13

The Laplace Transform in Circuit Analysis

Assessment Problems

AP 13.1 **[a]** $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$\frac{1}{RC} = \frac{10^6}{(500)(0.025)} = 80{,}000; \qquad \frac{1}{LC} = 25 \times 10^8$$

$$\text{Therefore} \quad Y = \frac{25 \times 10^{-9}(s^2 + 80{,}000s + 25 \times 10^8)}{s}$$

[b] $z_{1,2} = -40{,}000 \pm \sqrt{16 \times 10^8 - 25 \times 10^8} = -40{,}000 \pm j30{,}000$ rad/s

$-z_1 = -40{,}000 - j30{,}000$ rad/s

$-z_2 = -40{,}000 + j30{,}000$ rad/s

$p_1 = 0$ rad/s

AP 13.2 **[a]** $Z = 2000 + \frac{1}{Y} = 2000 + \frac{4 \times 10^7 s}{s^2 + 80{,}000s + 25 \times 10^8}$

$$= \frac{2000(s^2 + 10^5 s + 25 \times 10^8)}{s^2 + 80{,}000s + 25 \times 10^8} = \frac{2000(s + 50{,}000)^2}{s^2 + 80{,}000s + 25 \times 10^8}$$

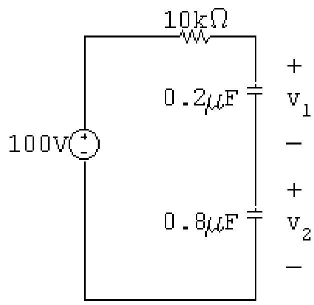
[b] $-z_1 = -z_2 = -50{,}000$ rad/s

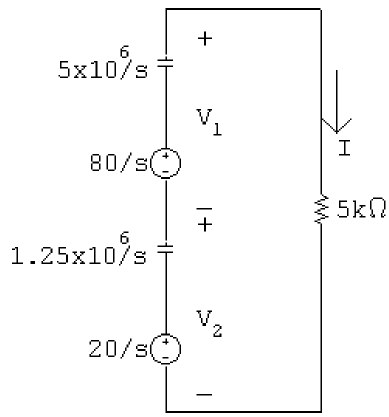
$-p_1 = -40{,}000 - j30{,}000$ rad/s

$-p_2 = -40{,}000 + j30{,}000$ rad/s

AP 13.3 **[a]** At $t = 0^-$, $0.2v_1 = (0.8)v_2$; $v_1 = 4v_2$; $v_1 + v_2 = 100\,\text{V}$

Therefore $v_1(0^-) = 80V = v_1(0^+)$; $v_2(0^-) = 20V = v_2(0^+)$





$$I = \frac{(80/s) + (20/s)}{5000 + [(5 \times 10^6)/s] + (1.25 \times 10^6/s)} = \frac{20 \times 10^{-3}}{s + 1250}$$

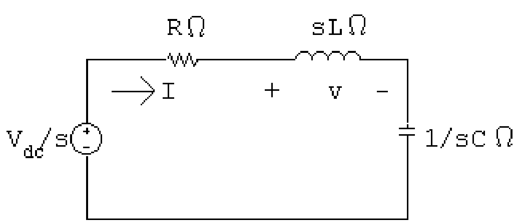
$$V_1 = \frac{80}{s} - \frac{5 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{80}{s + 1250}$$

$$V_2 = \frac{20}{s} - \frac{1.25 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{20}{s + 1250}$$

[b] $i = 20e^{-1250t}u(t)\,\text{mA}$; $v_1 = 80e^{-1250t}u(t)\,\text{V}$

$v_2 = 20e^{-1250t}u(t)\,\text{V}$

AP 13.4 **[a]**



$$I = \frac{V_{\text{dc}}/s}{R + sL + (1/sC)} = \frac{V_{\text{dc}}/L}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{V_{\text{dc}}}{L} = 40; \qquad \frac{R}{L} = 1.2; \qquad \frac{1}{LC} = 1.0$$

$$I = \frac{40}{(s + 0.6 - j0.8)(s + 0.6 + j0.8)} = \frac{K_1}{s + 0.6 - j0.8} + \frac{K_1^*}{s + 0.6 + j0.8}$$

$$K_1 = \frac{40}{j1.6} = -j25 = 25\underline{/-90^\circ}; \qquad K_1^* = 25\underline{/90^\circ}$$

[**b**] $i = 50e^{-0.6t}\cos(0.8t - 90^\circ) = [50e^{-0.6t}\sin 0.8t]u(t)\,\text{A}$

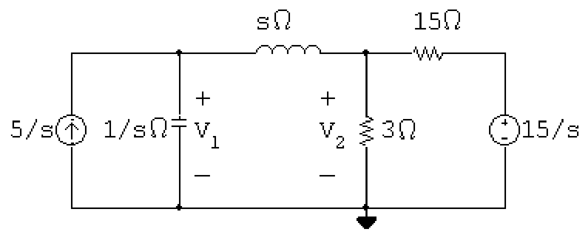
[**c**] $V = sLI = \dfrac{160s}{(s + 0.6 - j0.8)(s + 0.6 + j0.8)}$

$$= \frac{K_1}{s + 0.6 - j0.8} + \frac{K_1^*}{s + 0.6 + j0.8}$$

$$K_1 = \frac{160(-0.6 + j0.8)}{j1.6} = 100\underline{/36.87^\circ}$$

[**d**] $v(t) = [200e^{-0.6t}\cos(0.8t + 36.87^\circ)]u(t)\,\text{V}$

AP 13.5 [**a**]



The two node voltage equations are

$$\frac{V_1 - V_2}{s} + V_1 s = \frac{5}{s} \quad \text{and} \quad \frac{V_2}{3} + \frac{V_2 - V_1}{s} + \frac{V_2 - (15/s)}{15} = 0$$

Solving for V_1 and V_2 yields

$$V_1 = \frac{5(s + 3)}{s(s^2 + 2.5s + 1)}, \qquad V_2 = \frac{2.5(s^2 + 6)}{s(s^2 + 2.5s + 1)}$$

[**b**] The partial fraction expansions of V_1 and V_2 are

$$V_1 = \frac{15}{s} - \frac{50/3}{s + 0.5} + \frac{5/3}{s + 2} \quad \text{and} \quad V_2 = \frac{15}{s} - \frac{125/6}{s + 0.5} + \frac{25/3}{s + 2}$$

It follows that

$$v_1(t) = \left[15 - \frac{50}{3}e^{-0.5t} + \frac{5}{3}e^{-2t}\right]u(t)\,\text{V} \quad \text{and}$$

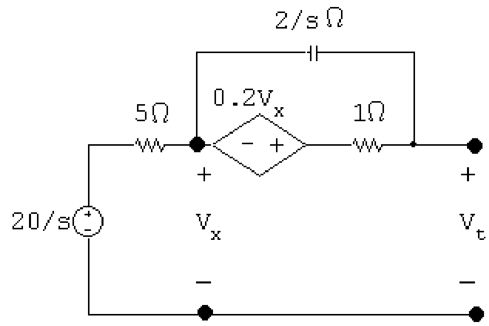
$$v_2(t) = \left[15 - \frac{125}{6}e^{-0.5t} + \frac{25}{3}e^{-2t}\right]u(t)\,\text{V}$$

[**c**] $v_1(0^+) = 15 - \dfrac{50}{3} + \dfrac{5}{3} = 0$

$$v_2(0^+) = 15 - \frac{125}{6} + \frac{25}{3} = 2.5\,\text{V}$$

[**d**] $v_1(\infty) = 15\,\text{V}; \qquad v_2(\infty) = 15\,\text{V}$

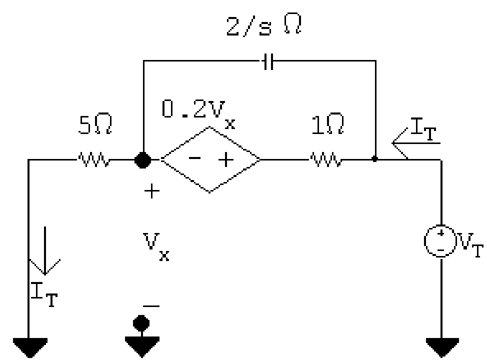
AP 13.6 [**a**]



With no load across terminals $a-b$ $\quad V_x = 20/s$:

$$\frac{1}{2}\left[\frac{20}{s} - V_{\text{Th}}\right]s + \left[1.2\left(\frac{20}{s}\right) - V_{\text{Th}}\right] = 0$$

therefore $\quad V_{\text{Th}} = \dfrac{20(s+2.4)}{s(s+2)}$



$V_x = 5I_T \quad$ and $\quad Z_{\text{Th}} = \dfrac{V_T}{I_T}$

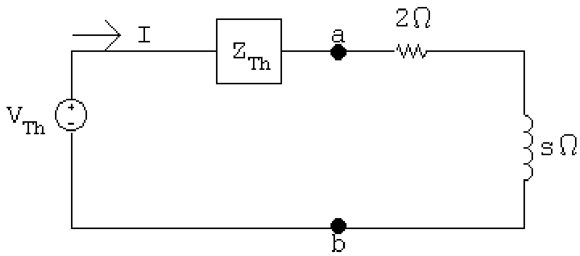
Solving for I_T gives

$$I_T = \frac{(V_T - 5I_T)s}{2} + V_T - 6I_T$$

Therefore

$$14I_T = V_T s + 5sI_T + 2V_T; \qquad \text{therefore} \quad Z_{\text{Th}} = \frac{5(s+2.8)}{s+2}$$

[b]



$$I = \frac{V_{\text{Th}}}{Z_{\text{Th}} + 2 + s} = \frac{20(s + 2.4)}{s(s + 3)(s + 6)}$$

AP 13.7 **[a]** $i_2 = 1.25e^{-t} - 1.25e^{-3t};$ therefore $\frac{di_2}{dt} = -1.25e^{-t} + 3.75e^{-3t}$

Therefore $\frac{di_2}{dt} = 0$ when

$1.25e^{-t} = 3.75e^{-3t}$ or $e^{2t} = 3,$ $t = 0.5(\ln 3) = 549.31\,\text{ms}$

$i_2(\text{max}) = 1.25[e^{-0.549} - e^{-3(0.549)}] = 481.13\,\text{mA}$

[b] From Eqs. 13.68 and 13.69, we have

$\Delta = 12(s^2 + 4s + 3) = 12(s + 1)(s + 3)$ and $N_1 = 60(s + 2)$

Therefore $I_1 = \frac{N_1}{\Delta} = \frac{5(s + 2)}{(s + 1)(s + 3)}$

A partial fraction expansion leads to the expression

$$I_1 = \frac{2.5}{s + 1} + \frac{2.5}{s + 3}$$

Therefore we get

$i_1 = 2.5[e^{-t} + e^{-3t}]u(t)\,\text{A}$

[c] $\frac{di_1}{dt} = -2.5[e^{-t} + 3e^{-3t}];$ $\frac{di_1(0.54931)}{dt} = -2.89\,\text{A/s}$

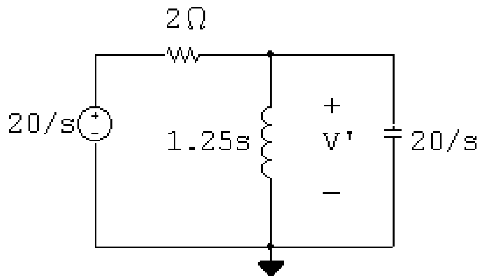
[d] When i_2 is at its peak value,

$$\frac{di_2}{dt} = 0$$

Therefore $L_2\left(\frac{di_2}{dt}\right) = 0$ and $i_2 = -\left(\frac{M}{12}\right)\left(\frac{di_1}{dt}\right)$

[e] $i_2(\text{max}) = \frac{-2(-2.89)}{12} = 481.13\,\text{mA}$ (checks)

AP 13.8 [**a**] The s-domain circuit with the voltage source acting alone is

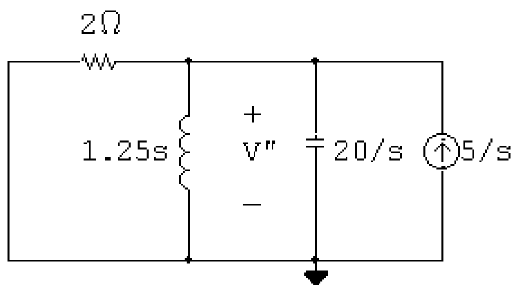


$$\frac{V' - (20/s)}{2} + \frac{V'}{1.25s} + \frac{V's}{20} = 0$$

$$V' = \frac{200}{(s+2)(s+8)} = \frac{100/3}{s+2} - \frac{100/3}{s+8}$$

$$v' = \frac{100}{3}[e^{-2t} - e^{-8t}]u(t)\,\text{V}$$

[**b**] With the current source acting alone,



$$\frac{V''}{2} + \frac{V''}{1.25s} + \frac{V''s}{20} = \frac{5}{s}$$

$$V'' = \frac{100}{(s+2)(s+8)} = \frac{50/3}{s+2} - \frac{50/3}{s+8}$$

$$v'' = \frac{50}{3}[e^{-2t} - e^{-8t}]u(t)\,\text{V}$$

[**c**] $v = v' + v'' = [50e^{-2t} - 50e^{-8t}]u(t)\,\text{V}$

AP 13.9 [**a**] $\dfrac{V_o}{s+2} + \dfrac{V_o s}{10} = I_g;$ therefore $\dfrac{V_o}{I_g} = H(s) = \dfrac{10(s+2)}{s^2+2s+10}$

[**b**] $-z_1 = -2$ rad/s; $-p_1 = -1 + j3$ rad/s; $-p_2 = -1 - j3$ rad/s

AP 13.10 [**a**]

$$V_o = \frac{10(s+2)}{s^2+2s+10} \cdot \frac{1}{s} = \frac{K_o}{s} + \frac{K_1}{s+1-j3} + \frac{K_1^*}{s+1+j3}$$

$K_o = 2;$ $K_1 = 5/3\underline{/-126.87^\circ};$ $K_1^* = 5/3\underline{/126.87^\circ}$

$$v_o = [2 + (10/3)e^{-t}\cos(3t - 126.87^\circ)]u(t)\,\text{V}$$

[b] $V_o = \frac{10(s+2)}{s^2+2s+10} \cdot 1 = \frac{K_2}{s+1-j3} + \frac{K_2^*}{s+1+j3}$

$K_2 = 5.27\underline{/-18.43^\circ}; \qquad K_2^* = 5.27\underline{/18.43^\circ}$

$v_o = [10.54e^{-t}\cos(3t - 18.43^\circ)]u(t)\,\text{V}$

AP 13.11 **[a]**

$H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{v_o(t)\}$

$$v_o(t) = 10{,}000\cos\theta e^{-70t}\cos 240t - 10{,}000\sin\theta e^{-70t}\sin 240t$$
$$= 9600e^{-70t}\cos 240t - 2800e^{-70t}\sin 240t$$

Therefore $H(s) = \frac{9600(s+70)}{(s+70)^2+(240)^2} - \frac{2800(240)}{(s+70)^2+(240)^2}$

$$= \frac{9600s}{s^2+140s+62{,}500}$$

[b] $V_o(s) = H(s)\cdot\frac{1}{s} = \frac{9600}{s^2+140s+62{,}500}$

$$= \frac{K_1}{s+70-j240} + \frac{K_1^*}{s+70+j240}$$

$K_1 = \frac{9600}{j480} = -j20 = 20\underline{/-90^\circ}$

Therefore

$v_o(t) = [40e^{-70t}\cos(240t - 90^\circ)]u(t)\,\text{V} = [40e^{-70t}\sin 240t]u(t)\,\text{V}$

AP 13.12 From Assessment Problem 13.9:

$H(s) = \frac{10(s+2)}{s^2+2s+10}$

Therefore $H(j4) = \frac{10(2+j4)}{10-16+j8} = 4.47\underline{/-63.43^\circ}$

Thus,

$v_o = (10)(4.47)\cos(4t - 63.43^\circ) = 44.7\cos(4t - 63.43^\circ)\,\text{V}$

AP 13.13 **[a]**

Let $R_1 = 10\,\text{k}\Omega, \quad R_2 = 50\,\text{k}\Omega, \quad C = 400\text{ pF}, \quad R_2C = 2\times 10^{-5}$

then $V_1 = V_2 = \frac{V_gR_2}{R_2 + (1/sC)}$

Also $\frac{V_1 - V_g}{R_1} + \frac{V_1 - V_o}{R_1} = 0$

therefore $V_o = 2V_1 - V_g$

Now solving for V_o/V_g, we get $H(s) = \frac{R_2Cs - 1}{R_2Cs + 1}$

It follows that $H(j50{,}000) = \frac{j-1}{j+1} = j1 = 1\underline{/90^\circ}$

Therefore $v_o = 10\cos(50{,}000t + 90^\circ)\,\text{V}$

[b] Replacing R_2 by R_x gives us $H(s) = \frac{R_xCs - 1}{R_xCs + 1}$

Therefore

$$H(j50{,}000) = \frac{j20 \times 10^{-6}R_x - 1}{j20 \times 10^{-6}R_x + 1} = \frac{R_x + j50{,}000}{R_x - j50{,}000}$$

Thus,

$$\frac{50{,}000}{R_x} = \tan 60^\circ = 1.7321, \qquad R_x = 28{,}867.51\,\Omega$$

Problems

P 13.1 $i = \frac{1}{L}\int_{0^-}^{t} v\,d\tau + I_0;$ therefore $I = \left(\frac{1}{L}\right)\left(\frac{V}{s}\right) + \frac{I_0}{s} = \frac{V}{sL} + \frac{I_0}{s}$

P 13.2 $V_{\text{Th}} = V_{\text{ab}} = CV_0\left(\frac{1}{sC}\right) = \frac{V_0}{s};$ $Z_{\text{Th}} = \frac{1}{sC}$

P 13.3 $I_{sc_{\text{ab}}} = I_{\text{N}} = \frac{-LI_0}{sL} = \frac{-I_0}{s};$ $Z_{\text{N}} = sL$

Therefore, the Norton equivalent is the same as the circuit in Fig. 13.4.

P 13.4 **[a]** $Z = R + sL + \frac{1}{sC} = \frac{L[s^2 + (R/L)s + (1/LC)]}{s}$

$$= \frac{[s^2 + 32{,}000s + 4\times 10^8]}{s}$$

[b] $s_{1,2} = -16{,}000 \pm j12{,}000$ rad/s

Zeros at $-16{,}000 + j12{,}000$ rad/s and $-16{,}000 - j12{,}000$ rad/s

Pole at 0.

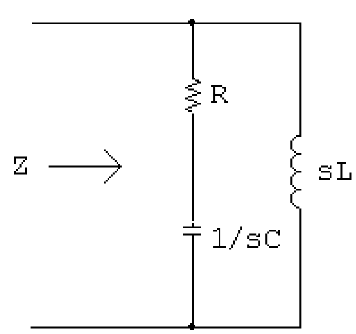
P 13.5 **[a]** $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$Z = \frac{1}{Y} = \frac{s/C}{s^2 + (1/RC)s + (1/LC)} = \frac{16\times 10^9 s}{s^2 + 2\times 10^6 s + 64\times 10^{10}}$$

[b] zero at $z_1 = 0$

poles at $-p_1 = -400$ krad/s and $-p_2 = -1600$ krad/s

P 13.6 **[a]**



$$Z = \frac{(R + 1/sC)(sL)}{R + sL + (1/sC)} = \frac{(Rs)(s + 1/RC)}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{R}{L} = 500; \qquad \frac{1}{RC} = 80; \qquad \frac{1}{LC} = 40\times 10^3$$

$$Z = \frac{200s(s+80)}{s^2 + 500s + 40\times 10^3}$$

[b] $Z = \dfrac{200s(s+80)}{(s+100)(s+400)}$

$z_1 = 0; \qquad -z_2 = -80$ rad/s

$-p_1 = -100$ rad/s; $\qquad -p_2 = -400$ rad/s

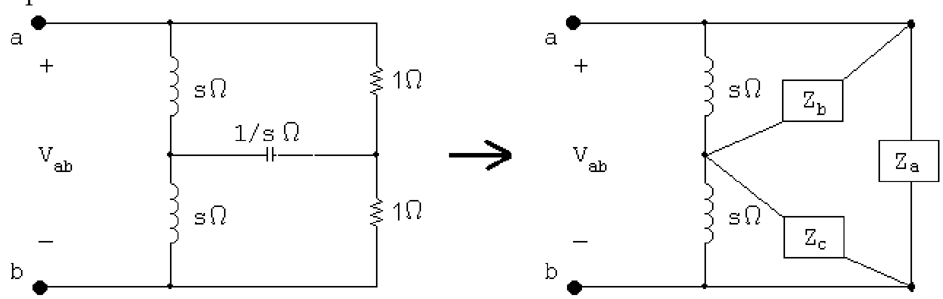
P 13.7 $\left(1\|s+\dfrac{1}{s}\right)\|1 = \left(\dfrac{s}{s+1}+\dfrac{1}{s}\right)\|1 = \dfrac{s^2+s+1}{s(s+1)}\|1$

$$= \frac{\dfrac{s^2+s+1}{s(s+1)}}{\dfrac{s^2+s+1}{s(s+1)}+1} = \frac{s^2+s+1}{2s^2+2s+1} = \frac{0.5(s^2+s+1)}{s^2+s+0.5}$$

$-z_1 = -0.5 + j0.866$ rad/s; $\qquad -z_2 = -0.5 - j0.866$ rad/s

$-p_1 = -0.5 + j0.5$ rad/s; $\qquad -p_2 = -0.5 - j0.5$ rad/s

P 13.8 Transform the Y-connection of the two resistors and the capacitor into the equivalent delta-connection:



where

$$Z_\text{a} = \frac{(1/s)(1)+(1)(1/s)+(1)(1)}{(1/s)} = s+2$$

$$Z_\text{b} = Z_\text{c} = \frac{(1/s)(1)+(1)(1/s)+(1)(1)}{1} = \frac{s+2}{s}$$

Then

$$Z_\text{ab} = Z_\text{a}\|[(s\|Z_\text{c})+(s\|Z_\text{b})] = Z_\text{a}\|2(s\|Z_\text{b})$$

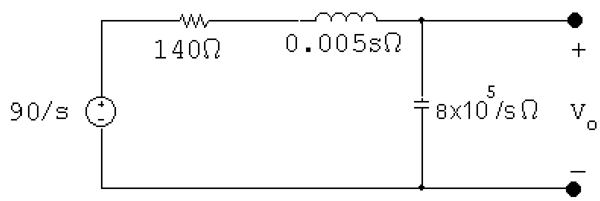
$$s\|Z_\text{b} = \frac{s[(s+2)/s]}{s+[(s+2)/s]} = \frac{s(s+2)}{s^2+s+2}$$

$$Z_{\rm ab} = (s+2)\|\frac{2s(s+2)}{s^2+s+2} = \frac{\dfrac{2s(s+2)^2}{s^2+s+2}}{s+2+\dfrac{2s(s+2)}{s^2+s+2}}$$

$$= \frac{2s(s+2)^2}{(s+2)(s^2+s+2)+2s(s+2)} = \frac{2s(s+2)}{s^2+3s+2} = \frac{2s}{s+1}$$

Zero at 0; pole at −1 rad/s.

P 13.9



$$V_o = \frac{(90/s)(8\times10^5/s)}{140+0.005s+(8\times10^5/s)}$$

$$= \frac{144\times10^8}{s(s^2+28{,}000s+16\times10^7)}$$

$$= \frac{144\times10^8}{s(s+8000)(s+20{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+8000} + \frac{K_3}{s+20{,}000}$$

$$K_1 = \frac{144\times10^8}{16\times10^7} = 90$$

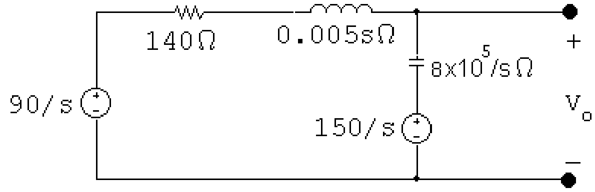
$$K_2 = \frac{144\times10^8}{(-8000)(12{,}000)} = -150$$

$$K_3 = \frac{144\times10^8}{(-12{,}000)(-20{,}000)} = 60$$

$$V_o = \frac{90}{s} - \frac{150}{s+8000} + \frac{60}{s+20{,}000}$$

$$v_o(t) = [90 - 150e^{-8000t} + 60e^{-20{,}000t}]u(t)\,\mathrm{V}$$

P 13.10 With a non-zero initial voltage on the capacitor, the s-domain circuit becomes:



$$\frac{V_o - 90/s}{0.005s + 140} + \frac{(V_o - 150/s)s}{8 \times 10^5} = 0$$

$$V_o\left[\frac{200}{s + 28{,}000} + \frac{s}{8 \times 10^5}\right] = \frac{150}{8 \times 10^5} + \frac{18{,}000}{s(s + 28{,}000)}$$

$$\therefore \quad V_o = \frac{150(s^2 + 28{,}000s + 96 \times 10^6}{s(s + 8000)(s + 20{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 8000} + \frac{K_3}{s + 20{,}000}$$

$$K_1 = \frac{144 \times 10^8}{160 \times 10^6} = 90$$

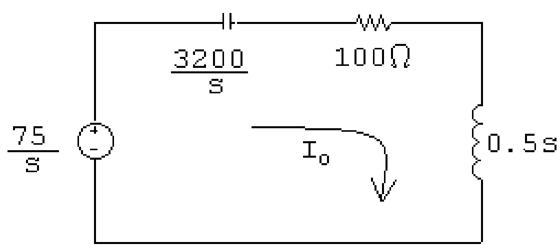
$$K_2 = \frac{150(s^2 + 28{,}000s + 96 \times 10^6}{s(s + 20{,}000)}\Big|_{s=-8000} = 100$$

$$K_3 = \frac{150(s^2 + 28{,}000s + 96 \times 10^6}{s(s + 8000)}\Big|_{s=-20{,}000} = -40$$

$$V_o = \frac{90}{s} + \frac{100}{s + 8000} - \frac{40}{s + 20{,}000}$$

$$v_o(t) = [90 + 100e^{-8000t} - 40e^{-20{,}000t}]u(t)\ \text{V}$$

P 13.11 **[a]** For $t > 0$:



[**b**] $$I_o = \frac{75/s}{(3200/s) + 100 + 0.5s}$$

$$= \frac{75}{0.5s^2 + 100s + 3200}$$

$$= \frac{150}{(s^2 + 200s + 6400)} = \frac{150}{(s + 40)(s + 160)}$$

[**c**] $$I_o = \frac{K_1}{s + 40} + \frac{K_2}{s + 160}$$

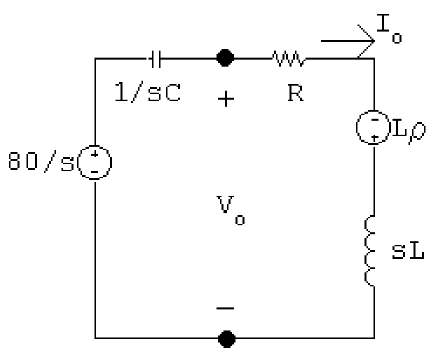
$$K_1 = \frac{150}{s + 160}\bigg|_{s=-40} = 1.25$$

$$K_2 = \frac{150}{s + 40}\bigg|_{s=-160} = -1.25$$

$$I_o = \frac{1.25}{s + 40} - \frac{1.25}{s + 160}$$

$$i_o(t) = (1.25e^{-40t} - 1.25e^{-160t})u(t)\,\text{A}$$

P 13.12 [**a**] $$i_o(0^-) = \frac{20}{4000} = 5\,\text{mA}$$



$$I_o = \frac{80/s + L\rho}{R + sL + 1/sC} = \frac{sC(80/s + L\rho)}{s^2LC + RsC + 1}$$

$$= \frac{80/L + s\rho}{s^2 + sR/L + 1/LC} = \frac{20{,}000 + s(0.1)}{s^2 + 200{,}000s + 10^{10}}$$

$$= \frac{0.1(s + 200{,}000)}{s^2 + 200{,}000s + 10^{10}} = \frac{K_1}{(s + 100{,}000)^2} + \frac{K_2}{s + 100{,}000}$$

$K_1 = 10{,}000;$ $\qquad K_2 = 0.1$

$$i_o(t) = [10{,}000te^{-100{,}000t} + 0.1e^{-100{,}000t}]u(t)\,\text{A}$$

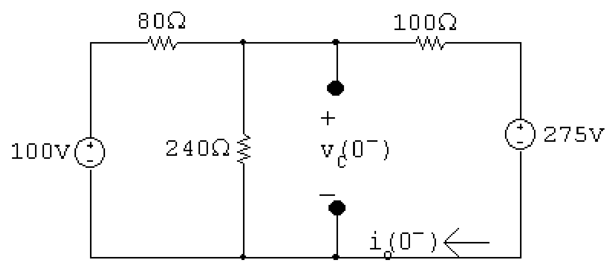
[b] $V_o = (R + sL)I_o - L\rho = \dfrac{(800 + 0.004s)(0.1s + 20{,}000)}{s^2 + 200{,}000s + 10^{10}} - 4 \times 10^{-4}$

$$= \frac{80(s + 150{,}000)}{(s + 100{,}000)^2} = \frac{K_1}{(s + 100{,}000)^2} + \frac{K_2}{s + 100{,}000}$$

$K_1 = 4 \times 10^6 \qquad K_2 = 80$

$v_o(t) = [4 \times 10^6 te^{-100{,}000t} + 80e^{-100{,}000t}]u(t)\,\text{A}$

P 13.13 **[a]** For $t < 0$:



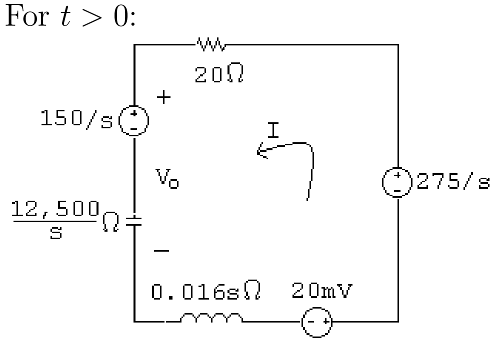
$$\frac{V_c - 100}{80} + \frac{V_c}{240} + \frac{V_c - 275}{100} = 0$$

$$V_c\left(\frac{1}{80} + \frac{1}{240} + \frac{1}{100}\right) = \frac{100}{80} + \frac{275}{100}$$

