10^{-8} m for $n_f = 2$, (the longest wavelength).

(b) 2.58×10^{-8} m for $n_f = 3$, (the second longest wavelength).

(c) 1.37×10^{-8} m for $n_f = 4$, (the third longest wavelength).

13. The position of maximum probability density corresponds to the center of the well: $x = L/2 = (200 \text{ pm})/2 = 100 \text{ pm}$.

(a) The probability of detection at x is given by Eq. 39-11:

$$p(x) = \psi_n^2(x)dx = \left[\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)\right]^2 dx = \frac{2}{L}\sin^2\left(\frac{n\pi}{L}x\right)dx$$

For $n = 3$, $L = 200 \text{ pm}$, and $dx = 2.00 \text{ pm}$ (width of the probe), the probability of detection at $x = L/2 = 100 \text{ pm}$ is

$$p(x = L/2) = \frac{2}{L}\sin^2\left(\frac{3\pi}{L}\cdot\frac{L}{2}\right)dx = \frac{2}{L}\sin^2\left(\frac{3\pi}{2}\right)dx = \frac{2}{L}dx = \frac{2}{200 \text{ pm}}(2.00 \text{ pm}) = 0.020.$$

(b) With $N = 1000$ independent insertions, the number of times we expect the electron to be detected is $n = Np = (1000)(0.020) = 20$.

14. From Eq. 39-11, the condition of zero probability density is given by

$$\sin\left(\frac{n\pi}{L}x\right) = 0 \;\Rightarrow\; \frac{n\pi}{L}x = m\pi$$

where m is an integer. The fact that $x = 0.300L$ and $x = 0.400L$ have zero probability density implies

$$\sin(0.300n\pi) = \sin(0.400n\pi) = 0$$

which can be satisfied for $n = 10m$, where $m = 1, 2, \ldots$ However, since the probability density is nonzero between $x = 0.300L$ and $x = 0.400L$, we conclude that the electron is in the $n = 10$ state. The change of energy after making a transition to $n' = 9$ is then equal to

$$|\Delta E| = \frac{h^2}{8mL^2}\left(n^2 - n'^2\right) = \frac{\left(6.63\times10^{-34} \text{ J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31} \text{ kg}\right)\left(2.00\times10^{-10} \text{ m}\right)^2}\left(10^2 - 9^2\right) = 2.86\times10^{-17} \text{ J}.$$

15. **THINK** The probability that the electron is found in any interval is given by $P = \int |\psi|^2 dx$, where the integral is over the interval.

EXPRESS If the interval width Δx is small, the probability can be approximated by $P = |\psi|^2 \Delta x$, where the wave function is evaluated for the center of the interval, say. For an electron trapped in an infinite well of width L, the ground state probability density is

$$|\psi|^2 = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right),$$

so

$$P = \left(\frac{2\Delta x}{L}\right)\sin^2\left(\frac{\pi x}{L}\right).$$

ANALYZE (a) We take L = 100 pm, x = 25 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(25\,\text{pm})}{100\,\text{pm}}\right] = 0.050.$$

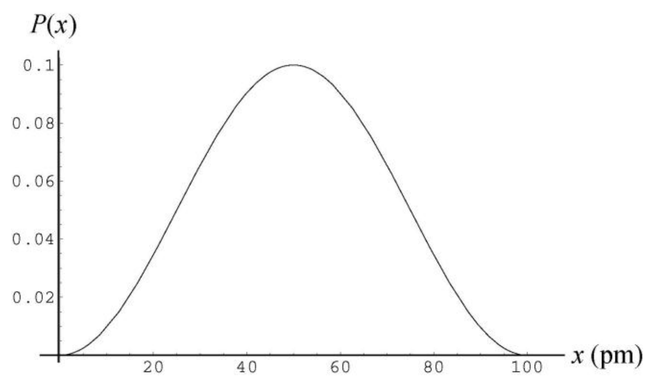
(b) We take L = 100 pm, x = 50 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(50\,\text{pm})}{100\,\text{pm}}\right] = 0.10.$$

(c) We take L = 100 pm, x = 90 pm, and Δx = 5.0 pm. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(90\,\text{pm})}{100\,\text{pm}}\right] = 0.0095.$$

LEARN The probability as a function of x is plotted next. As expected, the probability of detecting the electron is highest near the center of the well at $x = L/2$ = 50 pm.



16. We follow Sample Problem — "Detection potential in a 1D infinite potential well" in the presentation of this solution. The integration result quoted below is discussed in a little more detail in that Sample Problem. We note that the arguments of the sine functions used below are in radians.

(a) The probability of detecting the particle in the region $0 \le x \le L/4$ is

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_0^{\pi/4}\sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2}-\frac{\sin 2y}{4}\right)\Bigg|_0^{\pi/4} = 0.091.$$

(b) As expected from symmetry,

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{\pi}\sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2}-\frac{\sin 2y}{4}\right)\Bigg|_{\pi/4}^{\pi} = 0.091.$$

(c) For the region $L/4 \le x \le 3L/4$, we obtain

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{3\pi/4}\sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2}-\frac{\sin 2y}{4}\right)\Bigg|_{\pi/4}^{3\pi/4} = 0.82$$

which we could also have gotten by subtracting the results of part (a) and (b) from 1; that is, 1 – 2(0.091) = 0.82.

17. According to Fig. 39-9, the electron's initial energy is 106 eV. After the additional energy is absorbed, the total energy of the electron is 106 eV + 400 eV = 506 eV. Since it is in the region $x > L$, its potential energy is 450 eV, so its kinetic energy must be 506 eV – 450 eV = 56 eV.

18. From Fig. 39-9, we see that the sum of the kinetic and potential energies in that particular finite well is 233 eV. The potential energy is zero in the region $0 < x < L$. If the kinetic energy of the electron is detected while it is in that region (which is the only region where this is likely to happen), we should find K = 233 eV.

19. Using $E = hc/\lambda = (1240\,\text{eV}\cdot\text{nm})/\lambda$, the energies associated with λ_a, λ_b and λ_c are

$$E_a = \frac{hc}{\lambda_a} = \frac{1240\,\text{eV}\cdot\text{nm}}{14.588\text{ nm}} = 85.00\text{ eV}$$

$$E_b = \frac{hc}{\lambda_b} = \frac{1240\,\text{eV}\cdot\text{nm}}{4.8437\text{ nm}} = 256.0\text{ eV}$$

$$E_c = \frac{hc}{\lambda_c} = \frac{1240\,\text{eV}\cdot\text{nm}}{2.9108\text{ nm}} = 426.0\text{ eV}.$$

The ground-state energy is

$$E_1 = E_4 - E_c = 450.0\text{ eV} - 426.0\text{ eV} = 24.0\text{ eV}.$$

Since $E_a = E_2 - E_1$, the energy of the first excited state is

$$E_2 = E_1 + E_a = 24.0\text{ eV} + 85.0\text{ eV} = 109\text{ eV}.$$

20. The smallest energy a photon can have corresponds to a transition from the non-quantized region to E_3. Since the energy difference between E_3 and E_4 is

$$\Delta E = E_4 - E_3 = 9.0\text{ eV} - 4.0\text{ eV} = 5.0\text{ eV},$$

the energy of the photon is $E_{\text{photon}} = K + \Delta E = 2.00\text{ eV} + 5.00\text{ eV} = 7.00\text{ eV}$.

21. Schrödinger's equation for the region $x > L$ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(E - U_0\right)\psi = 0.$$

If $\psi = De^{2kx}$, then $d^2\psi/dx^2 = 4k^2De^{2kx} = 4k^2\psi$ and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(E - U_0\right)\psi = 4k^2\psi + \frac{8\pi^2 m}{h^2}\left(E - U_0\right)\psi.$$

This is zero provided

$$k = \frac{\pi}{h}\sqrt{2m\left(U_0 - E\right)}.$$

The proposed function satisfies Schrödinger's equation provided k has this value. Since U_0 is greater than E in the region $x > L$, the quantity under the radical is positive. This means k is real. If k is positive, however, the proposed function is physically unrealistic. It increases exponentially with x and becomes large without bound. The integral of the probability density over the entire x-axis must be unity. This is impossible if ψ is the proposed function.

22. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by writing Eq. 39-20 as

$$E_{nx,ny} = \frac{2h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right) = \frac{\left(hc\right)^2}{8\left(mc^2\right)}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right).$$

For $n_x = n_y = 1$, we obtain

$$E_{1,1} = \frac{\left(1240\text{eV}\cdot\text{nm}\right)^2}{8\left(511\times10^3\text{eV}\right)}\left(\frac{1}{\left(0.800\text{nm}\right)^2} + \frac{1}{\left(1.600\text{nm}\right)^2}\right) = 0.734\text{ eV}.$$

23. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by writing Eq. 39-21 as

$$E_{nx,ny,nz} = \frac{2h^2}{8m}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right) = \frac{(hc)^2}{8(mc^2)}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right).$$

For $n_x = n_y = n_z = 1$, we obtain

$$E_{1,1} = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8(511\times10^3\,\text{eV})}\left(\frac{1}{(0.800\,\text{nm})^2}+\frac{1}{(1.600\,\text{nm})^2}+\frac{1}{(0.390\,\text{nm})^2}\right) = 3.21\text{ eV}.$$

24. The statement that there are three probability density maxima along $x = L_x/2$ implies that $n_y = 3$ (see for example, Figure 39-6). Since the maxima are separated by 2.00 nm, the width of L_y is $L_y = n_y(2.00\text{ nm}) = 6.00\text{ nm}$. Similarly, from the information given along $y = L_y/2$, we find $n_x = 5$ and $L_x = n_x(3.00\text{ nm}) = 15.0\text{ nm}$. Thus, using Eq. 39-20, the energy of the electron is

$$E_{n_x,n_y} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}\right) = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{8(9.11\times10^{-31}\text{ kg})}\left[\frac{1}{(3.00\times10^{-9}\text{ m})^2}+\frac{1}{(2.00\times10^{-9}\text{ m})^2}\right]$$
$$= 2.2\times10^{-20}\text{ J}.$$

25. The discussion on the probability of detection for the one-dimensional case can be readily extended to two dimensions. In analogy to Eq. 39-10, the normalized wave function in two dimensions can be written as

$$\psi_{n_x,n_y}(x,y) = \psi_{n_x}(x)\psi_{n_y}(y) = \sqrt{\frac{2}{L_x}}\sin\left(\frac{n_x\pi}{L_x}x\right)\cdot\sqrt{\frac{2}{L_y}}\sin\left(\frac{n_y\pi}{L_y}y\right)$$
$$= \sqrt{\frac{4}{L_xL_y}}\sin\left(\frac{n_x\pi}{L_x}x\right)\sin\left(\frac{n_y\pi}{L_y}y\right).$$

The probability of detection by a probe of dimension $\Delta x\Delta y$ placed at (x, y) is

$$p(x,y) = |\psi_{n_x,n_y}(x,y)|^2\,\Delta x\Delta y = \frac{4(\Delta x\Delta y)}{L_xL_y}\sin^2\left(\frac{n_x\pi}{L_x}x\right)\sin^2\left(\frac{n_y\pi}{L_y}y\right).$$

With $L_x = L_y = L = 150$ pm and $\Delta x = \Delta y = 5.00$ pm, the probability of detecting an electron in $(n_x, n_y) = (1,3)$ state by placing a probe at $(0.200L, 0.800L)$ is

$$p=\frac{4(\Delta x\Delta y)}{L_xL_y}\sin^2\left(\frac{n_x\pi}{L_x}x\right)\sin^2\left(\frac{n_y\pi}{L_y}y\right)=\frac{4(5.00\text{ pm})^2}{(150\text{ pm})^2}\sin^2\left(\frac{\pi}{L}\cdot 0.200L\right)\sin^2\left(\frac{3\pi}{L}0.800L\right)$$
$$=4\left(\frac{5.00\text{ pm}}{150\text{ pm}}\right)^2\sin^2(0.200\pi)\sin^2(2.40\pi)=1.4\times10^{-3}.$$

26. We are looking for the values of the ratio

$$\frac{E_{nx,ny}}{h^2/8mL^2}=L^2\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}\right)=\left(n_x^2+\frac{1}{4}n_y^2\right)$$

and the corresponding differences.

(a) For $n_x = n_y = 1$, the ratio becomes $1+\frac{1}{4}=1.25$.

(b) For $n_x = 1$ and $n_y = 2$, the ratio becomes $1+\frac{1}{4}(4)=2.00$. One can check (by computing other (n_x, n_y) values) that this is the next to lowest energy in the system.

(c) The lowest set of states that are degenerate are (n_x, n_y) = (1, 4) and (2, 2). Both of these states have that ratio equal to $1+\frac{1}{4}(16)=5.00$.

(d) For $n_x = 1$ and $n_y = 3$, the ratio becomes $1+\frac{1}{4}(9)=3.25$. One can check (by computing other (n_x, n_y) values) that this is the lowest energy greater than that computed in part (b). The next higher energy comes from (n_x, n_y) = (2, 1) for which the ratio is $4+\frac{1}{4}(1)=4.25$. The difference between these two values is 4.25 – 3.25 = 1.00.

27. **THINK** The energy levels of an electron trapped in a regular corral with widths L_x and L_y are given by Eq. 39-20:

$$E_{n_x,n_y}=\frac{h^2}{8m}\left[\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}\right].$$

EXPRESS With $L_x = L$ and $L_y = 2L$, we have

$$E_{n_x,n_y}=\frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}\right)=\frac{h^2}{8mL^2}\left(n_x^2+\frac{n_y^2}{4}\right).$$

Thus, in units of $h^2/8mL^2$, the energy levels are given by $n_x^2+n_y^2/4$. The lowest five levels are $E_{1,1} = 1.25$, $E_{1,2} = 2.00$, $E_{1,3} = 3.25$, $E_{2,1} = 4.25$, and $E_{2,2} = E_{1,4} = 5.00$. It is clear that there are no other possible values for the energy less than 5.

The frequency of the light emitted or absorbed when the electron goes from an initial state i to a final state f is $f = (E_f - E_i)/h$, and in units of $h/8mL^2$ is simply the difference in the values of $n_x^2 + n_y^2/4$ for the two states. The possible frequencies are as follows:

$$0.75(1,2 \to 1,1), 2.00(1,3 \to 1,1), 3.00(2,1 \to 1,1),$$
$$3.75(2,2 \to 1,1), 1.25(1,3 \to 1,2), 2.25(2,1 \to 1,2), 3.00(2,2 \to 1,2), 1.00(2,1 \to 1,3),$$
$$1.75(2,2 \to 1,3), 0.75(2,2 \to 2,1),$$

all in units of $h/8mL^2$.

ANALYZE (a) From the above, we see that there are 8 different frequencies.

(b) The lowest frequency is, in units of $h/8mL^2$, 0.75 (2, 2$\to$2,1).

(c) The second lowest frequency is, in units of $h/8mL^2$, 1.00 (2, 1$\to$1,3).

(d) The third lowest frequency is, in units of $h/8mL^2$, 1.25 (1, 3$\to$1,2).

(e) The highest frequency is, in units of $h/8mL^2$, 3.75 (2, 2$\to$1,1).

(f) The second highest frequency is, in units of $h/8mL^2$, 3.00 (2, 2$\to$1,2) or (2, 1$\to$1,1).

(g) The third highest frequency is, in units of $h/8mL^2$, 2.25 (2, 1$\to$1,2).

LEARN In general, when the electron makes a transition from (n_x, n_y) to a higher level (n'_x, n'_y), the frequency of photon it emits or absorbs is given by

$$f = \frac{\Delta E}{h} = \frac{E_{n'_x,n'_y} - E_{n_x,n_y}}{h} = \frac{h}{8mL^2}\left(n_x'^2 + \frac{n_y'^2}{4}\right) - \frac{h}{8mL^2}\left(n_x^2 + \frac{n_y^2}{4}\right)$$
$$= \frac{h}{8mL^2}\left[\left(n_x'^2 - n_x^2\right) + \frac{1}{4}\left(n_y'^2 - n_y^2\right)\right].$$

28. We are looking for the values of the ratio

$$\frac{E_{n_x,n_y,n_z}}{h^2/8mL^2} = L^2\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \left(n_x^2 + n_y^2 + n_z^2\right)$$

and the corresponding differences.

(a) For $n_x = n_y = n_z = 1$, the ratio becomes 1 + 1 + 1 = 3.00.

(b) For $n_x = n_y = 2$ and $n_z = 1$, the ratio becomes 4 + 4 + 1 = 9.00. One can check (by computing other (n_x, n_y, n_z) values) that this is the third lowest energy in the system. One can also check that this same ratio is obtained for (n_x, n_y, n_z) = (2, 1, 2) and (1, 2, 2).

(c) For $n_x = n_y = 1$ and $n_z = 3$, the ratio becomes 1 + 1 + 9 = 11.00. One can check (by computing other (n_x, n_y, n_z) values) that this is three "steps" up from the lowest energy in the system. One can also check that this same ratio is obtained for (n_x, n_y, n_z) = (1, 3, 1) and (3, 1, 1). If we take the difference between this and the result of part (b), we obtain 11.0 – 9.00 = 2.00.

(d) For $n_x = n_y = 1$ and $n_z = 2$, the ratio becomes 1 + 1 + 4 = 6.00. One can check (by computing other (n_x, n_y, n_z) values) that this is the next to the lowest energy in the system. One can also check that this same ratio is obtained for (n_x, n_y, n_z) = (2, 1, 1) and (1, 2, 1). Thus, three states (three arrangements of (n_x, n_y, n_z) values) have this energy.

(e) For $n_x = 1$, $n_y = 2$ and $n_z = 3$, the ratio becomes 1 + 4 + 9 = 14.0. One can check (by computing other (n_x, n_y, n_z) values) that this is five "steps" up from the lowest energy in the system. One can also check that this same ratio is obtained for (n_x, n_y, n_z) = (1, 3, 2), (2, 3, 1), (2, 1, 3), (3, 1, 2) and (3, 2, 1). Thus, six states (six arrangements of (n_x, n_y, n_z) values) have this energy.

29. The ratios computed in Problem 39-28 can be related to the frequencies emitted using $f = \Delta E/h$, where each level E is equal to one of those ratios multiplied by $h^2/8mL^2$. This effectively involves no more than a cancellation of one of the factors of h. Thus, for a transition from the second excited state (see part (b) of Problem 39-28) to the ground state (treated in part (a) of that problem), we find

$$f = (9.00 - 3.00)\left(\frac{h}{8mL^2}\right) = (6.00)\left(\frac{h}{8mL^2}\right) .$$

In the following, we omit the $h/8mL^2$ factors. For a transition between the fourth excited state and the ground state, we have f = 12.00 – 3.00 = 9.00. For a transition between the third excited state and the ground state, we have f = 11.00 – 3.00 = 8.00. For a transition between the third excited state and the first excited state, we have f = 11.00 – 6.00 = 5.00. For a transition between the fourth excited state and the third excited state, we have f = 12.00 – 11.00 = 1.00. For a transition between the third excited state and the second excited state, we have f = 11.00 – 9.00 = 2.00. For a transition between the second excited state and the first excited state, we have f = 9.00 – 6.00 = 3.00, which also results from some other transitions.

(a) From the above, we see that there are 7 frequencies.

(b) The lowest frequency is, in units of $h/8mL^2$, 1.00.

(c) The second lowest frequency is, in units of $h/8mL^2$, 2.00.

(d) The third lowest frequency is, in units of $h/8mL^2$, 3.00.

(e) The highest frequency is, in units of $h/8mL^2$, 9.00.

(f) The second highest frequency is, in units of $h/8mL^2$, 8.00.

(g) The third highest frequency is, in units of $h/8mL^2$, 6.00.

30. In analogy to Eq. 39-10, the normalized wave function in two dimensions can be written as

$$\psi_{n_x,n_y}(x,y) = \psi_{n_x}(x)\psi_{n_y}(y) = \sqrt{\frac{2}{L_x}}\sin\left(\frac{n_x\pi}{L_x}x\right)\cdot\sqrt{\frac{2}{L_y}}\sin\left(\frac{n_y\pi}{L_y}y\right)$$
$$= \sqrt{\frac{4}{L_xL_y}}\sin\left(\frac{n_x\pi}{L_x}x\right)\sin\left(\frac{n_y\pi}{L_y}y\right).$$

The probability of detection by a probe of dimension $\Delta x\Delta y$ placed at (x,y) is

$$p(x,y) = \left|\psi_{n_x,n_y}(x,y)\right|^2\Delta x\Delta y = \frac{4(\Delta x\Delta y)}{L_xL_y}\sin^2\left(\frac{n_x\pi}{L_x}x\right)\sin^2\left(\frac{n_y\pi}{L_y}y\right).$$

A detection probability of 0.0450 of a ground-state electron ($n_x = n_y = 1$) by a probe of area $\Delta x\Delta y = 400\text{ pm}^2$ placed at $(x,y) = (L/8, L/8)$ implies

$$0.0450 = \frac{4(400\text{ pm}^2)}{L^2}\sin^2\left(\frac{\pi}{L}\cdot\frac{L}{8}\right)\sin^2\left(\frac{\pi}{L}\cdot\frac{L}{8}\right) = 4\left(\frac{20\text{ pm}}{L}\right)^2\sin^4\left(\frac{\pi}{8}\right).$$

Solving for L, we get $L = 27.6\text{ pm}$.

31. **THINK** The Lyman series is associated with transitions to or from the $n = 1$ level of the hydrogen atom, while the Balmer series is for transitions to or from the $n = 2$ level.

EXPRESS The energy E of the photon emitted when a hydrogen atom jumps from a state with principal quantum number n' to a state with principal quantum number $n < n'$ is given by

$$E = A\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

where A = 13.6 eV. The frequency f of the electromagnetic wave is given by $f = E/h$ and the wavelength is given by $\lambda = c/f$. Thus,

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E}{hc} = \frac{A}{hc}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right).$$

ANALYZE The shortest wavelength occurs at the series limit, for which $n' = \infty$. For the Balmer series, $n = 2$ and the shortest wavelength is $\lambda_B = 4hc/A$. For the Lyman series, $n = 1$ and the shortest wavelength is $\lambda_L = hc/A$. The ratio is $\lambda_B/\lambda_L = 4.0$.

LEARN The energy of the photon emitted associated with the transition of an electron from $n' = \infty \rightarrow n = 2$ (to become bound) is

$$E_{\infty\rightarrow 2} = \frac{13.6 \text{ eV}}{2^2} = 3.4 \text{ eV}.$$

Similarly, the energy associated with the transition of an electron from $n' = \infty \rightarrow n = 1$ (to become bound) is

$$E_{1\rightarrow\infty} = \frac{13.6 \text{ eV}}{1^2} = 13.6 \text{ eV}.$$

32. The difference between the energy absorbed and the energy emitted is

$$E_{\text{photon absorbed}} - E_{\text{photon emitted}} = \frac{hc}{\lambda_{\text{absorbed}}} - \frac{hc}{\lambda_{\text{emitted}}}.$$

Thus, using $hc = 1240$ eV · nm, the net energy absorbed is

$$hc\Delta\left(\frac{1}{\lambda}\right) = (1240 \text{ eV}\cdot\text{nm})\left(\frac{1}{375 \text{ nm}} - \frac{1}{580 \text{ nm}}\right) = 1.17 \text{ eV}.$$

33. (a) Since energy is conserved, the energy E of the photon is given by $E = E_i - E_f$, where E_i is the initial energy of the hydrogen atom and E_f is the final energy. The electron energy is given by $(-13.6 \text{ eV})/n^2$, where n is the principal quantum number. Thus,

$$E = E_3 - E_1 = \frac{-13.6 \text{ eV}}{(3)^2} - \frac{-13.6 \text{ eV}}{(1)^2} = 12.1 \text{ eV}.$$

(b) The photon momentum is given by

$$p = \frac{E}{c} = \frac{(12.1 \text{ eV})(1.60\times10^{-19} \text{ J/eV})}{3.00\times10^{8} \text{ m/s}} = 6.45\times10^{-27} \text{ kg}\cdot\text{m/s}.$$

(c) Using $hc = 1240$ eV · nm, the wavelength is $\lambda = \frac{hc}{E} = \frac{1240 \text{ eV}\cdot\text{nm}}{12.1 \text{ eV}} = 102 \text{ nm}.$

34. (a) We use Eq. 39-44. At $r = 0$, $P(r) \propto r^2 = 0$.

(b) At $r = a$, $P(r) = \frac{4}{a^3} a^2 e^{-2a/a} = \frac{4e^{-2}}{a} = \frac{4e^{-2}}{5.29\times10^{-2}\text{ nm}} = 10.2\text{ nm}^{-1}$.

(c) At $r = 2a$, $P(r) = \frac{4}{a^3}(2a)^2 e^{-4a/a} = \frac{16e^{-4}}{a} = \frac{16e^{-4}}{5.29\times10^{-2}\text{ nm}} = 5.54\text{ nm}^{-1}$.

35. (a) We use Eq. 39-39. At $r = a$,

$$\psi^2(r) = \left(\frac{1}{\sqrt{\pi}a^{3/2}} e^{-a/a}\right)^2 = \frac{1}{\pi a^3} e^{-2} = \frac{1}{\pi\left(5.29\times10^{-2}\text{ nm}\right)^3} e^{-2} = 291\text{ nm}^{-3}.$$

(b) We use Eq. 39-44. At $r = a$,

$$P(r) = \frac{4}{a^3} a^2 e^{-2a/a} = \frac{4e^{-2}}{a} = \frac{4e^{-2}}{5.29\times10^{-2}\text{ nm}} = 10.2\text{ nm}^{-1}.$$

36. (a) The energy level corresponding to the probability density distribution shown in Fig. 39-21 is the $n = 2$ level. Its energy is given by

$$E_2 = -\frac{13.6\text{ eV}}{2^2} = -3.4\text{ eV}.$$

(b) As the electron is removed from the hydrogen atom the final energy of the proton-electron system is zero. Therefore, one needs to supply at least 3.4 eV of energy to the system in order to bring its energy up from $E_2 = -3.4$ eV to zero. (If more energy is supplied, then the electron will retain some kinetic energy after it is removed from the atom.)

37. **THINK** The energy of the hydrogen atom is quantized.

EXPRESS If kinetic energy is not conserved, some of the neutron's initial kinetic energy could be used to excite the hydrogen atom. The least energy that the hydrogen atom can accept is the difference between the first excited state ($n = 2$) and the ground state ($n = 1$). Since the energy of a state with principal quantum number n is $-(13.6\text{ eV})/n^2$, the smallest excitation energy is

$$\Delta E = E_2 - E_1 = \frac{-13.6\text{ eV}}{(2)^2} - \frac{-13.6\text{ eV}}{(1)^2} = 10.2\text{ eV}.$$

ANALYZE The neutron, with a kinetic energy of 6.0 eV, does not have sufficient kinetic energy to excite the hydrogen atom, so the hydrogen atom is left in its ground state and all the initial kinetic energy of the neutron ends up as the final kinetic energies of the neutron and atom. The collision must be elastic.

LEARN The minimum kinetic energy the neutron must have in order to excite the hydrogen atom is 10.2 eV.

38. From Eq. 39-6, $\Delta E = hf = (4.14\times10^{-15}\ \text{eV}\cdot\text{s})(6.2\times10^{14}\ \text{Hz}) = 2.6\,\text{eV}$.

39. **THINK** The radial probability function for the ground state of hydrogen is

$$P(r) = (4r^2/a^3)e^{-2r/a},$$

where a is the Bohr radius.

EXPRESS We want to evaluate the integral $\int_0^\infty P(r)\,dr$. Equation 15 in the integral table of Appendix E is an integral of this form:

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}.$$

ANALYZE We set $n = 2$ and replace a in the given formula with $2/a$ and x with r. Then

$$\int_0^\infty P(r)dr = \frac{4}{a^3}\int_0^\infty r^2 e^{-2r/a}dr = \frac{4}{a^3}\frac{2}{(2/a)^3} = 1.$$

LEARN The integral over the radial probability function $P(r)$ must be equal to 1. This means that in a hydrogen atom, the electron must be somewhere in the space surrounding the nucleus.

40. (a) The calculation is shown in Sample Problem — "Light emission from a hydrogen atom." The difference in the values obtained in parts (a) and (b) of that Sample Problem is 122 nm – 91.4 nm ≈ 31 nm.

(b) We use Eq. 39-1. For the Lyman series,

$$\Delta f = \frac{2.998\times10^8\ \text{m/s}}{91.4\times10^{-9}\ \text{m}} - \frac{2.998\times10^8\ \text{m/s}}{122\times10^{-9}\ \text{m}} = 8.2\times10^{14}\ \text{Hz}.$$

(c) Figure 39-18 shows that the width of the Balmer series is 656.3 nm – 364.6 nm ≈ 292 nm $\approx 0.29\ \mu\text{m}$.

(d) The series limit can be obtained from the $\infty \to 2$ transition:

$$\Delta f = \frac{2.998\times10^8\ \text{m/s}}{364.6\times10^{-9}\ \text{m}} - \frac{2.998\times10^8\ \text{m/s}}{656.3\times10^{-9}\ \text{m}} = 3.65\times10^{14}\ \text{Hz} \approx 3.7\times10^{14}\ \text{Hz}.$$

41. Since Δr is small, we may calculate the probability using $p = P(r)\,\Delta r$, where $P(r)$ is the radial probability density. The radial probability density for the ground state of hydrogen is given by Eq. 39-44:

$$P(r) = \left(\frac{4r^2}{a^3}\right) e^{-2r/a}$$

where a is the Bohr radius.

(a) Here, $r = 0.500a$ and $\Delta r = 0.010a$. Then,

$$P = \left(\frac{4r^2\Delta r}{a^3}\right) e^{-2r/a} = 4(0.500)^2(0.010)e^{-1} = 3.68\times10^{-3} \approx 3.7\times10^{-3}.$$

(b) We set $r = 1.00a$ and $\Delta r = 0.010a$. Then,

$$P = \left(\frac{4r^2\Delta r}{a^3}\right) e^{-2r/a} = 4(1.00)^2(0.010)e^{-2} = 5.41\times10^{-3} \approx 5.4\times10^{-3}.$$

42. Conservation of linear momentum of the atom-photon system requires that

$$p_{\text{recoil}} = p_{\text{photon}} \Rightarrow m_p v_{\text{recoil}} = \frac{hf}{c}$$

where we use Eq. 39-7 for the photon and use the classical momentum formula for the atom (since we expect its speed to be much less than c). Thus, from Eq. 39-6 and Table 37-3,

$$v_{\text{recoil}} = \frac{\Delta E}{m_p c} = \frac{E_4 - E_1}{\left(m_p c^2\right)/c} = \frac{\left(-13.6\text{eV}\right)\left(4^{-2} - 1^{-2}\right)}{\left(938\times10^6\ \text{eV}\right)/\left(2.998\times10^8\ \text{m/s}\right)} = 4.1\ \text{m/s}\ .$$

43. (a) and (b) Letting $a = 5.292 \times 10^{-11}$ m be the Bohr radius, the potential energy becomes

$$U = -\frac{e^2}{4\pi\varepsilon_0 a} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602\times10^{-19}\ \text{C}\right)^2}{5.292\times10^{-11}\ \text{m}} = -4.36\times10^{-18}\ \text{J} = -27.2\,\text{eV}\ .$$

The kinetic energy is $K = E - U = (-13.6\text{ eV}) - (-27.2\text{ eV}) = 13.6$ eV.

44. (a) Since $E_2 = -0.85$ eV and $E_1 = -13.6\text{ eV} + 10.2\text{ eV} = -3.4$ eV, the photon energy is

$$E_{\text{photon}} = E_2 - E_1 = -0.85\text{ eV} - (-3.4\text{ eV}) = 2.6\text{ eV}.$$

(b) From

$$E_2 - E_1 = (-13.6\,\text{eV})\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = 2.6\ \text{eV}$$

we obtain

$$\frac{1}{n_2^2} - \frac{1}{n_1^2} = \frac{2.6\ \text{eV}}{13.6\,\text{eV}} \approx -\frac{3}{16} = \frac{1}{4^2} - \frac{1}{2^2}.$$

Thus, $n_2 = 4$ and $n_1 = 2$. So the transition is from the $n = 4$ state to the $n = 2$ state. One can easily verify this by inspecting the energy level diagram of Fig. 39-18. Thus, the higher quantum number is $n_2 = 4$.

(c) The lower quantum number is $n_1 = 2$.

45. **THINK** The probability density is given by $|\psi_{n\ell m_\ell}(r,\theta)|^2$, where $\psi_{n\ell m_\ell}(r,\theta)$ is the wave function.

EXPRESS To calculate $|\psi_{n\ell m_\ell}|^2 = \psi^*_{n\ell m_\ell}\psi_{n\ell m_\ell}$, we multiply the wave function by its complex conjugate. If the function is real, then $\psi^*_{n\ell m_\ell} = \psi_{n\ell m_\ell}$. Note that $e^{+i\phi}$ and $e^{-i\phi}$ are complex conjugates of each other, and $e^{i\phi}e^{-i\phi} = e^0 = 1$.

ANALYZE (a) ψ_{210} is real. Squaring it gives the probability density:

$$|\psi_{210}|^2 = \frac{r^2}{32\pi a^5} e^{-r/a}\cos^2\theta.$$

(b) Similarly,

$$|\psi_{21+1}|^2 = \frac{r^2}{64\pi a^5} e^{-r/a}\sin^2\theta$$

and

$$|\psi_{21-1}|^2 = \frac{r^2}{64\pi a^5} e^{-r/a}\sin^2\theta.$$

The last two functions lead to the same probability density.

(c) For $m_\ell = 0$, the probability density $|\psi_{210}|^2$ decreases with radial distance from the nucleus. With the $\cos^2\theta$ factor, $|\psi_{210}|^2$ is greatest along the z axis where $\theta = 0$. This is consistent with the dot plot of Fig. 39-23(a).

Similarly, for $m_\ell = \pm 1$, the probability density $|\psi_{21\pm1}|^2$ decreases with radial distance from the nucleus. With the $\sin^2\theta$ factor, $|\psi_{21\pm1}|^2$ is greatest in the *xy*-plane where θ = 90°. This is consistent with the dot plot of Fig. 39-23(b).

(d) The total probability density for the three states is the sum:

$$|\psi_{210}|^2 + |\psi_{21+1}|^2 + |\psi_{21-1}|^2 = \frac{r^2}{32\pi a^5} e^{-r/a}\left[\cos^2\theta + \frac{1}{2}\sin^2\theta + \frac{1}{2}\sin^2\theta\right] = \frac{r^2}{32\pi a^5} e^{-r/a}.$$

The trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. We note that the total probability density does not depend on θ or ϕ; it is spherically symmetric.

LEARN The wave functions discussed above are for the hydrogen states with $n = 2$ and $\ell = 1$. Since the angular momentum is nonzero, the probability densities are not spherically symmetric, but depend on both r and θ.

46. From Sample Problem — " Probability of detection of the electron in a hydrogen atom," we know that the probability of finding the electron in the ground state of the hydrogen atom inside a sphere of radius r is given by

$$p(r) = 1 - e^{-2x}\left(1 + 2x + 2x^2\right)$$

where $x = r/a$. Thus the probability of finding the electron between the two shells indicated in this problem is given by

$$p(a < r < 2a) = p(2a) - p(a) = \left[1 - e^{-2x}\left(1 + 2x + 2x^2\right)\right]_{x=2} - \left[1 - e^{-2x}\left(1 + 2x + 2x^2\right)\right]_{x=1}$$
$$= 0.439.$$

47. As illustrated in Fig. 39-24, the quantum number n in question satisfies $r = n^2 a$. Letting r = 1.0 mm, we solve for n:

$$n = \sqrt{\frac{r}{a}} = \sqrt{\frac{1.0 \times 10^{-3}\ \text{m}}{5.29 \times 10^{-11}\ \text{m}}} \approx 4.3 \times 10^3.$$

48. Using Eq. 39-6 and hc = 1240 eV · nm, we find

$$\Delta E = E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{121.6\,\text{nm}} = 10.2\,\text{eV}.$$

Therefore, $n_{\text{low}} = 1$, but what precisely is n_{high}?

$$E_{\text{high}} = E_{\text{low}} + \Delta E \quad \Rightarrow \quad -\frac{13.6\text{eV}}{n^2} = -\frac{13.6\text{eV}}{1^2} + 10.2\text{eV}$$

which yields $n = 2$ (this is confirmed by the calculation found from Sample Problem — "Light emission from a hydrogen atom). Thus, the transition is from the $n = 2$ to the $n = 1$ state.

(a) The higher quantum number is $n = 2$.

(b) The lower quantum number is $n = 1$.

(c) Referring to Fig. 39-18, we see that this must be one of the Lyman series transitions.

49. (a) We take the electrostatic potential energy to be zero when the electron and proton are far removed from each other. Then, the final energy of the atom is zero and the work done in pulling it apart is $W = -E_i$, where E_i is the energy of the initial state. The energy of the initial state is given by $E_i = (-13.6 \text{ eV})/n^2$, where n is the principal quantum number of the state. For the ground state, $n = 1$ and $W = 13.6$ eV.

(b) For the state with $n = 2$, $W = (13.6 \text{ eV})/(2)^2 = 3.40$ eV.

50. Using Eq. 39-6 and $hc = 1240$ eV · nm, we find

$$\Delta E = E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{106.6 \text{ nm}} = 12.09 \text{ eV}.$$

Therefore, $n_{\text{low}} = 1$, but what precisely is n_{high}?

$$E_{\text{high}} = E_{\text{low}} + \Delta E \quad \Rightarrow \quad -\frac{13.6 \text{ eV}}{n^2} = -\frac{13.6 \text{ eV}}{1^2} + 12.09 \text{ eV}$$

which yields $n = 3$. Thus, the transition is from the $n = 3$ to the $n = 1$ state.

(a) The higher quantum number is $n = 3$.

(b) The lower quantum number is $n = 1$.

(c) Referring to Fig. 39-18, we see that this must be one of the Lyman series transitions.

51. According to Sample Problem — " Probability of detection of the electron in a hydrogen atom," the probability the electron in the ground state of a hydrogen atom can be found inside a sphere of radius r is given by

$$p(r) = 1 - e^{-2x}\left(1 + 2x + 2x^2\right)$$

where $x = r/a$ and a is the Bohr radius. We want $r = a$, so $x = 1$ and

$$p(a) = 1 - e^{-2}(1+2+2) = 1 - 5e^{-2} = 0.323.$$

The probability that the electron can be found outside this sphere is 1 – 0.323 = 0.677. It can be found outside about 68% of the time.

52. (a) $\Delta E = -(13.6\text{ eV})(4^{-2} - 1^{-2}) = 12.8$ eV.

(b) There are 6 possible energies associated with the transitions 4 $\to$ 3, 4 $\to$ 2, 4 $\to$ 1, 3 $\to$ 2, 3 $\to$ 1 and 2 $\to$ 1.

(c) The greatest energy is $E_{4\to1} = 12.8$ eV.

(d) The second greatest energy is $E_{3\to1} = -(13.6\text{eV})\left(3^{-2} - 1^{-2}\right) = 12.1$ eV .

(e) The third greatest energy is $E_{2\to1} = -(13.6\text{eV})\left(2^{-2} - 1^{-2}\right) = 10.2$ eV .

(f) The smallest energy is $E_{4\to3} = -(13.6\text{eV})\left(4^{-2} - 3^{-2}\right) = 0.661$ eV .

(g) The second smallest energy is $E_{3\to2} = -(13.6\text{eV})\left(3^{-2} - 2^{-2}\right) = 1.89$ eV .

(h) The third smallest energy is $E_{4\to2} = -(13.6\text{eV})\left(4^{-2} - 2^{-2}\right) = 2.55$ eV.

53. **THINK** The ground state of the hydrogen atom corresponds to n = 1, $\ell = 0$, and $m_\ell = 0$.

EXPRESS The proposed wave function is

$$\psi = \frac{1}{\sqrt{\pi} a^{3/2}} e^{-r/a}$$

where a is the Bohr radius. Substituting this into the right side of Schrödinger's equation, our goal is to show that the result is zero.

ANALYZE The derivative is

$$\frac{d\psi}{dr} = -\frac{1}{\sqrt{\pi} a^{5/2}} e^{-r/a}$$

so

$$r^2 \frac{d\psi}{dr} = -\frac{r^2}{\sqrt{\pi} a^{5/2}} e^{-r/a}$$

and

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) = \frac{1}{\sqrt{\pi} a^{5/2}}\left(-\frac{2}{r} + \frac{1}{a}\right) e^{-r/a} = \frac{1}{a}\left(-\frac{2}{r} + \frac{1}{a}\right)\psi.$$

The energy of the ground state is given by $E = -me^4/8\varepsilon_0^2 h^2$ and the Bohr radius is given by $a = h^2\varepsilon_0/\pi me^2$, so $E = -e^2/8\pi\varepsilon_0 a$. The potential energy is given by

$$U = -e^2/4\pi\varepsilon_0 r,$$

so

$$\frac{8\pi^2 m}{h^2}(E-U)\psi = \frac{8\pi^2 m}{h^2}\left(-\frac{e^2}{8\pi\varepsilon_0 a}+\frac{e^2}{4\pi\varepsilon_0 r}\right)\psi = \frac{8\pi^2 m}{h^2}\frac{e^2}{8\pi\varepsilon_0}\left(-\frac{1}{a}+\frac{2}{r}\right)\psi$$
$$= \frac{\pi me^2}{h^2\varepsilon_0}\left(-\frac{1}{a}+\frac{2}{r}\right)\psi = \frac{1}{a}\left(-\frac{1}{a}+\frac{2}{r}\right)\psi.$$

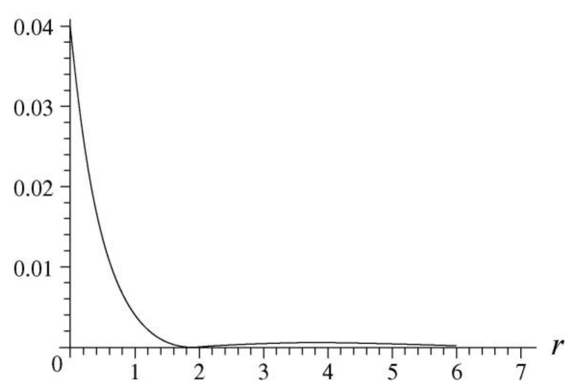
The two terms in Schrödinger's equation cancel, and the proposed function ψ satisfies that equation.

LEARN The radial probability density of the ground state of hydrogen atom is given by Eq. 39-44:

$$P(r) = |\psi|^2(4\pi r^2) = \frac{1}{\pi a^3}e^{-2r/a}(4\pi r^2) = \frac{4}{a^3}r^2 e^{-2r/a}.$$

A plot of $P(r)$ is shown in Fig. 39-20.

54. (a) The plot shown below for $|\psi_{200}(r)|^2$ is to be compared with the dot plot of Fig. 39-21. We note that the horizontal axis of our graph is labeled "*r*," but it is actually *r*/*a* (that is, it is in units of the parameter *a*). Now, in the plot below there is a high central peak between $r = 0$ and $r \sim 2a$, corresponding to the densely dotted region around the center of the dot plot of Fig. 39-21. Outside this peak is a region of near-zero values centered at $r = 2a$, where $\psi_{200} = 0$. This is represented in the dot plot by the empty ring surrounding the central peak. Further outside is a broader, flatter, low peak that reaches its maximum value at $r = 4a$. This corresponds to the outer ring with near-uniform dot density, which is lower than that of the central peak.



(b) The extrema of $\psi^2(r)$ for $0 < r < \infty$ may be found by squaring the given function, differentiating with respect to r, and setting the result equal to zero:

$$-\frac{1}{32}\frac{(r-2a)(r-4a)}{a^6\pi}e^{-r/a}=0$$

which has roots at $r = 2a$ and $r = 4a$. We can verify directly from the plot above that $r = 4a$ is indeed a local maximum of $\psi^2_{200}(r)$. As discussed in part (a), the other root ($r = 2a$) is a local minimum.

(c) Using Eq. 39-43 and Eq. 39-41, the radial probability is

$$P_{200}(r)=4\pi r^2\psi^2_{200}(r)=\frac{r^2}{8a^3}\left(2-\frac{r}{a}\right)^2 e^{-r/a}.$$

(d) Let $x = r/a$. Then

$$\int_0^\infty P_{200}(r)\,dr=\int_0^\infty \frac{r^2}{8a^3}\left(2-\frac{r}{a}\right)^2 e^{-r/a}dr=\frac{1}{8}\int_0^\infty x^2(2-x)^2e^{-x}dx=\int_0^\infty (x^4-4x^3+4x^2)e^{-x}dx$$
$$=\frac{1}{8}[4!-4(3!)+4(2!)]=1$$

where we have used the integral formula $\int_0^\infty x^n e^{-x}dx = n!$.

55. The radial probability function for the ground state of hydrogen is

$$P(r) = (4r^2/a^3)e^{-2r/a},$$

where a is the Bohr radius. (See Eq. 39-44.) The integral table of Appendix E may be used to evaluate the integral $r_{\text{avg}}=\int_0^\infty rP(r)\,dr$. Setting $n = 3$ and replacing a in the given formula with $2/a$ (and x with r), we obtain

$$r_{\text{avg}}=\int_0^\infty rP(r)\,dr=\frac{4}{a^3}\int_0^\infty r^3e^{-2r/a}dr=\frac{4}{a^3}\frac{6}{(2/a)^4}=1.5a.$$

56. (a) The allowed energy values are given by $E_n = n^2h^2/8mL^2$. The difference in energy between the state n and the state $n + 1$ is

$$\Delta E_{\text{adj}}=E_{n+1}-E_n=\left[(n+1)^2-n^2\right]\frac{h^2}{8mL^2}=\frac{(2n+1)h^2}{8mL^2}$$

and

$$\frac{\Delta E_{\text{adj}}}{E} = \left[\frac{(2n+1)h^2}{8mL^2}\right]\left(\frac{8mL^2}{n^2h^2}\right) = \frac{2n+1}{n^2}.$$

As n becomes large, $2n+1 \to 2n$ and $(2n+1)/n^2 \to 2n/n^2 = 2/n$.

(b) No. As $n \to \infty$, ΔE_{adj} and E do not approach 0, but $\Delta E_{\text{adj}}/E$ does.

(c) No. See part (b).

(d) Yes. See part (b).

(e) $\Delta E_{\text{adj}}/E$ is a better measure than either ΔE_{adj} or E alone of the extent to which the quantum result is approximated by the classical result.

57. From Eq. 39-4,

$$E_{n+2} - E_n = \left(\frac{h^2}{8mL^2}\right)(n+2)^2 - \left(\frac{h^2}{8mL^2}\right)n^2 = \left(\frac{h^2}{2mL^2}\right)(n+1).$$

58. (a) and (b) In the region $0 < x < L$, $U_0 = 0$, so Schrödinger's equation for the region is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

where $E > 0$. If $\psi^2(x) = B\sin^2 kx$, then $\psi(x) = B'\sin kx$, where B' is another constant satisfying $B'^2 = B$. Thus,

$$\frac{d^2\psi}{dx^2} = -k^2 B'\sin kx = -k^2\psi(x)$$

and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = -k^2\psi + \frac{8\pi^2 m}{h^2}E\psi.$$

This is zero provided that $k^2 = \dfrac{8\pi^2 mE}{h^2}$. The quantity on the right-hand side is positive, so k is real and the proposed function satisfies Schrödinger's equation. In this case, there exists no physical restriction as to the sign of k. It can assume either positive or negative values. Thus, $k = \pm\dfrac{2\pi}{h}\sqrt{2mE}$.

59. **THINK** For a finite well, the electron matter wave can penetrate the walls of the well. Thus, the wave function outside the well is not zero, but decreases exponentially with distance.

EXPRESS Schrödinger's equation for the region $x > L$ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(E - U_0\right)\psi = 0,$$

where $E - U_0 < 0$. If $\psi^2(x) = Ce^{-2kx}$, then $\psi(x) = \sqrt{C}\,e^{-kx}$.

ANALYZE (a) and (b) Thus,

$$\frac{d^2\psi}{dx^2} = 4k^2\sqrt{C}e^{-kx} = 4k^2\psi$$

and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(E - U_0\right)\psi = k^2\psi + \frac{8\pi^2 m}{h^2}\left(E - U_0\right)\psi.$$

This is zero provided that $k^2 = \frac{8\pi^2 m}{h^2}\left(U_0 - E\right)$. Choosing the positive root, we have

$$k = \frac{2\pi}{h}\sqrt{2m\left(U_0 - E\right)}.$$

LEARN Note that the quantity $U_0 - E$ is positive, so k is real and the proposed function satisfies Schrödinger's equation. If k is negative, however, the proposed function would be physically unrealistic. It would increase exponentially with x. Since the integral of the probability density over the entire x axis must be finite, ψ diverging as $x \to \infty$ would be unacceptable.

