16 Fourier Series

Assessment Problems

AP 16.1

$$a_v = \frac{1}{T}\int_0^{2T/3} V_m\,dt + \frac{1}{T}\int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\,dt = \frac{7}{9}V_m = 7\pi\,\text{V}$$

$$a_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \cos\,k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\cos\,\text{k}\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\sin\left(\frac{4k\pi}{3}\right) = \left(\frac{6}{k}\right)\sin\left(\frac{4k\pi}{3}\right)$$

$$b_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \sin k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\sin k\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\left[1-\cos\left(\frac{4k\pi}{3}\right)\right] = \left(\frac{6}{k}\right)\left[1-\cos\left(\frac{4k\pi}{3}\right)\right]$$

AP 16.2 **[a]** $a_v = 7\pi = 21.99\,\text{V}$

[b] $a_1 = -5.196$ $\quad a_2 = 2.598$ $\quad a_3 = 0$ $\quad a_4 = -1.299$ $\quad a_5 = 1.039$

$b_1 = 9$ $\quad b_2 = 4.5$ $\quad b_3 = 0$ $\quad b_4 = 2.25$ $\quad b_5 = 1.8$

[c] $w_0 = \left(\frac{2\pi}{T}\right) = 50\,\text{rad/s}$

[d] $f_3 = 3f_0 = 23.87\,\text{Hz}$

[e] $v(t) = 21.99 - 5.2\cos 50t + 9\sin 50t + 2.6\sin 100t + 4.5\cos 100t$

$-1.3\sin 200t + 2.25\cos 200t + 1.04\sin 250t + 1.8\cos 250t + \cdots\,\text{V}$

AP 16.3 Odd function with both half- and quarter-wave symmetry.

$$v_g(t) = \left(\frac{6V_m}{T}\right)t, \qquad 0 \le t \le T/6; \qquad a_v = 0, \qquad a_k = 0 \quad \text{for all } k$$

$$b_k = 0 \quad \text{for } k \text{ even}$$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \qquad k \text{ odd}$$

$$= \frac{8}{T}\int_0^{T/6}\left(\frac{6V_m}{T}\right) t\sin k\omega_0 t\,dt + \frac{8}{T}\int_{T/6}^{T/4} V_m \sin k\omega_0 t\,dt$$

$$= \left(\frac{12V_m}{k^2\pi^2}\right)\sin\left(\frac{k\pi}{3}\right)$$

$$v_g(t) = \frac{12V_m}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{3}\sin n\omega_0 t\,\text{V}$$

AP 16.4 **[a]** $A_1 = -5.2 - j9 = 10.4\underline{/-120^\circ}$; $\qquad A_2 = 2.6 - j4.5 = 5.2\underline{/-60^\circ}$

$A_3 = 0$; $\qquad A_4 = -1.3 - j2.25 = 2.6\underline{/-120^\circ}$

$A_5 = 1.04 - j1.8 = 2.1\underline{/-60^\circ}$

$\theta_1 = -120^\circ$; $\qquad \theta_2 = -60^\circ$; $\qquad \theta_3$ not defined;

$\theta_4 = -120^\circ$; $\qquad \theta_5 = -60^\circ$

[b] $v(t) = 21.99 + 10.4\cos(50t - 120^\circ) + 5.2\cos(100t - 60^\circ)$
$+2.6\cos(200t - 120^\circ) + 2.1\cos(250t - 60^\circ) + \cdots\,\text{V}$

AP 16.5 The Fourier series for the input voltage is

$$v_i = \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n^2}\sin\frac{n\pi}{2}\right)\sin n\omega_0(t + T/4)$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n^2}\sin^2\frac{n\pi}{2}\right)\cos n\omega_0 t$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_0 t$$

$$\frac{8A}{\pi^2} = \frac{8(281.25\pi^2)}{\pi^2} = 2250\,\text{mV}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{200\pi}\times 10^3 = 10$$

$$\therefore \qquad v_i = 2250 \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} \cos 10nt \, \text{mV}$$

From the circuit we have

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + (1/j\omega C)} \cdot \frac{1}{j\omega C} = \frac{\mathbf{V}_i}{1 + j\omega RC}$$

$$\mathbf{V}_o = \frac{1/RC}{1/RC + j\omega}\mathbf{V}_i = \frac{100}{100 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 2250\underline{/0^\circ}\, \text{mV}; \qquad \omega_0 = 10 \text{ rad/s}$$

$$\mathbf{V}_{i3} = \frac{2250}{9}\underline{/0^\circ} = 250\underline{/0^\circ}\, \text{mV}; \qquad 3\omega_0 = 30 \text{ rad/s}$$

$$\mathbf{V}_{i5} = \frac{2250}{25}\underline{/0^\circ} = 90\underline{/0^\circ}\, \text{mV}; \qquad 5\omega_0 = 50 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{100}{100 + j10}(2250\underline{/0^\circ}) = 2238.83\underline{/-5.71^\circ}\, \text{mV}$$

$$\mathbf{V}_{o3} = \frac{100}{100 + j30}(250\underline{/0^\circ}) = 239.46\underline{/-16.70^\circ}\, \text{mV}$$

$$\mathbf{V}_{o5} = \frac{100}{100 + j50}(90\underline{/0^\circ}) = 80.50\underline{/-26.57^\circ}\, \text{mV}$$

$$\therefore \qquad v_o = 2238.33 \cos(10t - 5.71^\circ) + 239.46 \cos(30t - 16.70^\circ)$$

$$+80.50 \cos(50t - 26.57^\circ) + \ldots \text{ mV}$$

AP 16.6 **[a]** $\omega_o = \frac{2\pi}{T} = \frac{2\pi}{0.2\pi}(10^3) = 10^4$ rad/s

$$v_g(t) = 840 \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} \cos n10{,}000t \text{ V}$$

$$= 840 \cos 10{,}000t - 280 \cos 30{,}000t + 168 \cos 50{,}000t$$

$$- 120 \cos 70{,}000t + \cdots \text{ V}$$

$$\mathbf{V}_{g1} = 840\underline{/0^\circ}\text{ V}; \qquad \mathbf{V}_{g3} = 280\underline{/180^\circ}\text{ V}$$

$$\mathbf{V}_{g5} = 168\underline{/0^\circ}\text{ V}; \qquad \mathbf{V}_{g7} = 120\underline{/180^\circ}\text{ V}$$

$$H(s) = \frac{V_o}{V_g} = \frac{\beta s}{s^2 + \beta s + \omega_c^2}$$

$$\beta = \frac{1}{RC} = \frac{10^9}{10^4(20)} = 5000 \text{ rad/s}$$

$$\omega_c^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{400} = 25 \times 10^8$$

$$H(s) = \frac{5000s}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{j5000\omega}{25 \times 10^8 - \omega^2 + j5000\omega}$$

$$H_1 = \frac{j5 \times 10^7}{24 \times 10^8 + j5 \times 10^7} = 0.02\underline{/88.81^\circ}$$

$$H_3 = \frac{j15 \times 10^7}{16 \times 10^8 + j15 \times 10^7} = 0.09\underline{/84.64^\circ}$$

$$H_5 = \frac{j25 \times 10^7}{25 \times 10^7} = 1\underline{/0^\circ}$$

$$H_7 = \frac{j35 \times 10^7}{-24 \times 10^8 + j35 \times 10^7} = 0.14\underline{/-81.70^\circ}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{g1}H_1 = 17.50\underline{/88.81^\circ}\text{ V}$$

$$\mathbf{V}_{o3} = \mathbf{V}_{g3}H_3 = 26.14\underline{/-95.36^\circ}\text{ V}$$

$$\mathbf{V}_{o5} = \mathbf{V}_{g5}H_5 = 168\underline{/0^\circ}\text{ V}$$

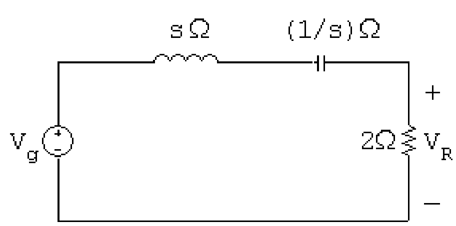
$$\mathbf{V}_{o7} = \mathbf{V}_{g7}H_7 = 17.32\underline{/98.30^\circ}\text{ V}$$

$$v_o = 17.50\cos(10{,}000t + 88.81^\circ) + 26.14\cos(30{,}000t - 95.36^\circ) + 168\cos(50{,}000t) + 17.32\cos(70{,}000t + 98.30^\circ) + \cdots \text{ V}$$

[b] The 5th harmonic because the circuit is a passive bandpass filter with a Q of 10 and a center frequency of 50 krad/s.

AP 16.7

$$w_0 = \frac{2\pi \times 10^3}{2094.4} = 3\,\text{rad/s}$$



$$j\omega_0 k = j3k$$

$$V_R = \frac{2}{2+s+1/s}(V_g) = \frac{2sV_g}{s^2+2s+1}$$

$$H(s) = \left(\frac{V_R}{V_g}\right) = \frac{2s}{s^2+2s+1}$$

$$H(j\omega_0 k) = H(j3k) = \frac{j6k}{(1-9k^2)+j6k}$$

$$v_{g_1} = 25.98 \sin \omega_0 t \text{ V}; \qquad V_{g_1} = 25.98\underline{/0^\circ} \text{ V}$$

$$H(j3) = \frac{j6}{-8+j6} = 0.6\underline{/-53.13^\circ}; \qquad V_{R_1} = 15.588\underline{/-53.13^\circ} \text{ V}$$

$$P_1 = \frac{(15.588/\sqrt{2})^2}{2} = 60.75 \text{ W}$$

$$v_{g_3} = 0, \quad \text{therefore} \quad P_3 = 0 \text{ W}$$

$$v_{g_5} = -1.04 \sin 5\omega_0 t \text{ V}; \qquad V_{g_5} = 1.04\underline{/180^\circ}$$

$$H(j15) = \frac{j30}{-224+j30} = 0.1327\underline{/-82.37^\circ}$$

$$V_{R_5} = (1.04\underline{/180^\circ})(0.1327\underline{/-82.37^\circ}) = 138\underline{/97.63^\circ} \text{ mV}$$

$$P_5 = \frac{(0.1396/\sqrt{2})^2}{2} = 4.76 \text{ mW}; \qquad \text{therefore} \quad P \cong P_1 \cong 60.75 \text{ W}$$

AP 16.8 Odd function with half- and quarter-wave symmetry, therefore $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for k even; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/8} 2 \sin \text{k}\omega_0 t\, dt + \frac{8}{T}\int_{T/8}^{T/4} 8 \sin k\omega_0 t\, dt$$
$$= \left(\frac{8}{\pi k}\right)\left[1 + 3\cos\left(\frac{k\pi}{4}\right)\right], \quad k \text{ odd}$$

$$\text{Therefore} \quad C_n = \left(\frac{-j4}{n\pi}\right)\left[1 + 3\cos\left(\frac{n\pi}{4}\right)\right], \quad n \text{ odd}$$

AP 16.9 **[a]** $I_{\text{rms}} = \sqrt{\frac{2}{T}\left[(2)^2\left(\frac{T}{8}\right)(2) + (8)^2\left(\frac{3T}{8} - \frac{T}{8}\right)\right]} = \sqrt{34} = 5.7683 \text{ A}$

[b] $C_1 = \dfrac{-j12.5}{\pi}; \quad C_3 = \dfrac{j1.5}{\pi}; \quad C_5 = \dfrac{j0.9}{\pi};$

$C_7 = \dfrac{-j1.8}{\pi}; \quad C_9 = \dfrac{-j1.4}{\pi}; \quad C_{11} = \dfrac{j0.4}{\pi}$

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5,\ldots}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}\,(12.5^2 + 1.5^2 + 0.9^2 + 1.8^2 + 1.4^2 + 0.4^2)}$$

$$\cong 5.777\,\text{A}$$

[c] $\%\ \text{Error} = \dfrac{5.777 - 5.831}{5.831} \times 100 = -0.93\%$

[d] Using just the terms $C_1 - C_9$,

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5,\ldots}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}\,(12.5^2 + 1.5^2 + 0.9^2 + 1.8^2 + 1.4^2)}$$

$$\cong 5.774\,\text{A}$$

$$\%\ \text{Error} = \frac{5.774 - 5.831}{5.831} \times 100 = -0.98\%$$

Thus, the % error is still less than 1%.