$V_c = 150\,\text{V}$

$$i_L(0^-) = \frac{150 - 275}{100} = -1.25\,\text{A}$$

For $t > 0$:



[b] $V_o = \dfrac{12{,}500}{s}I + \dfrac{150}{s}$

$$0 = -\frac{275}{s} + 20I + \frac{12{,}500}{s}I + \frac{150}{s} - 20 \times 10^{-3} + 0.016sI$$

$$I\left(20 + \frac{12{,}500}{s} + 0.016s\right) = \frac{125}{s} + 20 \times 10^{-3}$$

$$\therefore \quad I = \frac{7812.5 + 1.25s}{s^2 + 1250s + 781{,}250}$$

$$V_o = \frac{12{,}500}{s}\left(\frac{7812.5 + 1.25s}{s^2 + 1250s + 781{,}250}\right) + \frac{150}{s}$$

$$= \frac{150s^2 + 203{,}125s + 214{,}843{,}750}{s(s^2 + 1250s + 781{,}250)}$$

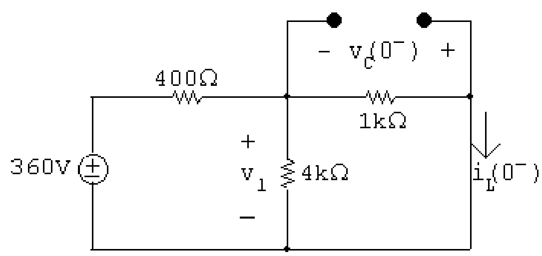
[c] $V_o = \frac{K_1}{s} + \frac{K_2}{s + 625 - j625} + \frac{K_2^*}{s + 625 + j625}$

$$K_1 = \frac{150s^2 + 203{,}125s + 214{,}843{,}750}{s^2 + 1250s + 781{,}250}\bigg|_{s=0} = 275$$

$$K_2 = \frac{150s^2 + 203{,}125s + 214{,}843{,}750}{s(s + 625 + j625)}\bigg|_{s=-625+j625} = 80.04\underline{/141.34^\circ}$$

$$v_o(t) = [2755 + 160.08e^{-625t}\cos(625t + 141.34^\circ)]u(t)\,\text{V}$$

P 13.14 **[a]** For $t < 0$:

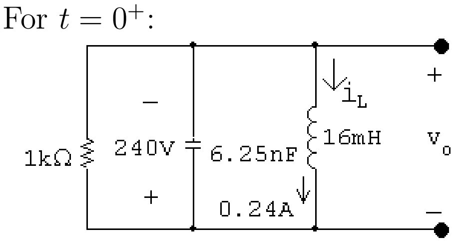


$$v_1 = \frac{4000\|1000}{400 + 4000\|1000}(360) = 240\,\text{V}$$

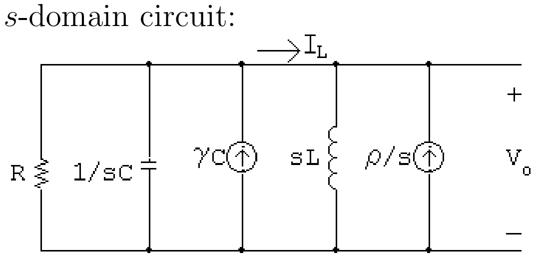
$$i_\text{L}(0^-) = \frac{240}{1000} = 0.24\,\text{A}$$

$$v_\text{C}(0^-) = -v_1 = -240\,\text{V}$$

For $t = 0^+$:



s-domain circuit:



where

$R = 1\,\text{k}\Omega; \qquad C = 6.25\,\text{nF}; \qquad \gamma = -240\,\text{V};$

$L = 16\,\text{mH}; \qquad \text{and} \qquad \rho = -0.24\,\text{A}$

[b] $\dfrac{V_o}{R} + V_o sC - \gamma C + \dfrac{V_o}{sL} - \dfrac{\rho}{s} = 0$

$$\therefore \quad V_o = \frac{\gamma[s + (\rho/\gamma C)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{-0.24}{(-240)(6.25 \times 10^{-9})} = 160{,}000$$

$$\frac{1}{RC} = \frac{1}{(1000)(6.25 \times 10^{-9})} = 160{,}000$$

$$\frac{1}{LC} = \frac{1}{(16 \times 10^{-3})(6.25 \times 10^{-9})} = 10^{10}$$

$$V_o = \frac{-240(s + 160{,}000)}{s^2 + 160{,}000s + 10^{10}}$$

[c] $I_\text{L} = \dfrac{V_o}{sL} - \dfrac{\rho}{s} = \dfrac{V_o}{0.016s} + \dfrac{0.24}{s}$

$$= \frac{-15{,}000(s + 160{,}000)}{s(s^2 + 160{,}000s + 10^{10})} + \frac{0.24}{s} = \frac{0.24(s + 97{,}500)}{(s^2 + 160{,}000s + 10^{10})}$$

[d] $V_o = \dfrac{-240(s + 160{,}000)}{s^2 + 160{,}000s + 10^{10}}$

$$= \frac{K_1}{s + 80{,}000 - j60{,}000} + \frac{K_1^*}{s + 80{,}000 + j60{,}000}$$

$$K_1 = \frac{-240(s + 160{,}000)}{s + 80{,}000 + j60{,}000}\bigg|_{s=-80{,}000+j60{,}000} = 200\underline{/126.87^\circ}$$

$$v_o(t) = [400e^{-80{,}000t}\cos(60{,}000t + 126.87^\circ)]u(t)\,\text{V}$$

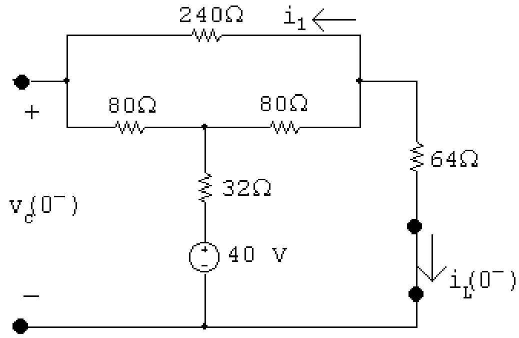
[e] $I_\text{L} = \dfrac{0.24(s + 97{,}500)}{s^2 + 160{,}000s + 10^{10}}$

$$= \frac{K_1}{s + 80{,}000 - j60{,}000} + \frac{K_1^*}{s + 80{,}000 + j60{,}000}$$

$$K_1 = \frac{0.24(s + 97{,}500)}{s + 80{,}000 + j60{,}000}\bigg|_{s=-80{,}000+j60{,}000} = 0.125\underline{/-16.26^\circ}$$

$$i_\text{L}(t) = [0.5e^{-80{,}000t}\cos(60{,}000t - 16.26^\circ)]u(t)\,\text{A}$$

P 13.15 **[a]** For $t < 0$:

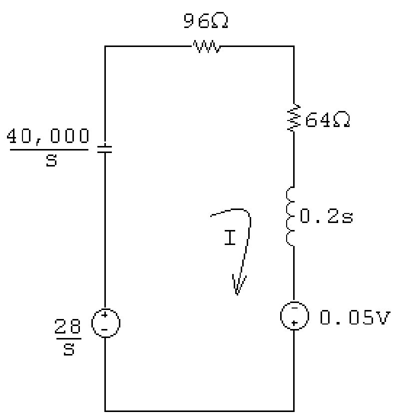


$$i_L(0^-) = \frac{40}{32 + 80\|320 + 64} = \frac{40}{160} = 0.25\,\text{A}$$

$$i_1 = \frac{80}{400}(-0.25) = -0.05\,\text{A}$$

$$v_C(0^-) = 80(-0.05) + 32(-0.25) + 40 = 28\,\text{V}$$

For $t > 0$:



[b] $(160 + 0.2s + 40{,}000/s)I = 0.05 + \dfrac{28}{s}$

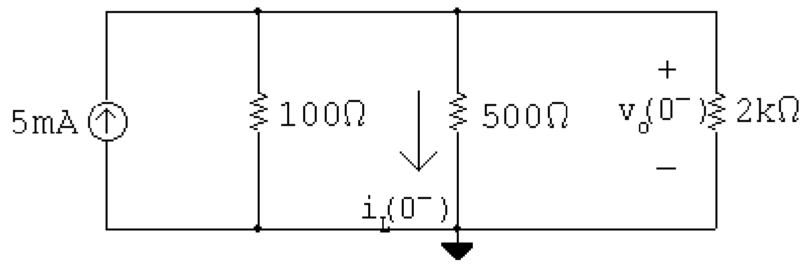
$$\therefore \qquad I = \frac{0.25(s + 560)}{s^2 + 800s + 200{,}000}$$

$$= \frac{K_1}{s + 400 - j200} + \frac{K_1^*}{s + 400 + j200}$$

$$K_1 = \frac{0.25(s + 560)}{s + 400 + j200}\bigg|_{s=-400+j200} = 0.16\underline{/-38.66^\circ}$$

[c] $i_o(t) = 0.32e^{-400t}\cos(200t - 38.66^\circ)u(t)\,\text{A}$

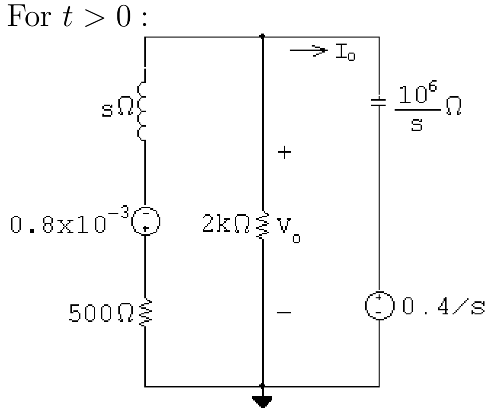
P 13.16 For $t < 0$:



$$v_o(0^-) = (100\|500\|2000)(0.005) = 0.4\,\text{V}$$

$$i_L(0^-) = \frac{v_o(0^-)}{500} = 0.8\,\text{mA}$$

For $t > 0$:



$$\frac{V_o + 0.8 \times 10^{-3}}{500 + s} + \frac{V_o}{2000} + \frac{V_o - (0.4/s)}{10^6/s} = 0$$

$$V_o\left(\frac{1}{500 + s} + \frac{1}{2000} + \frac{s}{10^6}\right) = 0.4 \times 10^{-6} - \frac{0.8 \times 10^{-3}}{500 + s}$$

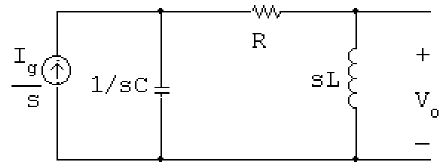
$$\therefore \qquad V_o = \frac{0.4(s - 1500)}{s^2 + 1000s + 125 \times 10^4}$$

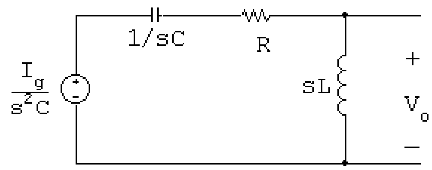
$$= \frac{K_1}{s + 500 - j1000} + \frac{K_1^*}{s + 500 + j1000}$$

$$K_1 = \frac{0.4(s - 1500)}{s + 500 + j1000}\bigg|_{s=-500+j1000} = 0.447\underline{/63.43^\circ}$$

$$v_o(t) = [0.894e^{-500t}\cos(1000t + 63.43^\circ)]u(t)\,\text{V}$$

P 13.17 [**a**]





$$V_o = \frac{sL(I_g/sC)}{R + sL + (1/sC)} = \frac{I_g/C}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{I_g}{C} = \frac{0.06}{0.001} = 60$$

$$\frac{R}{L} = 140; \qquad \frac{1}{LC} = 4000$$

$$V_o = \frac{60}{s^2 + 140s + 4000}$$

[**b**] $sV_o = \dfrac{60s}{s^2 + 140s + 4000}$

$$\lim_{s\to 0} sV_o = 0; \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to\infty} sV_o = 0; \qquad \therefore \quad v_o(0^+) = 0$$

[**c**] $V_o = \dfrac{60}{(s+40)(s+100)} = \dfrac{1}{s+40} + \dfrac{-1}{s+100}$

$$v_o = [e^{-40t} - e^{-100t}]u(t)\,\mathrm{V}$$

P 13.18 $I_C = \dfrac{I_g}{s} - \dfrac{V_o}{sL} = \dfrac{0.06}{s} - \dfrac{240}{s(s^2 + 140s + 4000)}$

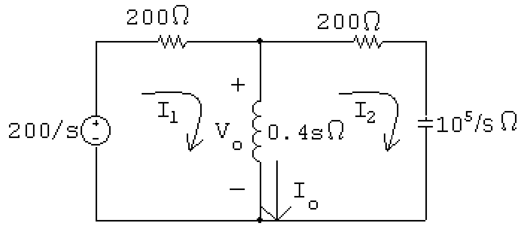
$$= \frac{0.06}{s} - \frac{0.06}{s} + \frac{0.1}{s+40} - \frac{0.04}{s+100}$$

$$\therefore \qquad i_C(t) = [100e^{-40t} - 40e^{-100t}]u(t)\,\mathrm{mA}$$

Check:

$i_C(0^+) = 60\,\mathrm{mA}$ (ok); $\qquad i_C(\infty) = 0$ (ok)

P 13.19 **[a]**



$$200i_1 + 0.4s(I_1 - I_2) = \frac{200}{s}; \qquad 200I_2 + \frac{10^5}{s}I_2 + 0.4s(I_2 - I_1) = 0$$

Solving the second equation for I_1:

$$I_1 = \frac{s^2 + 500s + 25 \times 10^4}{s^2} I_2$$

Substituting into the first equation and solving for I_2:

$$(0.4s + 200)\frac{s^2 + 500s + 25 \times 10^4}{s^2} - 0.4s = \frac{200}{s}$$

$$\therefore \qquad I_2 = \frac{0.5s}{s^2 + 500s + 125{,}000}$$

$$\therefore \qquad I_1 = \frac{s^2 + 500s + 25 \times 10^4}{s^2} \cdot \frac{0.5s}{s^2 + 500s + 125{,}000}$$

$$= \frac{0.5(s^2 + 500s + 25 \times 10^4)}{s(s^2 + 500s + 125{,}000)}$$

$$I_o = I_1 - I_2 = \frac{0.5(s^2 + 500s + 25 \times 10^4)}{s(s^2 + 500s + 125{,}000)} - \frac{0.5s}{s^2 + 500s + 125{,}000}$$

$$= \frac{250(s + 500)}{s(s^2 + 500s + 125{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 250 - j250} + \frac{K_2^*}{s + 250 + j250}$$

$K_1 = 1; \qquad K_2 = 0.5\underline{/-180^\circ} = -0.5$

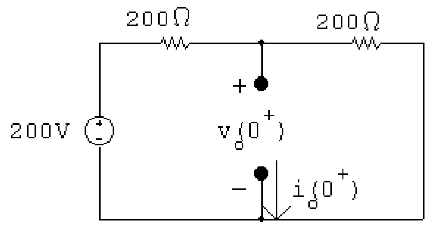
$$\therefore \qquad i_o(t) = [1 - 1e^{-250t}\cos 250t]u(t)\,\text{A}$$

[b] $V_o = 0.4sI_o = \dfrac{100(s + 500)}{s^2 + 500s + 125{,}000} = \dfrac{K_1}{s + 250 - j250} + \dfrac{K_1^*}{s + 250 + j250}$

$K_1 = 70.71\underline{/-45^\circ}$

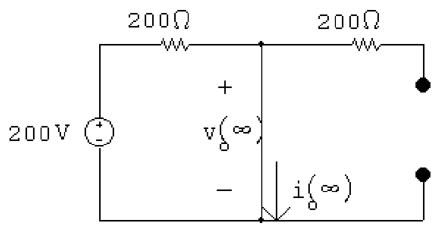
$$\therefore \qquad v_o(t) = 141.42e^{-250t}\cos(250t - 45^\circ)u(t)\,\text{V}$$

[c] At $t = 0^+$ the circuit is



$$\therefore \quad v_o(0^+) = 100\,\text{V} = 141.42\cos(-45^\circ); \qquad I_o(0^+) = 0$$

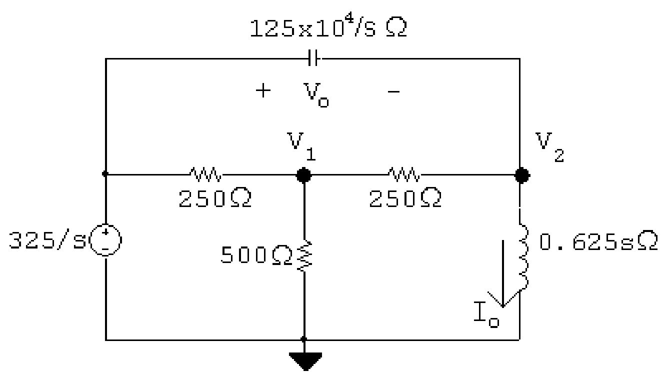
Both values agree with our solutions for v_o and i_o.
At $t = \infty$ the circuit is



$$\therefore \quad v_o(\infty) = 0; \qquad i_o(\infty) = 1\,\text{A}$$

Both values agree with our solutions for v_o and i_o.

P 13.20 **[a]**



$$\frac{V_1 - 325/s}{250} + \frac{V_1}{500} + \frac{V_1 - V_2}{250} = 0$$

$$\frac{V_2}{0.625s} + \frac{V_2 - V_1}{250} + \frac{(V_2 - 325/s)s}{125 \times 10^4} = 0$$

Thus,

$$5V_1 - 2V_2 = \frac{650}{s}$$

$$-5000sV_1 + (s^2 + 5000s + 2 \times 10^6)V_2 = 325s$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -5000s & s^2 + 5000s + 2 \times 10^6 \end{vmatrix} = 5(s + 1000)(s + 2000)$$

$$N_2 = \begin{vmatrix} 5 & 650/s \\ -5000s & 325s \end{vmatrix} = 1625(s + 2000)$$

$$V_2 = \frac{N_2}{\Delta} = \frac{1625(s + 2000)}{5(s + 1000)(s + 2000)} = \frac{325}{s + 1000}$$

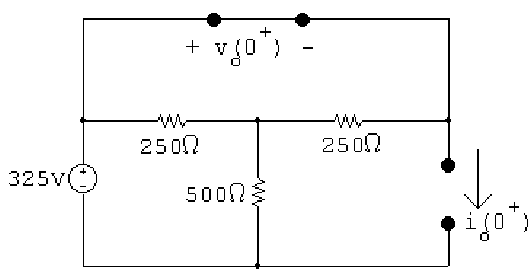
$$V_o = \frac{325}{s} - \frac{325}{s + 1000} = \frac{325{,}000}{s(s + 1000)}$$

$$I_o = \frac{V_2}{0.625s} = \frac{520}{s(s + 1000)} = \frac{0.52}{s} - \frac{0.52}{s + 1000}$$

[b] $v_o(t) = (325 - 325e^{-1000t})u(t)\,\mathrm{V}$

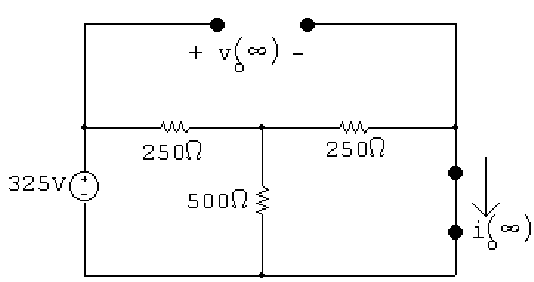
$i_o(t) = (520 - 520e^{-1000t})u(t)\,\mathrm{mA}$

[c] At $t = 0^+$ the circuit is



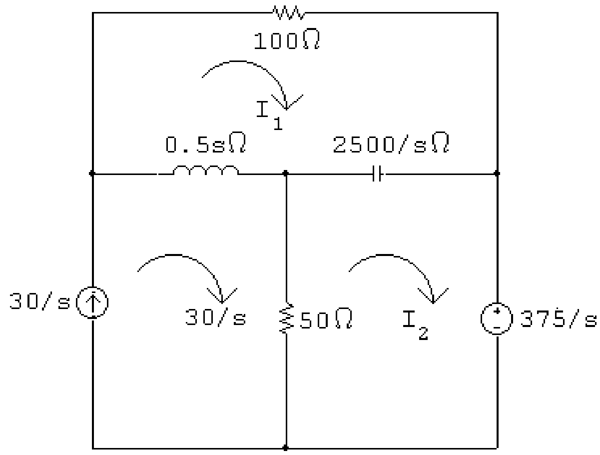
$v_o(0^+) = 0$; $\quad i_o(0^+) = 0 \quad$ Checks

At $t = \infty$ the circuit is



$v_o(\infty) = 325\,\mathrm{V}$; $\quad i_o(\infty) = \dfrac{325}{250 + (500\|250)} \cdot \dfrac{500}{750} = 0.52\,\mathrm{A} \quad$ Checks

P 13.21 **[a]**



$$0 = 0.5s(I_1 - 30/s) + \frac{2500}{s}(I_1 - I_2) + 100I_1$$

$$\frac{-375}{s} = \frac{2500}{s}(I_2 - I_1) + 50(I_2 - 30/s)$$

or

$$(s^2 + 200s + 5000)I_1 - 5000I_2 = 30s$$

$$-50I_1 + (s + 50)I_2 = 22.5$$

$$\Delta = \begin{vmatrix} (s^2 + 200s + 5000) & -5000 \\ -50 & (s + 50) \end{vmatrix} = s(s + 100)(s + 150)$$

$$N_1 = \begin{vmatrix} 30s & -5000 \\ 22.5 & (s + 50) \end{vmatrix} = 30(s^2 + 50s + 3750)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{30(s^2 + 50s + 3750)}{s(s + 100)(s + 150)}$$

$$N_2 = \begin{vmatrix} (s^2 + 200s + 5000) & 30s \\ -50 & 22.5 \end{vmatrix} = 22.5s^2 + 6000s + 112{,}500$$

$$I_2 = \frac{N_2}{\Delta} = \frac{22.5s^2 + 6000s + 112{,}500}{s(s + 100)(s + 150)}$$

[b] $sI_1 = \dfrac{30(s^2 + 50s + 3750)}{(s + 100)(s + 150)}$

$$\lim_{s\to\infty} sI_1 = i_1(0^+) = 30\,\text{A}; \qquad \lim_{s\to 0} sI_1 = i_1(\infty) = 7.5\,\text{A}$$

$$sI_2 = \frac{22.5s^2 + 6000s + 112{,}500}{(s+100)(s+150)}$$

$$\lim_{s\to\infty} sI_2 = i_2(0^+) = 22.5\,\text{A}; \qquad \lim_{s\to 0} sI_2 = i_2(\infty) = 7.5\,\text{A}$$

[c] $$I_1 = \frac{30(s^2 + 50s + 3750)}{s(s+100)(s+150)} = \frac{K_1}{s} + \frac{K_2}{s+100} + \frac{K_3}{s+150}$$

$$K_1 = 7.5; \qquad K_2 = -52.5; \qquad K_3 = 75$$

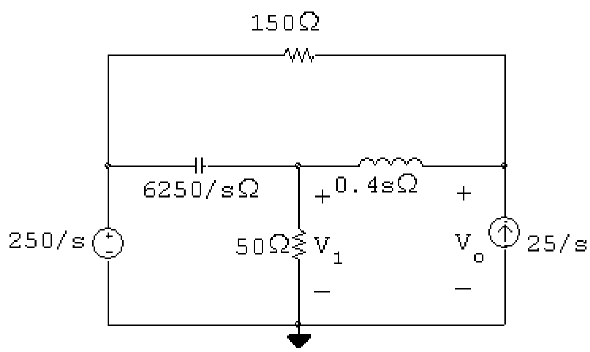
$$i_1(t) = [7.5 - 52.5e^{-100t} + 75e^{-150t}]u(t)\,\text{A}$$

$$I_2 = \frac{22.5s^2 + 6000s + 112{,}500}{s(s+100)(s+150)} = \frac{K_1}{s} + \frac{K_2}{s+100} + \frac{K_3}{s+150}$$

$$K_1 = 7.5; \qquad K_2 = 52.5; \qquad K_3 = -37.5$$

$$i_2(t) = [7.5 + 52.5e^{-100t} - 37.5e^{-250t}]u(t)\,\text{A}$$

P 13.22 **[a]**



$$\frac{V_1}{50} + \frac{V_1 - 250/s}{6250/s} + \frac{V_1 - V_o}{0.4s} = 0$$

$$\frac{-25}{s} + \frac{V_o - V_1}{0.4s} + \frac{V_o - 250/s}{150} = 0$$

Simplfying,

$$(s^2 + 125s + 15{,}625)V_1 - 15{,}625V_o = 250s$$

$$-375V_1 + (s+375)V_o = 4000$$

$$\Delta = \begin{vmatrix} (s^2 + 125s + 15{,}625) & -15{,}625 \\ -375 & (s+375) \end{vmatrix} = s(s+250)^2$$

$$N_o = \begin{vmatrix} (s^2 + 125s + 15{,}625) & 250s \\ -375 & 4000 \end{vmatrix} = 4000s^2 + 593{,}750s + 625 \times 10^5$$

$$V_o = \frac{N_o}{\Delta} = \frac{4000s^2 + 593{,}750s + 625 \times 10^5}{s(s+250)^2} = \frac{K_1}{s} + \frac{K_2}{(s+250)^2} + \frac{K_3}{s+250}$$

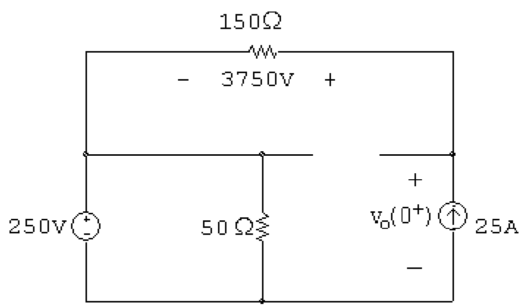
$K_1 = 1000; \quad K_2 = -656{,}250$

$$K_3 = 25\frac{d}{ds}\left[\frac{4000s^2 + 593{,}750s + 625 \times 10^5}{s}\right]_{s=-250} = 3000$$

$$\therefore \quad V_o = \frac{1000}{s} - \frac{656{,}250}{(s+250)^2} + \frac{3000}{s+250}$$

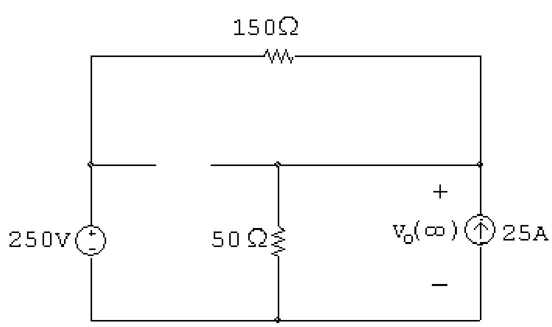
[b] $v_o(t) = [1 - 656.25te^{-250t} + 3e^{-250t}]u(t)\,\text{kV}$

[c] At $t = 0^+$:



$v_o(0^+) = 250 + 3750 = 4\,\text{kV(checks)}$

At $t = \infty$:

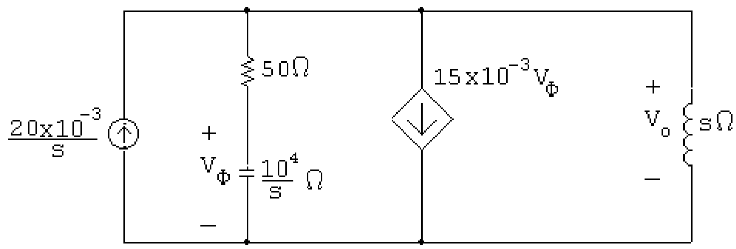


$$\frac{v_o(\infty)}{50} - 25 + \frac{v_o(\infty) - 250}{150} = 0$$

$\therefore \quad 3v_o(\infty) - 3750 + v_o(\infty) - 250 = 0; \qquad \therefore \quad 4v_o(\infty) = 4000$

$\therefore \qquad v_o(\infty) = 1\,\text{kV(checks)}$

P 13.23



$$\frac{20\times10^{-3}}{s} = \frac{V_o}{50 + 10^4/s} + 15\times10^{-3}V_\phi + \frac{V_o}{s}$$

$$V_\phi = \frac{10^4/s}{50+10^4/s}V_o = \frac{10^4V_o}{50s+10^4}$$

$$\therefore \qquad \frac{20\times10^{-3}}{s} = \frac{V_os}{50s+10^4} + \frac{150V_o}{50s+10^4} + \frac{V_o}{s}$$

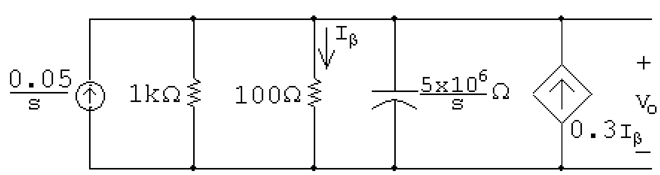
$$\therefore \quad V_o = \frac{s+200}{s^2+200s+10^4} = \frac{K_1}{(s+100)^2} + \frac{K_2}{s+100}$$

$$K_1 = 100; \qquad K_2 = 1$$

$$V_o = \frac{100}{(s+100)^2} + \frac{1}{s+100}$$

$$v_o(t) = [100te^{-100t} + e^{-100t}]u(t)\,\mathrm{V}$$

P 13.24 $v_C(0^-) = v_C(0^+) = 0$



$$\frac{0.05}{s} = \frac{V_o}{1000} + \frac{V_o}{100} + \frac{V_os}{5\times10^6} - \frac{0.3V_o}{100}$$