60. We can use the mc^2 value for an electron from Table 37-3 (511×10^3 eV) and hc = 1240 eV · nm by writing Eq. 39-4 as

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (hc)^2}{8(mc^2)L^2}.$$

(a) With $L = 3.0 \times 10^9$ nm, the energy difference is

$$E_2 - E_1 = \frac{1240^2}{8(511 \times 10^3)(3.0 \times 10^9)^2}\left(2^2 - 1^2\right) = 1.3 \times 10^{-19}\ \text{eV}.$$

(b) Since $(n + 1)^2 - n^2 = 2n + 1$, we have

$$\Delta E = E_{n+1} - E_n = \frac{h^2}{8mL^2}(2n+1) = \frac{(hc)^2}{8(mc^2)L^2}(2n+1).$$

Setting this equal to 1.0 eV, we solve for n:

$$n = \frac{4(mc^2)L^2\Delta E}{(hc)^2} - \frac{1}{2} = \frac{4(511\times10^3\,\text{eV})(3.0\times10^9\,\text{nm})^2\,(1.0\text{eV})}{(1240\text{eV}\cdot\text{nm})^2} - \frac{1}{2} \approx 1.2\times10^{19}.$$

(c) At this value of n, the energy is

$$E_n = \frac{1240^2}{8(511\times10^3)(3.0\times10^9)^2}(6\times10^{18})^2 \approx 6\times10^{18}\text{ eV}.$$

Thus,

$$\frac{E_n}{mc^2} = \frac{6\times10^{18}\,\text{eV}}{511\times10^3\,\text{eV}} = 1.2\times10^{13}.$$

(d) Since $E_n / mc^2 \gg 1$, the energy is indeed in the relativistic range.

61. (a) We recall that a derivative with respect to a dimensional quantity carries the (reciprocal) units of that quantity. Thus, the first term in Eq. 39-18 has dimensions of ψ multiplied by dimensions of x^{-2}. The second term contains no derivatives, does contain ψ, and involves several other factors that turn out to have dimensions of x^{-2}:

$$\frac{8\pi^2 m}{h^2}\left[E - U(x)\right] \;\Rightarrow\; \frac{\text{kg}}{(\text{J}\cdot\text{s})^2}[\text{J}]$$

assuming SI units. Recalling from Eq. 7-9 that J = kg·m^2/s^2, then we see the above is indeed in units of m^{-2} (which means dimensions of x^{-2}).

(b) In one-dimensional quantum physics, the wave function has units of m$^{-1/2}$, as shown in Eq. 39-17. Thus, since each term in Eq. 39-18 has units of ψ multiplied by units of x^{-2}, then those units are m$^{-1/2}$· m^{-2} = m$^{-2.5}$.

62. (a) The "home-base" energy level for the Balmer series is n = 2. Thus the transition with the least energetic photon is the one from the n = 3 level to the n = 2 level. The energy difference for this transition is

$$\Delta E = E_3 - E_2 = -(13.6\text{eV})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.889\,\text{eV}\;.$$

Using hc = 1240 eV · nm, the corresponding wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.889\,\text{eV}} = 658\,\text{nm}\;.$$

(b) For the series limit, the energy difference is

$$\Delta E = E_\infty - E_2 = -(13.6\,\text{eV})\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right) = 3.40\,\text{eV} .$$

The corresponding wavelength is then $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240\,\text{eV}\cdot\text{nm}}{3.40\,\text{eV}} = 366\,\text{nm}$.

63. (a) The allowed values of ℓ for a given n are 0, 1, 2, ..., n – 1. Thus there are n different values of ℓ .

(b) The allowed values of m_ℓ for a given ℓ are $-\ell$, $-\ell$ + 1, ..., ℓ . Thus there are 2ℓ + 1 different values of m_ℓ .

(c) According to part (a) above, for a given n there are n different values of ℓ . Also, each of these ℓ 's can have 2ℓ + 1 different values of m_ℓ [see part (b) above]. Thus, the total number of m_ℓ 's is

$$\sum_{\ell=0}^{n-1}(2\ell+1) = n^2.$$

64. For n = 1

$$E_1 = -\frac{m_e e^4}{8\varepsilon_0^2 h^2} = -\frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.6\times10^{-19}\,\text{C}\right)^4}{8\left(8.85\times10^{-12}\,\text{F/m}\right)^2\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2\left(1.60\times10^{-19}\,\text{J/eV}\right)} = -13.6\,\text{eV} .$$

65. (a) The angular momentum of the diatomic gas is

$$L = I\omega = 2\times m(d/2)^2\omega = \frac{1}{2}md^2\omega .$$

If its angular momentum is quantized, i.e., restricted to $L = n\hbar,\ n = 1, 2, \ldots$ then

$$\frac{1}{2}md^2\omega = n\hbar = \frac{nh}{2\pi} \quad\Rightarrow\quad \omega = \frac{nh}{\pi md^2}$$

(b) The quantized rotational energies are

$$E_n = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{md^2}{2}\right)\left(\frac{nh}{\pi md^2}\right)^2 = \frac{n^2h^2}{4\pi^2 md^2}$$

66. The expression for the probability of detecting an electron in the ground state of hydrogen atom inside a sphere of radius r is given in Sample Problem 39.07:

$$p(x) = 1 - e^{-2x}(1 + 2x + 2x^2)$$

where $x = r/a_0$, with $a_0 = 5.292\times10^{-11}$m. Given that $r = 1.1\times10^{-15}$m,

$$x = (1.1\times10^{-15}\text{ m})/(5.292\times10^{-11}\text{ m}) = 2.079\times10^{-5}.$$

For small x, $p(x)$ can be simplified as

$$p(x) = 1 - e^{-2x}\left(1 + 2x + 2x^2\right) \approx 1 - \left(1 - 2x + 2x^2 - \frac{4}{3}x^3 + \cdots\right)\left(1 + 2x + 2x^2\right) = \frac{4}{3}x^3$$
$$= \frac{4}{3}\left(2.079\times10^{-5}\right)^3 = 1.2\times10^{-14}.$$

67. (a) For a particle of mass m trapped inside a container of length L, he allowed energy values are given by $E_n = n^2h^2/8mL^2$. With an argon atom and L = 0.20 m, the energy difference between the lowest two levels is

$$\Delta E = E_2 - E_1 = \frac{h^2}{8mL^2}\left(2^2 - 1^2\right) = \frac{3h^2}{8mL^2} = \frac{3(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{8(0.0399\text{ kg}/6.02\times10^{23})(0.20\text{ m})^2}$$
$$= 6.21\times10^{-41}\text{ J} = 3.88\times10^{-22}\text{ eV}.$$

(b) The thermal energy at T = 300 K is its average kinetic energy:

$$\bar{K} = \frac{3}{2}kT = (1.38\times10^{-23}\text{ J/K})(300\text{ K}) = 6.21\times10^{-21}\text{ J} = 3.88\times10^{-2}\text{ eV}.$$

Thus, the ratio is

$$\frac{\bar{K}}{\Delta E} = \frac{3.88\times10^{-2}\text{ eV}}{3.9\times10^{-22}\text{ eV}} = 10^{20}.$$

(c) The temperature at which $\bar{K} = \frac{3}{2}kT = \Delta E$ is

$$T = \frac{2(\Delta E)}{3k} = \frac{2(6.21\times10^{-41}\text{ J})}{3(1.38\times10^{-23}\text{ J/K})} = 3.0\times10^{-18}\text{ K}.$$

68. The muon orbits the He^+ nucleus at a speed given by ($k = 1/4\pi\varepsilon_0$)

$$\frac{mv^2}{r} = \frac{Zke^2}{r^2} \quad\Rightarrow\quad v = \sqrt{\frac{Zke^2}{mr}}$$

With quantization condition $L = mvr = n\hbar$, the allowed values of the radius is

$$r_n = \frac{n^2\hbar^2}{Zke^2m}$$

Its total energy is

$$E = K + U = \frac{1}{2}mv^2 - \frac{Zke^2}{r} = -\frac{Zke^2}{2r}$$

The energy of the muon ground state is given by

$$E_n = -\frac{Zke^2}{2r_n} = -\frac{m(Ze^2)^2}{8\varepsilon_0^2 h^2}\frac{1}{n^2}$$

Evaluating the constants gives

$$\begin{aligned} E_n &= -\frac{m(Ze^2)^2}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -\frac{(207\times 9.11\times 10^{-31}\text{ kg})(2)^2(1.6\times 10^{-19}\text{ C})^4}{8(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)^2(6.626\times 10^{-34}\text{J}\cdot\text{s})^2}\frac{1}{n^2} \\ &= -\frac{1.8\times 10^{-15}\text{J}}{n^2} = -\frac{11.3\text{ keV}}{n^2}. \end{aligned}$$

69. The Ritz combination principle can be readily understood by noting that the transition from $n = n_i$ to $n = n_f < n_i$ can be done in two steps, with an intermediate state n':

$$\Delta E = E_{n_f} - E_{n_i} = (-13.6\,\text{eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = (-13.6\,\text{eV})\left(\frac{1}{n_f^2} - \frac{1}{n'^2}\right) + (-13.6\,\text{eV})\left(\frac{1}{n'^2} - \frac{1}{n_i^2}\right)$$

The transition $n_i = 3 \to n_f = 1$ associated with the second Lyman-series line can be thought of as $n_i = 3 \to n' = 2$ (first Balmer) followed by $n' = 2 \to n_f = 1$ (first Lyman). Another example would be $n_i = 4 \to n_f = 2$ (second Balmer), which can be thought of as $n_i = 4 \to n' = 3$ (first Paschen) followed by $n' = 3 \to n_f = 2$ (first Balmer).

70. (a) We use e_0 to denote the electric charge. The constant A can be calculated by integrating the charge density distribution:

$$-e_0 = \int \rho(r)dV = \int_0^\infty (Ae^{-2r/a_0})4\pi r^2 dr = 4\pi A a_0^3 \int_0^\infty x^2 e^{-2x} dx = \pi A a_0^3$$

which gives $A = -e_0 / \pi a_0^3$.

(b) We apply Gauss's to calculate the electric field at a distance r from the center of the atom. The charge enclosed by a Gaussian sphere of radius $r = a_0$, including the proton charge $+e_0$ at the center, is

$$q_{\text{enc}} = e_0 + \int \rho(r)dV = e_0 + \int_0^{a_0} (Ae^{-2r/a_0})4\pi r^2 dr = e_0 + 4\pi A a_0^3 \int_0^1 x^2 e^{-2x} dx$$
$$= e_0 + \pi A a_0^3 \left(1 - \frac{5}{e^2}\right) = e_0 + (-e_0)\left(1 - \frac{5}{e^2}\right) = (5e^{-2})e_0$$

Using Gauss's law, $\int \vec{E} \cdot d\vec{a} = q_{\text{enc}} / \varepsilon_0$, we obtain

$$E(4\pi a_0^2) = \frac{(5e^{-2})e_0}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{(5e^{-2})e_0}{4\pi\varepsilon_0 a_0^2}$$

(c) The net charge enclosed is positive, so the direction is radially outward.

71. (a) The charge enclosed by a sphere of radius r due to the uniform positive charge distribution is proportional to the volume: $q_{\text{enc}} = e(r/a_0)^3$. Using Gauss's law, $\int \vec{E} \cdot d\vec{a} = q_{\text{enc}} / \varepsilon_0$, the electric field at a radial distance r from the center of the atom is

$$E(4\pi r^2) = \frac{e}{\varepsilon_0}\left(\frac{r}{a_0}\right)^3 \quad \Rightarrow \quad E = \frac{e}{4\pi\varepsilon_0 a_0^3} r$$

and the force on the electron is $F = -eE = \dfrac{-e^2}{4\pi\varepsilon_0 a_0^3} r$. The negative sign means that the force points toward the center.

(b) Since $F = ma = md^2r/dt^2$,

$$m\frac{d^2r}{dt^2} = \frac{-e^2}{4\pi\varepsilon_0 a_0^3} r \quad \Rightarrow \quad \frac{d^2r}{dt^2} + \omega^2 r = 0$$

and the angular frequency is

$$\omega = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m a_0^3}} = \frac{e}{\sqrt{4\pi\varepsilon_0 m a_0^3}}.$$

72. (a) The electric potential is

$$V = \frac{kq}{r} = \frac{ke}{a_0} = \frac{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}{5.29\times10^{-11}\text{ m}} = 27.22\text{ V}$$

(b) The electric potential energy of the atom is

$$U = qV = -eV = -e(27.22\text{ V}) = -27.22\text{ eV}$$

(c) The electron moves in a circular orbit with

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \quad \Rightarrow \quad v = \sqrt{\frac{ke^2}{mr}}$$

Its kinetic energy at $r = a_0$ is

$$K = \frac{1}{2}mv^2 = \frac{ke^2}{2a_0} = \frac{1}{2}(27.22 \text{ eV}) = 13.6 \text{ eV} .$$

(d) The total energy of the system is

$$E = K + U = \frac{1}{2}mv^2 - \frac{ke^2}{a_0} = -\frac{ke^2}{2a_0} = -13.6 \text{ eV} .$$

Therefore, the energy required to ionize the atom is +13.6 eV.

73. The energy is, after evaluating the constants,

$$E_{n_1,n_2,n_3} = \frac{h^2}{8mL^2}\left(n_1^2 + n_2^2 + n_3^2\right) = \frac{(6.626\times10^{-34}\,\text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\text{ kg})(0.25\times10^{-6}\text{m})^2}\left(n_1^2 + n_2^2 + n_3^2\right)$$
$$= (6.024\ \mu\text{eV})\left(n_1^2 + n_2^2 + n_3^2\right)$$

The lowest five states correspond to $(n_1, n_2, n_3) = (1, 1, 1), (1, 2, 1), (1, 2, 2), (1, 3, 1)$ and $(2, 2, 2)$, and the energies are

$$E_{111} = \frac{h^2}{8mL^2}\left(1^2 + 1^2 + 1^2\right) = 3(6.024\ \mu\text{eV}) = 18.1\ \mu\text{eV}$$
$$E_{121} = \frac{h^2}{8mL^2}\left(1^2 + 2^2 + 1^2\right) = 6(6.024\ \mu\text{eV}) = 36.2\ \mu\text{eV}$$
$$E_{122} = \frac{h^2}{8mL^2}\left(1^2 + 2^2 + 2^2\right) = 9(6.024\ \mu\text{eV}) = 54.3\ \mu\text{eV}$$
$$E_{131} = \frac{h^2}{8mL^2}\left(1^2 + 3^2 + 1^2\right) = 11(6.024\ \mu\text{eV}) = 66.3\ \mu\text{eV}$$
$$E_{222} = \frac{h^2}{8mL^2}\left(2^2 + 2^2 + 2^2\right) = 12(6.024\ \mu\text{eV}) = 72.4\ \mu\text{eV}$$

Chapter 40

1. The magnitude L of the orbital angular momentum $\vec{L}$ is given by Eq. 40-2: $L=\sqrt{\ell(\ell+1)}\hbar$. On the other hand, the components L_z are $L_z = m_\ell \hbar$, where $m_\ell = -\ell, ... +\ell$. Thus, the semi-classical angle is $\cos\theta = L_z / L$. The angle is the smallest when $m=\ell$, or

$$\cos\theta = \frac{\ell\hbar}{\sqrt{\ell(\ell+1)}\hbar} \quad\Rightarrow\quad \theta = \cos^{-1}\left(\frac{\ell}{\sqrt{\ell(\ell+1)}}\right).$$

With $\ell = 5$, we have $\theta = \cos^{-1}(5/\sqrt{30}) = 24.1°$.

2. For a given quantum number n there are n possible values of ℓ, ranging from 0 to $n-1$. For each ℓ the number of possible electron states is $N_\ell = 2(2\ell + 1)$. Thus the total number of possible electron states for a given n is

$$N_n = \sum_{\ell=0}^{n-1} N_\ell = 2\sum_{\ell=0}^{n-1} (2\ell+1) = 2n^2.$$

Thus, in this problem, the total number of electron states is $N_n = 2n^2 = 2(5)^2 = 50$.

3. (a) We use Eq. 40-2:

$$L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{3(3+1)}\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right) = 3.65\times10^{-34}\ \text{J}\cdot\text{s}.$$

(b) We use Eq. 40-7: $L_z = m_\ell \hbar$. For the maximum value of L_z set $m_\ell = \ell$. Thus

$$[L_z]_{\text{max}} = \ell\hbar = 3\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right) = 3.16\times10^{-34}\ \text{J}\cdot\text{s}.$$

4. For a given quantum number n there are n possible values of ℓ, ranging from 0 to $n - 1$. For each ℓ the number of possible electron states is $N_\ell = 2(2\ell + 1)$. Thus, the total number of possible electron states for a given n is

$$N_n = \sum_{l=0}^{n-1} N_\ell = 2\sum_{l=0}^{n-1} (2\ell+1) = 2n^2.$$

(a) In this case $n = 4$, which implies $N_n = 2(4^2) = 32$.

(b) Now $n = 1$, so $N_n = 2(1^2) = 2$.

(c) Here $n = 3$, and we obtain $N_n = 2(3^2) = 18$.

(d) Finally, $n = 2 \rightarrow N_n = 2(2^2) = 8$.

5. (a) For a given value of the principal quantum number n, the orbital quantum number ℓ ranges from 0 to $n - 1$. For $n = 3$, there are three possible values: 0, 1, and 2.

(b) For a given value of ℓ, the magnetic quantum number m_ℓ ranges from $-\ell$ to $+\ell$. For $\ell = 1$, there are three possible values: – 1, 0, and +1.

6. For a given quantum number ℓ there are $(2\ell + 1)$ different values of m_ℓ. For each given m_ℓ the electron can also have two different spin orientations. Thus, the total number of electron states for a given ℓ is given by $N_\ell = 2(2\ell + 1)$.

(a) Now $\ell = 3$, so $N_\ell = 2(2 \times 3 + 1) = 14$.

(b) In this case, $\ell = 1$, which means $N_\ell = 2(2 \times 1 + 1) = 6$.

(c) Here $\ell = 1$, so $N_\ell = 2(2 \times 1 + 1) = 6$.

(d) Now $\ell = 0$, so $N_\ell = 2(2 \times 0 + 1) = 2$.

7. (a) Using Table 40-1, we find $\ell = [m_\ell]_{\max} = 4$.

(b) The smallest possible value of n is $n = \ell_{\max} + 1 \geq \ell + 1 = 5$.

(c) As usual, $m_s = \pm\frac{1}{2}$, so two possible values.

8. (a) For $\ell = 3$, the greatest value of m_ℓ is $m_\ell = 3$.

(b) Two states ($m_s = \pm\frac{1}{2}$) are available for $m_\ell = 3$.

(c) Since there are 7 possible values for m_ℓ : +3, +2, +1, 0, – 1, – 2, – 3, and two possible values for m_s, the total number of state available in the subshell $\ell = 3$ is 14.

9. **THINK** Knowing the value of ℓ, the orbital quantum number, allows us to determine the magnitudes of the angular momentum and the magnetic dipole moment.

EXPRESS The magnitude of the orbital angular momentum is

$$L = \sqrt{\ell(\ell+1)}\hbar.$$

Similarly, with $\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}$, the magnitude of $\vec{\mu}_{\text{orb}}$ is

$$\mu_{\text{orb}} = \frac{e\hbar}{2m}\sqrt{\ell(\ell+1)} = \mu_{\text{B}},$$

where $\mu_{\text{B}} = e\hbar/2m$ is the Bohr magneton.

ANALYZE (a) For $\ell = 3$, we have

$$L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{3(3+1)}\hbar = \sqrt{12}\hbar.$$

So the multiple is $\sqrt{12} \approx 3.46$.

(b) The magnitude of the orbital dipole moment is

$$\mu_{\text{orb}} = \sqrt{\ell(\ell+1)}\mu_B = \sqrt{12}\mu_B.$$

So the multiple is $\sqrt{12} \approx 3.46$.

(c) The largest possible value of m_ℓ is $m_\ell = \ell = 3$.

(d) We use $L_z = m_\ell\hbar$ to calculate the z component of the orbital angular momentum. The multiple is $m_\ell = 3$.

(e) We use $\mu_z = -m_\ell\mu_B$ to calculate the z component of the orbital magnetic dipole moment. The multiple is $-m_\ell = -3$.

(f) We use $\cos\theta = m_\ell\big/\sqrt{\ell(\ell+1)}$ to calculate the angle between the orbital angular momentum vector and the z axis. For $\ell = 3$ and $m_\ell = 3$, we have $\cos\theta = 3/\sqrt{12} = \sqrt{3}/2$, or $\theta = 30.0°$.

(g) For $\ell = 3$ and $m_\ell = 2$, we have $\cos\theta = 2/\sqrt{12} = 1/\sqrt{3}$, or $\theta = 54.7°$.

(h) For $\ell = 3$ and $m_\ell = -3$, $\cos\theta = -3/\sqrt{12} = -\sqrt{3}/2$, or $\theta = 150°$.

LEARN Neither $\vec{L}$ nor $\vec{\mu}_{\text{orb}}$ can be measured in any way. We can, however, measure their z components.

10. (a) For $n = 3$ there are 3 possible values of ℓ: 0, 1, and 2.

(b) We interpret this as asking for the number of distinct values for m_ℓ (this ignores the multiplicity of any particular value). For each ℓ there are $2\ell + 1$ possible values of m_ℓ. Thus the number of possible m_ℓ's for $\ell = 2$ is $(2\ell + 1) = 5$. Examining the $\ell = 1$ and $\ell = 0$ cases cannot lead to any new (distinct) values for m_ℓ, so the answer is 5.

(c) Regardless of the values of n, ℓ and m_ℓ, for an electron there are always two possible values of m_s: $\pm\frac{1}{2}$.

(d) The population in the $n = 3$ shell is equal to the number of electron states in the shell, or $2n^2 = 2(3^2) = 18$.

(e) Each subshell has its own value of ℓ. Since there are three different values of ℓ for $n = 3$, there are three subshells in the $n = 3$ shell.

11. **THINK** We can only measure one component of $\vec{L}$, say L_z, but not all three components.

EXPRESS Since $L^2 = L_x^2 + L_y^2 + L_z^2, \sqrt{L_x^2 + L_y^2} = \sqrt{L^2 - L_z^2}$. Replacing L^2 with $\ell(\ell+1)\hbar^2$ and L_z with $m_\ell\hbar$, we obtain

$$\sqrt{L_x^2 + L_y^2} = \hbar\sqrt{\ell(\ell+1) - m_\ell^2} .$$

ANALYZE For a given value of ℓ, the greatest that m_ℓ can be is ℓ, so the smallest that $\sqrt{L_x^2 + L_y^2}$ can be is $\hbar\sqrt{\ell(\ell+1) - \ell^2} = \hbar\sqrt{\ell}$. The smallest possible magnitude of m_ℓ is zero, so the largest $\sqrt{L_x^2 + L_y^2}$ can be is $\hbar\sqrt{\ell(\ell+1)}$. Thus,

$$\hbar\sqrt{\ell} \le \sqrt{L_x^2 + L_y^2} \le \hbar\sqrt{\ell(\ell+1)} .$$

LEARN Once we have chosen to measure $\vec{L}$ along the z axis, the x- and y-components cannot be measured with infinite certainty.

12. The angular momentum of the rotating sphere, $\vec{L}_{\text{sphere}}$, is equal in magnitude but in opposite direction to $\vec{L}_{\text{atom}}$, the angular momentum due to the aligned atoms. The number of atoms in the sphere is $N = \dfrac{N_A m}{M}$, where $N_A = 6.02\times10^{23}\,/\text{mol}$ is Avogadro's number and $M = 0.0558\,\text{kg/mol}$ is the molar mass of iron. The angular momentum due to the aligned atoms is

$$L_{\text{atom}} = 0.12N(m_s\hbar) = 0.12\frac{N_A m}{M}\frac{\hbar}{2}.$$

On the other hand, the angular momentum of the rotating sphere is (see Table 10-2 for *I*)

$$L_{\text{sphere}} = I\omega = \left(\frac{2}{5}mR^2\right)\omega.$$

Equating the two expressions, the mass *m* cancels out and the angular velocity is

$$\omega = 0.12\frac{5N_A\hbar}{4MR^2} = 0.12\frac{5(6.02\times10^{23}/\text{mol})(6.63\times10^{-34}\ \text{J}\cdot\text{s}/2\pi)}{4(0.0558\ \text{kg/mol})(2.00\times10^{-3}\ \text{m})^2} = 4.27\times10^{-5}\ \text{rad/s}.$$

13. **THINK** A gradient magnetic field gives rise to a magnetic force on the silver atom.

EXPRESS The force on the silver atom is given by

$$F_z = -\frac{dU}{dz} = -\frac{d}{dz}\left(-\mu_z B\right) = \mu_z\frac{dB}{dz}$$

where μ_z is the *z* component of the magnetic dipole moment of the silver atom, and *B* is the magnetic field. The acceleration is

$$a = \frac{F_z}{M} = \frac{\mu_z(dB/dz)}{M},$$

where *M* is the mass of a silver atom.

ANALYZE Using the data given in Sample Problem —"Beam separation in a Stern-Gerlach experiment," we obtain

$$a = \frac{\left(9.27\times10^{-24}\ \text{J/T}\right)\left(1.4\times10^{3}\ \text{T/m}\right)}{1.8\times10^{-25}\,\text{kg}} = 7.2\times10^{4}\ \text{m/s}^2.$$

LEARN The deflection of the silver atom is due to the interaction between the magnetic dipole moment of the atom and the magnetic field. However, if the field is uniform, then $dB/dz = 0$, and the silver atom will pass the poles undeflected.

14. (a) From Eq. 40-19,

$$F = \mu_B\left|\frac{dB}{dz}\right| = \left(9.27\times10^{-24}\ \text{J/T}\right)\left(1.6\times10^{2}\ \text{T/m}\right) = 1.5\times10^{-21}\ \text{N}.$$

(b) The vertical displacement is

$$\Delta x = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{l}{v}\right)^2 = \frac{1}{2}\left(\frac{1.5\times10^{-21}\,\text{N}}{1.67\times10^{-27}\,\text{kg}}\right)\left(\frac{0.80\text{m}}{1.2\times10^{5}\ \text{m/s}}\right)^2 = 2.0\times10^{-5}\,\text{m}.$$

15. The magnitude of the spin angular momentum is

$$S = \sqrt{s(s+1)}\hbar = \left(\sqrt{3}/2\right)\hbar,$$

where $s = \frac{1}{2}$ is used. The z component is either $S_z = \hbar/2$ or $-\hbar/2$.

(a) If $S_z = +\hbar/2$ the angle θ between the spin angular momentum vector and the positive z axis is

$$\theta = \cos^{-1}\left(\frac{S_z}{S}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°\ .$$

(b) If $S_z = -\hbar/2$, the angle is $\theta = 180° - 54.7° = 125.3° \approx 125°$.

16. (a) From Fig. 40-10 and Eq. 40-18,

$$\Delta E = 2\mu_B B = \frac{2\left(9.27\times10^{-24}\ \text{J/T}\right)\left(0.50\,\text{T}\right)}{1.60\times10^{-19}\ \text{J/eV}} = 58\,\mu\text{eV}\ .$$

(b) From $\Delta E = hf$ we get

$$f = \frac{\Delta E}{h} = \frac{9.27\times10^{-24}\ \text{J}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}} = 1.4\times10^{10}\ \text{Hz} = 14\ \text{GHz}\ .$$

(c) The wavelength is

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\ \text{m/s}}{1.4\times10^{10}\,\text{Hz}} = 2.1\ \text{cm}.$$

(d) The wave is in the short radio wave region.

17. The total magnetic field, $B = B_{\text{local}} + B_{\text{ext}}$, satisfies $\Delta E = hf = 2\mu B$ (see Eq. 40-22). Thus,

$$B_{\text{local}} = \frac{hf}{2\mu} - B_{\text{ext}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(34\times10^{6}\ \text{Hz}\right)}{2\left(1.41\times10^{-26}\ \text{J/T}\right)} - 0.78\ \text{T} = 19\ \text{mT}\ .$$

18. We let $\Delta E = 2\mu_B B_{\text{eff}}$ (based on Fig. 40-10 and Eq. 40-18) and solve for B_{eff}:

$$B_{\text{eff}} = \frac{\Delta E}{2\mu_B} = \frac{hc}{2\lambda\mu_B} = \frac{1240\,\text{nm}\cdot\text{eV}}{2\left(21\times10^{-7}\text{ nm}\right)\left(5.788\times10^{-5}\text{ eV/T}\right)} = 51\text{ mT}.$$

19. The energy of a magnetic dipole in an external magnetic field $\vec{B}$ is $U = -\vec{\mu}\cdot\vec{B} = -\mu_z B$, where $\vec{\mu}$ is the magnetic dipole moment and μ_z is its component along the field. The energy required to change the moment direction from parallel to antiparallel is $\Delta E = \Delta U = 2\mu_z B$. Since the z component of the spin magnetic moment of an electron is the Bohr magneton μ_B,

$$\Delta E = 2\mu_B B = 2\left(9.274\times10^{-24}\text{ J/T}\right)\left(0.200\text{ T}\right) = 3.71\times10^{-24}\text{ J}.$$

The photon wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.998\times10^{8}\text{ m/s}\right)}{3.71\times10^{-24}\text{ J}} = 5.35\times10^{-2}\text{ m}.$$

20. Using Eq. 39-20 we find that the lowest four levels of the rectangular corral (with this specific "aspect ratio") are nondegenerate, with energies $E_{1,1}$ = 1.25, $E_{1,2}$ = 2.00, $E_{1,3}$ = 3.25, and $E_{2,1}$ = 4.25 (all of these understood to be in "units" of $h^2/8mL^2$). Therefore, obeying the Pauli principle, we have

$$E_{\text{ground}} = 2E_{1,1} + 2E_{1,2} + 2E_{1,3} + E_{2,1} = 2(1.25) + 2(2.00) + 2(3.25) + 4.25$$

which means (putting the "unit" factor back in) that the lowest possible energy of the system is $E_{\text{ground}} = 17.25(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 17.25.

21. Because of the Pauli principle (and the requirement that we construct a state of lowest possible total energy), two electrons fill the n = 1, 2, 3 levels and one electron occupies the n = 4 level. Thus, using Eq. 39-4,

$$\begin{aligned}E_{\text{ground}} &= 2E_1 + 2E_2 + 2E_3 + E_4\\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + \left(\frac{h^2}{8mL^2}\right)(4)^2\\ &= \left(2+8+18+16\right)\left(\frac{h^2}{8mL^2}\right) = 44\left(\frac{h^2}{8mL^2}\right).\end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 44.

22. Due to spin degeneracy ($m_s = \pm1/2$), each state can accommodate two electrons. Thus, in the energy-level diagram shown, two electrons can be placed in the ground state with energy $E_1 = 4(h^2/8mL^2)$, six can occupy the "triple state" with $E_2 = 6(h^2/8mL^2)$,

and so forth. With 11 electrons, the lowest energy configuration consists of two electrons with $E_1 = 4(h^2/8mL^2)$, six electrons with $E_2 = 6(h^2/8mL^2)$, and three electrons with $E_3 = 7(h^2/8mL^2)$. Thus, we find the ground-state energy of the 11-electron system to be

$$E_{\text{ground}} = 2E_1 + 6E_2 + 3E_3 = 2\left(\frac{4h^2}{8mL^2}\right) + 6\left(\frac{6h^2}{8mL^2}\right) + 3\left(\frac{7h^2}{8mL^2}\right)$$
$$= \left[(2)(4) + (6)(6) + (3)(7)\right]\left(\frac{h^2}{8mL^2}\right) = 65\left(\frac{h^2}{8mL^2}\right).$$

The first excited state of the 11-electron system consists of two electrons with $E_1 = 4(h^2/8mL^2)$, five electrons with $E_2 = 6(h^2/8mL^2)$, and four electrons with $E_3 = 7(h^2/8mL^2)$. Thus, its energy is

$$E_{\text{1st excited}} = 2E_1 + 5E_2 + 4E_3 = 2\left(\frac{4h^2}{8mL^2}\right) + 5\left(\frac{6h^2}{8mL^2}\right) + 4\left(\frac{7h^2}{8mL^2}\right)$$
$$= \left[(2)(4) + (5)(6) + (4)(7)\right]\left(\frac{h^2}{8mL^2}\right) = 66\left(\frac{h^2}{8mL^2}\right).$$

Thus, the multiple of $h^2/8mL^2$ is 66.

23. **THINK** With eight electrons, the ground-state energy of the system is the sum of the energies of the individual electrons in the system's ground-state configuration.

EXPRESS In terms of the quantum numbers n_x, n_y, and n_z, the single-particle energy levels are given by

$$E_{n_x,n_y,n_z} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right).$$

The lowest single-particle level corresponds to $n_x = 1$, $n_y = 1$, and $n_z = 1$ and is $E_{1,1,1} = 3(h^2/8mL^2)$. There are two electrons with this energy, one with spin up and one with spin down. The next lowest single-particle level is three-fold degenerate in the three integer quantum numbers. The energy is

$$E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2).$$

Each of these states can be occupied by a spin up and a spin down electron, so six electrons in all can occupy the states. This completes the assignment of the eight electrons to single-particle states.

ANALYZE The ground state energy of the system is

$$E_{\text{gr}} = (2)(3)(h^2/8mL^2) + (6)(6)(h^2/8mL^2) = 42(h^2/8mL^2).$$

Thus, the multiple of $h^2/8mL^2$ is 42.

LEARN We summarize the ground-state configuration and the energies (in multiples of $h^2/8mL^2$) in the chart below:

n_x	n_y	n_z	m_s	energy
1	1	1	–1/2, + 1/2	3 + 3
1	1	2	–1/2, + 1/2	6 + 6
1	2	1	–1/2, + 1/2	6 + 6
2	1	1	–1/2, + 1/2	6 + 6
			total	42

24. (a) Using Eq. 39-20 we find that the lowest five levels of the rectangular corral (with this specific "aspect ratio") have energies

$$E_{1,1} = 1.25,\ E_{1,2} = 2.00,\ E_{1,3} = 3.25,\ E_{2,1} = 4.25,\ E_{2,2} = 5.00$$

(all of these understood to be in "units" of $h^2/8mL^2$). It should be noted that the energy level we denote $E_{2,2}$ actually corresponds to two energy levels ($E_{2,2}$ and $E_{1,4}$; they are degenerate), but that will not affect our calculations in this problem. The configuration that provides the lowest system energy higher than that of the ground state has the first three levels filled, the fourth one empty, and the fifth one half-filled:

$$E_{\text{first excited}} = 2E_{1,1} + 2E_{1,2} + 2E_{1,3} + E_{2,2} = 2(1.25) + 2(2.00) + 2(3.25) + 5.00$$

which means (putting the "unit" factor back in) the energy of the first excited state is $E_{\text{first excited}} = 18.00(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 18.00.

(b) The configuration that provides the next higher system energy has the first two levels filled, the third one half-filled, and the fourth one filled:

$$E_{\text{second excited}} = 2E_{1,1} + 2E_{1,2} + E_{1,3} + 2E_{2,1} = 2(1.25) + 2(2.00) + 3.25 + 2(4.25)$$

which means (putting the "unit" factor back in) the energy of the second excited state is

$$E_{\text{second excited}} = 18.25(h^2/8mL^2).$$

Thus, the multiple of $h^2/8mL^2$ is 18.25.

(c) Now, the configuration that provides the *next* higher system energy has the first two levels filled, with the next three levels half-filled:

$$E_{\text{third excited}} = 2E_{1,1} + 2E_{1,2} + E_{1,3} + E_{2,1} + E_{2,2} = 2(1.25) + 2(2.00) + 3.25 + 4.25 + 5.00$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 19.00(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 19.00.

(d) The energy states of this problem and Problem 40-22 are suggested below:

____________________ third excited $19.00(h^2/8mL^2)$

____________________ second excited $18.25(h^2/8mL^2)$

____________________ first excited $18.00(h^2/8mL^2)$

____________________ ground state $17.25(h^2/8mL^2)$

25. (a) Promoting one of the electrons (described in Problem 40-21) to a not-fully occupied higher level, we find that the configuration with the least total energy greater than that of the ground state has the $n = 1$ and 2 levels still filled, but now has only one electron in the $n = 3$ level; the remaining two electrons are in the $n = 4$ level. Thus,

$$\begin{aligned}E_{\text{first excited}} &= 2E_1 + 2E_2 + E_3 + 2E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + \left(\frac{h^2}{8mL^2}\right)(3)^2 + 2\left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2 + 8 + 9 + 32)\left(\frac{h^2}{8mL^2}\right) = 51\left(\frac{h^2}{8mL^2}\right).\end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 51.

(b) Now, the configuration which provides the next higher total energy, above that found in part (a), has the bottom three levels filled (just as in the ground state configuration) and has the seventh electron occupying the $n = 5$ level:

$$\begin{aligned}E_{\text{second excited}} &= 2E_1 + 2E_2 + 2E_3 + E_5 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + \left(\frac{h^2}{8mL^2}\right)(5)^2 \\ &= (2 + 8 + 18 + 25)\left(\frac{h^2}{8mL^2}\right) = 53\left(\frac{h^2}{8mL^2}\right).\end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 53.

(c) The third excited state has the n = 1, 3, 4 levels filled, and the n = 2 level half-filled:

$$\begin{aligned} E_{\text{third excited}} &= 2E_1 + E_2 + 2E_3 + 2E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + \left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + 2\left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2+4+18+32)\left(\frac{h^2}{8mL^2}\right) = 56\left(\frac{h^2}{8mL^2}\right). \end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 56.

(d) The energy states of this problem and Problem 40-21 are suggested below:

________________________ third excited $56(h^2/8mL^2)$

________________________ second excited $53(h^2/8mL^2)$

________________________ first excited $51(h^2/8mL^2)$

________________________ ground state $44(h^2/8mL^2)$

26. The energy levels are given by

$$E_{n_x,n_y,n_z} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right).$$

The Pauli principle requires that no more than two electrons be in the lowest energy level (at $E_{1,1,1} = 3(h^2/8mL^2)$ with $n_x = n_y = n_z = 1$), but — due to their degeneracies — as many as six electrons can be in the next three levels,

$$\begin{aligned} E' &= E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2) \\ E'' &= E_{1,2,2} = E_{2,2,1} = E_{2,1,2} = 9(h^2/8mL^2) \\ E''' &= E_{1,1,3} = E_{1,3,1} = E_{3,1,1} = 11(h^2/8mL^2). \end{aligned}$$

Using Eq. 39-21, the level above those can only hold two electrons:

$$E_{2,2,2} = (2^2 + 2^2 + 2^2)(h^2/8mL^2) = 12(h^2/8mL^2).$$

And the next higher level can hold as much as twelve electrons and has energy

$$E'''' = 14(h^2/8mL^2).$$

(a) The configuration that provides the lowest system energy higher than that of the ground state has the first level filled, the second one with one vacancy, and the third one with one occupant:

$$E_{\text{first excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 9$$

which means (putting the "unit" factor back in) the energy of the first excited state is

$$E_{\text{first excited}} = 45(h^2/8mL^2).$$

Thus, the multiple of $h^2/8mL^2$ is 45.

(b) The configuration that provides the next higher system energy has the first level filled, the second one with one vacancy, the third one empty, and the fourth one with one occupant:

$$E_{\text{second excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 11$$

which means (putting the "unit" factor back in) the energy of the second excited state is $E_{\text{second excited}} = 47(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 47.

(c) Now, there are a couple of configurations that provide the *next* higher system energy. One has the first level filled, the second one with one vacancy, the third and fourth ones empty, and the fifth one with one occupant:

$$E_{\text{third excited}} = 2E_{1,1,1} + 5E' + E''' = 2(3) + 5(6) + 12$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 48(h^2/8mL^2)$. Thus, the multiple of $h^2/8mL^2$ is 48. The other configuration with this same total energy has the first level filled, the second one with two vacancies, and the third one with one occupant.

(d) The energy states of this problem and Problem 40-25 are suggested below:

____________________ third excited $48(h^2/8mL^2)$

____________________ second excited $47(h^2/8mL^2)$

____________________ first excited $45(h^2/8mL^2)$

____________________ ground state $42(h^2/8mL^2)$

27. **THINK** The four quantum numbers (n, ℓ, m_ℓ, m_s) identify the quantum states of individual electrons in a multi-electron atom.

EXPRESS A lithium atom has three electrons. The first two electrons have quantum numbers (1, 0, 0, $\pm 1/2$). All states with principal quantum number n = 1 are filled. The next lowest states have n = 2.

The orbital quantum number can have the values $\ell = 0$ or 1 and of these, the $\ell = 0$ states have the lowest energy. The magnetic quantum number must be $m_\ell = 0$ since this is the only possibility if $\ell = 0$. The spin quantum number can have either of the values $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Since there is no external magnetic field, the energies of these two states are the same.

ANALYZE (a) Therefore, in the ground state, the quantum numbers of the third electron are either $n = 2, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2}$ or $n = 2, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2}$. That is, (n, ℓ, m_ℓ, m_s) = (2,0,0, +1/2) and (2,0,0, −1/2).

(b) The next lowest state in energy is an n = 2, $\ell = 1$ state. All n = 3 states are higher in energy. The magnetic quantum number can be $m_\ell = -1, 0, \text{or} +1$; the spin quantum number can be $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Thus, (n, ℓ, m_ℓ, m_s) = (2,1,1, +1/2), (2,1,1, −1/2), $(2,1,0,+1/2)$, $(2,1,0,-1/2)$, $(2,1,-1,+1/2)$ and $(2,1,-1,-1/2)$.

LEARN No two electrons can have the same set of quantum numbers, as required by the Pauli exclusion principle.

28. For a given value of the principal quantum number n, there are n possible values of the orbital quantum number ℓ, ranging from 0 to n – 1. For any value of ℓ, there are $2\ell + 1$ possible values of the magnetic quantum number m_ℓ, ranging from $-\ell$ to $+\ell$. Finally, for each set of values of ℓ and m_ℓ, there are two states, one corresponding to the spin quantum number $m_s = -\frac{1}{2}$ and the other corresponding to $m_s = +\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N = 2\sum_{\ell=0}^{n-1}(2\ell + 1).$$

Now

$$\sum_{\ell=0}^{n-1} 2\ell = 2\sum_{\ell=0}^{n-1} \ell = 2\frac{n}{2}(n-1) = n(n-1),$$

since there are n terms in the sum and the average term is $(n - 1)/2$. Furthermore,

$$\sum_{\ell=0}^{n-1} 1 = n\ .$$

Thus, $N = 2\left[n(n-1)+n\right] = 2n^2$.

29. The total number of possible electron states for a given quantum number *n* is

$$N_n = \sum_{\ell=0}^{n-1} N_\ell = 2\sum_{\ell=0}^{n-1}(2\ell+1) = 2n^2.$$

Thus, if we ignore any electron-electron interaction, then with 110 electrons, we would have two electrons in the $n=1$ shell, eight in the $n=2$ shell, 18 in the $n=3$ shell, 32 in the $n=4$ shell, and the remaining 50 ($=110-2-8-18-32$) in the $n=5$ shell. The 50 electrons would be placed in the subshells in the order $s, p, d, f, g, h, \ldots$ and the resulting configuration is $5s^2 5p^6 5d^{10} 5f^{14} 5g^{18}$. Therefore, the spectroscopic notation for the quantum number ℓ of the last electron would be *g*.

Note, however, when the electron-electron interaction is considered, the ground-state electronic configuration of darmstadtium actually is $[\mathrm{Rn}]5f^{14}6d^9 7s^1$, where

$$[\mathrm{Rn}]: 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6$$

represents the inner-shell electrons.

30. When a helium atom is in its ground state, both of its electrons are in the 1*s* state. Thus, for each of the electrons, *n* = 1, ℓ = 0, and m_ℓ = 0. One of the electrons is spin up $\left(m_s = +\tfrac{1}{2}\right)$ while the other is spin down $\left(m_s = -\tfrac{1}{2}\right)$. Thus,

(a) the quantum numbers (n, ℓ, m_ℓ, m_s) for the spin-up electron are (1,0,0,+1/2), and

(b) the quantum numbers (n, ℓ, m_ℓ, m_s) for the spin-down electron are (1,0,0,–1/2).

31. The first three shells (*n* = 1 through 3), which can accommodate a total of 2 + 8 + 18 = 28 electrons, are completely filled. For selenium (*Z* = 34) there are still 34 – 28 = 6 electrons left. Two of them go to the 4*s* subshell, leaving the remaining four in the highest occupied subshell, the 4*p* subshell.

(a) The highest occupied subshell is 4*p*.

(b) There are four electrons in the 4*p* subshell.

For bromine (*Z* = 35) the highest occupied subshell is also the 4*p* subshell, which contains five electrons.

(c) The highest occupied subshell is 4*p*.

(d) There are five electrons in the 4*p* subshell.

For krypton (Z = 36) the highest occupied subshell is also the 4*p* subshell, which now accommodates six electrons.

(e) The highest occupied subshell is 4*p*.

(f) There are six electrons in the 4*p* subshell.

32. (a) The number of different m_ℓ's is $2\ell+1=3$, ($m_\ell = 1, 0, -1$) and the number of different m_s's is 2, which we denote as +1/2 and −1/2. The allowed states are $(m_{\ell 1}, m_{s1}, m_{\ell 2}, m_{s2})$= (1, +1/2, 1, −1/2), (1, +1/2, 0, +1/2), (1, +1/2, 0, −1/2), (1, +1/2, −1, +1/2), (1, +1/2, −1, −1/2), (1, −1/2, 0, +1/2), (1, −1/2, 0, −1/2), (1, −1/2, −1, +1/2), (1, −1/2, −1, −1/2), (0, +1/2, 0, −1/2), (0, +1/2, −1, +1/2), (0, +1/2, −1, −1/2), (0, −1/2, −1, + 1/2), (0, −1/2, −1, −1/2), (−1, +1/2, −1, −1/2). So, there are 15 states.

(b) There are six states disallowed by the exclusion principle, in which both electrons share the quantum numbers: $(m_{\ell 1}, m_{s1}, m_{\ell 2}, m_{s2})$=(1, +1/2, 1, +1/2), (1, −1/2, 1, −1/2), (0, +1/2, 0, +1/2), (0, −1/2, 0, −1/2), (−1, +1/2, −1, +1/2), (−1, −1/2, −1, −1/2). So, if the Pauli exclusion principle is not applied, then there would be 15 + 6 = 21 allowed states.

33. The kinetic energy gained by the electron is eV, where V is the accelerating potential difference. A photon with the minimum wavelength (which, because of $E = hc/\lambda$, corresponds to maximum photon energy) is produced when all of the electron's kinetic energy goes to a single photon in an event of the kind depicted in Fig. 40-15. Thus, with $hc = 1240\,\text{eV}\cdot\text{nm}$,

$$eV = \frac{hc}{\lambda_{\min}} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.10\,\text{nm}} = 1.24\times 10^4\ \text{eV}\ .$$

Therefore, the accelerating potential difference is $V = 1.24 \times 10^4$ V = 12.4 kV.

34. With hc = 1240 eV·nm = 1240 keV·pm, for the K_α line from iron, the energy difference is

$$\Delta E = \frac{hc}{\lambda} = \frac{1240\text{keV}\cdot\text{pm}}{193\text{pm}} = 6.42\ \text{keV}.$$

We remark that for the hydrogen atom the corresponding energy difference is

$$\Delta E_{12} = -(13.6\,\text{eV})\left(\frac{1}{2^2} - \frac{1}{1^1}\right) = 10\,\text{eV}\ .$$

That this difference is much greater in iron is due to the fact that its atomic nucleus contains 26 protons, exerting a much greater force on the K- and L-shell electrons than that provided by the single proton in hydrogen.

35. **THINK** X-rays are produced when a solid target (silver in this case) is bombarded with electrons whose kinetic energies are in the keV range.

EXPRESS The wavelength is $\lambda_{\min} = hc/K_0$, where K_0 is the initial kinetic energy of the incident electron.

ANALYZE (a) With $hc = 1240\ \text{eV}\cdot\text{nm}$, we obtain

$$\lambda_{\min} = \frac{hc}{K_0} = \frac{1240\,\text{eV}\cdot\text{nm}}{35\times10^3\ \text{eV}} = 3.54\times10^{-2}\ \text{nm} = 35.4\,\text{pm}\ .$$

(b) A K_α photon results when an electron in a target atom jumps from the *L*-shell to the *K*-shell. The energy of this photon is

$$E = 25.51\ \text{keV} - 3.56\ \text{keV} = 21.95\ \text{keV}$$

and its wavelength is

$$\lambda_{K\alpha} = hc/E = (1240\ \text{eV}\cdot\text{nm})/(21.95\times10^3\ \text{eV}) = 5.65\times10^{-2}\ \text{nm} = 56.5\ \text{pm}.$$

(c) A K_β photon results when an electron in a target atom jumps from the *M*-shell to the *K*-shell. The energy of this photon is 25.51 keV – 0.53 keV = 24.98 keV and its wavelength is

$$\lambda_{K\beta} = (1240\,\text{eV}\cdot\text{nm})/(24.98\times10^3\ \text{eV}) = 4.96\times10^{-2}\ \text{nm} = 49.6\ \text{pm}.$$

LEARN Note that the cut-off wavelength $\lambda_{\min}$ is characteristic of the incident electrons, not of the target material.

36. (a) We use $eV = hc/\lambda_{\min}$ (see Eq. 40-23 and Eq. 38-4). With hc = 1240 eV·nm = 1240 keV·pm, the mean value of $\lambda_{\min}$ is

$$\lambda_{\min} = \frac{hc}{eV} = \frac{1240\,\text{keV}\cdot\text{pm}}{50.0\,\text{keV}} = 24.8\,\text{pm}\ .$$

(b) The values of λ for the K_α and K_β lines do not depend on the external potential and are therefore unchanged.

37. Suppose an electron with total energy *E* and momentum *p* spontaneously changes into a photon. If energy is conserved, the energy of the photon is *E* and its momentum has magnitude *E*/*c*. Now the energy and momentum of the electron are related by

$$E^2 = (pc)^2 + (mc^2)^2 \quad \Rightarrow \quad pc = \sqrt{E^2 - (mc^2)^2}\ .$$

Since the electron has nonzero mass, E/c and p cannot have the same value. Hence, momentum cannot be conserved. A third particle must participate in the interaction, primarily to conserve momentum. It does, however, carry off some energy.

