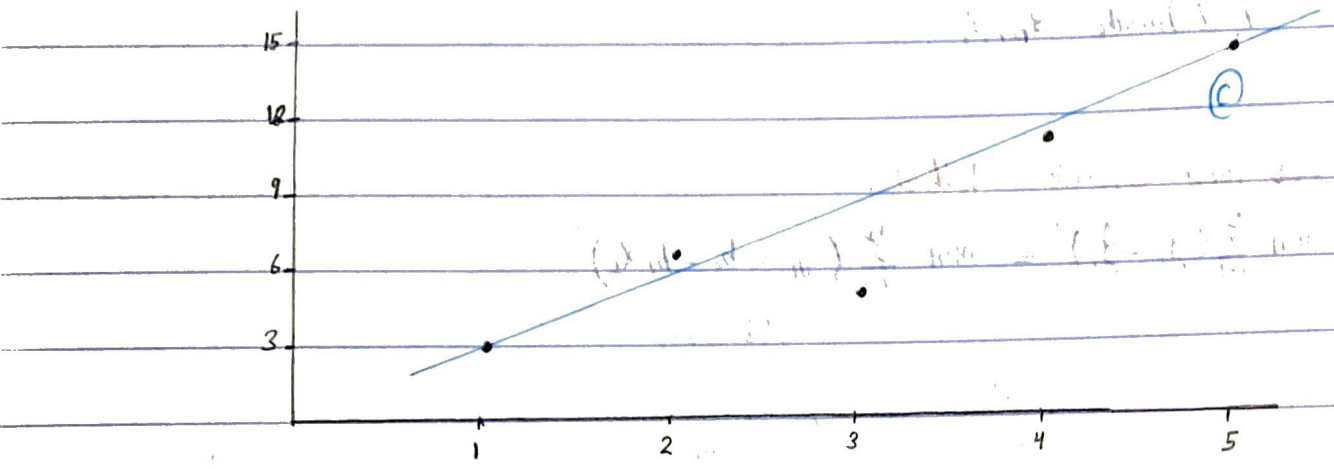
Exercises:

1. $x_i$ : 1 2 3 4 5 $\rightarrow \bar{x} = 3$

   $y_i$ : 3 7 5 11 14 $\rightarrow \bar{y} = 8$

| $(x_i - \bar{x})$ | $(y_i - \bar{y})$ | $(x_i - \bar{x})(y_i - \bar{y})$ | $(x_i - \bar{x})^2$ |
|---|---|---|---|
| -2 | -5 | 10 | 4 |
| -1 | -1 | 1 | 1 |
| 0 | -3 | 0 | 0 |
| 1 | 3 | 3 | 1 |
| 2 | 6 | 12 | 4 |
| 0 | 0 | 26 | 10 |

a. develope a scatter diagram for these data.




b. There appears to be a linear relationship between x and y.

c. Many different straight lines can be drawn to provied a linear approximation between x and y.

d. develope the estimate regression equation by computing the values of $b_0$ and $b_1$.

$$b_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = \frac{26}{10} = 2.6$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$b_0 = 8 - (2.6 \times 3) = 0.2$$

So $\hat{y} = 0.2 + 2.6x$.

e. Value of Y when X=4:

$$\hat{y} = 0.2 + 2.6(4)$$

$$\hat{y} = 10.6$$

2.

| $x_i$ | $y_i$ | $(x_i - \bar{x})$ | $(x_i - \bar{x})^2$ | $(y_i - \bar{y})$ | $(x_i - \bar{x})(y_i - \bar{y})$ |
|---|---|---|---|---|---|
| 2 | 25 | -1.8 | 3.24 | 1.8 | -3.24 |
| 3 | 25 | -0.8 | 0.64 | 1.8 | -1.44 |
| 5 | 20 | 1.2 | 1.44 | -3.2 | -3.84 |
| 1 | 30 | -2.8 | 7.84 | 6.8 | -19.04 |
| 8 | 16 | 4.2 | 17.64 | -7.2 | -30.24 |
| $\bar{x} = 3.8$ | 23.2 | | 30.8 | | -57.8 |

a, b, c : seem ok.

d. develope the estimated regression equation by computing the value of $b_0$, $b_1$.

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{-57.8}{30.8} = -1.8766$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$= 23.2 - (-1.8766 \times 3.8)$$

$$b_0 = 30.3311$$

So $\hat{y} = 30.33 - 1.88X$

e. predict the value of Y when $X = 6$.

$$\hat{y} = 30.33 - (1.88 \times 6)$$

$$= 19.05$$

3.

| $x_i$ | $y_i$ | $(x_i - \bar{x})$ | $(y_i - \bar{y})$ | $(x_i - \bar{x})(y_i - \bar{y})$ | $(x_i - \bar{x})^2$ |
|---|---|---|---|---|---|
| 2 | 2 | -3.2 | -1.4 | 4.48 | 10.24 |
| 4 | 3 | -1.2 | -0.4 | 0.48 | 1.44 |
| 5 | 2 | -0.2 | -1.4 | 0.28 | 0.04 |
| 7 | 6 | 1.8 | 2.6 | 4.68 | 3.24 |
| 8 | 4 | 2.8 | 0.6 | 1.68 | 7.84 |
| | | | | 11.6 | 22.8 |

$\bar{x} = 5.2 \quad \bar{y} = 3.4$

b. develope the estimated regression equation for these data.

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{11.6}{22.8} = 0.51.$$

$$b_0 = \bar{y} - b_1 \bar{x}$$
$$= 3.4 - (0.51 \times 5.2)$$
$$= 0.75$$

$$\rightarrow \hat{y} = 0.75 + 0.51 X.$$

c. predict the value of y when $X = 4$.

$$\hat{y} = 0.75 + 0.51(4)$$
$$= 2.79$$