10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 [a] $\mathbf{V} = 100\underline{/-45^\circ}\,\text{V}, \qquad \mathbf{I} = 20\underline{/15^\circ}\,\text{A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45 - (15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\underline{/0^\circ}, \qquad \mathbf{I} = 20\underline{/120^\circ}$

$$P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

AP 10.2

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos(-60^\circ) = 0.5 \text{ leading}$$

$$\text{rf} = \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$$

AP 10.3

From Ex. 9.4 $I_{\text{eff}} = \frac{I_\rho}{\sqrt{3}} = \frac{0.18}{\sqrt{3}}$ A

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54\,\text{W}$$

AP 10.4 **[a]** $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

Therefore $\mathbf{I}_\ell = \frac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}$ A (rms)

$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}$ V (rms)

$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}$ A (rms)

[b] $S_\text{L} = \mathbf{V}_L\mathbf{I}_L^* = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)$ VA

$P_\text{L} = 1129.09$ W; $Q_\text{L} = 752.73$ VAR

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52$ W; $Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09$ VAR

[d] S_g(delivering) $= 250\mathbf{I}_\ell^* = (1152.62 - j376.36)$ VA

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_\text{cap} = \frac{|\mathbf{V}_\text{L}|^2}{-52} = \frac{(252.20)^2}{-52} = -1223.18$ VAR

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $94.09 + 752.73 + 376.36 = 1223.18$ VAR and

$1129.09 + 23.52 = 1152.62$ W

AP 10.5 Series circuit derivation:

$S = 250\mathbf{I}^* = (40{,}000 - j30{,}000)$

Therefore $\mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}$ A (rms)

$\mathbf{I} = 200\underline{/36.87^\circ}$ A (rms)

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

Therefore $R = 1\,\Omega$, $X_\text{C} = -0.75\,\Omega$

Parallel circuit derivation

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_C}; \qquad \text{therefore} \quad X_C = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

AP 10.6

$$S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\text{VA}$$

$$S_2 = 6000(0.8) - j6000(0.6) = 4800 - j3600\,\text{VA}$$

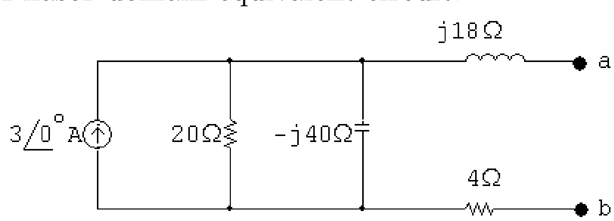
$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\text{VA}$$

$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\text{A}$$

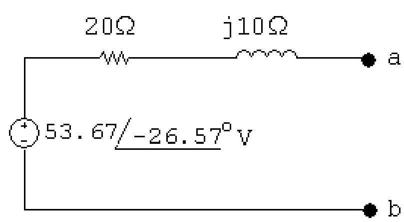
$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\text{V (rms)}$$

AP 10.7 **[a]** The phasor domain equivalent circuit and the Thévenin equivalent are shown below:

Phasor domain equivalent circuit:



Thévenin equivalent:



$$\mathbf{V}_{\text{Th}} = 3\frac{-j800}{20 - j40} = 48 - j24 = 53.67\underline{/-26.57^\circ}\,\text{V}$$

$$Z_{\text{Th}} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36\underline{/26.57^\circ}\,\Omega$$

For maximum power transfer, $Z_L = (20 - j10)\,\Omega$

[b] $\mathbf{I} = \dfrac{53.67\underline{/-26.57^\circ}}{40} = 1.34\underline{/-26.57^\circ}\,\text{A}$

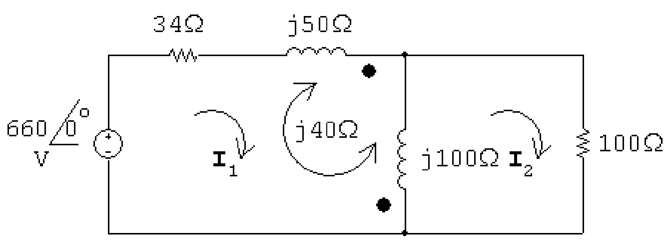
Therefore $\quad P = \left(\dfrac{1.34}{\sqrt{2}}\right)^2 20 = 17.96\,\text{W}$

[c] $R_\text{L} = |Z_\text{Th}| = 22.36\,\Omega$

[d] $\mathbf{I} = \dfrac{53.67\underline{/-26.57^\circ}}{42.36 + j10} = 1.23\underline{/-39.85^\circ}\,\text{A}$

Therefore $\quad P = \left(\dfrac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$

AP 10.8



Mesh current equations:

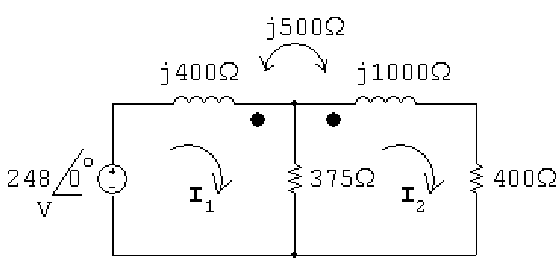
$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$

$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$

Solving,

$\mathbf{I}_2 = 3.5\underline{/0^\circ}\,\text{A}; \qquad \therefore \quad P = \dfrac{1}{2}(3.5)^2(100) = 612.50\,\text{W}$

AP 10.9 **[a]**



$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$

$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$

Solving,

$\mathbf{I}_1 = 0.80 - j0.62\,\text{A}; \qquad \mathbf{I}_2 = 0.4 - j0.3 = 0.5\underline{/-36.87^\circ}$

$\therefore \quad P = \dfrac{1}{2}(0.25)(400) = 50\,\text{W}$

[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32\,\text{A}$

$$P_{375} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2(375) = 49.20\,\text{W}$$

[c] $P_g = \frac{1}{2}(248)(0.8) = 99.20\,\text{W}$

$$\sum P_{\text{abs}} = 50 + 49.2 = 99.20\,\text{W} \quad \text{(checks)}$$

AP 10.10 **[a]** $V_{\text{Th}} = 210\,\text{V}; \qquad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$

Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = 14\,\text{A}; \qquad R_{\text{Th}} = \frac{210}{14} = 15\,\Omega$$

[b] $P_{\text{max}} = \left(\frac{210}{30}\right)^2 15 = 735\,\text{W}$

AP 10.11 **[a]** $\mathbf{V}_{\text{Th}} = -4(146\underline{/0^\circ}) = -584\underline{/0^\circ}\,\text{V (rms)}$

$$\mathbf{V}_2 = 4\mathbf{V}_1; \qquad \mathbf{I}_1 = -4\mathbf{I}_2$$

Short circuit equations:

$$146\underline{/0^\circ} = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = -146/365 = -0.40\,\text{A}; \qquad R_{\text{Th}} = \frac{-584}{-0.4} = 1460\,\Omega$$

[b] $P = \left(\frac{-584}{2920}\right)^2 1460 = 58.40\,\text{W}$

Problems

P 10.1 **[a]** $P = \frac{1}{2}(250)(4)\cos(45+30) = 500\cos 75^\circ = 129.41\,\text{W} \quad \text{(abs)}$

$Q = 500\sin 75^\circ = 482.96\ \text{VAR} \quad \text{(abs)}$

[b] $P = \frac{1}{2}(18)(5)\cos(30+75) = 45\cos(105^\circ) = -11.65\,\text{W} \quad \text{(del)}$

$Q = 45\sin(105^\circ) = 43.47\ \text{VAR} \quad \text{(abs)}$

[c] $P = \frac{1}{2}(150)(2)\cos(-65-50) = 150\cos(-115^\circ) = -63.39\,\text{W} \quad \text{(del)}$

$Q = 150\sin(-115^\circ) = -135.95\ \text{VAR} \quad \text{(del)}$

[d] $P = \frac{1}{2}(80)(10)\cos(120-170) = 400\cos(-50^\circ) = 257.12\,\text{W} \quad \text{(abs)}$

$Q = 400\sin(-50^\circ) = -306.42\ \text{VAR} \quad \text{(del)}$

P 10.2 **[a]** coffee maker = 1200 W frying pan = 1196 W

microwave oven = 1450 W toaster = 1146 W

$\sum P = 4992\,\text{W}$

Therefore $I_{\text{eff}} = \frac{4992}{120} = 41.6\,\text{A}$

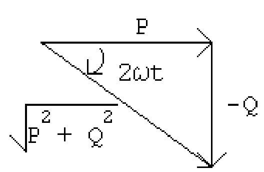
The breaker will not trip.

[b] $\sum P = 4992 + 860 + 630 = 6482\,\text{W}; \qquad I_{\text{eff}} = \frac{6482}{120} = 54.02\,\text{A}$

The breaker will trip because the current is greater than 20 A.

P 10.3 $p = P + P\cos 2\omega t - Q\sin 2\omega t; \qquad \frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$\frac{dp}{dt} = 0$ when $-2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t$ or $\tan 2\omega t = -\frac{Q}{P}$



$\cos 2\omega t = \frac{P}{\sqrt{P^2+Q^2}}; \qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2+Q^2}}$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

Therefore $\quad p_{\max} = P + P \cdot \dfrac{P}{\sqrt{P^2+Q^2}} - \dfrac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$

and $\quad p_{\min} = P - P \cdot \dfrac{P}{\sqrt{P^2+Q^2}} - Q \cdot \dfrac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$

P 10.4 **[a]** $P = \dfrac{1}{2}\dfrac{(240)^2}{480} = 60\,\text{W}$

$$-\frac{1}{\omega C} = \frac{-9\times 10^6}{(5000)(5)} = -360\,\Omega$$

$$Q = \frac{1}{2}\frac{(240)^2}{(-360)} = -80\,\text{VAR}$$

$$p_{\max} = P + \sqrt{P^2+Q^2} = 60 + \sqrt{(60)^2+(80)^2} = 160\,\text{W (del)}$$

[b] $p_{\min} = 60 - \sqrt{60^2+80^2} = -40\,\text{W (abs)}$

[c] $P = 60\,\text{W}$ from (a)

[d] $Q = -80\,\text{VAR}$ from (a)

[e] generates, because $Q < 0$

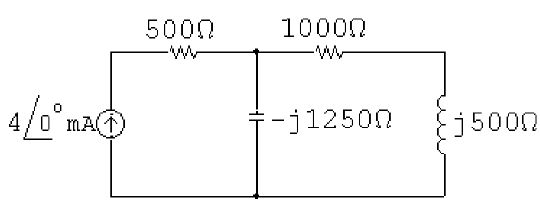
[f] pf $= \cos(\theta_v - \theta_i)$

$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\,\text{A}$$

$$\therefore \quad \text{pf} = \cos(0 - 53.13^\circ) = 0.6 \text{ leading}$$

[g] rf $= \sin(-53.13^\circ) = -0.8$

P 10.5 $\mathbf{I}_g = 4\underline{/0^\circ}\,\text{mA}; \qquad \dfrac{1}{j\omega C} = -j1250\,\Omega; \qquad j\omega L = j500\,\Omega$

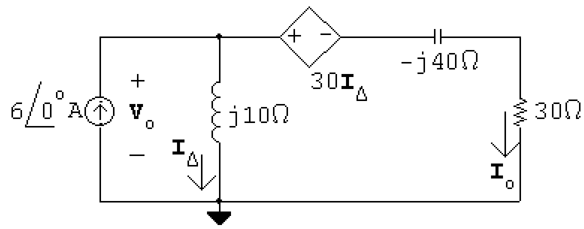


$$Z_{\text{eq}} = 500 + [-j1250 \| (1000 + j500)] = 1500 - j500\,\Omega$$

$$P_g = -\frac{1}{2}|I|^2\text{Re}\{Z_{\text{eq}}\} = -\frac{1}{2}(0.004)^2(1500) = -12\,\text{mW}$$

The source delivers 12 mW of power to the circuit.

P 10.6 $j\omega L = j20{,}000(0.5 \times 10^{-3}) = j10\,\Omega;\qquad \frac{1}{j\omega C} = \frac{10^6}{j20{,}000(1.25)} = -j40\,\Omega$



$$-6 + \frac{\mathbf{V}_o}{j10} + \frac{\mathbf{V}_o - 30(\mathbf{V}_o/j10)}{30 - j40} = 0$$

$$\therefore\quad \mathbf{V}_o\left[\frac{1}{j10} + \frac{1 + j3}{30 - j40}\right] = 6$$

$$\therefore\quad \mathbf{V}_o = 100\underline{/126.87^\circ}\,\text{V}$$

$$\therefore\quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{j10} = 10\underline{/36.87^\circ}\,\text{A}$$

$$\mathbf{I}_o = 6\underline{/0^\circ} - \mathbf{I}_\Delta = 6 - 8 - j6 = -2 - j6 = 6.32\underline{/-108.43^\circ}\,\text{A}$$

$$P_{30\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 30 = 600\,\text{W}$$

P 10.7 $Z_\text{f} = -j10{,}000\|20{,}000 = 4000 - j8000\,\Omega$

$$Z_\text{i} = 2000 - j2000\,\Omega$$

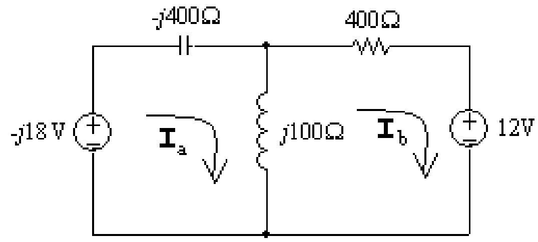
$$\therefore\quad \frac{Z_\text{f}}{Z_\text{i}} = \frac{4000 - j8000}{2000 - j2000} = 3 - j1$$

$$\mathbf{V}_o = -\frac{Z_\text{f}}{Z_\text{i}}\mathbf{V}_g;\qquad \mathbf{V}_g = 1\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_o = -(3 - j1)(1) = -3 + j1 = 3.16\underline{/161.57^\circ}\,\text{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(10)}{1000} = 5 \times 10^{-3} = 5\,\text{mW}$$

P 10.8 **[a]** From the solution to Problem 9.63 we have:



$$\mathbf{I}_a = 67.5 - j7.5\,\text{mA} \qquad \text{and} \qquad \mathbf{I}_b = -22.5 + j22.5\,\text{mA}$$

$$S_{-j18\text{V}} = -\frac{1}{2}(-j18)(0.0675 + j0.0075) = -67.5 + j607.5 \text{ mVA}$$

$$P_{-j18\text{V}} = -67.5 \text{ mW}; \qquad \text{and} \qquad Q_{-j18\text{V}} = 607.5 \text{ mVAR}$$

$$S_{12\text{V}} = \frac{1}{2}(12)(-0.0225 - j0.0225) = -135 - j135 \text{ mVA}$$

$$P_{12\text{V}} = -135 \text{ mW}; \qquad \text{and} \qquad Q_{12V} = -135 \text{ mVAR}$$

$$P_{-j400\Omega} = 0\,\text{W}; \qquad \text{and} \qquad Q_{-j400\Omega} = -\frac{1}{2}(400)|\mathbf{I}_a|^2 = -922.5 \text{ mVAR}$$

$$P_{400\Omega} = \frac{1}{2}(400)|\mathbf{I}_b|^2 = 202.5\,\text{mW}; \qquad \text{and} \qquad Q_{400\Omega} = 0 \text{ mVAR}$$

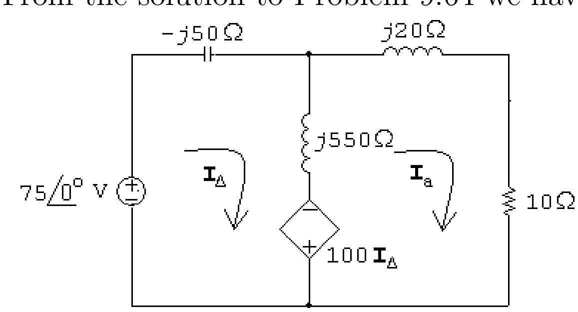
$$P_{j100\Omega} = 0\,\text{W}; \qquad \text{and} \qquad Q_{j100\Omega} = \frac{1}{2}(100)|\mathbf{I}_a - \mathbf{I}_b|^2 = 450 \text{ mVAR}$$