38. From the data given in the problem, we calculate frequencies (using Eq. 38-1), take their square roots, look up the atomic numbers (see Appendix F), and do a least-squares fit to find the slope: the result is 5.02×10^7 with the odd-sounding unit of a square root of a hertz. We remark that the least squares procedure also returns a value for the y-intercept of this statistically determined "best-fit" line; that result is negative and would appear on a graph like Fig. 40-17 to be at about – 0.06 on the vertical axis. Also, we can estimate the slope of the Moseley line shown in Fig. 40-17:

$$\frac{(1.95-0.50)10^9\,\text{Hz}^{1/2}}{40-11} \approx 5.0\times 10^7\,\text{Hz}^{1/2} .$$

39. **THINK** The frequency of an x-ray emission is proportional to $(Z - 1)^2$, where Z is the atomic number of the target atom.

EXPRESS The ratio of the wavelength λ_{Nb} for the K_α line of niobium to the wavelength λ_{Ga} for the K_α line of gallium is given by

$$\lambda_{\text{Nb}}/\lambda_{\text{Ga}} = (Z_{\text{Ga}}-1)^2/(Z_{\text{Nb}}-1)^2 ,$$

where Z_{Nb} is the atomic number of niobium (41) and Z_{Ga} is the atomic number of gallium (31). Thus, $\lambda_{\text{Nb}}/\lambda_{\text{Ga}} = (30)^2/(40)^2 = 9/16 \approx 0.563$.

LEARN The frequency of the K_α line is given by Eq. 40-26:

$$f = (2.46\times 10^{15}\ \text{Hz})(Z-1)^2 .$$

40. (a) According to Eq. 40-26, $f \propto (Z-1)^2$, so the ratio of energies is (using Eq. 38-2)

$$\frac{f}{f'} = \left(\frac{Z-1}{Z'-1}\right)^2 .$$

(b) We refer to Appendix F. Applying the formula from part (a) to $Z = 92$ and $Z' = 13$, we obtain

$$\frac{E}{E'} = \frac{f}{f'} = \left(\frac{Z-1}{Z'-1}\right)^2 = \left(\frac{92-1}{13-1}\right)^2 = 57.5 .$$

(c) Applying this to $Z = 92$ and $Z' = 3$, we obtain

$$\frac{E}{E'} = \left(\frac{92-1}{3-1}\right)^2 = 2.07\times10^3 .$$

41. We use Eq. 36-31, Eq. 39-6, and hc = 1240 eV·nm = 1240 keV·pm. Letting $2d\sin\theta = m\lambda = mhc/\Delta E$, where θ = 74.1°, we solve for d:

$$d = \frac{mhc}{2\Delta E\sin\theta} = \frac{(1)(1240\ \text{keV}\cdot\text{nm})}{2(8.979\ \text{keV} - 0.951\ \text{keV})(\sin 74.1°)} = 80.3\ \text{pm} .$$

42. Using hc = 1240 eV·nm = 1240 keV·pm, the energy difference $E_L - E_M$ for the x-ray atomic energy levels of molybdenum is

$$\Delta E = E_L - E_M = \frac{hc}{\lambda_L} - \frac{hc}{\lambda_M} = \frac{1240\ \text{keV}\cdot\text{pm}}{63.0\ \text{pm}} - \frac{1240\ \text{keV}\cdot\text{pm}}{71.0\ \text{pm}} = 2.2\ \text{keV} .$$

43. (a) An electron must be removed from the K-shell, so that an electron from a higher energy shell can drop. This requires an energy of 69.5 keV. The accelerating potential must be at least 69.5 kV.

(b) After it is accelerated, the kinetic energy of the bombarding electron is 69.5 keV. The energy of a photon associated with the minimum wavelength is 69.5 keV, so its wavelength is

$$\lambda_{\min} = \frac{1240\ \text{eV}\cdot\text{nm}}{69.5\times10^3\ \text{eV}} = 1.78\times10^{-2}\ \text{nm} = 17.8\ \text{pm} .$$

(c) The energy of a photon associated with the K_α line is 69.5 keV – 11.3 keV = 58.2 keV and its wavelength is

$$\lambda_{K\alpha} = (1240\ \text{eV}\cdot\text{nm})/(58.2 \times 10^3\ \text{eV}) = 2.13 \times 10^{-2}\ \text{nm} = 21.3\ \text{pm}.$$

(d) The energy of a photon associated with the K_β line is

$$E = 69.5\ \text{keV} - 2.30\ \text{keV} = 67.2\ \text{keV}$$

and its wavelength is, using hc = 1240 eV·nm,

$$\lambda_{K\beta} = hc/E = (1240\ \text{eV}\cdot\text{nm})/(67.2 \times 10^3\ \text{eV}) = 1.85 \times 10^{-2}\ \text{nm} = 18.5\ \text{pm}.$$

44. (a) and (b) Let the wavelength of the two photons be λ_1 and $\lambda_2 = \lambda_1 + \Delta\lambda$. Then,

$$eV = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_1 + \Delta\lambda} \quad \Rightarrow \quad \lambda_1 = \frac{-(\Delta\lambda/\lambda_0 - 2) \pm \sqrt{(\Delta\lambda/\lambda_0)^2 + 4}}{2/\Delta\lambda} .$$

Here, $\Delta\lambda$ = 130 pm and

$$\lambda_0 = hc/eV = 1240 \text{ keV}\cdot\text{pm}/20 \text{ keV} = 62 \text{ pm},$$

where we have used hc = 1240 eV·nm = 1240 keV·pm. We choose the plus sign in the expression for λ_1 (since $\lambda_1 > 0$) and obtain

$$\lambda_1 = \frac{-\left(130\,\text{pm}/62\,\text{pm} - 2\right) + \sqrt{\left(130\,\text{pm}/62\,\text{pm}\right)^2 + 4}}{2/62 \text{ pm}} = 87 \text{ pm}.$$

The energy of the electron after its first deceleration is

$$K = K_i - \frac{hc}{\lambda_1} = 20 \text{ keV} - \frac{1240 \text{ keV}\cdot\text{pm}}{87 \text{ pm}} = 5.7 \text{ keV}.$$

(c) The energy of the first photon is $E_1 = \frac{hc}{\lambda_1} = \frac{1240 \text{ keV}\cdot\text{pm}}{87 \text{ pm}} = 14 \text{ keV}$.

(d) The wavelength associated with the second photon is

$$\lambda_2 = \lambda_1 + \Delta\lambda = 87\,\text{pm} + 130\,\text{pm} = 2.2\times10^2 \text{ pm}.$$

(e) The energy of the second photon is $E_2 = \frac{hc}{\lambda_2} = \frac{1240 \text{ keV}\cdot\text{pm}}{2.2\times10^2\,\text{pm}} = 5.7 \text{ keV}$.

45. The initial kinetic energy of the electron is K_0 = 50.0 keV. After the first collision, the kinetic energy is K_1 = 25 keV; after the second, it is K_2 = 12.5 keV; and after the third, it is zero.

(a) The energy of the photon produced in the first collision is 50.0 keV – 25.0 keV = 25.0 keV. The wavelength associated with this photon is

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{25.0\times10^3 \text{ eV}} = 4.96\times10^{-2} \text{ nm} = 49.6 \text{ pm}$$

where we have used hc = 1240 eV·nm.

(b) The energies of the photons produced in the second and third collisions are each 12.5 keV and their wavelengths are

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{12.5\times10^3 \text{ eV}} = 9.92\times10^{-2} \text{ nm} = 99.2\,\text{pm}.$$

46. The transition is from $n = 2$ to $n = 1$, so Eq. 40-26 combined with Eq. 40-24 yields

$$f = \left(\frac{m_e e^4}{8\varepsilon_0^2 h^3}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(Z-1)^2$$

so that the constant in Eq. 40-27 is

$$C = \sqrt{\frac{3m_e e^4}{32\varepsilon_0^2 h^3}} = 4.9673 \times 10^7 \text{ Hz}^{1/2}$$

using the values in the next-to-last column in the table in Appendix B (but note that the power of ten is given in the middle column).

We are asked to compare the results of Eq. 40-27 (squared, then multiplied by the accurate values of h/e found in Appendix B to convert to x-ray energies) with those in the table of K_α energies (in eV) given at the end of the problem. We look up the corresponding atomic numbers in Appendix F.

(a) For Li, with $Z = 3$, we have

$$E_{\text{theory}} = \frac{h}{e}C^2(Z-1)^2 = \frac{6.6260688 \times 10^{-34}\text{ J}\cdot\text{s}}{1.6021765 \times 10^{-19}\text{ J/eV}}\left(4.9673 \times 10^7\text{ Hz}^{1/2}\right)^2(3-1)^2 = 40.817\,\text{eV}.$$

The percentage deviation is

$$\text{percentage deviation} = 100\left(\frac{E_{\text{theory}} - E_{\text{exp}}}{E_{\text{exp}}}\right) = 100\left(\frac{40.817 - 54.3}{54.3}\right) = -24.8\% \approx -25\%.$$

In subsequent calculations, we use the steps outlined above.

(b) For Be, with $Z = 4$, the percentage deviation is –15%.

(c) For B, with $Z = 5$, the percentage deviation is –11%.

(d) For C, with $Z = 6$, the percentage deviation is –7.9%.

(e) For N, with $Z = 7$, the percentage deviation is –6.4%.

(f) For O, with $Z = 8$, the percentage deviation is –4.7%.

(g) For F, with $Z = 9$, the percentage deviation is –3.5%.

(h) For Ne, with $Z = 10$, the percentage deviation is –2.6%.

(i) For Na, with Z = 11, the percentage deviation is –2.0%.

(j) For Mg, with Z = 12, the percentage deviation is –1.5%.

Note that the trend is clear from the list given above: the agreement between theory and experiment becomes better as Z increases. One might argue that the most questionable step in Section 40-10 is the replacement $e^4 \to (Z-1)^2 e^4$ and ask why this could not equally well be $e^4 \to (Z-.9)^2 e^4$ or $e^4 \to (Z-.8)^2 e^4$. For large Z, these subtleties would not matter so much as they do for small Z, since $Z - \xi \approx Z$ for $Z >> \xi$.

47. Let the power of the laser beam be P and the energy of each photon emitted be E. Then, the rate of photon emission is

$$R = \frac{P}{E} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} = \frac{(5.0\times10^{-3}\text{ W})(0.80\times10^{-6}\text{ m})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})} = 2.0\times10^{16}\text{ s}^{-1}.$$

48. The Moon is a distance $R = 3.82 \times 10^8$ m from Earth (see Appendix C). We note that the "cone" of light has apex angle equal to 2θ. If we make the small angle approximation (equivalent to using Eq. 36-14), then the diameter D of the spot on the Moon is

$$D = 2R\theta = 2R\left(\frac{1.22\lambda}{d}\right) = \frac{2(3.82\times10^{8}\text{ m})(1.22)(600\times10^{-9}\text{ m})}{0.12\text{ m}} = 4.7\times10^{3}\text{ m} = 4.7\text{ km}.$$

49. Let the range of frequency of the microwave be Δf. Then the number of channels that could be accommodated is

$$N = \frac{\Delta f}{10\text{ MHz}} = \frac{(2.998\times10^{8}\text{ m/s})[(450\text{ nm})^{-1} - (650\text{ nm})^{-1}]}{10\text{ MHz}} = 2.1\times10^{7}.$$

The higher frequencies of visible light would allow many more channels to be carried compared with using the microwave.

50. From Eq. 40-29, $N_2/N_1 = e^{-(E_2-E_1)/kT}$. We solve for T:

$$T = \frac{E_2 - E_1}{k\ln(N_1/N_2)} = \frac{3.2\text{ eV}}{(1.38\times10^{-23}\text{ J/K})\ln(2.5\times10^{15}/6.1\times10^{13})} = 1.0\times10^{4}\text{ K}.$$

51. **THINK** The number of atoms in a state with energy E is proportional to $e^{-E/kT}$, where T is the temperature on the Kelvin scale and k is the Boltzmann constant.

EXPRESS Thus, the ratio of the number of atoms in the thirteenth excited state to the number in the eleventh excited state is

$$\frac{n_{13}}{n_{11}} = \frac{e^{-E_{13}/kT}}{e^{-E_{11}/kT}} = e^{-(E_{13}-E_{11})/kT} = e^{-\Delta E/kT},$$

where $\Delta E = E_{13} - E_{11}$ is the difference in the energies:

$$\Delta E = E_{13} - E_{11} = 2(1.2 \text{ eV}) = 2.4 \text{ eV}.$$

ANALYZE For the given temperature, $kT = (8.62 \times 10^{-2}$ eV/K)(2000 K) = 0.1724 eV. Hence,

$$\frac{n_{13}}{n_{11}} = e^{-2.4/0.1724} = 9.0\times10^{-7}.$$

LEARN The 13th excited state has higher energy than the 11th excited state. Therefore, we expect fewer atoms to be in the 13th excited state.

52. The energy of the laser pulse is

$$E_p = P\Delta t = (2.80\times10^6 \text{ J/s})(0.500\times10^{-6} \text{ s}) = 1.400 \text{ J}.$$

Since the energy carried by each photon is

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^8 \text{ m/s})}{424\times10^{-9} \text{ m}} = 4.69\times10^{-19} \text{ J},$$

the number of photons emitted in each pulse is

$$N = \frac{E_p}{E} = \frac{1.400 \text{J}}{4.69\times10^{-19} \text{ J}} = 3.0\times10^{18} \text{ photons.}$$

With each atom undergoing stimulated emission only once, the number of atoms contributed to the pulse is also 3.0×10^{18}.

53. Let the power of the laser beam be P and the energy of each photon emitted be E. Then, the rate of photon emission is

$$R = \frac{P}{E} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} = \frac{\left(2.3\times10^{-3} \text{ W}\right)\left(632.8\times10^{-9} \text{ m}\right)}{\left(6.63\times10^{-34} \text{ J}\cdot\text{s}\right)\left(2.998\times10^8 \text{ m/s}\right)} = 7.3\times10^{15} \text{ s}^{-1}.$$

54. According to Sample Problem — "Population inversion in a laser," the population ratio at room temperature is $N_x/N_0 = 1.3 \times 10^{-38}$. Let the number of moles of the lasing material needed be n; then $N_0 = nN_A$, where N_A is the Avogadro constant. Also $N_x = 10$. We solve for n:

$$n = \frac{N_x}{(1.3\times10^{-38})N_A} = \frac{10}{(1.3\times10^{-38})(6.02\times10^{23})} = 1.3\times10^{15}\ \text{mol}.$$

55. (a) If t is the time interval over which the pulse is emitted, the length of the pulse is

$$L = ct = (3.00 \times 10^{8}\ \text{m/s})(1.20 \times 10^{-11}\ \text{s}) = 3.60 \times 10^{-3}\ \text{m}.$$

(b) If E_p is the energy of the pulse, E is the energy of a single photon in the pulse, and N is the number of photons in the pulse, then $E_p = NE$. The energy of the pulse is

$$E_p = (0.150\ \text{J})/(1.602 \times 10^{-19}\ \text{J/eV}) = 9.36 \times 10^{17}\ \text{eV}$$

and the energy of a single photon is $E = (1240\ \text{eV}\cdot\text{nm})/(694.4\ \text{nm}) = 1.786\ \text{eV}$. Hence,

$$N = \frac{E_p}{E} = \frac{9.36\times10^{17}\ \text{eV}}{1.786\ \text{eV}} = 5.24\times10^{17}\ \text{photons}.$$

56. Consider two levels, labeled 1 and 2, with $E_2 > E_1$. Since $T = -|T| < 0$,

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} = e^{-|E_2-E_1|/(-k|T|)} = e^{|E_2-E_1|/k|T|} > 1.$$

Thus, $N_2 > N_1$; this is population inversion. We solve for T:

$$T = -|T| = -\frac{E_2-E_1}{k\ln(N_2/N_1)} = -\frac{2.26\ \text{eV}}{(8.62\times10^{-5}\ \text{eV/K})\ln(1+0.100)} = -2.75\times10^{5}\ \text{K}.$$

57. (a) We denote the upper level as level 1 and the lower one as level 2. From $N_1/N_2 = e^{-(E_2-E_1)/kT}$ we get (using $hc = 1240\ \text{eV}\cdot\text{nm}$)

$$N_1 = N_2 e^{-(E_1-E_2)/kT} = N_2 e^{-hc/\lambda kT} = (4.0\times10^{20})\exp\left[-\frac{1240\,\text{eV}\cdot\text{nm}}{(580\text{nm})(8.62\times10^{-5}\text{eV/K})(300\text{K})}\right]$$
$$= 5.0\times10^{-16} << 1,$$

so practically no electron occupies the upper level.

(b) With $N_1 = 3.0 \times 10^{20}$ atoms emitting photons and $N_2 = 1.0 \times 10^{20}$ atoms absorbing photons, then the net energy output is

$$E=(N_1-N_2)E_{\text{photon}}=(N_1-N_2)\frac{hc}{\lambda}=(2.0\times10^{20})\frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{580\times10^{-9}\,\text{m}}$$
$$=68\,\text{J}.$$

58. For the nth harmonic of the standing wave of wavelength λ in the cavity of width L we have $n\lambda = 2L$, so $n\Delta\lambda + \lambda\Delta n = 0$. Let $\Delta n = \pm1$ and use $\lambda = 2L/n$ to obtain

$$|\Delta\lambda|=\frac{\lambda|\Delta n|}{n}=\frac{\lambda}{n}=\lambda\left(\frac{\lambda}{2L}\right)=\frac{(533\ \text{nm})^2}{2(8.0\times10^{7}\ \text{nm})}=1.8\times10^{-12}\ \text{m}=1.8\ \text{pm}.$$

59. For stimulated emission to take place, we need a long-lived state above a short-lived state in both atoms. In addition, for the light emitted by A to cause stimulated emission of B, an energy match for the transitions is required. The above conditions are fulfilled for the transition from the 6.9 eV state (lifetime 3 ms) to 3.9 eV state (lifetime 3 μs) in A, and the transition from 10.8 eV (lifetime 3 ms) to 7.8 eV (lifetime 3 μs) in B. Thus, the energy per photon of the stimulated emission of B is $10.8\,\text{eV}-7.8\,\text{eV}=3.0\,\text{eV}$.

60. (a) The radius of the central disk is

$$R=\frac{1.22f\lambda}{d}=\frac{(1.22)(3.50\ \text{cm})(515\ \text{nm})}{3.00\ \text{mm}}=7.33\ \mu\text{m}.$$

(b) The average power flux density in the incident beam is

$$\frac{P}{\pi d^2/4}=\frac{4(5.00\,\text{W})}{\pi(3.00\,\text{mm})^2}=7.07\times10^{5}\,\text{W/m}^2.$$

(c) The average power flux density in the central disk is

$$\frac{(0.84)P}{\pi R^2}=\frac{(0.84)(5.00\,\text{W})}{\pi(7.33\,\mu\text{m})^2}=2.49\times10^{10}\,\text{W/m}^2.$$

61. (a) If both mirrors are perfectly reflecting, there is a node at each end of the crystal. With one end partially silvered, there is a node very close to that end. We assume nodes at both ends, so there are an integer number of half-wavelengths in the length of the crystal. The wavelength in the crystal is $\lambda_c = \lambda/n$, where λ is the wavelength in a vacuum and n is the index of refraction of ruby. Thus $N(\lambda/2n) = L$, where N is the number of standing wave nodes, so

$$N=\frac{2nL}{\lambda}=\frac{2(1.75)(0.0600\ \text{m})}{694\times10^{-9}\ \text{m}}=3.03\times10^{5}.$$

(b) Since $\lambda = c/f$, where f is the frequency, $N = 2nLf/c$ and $\Delta N = (2nL/c)\Delta f$. Hence,

$$\Delta f = \frac{c\Delta N}{2nL} = \frac{(2.998\times10^8 \text{ m/s})(1)}{2(1.75)(0.0600 \text{ m})} = 1.43\times10^9 \text{ Hz}.$$

(c) The speed of light in the crystal is c/n and the round-trip distance is $2L$, so the round-trip travel time is $2nL/c$. This is the same as the reciprocal of the change in frequency.

(d) The frequency is

$$f = c/\lambda = (2.998 \times 10^8 \text{ m/s})/(694 \times 10^{-9} \text{ m}) = 4.32 \times 10^{14} \text{ Hz}$$

and the fractional change in the frequency is

$$\Delta f/f = (1.43 \times 10^9 \text{ Hz})/(4.32 \times 10^{14} \text{ Hz}) = 3.31 \times 10^{-6}.$$

62. The energy carried by each photon is

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^8 \text{ m/s})}{694\times10^{-9} \text{ m}} = 2.87\times10^{-19} \text{ J}.$$

Now, the photons emitted by the Cr ions in the excited state can be absorbed by the ions in the ground state. Thus, the average power emitted during the pulse is

$$P = \frac{(N_1 - N_0)E}{\Delta t} = \frac{(0.600-0.400)(4.00\times10^{19})(2.87\times10^{-19} \text{ J})}{2.00\times10^{-6} \text{ s}} = 1.1\times10^6 \text{ J/s}$$

or 1.1×10^6 W.

63. Due to spin degeneracy ($m_s = \pm1/2$), each state can accommodate two electrons. Thus, in the energy-level diagram shown, two electrons can be placed in the ground state with energy $E_1 = 3(h^2/8mL^2)$, six can occupy the “triple state” with $E_2 = 6(h^2/8mL^2)$, and so forth. With 22 electrons in the system, the lowest energy configuration consists of two electrons with $E_1 = 3(h^2/8mL^2)$, six electrons with $E_2 = 6(h^2/8mL^2)$, six electrons with $E_3 = 9(h^2/8mL^2)$, six electrons with $E_4 = 11(h^2/8mL^2)$, and two electrons with $E_5 = 12(h^2/8mL^2)$. Thus, we find the ground-state energy of the 22-electron system to be

$$\begin{aligned}E_{\text{ground}} &= 2E_1 + 6E_2 + 6E_3 + 6E_4 + 2E_5 \\ &= 2\left(\frac{3h^2}{8mL^2}\right) + 6\left(\frac{6h^2}{8mL^2}\right) + 6\left(\frac{9h^2}{8mL^2}\right) + 6\left(\frac{11h^2}{8mL^2}\right) + 2\left(\frac{12h^2}{8mL^2}\right) \\ &= \left[(2)(3) + (6)(6) + (6)(9) + (6)(11) + (2)(12)\right]\left(\frac{h^2}{8mL^2}\right) \\ &= 186\left(\frac{h^2}{8mL^2}\right).\end{aligned}$$

Thus, the multiple of $h^2/8mL^2$ is 186.

64. (a) In the lasing action the molecules are excited from energy level E_0 to energy level E_2. Thus the wavelength λ of the sunlight that causes this excitation satisfies

$$\Delta E = E_2 - E_0 = \frac{hc}{\lambda},$$

which gives (using hc = 1240 eV·nm)

$$\lambda = \frac{hc}{E_2 - E_0} = \frac{1240 \text{ eV}\cdot\text{nm}}{0.289 \text{ eV} - 0} = 4.29\times10^3 \text{ nm} = 4.29\ \mu\text{m}.$$

(b) Lasing occurs as electrons jump down from the higher energy level E_2 to the lower level E_1. Thus the lasing wavelength λ' satisfies

$$\Delta E' = E_2 - E_1 = \frac{hc}{\lambda'},$$

which gives

$$\lambda' = \frac{hc}{E_2 - E_1} = \frac{1240 \text{ eV}\cdot\text{nm}}{0.289 \text{ eV} - 0.165 \text{ eV}} = 1.00\times10^4 \text{ nm} = 10.0\ \mu\text{m}.$$

(c) Both λ and λ' belong to the infrared region of the electromagnetic spectrum.

65. (a) Using hc = 1240 eV·nm,

$$\Delta E = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = (1240 \text{eV}\cdot\text{nm})\left(\frac{1}{588.995 \text{nm}} - \frac{1}{589.592 \text{nm}}\right) = 2.13 \text{meV} .$$

(b) From $\Delta E = 2\mu_B B$ (see Fig. 40-10 and Eq. 40-18), we get

$$B = \frac{\Delta E}{2\mu_B} = \frac{2.13\times10^{-3} \text{ eV}}{2(5.788\times10^{-5} \text{ eV/T})} = 18 \text{T} .$$

66. (a) The energy difference between the two states 1 and 2 was equal to the energy of the photon emitted. Since the photon frequency was f = 1666 MHz, its energy was given by

$$hf = (4.14 \times 10^{-15}\ \text{eV}\cdot\text{s})(1666\ \text{MHz}) = 6.90 \times 10^{-6}\ \text{eV}.$$

Thus,

$$E_2 - E_1 = hf = 6.90\times10^{-6}\ \text{eV} = 6.90\ \mu\text{eV}.$$

(b) The emission was in the *radio* region of the electromagnetic spectrum.

67. Letting $eV = hc/\lambda_{\min}$ (see Eq. 40-23 and Eq. 38-4), we get

$$\lambda_{\min} = \frac{hc}{eV} = \frac{1240\,\text{nm}\cdot\text{eV}}{eV} = \frac{1240\,\text{pm}\cdot\text{keV}}{eV} = \frac{1240\,\text{pm}}{V}$$

where V is measured in kV.

68. (a) The distance from the Earth to the Moon is $d_{em} = 3.82 \times 10^8$ m (see Appendix C). Thus, the time required is given by

$$t = \frac{2d_{em}}{c} = \frac{2(3.82\times10^8\ \text{m})}{2.998\times10^8\ \text{m/s}} = 2.55\ \text{s}.$$

(b) We denote the uncertainty in time measurement as δt and let $2\delta d_{es}$ = 15 cm. Then, since $d_{em} \propto t$, $\delta t/t = \delta d_{em}/d_{em}$. We solve for δt:

$$\delta t = \frac{t\delta d_{em}}{d_{em}} = \frac{(2.55\ \text{s})(0.15\ \text{m})}{2(3.82\times10^8\ \text{m})} = 5.0\times10^{-10}\ \text{s}.$$

(c) The angular divergence of the beam is

$$\theta = 2\tan^{-1}\left(\frac{1.5\times10^3}{d_{em}}\right) = 2\tan^{-1}\left(\frac{1.5\times10^3}{3.82\times10^8}\right) = (4.5\times10^{-4})^\circ.$$

69. **THINK** The intensity at the target is given by $I = P/A$, where P is the power output of the source and A is the area of the beam at the target. We want to compute I and compare the result with 10^8 W/m^2.

EXPRESS The laser beam spreads because diffraction occurs at the aperture of the laser. Consider the part of the beam that is within the central diffraction maximum. The angular position of the edge is given by $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. At the target, a distance D away, the radius of the beam is

$r = D\tan\theta$. Since θ is small, we may approximate both $\sin\theta$ and $\tan\theta$ by θ, in radians. Then,

$$r = D\theta = 1.22D\lambda/d.$$

ANALYZE (a) Thus, we find the intensity to be

$$I = \frac{P}{\pi r^2} = \frac{Pd^2}{\pi(1.22D\lambda)^2} = \frac{(5.0\times10^6\text{ W})(4.0\text{ m})^2}{\pi\left[1.22(3000\times10^3\text{ m})(3.0\times10^{-6}\text{ m})\right]^2} = 2.1\times10^5\ \text{W/m}^2,$$

not great enough to destroy the missile.

(b) We solve for the wavelength in terms of the intensity and substitute $I = 1.0\times10^8$ W/m^2:

$$\lambda = \frac{d}{1.22D}\sqrt{\frac{P}{\pi I}} = \frac{4.0\text{ m}}{1.22(3000\times10^3\text{ m})}\sqrt{\frac{5.0\times10^6\text{ W}}{\pi(1.0\times10^8\text{ W/m}^2)}} = 1.40\times10^{-7}\text{ m} = 140\text{ nm}.$$

LEARN The wavelength corresponds to the x-rays on the electromagnetic spectrum.

70. (a) From Fig. 40-14 we estimate the wavelengths corresponding to the K_β line to be $\lambda_\beta = 63.0$ pm. Using $hc = 1240$ eV·nm = 1240 keV·pm, we have

$$E_\beta = (1240\text{ keV·pm})/(63.0\text{ pm}) = 19.7\text{ keV} \approx 20\text{ keV}.$$

(b) For K_α, with $\lambda_\alpha = 70.0$ pm, $E_\alpha = \dfrac{hc}{\lambda_\alpha} = \dfrac{1240\text{keV}\cdot\text{pm}}{70.0\text{pm}} = 17.7\text{keV} \approx 18\text{ keV}$.

(c) Both Zr and Nb can be used, since $E_\alpha < 18.00$ eV $< E_\beta$ and $E_\alpha < 18.99$ eV $< E_\beta$. According to the hint given in the problem statement, Zr is the best choice.

(d) Nb is the second best choice.

71. The principal quantum number n must be greater than 3. The magnetic quantum number m_ℓ can have any of the values – 3, – 2, – 1, 0, +1, +2, or +3. The spin quantum number can have either of the values $-\frac{1}{2}$ or $+\frac{1}{2}$.

72. For a given shell with quantum number n the total number of available electron states is $2n^2$. Thus, for the first four shells (n = 1 through 4) the numbers of available states are 2, 8, 18, and 32 (see Appendix G). Since 2 + 8 + 18 + 32 = 60 < 63, according to the "logical" sequence the first four shells would be completely filled in an europium atom, leaving 63 – 60 = 3 electrons to partially occupy the n = 5 shell. Two of these three electrons would fill up the 5s subshell, leaving only one remaining electron in the only partially filled subshell (the 5p subshell). In chemical reactions this electron would have the tendency to be transferred to another element, leaving the remaining 62 electrons in

chemically stable, completely filled subshells. This situation is very similar to the case of sodium, which also has only one electron in a partially filled shell (the 3*s* shell).

73. **THINK** One femtosecond (fs) is equal to 10^{-15} s.

EXPRESS The length of the pulse's wave train is given by $L = c\Delta t$, where Δt is the duration of the laser. Thus, the number of wavelengths contained in the pulse is

$$N = \frac{L}{\lambda} = \frac{c\Delta t}{\lambda}.$$

ANALYZE (a) With λ = 500 nm and $\Delta t = 10\times10^{-15}$ s, we have

$$N = \frac{L}{\lambda} = \frac{(3.0\times10^{8}\text{ m/s})(10\times10^{-15}\text{ s})}{500\times10^{-9}\text{ m}} = 6.0.$$

(b) We solve for X from 10 fm/1 m = 1 s/X:

$$X = \frac{(1\text{ s})(1\text{ m})}{10\times10^{-15}\text{ m}} = \frac{1\text{ s}}{(10\times10^{-15})(3.15\times10^{7}\text{ s/y})} = 3.2\times10^{6}\text{ y}.$$

LEARN Femtosecond lasers have important applications in areas such as micro-machining and optical data storage.

74. One way to think of the units of h is that, because of the equation $E = hf$ and the fact that f is in cycles/second, then the "explicit" units for h should be J·s/cycle. Then, since 2π rad/cycle is a conversion factor for $\text{cycles}\to\text{radians}$, $\hbar = h/2\pi$ can be thought of as the Planck constant expressed in terms of radians instead of cycles. Using the precise values stated in Appendix B,

$$\hbar = \frac{h}{2\pi} = \frac{6.62606876\times10^{-34}\text{ J}\cdot\text{s}}{2\pi} = 1.05457\times10^{-34}\text{ J}\cdot\text{s} = \frac{1.05457\times10^{-34}\text{ J}\cdot\text{s}}{1.6021765\times10^{-19}\text{ J/eV}}$$
$$= 6.582\times10^{-16}\text{ eV}\cdot\text{s}.$$

75. Without the spin degree of freedom the number of available electron states for each shell would be reduced by half. So the values of Z for the noble gas elements would become half of what they are now: Z = 1, 5, 9, 18, 27, and 43. Of this set of numbers, the only one that coincides with one of the familiar noble gas atomic numbers (Z = 2, 10, 18, 36, 54, and 86) is 18. Thus, argon would be the only one that would remain "noble."

76. (a) The value of ℓ satisfies $\sqrt{\ell(\ell+1)}\hbar \approx \sqrt{\ell^2}\hbar = \ell\hbar = L$, so $\ell \simeq L/\hbar \simeq 3\times10^{74}$.

(b) The number is $2\ell + 1 \approx 2(3 \times 10^{74}) = 6 \times 10^{74}$.

(c) Since

$$\cos\theta_{\min} = \frac{m_{\ell\,\max}\hbar}{\sqrt{\ell(\ell+1)}\hbar} = \frac{1}{\sqrt{\ell(\ell+1)}} \approx 1 - \frac{1}{2\ell} = 1 - \frac{1}{2(3\times 10^{74})}$$

or $\cos\theta_{\min} \simeq 1 - \theta_{\min}^2/2 \approx 1 - 10^{-74}/6$, we have

$$\theta_{\min} \simeq \sqrt{10^{-74}/3} = 6\times 10^{-38}\ \text{rad}.$$

The correspondence principle requires that all the quantum effects vanish as $\hbar \to 0$. In this case $\hbar/L$ is extremely small so the quantization effects are barely existent, with $\theta_{\min} \simeq 10^{-38}\ \text{rad} \simeq 0$.

77. We use $eV = hc/\lambda_{\min}$ (see Eq. 40-23 and Eq. 38-4):

$$h = \frac{eV\lambda_{\min}}{c} = \frac{(1.60\times 10^{-19}\ \text{C})(40.0\times 10^3\ \text{eV})(31.1\times 10^{-12}\ \text{m})}{2.998\times 10^8\ \text{m/s}} = 6.63\times 10^{-34}\ \text{J}\cdot\text{s}.$$

78. Using $hc = 1240$ eV·nm, we find the energy difference to be

$$\Delta E = hc\left(\frac{1}{\lambda_A} - \frac{1}{\lambda_B}\right) = (1240\,\text{eV}\cdot\text{nm})\left(\frac{1}{500\,\text{nm}} - \frac{1}{510\,\text{nm}}\right) = 0.049\,\text{eV}.$$

79. (a) Using $E = -\partial V/\partial r$, we find the electric field to be

$$E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left[\frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right)\right] = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

(b) The electric field at $r = R$ vanishes: $E(r = R) = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{R^2} - \frac{R}{R^3}\right) = 0$. Since $V = 0$ outside the sphere, we conclude that the electric field is zero in the region $r \geq R$.

(c) At $r = R$, the electric potential is

$$V(r = R) = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{3}{2R} + \frac{R^2}{2R^3}\right) = 0$$

The electric potential outside the sphere is also zero.

Chapter 41

1. According to Eq. 41-9, the Fermi energy is given by

$$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} n^{2/3}$$

where n is the number of conduction electrons per unit volume, m is the mass of an electron, and h is the Planck constant. This can be written $E_F = An^{2/3}$, where

$$A = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{9.109\times10^{-31}\ \text{kg}} = 5.842\times10^{-38}\ \text{J}^2\cdot\text{s}^2/\text{kg}\ .$$

Since $1\ \text{J} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$, the units of A can be taken to be $\text{m}^2\cdot\text{J}$. Dividing by 1.602×10^{-19} J/eV, we obtain $A = 3.65\times10^{-19}\ \text{m}^2\cdot\text{eV}$.

2. Equation 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}$$

for the density of states associated with the conduction electrons of a metal. This can be written

$$N(E) = CE^{1/2}$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi(9.109\times10^{-31}\ \text{kg})^{3/2}}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^3} = 1.062\times10^{56}\ \text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3$$
$$= 6.81\times10^{27}\ \text{m}^{-3}\cdot(\text{eV})^{-2/3}.$$

Thus,

$$N(E) = CE^{1/2} = \left[6.81\times10^{27}\ \text{m}^{-3}\cdot(\text{eV})^{-2/3}\right](8.0\,\text{eV})^{1/2} = 1.9\times10^{28}\ \text{m}^{-3}\cdot\text{eV}^{-1}\ .$$

This is consistent with that shown in Fig. 41-6.

3. The number of atoms per unit volume is given by $n = d/M$, where d is the mass density of copper and M is the mass of a single copper atom. Since each atom contributes one conduction electron, n is also the number of conduction electrons per unit volume. Since the molar mass of copper is $A = 63.54\,\text{g/mol}$,

$$M = A/N_A = (63.54\,\text{g/mol})/(6.022\times10^{23}\ \text{mol}^{-1}) = 1.055\times10^{-22}\ \text{g}\,.$$

Thus,

$$n = \frac{8.96\,\text{g}/\text{cm}^3}{1.055\times 10^{-22}\,\text{g}} = 8.49\times 10^{22}\,\text{cm}^{-3} = 8.49\times 10^{28}\,\text{m}^{-3}\ .$$

4. Let E_1 = 63 meV + E_F and E_2 = – 63 meV + E_F. Then according to Eq. 41-6,

$$P_1 = \frac{1}{e^{(E_1-E_F)/kT}+1} = \frac{1}{e^x+1}$$

where $x = (E_1 - E_F)/kT$. We solve for e^x:

$$e^x = \frac{1}{P_1} - 1 = \frac{1}{0.090} - 1 = \frac{91}{9}\ .$$

Thus,

$$P_2 = \frac{1}{e^{(E_2-E_F)/kT}+1} = \frac{1}{e^{-(E_1-E_F)/kT}+1} = \frac{1}{e^{-x}+1} = \frac{1}{(91/9)^{-1}+1} = 0.91,$$

where we use $E_2 - E_F$ = – 63 meV = $E_F - E_1$ = – ($E_1 - E_F$).

5. (a) Equation 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}$$

for the density of states associated with the conduction electrons of a metal. This can be written

$$N(E) = CE^{1/2}$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi(9.109\times 10^{-31}\,\text{kg})^{3/2}}{(6.626\times 10^{-34}\,\text{J}\cdot\text{s})^3} = 1.062\times 10^{56}\,\text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3.$$

(b) Now, $1\,\text{J} = 1\text{kg}\cdot\text{m}^2/\text{s}^2$ (think of the equation for kinetic energy $K = \frac{1}{2}mv^2$), so 1 kg = 1 J·s^2·m^{-2}. Thus, the units of C can be written as

$$(\text{J}\cdot\text{s}^2)^{3/2}\cdot(\text{m}^{-2})^{3/2}\cdot\text{J}^{-3}\cdot\text{s}^{-3} = \text{J}^{-3/2}\cdot\text{m}^{-3}.$$

This means

$$C = (1.062\times 10^{56}\,\text{J}^{-3/2}\cdot\text{m}^{-3})(1.602\times 10^{-19}\,\text{J}/\text{eV})^{3/2} = 6.81\times 10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2}\ .$$

(c) If E = 5.00 eV, then

$$N(E) = (6.81\times 10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2})(5.00\,\text{eV})^{1/2} = 1.52\times 10^{28}\,\text{eV}^{-1}\cdot\text{m}^{-3}\ .$$

6. We note that $n = 8.43 \times 10^{28}$ m^{-3} = 84.3 nm^{-3}. From Eq. 41-9,

$$E_F = \frac{0.121(hc)^2}{m_e c^2} n^{2/3} = \frac{0.121(1240\,\text{eV}\cdot\text{nm})^2}{511\times 10^3\ \text{eV}}(84.3\,\text{nm}^{-3})^{2/3} = 7.0\ \text{eV}$$

where we have used $hc = 1240\,\text{eV}\cdot\text{nm}$.

7. **THINK** This problem deals with occupancy probability $P(E)$, the probability that an energy level will be occupied by an electron.

EXPRESS A plot of $P(E)$ as a function of E is shown in Fig. 41-7. From the figure, we see that at T = 0 K, $P(E)$ is unity for $E \le E_F$, where E_F is the Fermi energy, and zero for $E > E_F$. On the other hand, the probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and E_F is the Fermi energy.

ANALYZE (a) At absolute temperature T = 0, the probability is zero that any state with energy above the Fermi energy is occupied.

(b) Now, $E - E_F$ = 0.0620 eV, and

$$(E - E_F)/kT = (0.0620\text{eV})/(8.62\times 10^{-5}\ \text{eV/K})(320\ \text{K}) = 2.248\,.$$

We find $P(E)$ to be

$$P(E) = \frac{1}{e^{2.248} + 1} = 0.0955\,.$$

See Appendix B for the value of k.

LEARN When $E = E_F$, the occupancy probability is $P(E_F) = 0.5$. Thus, one may think of the Fermi energy as the energy of a quantum state that has a probability 0.5 of being occupied by an electron.

8. We note that there is one conduction electron per atom and that the molar mass of gold is $197\,\text{g/mol}$. Therefore, combining Eqs. 41-2, 41-3, and 41-4 leads to

$$n = \frac{(19.3\,\text{g/cm}^3)(10^6\ \text{cm}^3/\text{m}^3)}{(197\,\text{g/mol})/(6.02\times 10^{23}\ \text{mol}^{-1})} = 5.90\times 10^{28}\ \text{m}^{-3}\ .$$

9. **THINK** According to Appendix F the molar mass of silver is M = 107.870 g/mol and the density is ρ = 10.49 g/cm^3. Silver is monovalent.

EXPRESS The mass of a silver atom is, dividing the molar mass by Avogadro's number:

$$M_0 = \frac{M}{N_A} = \frac{107.870\times10^{-3}\ \text{kg/mol}}{6.022\times10^{23}\ \text{mol}^{-1}} = 1.791\times10^{-25}\ \text{kg}\ .$$

Since silver is monovalent, there is one valence electron per atom (see Eq. 41-2).

ANALYZE (a) The number density is

$$n = \frac{\rho}{M_0} = \frac{10.49\times10^{-3}\ \text{kg/m}^3}{1.791\times10^{-25}\ \text{kg}} = 5.86\times10^{28}\ \text{m}^{-3}\ .$$

This is the same as the number density of conduction electrons.

(b) The Fermi energy is

$$E_F = \frac{0.121h^2}{m}n^{2/3} = \frac{(0.121)(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{9.109\times10^{-31}\text{kg}} = (5.86\times10^{28}\ \text{m}^{-3})^{2/3}$$
$$= 8.80\times10^{-19}\ \text{J} = 5.49\text{eV}.$$

(c) Since $E_F = \frac{1}{2}mv_F^2$,

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2(8.80\times10^{-19}\ \text{J})}{9.109\times10^{-31}\ \text{kg}}} = 1.39\times10^6\ \text{m/s}\ .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{mv_F} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.109\times10^{-31}\ \text{kg})(1.39\times10^6\text{m/s})} = 5.22\times10^{-10}\,\text{m}.$$

LEARN Once the number density of conduction electrons is known, the Fermi energy for a particular metal can be calculated using Eq. 41-9.

10. The probability P_h that a state is occupied by a hole is the same as the probability the state is *unoccupied* by an electron. Since the total probability that a state is either occupied or unoccupied is 1, we have $P_h + P = 1$. Thus,

$$P_h = 1 - \frac{1}{e^{(E-E_F)/kT}+1} = \frac{e^{(E-E_F)/kT}}{1+e^{(E-E_F)/kT}} = \frac{1}{e^{-(E-E_F)/kT}+1}\ .$$

11. We use

$$N_O(E) = N(E)P(E) = CE^{1/2}\left[e^{(E-E_F)/kT}+1\right]^{-1},$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi(9.109\times10^{-31}\,\text{kg})^{3/2}}{(6.626\times10^{-34}\,\text{J}\cdot\text{s})^3} = 1.062\times10^{56}\,\text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3$$
$$= 6.81\times10^{27}\,\text{m}^{-3}\cdot(\text{eV})^{-3/2}.$$

(a) At E = 4.00 eV,

$$N_O = \frac{\left(6.81\times10^{27}\,\text{m}^{-3}\cdot(\text{eV})^{-3/2}\right)(4.00\,\text{eV})^{1/2}}{\exp\left((4.00\,\text{eV}-7.00\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})]\right)+1} = 1.36\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$$

(b) At E = 6.75 eV,

$$N_O = \frac{\left(6.81\times10^{27}\,\text{m}^{-3}\cdot(\text{eV})^{-3/2}\right)(6.75\,\text{eV})^{1/2}}{\exp\left((6.75\,\text{eV}-7.00\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})]\right)+1} = 1.68\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$$

(c) Similarly, at E = 7.00 eV, the value of $N_o(E)$ is $9.01\times10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-1}$.

(d) At E = 7.25 eV, the value of $N_o(E)$ is $9.56\times10^{26}\,\text{m}^{-3}\cdot\text{eV}^{-1}$.

(e) At E = 9.00 eV, the value of $N_o(E)$ is $1.71\times10^{18}\,\text{m}^{-3}\cdot\text{eV}^{-1}$.

12. The molar mass of carbon is m = 12.01115 g/mol and the mass of the Earth is M_e = 5.98×10^{24} kg. Thus, the number of carbon atoms in a diamond as massive as the Earth is $N = (M_e/m)N_A$, where N_A is the Avogadro constant. From the result of Sample Problem – "Probability of electron excitation in an insulator," the probability in question is given by

$$P = N_e^{-E_g/kT} = \left(\frac{M_e}{m}\right)N_A e^{-E_g/kT} = \left(\frac{5.98\times10^{24}\,\text{kg}}{12.01115\,\text{g/mol}}\right)(6.02\times10^{23}/\text{mol})(3\times10^{-93})$$
$$= 9\times10^{-43} \approx 10^{-42}\;.$$

13. (a) Equation 41-6 leads to

$$E = E_F + kT\ln(P^{-1}-1) = 7.00\,\text{eV} + (8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})\ln\left(\frac{1}{0.900}-1\right) = 6.81\,\text{eV}.$$

(b) $N(E) = CE^{1/2} = \left(6.81\times10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2}\right)(6.81\,\text{eV})^{1/2} = 1.77\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$

(c) $N_O(E) = P(E)N(E) = (0.900)(1.77\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}) = 1.59\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}$.

14. (a) The volume per cubic meter of sodium occupied by the sodium ions is

$$V_{\text{Na}} = \frac{(971\text{kg})(6.022\times10^{23}/\text{mol})(4\pi/3)(98.0\times10^{-12}\,\text{m})^3}{(23.0\,\text{g}/\text{mol})} = 0.100\,\text{m}^3,$$

so the fraction available for conduction electrons is $1-(V_{\text{Na}}/1.00\,\text{m}^3) = 1-0.100 = 0.900$, or 90.0%.

(b) For copper, we have

$$V_{\text{Cu}} = \frac{(8960\,\text{kg})(6.022\times10^{23}/\text{mol})(4\pi/3)(135\times10^{-12}\,\text{m})^3}{(63.5\,\text{g}/\text{mol})} = 0.1876\,\text{m}^{-3}\ .$$

Thus, the fraction is $1-(V_{\text{Cu}}/1.00\,\text{m}^3) = 1-0.876 = 0.124$, or 12.4%.

(c) Sodium, because the electrons occupy a greater portion of the space available.

15. **THINK** The Fermi-Dirac occupation probability is given by $P_{\text{FD}} = 1/\left(e^{\Delta E/kT}+1\right)$, and the Boltzmann occupation probability is given by $P_{\text{B}} = e^{-\Delta E/kT}$.

EXPRESS Let *f* be the fractional difference. Then

$$f = \frac{P_{\text{B}} - P_{\text{FD}}}{P_{\text{B}}} = \frac{e^{-\Delta E/kT} - \frac{1}{e^{\Delta E/kT}+1}}{e^{-\Delta E/kT}}\ .$$

Using a common denominator and a little algebra yields $f = \dfrac{e^{-\Delta E/kT}}{e^{-\Delta E/kT}+1}$. The solution for $e^{-\Delta E/kT}$ is

$$e^{-\Delta E/kT} = \frac{f}{1-f}\ .$$

We take the natural logarithm of both sides and solve for *T*. The result is

$$T = \frac{\Delta E}{k\ln\left(\frac{f}{1-f}\right)}\ .$$

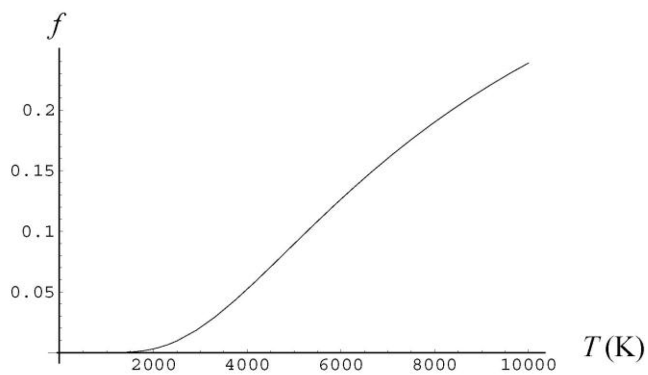
ANALYZE (a) Letting *f* equal 0.01, we evaluate the expression for *T*:

$$T = \frac{(1.00\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.38\times10^{-23}\,\text{J/K})\ln\left(\dfrac{0.010}{1-0.010}\right)} = 2.50\times10^{3}\,\text{K}.$$

(b) We set f equal to 0.10 and evaluate the expression for T:

$$T = \frac{(1.00\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.38\times10^{-23}\,\text{J/K})\ln\left(\dfrac{0.10}{1-0.10}\right)} = 5.30\times10^{3}\,\text{K}.$$

LEARN The fractional difference as a function of T is plotted below:



With a given ΔE, the difference increases with T.