AP 16.10 $T = 32\,\text{ms}$, therefore 8 ms requires shifting the function $T/4$ to the right.

$$i = \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} -j\frac{4}{n\pi}\left(1 + 3\cos\frac{n\pi}{4}\right)e^{jn\omega_0(t-T/4)}$$

$$= \frac{4}{\pi}\sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} \frac{1}{n}\left(1 + 3\cos\frac{n\pi}{4}\right)e^{-j(n+1)(\pi/2)}e^{jn\omega_0 t}$$

Problems

P 16.1 [a] $\omega_{\rm oa} = \dfrac{2\pi}{8\times10^{-3}} = 785.4$ rad/s

$$\omega_{\rm ob} = \frac{2\pi}{80\times10^{-3}} = 78.54 \text{ krad/s}$$

[b] $f_{\rm oa} = \dfrac{1}{T} = \dfrac{1}{8\times10^{-3}} = 125\,\text{Hz}; \qquad f_{\rm ob} = \dfrac{1}{80\times10^{-3}} = 12.5\,\text{Hz}$

[c] $a_{\rm va} = \dfrac{50(4\times10^{-3})}{8\times10^{-3}} = 25\,\text{V}; \qquad a_{\rm vb} = 0$

[d] The periodic function in Fig. P16.1(a):

$$a_{\rm va} = 25\,\text{V}$$

$$a_{\rm ka} = \frac{2}{T}\int_{-T/4}^{T/4} 50\cos\frac{2\pi kt}{T}\,dt$$

$$= \frac{100}{T}\frac{T}{2\pi k}\sin\frac{2\pi k}{T}t\,\Big|_{-T/4}^{T/4}$$

$$= \frac{100}{\pi k}\sin\frac{\pi k}{2}$$

$$b_{\rm ka} = \frac{2}{T}\int_{-T/4}^{T/4} 50\sin\frac{2\pi kt}{T}\,dt$$

$$= \frac{-100}{T}\frac{T}{2\pi k}\cos\frac{2\pi k}{T}t\,\Big|_{-T/4}^{T/4}$$

$$= 0$$

The periodic function in Fig. P16.1(b):

$$a_{\rm vb} = 0$$

$$a_{\rm kb} = \frac{2}{T}\left[\int_0^{T/4} 90\cos\frac{2\pi kt}{T}\,dt + \int_{T/4}^{T/2} 30\cos\frac{2\pi kt}{T}\,dt\right]$$

$$-\frac{2}{T}\left[\int_{T/2}^{3T/4} 90\cos\frac{2\pi kt}{T}\,dt + \int_{3T/4}^{T} 30\cos\frac{2\pi kt}{T}\,dt\right]$$

$$= \frac{60}{T}\frac{T}{2\pi k}\left[3\sin\frac{2\pi kt}{T}\,\Big|_0^{T/4} + \sin\frac{2\pi kt}{T}\,\Big|_{T/4}^{T/2}\right]$$

$$-\frac{60}{T}\frac{T}{2\pi k}\left[3\sin\frac{2\pi kt}{T}\,\Big|_{T/2}^{3T/4} + \sin\frac{2\pi kt}{T}\,\Big|_{3T/4}^{T}\right]$$

$$= \frac{30}{\pi k}\left[2\sin\frac{\pi k}{2} - 2\sin\frac{3\pi k}{2}\right] = \frac{120}{\pi k}\sin\frac{\pi k}{2}$$

Note that $a_{\rm kb}$ is 0 for even values of k.

$$b_{\rm kb} = \frac{2}{T}\left[\int_0^{T/4} 90\sin\frac{2\pi kt}{T}\,dt + \int_{T/4}^{T/2} 30\sin\frac{2\pi kt}{T}\,dt\right]$$

$$-\frac{2}{T}\left[\int_{T/2}^{3T/4} 90\sin\frac{2\pi kt}{T}\,dt + \int_{3T/4}^{T} 30\sin\frac{2\pi kt}{T}\,dt\right]$$

$$= \frac{-60}{T}\frac{T}{2\pi k}\left[3\cos\frac{2\pi kt}{T}\Big|_0^{T/4} + \cos\frac{2\pi kt}{T}\Big|_{T/4}^{T/2}\right]$$

$$-\frac{-60}{T}\frac{T}{2\pi k}\left[3\cos\frac{2\pi kt}{T}\Big|_{T/2}^{3T/4} + \cos\frac{2\pi kt}{T}\Big|_{3T/4}^{T}\right]$$

$$= \frac{120}{\pi k}[1 - \cos(k\pi)]$$

Note that $b_{\rm kb}$ is 0 for even values of k and equal to $120(2)/k\pi$ for odd values of k.

[e] For the periodic function in Fig. P16.1(a),

$$v(t) = 25 + \frac{100}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\cos n\omega_o t\right)\text{ V}$$

For the periodic function in Fig. P16.1(b),

$$v(t) = \frac{120}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\left(\sin\frac{n\pi}{2}\cos n\omega_o t + 2\sin n\omega_o t\right)\text{ V}$$

P 16.2 **[a]** Odd function with half- and quarter-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for even k; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/4} V_m\sin k\omega_0 t\,dt = \frac{4V_m}{k\pi}, \qquad k\text{ odd}$$

$$\text{and}\quad v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin n\omega_0 t\text{ V}$$

[b] Even function: $b_k = 0$ for k

$$a_v = \frac{2}{T}\int_0^{T/2} V_m\sin\frac{\pi}{T}t\,dt = \frac{2V_m}{\pi}$$

$$a_k = \frac{4}{T}\int_0^{T/2} V_m\sin\frac{\pi}{T}t\cos k\omega_0 t\,dt = \frac{2V_m}{\pi}\left(\frac{1}{1-2k} + \frac{1}{1+2k}\right)$$

$$= \frac{4V_m/\pi}{1-4k^2}$$

and $\quad v(t) = \frac{2V_m}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\frac{1}{1-4n^2}\cos n\omega_0 t\right]\text{V}$

[c] $a_v = \frac{1}{T}\int_0^{T/2} V_m \sin\left(\frac{2\pi}{T}\right)t\,dt = \frac{V_m}{\pi}$

$$a_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\cos k\omega_0 t\,dt = \frac{V_m}{\pi}\left(\frac{1+\cos k\pi}{1-k^2}\right)$$

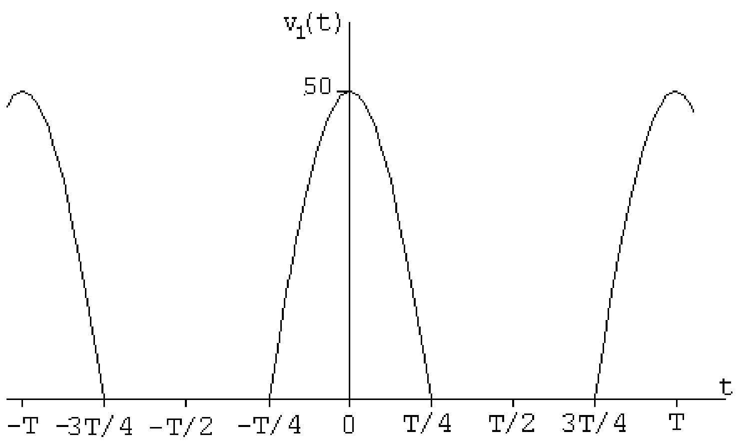
Note: $\quad a_k = 0 \quad$ for k-odd, $\quad a_k = \frac{2V_m}{\pi(1-k^2)} \quad$ for k even,

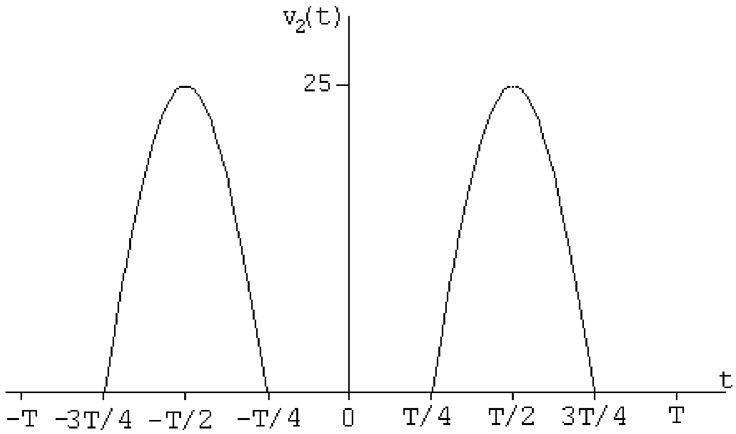
$$b_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\sin k\omega_0 t\,dt = 0 \quad \text{for} \quad k = 2, 3, 4, \ldots$$

For $k = 1$, we have $\quad b_1 = \frac{V_m}{2}; \qquad$ therefore

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega_0 t + \frac{2V_m}{\pi}\sum_{n=2,4,6,\ldots}^{\infty}\frac{1}{1-n^2}\cos n\omega_0 t\,\text{V}$$

P 16.3 In studying the periodic function in Fig. P16.3 note that it can be visualized as the combination of two half-wave rectified sine waves, as shown in the figure below. Hence we can use the Fourier series for a half-wave rectified sine wave which is given as the answer to Problem 16.2(c).





In using the previously derived Fourier series for the half-wave rectified sine wave we note $v_1(t)$ has been shifted $T/4$ units to the left and $v_2(t)$ has been shifted $T/4$ units to the right. Thus,

$$v_1(t) = \frac{50}{\pi} + 25\sin\omega_o(t + T/4) - \frac{100}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos n\omega_o(t + T/4)}{(1 - n^2)}\ \text{V}$$

Now observe the following:

$$\sin\omega_o(t + T/4) = \sin(\omega_o t + \pi/2) = \cos\omega_o t$$

$$\cos n\omega_o(t + T/4) = \cos(n\omega_o t + n\pi/2) = \cos\frac{n\pi}{2}\cos n\omega_o t \quad \text{because } n \text{ is even.}$$

$$\therefore \quad v_1(t) = \frac{50}{\pi} + 25\cos\omega_o t - \frac{100}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(1 - n^2)}\ \text{V}$$

Also,

$$v_2(t) = \frac{25}{\pi} + 12.5\sin\omega_o(t - T/4) - \frac{50}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos n\omega_o(t - T/4)}{(1 - n^2)}\ \text{V}$$

Again, observe the following:

$$\sin(\omega_o t - \pi/2) = -\cos\omega_o t$$

$$\cos(n\omega_o t - n\pi/2) = \cos(n\pi/2)\cos n\omega_o t \quad \text{because } n \text{ is even.}$$

$$\therefore \quad v_2(t) = \frac{25}{\pi} - 12.5\cos\omega_o t - \frac{50}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(1 - n^2)}\ \text{V}$$

Thus: $\quad v = v_1 + v_2$

$$\therefore \quad v(t) = \frac{75}{\pi} + 12.5\cos\omega_o t - \frac{150}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos(n\pi/2)\cos(n\omega_o t)}{(1 - n^2)}\ \text{V}$$

P 16.4 $a_{\text{v}} = \frac{1}{T}\int_0^{T/4} V_m\,dt + \frac{1}{T}\int_{T/4}^{T} \frac{V_m}{2}\,dt = \frac{5}{8}V_m = 62.5\pi\ \text{V}$

$$a_{\text{k}} = \frac{2}{T}\left[\int_0^{T/4} V_m \cos k\omega_0 t\,dt + \int_{T/4}^{T} \frac{V_m}{2}\cos k\omega_0 t\,dt\right]$$

$$= \frac{V_m}{k\omega_0 T}\sin\frac{k\pi}{2} = \frac{50}{k}\sin\frac{k\pi}{2}$$

$$b_{\text{k}} = \frac{2}{T}\left[\int_0^{T/4} V_m \sin k\omega_0 t\,dt + \int_{T/4}^{T} \frac{V_m}{2}\sin k\omega_0 t\,dt\right]$$

$$= \frac{V_m}{k\omega_0 T}\left[1 - \cos\frac{k\pi}{2}\right] = \frac{50}{k}\left[1 - \cos\frac{k\pi}{2}\right]$$

P 16.5 **[a]** $I_6 = \int_{t_o}^{t_o+T} \sin m\omega_0 t\,dt = -\frac{1}{m\omega_0}\cos m\omega_0 t\,\Big|_{t_o}^{t_o+T}$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0(t_o+T) - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o \cos m\omega_0 T - \sin m\omega_0 t_o \sin m\omega_0 T - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o - 0 - \cos m\omega_0 t_o] = 0 \quad \text{for all } m,$$

$$I_7 = \int_{t_o}^{t_o+T} \cos m\omega_0 t_o\,dt = \frac{1}{m\omega_0}[\sin m\omega_0 t]\,\Big|_{t_o}^{t_o+T}$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0(t_o+T) - \sin m\omega_0 t_o]$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0 t_o - \sin m\omega_0 t_o] = 0 \quad \text{for all } m$$

[b] $I_8 = \int_{t_o}^{t_o+T} \cos m\omega_0 t \sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T}[\sin(m+n)\omega_0 t - \sin(m-n)\omega_0 t]\,dt$

But $(m+n)$ and $(m-n)$ are integers, therefore from I_6 above, $I_8 = 0$ for all m, n.