[b] $\sum P_{\text{gen}} = 67.5 + 135 = 202.5\,\text{mW} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{gen}} = 135 + 922.5 = 1057.5\,\text{mVAR}$

$$\sum Q_{\text{abs}} = 607.5 + 450 = 1057.5\,\text{mVAR (check)}$$

P 10.9 **[a]** From the solution to Problem 9.64 we have



$$\mathbf{I}_\Delta = 0.5 + j2.5\,\text{A}; \qquad \text{and} \qquad \mathbf{I}_a = j2.5\,\text{A}$$

$$S_{75\text{V}} = -\frac{1}{2}(75)(0.5 - j2.5) = -18.75 + j93.75 \text{ VA}$$

$$P_{75\text{V}} = -18.75\,\text{W}; \qquad \text{and} \qquad Q_{75\text{V}} = 93.75\ \text{VAR}$$

$$S_{\text{dep.source}} = \frac{1}{2}(100\mathbf{I}_\Delta)(\mathbf{I}_a - \mathbf{I}_\Delta)^* = -12.5 - j62.5\ \text{VA}$$

$$P_{\text{dep.source}} = -12.5\,\text{W}; \qquad \text{and} \qquad Q_{\text{dep.source}} = -62.5\ \text{VAR}$$

$$P_{-j50\Omega} = 0\,\text{W}; \qquad \text{and} \qquad Q_{-j50\Omega} = -\frac{1}{2}(50)|\mathbf{I}_\Delta|^2 = -162.5\ \text{VAR}$$

$$P_{j20\Omega} = 0\,\text{W}; \qquad \text{and} \qquad Q_{j20\Omega} = \frac{1}{2}(20)|\mathbf{I}_a|^2 = 62.5\ \text{VAR}$$

$$P_{10\Omega} = \frac{1}{2}(10)|\mathbf{I}_a|^2 = 31.25\,\text{W}; \qquad \text{and} \qquad Q_{10\Omega} = 0\ \text{VAR}$$

$$P_{j550\Omega} = 0,\text{W}; \qquad \text{and} \qquad Q_{j550\Omega} = \frac{1}{2}(550)|\mathbf{I}_\Delta - \mathbf{I}_a|^2 = 68.75\ \text{VAR}$$

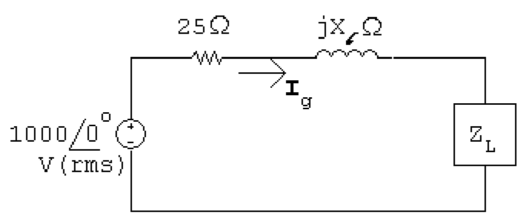
[b] $\sum P_{\text{dev}} = 18.75 + 12.5 = 31.25\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{dev}} = 162.5 + 62.5 = 225\,\text{VAR}$

$$\sum Q_{\text{abs}} = 93.75 + 62.5 + 68.5 = 225\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.10 **[a]** line loss $= 8000 - 6000 = 2000\,\text{W}$

line loss $= |\mathbf{I}_g|^2 25 \qquad \therefore \quad |\mathbf{I}_g|^2 = 80$

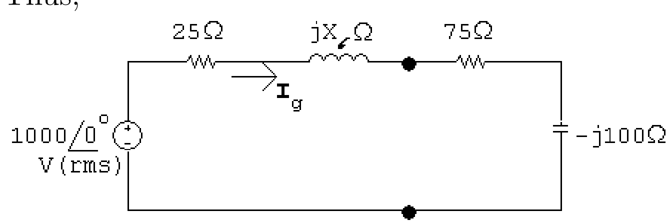


$|\mathbf{I}_g| = \sqrt{80}\,\text{A}$

$|\mathbf{I}_g|^2 R_\text{L} = 6000 \qquad \therefore \quad R_\text{L} = 75\,\Omega$

$|\mathbf{I}_g|^2 X_\text{L} = -8000 \qquad \therefore \quad X_\text{L} = -100\,\Omega$

Thus,



$$|Z| = \sqrt{(100)^2 + (X_\ell - 100)^2} \qquad |\mathbf{I}_g| = \frac{1000}{\sqrt{10{,}000 + (X_\ell - 100)^2}}$$

$$\therefore \quad 10{,}000 + (X_\ell - 100)^2 = \frac{10^6}{80} = 12{,}500$$

Solving, $(X_\ell - 100) = \pm 50.$

Thus, $X_\ell = 150\,\Omega$ or $X_\ell = 50\,\Omega$

[b] If $X_\ell = 150\,\Omega$:

$$\mathbf{I}_g = \frac{1000}{100 + j50} = 8 - j4\,\text{A}$$

$$S_g = -1000\mathbf{I}_g^* = -8000 - j4000\,\text{VA}$$

Thus, the voltage source is delivering 8 kW and 4 kvars.

$$Q_{j150} = |\mathbf{I}_g|^2 X_\ell = 80(150) = 12 \text{ kvars}$$

Therefore the line reactance is absorbing 12 kvars.

$$Q_{-j100} = |\mathbf{I}_g|^2 X_\text{L} = 80(-100) = -8\,\text{kvars}$$

Therefore the load reactance is generating 8 kvars.

$$\sum Q_\text{gen} = 12\,\text{kvars} = \sum Q_\text{abs}$$

If $X_\ell = 50\,\Omega$:

$$\mathbf{I}_g = \frac{1000}{100 - j50} = 8 + j4\,\text{A}$$

$$S_g = -1000\mathbf{I}_g^* = -8000 + j4000\,\text{VA}$$

Thus, the voltage source is delivering 8 kW and absorbing 4 kvars.

$$Q_{j50} = |\mathbf{I}_g|^2(50) = 80(50) = 4\,\text{kvars}$$

Therefore the line reactance is absorbing 4 kvars. The load continues to deliver 8 kvars.

$$\sum Q_\text{gen} = 8\,\text{kvars} = \sum Q_\text{abs}$$

P 10.11 **[a]** $I_\text{eff} = 40/115 \cong 0.35\,\text{A}$

[b] $I_\text{eff} = 130/115 \cong 1.13\,\text{A}$

P 10.12 $i(t) = 250t \qquad 0 \le t \le 80\,\text{ms}$

$$i(t) = 100 - 1000t \qquad 80\,\text{ms} \le t \le 100\,\text{ms}$$

$$\therefore \quad I_\text{rms} = \sqrt{\frac{1}{0.1}\left\{\int_0^{0.08}(250)^2 t^2\,dt + \int_{0.08}^{0.1}(100-1000t)^2\,dt\right\}}$$

$$\int_0^{0.08}(250)^2 t^2\,dt = (250)^2\frac{t^3}{3}\Big|_0^{0.08} = \frac{32}{3}$$

$$(100-1000t)^2 = 10^4 - 2\times 10^5 t + 10^6 t^2$$

$$\int_{0.08}^{0.1} 10^4\,dt = 200$$

$$\int_{0.08}^{0.1} 2\times 10^5 t\,dt = 10^5 t^2\Big|_{0.08}^{0.1} = 360$$

$$10^6\int_{0.08}^{0.1} t^2\,dt = \frac{10^6}{3}t^3\Big|_{0.08}^{0.1} = \frac{488}{3}$$

$$\therefore\quad I_{\text{rms}} = \sqrt{10\{(32/3)+225-360+(488/3)\}} = 11.55\,\text{A}$$

P 10.13 $P = I_{\text{rms}}^2 R \qquad \therefore\quad R = \dfrac{1280}{(11.55)^2} = 9.6\,\Omega$

P 10.14 **[a]** $A = 40^2(5) + (-40)^2)(5) = 16{,}000$

$$\text{mean value} = \frac{A}{20} = 800$$

$$V_{\text{rms}} = \sqrt{800} = 28.28\ \text{V(rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{800}{40} = 20\,\text{W}$

[c] $R = \dfrac{V_{\text{rms}}^2}{P} = \dfrac{800}{0.01} = 80\,\text{k}\Omega$

P 10.15 **[a]** Area under one cycle of v_g^2:

$$A = (100)(25\times 10^{-6}) + 400(25\times 10^{-6}) + 400(25\times 10^{-6}) + 100(25\times 10^{-6})$$
$$= 1000(25\times 10^{-6})$$

Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{100\times 10^{-6}} = \frac{1000(25\times 10^{-6})}{100\times 10^{-6}} = 250$$

$$\therefore\quad V_{\text{rms}} = \sqrt{250} = 15.81\,\text{V (rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{250}{4} = 62.5\,\text{W}$

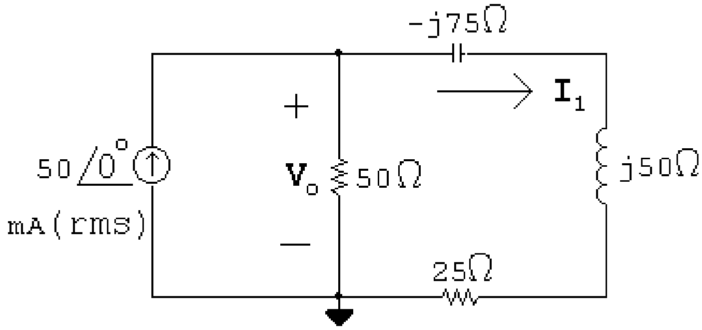
P 10.16 $W_{\rm dc} = \dfrac{V_{\rm dc}^2}{R}T; \qquad W_s = \int_{t_o}^{t_o+T} \dfrac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\rm dc}^2}{R}T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$$

$$V_{\rm dc}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\rm dc} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\rm rms} = V_{\rm eff}$$

P 10.17 [**a**]



$$Z_{\rm eq} = 50\|(25 - j25) = 20 - j10\,\Omega$$

$$\therefore \qquad \mathbf{V}_o = 0.05Z_{\rm eq} = 1 - j0.5 \text{ V(rms)}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{25 - j25} = 30 + j10 \text{ mA(rms)}$$

$$S_g = -\mathbf{V}_g\mathbf{I}_g^* = -(1 - j0.5)(0.05)$$
$$= -50 + j25\,\text{mVA}$$

[**b**] Source is delivering 50 mW.

[**c**] Source is absorbing 25 mVAR.

[**d**] $Q_{\rm cap} = -|\mathbf{I}_1|^2(75) = -75$ mVAR

$$P_{50\Omega} = \frac{|\mathbf{V}_o|^2}{50} = 25\,\text{mW}$$

$$P_{25\Omega} = |\mathbf{I}_1|^2(25) = 25\,\text{mW}$$

$$Q_{\rm ind} = |\mathbf{I}_1|^2(50) = 50 \text{ mVAR}$$

$P_{\text{middle branch}} = 25\,\text{mW}; \qquad Q_{\text{middle branch}} = 0$

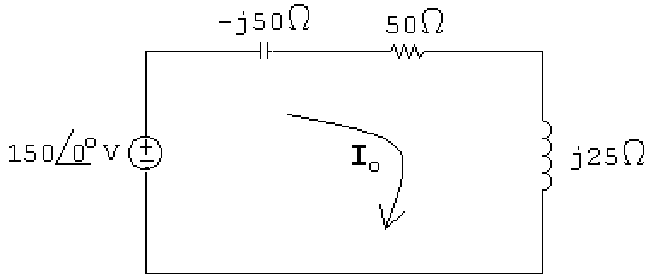
$P_{\text{right branch}} = 25\,\text{mW}; \qquad Q_{\text{right branch}} = -75 + 50 = -25$ mVAR

[e] $\sum P_{\text{del}} = 50\,\text{mW}$

$$\sum P_{\text{diss}} = 25 + 25 = 50\,\text{mW} = \sum P_{\text{del}}$$

[f] $\sum Q_{\text{abs}} = 25 + 50 = 75\,\text{mVAR} = \sum Q_{\text{dev}}$

P 10.18 $j\omega L = j25\,\Omega; \qquad \frac{1}{j\omega C} = -j75\,\Omega$



$$\mathbf{I}_o = \frac{j150}{50 - j25} = 2.4 + j1.2\,\text{A}$$

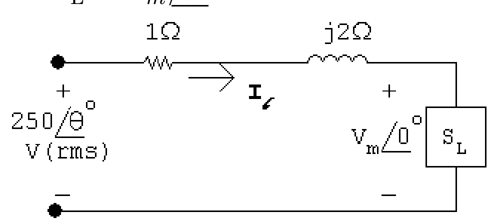
$$P = \frac{1}{2}|\mathbf{I}_o|^2(50) = \frac{1}{2}(7.2)(50) = 180\,\text{W}$$

$$Q = \frac{1}{2}|\mathbf{I}_o|^2(25) = 90\,\text{VAR}$$

$$S = P + jQ = 180 + j90\,\text{VA}$$

$$|S| = 201.25\,\text{VA}$$

P 10.19 **[a]** Let $\mathbf{V}_\text{L} = V_m\underline{/0^\circ}$:



$$S_\text{L} = 2500(0.8 + j0.6) = 2000 + j1500\,\text{VA}$$

$$\mathbf{I}_\ell^* = \frac{2000}{V_m} + j\frac{1500}{V_m}; \qquad \mathbf{I}_\ell = \frac{2000}{V_m} - j\frac{1500}{V_m}$$

$$250\underline{/\theta} = V_m + \left(\frac{2000}{V_m} - j\frac{1500}{V_m}\right)(1 + j2)$$

$$250V_m\underline{/\theta} = V_m^2 + (2000 - j1500)(1 + j2) = V_m^2 + 5000 + j2500$$

$$250V_m \cos\theta = V_m^2 + 5000; \qquad 250V_m \sin\theta = 2500$$

$$(250)^2V_m^2 = (V_m^2 + 5000)^2 + 2500^2$$

$$62{,}500V_m^2 = V_m^4 + 10{,}000V_m^2 + 31.25 \times 10^6$$

or

$$V_m^4 - 52{,}500V_m^2 + 31.25 \times 10^6 = 0$$

Solving,

$$V_m^2 = 26{,}250 \pm 25{,}647.86; \qquad V_m = 227.81\,\text{V and } V_m = 24.54\,\text{V}$$

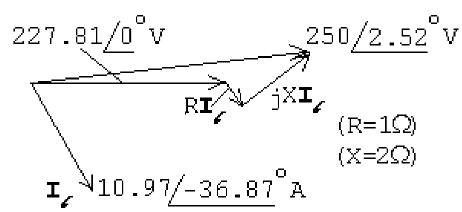
If $V_m = 227.81$ V:

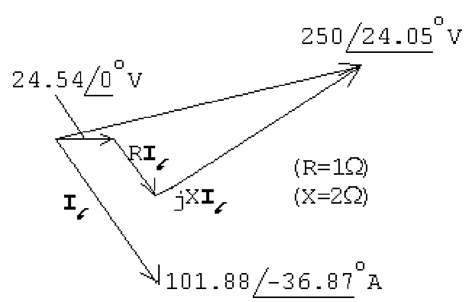
$$\sin\theta = \frac{2500}{(227.81)(250)} = 0.044; \qquad \therefore \quad \theta = 2.52^\circ$$

If $V_m = 24.54$ V:

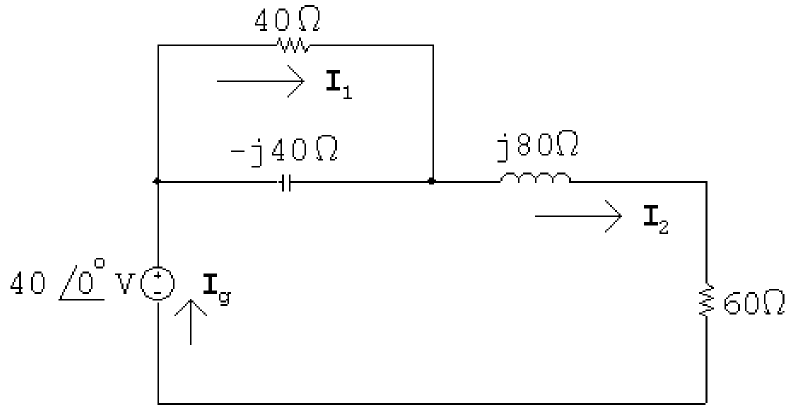
$$\sin\theta = \frac{2500}{(24.54)(250)} = 0.4075; \qquad \therefore \quad \theta = 24.05^\circ$$

[**b**]