16. (a) The ideal gas law in the form of Eq. 20-9 leads to $p = NkT/V = n_0kT$. Thus, we solve for the molecules per cubic meter:

$$n_0 = \frac{p}{kT} = \frac{(1.0\,\text{atm})(1.0\times10^{5}\,\text{Pa/atm})}{(1.38\times10^{-23}\,\text{J/K})(273\,\text{K})} = 2.7\times10^{25}\,\text{m}^{-3}\ .$$

(b) Combining Eqs. 41-2, 41-3, and 41-4 leads to the conduction electrons per cubic meter in copper:

$$n = \frac{8.96\times10^{3}\,\text{kg/m}^3}{(63.54)(1.67\times10^{-27}\,\text{kg})} = 8.43\times10^{28}\,\text{m}^{-3}\ .$$

(c) The ratio is $n/n_0 = (8.43\times10^{28}\,\text{m}^{-3})/(2.7\times10^{25}\,\text{m}^{-3}) = 3.1\times10^{3}$.

(d) We use $d_{avg} = n^{-1/3}$. For case (a), $d_{avg,\,0} = (2.7\times10^{25}\,\text{m}^{-3})^{-1/3} = 3.3$ nm.

(e) For case (b), $d_{avg} = (8.43\times10^{28}\,\text{m}^{-3})^{-1/3} = 0.23$ nm.

17. Let N be the number of atoms per unit volume and n be the number of free electrons per unit volume. Then, the number of free electrons per atom is n/N. We use the result of Problem 41-1 to find n: $E_F = An^{2/3}$, where $A = 3.65 \times 10^{-19}\ \text{m}^2 \cdot \text{eV}$. Thus,

$$n=\left(\frac{E_F}{A}\right)^{3/2}=\left(\frac{11.6\,\text{eV}}{3.65\times10^{-19}\,\text{m}^2\cdot\text{eV}}\right)^{3/2}=1.79\times10^{29}\,\text{m}^{-3}\ .$$

If M is the mass of a single aluminum atom and d is the mass density of aluminum, then $N = d/M$. Now,

$$M = (27.0\ \text{g/mol})/(6.022 \times 10^{23}\ \text{mol}^{-1}) = 4.48 \times 10^{-23}\ \text{g},$$

so

$$N = (2.70\ \text{g/cm}^3)/(4.48 \times 10^{-23}\ \text{g}) = 6.03 \times 10^{22}\ \text{cm}^{-3} = 6.03 \times 10^{28}\ \text{m}^{-3}.$$

Thus, the number of free electrons per atom is

$$\frac{n}{N}=\frac{1.79\times10^{29}\,\text{m}^{-3}}{6.03\times10^{28}\,\text{m}^{-3}}=2.97\approx3.$$

18. The mass of the sample is

$$m=\rho V=(9.0\ \text{g/cm}^3)(40.0\ \text{cm}^3)=360\ \text{g}\,,$$

which is equivalent to

$$n=\frac{m}{M}=\frac{360\ \text{g}}{60\ \text{g/mol}}=6.0\ \text{mol}\,.$$

Since the atoms are bivalent (each contributing two electrons), there are 12.0 moles of conduction electrons, or

$$N=nN_\text{A}=(12.0\ \text{mol})(6.02\times10^{23}\,/\,\text{mol})=7.2\times10^{24}\,.$$

19. (a) We evaluate $P(E) = 1/(e^{(E-E_F)/kT}+1)$ for the given value of E, using

$$kT=\frac{(1.381\times10^{-23}\ \text{J / K})(273\,\text{K})}{1.602\times10^{-19}\ \text{J / eV}}=0.02353\,\text{eV}\ .$$

For $E = 4.4$ eV, $(E - E_F)/kT = (4.4\ \text{eV} - 5.5\ \text{eV})/(0.02353\ \text{eV}) = -46.25$ and

$$P(E)=\frac{1}{e^{-46.25}+1}=1.0.$$

(b) Similarly, for $E = 5.4$ eV, $P(E) = 0.986 \approx 0.99$.

(c) For E = 5.5 eV, $P(E)$ = 0.50.

(d) For E = 5.6 eV, $P(E)$ = 0.014.

(e) For E = 6.4 eV, $P(E) = 2.447 \times 10^{-17} \approx 2.4 \times 10^{-17}$.

(f) Solving $P = 1/(e^{\Delta E/kT} + 1)$ for $e^{\Delta E/kT}$, we get

$$e^{\Delta E/kT} = \frac{1}{P} - 1 .$$

Now, we take the natural logarithm of both sides and solve for T. The result is

$$T = \frac{\Delta E}{k \ln\left(\frac{1}{P} - 1\right)} = \frac{(5.6\,\text{eV} - 5.5\,\text{eV})(1.602\times10^{-19}\text{ J/eV})}{(1.381\times10^{-23}\text{ J/K})\ln\left(\frac{1}{0.16} - 1\right)} = 699\,\text{K} \approx 7.0\times10^{2}\,\text{K}.$$

20. The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and E_F is the Fermi energy. Now,

$$E - E_F = 6.10\text{ eV} - 5.00\text{ eV} = 1.10\text{ eV}$$

and

$$\frac{E - E_F}{kT} = \frac{1.10\,\text{eV}}{(8.62\times10^{-5}\,\text{eV/K})(1500\,\text{K})} = 8.51 ,$$

so

$$P(E) = \frac{1}{e^{8.51} + 1} = 2.01\times10^{-4} .$$

From Fig. 41-6, we find the density of states at 6.0 eV to be about $N(E) = 1.7\times10^{28}\,/\text{m}^3\cdot\text{eV}.$ Thus, using Eq. 41-7, the density of occupied states is

$$N_{\text{O}}(E) = N(E)P(E) = (1.7\times10^{28}\,/\text{m}^3\cdot\text{eV})(2.01\times10^{-4}) = 3.42\times10^{24}\,/\text{m}^3\cdot\text{eV}.$$

Within energy range of $\Delta E = 0.0300\,\text{eV}$ and a volume $V = 5.00\times10^{-8}\ \text{m}^3,$ the number of occupied states is

$$\begin{pmatrix}\text{number}\\ \text{states}\end{pmatrix} = N_{\text{O}}(E)V\Delta E = (3.42\times10^{24}\,/\text{m}^3\cdot\text{eV})(5.00\times10^{-8}\ \text{m}^3)(0.0300\,\text{eV})$$
$$= 5.1\times10^{15} .$$

21. (a) At T = 300 K, $f = \dfrac{3kT}{2E_F} = \dfrac{3(8.62\times10^{-5}\ \text{eV/K})(300\,\text{K})}{2(7.0\,\text{eV})} = 5.5\times10^{-3}$.

(b) At T = 1000 K, $f = \dfrac{3kT}{2E_F} = \dfrac{3(8.62\times10^{-5}\ \text{eV/K})(1000\,\text{K})}{2(7.0\,\text{eV})} = 1.8\times10^{-2}$.

(c) Many calculators and most math software packages (here we use MAPLE) have built-in numerical integration routines. Setting up ratios of integrals of Eq. 41-7 and canceling common factors, we obtain

$$frac = \frac{\int_{E_F}^{\infty} \sqrt{E}\,/\,(e^{(E-E_F)/kT}+1)dE}{\int_{0}^{\infty} \sqrt{E}\,/\,(e^{(E-E_F)/kT}+1)dE}$$

where $k = 8.62 \times 10^{-5}$ eV/K. We use the Fermi energy value for copper (E_F = 7.0 eV) and evaluate this for T = 300 K and T = 1000 K; we find *frac* = 0.00385 and *frac* = 0.0129, respectively.

22. The fraction f of electrons with energies greater than the Fermi energy is (approximately) given in Problem 41-21:

$$f = \frac{3kT/2}{E_F}$$

where T is the temperature on the Kelvin scale, k is the Boltzmann constant, and E_F is the Fermi energy. We solve for T:

$$T = \frac{2fE_F}{3k} = \frac{2(0.013)(4.70\,\text{eV})}{3(8.62\times10^{-5}\,\text{eV/K})} = 472\ \text{K}.$$

23. The average energy of the conduction electrons is given by

$$E_{\text{avg}} = \frac{1}{n}\int_0^{\infty} EN(E)P(E)dE$$

where n is the number of free electrons per unit volume, $N(E)$ is the density of states, and $P(E)$ is the occupation probability. The density of states is proportional to $E^{1/2}$, so we may write $N(E) = CE^{1/2}$, where C is a constant of proportionality. The occupation probability is one for energies below the Fermi energy and zero for energies above. Thus,

$$E_{\text{avg}} = \frac{C}{n}\int_0^{E_F} E^{3/2}dE = \frac{2C}{5n}E_F^{5/2} .$$

Now

$$n=\int_0^\infty N(E)P(E)dE=C\int_0^{E_F}E^{1/2}dE=\frac{2C}{3}E_F^{3/2}.$$

We substitute this expression into the formula for the average energy and obtain

$$E_{\text{avg}}=\left(\frac{2C}{5}\right)E_F^{5/2}\left(\frac{3}{2CE_F^{3/2}}\right)=\frac{3}{5}E_F\ .$$

24. From Eq. 41-9, we find the number of conduction electrons per unit volume to be

$$n=\frac{16\sqrt{2}\pi}{3}\left(\frac{m_eE_F}{h^2}\right)^{3/2}=\frac{16\sqrt{2}\pi}{3}\left(\frac{(m_ec^2)E_F}{(hc)^2}\right)^{3/2}=\frac{16\sqrt{2}\pi}{3}\left(\frac{(0.511\times10^6\text{ eV})(5.0\text{ eV})}{(1240\text{ eV}\cdot\text{ nm})^2}\right)^{3/2}$$
$$=50.9/\text{nm}^3=5.09\times10^{28}/\text{m}^3$$
$$\approx8.4\times10^4\text{ mol/m}^3.$$

Since the atom is bivalent, the number density of the atom is

$$n_{\text{atom}}=n/2=4.2\times10^4\text{ mol/m}^3.$$

Thus, the mass density of the atom is

$$\rho=n_{\text{atom}}M=(4.2\times10^4\text{ mol/m}^3)(20.0\text{ g/mol})=8.4\times10^5\text{ g/m}^3=0.84\text{ g/cm}^3.$$

25. (a) Using Eq. 41-4, the energy released would be

$$E=NE_{\text{avg}}=\frac{(3.1\text{g})}{(63.54\text{g}/\text{mol})/(6.02\times10^{23}/\text{mol})}\left(\frac{3}{5}\right)(7.0\text{eV})(1.6\times10^{-19}\text{ J/eV})$$
$$=1.97\times10^4\text{J}.$$

(b) Keeping in mind that a watt is a joule per second, we have

$$t=\frac{E}{P}=\frac{1.97\times10^4\text{ J}}{100\text{J/s}}=197\text{s}.$$

26. Let the energy of the state in question be an amount ΔE above the Fermi energy E_F. Then, Eq. 41-6 gives the occupancy probability of the state as

$$P=\frac{1}{e^{(E_F+\Delta E-E_F)/kT}+1}=\frac{1}{e^{\Delta E/kT}+1}\ .$$

We solve for ΔE to obtain

$$\Delta E = kT\ln\left(\frac{1}{P}-1\right) = (1.38\times 10^{23}\ \text{J/K})(300\ \text{K})\ln\left(\frac{1}{0.10}-1\right) = 9.1\times 10^{-21}\ \text{J}\ ,$$

which is equivalent to 5.7×10^{-2} eV = 57 meV.

27. (a) Combining Eqs. 41-2, 41-3, and 41-4 leads to the conduction electrons per cubic meter in zinc:

$$n = \frac{2(7.133\ \text{g/cm}^3)}{(65.37\ \text{g/mol})/(6.02\times 10^{23}\ \text{mol})} = 1.31\times 10^{23}\ \text{cm}^{-3} = 1.31\times 10^{29}\ \text{m}^{-3}\ .$$

(b) From Eq. 41-9,

$$E_F = \frac{0.121h^2}{m_e}n^{2/3} = \frac{0.121(6.63\times 10^{-34}\ \text{J}\cdot\text{s})^2(1.31\times 10^{29}\ \text{m}^{-3})^{2/3}}{(9.11\times 10^{-31}\ \text{kg})(1.60\times 10^{-19}\ \text{J/eV})} = 9.43\ \text{eV}.$$

(c) Equating the Fermi energy to $\frac{1}{2}m_e v_F^2$ we find (using the $m_e c^2$ value in Table 37-3)

$$v_F = \sqrt{\frac{2E_F c^2}{m_e c^2}} = \sqrt{\frac{2(9.43\ \text{eV})(2.998\times 10^8\ \text{m/s})^2}{511\times 10^3\ \text{eV}}} = 1.82\times 10^6\ \text{m/s}\ .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{m_e v_F} = \frac{6.63\times 10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times 10^{-31}\ \text{kg})(1.82\times 10^6\ \text{m/s})} = 0.40\ \text{nm}\ .$$

28. Combining Eqs. 41-2, 41-3, and 41-4, the number density of conduction electrons in gold is

$$n = \frac{(19.3\ \text{g/cm}^3)(6.02\times 10^{23}\ /\text{mol})}{(197\ \text{g/mol})} = 5.90\times 10^{22}\ \text{cm}^{-3} = 59.0\ \text{nm}^{-3}\ .$$

Now, using $hc = 1240\ \text{eV}\cdot\text{nm}$, Eq. 41-9 leads to

$$E_F = \frac{0.121(hc)^2}{(m_e c^2)}n^{2/3} = \frac{0.121(1240\ \text{eV}\cdot\text{nm})^2}{511\times 10^3\ \text{eV}}(59.0\ \text{nm}^{-3})^{2/3} = 5.52\ \text{eV}\ .$$

29. Let the volume be $v = 1.00 \times 10^{-6}$ m^3. Then,

$$K_{\text{total}} = NE_{\text{avg}} = n\nu E_{\text{avg}} = (8.43\times10^{28}\,\text{m}^{-3})(1.00\times10^{-6}\,\text{m}^3)\left(\frac{3}{5}\right)(7.00\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})$$
$$= 5.71\times10^4\,\text{J} = 57.1\text{ kJ}.$$

30. The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

where k is the Boltzmann constant and

$$E_F = \frac{0.121h^2}{m_e}n^{2/3} = \frac{0.121(6.626\times10^{-34}\,\text{J}\cdot\text{s})^2}{9.11\times10^{-31}\,\text{kg}}(1.70\times10^{28}\,\text{m}^{-3})^{2/3} = 3.855\times10^{-19}\,\text{J}$$

is the Fermi energy. Now,

$$E - E_F = 4.00\times10^{-19}\text{ J} - 3.855\times10^{-19}\text{ J} = 1.45\times10^{-20}\text{ J}$$

and

$$\frac{E-E_F}{kT} = \frac{1.45\times10^{-20}\text{ J}}{(1.38\times10^{-23}\,\text{J/K})(200\text{K})} = 5.2536,$$

so

$$P(E) = \frac{1}{e^{5.2536}+1} = 5.20\times10^{-3}.$$

Next, for the density of states associated with the conduction electrons of a metal, Eq. 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2} = \frac{8\sqrt{2}\pi(9.109\times10^{-31}\,\text{kg})^{3/2}}{(6.626\times10^{-34}\,\text{J}\cdot\text{s})^3}\left(4.00\times10^{-19}\,\text{J}\right)^{1/2}$$
$$= \left(1.062\times10^{56}\,\text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3\right)\left(4.00\times10^{-19}\,\text{J}\right)^{1/2}$$
$$= 6.717\times10^{46}\ /\text{m}^3\cdot\text{J}$$

where we have used 1 kg =1 $\text{J}\cdot\text{s}^2\cdot\text{m}^{-2}$ for unit conversion. Thus, using Eq. 41-7, the density of occupied states is

$$N_{\text{O}}(E) = N(E)P(E) = (6.717\times10^{46}/\text{m}^3\cdot\text{J})(5.20\times10^{-3}) = 3.49\times10^{44}/\text{m}^3\cdot\text{J}.$$

Within energy range of $\Delta E = 3.20\times10^{-20}$ J and a volume $V = 6.00\times10^{-6}$ m^3, the number of occupied states is

$$\begin{pmatrix}\text{number}\\ \text{states}\end{pmatrix} = N_O(E)V\Delta E = (3.49\times10^{44}/\text{m}^3\cdot\text{ J})(6.00\times10^{-6}\text{ m}^3)(3.20\times10^{-20}\text{ J})$$
$$= 6.7\times10^{19}.$$

31. **THINK** The valence band and the conduction band are separated by an energy gap.

EXPRESS Since the electron jumps from the conduction band to the valence band, the energy of the photon equals the energy gap between those two bands. The photon energy is given by $hf = hc/\lambda$, where f is the frequency of the electromagnetic wave and λ is its wavelength.

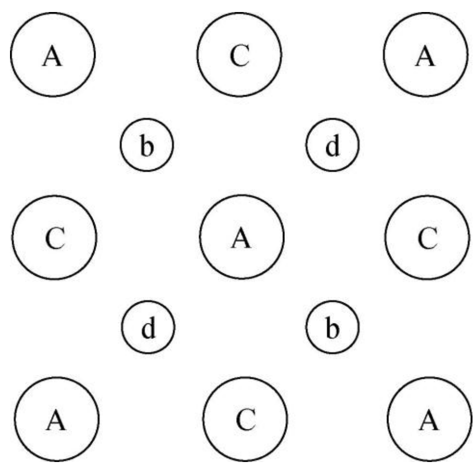
ANALYZE (a) Thus, $E_g = hc/\lambda$ and

$$\lambda = \frac{hc}{E_g} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(2.998\times10^{8}\,\text{m/s})}{(5.5\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})} = 2.26\times10^{-7}\text{ m} = 226\,\text{nm}.$$

(b) These photons are in the ultraviolet portion of the electromagnetic spectrum.

LEARN Note that photons from other transitions have a greater energy, so their waves have shorter wavelengths.

32. Each arsenic atom is connected (by covalent bonding) to four gallium atoms, and each gallium atom is similarly connected to four arsenic atoms. The "depth" of their very nontrivial lattice structure is, of course, not evident in a flattened-out representation such as shown for silicon in Fig. 41-10.



Still we try to convey some sense of this (in the [1, 0, 0] view shown — for those who might be familiar with Miller indices) by using letters to indicate the depth: A for the closest atoms (to the observer), b for the next layer deep, C for further into the page, d for the last layer seen, and E (not shown) for the atoms that are at the deepest layer (and are behind the A's) needed for our description of the structure. The capital letters are used for the gallium atoms, and the small letters for the arsenic.

Consider the arsenic atom (with the letter b) near the upper left; it has covalent bonds with the two A's and the two C's near it. Now consider the arsenic atom (with the letter d) near the upper right; it has covalent bonds with the two C's, which are near it, and with the two E's (which are behind the A's which are near :+).

(a) The 3p, 3d, and 4s subshells of both arsenic and gallium are filled. They both have partially filled 4p subshells. An isolated, neutral arsenic atom has three electrons in the 4p subshell, and an isolated, neutral gallium atom has one electron in the 4p subshell. To supply the total of eight shared electrons (for the four bonds connected to each ion in the lattice), not only the electrons from 4p must be shared but also the electrons from 4s. The core of the gallium ion has charge $q = +3e$ (due to the "loss" of its single 4p and two 4s electrons).

(b) The core of the arsenic ion has charge $q = +5e$ (due to the "loss" of the three 4p and two 4s electrons).

(c) As remarked in part (a), there are two electrons shared in each of the covalent bonds. This is the same situation that one finds for silicon (see Fig. 41-10).

33. (a) At the bottom of the conduction band E = 0.67 eV. Also E_F = 0.67 eV/2 = 0.335 eV. So the probability that the bottom of the conduction band is occupied is

$$P(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1} = \frac{1}{\exp\left(\frac{0.67\,\text{eV} - 0.335\,\text{eV}}{(8.62\times10^{-5}\,\text{eV/K})(290\,\text{K})}\right) + 1} = 1.5\times10^{-6}\,.$$

(b) At the top of the valence band E = 0, so the probability that the state is *unoccupied* is given by

$$1 - P(E) = 1 - \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^{-(E-E_F)/kT} + 1} = \frac{1}{e^{-(0-0.335\,\text{eV})/\left[(8.62\times10^{-5}\,\text{eV/K})(290\,\text{K})\right]} + 1}$$
$$= 1.5\times10^{-6}\,.$$

34. (a) The number of electrons in the valence band is

$$N_{ev} = N_v P(E_v) = \frac{N_v}{e^{(E_v - E_F)/kT} + 1}\,.$$

Since there are a total of N_v states in the valence band, the number of holes in the valence band is

$$N_{hv} = N_v - N_{ev} = N_v\left[1 - \frac{1}{e^{(E_v - E_F)/kT} + 1}\right] = \frac{N_v}{e^{-(E_v - E_F)/kT} + 1}\,.$$

Now, the number of electrons in the conduction band is

$$N_{ec} = N_c P(E_c) = \frac{N_c}{e^{(E_c - E_F)/kT} + 1},$$

Hence, from $N_{ev} = N_{hc}$, we get

$$\frac{N_v}{e^{-(E_v - E_F)/kT} + 1} = \frac{N_c}{e^{(E_c - E_F)/kT} + 1}.$$

(b) In this case, $e^{(E_c - E_F)/kT} \gg 1$ and $e^{-(E_v - E_F)/kT} \gg 1$. Thus, from the result of part (a),

$$\frac{N_c}{e^{(E_c - E_F)/kT}} \approx \frac{N_v}{e^{-(E_v - E_F)/kT}},$$

or $e^{(E_v - E_c + 2E_F)/kT} \approx N_v / N_c$. We solve for E_F:

$$E_F \approx \frac{1}{2}\left(E_c + E_v\right) + \frac{1}{2} kT \ln\left(\frac{N_v}{N_c}\right).$$

35. **THINK** Doping silicon with phosphorus increases the number of electrons in the conduction band.

EXPRESS Sample Problem — "Doping silicon with phosphorus" gives the fraction of silicon atoms that must be replaced by phosphorus atoms. We find the number the silicon atoms in 1.0 g, then the number that must be replaced, and finally the mass of the replacement phosphorus atoms. The molar mass of silicon is $M_{\text{Si}} = 28.086$ g/mol, so the mass of one silicon atom is

$$m_{0,\text{Si}} = M_{\text{Si}} / N_A = (28.086 \text{ g/mol})/(6.022 \times 10^{23} \text{ mol}^{-1}) = 4.66 \times 10^{-23} \text{ g}$$

and the number of atoms in 1.0 g is

$$N_{\text{Si}} = m_{\text{Si}} / m_{0,\text{Si}} = (1.0 \text{ g})/(4.66 \times 10^{-23} \text{ g}) = 2.14 \times 10^{22}.$$

According to the Sample Problem, one of every 5×10^6 silicon atoms is replaced with a phosphorus atom. This means there will be

$$N_{\text{P}} = (2.14 \times 10^{22})/(5 \times 10^6) = 4.29 \times 10^{15}$$

phosphorus atoms in 1.0 g of silicon.

ANALYZE The molar mass of phosphorus is $M_{\text{P}} = 30.9758$ g/mol so the mass of a phosphorus atom is

$$m_{0,\mathrm{P}} = M_\mathrm{P} / N_A = (30.9758 \text{ g/mol})/(6.022 \times 10^{-23} \text{ mol}^{-1}) = 5.14 \times 10^{-23} \text{ g}.$$

The mass of phosphorus that must be added to 1.0 g of silicon is

$$m_\mathrm{P} = N_\mathrm{P} m_{0,\mathrm{P}} = (4.29 \times 10^{15})(5.14 \times 10^{-23} \text{ g}) = 2.2 \times 10^{-7} \text{ g}.$$

LEARN The phosphorus atom is a *donor* atom since it donates an electron to the conduction band. Semiconductors doped with donor atoms are called *n*-type semiconductors.

36. (a) The Fermi level is above the top of the silicon valence band.

(b) Measured from the top of the valence band, the energy of the donor state is

$$E = 1.11 \text{ eV} - 0.11 \text{ eV} = 1.0 \text{ eV}.$$

We solve E_F from Eq. 41-6:

$$E_F = E - kT \ln\left[P^{-1} - 1\right] = 1.0\,\text{eV} - \left(8.62\times10^{-5} \text{ eV/K}\right)\left(300\text{K}\right)\ln\left[\left(5.00\times10^{-5}\right)^{-1} - 1\right]$$
$$= 0.744\,\text{eV}.$$

(c) Now $E = 1.11$ eV, so

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^{(1.11\text{eV}-0.744\text{eV})/\left[\left(8.62\times10^{-5}\text{ eV/K}\right)(300\text{K})\right]} + 1} = 7.13\times10^{-7}.$$

37. (a) The probability that a state with energy E is occupied is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where E_F is the Fermi energy, T is the temperature on the Kelvin scale, and k is the Boltzmann constant. If energies are measured from the top of the valence band, then the energy associated with a state at the bottom of the conduction band is $E = 1.11$ eV. Furthermore,

$$kT = (8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K}) = 0.02586 \text{ eV}.$$

For pure silicon, $E_F = 0.555$ eV and

$$(E - E_F)/kT = (0.555 \text{ eV})/(0.02586 \text{ eV}) = 21.46.$$

Thus,

$$P(E) = \frac{1}{e^{21.46} + 1} = 4.79 \times 10^{-10}.$$

(b) For the doped semiconductor,

$$(E - E_F)/kT = (0.11 \text{ eV})/(0.02586 \text{ eV}) = 4.254$$

and

$$P(E) = \frac{1}{e^{4.254}+1} = 1.40 \times 10^{-2}.$$

(c) The energy of the donor state, relative to the top of the valence band, is 1.11 eV – 0.15 eV = 0.96 eV. The Fermi energy is 1.11 eV – 0.11 eV = 1.00 eV. Hence,

$$(E - E_F)/kT = (0.96 \text{ eV} - 1.00 \text{ eV})/(0.02586 \text{ eV}) = -1.547$$

and

$$P(E) = \frac{1}{e^{-1.547}+1} = 0.824.$$

38. (a) The semiconductor is *n*-type, since each phosphorus atom has one more valence electron than a silicon atom.

(b) The added charge carrier density is

$$n_P = 10^{-7}\, n_{Si} = 10^{-7}\,(5 \times 10^{28} \text{ m}^{-3}) = 5 \times 10^{21} \text{ m}^{-3}.$$

(c) The ratio is

$$(5 \times 10^{21} \text{ m}^{-3})/[2(5 \times 10^{15} \text{ m}^{-3})] = 5 \times 10^{5}.$$

Here the factor of 2 in the denominator reflects the contribution to the charge carrier density from *both* the electrons in the conduction band *and* the holes in the valence band.

39. **THINK** The valence band and the conduction band are separated by an energy gap E_g. An electron must acquire E_g in order to make the transition to the conduction band.

EXPRESS Since the energy received by each electron is exactly E_g, the difference in energy between the bottom of the conduction band and the top of the valence band, the number of electrons that can be excited across the gap by a single photon of energy E is

$$N = E/E_g.$$

ANALYZE With E_g = 1.1 eV and E = 662 keV, we obtain

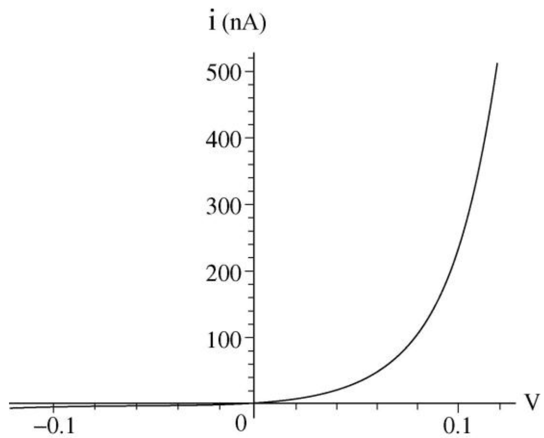
$$N = (662 \times 10^{3} \text{ eV})/(1.1 \text{ eV}) = 6.0 \times 10^{5}.$$

Since each electron that jumps the gap leaves a hole behind, this is also the number of electron-hole pairs that can be created.

LEARN The wavelength of the photon is

$$\lambda = \frac{hc}{E} = \frac{1240\ \text{nm}\cdot\text{eV}}{662\times10^{3}\ \text{eV}} = 1.87\times10^{-3}\ \text{nm} = 1.87\ \text{pm}.$$

40. (a) The vertical axis in the graph below is the current in nanoamperes:



(b) The ratio is

$$\frac{I\big|_{v=+0.50\text{V}}}{I\big|_{v=-0.50\text{V}}} = \frac{I_0\left[\exp\left(\dfrac{+0.50\,\text{eV}}{(8.62\times10^{-5}\ \text{eV/K})(300\,\text{K})}\right)-1\right]}{I_0\left[\exp\left(\dfrac{-0.50\,\text{eV}}{(8.62\times10^{-5}\ \text{eV/K})(300\,\text{K})}\right)-1\right]} = 2.5\times10^{8}.$$

41. The valence band is essentially filled and the conduction band is essentially empty. If an electron in the valence band is to absorb a photon, the energy it receives must be sufficient to excite it across the band gap. Photons with energies less than the gap width are not absorbed and the semiconductor is transparent to this radiation. Photons with energies greater than the gap width are absorbed and the semiconductor is opaque to this radiation. Thus, the width of the band gap is the same as the energy of a photon associated with a wavelength of 295 nm. Noting that $hc = 1240\,\text{eV}\cdot\text{nm}$, we obtain

$$E_{\text{gap}} = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{295\,\text{nm}} = 4.20\,\text{eV}.$$

42. Since (using $hc = 1240\,\text{eV}\cdot\text{nm}$)

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{140\,\text{nm}} = 8.86\,\text{eV} > 7.6\,\text{eV},$$

the light will be absorbed by the KCI crystal. Thus, the crystal is opaque to this light.

43. We denote the maximum dimension (side length) of each transistor as $\ell_{\max}$, the size of the chip as A, and the number of transistors on the chip as N. Then $A = N\ell_{\max}^2$. Therefore,

$$\ell_{\max} = \sqrt{\frac{A}{N}} = \sqrt{\frac{(1.0\,\text{in.} \times 0.875\,\text{in.})(2.54\times 10^{-2}\,\text{m/in.})^2}{3.5\times 10^6}} = 1.3\times 10^{-5}\,\text{m} = 13\,\mu\text{m}.$$

44. (a) According to Chapter 25, the capacitance is $C = \kappa\varepsilon_0 A/d$. In our case $\kappa = 4.5$, $A = (0.50\ \mu\text{m})^2$, and $d = 0.20\ \mu\text{m}$, so

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{(4.5)(8.85\times 10^{-12}\,\text{F/m})(0.50\,\mu\text{m})^2}{0.20\,\mu\text{m}} = 5.0\times 10^{-17}\,\text{F}.$$

(b) Let the number of elementary charges in question be N. Then, the total amount of charges that appear in the gate is $q = Ne$. Thus, $q = Ne = CV$, which gives

$$N = \frac{CV}{e} = \frac{(5.0\times 10^{-17}\,\text{F})(1.0\,\text{V})}{1.6\times 10^{-19}\,\text{C}} = 3.1\times 10^2.$$

45. **THINK** We differentiate the occupancy probability $P(E)$ with respect to E to explore the properties of $P(E)$.

EXPRESS The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

where k is the Boltzmann constant and E_F is the Fermi energy.

ANALYZE (a) The derivative of $P(E)$ is

$$\frac{dP}{dE} = \frac{-1}{[e^{(E-E_F)/kT}+1]^2}\frac{d}{dE}e^{(E-E_F)/kT} = \frac{-1}{[e^{(E-E_F)/kT}+1]^2}\frac{1}{kT}e^{(E-E_F)/kT}.$$

For $E = E_F$, we readily obtain the desired result:

$$\left.\frac{dP}{dE}\right|_{E=E_F} = \frac{-1}{[e^{(E_E-E_F)/kT}+1]^2}\frac{1}{kT}e^{(E_F-E_F)/kT} = -\frac{1}{4kT}.$$

(b) The equation of a line may be written as $y = m(x - x_0)$ where $m = -1/4kT$ is the slope, and x_0 is the x-intercept (which is what we are asked to solve for). It is clear that $P(E_F) = 1/2$, so our equation of the line, evaluated at $x = E_F$, becomes

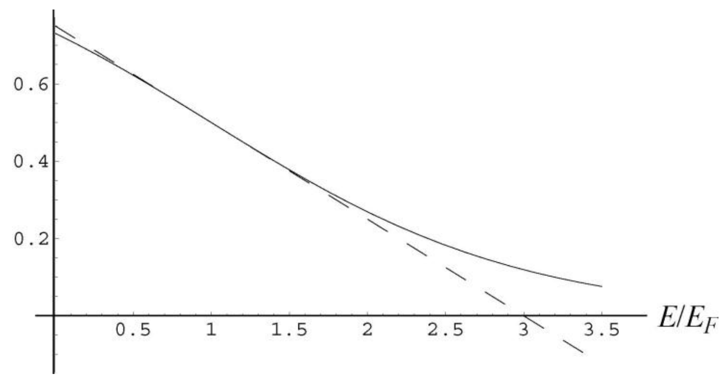
$$1/2 = (-\,1/4kT)(E_F - x_0),$$

which leads to $x_0 = E_F + 2kT$.

LEARN The straight line can be rewritten as

$$y = \frac{1}{2} - \frac{1}{4kT}\left(E - E_F\right).$$

A plot of $P(E)$ (solid line) and $y(E)$ (dashed line) in units of E_F / kT. The straight line passes the horizontal axis at $E/E_F = 3$.



46. (a) For copper, Eq. 41-10 leads to

$$\frac{d\rho}{dT} = [\rho\alpha]_{\text{Cu}} = (2\times10^{-8}\,\Omega\cdot\text{m})(4\times10^{-3}\,\text{K}^{-1}) = 8\times10^{-11}\,\Omega\cdot\text{m/K}.$$

(b) For silicon,

$$\frac{d\rho}{dT} = [\rho\alpha]_{\text{Si}} = (3\times10^{3}\,\Omega\cdot\text{m})(-70\times10^{-3}\,\text{K}^{-1}) = -2.1\times10^{2}\,\Omega\cdot\text{m/K}.$$

47. The description in the problem statement implies that an atom is at the center point C of the regular tetrahedron, since its four *neighbors* are at the four vertices. The side length for the tetrahedron is given as a = 388 pm. Since each face is an equilateral triangle, the "altitude" of each of those triangles (which is not to be confused with the altitude of the tetrahedron itself) is $h' = \frac{1}{2}a\sqrt{3}$ (this is generally referred to as the "slant height" in the solid geometry literature). At a certain location along the line segment representing the "slant height" of each face is the center C' of the face. Imagine this line segment starting at atom A and ending at the midpoint of one of the sides. Knowing that this line segment bisects the 60° angle of the equilateral face, it is easy to see that C' is a distance $AC' = a/\sqrt{3}$. If we draw a line from C' all the way to the farthest point on the

tetrahedron (this will land on an atom we label *B*), then this new line is the altitude *h* of the tetrahedron. Using the Pythagorean theorem,

$$h = \sqrt{a^2 - (AC')^2} = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = a\sqrt{\frac{2}{3}} .$$

Now we include coordinates: imagine atom *B* is on the +*y* axis at $y_b = h = a\sqrt{2/3}$, and atom *A* is on the +*x* axis at $x_a = AC' = a/\sqrt{3}$. Then point *C′* is the origin. The tetrahedron center point *C* is on the *y* axis at some value y_c, which we find as follows: *C* must be equidistant from *A* and *B*, so

$$y_b - y_c = \sqrt{x_a^2 + y_c^2} \quad \Rightarrow \quad a\sqrt{\frac{2}{3}} - y_c = \sqrt{\left(\frac{a}{\sqrt{3}}\right)^2 + y_c^2}$$

which yields $y_c = a/2\sqrt{6}$.

(a) In unit vector notation, using the information found above, we express the vector starting at *C* and going to *A* as

$$\vec{r}_{ac} = x_a\hat{\mathrm{i}} + (-y_c)\hat{\mathrm{j}} = \frac{\mathrm{a}}{\sqrt{3}}\hat{\mathrm{i}} - \frac{a}{2\sqrt{6}}\hat{\mathrm{j}} .$$

Similarly, the vector starting at *C* and going to *B* is

$$\vec{r}_{bc} = (y_b - y_c)\hat{\mathrm{j}} = \tfrac{a}{2}\sqrt{3/2}\,\hat{\mathrm{j}} .$$

Therefore, using Eq. 3-20,

$$\theta = \cos^{-1}\left(\frac{\vec{r}_{ac}\cdot\vec{r}_{bc}}{|\vec{r}_{ac}||\vec{r}_{bc}|}\right) = \cos^{-1}\left(-\frac{1}{3}\right)$$

which yields $\theta = 109.5°$ for the angle between adjacent bonds.

(b) The length of vector $\vec{r}_{bc}$ (which is, of course, the same as the length of $\vec{r}_{ac}$) is

$$|\vec{r}_{bc}| = \frac{a}{2}\sqrt{\frac{3}{2}} = \frac{388\text{pm}}{2}\sqrt{\frac{3}{2}} = 237.6 \text{ pm} \approx 238 \text{ pm}.$$

We note that in the solid geometry literature, the distance $\frac{a}{2}\sqrt{\frac{3}{2}}$ is known as the circumradius of the regular tetrahedron.

48. According to Eq. 41-6,

$$P(E_F+\Delta E)=\frac{1}{e^{(E_F+\Delta E-E_F)/kT}+1}=\frac{1}{e^{\Delta E/kT}+1}=\frac{1}{e^x+1}$$

where $x=\Delta E/kT$. Also,

$$P(E_F+\Delta E)=\frac{1}{e^{(E_F-\Delta E-E_F)/kT}+1}=\frac{1}{e^{-\Delta E/kT}+1}=\frac{1}{e^{-x}+1}\ .$$

Thus,

$$P(E_F+\Delta E)+P(E_F-\Delta E)=\frac{1}{e^x+1}+\frac{1}{e^{-x}+1}=\frac{e^x+1+e^{-x}+1}{(e^{-x}+1)(e^x+1)}=1\ .$$

A special case of this general result can be found in Problem 41-4, where ΔE = 63 meV and

$$P(E_F + 63\text{ meV}) + P(E_F - 63\text{ meV}) = 0.090 + 0.91 = 1.0.$$

49. (a) Setting $E = E_F$ (see Eq. 41-9), Eq. 41-5 becomes

$$N(E_F)=\frac{8\pi m\sqrt{2m}}{h^3}\left(\frac{3}{16\pi\sqrt{2}}\right)^{1/3}\frac{h}{\sqrt{m}}n^{1/3}\ .$$

Noting that $16\sqrt{2}=2^4 2^{1/2}=2^{9/2}$ so that the cube root of this is $2^{3/2}=2\sqrt{2}$, we are able to simplify the above expression and obtain

$$N(E_F)=\frac{4m}{h^2}\sqrt[3]{3\pi^2 n}$$

which is equivalent to the result shown in the problem statement. Since the desired numerical answer uses eV units, we multiply numerator and denominator of our result by c^2 and make use of the mc^2 value for an electron in Table 37-3 as well as the value $hc=1240\,\text{eV}\cdot\text{nm}$:

$$N(E_F)=\left(\frac{4mc^2}{(hc)^2}\sqrt[3]{3\pi^2}\right)n^{1/3}=\left(\frac{4(511\times10^3\text{ eV})}{(1240\,\text{eV}\cdot\text{nm})^2}\sqrt[3]{3\pi^2}\right)n^{1/3}=(4.11\,\text{nm}^{-2}\cdot\text{eV}^{-1})\,n^{1/3}$$

which is equivalent to the value indicated in the problem statement.

(b) Since there are 10^{27} cubic nanometers in a cubic meter, then the result of Problem 41-3 may be written as

$$n=8.49\times10^{28}\,\text{m}^{-3}=84.9\,\text{nm}^{-3}\ .$$

The cube root of this is $n^{1/3}\approx$ 4.4/nm. Hence, the expression in part (a) leads to

$$N(E_F)=(4.11\text{nm}^{-2}\cdot\text{eV}^{-1})(4.4\text{nm}^{-1})=18\text{nm}^{-3}\cdot\text{eV}^{-1}=1.8\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$$

If we multiply this by 10^{27} m^3/nm^3, we see this compares very well with the curve in Fig. 41-6 evaluated at 7.0 eV.

50. If we use the approximate formula discussed in Problem 41-21, we obtain

$$frac = \frac{3(8.62\times10^{-5}\text{ eV / K})(961+273\text{ K})}{2(5.5\text{ eV})} \approx 0.03 .$$

The numerical approach is briefly discussed in part (c) of Problem 41-21. Although the problem does not ask for it here, we remark that numerical integration leads to a fraction closer to 0.02.

51. We equate E_F with $\frac{1}{2}m_e v_F^2$ and write our expressions in such a way that we can make use of the electron mc^2 value found in Table 37-3:

$$v_F = \sqrt{\frac{2E_F}{m}} = c\sqrt{\frac{2E_F}{mc^2}} = (3.0\times10^5\text{ km / s})\sqrt{\frac{2(7.0\text{ eV})}{5.11\times10^5\text{ eV}}} = 1.6\times10^3\text{ km / s} .$$

52. The numerical factor $\left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3}$ is approximately equal to 0.121.

53. We use the ideal gas law in the form of Eq. 20-9:

$$p = nkT = (8.43\times10^{28}\text{ m}^{-3})(1.38\times10^{-23}\text{ J/K})(300\text{ K}) = 3.49\times10^8\text{Pa} = 3.49\times10^3\text{ atm} .$$

Chapter 42

1. Kinetic energy (we use the classical formula since v is much less than c) is converted into potential energy (see Eq. 24-43). From Appendix F or G, we find $Z = 3$ for lithium and $Z = 90$ for thorium; the charges on those nuclei are therefore $3e$ and $90e$, respectively. We manipulate the terms so that one of the factors of e cancels the "e" in the kinetic energy unit MeV, and the other factor of e is set to be 1.6×10^{-19} C. We note that $k = 1/4\pi\varepsilon_0$ can be written as 8.99×10^9 V·m/C. Thus, from energy conservation, we have

$$K = U \Rightarrow \quad r = \frac{kq_1q_2}{K} = \frac{\left(8.99\times10^9\ \frac{\text{V}\cdot\text{m}}{\text{C}}\right)\left(3\times1.6\times10^{-19}\ \text{C}\right)\left(90e\right)}{3.00\times10^6\ \text{eV}}$$

which yields $r = 1.3 \times 10^{-13}$ m (or about 130 fm).

2. Our calculation is similar to that shown in Sample Problem — "Rutherford scattering of an alpha particle by a gold nucleus." We set

$$K = 5.30\,\text{MeV} = U = \left(1/4\pi\varepsilon_0\right)\left(q_\alpha q_{\text{Cu}} / r_{\min}\right)$$

and solve for the closest separation, $r_{\min}$:

$$r_{\min} = \frac{q_\alpha q_{\text{Cu}}}{4\pi\varepsilon_0 K} = \frac{kq_\alpha q_{\text{Cu}}}{4\pi\varepsilon_0 K} = \frac{(2e)(29)\left(1.60\times10^{-19}\ \text{C}\right)\left(8.99\times10^9\ \text{V}\cdot\text{m/C}\right)}{5.30\times10^6\ \text{eV}}$$
$$= 1.58\times10^{-14}\ \text{m} = 15.8\ \text{fm}.$$

We note that the factor of e in $q_\alpha = 2e$ was not set equal to 1.60×10^{-19} C, but was instead allowed to cancel the "e" in the non-SI energy unit, electron-volt.

3. Kinetic energy (we use the classical formula since v is much less than c) is converted into potential energy. From Appendix F or G, we find $Z = 3$ for lithium and $Z = 110$ for Ds; the charges on those nuclei are therefore $3e$ and $110e$, respectively. From energy conservation, we have

$$K = U = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{Li}}q_{\text{Ds}}}{r}$$

which yields

$$r = \frac{1}{4\pi\varepsilon_0}\frac{q_{\rm Li}q_{\rm Ds}}{K} = \frac{(8.99\times10^9\ {\rm N\cdot m^2/C^2})(3\times1.6\times10^{-19}\ {\rm C})(110\times1.6\times10^{-19}\ {\rm C})}{(10.2\ {\rm MeV})(1.60\times10^{-13}\ {\rm J/MeV})}$$
$$= 4.65\times10^{-14}\ {\rm m} = 46.5\ {\rm fm}.$$

4. In order for the α particle to penetrate the gold nucleus, the separation between the centers of mass of the two particles must be no greater than

$$r = r_{\rm Cu} + r_\alpha = 6.23\ {\rm fm} + 1.80\ {\rm fm} = 8.03\ {\rm fm}.$$

Thus, the minimum energy K_α is given by

$$K_\alpha = U = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\rm Au}}{r} = \frac{kq_\alpha q_{\rm Au}}{r}$$
$$= \frac{\left(8.99\times10^9\ {\rm V\cdot m/C}\right)\left(2e\right)\left(79\right)\left(1.60\times10^{-19}\ {\rm C}\right)}{8.03\times10^{-15}\ {\rm m}} = 28.3\times10^6\ {\rm eV}.$$

We note that the factor of e in $q_\alpha = 2e$ was not set equal to 1.60×10^{-19} C, but was instead carried through to become part of the final units.

5. The conservation laws of (classical kinetic) energy and (linear) momentum determine the outcome of the collision (see Chapter 9). The final speed of the α particle is

$$v_{\alpha f} = \frac{m_\alpha - m_{\rm Au}}{m_\alpha + m_{\rm Au}} v_{\alpha i},$$

and that of the recoiling gold nucleus is

$$v_{{\rm Au},f} = \frac{2m_\alpha}{m_\alpha + m_{\rm Au}} v_{\alpha i}.$$

(a) Therefore, the kinetic energy of the recoiling nucleus is

$$K_{{\rm Au},f} = \frac{1}{2}m_{\rm Au}v_{{\rm Au},f}^2 = \frac{1}{2}m_{\rm Au}\left(\frac{2m_\alpha}{m_\alpha + m_{\rm Au}}\right)^2 v_{\alpha i}^2 = K_{\alpha i}\,\frac{4m_{\rm Au}m_\alpha}{\left(m_\alpha + m_{\rm Au}\right)^2}$$
$$= \left(5.00\ {\rm MeV}\right)\frac{4\left(197\ {\rm u}\right)\left(4.00\ {\rm u}\right)}{\left(4.00\ {\rm u} + 197\ {\rm u}\right)^2}$$
$$= 0.390\ {\rm MeV}.$$

(b) The final kinetic energy of the alpha particle is

$$K_{\alpha f} = \frac{1}{2} m_\alpha v_{\alpha f}^2 = \frac{1}{2} m_\alpha \left(\frac{m_\alpha - m_{\text{Au}}}{m_\alpha + m_{\text{Au}}} \right)^2 v_{\alpha i}^2 = K_{\alpha i} \left(\frac{m_\alpha - m_{\text{Au}}}{m_\alpha + m_{\text{Au}}} \right)^2$$

$$= (5.00 \text{ MeV}) \left(\frac{4.00 \text{ u} - 197 \text{ u}}{4.00 \text{ u} + 197 \text{ u}} \right)^2$$

$$= 4.61 \text{ MeV}.$$

We note that $K_{af} + K_{\text{Au},f} = K_{\alpha i}$ is indeed satisfied.

6. (a) The atomic number Z = 39 corresponds to the element yttrium (see Appendix F and/or Appendix G).

(b) The atomic number Z = 53 corresponds to iodine.

(c) A detailed listing of stable nuclides (such as the Web site http://nucleardata. nuclear.lu.se/nucleardata) shows that the stable isotope of yttrium has 50 neutrons (this can also be inferred from the Molar Mass values listed in Appendix F).

(d) Similarly, the stable isotope of iodine has 74 neutrons.

(e) The number of neutrons left over is 235 – 127 – 89 = 19.

7. For ^{55}Mn the mass density is

$$\rho_m = \frac{M}{V} = \frac{0.055 \text{kg/mol}}{(4\pi/3)\left[(1.2\times10^{-15}\,\text{m})(55)^{1/3}\right]^3 (6.02\times10^{23}/\text{mol})} = 2.3\times10^{17}\,\text{kg/m}^3.$$

(b) For ^{209}Bi,

$$\rho_m = \frac{M}{V} = \frac{0.209\,\text{kg/mol}}{(4\pi/3)\left[(1.2\times10^{-15}\,\text{m})(209)^{1/3}\right]^3 (6.02\times10^{23}/\text{mol})} = 2.3\times10^{17}\,\text{kg/m}^3.$$

(c) Since $V \propto r^3 = (r_0 A^{1/3})^3 \propto A,$ we expect $\rho_m \propto A/V \propto A/A \approx \text{const.}$ for all nuclides.

(d) For ^{55}Mn, the charge density is

$$\rho_q = \frac{Ze}{V} = \frac{(25)(1.6\times10^{-19}\,\text{C})}{(4\pi/3)\left[(1.2\times10^{-15}\,\text{m})(55)^{1/3}\right]^3} = 1.0\times10^{25}\,\text{C/m}^3.$$

(e) For ^{209}Bi, the charge density is

$$\rho_q = \frac{Ze}{V} = \frac{(83)(1.6\times10^{-19}\ \text{C})}{(4\pi/3)\left[(1.2\times10^{-15}\ \text{m})(209)^{1/3}\right]^3} = 8.8\times10^{24}\ \text{C/m}^3.$$

Note that $\rho_q \propto Z/V \propto Z/A$ should gradually decrease since $A > 2Z$ for large nuclides.

8. (a) The mass number A is the number of nucleons in an atomic nucleus. Since $m_p \approx m_n$ the mass of the nucleus is approximately Am_p. Also, the mass of the electrons is negligible since it is much less than that of the nucleus. So $M \approx Am_p$.