[c] $I_9 = \int_{t_o}^{t_o+T} \sin m\omega_0 t \sin n\omega_0 t\,dt = \frac{1}{2}\int_{t_o}^{t_o+T}[\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t]\,dt$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we get

$$I_9 = \frac{1}{2}\int_{t_o}^{t_o+T} dt - \frac{1}{2}\int_{t_o}^{t_o+T}\cos 2m\omega_0 t\,dt = \frac{T}{2} - 0 = \frac{T}{2}$$

[d] $I_{10} = \int_{t_o}^{t_o+T} \cos m\omega_0 t \cos n\omega_0 t\, dt$

$$= \frac{1}{2}\int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t + \cos(m+n)\omega_0 t]\, dt$$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we have

$$I_{10} = \frac{1}{2}\int_{t_o}^{t_o+T} dt + \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\, dt = \frac{T}{2} + 0 = \frac{T}{2}$$

P 16.6 $f(t)\sin k\omega_0 t = a_v \sin k\omega_0 t + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t \sin k\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \sin k\omega_0 t$

Now integrate both sides from t_o to $t_o + T$. All the integrals on the right-hand side reduce to zero except in the last summation when $n = k$, therefore we have

$$\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t\, dt = 0 + 0 + b_k\left(\frac{T}{2}\right) \quad \text{or} \quad b_k = \frac{2}{T}\int_{t_o}^{t_o+T} f(t)\sin k\omega_0 t\, dt$$

P 16.7 $a_v = \frac{1}{T}\int_{t_o}^{t_o+T} f(t)\, dt = \frac{1}{T}\left\{\int_{-T/2}^{0} f(t)\, dt + \int_{0}^{T/2} f(t)\, dt\right\}$

Let $t = -x, \quad dt = -dx, \quad x = \frac{T}{2} \quad \text{when} \quad t = \frac{-T}{2}$

and $x = 0$ when $t = 0$

Therefore $\frac{1}{T}\int_{-T/2}^{0} f(t)\, dt = \frac{1}{T}\int_{T/2}^{0} f(-x)(-dx) = -\frac{1}{T}\int_{0}^{T/2} f(x)\, dx$

Therefore $a_v = -\frac{1}{T}\int_{0}^{T/2} f(t)\, dt + \frac{1}{T}\int_{0}^{T/2} f(t)\, dt = 0$

$$a_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\, dt + \frac{2}{T}\int_{0}^{T/2} f(t)\cos k\omega_0 t\, dt$$

Again, let $t = -x$ in the first integral and we get

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\, dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\cos k\omega_0 x\, dx$$

Therefore $a_k = 0$ for all k.

$$b_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\, dt + \frac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\, dt$$

Using the substitution $t = -x$, the first integral becomes

$$\frac{2}{T}\int_0^{T/2} f(x)\sin k\omega_0 x\, dx$$

Therefore we have $\quad b_k = \dfrac{4}{T}\displaystyle\int_0^{T/2} f(t)\sin k\omega_0 t\, dt$

P 16.8 $\quad b_k = \dfrac{2}{T}\displaystyle\int_{-T/2}^{0} f(t)\sin k\omega_0 t\, dt + \dfrac{2}{T}\displaystyle\int_0^{T/2} f(t)\sin k\omega_0 t\, dt$

Now let $t = x - T/2$ in the first integral, then $dt = dx$, $x = 0$ when $t = -T/2$ and $x = T/2$ when $t = 0$, also
$\sin k\omega_0(x - T/2) = \sin(k\omega_0 x - k\pi) = \sin k\omega_0 x \cos k\pi$. Therefore

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\, dt = -\frac{2}{T}\int_0^{T/2} f(x)\sin k\omega_0 x \cos k\pi\, dx \quad \text{and}$$

$$b_k = \frac{2}{T}(1 - \cos k\pi)\int_0^{T/2} f(x)\sin k\omega_0 t\, dt$$

Now note that $1 - \cos k\pi = 0$ when k is even, and $1 - \cos k\pi = 2$ when k is odd. Therefore $b_k = 0$ when k is even, and

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_0 t\, dt \quad \text{when } k \text{ is odd}$$

P 16.9 Because the function is even and has half-wave symmetry, we have $a_v = 0$, $a_k = 0$ for k even, $b_k = 0$ for all k and

$$a_k = \frac{4}{T}\int_0^{T/2} f(t)\cos k\omega_0 t\, dt, \qquad k \text{ odd}$$

The function also has quarter-wave symmetry;
therefore $f(t) = -f(T/2 - t)$ in the interval $T/4 \le t \le T/2$;
thus we write

$$a_k = \frac{4}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\, dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\, dt$$

Now let $t = (T/2 - x)$ in the second integral, then $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$. Therefore we get

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\, dt = -\frac{4}{T}\int_0^{T/4} f(x)\cos k\pi \cos k\omega_0 x\, dx$$

Therefore we have

$$a_k = \frac{4}{T}(1 - \cos k\pi)\int_0^{T/4} f(t)\cos k\omega_0 t\, dt$$

But k is odd, hence

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\, dt, \qquad k \text{ odd}$$

P 16.10 Because the function is odd and has half-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, and $b_k = 0$ for k even. For k odd we have

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_0 t\, dt$$

The function also has quarter-wave symmetry, therefore $f(t) = f(T/2 - t)$ in the interval $T/4 \le t \le T/2$. Thus we have

$$b_k = \frac{4}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\, dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\, dt$$

Now let $t = (T/2 - x)$ in the second integral and note that $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$, thus

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\, dt = -\frac{4}{T}\cos k\pi \int_0^{T/4} f(x)(\sin k\omega_0 x)\, dx$$

But k is odd, therefore the expression for b_k becomes

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\, dt$$

P 16.11 **[a]** $\omega_o = \dfrac{2\pi}{T} = \pi$ rad/s

[b] yes

[c] no

[d] yes

P 16.12 **[a]** $f = \dfrac{1}{T} = \dfrac{1}{40 \times 10^{-3}} = 25\,\text{Hz}$

[b] no

[c] yes

[d] yes

[e] yes

[f] $a_v = 0,\qquad$ function is odd

$a_k = 0,\qquad$ for all k; the function is odd

$b_k = 0,\qquad$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t,\qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} -8t\sin k\omega_o t\,dt + \int_{T/8}^{T/4} -0.04\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = -8\int_0^{T/8} t\sin k\omega_o t\,dt$$

$$= -8\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/8}\right]$$

$$= \frac{-8}{k^2\omega_o^2}\sin\frac{k\pi}{4} + \frac{T}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int2} = -0.04\int_{T/8}^{T/4}\sin k\omega_o t\,dt = \frac{0.04}{k\omega_o}\cos k\omega_o t\,\Big|_{T/8}^{T/4} = \frac{-0.04}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int1} + \text{Int2} = \frac{-8}{k^2\omega_o^2}\sin\frac{k\pi}{4} + \left(\frac{-0.04}{k\omega_o} + \frac{T}{k\omega_o}\right)\cos\frac{k\pi}{4}$$

$T = 0.04\,\text{s}$

$$\therefore\quad \text{Int1} + \text{Int2} = \frac{-8}{k^2\omega_o^2}\sin\frac{k\pi}{4}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{-8}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{4} = \frac{-0.64}{\pi^2k^2}\sin\frac{k\pi}{4},\qquad k \text{ odd}$$

$$i(t) = \frac{-640}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\pi/4)}{n^2}\sin n\omega_o t\,\text{mA}$$

P 16.13 **[a]** $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for all k, $a_k = 0$ for k-even; for odd k we have

$$a_k = \frac{8}{T}\int_0^{T/4} V_m\cos k\omega_0 t\,dt = \frac{4V_m}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

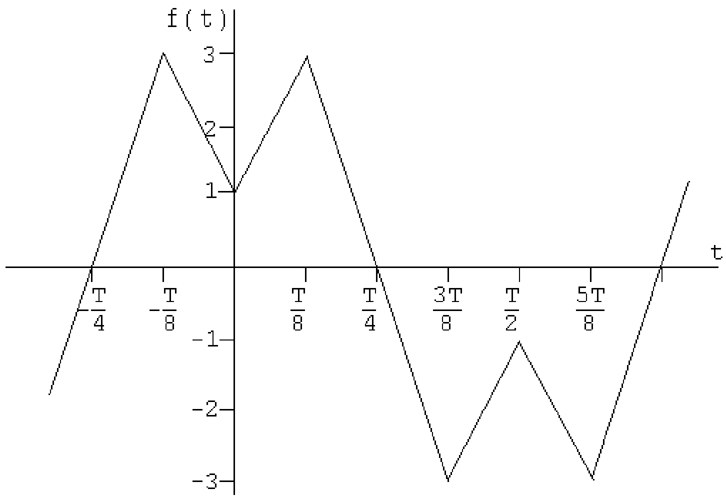
$$v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t\,\text{V}$$

[b] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for k-even, $a_k = 0$ for all k; for k-odd we have

$$a_k = \frac{8}{T}\int_0^{T/4}\left(\frac{4V_p}{T}t - V_p\right)\cos k\omega_0 t\,dt = \frac{-8V_p}{\pi^2 k^2}$$

$$\text{Therefore}\quad v(t) = \frac{-8V_p}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_0 t\ \text{V}$$

P 16.14 **[a]**



[b] $a_v = 0$; $\qquad a_k = 0$ for all k even; $\qquad b_k = 0$ for all k

$$\text{For } k \text{ odd,}\qquad a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_o t\,dt$$

$$a_k = \frac{8}{T}\int_0^{T/8}\left(1 + \frac{16t}{T}\right)\cos k\omega_o t\,dt + \frac{8}{T}\int_{T/8}^{T/4}\left(6 - \frac{24t}{T}\right)\cos k\omega_o t\,dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{8}{T}\int_0^{T/8}\cos k\omega_o t\,dt + \frac{128}{T^2}\int_0^{T/8} t\cos k\omega_o t\,dt$$

$$= \frac{8}{T}\frac{\sin k\omega_o t}{k\omega_o}\Big|_0^{T/8} + \frac{128}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o}\sin k\omega_o t\right]_0^{T/8}$$

$$k\omega_o T = 2k\pi;\qquad (k\omega_o T)^2 = 4k^2\pi^2$$

$$\text{Int1} = \frac{12}{k\pi}\sin\frac{k\pi}{4} + \frac{32}{k^2\pi^2}\left[\cos\left(\frac{k\pi}{4}\right) - 1\right]\qquad k \text{ odd}$$

$$\text{Int2} = \frac{48}{T}\int_{T/8}^{T/4}\cos k\omega_o t\,dt - \frac{192}{T^2}\int_{T/8}^{T/4} t\cos k\omega_o t\,dt$$

$$= \frac{48}{T}\frac{\sin k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4} - \frac{192}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o}\sin k\omega_o t\right]_{T/8}^{T/4}$$

$$\text{Int2} = \frac{-12}{k\pi}\sin\frac{k\pi}{4} + \frac{48}{k^2\pi^2}\cos\frac{k\pi}{4} \qquad k \text{ odd}$$

$$a_k = \text{Int1} + \text{Int2}$$
$$= \frac{80}{k^2\pi^2}\cos\frac{k\pi}{4} - \frac{32}{k^2\pi^2}$$