P 10.20 **[a]** $\frac{1}{j\omega C} = -j40\,\Omega; \qquad j\omega L = j80\,\Omega$



$$Z_{\text{eq}} = 40\| - j40 + j80 + 60 = 80 + j60\,\Omega$$

$$\mathbf{I}_g = \frac{40/0^\circ}{80 + j60} = 0.32 - j0.24\,\text{A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = -\frac{1}{2}40(0.32 + j0.24) = -6.4 - j4.8\,\text{VA}$$

$$P = 6.4\,\text{W (del)}; \qquad Q = 4.8\,\text{VAR (del)}$$

$$|S| = |S_g| = 8\,\text{VA}$$

[b] $\mathbf{I}_1 = \frac{-j40}{40 - j40}\mathbf{I}_g = 0.04 - j0.28\,\text{A}$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(40) = 1.6\,\text{W}$$

$$P_{60\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(60) = 4.8\,\text{W}$$

$$\sum P_{\text{diss}} = 1.6 + 4.8 = 6.4\,\text{W} = \sum P_{\text{dev}}$$

[c] $\mathbf{I}_{-j40\Omega} = \mathbf{I}_g - \mathbf{I}_1 = 0.28 + j0.04\,\text{A}$

$$Q_{-j40\Omega} = \frac{1}{2}|\mathbf{I}_{-j40\Omega}|^2(-40) = -1.6\,\text{VAR (del)}$$

$$Q_{j80\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(80) = 6.4\,\text{VAR (abs)}$$

$$\sum Q_{\text{abs}} = 6.4 - 1.6 = 4.8\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.21 $S_T = 40{,}800 + j30{,}600\,\text{VA}$

$$S_1 = 20{,}000(0.96 - j0.28) = 19{,}200 - j5600\,\text{VA}$$

$$S_2 = S_T - S_1 = 21{,}600 + j36{,}200 = 42{,}154.48\underline{/59.176^\circ}\,\text{VA}$$

$$\text{rf} = \sin(59.176^\circ) = 0.8587$$

$$\text{pf} = \cos(59.176^\circ) = 0.5124 \text{ lagging}$$

P 10.22 **[a]** $S_1 = 10{,}000 - j4000\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_L|^2}{Z_2^*} = \frac{(1000)^2}{60 - j80} = 6 + j8\,\text{kVA}$$

$$S_1 + S_2 = 16 + j4\,\text{kVA}$$

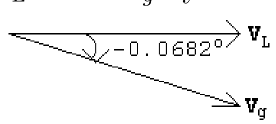
$$1000\mathbf{I}_L^* = 16{,}000 + j4000; \qquad \therefore \quad \mathbf{I}_L = 16 - j4\,\text{A(rms)}$$

$$\mathbf{V}_g = \mathbf{V}_L + \mathbf{I}_L(0.5 + j0.05) = 1000 + (16 - j4)(0.5 + j0.05)$$
$$= 1008.2 - j1.2 = 1008.2\underline{/-0.0682^\circ}\,\text{Vrms}$$

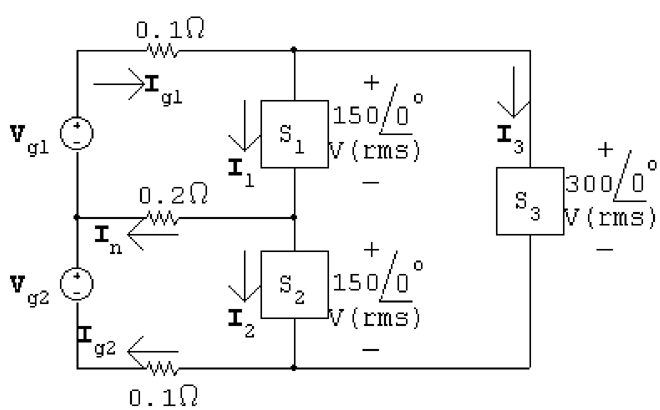
[b] $T = \dfrac{1}{f} = \dfrac{1}{50} = 20\,\text{ms}$

$$\frac{-0.0682^\circ}{360^\circ} = \frac{t}{20\text{ ms}}; \qquad \therefore \quad t = -3.79\,\mu\text{s}$$

[c] $\mathbf{V}_L$ leads $\mathbf{V}_g$ by 0.0682° or $3.79\,\mu$s



P 10.23 **[a]**



$$\mathbf{I}_1 = \frac{6000 - j3000}{150} = 40 - j20\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{7500 + j4500}{150} = 50 + j30\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{12{,}000 - j9000}{300} = 40 - j30\,\text{A (rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 80 - j50\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = -10 - j50\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 90 + j0\,\text{A}$$

$$\mathbf{V}_{g1} = 0.1\mathbf{I}_{g1} + 150 + 0.2\mathbf{I}_n = 156 - j15\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.2\mathbf{I}_n + 150 + 0.1\mathbf{I}_{g2} = 161 + j10\,\text{V(rms)}$$

$$S_{g1} = -[(156 - j15)(80 + j50)] = -[13{,}230 + j6600]\,\text{VA}$$

$$S_{g2} = -[(161 + j10)(90 + j0)] = -[14{,}490 + j900]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b] $P_{0.1} = |\mathbf{I}_{g1}|^2(0.1) = 890\,\text{W}$

$$P_{0.2} = |\mathbf{I}_n|^2(0.2) = 520\,\text{W}$$

$$P_{0.1} = |\mathbf{I}_{g2}|^2(0.1) = 810\,\text{W}$$

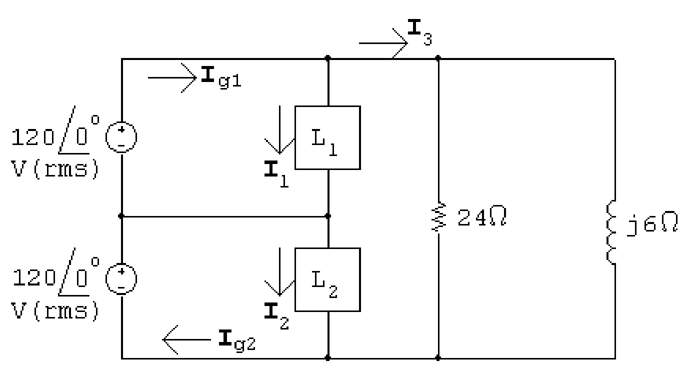
$$\sum P_{\text{dis}} = 890 + 520 + 810 + 6000 + 7500 + 12{,}000 = 27{,}720\,\text{W}$$

$$\sum P_{\text{dev}} = 13{,}230 + 14{,}490 = 27{,}720\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 3000 + 9000 = 12{,}000\,\text{VAR}$$

$$\sum Q_{\text{del}} = 4500 + 6600 + 900 = 12{,}000\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.24 **[a]**



$$120\mathbf{I}_1^* = 4800 - j2400; \qquad \therefore \quad \mathbf{I}_1 = 40 + j20\,\text{A(rms)}$$

$120\mathbf{I}_2^* = 4800 + j3600; \qquad \therefore \quad \mathbf{I}_2 = 40 - j30\,\text{A(rms)}$

$$\mathbf{I}_3 = \frac{240}{24} + \frac{240}{j6} = 10 - j40\,\text{A(rms)}$$

$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 50 - j20\,\text{A}$

$S_{g1} = -120(50 + j20) = -6000 - j2400\,\text{VA}$

Thus the $\mathbf{V}_{g1}$ source is delivering 6 kW and 2.4 kvars.

$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 50 - j70\,\text{A(rms)}$

$S_{g2} = -120(50 + j70) = -6000 - j8400\,\text{VA}$

Thus the $\mathbf{V}_{g2}$ source is delivering 6 kW and 8.4 kvars.

[b] $\sum P_{\text{gen}} = 6 + 6 = 12\,\text{kW}$

$$\sum P_{\text{abs}} = 4800 + 4800 + \frac{(240)^2}{24} = 12\,\text{kW} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 2400 + 8400 + 2400 = 13.2\,\text{kVAR}$$

$$\sum Q_{\text{abs}} = 3600 + \frac{(240)^2}{6} = 13.2\,\text{kVAR} = \sum Q_{\text{del}}$$

P 10.25 $S_1 = 1146 + 1200 + 1450 = 3796 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_1 = \frac{3796}{110} = 34.51\,\text{A}$$

$S_2 = 145 + 630 + 1322 = 2097 + j0\,\text{VA}$

$$\therefore \quad \mathbf{I}_2 = \frac{2097}{1!0} = 19.064\,\text{A}$$

$S_3 = 512 + 4856 = 5368 + j0\,\text{VA}$

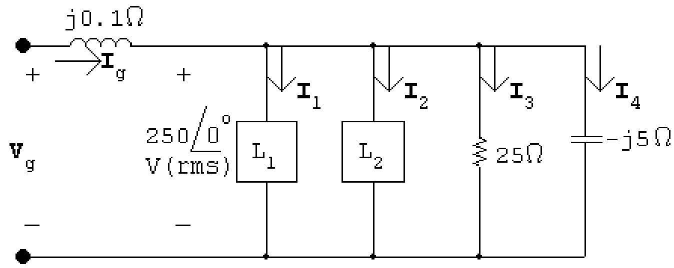
$$\therefore \quad \mathbf{I}_3 = \frac{5368}{220} = 24.4\,\text{A}$$

$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 58.91\,\text{A}$

$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 43.464\,\text{A}$

The breaker protecting the upper service conductor will trip because its feeder current exceeds 50 A. The breaker protecting the lower service conductor will not trip because its feeder current is less than 50 A. Thus, service to Loads 1 and 3 will be interrupted.

P 10.26



$$250\mathbf{I}_1^* = 6000 - j8000$$

$$\mathbf{I}_1^* = 24 - j32; \qquad \therefore \quad \mathbf{I}_1 = 24 + j32\,\text{A(rms)}$$

$$250\mathbf{I}_2^* = 9000 + j3000$$

$$\mathbf{I}_2^* = 36 + j12; \qquad \therefore \quad \mathbf{I}_2 = 36 - j12\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{250\angle 0^\circ}{25} = 10 + j0\,\text{A}; \qquad \mathbf{I}_4 = \frac{250\angle 0^\circ}{-j5} = 0 + j50\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 70 + j70\,\text{A}$$

$$\mathbf{V}_g = 250 + (70 + j70)(j0.1) = 243 + j7 = 243.1\angle 1.65^\circ\,\text{V (rms)}$$

P 10.27 **[a]** From Problem 9.78,

$$Z_{\text{ab}} = 20 + j27.25 \quad \text{so}$$

$$\mathbf{I}_1 = \frac{75}{10 + j12.75 + 20 + 27.25} = \frac{75}{30 + j40} = 0.9 - j1.2\,\text{A}$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{22}}\mathbf{I}_1 = \frac{j54}{160 + j120}(0.9 - j1.2) = 405 + j0\,\text{mA}$$

$$\mathbf{V}_L = (60 + j20)(0.405 + j0) = 24.3 + j8.1$$

$$|\mathbf{V}_L| = 25.61\,\text{V}$$

[b] $P_g(\text{ideal}) = 75(0.9) = 67.5\,\text{W}$

$$P_g(\text{practical}) = 67.5 - |\mathbf{I}_1|^2(10) = 45\,\text{W}$$

$$P_{\text{L}} = |\mathbf{I}_2|^2(60) = 9.8415\,\text{W}$$

$$\%\ \text{delivered} = \frac{9.8415}{45}(100) = 21.87\%$$

P 10.28 **[a]** $S_1 = 3 + j0\,\text{kVA}; \qquad S_2 = 4 - j3\,\text{kVA}; \qquad S_3 = 5 - j6\,\text{kVA}$

$$S_\text{T} = S_1 + S_2 + S_3 = 12 - j9\,\text{kVA}$$

$$300\mathbf{I}^* = (12 - j9) \times 10^3; \qquad \therefore \quad \mathbf{I} = 40 + j30\,\text{A}$$

$$Z = \frac{300}{40 + j30} = 4.8 - j3.6\,\Omega = 6\underline{/-36.87^\circ}\,\Omega$$

[b] pf $= \cos(-36.87^\circ) = 0.8$ leading

P 10.29 **[a]** From the solution to Problem 10.28 we have

$$\mathbf{I}_\text{L} = 40 + j30\,\text{A (rms)}$$

$$\therefore \quad \mathbf{V}_s = 300\underline{/0^\circ} + (40 + j30)(0.2 + j0.05) = 306.5 + j8$$

$$= 306.6\underline{/1.5^\circ}\,\text{V (rms)}$$

[b] $|\mathbf{I}_\text{L}| = \sqrt{16{,}000}$

$$P_\ell = (2500)(0.2) = 500\,\text{W} \qquad Q_\ell = (2500)(0.05) = 125\,\text{VAR}$$

[c] $P_s = 12{,}000 + 500 = 12.5\,\text{kW} \qquad Q_s = -9000 + 125 = -8.875\,\text{kVAR}$

[d] $\eta = \frac{12}{12.5}(100) = 96\%$

P 10.30 **[a]** $Z_1 = 240 + j70 = 250\underline{/16.26^\circ}\,\Omega$

pf $= \cos(16.26^\circ) = 0.96$ lagging

rf $= \sin(16.26^\circ) = 0.28$

$$Z_2 = 160 - j120 = 200\underline{/-36.87^\circ}\,\Omega$$

pf $= \cos(-36.87^\circ) = 0.8$ leading

rf $= \sin(-36.87^\circ) = -0.6$

$$Z_3 = 30 - j40 = 50\underline{/-53.13^\circ}\,\Omega$$

pf $= \cos(-53.13^\circ) = 0.6$ leading

rf $= \sin(-53.13^\circ) = -0.8$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250\underline{/16.26^\circ}}; \qquad Y_2 = \frac{1}{200\underline{/-36.87^\circ}}; \qquad Y_3 = \frac{1}{50\underline{/-53.13^\circ}}$$

$$Y = 19.84 + j17.88\,\text{mS}$$

$$Z = \frac{1}{Y} = 37.44\underline{/-42.03^\circ}\,\Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 10.31 [**a**] $\mathbf{I} = \dfrac{270\underline{/0^\circ}}{36 + j48} = 2.7 - j3.6 = 4.5\underline{/-53.13^\circ}\,\text{A(rms)}$

$$P = (4.5)^2(6) = 121.5\,\text{W}$$

[**b**] $Y_\text{L} = \dfrac{1}{30 + j40} = 12 - j16 \text{ mS}$

$$\therefore \quad X_\text{C} = \frac{1}{-16 \times 10^{-3}} = -62.5\,\Omega$$

[**c**] $Z_\text{L} = \dfrac{1}{12 \times 10^{-3}} = 83.33\,\Omega$

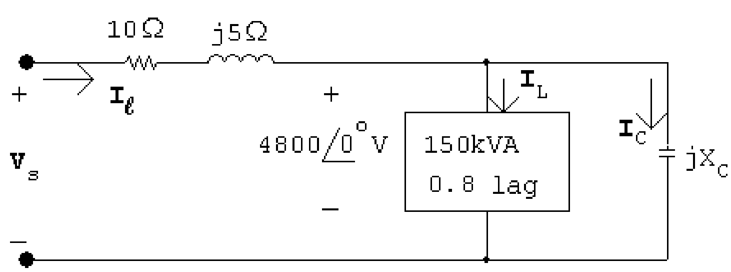
[**d**] $\mathbf{I} = \dfrac{270\underline{/0^\circ}}{89.33 + j8} = 3.01\underline{/-5.12^\circ}\,\text{A}$

$$P = (3.01)^2(6) = 54.37\,\text{W}$$

[**e**] $\% = \dfrac{54.37}{121.5}(100) = 44.75\%$

Thus the power loss after the capacitor is added is 44.75% of the power loss before the capacitor is added.

