14 a.  $F = \frac{s_1^2}{s_2^2} = \frac{5.8}{2.4} = 2.4$ , degrees of freedom 15 and 20

Using F table, p-value is between 0.025 and 0.05. (Actual p-value = 0.0334.)

p-value < 0.05, reject  $H_0$ . Conclude  $\sigma_1^2 > \sigma_2^2$ 

- b.  $F_{0.05} = 2.20$ , reject  $H_0$  if  $F \ge 2.20$ 2.4 > 2.20, reject  $H_0$ . Conclude  $\sigma_1^2 > \sigma_2^2$
- 16 a.  $H_0: \sigma_1^2 \le \sigma_2^2$  (population 1 is four-year-old automobiles),  $H_1: \sigma_1^2 > \sigma_2^2$ 
  - b. F = 2.89, degrees of freedom 25 and 24, p-value is less than 0.01.

Reject H<sub>0</sub>. Conclude that four-year-old cars have a larger variance in annual repair costs compared to two-year-old cars. This is expected due to the fact that older cars are more likely to have more expensive repairs that lead to greater variance in the annual repair costs.

18 F = 2.15, degrees of freedom 25 and 25, one-tailed p-value is between 0.05 and 0.025.

p-value  $\leq 0.05$ , reject  $H_0$ . Conclude that the small cap fund is riskier than the large cap fund.

20 a.  $H_0: \sigma_1^2 \le \sigma_2^2$  (Population 1 is wet roads),  $H_1: \sigma_1^2 > \sigma_2^2$   $F = \frac{s_1^2}{s_2^2} = \frac{32^2}{16^2} = 4.00$ , degrees of freedom 15 and 15

Using F table, p-value is less than 0.01. (Actual p-value = 0.0054.)

p-value < 0.05, reject  $H_0$ . Conclude that there is greater variability in stopping distances on wet roads.

- Drive carefully on wet roads because of the uncertainty in stopping distances.
- **22**  $H_0: \sigma_1^2 = \sigma_2^2, H_1: \sigma_1^2 \neq \sigma_2^2$

F = 8.28, degrees of freedom 24 and 21, two-tailed *p*-value is less than 0.02.

p-value < 0.05, reject  $H_0$ . The process variances are significantly different. Machine 1 offers the greater opportunity for process quality improvements.

#### Chapter 12

#### Solutions

Expected frequencies:  $e_1 = 300(0.25) = 75$ ,  $e_2 = 300(0.25) = 75$ ,  $e_3 = 300(0.25) = 75$ ,  $e_4 = 300(0.25) = 75$ . Actual frequencies:  $f_1 = 85$ ,  $f_2 = 95$ ,  $f_3 = 50$ ,  $f_4 = 70$ .

 $\chi^{2} = \frac{(85 - 75)^{2}}{75} + \frac{(95 - 75)^{2}}{75} + \frac{(50 - 75)^{2}}{75} + \frac{(70 - 75)^{2}}{75}$   $= \frac{100}{75} + \frac{400}{75} + \frac{625}{75} + \frac{25}{75}$   $= \frac{1150}{75}$  = 15.33

k-1=3 degrees of freedom. Chi-squared table shows p-value less than 0.005. (Actual p-value = 0.0016.) p-value < 0.05, reject  $H_0$ , conclude that the population proportions are not the same.

- 4 χ² = 16.3, df = 3, p-value < 0.005, reject H₀. Conclude that the ratings differ. A comparison of observed and expected frequencies shows telephone service has more excellent and good ratings.
- 6  $\chi^2 = 21.7$ , df = 6, p-value < 0.005, reject  $H_0$ . The park manager should not plan on the same number attending each day. Plan on a larger staff for Sundays.
- 8 H<sub>0</sub> = The column variable is independent of the row variable H<sub>1</sub> = The column variable is not independent of the row variable

Expected Frequencies:

	Α	В	C
Р	28.5	39.9	45.6
Q	21.5	30.1	34.4

$$\chi^2 = \frac{(20 - 28.5)^2}{28.5} + \frac{(44 - 39.9)^2}{39.9} + \frac{(50 - 45.6)^2}{45.6} + \frac{(30 - 21.5)^2}{21.5} + \frac{(26 - 30.1)^2}{30.1} + \frac{(30 - 34.4)^2}{34.4}$$

Degrees of freedom = (2-1)(3-1) = 2. Using  $\chi^2$  table,  $\chi^2 = 7.86$  provides a p-value between 0.01 and 0.025. (Actual p-value = 0.0196.) p-value < 0.05, reject  $H_0$ . Conclude that the column variable is not independent of the row variable.