(b) For ^{1}H, the approximate formula gives

$$M \approx Am_p = (1)(1.007276\ \text{u}) = 1.007276\ \text{u}.$$

The actual mass is (see Table 42-1) 1.007825 u. The percentage deviation committed is then

$$\delta = (1.007825\ \text{u} - 1.007276\ \text{u})/1.007825\ \text{u} = 0.054\% \approx 0.05\%.$$

(c) Similarly, for ^{31}P, $\delta = 0.81\%$.

(d) For ^{120}Sn, $\delta = 0.81\%$.

(e) For ^{197}Au, $\delta = 0.74\%$.

(f) For ^{239}Pu, $\delta = 0.71\%$.

(g) No. In a typical nucleus the binding energy per nucleon is several MeV, which is a bit less than 1% of the nucleon mass times c^2. This is comparable with the percent error calculated in parts (b) – (f) , so we need to use a more accurate method to calculate the nuclear mass.

9. (a) 6 protons, since $Z = 6$ for carbon (see Appendix F).

(b) 8 neutrons, since $A - Z = 14 - 6 = 8$ (see Eq. 42-1).

10. (a) Table 42-1 gives the atomic mass of ^{1}H as $m = 1.007825$ u. Therefore, the *mass excess* for ^{1}H is

$$\Delta = (1.007825\ \text{u} - 1.000000\ \text{u}) = 0.007825\ \text{u}.$$

(b) In the unit MeV/c^2,

$$\Delta = (1.007825\ \text{u} - 1.000000\ \text{u})(931.5\ \text{MeV}/c^2\cdot\text{u}) = +7.290\ \text{MeV}/c^2.$$

(c) The mass of the neutron is $m_n = 1.008665$ u. Thus, for the neutron,

$$\Delta = (1.008665 \text{ u} - 1.000000 \text{ u}) = 0.008665 \text{ u}.$$

(d) In the unit MeV/c^2,

$$\Delta = (1.008665 \text{ u} - 1.000000 \text{ u})(931.5 \text{ MeV}/ c^2\cdot\text{u}) = +8.071 \text{ MeV}/c^2.$$

(e) Appealing again to Table 42-1, we obtain, for ^{120}Sn,

$$\Delta = (119.902199 \text{ u} - 120.000000 \text{ u}) = -0.09780 \text{ u}.$$

(f) In the unit MeV/c^2,

$$\Delta = (119.902199 \text{ u} - 120.000000 \text{ u}) (931.5 \text{ MeV}/ c^2\cdot\text{u}) = -91.10 \text{ MeV}/c^2.$$

11. **THINK** To resolve the detail of a nucleus, the de Broglie wavelength of the probe must be smaller than the size of the nucleus.

EXPRESS The de Broglie wavelength is given by $\lambda = h/p$, where p is the magnitude of the momentum. Since the kinetic energy K of the electron is much greater than its rest energy, relativistic formulation must be used. The kinetic energy and the momentum are related by Eq. 37-54:

$$pc = \sqrt{K^2 + 2Kmc^2}.$$

ANALYZE (a) With K = 200 MeV and mc^2 = 0.511 MeV, we obtain

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(200\,\text{MeV})^2 + 2(200\,\text{MeV})(0.511\,\text{MeV})} = 200.5\,\text{MeV}.$$

Thus,

$$\lambda = \frac{hc}{pc} = \frac{1240 \text{ eV}\cdot\text{nm}}{200.5\times10^6\,\text{eV}} = 6.18\times10^{-6}\,\text{nm} \approx 6.2 \text{ fm}.$$

(b) The diameter of a copper nucleus, for example, is about 8.6 fm, just a little larger than the de Broglie wavelength of a 200-MeV electron. To resolve detail, the wavelength should be smaller than the target, ideally a tenth of the diameter or less. 200-MeV electrons are perhaps at the lower limit in energy for useful probes.

LEARN The more energetic the incident particle, the finer the details of the target that can be probed.

12. (a) Since $U > 0$, the energy represents a tendency for the sphere to blow apart.

(b) For ^{239}Pu, $Q = 94e$ and R = 6.64 fm. Including a conversion factor for $\text{J} \to \text{eV}$ we obtain

$$U=\frac{3Q^2}{20\pi\varepsilon_0 r}=\frac{3\,94^2\left(1.60\times10^{-19}\text{ C}\right)^2\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)}{5\left(6.64\times10^{-15}\text{ m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\text{ J}}\right)$$
$$=1.15\times10^9\text{ eV}=1.15\,\text{GeV}.$$

(c) Since $Z = 94$, the electrostatic potential per proton is 1.15 GeV/94 = 12.2 MeV/proton.

(d) Since $A = 239$, the electrostatic potential per nucleon is 1.15 GeV/239 = 4.81 MeV/nucleon.

(e) The strong force that binds the nucleus is very strong.

13. We note that the mean density and mean radius for the Sun are given in Appendix C. Since $\rho = M/V$ where $V \propto r^3$, we get $r \propto \rho^{-1/3}$. Thus, the new radius would be

$$r = R_s\left(\frac{\rho_s}{\rho}\right)^{1/3}=\left(6.96\times10^8\text{ m}\right)\left(\frac{1410\text{ kg/m}^3}{2\times10^{17}\text{ kg/m}^3}\right)^{1/3}=1.3\times10^4\text{ m}.$$

14. The binding energy is given by

$$\Delta E_{\text{be}}=\left[Zm_H+\left(A-Z\right)m_n-M_{\text{Am}}\right]c^2,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Am} is the mass of a ${}^{244}_{95}\text{Am}$ atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Am}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (95)(1.007825\text{ u}) + (244 - 95)(1.008665\text{ u}) - (244.064279\text{ u}) = 1.970181\text{ u}.$$

Since 1 u is equivalent to 931.494013 MeV,

$$\Delta E_{\text{be}} = (1.970181\text{ u})(931.494013\text{ MeV/u}) = 1835.212\text{ MeV}.$$

Since there are 244 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1835.212\text{ MeV})/244 = 7.52\text{ MeV}.$$

15. (a) Since the nuclear force has a short range, any nucleon interacts only with its nearest neighbors, not with more distant nucleons in the nucleus. Let N be the number of neighbors that interact with any nucleon. It is independent of the number A of nucleons in the nucleus. The number of interactions in a nucleus is approximately NA, so the energy

associated with the strong nuclear force is proportional to *NA* and, therefore, proportional to *A* itself.

(b) Each proton in a nucleus interacts electrically with every other proton. The number of pairs of protons is $Z(Z-1)/2$, where Z is the number of protons. The Coulomb energy is, therefore, proportional to $Z(Z-1)$.

(c) As A increases, Z increases at a slightly slower rate but Z^2 increases at a faster rate than A and the energy associated with Coulomb interactions increases faster than the energy associated with strong nuclear interactions.

16. The binding energy is given by

$$\Delta E_{\text{be}} = \left[Zm_H + \left(A - Z \right) m_n - M_{\text{Eu}} \right] c^2 ,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Eu} is the mass of a ${}^{152}_{63}\text{Eu}$ atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Eu}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (63)(1.007825\text{ u}) + (152 - 63)(1.008665\text{ u}) - (151.921742\text{ u}) = 1.342418\text{ u}.$$

Since 1 u is equivalent to 931.494013 MeV,

$$\Delta E_{\text{be}} = (1.342418\text{ u})(931.494013\text{ MeV/u}) = 1250.454\text{ MeV}.$$

Since there are 152 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1250.454\text{ MeV})/152 = 8.23\text{ MeV}.$$

17. It should be noted that when the problem statement says the “masses of the proton and the deuteron are …” they are actually referring to the corresponding atomic masses (given to very high precision). That is, the given masses include the “orbital” electrons. As in many computations in this chapter, this circumstance (of implicitly including electron masses in what should be a purely nuclear calculation) does not cause extra difficulty in the calculation. Setting the gamma ray energy equal to ΔE_{be}, we solve for the neutron mass (with each term understood to be in u units):

$$\begin{aligned} m_{\text{n}} &= M_{\text{d}} - m_{\text{H}} + \frac{E_\gamma}{c^2} = 2.013553212 - 1.007276467 + \frac{2.2233}{931.502} \\ &= 1.0062769 + 0.0023868 \end{aligned}$$

which yields m_n = 1.0086637 u $\approx$ 1.0087 u.

18. The binding energy is given by

$$\Delta E_{\text{be}} = \left[Zm_H + (A - Z)m_n - M_{\text{Rf}} \right] c^2 ,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Rf} is the mass of a ${}^{259}_{104}\text{Rf}$ atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Rf}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (104)(1.007825\text{ u}) + (259 - 104)(1.008665\text{ u}) - (259.10563\text{ u}) = 2.051245\text{ u}.$$

Since 1 u is equivalent to 931.494013 MeV,

$$\Delta E_{\text{be}} = (2.051245\text{ u})(931.494013\text{ MeV/u}) = 1910.722\text{ MeV}.$$

Since there are 259 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1910.722\text{ MeV})/259 = 7.38\text{ MeV}.$$

19. Let f_{24} be the abundance of ^{24}Mg, let f_{25} be the abundance of ^{25}Mg, and let f_{26} be the abundance of ^{26}Mg. Then, the entry in the periodic table for Mg is

$$24.312 = 23.98504 f_{24} + 24.98584 f_{25} + 25.98259 f_{26}.$$

Since there are only three isotopes, $f_{24} + f_{25} + f_{26} = 1$. We solve for f_{25} and f_{26}. The second equation gives $f_{26} = 1 - f_{24} - f_{25}$. We substitute this expression and f_{24} = 0.7899 into the first equation to obtain

$$24.312 = (23.98504)(0.7899) + 24.98584 f_{25} + 25.98259 - (25.98259)(0.7899) - 25.98259 f_{25}.$$

The solution is f_{25} = 0.09303. Then,

$$f_{26} = 1 - 0.7899 - 0.09303 = 0.1171.\ 78.99\%$$

of naturally occurring magnesium is ^{24}Mg.

(a) Thus, 9.303% is ^{25}Mg.

(b) 11.71% is ^{26}Mg.

20. From Appendix F and/or G, we find Z = 107 for bohrium, so this isotope has $N = A - Z = 262 - 107 = 155$ neutrons. Thus,

$$\Delta E_{\text{ben}} = \frac{\left(Zm_{\text{H}} + Nm_n - m_{\text{Bh}}\right)c^2}{A} = \frac{\left((107)(1.007825\,\text{u}) + (155)(1.008665\,\text{u}) - 262.1231\,\text{u}\right)(931.5\,\text{MeV/u})}{262}$$

which yields 7.31 MeV per nucleon.

21. **THINK** Binding energy is the difference in mass energy between a nucleus and its individual nucleons.

EXPRESS If a nucleus contains Z protons and N neutrons, its binding energy is given by Eq. 42-7:

$$\Delta E_{\text{be}} = \sum(mc^2) - Mc^2 = \left(Zm_H + Nm_n - M\right)c^2,$$

where m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M is the mass of the atom containing the nucleus of interest.

ANALYZE (a) If the masses are given in atomic mass units, then mass excesses are defined by $\Delta_H = (m_H - 1)c^2, \Delta_n = (m_n - 1)c^2$, and $\Delta = (M - A)c^2$. This means $m_H c^2 = \Delta_H + c^2$, $m_n c^2 = \Delta_n + c^2$, and $Mc^2 = \Delta + Ac^2$. Thus,

$$\Delta E_{\text{be}} = \left(Z\Delta_H + N\Delta_n - \Delta\right) + \left(Z + N - A\right)c^2 = Z\Delta_H + N\Delta_n - \Delta,$$

where $A = Z + N$ is used.

(b) For ${}^{197}_{79}\text{Au}$, Z = 79 and N = 197 – 79 = 118. Hence,

$$\Delta E_{\text{be}} = (79)(7.29\,\text{MeV}) + (118)(8.07\,\text{MeV}) - (-31.2\,\text{MeV}) = 1560\,\text{MeV}.$$

This means the binding energy per nucleon is $\Delta E_{\text{ben}} = (1560\,\text{MeV})/197 = 7.92\,\text{MeV}$.

LEARN Using mass excesses (Δ_H, Δ_n, and Δ) instead of actual masses provides another convenient way of calculating the binding energy of a nucleus.

22. (a) The first step is to add energy to produce ${}^4\text{He} \to p + {}^3\text{H}$, which — to make the electrons "balance" — may be rewritten as ${}^4\text{He} \to {}^1\text{H} + {}^3\text{H}$. The energy needed is

$$\Delta E_1 = \left(m_{{}^3\text{H}} + m_{{}^1\text{H}} - m_{{}^4\text{He}}\right)c^2 = \left(3.01605\,\text{u} + 1.00783\,\text{u} - 4.00260\,\text{u}\right)(931.5\,\text{MeV/u}) = 19.8\,\text{MeV}.$$

(b) The second step is to add energy to produce ${}^3\text{H} \to n + {}^2\text{H}$. The energy needed is

$$\Delta E_2 = \left(m_{{}^2\text{H}} + m_n - m_{{}^3\text{H}}\right)c^2 = \left(2.01410\,\text{u} + 1.00867\,\text{u} - 3.01605\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= 6.26\,\text{MeV}.$$

(c) The third step: ${}^2\text{H} \to p + n$, which — to make the electrons "balance" — may be rewritten as ${}^2\text{H} \to {}^1\text{H} + n$. The work required is

$$\Delta E_3 = \left(m_{{}^1\text{H}} + m_n - m_{{}^2\text{H}}\right)c^2 = \left(1.00783\,\text{u} + 1.00867\,\text{u} - 2.01410\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= 2.23\,\text{MeV}.$$

(d) The total binding energy is

$$\Delta E_{\text{be}} = \Delta E_1 + \Delta E_2 + \Delta E_3 = 19.8\,\text{MeV} + 6.26\,\text{MeV} + 2.23\,\text{MeV} = 28.3\,\text{MeV}.$$

(e) The binding energy per nucleon is

$$\Delta E_{\text{ben}} = \Delta E_{\text{be}} / A = 28.3\,\text{MeV} / 4 = 7.07\,\text{MeV}.$$

(f) No, the answers do not match.

23. **THINK** The binding energy is given by

$$\Delta E_{\text{be}} = \left[Zm_H + (A - Z)m_n - M_{\text{Pu}}\right]c^2,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Pu} is the mass of a ${}^{239}_{94}\text{Pu}$ atom.

EXPRESS In principle, nuclear masses should be used, but the mass of the Z electrons included in Zm_H is canceled by the mass of the Z electrons included in M_{Pu}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (94)(1.00783\text{ u}) + (239 - 94)(1.00867\text{ u}) - (239.05216\text{ u}) = 1.94101\text{ u}.$$

Since the mass energy of 1 u is equivalent to 931.5 MeV,

$$\Delta E_{\text{be}} = (1.94101\text{ u})(931.5\text{ MeV/u}) = 1808\text{ MeV}.$$

ANALYZE With 239 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1808\text{ MeV})/239 = 7.56\text{ MeV}.$$

The result is the same as that given in Table 42-1.

LEARN An alternative way to calculate binding energy is to use mass excesses, as discussed in Problem 21. The formula is

$$\Delta E_{\text{be}} = Z\Delta_H + N\Delta_n - \Delta_{239},$$

where $\Delta_H = (m_H - 1)c^2$, $\Delta_n = (m_n - 1)c^2$, and $\Delta_{239} = (M_{\text{Pu}} - 239\text{ u})c^2$.

24. We first "separate" all the nucleons in one copper nucleus (which amounts to simply calculating the nuclear binding energy) and then figure the number of nuclei in the penny (so that we can multiply the two numbers and obtain the result). To begin, we note that (using Eq. 42-1 with Appendix F and/or G) the copper-63 nucleus has 29 protons and 34 neutrons. Thus,

$$\begin{aligned}\Delta E_{\text{be}} &= \big(29(1.007825\,\text{u}) + 34(1.008665\,\text{u}) - 62.92960\,\text{u}\big)(931.5\,\text{MeV/u}) \\ &= 551.4\,\text{MeV}.\end{aligned}$$

To figure the number of nuclei (or, equivalently, the number of atoms), we adapt Eq. 42-21:

$$N_{\text{Cu}} = \left(\frac{3.0\,\text{g}}{62.92960\,\text{g/mol}}\right)\left(6.02\times10^{23}\text{ atoms/mol}\right) \approx 2.9\times10^{22}\text{ atoms}.$$

Therefore, the total energy needed is

$$N_{\text{Cu}}\Delta E_{\text{be}} = (551.4\,\text{MeV})\left(2.9\times10^{22}\right) = 1.6\times10^{25}\text{ MeV}.$$

25. The rate of decay is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. In terms of the half-life $T_{1/2}$, the disintegration constant is $\lambda = (\ln 2)/T_{1/2}$, so

$$\begin{aligned}N &= \frac{R}{\lambda} = \frac{RT_{1/2}}{\ln 2} = \frac{(6000\,\text{Ci})\left(3.7\times10^{10}\text{ s}^{-1}/\text{Ci}\right)(5.27\,\text{y})\left(3.16\times10^{7}\text{ s/y}\right)}{\ln 2} \\ &= 5.33\times10^{22}\text{ nuclei}.\end{aligned}$$

26. By the definition of half-life, the same has reduced to $\frac{1}{2}$ its initial amount after 140 d. Thus, reducing it to $\frac{1}{4} = \left(\frac{1}{2}\right)^2$ of its initial number requires that two half-lives have passed: $t = 2T_{1/2} = 280$ d.

27. (a) Since 60 y = 2(30 y) = $2T_{1/2}$, the fraction left is $2^{-2} = 1/4 = 0.250$.

(b) Since 90 y = 3(30 y) = $3T_{1/2}$, the fraction that remains is $2^{-3} = 1/8 = 0.125$.

28. (a) We adapt Eq. 42-21:

$$N_{\text{Pu}} = \left(\frac{0.002\,\text{g}}{239\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{nuclei/mol}\right) \approx 5.04\times10^{18}\,\text{nuclei}.$$

(b) Eq. 42-20 leads to

$$R = \frac{N\ln 2}{T_{1/2}} = \frac{5\times10^{18}\ln 2}{2.41\times10^{4}\,\text{y}} = 1.4\times10^{14}\,/\,\text{y}$$

which is equivalent to 4.60×10^6/s = 4.60×10^6 Bq (the unit becquerel is defined as 1 decay/s).

29. **THINK** Half-life is the time is takes for the number of radioactive nuclei to decrease to half of its initial value.

EXPRESS The half-life $T_{1/2}$ and the disintegration constant λ are related by

$$T_{1/2} = (\ln 2)/\lambda.$$

ANALYZE (a) With $\lambda = 0.0108\ \text{h}^{-1}$, we obtain

$$T_{1/2} = (\ln 2)/(0.0108\ \text{h}^{-1}) = 64.2\ \text{h}.$$

(b) At time t, the number of undecayed nuclei remaining is given by

$$N = N_0\, e^{-\lambda t} = N_0\, e^{-(\ln 2)t/T_{1/2}}.$$

We substitute $t = 3T_{1/2}$ to obtain

$$\frac{N}{N_0} = e^{-3\ln 2} = 0.125.$$

In each half-life, the number of undecayed nuclei is reduced by half. At the end of one half-life, $N = N_0/2$, at the end of two half-lives, $N = N_0/4$, and at the end of three half-lives, $N = N_0/8 = 0.125N_0$.

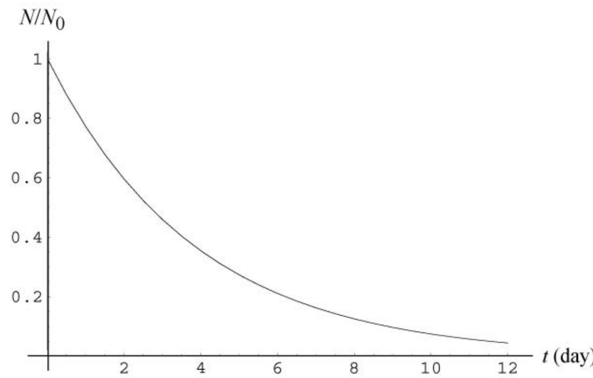
(c) We use

$$N = N_0\, e^{-\lambda t}.$$

Since 10.0 d is 240 h, $\lambda t = (0.0108\ \text{h}^{-1})(240\ \text{h}) = 2.592$ and

$$\frac{N}{N_0} = e^{-2.592} = 0.0749.$$

LEARN The fraction of the Hg sample remaining as a function of time (measured in days) is plotted below.



30. We note that t = 24 h is four times $T_{1/2}$ = 6.5 h. Thus, it has reduced by half, four-fold:

$$\left(\frac{1}{2}\right)^4 \left(48\times10^{19}\right) = 3.0\times10^{19}.$$

31. (a) The decay rate is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. Initially, $R = R_0 = \lambda N_0$, where N_0 is the number of undecayed nuclei at that time. One must find values for both N_0 and λ. The disintegration constant is related to the half-life $T_{1/2}$ by

$$\lambda = (\ln 2)/T_{1/2} = (\ln 2)/(78\text{h}) = 8.89\times10^{-3}\,\text{h}^{-1}.$$

If M is the mass of the sample and m is the mass of a single atom of gallium, then $N_0 = M/m$. Now,

$$m = (67\text{ u})(1.661 \times 10^{-24}\text{ g/u}) = 1.113 \times 10^{-22}\text{ g}$$

and

$$N_0 = (3.4\text{ g})/(1.113 \times 10^{-22}\text{ g}) = 3.05 \times 10^{22}.$$

Thus,

$$R_0 = (8.89 \times 10^{-3}\text{ h}^{-1})(3.05 \times 10^{22}) = 2.71 \times 10^{20}\text{ h}^{-1} = 7.53 \times 10^{16}\text{ s}^{-1}.$$

(b) The decay rate at any time t is given by

$$R = R_0\, e^{-\lambda t}$$

where R_0 is the decay rate at t = 0. At t = 48 h, $\lambda t = (8.89 \times 10^{-3}\text{ h}^{-1})(48\text{ h}) = 0.427$ and

$$R = \left(7.53\times10^{16}\ \mathrm{s}^{-1}\right)e^{-0.427} = 4.91\times10^{16}\ \mathrm{s}^{-1}.$$

32. Using Eq. 42-15 with Eq. 42-18, we find the fraction remaining:

$$\frac{N}{N_0} = e^{-t\ln 2/T_{1/2}} = e^{-30\ln 2/29} = 0.49.$$

33. We note that 3.82 days is 330048 s, and that a becquerel is a disintegration per second (see Section 42-3). From Eq. 34-19, we have

$$\frac{N}{\mathcal{V}} = \frac{R}{\mathcal{V}}\frac{T_{1/2}}{\ln 2} = \left(1.55\times10^{5}\ \frac{\mathrm{Bq}}{\mathrm{m}^3}\right)\frac{330048\ \mathrm{s}}{\ln 2} = 7.4\times10^{10}\ \frac{\mathrm{atoms}}{\mathrm{m}^3}$$

where we have divided by volume v. We estimate v (the volume breathed in 48 h = 2880 min) as follows:

$$\left(2\,\frac{\mathrm{liters}}{\mathrm{breath}}\right)\left(\frac{1\,\mathrm{m}^3}{1000\,\mathrm{L}}\right)\left(40\,\frac{\mathrm{breaths}}{\mathrm{min}}\right)\left(2880\,\mathrm{min}\right)$$

which yields $v \approx 200\ \mathrm{m}^3$. Thus, the order of magnitude of N is

$$\left(\frac{N}{\mathcal{V}}\right)(\mathcal{V}) \approx \left(7\times10^{10}\ \frac{\mathrm{atoms}}{\mathrm{m}^3}\right)\left(200\,\mathrm{m}^3\right) \approx 1\times10^{13}\ \mathrm{atoms}.$$

34. Combining Eqs. 42-20 and 42-21, we obtain

$$M_{\mathrm{sam}} = N\frac{M_{\mathrm{K}}}{M_{\mathrm{A}}} = \left(\frac{RT_{1/2}}{\ln 2}\right)\left(\frac{40\ \mathrm{g/mol}}{6.02\times10^{23}\ /\mathrm{mol}}\right)$$

which gives 0.66 g for the mass of the sample once we plug in 1.7×10^{5}/s for the decay rate and 1.28×10^{9} y = 4.04×10^{16} s for the half-life.

35. **THINK** We modify Eq. 42-11 to take into consideration the rate of production of the radionuclide.

EXPRESS If N is the number of undecayed nuclei present at time t, then

$$\frac{dN}{dt} = R - \lambda N$$

where R is the rate of production by the cyclotron and λ is the disintegration constant. The second term gives the rate of decay. Note the sign difference between R and λN.

ANALYZE (a) Rearrange the equation slightly and integrate:

$$\int_{N_0}^{N} \frac{dN}{R-\lambda N} = \int_0^t dt$$

where N_0 is the number of undecayed nuclei present at time $t = 0$. This yields

$$-\frac{1}{\lambda}\ln\frac{R-\lambda N}{R-\lambda N_0} = t.$$

We solve for N:

$$N = \frac{R}{\lambda} + \left(N_0 - \frac{R}{\lambda}\right)e^{-\lambda t}.$$

After many half-lives, the exponential is small and the second term can be neglected. Then, $N = R/\lambda$.

(b) The result $N = R/\lambda$ holds regardless of the initial value N_0, because the dependence on N_0 shows up only in the second term, which is exponentially suppressed at large t.

LEARN At times that are long compared to the half-life, the rate of production equals the rate of decay and N is a constant. The nuclide is in secular equilibrium with its source.

36. We have one alpha particle (helium nucleus) produced for every plutonium nucleus that decays. To find the number that have decayed, we use Eq. 42-15, Eq. 42-18, and adapt Eq. 42-21:

$$N_0 - N = N_0\left(1 - e^{-t\ln 2/T_{1/2}}\right) = N_A\frac{12.0\text{ g/mol}}{239\text{ g/mol}}\left(1 - e^{-20000\ln 2/24100}\right)$$

where N_A is the Avogadro constant. This yields 1.32×10^{22} alpha particles produced. In terms of the amount of helium gas produced (assuming the α particles slow down and capture the appropriate number of electrons), this corresponds to

$$m_{\text{He}} = \left(\frac{1.32\times 10^{22}}{6.02\times 10^{23}\text{ /mol}}\right)\left(4.0\text{ g/mol}\right) = 87.9\times 10^{-3}\text{ g}.$$

37. Using Eq. 42-15 and Eq. 42-18 (and the fact that mass is proportional to the number of atoms), the amount decayed is

$$\begin{aligned}|\Delta m| &= m\Big|_{t_f=16.0\text{ h}} - m\Big|_{t_f=14.0\text{ h}} = m_0\left(1-e^{-t_i\ln 2/T_{1/2}}\right) - m_0\left(1-e^{-t_f\ln 2/T_{1/2}}\right)\\ &= m_0\left(e^{-t_f\ln 2/T_{1/2}} - e^{-t_i\ln 2/T_{1/2}}\right) = (5.50\text{g})\left[e^{-(16.0h/12.7\text{h})\ln 2} - e^{-(14.0\text{h}/12.7\text{h})\ln 2}\right]\\ &= 0.265\text{g}.\end{aligned}$$

38. With $T_{1/2} = 3.0\text{ h} = 1.08\times10^4\text{ s}$, the decay constant is (using Eq. 42-18)

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.08\times10^4\text{ s}} = 6.42\times10^{-5}\,/\text{s}\,.$$

Thus, the number of isotope parents injected is

$$N = \frac{R}{\lambda} = \frac{(8.60\times10^{-6}\,\text{Ci})(3.7\times10^{10}\,\text{Bq/Ci})}{6.42\times10^{-5}\,/\text{s}} = 4.96\times10^9\,.$$

39. (a) The sample is in secular equilibrium with the source, and the decay rate equals the production rate. Let R be the rate of production of ^{56}Mn and let λ be the disintegration constant. According to the result of Problem 42-35, $R = \lambda N$ after a long time has passed. Now, $\lambda N = 8.88 \times 10^{10}\text{ s}^{-1}$, so $R = 8.88 \times 10^{10}\text{ s}^{-1}$.

(b) We use $N = R/\lambda$. If $T_{1/2}$ is the half-life, then the disintegration constant is

$$\lambda = (\ln 2)/T_{1/2} = (\ln 2)/(2.58\text{ h}) = 0.269\text{ h}^{-1} = 7.46 \times 10^{-5}\text{ s}^{-1},$$

so $N = (8.88 \times 10^{10}\text{ s}^{-1})/(7.46 \times 10^{-5}\text{ s}^{-1}) = 1.19 \times 10^{15}$.

(c) The mass of a ^{56}Mn nucleus is

$$m = (56\text{ u})\,(1.661 \times 10^{-24}\text{ g/u}) = 9.30 \times 10^{-23}\text{ g}$$

and the total mass of ^{56}Mn in the sample at the end of the bombardment is

$$Nm = (1.19 \times 10^{15})(9.30 \times 10^{-23}\text{ g}) = 1.11 \times 10^{-7}\text{ g}.$$

40. We label the two isotopes with subscripts 1 (for ^{32}P) and 2 (for ^{33}P). Initially, 10% of the decays come from ^{33}P, which implies that the initial rate $R_{02} = 9R_{01}$. Using Eq. 42-17, this means

$$R_{01} = \lambda_1 N_{01} = \frac{1}{9} R_{02} = \frac{1}{9}\lambda_2 N_{02}.$$

At time t, we have $R_1 = R_{01}e^{-\lambda_1 t}$ and $R_2 = R_{02}e^{-\lambda_2 t}$. We seek the value of t for which $R_1 = 9R_2$ (which means 90% of the decays arise from ^{33}P). We divide equations to obtain

$$\left(R_{01}/R_{02}\right)e^{-(\lambda_1-\lambda_2)t} = 9,$$

and solve for t:

$$t=\frac{1}{\lambda_1-\lambda_2}\ln\left(\frac{R_{01}}{9R_{02}}\right)=\frac{\ln\left(R_{01}/9R_{02}\right)}{\ln 2/T_{1/2_1}-\ln 2/T_{1/2_2}}=\frac{\ln\left[(1/9)^2\right]}{\ln 2\left[(14.3\text{d})^{-1}-(25.3\text{d})^{-1}\right]}$$
$$=209\text{d}.$$

41. The number N of undecayed nuclei present at any time and the rate of decay R at that time are related by $R = \lambda N$, where λ is the disintegration constant. The disintegration constant is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$, so $R = (N \ln 2)/T_{1/2}$ and

$$T_{1/2} = (N \ln 2)/R.$$

Since 15.0% by mass of the sample is ^{147}Sm, the number of ^{147}Sm nuclei present in the sample is

$$N=\frac{(0.150)(1.00\text{ g})}{(147\text{ u})(1.661\times10^{-24}\text{ g/u})}=6.143\times10^{20}.$$

Thus,

$$T_{1/2}=\frac{(6.143\times10^{20})\ln 2}{120\text{ s}^{-1}}=3.55\times10^{18}\text{ s}=1.12\times10^{11}\text{ y}.$$

42. Adapting Eq. 42-21, we have

$$N_{\text{Kr}}=\frac{M_{\text{sam}}}{M_{\text{Kr}}}N_A=\left(\frac{20\times10^{-9}\text{ g}}{92\text{ g/mol}}\right)(6.02\times10^{23}\text{ atoms/mol})=1.3\times10^{14}\text{ atoms}.$$

Consequently, Eq. 42-20 leads to

$$R=\frac{N\ln 2}{T_{1/2}}=\frac{(1.3\times10^{14})\ln 2}{1.84\text{ s}}=4.9\times10^{13}\text{ Bq}.$$

43. Using Eq. 42-16 with Eq. 42-18, we find the initial activity:

$$R_0=Re^{t\ln 2/T_{1/2}}=(7.4\times10^{8}\text{ Bq})e^{24\ln 2/83.61}=9.0\times10^{8}\text{ Bq}.$$

44. The number of atoms present initially at $t=0$ is $N_0=2.00\times10^6$. From Fig. 42-19, we see that the number is halved at $t=2.00$ s. Thus, using Eq. 42-15, we find the decay constant to be

$$\lambda=\frac{1}{t}\ln\left(\frac{N_0}{N}\right)=\frac{1}{2.00\text{ s}}\ln\left(\frac{N_0}{N_0/2}\right)=\frac{1}{2.00\text{ s}}\ln 2=0.3466\text{ s}^{-1}.$$

At $t=27.0$ s, the number of atoms remaining is

$$N = N_0 e^{-\lambda t} = (2.00\times10^6)e^{-(0.3466/\text{s})(27.0\,\text{s})} \approx 173\,.$$

Using Eq. 42-17, the decay rate is

$$R = \lambda N = (0.3466/\text{s})(173) \approx 60/\text{s} = 60\text{ Bq}\,.$$

45. (a) Equation 42-20 leads to

$$R = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2}{30.2\text{y}}\left(\frac{M_{\text{sam}}}{m_{\text{atom}}}\right) = \frac{\ln 2}{9.53\times10^8\text{ s}}\left(\frac{0.0010\text{kg}}{137\times1.661\times10^{-27}\text{ kg}}\right)$$
$$= 3.2\times10^{12}\text{ Bq.}$$

(b) Using the conversion factor $1\text{ Ci} = 3.7\times10^{10}$ Bq, $R = 3.2\times10^{12}\text{ Bq} = 86$ Ci.

46. (a) Molybdenum beta decays into technetium:

$$^{99}_{42}\text{Mo} \rightarrow ^{99}_{43}\text{Tc} + e^- + \nu$$

(b) Each decay corresponds to a photon produced when the technetium nucleus de-excites (note that the de-excitation half-life is much less than the beta decay half-life). Thus, the gamma rate is the same as the decay rate: 8.2×10^7/s.

(c) Equation 42-20 leads to

$$N = \frac{RT_{1/2}}{\ln 2} = \frac{(38/\text{s})(6.0\text{ h})(3600\text{ s/h})}{\ln 2} = 1.2\times10^6.$$

47. **THINK** The mass fraction of Ra in $RaCl_2$ is given by

$$\frac{M_{\text{Ra}}}{M_{\text{Ra}} + 2M_{\text{Cl}}}$$

where M_{Ra} is the molar mass of Ra and M_{Cl} is the molar mass of Cl.

EXPRESS We assume that the chlorine in the sample had the naturally occurring isotopic mixture, so the average molar mass is 35.453 g/mol, as given in Appendix F. Then, the mass of ^{226}Ra was

$$m = \frac{226}{226 + 2(35.453)}(0.10\text{ g}) = 76.1\times10^{-3}\text{ g.}$$

ANALYZE (a) The mass of a ^{226}Ra nucleus is $(226\text{ u})(1.661\times10^{-24}\text{ g/u}) = 3.75\times10^{-22}$ g, so the number of ^{226}Ra nuclei present was

$$N = (76.1 \times 10^{-3}\ \text{g})/(3.75 \times 10^{-22}\ \text{g}) = 2.03 \times 10^{20}.$$

(b) The decay rate is given by

$$R = N\lambda = (N \ln 2)/T_{1/2},$$

where λ is the disintegration constant, $T_{1/2}$ is the half-life, and N is the number of nuclei. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. Thus,

$$R = \frac{\left(2.03\times10^{20}\right)\ln 2}{\left(1600\ \text{y}\right)\left(3.156\times10^{7}\ \text{s/y}\right)} = 2.79\times10^{9}\ \text{s}^{-1}.$$

LEARN Radium has 33 different known isotopes, four of which naturally occurring. ^{226}Ra, with a half-life of 1600 years, is the most stable isotope of radium.

48. (a) The nuclear reaction is written as $^{238}\text{U}\rightarrow{}^{234}\text{Th} + {}^{4}\text{He}$. The energy released is

$$\begin{aligned}\Delta E_1 &= \left(m_{\text{U}} - m_{\text{He}} - m_{\text{Th}}\right)c^2\\ &= \left(238.05079\ \text{u} - 4.00260\ \text{u} - 234.04363\ \text{u}\right)\left(931.5\ \text{MeV/u}\right)\\ &= 4.25\ \text{MeV}.\end{aligned}$$

(b) The reaction series consists of $^{238}\text{U}\rightarrow{}^{237}\text{U}+\text{n}$, followed by

$$\begin{aligned}&^{237}\text{U}\rightarrow{}^{236}\text{Pa}+\text{p}\\ &^{236}\text{Pa}\rightarrow{}^{235}\text{Pa}+\text{n}\\ &^{235}\text{Pa}\rightarrow{}^{234}\text{Th}+\text{p}\end{aligned}$$

The net energy released is then

$$\begin{aligned}\Delta E_2 &= \left(m_{^{238}\text{U}} - m_{^{237}\text{U}} - m_n\right)c^2 + \left(m_{^{237}\text{U}} - m_{^{236}\text{Pa}} - m_p\right)c^2\\ &\quad + \left(m_{^{236}\text{Pa}} - m_{^{235}\text{Pa}} - m_n\right)c^2 + \left(m_{^{235}\text{Pa}} - m_{^{234}\text{Th}} - m_p\right)c^2\\ &= \left(m_{^{238}\text{U}} - 2m_n - 2m_p - m_{^{234}\text{Th}}\right)c^2\\ &= \left[238.05079\ \text{u} - 2\left(1.00867\ \text{u}\right) - 2\left(1.00783\ \text{u}\right) - 234.04363\ \text{u}\right]\left(931.5\ \text{MeV/u}\right)\\ &= -24.1\ \text{MeV}.\end{aligned}$$

(c) This leads us to conclude that the binding energy of the α particle is

$$\left|\left(2m_n + 2m_p - m_{\text{He}}\right)c^2\right| = \left|-24.1\ \text{MeV} - 4.25\ \text{MeV}\right| = 28.3\ \text{MeV}.$$

49. **THINK** The time for half the original ^{238}U nuclei to decay is equal to 4.5×10^9 y, which is the half-life of ^{238}U.

EXPRESS The fraction of undecayed nuclei remaining after time t is given by

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$$

where λ is the disintegration constant and $T_{1/2} = (\ln 2)/\lambda$ is the half-life.

(a) For ^{244}Pu at $t = 4.5 \times 10^9$ y,

$$\lambda t = \frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5\times 10^9\text{ y})}{8.0\times 10^7\text{ y}} = 39$$

and the fraction remaining is

$$\frac{N}{N_0} = e^{-39.0} \approx 1.2\times 10^{-17}.$$

(b) For ^{248}Cm at $t = 4.5 \times 10^9$ y,

$$\frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5\times 10^9\text{ y})}{3.4\times 10^5\text{ y}} = 9170$$

and the fraction remaining is

$$\frac{N}{N_0} = e^{-9170} = 3.31\times 10^{-3983}.$$

For any reasonably sized sample this is less than one nucleus and may be taken to be zero. A standard calculator probably cannot evaluate e^{-9170} directly. Our recommendation is to treat it as $(e^{-91.70})^{100}$.

LEARN Since $(T_{1/2})_{^{248}\text{Cm}} < (T_{1/2})_{^{244}\text{Pu}} < (T_{1/2})_{^{238}\text{U}}$, with $N/N_0 = e^{-(\ln 2)t/T_{1/2}}$, we have

$$(N/N_0)_{^{248}\text{Cm}} < (N/N_0)_{^{244}\text{Pu}} < (N/N_0)_{^{238}\text{U}}.$$

50. (a) The disintegration energy for uranium-235 "decaying" into thorium-232 is

$$Q_3 = \left(m_{^{235}\text{U}} - m_{^{232}\text{Th}} - m_{^{3}\text{He}}\right)c^2 = (235.0439\text{ u} - 232.0381\text{ u} - 3.0160\text{ u})(931.5\text{ MeV/u})$$
$$= -9.50\text{ MeV}.$$

(b) Similarly, the disintegration energy for uranium-235 decaying into thorium-231 is

$$Q_4 = \left(m_{^{235}\text{U}} - m_{^{231}\text{Th}} - m_{^{4}\text{He}}\right)c^2 = \left(235.0439\,\text{u} - 231.0363\,\text{u} - 4.0026\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= 4.66\,\text{MeV}.$$

(c) Finally, the considered transmutation of uranium-235 into thorium-230 has a Q-value of

$$Q_5 = \left(m_{^{235}\text{U}} - m_{^{230}\text{Th}} - m_{^{5}\text{He}}\right)c^2 = \left(235.0439\,\text{u} - 230.0331\,\text{u} - 5.0122\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= -1.30\,\text{MeV}.$$

Only the second decay process (the α decay) is spontaneous, as it releases energy.

51. Energy and momentum are conserved. We assume the residual thorium nucleus is in its ground state. Let K_α be the kinetic energy of the alpha particle and K_{Th} be the kinetic energy of the thorium nucleus. Then, $Q = K_\alpha + K_{\text{Th}}$. We assume the uranium nucleus is initially at rest. Then, conservation of momentum yields $0 = p_\alpha + p_{\text{Th}}$, where p_α is the momentum of the alpha particle and p_{Th} is the momentum of the thorium nucleus. Both particles travel slowly enough that the classical relationship between momentum and energy can be used. Thus $K_{\text{Th}} = p_{\text{Th}}^2 / 2m_{\text{Th}}$, where m_{Th} is the mass of the thorium nucleus. We substitute $p_{\text{Th}} = -p_\alpha$ and use $K_\alpha = p_\alpha^2 / 2m_\alpha$ to obtain $K_{\text{Th}} = (m_\alpha/m_{\text{Th}})K_\alpha$. Consequently,

$$Q = K_\alpha + \frac{m_\alpha}{m_{\text{Th}}}K_\alpha = \left(1 + \frac{m_\alpha}{m_{\text{Th}}}\right)K_\alpha = \left(1 + \frac{4.00\text{u}}{234\text{u}}\right)\left(4.196\text{MeV}\right) = 4.269\text{MeV}.$$

52. (a) For the first reaction

$$Q_1 = \left(m_{\text{Ra}} - m_{\text{Pb}} - m_{\text{C}}\right)c^2 = \left(223.01850\text{u} - 208.98107\text{u} - 14.00324\text{u}\right)\left(931.5\text{MeV/u}\right)$$
$$= 31.8\text{MeV}.$$

(b) For the second one

$$Q_2 = \left(m_{\text{Ra}} - m_{\text{Rn}} - m_{\text{He}}\right)c^2 = \left(223.01850\,\text{u} - 219.00948\,\text{u} - 4.00260\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= 5.98\,\text{MeV}.$$

(c) From $U \propto q_1 q_2/r$, we get

$$U_1 \approx U_2\left(\frac{q_{\text{Pb}}\,q_C}{q_{\text{Rn}}\,q_{\text{He}}}\right) = \left(30.0\,\text{MeV}\right)\frac{\left(82e\right)\left(6.0e\right)}{\left(86e\right)\left(2.0e\right)} = 86\,\text{MeV}.$$

53. **THINK** The energy released in the decay is the disintegration energy:

$$Q = M_i c^2 - M_f c^2 = (M_i - M_f)c^2 = -\Delta M\, c^2,$$

where $\Delta M = M_f - M_i$ is the change in mass due to the decay.

EXPRESS Let M_{Cs} be the mass of one atom of ${}^{137}_{55}\text{Cs}$ and M_{Ba} be the mass of one atom of ${}^{137}_{56}\text{Ba}$. The energy released is

$$Q = (M_{Cs} - M_{Ba})c^2 .$$

ANALYZE With M_{Cs} = 136.9071 u and M_{Ba} = 136.9058 u, we obtain

$$Q = [136.9071\,\text{u} - 136.9058\,\text{u}]c^2 = (0.0013\,\text{u})c^2 = (0.0013\,\text{u})(931.5\,\text{MeV/u})$$
$$= 1.21\,\text{MeV}.$$

LEARN In calculating Q above, we have used the atomic masses instead of nuclear masses. One can readily show that both lead to the same results. To obtain the nuclear masses, we subtract the mass of 55 electrons from M_{Cs} and the mass of 56 electrons from M_{Ba}. The energy released is

$$Q = [(M_{Cs} - 55m) - (M_{Ba} - 56m) - m]\, c^2,$$

where m is the mass of an electron (the last term in the bracket comes from the beta decay). Once cancellations have been made, $Q = (M_{Cs} - M_{Ba})c^2$, which is the same as before.

54. Assuming the neutrino has negligible mass, then

$$\Delta m c^2 = (\mathbf{m}_{\text{Ti}} - \mathbf{m}_{\text{V}} - m_e)c^2.$$

Now, since vanadium has 23 electrons (see Appendix F and/or G) and titanium has 22 electrons, we can add and subtract $22m_e$ to the above expression and obtain

$$\Delta m c^2 = (\mathbf{m}_{\text{Ti}} + 22m_e - \mathbf{m}_{\text{V}} - 23m_e)c^2 = (m_{\text{Ti}} - m_{\text{V}})c^2.$$

We note that our final expression for Δmc^2 involves the *atomic* masses, and that this assumes (due to the way they are usually tabulated) the atoms are in the ground states (which is certainly not the case here, as we discuss below). The question now is: do we set $Q = -\Delta mc^2$ as in Sample Problem —"Q value in a beta decay, suing masses?" The answer is "no." The atom is left in an excited (high energy) state due to the fact that an electron was captured from the lowest shell (where the absolute value of the energy, E_K, is quite large for large Z). To a very good approximation, the energy of the K-shell electron in Vanadium is equal to that in Titanium (where there is now a "vacancy" that must be filled by a readjustment of the whole electron cloud), and we write $Q = -\Delta m c^2 - E_K$ so that Eq. 42-26 still holds. Thus,

$$Q = \left(m_{\text{V}} - m_{\text{Ti}}\right)c^2 - E_{K.}$$

55. The decay scheme is $\text{n} \to \text{p} + \text{e}^- + \nu$. The electron kinetic energy is a maximum if no neutrino is emitted. Then,

$$K_{\text{max}} = (m_n - m_p - m_e)c^2,$$

where m_n is the mass of a neutron, m_p is the mass of a proton, and m_e is the mass of an electron. Since $m_p + m_e = m_{\text{H}}$, where m_{H} is the mass of a hydrogen atom, this can be written $K_{\text{max}} = (m_n - m_{\text{H}})c^2$. Hence,

$$K_{\text{max}} = (840 \times 10^{-6}\ \text{u})c^2 = (840 \times 10^{-6}\ \text{u})(931.5\ \text{MeV/u}) = 0.783\ \text{MeV}.$$

56. (a) We recall that $mc^2 = 0.511$ MeV from Table 37-3, and $hc = 1240$ MeV·fm. Using Eq. 37-54 and Eq. 38-13, we obtain

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}}$$
$$= \frac{1240\,\text{MeV} \cdot \text{fm}}{\sqrt{\left(1.0\,\text{MeV}\right)^2 + 2\left(1.0\,\text{MeV}\right)\left(0.511\,\text{MeV}\right)}} = 9.0 \times 10^2\ \text{fm}.$$

(b) $r = r_0 A^{1/3} = (1.2\ \text{fm})(150)^{1/3} = 6.4$ fm.

(c) Since $\lambda \gg r$ the electron cannot be confined in the nuclide. We recall that at least $\lambda/2$ was needed in any particular direction, to support a standing wave in an "infinite well." A finite well is able to support *slightly* less than $\lambda/2$ (as one can infer from the ground state wave function in Fig. 39-6), but in the present case λ/r is far too big to be supported.

(d) A strong case can be made on the basis of the remarks in part (c), above.

57. (a) Since the positron has the same mass as an electron, and the neutrino has negligible mass, then

$$\Delta mc^2 = \left(\mathbf{m}_{\text{B}} + m_e - \mathbf{m}_{\text{C}}\right)c^2.$$

Now, since carbon has 6 electrons (see Appendix F and/or G) and boron has 5 electrons, we can add and subtract $6m_e$ to the above expression and obtain

$$\Delta mc^2 = \left(\mathbf{m}_{\text{B}} + 7m_e - \mathbf{m}_{\text{C}} - 6m_e\right)c^2 = \left(m_{\text{B}} + 2m_e - m_{\text{C}}\right)c^2.$$

We note that our final expression for Δmc^2 involves the *atomic* masses, as well an "extra" term corresponding to two electron masses. From Eq. 37-50 and Table 37-3, we obtain

$$Q = \left(m_{\text{C}} - m_{\text{B}} - 2m_e\right)c^2 = \left(m_{\text{C}} - m_{\text{B}}\right)c^2 - 2\left(0.511\,\text{MeV}\right).$$

(b) The disintegration energy for the positron decay of carbon-11 is

$$Q = (11.011434\,\text{u} - 11.009305\,\text{u})(931.5\,\text{MeV/u}) - 1.022\,\text{MeV} = 0.961\,\text{MeV}.$$

58. (a) The rate of heat production is

$$\begin{aligned}\frac{dE}{dt} &= \sum_{i=1}^{3} R_i Q_i = \sum_{i=1}^{3} \lambda_1 N_i Q_i = \sum_{i=1}^{3} \left(\frac{\ln 2}{T_{1/2_i}}\right)\frac{(1.00\,\text{kg}) f_i}{m_i} Q_i \\ &= \frac{(1.00\,\text{kg})(\ln 2)(1.60\times10^{-13}\,\text{J / MeV})}{(3.15\times10^{7}\,\text{s / y})(1.661\times10^{-27}\,\text{kg / u})}\left[\frac{(4\times10^{-6})(51.7\,\text{MeV})}{(238\,\text{u})(4.47\times10^{9}\,\text{y})}\right. \\ &\quad \left. + \frac{(13\times10^{-6})(42.7\,\text{MeV})}{(232\,\text{u})(1.41\times10^{10}\,\text{y})} + \frac{(4\times10^{-6})(1.31\,\text{MeV})}{(40\,\text{u})(1.28\times10^{9}\,\text{y})}\right] \\ &= 1.0\times10^{-9}\,\text{W}.\end{aligned}$$

(b) The contribution to heating, due to radioactivity, is

$$P = (2.7 \times 10^{22}\,\text{kg})(1.0 \times 10^{-9}\,\text{W/kg}) = 2.7 \times 10^{13}\,\text{W},$$

which is very small compared to what is received from the Sun.