P 10.32



$$\mathbf{I}_\text{L} = \frac{120{,}000 - j90{,}000}{4800} = 25 - j18.75\,\text{A (rms)}$$

$$\mathbf{I}_\text{C} = \frac{4800}{jX_\text{C}} = -j\frac{4800}{X_\text{C}} = jI_\text{C}$$

$$\mathbf{I}_\ell = 25 - j18.75 + jI_\text{C} = 25 + j(I_\text{C} - 18.75)$$

$$\mathbf{V}_s = 4800 + (10 + j5)[25 + j(I_\text{C} - 18.75)]$$
$$= (5143.75 - 5I_\text{C}) + j(-62.5 + 10I_\text{C})$$

$$|\mathbf{V}_s|^2 = (5143.75 - 5I_\text{C})^2 + (-62.5 + 10I_\text{C})^2 = (4800)^2$$

$$\therefore \quad 125I_\text{C}^2 - 52{,}687.5I_\text{C} + 3{,}422{,}070.313 = 0$$

$$I_\text{C} = 341.284\,\text{A(rms)} \quad \text{and} \quad I_\text{C} = 80.2165\,\text{A(rms)}^*$$

*Select the smaller value of I_C to minimize the magnitude of I_ℓ.

$$\therefore \quad X_\text{C} = -\frac{4800}{80.2165} = -59.838$$

$$\therefore \quad C = \frac{1}{(59.838)(120\pi)} = 44.43\,\mu\text{F}$$

P 10.33 **[a]** $S_\text{L} = 20{,}000(0.85 + j0.53) = 17{,}000 + j10{,}535.65\,\text{VA}$

$$125\mathbf{I}_\text{L}^* = (17{,}000 + j10{,}535.65); \qquad \mathbf{I}_\text{L}^* = 136 + j84.29\,\text{A(rms)}$$

$$\therefore \quad \mathbf{I}_\text{L} = 136 - j84.29\,\text{A(rms)}$$

$$\mathbf{V}_s = 125 + (136 - j84.29)(0.01 + j0.08) = 133.10 + j10.04$$
$$= 133.48\underline{/4.31^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 133.48\,\text{V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.01) = (160)^2(0.01) = 256\,\text{W}$

[c] $\dfrac{(125)^2}{X_\text{C}} = -10{,}535.65; \qquad X_\text{C} = -1.48306\,\Omega$

$$-\frac{1}{\omega C} = -1.48306; \qquad C = \frac{1}{(1.48306)(120\pi)} = 1788.59\,\mu\text{F}$$

[d] $\mathbf{I}_\ell = 136 + j0\,\text{A(rms)}$

$$\mathbf{V}_s = 125 + 136(0.01 + j0.08) = 126.36 + j10.88$$
$$= 126.83\underline{/4.92^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 126.83\,\text{V(rms)}$$

[e] $P_\ell = (136)^2(0.01) = 184.96\,\text{W}$

P 10.34 **[a]** $S_o =$ original load $= 1600 + j\dfrac{1600}{0.8}(0.6) = 1600 + j1200\,\mathrm{kVA}$

$$S_f = \text{ final load } = 1920 + j\frac{1920}{0.96}(0.28) = 1920 + j560\,\mathrm{kVA}$$

$$\therefore\quad Q_{\text{added}} = 560 - 1200 = -640\,\mathrm{kVAR}$$

[b] deliver

[c] $S_{\mathrm{a}} =$ added load $= 320 - j640 = 715.54\underline{/-63.43^\circ}\,\mathrm{kVA}$

$$\text{pf } = \cos(-63.43) = 0.447 \text{ leading}$$

[d] $\mathbf{I}_{\mathrm{L}}^* = \dfrac{(1600 + j1200)\times 10^3}{2400} = 666.67 + j500\,\mathrm{A}$

$$\mathbf{I}_{\mathrm{L}} = 666.67 - j500 = 833.33\underline{/-36.87^\circ}\,\mathrm{A(rms)}$$

$$|\mathbf{I}_{\mathrm{L}}| = 833.33\,\mathrm{A(rms)}$$

[e] $\mathbf{I}_{\mathrm{L}}^* = \dfrac{(1920 + j560)\times 10^3}{2400} = 800 + j233.33$

$$\mathbf{I}_{\mathrm{L}} = 800 - j233.33 = 833.33\underline{/-16.26^\circ}\,\mathrm{A(rms)}$$

$$|\mathbf{I}_{\mathrm{L}}| = 833.33\,\mathrm{A(rms)}$$

P 10.35 **[a]** $P_{\text{before}} = P_{\text{after}} = (833.33)^2(0.25) = 173.611\,\mathrm{kW}$

[b] $\mathbf{V}_s(\text{before}) = 2400 + (666.67 - j500)(0.25 + j0.1)$

$$= 2616.67 - j58.33 = 2617.32\underline{/-1.28^\circ}\,\mathrm{V(rms)}$$

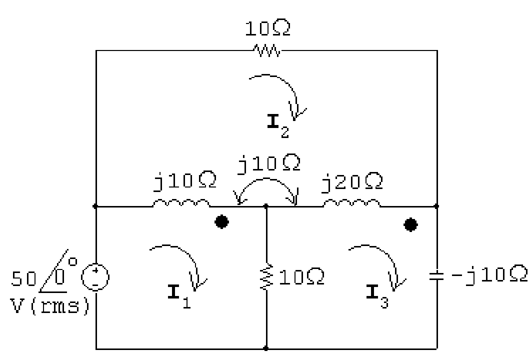
$$|\mathbf{V}_s(\text{before})| = 2617.32\,\mathrm{V(rms)}$$

$$\mathbf{V}_s(\text{after}) = 2400 + (800 - j233.33)(0.25 + j0.1)$$

$$= 2623.33 + j21.67 = 2623.42\underline{/0.47^\circ}\,\mathrm{V(rms)}$$

$$|\mathbf{V}_s(\text{after})| = 2623.42\,\mathrm{V(rms)}$$

P 10.36 **[a]**



$$50 = j10(\mathbf{I}_1 - \mathbf{I}_2) + j10(\mathbf{I}_3 - \mathbf{I}_2) + 10(\mathbf{I}_1 - \mathbf{I}_3)$$

$$0 = 10\mathbf{I}_2 + j20(\mathbf{I}_2 - \mathbf{I}_3) + j10(\mathbf{I}_2 - \mathbf{I}_1) + j10(\mathbf{I}_2 - \mathbf{I}_1) + j10(\mathbf{I}_2 - \mathbf{I}_3)$$

$$0 = -j10\mathbf{I}_3 + 10(\mathbf{I}_3 - \mathbf{I}_1) + j20(\mathbf{I}_3 - \mathbf{I}_2) + j10(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

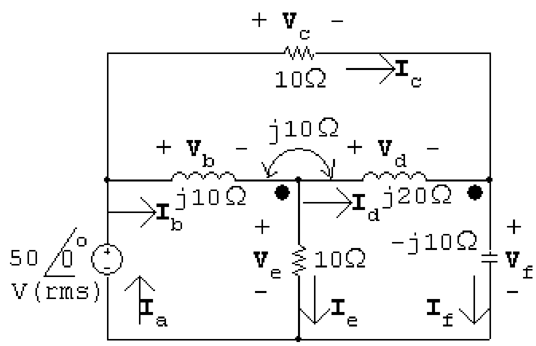
$\mathbf{I}_1 = 5.5 + j0.5\,\mathrm{A(rms)}$; $\mathbf{I}_2 = 3 + j2\,\mathrm{A(rms)}$; $\mathbf{I}_3 = 2 + j2\,\mathrm{A(rms)}$

$\mathbf{I}_\mathrm{a} = \mathbf{I}_1 = 5.5 + j0.5\,\mathrm{A}$ $\quad$ $\mathbf{I}_\mathrm{b} = \mathbf{I}_1 - \mathbf{I}_2 = 2.5 - j1.5\,\mathrm{A}$

$\mathbf{I}_\mathrm{c} = \mathbf{I}_2 = 3 + j2\,\mathrm{A}$ $\quad$ $\mathbf{I}_\mathrm{d} = \mathbf{I}_3 - \mathbf{I}_2 = -1\,\mathrm{A}$

$\mathbf{I}_\mathrm{e} = \mathbf{I}_1 - \mathbf{I}_3 = 3.5 - j1.5\,\mathrm{A}$ $\quad$ $\mathbf{I}_\mathrm{f} = \mathbf{I}_3 = 2 + j2\,\mathrm{A}$

[b]



$\mathbf{V}_\mathrm{a} = 50\,\mathrm{V}$ $\quad$ $\mathbf{V}_\mathrm{b} = j10\mathbf{I}_\mathrm{b} + j10\mathbf{I}_\mathrm{d} = 15 + j15\,\mathrm{V}$

$\mathbf{V}_\mathrm{c} = 10\mathbf{I}_\mathrm{c} = 30 + j20\,\mathrm{V}$ $\quad$ $\mathbf{V}_\mathrm{d} = j20\mathbf{I}_\mathrm{d} + j10\mathbf{I}_\mathrm{b} = 15 + j5\,\mathrm{V}$

$\mathbf{V}_\mathrm{e} = 10\mathbf{I}_\mathrm{e} = 35 - j15\,\mathrm{V}$ $\quad$ $\mathbf{V}_\mathrm{f} = -j10\mathbf{I}_\mathrm{f} = 20 - j20\,\mathrm{V}$

$$S_\mathrm{a} = -50\mathbf{I}_\mathrm{a}^* = -275 + j25\,\mathrm{VA}$$

$$S_\mathrm{b} = \mathbf{V}_\mathrm{b}\mathbf{I}_\mathrm{b}^* = 15 + j60\,\mathrm{VA}$$

$$S_\mathrm{c} = \mathbf{V}_\mathrm{c}\mathbf{I}_\mathrm{c}^* = 130 + j0\,\mathrm{VA}$$

$$S_\mathrm{d} = \mathbf{V}_\mathrm{d}\mathbf{I}_\mathrm{d}^* = -15 - j5\,\mathrm{VA}$$

$$S_\mathrm{e} = \mathbf{V}_\mathrm{e}\mathbf{I}_\mathrm{e}^* = 145 - j0\,\mathrm{VA}$$

$$S_\mathrm{f} = \mathbf{V}_\mathrm{f}\mathbf{I}_\mathrm{f}^* = 0 - j80\,\mathrm{VA}$$

[c] $\sum P_\mathrm{dev} = 275 + 15 = 290\,\mathrm{W}$

$$\sum P_\mathrm{abs} = 15 + 130 + 145 = 290\,\mathrm{W}$$

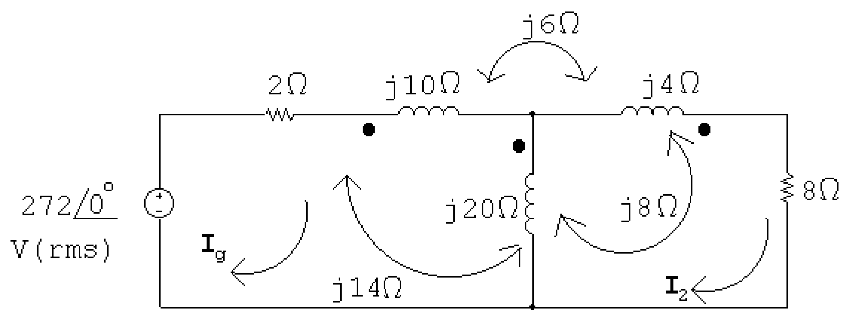
Note that the total power absorbed by the coupled coils is zero:
$15 - 15 = 0 = P_\mathrm{b} + P_\mathrm{d}$

[d] $\sum Q_{\text{dev}} = 5 + 80 = 85\,\text{VAR}$

$\sum Q_{\text{abs}} = 25 + 60 = 85\,\text{VAR}$

$\sum Q$ absorbed by the coupled coils is $Q_{\text{b}} + Q_{\text{d}} = 55$

P 10.37 **[a]**



$$272\underline{/0^\circ} = 2\mathbf{I}_g + j10\mathbf{I}_g + j14(\mathbf{I}_g - \mathbf{I}_2) - j6\mathbf{I}_2$$
$$+j14\mathbf{I}_g - j8\mathbf{I}_2 + j20(\mathbf{I}_g - \mathbf{I}_2)$$
$$0 = j20(\mathbf{I}_2 - \mathbf{I}_g) - j14\mathbf{I}_g + j8\mathbf{I}_2 + j4\mathbf{I}_2$$
$$+j8(\mathbf{I}_2 - \mathbf{I}_g) - j6\mathbf{I}_g + 8\mathbf{I}_2$$

Solving,

$I_g = 20 - j4\,\text{A(rms)}; \qquad \mathbf{I}_2 = 24\underline{/0^\circ}\,\text{A(rms)}$

$P_{8\Omega} = (24)^2(8) = 4608\,\text{W}$

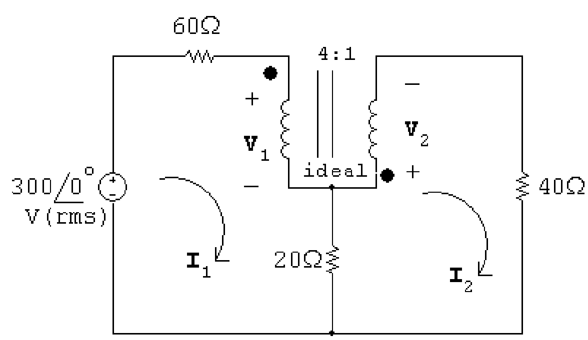
[b] $P_g(\text{developed}) = (272)(20) = 5440\,\text{W}$

[c] $Z_{\text{ab}} = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} - 2 = \dfrac{272}{20 - j4} - 2 = 11.08 + j2.62 = 11.38\underline{/13.28^\circ}\,\Omega$

[d] $P_{2\Omega} = |I_g|^2(2) = 832\,\text{W}$

$\sum P_{\text{diss}} = 832 + 4608 = 5440\,\text{W} = \sum P_{\text{dev}}$

P 10.38 **[a]**



$$300 = 60\mathbf{I}_1 + \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2)$$

$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2 + 40\mathbf{I}_2$

$\mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_2 = -4\mathbf{I}_1$

Solving,

$\mathbf{V}_1 = 260\,\text{V (rms)}; \qquad \mathbf{V}_2 = 65\,\text{V (rms)}$

$\mathbf{I}_1 = 0.25\,\text{A (rms)}; \qquad \mathbf{I}_2 = -1.0\,\text{A (rms)}$

$\mathbf{V}_{5\text{A}} = \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2) = 285\,\text{V (rms)}$

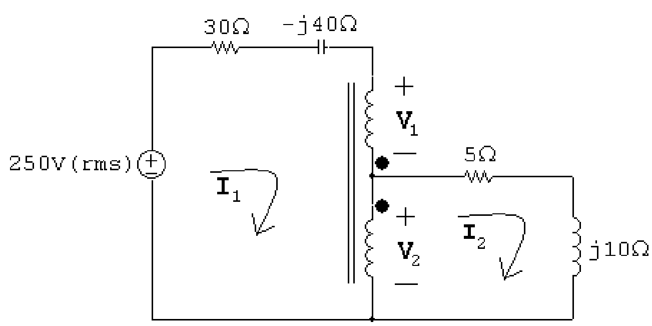
$\therefore \quad P = -(285)(5) = -1425\,\text{W}$

Thus 1425 W is delivered by the current source to the circuit.

[b] $\mathbf{I}_{20\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25\,\text{A(rms)}$

$\therefore \quad P_{20\Omega} = (1.25)^2(20) = 31.25\,\text{W}$

P 10.39 **[a]**



$(30 - j40)\mathbf{I}_1 + \mathbf{V}_1 + \mathbf{V}_2 = 250$

$(5 + j10)\mathbf{I}_2 - \mathbf{V}_2 = 0$

$\frac{\mathbf{V}_1}{900} = \frac{-\mathbf{V}_2}{300}$

$900\mathbf{I}_1 = 300(\mathbf{I}_1 - \mathbf{I}_2)$

Solving,

$\mathbf{V}_1 = 150 + j300\ \text{V(rms)}; \qquad \mathbf{V}_2 = -50 - j100\ \text{V(rms)}$

$\mathbf{I}_1 = 5 + j0\ \text{A(rms)}; \qquad \mathbf{I}_2 = -10 + j0\ \text{A(rms)}$

$P_{30\Omega} = (5)^2(30) = 750\,\text{W}; \qquad \text{and} \qquad P_{5\Omega} = (10)^2(5) = 500\,\text{W}$

[b] $P_g = -250(5\underline{/0^\circ}) = -1250\,\text{W}$

$\sum P_{\text{abs}} = 750 + 500 = 1250\,\text{W} = \sum P_{\text{dev}}$

P 10.40 **[a]** $25a_1^2 + 4a_2^2 = 500$

$\mathbf{I}_{25} = a_1\mathbf{I}; \qquad P_{25} = a_1^2\mathbf{I}^2(25)$

$\mathbf{I}_4 = a_2\mathbf{I}; \qquad P_4 = a_2^2\mathbf{I}^2(4)$

$P_4 = 4P_{25}; \qquad a_2^2\mathbf{I}^2 4 = 100a_1^2\mathbf{I}^2$

$\therefore \qquad 100a_1^2 = 4a_2^2$

$25a_1^2 + 100a_1^2 = 500; \qquad a_1 = 2$

$25(4) + 4a_2^2 = 500; \qquad a_2 = 10$

[b] $\mathbf{I} = \dfrac{2000/\underline{0^\circ}}{500 + 500} = 2/\underline{0^\circ}\,\text{A(rms)}$

$\mathbf{I}_{25} = a_1\mathbf{I} = 4\,\text{A}$

$P_{25\Omega} = (16)(25) = 400\,\text{W}$

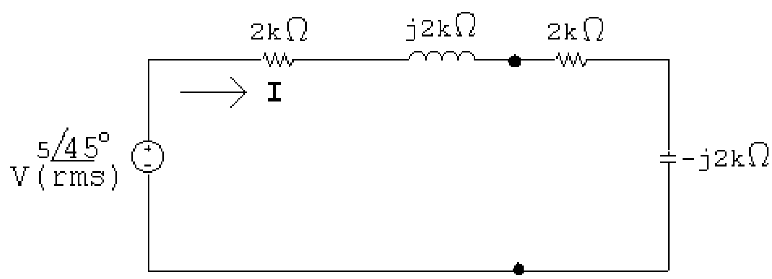
[c] $\mathbf{I}_4 = a_2\mathbf{I} = 10(2) = 20\,\text{A(rms)}$