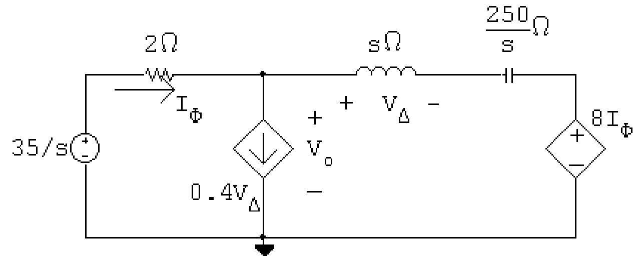
$$\frac{250\times10^3}{s} = (5000 + 50{,}000 + s - 15{,}000)V_o$$

$$V_o = \frac{250\times10^3}{s(s+40{,}000)} = \frac{K_1}{s} + \frac{K_2}{s+40{,}000}$$

$$= \frac{6.25}{s} - \frac{6.25}{s+40{,}000}$$

$$v_o(t) = [6.25 - 6.25e^{-40{,}000t}]u(t)\,\mathrm{V}$$

P 13.25 **[a]**



$$\frac{V_o - 35/s}{2} + 0.4V_\Delta + \frac{V_o - 8I_\phi}{s + (250/s)} = 0$$

$$V_\Delta = \left[\frac{V_o - 8I_\phi}{s + (250/s)}\right] s; \qquad I_\phi = \frac{(35/s) - V_o}{2}$$

Solving for V_o yields:

$$V_o = \frac{29.4s^2 + 56s + 1750}{s(s^2 + 2s + 50)} = \frac{29.4s^2 + 56s + 1750}{s(s + 1 - j7)(s + 1 + j7)}$$

$$V_o = \frac{K_1}{s} + \frac{K_2}{s + 1 - j7} + \frac{K_2^*}{s + 1 + j7}$$

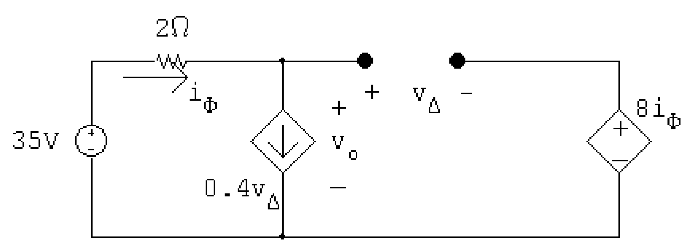
$$K_1 = \frac{29.4s^2 + 56s + 1750}{s^2 + 2s + 50}\bigg|_{s=0} = 35$$

$$K_2 = \frac{29.4s^2 + 56s + 1750}{s(s + 1 + j7)}\bigg|_{s=-1+j7}$$

$$= -2.8 + j0.6 = 2.86\underline{/167.91^\circ}$$

$$\therefore \quad v_o(t) = [35 + 5.73e^{-t}\cos(7t + 167.91^\circ)]u(t)\,\text{V}$$

[b] At $t = 0^+$ $\qquad v_o = 35 + 5.73\cos(167.91^\circ) = 29.4\,\text{V}$

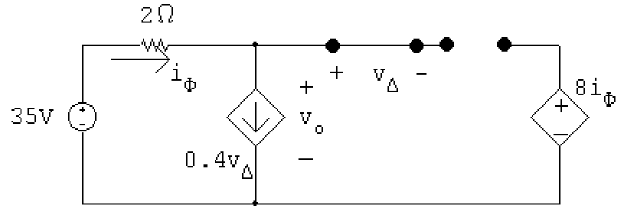


$$\frac{v_o - 35}{2} + 0.4v_\Delta = 0; \qquad v_o - 35 + 0.8v_\Delta = 0$$

$$v_o = v_\Delta + 8i_\phi = v_\Delta + 8(0.4v_\Delta) = 4.2\,\text{V}$$

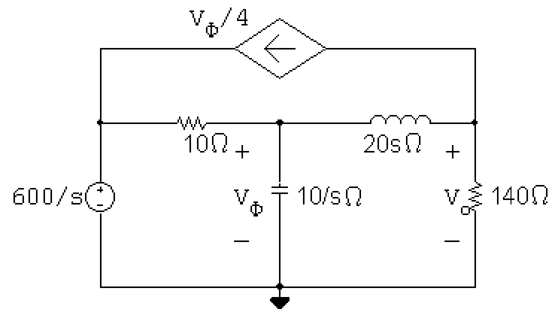
$$v_o + (0.8)\frac{v_o}{4.2} = 35; \qquad \therefore \quad v_o(0^+) = 29.4\,\text{V(checks)}$$

At $t = \infty$, the circuit is



$v_\Delta = 0, \quad i_\phi = 0 \qquad \therefore \quad v_o = 35\,\text{V(checks)}$

P 13.26 **[a]**



$$\frac{V_\phi}{10/s} + \frac{V_\phi - (600/s)}{10} + \frac{V_\phi - V_o}{20s} = 0$$

$$\frac{V_o}{140} + \frac{V_o - V_\phi}{20s} + \frac{V_\phi}{4} = 0$$

Simplfying,

$$(2s^2 + 2s + 1)V_\phi - V_o = 1200$$

$$(35s - 7)V_\phi + (s + 7)V_o = 0$$

$$\Delta = \begin{vmatrix} 2s^2 + 2s + 1 & -1 \\ 35s - 7 & s + 7 \end{vmatrix} = 2s(s^2 + 8s + 25)$$

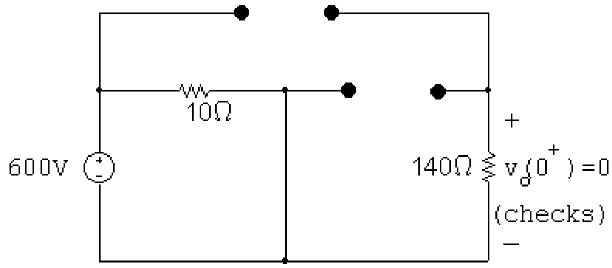
$$N_2 = \begin{vmatrix} 2s^2 + 2s + 1 & 1200 \\ 35s - 7 & 0 \end{vmatrix} = -42{,}000s + 8400$$

$$V_o = \frac{N_2}{\Delta} = \frac{-21{,}000s + 4200}{s(s^2 + 8s + 25)} = \frac{-4200(5s - 1)}{s(s^2 + 8s + 25)}$$

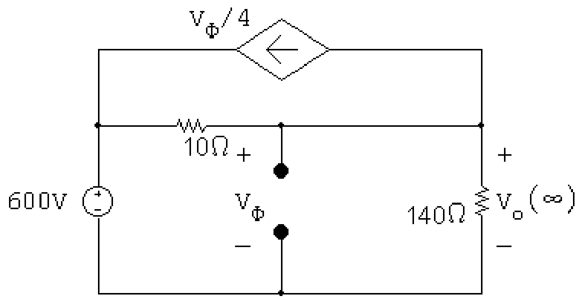
[b] $v_o(0^+) = \lim_{s\to\infty} sV_o = 0$

$$v_o(\infty) = \lim_{s\to 0} sV_o = \frac{4200}{25} = 168$$

[c] At $t = 0^+$ the circuit is



At $t = \infty$ the circuit is



$$\frac{V_\phi - 600}{10} + \frac{V_\phi}{140} + \frac{V_\phi}{4} = 0$$

$$\therefore \quad V_\phi = 168\,\text{V} = V_o(\infty) \qquad \text{(checks)}$$

[d] $$V_o = \frac{N_2}{\Delta} = \frac{-21{,}000s + 4200}{s(s^2 + 8s + 25)} = \frac{K_1}{s} + \frac{K_2}{s + 4 - j3} + \frac{K_2^*}{s + 4 + j3}$$

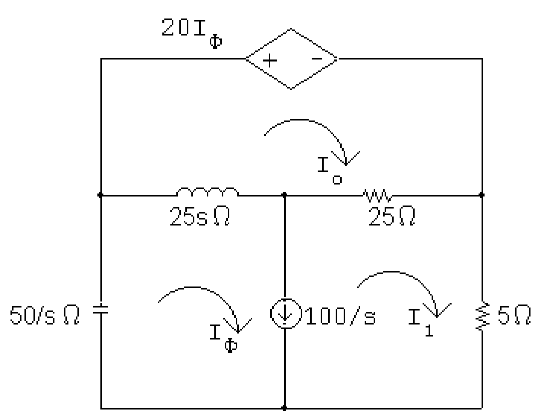
$$K_1 = \frac{4200}{25} = 168$$

$$K_2 = \frac{-21{,}000(-4 + j3) + 4200}{(-4 + j3)(j6)} = -84 + j3612 = 3612.98\underline{/91.33^\circ}$$

$$v_o(t) = [168 + 7225.95e^{-4t}\cos(3t + 91.33^\circ)]u(t)\,\text{V}$$

Check: $\quad v_o(0^+) = 0\,\text{V}; \qquad v_o(\infty) = 168\,\text{V}$

P 13.27 **[a]**



$$20I_\phi + 25s(I_o - I_\phi) + 25(I_o - I_1) = 0$$

$$25s(I_\phi - I_o) + \frac{50}{s}I_\phi + 5I_1 + 25(I_1 - I_o) = 0$$

$$I_\phi - I_1 = \frac{100}{s}; \qquad \therefore \quad I_1 = I_\phi - \frac{100}{s}$$

Simplifying,

$$(-5s - 1)I_\phi + (5s + 5)I_o = -500/s$$

$$(5s^2 + 6s + 10)I_\phi + (-5s^2 - 5s)I_o = 600$$

$$\Delta = \begin{vmatrix} -5s - 1 & 5s + 5 \\ 5s^2 + 6s + 10 & -5s^2 - 5s \end{vmatrix} = -25(s^2 + 3s + 2)$$

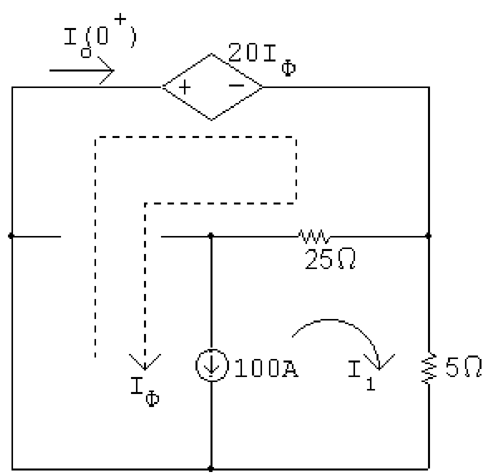
$$N_2 = \begin{vmatrix} -5s - 1 & -500/s \\ 5s^2 + 6s + 10 & 600 \end{vmatrix} = -\frac{500}{s}(s^2 - 4.8s - 10)$$

$$I_o = \frac{N_2}{\Delta} = \frac{20s^2 - 96s - 200}{s(s + 1)(s + 2)}$$

[b] $i_o(0^+) = \lim_{s\to\infty} sI_o = 20\,\text{A}$

$$i_o(\infty) = \lim_{s\to 0} sI_o = \frac{-200}{2} = -100\,\text{A}$$

[c] At $t = 0^+$ the circuit is

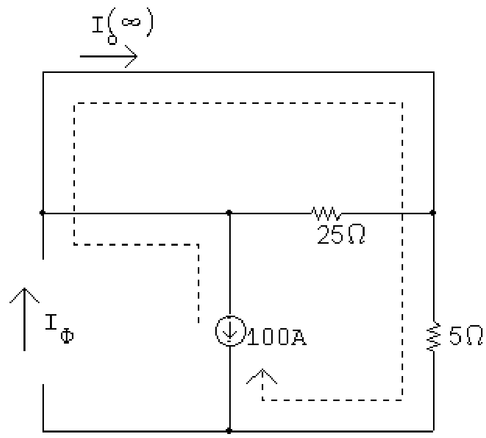


$$20I_\phi + 5I_1 = 0; \quad I_\phi - I_1 = 100$$

$$\therefore \quad 20I_\phi + 5(I_\phi - 100) = 0; \qquad 25I_\phi = 500$$

$$\therefore \quad I_\phi = I_o(0^+) = 20\,\text{A(checks)}$$

At $t + \infty$ the circuit is



$I_o(\infty) = -100\,\text{A}(\text{checks})$

[d] $I_o = \frac{20s^2 - 96s - 200}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$

$K_1 = \frac{-200}{(1)(2)} = -100; \qquad K_2 = \frac{20 + 96 - 200}{(-1)(1)} = 84$

$K_3 = \frac{80 + 192 - 200}{(-2)(-1)} = 36$

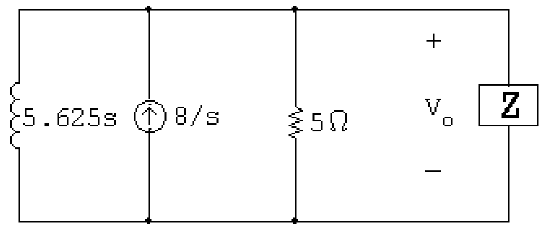
$I_o = \frac{-100}{s} + \frac{84}{s+1} + \frac{36}{s+2}$

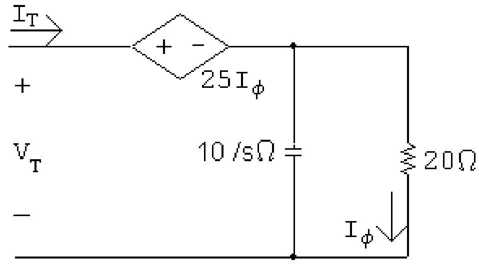
$i_o(t) = (-100 + 84e^{-t} + 36e^{-2t})u(t)\,\text{A}$

$i_o(\infty) = -100\,\text{A}(\text{checks})$

$i_o(0^+) = -100 + 84 + 36 = 20\,\text{A}(\text{checks})$

P 13.28 **[a]** $i_L(0^-) = i_L(0^+) = \dfrac{24}{3} = 8\,\text{A}$ directed upward





$$V_T = 25I_\phi + \left[\frac{20(10/s)}{20+(10/s)}\right] I_T = \frac{25I_T(10/s)}{20+(10/s)} + \left(\frac{200}{10+20s}\right) I_T$$

$$\frac{V_T}{I_T} = Z = \frac{250+200}{20s+10} = \frac{45}{2s+1}$$

$$\frac{V_o}{5} + \frac{V_o(2s+1)}{45} + \frac{V_o}{5.625s} = \frac{8}{s}$$

$$\frac{[9s+(2s+1)s+8]V_o}{45s} = \frac{8}{s}$$

$$V_o[2s^2+10s+8] = 360$$

$$V_o = \frac{360}{2s^2+10s+8} = \frac{180}{s^2+5s+4}$$

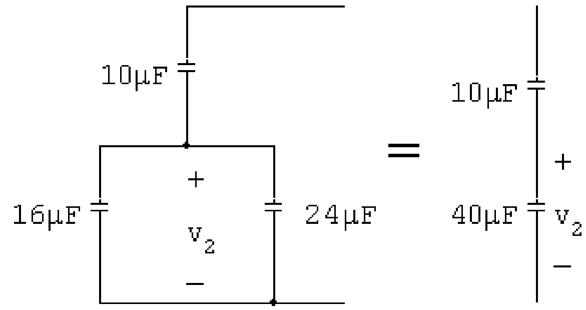
[b] $V_o = \dfrac{180}{(s+1)(s+4)} = \dfrac{K_1}{s+1} + \dfrac{K_2}{s+4}$

$$K_1 = \frac{180}{3} = 60; \qquad K_2 = \frac{180}{-3} = -60$$

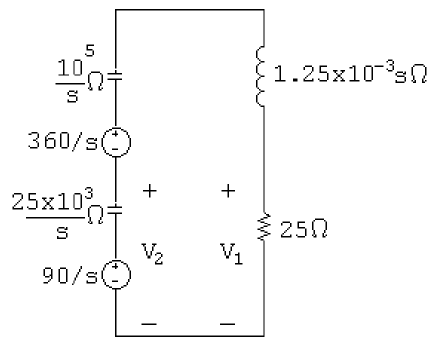
$$V_o = \frac{60}{s+1} - \frac{60}{s+4}$$

$$v_o(t) = [60e^{-t} - 60e^{-4t}]u(t)\,\text{V}$$

P 13.29 **[a]** For $t < 0$:



$$V_2 = \frac{10}{10+40}(450) = 90\,\text{V}$$



[b] $V_1 = \dfrac{25(450/s)}{(125{,}000/s) + 25 + 1.25 \times 10^{-3}s}$

$$= \frac{9 \times 10^6}{s^2 + 20,000s + 10^8} = \frac{9 \times 10^6}{(s + 10{,}000)^2}$$

$$v_1(t) = (9 \times 10^6 te^{-10{,}000t})u(t)\,\text{V}$$

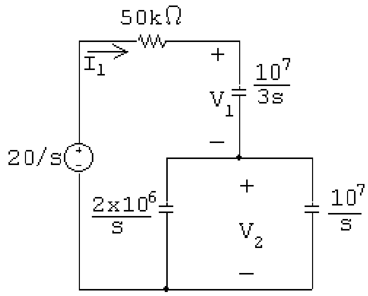
[c] $V_2 = \dfrac{90}{s} - \dfrac{(25{,}000/s)(450/s)}{(125{,}000/s) + 1.25 \times 10^{-3}s + 25}$

$$= \frac{90(s + 20{,}000)}{s^2 + 20{,}000s + 10^8}$$

$$= \frac{900{,}000}{(s + 10{,}000)^2} + \frac{90}{s + 10{,}000}$$

$$v_2(t) = [9 \times 10^5 te^{-10{,}000t} + 90e^{-10{,}000t}]u(t)\,\text{V}$$

P 13.30 **[a]**



[b] $Z_{\text{eq}} = 50{,}000 + \dfrac{10^7}{3s} + \dfrac{20 \times 10^{12}/s^2}{12 \times 10^6/s}$

$$= 50{,}000 + \frac{10^7}{3s} + \frac{20 \times 10^{12}}{12 \times 10^6 s}$$

$$= \frac{100{,}000s + 10^7}{2s}$$

$$I_1 = \frac{20/s}{Z_{\text{eq}}} = \frac{0.4 \times 10^{-3}}{s + 100}$$

$$V_1 = \frac{10^7}{3s} I_1 = \frac{4000/3}{s(s + 100)}$$

$$V_2 = \frac{10^7}{6s} \cdot \frac{0.4 \times 10^{-4}}{s + 100} = \frac{2000/3}{s(s + 100)}$$

[c] $i_1(t) = 0.4e^{-100t}u(t)\,\text{mA}$

$$V_1 = \frac{40/3}{s} - \frac{40/3}{s + 100}; \qquad v_1(t) = (40/3)(1 - 1e^{-100t})u(t)\,\text{V}$$

$$V_2 = \frac{20/3}{s} - \frac{20/3}{s + 100}; \qquad v_2(t) = (20/3)(1 - 1e^{-100t})u(t)\,\text{V}$$

[d] $i_1(0^+) = 0.4\,\text{mA}$

$$i_1(0^+) = \frac{20}{50} \times 10^{-3} = 0.44\,\text{mA(checks)}$$

$$v_1(0^+) = 0; \qquad v_2(0^+) = 0\text{(checks)}$$

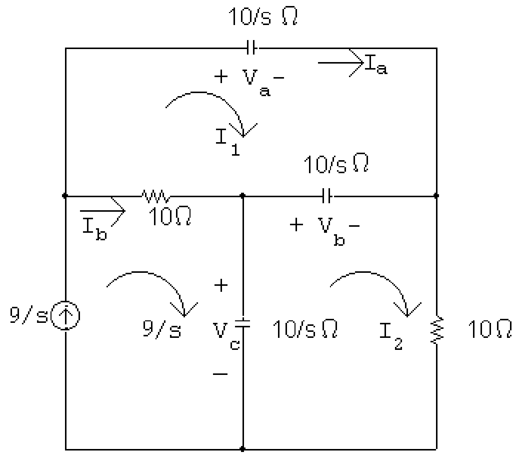
$$v_1(\infty) = 40/3\,\text{V}; \qquad v_2(\infty) = 20/3\,\text{V(checks)}$$

$$v_1(\infty) + v_2(\infty) = 20\,\text{V(checks)}$$

$$(0.3 \times 10^{-6})v_1(\infty) = 4\,\mu\text{C}$$

$$(0.6 \times 10^{-6})v_2(\infty) = 4\,\mu\text{C(checks)}$$

P 13.31 **[a]**



$$\frac{10}{s}I_1 + \frac{10}{s}(I_1 - I_2) + 10(I_1 - 9/s) = 0$$

$$\frac{10}{s}(I_2 - 9/s) + \frac{10}{s}(I_2 - I_1) + 10I_2 = 0$$

Simplifying,

$$(s+2)I_1 - I_2 = 9$$

$$-I_1 + (s+2)I_2 = \frac{9}{s}$$

$$\Delta = \begin{vmatrix} (s+2) & -1 \\ -1 & (s+2) \end{vmatrix} = s^2 + 4s + 3 = (s+1)(s+3)$$

$$N_1 = \begin{vmatrix} 9 & -1 \\ 9/s & (s+2) \end{vmatrix} = \frac{9s^2 + 18s + 9}{s} = \frac{9}{s}(s+1)^2$$

$$I_1 = \frac{N_1}{\Delta} = \frac{9}{s}\left[\frac{(s+1)^2}{(s+1)(s+3)}\right] = \frac{9(s+1)}{s(s+3)}$$

$$N_2 = \begin{vmatrix} (s+2) & 9 \\ -1 & 9/s \end{vmatrix} = \frac{18}{s}(s+1)$$

$$I_2 = \frac{N_2}{\Delta} = \frac{18(s+1)}{s(s+1)(s+3)} = \frac{18}{s(s+3)}$$

$$I_\text{a} = I_1 = \frac{9(s+1)}{s(s+3)} = \frac{3}{s} + \frac{6}{s+3}$$

$$I_\text{b} = \frac{9}{s} - I_1 = \frac{9}{s} - \frac{9(s+1)}{s(s+3)} = \frac{6}{s} - \frac{6}{s+3}$$

[b] $i_{\rm a}(t) = 3(1 + 2e^{-3t})u(t)\,{\rm A}$

$i_{\rm b}(t) = 6(1 - e^{-3t})u(t)\,{\rm A}$

[c] $V_{\rm a} = \frac{10}{s}I_{\rm b} = \frac{10}{s}\left(\frac{3}{s} + \frac{6}{s+3}\right)$

$= \frac{30}{s^2} + \frac{60}{s(s+3)} = \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$

$V_{\rm b} = \frac{10}{s}(I_2 - I_1) = \frac{10}{s}\left[\left(\frac{6}{s} - \frac{6}{s+3}\right) - \left(\frac{3}{s} + \frac{6}{s+3}\right)\right]$

$= \frac{10}{s}\left[\frac{3}{s} - \frac{12}{s+3}\right] = \frac{30}{s^2} - \frac{40}{s} + \frac{40}{s+3}$

$V_{\rm c} = \frac{10}{s}(9/s - I_2) = \frac{10}{s}\left(\frac{9}{s} - \frac{6}{s} + \frac{6}{s+3}\right)$

$= \frac{30}{s^2} + \frac{20}{s} - \frac{20}{s+3}$

[d] $v_{\rm a}(t) = [30t + 20 - 20e^{-3t}]u(t)\,{\rm V}$

$v_{\rm b}(t) = [30t - 40 + 40e^{-3t}]u(t)\,{\rm V}$

$v_{\rm c}(t) = [30t + 20 - 20e^{-3t}]u(t)\,{\rm V}$

[e] Calculating the time when the capacitor voltage drop first reaches 1000 V:

$30t + 20 - 20e^{-3t} = 1000 \quad \text{or} \quad 30t - 40 + 40e^{-3t} = 1000$

Note that in either of these expressions the exponential tem is negligible when compared to the other terms. Thus,

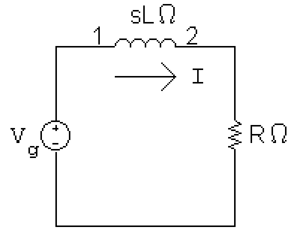
$30t + 20 = 1000 \quad \text{or} \quad 30t - 40 = 1000$

Thus,

$t = \frac{980}{30} = 32.67\,{\rm s} \quad \text{or} \quad t = \frac{1040}{30} = 34.67\,{\rm s}$

Therefore, the breakdown will occur at $t = 32.67$ s.

P 13.32 **[a]** The s-domain equivalent circuit is



$$I = \frac{V_g}{R + sL} = \frac{V_g/L}{s + (R/L)}, \qquad V_g = \frac{V_m(\omega\cos\phi + s\sin\phi)}{s^2 + \omega^2}$$

$$I = \frac{K_0}{s + R/L} + \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega}$$

$$K_0 = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}, \qquad K_1 = \frac{V_m\underline{/\phi - 90^\circ - \theta(\omega)}}{2\sqrt{R^2 + \omega^2 L^2}}$$

where $\tan\theta(\omega) = \omega L/R$. Therefore, we have

$$i(t) = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}e^{-(R/L)t} + \frac{V_m\sin[\omega t + \phi - \theta(\omega)]}{\sqrt{R^2 + \omega^2 L^2}}$$

[b] $i_{ss}(t) = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin[\omega t + \phi - \theta(\omega)]$

[c] $i_{\text{tr}} = \dfrac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}e^{-(R/L)t}$

[d] $\mathbf{I} = \dfrac{\mathbf{V}_g}{R + j\omega L}, \qquad \mathbf{V}_g = V_m\underline{/\phi - 90^\circ}$

Therefore $\quad \mathbf{I} = \dfrac{V_m\underline{/\phi - 90^\circ}}{\sqrt{R^2 + \omega^2 L^2}\underline{/\theta(\omega)}} = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\underline{/\phi - \theta(\omega) - 90^\circ}$

Therefore $\quad i_{ss} = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin[\omega t + \phi - \theta(\omega)]$

[e] The transient component vanishes when

$$\omega L\cos\phi = R\sin\phi \quad \text{or} \quad \tan\phi = \frac{\omega L}{R} \quad \text{or} \quad \phi = \theta(\omega)$$

P 13.33 $v_C = 12\times 10^5 te^{-5000t}\,\text{V}, \quad C = 5\,\mu\text{F}; \qquad$ therefore

$$i_C = C\left(\frac{dv_C}{dt}\right) = 6e^{-5000t}(1 - 5000t)\,\text{A}$$

$i_C > 0 \quad$ when $\quad 1 > 5000t \quad$ or $\quad i_C > 0 \quad$ when $\quad 0 < t < 200\,\mu\text{s}$

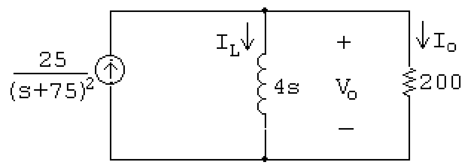
and $\quad i_C < 0 \quad$ when $\quad t > 200\,\mu\text{s}$

$i_C = 0 \quad$ when $\quad 1 - 5000t = 0, \quad$ or $\quad t = 200\,\mu\text{s}$

$$\frac{dv_C}{dt} = 12\times 10^5 e^{-5000t}[1 - 5000t]$$

$$\therefore \quad i_C = 0 \quad \text{when} \quad \frac{dv_C}{dt} = 0$$

P 13.34 **[a]**



$$200\|4s = \frac{800s}{4s+200} = \frac{200s}{s+50}$$

$$V_o = \frac{200s}{s+50}\left[\frac{25}{(s+75)^2}\right] = \frac{5000s}{(s+50)(s+75)^2}$$

$$I_o = \frac{V_o}{200} = \frac{25s}{(s+50)(s+75)^2}$$

$$I_L = \frac{V_o}{4s} = \frac{1250}{(s+50)(s+75)^2}$$

[b] $$V_o = \frac{K_1}{s+50} + \frac{K_2}{(s+75)^2} + \frac{K_3}{s+75}$$

$$K_1 = \frac{5000s}{(s+75)^2}\bigg|_{s=-50} = -400$$

$$K_2 = \frac{5000s}{(s+50)}\bigg|_{s=-75} = 15{,}000$$

$$K_3 = \frac{d}{ds}\left[\frac{5000s}{s+50}\right]_{s=-75} = \left[\frac{5000}{s+50} - \frac{5000s}{(s+50)^2}\right]_{s=-75} = 400$$

$$v_o(t) = [-400e^{-50t} + 15{,}000te^{-75t} + 400e^{-75t}]u(t)\,\text{V}$$

$$I_o = \frac{K_1}{s+50} + \frac{K_2}{(s+75)^2} + \frac{K_3}{s+75}$$

$$K_1 = \frac{25s}{(s+75)^2}\bigg|_{s=-50} = -2$$

$$K_2 = \frac{25s}{(s+50)}\bigg|_{s=-75} = 75$$

$$K_3 = \frac{d}{ds}\left[\frac{25s}{s+50}\right]_{s=-75} = \left[\frac{25}{s+50} - \frac{25s}{(s+50)^2}\right]_{s=-75} = 2$$

$$i_o(t) = [-2e^{-50t} + 75te^{-75t} + 2e^{-75t}]u(t)\,\text{V}$$

$$I_L = \frac{K_1}{s+50} + \frac{K_2}{(s+75)^2} + \frac{K_3}{s+75}$$

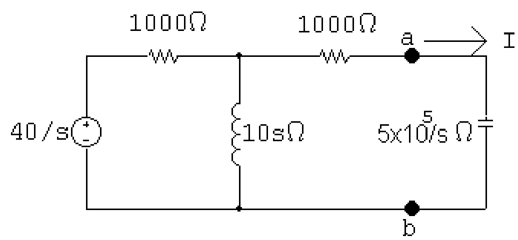
$$K_1 = \frac{1250}{(s+75)^2}\bigg|_{s=-50} = 2$$

$$K_2 = \frac{1250}{(s+50)}\bigg|_{s=-75} = -50$$

$$K_3 = \frac{d}{ds}\left[\frac{1250}{s+50}\right]_{s=-75} = \left[-\frac{1250}{(s+50)^2}\right]_{s=-75} = -2$$