[c] $a_1 = \frac{80}{\pi^2}\cos\frac{\pi}{4} - \frac{32}{\pi^2} = 2.489$

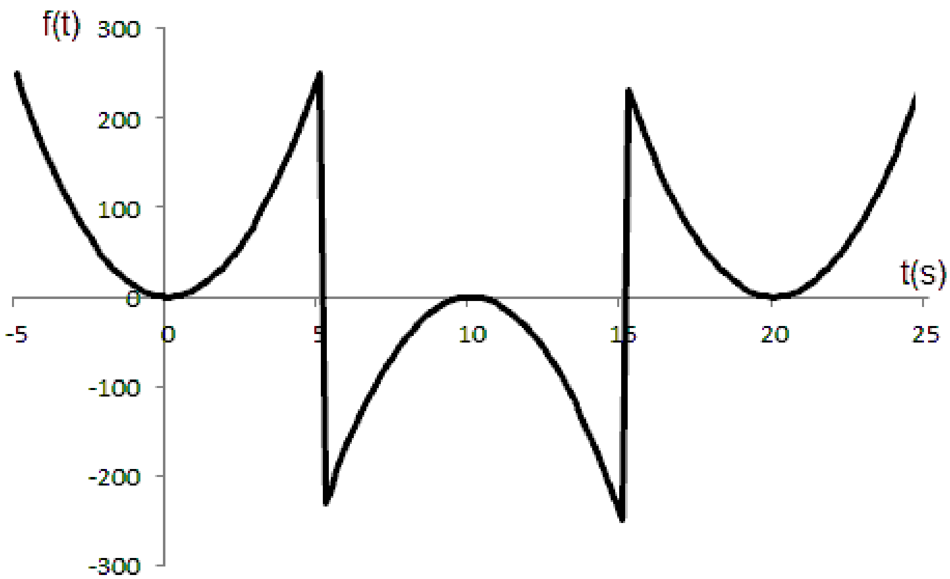
$$a_3 = \frac{80}{9\pi^2}\cos\frac{3\pi}{4} - \frac{32}{9\pi^2} = -0.9971$$

$$a_5 = \frac{80}{25\pi^2}\cos\frac{5\pi}{4} - \frac{32}{25\pi^2} = -0.359$$

$$f(t) = 2.489\cos\omega_o t - 0.9971\cos 3\omega_o t - 0.359\cos 5\omega_o t - \cdots$$

[d] $f(T/8) = 2.489\cos(\pi/4) - 0.9971\cos(3\pi/4) - 0.359\cos(5\pi/4) = 2.719$

P 16.15 [a]



[b] Even, since $f(t) = f(-t)$

[c] Yes, since $f(t) = -f(T/2 - t)$ in the interval $0 < t < 10$.

[d] $a_v = 0, \quad a_k = 0, \quad$ for k even (half-wave symmetry)

$b_k = 0, \quad$ for all k (function is even)

Because of the quarter-wave symmetry, the expression for a_k is

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt, \quad k \text{ odd}$$

$$= \frac{8}{20}\int_0^5 10t^2\cos k\omega_0 t\,dt = 4\left[\frac{2t}{k^2\omega_0^2}\cos k\omega_0 t + \frac{k^2\omega_0^2 t^2 - 2}{k^3\omega_0^3}\sin k\omega_0 t\right]_0^5$$

$$k\omega_0(5) = k\left(\frac{2\pi}{20}\right)(5) = \frac{k\pi}{2}$$

$$\cos(k\pi/2) = 0, \quad \text{since } k \text{ is odd}$$

$$\therefore \qquad a_k = \frac{2}{5}\left[0 + \frac{(k^2\pi^2/4) - 2}{k^3\omega_0^3}\sin(k\pi/2)\right] = \frac{k^2\omega_0^2 - 8}{10k^3\omega_0^3}\sin(k\pi/2)$$

$$\omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}; \qquad \omega_0^2 = \frac{\pi^2}{100}; \qquad \omega_0^3 = \frac{\pi^3}{1000}$$

$$a_k = \left(\frac{k^2\pi^2 - 800}{k^3\pi^3}\right)\sin(k\pi/2)$$

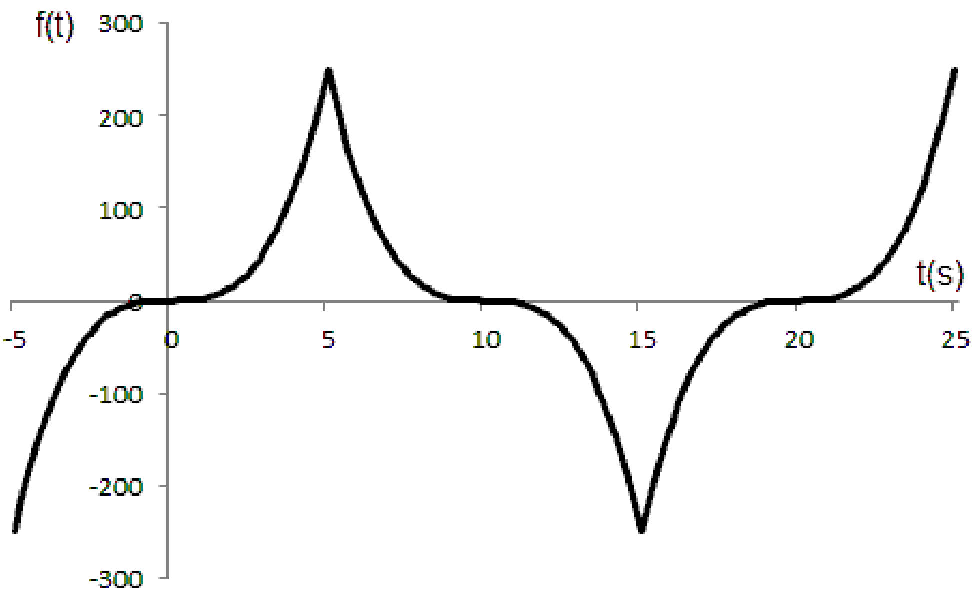
$$f(t) = \sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 800}{\pi^3 n^3}\right]\sin(n\pi/2)\cos(n\omega_0 t)$$

[e] $\cos n\omega_0(t-5) = \cos(n\omega_0 t - n\pi/2) = \sin(n\pi/2)\sin n\omega_0 t$

$$f(t) = \sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 800}{\pi^3 n^3}\right]\sin^2(n\pi/2)\sin(n\omega_0 t)$$

$$= \sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{n^2\pi^2 - 800}{\pi^3 n^3}\right]\sin(n\omega_0 t)$$

P 16.16 **[a]**



[b] Odd, since $f(-t) = -f(t)$

[c] $f(t)$ has quarter-wave symmetry, since $f(T/2 - t) = f(t)$ in the interval $0 < t < 4$.

[d] $a_v = 0$, (half-wave symmetry); $a_k = 0$, for all k (function is odd)

$b_k = 0$, for k even (half-wave symmetry)

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \quad k \text{ odd}$$

$$= \frac{16}{20}\int_0^5 t^3 \sin k\omega_0 t\,dt$$

$$= \frac{4}{5}\left[\frac{3t^2}{k^2\omega_0^2}\sin k\omega_0 t - \frac{6}{k^4\omega_0^4}\sin k\omega_0 t - \frac{t^3}{k\omega_0}\cos k\omega_0 t + \frac{6t}{k^3\omega_0^3}\cos k\omega_0 t\right]_0^5$$

$$k\omega_0(5) = k\left(\frac{2\pi}{20}\right)(5) = \frac{k\pi}{2}$$

$\cos(k\pi/2) = 0$, since k is odd

$$\therefore \quad b_k = \left[\frac{60}{k^2\omega_0^2}\sin(k\pi/2) - \frac{4.8}{k^4\omega_0^4}\sin(k\pi/2)\right]$$

$$k\omega_0 = k\left(\frac{2\pi}{20}\right) = \frac{k\pi}{10}; \qquad k^2\omega_0^2 = \frac{k^2\pi^2}{100}; \qquad k^4\omega_0^4 = \frac{k^4\pi^4}{10{,}000}$$

$$\therefore \quad b_k = \frac{6000}{\pi^2 k^2}\left[1 - \frac{8}{\pi^2 k^2}\right]\sin(k\pi/2), \quad k \text{ odd}$$

$$f(t) = \frac{6000}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n^2}\left(1 - \frac{8}{\pi^2 n^2}\right)\sin(n\pi/2)\right]\sin n\omega_0 t$$

[e] $\sin n\omega_0(t-2) = \sin(n\omega_0 t - n\pi/2) = -\cos n\omega_0 t \sin(n\pi/2)$

$$f(t) = \frac{-6000}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\left[\frac{1}{n^2}\left(1 - \frac{8}{\pi^2 n^2}\right)\right]\cos n\omega_0 t$$

P 16.17 **[a]** $i(t)$ is odd, therefore $a_v = 0$ and $b_k = 0$ for all k.

$$f(t) = i(t) = I_m - \frac{2I_m}{T}t, \quad 0 \le t \le T$$

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_o t\,dt$$

$$= \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\sin k\omega_o t\,dt$$

$$= \frac{4I_m}{T}\left[\int_0^{T/2}\sin k\omega_0 t\,dt - \frac{2}{T}\int_0^{T/2} t\sin k\omega_0 t\,dt\right]$$

$$= \frac{4I_m}{T}\left[\frac{-\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2} - \frac{2}{T}\left(\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t\cos k\omega_0 t}{k\omega_0}\right)\Big|_0^{T/2}\right]$$

$$= \frac{4I_m}{T}\left[\frac{1-\cos k\pi}{k\omega_0} + \frac{\cos k\pi}{k\omega_0}\right]$$

$$= \frac{4I_m}{k\omega_0 T} = \frac{2I_m}{k\pi}$$

$$\therefore \quad i(t) = \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_o t$$

[b] $i(t) = \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_o(t+T/2)$

$$= \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}\sin n\omega_o t$$

P 16.18 $v_2(t+T/8)$ is even, so $b_k = 0$ for all k.

$$a_v = \frac{(V_m/2)(T/4)}{T} = \frac{V_m}{8}$$

$$a_k = \frac{4}{T}\int_0^{T/8}\frac{V_m}{2}\cos k\omega_0 t\,dt = \frac{V_m}{k\pi}\sin\frac{k\pi}{4}$$

Therefore, $\quad v_2(t+T/8) = \frac{V_m}{8} + \frac{V_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_0 t$

so $\quad v_2(t) = \frac{V_m}{8} + \frac{V_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_0(t-T/8)$

$$\therefore \qquad v(t) = \frac{V_m}{2} + \frac{V_m}{8} + \frac{V_m}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{4}\cos\frac{n\pi}{4}\right)\cos n\omega_0 t + \left(\frac{1}{n}\sin^2\frac{n\pi}{4}\right)\sin n\omega_0 t$$

$$= \frac{5V_m}{8} + \frac{V_m}{2\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos n\omega_0 t + \left(1-\cos\frac{n\pi}{2}\right)\sin n\omega_0 t\,\mathrm{V}$$

Thus, since $a_v = 5V_m/8 = 37.5\pi\,\mathrm{V}$,

$$a_k = \frac{V_m}{2\pi k}\sin\frac{k\pi}{2} = \frac{30}{k}\sin\frac{k\pi}{2}$$

and

$$b_k = \frac{V_m}{2\pi k}\left[1-\cos\frac{k\pi}{2}\right] = \frac{30}{k}\left[1-\cos\frac{k\pi}{2}\right]$$

These equations match the equations for a_v, a_k, and b_k derived in Problem 16.4.