59. **THINK** The beta decay of ^{32}P is given by

$$^{32}\text{P} \rightarrow {}^{32}\text{S} + e^- + \bar{\nu}\,.$$

However, since the electron has the maximum possible kinetic energy, no (anti)neutrino is emitted.

EXPRESS Since momentum is conserved, the momentum of the electron and the momentum of the residual sulfur nucleus are equal in magnitude and opposite in direction. If p_e is the momentum of the electron and p_S is the momentum of the sulfur nucleus, then $p_S = -p_e$. The kinetic energy K_S of the sulfur nucleus is

$$K_S = p_S^2 / 2M_S = p_e^2 / 2M_S\,,$$

where M_S is the mass of the sulfur nucleus. Now, the electron's kinetic energy K_e is related to its momentum by the relativistic equation $(p_e c)^2 = K_e^2 + 2K_e mc^2$, where m is the mass of an electron.

ANALYZE With K_e = 1.71 MeV, the kinetic energy of the recoiling sulfur nucleus is

$$K_S = \frac{(p_e c)^2}{2M_S c^2} = \frac{K_e^2 + 2K_e mc^2}{2M_S c^2} = \frac{(1.71\,\text{MeV})^2 + 2(1.71\,\text{MeV})(0.511\,\text{MeV})}{2(32\,\text{u})(931.5\,\text{MeV/u})}$$
$$= 7.83\times 10^{-5}\,\text{MeV} = 78.3\text{ eV}$$

where mc^2 = 0.511 MeV is used for the electron (see Table 37-3).

LEARN The maximum kinetic energy of the electron is equal to the disintegration energy Q:

$$Q = K_{\text{max}}\,.$$

To show this, we use the following data: M_P = 31.97391 u and M_S = 31.97207 u. The result is

$$Q = [31.97391\,\text{u} - 31.97207\,\text{u}]c^2 = (0.00184\,\text{u})c^2 = (0.00184\,\text{u})(931.5\,\text{MeV/u})$$
$$= 1.71\,\text{MeV}.$$

60. We solve for t from $R = R_0 e^{-\lambda t}$:

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \left(\frac{5730\,\text{y}}{\ln 2}\right)\ln\left[\left(\frac{15.3}{63.0}\right)\left(\frac{5.00}{1.00}\right)\right] = 1.61\times 10^3\text{ y}.$$

61. (a) The mass of a ^{238}U atom is (238 u)(1.661×10^{-24} g/u) = 3.95×10^{-22} g, so the number of uranium atoms in the rock is

$$N_U = (4.20 \times 10^{-3}\text{ g})/(3.95 \times 10^{-22}\text{ g}) = 1.06 \times 10^{19}.$$

(b) The mass of a ^{206}Pb atom is (206 u)(1.661×10^{-24} g) = 3.42×10^{-22} g, so the number of lead atoms in the rock is

$$N_{Pb} = (2.135 \times 10^{-3}\text{ g})/(3.42 \times 10^{-22}\text{ g}) = 6.24 \times 10^{18}.$$

(c) If no lead was lost, there was originally one uranium atom for each lead atom formed by decay, in addition to the uranium atoms that did not yet decay. Thus, the original number of uranium atoms was

$$N_{U0} = N_U + N_{Pb} = 1.06 \times 10^{19} + 6.24 \times 10^{18} = 1.68 \times 10^{19}.$$

(d) We use

$$N_U = N_{U0} e^{-\lambda t}$$

where λ is the disintegration constant for the decay. It is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$. Thus,

$$t = -\frac{1}{\lambda}\ln\left(\frac{N_U}{N_{U0}}\right) = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_U}{N_{U0}}\right) = -\frac{4.47\times10^9\text{ y}}{\ln 2}\ln\left(\frac{1.06\times10^{19}}{1.68\times10^{19}}\right) = 2.97\times10^9\text{ y}.$$

62. The original amount of ^{238}U the rock contains is given by

$$m_0 = me^{\lambda t} = (3.70\text{ mg})e^{(\ln 2)(260\times10^6\text{ y})/(4.47\times10^9\text{ y})} = 3.85\text{ mg}.$$

Thus, the amount of lead produced is

$$m' = (m_0 - m)\left(\frac{m_{206}}{m_{238}}\right) = (3.85\text{ mg} - 3.70\text{ mg})\left(\frac{206}{238}\right) = 0.132\text{ mg}.$$

63. We can find the age t of the rock from the masses of ^{238}U and ^{206}Pb. The initial mass of ^{238}U is

$$m_{U_0} = m_U + \frac{238}{206}m_{Pb}.$$

Therefore,

$$m_U = m_{U_0}e^{-\lambda_U t} = \left(m_U + m_{^{238}Pb}/206\right)e^{-(t\ln 2)/T_{1/2_U}}.$$

We solve for t:

$$t = \frac{T_{1/2_U}}{\ln 2}\ln\left(\frac{m_U + (238/206)m_{Pb}}{m_U}\right) = \frac{4.47\times10^9\text{ y}}{\ln 2}\ln\left[1+\left(\frac{238}{206}\right)\left(\frac{0.15\text{mg}}{0.86\text{mg}}\right)\right]$$
$$= 1.18\times10^9\text{ y}.$$

For the β decay of ^{40}K, the initial mass of ^{40}K is

$$m_{K_0} = m_K + (40/40)m_{Ar} = m_K + m_{Ar},$$

so

$$m_K = m_{K_0}e^{-\lambda_K t} = (m_K + m_{Ar})e^{-\lambda_K t}.$$

We solve for m_K:

$$m_K = \frac{m_{Ar}e^{-\lambda_K t}}{1-e^{-\lambda_K t}} = \frac{m_{Ar}}{e^{\lambda_K t}-1} = \frac{1.6\text{mg}}{e^{(\ln 2)(1.18\times10^9\text{ y})/(1.25\times10^9\text{ y})}-1} = 1.7\text{mg}.$$

64. We note that every calcium-40 atom and krypton-40 atom found now in the sample was once one of the original numbers of potassium atoms. Thus, using Eq. 42-14 and Eq. 42-18, we find

$$\ln\left(\frac{N_K}{N_K + N_{Ar} + N_{Ca}}\right) = -\lambda t \quad \Rightarrow \quad \ln\left(\frac{1}{1+1+8.54}\right) = -\frac{\ln 2}{T_{1/2}} t$$

which (with $T_{1/2} = 1.26 \times 10^9$ y) yields $t = 4.28 \times 10^9$ y.

65. **THINK** The activity of a radioactive sample expressed in curie (Ci) can be converted to SI units (Bq) as

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq} = 3.7 \times 10^{10} \text{ disintegrations/s}.$$

EXPRESS The decay rate R is related to the number of nuclei N by $R = \lambda N$, where λ is the disintegration constant. The disintegration constant is related to the half-life $T_{1/2}$ by

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad \Rightarrow \quad N = \frac{R}{\lambda} = \frac{RT_{1/2}}{\ln 2} \; .$$

Since 1 Ci = 3.7×10^{10} disintegrations/s,

$$N = \frac{(250 \text{ Ci})(3.7 \times 10^{10} \text{ s}^{-1}/\text{Ci})(2.7 \text{ d})(8.64 \times 10^4 \text{ s/d})}{\ln 2} = 3.11 \times 10^{18}.$$

ANALYZE The mass of a ^{198}Au atom is

$$M_0 = (198 \text{ u})(1.661 \times 10^{-24} \text{ g/u}) = 3.29 \times 10^{-22} \text{ g},$$

so the mass required is

$$M = N M_0 = (3.11 \times 10^{18})(3.29 \times 10^{-22} \text{ g}) = 1.02 \times 10^{-3} \text{ g} = 1.02 \text{ mg}.$$

LEARN The ^{198}Au atom undergoes beta decay and emit an electron:

$$^{198}\text{Au} \rightarrow {}^{198}\text{Hg} + e^- + \bar{\nu} \; .$$

66. The becquerel (Bq) and curie (Ci) are defined in Section 42-3.

(a) $R = 8700/60 = 145$ Bq.

(b) $R = \dfrac{145 \text{ Bq}}{3.7 \times 10^{10} \text{ Bq/Ci}} = 3.92 \times 10^{-9}$ Ci.

67. The absorbed dose is

$$\text{absorbed dose} = \frac{2.00\times10^{-3}\ \text{J}}{4.00\ \text{kg}} = 5.00\times10^{-4}\ \text{J/kg} = 5.00\times10^{-4}\ \text{Gy}$$

where $1\ \text{J/kg} = 1\,\text{Gy}$. With $\text{RBE} = 5$, the dose equivalent is

$$\begin{aligned}\text{dose equivalent} &= \text{RBE}\cdot(5.00\times10^{-4}\ \text{Gy}) = 5(5.00\times10^{-4}\ \text{Gy}) = 2.50\times10^{-3}\ \text{Sv}\\ &= 2.50\ \text{mSv}.\end{aligned}$$

68. (a) Using Eq. 42-32, the energy absorbed is

$$\left(2.4\times10^{-4}\ \text{Gy}\right)\left(75\,\text{kg}\right) = 18\,\text{mJ}.$$

(b) The dose equivalent is

$$\left(2.4\times10^{-4}\,\text{Gy}\right)(12) = 2.9\times10^{-3}\,\text{Sv}.$$

(c) Using Eq. 42-33, we have $2.9\times10^{-3}\,\text{Sv} = 0.29\ \text{rem}$.

69. (a) Adapting Eq. 42-21, we find

$$N_0 = \frac{\left(2.5\times10^{-3}\ \text{g}\right)\left(6.02\times10^{23}\,/\,\text{mol}\right)}{239\,\text{g}\,/\,\text{mol}} = 6.3\times10^{18}.$$

(b) From Eq. 42-15 and Eq. 42-18,

$$|\Delta N| = N_0\left[1-e^{-t\ln 2/T_{1/2}}\right] = \left(6.3\times10^{18}\right)\left[1-e^{-(12\,\text{h})\ln 2/(24{,}100\,\text{y})(8760\,\text{h/y})}\right] = 2.5\times10^{11}.$$

(c) The energy absorbed by the body is

$$(0.95)\,E_\alpha\left|\Delta N\right| = (0.95)\left(5.2\,\text{MeV}\right)\left(2.5\times10^{11}\right)\left(1.6\times10^{-13}\ \text{J/MeV}\right) = 0.20\ \text{J}.$$

(d) On a per unit mass basis, the previous result becomes (according to Eq. 42-32)

$$\frac{0.20\,\text{mJ}}{85\,\text{kg}} = 2.3\times10^{-3}\ \text{J}\,/\,\text{kg} = 2.3\,\text{mGy}.$$

(e) Using Eq. 42-31, (2.3 mGy)(13) = 30 mSv.

70. From Eq. 19-24, we obtain

$$T = \frac{2}{3}\left(\frac{K_{avg}}{k}\right) = \frac{2}{3}\left(\frac{5.00\times10^6\ \text{eV}}{8.62\times10^{-5}\ \text{eV/K}}\right) = 3.87\times10^{10}\ \text{K}.$$

71. (a) Following Sample Problem — "Lifetime of a compound nucleus made by neutron capture," we compute

$$\Delta E \approx \frac{\hbar}{t_{avg}} = \frac{(4.14\times10^{-15}\ \text{eV}\cdot\text{fs})/2\pi}{1.0\times10^{-22}\ \text{s}} = 6.6\times10^6\ \text{eV}.$$

(b) In order to fully distribute the energy in a fairly large nucleus, and create a "compound nucleus" equilibrium configuration, about 10^{-15} s is typically required. A reaction state that exists no more than about 10^{-22} s does not qualify as a compound nucleus.

72. (a) We compare both the proton numbers (atomic numbers, which can be found in Appendix F and/or G) and the neutron numbers (see Eq. 42-1) with the magic nucleon numbers (special values of either Z or N) listed in Section 42-8. We find that ^{18}O, ^{60}Ni, ^{92}Mo, ^{144}Sm, and ^{207}Pb each have a filled shell for either the protons or the neutrons (two of these, ^{18}O and ^{92}Mo, are explicitly discussed in that section).

(b) Consider ^{40}K, which has $Z = 19$ protons (which is one less than the magic number 20). It has $N = 21$ neutrons, so it has one neutron outside a closed shell for neutrons, and thus qualifies for this list. Others in this list include ^{91}Zr, ^{121}Sb, and ^{143}Nd.

(c) Consider ^{13}C, which has $Z = 6$ and $N = 13 - 6 = 7$ neutrons. Since 8 is a magic number, then ^{13}C has a vacancy in an otherwise filled shell for neutrons. Similar arguments lead to inclusion of ^{40}K, ^{49}Ti, ^{205}Tl, and ^{207}Pb in this list.

73. **THINK**A generalized formation reaction can be written $X + x \rightarrow Y$, where X is the target nucleus, x is the incident light particle, and Y is the excited compound nucleus (^{20}Ne).

EXPRESS We assume X is initially at rest. Then, conservation of energy yields

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + K_Y + E_Y$$

where m_X, m_x, and m_Y are masses, K_x and K_Y are kinetic energies, and E_Y is the excitation energy of Y. Conservation of momentum yields $p_x = p_Y$. Now,

$$K_Y = \frac{p_Y^2}{2m_Y} = \frac{p_x^2}{2m_Y} = \left(\frac{m_x}{m_Y}\right)K_x$$

so

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + (m_x / m_Y) K_x + E_Y$$

and

$$K_x = \frac{m_Y}{m_Y - m_x} (m_Y - m_X - m_x) c^2 + E_Y \ .$$

ANALYZE (a) Let x represent the alpha particle and X represent the ^{16}O nucleus. Then,

$$\begin{aligned}(m_Y - m_X - m_x)c^2 &= (19.99244 \text{ u} - 15.99491 \text{ u} - 4.00260 \text{ u})(931.5 \text{ MeV/u}) \\ &= -4.722 \text{ MeV}\end{aligned}$$

and

$$K_\alpha = \frac{19.99244\text{u}}{19.99244\text{u} - 4.00260\text{u}}(-4.722\text{MeV} + 25.0\text{MeV}) = 25.35\text{MeV} \approx 25.4 \text{ MeV}.$$

(b) Let x represent the proton and X represent the ^{19}F nucleus. Then,

$$\begin{aligned}(m_Y - m_X - m_x)c^2 &= (19.99244 \text{ u} - 18.99841 \text{ u} - 1.00783 \text{ u})(931.5 \text{ MeV/u}) \\ &= -12.85 \text{ MeV}\end{aligned}$$

and

$$K_\alpha = \frac{19.99244\text{ u}}{19.99244\text{ u} - 1.00783\text{ u}}(-12.85\text{ MeV} + 25.0\text{ MeV}) = 12.80\text{ MeV}.$$

(c) Let x represent the photon and X represent the ^{20}Ne nucleus. Since the mass of the photon is zero, we must rewrite the conservation of energy equation: if E_γ is the energy of the photon, then

$$E_\gamma + m_X c^2 = m_Y c^2 + K_Y + E_Y.$$

Since $m_X = m_Y$, this equation becomes $E_\gamma = K_Y + E_Y$. Since the momentum and energy of a photon are related by $p_\gamma = E_\gamma/c$, the conservation of momentum equation becomes $E_\gamma/c = p_Y$. The kinetic energy of the compound nucleus is

$$K_Y = \frac{p_Y^2}{2m_Y} = \frac{E_\gamma^2}{2m_Y c^2} \ .$$

We substitute this result into the conservation of energy equation to obtain

$$E_\gamma = \frac{E_\gamma^2}{2m_Y c^2} + E_Y.$$

This quadratic equation has the solutions

$$E_\gamma = m_Y c^2 \pm \sqrt{(m_Y c^2)^2 - 2m_Y c^2 E_Y}.$$

If the problem is solved using the relativistic relationship between the energy and momentum of the compound nucleus, only one solution would be obtained, the one corresponding to the negative sign above. Since

$$m_Y c^2 = (19.99244 \text{ u})(931.5 \text{ MeV/u}) = 1.862 \times 10^4 \text{ MeV},$$

we have

$$\begin{aligned} E_\gamma &= \left(1.862 \times 10^4 \text{ MeV}\right) - \sqrt{\left(1.862 \times 10^4 \text{ MeV}\right)^2 - 2\left(1.862 \times 10^4 \text{ MeV}\right)\left(25.0 \text{ MeV}\right)} \\ &= 25.0 \text{ MeV}. \end{aligned}$$

LEARN In part (c), the kinetic energy of the compound nucleus is

$$K_Y = \frac{E_\gamma^2}{2m_Y c^2} = \frac{(25.0 \text{ MeV})^2}{2(1.862 \times 10^4 \text{ MeV})} = 0.0168 \text{ MeV}$$

which is very small compared to E_Y = 25.0 MeV. Essentially all of the photon energy goes to excite the nucleus.

74. Using Eq. 42-15, the amount of uranium atoms and lead atoms present in the rock at time t is

$$\begin{aligned} N_{\text{U}} &= N_0 e^{-\lambda t} \\ N_{\text{Pb}} &= N_0 - N_{\text{U}} = N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t}) \end{aligned}$$

and their ratio is

$$\frac{N_{\text{Pb}}}{N_{\text{U}}} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1 .$$

The age of the rock is

$$t = \frac{1}{\lambda} \ln\left(1 + \frac{N_{\text{Pb}}}{N_{\text{U}}}\right) = \frac{T_{1/2}}{\ln 2} \ln\left(1 + \frac{N_{\text{Pb}}}{N_{\text{U}}}\right) = \frac{4.47 \times 10^9 \text{ y}}{\ln 2} \ln\left(1 + 0.30\right) = 1.69 \times 10^9 \text{ y} .$$

75. **THINK** We represent the unknown nuclide as ${}^A_Z X$, where A and Z are its mass number and atomic number, respectively.

EXPRESS The reaction equation can be written as

$${}^A_Z X + {}^1_0\text{n} \rightarrow {}^0_{-1}\text{e} + 2\,{}^4_2\text{He}.$$

Conservation of charge yields $Z + 0 = -1 + 4$ or $Z = 3$. Conservation of mass number yields $A + 1 = 0 + 8$ or $A = 7$.

ANALYZE According to the periodic table in Appendix G (also see Appendix F), lithium has atomic number 3, so the nuclide must be ${}^{7}_{3}\mathrm{Li}$.

LEARN Charge and mass number are conserved in the neutron-capture process. The intermediate nuclide is ^{8}Li, which is unstable and decays (via α and β^- modes) into two ^{4}He's and an electron.

76. The dose equivalent is the product of the absorbed dose and the RBE factor, so the absorbed dose is

$$(\text{dose equivalent})/(\text{RBE}) = (250 \times 10^{-6}\ \text{Sv})/(0.85) = 2.94 \times 10^{-4}\ \text{Gy}.$$

But 1 Gy = 1 J/kg, so the absorbed dose is

$$\left(2.94\times10^{-4}\ \text{Gy}\right)\left(1\,\frac{\text{J}}{\text{kg}\cdot\text{Gy}}\right) = 2.94\times10^{-4}\ \text{J/kg}.$$

To obtain the total energy received, we multiply this by the mass receiving the energy:

$$E = (2.94 \times 10^{-4}\ \text{J/kg})(44\ \text{kg}) = 1.29 \times 10^{-2}\ \text{J} \approx 1.3 \times 10^{-2}\ \text{J}.$$

77. **THINK** The decay rate R is proportional to N, the number of radioactive nuclei.

EXPRESS According to Eq. 42-17, $R = \lambda N$, where λ is the decay constant. Since R is proportional to N, then $N/N_0 = R/R_0 = e^{-\lambda t}$. Since $\lambda = (\ln 2)/T_{1/2}$, the solution for t is

$$t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right) = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right).$$

ANALYZE With $T_{1/2}$ = 5730 y and R/R_0 = 0.020, we obtain

$$t = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right) = -\frac{5730\,\text{y}}{\ln 2}\ln\left(0.020\right) = 3.2\times10^{4}\ \text{y}.$$

LEARN Radiocarbon dating based on the decay of ^{14}C is one of the most widely used dating method in estimating the age of organic remains.

78. Let N_{AA0} be the number of element AA at $t = 0$. At a later time t, due to radioactive decay, we have

$$N_{\text{AA0}} = N_{\text{AA}} + N_{\text{BB}} + N_{\text{CC}}.$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{8.00\ \text{d}} = 0.0866/\text{d}.$$

Since $N_{BB}/N_{CC} = 2$, when $N_{CC}/N_{AA} = 1.50$, $N_{BB}/N_{AA} = 3.00$. Therefore, at time *t*,

$$N_{AA0} = N_{AA} + N_{BB} + N_{CC} = N_{AA} + 3.00N_{AA} + 1.50N_{AA} = 5.50N_{AA}.$$

Since $N_{AA} = N_{AA0}e^{-\lambda t}$, combining the two expressions leads to

$$\frac{N_{AA0}}{N_{AA}} = e^{\lambda t} = 5.50$$

which can be solved to give

$$t = \frac{\ln(5.50)}{\lambda} = \frac{\ln(5.50)}{0.0866/\text{d}} = 19.7\ \text{d}.$$

79. **THINK** The count rate in the area in question is given by $R = \lambda N$, where λ is the decay constant and N is the number of radioactive nuclei.

EXPRESS Since the spreading is assumed uniform, the count rate R = 74,000/s is given by

$$R = \lambda N = \lambda(M/m)(a/A),$$

where M is the mass of ^{90}Sr produced, m is the mass of a single ^{90}Sr nucleus, A is the area over which fall out occurs, and a is the area in question. Since $\lambda = (\ln 2)/T_{1/2}$, the solution for a is

$$a = A\left(\frac{m}{M}\right)\left(\frac{R}{\lambda}\right) = \frac{AmRT_{1/2}}{M\ln 2}.$$

ANALYZE The molar mass of ^{90}Sr is 90g/mol. With M = 400 g and A = 2000 km^2, we find the area to be

$$a = \frac{AmRT_{1/2}}{M\ln 2} = \frac{\left(2000\times10^6\ \text{m}^2\right)\left(90\ \text{g/mol}\right)\left(74{,}000/\text{s}\right)\left(29\ \text{y}\right)\left(3.15\times10^7\ \text{s/y}\right)}{\left(400\ \text{g}\right)\left(6.02\times10^{23}/\text{mol}\right)\left(\ln 2\right)}$$
$$= 7.3\times10^{-2}\ \text{m}^{-2} = 730\,\text{cm}^2.$$

LEARN The Chernobyl nuclear accident in 1986 contaminated a very large area with ^{90}Sr.

80. (a) Assuming a “target” area of one square meter, we establish a ratio:

$$\frac{\text{rate through you}}{\text{total rate upward}} = \frac{1\,\text{m}^2}{\left(2.6\times10^5\,\text{km}^2\right)\left(1000\,\text{m/km}\right)^2} = 3.8\times10^{-12}.$$

The SI unit becquerel is equivalent to a disintegration per second. With half the beta-decay electrons moving upward, we find

$$\text{rate through you} = \frac{1}{2}\left(1\times10^{16}/\text{s}\right)\left(3.8\times10^{-12}\right) = 1.9\times10^4/\text{s}$$

which implies (converting $\text{s}\rightarrow\text{h}$) that the rate of electrons you would intercept is $R_0 = 7 \times 10^7$/h. So in one hour, 7×10^7 electrons would be intercepted.

(b) Let D indicate the current year (2003, 2004, etc.). Combining Eq. 42-16 and Eq. 42-18, we find

$$R = R_0 e^{-t\ln 2/T_{1/2}} = \left(7\times10^7/\text{h}\right)e^{-(D-1996)\ln 2/(30.2\,\text{y})}.$$

81. The lines that lead toward the lower left are alpha decays, involving an atomic number change of $\Delta Z_\alpha = -2$ and a mass number change of $\Delta A_\alpha = -4$. The short horizontal lines toward the right are beta decays (involving electrons, not positrons) in which case A stays the same but the change in atomic number is $\Delta Z_\beta = +1$. Figure 42-20 shows three alpha decays and two beta decays; thus,

$$Z_f = Z_i + 3\Delta Z_\alpha + 2\Delta Z_\beta \text{ and } A_f = A_i + 3\Delta A_\alpha.$$

Referring to Appendix F or G, we find $Z_i = 93$ for neptunium, so

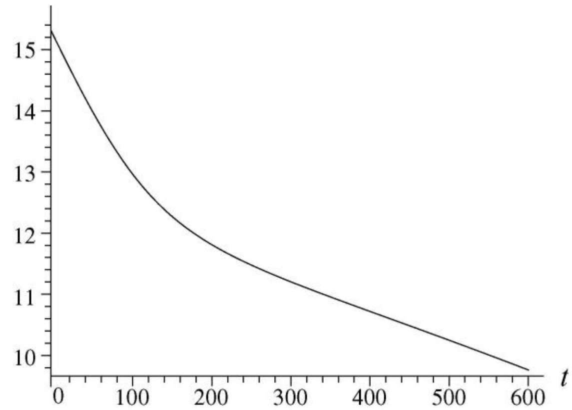
$$Z_f = 93 + 3(-2) + 2(1) = 89,$$

which indicates the element actinium. We are given $A_i = 237$, so $A_f = 237 + 3(-4) = 225$. Therefore, the final isotope is ^{225}Ac.

82. We note that 2.42 min = 145.2 s. We are asked to plot (with SI units understood)

$$\ln R = \ln\left(R_0 e^{-\lambda t} + R_0' e^{-\lambda' t}\right)$$

where $R_0 = 3.1 \times 10^5$, $R_0' = 4.1 \times 10^6$, $\lambda = \ln 2/145.2$, and $\lambda' = \ln 2/24.6$. Our plot is shown below.



We note that the magnitude of the slope for small t is λ' (the disintegration constant for ^{110}Ag), and for large t is λ (the disintegration constant for ^{108}Ag).

83. We note that hc = 1240 MeV·fm, and that the classical kinetic energy $\frac{1}{2}mv^2$ can be written directly in terms of the classical momentum $p = mv$ (see below). Letting

$$p \simeq \Delta p \simeq \Delta h / \Delta x \simeq h / r,$$

we get

$$E = \frac{p^2}{2m} \simeq \frac{(hc)^2}{2(mc^2)r^2} = \frac{(1240\,\text{MeV}\cdot\text{fm})^2}{2(938\,\text{MeV})\left[(1.2\,\text{fm})(100)^{1/3}\right]^2} \simeq 30\,\text{MeV}.$$

84. (a) The rate at which radium-226 is decaying is

$$R = \lambda N = \left(\frac{\ln 2}{T_{1/2}}\right)\left(\frac{M}{m}\right) = \frac{(\ln 2)(1.00\,\text{mg})(6.02\times10^{23}\,/\,\text{mol})}{(1600\,\text{y})(3.15\times10^7\,\text{s}/\text{y})(226\,\text{g}/\text{mol})} = 3.66\times10^7\,\text{s}^{-1}.$$

The activity is 3.66×10^7 Bq.

(b) The activity of ^{222}Rn is also 3.66×10^7 Bq.

(c) From $R_{\text{Ra}} = R_{\text{Rn}}$ and $R = \lambda N = (\ln 2/T_{1/2})(M/m)$, we get

$$M_{\text{Rn}} = \left(\frac{T_{1/2_{\text{Rn}}}}{T_{1/2_{\text{Ra}}}}\right)\left(\frac{m_{\text{Rn}}}{m_{\text{Ra}}}\right)M_{\text{Ra}} = \frac{(3.82\text{d})(1.00\times10^{-3}\,\text{g})(222\text{u})}{(1600\text{y})(365\text{d/y})(226\text{u})} = 6.42\times10^{-9}\,\text{g}.$$

85. Although we haven't drawn the requested lines in the following table, we can indicate their slopes: lines of constant A would have – 45° slopes, and those of constant $N – Z$ would have 45°. As an example of the latter, the $N – Z$ = 20 line (which is one of "eighteen-neutron excess") would pass through Cd-114 at the lower left corner up through Te-122 at the upper right corner. The first column corresponds to N = 66, and the

bottom row to $Z = 48$. The last column corresponds to $N = 70$, and the top row to $Z = 52$. Much of the information below (regarding values of $T_{1/2}$ particularly) was obtained from the Web sites http://nucleardata.nuclear.lu.se/nucleardata and http://www.nndc.bnl.gov/nndc/ensdf.

^{118}Te 6.0 days	^{119}Te 16.0 h	^{120}Te 0.1%	^{121}Te 19.4 days	^{122}Te 2.6%
^{117}Sb 2.8 h	^{118}Sb 3.6 min	^{119}Sb 38.2 s	^{120}Sb 15.9 min	^{121}Sb 57.2%
^{116}Sn 14.5%	^{117}Sn 7.7%	^{118}Sn 24.2%	^{119}Sn 8.6%	^{120}Sn 32.6%
^{115}In 95.7%	^{116}In 14.1 s	^{117}In 43.2 min	^{118}In 5.0 s	^{119}In 2.4 min
^{114}Cd 28.7%	^{115}Cd 53.5 h	^{116}Cd 7.5%	^{117}Cd 2.5 h	^{118}Cd 50.3 min

86. Using Eq. 42-3 ($r = r_0 A^{1/3}$), we estimate the nuclear radii of the alpha particle and Al to be

$$r_\alpha = (1.2\times10^{-15}\ \text{m})(4)^{1/3} = 1.90\times10^{-15}\ \text{m}$$
$$r_{\text{Al}} = (1.2\times10^{-15}\ \text{m})(27)^{1/3} = 3.60\times10^{-15}\ \text{m}.$$

The distance between the centers of the nuclei when their surfaces touch is

$$r = r_\alpha + r_{\text{Al}} = 1.90\times10^{-15}\ \text{m} + 3.60\times10^{-15}\ \text{m} = 5.50\times10^{-15}\ \text{m}.$$

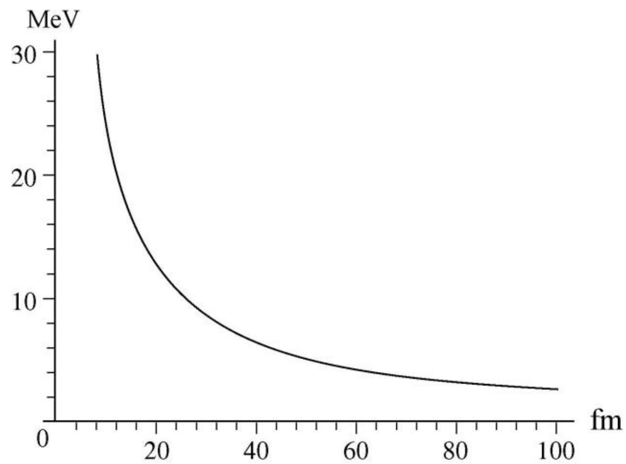
From energy conservation, the amount of energy required is

$$K = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{Al}}}{r} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\times1.6\times10^{-19}\ \text{C})(13\times1.6\times10^{-19}\ \text{C})}{5.50\times10^{-15}\ \text{m}}$$
$$= 1.09\times10^{-12}\ \text{J} = 6.79\times10^6\ \text{eV}$$

87. Equation 24-43 gives the electrostatic potential energy between two uniformly charged spherical charges (in this case $q_1 = 2e$ and $q_2 = 90e$) with r being the distance between their centers. Assuming the "uniformly charged spheres" condition is met in this instance, we write the equation in such a way that we can make use of $k = 1/4\pi\varepsilon_0$ and the electronvolt unit:

$$U = k\frac{(2e)(90e)}{r} = \left(8.99\times10^{9}\ \frac{\text{V}\cdot\text{m}}{\text{C}}\right)\frac{(3.2\times10^{-19}\ \text{C})(90e)}{r} = \frac{2.59\times10^{-7}}{r}\ \text{eV}$$

with r understood to be in meters. It is convenient to write this for r in femtometers, in which case $U = 259/r$ MeV. This is shown plotted below.



88. We take the speed to be constant, and apply the classical kinetic energy formula:

$$t = \frac{d}{v} = \frac{d}{\sqrt{2K/m}} = 2r\sqrt{\frac{m_n}{2K}} = \frac{r}{c}\sqrt{\frac{2mc^2}{K}}$$
$$\approx \frac{(1.2\times10^{-15}\ \text{m})(100)^{1/3}}{3.0\times10^{8}\ \text{m/s}}\sqrt{\frac{2(938\text{MeV})}{5\text{MeV}}}$$
$$\approx 4\times10^{-22}\ \text{s}.$$

89. We solve for A from Eq. 42-3:

$$A = \left(\frac{r}{r_0}\right)^3 = \left(\frac{3.6\ \text{fm}}{1.2\ \text{fm}}\right)^3 = 27.$$

90. The problem with Web-based services is that there are no guarantees of accuracy or that the Web page addresses will not change from the time this solution is written to the time someone reads this. Still, it is worth mentioning that a very accessible Web site for a wide variety of periodic table and isotope-related information is http://www.webelements.com. Two sites, http://nucleardata.nuclear.lu.se/nucleardata and http://www.nndc.bnl.gov/nndc/ensdf, are aimed more toward the nuclear professional. These are the sites where some of the information mentioned below was obtained.

(a) According to Appendix F, the atomic number 60 corresponds to the element neodymium (Nd). The first Web site mentioned above gives ^{142}Nd, ^{143}Nd, ^{144}Nd, ^{145}Nd, ^{146}Nd, ^{148}Nd, and ^{150}Nd in its list of naturally occurring isotopes. Two of these, ^{144}Nd and ^{150}Nd, are not perfectly stable, but their half-lives are much longer than the age of the universe (detailed information on their half-lives, modes of decay, etc. are available at the last two Web sites referred to, above).

(b) In this list, we are asked to put the nuclides that contain 60 neutrons and that are recognized to exist but not stable nuclei (this is why, for example, ^{108}Cd is not included here). Although the problem does not ask for it, we include the half-lives of the nuclides in our list, though it must be admitted that not all reference sources agree on those values (we picked ones we regarded as "most reliable"). Thus, we have ^{97}Rb (0.2 s), ^{98}Sr (0.7 s), ^{99}Y (2 s), ^{100}Zr (7 s), ^{101}Nb (7 s), ^{102}Mo (11 minutes), ^{103}Tc (54 s), ^{105}Rh (35 hours), ^{109}In (4 hours), ^{110}Sn (4 hours), ^{111}Sb (75 s), ^{112}Te (2 minutes), ^{113}I (7 s), ^{114}Xe (10 s), ^{115}Cs (1.4 s), and ^{116}Ba (1.4 s).

(c) We would include in this list: ^{60}Zn, ^{60}Cu, ^{60}Ni, ^{60}Co, ^{60}Fe, ^{60}Mn, ^{60}Cr, and ^{60}V.

91. (a) In terms of the original value of u, the newly defined u is greater by a factor of 1.007825. So the mass of ^{1}H would be 1.000000 u, the mass of ^{12}C would be

$$(12.000000/1.007825)\ \text{u} = 11.90683\ \text{u}.$$

(b) The mass of ^{238}U would be (238.050785/ 1.007825) u = 236.2025 u.

92. (a) The mass number A of a radionuclide changes by 4 in an α decay and is unchanged in a β decay. If the mass numbers of two radionuclides are given by $4n + k$ and $4n' + k$ (where $k = 0, 1, 2, 3$), then the heavier one can decay into the lighter one by a series of α (and β) decays, as their mass numbers differ by only an integer times 4. If $A = 4n + k$, then after α-decaying for m times, its mass number becomes

$$A = 4n + k - 4m = 4(n - m) + k,$$

still in the same chain.

(b) For ^{235}U, $235 = 58 \times 4 + 3 = 4n + 3$.

(c) For ^{236}U, $236 = 59 \times 4 = 4n$.

(d) For ^{238}U, $238 = 59 \times 4 + 2 = 4n + 2$.

(e) For ^{239}Pu, $239 = 59 \times 4 + 3 = 4n + 3$.

(f) For ^{240}Pu, $240 = 60 \times 4 = 4n$.

(g) For ^{245}Cm, $245 = 61 \times 4 + 1 = 4n + 1$.

(h) For ^{246}Cm, $246 = 61 \times 4 + 2 = 4n + 2$.

(i) For ^{249}Cf, $249 = 62 \times 4 + 1 = 4n + 1$.

(j) For ^{253}Fm, $253 = 63 \times 4 + 1 = 4n + 1$.

93. The disintegration energy is

$$\begin{aligned} Q &= (m_{\text{V}} - m_{\text{Ti}})c^2 - E_K \\ &= (48.94852\,\text{u} - 48.94787\,\text{u})(931.5\,\text{MeV/u}) - 0.00547\,\text{MeV} \\ &= 0.600\,\text{MeV}. \end{aligned}$$

94. We locate a nuclide from Table 42-1 by finding the coordinate (N, Z) of the corresponding point in Fig. 42-4. It is clear that all the nuclides listed in Table 42-1 are stable except the last two, ^{227}Ac and ^{239}Pu.

95. (a) We use $R = R_0 e^{-\lambda t}$ to find t:

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \frac{T_{1/2}}{\ln 2}\ln\frac{R_0}{R} = \frac{14.28\,\text{d}}{\ln 2}\ln\frac{3050}{170} = 59.5\,\text{d}.$$

(b) The required factor is

$$\frac{R_0}{R} = e^{\lambda t} = e^{t\ln 2/T_{1/2}} = e^{(3.48\text{d}/14.28\text{d})\ln 2} = 1.18.$$

96. (a) From the decay series, we know that N_{210}, the amount of ^{210}Pb nuclei, changes because of two decays: the decay from ^{226}Ra into ^{210}Pb at the rate $R_{226} = \lambda_{226}N_{226}$, and the decay from ^{210}Pb into ^{206}Pb at the rate $R_{210} = \lambda_{210}N_{210}$. The first of these decays causes N_{210} to increase while the second one causes it to decrease. Thus,

$$\frac{dN_{210}}{dt} = R_{226} - R_{210} = \lambda_{226}N_{226} - \lambda_{210}N_{210}.$$

(b) We set $dN_{210}/dt = R_{226} - R_{210} = 0$ to obtain $R_{226}/R_{210} = 1.00$.

(c) From $R_{226} = \lambda_{226}N_{226} = R_{210} = \lambda_{210}N_{210}$, we obtain

$$\frac{N_{226}}{N_{210}} = \frac{\lambda_{210}}{\lambda_{226}} = \frac{T_{1/2\,226}}{T_{1/2\,210}} = \frac{1.60\times 10^3\,\text{y}}{22.6\,\text{y}} = 70.8.$$

(d) Since only 1.00% of the ^{226}Ra remains, the ratio R_{226}/R_{210} is 0.00100 of that of the equilibrium state computed in part (b). Thus the ratio is (0.0100)(1) = 0.0100.

(e) This is similar to part (d) above. Since only 1.00% of the ^{226}Ra remains, the ratio N_{226}/N_{210} is 1.00% of that of the equilibrium state computed in part (c), or (0.0100)(70.8) = 0.708.

(f) Since the actual value of N_{226}/N_{210} is 0.09, which much closer to 0.0100 than to 1, the sample of the lead pigment cannot be 300 years old. So *Emmaus* is not a *Vermeer*.

97. (a) Replacing differentials with deltas in Eq. 42-12, we use the fact that $\Delta N = -12$ during $\Delta t = 1.0$ s to obtain

$$\frac{\Delta N}{N} = -\lambda \Delta t \quad \Rightarrow \quad \lambda = 4.8 \times 10^{-18}\ /\mathrm{s}$$

where $N = 2.5 \times 10^{18}$, mentioned at the second paragraph of Section 42-3, is used.

(b) Equation 42-18 yields $T_{1/2} = \ln 2/\lambda = 1.4 \times 10^{17}$ s, or about 4.6 billion years.

Chapter 43

1. (a) Using Eq. 42-20 and adapting Eq. 42-21 to this sample, the number of fission-events per second is

$$R_{\text{fission}} = \frac{N \ln 2}{T_{1/2_{\text{fission}}}} = \frac{M_{\text{sam}} N_A \ln 2}{M_{\text{U}} T_{1/2_{\text{fission}}}}$$
$$= \frac{(1.0 \text{ g})(6.02 \times 10^{23} / \text{mol}) \ln 2}{(235 \text{ g/mol})(3.0 \times 10^{17} \text{ y})(365 \text{ d/y})} = 16 \text{ fissions/day}.$$

(b) Since $R \propto 1/T_{1/2}$ (see Eq. 42-20), the ratio of rates is

$$\frac{R_\alpha}{R_{\text{fission}}} = \frac{T_{1/2_{\text{fission}}}}{\text{T}_{1/2_\alpha}} = \frac{3.0 \times 10^{17} \text{ y}}{7.0 \times 10^{8} \text{ y}} = 4.3 \times 10^{8}.$$

2. When a neutron is captured by ^{237}Np it gains 5.0 MeV, more than enough to offset the 4.2 MeV required for ^{238}Np to fission. Consequently, ^{237}Np is fissionable by thermal neutrons.

3. The energy transferred is

$$Q = (m_{\text{U238}} + m_n - m_{\text{U239}})c^2$$
$$= (238.050782 \text{ u} + 1.008664 \text{ u} - 239.054287 \text{ u})(931.5 \text{ MeV/u})$$
$$= 4.8 \text{ MeV}.$$

4. Adapting Eq. 42-21, there are

$$N_{\text{Pu}} = \frac{M_{\text{sam}}}{M_{\text{Pu}}} NA = \left(\frac{1000 \text{ g}}{239 \text{ g/mol}}\right)(6.02 \times 10^{23} / \text{mol}) = 2.5 \times 10^{24}$$

plutonium nuclei in the sample. If they all fission (each releasing 180 MeV), then the total energy release is 4.54×10^{26} MeV.

5. The yield of one warhead is 2.0 megatons of TNT, or

$$\text{yield} = 2(2.6 \times 10^{28} \text{ MeV}) = 5.2 \times 10^{28} \text{ MeV}.$$

Since each fission event releases about 200 MeV of energy, the number of fissions is

$$N = \frac{5.2\times10^{28}\ \text{MeV}}{200\ \text{MeV}} = 2.6\times10^{26}.$$

However, this only pertains to the 8.0% of Pu that undergoes fission, so the total number of Pu is

$$N_0 = \frac{N}{0.080} = \frac{2.6\times10^{26}}{0.080} = 3.25\times10^{27} = 5.4\times10^{3}\ \text{mol}.$$

With $M = 0.239$ kg/mol, the mass of the warhead is

$$m = (5.4\times10^{3}\ \text{mol})(0.239\ \text{kg/mol}) = 1.3\times10^{3}\ \text{kg}.$$

6. We note that the sum of superscripts (mass numbers A) must balance, as well as the sum of Z values (where reference to Appendix F or G is helpful). A neutron has $Z = 0$ and $A = 1$. Uranium has $Z = 92$.

(a) Since xenon has $Z = 54$, then "Y" must have $Z = 92 - 54 = 38$, which indicates the element strontium. The mass number of "Y" is 235 + 1 – 140 – 1 = 95, so "Y" is ^{95}Sr.

(b) Iodine has $Z = 53$, so "Y" has $Z = 92 - 53 = 39$, corresponding to the element yttrium (the symbol for which, coincidentally, is Y). Since 235 + 1 – 139 – 2 = 95, then the unknown isotope is ^{95}Y.

(c) The atomic number of zirconium is $Z = 40$. Thus, 92 – 40 – 2 = 52, which means that "X" has $Z = 52$ (tellurium). The mass number of "X" is 235 + 1 – 100 – 2 = 134, so we obtain ^{134}Te.

(d) Examining the mass numbers, we find $b = 235 + 1 - 141 - 92 = 3$.

7. If R is the fission rate, then the power output is $P = RQ$, where Q is the energy released in each fission event. Hence,

$$R = P/Q = (1.0\ \text{W})/(200\times10^{6}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV}) = 3.1\times10^{10}\ \text{fissions/s}.$$

8. (a) We consider the process $^{98}\text{Mo}\rightarrow{}^{49}\text{Sc}+{}^{49}\text{Sc}$. The disintegration energy is

$$Q = (m_{\text{Mo}} - 2m_{\text{Sc}})c^2 = [97.90541\ \text{u} - 2(48.95002\ \text{u})](931.5\ \text{MeV/u}) = +5.00\ \text{MeV}.$$

(b) The fact that it is positive does not necessarily mean we should expect to find a great deal of molybdenum nuclei spontaneously fissioning; the energy barrier (see Fig. 43-3) is presumably higher and/or broader for molybdenum than for uranium.

9. (a) The mass of a single atom of ^{235}U is

$$m_0 = (235\ \text{u})(1.661\times10^{-27}\ \text{kg/u}) = 3.90\times10^{-25}\ \text{kg},$$

so the number of atoms in m = 1.0 kg is

$$N = m/m_0 = (1.0 \text{ kg})/(3.90 \times 10^{-25} \text{ kg}) = 2.56 \times 10^{24} \approx 2.6 \times 10^{24}.$$

An alternate approach (but essentially the same once the connection between the "u" unit and N_A is made) would be to adapt Eq. 42-21.

(b) The energy released by N fission events is given by $E = NQ$, where Q is the energy released in each event. For 1.0 kg of ^{235}U,

$$E = (2.56 \times 10^{24})(200 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) = 8.19 \times 10^{13} \text{ J} \approx 8.2 \times 10^{13} \text{ J}.$$

(c) If P is the power requirement of the lamp, then

$$t = E/P = (8.19 \times 10^{13} \text{ J})/(100 \text{ W}) = 8.19 \times 10^{11} \text{ s} = 2.6 \times 10^4 \text{ y}.$$

The conversion factor 3.156×10^7 s/y is used to obtain the last result.

10. The energy released is

$$\begin{aligned} Q &= (m_U + m_n - m_{Cs} - m_{Rb} - 2m_n)c^2 \\ &= (235.04392 \text{ u} - 1.00867 \text{ u} - 140.91963 \text{ u} - 92.92157 \text{ u})(931.5 \text{ MeV/u}) \\ &= 181 \text{ MeV}. \end{aligned}$$

11. If M_{Cr} is the mass of a ^{52}Cr nucleus and M_{Mg} is the mass of a ^{26}Mg nucleus, then the disintegration energy is

$$Q = (M_{Cr} - 2M_{Mg})c^2 = [51.94051 \text{ u} - 2(25.98259 \text{ u})](931.5 \text{ MeV/u}) = -23.0 \text{ MeV}.$$

12. (a) Consider the process $^{239}\text{U} + \text{n} \rightarrow {}^{140}\text{Ce} + {}^{99}\text{Ru} + \text{Ne}.$ We have

$$Z_f - Z_i = Z_{Ce} + Z_{Ru} - Z_U = 58 + 44 - 92 = 10.$$

Thus the number of beta-decay events is 10.

(b) Using Table 37-3, the energy released in this fission process is

$$\begin{aligned} Q &= (m_U + m_n - m_{Ce} - m_{Ru} - 10m_e)c^2 \\ &= (238.05079 \text{ u} + 1.00867 \text{ u} - 139.90543 \text{ u} - 98.90594 \text{ u})(931.5 \text{ MeV/u}) - 10(0.511 \text{ MeV}) \\ &= 226 \text{ MeV}. \end{aligned}$$

13. (a) The electrostatic potential energy is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Z_{Xe}Z_{Sr}e^2}{r_{Xe}+r_{Sr}}$$

where Z_{Xe} is the atomic number of xenon, Z_{Sr} is the atomic number of strontium, r_{Xe} is the radius of a xenon nucleus, and r_{Sr} is the radius of a strontium nucleus. Atomic numbers can be found either in Appendix F or Appendix G. The radii are given by $r = (1.2 \text{ fm})A^{1/3}$, where A is the mass number, also found in Appendix F. Thus,

$$r_{Xe} = (1.2 \text{ fm})(140)^{1/3} = 6.23 \text{ fm} = 6.23 \times 10^{-15} \text{ m}$$

and

$$r_{Sr} = (1.2 \text{ fm})(96)^{1/3} = 5.49 \text{ fm} = 5.49 \times 10^{-15} \text{ m}.$$

Hence, the potential energy is

$$U = (8.99\times10^9 \text{ V}\cdot\text{m/C})\frac{(54)(38)(1.60\times10^{-19} \text{ C})^2}{6.23\times10^{-15} \text{ m}+5.49\times10^{-15} \text{ m}} = 4.08\times10^{-11} \text{ J}$$
$$= 251 \text{ MeV}.$$

(b) The energy released in a typical fission event is about 200 MeV, roughly the same as the electrostatic potential energy when the fragments are touching. The energy appears as kinetic energy of the fragments and neutrons produced by fission.