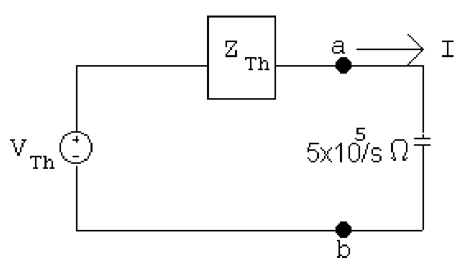
$$i_L(t) = [2e^{-50t} - 50te^{-75t} - 2e^{-75t}]u(t)\,\mathrm{V}$$

P 13.35



$$V_{\mathrm{Th}} = \frac{10s}{10s+1000}\cdot\frac{40}{s} = \frac{400}{10s+1000} = \frac{40}{s+100}$$

$$Z_{\mathrm{Th}} = 1000 + 1000\|10s = 1000 + \frac{10{,}000s}{10s+1000} = \frac{2000(s+50)}{s+100}$$



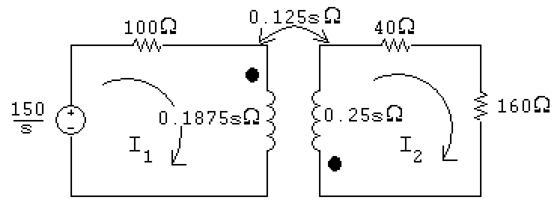
$$I = \frac{40/(s+100)}{(5\times10^5)/s + 2000(s+50)/(s+100)} = \frac{40s}{2000s^2 + 600{,}000s + 5\times10^7}$$

$$= \frac{0.02s}{s^2+300s+25{,}000} = \frac{K_1}{s+150-j50} + \frac{K_1^*}{s+150+j50}$$

$$K_1 = \frac{0.02s}{s+150+j50}\bigg|_{s=-150+j50} = 31.62\times10^{-3}\underline{/71.57^\circ}$$

$$i(t) = 63.25e^{-150t}\cos(50t+71.57^\circ)u(t)\,\mathrm{mA}$$

P 13.36 **[a]**



$$\frac{150}{s} = (100 + 0.1875s)I_1 + 0.125sI_2$$

$$0 = 0.125sI_1 + (200 + 0.25s)I_1$$

$$\Delta = \begin{vmatrix} 0.1875s + 100 & 0.125s \\ 0.125s & 0.25s + 200 \end{vmatrix} = 0.03125(s + 400)(s + 1600)$$

$$N_1 = \begin{vmatrix} 150/s & 0.125s \\ 0 & 0.25s + 200 \end{vmatrix} = \frac{37.5(s + 800)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{1200(s + 800)}{s(s + 400)(s + 1600)}$$

[b] $sI_1 = \dfrac{1200(s + 800)}{(s + 400)(s + 1600)}$

$$\lim_{s\to 0} sI_1 = i_1(\infty) = 1.5\,\text{A}$$

$$\lim_{s\to \infty} sI_1 = i_1(0) = 0$$

[c] $I_1 = \dfrac{K_1}{s} + \dfrac{K_2}{s + 400} + \dfrac{K_3}{s + 1600}$

$$K_1 = 1.5; \qquad K_2 = -1; \qquad K_3 = -0.5$$

$$i_1(t) = (1.5 - e^{-400t} - 0.5e^{-1600t})u(t)\,\text{A}$$

P 13.37 **[a]** From the solution to Problem 13.36 we have

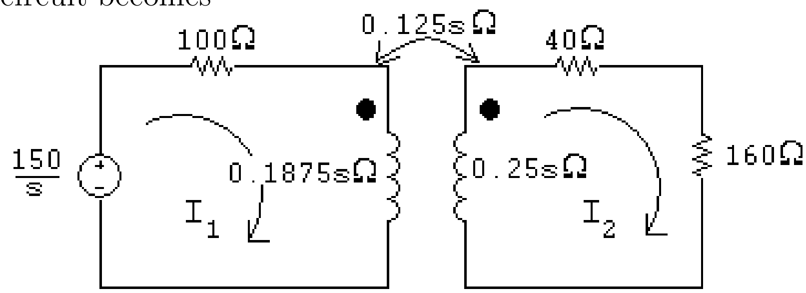
$$N_2 = \begin{vmatrix} 0.1875s + 100 & 150/s \\ 0.125s & 0 \end{vmatrix} = -18.75$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-600}{(s + 400)(s + 1600)} = \frac{K_1}{s + 400} + \frac{K_2}{s + 1600}$$

$$K_1 = \frac{-600}{1200} = -0.5; \qquad K_2 = \frac{-600}{-1200} = 0.5$$

$$i_2(t) = (-0.5e^{-400t} + 0.5e^{-1600t})u(t)\,\text{A}$$

[b] Reversing the dot on the 250 mH coil will reverse the sign of M, thus the circuit becomes



The two simulanteous equations are

$$\frac{150}{s} = (100 + 0.1875s)I_1 - 0.125sI_2$$

$$0 = -0.125sI_1 + (0.25s + 200)I_2$$

When these equations are compared to those derived in Problem 13.36 we see the only difference is the algebraic sign of the $0.125s$ term. Thus reversing the dot will have no effect on I_1 and will reverse the sign of I_2. Hence,

$$i_2(t) = (0.5e^{-400t} - 0.5e^{-1600t})u(t)\,\text{A}$$

P 13.38 **[a]** $W = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2$

$$W = 0.16(300)^2 + 0.36(200)^2 + 0.24(200)(300) = 43.2\,\text{kJ}$$

[b] $240i_1 + 0.32\frac{di_1}{dt} - 0.24\frac{di_2}{dt} = 0$

$$540i_2 + 0.72\frac{di_2}{dt} - 0.24\frac{di_1}{dt} = 0$$

Laplace transform the equations to get

$$240I_1 + 0.32(sI_1 - 300) - 0.24(sI_2 + 200) = 0$$

$$540I_2 + 0.72(sI_2 + 200) - 0.24(sI_1 - 300) = 0$$

In standard form,

$$(0.32s + 240)I_1 - 0.24sI_2 = 144$$

$$-0.24sI_1 + (0.72s + 540)I_2 = -216$$

$$\Delta = \begin{vmatrix} 0.32s + 240 & -0.24s \\ -0.24s & 0.72s + 540 \end{vmatrix} = 0.1728(s + 500)(s + 1500)$$

$$N_1 = \begin{vmatrix} 144 & -0.24s \\ -216 & 0.72s + 540 \end{vmatrix} = 51.84(s + 1500)$$

$$N_2 = \begin{vmatrix} 0.32s + 240 & 144 \\ -0.24s & -216 \end{vmatrix} = -103.68(s + 1500)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{51.84(s + 1500)}{0.1728(s + 500)(s + 1500)} = \frac{300}{s + 500}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-103.68(s + 1500)}{0.1728(s + 500)(s + 1500)} = \frac{-200}{s + 500}$$

[c] $i_1(t) = 300e^{-500t}u(t)\,\text{A}; \qquad i_2(t) = -200e^{-500t}u(t)\,\text{A}$

[d] $W_{240\Omega} = \int_0^\infty (9 \times 10^4 e^{-1000t})(240)\,dt = 216 \times 10^5 \frac{e^{-1000t}}{-1000}\Big|_0^\infty = 21{,}600\,\text{J}$

$$W_{540\Omega} = \int_0^\infty (4 \times 10^4 e^{-1000t})(540)\,dt = 216 \times 10^5 \frac{e^{-1000t}}{-1000}\Big|_0^\infty = 21{,}600\,\text{J}$$

$$W_{240\Omega} + W_{540\Omega} = 43.2\,\text{kJ}$$

[e] $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = 14{,}400 + 14{,}400 - 14{,}400 = 14.4\,\text{kJ}$

With the dot reversed the s-domain equations are

$$(0.32s + 240)I_1 + 0.24sI_2 = 48$$

$$0.24sI_1 + (0.72s + 540)I_2 = -72$$

As before, $\Delta = 0.1728(s + 500)(s + 1500)$. Now,

$$N_1 = \begin{vmatrix} 48 & -0.24s \\ -72 & 0.72s + 540 \end{vmatrix} = 51.84(s + 500)$$

$$N_2 = \begin{vmatrix} 0.32s + 240 & 48 \\ 0.24s & -72 \end{vmatrix} = -34.56(s + 500)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{300}{s + 1500}; \qquad I_2 = \frac{N_2}{\Delta} = \frac{-200}{s + 1500}$$

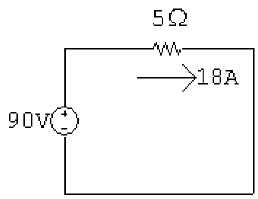
$$i_1(t) = 300e^{-1500t}u(t)\,\text{A}; \qquad i_2(t) = -200e^{-1500t}u(t)\,\text{A}$$

$$W_{240\Omega} = \int_0^\infty (9 \times 10^4 e^{-3000t})(240)\,dt = 7200\,\text{J}$$

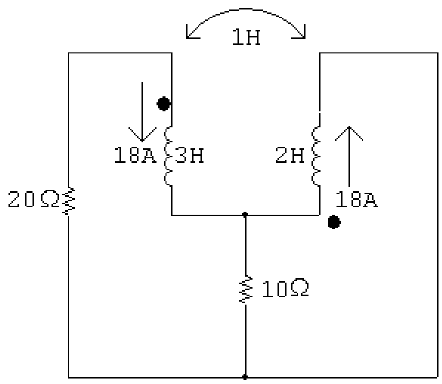
$$W_{540\Omega} = \int_0^{\infty} (4 \times 10^4 e^{-3000t})(540)\,dt = 7200\,\text{J}$$

$$W_{120\Omega} + W_{270\Omega} = 14.4\,\text{kJ}$$

P 13.39 For $t < 0$:



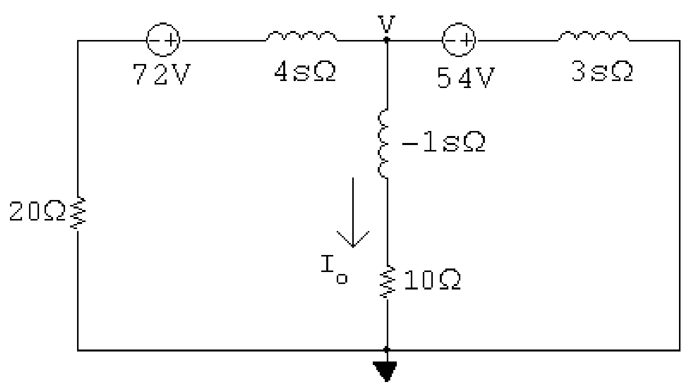
For $t > 0^+$:



Note that because of the dot locations on the coils, the sign of the mutual inductance is negative! (See Example C.1 in Appendix C.)

$L_1 - M = 3 + 1 = 4\,\text{H}; \qquad L_2 - M = 2 + 1 = 3\,\text{H}$

$18 \times 4 = 72; \qquad 18 \times 3 = 54$



$$\frac{V - 72}{4s + 20} + \frac{V}{-s + 10} + \frac{V + 54}{3s} = 0$$

$$V\left(\frac{1}{4s + 20} + \frac{1}{-s + 10} + \frac{1}{3s}\right) = \frac{72}{4s + 20} - \frac{54}{3s}$$

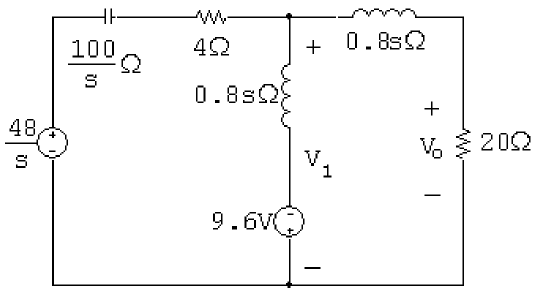
$$V\left[\frac{3s(-s+10)+3s(4s+20)+(4s+20)(-s+10)}{3s(-s+10)(4s+20)}\right] = \frac{72(3s)-54(4s+20)}{3s(4s+20)}$$

$$V = \frac{[72(3s)-54(4s+20)](-s+10)}{5s^2+110s+200}$$

$$I_o = \frac{V}{-s+10} = \frac{-108}{(s+2)(s+20)} = \frac{-1.2}{s+2} + \frac{1.2}{s+20}$$

$$i_o(t) = 1.2[e^{-20t} - e^{-2t}]u(t)\,\text{A}$$

P 13.40 The s-domain equivalent circuit is



$$\frac{V_1 - 48/s}{4+(100/s)} + \frac{V_1+9.6}{0.8s} + \frac{V_1}{0.8s+20} = 0$$

$$V_1 = \frac{-1200}{s^2+10s+125}$$

$$V_o = \frac{20}{0.8s+20}V_1 = \frac{-30{,}000}{(s+25)(s+5-j10)(s+5+j10)}$$

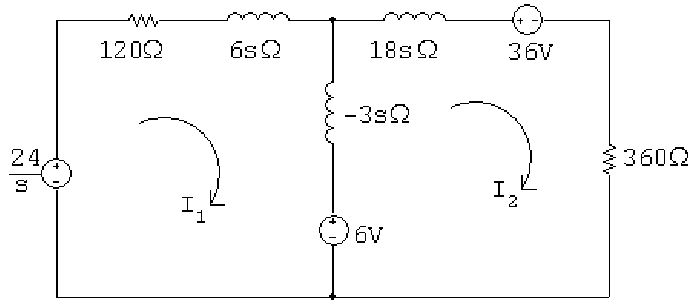
$$= \frac{K_1}{s+25} + \frac{K_2}{s+5-j10} + \frac{K_2^*}{s+5+j10}$$

$$K_1 = \frac{-30{,}000}{s^2+10s+125}\bigg|_{s=-25} = -60$$

$$K_2 = \frac{-30{,}000}{(s+25)(s+5+j10)}\bigg|_{s=-5+j10} = 67.08\underline{/63.43^\circ}$$

$$v_o(t) = [-60e^{-25t} + 134.16e^{-5t}\cos(10t+63.43^\circ)]u(t)\,\text{V}$$

P 13.41 **[a]** s-domain equivalent circuit is



Note: $\quad i_2(0^+) = -\frac{20}{10} = -2$ A

[b] $\frac{24}{s} = (120 + 3s)I_1 + 3sI_2 + 6$

$0 = -6 + 3sI_1 + (360 + 15s)I_2 + 36$

In standard form,

$(s + 40)I_1 + sI_2 = (8/s) - 2$

$sI_1 + (5s + 120)I_2 = -10$

$$\Delta = \begin{vmatrix} s+40 & s \\ s & 5s+120 \end{vmatrix} = 4(s + 20)(s + 60)$$

$$N_1 = \begin{vmatrix} (8/s)-2 & s \\ -10 & 5s+120 \end{vmatrix} = \frac{-200(s - 4.8)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{-50(s - 4.8)}{s(s + 20)(s + 60)}$$

[c] $sI_1 = \frac{-50(s - 4.8)}{(s + 20)(s + 60)}$

$$\lim_{s\to\infty} sI_1 = i_1(0^+) = 0\,\text{A}$$

$$\lim_{s\to 0} sI_1 = i_1(\infty) = \frac{(-50)(-4.8)}{(20)(60)} = 0.2\,\text{A}$$

[d] $I_1 = \frac{K_1}{s} + \frac{K_2}{s + 20} + \frac{K_3}{s + 60}$

$$K_1 = \frac{240}{1200} = 0.2; \qquad K_2 = \frac{-50(-20) + 240}{(-20)(40)} = -1.55$$

$$K_3 = \frac{-50(-60) + 240}{(-60)(-40)} = 1.35$$

$$i_1(t) = [0.2 - 1.55e^{-20t} + 1.35e^{-60t}]u(t)\,\text{A}$$

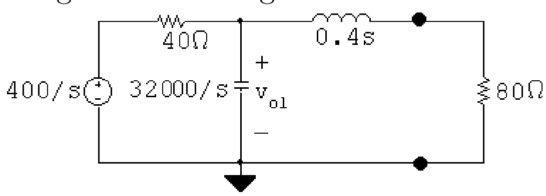
P 13.42 $\Delta = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}^2$

$$N_2 = \begin{vmatrix} Y_{11} & [(V_g/R_1) + \gamma C - (\rho/s)] \\ Y_{12} & (I_g - \gamma C) \end{vmatrix}$$

$$V_2 = \frac{N_2}{\Delta}$$

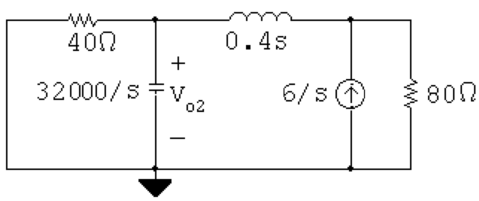
Substitution and simplification lead directly to Eq. 13.90.

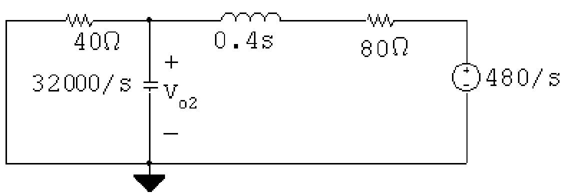
P 13.43 **[a]** Voltage source acting alone:



$$\frac{V_{o1} - 400/s}{40} + \frac{V_{01}s}{32{,}000} + \frac{V_{01}}{80 + 0.4s} = 0$$

$$\therefore \quad V_{01} = \frac{32 \times 10^4(s + 200)}{s(s + 400)(s + 600)}$$





$$\frac{V_{o2}}{40} + \frac{V_{02}s}{32{,}000} + \frac{V_{02} - 480/s}{0.4s + 80} = 0$$

$$\therefore \quad V_{02} = \frac{38.4 \times 10^6}{s(s + 400)(s + 600)}$$

$$V_o = V_{o1} + V_{o2} = \frac{32 \times 10^4(s + 200) + 38.4 \times 10^6}{s(s + 400)(s + 600)} = \frac{32 \times 10^4(s + 320)}{s(s + 400)(s + 600)}$$

[b] $V_o = \frac{K_1}{s} + \frac{K_2}{s+400} + \frac{K_3}{s+600}$

$$K_1 = \frac{(32\times 10^4)(320)}{(400)(600)} = 426.67; \qquad K_2 = \frac{32\times 10^4(-80)}{(-400)(200)} = 320;$$

$$K_3 = \frac{32\times 10^4(-280)}{(-600)(-200)} = -746.67$$

$$v_o(t) = [426.67 + 320e^{-400t} - 746.67e^{-600t}]u(t)\,\text{V}$$

P 13.44 **[a]** $V_o = -\frac{Z_f}{Z_i}V_g$

$$Z_f = \frac{2000(1.6\times 10^6/s)}{2000 + 1.6\times 10^6/s} = \frac{1.6\times 10^6}{s+800}$$

$$Z_i = 800 + \frac{10^6}{3.125s} = \frac{800(s+400)}{s}$$

$$V_g = \frac{5000}{s^2}$$

$$\therefore \quad V_o = \frac{-10^7}{s(s+400)(s+800)}$$

[b] $V_o = \frac{K_1}{s} + \frac{K_2}{s+400} + \frac{K_3}{s+800}$

$$K_1 = \frac{-10^7}{(400)(800)} = -31.25$$

$$K_2 = \frac{-10^7}{(-400)(400)} = 62.5$$

$$K_3 = \frac{-10^7}{(-800)(-400)} = -31.25$$

$$\therefore \quad v_o(t) = (-31.25 + 62.5e^{-400t} - 31.25e^{-800t})u(t)\,\text{V}$$

[c] $-31.25 + 62.5e^{-400t_s} - 31.25e^{-800t_s} = -10$

$$\therefore \quad 62.5e^{-400t_s} - 31.25e^{-800t_s} = 21.25$$

Let $x = e^{-400t_s}$. Then

$62.5x - 31.25x^2 = 21.25;$ or $x^2 - 2x + 0.68 = 0$

Solving,

$x = 0.4343$

$\therefore \quad e^{-400t_s} = 0.4343; \qquad \therefore \quad t_s = 2.1\,\text{ms}$

[d] $v_g = m\,tu(t); \qquad V_g = \dfrac{m}{s^2}$

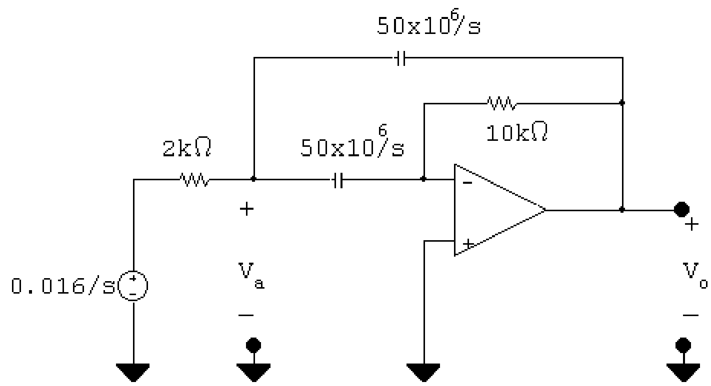
$$V_o = \frac{-2000m}{s(s+400)(s+800)}$$

$$K_1 = \frac{-2000m}{(400)(800)} = \frac{-2000m}{32\times10^4}$$

$$\therefore \quad -10 = \frac{-2000m}{32\times10^4} \qquad \therefore \quad m = 1600\,\text{V/s}$$

Thus, m must be less than or equal to 1600 V/s to avoid saturation.

P 13.45



$$\frac{V_a - 0.016/s}{2000} + \frac{V_a s}{50\times10^6} + \frac{(V_a - V_o)s}{50\times10^6} = 0$$

$$\frac{(0 - V_a)s}{50\times10^6} + \frac{(0 - V_o)}{10{,}000} = 0$$

$$V_a = \frac{-5000V_o}{s}$$

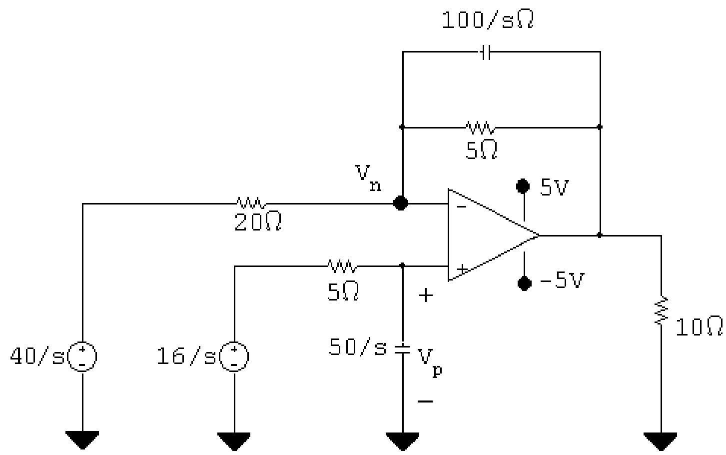
$$\therefore \quad \frac{-5000V_o}{s}(2s + 25{,}000) - sV_o = 25{,}000\left(\frac{0.016}{s}\right)$$

$$V_o = \frac{-4000}{(s + 5000 - j10{,}000)(s + 5000 + j10{,}000)}$$

$$K_1 = \frac{-400}{j10{,}000} = j0.02 = 0.02\underline{/90^\circ}$$

$$v_o(t) = 40e^{-5000t}\cos(10{,}000t + 90^\circ) = -40e^{-5000t}\sin(10{,}000t)u(t)\,\text{mV}$$

P 13.46 [**a**]



$$V_p = \frac{50/s}{5 + 50/s}V_{g2} = \frac{50}{5s + 50}V_{g2}$$

$$\frac{V_p - 40/s}{20} + \frac{V_p - V_o}{5} + \frac{V_p - V_o}{100/s} = 0$$

$$V_p\left(\frac{1}{20} + \frac{1}{5} + \frac{s}{100}\right) - V_o\left(\frac{1}{5} + \frac{s}{100}\right) = \frac{2}{s}$$

$$\frac{s + 25}{100}\left(\frac{50}{5s + 50}\right)\frac{16}{s} - \frac{2}{s} = V_o\left(\frac{1}{5} + \frac{s}{100}\right) = V_o\left(\frac{s + 20}{100}\right)$$

$$V_o = \frac{100}{s + 20}\left[\frac{16(s + 25)}{10(s + 10)(s)} - \frac{2}{s}\right] = \frac{-40s + 2000}{s(s + 10)(s + 20)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 10} + \frac{K_3}{s + 20}$$

$$K_1 = 10; \qquad K_2 = -24; \qquad K_3 = 14$$

$$\therefore \qquad v_o(t) = [10 - 24e^{-10t} + 14e^{-20t}]u(t)\,\text{V}$$

[**b**] $10 - 24x + 14x^2 = 5$

$$14x^2 - 24x + 5 = 0$$

$$x = 0 \quad \text{or} \quad 0.242691$$

$$e^{-10t} = 0.242691 \qquad \therefore \qquad t = 141.60\,\text{ms}$$

P 13.47 [**a**] Let v_a be the voltage across the $0.5\,\mu$F capacitor, positive at the upper terminal.
Let v_b be the voltage across the $100\,\text{k}\Omega$ resistor, positive at the upper terminal.

Also note

$$\frac{10^6}{0.5s} = \frac{2\times 10^6}{s} \quad \text{and} \quad \frac{10^6}{0.25s} = \frac{4\times 10^6}{s}; \qquad V_g = \frac{0.5}{s}$$

$$\frac{sV_\text{a}}{s\times 10^6} + \frac{V_\text{a} - (0.5/s)}{200{,}000} + \frac{V_\text{a}}{200{,}000} = 0$$

$$sV_\text{a} + 10V_\text{a} - \frac{5}{s} + 10V_\text{a} = 0$$

$$V_\text{a} = \frac{5}{s(s+20)}$$

$$\frac{0 - V_\text{a}}{200{,}000} + \frac{(0 - V_\text{b})s}{4\times 10^6} = 0$$

$$\therefore \qquad V_\text{b} = -\frac{20}{s}V_\text{a} = \frac{-100}{s^2(s+20)}$$

$$\frac{V_\text{b}}{100{,}000} + \frac{(V_\text{b} - 0)s}{4\times 10^6} + \frac{(V_\text{b} - V_o)s}{4\times 10^6} = 0$$

$$40V_\text{b} + sV_\text{b} + sV_\text{b} = sV_o$$

$$\therefore \qquad V_o = \frac{2(s+20)V_\text{b}}{s}; \qquad V_o = 2\left(\frac{-100}{s^3}\right) = \frac{-200}{s^3}$$

[b] $v_o(t) = -100t^2u(t)\,\text{V}$

[c] $-100t^2 = -4; \qquad t = 0.2\,\text{s} = 200\,\text{ms}$

P 13.48 **[a]** $\dfrac{1/sC}{R + 1/sC} = \dfrac{1}{RsC + 1} = \dfrac{1/RC}{s + 1/RC}$

There are no zeros, and a single pole at $-1/RC$ rad/sec.

[b] $\dfrac{R}{R + sL} = \dfrac{R/L}{s + R/L}$

There are no zeros, and a single pole at $-R/L$ rad/sec.

[c] There are several possible solutions. One is

$$R = 10\,\Omega; \qquad L = 10\,\text{mH}; \qquad C = 100\,\mu\text{F}$$

P 13.49 **[a]** $\dfrac{R}{R + 1/sC} = \dfrac{RsC}{RsC + 1} = \dfrac{s}{s + 1/RC}$

There is a single zero at 0 rad/sec, and a single pole at $-1/RC$ rad/sec.

[b] $\dfrac{sL}{R + sL} = \dfrac{s}{s + R/L}$

There is a single zero at 0 rad/sec, and a single pole at $-R/L$ rad/sec.