$\mathbf{V}_4 = (20)(4) = 80/\underline{0^\circ}\,\text{V(rms)}$

P 10.41 **[a]** $Z_{\text{Th}} = j4000 + \dfrac{(4000)(-j4000)}{4000 - j4000} = 2000 + j2000\,\Omega$

$\therefore \quad Z_{\text{L}} = Z_{\text{Th}}^* = 2000 - j2000\,\Omega$

[b] $\mathbf{V}_{\text{Th}} = \dfrac{10/\underline{0^\circ}(4000)}{4000 - j4000} = 5 + j5 = 5\sqrt{2}/\underline{45^\circ}\,\text{V}$



$\mathbf{I} = \dfrac{5\sqrt{2}/\underline{45^\circ}}{4000} = 1.25\sqrt{2}/\underline{45^\circ}\,\text{mA}$

$|\mathbf{I}_{\text{rms}}| = 1.25\,\text{mA}$

$P_{\text{load}} = (0.00125)^2(2000) = 3.125\,\text{mW}$

[c] The closest resistor values from Appendix H are 1.8 kΩ and 2.2 kΩ. Find the capacitor value:

$$\frac{1}{8000C} = 2000 \qquad \text{so} \qquad C = 62.5\,\text{nF}$$

The closest capacitor value is 47 nF. Try $R = 1.8\,\text{k}\Omega$:

$$\mathbf{I} = \frac{5\underline{/45^\circ}}{2000 + j2000 + 1800 - j2659.57} = 0.7462 + j1.06 \text{ mA(rms)}$$

$$= 1.3\underline{/54.85^\circ} \text{ mA(rms)}$$

$$P_{\text{load}} = (0.0013)^2(1800) = 3.03\,\text{mW} \qquad \text{(instead of 3.125 mW)}$$

Try $R = 2.2\,\text{k}\Omega$:

$$\mathbf{I} = \frac{5\underline{/45^\circ}}{2000 + j2000 + 2200 - j2659.57} = 0.6925 + j0.9505 \text{ mA(rms)}$$

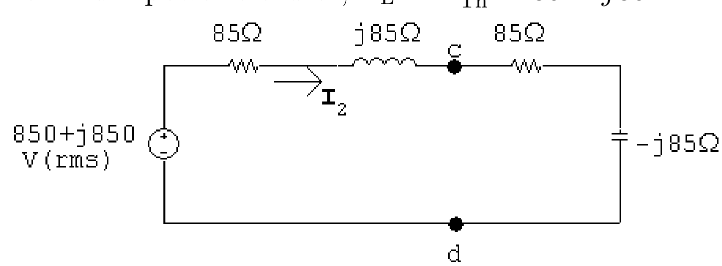
$$= 1.176\underline{/53.92^\circ} \text{ mA(rms)}$$

$$P_{\text{load}} = (0.001176)^2(2200) = 3.04\,\text{mW} \qquad \text{(instead of 3.125 mW)}$$

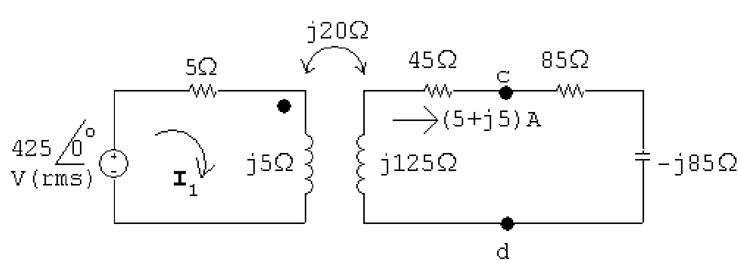
Therefore, use the 2.2 kΩ resistor to give a load impedance of

$Z_{\text{L}} = 2200 - j2659.57\,\Omega$.

P 10.42 **[a]** From Problem 9.75, $Z_{\text{Th}} = 85 + j85\,\Omega$ and $\mathbf{V}_{\text{Th}} = 850 + j850\,\text{V}$. Thus, for maximum power transfer, $Z_{\text{L}} = Z_{\text{Th}}^* = 85 - j85\,\Omega$:



$$\mathbf{I}_2 = \frac{850 + j850}{170} = 5 + j5\,\text{A}$$



$$425\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - j20(5 + j5)$$

$$\therefore \quad \mathbf{I}_1 = \frac{325 + j100}{5 + j5} = 42.5 - j22.5\,\text{A}$$

$$S_g(\text{del}) = 425(42.5 + j22.5) = 18{,}062.5 + j9562.5\,\text{VA}$$

$$P_g = 18{,}062.5\,\text{W}$$

[b] $P_{\text{loss}} = |\mathbf{I}_1|^2(5) + |\mathbf{I}_2|^2(45) = 11{,}562.5 + 2250 = 13{,}812.5\,\text{W}$

$$\%\text{ loss in transformer} = \frac{13{,}812.5}{18{,}062.5}(100) = 76.47\%$$

P 10.43 **[a]** $\dfrac{156 - j42 - 300}{Z_{\text{Th}}} + \dfrac{156 - j42}{200 - j500} = 0$

$$\therefore \quad Z_{\text{Th}} = \frac{300 - 156 + j42}{0.18 + j0.24} = 400 - j300\,\Omega$$

$$\therefore \quad Z_{\text{L}} = 400 + j300\,\Omega$$

[b] $\mathbf{I} = \dfrac{300\underline{/0^\circ}}{800\underline{/0^\circ}} = 0.375\underline{/0^\circ}\,\text{A(rms)}$

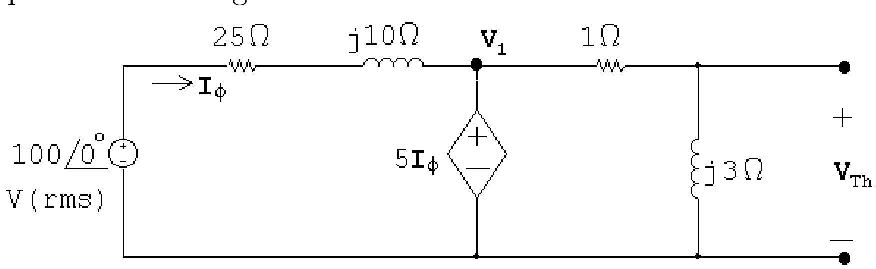
$$P = (0.375)^2(400) = 56.25\,\text{W}$$

[c] Let $R = 180\,\Omega + 220\,\Omega = 400\,\Omega$

$$2\pi(500)L = 300 \qquad \text{so} \qquad L = \frac{300}{1000\pi} = 95.5\,\text{mH}$$

Use 9 series-connected 10 mH inductors to get 90 mH. Use 2 parallel-connected 10 mH inductors to get 5 mH. Use 2 parallel-connected 1 mH inductors to get 0.5 mH. Combine the 90 mH, the 5 mH, and the 0.5 mH in series to get 95.5 mH.

P 10.44 **[a]** Open circuit voltage:



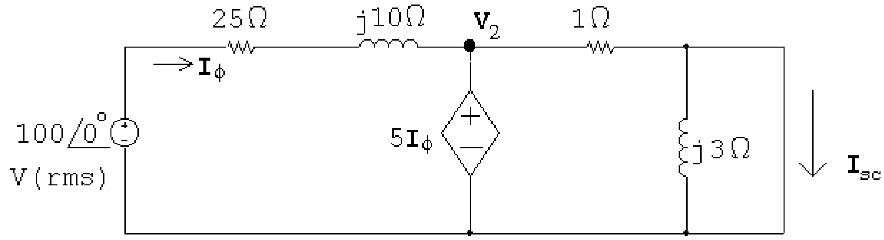
$$\mathbf{V}_1 = 5\mathbf{I}\phi = 5\frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$(25 + j10)\mathbf{I}_\phi = 100 - 5\mathbf{I}\phi$$

$$\mathbf{I}_\phi = \frac{100}{30 + j10} = 3 - j\,\text{A}$$

$$\mathbf{V}_{\rm Th} = \frac{j3}{1+j3}(5\mathbf{I}_\phi) = 15\,\mathrm{V}$$

Short circuit current:



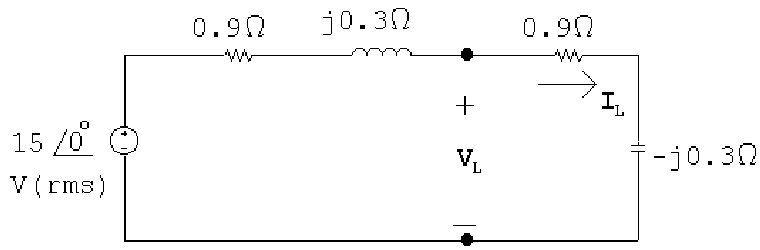
$$\mathbf{V}_2 = 5\mathbf{I}_\phi = \frac{100-5\mathbf{I}_\phi}{25+j10}$$

$$\mathbf{I}_\phi = 3 - j1\,\mathrm{A}$$

$$\mathbf{I}_{\rm sc} = \frac{5\mathbf{I}_\phi}{1} = 15 - j5\,\mathrm{A}$$

$$Z_{\rm Th} = \frac{15}{15-j5} = 0.9 + j0.3\,\Omega$$

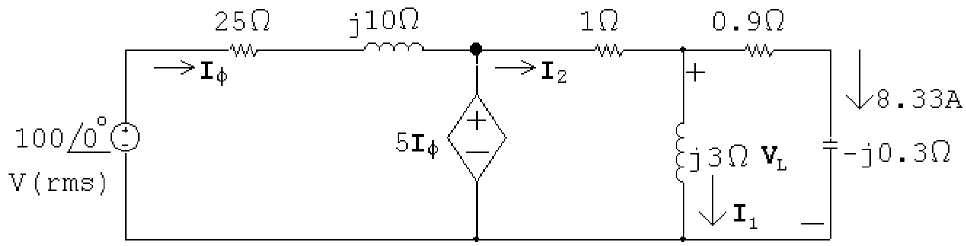
$$Z_L = Z_{\rm Th}^* = 0.9 - j0.3\,\Omega$$



$$\mathbf{I}_{\rm L} = \frac{0.3}{1.8} = 8.33\,\mathrm{A(rms)}$$

$$P = |\mathbf{I}_L|^2(0.9) = 62.5\,\mathrm{W}$$

[b] $\mathbf{V}_{\rm L} = (0.9 - j0.3)(8.33) = 7.5 - j2.5\,\mathrm{V(rms)}$



$$\mathbf{I}_1 = \frac{\mathbf{V}_{\rm L}}{j3} = -0.833 - j2.5\,\mathrm{A(rms)}$$

$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_\text{L} = 7.5 - j2.5\,\text{A(rms)}$

$5\mathbf{I}_\phi = \mathbf{I}_2 + \mathbf{V}_L \quad \therefore \quad \mathbf{I}_\phi = 3 - j1\,\text{A}$

$\mathbf{I}_\text{d.s.} = \mathbf{I}_\phi - \mathbf{I}_2 = -4.5 + j1.5\,\text{A}$

$S_g = -100(3 + j1) = -300 - j100\,\text{VA}$

$S_\text{d.s.} = 5(3 - j1)(-4.5 - j1.5) = -75 + j0\,\text{VA}$

$P_\text{dev} = 300 + 75 = 375\,\text{W}$

$\%\text{ developed} = \frac{62.5}{375}(100) = 16.67\%$

Checks:

$P_{25\Omega} = (10)(25) = 250\,\text{W}$

$P_{1\Omega} = (67.5)(1) = 67.5\,\text{W}$

$P_{0.9\Omega} = 62.5\,\text{W}$

$\sum P_\text{abs} = 230 + 62.5 + 67.5 = 375 = \sum P_\text{dev}$

$Q_{j10} = (10)(10) = 100\,\text{VAR}$

$Q_{j3} = (6.94)(3) = 20.82\,\text{VAR}$

$Q_{-j0.3} = (69.4)(-0.3) = -20.82\,\text{VAR}$

$Q_\text{source} = -100\,\text{VAR}$

$\sum Q = 100 + 20.82 - 20.82 - 100 = 0$

P 10.45 $Z_\text{L} = |Z_\text{L}|\underline{/\theta^\circ} = |Z_\text{L}|\cos\theta^\circ + j|Z_\text{L}|\sin\theta^\circ$

Thus $$|\mathbf{I}| = \frac{|\mathbf{V}_\text{Th}|}{\sqrt{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}}$$

Therefore $$P = \frac{0.5|\mathbf{V}_\text{Th}|^2|Z_\text{L}|\cos\theta}{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}$$

Let D = demoninator in the expression for P, then

$$\frac{dP}{d|Z_\text{L}|} = \frac{(0.5|\mathbf{V}_\text{Th}|^2\cos\theta)(D\cdot 1 - |Z_\text{L}|dD/d|Z_\text{L}|)}{D^2}$$

$$\frac{dD}{d|Z_\text{L}|} = 2(R_\text{Th} + |Z_\text{L}|\cos\theta)\cos\theta + 2(X_\text{Th} + |Z_\text{L}|\sin\theta)\sin\theta$$

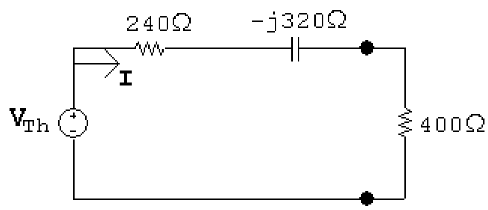
$$\frac{dP}{d|Z_L|} = 0 \quad \text{when} \quad D = |Z_L|\left(\frac{dD}{d|Z_L|}\right)$$

Substituting the expressions for D and $(dD/d|Z_L|)$ into this equation gives us the relationship $R_{Th}^2 + X_{Th}^2 = |Z_L|^2$ or $|Z_{Th}| = |Z_L|$.

P 10.46 **[a]** $Z_{Th} = 200 - j480 + \frac{(j200)(500 + j300)}{500 + j500} = 240 - j320 = 400\underline{/-53.13^\circ}\,\Omega$

$\therefore \quad R = |Z_{Th}| = 400\,\Omega$

[b] $\mathbf{V}_{Th} = \frac{j200}{500 + j300 + j200}(300\underline{/0^\circ}) = 60 + j60\,\text{V(rms)}$



$\mathbf{I} = \frac{60 + j60}{640 - j320} = 37.5 + j112.5\text{ mA(rms)} = 118.59\underline{/71.57^\circ}\text{ mA(rms)}$

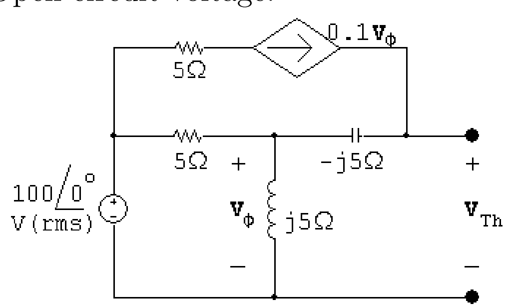
$P = (0.11859)^2(400) = 5.625\,\text{W}$

[c] Pick the $390\,\Omega$ resistor from Appendix H for the closest match:

$\mathbf{I} = \frac{60 + j60}{630 - j320} = 120.084\underline{/71.93^\circ}\text{ mA(rms)}$

$P = (0.120084)^2(390) = 5.624\,\text{W}$

P 10.47 **[a]** Open circuit voltage:

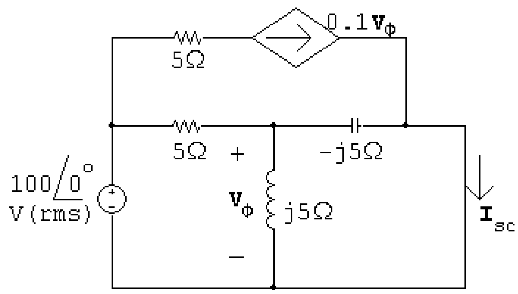


$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$\therefore \quad \mathbf{V}_\phi = 40 + j80\,\text{V(rms)}$

$\mathbf{V}_{Th} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60\,\text{V(rms)}$

Short circuit current:



$$\mathbf{I}_{\rm sc} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

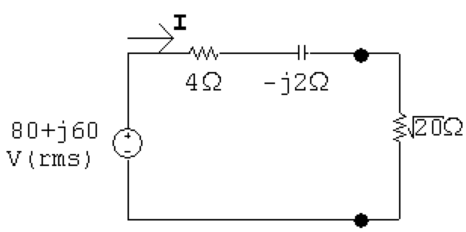
$$\therefore \quad \mathbf{V}_\phi = 100\,\mathrm{V(rms)}$$

$$\mathbf{I}_{\rm sc} = (0.1 + j0.2)(100) = 10 + j20\,\mathrm{A(rms)}$$

$$Z_{\rm Th} = \frac{\mathbf{V}_{\rm Th}}{\mathbf{I}_{\rm sc}} = \frac{80 + j60}{10 + j20} = 4 - j2\,\Omega$$

$$\therefore \quad R_o = |Z_{\rm Th}| = 4.47\,\Omega$$

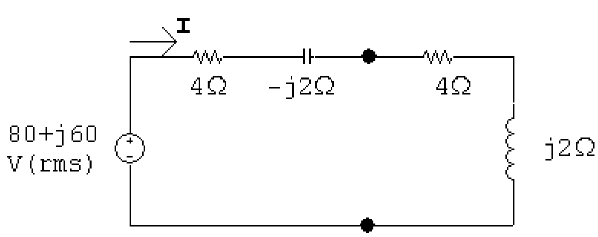
[**b**]