14. (a) The surface area a of a nucleus is given by

$$a \simeq 4\pi R^2 \simeq 4\pi\left(R_0A^{1/3}\right)^2 \propto A^{2/3}.$$

Thus, the fractional change in surface area is

$$\frac{\Delta a}{a_i} = \frac{a_f - a_i}{a_i} = \frac{(140)^{2/3}+(96)^{2/3}}{(236)^{2/3}} - 1 = +0.25.$$

(b) Since $V \propto R^3 \propto (A^{1/3})^3 = A$, we have

$$\frac{\Delta V}{V} = \frac{V_f}{V_i} - 1 = \frac{140+96}{236} - 1 = 0.$$

(c) The fractional change in potential energy is

$$\frac{\Delta U}{U} = \frac{U_f}{U_i} - 1 = \frac{Q_{Xe}^2/R_{Xe}+Q_{Sr}^2/R_{Sr}}{Q_U^2/R_U} - 1 = \frac{(54)^2(140)^{-1/3}+(38)^2(96)^{-1/3}}{(92)^2(236)^{-1/3}} - 1$$
$$= -0.36.$$

15. **THINK** One megaton of TNT releases 2.6×10^{28} MeV of energy. The energy released in each fission event is about 200 MeV.

EXPRESS The energy yield of the bomb is

$$E = (66 \times 10^{-3} \text{ megaton})(2.6 \times 10^{28} \text{ MeV/ megaton}) = 1.72 \times 10^{27} \text{ MeV}.$$

At 200 MeV per fission event, the total number of fission events taking place is

$$(1.72 \times 10^{27} \text{ MeV})/(200 \text{ MeV}) = 8.58 \times 10^{24}.$$

Now, since only 4.0% of the ^{235}U nuclei originally present undergo fission, there must have been $(8.58 \times 10^{24})/(0.040) = 2.14 \times 10^{26}$ nuclei originally present.

ANALYZE (a) The mass of ^{235}U originally present was

$$(2.14 \times 10^{26})(235 \text{ u})(1.661 \times 10^{-27} \text{ kg/u}) = 83.7 \text{ kg} \approx 84 \text{ kg}.$$

(b) Two fragments are produced in each fission event, so the total number of fragments is

$$2(8.58 \times 10^{24}) = 1.72 \times 10^{25} \approx 1.7 \times 10^{25}.$$

(c) One neutron produced in a fission event is used to trigger the next fission event, so the average number of neutrons released to the environment in each event is 1.5. The total number released is

$$(8.58 \times 10^{24})(1.5) = 1.29 \times 10^{25} \approx 1.3 \times 10^{25}.$$

LEARN When one ^{235}U nucleus undergoes fission, the neutrons it produces (an average number of 2.5 neutrons per fission) can trigger other ^{235}U nuclei to fission, thereby setting up a chain reaction that allows an enormous amount of energy to be released.

16. (a) Using the result of Problem 43-4, the TNT equivalent is

$$\frac{(2.50 \text{ kg})(4.54 \times 10^{26} \text{ MeV/kg})}{2.6 \times 10^{28} \text{ MeV}/10^{6} \text{ ton}} = 4.4 \times 10^{4} \text{ ton} = 44 \text{ kton}.$$

(b) Assuming that this is a fairly inefficiently designed bomb, then much of the remaining 92.5 kg is probably “wasted” and was included perhaps to make sure the bomb did not “fizzle.” There is also an argument for having more than just the critical mass based on the short assembly time of the material during the implosion, but this so-called “super-critical mass,” as generally quoted, is much less than 92.5 kg, and does not necessarily have to be purely plutonium.

17. **THINK** We represent the unknown fragment as ${}^{A}_{Z}X$, where A and Z are its mass number and atomic number, respectively. Charge and mass number are conserved in the neutron-capture process.

EXPRESS The reaction can be written as

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{82}_{32}\text{Ge} + {}^{A}_{Z}X \,.$$

Conservation of charge yields 92 + 0 = 32 + Z, so Z = 60. Conservation of mass number yields 235 + 1 = 83 + A, so A = 153.

ANALYZE (a) Looking in Appendix F or G for nuclides with Z = 60, we find that the unknown fragment is ${}^{153}_{60}\text{Nd}$.

(b) We neglect the small kinetic energy and momentum carried by the neutron that triggers the fission event. Then,

$$Q = K_{\text{Ge}} + K_{\text{Nd}},$$

where K_{Ge} is the kinetic energy of the germanium nucleus and K_{Nd} is the kinetic energy of the neodymium nucleus. Conservation of momentum yields $\vec{p}_{\text{Ge}} + \vec{p}_{\text{Nd}} = 0$. Now, we can write the classical formula for kinetic energy in terms of the magnitude of the momentum vector:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

which implies that

$$K_{\text{Nd}} = \frac{p_{\text{Nd}}^2}{2M_{\text{Nd}}} = \frac{p_{\text{Ge}}^2}{2M_{\text{Nd}}} = \frac{M_{\text{Ge}}}{M_{\text{Nd}}}\frac{p_{\text{Ge}}^2}{2M_{\text{Ge}}} = \frac{M_{\text{Ge}}}{M_{\text{Nd}}}K_{\text{Ge}} \,.$$

Thus, the energy equation becomes

$$Q = K_{\text{Ge}} + \frac{M_{\text{Ge}}}{M_{\text{Nd}}}K_{\text{Ge}} = \frac{M_{\text{Nd}} + M_{\text{Ge}}}{M_{\text{Nd}}}K_{\text{Ge}}$$

and

$$K_{\text{Ge}} = \frac{M_{\text{Nd}}}{M_{\text{Nd}} + M_{\text{Ge}}}Q = \frac{153\text{ u}}{153\text{ u} + 83\text{ u}}(170\text{ MeV}) = 110\text{ MeV}.$$

(c) Similarly,

$$K_{\text{Nd}} = \frac{M_{\text{Ge}}}{M_{\text{Nd}} + M_{\text{Ge}}}Q = \frac{83\text{ u}}{153\text{ u} + 83\text{ u}}(170\text{ MeV}) = 60\text{ MeV}.$$

(d) The initial speed of the germanium nucleus is

$$v_{\text{Ge}} = \sqrt{\frac{2K_{\text{Ge}}}{M_{\text{Ge}}}} = \sqrt{\frac{2(110\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(83\ \text{u})(1.661\times10^{-27}\ \text{kg/u})}} = 1.60\times10^7\ \text{m/s}.$$

(e) The initial speed of the neodymium nucleus is

$$v_{\text{Nd}} = \sqrt{\frac{2K_{\text{Nd}}}{M_{\text{ND}}}} = \sqrt{\frac{2(60\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(153\ \text{u})(1.661\times10^{-27}\ \text{kg/u})}} = 8.69\times10^6\ \text{m/s}.$$

LEARN By momentum conservation, the two fragments fly apart in opposite directions.

18. If P is the power output, then the energy E produced in the time interval Δt (= 3 y) is

$$\begin{aligned} E = P\,\Delta t &= (200\times10^6\ \text{W})(3\ \text{y})(3.156\times10^7\ \text{s/y}) = 1.89\times10^{16}\ \text{J} \\ &= (1.89\times10^{16}\ \text{J})/(1.60\times10^{-19}\ \text{J/eV}) = 1.18\times10^{35}\ \text{eV} \\ &= 1.18\times10^{29}\ \text{MeV}. \end{aligned}$$

At 200 MeV per event, this means $(1.18\times10^{29})/200 = 5.90\times10^{26}$ fission events occurred. This must be half the number of fissionable nuclei originally available. Thus, there were $2(5.90\times10^{26}) = 1.18\times10^{27}$ nuclei. The mass of a ^{235}U nucleus is

$$(235\ \text{u})(1.661\times10^{-27}\ \text{kg/u}) = 3.90\times10^{-25}\ \text{kg},$$

so the total mass of ^{235}U originally present was $(1.18\times10^{27})(3.90\times10^{-25}\ \text{kg}) = 462$ kg.

19. After each time interval t_{gen} the number of nuclides in the chain reaction gets multiplied by k. The number of such time intervals that has gone by at time t is t/t_{gen}. For example, if the multiplication factor is 5 and there were 12 nuclei involved in the reaction to start with, then after one interval 60 nuclei are involved. And after another interval 300 nuclei are involved. Thus, the number of nuclides engaged in the chain reaction at time t is $N(t) = N_0 k^{t/t_{\text{gen}}}$. Since $P \propto N$ we have

$$P(t) = P_0 k^{t/t_{\text{gen}}}.$$

20. We use the formula from Problem 43-19:

$$P(t) = P_0 k^{t/t_{\text{gen}}} = (400\,\text{MW})(1.0003)^{(5.00\ \text{min})(60\ \text{s/min})/(0.00300\,\text{s})} = 8.03\times10^3\,\text{MW}.$$

21. If R is the decay rate then the power output is $P = RQ$, where Q is the energy produced by each alpha decay. Now

$$R = \lambda N = N\ln 2/T_{1/2},$$

where λ is the disintegration constant and $T_{1/2}$ is the half-life. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. If M is the total mass of material and m is the mass of a single ^{238}Pu nucleus, then

$$N = \frac{M}{m} = \frac{1.00 \text{ kg}}{(238 \text{ u})(1.661\times 10^{-27} \text{ kg/u})} = 2.53\times 10^{24}.$$

Thus,

$$P = \frac{NQ\ln 2}{T_{1/2}} = \frac{(2.53\times 10^{24})(5.50\times 10^{6} \text{ eV})(1.60\times 10^{-19} \text{ J/eV})(\ln 2)}{(87.7 \text{ y})(3.156\times 10^{7} \text{ s/y})} = 557 \text{ W}.$$

22. We recall Eq. 43-6:

$$Q \approx 200 \text{ MeV} = 3.2 \times 10^{-11} \text{ J}.$$

It is important to bear in mind that watts multiplied by seconds give joules. From $E = Pt_{\text{gen}} = NQ$ we get the number of free neutrons:

$$N = \frac{Pt_{\text{gen}}}{Q} = \frac{(500\times 10^{6} \text{ W})(1.0\times 10^{-3} \text{ s})}{3.2\times 10^{-11} \text{ J}} = 1.6\times 10^{16}.$$

23. **THINK** The neutron generation time t_{gen} in a reactor is the average time needed for a fast neutron emitted in a fission event to be slowed to thermal energies by the moderator and then initiate another fission event.

EXPRESS Let P_0 be the initial power output, P be the final power output, k be the multiplication factor, t be the time for the power reduction, and t_{gen} be the neutron generation time. Then, according to the result of Problem 43-19,

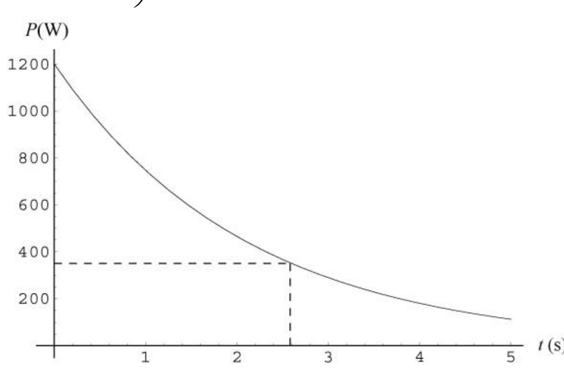
$$P = P_0 k^{t/t_{\text{gen}}}.$$

ANALYZE We divide by P_0, take the natural logarithm of both sides of the equation and solve for ln k:

$$\ln k = \frac{t_{\text{gen}}}{t}\ln\left(\frac{P}{P_0}\right) = \frac{1.3\times 10^{-3} \text{ s}}{2.6 \text{ s}}\ln\left(\frac{350 \text{ MW}}{1200 \text{ MW}}\right) = -0.0006161.$$

Hence, $k = e^{-0.0006161} = 0.99938$.

LEARN The power output as a function of time is shown to the right. Since the multiplication factor k is smaller than 1, the output decreases with time.



24. (a) We solve Q_{eff} from $P = RQ_{\text{eff}}$:

$$Q_{\text{eff}} = \frac{P}{R} = \frac{P}{N\lambda} = \frac{mPT_{1/2}}{M\ln 2}$$
$$= \frac{(90.0 \text{ u})(1.66\times10^{-27} \text{ kg/u})(0.93 \text{ W})(29 \text{ y})(3.15\times10^{7} \text{ s/y})}{(1.00\times10^{-3} \text{ kg})(\ln 2)(1.60\times10^{-13} \text{ J/MeV})}$$
$$= 1.2 \text{ MeV}.$$

(b) The amount of ^{90}Sr needed is

$$M = \frac{150 \text{ W}}{(0.050)(0.93 \text{ W/g})} = 3.2 \text{ kg}.$$

25. **THINK** Momentum is conserved in the collision process. In addition, energy is also conserved since the collision is elastic.

EXPRESS Let v_{ni} be the initial velocity of the neutron, v_{nf} be its final velocity, and v_f be the final velocity of the target nucleus. Then, since the target nucleus is initially at rest, conservation of momentum yields

$$m_n v_{ni} = m_n v_{nf} + m v_f$$

and conservation of energy yields

$$\frac{1}{2}m_n v_{ni}^2 = \frac{1}{2}m_n v_{nf}^2 + \frac{1}{2}m v_f^2.$$

We solve these two equations simultaneously for v_f. This can be done, for example, by using the conservation of momentum equation to obtain an expression for v_{nf} in terms of v_f and substituting the expression into the conservation of energy equation. We solve the resulting equation for v_f. We obtain $v_f = 2m_n v_{ni}/(m + m_n)$.

ANALYZE (a) The energy lost by the neutron is the same as the energy gained by the target nucleus, so

$$\Delta K = \frac{1}{2}m v_f^2 = \frac{1}{2}\frac{4m_n^2 m}{(m+m_n)^2}v_{ni}^2.$$

The initial kinetic energy of the neutron is $K = \frac{1}{2}m_n v_{ni}^2$, so

$$\frac{\Delta K}{K} = \frac{4m_n m}{(m+m_n)^2}.$$

(b) The mass of a neutron is 1.0 u and the mass of a hydrogen atom is also 1.0 u. (Atomic masses can be found in Appendix G.) Thus,

$$\frac{\Delta K}{K}=\frac{4(1.0\text{ u})(1.0\text{ u})}{(1.0\text{ u}+1.0\text{ u})^2}=1.0.$$

(c) Similarly, the mass of a deuterium atom is 2.0 u, so

$$(\Delta K)/K = 4(1.0\text{ u})(2.0\text{ u})/(2.0\text{ u} + 1.0\text{ u})^2 = 0.89.$$

(d) The mass of a carbon atom is 12 u, so

$$(\Delta K)/K = 4(1.0\text{ u})(12\text{ u})/(12\text{ u} + 1.0\text{ u})^2 = 0.28.$$

(e) The mass of a lead atom is 207 u, so

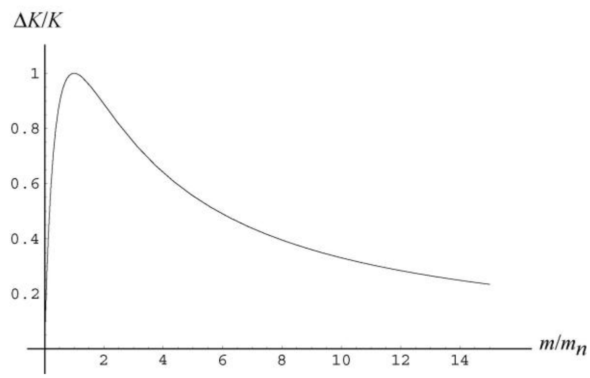
$$(\Delta K)/K = 4(1.0\text{ u})(207\text{ u})/(207\text{ u} + 1.0\text{ u})^2 = 0.019.$$

(f) During each collision, the energy of the neutron is reduced by the factor 1 – 0.89 = 0.11. If E_i is the initial energy, then the energy after n collisions is given by $E = (0.11)^n E_i$. We take the natural logarithm of both sides and solve for n. The result is

$$n=\frac{\ln(E/E_i)}{\ln 0.11}=\frac{\ln(0.025\text{ eV}/1.00\text{ eV})}{\ln 0.11}=7.9\approx 8.$$

The energy first falls below 0.025 eV on the eighth collision.

LEARN The fractional kinetic energy loss as a function of the mass of the stationary atom (in units of m/m_n) is plotted below.



From the plot, it is clear that the energy loss is greatest ($\Delta K/K = 1$) when the atom has the same as the neutron.

26. The ratio is given by

$$\frac{N_5(t)}{N_8(t)} = \frac{N_5(0)}{N_8(0)} e^{-(\lambda_5-\lambda_8)t},$$

or

$$t = \frac{1}{\lambda_8 - \lambda_5} \ln\left[\left(\frac{N_5(t)}{N_8(t)}\right)\left(\frac{N_8(0)}{N_5(0)}\right)\right] = \frac{1}{(1.55-9.85)10^{-10}\ \text{y}^{-1}} \ln[(0.0072)(0.15)^{-1}]$$
$$= 3.6\times10^9\ \text{y}.$$

27. (a) $P_{avg} = (15 \times 10^9$ W·y)/(200,000 y) $= 7.5 \times 10^4$ W = 75 kW.

(b) Using the result of Eq. 43-6, we obtain

$$M = \frac{m_U E_{total}}{Q} = \frac{(235\text{u})(1.66\times10^{-27}\,\text{kg/u})(15\times10^9\,\text{W}\cdot\text{y})(3.15\times10^7\,\text{s/y})}{(200\,\text{MeV})(1.6\times10^{-13}\,\text{J/MeV})} = 5.8\times10^3\ \text{kg}.$$

28. The nuclei of ^{238}U can capture neutrons and beta-decay. With a large amount of neutrons available due to the fission of ^{235}U, the probability for this process is substantially increased, resulting in a much higher decay rate for ^{238}U and causing the depletion of ^{238}U (and relative enrichment of ^{235}U).

29. **THINK** With a shorter half-life, ^{235}U has a greater decay rate than ^{238}U. Thus, if the ore contains only 0.72% of ^{235}U today, then the concentration must be higher in the far distant past.

EXPRESS Let t be the present time and t = 0 be the time when the ratio of ^{235}U to ^{238}U was 3.0%. Let N_{235} be the number of ^{235}U nuclei present in a sample now and $N_{235,0}$ be the number present at t = 0. Let N_{238} be the number of ^{238}U nuclei present in the sample now and $N_{238,0}$ be the number present at t = 0. The law of radioactive decay holds for each species, so

$$N_{235} = N_{235,0} e^{-\lambda_{235} t}$$

and

$$N_{238} = N_{238,0} e^{-\lambda_{238} t}.$$

Dividing the first equation by the second, we obtain

$$r = r_0 e^{-(\lambda_{235}-\lambda_{238})t}$$

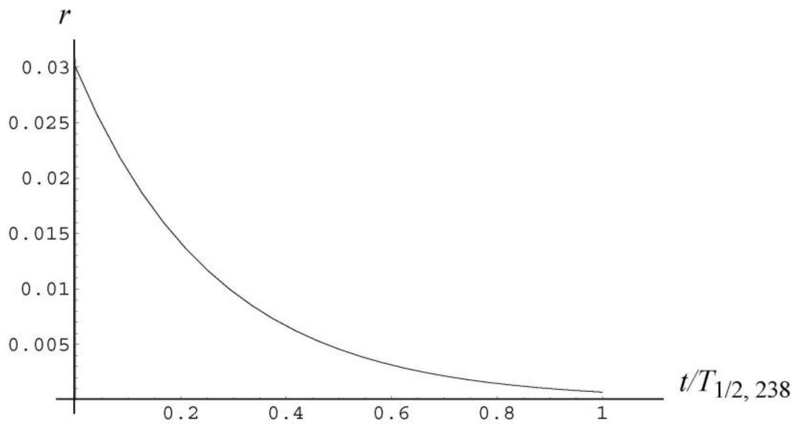
where $r = N_{235}/N_{238}$ (= 0.0072) and $r_0 = N_{235,0}/N_{238,0}$ (= 0.030). We solve for t:

$$t = -\frac{1}{\lambda_{235} - \lambda_{238}} \ln\left(\frac{r}{r_0}\right).$$

ANALYZE Now we use $\lambda_{235} = (\ln 2)/T_{1/2_{235}}$ and $\lambda_{238} = (\ln 2)/T_{1/2_{238}}$ to obtain

$$t = \frac{T_{1/2_{235}} T_{1/2_{238}}}{(T_{1/2_{238}} - T_{1/2_{235}})\ln 2} \ln\left(\frac{r}{r_0}\right) = -\frac{(7.0\times10^8\text{ y})(4.5\times10^9\text{ y})}{(4.5\times10^9\text{ y} - 7.0\times10^8\text{ y})\ln 2} \ln\left(\frac{0.0072}{0.030}\right)$$
$$= 1.7\times10^9\text{ y}.$$

LEARN How the ratio $r = N_{235}/N_{238}$ changes with time is plotted below. In the plot, we take the ratio to be 0.03 at t = 0. At $t = 1.7\times10^9$ y or $t/T_{1/2,238} = 0.378$, r is reduced to 0.072.



30. We are given the energy release per fusion (Q = 3.27 MeV = 5.24 × 10^{-13} J) and that a pair of deuterium atoms is consumed in each fusion event. To find how many pairs of deuterium atoms are in the sample, we adapt Eq. 42-21:

$$N_{d\,\text{pairs}} = \frac{M_{\text{sam}}}{2M_d} N_A = \left(\frac{1000\text{ g}}{2(2.0\text{ g/mol})}\right)(6.02\times10^{23}\text{/mol}) = 1.5\times10^{26}.$$

Multiplying this by Q gives the total energy released: 7.9 × 10^{13} J. Keeping in mind that a watt is a joule per second, we have

$$t = \frac{7.9\times10^{13}\text{ J}}{100\text{ W}} = 7.9\times10^{11}\text{ s} = 2.5\times10^4\text{ y}.$$

31. **THINK** Coulomb repulsion acts to prevent two charged particles from coming close enough to be within the range of their attractive nuclear force.

EXPRESS We take the height of the Coulomb barrier to be the value of the kinetic energy K each deuteron must initially have if they are to come to rest when their surfaces touch. If r is the radius of a deuteron, conservation of energy yields

$$2K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}.$$

ANALYZE With r = 2.1 fm, we have

$$K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{4r} = (8.99\times10^9\ \text{V}\cdot\text{m/C})\frac{(1.60\times10^{-19}\ \text{C})^2}{4(2.1\times10^{-15}\ \text{m})} = 2.74\times10^{-14}\ \text{J} = 170\ \text{keV}.$$

LEARN The height of the Coulomb barrier depends on the charges and radii of the two interacting nuclei. Increasing the charge raises the barrier.

32. (a) Our calculation is identical to that in Sample Problem — "Fusion in a gas of protons and required temperature" except that we are now using R appropriate to two deuterons coming into "contact," as opposed to the R = 1.0 fm value used in the Sample Problem. If we use R = 2.1 fm for the deuterons, then our K is simply the K calculated in the Sample Problem, divided by 2.1:

$$K_{d+d} = \frac{K_{p+p}}{2.1} = \frac{360\ \text{keV}}{2.1} \approx 170\ \text{keV}.$$

Consequently, the voltage needed to accelerate each deuteron from rest to that value of K is 170 kV.

(b) Not all deuterons that are accelerated toward each other will come into "contact" and not all of those that do so will undergo nuclear fusion. Thus, a great many deuterons must be repeatedly encountering other deuterons in order to produce a macroscopic energy release. An accelerator needs a fairly good vacuum in its beam pipe, and a very large number flux is either impractical and/or very expensive. Regarding expense, there are other factors that have dissuaded researchers from using accelerators to build a controlled fusion "reactor," but those factors may become less important in the future — making the feasibility of accelerator "add-ons" to magnetic and inertial confinement schemes more cost-effective.

33. Our calculation is very similar to that in Sample Problem – "Fusion in a gas of protons and required temperature" except that we are now using R appropriate to two lithium-7 nuclei coming into "contact," as opposed to the R = 1.0 fm value used in the Sample Problem. If we use

$$R = r = r_0 A^{1/3} = (1.2\ \text{fm})\sqrt[3]{7} = 2.3\ \text{fm}$$

and $q = Ze = 3e$, then our K is given by (see the Sample Problem)

$$K = \frac{Z^2e^2}{16\pi\varepsilon_0 r} = \frac{3^2(1.6\times10^{-19}\ \text{C})^2}{16\pi(8.85\times10^{-12}\ \text{F/m})(2.3\times10^{15}\ \text{m})}$$

which yields 2.25×10^{-13} J = 1.41 MeV. We interpret this as the answer to the problem, though the term "Coulomb barrier height" as used here may be open to other interpretations.

34. From the expression for $n(K)$ given we may write $n(K) \propto K^{1/2}e^{-K/kT}$. Thus, with

$$k = 8.62 \times 10^{-5} \text{ eV/K} = 8.62 \times 10^{-8} \text{ keV/K},$$

we have

$$\frac{n(K)}{n(K_{\text{avg}})} = \left(\frac{K}{K_{\text{avg}}}\right)^{1/2} e^{-(K-K_{\text{avg}})/kT} = \left(\frac{5.00\,\text{keV}}{1.94\,\text{keV}}\right)^{1/2} \exp\left(-\frac{5.00\,\text{keV}-1.94\,\text{keV}}{(8.62\times10^{-8}\,\text{keV})(1.50\times10^{7}\,\text{K})}\right) = 0.151.$$

35. The kinetic energy of each proton is

$$K = k_B T = (1.38\times10^{-23}\ \text{J/K})(1.0\times10^{7}\ \text{K}) = 1.38\times10^{-16}\ \text{J}.$$

At the closest separation, r_{min}, all the kinetic energy is converted to potential energy:

$$K_{\text{tot}} = 2K = U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_{\text{min}}}\ .$$

Solving for r_{min}, we obtain

$$r_{\text{min}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2K} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{2(1.38\times10^{-16}\,\text{J})} = 8.33\times10^{-13}\ \text{m} \approx 1\ \text{pm}.$$

36. The energy released is

$$\begin{aligned} Q &= -\Delta mc^2 = -(m_{\text{He}} - m_{\text{H2}} - m_{\text{H1}})c^2 \\ &= -(3.016029\ \text{u} - 2.014102\ \text{u} - 1.007825\ \text{u})(931.5\ \text{MeV/u}) \\ &= 5.49\ \text{MeV}. \end{aligned}$$

37. (a) Let M be the mass of the Sun at time t and E be the energy radiated to that time. Then, the power output is

$$P = dE/dt = (dM/dt)c^2,$$

where $E = Mc^2$ is used. At the present time,

$$\frac{dM}{dt} = \frac{P}{c^2} = \frac{3.9\times10^{26}\,\text{W}}{\left(2.998\times10^{8}\ \text{m/s}\right)^2} = 4.3\times10^{9}\ \text{kg/s}.$$

(b) We assume the rate of mass loss remained constant. Then, the total mass loss is

$$\begin{aligned} \Delta M &= (dM/dt)\,\Delta t = (4.33 \times 10^{9}\ \text{kg/s})\,(4.5 \times 10^{9}\ \text{y})\,(3.156 \times 10^{7}\ \text{s/y}) \\ &= 6.15 \times 10^{26}\ \text{kg}. \end{aligned}$$

The fraction lost is

$$\frac{\Delta M}{M+\Delta M}=\frac{6.15\times10^{26}\,\text{kg}}{2.0\times10^{30}\,\text{kg}+6.15\times10^{26}\,\text{kg}}=3.1\times10^{-4}\,.$$

38. In Fig. 43-10, let Q_1 = 0.42 MeV, Q_2 = 1.02 MeV, Q_3 = 5.49 MeV, and Q_4 = 12.86 MeV. For the overall proton-proton cycle

$$\begin{aligned} Q &= 2Q_1+2Q_2+2Q_3+Q_4 \\ &= 2(0.42\text{ MeV}+1.02\text{ MeV}+5.49\text{ MeV})+12.86\text{ MeV}=26.7\text{ MeV}. \end{aligned}$$

39. If M_{He} is the mass of an atom of helium and M_{C} is the mass of an atom of carbon, then the energy released in a single fusion event is

$$Q=\left(3M_{\text{He}}-M_{\text{C}}\right)c^2=[3(4.0026\text{ u})-(12.0000\text{ u})](931.5\text{ MeV/u})=7.27\text{ MeV}.$$

Note that $3M_{\text{He}}$ contains the mass of six electrons and so does M_{C}. The electron masses cancel and the mass difference calculated is the same as the mass difference of the nuclei.

40. (a) We are given the energy release per fusion (Q = 26.7 MeV = 4.28×10^{-12} J) and that four protons are consumed in each fusion event. To find how many sets of four protons are in the sample, we adapt Eq. 42-21:

$$N_{4p}=\frac{M_{\text{sam}}}{4M_{\text{H}}}N_{\text{A}}=\left(\frac{1000\,\text{g}}{4(1.0\,\text{g/mol})}\right)\left(6.02\times10^{23}/\text{mol}\right)=1.5\times10^{26}\,.$$

Multiplying this by Q gives the total energy released: 6.4×10^{14} J. It is not required that the answer be in SI units; we could have used MeV throughout (in which case the answer is 4.0×10^{27} MeV).

(b) The number of ^{235}U nuclei is

$$N_{235}=\left(\frac{1000\,\text{g}}{235\,\text{g/mol}}\right)\left(6.02\times10^{23}/\text{mol}\right)=2.56\times10^{24}\,.$$

If all the U-235 nuclei fission, the energy release (using the result of Eq. 43-6) is

$$N_{235}Q_{\text{fission}}=\left(2.56\times10^{22}\right)\left(200\,\text{MeV}\right)=5.1\times10^{26}\text{ MeV}=8.2\times10^{13}\text{ J}\,.$$

We see that the fusion process (with regard to a unit mass of fuel) produces a larger amount of energy (despite the fact that the Q value per event is smaller).

41. Since the mass of a helium atom is

$$(4.00 \text{ u})(1.661 \times 10^{-27} \text{ kg/u}) = 6.64 \times 10^{-27} \text{ kg},$$

the number of helium nuclei originally in the star is

$$(4.6 \times 10^{32} \text{ kg})/(6.64 \times 10^{-27} \text{ kg}) = 6.92 \times 10^{58}.$$

Since each fusion event requires three helium nuclei, the number of fusion events that can take place is

$$N = 6.92 \times 10^{58}/3 = 2.31 \times 10^{58}.$$

If Q is the energy released in each event and t is the conversion time, then the power output is $P = NQ/t$ and

$$t = \frac{NQ}{P} = \frac{(2.31\times10^{58})(7.27\times10^{6}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{5.3\times10^{30}\text{ W}} = 5.07\times10^{15}\text{ s}$$
$$= 1.6\times10^{8}\text{ y}.$$

42. We assume the neutrino has negligible mass. The photons, of course, are also taken to have zero mass.

$$\begin{aligned}
Q_1 &= (2m_p - m_2 - m_e)c^2 = [2(m_1 - m_e) - (m_2 - m_e) - m_e]c^2 \\
&= [2(1.007825\text{ u}) - 2.014102\text{ u} - 2(0.0005486\text{ u})](931.5\text{ MeV/u}) \\
&= 0.42\text{ MeV} \\
Q_2 &= (m_2 + m_p - m_3)c^2 = (m_2 + m_p - m_3)c^2 \\
&= [(2.014102\text{ u}) + 1.007825\text{ u} - 3.016029\text{ u})](931.5\text{ MeV/u}) \\
&= 5.49\text{ MeV} \\
Q_3 &= (2m_3 - m_4 - 2m_p)c^2 = (2m_3 - m_4 - 2m_p)c^2 \\
&= [2(3.016029\text{ u}) - 4.002603\text{ u} - 2(1.007825\text{ u})](931.5\text{ MeV/u}) \\
&= 12.86\text{ MeV}.
\end{aligned}$$

43. (a) The energy released is

$$\begin{aligned}
Q &= (5m_{^2\text{H}} - m_{^3\text{He}} - m_{^4\text{He}} - m_{^1\text{H}} - 2m_n)c^2 \\
&= [5(2.014102\text{ u}) - 3.016029\text{ u} - 4.002603\text{ u} - 1.007825\text{ u} - 2(1.008665\text{ u})](931.5\text{ MeV/u}) \\
&= 24.9\text{ MeV}.
\end{aligned}$$

(b) Assuming 30.0% of the deuterium undergoes fusion, the total energy released is

$$E = NQ = \left(\frac{0.300\,M}{5m_{^2\text{H}}}\right)Q\ .$$

Thus, the rating is

$$\begin{aligned} R &= \frac{E}{2.6\times10^{28}\ \text{MeV/megaton TNT}} \\ &= \frac{(0.300)(500\ \text{kg})(24.9\ \text{MeV})}{5(2.0\ \text{u})(1.66\times10^{-27}\ \text{kg/u})(2.6\times10^{28}\ \text{MeV/megaton TNT})} \\ &= 8.65\ \text{megaton TNT}\ . \end{aligned}$$

44. The mass of the hydrogen in the Sun's core is $m_{\text{H}} = 0.35\left(\frac{1}{8}M_{\text{Sun}}\right)$. The time it takes for the hydrogen to be entirely consumed is

$$t = \frac{M_{\text{H}}}{dm/dt} = \frac{(0.35)(\frac{1}{8})(2.0\times10^{30}\ \text{kg})}{(6.2\times10^{11}\ \text{kg/s})(3.15\times10^{7}\ \text{s/y})} = 5\times10^{9}\ \text{y}\ .$$

45. (a) Since two neutrinos are produced per proton-proton cycle (see Eq. 43-10 or Fig. 43-10), the rate of neutrino production R_ν satisfies

$$R_\nu = \frac{2P}{Q} = \frac{2(3.9\times10^{26}\ \text{W})}{(26.7\ \text{MeV})(1.6\times10^{-13}\ \text{J/MeV})} = 1.8\times10^{38}\ \text{s}^{-1}\ .$$

(b) Let d_{es} be the Earth to Sun distance, and R be the radius of Earth (see Appendix C). Earth represents a small cross section in the "sky" as viewed by a fictitious observer on the Sun. The rate of neutrinos intercepted by that area (very small, relative to the area of the full "sky") is

$$R_{\nu,\text{Earth}} = R_\nu\left(\frac{\pi R_e^2}{4\pi d_{es}^2}\right) = \frac{(1.8\times10^{38}\ \text{s}^{-1})}{4}\left(\frac{6.4\times10^{6}\ \text{m}}{1.5\times10^{11}\ \text{m}}\right)^2 = 8.2\times10^{28}\ \text{s}^{-1}\ .$$

46. (a) The products of the carbon cycle are $2e^+ + 2\nu + {}^4\text{He}$, the same as that of the proton-proton cycle (see Eq. 43-10). The difference in the number of photons is not significant.

(b) We have

$$\begin{aligned} Q_{\text{carbon}} &= Q_1 + Q_2 + \cdots + Q_6 \\ &= (1.95\times1.19 + 7.55 + 7.30 + 1.73 + 4.97)\,\text{MeV} \\ &= 24.7\,\text{MeV} \end{aligned}$$

which is the same as that for the proton-proton cycle (once we subtract out the electron-positron annihilations; see Fig. 43-10):

$$Q_{p-p} = 26.7\text{ MeV} - 2(1.02\text{ MeV}) = 24.7\text{ MeV}.$$

47. **THINK** The energy released by burning 1 kg of carbon is 3.3×10^7 J.

EXPRESS The mass of a carbon atom is $(12.0\text{ u})(1.661 \times 10^{-27}\text{ kg/u}) = 1.99 \times 10^{-26}$ kg, so the number of carbon atoms in 1.00 kg of carbon is

$$(1.00\text{ kg})/(1.99 \times 10^{-26}\text{ kg}) = 5.02 \times 10^{25}.$$

ANALYZE (a) The heat of combustion per atom is

$$(3.3 \times 10^7\text{ J/kg})/(5.02 \times 10^{25}\text{ atom/kg}) = 6.58 \times 10^{-19}\text{ J/atom}.$$

This is 4.11 eV/atom.

(b) In each combustion event, two oxygen atoms combine with one carbon atom, so the total mass involved is 2(16.0 u) + (12.0 u) = 44 u. This is

$$(44\text{ u})(1.661 \times 10^{-27}\text{ kg/u}) = 7.31 \times 10^{-26}\text{ kg}.$$

Each combustion event produces 6.58×10^{-19} J so the energy produced per unit mass of reactants is $(6.58 \times 10^{-19}\text{ J})/(7.31 \times 10^{-26}\text{ kg}) = 9.00 \times 10^6$ J/kg.

(c) If the Sun were composed of the appropriate mixture of carbon and oxygen, the number of combustion events that could occur before the Sun burns out would be

$$(2.0 \times 10^{30}\text{ kg})/(7.31 \times 10^{-26}\text{ kg}) = 2.74 \times 10^{55}.$$

The total energy released would be

$$E = (2.74 \times 10^{55})(6.58 \times 10^{-19}\text{ J}) = 1.80 \times 10^{37}\text{ J}.$$

If P is the power output of the Sun, the burn time would be

$$t = \frac{E}{P} = \frac{1.80\times10^{37}\text{ J}}{3.9\times10^{26}\text{ W}} = 4.62\times10^{10}\text{ s} = 1.46\times10^{3}\text{ y},$$

or 1.5×10^3 y, to two significant figures.

LEARN The Sun burns not coal but hydrogen via the proton-proton cycle in which the fusion of hydrogen nuclei into helium nuclei take place. The mechanism of thermonuclear fusion reactions allows the Sun to radiate energy at a rate of 3.9×10^{26} W for several billion years.

48. In Eq. 43-13,

$$Q=\left(2m_{^2\text{H}}-m_{^3\text{He}}-m_n\right)c^2=\left[2\left(2.014102\,\text{u}\right)-3.016049\,\text{u}-1.008665\,\text{u}\right]\left(931.5\,\text{MeV/u}\right)$$
$$=3.27\text{MeV}\ .$$

In Eq. 43-14,

$$Q=\left(2m_{^2\text{H}}-m_{^3\text{H}}-m_{^1\text{H}}\right)c^2=\left[2\left(2.014102\,\text{u}\right)-3.016049\,\text{u}-1.007825\,\text{u}\right]\left(931.5\,\text{MeV/u}\right)$$
$$=4.03\,\text{MeV}\ .$$

Finally, in Eq. 43-15,

$$\begin{aligned}Q&=\left(m_{^2\text{H}}+m_{^3\text{H}}-m_{^4\text{He}}-m_n\right)c^2\\&=\left(2.014102\,\text{u}+3.016049\,\text{u}-4.002603\,\text{u}-1.008665\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)\\&=17.59\,\text{MeV}\ .\end{aligned}$$

49. Since 1.00 L of water has a mass of 1.00 kg, the mass of the heavy water in 1.00 L is 0.0150×10^{-2} kg = 1.50×10^{-4} kg. Since a heavy water molecule contains one oxygen atom, one hydrogen atom and one deuterium atom, its mass is

$$(16.0\text{ u} + 1.00\text{ u} + 2.00\text{ u}) = 19.0\text{ u} = (19.0\text{ u})(1.661 \times 10^{-27}\text{ kg/u}) = 3.16 \times 10^{-26}\text{ kg.}$$

The number of heavy water molecules in a liter of water is

$$(1.50 \times 10^{-4}\text{ kg})/(3.16 \times 10^{-26}\text{ kg}) = 4.75 \times 10^{21}.$$

Since each fusion event requires two deuterium nuclei, the number of fusion events that can occur is $N = 4.75 \times 10^{21}/2 = 2.38 \times 10^{21}$. Each event releases energy

$$Q = (3.27 \times 10^{6}\text{ eV})(1.60 \times 10^{-19}\text{ J/eV}) = 5.23 \times 10^{-13}\text{ J.}$$

Since all events take place in a day, which is 8.64×10^4 s, the power output is

$$P=\frac{NQ}{t}=\frac{\left(2.38\times10^{21}\right)\left(5.23\times10^{-13}\text{ J}\right)}{8.64\times10^{4}\text{ s}}=1.44\times10^{4}\text{ W}=14.4\,\text{kW}\ .$$

50. (a) From $E = NQ = (M_{\text{sam}}/4m_p)Q$ we get the energy per kilogram of hydrogen consumed:

$$\frac{E}{M_{\text{sam}}}=\frac{Q}{4m_p}=\frac{(26.2\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{4(1.67\times10^{-27}\text{ kg})}=6.3\times10^{14}\text{ J/kg}\ .$$

(b) Keeping in mind that a watt is a joule per second, the rate is

$$\frac{dm}{dt}=\frac{3.9\times10^{26}\text{ W}}{6.3\times10^{14}\text{ J/kg}}=6.2\times10^{11}\text{ kg/s}\ .$$

This agrees with the computation shown in Sample Problem — "Consumption rate of hydrogen in the Sun."

(c) From the Einstein relation $E = Mc^2$ we get $P = dE/dt = c^2 dM/dt$, or

$$\frac{dM}{dt}=\frac{P}{c^2}=\frac{3.9\times10^{26}\text{ W}}{(3.0\times10^{8}\text{ m/s})^2}=4.3\times10^{9}\text{ kg/s}\ .$$

(d) This finding, that $dm/dt > dM/dt$, is in large part due to the fact that, as the protons are consumed, their mass is mostly turned into alpha particles (helium), which remain in the Sun.

(e) The time to lose 0.10% of its total mass is

$$t=\frac{0.0010\,M}{dM/dt}=\frac{(0.0010)(2.0\times10^{30}\text{ kg})}{(4.3\times10^{9}\text{ kg/s})(3.15\times10^{7}\text{ s/y})}=1.5\times10^{10}\text{ y}\ .$$

51. Since plutonium has $Z = 94$ and uranium has $Z = 92$, we see that (to conserve charge) two electrons must be emitted so that the nucleus can gain a $+2e$ charge. In the beta decay processes described in Chapter 42, electrons and neutrinos are emitted. The reaction series is as follows:

$$^{238}\text{U}+\text{n}\rightarrow\,^{239}\text{Np}+\,^{239}\text{U}+e+\nu$$
$$^{239}\text{Np}\rightarrow\,^{239}\text{Pu}+e+\nu$$

52. Conservation of energy gives $Q = K_\alpha + K_n$, and conservation of linear momentum (due to the assumption of negligible initial velocities) gives $|p_\alpha| = |p_n|$. We can write the classical formula for kinetic energy in terms of momentum:

$$K=\frac{1}{2}mv^2=\frac{p^2}{2m}$$

which implies that $K_n = (m_\alpha/m_n)K_\alpha$.

(a) Consequently, conservation of energy and momentum allows us to solve for kinetic energy of the alpha particle, which results from the fusion:

$$K_\alpha = \frac{Q}{1+(m_\alpha / m_n)} = \frac{17.59\,\text{MeV}}{1+(4.0015\,\text{u}/1.008665\,\text{u})} = 3.541\,\text{MeV}$$

where we have found the mass of the alpha particle by subtracting two electron masses from the ^{4}He mass (quoted several times in this Chapter 42).

(b) Then, $K_n = Q - K_\alpha$ yields 14.05 MeV for the neutron kinetic energy.

53. At T = 300 K, the average kinetic energy of the neutrons is (using Eq. 20-24)

$$K_{\text{avg}} = \frac{3}{2}KT = \frac{3}{2}(8.62\times10^{-5}\ \text{eV/K})(300\ \text{K}) \approx 0.04\ \text{eV}.$$

54. First, we figure out the mass of U-235 in the sample (assuming "3.0%" refers to the proportion by weight as opposed to proportion by number of atoms):

$$\begin{aligned} M_{\text{U}-235} &= (3.0\%)M_{\text{sam}}\left(\frac{(97\%)m_{238}+(3.0\%)m_{235}}{(97\%)m_{238}+(3.0\%)m_{235}+2m_{16}}\right) \\ &= (0.030)(1000\ \text{g})\left(\frac{0.97(238)+0.030(235)}{0.97(238)+0.030(235)+2(16.0)}\right) \\ &= 26.4\ \text{g}. \end{aligned}$$

Next, the number of ^{235}U nuclei is

$$N_{235} = \frac{(26.4\ \text{g})(6.02\times10^{23}/\text{mol})}{235\ \text{g/mol}} = 6.77\times10^{22}.$$

If all the U-235 nuclei fission, the energy release (using the result of Eq. 43-6) is

$$N_{235}Q_{\text{fission}} = (6.77\times10^{22})(200\ \text{MeV}) = 1.35\times10^{25}\ \text{MeV} = 2.17\times10^{12}\ \text{J}.$$

Keeping in mind that a watt is a joule per second, the time that this much energy can keep a 100-W lamp burning is found to be

$$t = \frac{2.17\times10^{12}\ \text{J}}{100\ \text{W}} = 2.17\times10^{10}\ \text{s} \approx 690\ \text{y}.$$

If we had instead used the Q = 208 MeV value from Sample Problem — "Q value in a fission of uranium-235," then our result would have been 715 y, which perhaps suggests that our result is meaningful to just one significant figure ("roughly 700 years").

55. (a) From $\rho_H = 0.35\rho = n_p m_p$, we get the proton number density n_p:

$$n_p = \frac{0.35\rho}{m_p} = \frac{(0.35)(1.5\times10^5 \text{ kg/m}^3)}{1.67\times10^{-27}\text{ kg}} = 3.1\times10^{31}\text{ m}^{-3}.$$

(b) From Chapter 19 (see Eq. 19-9), we have

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.01\times10^5 \text{ Pa}}{(1.38\times10^{-23}\text{ J/K})(273\text{ K})} = 2.68\times10^{25}\text{ m}^{-3}$$

for an ideal gas under "standard conditions." Thus,

$$\frac{n_p}{(N/V)} = \frac{3.14\times10^{31}\text{ m}^{-3}}{2.44\times10^{25}\text{ m}^{-3}} = 1.2\times10^6 .$$

56. (a) Rather than use $P(v)$ as it is written in Eq. 19-27, we use the more convenient nK expression given in Problem 43-34. The $n(K)$ expression can be derived from Eq. 19-27, but we do not show that derivation here. To find the most probable energy, we take the derivative of $n(K)$ and set the result equal to zero:

$$\left.\frac{dn(K)}{dK}\right|_{K=K_p} = \frac{1.13n}{(kT)^{3/2}}\left(\frac{1}{2K^{1/2}} - \frac{K^{3/2}}{kT}\right)e^{-K/kT}\bigg|_{K=K_p} = 0,$$

which gives $K_p = \frac{1}{2}kT$. Specifically, for $T = 1.5\times10^7$ K we find

$$K_p = \frac{1}{2}kT = \frac{1}{2}(8.62\times10^{-5}\text{ eV/K})(1.5\times10^7\text{ K}) = 6.5\times10^2\text{ eV}$$

or 0.65 keV, in good agreement with Fig. 43-10.

(b) Equation 19-35 gives the most probable speed in terms of the molar mass M, and indicates its derivation. Since the mass m of the particle is related to M by the Avogadro constant, then using Eq. 19-7,

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2RT}{mN_A}} = \sqrt{\frac{2kT}{m}} .$$

With $T = 1.5\times10^7$ K and $m = 1.67\times10^{-27}$ kg, this yields $v_p = 5.0\times10^5$ m/s.

(c) The corresponding kinetic energy is

$$K_{v,p} = \frac{1}{2}mv_p^2 = \frac{1}{2}m\left(\sqrt{\frac{2kT}{m}}\right)^2 = kT$$

which is twice as large as that found in part (a). Thus, at $T = 1.5 \times 10^7$ K we have $K_{v,p}$ = 1.3 keV, which is indicated in Fig. 43-10 by a single vertical line.

57. (a) The mass of each DT pellet is

$$m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi(20\times10^{-6}\,\text{m})^3(200\,\text{kg/m}^3) = 6.7\times10^{-12}\,\text{kg}$$

Since there are equal number of ^2H and ^3H present, we have

$$N_{^2\text{H}} = N_{^3\text{H}} = \frac{mN_A}{M_{^2\text{H}} + M_{^3\text{H}}} = \frac{(6.7\times10^{-12}\,\text{kg})(6.02\times10^{23})}{(0.020\,\text{kg}) + (0.030\,\text{kg})} = 8.07\times10^{14}$$

Each fusion reaction releases 17.59 MeV of energy, with 10% efficiency, the total energy released by the pellet is

$$E = (0.10)(8.07\times10^{14})(17.59\,\text{MeV}) = 1.42\times10^{15}\ \text{MeV} = 227\ \text{J}$$

or about 230 J.

(b) Since 1.0 kg of TNT gives off 4.6 MJ, the TNT equivalent of the pellet is

$$m = \frac{227\ \text{J}}{4.6\times10^6\ \text{J}} = 4.93\times10^{-5}\ \text{kg}.$$

(c) The power generated is

$$P = \left(\frac{dN}{dt}\right)E = (100\,/\,\text{s})(227\ \text{J}) = 2.3\times10^4\ \text{W}$$

58. (a) Equation 19-35 gives the most probable speed in terms of the molar mass M: $v_p = \sqrt{2RT/M}$. With $T = 1 \times 10^8$ K and $M = 2.0 \times 10^{-3}$ kg/mol, this yields

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2(8.314\ \text{J/mol}\cdot\text{K})(108\ \text{K})}{2.0\times10^{-3}\ \text{kg}}} = 9.1\times10^5\,\text{m/s}.$$

(b) The distance moved is $r = v_p\Delta t = (9.1\times10^5\,\text{m/s})(1\times10^{-12}\text{s}) = 9.1\times10^{-7}\,\text{m}$.

Chapter 44

1. By charge conservation, it is clear that reversing the sign of the pion means we must reverse the sign of the muon. In effect, we are replacing the charged particles by their antiparticles. Less obvious is the fact that we should now put a "bar" over the neutrino (something we should also have done for some of the reactions and decays discussed in Chapters 42 and 43, except that we had not yet learned about antiparticles, which are usually denoted with a "bar." The decay of the negative pion is $\pi^- \to \mu^- + \bar{\nu}$. A subscript can be added to the antineutrino to clarify what "type" it is.