[c] There are several possible solutions. One is

$$R = 100\,\Omega; \qquad L = 10\,\text{mH}; \qquad C = 1\,\mu\text{F}$$

P 13.50 **[a]** $\dfrac{R}{1/sC + sL + R} = \dfrac{(R/L)s}{s^2 + (R/L)s + 1/LC}$

There is a single zero at 0 rad/sec, and two poles:

$p_1 = -(R/2L) + \sqrt{(R/2L)^2 - (1/LC)}; \qquad p_2 = -(R/2L) - \sqrt{(R/2L)^2 - (1/LC)}$

[b] There are several possible solutions. One is

$R = 250\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$

These component values yield the following poles:

$-p_1 = -5000$ rad/sec and $-p_2 = -20{,}000$ rad/sec

[c] There are several possible solutions. One is

$R = 200\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$

These component values yield the following poles:

$-p_1 = -10{,}000$ rad/sec and $-p_2 = -10{,}000$ rad/sec

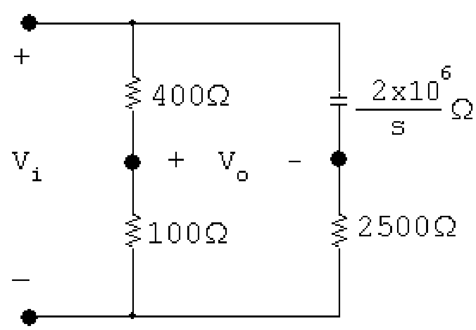
[d] There are several possible solutions. One is

$R = 120\,\Omega; \quad L = 10\,\text{mH}; \quad C = 1\,\mu\text{F}$

These component values yield the following poles:

$-p_1 = -6000 + j8000$ rad/sec and $-p_2 = -6000 - j8000$ rad/sec

P 13.51 **[a]**



$$\frac{100}{500}V_i = V_o + \frac{2500V_i}{2500 + (2 \times 10^6/s)}$$

$$0.2V_i - \frac{sV_i}{s+800} = V_o$$

$$\therefore \quad \frac{V_o}{V_i} = H(s) = \frac{-0.8(s-200)}{(s+800)}$$

[b] $-z_1 = 200\,\text{rad/s}$

$-p_1 = -800\,\text{rad/s}$

P 13.52 **[a]** $\frac{V_o}{V_i} = \frac{1/sC}{R + 1/sC} = \frac{1}{RCs + 1}$

$$H(s) = \frac{(1/RC)}{s + (1/RC)} = \frac{25}{s + 25}; \qquad -p_1 = -25\,\text{rad/s}$$

[b] $\frac{V_o}{V_i} = \frac{R}{R + 1/sC} = \frac{RCs}{RCs + 1} = \frac{s}{s + (1/RC)}$

$$= \frac{s}{s + 25}; \qquad z_1 = 0, \quad -p_1 = -25\,\text{rad/s}$$

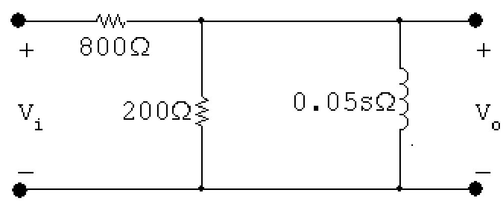
[c] $\frac{V_o}{V_i} = \frac{sL}{R + sL} = \frac{s}{s + R/L} = \frac{s}{s + 2000}$

$$z_1 = 0; \qquad -p_1 = -2000\,\text{rad/s}$$

[d] $\frac{V_o}{V_i} = \frac{R}{R + sL} = \frac{R/L}{s + (R/L)} = \frac{2000}{s + 2000}$

$$-p_1 = -2000\,\text{rad/s}$$

[e]



$$\frac{V_o}{0.05s} + \frac{V_o}{200} + \frac{V_o - V_i}{800} = 0$$

$$sV_o + 4sV_o + 16{,}000V_o = sV_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{0.2s}{s + 3200}$$

$$z_1 = 0; \qquad -p_1 = -3200\,\text{rad/s}$$

P 13.53 **[a]** Let $R_1 = 250\,\text{k}\Omega$; $R_2 = 125\,\text{k}\Omega$; $C_2 = 1.6\,\text{nF}$; and $C_f = 0.4\,\text{nF}$. Then

$$Z_f = \frac{(R_2 + 1/sC_2)1/sC_f}{\left(R_2 + \frac{1}{sC_2} + \frac{1}{sC_f}\right)} = \frac{(s + 1/R_2C_2)}{C_f s\left(s + \frac{C_2 + C_f}{C_2C_fR_2}\right)}$$

$$\frac{1}{C_f} = 2.5 \times 10^9$$

$$\frac{1}{R_2C_2} = \frac{62.5 \times 10^7}{125 \times 10^3} = 5000\,\text{rad/s}$$

$$\frac{C_2 + C_f}{C_2C_fR_2} = \frac{2 \times 10^{-9}}{(0.64 \times 10^{-18})(125 \times 10^3)} = 25{,}000\,\text{rad/s}$$

$$\therefore \quad Z_f = \frac{2.5 \times 10^9(s + 5000)}{s(s + 25{,}000)}\Omega$$

$$Z_i = R_1 = 250 \times 10^3\,\Omega$$

$$H(s) = \frac{V_o}{V_g} = \frac{-Z_f}{Z_i} = \frac{-10^4(s + 5000)}{s(s + 25{,}000)}$$

[b] $-z_1 = -5000\,\text{rad/s}$

$-p_1 = 0; \qquad -p_2 = -25{,}000\,\text{rad/s}$

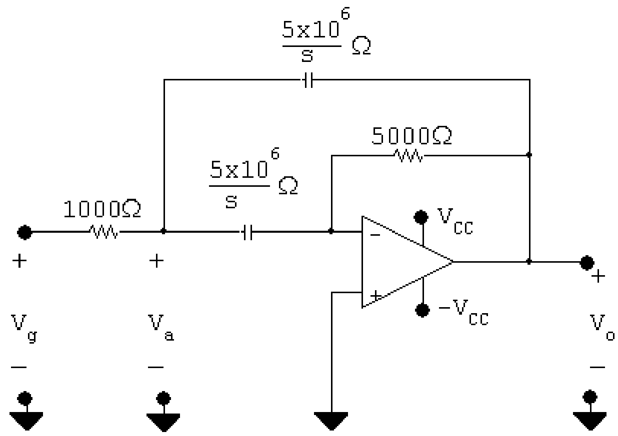
P 13.54 **[a]** $Z_i = 250 + \dfrac{31{,}250}{s} = \dfrac{250(s + 125)}{s}$

$$Z_f = \frac{25 \times 10^4}{s} \| 10{,}000 = \frac{25 \times 10^4}{s + 25}$$

$$H(s) = -\frac{Z_f}{Z_i} = \frac{-25 \times 10^4/(s + 25)}{250(s + 125)/s} = \frac{-1000s}{(s + 25)(s + 125)}$$

[b] Zero at $z_1 = 0$; Poles at $-p_1 = -25$ rad/s and $-p_2 = -125$ rad/s

P 13.55 **[a]**



$$\frac{V_a - V_g}{1000} + \frac{sV_a}{5 \times 10^6} + \frac{(V_a - V_o)s}{5 \times 10^6} = 0$$

$$5000V_a - 5000V_g + 2sV_a - sV_o = 0$$

$$(5000 + 2s)V_a - sV_o = 5000V_g$$

$$\frac{(0 - V_a)s}{5 \times 10^6} + \frac{0 - V_o}{5000} = 0$$

$$-sV_a - 1000V_o = 0; \quad \therefore \qquad V_a - \frac{-1000}{s}V_o$$

$$(2s + 5000)\left(\frac{-1000}{s}\right)V_o - sV_o = 5000V_g$$

$$1000V_o(2s + 5000) + s^2V_o = -5000sV_g$$

$$V_o(s^2 + 2000s + 5 \times 10^6) = -5000sV_g$$

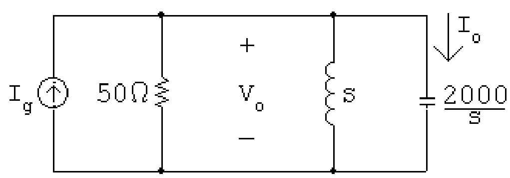
$$\frac{V_o}{V_g} = \frac{-5000s}{s^2 + 2000s + 5 \times 10^6}$$

$$s_{1,2} = -1000 \pm \sqrt{10^6 - 5 \times 10^6} = -1000 \pm j2000$$

$$\frac{V_o}{V_g} = \frac{-5000s}{(s + 1000 - j2000)(s + 1000 + j2000)}$$

[b] $z_1 = 0; \qquad -p_1 = -1000 + j2000; \qquad -p_2 = -1000 - j2000$

P 13.56 **[a]**



$$\frac{V_o}{50} + \frac{V_o}{s} + \frac{V_o s}{2000} = I_g$$

$$V_o = \frac{2000s}{s^2 + 40s + 2000} \cdot I_g \qquad \text{and} \qquad I_o = 5 \times 10^{-4} sV_o$$

$$\therefore \quad H(s) = \frac{I_o}{I_g} = \frac{s^2}{s^2 + 40s + 2000}$$

[b] $I_g = \dfrac{0.025s}{s^2 + 40{,}000}$ so $I_o = \dfrac{(s^2)(0.025s)}{(s + 20 - j40)(s + 20 + j40)(s^2 + 40{,}000)}$

$$I_o = \frac{0.025s^3}{(s + 20 - j40)(s + 20 + j40)(s + j200)(s - j200)}$$

[c] Damped sinusoid of the form

$$Me^{-20t}\cos(40t + \theta_1)$$

[d] Steady-state sinusoid of the form

$$N\cos(200t + \theta_2)$$

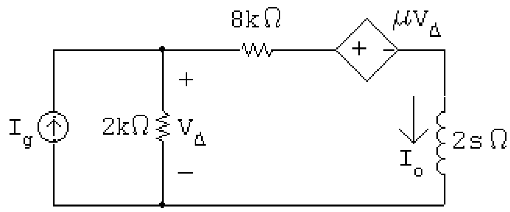
[e] $I_o = \dfrac{K_1}{s + 20 - j40} + \dfrac{K_1^*}{s + 20 + j40} + \dfrac{K_2}{s - j200} + \dfrac{K_2^*}{s + j200}$

$$K_1 = \frac{0.025(-20 + j40)^3}{(j80)(-20 - j160)(-20 + j240)} = 719.77 \times 10^{-6}\underline{/-97.94^\circ}$$

$$K_2 = \frac{0.025(j200)^3}{(j400)(20 + j160)(20 + j240)} = 12.88 \times 10^{-3}\underline{/11.89^\circ}$$

$$i_o(t) = [1.44e^{-20t}\cos(40t - 97.94^\circ) + 25.76\cos(200t + 11.89^\circ)]\,\text{mA}$$

P 13.57 [**a**]



$$2000(I_o - I_g) + 8000I_o + \mu(I_g - I_o)(2000) + 2sI_o = 0$$

$$\therefore \quad I_o = \frac{1000(1-\mu)}{s + 1000(5-\mu)} I_g$$

$$\therefore \quad H(s) = \frac{1000(1-\mu)}{s + 1000(5-\mu)}$$

[**b**] $\mu < 5$

[**c**]

μ	$H(s)$	I_o
-3	$4000/(s+8000)$	$20{,}000/s(s+8000)$
0	$1000/(s+5000)$	$5000/s(s+5000)$
4	$-3000/(s+1000)$	$-15{,}000/s(s+1000)$
5	$-4000/s$	$-20{,}000/s^2$
6	$-5000/(s-1000)$	$-25{,}000/s(s-1000)$

$\mu = -3$:

$$I_o = \frac{2.5}{s} - \frac{2.5}{(s+8000)}; \qquad i_o = [2.5 - 2.5e^{-8000t}]u(t)\,\text{A}$$

$\mu = 0$:

$$I_o = \frac{1}{s} - \frac{1}{s+5000}; \qquad i_o = [1 - e^{-5000t}]u(t)\,\text{A}$$

$\mu = 4$:

$$I_o = \frac{-15}{s} - \frac{15}{s+1000}; \qquad i_o = [-15 + 15e^{-1000t}]u(t)\,\text{A}$$

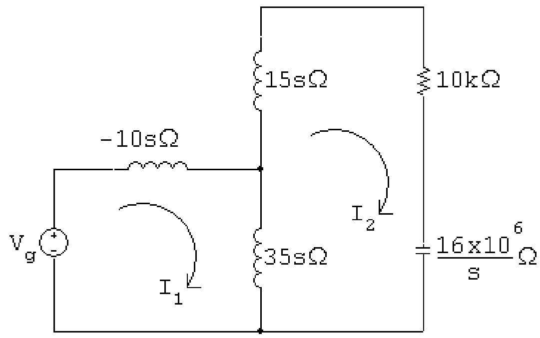
$\mu = 5$:

$$I_o = \frac{-20{,}000}{s^2}; \qquad i_o = -20{,}000t\,u(t)\,\text{A}$$

$\mu = 6$:

$$I_o = \frac{25}{s} - \frac{25}{s-1000}; \qquad i_o = 25[1 - e^{1000t}]u(t)\,\text{A}$$

P 13.58



$$V_g = 25sI_1 - 35sI_2$$

$$0 = -35sI_1 + \left(50s + 10{,}000 + \frac{16\times10^6}{s}\right)I_2$$

$$\Delta = \begin{vmatrix} 25s & -35s \\ -35s & 50s + 10{,}000 + 16\times10^6/s \end{vmatrix} = 25(s+2000)(s+8000)$$

$$N_2 = \begin{vmatrix} 25s & V_g \\ -35s & 0 \end{vmatrix} = 35sV_g$$

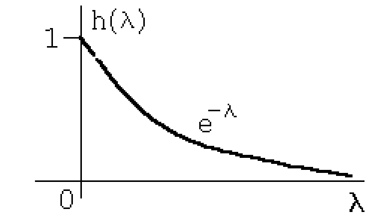
$$I_2 = \frac{N_2}{\Delta} = \frac{35sV_g}{25(s+2000)(s+8000)}$$

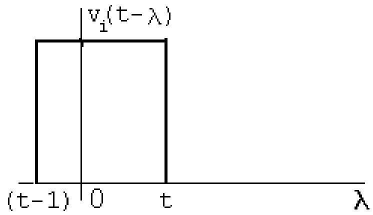
$$H(s) = \frac{I_2}{V_g} = \frac{1.4s}{(s+2000)(s+8000)}$$

$\therefore \quad z_1 = 0; \qquad -p_1 = -2000$ rad/s; $\qquad -p_2 = -8000$ rad/s

P 13.59 $H(s) = \frac{V_o}{V_i} = \frac{1}{s+1}; \qquad h(t) = e^{-t}$

For $\quad 0 \le t \le 1$:





$$v_o = \int_0^t e^{-\lambda}\, d\lambda = (1 - e^{-t})\,\text{V}$$

For $\quad 1 \le t \le \infty$:

$$v_o = \int_{t-1}^t e^{-\lambda}\, d\lambda = (e-1)e^{-t}\,\text{V}$$

P 13.60 $H(s) = \frac{V_o}{V_i} = \frac{s}{s+1} = 1 - \frac{1}{s+1}; \qquad h(t) = \delta(t) - e^{-t}$

$$h(\lambda) = \delta(\lambda) - e^{-\lambda}$$

For $\quad 0 \le t \le 1$:

$$v_o = \int_0^t [\delta(\lambda) - e^{-\lambda}]\, d\lambda = [1 + e^{-\lambda}]\, \Big|_0^t = e^{-t}\,\text{V}$$

For $\quad 1 \le t \le \infty$:

$$v_o = \int_{t-1}^t (-e^{-\lambda})\, d\lambda = e^{-\lambda}\, \Big|_{t-1}^t = (1-e)e^{-t}\,\text{V}$$

P 13.61 **[a]** From Problem 13.52(a)

$$H(s) = \frac{25}{s+25}$$

$$h(\lambda) = 25e^{-25\lambda}$$

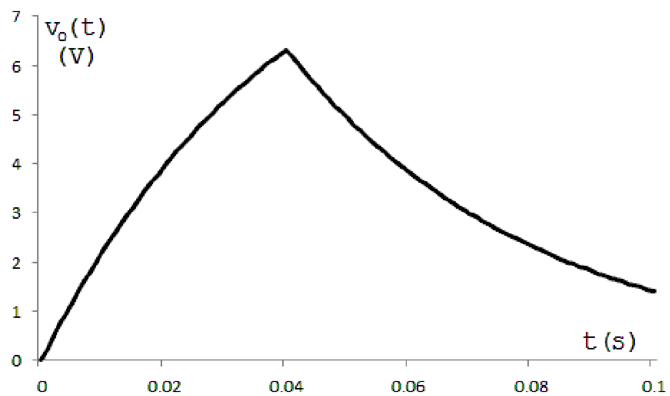
$0 \le t \le 40\,\text{ms}$:

$$v_o = \int_0^t 10(25)e^{-25\lambda}\,d\lambda = 10(1 - e^{-25t})\,\text{V}$$

$40\,\text{ms} \le t \le \infty$:

$$v_o = \int_{t-0.04}^t 10(25)e^{-25\lambda}\,d\lambda = 10(e - 1)e^{-25t}\,\text{V}$$

[**b**]



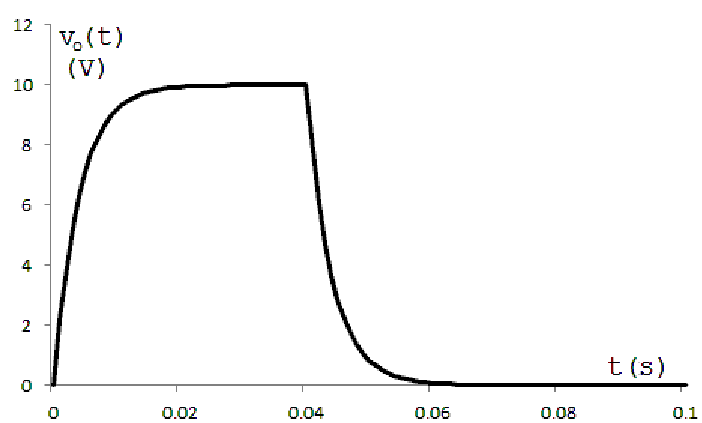
P 13.62 [**a**] $H(s) = \dfrac{250}{s + 250} \qquad \therefore \quad h(\lambda) = 250e^{-250\lambda}$

$0 \le t \le 40\,\text{ms}$:

$$v_o = \int_0^t 10(250)e^{-250\lambda}\,d\lambda = 10(1 - e^{-250t})\,\text{V}$$

$40\,\text{ms} \le t \le \infty$:

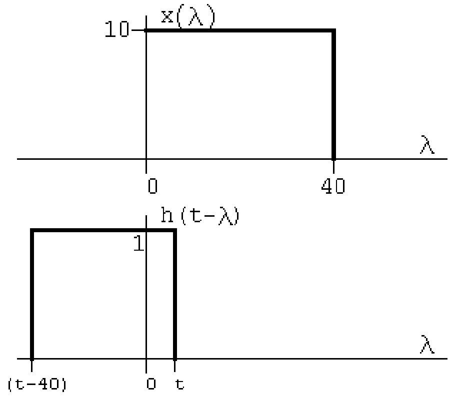
$$v_o = \int_{t-0.04}^t 10(250)e^{-250\lambda}\,d\lambda = 10(e^{10} - 1)e^{-250t}\,\text{V}$$



[**b**] decrease

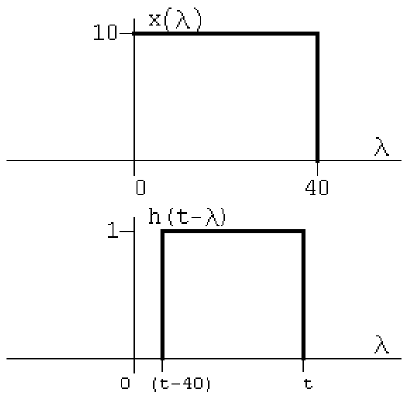
[**c**] The circuit with $R = 200\,\Omega$.

P 13.63 **[a]** $0 \leq t \leq 40$:



$$y(t) = \int_0^t (10)(1)(d\lambda) = 10\lambda \Big|_0^t = 10t$$

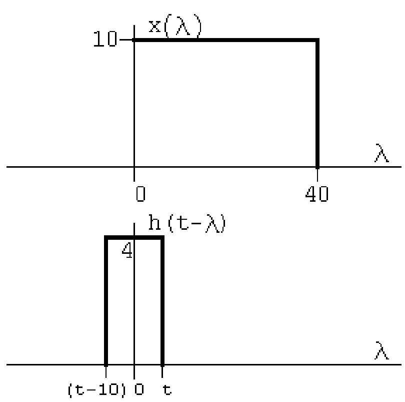
$40 \leq t \leq 80$:



$$y(t) = \int_{t-40}^{40} (10)(1)(d\lambda) = 10\lambda \Big|_{t-40}^{40} = 10(80 - t)$$

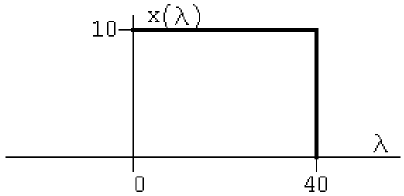
$t \geq 80$: $\quad y(t) = 0$

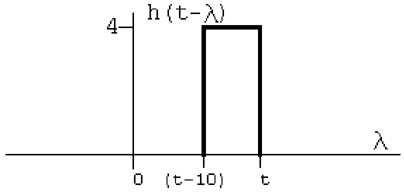
[b] $0 \leq t \leq 10$:



$$y(t) = \int_0^t 40\, d\lambda = 40\lambda \Big|_0^t = 40t$$

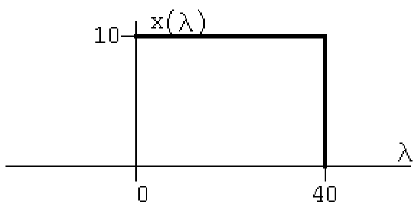
$10 \le t \le 40$:

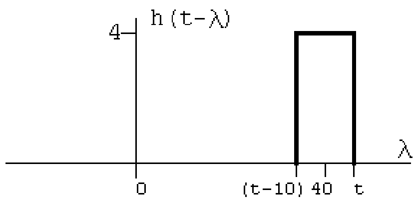




$$y(t) = \int_{t-10}^{t} 40\,d\lambda = 40\lambda \Big|_{t-10}^{t} = 400$$

$40 \le t \le 50$:





$$y(t) = \int_{t-10}^{40} 40\,d\lambda = 40\lambda \Big|_{t-10}^{40} = 40(50-t)$$

$t \ge 50: \qquad y(t) = 0$

[**c**] The expressions are

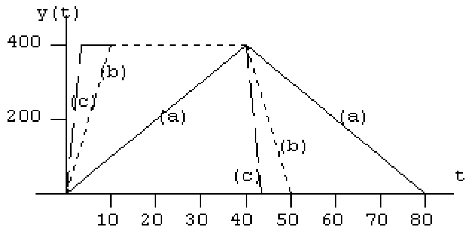
$$0 \le t \le 1: \qquad y(t) = \int_{0}^{t} 400\,d\lambda = 400\lambda \Big|_{0}^{t} = 400t$$

$$1 \le t \le 40: \qquad y(t) = \int_{t-1}^{t} 400\,d\lambda = 400\lambda \Big|_{t-1}^{t} = 400$$

$$40 \le t \le 41: \qquad y(t) = \int_{t-1}^{40} 400\,d\lambda = 400\lambda \Big|_{t-1}^{40} = 400(41-t)$$

$$41 \le t < \infty: \qquad y(t) = 0$$

[**d**]



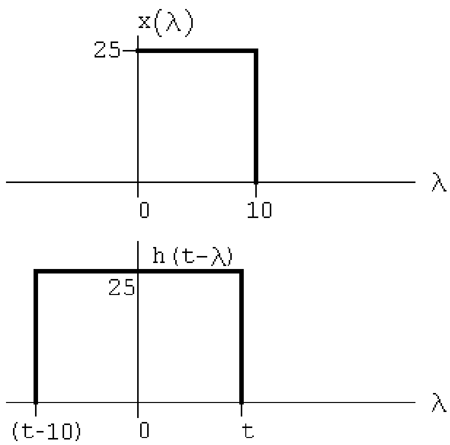
[**e**] Yes, note that $h(t)$ is approaching $40\delta(t)$, therefore $y(t)$ must approach $40x(t)$, i.e.

$$y(t) = \int_0^t h(t-\lambda)x(\lambda)\,d\lambda \to \int_0^t 40\delta(t-\lambda)x(\lambda)\,d\lambda$$

$$\to 40x(t)$$

This can be seen in the plot, e.g., in part (c), $y(t) \cong 40x(t)$.

P 13.64 [**a**]

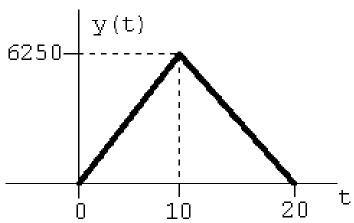


$y(t) = 0 \qquad t < 0$

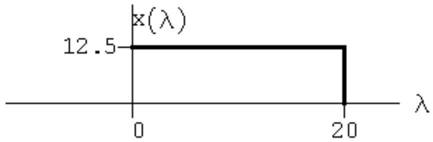
$0 \le t \le 10: \qquad y(t) = \int_0^t 625\,d\lambda = 625t$

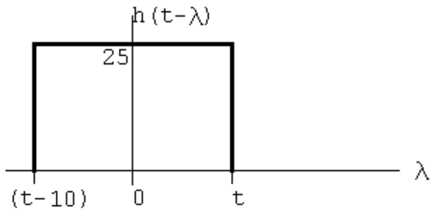
$10 \le t \le 20: \qquad y(t) = \int_{t-10}^{10} 625\,d\lambda = 625(10 - t + 10) = 625(20 - t)$

$20 \le t < \infty: \qquad y(t) = 0$



[**b**]





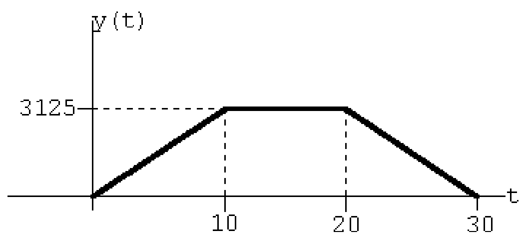
$y(t) = 0 \qquad t < 0$

$0 \le t \le 10: \qquad y(t) = \int_0^t 312.5\,d\lambda = 312.5t$

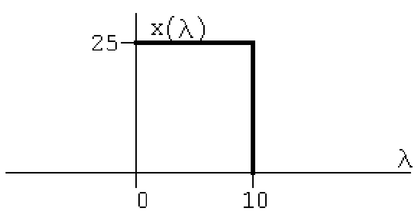
$10 \le t \le 20: \qquad y(t) = \int_0^{10} 312.5\,d\lambda = 3125$

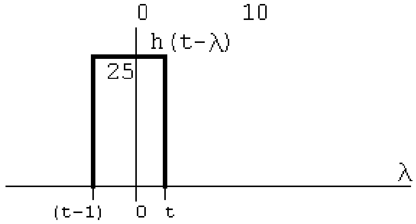
$20 \le t \le 30: \qquad y(t) = \int_{t-20}^{10} 312.5\,d\lambda = 312.5(30-t)$

$30 \le t < \infty: \qquad y(t) = 0$



[**c**]





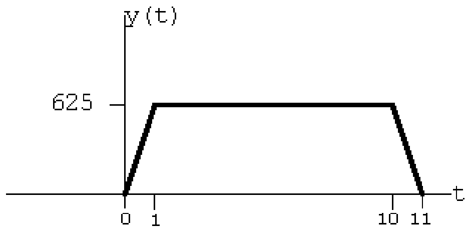
$y(t) = 0 \qquad t < 0$

$0 \le t \le 1: \qquad y(t) = \int_0^t 625\,d\lambda = 625t$

$$1 \le t \le 10: \qquad y(t) = \int_0^1 625\,d\lambda = 625$$

$$10 \le t \le 11: \qquad y(t) = \int_{t-10}^1 625\,d\lambda = 625(11-t)$$

$$11 \le t < \infty: \qquad y(t) = 0$$



P 13.65 **[a]** $-1 \le t \le 4$:

$$v_o = \int_0^{t+1} 10\lambda\,d\lambda = 5\lambda^2 \Big|_0^{t+1} = 5t^2 + 10t + 5\,\text{V}$$

$4 \le t \le 9$:

$$v_o = \int_{t-4}^{t+1} 10\lambda\,d\lambda = 5\lambda^2 \Big|_{t-4}^{t+1} = 50t - 75\,\text{V}$$

$9 \le t \le 14$:

$$v_o = 10\int_{t-4}^{10} \lambda\,d\lambda + 10\int_{10}^{t+1} 10\,d\lambda$$

$$= 5\lambda^2 \Big|_{t-4}^{10} + 100\lambda \Big|_{10}^{t+1} = -5t^2 + 140t - 480\,\text{V}$$

$14 \le t \le 19$:

$$v_o = 100\int_{t-4}^{t+1} d\lambda = 500\,\text{V}$$

$19 \le t \le 24$:

$$v_o = \int_{t-4}^{20} 100\lambda\,d\lambda + \int_{20}^{t+2} 10(30-\lambda)\,d\lambda$$

$$= 100\lambda \Big|_{t-2}^{20} + 300\lambda \Big|_{20}^{t+1} - 5\lambda^2 \Big|_{20}^{t+2}$$

$$= -5t^2 + 190t - 1305\,\text{V}$$

$24 \le t \le 29$:

$$v_o = 10\int_{t-4}^{t+1} (30-\lambda)\,d\lambda = 300\lambda \Big|_{t-4}^{t+1} - 5\lambda^2 \Big|_{t-4}^{t+1}$$

$$= 1575 - 50t\,\text{V}$$

$29 \leq t \leq 34$:

$$v_o = 10\int_{t-4}^{30}(30-\lambda)\,d\lambda = 300\lambda\,\Big|_{t-4}^{30} - 5\lambda^2\,\Big|_{t-2}^{30}$$

$$= 5t^2 - 340t + 5780\,\text{V}$$

Summary:

$v_o = 0 \qquad -\infty \leq t \leq -1$

$v_o = 5t^2 + 10t + 5\,\text{V} \qquad -1 \leq t \leq 4$

$v_o = 50t - 75\,\text{V} \qquad 4 \leq t \leq 9$

$v_o = -5t^2 + 140t - 480\,\text{V} \qquad 9 \leq t \leq 14$

$v_o = 500\,\text{V} \qquad 14 \leq t \leq 19$

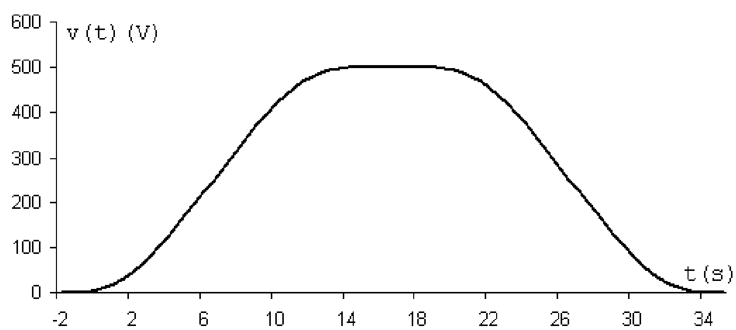
$v_o = -5t^2 + 190t - 1305\,\text{V} \qquad 19 \leq t \leq 24$

$v_o = 1575 - 50t\,\text{V} \qquad 24 \leq t \leq 29$

$v_o = 5t^2 - 340t + 5780\,\text{V} \qquad 29 \leq t \leq 34$

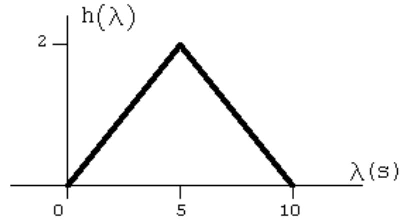
$v_o = 0 \qquad 34 \leq t \leq \infty$

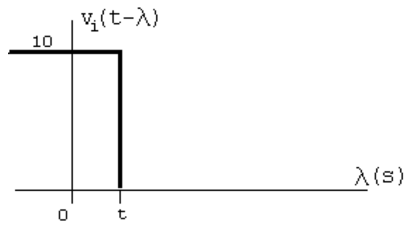
[b]