$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82\,\mathrm{A\ (rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17\,\mathrm{W}$$

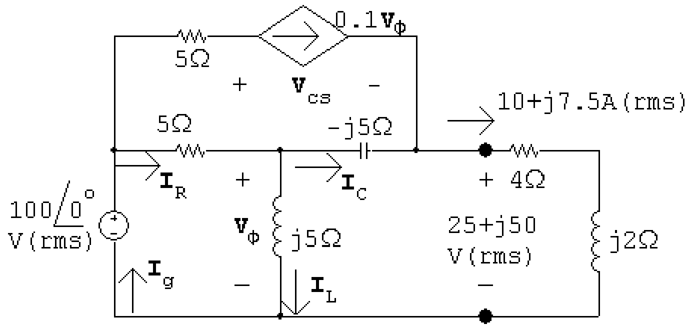
[**c**]



$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5\,\mathrm{A\ (rms)}$$

$$P = (10^2 + 7.5^2)(4) = 625\,\mathrm{W}$$

[**d**]



$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_o - (25 + j50)}{-j5} = 0$$

$\mathbf{V}_\phi = 50 + j25\,\text{V (rms)}$

$0.1\mathbf{V}_\phi = 5 + j2.5\,\text{V (rms)}$

$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$

$\mathbf{I}_C = 5 + j5\,\text{A (rms)}$

$$\mathbf{I}_L = \frac{\mathbf{V}_\phi}{j5} = 5 - j10\,\text{A (rms)}$$

$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5\,\text{A (rms)}$

$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5\,\text{A (rms)}$

$S_g = -100\mathbf{I}_g^* = -1500 - j250\,\text{VA}$

$100 = 5(5 + j2.5) + \mathbf{V}_{\text{cs}} + 25 + j50 \quad \therefore \quad \mathbf{V}_{\text{cs}} = 50 - j62.5\,\text{V (rms)}$

$S_{\text{cs}} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5\,\text{VA}$

Thus,

$\sum P_{\text{dev}} = 1500$

$$\%\text{ delivered to } R_o = \frac{625}{1500}(100) = 41.67\%$$

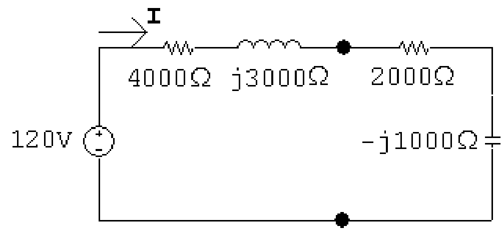
P 10.48 [**a**] First find the Thévenin equivalent:

$j\omega L = j3000\,\Omega$

$Z_{\text{Th}} = 6000\|12{,}000 + j3000 = 4000 + j3000\,\Omega$

$$\mathbf{V}_{\text{Th}} = \frac{12{,}000}{6000 + 12{,}000}(180) = 120\,\text{V}$$

$$\frac{-j}{\omega C} = -j1000\,\Omega$$



$$\mathbf{I} = \frac{120}{6000 + j2000} = 18 - j6\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(2000) = 360\,\text{mW}$$

[b] Set $C_o = 0.1\,\mu\text{F}$ so $-j/\omega C = -j2000\,\Omega$

Set R_o as close as possible to

$$R_o = \sqrt{4000^2 + (3000 - 2000)^2} = 4123.1\,\Omega$$

$$\therefore\quad R_o = 4000\,\Omega$$

[c] $\mathbf{I} = \dfrac{120}{8000 + j1000} = 14.77 - j1.85\,\text{mA}$

$$P = \frac{1}{2}|\mathbf{I}|^2(4000) = 443.1\,\text{mW}$$

Yes; $443.1\,\text{mW} > 360\,\text{mW}$

[d] $\mathbf{I} = \dfrac{120}{8000} = 15\,\text{mA}$

$$P = \frac{1}{2}(0.015)^2(4000) = 450\,\text{mW}$$

[e] $R_o = 4000\,\Omega$; $C_o = 66.67\,\text{nF}$

[f] Yes; $450\,\text{mW} > 443.1\,\text{mW}$

P 10.49 **[a]** Set $C_o = 0.1\,\mu\text{F}$, so $-j/\omega C = -j2000\,\Omega$; also set $R_o = 4123.1\,\Omega$

$$\mathbf{I} = \frac{120}{8123.1 + j1000} = 14.55 - j1.79\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4123.1) = 443.18\,\text{mW}$$

[b] Yes; $443.18\,\text{mW} > 360\,\text{mW}$

[c] Yes; $443.18\,\text{mW} < 450\,\text{mW}$

P 10.50 **[a]** $\dfrac{1}{\omega C} = 100\,\Omega$; $C = \dfrac{1}{(60)(200\pi)} = 26.53\,\mu\text{F}$

[b] $\mathbf{V}_{\text{swo}} = 4000 + (40)(1.25 + j10) = 4050 + j400$

$= 4069.71\underline{/5.64^\circ}$ V(rms)

$\mathbf{V}_{\text{sw}} = 4000 + (40 - j40)(1.25 + j10) = 4450 + j350 = 4463.73\underline{/4.50^\circ}$ V(rms)

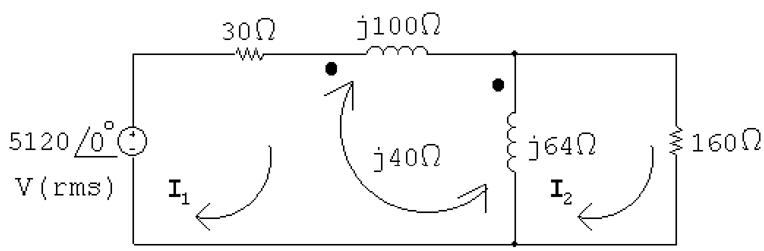
$$\%\text{ increase } = \left(\frac{4463.73}{4069.71} - 1\right)(100) = 9.68\%$$

[c] $P_{\ell\text{wo}} = (40\sqrt{2})^2(1.25) = 4000$ W

$P_{\ell\text{w}} = 40^2(1.25) = 2000$ W

$$\%\text{ increase } = \left(\frac{4000}{2000} - 1\right)(100) = 100\%$$

P 10.51 **[a]**



$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2) + j64(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1$

$0 = 160\mathbf{I}_2 + j64(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1$

Solving,

$\mathbf{I}_1 = 8 - j20$ A(rms); $\quad \mathbf{I}_2 = 13\underline{/0^\circ}$ A(rms)

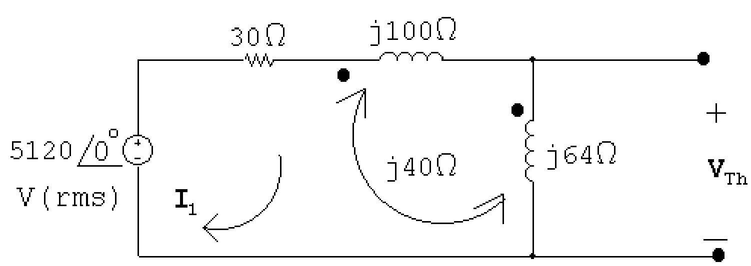
$\therefore \quad \mathbf{V}_o = (13)(160) = 2080$ V(rms)

[b] $P = (13)^2(160) = 27{,}040$ W

[c] $S_g = -(5120)(8 + j20) = -40{,}960 - j102{,}400$ VA $\quad \therefore \quad P_g = -40{,}960$ W

$$\%\text{ delivered } = \frac{27{,}040}{40{,}960}(100) = 66\%$$

P 10.52 **[a]** Open circuit voltage:

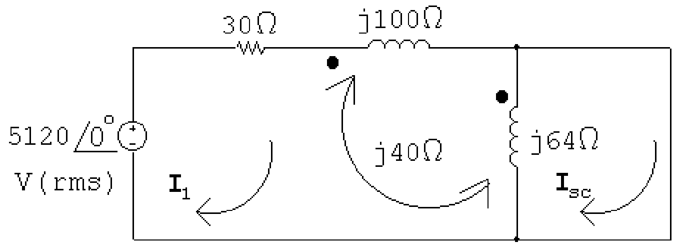


$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40\mathbf{I}_1 + j64\mathbf{I}_1 + j40\mathbf{I}_1$

$$\therefore \quad \mathbf{I}_1 = \frac{5120}{30 + j244} = 2.54 - j20.67\,\mathrm{A(rms)}$$

$$\mathbf{V}_{\mathrm{Th}} = j64\mathbf{I}_1 + j40\mathbf{I}_1 = j104\mathbf{I}_1 = 2149.8 + j264.32\,\mathrm{V}$$

Short circuit current:



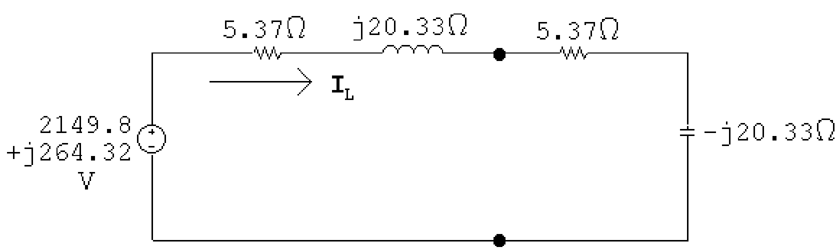
$$5120 = 30\mathbf{I}_1 + j100\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_{\mathrm{sc}}) + j64(\mathbf{I}_1 - \mathbf{I}_{\mathrm{sc}}) + j40\mathbf{I}_1$$

$$0 = j64(\mathbf{I}_{\mathrm{sc}} - \mathbf{I}_1) - j40\mathbf{I}_1$$

Solving,

$$\mathbf{I}_{\mathrm{sc}} = 38.25 - j95.63\,\mathrm{A}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{2149.8 + j264.32}{38.25 - j95.63} = 5.37 + j20.33\,\Omega$$



$$\mathbf{I}_{\mathrm{L}} = \frac{2149.8 + j264.32}{10.74} = 200.17 + j24.61 = 201.67\underline{/7.01^\circ}\,\mathrm{A}$$

$$P_{\mathrm{L}} = (201.67)^2(5.37) = 218.4\,\mathrm{kW}$$

[b] $Z_o\mathbf{I}_2 + j64(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 = 0 \qquad \text{so} \qquad \mathbf{I}_1 = \dfrac{Z_o + j64}{j104}\mathbf{I}_2$

$$\therefore \qquad \mathbf{I}_1 = \frac{5.37 - j20.33 + j64}{j104}(200.17 + j24.61) = 85.32\underline{/0^\circ}\,\mathrm{A(rms)}$$

$$P_{\mathrm{dev}} = (5120)(85.32) = 436.8\,\mathrm{kW}$$

[c] Begin by choosing the capacitor value from Appendix H that is closest to the required reactive impedance, assuming the frequency of the source is 60 Hz:

$$20.33 = \frac{1}{2\pi(60)C} \qquad \text{so} \qquad C = \frac{1}{2\pi(60)(20.33)} = 130.48\,\mu\mathrm{F}$$

Choose the capacitor value closest to this capacitance from Appendix H, which is 100 μF. Then,

$$X_L = -\frac{1}{2\pi(60)(100\times10^{-6})} = -26.5258\,\Omega$$

Now set R_L as close as possible to $\sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$:

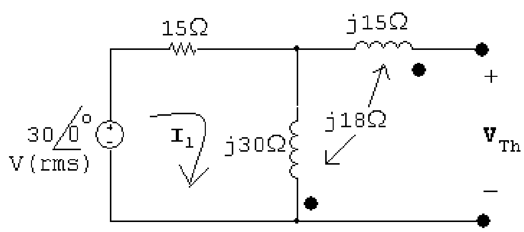
$$R_L = \sqrt{5.37^2 + (20.33 - 26.5258)^2} = 8.2\,\Omega$$

The closest single resistor value from Appendix H is 10 Ω. The resulting real power developed by the source is calculated below, using the Thévenin equivalent circuit:

$$\mathbf{I} = \frac{2149.8 + j264.32}{5.37 + j20.33 + 10 - j26.5258} = 130.7\underline{/28.96^\circ}$$

$$P = |2149.8 + j264.32|(130.7) = 283.1\,\text{kW} \qquad \text{(instead of 436.8 kW)}$$

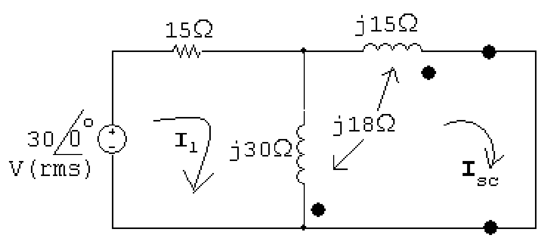
P 10.53 Open circuit voltage:



$$\mathbf{I}_1 = \frac{30\underline{/0^\circ}}{15 + j30} = 0.4 - j0.8\,\text{A}$$

$$\mathbf{V}_{Th} = j30\mathbf{I}_1 - j18\mathbf{I}_1 = j12\mathbf{I}_1 = 9.6 + j4.8 = 10.73\underline{/26.57^\circ}$$

Short circuit current:



$$30\underline{/0^\circ} = 15\mathbf{I}_1 + j30(\mathbf{I}_1 - \mathbf{I}_{sc}) + j18\mathbf{I}_{sc}$$

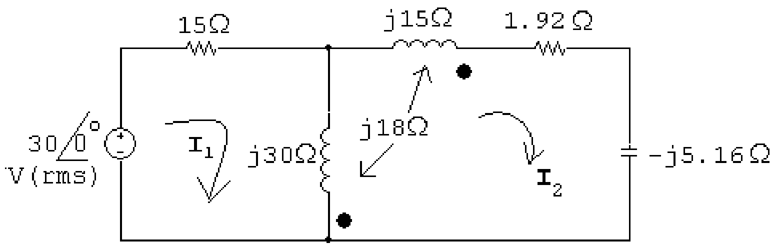
$$0 = j15\mathbf{I}_{sc} - j18(\mathbf{I}_{sc} - \mathbf{I}_1) + j30(\mathbf{I}_{sc} - \mathbf{I}_1) - j18\mathbf{I}_{sc}$$

Solving,

$$\mathbf{I}_{sc} = 1.95\underline{/-43.025^\circ}\,\text{A}$$

$$Z_{\text{Th}} = \frac{9.6 + j4.8}{1.95\underline{/-43.025^\circ}} = 1.92 + j5.16\,\Omega$$

$$\therefore \quad \mathbf{I}_2 = \frac{9.6 + j4.8}{3.84} = 2.795\underline{/26.57^\circ}\,\text{A}$$

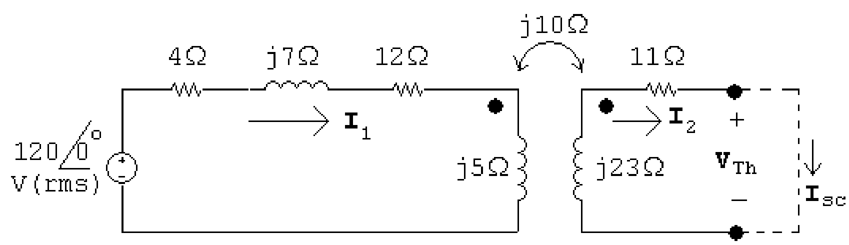


$$30\underline{/0^\circ} = 15\mathbf{I}_1 + j30(\mathbf{I}_1 - \mathbf{I}_2) + j18\mathbf{I}_2$$

$$\therefore \qquad \mathbf{I}_1 = \frac{30 + j12\mathbf{I}_2}{15 + j30} = \frac{30 + j12(2.795\underline{/26.57^\circ})}{15 + j30} = 1\,\text{A}$$

$$Z_g = \frac{30\underline{/0^\circ}}{1} = 30 + j0 = 30\underline{/0^\circ}\,\Omega$$