2. Since the density of water is ρ = 1000 kg/m^3 = 1 kg/L, then the total mass of the pool is $\rho\mathcal{V}$ = 4.32 × 10^5 kg, where $\mathcal{V}$ is the given volume. Now, the fraction of that mass made up by the protons is 10/18 (by counting the protons versus total nucleons in a water molecule). Consequently, if we ignore the effects of neutron decay (neutrons can beta decay into protons) in the interest of making an order-of-magnitude calculation, then the number of particles susceptible to decay via this $T_{1/2} = 10^{32}$ y half-life is

$$N = \frac{(10/18)M_{\text{pool}}}{m_p} = \frac{(10/18)\left(4.32\times10^5 \text{ kg}\right)}{1.67\times10^{-27} \text{ kg}} = 1.44\times10^{32}.$$

Using Eq. 42-20, we obtain

$$R = \frac{N\ln 2}{T_{1/2}} = \frac{\left(1.44\times10^{32}\right)\ln 2}{10^{32}\,\text{y}} \approx 1\,\text{decay/y}.$$

3. The total rest energy of the electron-positron pair is

$$E = m_ec^2 + m_ec^2 = 2m_ec^2 = 2(0.511 \text{ MeV}) = 1.022 \text{ MeV}.$$

With two gamma-ray photons produced in the annihilation process, the wavelength of each photon is (using $hc = 1240\,\text{eV}\cdot\text{nm}$)

$$\lambda = \frac{hc}{E/2} = \frac{1240\text{ eV}\cdot\text{nm}}{0.511\times10^6\text{eV}} = 2.43\times10^{-3}\text{ nm} = 2.43\text{ pm}.$$

4. Conservation of momentum requires that the gamma ray particles move in opposite directions with momenta of the same magnitude. Since the magnitude p of the momentum of a gamma ray particle is related to its energy by $p = E/c$, the particles have the same energy E. Conservation of energy yields $m_\pi c^2 = 2E$, where m_π is the mass of a neutral pion. The rest energy of a neutral pion is $m_\pi c^2$ = 135.0 MeV, according to Table

44-4. Hence, E = (135.0 MeV)/2 = 67.5 MeV. We use $hc = 1240\,\text{eV}\cdot\text{nm}$ to obtain the wavelength of the gamma rays:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{67.5\times10^{6}\,\text{eV}} = 1.84\times10^{-5}\ \text{nm} = 18.4\ \text{fm}.$$

5. We establish a ratio, using Eq. 22-4 and Eq. 14-1:

$$\frac{F_{\text{gravity}}}{F_{\text{electric}}} = \frac{Gm_e^2/r^2}{ke^2/r^2} = \frac{4\pi\varepsilon_0 G m_e^2}{e^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(9.11\times10^{-31}\ \text{kg}\right)^2}{\left(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}$$
$$= 2.4\times10^{-43}.$$

Since $F_{\text{gravity}} \ll F_{\text{electric}}$, we can neglect the gravitational force acting between particles in a bubble chamber.

6. (a) Conservation of energy gives

$$Q = K_2 + K_3 = E_1 - E_2 - E_3$$

where E refers here to the *rest* energies (mc^2) instead of the total energies of the particles. Writing this as

$$K_2 + E_2 - E_1 = -(K_3 + E_3)$$

and squaring both sides yields

$$K_2^2 + 2K_2E_2 - 2K_2E_1 + \left(E_1 - E_2\right)^2 = K_3^2 + 2K_3E_3 + E_3^2.$$

Next, conservation of linear momentum (in a reference frame where particle 1 was at rest) gives $|p_2| = |p_3|$ (which implies $(p_2c)^2 = (p_3c)^2$). Therefore, Eq. 37-54 leads to

$$K_2^2 + 2K_2E_2 = K_3^2 + 2K_3E_3$$

which we subtract from the above expression to obtain

$$-2K_2E_1 + \left(E_1 - E_2\right)^2 = E_3^2.$$

This is now straightforward to solve for K_2 and yields the result stated in the problem.

(b) Setting $E_3 = 0$ in

$$K_2 = \frac{1}{2E_1}\left[\left(E_1 - E_2\right)^2 - E_3^2\right]$$

and using the rest energy values given in Table 44-1 readily gives the same result for K_μ as computed in Sample Problem – "Momentum and kinetic energy in a pion decay."

7. Table 44-4 gives the rest energy of each pion as 139.6 MeV. The magnitude of the momentum of each pion is p_π = (358.3 MeV)/c. We use the relativistic relationship between energy and momentum (Eq. 37-54) to find the total energy of each pion:

$$E_\pi = \sqrt{(p_\pi c)^2 + (m_\pi c^2)^2} = \sqrt{(358.3\text{ MeV})^2 + (139.6\text{ Mev})} = 384.5\text{ MeV}.$$

Conservation of energy yields

$$m_\rho c^2 = 2E_\pi = 2(384.5\text{ MeV}) = 769\text{ MeV}.$$

8. (a) In SI units, the kinetic energy of the positive tau particle is

$$K = (2200\text{ MeV})(1.6 \times 10^{-13}\text{ J/MeV}) = 3.52 \times 10^{-10}\text{ J}.$$

Similarly, $mc^2 = 2.85 \times 10^{-10}$ J for the positive tau. Equation 37-54 leads to the relativistic momentum:

$$p = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{1}{2.998 \times 10^8\text{ m/s}}\sqrt{\left(3.52\times10^{-10}\text{ J}\right)^2 + 2\left(3.52\times10^{-10}\text{ J}\right)\left(2.85\times10^{-10}\text{ J}\right)}$$

which yields $p = 1.90 \times 10^{-18}$ kg·m/s.

(b) The radius should be calculated with the relativistic momentum:

$$r = \frac{\gamma mv}{|q|B} = \frac{p}{eB}$$

where we use the fact that the positive tau has charge $e = 1.6 \times 10^{-19}$ C. With B = 1.20 T, this yields r = 9.90 m.

9. From Eq. 37-48, the Lorentz factor would be

$$\gamma = \frac{E}{mc^2} = \frac{1.5\times10^6\text{ eV}}{20\text{ eV}} = 75000.$$

Solving Eq. 37-8 for the speed, we find

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \Rightarrow v = c\sqrt{1-\frac{1}{\gamma^2}}$$

which implies that the difference between v and c is

$$c - v = c\left(1 - \sqrt{1 - \frac{1}{\gamma^2}}\right) \approx c\left(1 - \left(1 - \frac{1}{2\gamma^2} + \cdots\right)\right)$$

where we use the binomial expansion (see Appendix E) in the last step. Therefore,

$$c - v \approx c\left(\frac{1}{2\gamma^2}\right) = (299792458\,\text{m/s})\left(\frac{1}{2(75000)^2}\right) = 0.0266\,\text{m/s} \approx 2.7\,\text{cm/s}.$$

10. From Eq. 37-52, the Lorentz factor is

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{80\text{ MeV}}{135\text{ MeV}} = 1.59.$$

Solving Eq. 37-8 for the speed, we find

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \Rightarrow v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

which yields $v = 0.778c$ or $v = 2.33 \times 10^8$ m/s. Now, in the reference frame of the laboratory, the lifetime of the pion is not the given τ value but is "dilated." Using Eq. 37-9, the time in the lab is

$$t = \gamma\tau = (1.59)\left(8.3 \times 10^{-17}\text{ s}\right) = 1.3 \times 10^{-16}\text{ s}.$$

Finally, using Eq. 37-10, we find the distance in the lab to be

$$x = vt = \left(2.33 \times 10^8\text{ m/s}\right)\left(1.3 \times 10^{-16}\text{ s}\right) = 3.1 \times 10^{-8}\text{ m}.$$

11. **THINK** The conservation laws we shall examine are associated with energy, momentum, angular momentum, charge, baryon number, and the three lepton numbers.

EXPRESS In all particle interactions, the net lepton number for each family (L_e for electron, L_μ for muon, and L_τ for tau) is separately conserved. Conservation of baryon number implies that a process cannot occur if the net baryon number is changed.

ANALYZE (a) For the process $\mu^- \to e^- + \nu_\mu$, the rest energy of the muon is 105.7 MeV, the rest energy of the electron is 0.511 MeV, and the rest energy of the neutrino is zero. Thus, the total rest energy before the decay is greater than the total rest energy after. The excess energy can be carried away as the kinetic energies of the decay products and energy can be conserved. Momentum is conserved if the electron and neutrino move away from the decay in opposite directions with equal magnitudes of momenta. Since the orbital angular momentum is zero, we consider only spin angular momentum. All the particles have spin $\hbar/2$. The total angular momentum after the decay must be either $\hbar$ (if the spins are aligned) or zero (if the spins are anti-aligned). Since the spin before the decay is $\hbar/2$ angular momentum cannot be conserved. The muon has charge $-e$, the electron has charge $-e$, and the neutrino has charge zero, so the total charge before the decay is $-e$ and the total charge after is $-e$. Charge is conserved. All particles have baryon number zero, so baryon number is conserved. The muon lepton number of the muon is +1, the muon lepton number of the muon neutrino is +1, and the muon lepton number of the electron is 0. Muon lepton number is conserved. The electron lepton numbers of the muon and muon neutrino are 0 and the electron lepton number of the electron is +1. Electron lepton number is not conserved. The laws of conservation of angular momentum and electron lepton number are not obeyed and this decay does not occur.

(b) We analyze the decay $\mu^- \to e^+ + \nu_e + \bar{\nu}_\mu$ in the same way. We find that charge and the muon lepton number L_μ are not conserved.

(c) For the process $\mu^+ \to \pi^+ + \nu_\mu$, we find that energy cannot be conserved because the mass of muon is less than the mass of a pion. Also, muon lepton number L_μ is not conserved.

LEARN In all three processes considered, since the initial particle is stationary, the question associated with energy conservation amounts to asking whether the initial mass energy is sufficient to produce the mass energies and kinetic energies of the decayed products.

12. (a) Noting that there are two positive pions created (so, in effect, its decay products are doubled), then we count up the electrons, positrons, and neutrinos: $2\mathrm{e}^+ + \mathrm{e}^- + 5\nu + 4\bar{\nu}$.

(b) The final products are all leptons, so the baryon number of A_2^+ is zero. Both the pion and rho meson have integer-valued spins, so A_2^+ is a boson.

(c) A_2^+ is also a meson.

(d) As stated in (b), the baryon number of A_2^+ is zero.

13. The formula for T_z as it is usually written to include strange baryons is $T_z = q - (S + B)/2$. Also, we interpret the symbol q in the T_z formula in terms of elementary charge units; this is how q is listed in Table 44-3. In terms of charge q as we have used it in previous chapters, the formula is

$$T_z = \frac{q}{e} - \frac{1}{2}(B + S).$$

For instance, $T_z = +\frac{1}{2}$ for the proton (and the neutral Xi) and $T_z = -\frac{1}{2}$ for the neutron (and the negative Xi). The baryon number B is +1 for all the particles in Fig. 44-4(a). Rather than use a sloping axis as in Fig. 44-4 (there it is done for the q values), one reproduces (if one uses the "corrected" formula for T_z mentioned above) exactly the same pattern using regular rectangular axes (T_z values along the horizontal axis and Y values along the vertical) with the neutral lambda and sigma particles situated at the origin.

14. (a) From Eq. 37-50,

$$\begin{aligned} Q &= -\Delta mc^2 = (m_{\Sigma^+} + m_{K^+} - m_{\pi^+} - m_p)c^2 \\ &= 1189.4\,\text{MeV} + 493.7\,\text{MeV} - 139.6\,\text{MeV} - 938.3\,\text{MeV} \\ &= 605\,\text{MeV}. \end{aligned}$$

(b) Similarly,

$$\begin{aligned} Q &= -\Delta mc^2 = (m_{\Lambda^0} + m_{\pi^0} - m_{K^-} - m_p)c^2 \\ &= 1115.6\,\text{MeV} + 135.0\,\text{MeV} - 493.7\,\text{MeV} - 938.3\,\text{MeV} \\ &= -181\,\text{MeV}. \end{aligned}$$

15. (a) The lambda has a rest energy of 1115.6 MeV, the proton has a rest energy of 938.3 MeV, and the kaon has a rest energy of 493.7 MeV. The rest energy before the decay is less than the total rest energy after, so energy cannot be conserved. Momentum can be conserved. The lambda and proton each have spin $\hbar/2$ and the kaon has spin zero, so angular momentum can be conserved. The lambda has charge zero, the proton has charge $+e$, and the kaon has charge $-e$, so charge is conserved. The lambda and proton each have baryon number +1, and the kaon has baryon number zero, so baryon number is conserved. The lambda and kaon each have strangeness –1 and the proton has strangeness zero, so strangeness is conserved. Only energy cannot be conserved.

(b) The omega has a rest energy of 1680 MeV, the sigma has a rest energy of 1197.3 MeV, and the pion has a rest energy of 135 MeV. The rest energy before the decay is greater than the total rest energy after, so energy can be conserved. Momentum can be conserved. The omega and sigma each have spin $\hbar/2$ and the pion has spin zero, so angular momentum can be conserved. The omega has charge $-e$, the sigma has charge $-e$, and the pion has charge zero, so charge is conserved. The omega and sigma have baryon number +1 and the pion has baryon number 0, so baryon number is conserved. The omega has strangeness –3, the sigma has strangeness –1, and the pion has strangeness zero, so strangeness is not conserved.

(c) The kaon and proton can bring kinetic energy to the reaction, so energy can be conserved even though the total rest energy after the collision is greater than the total rest energy before. Momentum can be conserved. The proton and lambda each have spin $\hbar/2$ and the kaon and pion each have spin zero, so angular momentum can be conserved. The kaon has charge $-e$, the proton has charge $+e$, the lambda has charge zero, and the pion has charge +*e*, so charge is not conserved. The proton and lambda each have baryon number +1, and the kaon and pion each have baryon number zero; baryon number is conserved. The kaon has strangeness –1, the proton and pion each have strangeness zero, and the lambda has strangeness –1, so strangeness is conserved. Only charge is not conserved.

16. To examine the conservation laws associated with the proposed reaction $\mathrm{p}+\overline{\mathrm{p}} \to \Lambda^0 + \Sigma^+ + e^-$, we make use of particle properties found in Tables 44-3 and 44-4.

(a) With $q(\mathrm{p}) = +1, q(\overline{\mathrm{p}}) = -1, \quad q(\Lambda^0) = 0, \quad q(\Sigma^+) = +1,$ and $q(e^-) = -1$, we have $1+(-1) = 0+1+(-1)$. Thus, the process conserves charge.

(b) With $B(\mathrm{p}) = +1, B(\overline{\mathrm{p}}) = -1, \quad B(\Lambda^0) = 1, \quad B(\Sigma^+) = +1,$ and $B(e^-) = 0$, we have $1+(-1) \neq 1+1+0$. Thus, the process does not conserve baryon number.

(c) With $L_e(\mathrm{p}) = L_e(\overline{\mathrm{p}}) = 0, \quad L_e(\Lambda^0) = L_e(\Sigma^+) = 0,$ and $L_e(e^-) = 1$, we have $0+0 \neq 0+0+1$, so the process does not conserve electron lepton number.

(d) All the particles on either side of the reaction equation are fermions with $s = 1/2$. Therefore, $(1/2)+(1/2) \neq (1/2)+(1/2)+(1/2)$ and the process does not conserve spin angular momentum.

(e) With $S(\mathrm{p}) = S(\overline{\mathrm{p}}) = 0, \quad S(\Lambda^0) = 1, \quad S(\Sigma^+) = +1,$ and $S(e^-) = 0$, we have $0+0 \neq 1+1+0$, so the process does not conserve strangeness.

(f) The process does conserve muon lepton number since all the particles involved have muon lepton number of zero.

17. To examine the conservation laws associated with the proposed decay process $\Xi^- \to \pi^- + \mathrm{n} + \mathrm{K}^- + \mathrm{p}$, we make use of particle properties found in Tables 44-3 and 44-4.

(a) With $q(\Xi^-) = -1$, $q(\pi^-) = -1, q(\mathrm{n}) = 0, q(\mathrm{K}^-) = -1,$ and $q(\mathrm{p}) = +1,$ we have $-1 = -1+0+(-1)+1$. Thus, the process conserves charge.

(b) Since $B(\Xi^-) = +1$, $B(\pi^-) = 0, \quad B(\mathrm{n}) = +1, B(\mathrm{K}^-) = 0,$ and $B(\mathrm{p}) = +1,$ we have $+1 \neq 0+1+0+1 = 2$. Thus, the process does not conserve baryon number.

(c) Ξ^-, n and p are fermions with $s = 1/2$, while π^- and K^- are mesons with spin zero. Therefore, $+1/2 \neq 0+(1/2)+0+(1/2)$ and the process does not conserve spin angular momentum.

(d) Since $S(\Xi^-) = -2$, $S(\pi^-) = 0,$ $S(\mathrm{n}) = 0, S(\mathrm{K}^-) = -1,$ and $S(\mathrm{p}) = 0,$ we have $-2 \neq 0 + 0 + (-1) + 0,$ so the process does not conserve strangeness.

18. (a) Referring to Tables 44-3 and 44-4, we find that the strangeness of K^0 is +1, while it is zero for both π^+ and π^-. Consequently, strangeness is not conserved in this decay; $\mathrm{K}^0 \to \pi^+ + \pi^-$ does not proceed via the strong interaction.

(b) The strangeness of each side is –1, which implies that the decay is governed by the strong interaction.

(c) The strangeness or Λ^0 is –1 while that of p + π^- is zero, so the decay is not via the strong interaction.

(d) The strangeness of each side is –1; it proceeds via the strong interaction.

19. For purposes of deducing the properties of the antineutron, one may cancel a proton from each side of the reaction and write the equivalent reaction as $\pi^+ \to \mathrm{p} + \overline{\mathrm{n}}$.

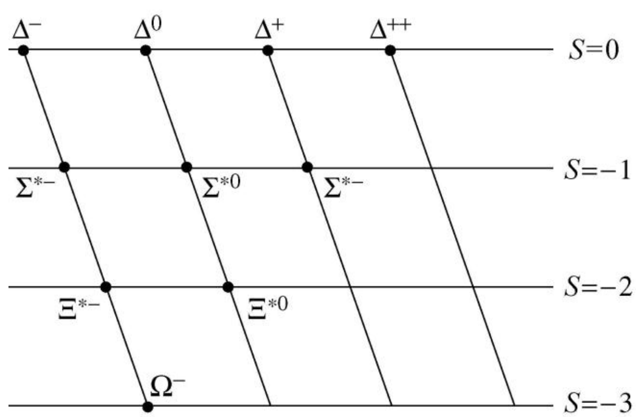
Particle properties can be found in Tables 44-3 and 44-4. The pion and proton each have charge +*e*, so the antineutron must be neutral. The pion has baryon number zero (it is a meson) and the proton has baryon number +1, so the baryon number of the antineutron must be –1. The pion and the proton each have strangeness zero, so the strangeness of the antineutron must also be zero. In summary, for the antineutron,

(a) $q = 0$,

(b) $B = -1$,

(c) and $S = 0$.

20. If we were to use regular rectangular axes, then this would appear as a right triangle. Using the sloping q axis as the problem suggests, it is similar to an "upside down" equilateral triangle as we show below.



The leftmost slanted line is for the –1 charge, and the rightmost slanted line is for the +2 charge.

21. (a) As far as the conservation laws are concerned, we may cancel a proton from each side of the reaction equation and write the reaction as $p \to \Lambda^0 + x$. Since the proton and the lambda each have a spin angular momentum of $\hbar/2$, the spin angular momentum of x must be either zero or $\hbar$. Since the proton has charge +*e* and the lambda is neutral, x must have charge +*e*. Since the proton and the lambda each have a baryon number of +1, the baryon number of x is zero. Since the strangeness of the proton is zero and the strangeness of the lambda is –1, the strangeness of x is +1. We take the unknown particle to be a spin zero meson with a charge of +*e* and a strangeness of +1. Look at Table 44-4 to identify it as a K^+ particle.

(b) Similar analysis tells us that x is a spin-$\frac{1}{2}$ antibaryon (B = –1) with charge and strangeness both zero. Inspection of Table 44-3 reveals that it is an antineutron.

(c) Here x is a spin-0 (or spin-1) meson with charge zero and strangeness +1. According to Table 44-4, it could be a K^0 particle.

22. Conservation of energy (see Eq. 37-47) leads to

$$\begin{aligned} K_f &= -\Delta mc^2 + K_i = (m_{\Sigma^-} - m_{\pi^-} - m_n)c^2 + K_i \\ &= 1197.3\,\text{MeV} - 139.6\,\text{MeV} - 939.6\,\text{MeV} + 220\,\text{MeV} \\ &= 338\,\text{MeV}. \end{aligned}$$

23. (a) From Eq. 37-50,

$$\begin{aligned} Q &= -\Delta mc^2 = (m_{\Lambda^0} - m_p - m_{\pi^-})c^2 \\ &= 1115.6\,\text{MeV} - 938.3\,\text{MeV} - 139.6\,\text{MeV} = 37.7\,\text{MeV}. \end{aligned}$$

(b) We use the formula obtained in Problem 44-6 (where it should be emphasized that E is used to mean the rest energy, not the total energy):

$$\begin{aligned} K_p &= \frac{1}{2E_\Lambda}\left[(E_\Lambda - E_p)^2 - E_\pi^2\right] \\ &= \frac{(1115.6\,\text{MeV} - 938.3\,\text{MeV})^2 - (139.6\,\text{MeV})^2}{2(1115.6\,\text{MeV})} = 5.35\,\text{MeV}. \end{aligned}$$

(c) By conservation of energy,

$$K_{\pi^-} = Q - K_p = 37.7\,\text{MeV} - 5.35\,\text{MeV} = 32.4\,\text{MeV}.$$

24. From $\gamma = 1 + K/mc^2$ (see Eq. 37-52) and $v = \beta c = c\sqrt{1-\gamma^{-2}}$ (see Eq. 37-8), we get

$$v = c\sqrt{1-\left(1+\frac{K}{mc^2}\right)^{-2}}.$$

(a) Therefore, for the Σ^{*0} particle,

$$v = (2.9979\times 10^8\ \text{m/s})\sqrt{1-\left(1+\frac{1000\,\text{MeV}}{1385\,\text{MeV}}\right)^{-2}} = 2.4406\times 10^8\ \text{m/s}.$$

For Σ^0,

$$v' = (2.9979\times 10^8\ \text{m/s})\sqrt{1-\left(1+\frac{1000\,\text{MeV}}{1192.5\,\text{MeV}}\right)^{-2}} = 2.5157\times 10^8\ \text{m/s}.$$

Thus Σ^0 moves faster than Σ^{*0}.

(b) The speed difference is

$$\Delta v = v' - v = (2.5157 - 2.4406)(10^8\ \text{m/s}) = 7.51\times 10^6\ \text{m/s}.$$

25. (a) We indicate the antiparticle nature of each quark with a "bar" over it. Thus, $\bar{\text{u}}\bar{\text{u}}\bar{\text{d}}$ represents an antiproton.

(b) Similarly, $\bar{\text{u}}\bar{\text{d}}\bar{\text{d}}$ represents an antineutron.

26. (a) The combination ddu has a total charge of $\left(-\frac{1}{3}-\frac{1}{3}+\frac{2}{3}\right)=0$, and a total strangeness of zero. From Table 44-3, we find it to be a neutron (n).

(b) For the combination uus, we have $Q = +\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=1$ and $S = 0 + 0 - 1 = -1$. This is the Σ^+ particle.

(c) For the quark composition ssd, we have $Q = -\frac{1}{3}-\frac{1}{3}-\frac{1}{3}=-1$ and $S = -1 - 1 + 0 = -2$. This is a Ξ^-.

27. The meson $\bar{\text{K}}^0$ is made up of a quark and an anti-quark, with net charge zero and strangeness $S=-1$. The quark with $S=-1$ is s. By charge neutrality condition, the anti-quark must be $\bar{\text{d}}$. Therefore, the constituents of $\bar{\text{K}}^0$ are s and $\bar{\text{d}}$.

28. (a) Using Table 44-3, we find $q = 0$ and $S = -1$ for this particle (also, $B = 1$, since that is true for all particles in that table). From Table 44-5, we see it must therefore contain a strange quark (which has charge –1/3), so the other two quarks must have charges to add

to zero. Assuming the others are among the lighter quarks (none of them being an anti-quark, since B = 1), then the quark composition is sud.

(b) The reasoning is very similar to that of part (a). The main difference is that this particle must have two strange quarks. Its quark combination turns out to be uss.

29. (a) The combination ssu has a total charge of $\left(-\frac{1}{3}-\frac{1}{3}+\frac{2}{3}\right)=0$, and a total strangeness of – 2. From Table 44-3, we find it to be the Ξ^0 particle.

(b) The combination dds has a total charge of $\left(-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}\right)=-1$, and a total strangeness of –1. From Table 44-3, we find it to be the Σ^- particle.

30. **THINK** A baryon is made up of three quarks.

EXPRESS The quantum numbers of the up, down, and strange quarks are (see Table 44-5) as follows:

Particle	Charge *q*	Strangeness S	Baryon number B
Up (u)	+2/3	0	+1/3
Down (d)	–1/3	0	+1/3
Strange (s)	–1/3	–1	+1/3

ANALYZE (a) To obtain a strangeness of –2, two of them must be s quarks. Each of these has a charge of $-e/3$, so the sum of their charges is $-2e/3$. To obtain a total charge of e, the charge on the third quark must be $5e/3$. There is no quark with this charge, so the particle cannot be constructed. In fact, such a particle has never been observed.

(b) Again the particle consists of three quarks (and no antiquarks). To obtain a strangeness of zero, none of them may be s quarks. We must find a combination of three u and d quarks with a total charge of $2e$. The only such combination consists of three u quarks.

LEARN The baryon with three u quarks is Δ^{++}.

31. First, we find the speed of the receding galaxy from Eq. 37-31:

$$\beta=\frac{1-(f/f_0)^2}{1+(f/f_0)^2}=\frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2}$$
$$=\frac{1-(590.0\text{ nm}/602.0\text{ nm})^2}{1+(590.0\text{ nm}/602.0\text{ nm})^2}=0.02013$$

where we use $f = c/\lambda$ and $f_0 = c/\lambda_0$. Then from Eq. 44-19,

$$r=\frac{v}{H}=\frac{\beta c}{H}=\frac{(0.02013)\left(2.998\times10^{8}\text{ m/s}\right)}{0.0218\text{ m/s}\cdot\text{ly}}=2.77\times10^{8}\text{ ly}.$$

32. Since

$$\lambda=\lambda_0\sqrt{\frac{1+\beta}{1-\beta}}=2\lambda_0\quad\Rightarrow\quad\sqrt{\frac{1+\beta}{1-\beta}}=2,$$

the speed of the receding galaxy is $v=\beta c=3c/5$. Therefore, the distance to the galaxy when the light was emitted is

$$r=\frac{v}{H}=\frac{\beta c}{H}=\frac{(3/5)c}{H}=\frac{(0.60)(2.998\times10^{8}\text{ m/s})}{0.0218\text{ m/s}\cdot\text{ly}}=8.3\times10^{9}\text{ ly}.$$

33. We apply Eq. 37-36 for the Doppler shift in wavelength:

$$\frac{\Delta\lambda}{\lambda}=\frac{v}{c}$$

where v is the recessional speed of the galaxy. We use Hubble's law to find the recessional speed: $v = Hr$, where r is the distance to the galaxy and H is the Hubble constant $\left(21.8\times10^{-3}\,\frac{\text{m}}{\text{s}\cdot\text{ly}}\right)$. Thus,

$$v=\left(21.8\times10^{-3}\text{ m/s}\cdot\text{ly}\right)\left(2.40\times10^{8}\text{ ly}\right)=5.23\times10^{6}\text{ m/s}$$

and

$$\Delta\lambda=\frac{v}{c}\lambda=\left(\frac{5.23\times10^{6}\text{ m/s}}{3.00\times10^{8}\text{ m/s}}\right)(656.3\text{ nm})=11.4\text{ nm}.$$

Since the galaxy is receding, the observed wavelength is longer than the wavelength in the rest frame of the galaxy. Its value is

$$656.3\text{ nm} + 11.4\text{ nm} = 667.7\text{ nm} \approx 668\text{ nm}.$$

34. (a) Using Hubble's law given in Eq. 44-19, the speed of recession of the object is

$$v=Hr=\left(0.0218\text{ m/s}\cdot\text{ly}\right)\left(1.5\times10^{4}\text{ly}\right)=327\text{ m/s}.$$

Therefore, the extra distance of separation one year from now would be

$$d=vt=(327\text{ m/s})(365\text{ d})(86400\text{ s/d})=1.0\times10^{10}\text{ m}.$$

(b) The speed of the object is $v = 327 \text{ m/s} \approx 3.3\times10^2$ m/s.

35. Letting $v = Hr = c$, we obtain

$$r = \frac{c}{H} = \frac{3.0\times10^8 \text{ m/s}}{0.0218 \text{ m/s}\cdot\text{ly}} = 1.376\times10^{10} \text{ ly} \approx 1.4\times10^{10} \text{ ly}.$$

36. From $F_{\text{grav}} = GMm/r^2 = mv^2/r$ we find $M \propto v^2$. Thus, the mass of the Sun would be

$$M_s' = \left(\frac{v_{\text{Mercury}}}{v_{\text{Pluto}}}\right)^2 M_s = \left(\frac{47.9 \text{ km/s}}{4.74 \text{ km/s}}\right)^2 M_s = 102 M_s.$$

37. (a) For the universal microwave background, Wien's law leads to

$$T = \frac{2898\,\mu\text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{2898 \text{ mm}\cdot\text{K}}{1.1 \text{ mm}} = 2.6 \text{ K}.$$

(b) At "decoupling" (when the universe became approximately "transparent"),

$$\lambda_{\max} = \frac{2898\,\mu\text{m}\cdot\text{K}}{T} = \frac{2898\,\mu\text{m}\cdot\text{K}}{2970\text{K}} = 0.976\ \mu\text{m} = 976 \text{ nm}.$$

38. (a) We substitute $\lambda = (2898\ \mu\text{m}\cdot\text{K})/T$ into the expression:

$$E = hc/\lambda = (1240 \text{ eV}\cdot\text{nm})/\lambda.$$

First, we convert units:

$$2898\ \mu\text{m}\cdot\text{K} = 2.898\times10^6 \text{ nm}\cdot\text{K and } 1240 \text{ eV}\cdot\text{nm} = 1.240\times10^{-3} \text{ MeV}\cdot\text{nm}.$$

Thus,

$$E = \frac{(1.240\times10^{-3} \text{ MeV}\cdot\text{nm})T}{2.898\times10^6 \text{ nm}\cdot\text{K}} = (4.28\times10^{-10} \text{ MeV/K})T.$$

(b) The minimum energy required to create an electron-positron pair is twice the rest energy of an electron, or 2(0.511 MeV) = 1.022 MeV. Hence,

$$T = \frac{E}{4.28\times10^{-10} \text{ MeV/K}} = \frac{1.022 \text{ MeV}}{4.28\times10^{-10} \text{ MeV/K}} = 2.39\times10^9 \text{ K}.$$

39. (a) Letting $v(r) = Hr \le v_e = \sqrt{2GM/r}$, we get $M/r^3 \ge H^2/2G$. Thus,

$$\rho = \frac{M}{4\pi r^2/3} = \frac{3}{4\pi}\frac{M}{r^3} \geq \frac{3H^2}{8\pi G}.$$

(b) The density being expressed in H-atoms/m^3 is equivalent to expressing it in terms of $\rho_0 = m_H/\text{m}^3 = 1.67 \times 10^{-27}$ kg/m^3. Thus,

$$\rho = \frac{3H^2}{8\pi G\rho_0}\left(\text{H atoms/m}^3\right) = \frac{3\left(0.0218\,\text{m/s}\cdot\text{ly}\right)^2\left(1.00\,\text{ly}/9.460\times10^{15}\text{ m}\right)^2\left(\text{H atoms/m}^3\right)}{8\pi\left(6.67\times10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2\right)\left(1.67\times10^{-27}\,\text{kg/m}^3\right)}$$
$$= 5.7 \text{ H atoms/m}^3.$$

40. (a) From $f = c/\lambda$ and Eq. 37-31, we get

$$\lambda_0 = \lambda\sqrt{\frac{1-\beta}{1+\beta}} = (\lambda_0 + \Delta\lambda)\sqrt{\frac{1-\beta}{1+\beta}}.$$

Dividing both sides by λ_0 leads to

$$1 = (1+z)\sqrt{\frac{1-\beta}{1+\beta}}$$

where $z = \Delta\lambda/\lambda_0$. We solve for β:

$$\beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \frac{z^2 + 2z}{z^2 + 2z + 2}.$$

(b) Now $z = 4.43$, so

$$\beta = \frac{(4.43)^2 + 2(4.43)}{(4.43)^2 + 2(4.43) + 2} = 0.934 .$$

(c) From Eq. 44-19,

$$r = \frac{v}{H} = \frac{\beta c}{H} = \frac{(0.934)\left(3.0\times10^8\text{ m/s}\right)}{0.0218\,\text{m/s}\cdot\text{ly}} = 1.28\times10^{10}\text{ ly} .$$

41. Using Eq. 39-33, the energy of the emitted photon is

$$E = E_3 - E_2 = -(13.6\text{ eV})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.89\text{ eV}$$

and its wavelength is

$$\lambda_0 = \frac{hc}{E} = \frac{1240\text{ eV}\cdot\text{ nm}}{1.89\text{ eV}} = 6.56\times10^{-7}\text{ m} .$$

Given that the detected wavelength is $\lambda = 3.00\times10^{-3}$ m, we find

$$\frac{\lambda}{\lambda_0} = \frac{3.00\times10^{-3}\ \text{m}}{6.56\times10^{-7}\ \text{m}} = 4.57\times10^{3}.$$

42. (a) From Eq. 41-29, we know that $N_2/N_1 = e^{-\Delta E/kT}$. We solve for ΔE:

$$\Delta E = kT\ln\frac{N_1}{N_2} = \left(8.62\times10^{-5}\ \text{eV/K}\right)\left(2.7\text{K}\right)\ln\left(\frac{1-0.25}{0.25}\right)$$
$$= 2.56\times10^{-4}\ \text{eV} \approx 0.26\ \text{meV}.$$

(b) Using $hc = 1240\text{eV}\cdot\text{nm},$ we get

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\text{eV}\cdot\text{nm}}{2.56\times10^{-4}\text{eV}} = 4.84\times10^{6}\ \text{nm} \approx 4.8\text{mm}.$$

43. **THINK** The radius of the orbit is still given by 1.50×10^{11} km, the original Earth-Sun distance.

EXPRESS The gravitational force on Earth is only due to the mass M within Earth's orbit. If r is the radius of the orbit, R is the radius of the new Sun, and M_S is the mass of the Sun, then

$$M = \left(\frac{r}{R}\right)^3 M_s = \left(\frac{1.50\times10^{11}\ \text{m}}{5.90\times10^{12}\ \text{m}}\right)^3 \left(1.99\times10^{30}\ \text{kg}\right) = 3.27\times10^{25}\ \text{kg}.$$

The gravitational force on Earth is given by GMm/r^2, where m is the mass of Earth and G is the universal gravitational constant. Since the centripetal acceleration is given by v^2/r, where v is the speed of Earth, $GMm/r^2 = mv^2/r$ and

$$v = \sqrt{\frac{GM}{r}}.$$

ANALYZE (a) Substituting the values given, we obtain

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(3.27\times10^{25}\ \text{kg}\right)}{1.50\times10^{11}\ \text{m}}} = 1.21\times10^{2}\ \text{m/s}.$$

(b) The ratio of the speeds is

$$\frac{v}{v_0} = \frac{1.21\times10^{2}\ \text{m/s}}{2.98\times10^{4}\ \text{m/s}} = 0.00405.$$

(c) The period of revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(1.50\times10^{11}\ \text{m}\right)}{1.21\times10^{2}\ \text{m/s}} = 7.82\times10^{9}\ \text{s} = 247\ \text{y}\ .$$

LEARN An alternative ways to calculate the speed ratio and the periods are as follows. Since $v \sim \sqrt{M}$, the ratio of the speeds can be obtained as

$$\frac{v}{v_0} = \sqrt{\frac{M}{M_S}} = \left(\frac{r}{R}\right)^{3/2} = \left(\frac{1.50\times10^{11}\ \text{m}}{5.90\times10^{12}\ \text{m}}\right)^{3/2} = 0.00405.$$

In addition, since $T \sim 1/v \sim 1/\sqrt{M}$, we have

$$T = T_0\sqrt{\frac{M_S}{M}} = T_0\left(\frac{R}{r}\right)^{3/2} = (1\ \text{y})\left(\frac{5.90\times10^{12}\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^{3/2} = 247\ \text{y}.$$

44. (a) The mass of the portion of the galaxy within the radius r from its center is given by $M' = (r/R)^3 M$. Thus, from $GM'm/r^2 = mv^2/r$ (where m is the mass of the star) we get

$$v = \sqrt{\frac{GM'}{r}} = \sqrt{\frac{GM}{r}\left(\frac{r}{R}\right)^3} = r\sqrt{\frac{GM}{R^3}}\ .$$

(b) In the case where $M' = M$, we have

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}} = \frac{2\pi r^{3/2}}{\sqrt{GM}}\ .$$

45. **THINK** A meson is made up of a quark and an antiquark.

EXPRESS Only the strange quark has nonzero strangeness; an s quark has strangeness $S = -1$ and charge $q = -1/3$, while an $\bar{\text{s}}$ quark has strangeness $S = +1$ and charge $q = +1/3$.

ANALYZE (a) In order to obtain $S = -1$ we need to combine s with some non-strange antiquark (which would have the negative of the quantum numbers listed in Table 44-5). The difficulty is that the charge of the strange quark is –1/3, which means that (to obtain a total charge of +1) the antiquark would have to have a charge of $+\frac{4}{3}$. Clearly, there are no such antiquarks in our list. Thus, a meson with $S = -1$ and $q = +1$ cannot be formed with the quarks/antiquarks of Table 44-5.

(b) Similarly, one can show that, since no quark has $q = -\frac{4}{3}$, there cannot be a meson with $S = +1$ and $q = -1$.

LEARN Quarks and antiquarks can be combined to form baryons and mesons, but not all combinations are allowed because of the constraint from the quantum numbers.

46. Assuming the line passes through the origin, its slope is $0.40c/(5.3 \times 10^9$ ly). Then,

$$T = \frac{1}{H} = \frac{1}{\text{slope}} = \frac{5.3\times10^9 \text{ ly}}{0.40c} = \frac{5.3\times10^9 \text{ y}}{0.40} \approx 13\times10^9 \text{ y}.$$

47. **THINK** Pair annihilation is a process in which a particle and its antiparticle collide and annihilate each other.

EXPRESS The energy released would be twice the rest energy of Earth, or $E = 2M_Ec^2$.

ANALYZE The mass of the Earth is $M_E = 5.98 \times 10^{24}$ kg (found in Appendix C). Thus, the energy released is

$$E = 2M_Ec^2 = 2(5.98 \times 10^{24} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 1.08 \times 10^{42} \text{ J}.$$

LEARN As in the case of annihilation between an electron and a positron, the total energy of the Earth and the anti-Earth after the annihilation would appear as electromagnetic radiation.

48. We note from track 1, and the quantum numbers of the original particle (A), that positively charged particles move in counterclockwise curved paths, and — by inference — negatively charged ones move along clockwise arcs. This immediately shows that tracks 1, 2, 4, 6, and 7 belong to positively charged particles, and tracks 5, 8 and 9 belong to negatively charged ones. Looking at the fictitious particles in the table (and noting that each appears in the cloud chamber once [or not at all]), we see that this observation (about charged particle motion) greatly narrows the possibilities:

$$\text{tracks } 2,4,6,7, \leftrightarrow \text{ particles } C,F,H,J$$
$$\text{tracks } 5,8,9 \leftrightarrow \text{ particles } D,E,G$$

This tells us, too, that the particle that does not appear at all is either B or I (since only one neutral particle "appears"). By charge conservation, tracks 2, 4 and 6 are made by particles with a single unit of positive charge (note that track 5 is made by one with a single unit of negative charge), which implies (by elimination) that track 7 is made by particle H. This is confirmed by examining charge conservation at the end-point of track 6. Having exhausted the charge-related information, we turn now to the fictitious quantum numbers. Consider the vertex where tracks 2, 3, and 4 meet (the Whimsy number is listed here as a subscript):

$$\text{tracks } 2,4 \leftrightarrow \text{ particles } C_2, F_0, J_{-6}$$
$$\text{tracks } 3 \leftrightarrow \text{ particle } B_4 \text{ or } I_6$$

The requirement that the Whimsy quantum number of the particle making track 4 must equal the sum of the Whimsy values for the particles making tracks 2 and 3 places a powerful constraint (see the subscripts above). A fairly quick trial and error procedure leads to the assignments: particle *F* makes track 4, and particles *J* and *I* make tracks 2 and 3, respectively. Particle *B*, then, is irrelevant to this set of events. By elimination, the particle making track 6 (the only positively charged particle not yet assigned) must be *C*. At the vertex defined by

$$A \to F + C + (\text{track } 5)_- \,,$$

where the charge of that particle is indicated by the subscript, we see that Cuteness number conservation requires that the particle making track 5 has Cuteness = –1, so this must be particle *G*. We have only one decision remaining:

$$\text{tracks } 8,9, \leftrightarrow \text{ particles } D, E$$

Re-reading the problem, one finds that the particle making track 8 must be particle *D* since it is the one with seriousness = 0. Consequently, the particle making track 9 must be *E*.

Thus, we have the following:

(a) Particle *A* is for track 1.

(b) Particle *J* is for track 2.

(c) Particle *I* is for track 3.

(d) Particle *F* is for track 4.

(e) Particle *G* is for track 5.

(f) Particle *C* is for track 6.

(g) Particle *H* is for track 7.

(h) Particle *D* is for track 8.

(i) Particle *E* is for track 9.

49. (a) We use the relativistic relationship between speed and momentum:

$$p = \gamma mv = \frac{mv}{\sqrt{1-(v/c)^2}} ,$$

which we solve for the speed *v*:

$$\frac{v}{c} = \sqrt{1 - \frac{1}{(pc/mc^2)^2 + 1}} .$$

For an antiproton mc^2 = 938.3 MeV and pc = 1.19 GeV = 1190 MeV, so

$$v = c\sqrt{1 - \frac{1}{(1190\,\text{MeV}/938.3\,\text{MeV})^2 + 1}} = 0.785c .$$

(b) For the negative pion mc^2 = 193.6 MeV, and *pc* is the same. Therefore,

$$v = c\sqrt{1 - \frac{1}{(1190\,\text{MeV}/193.6\,\text{MeV})^2 + 1}} = 0.993c .$$

(c) Since the speed of the antiprotons is about 0.78*c* but not over 0.79*c*, an antiproton will trigger C2.

(d) Since the speed of the negative pions exceeds 0.79*c*, a negative pion will trigger C1.

(e) We use $\Delta t = d/v$, where d = 12 m. For an antiproton

$$\Delta t = \frac{1}{0.785(2.998 \times 10^8 \text{ m/s})} = 5.1 \times 10^{-8}\,\text{s} = 51\,\text{ns}.$$

(f) For a negative pion

$$\Delta t = \frac{12\,\text{m}}{0.993(2.998 \times 10^8 \text{ m/s})} = 4.0 \times 10^{-8}\,\text{s} = 40\,\text{ns} .$$

50. (a) Eq. 44-14 conserves charge since both the proton and the positron have $q = +e$ (and the neutrino is uncharged).

(b) Energy conservation is not violated since $m_p c^2 > m_e c^2 + m_\nu c^2$.

(c) We are free to view the decay from the rest frame of the proton. Both the positron and the neutrino are able to carry momentum, and so long as they travel in opposite directions with appropriate values of *p* (so that $\sum \vec{p} = 0$) then linear momentum is conserved.

(d) If we examine the spin angular momenta, there does seem to be a violation of angular momentum conservation (Eq. 44-14 shows a spin-one-half particle decaying into two spin-one-half particles).

51. (a) During the time interval Δt, the light emitted from galaxy A has traveled a distance $c\Delta t$. Meanwhile, the distance between Earth and the galaxy has expanded from r to $r' = r + r\alpha\,\Delta t$. Let $c\Delta t = r' = r + r\alpha\Delta t$, which leads to

$$\Delta t = \frac{r}{c - r\alpha}\ .$$

(b) The detected wavelength λ' is longer than λ by $\lambda\alpha\Delta t$ due to the expansion of the universe: $\lambda' = \lambda + \lambda\alpha\Delta t$. Thus,

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \alpha\Delta t = \frac{\alpha r}{c - \alpha r}\ .$$

(c) We use the binomial expansion formula (see Appendix E):

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \quad (x^2 < 1)$$

to obtain

$$\frac{\Delta\lambda}{\lambda} = \frac{\alpha r}{c - \alpha r} = \frac{\alpha r}{c}\left(1 - \frac{\alpha r}{c}\right)^{-1} = \frac{\alpha r}{c}\left[1 + \frac{-1}{1!}\left(-\frac{\alpha r}{c}\right) + \frac{(-1)(-2)}{2!}\left(-\frac{\alpha r}{c}\right)^2 + \cdots\right]$$
$$\approx \frac{\alpha r}{c} + \left(\frac{\alpha r}{c}\right)^2 + \left(\frac{\alpha r}{c}\right)^3\ .$$

(d) When only the first term in the expansion for $\Delta\lambda/\lambda$ is retained we have

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\alpha r}{c}\ .$$

(e) We set

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = \frac{Hr}{c}$$

and compare with the result of part (d) to obtain $\alpha = H$.

(f) We use the formula $\Delta\lambda/\lambda = \alpha r/(c - \alpha r)$ to solve for r:

$$r = \frac{c(\Delta\lambda/\lambda)}{\alpha(1 + \Delta\lambda/\lambda)} = \frac{(2.998\times10^8\ \text{m/s})(0.050)}{(0.0218\ \text{m/s}\cdot\text{ly})(1 + 0.050)} = 6.548\times10^8\ \text{ly} \approx 6.5\times10^8\ \text{ly}.$$

(g) From the result of part (a),

$$\Delta t = \frac{r}{c-\alpha r} = \frac{\left(6.5\times10^{8}\text{ ly}\right)\left(9.46\times10^{15}\text{ m/ly}\right)}{2.998\times10^{8}\text{ m/s}-\left(0.0218\text{ m/s}\cdot\text{ly}\right)\left(6.5\times10^{8}\text{ ly}\right)} = 2.17\times10^{16}\text{ s},$$

which is equivalent to 6.9×10^8 y.

(h) Letting $r = c\Delta t$, we solve for Δt:

$$\Delta t = \frac{r}{c} = \frac{6.5\times10^{8}\text{ ly}}{c} = 6.5\times10^{8}\text{ y}.$$

(i) The distance is given by

$$r = c\Delta t = c\left(6.9\times10^{8}\text{ y}\right) = 6.9\times10^{8}\text{ ly}.$$

(j) From the result of part (f),

$$r_B = \frac{c\left(\Delta\lambda/\lambda\right)}{\alpha\left(1+\Delta\lambda/\lambda\right)} = \frac{\left(2.998\times10^{8}\text{ m/s}\right)\left(0.080\right)}{\left(0.0218\text{ mm/s}\cdot\text{ly}\right)\left(1+0.080\right)} = 1.018\times10^{9}\text{ ly} \approx 1.0\times10^{9}\text{ ly}.$$

(k) From the formula obtained in part (a),

$$\Delta t_{\text{B}} = \frac{r_{\text{B}}}{c-r_{\text{B}}\alpha} = \frac{\left(1.0\times10^{9}\text{ ly}\right)\left(9.46\times10^{15}\text{ m/ly}\right)}{2.998\times10^{8}\text{ m/s}-\left(1.0\times10^{9}\text{ ly}\right)\left(0.0218\text{ m/s}\cdot\text{ly}\right)} = 3.4\times10^{16}\text{ s},$$

which is equivalent to 1.1×10^9 y.

(l) At the present time, the separation between the two galaxies A and B is given by $r_{\text{now}} = c\Delta t_{\text{B}} - c\Delta t_{\text{A}}$. Since $r_{\text{now}} = r_{\text{then}} + r_{\text{then}}\alpha\Delta t$, we get

$$r_{\text{then}} = \frac{r_{\text{now}}}{1+\alpha\Delta t} = 3.9\times10^{8}\text{ ly}.$$

52. Using Table 44-1, the difference in mass between the muon and the pion is

$$\Delta m = \left(139.6\text{ MeV}/c^2 - 105.7\text{ MeV}/c^2\right) = \frac{(33.9\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)}{\left(2.998\times10^{8}\text{ m/s}\right)^2}$$

$$= 6.03\times10^{-29}\text{ kg}.$$

53. (a) The quark composition for Σ^- is dss.

(b) The quark composition for $\overline{\Sigma}^-$ is $\overline{\text{d}}\,\overline{\text{s}}\,\overline{\text{s}}$.

54. The speed of the electron is relativistic, so we first calculated the Lorentz factor:

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{2.5\text{ MeV}}{0.511\text{ MeV}} = 5.892$$

The total energy carried by the electron or the positron is

$$E = \gamma mc^2 = (5.892)(0.511\text{ MeV}) = 3.011\text{ MeV} = 4.82\times10^{-13}\text{ J}$$

The corresponding frequency of the photons produced is

$$f = \frac{E}{h} = \frac{4.82\times10^{-13}\text{ J}}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 7.3\times10^{20}\text{ Hz}.$$