P 13.66 **[a]** $h(\lambda) = \frac{2}{5}\lambda \qquad 0 \leq \lambda \leq 5$

$h(\lambda) = \left(4 - \frac{2}{5}\lambda\right) \qquad 5 \leq \lambda \leq 10$





$0 \leq t \leq 5$:

$$v_o = 10\int_0^t \frac{2}{5}\lambda\,d\lambda = 2t^2$$

$5 \leq t \leq 10$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\,d\lambda + 10\int_5^t \left(4 - \frac{2}{5}\lambda\right)\,d\lambda$$

$$= \frac{4\lambda^2}{2}\,\Big|_0^5 + 40\lambda\,\Big|_5^t - \frac{4\lambda^2}{2}\,\Big|_5^t$$

$$= -100 + 40t - 2t^2$$

$10 \leq t \leq \infty$:

$$v_o = 10\int_0^5 \frac{2}{5}\lambda\,d\lambda + 10\int_5^{10} \left(4 - \frac{2}{5}\lambda\right)\,d\lambda$$

$$= \frac{4\lambda^2}{2}\,\Big|_0^5 + 40\lambda\,\Big|_5^{10} - \frac{4\lambda^2}{2}\,\Big|_5^{10}$$

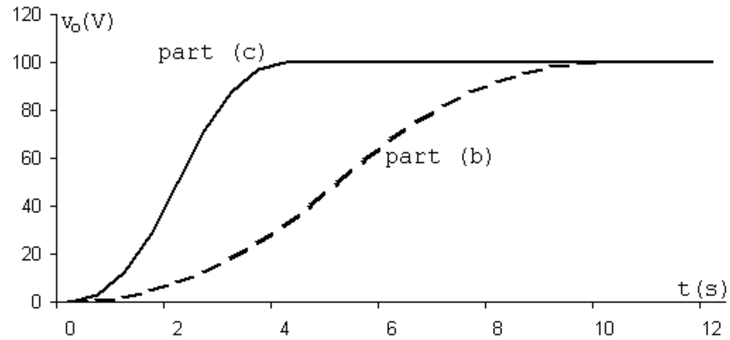
$$= 50 + 200 - 150 = 100$$

$v_o = 2t^2\,\text{V} \qquad 0 \leq t \leq 5$

$v_o = 40t - 100 - 2t^2\,\text{V} \qquad 5 \leq t \leq 10$

$v_o = 100\,\text{V} \qquad 10 \leq t \leq \infty$

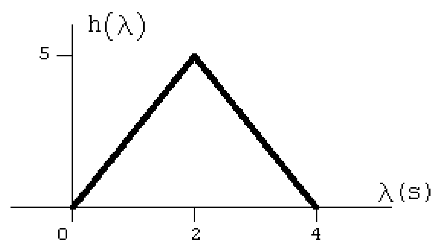
[**b**]

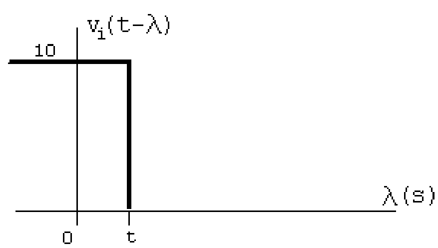


[**c**] Area $= \frac{1}{2}(10)(2) = 10 \quad \therefore \quad \frac{1}{2}(4)h = 10 \quad$ so $\quad h = 5$

$$h(\lambda) = \frac{5}{2}\lambda \qquad 0 \le \lambda \le 2$$

$$h(\lambda) = \left(10 - \frac{5}{2}\lambda\right) \qquad 2 \le \lambda \le 4$$





$0 \le t \le 2$:

$$v_o = 10\int_0^t \frac{5}{2}\lambda\, d\lambda = 12.5t^2$$

$2 \le t \le 4$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\, d\lambda + 10\int_2^t \left(10 - \frac{5}{2}\lambda\right)\, d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^t - \frac{25\lambda^2}{2}\Big|_2^t$$

$$= -100 + 100t - 12.5t^2$$

$4 \le t \le \infty$:

$$v_o = 10\int_0^2 \frac{5}{2}\lambda\, d\lambda + 10\int_2^4 \left(10 - \frac{5}{2}\lambda\right)\, d\lambda$$

$$= \frac{25\lambda^2}{2}\Big|_0^2 + 100\lambda\Big|_2^4 - \frac{25\lambda^2}{2}\Big|_2^4$$

$$= 50 + 200 - 150 = 100$$

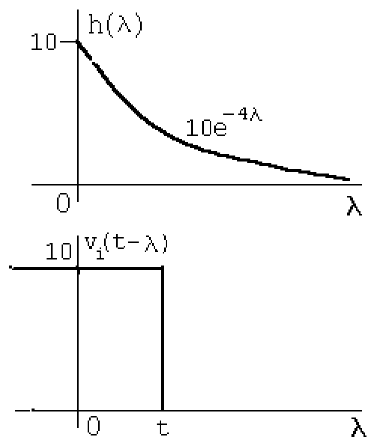
$$v_o = 12.5t^2\,\text{V} \qquad 0 \le t \le 2$$

$$v_o = 100t - 100 - 12.5t^2\,\text{V} \qquad 2 \le t \le 4$$

$$v_o = 100\,\text{V} \qquad 4 \le t \le \infty$$

[d] The waveform in part (c) is closer to replicating the input waveform because in part (c) $h(\lambda)$ is closer to being an ideal impulse response. That is, the area was preserved as the base was shortened.

P 13.67 **[a]**

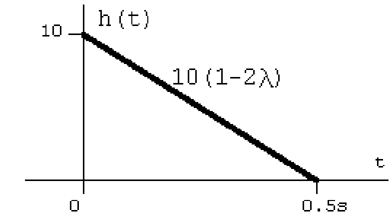


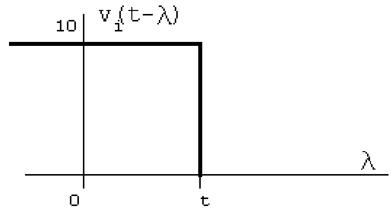
$$v_o = \int_0^t 10(10e^{-4\lambda})\, d\lambda$$

$$= 100\frac{e^{-4\lambda}}{-4}\Big|_0^t = -25[e^{-4t} - 1]$$

$$= 25(1 - e^{-4t})\,\text{V}, \qquad 0 \le t \le \infty$$

[b]





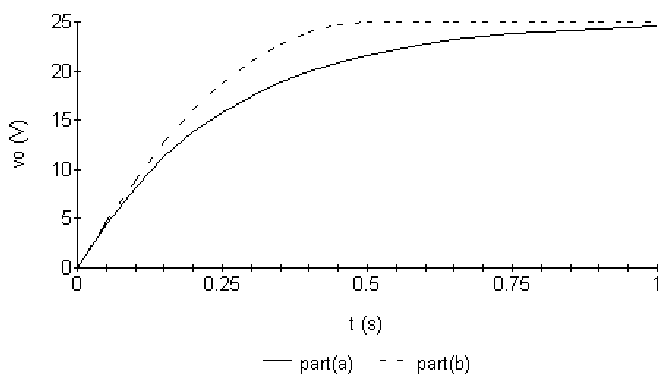
$0 \leq t \leq 0.5$:

$$v_o = \int_0^t 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^t = 100t(1-t)$$

$0.5 \leq t \leq \infty$:

$$v_o = \int_0^{0.5} 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^{0.5} = 25$$

[c]



P 13.68 $H(s) = \dfrac{16s}{40+4s+16s} = \dfrac{0.8s}{s+2} = 0.8\left(1 - \dfrac{2}{s+2}\right) = 0.8 - \dfrac{1.6}{s+2}$

$$h(\lambda) = 0.8\delta(\lambda) - 1.6e^{-2\lambda}u(\lambda)$$

$$v_o = \int_0^t 75[0.8\delta(\lambda) - 1.6e^{-2\lambda}]\,d\lambda = \int_0^t 60\delta(\lambda)\,d\lambda - 120\int_0^t e^{-2\lambda}\,d\lambda$$

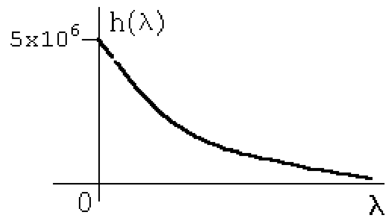
$$= 60 - 120\frac{e^{-2\lambda}}{-2}\Big|_0^t = 60 + 60(e^{-2t} - 1)$$

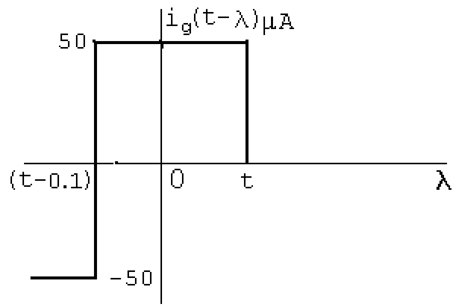
$$= 60e^{-2t}u(t)\,\text{V}$$

P 13.69 **[a]** $I_o = \frac{V_o}{10^5} + \frac{V_o s}{5 \times 10^6} = \frac{V_o(s+50)}{5 \times 10^6}$

$$\frac{V_o}{I_g} = H(s) = \frac{5 \times 10^6}{s+50}$$

$$h(\lambda) = 5 \times 10^6 e^{-50\lambda} u(\lambda)$$

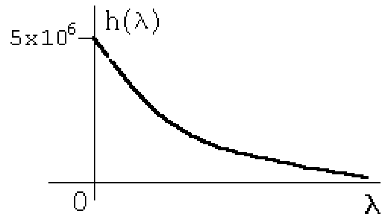


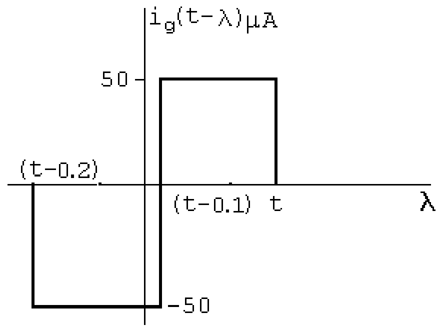


$0 \le t \le 0.1\,\text{s}$:

$$v_o = \int_0^t (50 \times 10^{-6})(5 \times 10^6) e^{-50\lambda}\, d\lambda = 250 \frac{e^{-50\lambda}}{-50} \Big|_0^t = 5(1 - e^{-50t})\,\text{V}$$

$0.1\,\text{s} \leq t \leq 0.2\,\text{s}$:





$$v_o = \int_0^{t-0.1}(-50\times10^{-6})(5\times10^6 e^{-50\lambda}\,d\lambda)$$

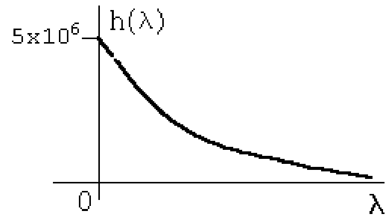
$$+\int_{t-0.1}^{t}(50\times10^{-6})(5\times10^6 e^{-50\lambda}\,d\lambda)$$

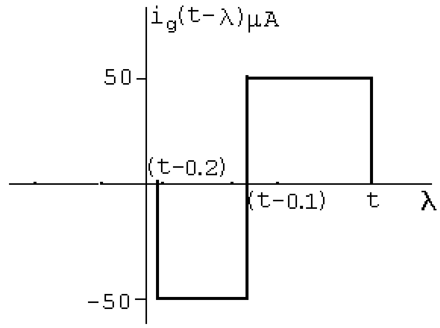
$$= -250\frac{e^{-50\lambda}}{-50}\Big|_0^{t-0.1} + 250\frac{e^{-50\lambda}}{-50}\Big|_{t-0.1}^{t}$$

$$= 5\left[e^{-50(t-0.1)} - 1\right] - 5\left[e^{-50t} - e^{-50(t-0.1)}\right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50t} - 5]\,\text{V}$$

$0.2\,\text{s} \leq t \leq \infty$:





$$v_o = \int_{t-0.2}^{t-0.1} -250e^{-50\lambda}\,d\lambda + \int_{t-0.1}^{t} 250e^{-50\lambda}\,d\lambda$$

$$= \left[5e^{-50\lambda}\Big|_{t-0.2}^{t-0.1} - 5e^{-50\lambda}\Big|_{t-0.1}^{t}\right]$$

$$v_o = [10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}]\,\text{V}$$

[b] $I_o = \dfrac{V_o s}{5\times10^6} = \dfrac{s}{5\times10^6}\cdot\dfrac{5\times10^6 I_g}{s+50}$

$$\frac{I_o}{I_g} = H(s) = \frac{s}{s+50} = 1 - \frac{50}{s+50}$$

$$h(\lambda) = \delta(\lambda) - 50e^{-50\lambda}$$

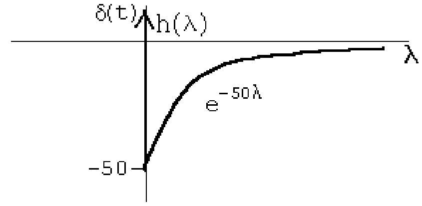
$0 < t < 0.1\,\text{s}$:

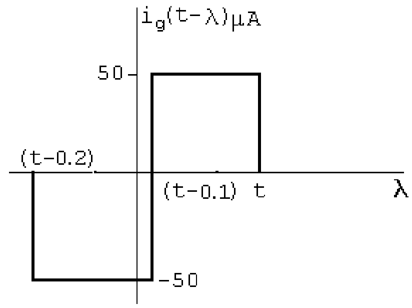
$$i_o = \int_0^t (50\times10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

$$= 50\times10^{-6} - 25\times10^{-3}\frac{e^{-50\lambda}}{-50}\Big|_0^t$$

$$= 50\times10^{-6} + 50\times10^{-6}[e^{-50t} - 1] = 50e^{-50t}\,\mu\text{A}$$

$0.1\,\text{s} < t < 0.2\,\text{s}$:





$$i_o = \int_0^{t-0.1} (-50\times10^{-6})[\delta(\lambda) - 50e^{-50\lambda}]\,d\lambda$$

$$+\int_{t-0.1}^{t} (50\times10^{-6})(-50e^{-50\lambda})\,d\lambda$$

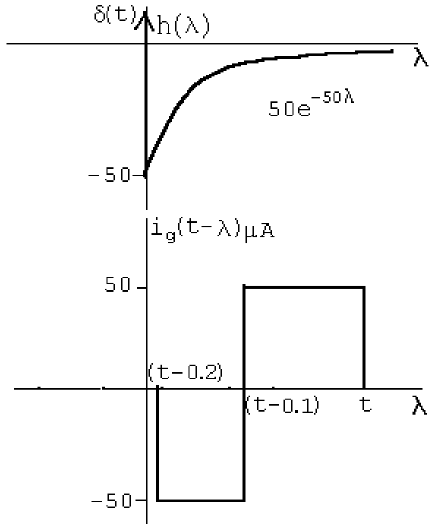
$$= -50\times10^{-6} + 2.5\times10^{-3}\frac{e^{-50\lambda}}{-50}\Bigg|_0^{t-0.1} - 2.5\times10^{-3}\frac{e^{-50\lambda}}{-50}\Bigg|_{t-0.1}^{t}$$

$$= -50\times10^{-6} - 50\times10^{-6}e^{-50(t-0.1)} + 50\times10^{-6}$$

$$+50\times10^{-6}e^{-50t} - 50\times10^{-6}e^{-50(t-0.1)}$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)}\,\mu\text{A}$$

$0.2\,\mathrm{s} < t < \infty$:



$$i_o = \int_{t-0.2}^{t-0.1}(-50\times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$+\int_{t-0.1}^{t}(50\times 10^{-6})(-50e^{-50\lambda})\,d\lambda$$

$$= 50e^{-50t} - 100e^{-50(t-0.1)} + 50e^{-50(t-0.2)}\,\mu\mathrm{A}$$

[c] At $t = 0.1^-$:

$$v_o = 5(1-e^{-5}) = 4.97\,\mathrm{V}; \qquad i_{100\mathrm{k}\Omega} = \frac{4.97}{0.1} = 49.66\,\mu\mathrm{A}$$

$\therefore \quad i_o = 50 - 49.66 = 0.34\,\mu\mathrm{A}$

From the solution for i_o we have

$i_o(0.1^-) = 50e^{-5} = 0.34\,\mu\mathrm{A}$ (checks)

At $t = 0.1^+$:

$v_o(0.1^+) = v_o(0.1^-) = 4.97\,\mathrm{V}$

$i_{100\mathrm{k}\Omega} = 49.66\,\mu\mathrm{A}$

$\therefore \quad i_o(0.1^+) = -(50 + 49.66) = -99.66\,\mu\mathrm{A}$

From the solution for i_o we have

$i_o(0.1^+) = 50e^{-5} - 100 = 99.66\,\mu\mathrm{A}$ (checks)

At $t = 0.2^-$:

$v_o = 10e^{-5} - 5e^{-10} - 5 = -4.93\,\mathrm{V}$

$i_{100\text{k}\Omega} = 49.33\,\mu\text{A}$

$i_o = -50 + 49.33 = -0.67\,\mu\text{A}$

From the solution for i_o,

$v_o(0.2^-) = 50e^{-10} - 100e^{-5} = -0.67\,\mu\text{A} \quad \text{(checks)}$

At $\quad t = 0.2^+$:

$v_o(0.2^+) = v_o(0.2^-) = -4.93\,\text{V}; \qquad i_{100\text{k}\Omega} = -49.33\,\mu\text{A}$

$i_o = 0 + 49.33 = 49.33\,\mu\text{A}$

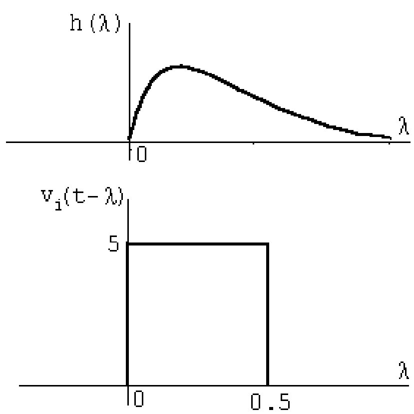
From the solution for i_o,

$i_o(0.2^+) = 50e^{-10} - 100e^{-5} + 50 = 49.33\,\mu\text{A}\text{(checks)}$

P 13.70 **[a]** $H(s) = \dfrac{V_o}{V_i} = \dfrac{1/LC}{s^2 + (R/L)s + (1/LC)}$

$$= \frac{100}{s^2 + 20s + 100} = \frac{100}{(s+10)^2}$$

$h(\lambda) = 100\lambda e^{-10\lambda}u(\lambda)$



$0 \le t \le 0.5$:

$$v_o = 500\int_0^t \lambda e^{-10\lambda}\,d\lambda$$

$$= 500\left\{\frac{e^{-10\lambda}}{100}(-10\lambda - 1)\,\Big|_0^t\right\}$$

$$= 5[1 - e^{-10t}(10t + 1)]$$

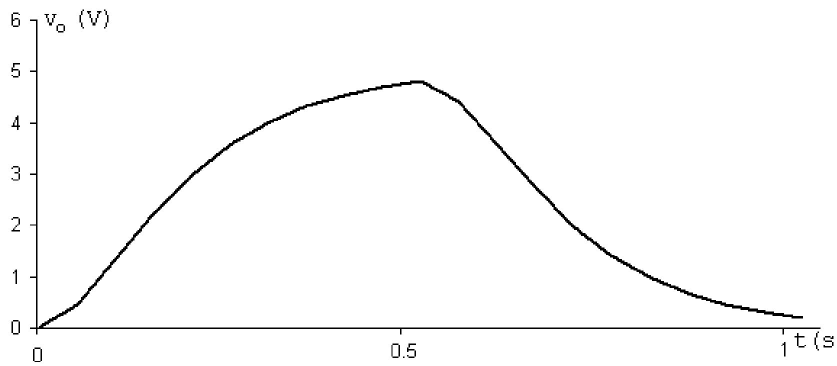
$0.5 \leq t \leq \infty$:

$$v_o = 500\int_{t-0.5}^{t} \lambda e^{-10\lambda}\, d\lambda$$

$$= 500\left\{\frac{e^{-10\lambda}}{100}(-10\lambda - 1)\Big|_{t-0.5}^{t}\right\}$$

$$= 5e^{-10t}[e^{5}(10t - 4) - 10t - 1]$$

[b]

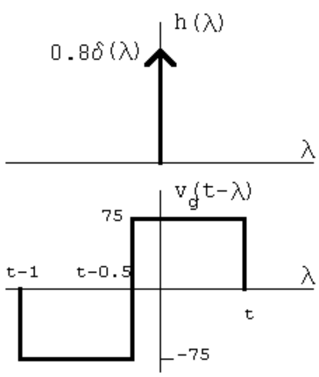


P 13.71 **[a]** $V_o = \dfrac{16}{20}V_g$

$$\therefore \qquad H(s) = \frac{V_o}{V_g} = \frac{4}{5}$$

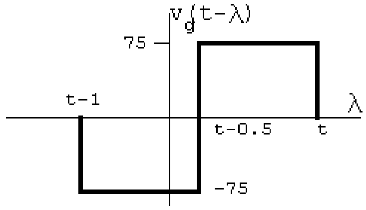
$$h(\lambda) = 0.8\delta(\lambda)$$

[b]



$$0 < t < 0.5\,\text{s}: \qquad v_o = \int_0^t 75[0.8\delta(\lambda)]\, d\lambda = 60\,\text{V}$$

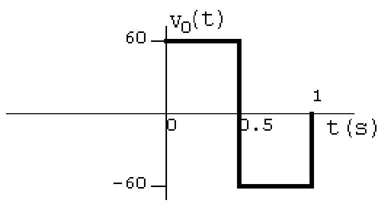
$0.5\,\mathrm{s} \le t \le 1.0\,\mathrm{s}$:



$$v_o = \int_0^{t-0.5} -75[0.8\delta(\lambda)]\,d\lambda = -60\,\mathrm{V}$$

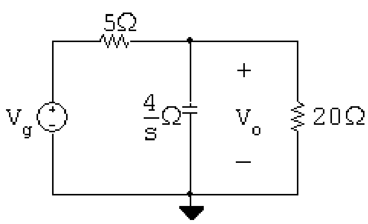
$1\,\mathrm{s} < t < \infty: \qquad v_o = 0$

[c]



Yes, because the circuit has no memory.

P 13.72 **[a]**

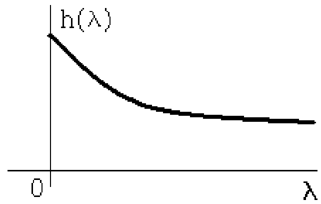


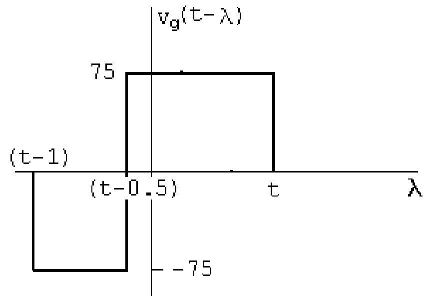
$$\frac{V_o - V_g}{5} + \frac{V_o s}{4} + \frac{V_o}{20} = 0$$

$$(5s+5)V_o = 4V_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{0.8}{s+1}; \qquad h(\lambda) = 0.8e^{-\lambda}u(\lambda)$$

[b]



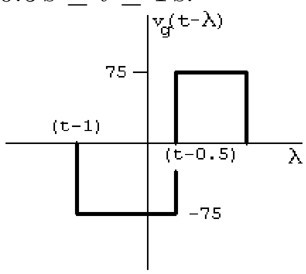


$0 \le t \le 0.5\,\text{s}$;

$$v_o = \int_0^t 75(0.8e^{-\lambda})\,d\lambda = 60\frac{e^{-\lambda}}{-1}\Big|_0^t$$

$$v_o = 60 - 60e^{-t}\,\text{V}, \qquad 0 \le t \le 0.5\,\text{s}$$

$0.5\,\text{s} \le t \le 1\,\text{s}$:



$$v_o = \int_0^{t-0.5}(-75)(0.8e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 75(0.8e^{-\lambda})\,d\lambda$$

$$= -60\frac{e^{-\lambda}}{-1}\Big|_0^{t-0.5} + 60\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

$$= 120e^{-(t-0.5)} - 60e^{-t} - 60\,\text{V}, \qquad 0.5\,\text{s} \le t \le 1\,\text{s}$$

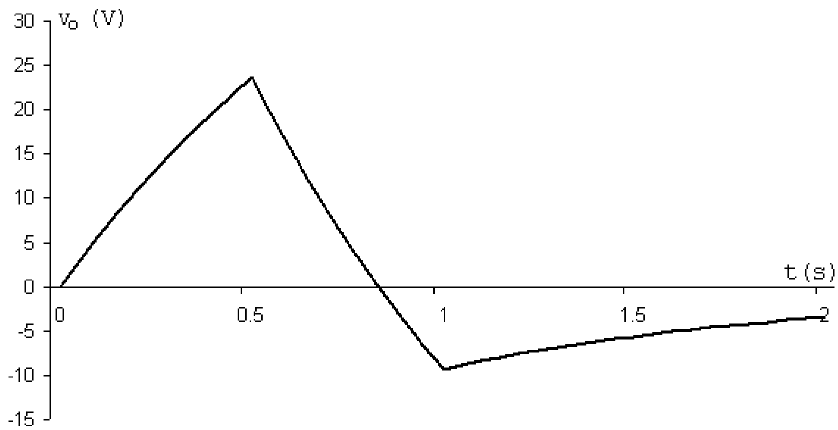
$1\,\text{s} \le t \le \infty$;

$$v_o = \int_{t-1}^{t-0.5}(-75)(0.8e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 75(0.8e^{-\lambda})\,d\lambda$$

$$= -60\frac{e^{-\lambda}}{-1}\Big|_{t-1}^{t-0.5} + 60\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

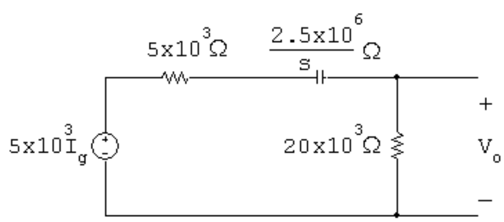
$$= 120e^{-(t-0.5)} - 60e^{-(t-1)} - 60e^{-t}\,\text{V}, \qquad 1\,\text{s} \le t \le \infty$$

[**c**]



[**d**] No, the circuit has memory because of the capacitive storage element.

P 13.73



$$V_o = \frac{5\times10^3 I_g}{25\times10^3 + 2.5\times10^6/s}(20\times10^3)$$

$$\frac{V_o}{I_g} = H(s) = \frac{4000s}{s+100}$$

$$H(s) = 4000\left[1 - \frac{100}{s+100}\right] = 4000 - \frac{4\times10^5}{s+100}$$

$$h(t) = 4000\delta(t) - 4\times10^5 e^{-100t}$$

$$v_o = \int_0^{10^{-3}} (-20\times10^{-3})[4000\delta(\lambda) - 4\times10^5 e^{-100\lambda}]\,d\lambda$$

$$+\int_{10^{-3}}^{5\times10^{-3}} (10\times10^{-3})[-4\times10^5 e^{-100\lambda}]\,d\lambda$$

$$= -80 + 8000\int_0^{10^{-3}} e^{-100\lambda}\,d\lambda - 4000\int_{10^{-3}}^{5\times10^{-3}} e^{-100\lambda}\,d\lambda$$

$$= -80 - 80(e^{-0.1} - 1) + 40(e^{-0.5} - e^{-0.1})$$

$$= 40e^{-0.5} - 120e^{-0.1} = -84.32\,\text{V}$$

Alternate:

$$I_g = \int_0^{4\times10^{-3}} (10 \times 10^{-3})e^{-st}\,dt + \int_{4\times10^{-3}}^{6\times10^{-3}} (-20 \times 10^{-3})e^{-st}\,dt$$

$$= \left[\frac{10}{s} - \frac{30}{s}e^{-4\times10^{-3}s} + \frac{20}{s}e^{-6\times10^{-3}s}\right] \times 10^{-3}$$

$$V_o = I_g H(s) = \frac{40}{s+100}[1 - 3e^{-4\times10^{-3}s} + 2e^{-6\times10^{-3}s}]$$

$$= \frac{40}{s+100} - \frac{120e^{-4\times10^{-3}s}}{s+100} + \frac{80e^{-6\times10^{-3}s}}{s+100}]$$

$$v_o(t) = 40e^{-100t} - 120e^{-100(t-4\times10^{-3})}u(t - 4 \times 10^{-3})$$
$$+80e^{-100(t-6\times10^{-3})}u(t - 6 \times 10^{-3})$$

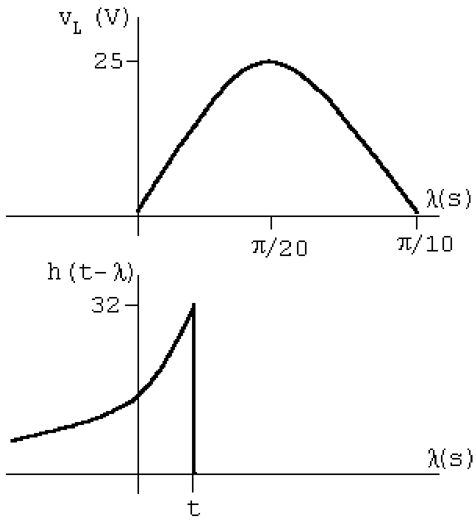
$$v_o(5 \times 10^{-3}) = 40e^{-0.5} - 120e^{-0.1} + 80(0) = -84.32\,\text{V} \quad \text{(checks)}$$

P 13.74 $v_i = 25\,\sin 10\lambda\,[u(\lambda) - u(\lambda - \pi/10)]$

$$H(s) = \frac{32}{s+32}$$

$$h(\lambda) = 32e^{-32\lambda}$$

$$h(t-\lambda) = 32e^{-32(t-\lambda)} = 32e^{-32t}e^{32\lambda}$$



$$v_o = 800e^{-32t}\int_0^t e^{32\lambda}\sin 10\lambda\, d\lambda$$

$$= 800e^{-32t}\left[\frac{e^{32\lambda}}{32^2+10^2}(32\sin 10\lambda - 10\cos 10\lambda \Big|_0^t\right]$$

$$= \frac{800e^{-32t}}{1124}[e^{32t}(32\sin 10t - 10\cos 10t) + 10]$$

$$= \frac{800}{1124}[32\sin 10t - 10\cos 10t + 10e^{-32t}]$$

$$v_o(0.075) = 10.96\,\text{V}$$

P 13.75 **[a]** $Y(s) = \int_0^\infty y(t)e^{-st}\, dt$

$$Y(s) = \int_0^\infty e^{-st}\left[\int_0^\infty h(\lambda)x(t-\lambda)\, d\lambda\right] dt$$

$$= \int_0^\infty\int_0^\infty e^{-st}h(\lambda)x(t-\lambda)\, d\lambda\, dt$$

$$= \int_0^\infty h(\lambda)\int_0^\infty e^{-st}x(t-\lambda)\, dt\, d\lambda$$

But $x(t-\lambda) = 0$ when $t < \lambda$.