P 10.54 **[a]**



Open circuit:

$$\mathbf{V}_{\text{Th}} = \frac{120}{16 + j12}(j10) = 36 + j48\,\text{V}$$

Short circuit:

$$(16 + j12)\mathbf{I}_1 - j10\mathbf{I}_{\text{sc}} = 120$$

$$-j10\mathbf{I}_1 + (11 + j23)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 2.4\,\text{A}$$

$$Z_{\text{Th}} = \frac{36 + j48}{2.4} = 15 + j20\,\Omega$$

$$\therefore \quad Z_L = Z^*_{Th} = 15 - j20\,\Omega$$

$$\mathbf{I}_L = \frac{\mathbf{V}_{Th}}{Z_{Th} + Z_L} = \frac{36 + j48}{30} = 1.2 + j1.6\,\text{A(rms)}$$

$$P_L = |\mathbf{I}_L|^2(15) = 60\,\text{W}$$

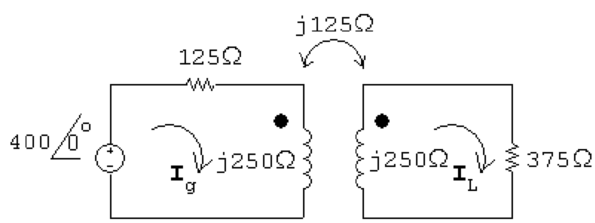
[b] $\mathbf{I}_1 = \frac{Z_{22}\mathbf{I}_2}{j\omega M} = \frac{26 + j3}{j10}(1.2 + j1.6) = 5.23\underline{/-30.29^\circ}\,\text{A (rms)}$

$$P_{\text{transformer}} = (120)(5.23)\cos(-30.29^\circ) - (5.23)^2(4) = 432.8\,\text{W}$$

$$\%\ \text{delivered} = \frac{60}{432.8}(100) = 13.86\%$$

P 10.55 **[a]** $j\omega L_1 = j\omega L_2 = j(400)(625 \times 10^{-3}) = j250\,\Omega$

$$j\omega M = j(400)(312.5 \times 10^{-3}) = j125\,\Omega$$



$$400 = (125 + j250)\mathbf{I}_g - j125\mathbf{I}_L$$

$$0 = -j125\mathbf{I}_g + (375 + j250)\mathbf{I}_L$$

Solving,

$\mathbf{I}_g = 0.8 - j1.2\,\text{A}; \qquad \mathbf{I}_L = 0.4\,\text{A}$

Thus,

$$i_g = 1.44\cos(400t - 56.31^\circ)\,\text{A}$$

$$i_L = 0.4\cos 400t\,\text{A}$$

[b] $k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.3125}{0.625} = 0.5$

[c] When $t = 1.25\pi$ ms:

$$400t = (400)(1.25\pi) \times 10^{-3} = 0.5\pi \text{ rad} = 90^\circ$$

$$i_g(1.25\pi\,\text{ms}) = 1.44\cos(90^\circ - 56.31^\circ) = 1.2\,\text{A}$$

$$i_L(1.25\pi\,\text{ms}) = 0.4\cos(90^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 = \frac{1}{2}(0.625)(1.2)^2 + 0 - 0 = 450\,\text{mJ}$$

When $t = 2.5\pi$ ms:

$400t = (400)(2.5\pi) \times 10^{-3} = \pi = 180^\circ$

$i_g(2.5\pi\,\text{ms}) = 1.44\cos(180 - 56.31^\circ) = -0.8\,\text{A}$

$i_\text{L}(2.5\pi\,\text{ms}) = 0.4\cos(180) = -0.4\,\text{A}$

$$w = \frac{1}{2}(0.625)(0.8)^2 + \frac{1}{2}(0.625)(-0.4)^2 - (0.3125)(-0.8)(-0.4) = 150\,\text{mJ}$$

[d] From (a), $I_L = 0.4$ A,

$$\therefore \quad P = \frac{1}{2}(0.4)^2(375) = 30\,\text{W}$$

[e] Open circuit:

$$\mathbf{V}_\text{Th} = \frac{400}{125 + j250}(j125) = 160 + j80\,\text{V}$$

Short circuit:

$400 = (125 + j250)\mathbf{I}_1 - j125\mathbf{I}_\text{sc}$

$0 = -j125\mathbf{I}_1 + j250\mathbf{I}_\text{sc}$

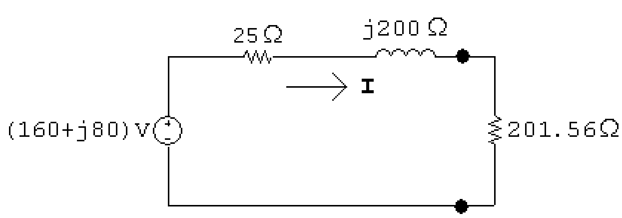
Solving,

$\mathbf{I}_\text{sc} = 0.4923 - j0.7385$

$$Z_\text{Th} = \frac{\mathbf{V}_\text{Th}}{\mathbf{I}_\text{sc}} = \frac{160 + j80}{0.4923 - j0.7385} = 25 + j200\,\Omega$$

$\therefore \quad R_\text{L} = 201.56\,\Omega$

[f]



$$\mathbf{I} = \frac{160 + j80}{226.56 + j200} = 0.592\underline{/-14.87^\circ}\,\text{A}$$

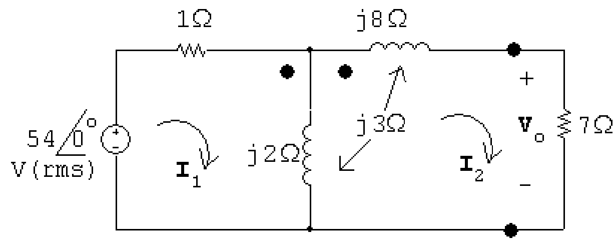
$$P = \frac{1}{2}(0.592)^2(201.56) = 35.3\,\text{W}$$

[g] $Z_\text{L} = Z^*_\text{Th} = 25 - j200\,\Omega$

[h] $\mathbf{I} = \dfrac{160 + j80}{50} = 3.58\underline{/26.57^\circ}$

$$P = \frac{1}{2}(3.58)^2(25) = 160\,\text{W}$$

P 10.56 **[a]**



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j3\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j3\mathbf{I}_2 + j8\mathbf{I}_2 + j3(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$\mathbf{I}_1 = 12 - j21\,\text{A (rms)}; \qquad \mathbf{I}_2 = -3\,\text{A (rms)}$

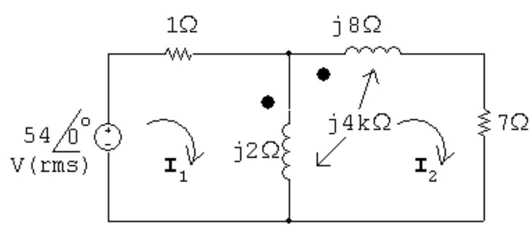
$\mathbf{V}_o = 7\mathbf{I}_2 = -21\underline{/0^\circ}\,\text{V(rms)}$

[b] $P = |\mathbf{I}_2|^2(7) = 63\,\text{W}$

[c] $P_g = (54)(12) = 648\,\text{W}$

$$\%\text{ delivered } = \frac{63}{648}(100) = 9.72\%$$

P 10.57 **[a]**



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j4k\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j4k\mathbf{I}_2 + j8\mathbf{I}_2 + j4k(\mathbf{I}_1 - \mathbf{I}_2)$$

Place the equations in standard form:

$$54 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$$

$$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{54 - \mathbf{I}_2 j(4k - 2)}{(1 + j2)}$$

Substituting,

$$\mathbf{I}_2 = \frac{j54(4k - 2)}{[7 + j(10 - 8k)](1 + j2) - (4k - 2)}$$

For $\mathbf{V}_o = 0, \mathbf{I}_2 = 0$, so if $4k - 2 = 0$, then $k = 0.5$.

[b] When $\mathbf{I}_2 = 0$

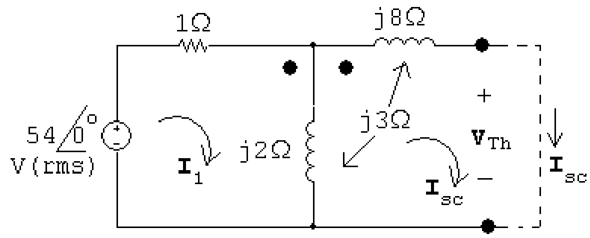
$$\mathbf{I}_1 = \frac{54}{1+j2} = 10.8 - j21.6\,\text{A(rms)}$$

$$P_g = (54)(10.8) = 583.2\,\text{W}$$

Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(1) = 583.2\,\text{W}$$

P 10.58 **[a]**



Open circuit:

$$\mathbf{V}_{\text{Th}} = -j3\mathbf{I}_1 + j2\mathbf{I}_1 = -j\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{54}{1+j2} = 10.8 - j21.6$$

$$\mathbf{V}_{\text{Th}} = -21.6 - j10.8\,\text{V}$$

Short circuit:

$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j3\mathbf{I}_{\text{sc}}$$

$$0 = j2(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) - j3\mathbf{I}_{\text{sc}} + j8\mathbf{I}_{\text{sc}} + j3(\mathbf{I}_1 - \mathbf{I}_{\text{sc}})$$

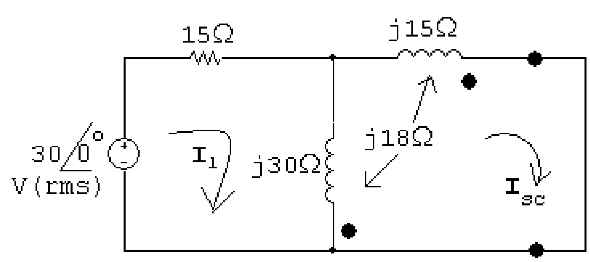
Solving,

$$\mathbf{I}_{\text{sc}} = -3.32 + j5.82$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-21.6 - j10.8}{-3.32 + j5.82} = 0.2 + j3.6 = 3.6\underline{/86.86^\circ}\,\Omega$$

$$\therefore\quad R_{\text{L}} = |Z_{\text{Th}}| = 3.6\,\Omega$$

[b]



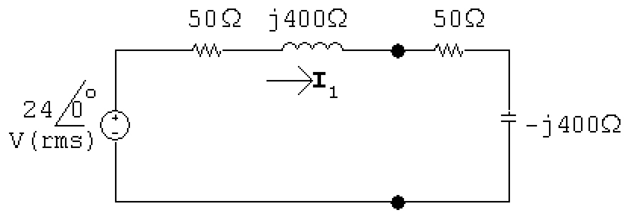
$$\mathbf{I} = \frac{-21.6 - j10.8}{3.8 + j3.6} = 4.614\underline{/163.1^\circ}$$

$$P = |\mathbf{I}|^2(3.6) = 76.6\,\text{W}, \quad \text{which is greater than when } R_L = 7\,\Omega$$

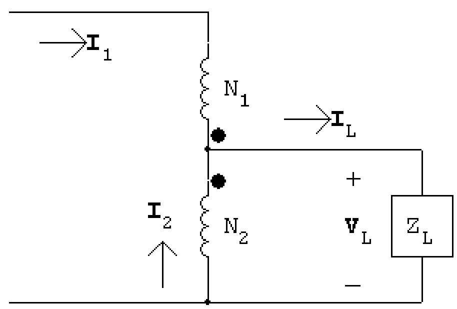
P 10.59 **[a]** $Z_{\text{ab}} = 50 - j400 = \left(1 - \frac{N_1}{N_2}\right)^2 Z_{\text{L}}$

$$\therefore \quad Z_{\text{L}} = \frac{1}{(1-6)^2}(50 - j400) = 2 - j16\,\Omega$$

[b]



$$\mathbf{I}_1 = \frac{24}{100} = 240/\underline{0^\circ}\,\text{mA}$$



$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -6\mathbf{I}_1 = -1.44/\underline{0^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{L}} = \mathbf{I}_1 + \mathbf{I}_2 = -1.68/\underline{0^\circ}\,\text{A}$$

$$\mathbf{V}_{\text{L}} = (2 - j16)\mathbf{I}_{\text{L}} = -3.36 + j26.88 = 27.1/\underline{97.13^\circ}\,\text{V(rms)}$$

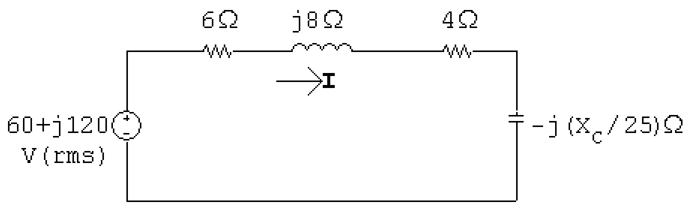
P 10.60 **[a]** Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_{\text{Th}} = (15)(20\|j10) = 60 + j120\,\text{V}$$

$$Z_{\text{Th}} = 2 + 20\|j10 = 6 + j8\,\Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{25}(100 - jX_{\text{C}}) = 4 - j\frac{X_{\text{C}}}{25}\,\Omega$$



Now maximize **I** by setting $(X_C/25) = 8\,\Omega$:

$$\therefore \quad C = \frac{1}{200(20 \times 10^3)} = 0.25\,\mu\text{F}$$

[b] $\mathbf{I} = \dfrac{60 + j120}{10} = 6 + j12\,\text{A}$

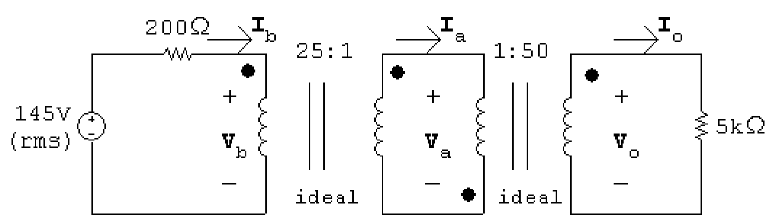
$$P = |\mathbf{I}|^2(4) = 720\,\text{W}$$

[c] $\dfrac{R_o}{25} = 6\,\Omega; \qquad \therefore \quad R_o = 150\,\Omega$

[d] $\mathbf{I} = \dfrac{60 + j120}{12} = 5 + j10\,\text{A}$

$$P = |\mathbf{I}|^2(6) = 750\,\text{W}$$

P 10.61



$$\frac{\mathbf{V}_a}{1} = \frac{-\mathbf{V}_o}{50}; \qquad 1\mathbf{I}_a = -50\mathbf{I}_o$$

$$\therefore \qquad \frac{\mathbf{V}_a}{\mathbf{I}_a} = \frac{-\mathbf{V}_o/50}{-50\mathbf{I}_o} = \frac{\mathbf{V}_o/\mathbf{I}_o}{50^2} = \frac{5000}{50^2} = 2\,\Omega$$

$$\frac{\mathbf{V}_b}{25} = \frac{\mathbf{V}_a}{1}; \qquad 25\mathbf{I}_b = 1\mathbf{I}_a$$

$$\therefore \qquad \frac{\mathbf{V}_b/25}{25\mathbf{I}_b} = \frac{\mathbf{V}_b/\mathbf{I}_b}{25^2} = \frac{\mathbf{V}_a}{\mathbf{I}_a}$$

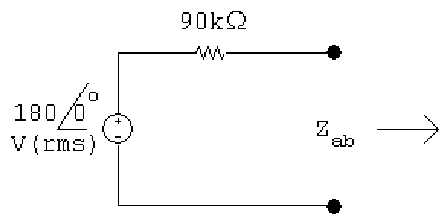
$$\therefore \qquad \frac{\mathbf{V}_b}{\mathbf{I}_b} = 25^2\frac{\mathbf{V}_a}{\mathbf{I}_a} = 25^2(2) = 1250\,\Omega$$

Thus $\mathbf{I}_\text{b} = [145/(200 + 1250)] = 100\,\text{mA}$ (rms); since the ideal transformers are lossless, $P_{5\text{k}\Omega} = P_{1250\Omega}$, and the power delivered to the $1250\,\Omega$ resistor is $(0.1)^2(1250)$ or $12.5\,\text{W}$.