P 16.19 From Problem 16.1(a),

$$a_v = 25\,\text{V} = A_0$$

$$a_n = \frac{100}{n\pi}\sin\frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{100}{n\pi}\sin\frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

Thus, $v(t) = 25 + \dfrac{100}{\pi}\sum_{n=1}^{\infty}\dfrac{1}{n}\sin\dfrac{n\pi}{2}\cos n\omega_o t\,\text{V}$

For the periodic function in Fig. P16.1(b):

$$a_n = \frac{120}{\pi n}\sin\frac{\pi n}{2} \qquad \text{and} \qquad b_n = \frac{240}{\pi n} \quad \text{for } n \text{ odd.}$$

$$A_n\underline{/-\theta_n} = a_n - jb_n = \frac{120}{\pi n}\sin\frac{\pi n}{2} - j\frac{240}{\pi n}, \qquad n \text{ odd}$$

Therefore,

$$A_n = \frac{120\sqrt{5}}{n\pi}, \qquad n \text{ odd}$$

and

$$\theta_n = \tan^{-1}(-240/120) = -63.43^\circ, \quad n = 1, 5, 9, \ldots$$

and

$$\theta_n = \tan^{-1}(-240/-120) = 63.43^\circ, \quad n = 3, 7, 11, \ldots$$

Thus, $v(t) = \dfrac{120\sqrt{5}}{\pi}\sum_{n=1,5,9,\ldots}^{\infty}\dfrac{1}{n}\cos(n\omega_o t - 63.43^\circ)$

$$+\frac{120\sqrt{5}}{\pi}\sum_{n=3,7,11,\ldots}^{\infty}\frac{1}{n}\cos(n\omega_o t + 63.43^\circ)\,\text{V}$$

P 16.20 The periodic function in Problem 16.12 is odd, so $a_v = 0$ and $a_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = 0 - jb_n = b_n\underline{/-90^\circ}$$

From Problem 16.12,

$$b_k = \frac{-0.64}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

Therefore,

$$A_n = \frac{-0.64}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

and

$$-\theta_n = -90^\circ, \qquad n \text{ odd}$$

Thus, $i(t) = \frac{640}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\pi/4)}{n^2}\cos(n\omega_o t + 90^\circ)\,\text{mA}$

P 16.21 The periodic function in Problem 16.15 is even, so $b_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = a_n = a_n\underline{/0^\circ}$$

From Problem 16.15,

$$a_v = 0 = A_0$$

$$a_n = \frac{n^2\pi^2 - 800}{\pi^3 n^3}\sin\frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{n^2\pi^2 - 800}{\pi^3 n^3}\sin\frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

Thus, $f(t) = \frac{1}{\pi^3}\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{n^2\pi^2 - 800}{n^3}\right)\sin\frac{n\pi}{2}\cos n\omega_o t$

P 16.22 **[a]** The current has half-wave symmetry. Therefore,

$$a_v = 0; \qquad a_k = b_k = 0, \quad k \text{ even}$$

For k odd,

$$a_k = \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\cos k\omega_o t\,dt$$

$$= \frac{4}{T}\int_0^{T/2} I_m \cos k\omega_0 t\,dt - \frac{8I_m}{T^2}\int_0^{T/2} t\cos k\omega_0 t\,dt$$

$$= \frac{4I_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2} - \frac{8I_m}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 t\right]_0^{T/2}$$

$$= 0 - \frac{8I_m}{T^2}\left[\frac{\cos k\pi}{k^2\omega_0^2} - \frac{1}{k^2\omega_0^2}\right]$$

$$= \left(\frac{8I_m}{T^2}\right)\left(\frac{1}{k^2\omega_0^2}\right)(1 - \cos k\pi)$$

$$= \frac{4I_m}{\pi^2 k^2} = \frac{20}{k^2}, \quad \text{for } k \text{ odd}$$

$$b_k = \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\sin k\omega_o t\,dt$$

$$= \frac{4I_m}{T}\int_0^{T/2}\sin k\omega_0 t\,dt - \frac{8I_m}{T^2}\int_0^{T/2} t\sin k\omega_0 t\,dt$$

$$= \frac{4I_m}{T}\left[\frac{-\cos k\omega_0 t}{k\omega_0}\right]_0^{T/2} - \frac{8I_m}{T^2}\left[\frac{\sin k\omega_o t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t\right]_0^{T/2}$$

$$= \frac{4I_m}{T}\left[\frac{1 - \cos k\pi}{k\omega_0}\right] - \frac{8I_m}{T^2}\left[\frac{-T\cos k\pi}{2k\omega_0}\right]$$

$$= \frac{8I_m}{k\omega_0 T}\left[\frac{1}{2}\right]$$

$$= \frac{2I_m}{\pi k} = \frac{10\pi}{k}, \quad \text{for } k \text{ odd}$$

$$a_k - jb_k = \frac{20}{k^2} - j\frac{10\pi}{k} = \frac{10}{k}\left(\frac{2}{k} - j\pi\right) = \frac{10}{k^2}\sqrt{\pi^2 k^2 + 4}\,\underline{/-\theta_k}$$

where $\quad \tan\theta_k = \dfrac{\pi k}{2}$

$$i(t) = 10\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sqrt{(n\pi)^2 + 4}}{n^2}\cos(n\omega_0 t - \theta_n), \qquad \theta_n = \tan^{-1}\frac{n\pi}{2}$$

[b] $A_1 = 10\sqrt{4+\pi^2} \cong 37.24\,\text{A} \qquad \tan\theta_1 = \frac{\pi}{2} \qquad \theta_1 \cong 57.52^\circ$

$$A_3 = \frac{10}{9}\sqrt{4+9\pi^2} \cong 10.71\,\text{A} \qquad \tan\theta_3 = \frac{3\pi}{2} \qquad \theta_3 \cong 78.02^\circ$$

$$A_5 = \frac{10}{25}\sqrt{4+25\pi^2} \cong 6.33\,\text{A} \qquad \tan\theta_5 = \frac{5\pi}{2} \qquad \theta_5 \cong 82.74^\circ$$

$$A_7 = \frac{10}{49}\sqrt{4+49\pi^2} \cong 4.51\,\text{A} \qquad \tan\theta_7 = \frac{7\pi}{2} \qquad \theta_7 \cong 84.80^\circ$$

$$A_9 = \frac{10}{81}\sqrt{4+81\pi^2} \cong 3.50\,\text{A} \qquad \tan\theta_9 = \frac{9\pi}{2} \qquad \theta_9 \cong 85.95^\circ$$

$$i(t) \cong 37.24\cos(\omega_o t - 57.52^\circ) + 10.71\cos(3\omega_o t - 78.02^\circ)$$

$$+6.33\cos(5\omega_o t - 82.74^\circ) + 4.51\cos(7\omega_o t - 84.80^\circ)$$

$$+3.50\cos(9\omega_o t - 85.95^\circ) + \ldots$$

$$i(T/4) \cong 37.24\cos(90 - 57.52^\circ) + 10.71\cos(270 - 78.02^\circ)$$

$$+6.33\cos(450 - 82.74^\circ) + 4.51\cos(630 - 84.80^\circ)$$

$$+3.50\cos(810 - 85.95^\circ) \cong 26.23\,\text{A}$$

Actual value:

$$i\left(\frac{T}{4}\right) = \frac{1}{2}(5\pi^2) \cong 24.67\,\text{A}$$

P 16.23 The function has half-wave symmetry, thus $a_k = b_k = 0$ for k-even, $a_v = 0$; for k-odd

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \cos k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\cos k\omega_0 t\,dt$$

where $\rho = \left[1 + e^{-T/2RC}\right]$.

Upon integrating we get

$$a_k = \frac{4V_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\left\{\frac{e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2}\cdot\left[\frac{-\cos k\omega_0 t}{RC} + k\omega_0 \sin k\omega_0 t\right]\Big|_0^{T/2}\right\}$$

$$= \frac{-8V_m RC}{T[1 + (k\omega_0 RC)^2]}$$

$$b_k = \frac{4}{T}\int_0^{T/2} V_m \sin k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\sin k\omega_0 t\,dt$$

$$= -\frac{4V_m}{T}\frac{\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\left\{\frac{-e^{-t/RC}}{(1/RC)^2+(k\omega_0)^2}\cdot\left[\frac{\sin k\omega_0 t}{RC}+k\omega_0\cos k\omega_0 t\right]\Big|_0^{T/2}\right\}$$

$$= \frac{4V_m}{\pi k} - \frac{8k\omega_0 V_m R^2C^2}{T[1+(k\omega_0 RC)^2]}$$

P 16.24 **[a]** $a_k^2 + b_k^2 = a_k^2 + \left(\frac{4V_m}{\pi k} + k\omega_0 RC a_k\right)^2$

$$= a_k^2\left[1+(k\omega_0 RC)^2\right] + \tfrac{8V_m}{\pi k}\left[\tfrac{2V_m}{\pi k} + k\omega_0 RC a_k\right]$$

But $\quad a_k = \left\{\frac{-8V_m RC}{T\left[1+(k\omega_0 RC)^2\right]}\right\}$

Therefore $\quad a_k^2 = \left\{\frac{64V_m^2 R^2C^2}{T^2[1+(k\omega_0 RC)^2]^2}\right\},\quad$ thus we have

$$a_k^2 + b_k^2 = \frac{64V_m^2R^2C^2}{T^2[1+(k\omega_0RC)^2]} + \frac{16V_m^2}{\pi^2k^2} - \frac{64V_m^2 k\omega_0 R^2C^2}{\pi kT[1+(k\omega_0RC)^2]}$$

Now let $\alpha = k\omega_0 RC$ and note that $T = 2\pi/\omega_0$, thus the expression for $a_k^2 + b_k^2$ reduces to $a_k^2 + b_k^2 = 16V_m^2/\pi^2k^2(1+\alpha^2)$. It follows that

$$\sqrt{a_k^2+b_k^2} = \frac{4V_m}{\pi k\sqrt{1+(k\omega_0RC)^2}}$$

[b] $b_k = k\omega_0 RC a_k + \frac{4V_m}{\pi k}$

Thus $\quad \frac{b_k}{a_k} = k\omega_0 RC + \frac{4V_m}{\pi k a_k} = \alpha - \frac{1+\alpha^2}{\alpha} = -\frac{1}{\alpha}$

Therefore $\quad \frac{a_k}{b_k} = -\alpha = -k\omega_0 RC$

P 16.25 Since $a_v = 0$ (half-wave symmetry), Eq. 16.38 gives us

$$v_o(t) = \sum_{1,3,5,\ldots}^{\infty}\frac{4V_m}{n\pi}\frac{1}{\sqrt{1+(n\omega_0RC)^2}}\cos(n\omega_0 t - \theta_n)\quad\text{where}\quad \tan\theta_n = \frac{b_n}{a_n}$$

But from Eq. 16.57, we have $\tan\beta_k = k\omega_0 RC$. It follows from Eq. 16.72 that $\tan\beta_k = -a_k/b_k$ or $\tan\theta_n = -\cot\beta_n$. Therefore $\theta_n = 90^\circ + \beta_n$ and

$\cos(n\omega_0 t - \theta_n) = \cos(n\omega_0 t - \beta_n - 90°) = \sin(n\omega_0 t - \beta_n)$, thus our expression for v_o becomes

$$v_o = \frac{4V_m}{\pi} \sum_{n=1,3,5,...}^{\infty} \frac{\sin(n\omega_0 t - \beta_n)}{n\sqrt{1 + (n\omega_0 RC)^2}}$$

P 16.26 **[a]** $e^{-x} \cong 1 - x$ for small x; therefore

$$e^{-t/RC} \cong \left(1 - \frac{t}{RC}\right) \quad \text{and} \quad e^{-T/2RC} \cong \left(1 - \frac{T}{2RC}\right)$$

$$v_o \cong V_m - \frac{2V_m[1-(t/RC)]}{2-(T/2RC)} = \left(\frac{V_m}{RC}\right)\left[\frac{2t-(T/2)}{2-(T/2RC)}\right]$$

$$\cong \left(\frac{V_m}{RC}\right)\left(t - \frac{T}{4}\right) = \left(\frac{V_m}{RC}\right)t - \frac{V_m T}{4RC} \qquad \text{for} \quad 0 \le t \le \frac{T}{2}$$

[b] $a_k = \left(\frac{-8}{\pi^2 k^2}\right)V_p = \left(\frac{-8}{\pi^2 k^2}\right)\left(\frac{V_m T}{4RC}\right) = \frac{-4V_m}{\pi\omega_0 RCk^2}$

P 16.27 **[a]** Express v_g as a constant plus a symmetrical square wave. The constant is $V_m/2$ and the square wave has an amplitude of $V_m/2$, is odd, and has half- and quarter-wave symmetry. Therefore the Fourier series for v_g is

$$v_g = \frac{V_m}{2} + \frac{2V_m}{\pi} \sum_{n=1,3,5,...}^{\infty} \frac{1}{n}\sin n\omega_0 t$$

The dc component of the current is $V_m/2R$ and the kth harmonic phase current is