Therefore $Y(s) = \int_0^\infty h(\lambda)\int_\lambda^\infty e^{-st}x(t-\lambda)\, dt\, d\lambda$

Let $u = t-\lambda$; $du = dt$; $u = 0$ when $t = \lambda$; $u = \infty$ when $t = \infty$.

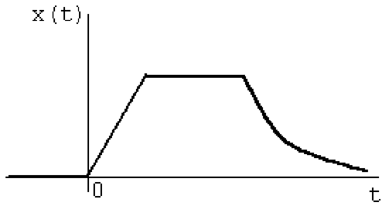
$$Y(s) = \int_0^\infty h(\lambda)\int_0^\infty e^{-s(u+\lambda)}x(u)\,du\,d\lambda$$

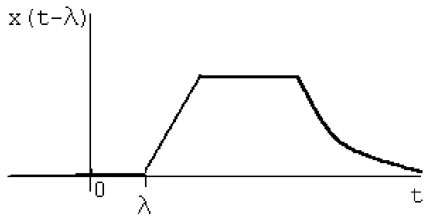
$$= \int_0^\infty h(\lambda)e^{-s\lambda}\int_0^\infty e^{-su}x(u)\,du\,d\lambda$$

$$= \int_0^\infty h(\lambda)e^{-s\lambda}X(s)\,d\lambda = H(s)\,X(s)$$

Note on $\quad x(t-\lambda) = 0, \qquad t < \lambda$

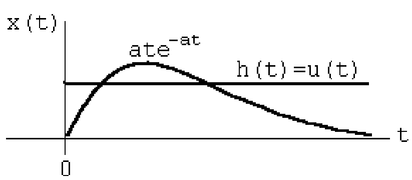
We are using one-sided Laplace transforms; therefore $h(t)$ and $x(t)$ are assumed zero for $t < 0$.





[b] $F(s) = \dfrac{a}{s(s+a)^2} = \dfrac{1}{s}\cdot\dfrac{a}{(s+a)^2} = H(s)X(s)$

$\therefore \quad h(t) = u(t), \qquad x(t) = at\,e^{-at}u(t)$



$$\therefore \quad f(t) = \int_0^t (1)a\lambda e^{-a\lambda}\,d\lambda = a\left[\frac{e^{-a\lambda}}{a^2}(-a\lambda - 1)\right|_0^t$$

$$= \frac{1}{a}[e^{-at}(-at-1) - 1(-1)] = \frac{1}{a}[1 - e^{-at} - ate^{-at}]$$

$$= \left[\frac{1}{a} - \frac{1}{a}e^{-at} - te^{-at}\right]u(t)$$

Check:

$$F(s) = \frac{a}{s(s+a)^2} = \frac{K_0}{s} + \frac{K_1}{(s+a)^2} + \frac{K_2}{s+a}$$

$$K_0 = \frac{1}{a}; \qquad K_1 = -1; \qquad K_2 = \frac{d}{ds}\left(\frac{a}{s}\right)_{s=-a} = -\frac{1}{a}$$

$$f(t) = \left[\frac{1}{a} - te^{-at} - \frac{1}{a}e^{-at}\right] u(t)$$

P 13.76 **[a]** $H(s) = \frac{-Z_f}{Z_i}$

$$Z_f = \frac{(1/C_f)}{s + (1/R_fC_f)} = \frac{10^7}{s + 5000}$$

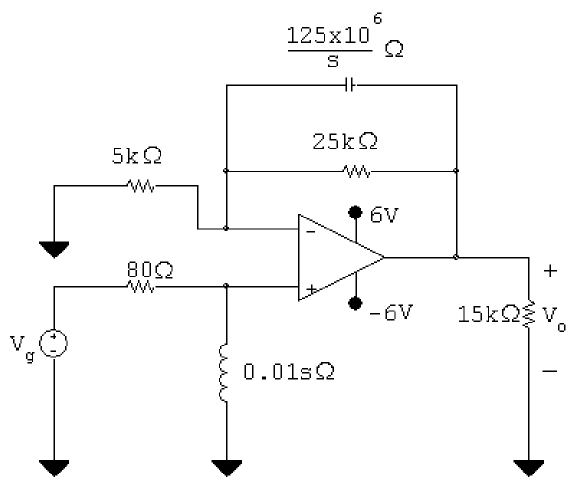
$$Z_i = \frac{R_i[s + (1/R_iC_i)]}{s} = \frac{200(s + 2000)}{s}$$

$$H(s) = \frac{-50{,}000s}{(s + 2000)(s + 5000)}$$

[b] $H(j3000) = \frac{-50{,}000(j3000)}{(2000 + j3000)(5000 + j3000)} = 7.135\underline{/-177.27^\circ}$

$$v_o(t) = 7.135\cos(3000t - 177.27^\circ)\text{ V}$$

P 13.77 **[a]**



$$V_p = \frac{0.01s}{80 + 0.01s}V_g = \frac{s}{s + 8000}V_g$$

$$V_n = V_p$$

$$\frac{V_n}{5000} + \frac{V_n - V_o}{25{,}000} + (V_n - V_o)8 \times 10^{-9}s = 0$$

$$5V_n + V_n - V_o + (V_n - V_o)2 \times 10^{-4}s = 0$$

$$6V_n + 2 \times 10^{-4}sV_n = V_o + 2 \times 10^{-4}sV_o$$

$$2 \times 10^{-4} V_n(s + 30{,}000) = 2 \times 10^{-4} V_o(s + 5000)$$

$$V_o = \frac{s + 30{,}000}{s + 5000} V_i = \left(\frac{s + 30{,}000}{s + 5000}\right)\left(\frac{sV_g}{s + 8000}\right)$$

$$H(s) = \frac{V_o}{V_g} = \frac{s(s + 30{,}000)}{(s + 5000)(s + 8000)}$$

[b] $v_g = 0.6u(t); \qquad V_g = \dfrac{0.6}{s}$

$$V_o = \frac{0.6(s + 30{,}000)}{(s + 5000)(s + 8000)} = \frac{K_1}{s + 5000} + \frac{K_2}{s + 8000}$$

$$K_1 = \frac{0.6(25{,}000)}{3000} = 5; \qquad K_2 = \frac{0.6(22{,}000)}{-3000} = -4.4$$

$$\therefore \quad v_o(t) = (5e^{-5000t} - 4.4e^{-8000t})u(t)\,\text{V}$$

[c] $V_g = 2\cos 10{,}000t\,\text{V}$

$$H(j\omega) = \frac{j10{,}000(30{,}000 + j10{,}000)}{(5000 + j10{,}000)(8000 + j10{,}000)} = 2.21\underline{/-6.34^\circ}$$

$$\therefore \qquad v_o = 4.42\cos(10{,}000t - 6.34^\circ)\,\text{V}$$

P 13.78 $H(j20) = \dfrac{25(8 + j20)}{-400 + j1200 + 150} = 0.44\underline{/-33.57^\circ}$

$$\therefore \quad v_o(t) = 4.4\cos(20t - 33.57^\circ)\,\text{V}$$

P 13.79 $H(j500) = \dfrac{125(400 + j500)}{(j500)(10^4 - 500^2 + j10^5)} = 0.6157 \times 10^{-3}\underline{/163.96^\circ}$

$$\therefore \quad i_o(t) = 49.3\cos(500t + 163.96^\circ)\,\text{mA}$$

P 13.80 $V_o = \dfrac{50}{s + 8000} - \dfrac{20}{s + 5000} = \dfrac{30(s + 3000)}{(s + 5000)(s + 8000)}$

$$V_o = H(s)V_g = H(s)\left(\frac{30}{s}\right)$$

$$\therefore \quad H(s) = \frac{s(s + 3000)}{(s + 5000)(s + 8000)}$$

$$H(j6000) = \frac{(j6000)(3000 + j6000)}{(5000 + j6000)(8000 + j6000)} = 0.52\underline{/66.37^\circ}$$

$$\therefore \quad v_o(t) = 61.84\cos(6000t + 66.37^\circ)\,\text{V}$$

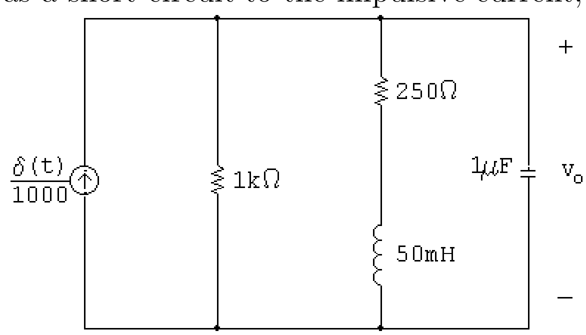
P 13.81 Original charge on C_1; $q_1 = V_0C_1$

The charge transferred to C_2; $q_2 = V_0C_e = \frac{V_0C_1C_2}{C_1 + C_2}$

The charge remaining on C_1; $q_1' = q_1 - q_2 = \frac{V_0C_1^2}{C_1 + C_2}$

Therefore $V_2 = \frac{q_2}{C_2} = \frac{V_0C_1}{C_1 + C_2}$ and $V_1 = \frac{q_1'}{C_1} = \frac{V_0C_1}{C_1 + C_2}$

P 13.82 **[a]** After making a source transformation, the circuit is as shown. The impulse current will pass through the capacitive branch since it appears as a short circuit to the impulsive current,



Therefore $v_o(0^+) = 10^6 \int_{0^-}^{0^+} \left[\frac{\delta(t)}{1000}\right] dt = 1000\,\text{V}$

Therefore $w_C = (0.5)Cv^2 = 0.5\,\text{J}$

[b] $i_L(0^+) = 0;$ therefore $w_L = 0\,\text{J}$

[c] $V_o(10^{-6})s + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$

Therefore

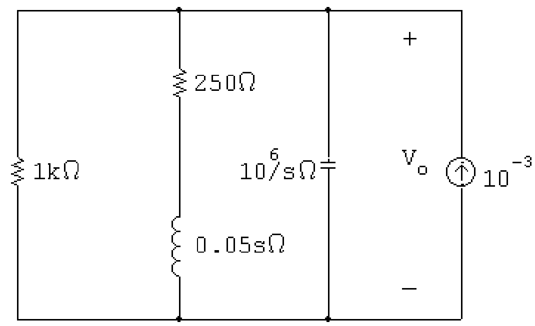
$$V_o = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000}$$

$K_1 = 559.02\underline{/-26.57^\circ}$; $K_1^* = 559.02\underline{/26.57^\circ}$

$v_o = [1118.03e^{-3000t}\cos(4000t - 26.57^\circ)]u(t)\,\text{V}$

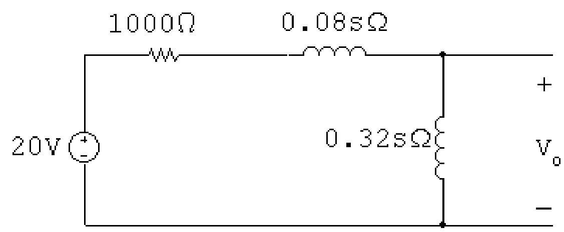
[d] The s-domain circuit is



$$\frac{V_o s}{10^6} + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$$

Note that this equation is identical to that derived in part [c], therefore the solution for V_o will be the same.

P 13.83 **[a]**



$$V_o = \frac{20}{1000 + 0.4s} \cdot 0.32s$$

$$= \frac{16s}{s + 2500} = 16 - \frac{40{,}000}{s + 2500}$$

$$v_o(t) = 16\delta(t) - 40{,}000e^{-2500t}u(t)\,\text{V}$$

[b] At $t = 0$ the voltage impulse establishes a current in the inductors; thus

$$i_L(0) = \frac{10^3}{400}\int_{0^-}^{0^+} 20\delta(t)\,dt = 50\,\text{A}$$

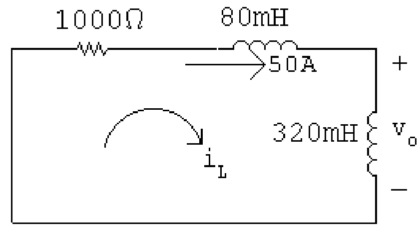
It follows that since $i_L(0^-) = 0$,

$$\frac{di_L}{dt}(0) = 50\delta(t)$$

$$\therefore \quad v_o(0) = (0.32)(50\delta(t)) = 16\delta(t)$$

This agrees with our solution.

At $t = 0^+$ our circuit is



$$\therefore \quad i_L(t) = 50e^{-t/\tau}\,\text{A}, \qquad t \geq 0^+$$

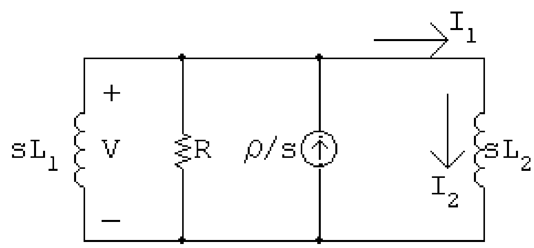
$$\tau = L/R = 0.4\,\text{ms}$$

$$\therefore \quad i_L(t) = 50e^{-2500t}\,\text{A}, \qquad t \geq 0^+$$

$$v_o(t) = 0.32\frac{di_L}{dt} = -40{,}000e^{-2500t}\,\text{V}, \qquad t \geq 0^+$$

which agrees with our solution.

P 13.84 **[a]** The s-domain circuit is



The node-voltage equation is $\quad \dfrac{V}{sL_1} + \dfrac{V}{R} + \dfrac{V}{sL_2} = \dfrac{\rho}{s}$

Therefore $\quad V = \dfrac{\rho R}{s + (R/L_e)} \quad$ where $\quad L_e = \dfrac{L_1 L_2}{L_1 + L_2}$

Therefore $\quad v = \rho R e^{-(R/L_e)t} u(t)\,\text{V}$

[b] $I_1 = \dfrac{V}{R} + \dfrac{V}{sL_2} = \dfrac{\rho[s + (R/L_2)]}{s[s + (R/L_e)]} = \dfrac{K_0}{s} + \dfrac{K_1}{s + (R/L_e)}$

$$K_0 = \frac{\rho L_1}{L_1 + L_2}; \qquad K_1 = \frac{\rho L_2}{L_1 + L_2}$$

Thus we have $\quad i_1 = \dfrac{\rho}{L_1 + L_2}[L_1 + L_2 e^{-(R/L_e)t}]u(t)\,\text{A}$

[c] $I_2 = \dfrac{V}{sL_2} = \dfrac{(\rho R/L_2)}{s[s + (R/L_e)]} = \dfrac{K_2}{s} + \dfrac{K_3}{s + (R/L_e)}$

$$K_2 = \frac{\rho L_1}{L_1 + L_2}; \qquad K_3 = \frac{-\rho L_1}{L_1 + L_2}$$

Therefore $\quad i_2 = \dfrac{\rho L_1}{L_1 + L_2}[1 - e^{-(R/L_e)t}]u(t)$

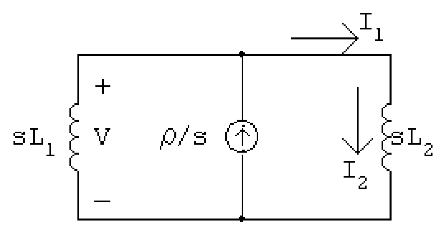
[d] $\lambda(t) = L_1 i_1 + L_2 i_2 = \rho L_1$

P 13.85 **[a]** As $R \to \infty$, $v(t) \to \rho L_e \delta(t)$ since the area under the impulse generating function is ρL_e.

$$i_1(t) \to \frac{\rho L_1}{L_1 + L_2} u(t)\,\text{A} \quad \text{as} \quad R \to \infty$$

$$i_2(t) \to \frac{\rho L_1}{L_1 + L_2} u(t)\,\text{A} \quad \text{as} \quad R \to \infty$$

[b] The s-domain circuit is



$$\frac{V}{sL_1} + \frac{V}{sL_2} = \frac{\rho}{s}; \qquad \text{therefore} \quad V = \frac{\rho L_1 L_2}{L_1 + L_2} = \rho L_e$$

Therefore $\quad v(t) = \rho L_e \delta(t)$

$$I_1 = I_2 = \frac{V}{sL_2} = \left(\frac{\rho L_1}{L_1 + L_2}\right)\left(\frac{1}{s}\right)$$

Therefore $\quad i_1 = i_2 = \dfrac{\rho L_1}{L_1 + L_2} u(t)\,\text{A}$

P 13.86 **[a]** $Z_1 = \dfrac{1/C_1}{s + 1/R_1 C_1} = \dfrac{25 \times 10^{10}}{s + 20 \times 10^4}\Omega$

$$Z_2 = \frac{1/C_2}{s + 1/R_2 C_2} = \frac{6.25 \times 10^{10}}{s + 12{,}500}\Omega$$

$$\frac{V_0}{Z_2} + \frac{V_0 - 10/s}{Z_1} = 0$$

$$\frac{V_0(s + 12{,}500)}{6.25 \times 10^{10}} + \frac{V_0(s + 20 \times 10^4)}{25 \times 10^{10}} = \frac{10}{s}\,\frac{(s + 20 \times 10^4)}{25 \times 10^{10}}$$

$$V_0 = \frac{2(s + 200{,}000)}{s(s + 50{,}000)} = \frac{K_1}{s} + \frac{K_2}{s + 50{,}000}$$

$$K_1 = \frac{2(200{,}000)}{50{,}000} = 8$$

$$K_2 = \frac{2(150{,}000)}{-50{,}000} = -6$$

$$\therefore \quad v_o = [8 - 6e^{-50{,}000t}]u(t)\,\text{V}$$

[b] $$I_0 = \frac{V_0}{Z_2} = \frac{2(s+200{,}000)(s+12{,}500)}{s(s+50{,}000)6.25\times10^{10}}$$

$$= 32\times10^{-12}\left[1 + \frac{162{,}500s + 25\times10^8}{s(s+50{,}000)}\right]$$

$$= 32\times10^{-12}\left[1 + \frac{K_1}{s} + \frac{K_2}{s+50{,}000}\right]$$

$$K_1 = 50{,}000; \qquad K_2 = 112{,}500$$

$$i_o = 32\delta(t) + [1.6\times10^6 + 3.6\times10^6e^{-50{,}000t}]u(t)\ \text{pA}$$

[c] When $C_1 = 64\,\text{pF}$

$$Z_1 = \frac{156.25\times10^8}{s+12{,}500}\Omega$$

$$\frac{V_0(s+12{,}500)}{625\times10^8} + \frac{V_0(s+12{,}500)}{156.25\times10^8} = \frac{10}{s}\,\frac{(s+12{,}500)}{156.25\times10^8}$$

$$\therefore \quad V_0 + 4V_0 = \frac{40}{s}$$

$$V_0 = \frac{8}{s}$$

$$v_o = 8u(t)\,\text{V}$$

$$I_0 = \frac{V_0}{Z_2} = \frac{8}{s}\,\frac{(s+12{,}500)}{6.25\times10^{10}} = 128\times10^{-12}\left[1 + \frac{12{,}500}{s}\right]$$

$$i_o(t) = 128\delta(t) + 1.6\times10^6u(t)\,\text{pA}$$

P 13.87 Let $a = \dfrac{1}{R_1C_1} = \dfrac{1}{R_2C_2}$

Then $Z_1 = \dfrac{1}{C_1(s+a)}$ and $Z_2 = \dfrac{1}{C_2(s+a)}$

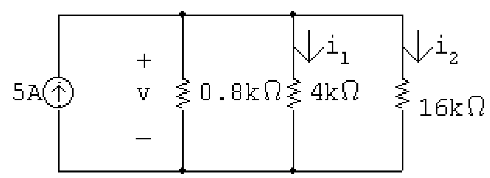
$$\frac{V_o}{Z_2} + \frac{V_o}{Z_1} = \frac{10/s}{Z_1}$$

$$V_oC_2(s+a) + V_0C_1(s+a) = (10/s)C_1(s+a)$$

$$V_o = \frac{10}{s}\left(\frac{C_1}{C_1 + C_2}\right)$$

Thus, v_o is the input scaled by the factor $\dfrac{C_1}{C_1 + C_2}$.

P 13.88 **[a]** For $t < 0$:



$$R_{eq} = 0.8\,\text{k}\Omega \| 4\,\text{k}\Omega \| 16\,\text{k}\Omega = 0.64\,\text{k}\Omega; \qquad v = 5(640) = 3200\,\text{V}$$

$$i_1(0^-) = \frac{3200}{4000} = 0.8\,\text{A}; \qquad i_2(0^-) = \frac{3200}{1600} = 0.2\,\text{A}$$

[b] For $t > 0$:

$$i_1 + i_2 = 0$$

$$8(\Delta i_1) = 2(\Delta i_2)$$

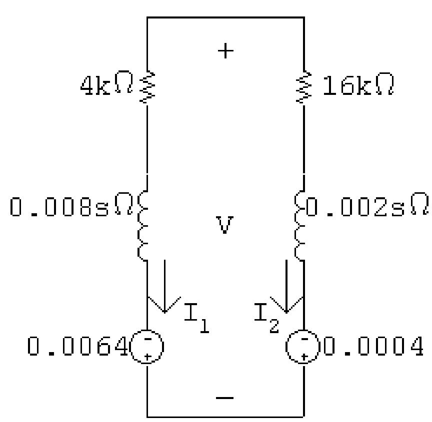
$$i_1(0^-) + \Delta i_1 + i_2(0^-) + \Delta i_2 = 0; \qquad \text{therefore} \quad \Delta i_1 = -0.2\,\text{A}$$

$$\Delta i_2 = -0.8\,\text{A}; \qquad i_1(0^+) = 0.8 - 0.2 = 0.6\,\text{A}$$

[c] $i_2(0^-) = 0.2\,\text{A}$

[d] $i_2(0^+) = 0.2 - 0.8 = -0.6\,\text{A}$

[e] The s-domain equivalent circuit for $t > 0$ is



$$I_1 = \frac{0.006}{0.01s + 20{,}000} = \frac{0.6}{s + 2 \times 10^6}$$

$$i_1(t) = 0.6e^{-2\times 10^6 t}u(t)\,\text{A}$$

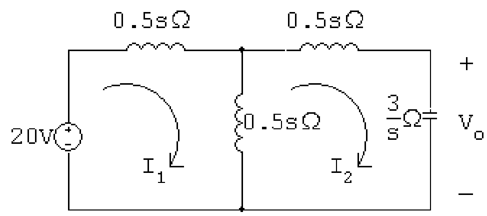
[f] $i_2(t) = -i_1(t) = -0.6e^{-2\times 10^6 t}u(t)\,\text{A}$

[g] $V = -0.0064 + (0.008s + 4000)I_1 = \dfrac{-0.0016(s + 6.5\times 10^6)}{s + 2\times 10^6}$

$$= -1.6\times 10^{-3} - \frac{7200}{s + 2\times 10^6}$$

$$v(t) = [-1.6\times 10^{-3}\delta(t)] - [7200e^{-2\times 10^6 t}u(t)]\,\text{V}$$

P 13.89 **[a]**



$$20 = sI_1 - 0.5sI_2$$

$$0 = -0.5sI_1 + \left(s + \frac{3}{s}\right)I_2$$

$$\Delta = \begin{vmatrix} s & -0.5s \\ -0.5s & (s+3/s) \end{vmatrix} = s^2 + 3 - 0.25s^2 = 0.75(s^2+4)$$

$$N_1 = \begin{vmatrix} 20 & -0.5s \\ 0 & (s+3/s) \end{vmatrix} = 20s + \frac{60}{s} = \frac{20s^2+60}{s} = \frac{20(s^2+3)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{20(s^2+3)}{s(0.75)(s^2+4)} = \frac{80}{3}\cdot\frac{s^2+3}{s(s^2+4)}$$

$$= \frac{K_0}{s} + \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_0 = \frac{80}{3}\left(\frac{3}{4}\right) = 20; \qquad K_1 = \frac{80}{3}\left[\frac{-4+3}{(j2)(j4)}\right] = \frac{10}{3}\angle 0^\circ$$

$$\therefore \quad i_1 = \left[20 + \frac{20}{3}\cos 2t\right]u(t)\,\text{A}$$

[b] $N_2 = \begin{vmatrix} s & 20 \\ -0.5s & 0 \end{vmatrix} = 10s$

$$I_2 = \frac{N_2}{\Delta} = \frac{10s}{0.75(s^2+4)} = \frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_1 = \frac{40}{3}\left(\frac{j2}{j4}\right) = \frac{20}{3}\underline{/0^\circ}$$

$$i_2 = \left[\frac{40}{3}\cos 2t\right]u(t)\,\text{A}$$

[c] $V_0 = \frac{3}{s}I_2 = \left(\frac{3}{s}\right)\frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{40}{s^2+4} = \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$

$$K_1 = \frac{40}{j4} = -j10 = 10\underline{/90^\circ}$$

$$v_o = 20\cos(2t - 90^\circ) = 20\sin 2t$$

$$v_o = [20\sin 2t]u(t)\,\text{V}$$

[d] Let us begin by noting i_1 jumps from 0 to (80/3) A between 0^- and 0^+ and in this same interval i_2 jumps from 0 to (40/3) A. Therefore in the derivatives of i_1 and i_2 there will be impulses of $(80/3)\delta(t)$ and $(40/3)\delta(t)$, respectively. Thus

$$\frac{di_1}{dt} = \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t\,\text{A/s}$$

$$\frac{di_2}{dt} = \frac{40}{3}\delta(t) - \frac{80}{3}\sin 2t\,\text{A/s}$$

From the circuit diagram we have

$$20\delta(t) = 1\frac{di_1}{dt} - 0.5\frac{di_2}{dt}$$

$$= \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t - \frac{20\delta(t)}{3} + \frac{40}{3}\sin 2t$$

$$= 20\delta(t)$$

Thus our solutions for i_1 and i_2 are in agreement with known circuit behavior.

Let us also note the impulsive voltage will impart energy into the circuit. Since there is no resistance in the circuit, the energy will not dissipate. Thus the fact that i_1, i_2, and v_o exist for all time is consistent with known circuit behavior.

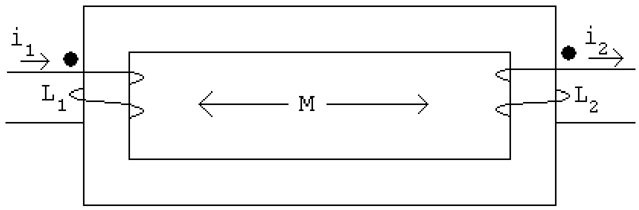
Also note that although i_1 has a dc component, i_2 does not. This follows from known transformer behavior.