P 10.62 **[a]** $\frac{\mathbf{V}_b}{\mathbf{I}_b} = \frac{25^2(5000)}{a^2} = 200\,\Omega;$ therefore $a^2 = 15{,}625,$ $a = 125$

[b] $\mathbf{I}_b = \frac{145}{400} = 362.5\,\text{mA};$ $P = (0.3625)^2(200) = 26.28125\,\text{W}$

P 10.63 **[a]**



For maximum power transfer, $Z_{ab} = 90\,\text{k}\Omega$

$$Z_{ab} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_L$$

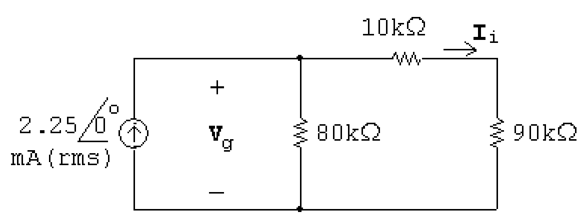
$$\therefore \quad \left(1 - \frac{N_1}{N_2}\right)^2 = \frac{90{,}000}{400} = 225$$

$$1 - \frac{N_1}{N_2} = \pm 15; \qquad \frac{N_1}{N_2} = 15 + 1 = 16$$

[b] $P = |\mathbf{I}_i|^2(90{,}000) = \left(\frac{180}{180{,}000}\right)^2(90{,}000) = 90\,\text{mW}$

[c] $\mathbf{V}_1 = R_i\mathbf{I}_i = (90{,}000)\left(\frac{180}{180{,}000}\right) = 90\,\text{V}$

[d]



$$\mathbf{V}_g = (2.25 \times 10^{-3})(100{,}000 \| 80{,}000) = 100\,\text{V}$$

$$P_g(\text{del}) = (2.25 \times 10^{-3})(100) = 225\,\text{mW}$$

$$\%\ \text{delivered} = \frac{90}{225}(100) = 40\%$$

P 10.64 **[a]** $Z_{Th} = 720 + j1500 + \left(\frac{200}{50}\right)^2(40 - j30) = 1360 + j1020 = 1700\underline{/36.87^\circ}\,\Omega$

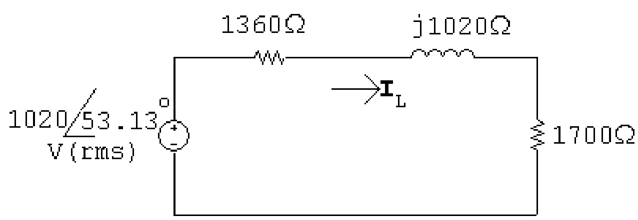
$$\therefore \quad Z_{ab} = 1700\,\Omega$$

$$Z_{\rm ab} = \frac{Z_{\rm L}}{(1 + N_1/N_2)^2}$$

$$(1 + N_1/N_2)^2 = 6800/1700 = 4$$

$$\therefore \quad N_1/N_2 = 1 \quad \text{or} \quad N_2 = N_1 = 1000 \text{ turns}$$

[b] $\mathbf{V}_{\rm Th} = \dfrac{255\underline{/0^\circ}}{40 + j30}(j200) = 1020\underline{/53.13^\circ}\,\text{V}$

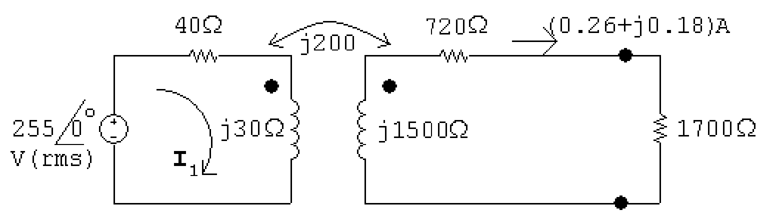


$$\mathbf{I}_L = \frac{1020\underline{/53.13^\circ}}{3060 + j1020} = 0.316\underline{/34.7^\circ}\,\text{A(rms)}$$

Since the transformer is ideal, $P_{6800} = P_{1700}$.

$P = |\mathbf{I}|^2(1700) = 170\,\text{W}$

[c]



$$255\underline{/0^\circ} = (40 + j30)\mathbf{I}_1 - j200(0.26 + j0.18)$$

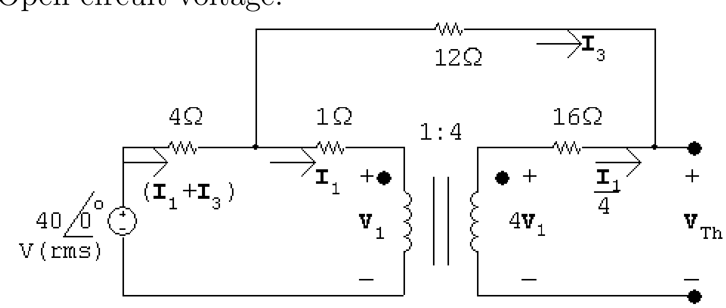
$$\therefore \quad \mathbf{I}_1 = 4.13 - j1.80\,\text{A(rms)}$$

$$P_{\rm gen} = (255)(4.13) = 1053\,\text{W}$$

$$P_{\rm diss} = 1053 - 170 = 883\,\text{W}$$

$$\%\text{ dissipated} = \frac{883}{1053}(100) = 83.85\%$$

P 10.65 **[a]** Open circuit voltage:



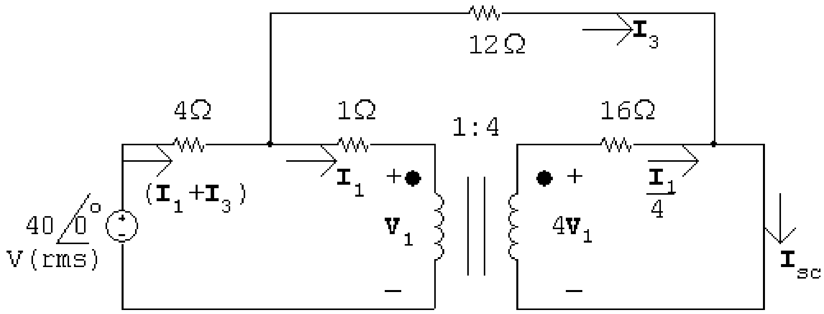
$$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + \mathbf{V}_{\rm Th}$$

$\frac{\mathbf{I}_1}{4} = -\mathbf{I}_3; \qquad \mathbf{I}_1 = -4\mathbf{I}_3$

Solving,

$\mathbf{V}_{\text{Th}} = 40\underline{/0^\circ}\,\text{V}$

Short circuit current:



$40\underline{/0^\circ} = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$

$4\mathbf{V}_1 = 16(\mathbf{I}_1/4) = 4\mathbf{I}_1; \qquad \therefore \quad \mathbf{V}_1 = \mathbf{I}_1$

$\therefore \quad 40\underline{/0^\circ} = 6\mathbf{I}_1 + 4\mathbf{I}_3$

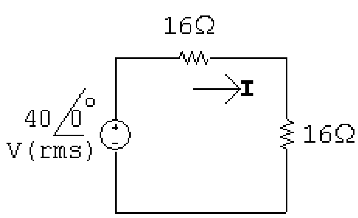
Also,

$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = 1\,\text{A}; \qquad \mathbf{I}_{\text{sc}} = \mathbf{I}_1/4 + \mathbf{I}_3 = 2.5\,\text{A}$

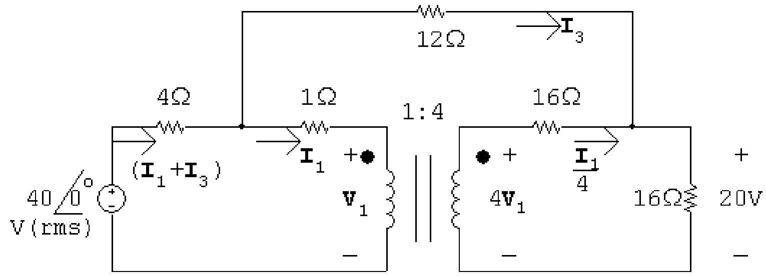
$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{40}{2.5} = 16\,\Omega$



$\mathbf{I} = \frac{40\underline{/0^\circ}}{32} = 1.25\underline{/0^\circ}\,\text{A(rms)}$

$P = (1.25)^2(16) = 25\,\text{W}$

[**b**]



$$40 = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + 20$$

$$4\mathbf{V}_1 = 4\mathbf{I}_1 + 16(\mathbf{I}_1/4 + \mathbf{I}_3); \qquad \therefore \quad \mathbf{V}_1 = 2\mathbf{I}_1 + 4\mathbf{I}_3$$

$$40 = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$\therefore \quad \mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = -0.25\,\text{A}; \qquad \mathbf{I}_1 + \mathbf{I}_3 = 5.75\underline{/0^\circ}\,\text{A}$$

$$P_{40\text{V}}(\text{developed}) = 40(5.75) = 230\,\text{W}$$

$$\therefore \quad \%\ \text{delivered} = \frac{25}{230}(100) = 10.87\%$$

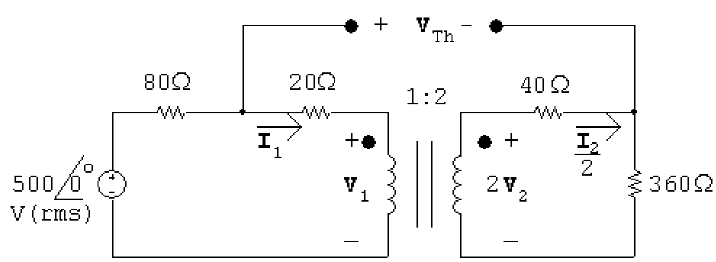
[c] $P_{R_L} = 25\,\text{W}; \qquad P_{16\Omega} = (1.5)^2(16) = 36\,\text{W}$

$$P_{4\Omega} = (5.75)^2(4) = 132.25\,\text{W}; \qquad P_{1\Omega} = (6)^2(1) = 36\,\text{W}$$

$$P_{12\Omega} = (-0.25)^2(12) = 0.75\,\text{W}$$

$$\sum P_{\text{abs}} = 25 + 36 + 132.25 + 36 + 0.75 = 230\,\text{W} = \sum P_{\text{dev}}$$

P 10.66 **[a]** Open circuit voltage:



$$500 = 100\mathbf{I}_1 + \mathbf{V}_1$$

$$\mathbf{V}_2 = 400\mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{1} = \frac{\mathbf{V}_2}{2} \qquad \therefore \qquad \mathbf{V}_2 = 2\mathbf{V}_1$$

$$\mathbf{I}_1 = 2\mathbf{I}_2$$

Substitute and solve:

$$2\mathbf{V}_1 = 400\mathbf{I}_1/2 = 200\mathbf{I}_1 \qquad \therefore \qquad \mathbf{V}_1 = 100\mathbf{I}_1$$

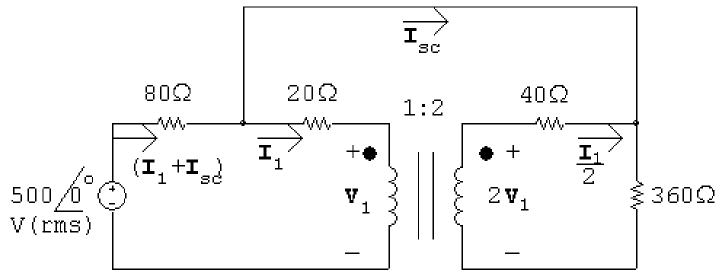
$$500 = 100\mathbf{I}_1 + 100\mathbf{I}_1 \quad \therefore \quad \mathbf{I}_1 = 500/200 = 2.5\,\text{A}$$

$$\therefore \quad \mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = 1.25\,\text{A}$$

$$\mathbf{V}_1 = 100(2.5) = 250\,\text{V}; \qquad \mathbf{V}_2 = 2\mathbf{V}_1 = 500\,\text{V}$$

$$V_{\text{Th}} = 20\mathbf{I}_1 + \mathbf{V}_1 - \mathbf{V}_2 + 40\mathbf{I}_2 = -150\,\text{V(rms)}$$

Short circuit current:



$$500 = 80(\mathbf{I}_{\rm sc} + \mathbf{I}_1) + 360(\mathbf{I}_{\rm sc} + 0.5\mathbf{I}_1)$$

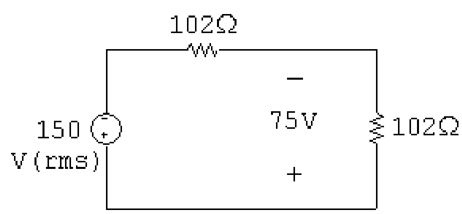
$$2\mathbf{V}_1 = 40\frac{\mathbf{I}_1}{2} + 360(\mathbf{I}_{\rm sc} + 0.5\mathbf{I}_1)$$

$$500 = 80(\mathbf{I}_1 + \mathbf{I}_{\rm sc}) + 20\mathbf{I}_1 + \mathbf{V}_1$$

Solving,

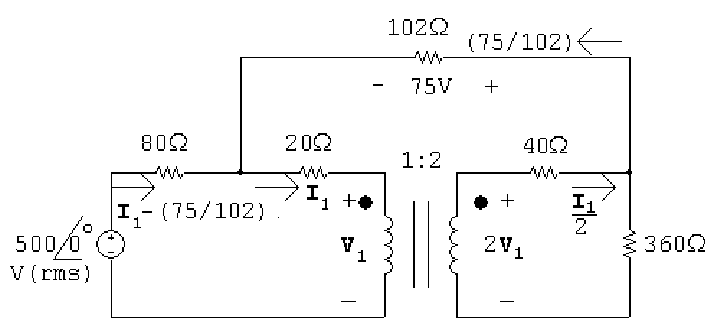
$$\mathbf{I}_{\rm sc} = -1.47\,\text{A}$$

$$R_{\rm Th} = \frac{\mathbf{V}_{\rm Th}}{\mathbf{I}_{\rm sc}} = \frac{-150}{-1.47} = 102\,\Omega$$



$$P = \frac{75^2}{102} = 55.15\,\text{W}$$

[**b**]



$$500 = 80[\mathbf{I}_1 - (75/102)] - 75 + 360[\mathbf{I}_2 - (75/102)]$$

$$575 + \frac{6000}{102} + \frac{27{,}000}{102} = 80\mathbf{I}_1 + 180\mathbf{I}_2$$

$$\therefore \qquad \mathbf{I}_1 = 3.456\,\text{A}$$

$$P_{\text{source}} = (500)[3.456 - (75/102)] = 1360.35\,\text{W}$$

$$\%\text{ delivered} = \frac{55.15}{1360.35}(100) = 4.05\%$$

[c] $P_{80\Omega} = 80(\mathbf{I}_1 + \mathbf{I}_L)^2 = 592.13\,\text{W}$

$$P_{20\Omega} = 20\mathbf{I}_1^2 = 238.86\,\text{W}$$

$$P_{40\Omega} = 40\mathbf{I}_2^2 = 119.43\,\text{W}$$

$$P_{102\Omega} = 102\mathbf{I}_L^2 = 55.15\,\text{W}$$

$$P_{360\Omega} = 360(\mathbf{I}_2 + \mathbf{I}_L)^2 = 354.73\,\text{W}$$

$$\sum P_{\text{abs}} = 592.13 + 238.86 + 119.43 + 55.15 + 354.73 = 1360.3\,\text{W} = \sum P_{\text{dev}}$$

P 10.67 **[a]** $\dfrac{30[5(44.28) + 19(15.77)]}{1000} = 15.63\text{ kWh}$

[b] $\dfrac{30[5(44.28) + 19(8.9)]}{1000} = 11.72\text{ kWh}$

[c] $\dfrac{30[5(44.28) + 19(4.42)]}{1000} = 9.16\text{ kWh}$

[d] $\dfrac{30[5(44.28) + 19(0)]}{1000} = 6.64\text{ kWh}$

Note that this is about 40 % of the amount of total power consumed in part (a).

P 10.68 **[a]** $\dfrac{30[0.2(1433) + 23.8(3.08)]}{1000} = 10.8\text{ kWh}$

[b] The standby power consumed in one month by the microwave oven when in the ready state is

$$\frac{30[23.8(3.08)]}{1000} = 2.2\text{ kWh}$$

This is $(2.2/10.8) * 100 = 20.4\%$ of the total power consumed by the microwave in one month. Since it is not practical to unplug the microwave when you are not using it, this is the cost associated with having a microwave oven.

P 10.69 $j\omega L_1 = j(2\pi)(60)(0.25) = j94.25\,\Omega$

$$\mathbf{I} = \frac{120}{5 + j94.25} = 1.27\underline{/-86.96^\circ}\text{ A(rms)}$$

$$P = R_1|\mathbf{I}|^2 = 5(1.27)^2 = 8.06\,\text{W}$$

P 10.70 $j\omega L_1 = j(2\pi)(60)(0.25) = j94.25\,\Omega$

$$\mathbf{I} = \frac{120}{0.05 + j94.25} = 1.27\underline{/-86.97^\circ}\ \text{A(rms)}$$

$$P = R_1|\mathbf{I}|^2 = 0.05(1.27)^2 = 81.1\,\text{mW}$$

Note that while the current supplied by the voltage source is virtually identical to that calculated in Problem 10.69, the much smaller value of transformer resistance results in a much smaller value of real power consumed by the transformer.

P 10.71 An ideal transformer has no resistance, so consumes no real power. This is one of the important characteristics of ideal transformers.