- 10  $\chi^2 = 100.4$ , df = 2, p-value is between 0.025 and 0.05, reject  $H_0$ . Conclude that the type of ticket purchased is not independent of the type of flight.
- 12 a. Observed Frequency (f;;)

	Pharm	Consumer	Computer	Telecom	Total
Correct	207	136	151	178	672
Incorrect	3	4	9	12	28
Total	210	140	160	190	700

## Expected Frequency $(e_{ii})$

	Pharm	Consumer	Computer	Telecom	Total
Correct	201.6	134.4	153.6	182.4	, 5 cm
Incorrect	8.4	5.6	6.4		6/2
Total	210	140		7.6	28
	210	170	160	190	700

## Chi-squared $(f_{ij} - e_{ij})^2 / e_{ij}$

	Pharm	Consumer	Computer	Telecom	Total
Correct	0.14	0.02	0.04	0.11	
Incorrect	3.47	0.46	1.06	2.55	7.53
					$\chi^2 = 7.8$

Degrees of freedom = (2-1)(4-1) = 3. Using  $\chi^2$  table,  $\chi^2 = 7.85$  shows p-value is between 0.025 and 0.05. (Actual p-value = 0.0493.) p-value < 0.05, reject  $H_0$ . Conclude that fulfilment of orders is not independent of industry.

- b. The pharmaceutical industry is doing the best with 207 of 210 (98.6 per cent) correctly filled orders.
- 14  $\chi^2 = 8.10$ , df = 23, p-value is between 0.01 and 0.025, reject  $H_0$ . Conclude that shift and quality are not independent.
- 16  $\chi^2 = 9.76$ , df = 4, p-value is between 0.025 and 0.05, reject  $H_0$ . We can conclude that industry type and P/E ratio are related. Banking tends to have lower P/E ratios.
- 18 First estimate  $\mu$  from the sample data. Sample size = 120.

$$\bar{x} = \frac{0(39) + 1(30) + 2(30) + 3(18) + 4(3)}{120} = \frac{156}{120} = 1.3$$

Therefore, we use Poisson probabilities with  $\mu=1.3$  to compute expected frequencies.

$$\chi^2 = \frac{(6.30)^2}{32.70} + \frac{(-12.51)^2}{42.51} + \frac{(2.37)^2}{27.63} + \frac{(6.02)^2}{11.98} + \frac{(-2.17)^2}{5.16} = 9.04$$

x	Observed frequency	Poisson probabili <u>t</u> y	Expected frequency	Difference $(f_i - e_i)$
0	39	0.2725	32.70	6.30
1	30	0.3543	42.51	-12.51
2	30	0.2303	27.63	2.37
3	18	0.0998	11.98	6.02
4	3	0.0431	5.16	-2.17

Degrees of freedom = 5 - 1 - 1 = 3. Using  $\chi^2$  table,  $\chi^2 = 9.04$  shows p-value is between 0.025 and 0.05. (Actual p-value = 0.0287.) p-value < 0.05, reject  $H_0$ . Conclude that the data do not follow a Poisson probability distribution.

# **20** $\bar{x} = 24.5$ , s = 3, n = 30. Use 6 classes.

Interval	Observed frequency	Expected frequency
less than 21.56	5	5
21.56-23.20	4	5
23.21-24.49	3	5
24.50-25.78	7	5
25.79-27.40	7	5
27.41 upwards	4	5

- $\chi^2 = 2.8$ . Degrees of freedom = 6 2 1 = 3. Using  $\chi^2$  table,  $\chi^2 = 2.8$  shows *p*-value is greater than 0.10. (Actual *p*-value = 0.4235.) *p*-value > 0.10, do not reject  $H_0$ . The assumption of a normal distribution cannot be rejected.
- 22  $\chi^2 = 4.30$ , df = 2. p-value greater than 0.10. Do not reject  $H_0$ .