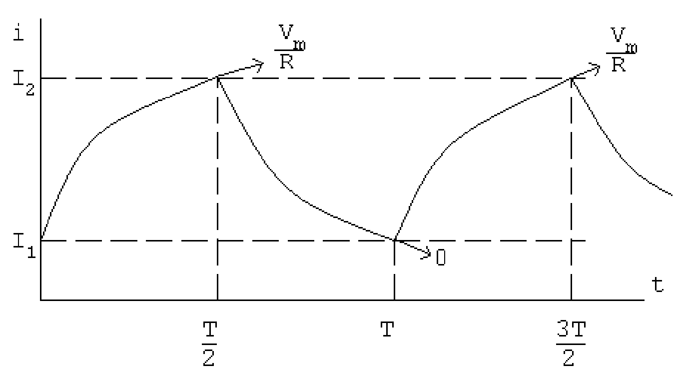
$$\mathbf{I}_k = \frac{2V_m/k\pi}{R + jk\omega_0 L} = \frac{2V_m}{k\pi\sqrt{R^2 + (k\omega_0 L)^2}} \angle -\theta_k$$

where $\theta_k = \tan^{-1}\left(\frac{k\omega_0 L}{R}\right)$

Thus the Fourier series for the steady-state current is

$$i = \frac{V_m}{2R} + \frac{2V_m}{\pi} \sum_{n=1,3,5,...}^{\infty} \frac{\sin(n\omega_0 t - \theta_n)}{n\sqrt{R^2 + (n\omega_0 L)^2}} \text{ A}$$

[b]



The steady-state current will alternate between I_1 and I_2 in exponential traces as shown. Assuming $t = 0$ at the instant i increases toward (V_m/R), we have

$$i = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right) e^{-t/\tau} \quad \text{for} \quad 0 \le t \le \frac{T}{2}$$

and $i = I_2 e^{-[t-(T/2)]/\tau}$ for $T/2 \le t \le T$, where $\tau = L/R$. Now we solve for I_1 and I_2 by noting that

$$I_1 = I_2 e^{-T/2\tau} \quad \text{and} \quad I_2 = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right) e^{-T/2\tau}$$

These two equations are now solved for I_1. Letting $x = T/2\tau$, we get

$$I_1 = \frac{(V_m/R)e^{-x}}{1 + e^{-x}}$$

Therefore the equations for i become

$$i = \frac{V_m}{R} - \left[\frac{V_m}{R(1 + e^{-x})}\right] e^{-t/\tau} \qquad \text{for} \quad 0 \le t \le \frac{T}{2} \quad \text{and}$$

$$i = \left[\frac{V_m}{R(1 + e^{-x})}\right] e^{-[t-(T/2)]/\tau} \qquad \text{for} \quad \frac{T}{2} \le t \le T$$

A check on the validity of these expressions shows they yield an average value of $(V_m/2R)$:

$$I_{\text{avg}} = \frac{1}{T}\left\{\int_0^{T/2} \left[\frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right) e^{-t/\tau}\right] dt + \int_{T/2}^{T} I_2 e^{-[t-(T/2)]/\tau}\, dt\right\}$$

$$= \frac{1}{T}\left\{\frac{V_m T}{2R} + \tau(1 - e^{-x})\left(I_1 - \frac{V_m}{R} + I_2\right)\right\}$$

$$= \frac{V_m}{2R} \quad \text{since} \quad I_1 + I_2 = \frac{V_m}{R}$$

P 16.28 **[a]** From the solution to Problem 16.13(a) the Fourier series for the input voltage is

$$v_g = \frac{4V_m}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \left[\frac{1}{n} \sin \frac{n\pi}{2}\right] \cos n\omega_0 t \text{ V}$$

Since $V_m = 0.5\pi$ V and $T = 10\pi$ ms, we can write the input voltage as

$$v_g = 2 \sum_{n=1,3,5,\ldots}^{\infty} \left[\frac{1}{n} \sin\left(\frac{n\pi}{2}\right)\right] \cos 200nt \text{ V}$$

$$= 2\cos 200t - \frac{2}{3}\cos 600t + \frac{2}{5}\cos 1000t - \frac{2}{7}\cos 1400t + \cdots$$

We can phasor transform this Fourier series to get

$$\mathbf{V}_{g1} = 2\underline{/0^\circ} \qquad \omega_0 = 200 \text{ rad/s}$$

$\mathbf{V}_{g3} = 0.667\underline{/180^\circ} \qquad 3\omega_0 = 600$ rad/s

$\mathbf{V}_{g5} = 0.4\underline{/0^\circ} \qquad 5\omega_0 = 1000$ rad/s

$\mathbf{V}_{g7} = 0.286\underline{/180^\circ} \qquad 7\omega_0 = 1400$ rad/s

From the circuit in Fig. P16.28 we have

$$\frac{V_o}{R} + \frac{V_o - V_g}{sL} + (V_o - V_g)sC = 0$$

$$\therefore \quad \frac{V_o}{V_g} = H(s) = \frac{s^2 + 1/LC}{s^2 + (s/RC) + (1/LC)}$$

Substituting in the numerical values gives

$$H(s) = \frac{s^2 + 10^6}{s^2 + 40s + 10^6}$$

$$H(j200) = \frac{96}{96 + j0.8} = 0.99997\underline{/-0.48^\circ}$$

$$H(j600) = \frac{64}{64 + j2.4} = 0.9993\underline{/-2.15^\circ}$$

$$H(j1000) = 0$$

$$H(j1400) = \frac{-96}{-96 + j5.6} = 0.9983\underline{/3.34^\circ}$$

$$\mathbf{V}_{o1} = (2\underline{/0^\circ})(0.99997\underline{/-0.48^\circ}) = 1.9999\underline{/-0.48^\circ}\text{ V}$$

$$\mathbf{V}_{o3} = (0.667\underline{/180^\circ})(0.9993\underline{/-2.15^\circ}) = 0.6662\underline{/177.85^\circ}\text{ V}$$

$$\mathbf{V}_{o5} = 0\text{ V}$$

$$\mathbf{V}_{o7} = (0.286\underline{/180^\circ})(0.9983\underline{/3.34^\circ}) = 0.286\underline{/-176.66^\circ}\text{ V}$$

$$v_o = 1.9999\cos(200t - 0.48^\circ) + 0.6662\cos(600t + 177.85^\circ)$$
$$+0.286\cos(1400t - 176.66^\circ) + \ldots \text{ V}$$

[b] The 5th harmonic at the frequency $\sqrt{1/LC} = 1000$ rad/s has been eliminated from the output voltage by the circuit, which is a band reject filter with a center frequency of 1000 rad/s.

P 16.29 $v_i = \dfrac{4A}{\pi} \displaystyle\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin n\omega_0(t + T/4)$

$$= \frac{4A}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos n\omega_0 t$$

$$\omega_0 = \frac{2\pi}{4\pi} \times 10^6 = 10{,}000 \text{ rad/s}; \qquad \frac{4A}{\pi} = 120$$

$$v_i = 120 \sum_{n=1,3,5,\ldots}^{\infty} \left(\frac{1}{n}\sin\frac{n\pi}{2}\right)\cos 10{,}000nt \text{ V}$$

From the circuit

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + j\omega L}\cdot j\omega L = \frac{j\omega}{R/L + j\omega}\mathbf{V}_i = \frac{j\omega}{30{,}000 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 120\angle 0^\circ \text{ V}; \qquad \omega = 10{,}000 \text{ rad/s}$$

$$\mathbf{V}_{i3} = -40\angle 0^\circ = 40\angle 180^\circ \text{ V}; \qquad 3\omega = 30{,}000 \text{ rad/s}$$

$$\mathbf{V}_{i5} = 24\angle 0^\circ \text{ V}; \qquad 5\omega = 50{,}000 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{j10{,}000}{30{,}000 + j10{,}000}(120\angle 0^\circ) = 37.95\angle 71.57^\circ \text{ V}$$

$$\mathbf{V}_{o3} = \frac{j30{,}000}{30{,}000 + j30{,}000}(40\angle 180^\circ) = 28.28\angle -135^\circ \text{ V}$$

$$\mathbf{V}_{o5} = \frac{j50{,}000}{30{,}000 + j50{,}000}(24\angle 0^\circ) = 20.58\angle 30.96^\circ \text{ V}$$

$$\therefore \qquad v_o = 37.95\cos(10{,}000t + 71.57^\circ) + 28.28\cos(30{,}000t - 135^\circ)$$

$$+20.58\cos(50{,}000t + 30.96^\circ) + \ldots \text{ V}$$

P 16.30 **[a]** $\displaystyle \frac{V_0 - V_g}{16s} + V_0(12.5 \times 10^{-6}s) + \frac{V_0}{1000} = 0$

$$V_0\left[\frac{1}{16s} + 12.6 \times 10^{-6}s + \frac{1}{1000}\right] = \frac{V_g}{16s}$$

$$V_0(1000 + 0.2s^2 + 16s) = 1000V_g$$

$$V_0 = \frac{5000V_g}{s^2 + 80s + 5000}$$

$$I_0 = \frac{V_0}{1000} = \frac{5V_g}{s^2 + 80s + 5000}$$

$$H(s) = \frac{I_0}{V_g} = \frac{5}{s^2 + 80s + 5000}$$

$$H(nj\omega_0) = \frac{5}{(5000 - n^2\omega_0^2) + j80n\omega_0}$$

$$\omega_0 = \frac{2\pi}{T} = 240\pi; \qquad \omega_0^2 = 57{,}600\pi^2; \qquad 80\omega_0 = 19{,}200\pi$$

$$H(jn\omega_0) = \frac{5}{(5000 - 57{,}600\pi^2n^2) + j19{,}200\pi n}$$

$$H(0) = 10^{-3}$$

$$H(j\omega_0) = 8.82 \times 10^{-6}\underline{/-173.89^\circ}$$

$$H(j2\omega_0) = 2.20 \times 10^{-6}\underline{/-176.96^\circ}$$

$$H(j3\omega_0) = 9.78 \times 10^{-7}\underline{/-177.97^\circ}$$

$$H(j4\omega_0) = 5.5 \times 10^{-7}\underline{/-178.48^\circ}$$

$$v_g = \frac{680}{\pi} - \frac{1360}{\pi}\left[\frac{1}{3}\cos\omega_0 t + \frac{1}{15}\cos 2\omega_0 t + \frac{1}{35}\cos 3\omega_0 t + \frac{1}{63}\cos 4\omega_0 t + \ldots\right]$$

$$i_0 = \frac{680}{\pi} \times 10^{-3} - \frac{1360}{3\pi}(8.82 \times 10^{-6})\cos(\omega_0 t - 173.89^\circ)$$

$$-\frac{1360}{15\pi}(2.20 \times 10^{-6})\cos(2\omega_0 t - 176.96^\circ)$$

$$-\frac{1360}{35\pi}(9.78 \times 10^{-7})\cos(3\omega_0 t - 177.97^\circ)$$

$$-\frac{1360}{63\pi}(5.5 \times 10^{-7})\cos(4\omega_0 t - 178.48^\circ) - \ldots$$

$$= 216.45 \times 10^{-3} + 1.27 \times 10^{-3}\cos(240\pi t + 6.11^\circ)$$

$$+6.35 \times 10^{-5}\cos(480\pi t + 3.04^\circ)$$

$$+1.21 \times 10^{-5}\cos(720\pi t + 2.03^\circ)$$

$$+3.8 \times 10^{-6}\cos(960\pi t + 1.11^\circ) - \ldots$$

$$i_0 \cong 216.45 + 1.27\cos(240\pi t + 6.11^\circ)\,\text{mA}$$

Note that the sinusoidal component is very small compared to the dc component, so

$i_0 \cong 216.45\,\text{mA}$ (a dc current)

[b] The circuit is a low pass filter, so the harmonic terms are greatly reduced in the output.