Finally we note the flux linkage prior to the appearance of the impulsive voltage is zero. Now since $v = d\lambda/dt$, the impulsive voltage source must be matched to an instantaneous change in flux linkage at $t = 0^+$ of 20. For the given polarity dots and reference directions of i_1 and i_2 we have

$$\lambda(0^+) = L_1 i_1(0^+) + M i_1(0^+) - L_2 i_2(0^+) - M i_2(0^+)$$

$$\lambda(0^+) = 1\left(\frac{80}{3}\right) + 0.5\left(\frac{80}{3}\right) - 1\left(\frac{40}{3}\right) - 0.5\left(\frac{40}{3}\right)$$

$$= \frac{120}{3} - \frac{60}{3} = 20 \quad \text{(checks)}$$



P 13.90 **[a]** For $t < 0$, $0.5v_1 = 2v_2$; therefore $v_1 = 4v_2$

$v_1 + v_2 = 100$; therefore $v_1(0^-) = 80\,\text{V}$

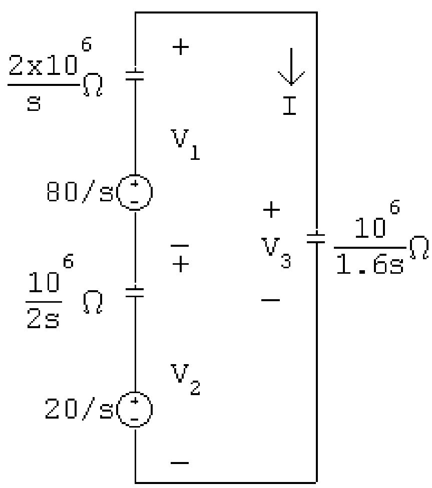
[b] $v_2(0^-) = 20\,\text{V}$

[c] $v_3(0^-) = 0\,\text{V}$

[d] For $t > 0$:

$$I = \frac{100/s}{3.125/s} \times 10^{-6} = 32 \times 10^{-6}$$

$i(t) = 32\delta(t)\,\mu\text{A}$



[e] $v_1(0^+) = -\frac{10^6}{0.5}\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 80 = -64 + 80 = 16\,\text{V}$

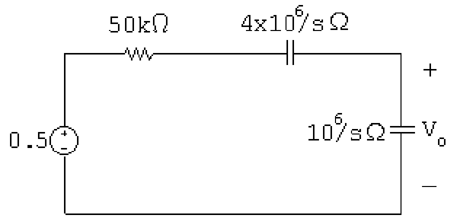
[f] $v_2(0^+) = -\frac{10^6}{2}\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 20 = -16 + 20 = 4\,\text{V}$

[g] $V_3 = \frac{0.625 \times 10^6}{s} \cdot 32 \times 10^{-6} = \frac{20}{s}$

$v_3(t) = 20u(t)\,\text{V}$; $v_3(0^+) = 20\,\text{V}$

Check: $v_1(0^+) + v_2(0^+) = v_3(0^+)$

P 13.91 **[a]**



$$V_o = \frac{0.5}{50{,}000 + 5 \times 10^6/s} \cdot \frac{10^6}{s}$$

$$\frac{500{,}000}{50{,}000s + 5 \times 10^6} = \frac{10}{s + 100}$$

$$v_o = 10e^{-100t}u(t)\,\text{V}$$

[b] At $t = 0$ the current in the $1\,\mu$F capacitor is $10\delta(t)\,\mu$A

$$\therefore \qquad v_o(0^+) = 10^6 \int_{0^-}^{0^+} 10 \times 10^{-6}\delta(t)\,dt = 10\,\text{V}$$

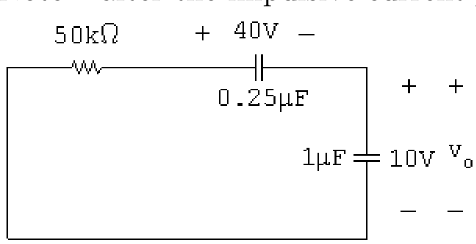
After the impulsive current has charged the $1\,\mu$F capacitor to 10 V it discharges through the 50 kΩ resistor.

$$C_e = \frac{C_1 C_2}{C_1 + C_2} = \frac{0.25}{1.25} = 0.2\,\mu\text{F}$$

$$\tau = (50{,}000)(0.2 \times 10^{-6}) = 10^{-2}$$

$$\frac{1}{\tau} = 100 \text{ (checks)}$$

Note – after the impulsive current passes the circuit becomes



The solution for v_o in this circuit is also

$$v_o = 10e^{-100t}u(t)\,\text{V}$$

P 13.92 **[a]** The circuit parameters are

$$R_\text{a} = \frac{120^2}{1200} = 12\,\Omega \qquad R_\text{b} = \frac{120^2}{1800} = 8\,\Omega \qquad X_\text{a} = \frac{120^2}{350} = \frac{288}{7}\,\Omega$$

The branch currents are

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{12} = 10\underline{/0^\circ}\text{ A(rms)} \qquad \mathbf{I}_2 = \frac{120\underline{/0^\circ}}{j1440/35} = -j\frac{35}{12} = \frac{35}{12}\underline{/-90^\circ}\text{ A(rms)}$$

$$\mathbf{I}_3 = \frac{120\underline{/0^\circ}}{8} = 15\underline{/0^\circ}\ \text{A(rms)}$$

$$\therefore\quad \mathbf{I}_o = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 25 - j\frac{35}{12} = 25.17\underline{/-6.65^\circ}\ \text{A(rms)}$$

Therefore,

$$i_2 = \left(\frac{35}{12}\right)\sqrt{2}\cos(\omega t - 90^\circ)\,\text{A} \qquad \text{and} \qquad i_o = 25.17\sqrt{2}\cos(\omega t - 6.65^\circ)\,\text{A}$$

Thus,

$$i_2(0^-) = i_2(0^+) = 0\,\text{A} \qquad \text{and} \qquad i_o(0^-) = i_o(0^+) = 25\sqrt{2}\,\text{A}$$

[b] Begin by using the s-domain circuit in Fig. 13.60 to solve for V_0 symbolically. Write a single node voltage equation:

$$\frac{V_0 - (V_g + L_\ell I_o)}{sL_\ell} + \frac{V_0}{R_a} + \frac{V_0}{sL_a} = 0$$

$$\therefore\quad V_0 = \frac{(R_a/L_\ell)V_g + I_oR_a}{s + [R_a(L_a + L_\ell)]/L_aL_\ell}$$

where $L_\ell = 1/120\pi$ H, $L_a = 12/35\pi$ H, $R_a = 12\,\Omega$, and $I_0R_a = 300\sqrt{2}$ V. Thus,

$$V_0 = \frac{1440\pi(122.92\sqrt{2}s - 3000\pi\sqrt{2})}{(s + 1475\pi)(s^2 + 14{,}400\pi^2)} + \frac{300\sqrt{2}}{s + 1475\pi}$$

$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi} + \frac{300\sqrt{2}}{s + 1475\pi}$$

The coefficients are

$$K_1 = -121.18\sqrt{2}\,\text{V} \qquad K_2 = 61.03\sqrt{2}\underline{/6.85^\circ}\,\text{V} \qquad K_2^* = 61.03\sqrt{2}\underline{/-6.85^\circ}$$

Note that $K_1 + 300\sqrt{2} = 178.82\sqrt{2}$ V. Thus, the inverse transform of V_0 is

$$v_0 = 178.82\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)\,\text{V}$$

Initially,

$$v_0(0^+) = 178.82\sqrt{2} + 122.06\sqrt{2}\cos 6.85^\circ = 300\sqrt{2}\,\text{V}$$

Note that at $t = 0^+$ the initial value of i_L, which is $25\sqrt{2}$ A, exists in the $12\,\Omega$ resistor R_a. Thus, the initial value of V_0 is $(25\sqrt{2})(12) = 300\sqrt{2}$ V.

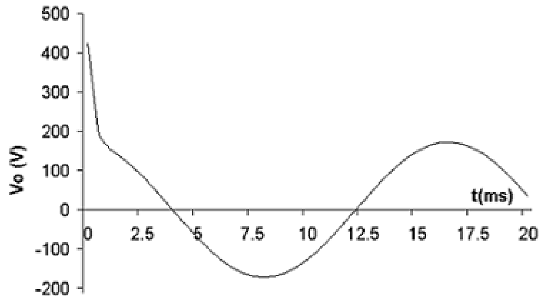
[c] The phasor domain equivalent circuit has a $j1\,\Omega$ inductive impedance in series with the parallel combination of a $12\,\Omega$ resistive impedance and a $j1440/35\,\Omega$ inductive impedance (remember that $\omega = 120\pi$ rad/s). Note that $\mathbf{V}_g = 120\underline{/0^\circ} + (25.17\underline{/-6.65^\circ})(j1) = 125.43\underline{/11.50^\circ}$ V(rms). The node voltage equation in the phasor domain circuit is

$$\frac{\mathbf{V}_0 - 125.43\underline{/11.50^\circ}}{j1} + \frac{\mathbf{V}_0}{12} + \frac{35\mathbf{V}_0}{1440} = 0$$

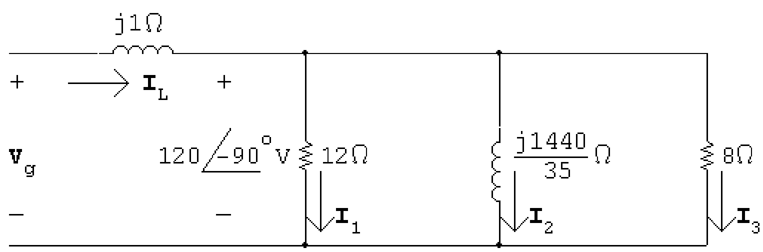
$\therefore \quad \mathbf{V}_0 = 122.06\underline{/6.85^\circ}$ V(rms)

Therefore, $v_0 = 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)$ V, agreeing with the steady-state component of the result in part (b).

[d] A plot of v_0, generated in Excel, is shown below.



P 13.93 **[a]** At $t = 0^-$ the phasor domain equivalent circuit is



$$\mathbf{I}_1 = \frac{-j120}{12} = -j10 = 10\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{-j120(35)}{j1440} = -\frac{35}{12} = \frac{35}{12}\underline{/180^\circ}\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{-j120}{8} = -j15 = 15\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = -\frac{35}{12} - j25 = 25.17\underline{/-96.65^\circ}\text{A (rms)}$$

$$i_L = 25.17\sqrt{2}\cos(120\pi t - 96.65^\circ)\text{A}$$

$$i_L(0^-) = i_L(0^+) = -2.92\sqrt{2}\text{A}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + 180^\circ)\text{A}$$

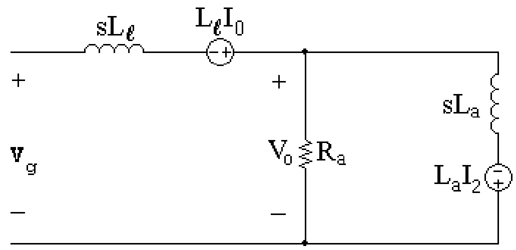
$$i_2(0^-) = i_2(0^+) = -\frac{35}{12}\sqrt{2} = -2.92\sqrt{2}\text{A}$$

$$\mathbf{V}_g = \mathbf{V}_o + j1\mathbf{I}_L$$

$$\mathbf{V}_g = -j120 + 25 - j\frac{35}{12}$$
$$= 25 - j122.92 = 125.43\underline{/-78.50^\circ}\text{V (rms)}$$
$$v_g = 125.43\sqrt{2}\cos(120\pi t - 78.50^\circ)\text{V}$$
$$= 125.43\sqrt{2}[\cos 120\pi t \cos 78.50^\circ + \sin 120\pi t \sin 78.50^\circ]$$
$$= 25\sqrt{2}\cos 120\pi t + 122.92\sqrt{2}\sin 120\pi t$$

$$\therefore \quad V_g = \frac{25\sqrt{2}s + 122.92\sqrt{2}(120\pi)}{s^2 + (120\pi)^2}$$

s-domain circuit:



where

$$L_l = \frac{1}{120\pi}\text{ H}; \qquad L_a = \frac{12}{35\pi}\text{ H}; \qquad R_a = 12\,\Omega$$

$$i_L(0) = -2.92\sqrt{2}\,\text{A}; \qquad i_2(0) = -2.92\sqrt{2}\,\text{A}$$

The node voltage equation is

$$0 = \frac{V_o - (V_g + i_L(0)L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + i_2(0)L_a}{sL_a}$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l}{[s + R_a(L_l + L_a)/L_a L_l]} + \frac{R_a[i_L(0) - i_2(0)]}{[s + R_a(L_l + L_a)/L_l L_a]}$$

$$\frac{R_a}{L_l} = 1440\pi$$

$$\frac{R_a(L_l + L_a)}{L_l L_a} = \frac{12(\frac{1}{120\pi} + \frac{12}{35\pi})}{(\frac{12}{35\pi})(\frac{1}{120\pi})} = 1475\pi$$

$$i_L(0) - i_2(0) = -2.92\sqrt{2} + 2.92\sqrt{2} = 0$$

$$\therefore \quad V_o = \frac{1440\pi[25\sqrt{2}s + 122.92\sqrt{2}(120\pi)]}{(s + 1475\pi)[s^2 + (120\pi)^2]}$$
$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

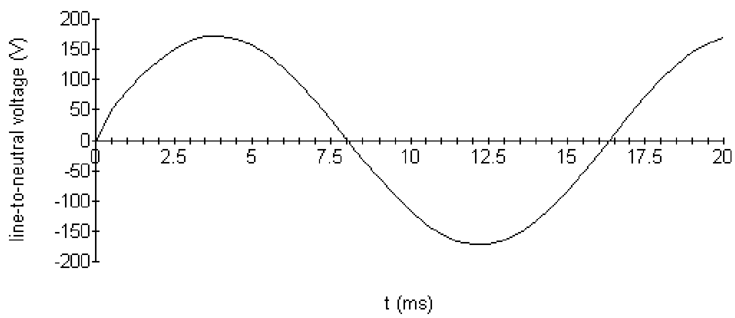
$K_1 = -14.55\sqrt{2} \qquad K_2 = 61.03\sqrt{2}\underline{/-83.15^\circ}$

$\therefore \quad v_o(t) = -14.55\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t - 83.15^\circ)\text{V}$

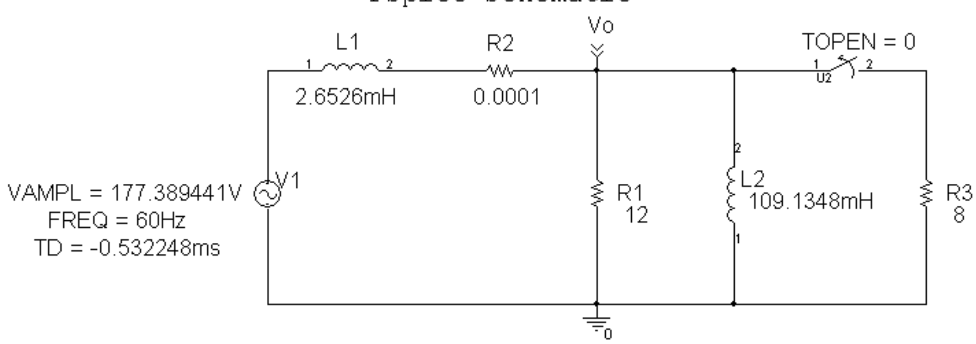
Check:

$v_o(0) = (-14.55 + 14.55)\sqrt{2} = 0$

[**b**]



PSpice schematic



PSpice output file

```
**** 07/15/01 07:40:45 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-tran"  [ C:\shortcircuits\solutions\p9_76-SCHEMATIC1-tran.sim ]


 ****     CIRCUIT DESCRIPTION


******************************************************************************
```

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** Creating circuit file "p9_76-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_76-SCHEMATIC1.net"



**** INCLUDING p9_76-SCHEMATIC1.net ****
* source P9_76
V_V1         N00637 0
+SIN 0 177.389441V 60Hz -0.532248ms 0 0
L_L1         N00637 N01311  2.6526mH IC=0
L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  12
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg

**** RESUMING p9_76-SCHEMATIC1-tran.sim.cir ****
.END
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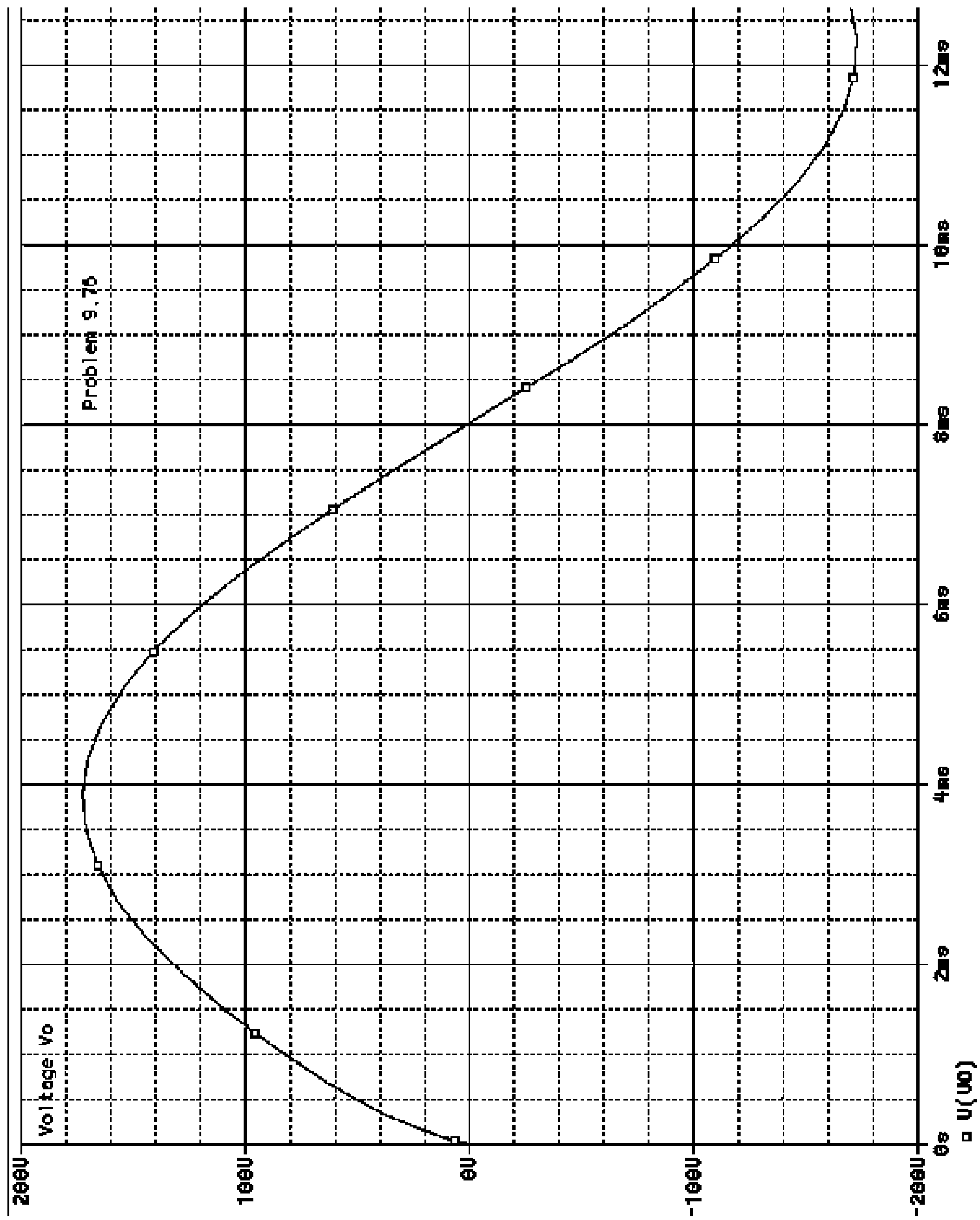
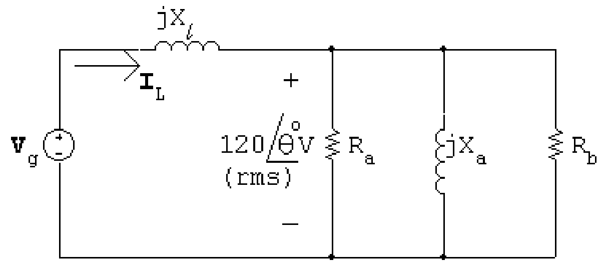


[c] In Problem 13.92, the line-to-neutral voltage spikes at $300\sqrt{2}$ V. Here the line-to-neutral voltage has no spike. Thus the amount of voltage disturbance depends on what part of the cycle the sinusoidal steady-state voltage is switched.

P 13.94 **[a]** First find V_g before R_b is disconnected. The phasor domain circuit is



$$\mathbf{I}_L = \frac{120/\theta^\circ}{R_a} + \frac{120/\theta^\circ}{R_b} + \frac{120/\theta^\circ}{jX_a}$$
$$= \frac{120/\theta^\circ}{R_aR_bX_a}[(R_a + R_b)X_a = jR_aR_b]$$

Since $X_l = 1\,\Omega$ we have

$$\mathbf{V}_g = 120/\theta^\circ + \frac{120/\theta^\circ}{R_aR_bX_a}[R_aR_b + j(R_a + R_b)X_a]$$

$$R_a = 12\,\Omega; \qquad R_b = 8\,\Omega; \qquad X_a = \frac{1440}{35}\,\Omega$$

$$\mathbf{V}_g = \frac{120/\theta^\circ}{1400}(1475 + j300)$$
$$= \frac{25}{12}/\theta^\circ(59 + j12) = 125.43/(\theta + 11.50)^\circ$$

$$v_g = 125.43\sqrt{2}\cos(120\pi t + \theta + 11.50^\circ)\text{V}$$

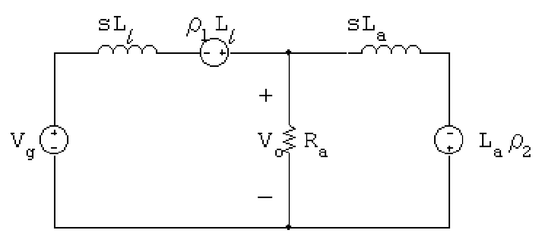
Let $\beta = \theta + 11.50^\circ$. Then

$$v_g = 125.43\sqrt{2}(\cos 120\pi t \cos\beta - \sin 120\pi t \sin\beta)\text{V}$$

Therefore

$$V_g = \frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + (120\pi)^2}$$

The s-domain circuit becomes



where $\rho_1 = i_L(0^+)$ and $\rho_2 = i_2(0^+)$.
The s-domain node voltage equation is

$$\frac{V_o - (V_g + \rho_1 L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + \rho_2 L_a}{sL_a} = 0$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l + (\rho_1 - \rho_2)R_a}{[s + \frac{(L_a+L_l)R_a}{L_a L_l}]}$$

Substituting the numerical values

$$L_l = \frac{1}{120\pi}\ \text{H}; \qquad L_a = \frac{12}{35\pi}\ \text{H}; \qquad R_a = 12\,\Omega; \qquad R_b = 8\,\Omega;$$

gives

$$V_o = \frac{1440\pi V_g + 12(\rho_1 - \rho_2)}{(s + 1475\pi)}$$

Now determine the values of ρ_1 and ρ_2.

$$\rho_1 = i_L(0^+) \quad \text{and} \quad \rho_2 = i_2(0^+)$$

$$\mathbf{I}_L = \frac{120/\underline{\theta^\circ}}{R_a R_b X_a}[(R_a + R_b)X_a - jR_a R_b]$$

$$= \frac{120/\underline{\theta^\circ}}{96(1440/35)}\left[\frac{(20)(1440)}{35} - j96\right]$$

$$= 25.17/\underline{(\theta - 6.65)^\circ}\text{A(rms)}$$

$$\therefore \quad i_L = 25.17\sqrt{2}\cos(120\pi t + \theta - 6.65^\circ)\text{A}$$

$$i_L(0^+) = \rho_1 = 25.17\sqrt{2}\cos(\theta - 6.65^\circ)\text{A}$$

$$\therefore \quad \rho_1 = 25\sqrt{2}\cos\theta + 2.92\sqrt{2}\sin\theta\text{A}$$

$$\mathbf{I}_2 = \frac{120/\underline{\theta^\circ}}{j(1440/35)} = \frac{35}{12}/\underline{(\theta - 90)^\circ}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + \theta - 90^\circ)\text{A}$$

$$\rho_2 = i_2(0^+) = \frac{35}{12}\sqrt{2}\sin\theta = 2.92\sqrt{2}\sin\theta\text{A}$$

$$\therefore \quad \rho_1 = \rho_2 = 25\sqrt{2}\cos\theta$$

$$(\rho_1 - \rho_2)R_a = 300\sqrt{2}\cos\theta$$

$$\therefore \quad \text{V}_o = \frac{1440\pi}{s + 1475\pi}\cdot V_g + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{1440\pi}{s + 1475\pi}\left[\frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + 14{,}400\pi^2}\right] + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{K_1 + 300\sqrt{2}\cos\theta}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

Now

$$K_1 = \frac{(1440\pi)(125.43\sqrt{2})[-1475\pi\cos\beta - 120\pi\sin\beta]}{1475^2\pi^2 + 14{,}400\pi^2}$$

$$= \frac{-1440(125.43\sqrt{2})[1475\cos\beta + 120\sin\beta]}{1475^2 + 14{,}000}$$

Since $\beta = \theta + 11.50°$, K_1 reduces to

$K_1 = -121.18\sqrt{2}\cos\theta + 14.55\sqrt{2}\sin\theta$

From the partial fraction expansion for V_o we see $v_o(t)$ will go directly into steady state when $K_1 = -300\sqrt{2}\cos\theta$. It follow that

$14.55\sqrt{2}\sin\theta = -178.82\sqrt{2}\cos\theta$

or $\tan\theta = -12.29$

Therefore, $\theta = -85.35°$

[b] When $\theta = -85.35°$, $\beta = -73.85°$

$$\therefore\quad K_2 = \frac{1440\pi(125.43\sqrt{2})[-120\pi\sin(-73.85°) + j120\pi\cos(-73.85°)}{(1475\pi + j120\pi)(j240\pi)}$$

$$= \frac{720\sqrt{2}(120.48 + j34.88)}{-120 + j1475}$$

$$= 61.03\sqrt{2}\underline{/-78.50°}$$

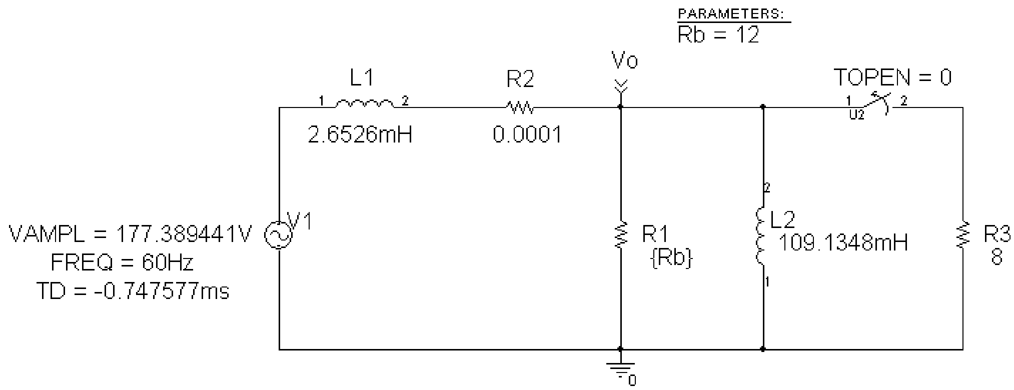
$$\therefore\quad v_o = 122.06\sqrt{2}\cos(120\pi t - 78.50°)\text{ V}\quad t > 0$$

$$= 172.61\cos(120\pi t - 78.50°)\text{ V}\quad t > 0$$

[c] $v_{o1} = 169.71\cos(120\pi t - 85.35°)\text{V}\qquad t < 0$

$v_{o2} = 172.61\cos(120\pi t - 78.50°)\text{V}\qquad t > 0$

PSpice schematic



PSpice output file

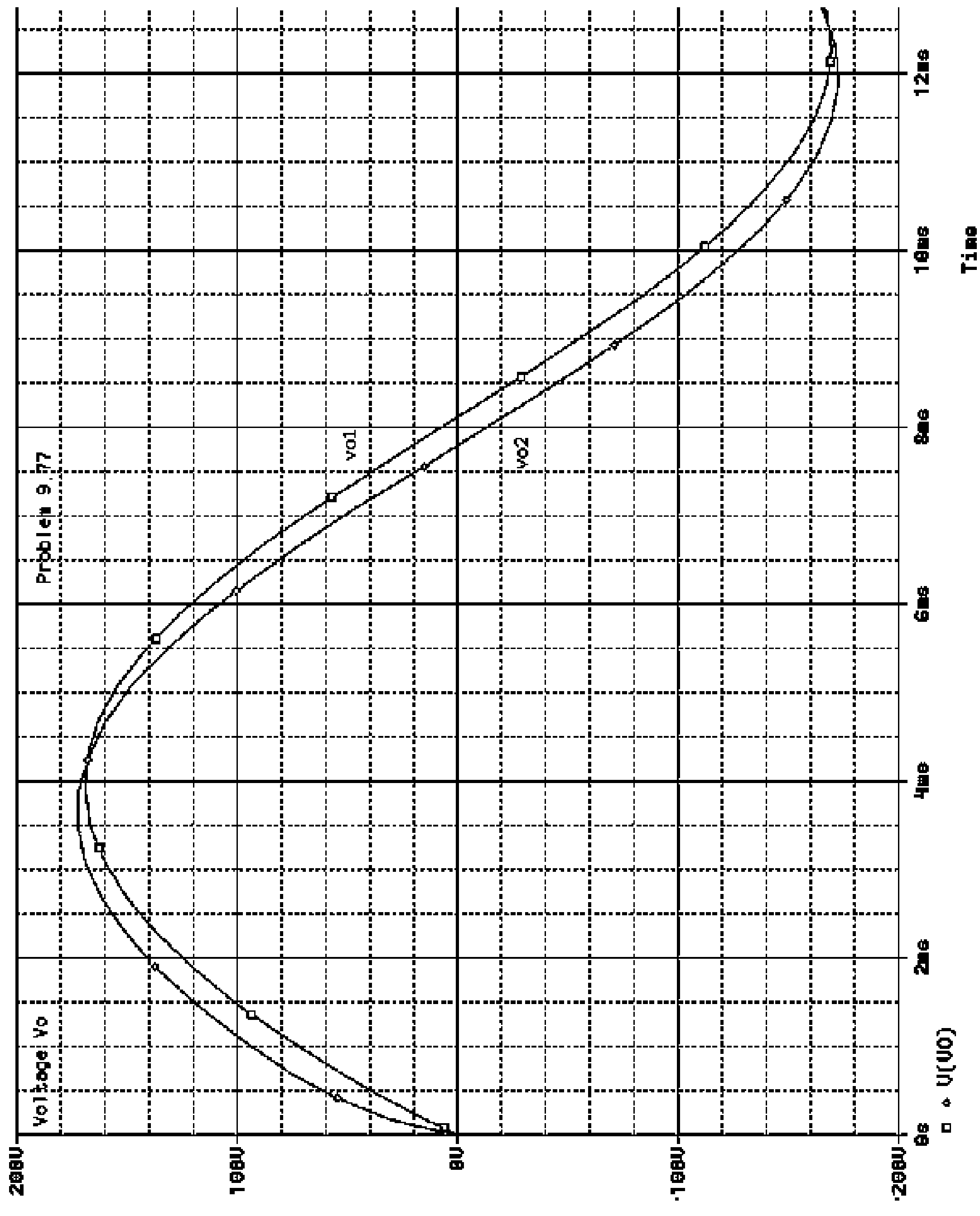
```
** Creating circuit file "p9_77-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.STEP PARAM Rb LIST 4.8  12
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_77-SCHEMATIC1.net"

**** INCLUDING p9_77-SCHEMATIC1.net ****
* source P9_77
V_V1         N00637 0
+SIN 0 177.389441V 60Hz -0.747577ms 0 0
L_L1         N00637 N01311  2.6526mH IC=0
L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  {Rb}
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg
.PARAM  Rb=12

**** RESUMING p9_77-SCHEMATIC1-tran.sim.cir ****
.END
```


Voltage Vo
Problem 9.77
vo1
vo2
200V
100V
0V
-100V
-200V
0s
2ms
4ms
6ms
8ms
10ms
12ms
Time
V(VO)