P 16.31 The function is odd with half-wave and quarter-wave symmetry. Therefore,

$a_k = 0, \qquad$ for all k; the function is odd

$b_k = 0, \qquad$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/10} 100t\sin k\omega_o t\,dt + \int_{T/10}^{T/4} 5\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = 100\int_0^{T/10} t\sin k\omega_o t\,dt$$

$$= 100\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/10}\right]$$

$$= \frac{100}{k^2\omega_o^2}\sin\frac{k\pi}{5} - \frac{10T}{k\omega_o}\cos\frac{k\pi}{5}$$

$$\text{Int2} = \int_{T/10}^{T/4} 5\sin k\omega_o t\,dt = \frac{-5}{k\omega_o}\cos k\omega_o t\,\Big|_{T/10}^{T/4} = \frac{5}{k\omega_o}\cos\frac{k\pi}{5}$$

$$\text{Int1} + \text{Int2} = \frac{100}{k^2\omega_o^2}\sin\frac{k\pi}{5} + \left(\frac{5}{k\omega_o} - \frac{10T}{k\omega_o}\right)\cos\frac{k\pi}{5}$$

$$10T = 10(0.5) = 5$$

$$\therefore \quad \text{Int1} + \text{Int2} = \frac{100}{k^2\omega_o^2}\sin\frac{k\pi}{5}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{100}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{5} = \frac{100}{\pi^2k^2}\sin\frac{k\pi}{5}, \qquad k \text{ odd}$$

$$i(t) = \frac{100}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\sin(n\pi/5)}{n^2}\sin n\omega_o t \text{ A}$$

From the circuit,

$$H(s) = \frac{V_o}{I_g} = Z_{\text{eq}}$$

$$Y_{\rm eq} = \frac{1}{R_1} + \frac{1}{R_2 + sL} + sC$$

$$Z_{\rm eq} = \frac{1/C(s + R_2/L)}{s^2 + s(R_1R_2C + L)/R_1LC + (R_1 + R_2)/R_1LC}$$

Therefore,

$$H(s) = \frac{20{,}000(s + 400)}{s^2 + 10{,}400s + 450 \times 10^4}$$

We want the output for the third harmonic:

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi; \qquad 3\omega_0 = 12\pi$$

$$I_{g3} = \frac{100}{\pi^2}\frac{1}{9}\sin\frac{3\pi}{5} = 1.07\underline{/0^\circ}$$

$$H(j12\pi) = \frac{20{,}000(j12\pi + 400)}{(j12\pi)^2 + 10{,}400(j12\pi) + 450 \times 10^4} = 1.78\underline{/0.403^\circ}$$

Therefore,

$$V_{o3} = H(j12\pi)I_{g3} = (1.78\underline{/0.403^\circ})(1.07\underline{/0^\circ}) = 1.9\underline{/0.403^\circ}\,\text{V}$$

$$v_{o3} = 1.9\sin(12\pi t + 0.403^\circ)\,\text{V}$$

P 16.32 $\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 200$ krad/s

$$\therefore \quad n = \frac{3 \times 10^6}{0.2 \times 10^6} = 15; \qquad n = \frac{5 \times 10^6}{0.2 \times 10^6} = 25$$

$$H(s) = \frac{V_o}{V_g} = \frac{(1/RC)s}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{RC} = \frac{10^{12}}{(250 \times 10^3)(4)} = 10^6; \qquad \frac{1}{LC} = \frac{(10^3)(10^{12})}{10)(4)} = 25 \times 10^{12}$$

$$H(s) = \frac{10^6 s}{s^2 + 10^6 s + 25 \times 10^{12}}$$

$$H(j\omega) = \frac{j\omega \times 10^6}{(25 \times 10^{12} - \omega^2) + j10^6\omega}$$

15th harmonic input:

$v_{g15} = (150)(1/15)\sin(15\pi/2)\cos 15\omega_o t = -10\cos 3\times 10^6 t\,\text{V}$

$\therefore\quad \mathbf{V}_{g15} = 10\underline{/-180^\circ}\,\text{V}$

$$H(j3\times 10^6) = \frac{j3}{16 + j3} = 0.1843\underline{/79.38^\circ}$$

$\mathbf{V}_{o15} = (10)(0.1843)\underline{/-100.62^\circ}\,\text{V}$

$v_{o15} = 1.84\cos(3\times 10^6 t - 100.62^\circ)\,\text{V}$

25th harmonic input:

$v_{g25} = (150)(1/25)\sin(25\pi/2)\cos 5\times 10^6 t = 6\cos 5\times 10^6 t\,\text{V}$

$\therefore\quad \mathbf{V}_{g25} = 6\underline{/0^\circ}\,\text{V}$

$$H(j5\times 10^6) = \frac{j5}{0 + j5} = 1\underline{/0^\circ}$$

$\mathbf{V}_{o25} = 6\underline{/0^\circ}\,\text{V}$

$v_{o25} = 6\cos 5\times 10^6 t\,\text{V}$

P 16.33 **[a]** $a_v = \frac{1}{T}\left[\frac{1}{2}\left(\frac{T}{2}\right)I_m + \frac{T}{2}I_m\right] = \frac{3I_m}{4}$

$$i(t) = \frac{2I_m}{T}t, \qquad 0 \le t \le T/2$$

$$i(t) = I_m, \qquad T/2 \le t \le T$$

$$a_k = \frac{2}{T}\int_0^{T/2}\frac{2I_m}{T}t\cos k\omega_o t\,dt + \frac{2}{T}\int_{T/2}^{T} I_m\cos k\omega_o t\,dt$$

$$= \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1)$$

$$b_k = \frac{2}{T}\int_0^{T/2}\frac{2I_m}{T}t\sin k\omega_o t\,dt + \frac{2}{T}\int_{T/2}^{T} I_m\sin k\omega_o t\,dt$$

$$= -\frac{I_m}{\pi k}$$

$$a_v = \frac{3I_m}{4}, \qquad a_1 = \frac{-2I_m}{\pi^2}, \quad a_2 = 0,$$

$$b_1 = -\frac{I_m}{\pi}, \quad b_2 = -\frac{I_m}{2\pi}$$

$$\therefore \quad I_{\text{rms}} = I_m\sqrt{\frac{9}{16} + \frac{2}{\pi^4} + \frac{1}{2\pi^2} + \frac{1}{8\pi^2}} = 0.8040I_m$$

$$I_{\text{rms}} = 4.02\,\text{A}$$

$$P = (4.02)^2(2500) = 40.4\,\text{kW}$$

[b] Area under i^2:

$$A = \int_0^{T/2} \frac{4I_m^2}{T^2}t^2\,dt + I_m^2\frac{T}{2}$$

$$= \frac{4I_m^2}{T^2}\frac{t^3}{3}\bigg|_0^{T/2} + I_m^2\frac{T}{2}$$

$$= I_m^2T\left[\frac{1}{6} + \frac{3}{6}\right] = \frac{2}{3}TI_m^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\cdot\frac{2}{3}TI_m^2} = \sqrt{\frac{2}{3}}I_m = 4.0825\,\text{A}$$

$$P = (4.0825)^2(2500) = 41.67\,\text{kW}$$

[c] Error $= \left(\frac{40.4}{41.67} - 1\right)100 = -3.05\%$

P 16.34 **[a]** $a_v = \frac{2\left(\frac{1}{2}\,\frac{T}{4}V_m\right)}{T} = \frac{V_m}{4}$

$$a_k = \frac{4}{T}\int_0^{T/4}\left[V_m - \frac{4V_m}{T}t\right]\cos k\omega_o t\,dt$$

$$= \frac{4V_m}{\pi^2k^2}\left[1 - \cos\frac{k\pi}{2}\right]$$

$$b_k = 0, \quad \text{all } k$$

$$a_v = \frac{200}{4} = 50\,\text{V}$$

$$a_1 = \frac{800}{\pi^2}$$

$$a_2 = \frac{800}{4\pi^2}(1 - \cos\pi) = \frac{400}{\pi^2}$$

$$V_{\rm rms} = \sqrt{(50)^2 + \frac{1}{2}\left[\left(\frac{800}{\pi^2}\right)^2 + \left(\frac{400}{\pi^2}\right)^2\right]} = 81.28\,\text{V}$$

$$P = \frac{(81.28)^2}{400} = 16.516\,\text{W}$$

[b] Area under v^2; $\qquad 0 \le t \le T/4$

$$v^2 = 40{,}000 - \frac{320{,}000}{T}t + \frac{640{,}000}{T^2}t^2$$

$$A = 2\int_0^{T/4}\left[40{,}000 - \frac{320{,}000}{T}t + \frac{640{,}000}{T^2}t^2\right]dt = 6666.67T$$

$$V_{\rm rms} = \sqrt{\frac{1}{T}6666.67T} = \sqrt{6666.67} = 81.65\,\text{V}$$

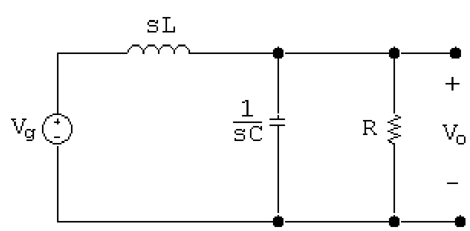
$$P = \sqrt{6666.67}^2/400 = 16.667\,\text{W}$$

[c] Error $= \left(\frac{16.516}{16.667} - 1\right)100 = -0.904\%$

P 16.35 $v_g = 10 - \frac{80}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n^2}\cos n\omega_o t\,\text{V}$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{4\pi}\times 10^3 = 500 \text{ rad/s}$$

$$v_g = 10 - \frac{80}{\pi^2}\cos 500t - \frac{80}{9\pi^2}\cos 1500t + \ldots$$



$$\frac{V_o - V_g}{sL} + sCV_o + \frac{V_o}{R} = 0$$

$$V_o(RLCs^2 + Ls + R) = RV_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{1/LC}{s^2 + s/RC + 1/LC}$$

$$\frac{1}{LC} = \frac{10^6}{(0.1)(10)} = 10^6$$

$$\frac{1}{RC} = \frac{10^6}{(50\sqrt{2})(10)} = 1000\sqrt{2}$$

$$H(s) = \frac{10^6}{s^2 + 1000\sqrt{2}s + 10^6}$$

$$H(j\omega) = \frac{10^6}{10^6 - \omega^2 + j1000\omega\sqrt{2}}$$

$$H(j0) = 1$$

$$H(j500) = 0.9701\underline{/-43.31^\circ}$$

$$H(j1500) = 0.4061\underline{/-120.51^\circ}$$

$$v_o = 10(1) + \frac{80}{\pi^2}(0.9701)\cos(500t - 43.31^\circ)$$

$$+\frac{80}{9\pi^2}(0.4061)\cos(1500t - 120.51^\circ) + \ldots$$

$$v_o = 10 + 7.86\cos(500t - 43.31^\circ) + 0.3658\cos(1500t - 120.51^\circ) + \ldots$$

$$V_{\text{rms}} \cong \sqrt{10^2 + \left(\frac{7.86}{\sqrt{2}}\right)^2 + \left(\frac{0.3658}{\sqrt{2}}\right)^2} = 11.44\,\text{V}$$

$$P \cong \frac{V_{\text{rms}}^2}{50\sqrt{2}} = 1.85\,\text{W}$$

Note – the higher harmonics are severely attenuated and can be ignored. For example, the 5th harmonic component of v_o is

$$v_{o5} = (0.1580)\left(\frac{80}{25\pi^2}\right)\cos(2500t - 146.04^\circ) = 0.0512\cos(2500t - 146.04^\circ)\,\text{V}$$

P 16.36 **[a]** $v = 30 + 60\cos 2000t + 20\cos(8000t - 90^\circ)\,\text{V}$

$$i = 3 + 4\cos(2000t - 25^\circ) + \cos(8000t - 45^\circ)\,\text{A}$$

$$P = (30)(3) + \frac{1}{2}(60)(4)\cos(25^\circ) + \frac{1}{2}(20)(1)\cos(-45^\circ) = 205.83\,\text{W}$$

[**b**] $V_{\rm rms} = \sqrt{(30)^2 + \left(\frac{60}{\sqrt{2}}\right)^2 + \left(\frac{20}{\sqrt{2}}\right)^2} = 53.85\,\text{V}$

[**c**] $I_{\rm rms} = \sqrt{(3)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 4.18\,\text{A}$

P 16.37 [**a**] Area under $v^2 = A = 4\int_0^{T/6} \frac{36V_m^2}{T^2}t^2\,dt + 2V_m^2\left(\frac{T}{3} - \frac{T}{6}\right)$

$$= \frac{2V_m^2T}{9} + \frac{V_m^2T}{3}$$

Therefore $V_{\rm rms} = \sqrt{\frac{1}{T}\left(\frac{2V_m^2T}{9} + \frac{V_m^2T}{3}\right)} = V_m\sqrt{\frac{2}{9} + \frac{1}{3}} = 74.5356\,\text{V}$

[**b**] From Asssessment Problem 16.3,

$v_g = 105.30\sin\omega_0 t - 4.21\sin 5\omega_0 t + 2.15\sin 7\omega_0 t + \cdots\,\text{V}$

Therefore $V_{\rm rms} \cong \sqrt{\frac{(105.30)^2 + (4.21)^2 + (2.15)^2}{2}} = 74.5306\,\text{V}$

P 16.38 [**a**] v_g has half-wave symmetry, quarter-wave symmetry, and is odd

$\therefore\quad a_v = 0,\ a_k = 0 \text{ all } k,\ b_k = 0\ k\text{-even}$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\,dt, \quad k\text{-odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} V_m \sin k\omega_o t\,dt + \int_{T/8}^{T/4}\frac{V_m}{2}\sin k\omega_o t\,dt\right\}$$

$$= \frac{8V_m}{T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\right]_0^{T/8} + \frac{8V_m}{2T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\right]_{T/8}^{T/4}$$

$$= \frac{8V_m}{k\omega_o T}\left[1 - \cos\frac{k\pi}{4}\right] + \frac{8V_m}{2Tk\omega_o}\left[\cos\frac{k\pi}{4} - 0\right]$$

$$= \frac{8V_m}{k\omega_o T}\left\{1 - \cos\frac{k\pi}{4} + \frac{1}{2}\cos\frac{k\pi}{4}\right\}$$

$$= \frac{4V_m}{\pi k}\left\{1 - 0.5\cos\frac{k\pi}{4}\right\} = \frac{1}{k}[16 - 8\cos(k\pi/4)]$$

$$b_1 = 16 - 8\cos(\pi/4) = 10.34$$
$$b_3 = \frac{1}{3}[16 - 8\cos(3\pi/4)] = 7.22$$
$$b_5 = \frac{1}{5}[16 - 8\cos(5\pi/4)] = 4.33$$

$$V_g(\text{rms}) \approx \mathbf{V}_m\sqrt{\frac{10.34^2 + 7.22^2 + 4.33^2}{2}} = 9.43$$

[b] $\text{Area} = 2\left[2(4\pi)^2\left(\frac{T}{8}\right) + (2\pi)^2\left(\frac{T}{4}\right)\right] = 10\pi^2 T$

$$V_g(\text{rms}) = \sqrt{\frac{1}{T}(10\pi^2)T} = \sqrt{10}\pi = 9.935$$

[c] $\%\ \text{Error} = \left(\frac{9.43}{9.935} - 1\right)(100) = -5.08\%$

P 16.39 **[a]** $v(t) = \frac{480}{\pi}\{\sin\omega_o t + \frac{1}{3}\sin 3\omega_o t + \frac{1}{5}\sin 5\omega_o t + \frac{1}{7}\sin 7\omega_o t + \frac{1}{9}\sin 9\omega_o t + \cdots$

$$\begin{aligned} V_{\text{rms}} &= \frac{480}{\pi}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{1}{7\sqrt{2}}\right)^2 + \left(\frac{1}{9\sqrt{2}}\right)^2} \\ &= \frac{480}{\pi\sqrt{2}}\sqrt{1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}} \\ &= 117.55\,\text{V} \end{aligned}$$

[b] $\%\ \text{error} = \left(\frac{117.55}{120} - 1\right)(100) = -2.04\%$

[c] $v(t) = \frac{960}{\pi^2}\left\{\sin\omega_o t + \frac{1}{9}\sin 3\omega_o t + \frac{1}{25}\sin 5\omega_o t\right.$

$$\left. - \frac{1}{49}\sin 7\omega_o t + \frac{1}{81}\sin 9\omega_o t - \cdots\right\}$$

$$V_{\text{rms}} \cong \frac{960}{\pi^2\sqrt{2}}\sqrt{1 + \frac{1}{81} + \frac{1}{625} + \frac{1}{2401} + \frac{1}{6561}}$$

$$\cong 69.2765\,\text{V}$$

$$V_{\text{rms}} = \frac{120}{\sqrt{3}} = 69.2820\,\text{V}$$

$$\%\ \text{error} = \left(\frac{69.2765}{69.2820} - 1\right)(100) = -0.0081\%$$

P 16.40 **[a]** $v(t) \approx \frac{340}{\pi} - \frac{680}{\pi}\left\{\frac{1}{3}\cos\omega_o t + \frac{1}{15}\cos 2\omega_o t + \cdots\right\}$

$$V_{\rm rms} \approx \sqrt{\left(\frac{340}{\pi}\right)^2 + \left(\frac{680}{\pi}\right)^2\left[\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{15\sqrt{2}}\right)^2\right]}$$

$$= \frac{340}{\pi}\sqrt{1 + 4\left(\frac{1}{18} + \frac{1}{450}\right)} = 120.0819\,\text{V}$$

[b] $V_{\rm rms} = \frac{170}{\sqrt{2}} = 120.2082$

$$\%\ \text{error}\ = \left(\frac{120.0819}{120.2082} - 1\right)(100) = -0.11\%$$

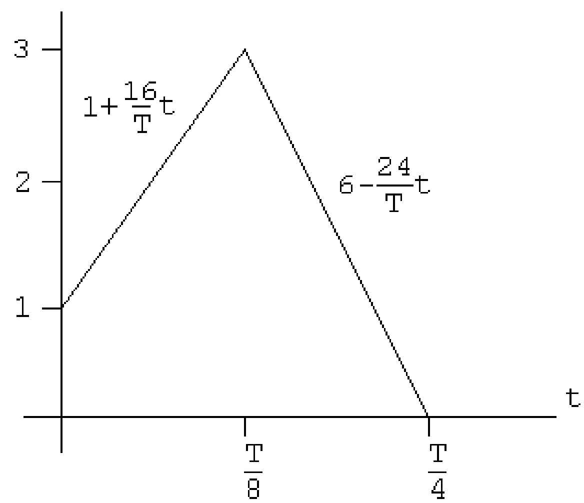
[c] $v(t) \approx \frac{170}{\pi} + 85\sin\omega_o t - \frac{340}{3\pi}\cos 2\omega_o t$

$$V_{\rm rms} \approx \sqrt{\left(\frac{170}{\pi}\right)^2 + \left(\frac{85}{\sqrt{2}}\right)^2 + \left(\frac{340}{3\sqrt{2}\pi}\right)^2} \approx 84.8021\,\text{V}$$

$$V_{\rm rms} = \frac{170}{2} = 85\,\text{V}$$

$$\%\ \text{error}\ = -0.23\%$$

P 16.41 **[a]**



Area under i^2:

$$\text{A}\ = 4\left[\int_0^{T/8}\left(1 + \frac{16}{T}t\right)^2 dt + \int_{T/8}^{T/4}\left(6 - \frac{24}{T}t\right)^2 dt\right]$$

$$= 4\left[\frac{T}{8} + \frac{T}{4} + \frac{T}{6} + 9T - 4.5T - 9T + +2.25T + 3T - 0.0375T\right]$$

$$= \frac{11T}{3}$$

$$I_{\rm rms} = \sqrt{\frac{1}{T}\left(\frac{11T}{3}\right)} = \sqrt{\frac{11}{3}} = 1.915$$

[b] $P = I_{\rm rms}^2(100) = 366.7\,\text{W}$

[c] From Problem 16.14:

$$a_1 = 2.489\text{A}$$

$$i_g \approx 2.489\cos\omega_o t\,\text{A}$$

$$P = \left(\frac{2.489}{\sqrt{2}}\right)^2(100) = 309.76\,\text{W}$$

[d] $\%\text{ error } = \left(\frac{308.76}{366.7} - 1\right) = -15.52\%$

P 16.42 **[a]** Half-wave symmetry $a_v = 0$, $a_k = b_k = 0$, even k

$$a_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\cos k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4}t\cos k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 t\,\Big|_0^{T/4}\right\}$$

$$= \frac{16I_m}{T^2}\left\{0 + \frac{T}{4k\omega_0}\sin\frac{k\pi}{2} - \frac{1}{k^2\omega_0^2}\right\}$$

$$a_k = \frac{2I_m}{\pi k}\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right], \qquad k\text{—odd}$$

$$b_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\sin k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4}t\sin k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t\,\Big|_0^{T/4}\right\} = \frac{4I_m}{\pi^2k^2}\sin\left(\frac{k\pi}{2}\right)$$

[b] $a_k - jb_k = \frac{2I_m}{\pi k}\left\{\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right] - \left[j\frac{2}{\pi k}\sin\left(\frac{k\pi}{2}\right)\right]\right\}$

$$a_1 - jb_1 = \frac{2I_m}{\pi}\left\{\left(1 - \frac{2}{\pi}\right) - j\frac{2}{\pi}\right\} = 0.47I_m\underline{/-60.28^\circ}$$

$$a_3 - jb_3 = \frac{2I_m}{3\pi}\left\{\left(-1 - \frac{2}{3\pi}\right) + j\left(\frac{2}{3\pi}\right)\right\} = 0.26I_m\underline{/170.07^\circ}$$

$$a_5 - jb_5 = \frac{2I_m}{5\pi}\left\{\left(1 - \frac{2}{5\pi}\right) - j\left(\frac{2}{5\pi}\right)\right\} = 0.11I_m\underline{/-8.30^\circ}$$

$$a_7 - jb_7 = \frac{2I_m}{7\pi}\left\{\left(-1 - \frac{2}{7\pi}\right) + j\left(\frac{2}{7\pi}\right)\right\} = 0.10I_m\underline{/175.23^\circ}$$

$$i_g = 0.47I_m\cos(\omega_0 t - 60.28^\circ) + 0.26I_m\cos(3\omega_0 t + 170.07^\circ)$$
$$+0.11I_m\cos(5\omega_0 t - 8.30^\circ) + 0.10I_m\cos(7\omega_0 t + 175.23^\circ) + \cdots$$

[c] $$I_g = \sqrt{\sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{A_n^2}{2}\right)}$$
$$\cong I_m\sqrt{\frac{(0.47)^2 + (0.26)^2 + (0.11)^2 + (0.10)^2}{2}} = 0.39I_m$$

[d] $$\text{Area} = 2\int_0^{T/4}\left(\frac{4I_m}{T}t\right)^2 dt = \left(\frac{32I_m^2}{T^2}\right)\left(\frac{t^3}{3}\right)\Big|_0^{T/4} = \frac{I_m^2 T}{6}$$

$$I_g = \sqrt{\frac{1}{T}\left(\frac{I_m^2 T}{6}\right)} = \frac{I_m}{\sqrt{6}} = 0.41I_m$$

[e] $$\%\ \text{error} = \left(\frac{\text{estimated}}{\text{exact}} - 1\right)100 = \left(\frac{0.3927I_m}{(I_m/\sqrt{6})} - 1\right)100 = -3.8\%$$

P 16.43 Figure P16.43(b): $t_a = 0.2s$; $t_b = 0.6s$

$$v = 50t \quad 0 \le t \le 0.2$$

$$v = -50t + 20 \quad 0.2 \le t \le 0.6$$

$$v = 25t - 25 \quad 0.6 \le t \le 1.0$$

$$\text{Area 1} = A_1 = \int_0^{0.2} 2500t^2\,dt = \frac{20}{3}$$

$$\text{Area 2} = A_2 = \int_{0.2}^{0.6} 100(4 - 20t + 25t^2)\,dt = \frac{40}{3}$$

$$\text{Area 3} = A_3 = \int_{0.6}^{1.0} 625(t^2 - 2t + 1)\,dt = \frac{40}{3}$$

$$A_1 + A_2 + A_3 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\ \text{V}.$$

Figure P16.43(c): $t_a = t_b = 0.4s$

$$v(t) = 25t \quad 0 \le t \le 0.4$$

$$v(t) = \frac{50}{3}(t-1) \quad 0.4 \le t \le 1$$

$$A_1 = \int_0^{0.4} 625t^2\,dt = \frac{40}{3}$$

$$A_2 = \int_{0.4}^{1.0} \frac{2500}{9}(t^2 - 2t + 1)\,dt = \frac{60}{3}$$

$$A_1 + A_2 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}(A_1 + A_2)} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

Figure P16.43(d): $t_a = t_b = 1$

$$v = 10t \quad 0 \le t \le 1$$

$$A_1 = \int_0^1 100t^2\,dt = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

P 16.44 $C_o = A_v = \dfrac{V_m T}{2} \cdot \dfrac{1}{T} = \dfrac{V_m}{2}$

$$C_n = \frac{1}{T}\int_0^T \frac{V_m}{T} t e^{-jn\omega_o t}\,dt$$

$$= \frac{V_m}{T^2}\left[\frac{e^{-jn\omega_0 t}}{-n^2\omega_0^2}(-jn\omega_0 t - 1)\right]_0^T$$

$$= \frac{V_m}{T^2}\left[\frac{e^{-jn2\pi T/T}}{-n^2\omega_0^2}\left(-jn\frac{2\pi}{T}T - 1\right) - \frac{1}{-n^2\omega_0^2}(-1)\right]$$

$$= \frac{V_m}{T^2}\left[\frac{1}{n^2\omega_0^2}(1 + jn2\pi) - \frac{1}{n^2\omega_0^2}\right]$$

$$= j\frac{V_m}{2n\pi}, \quad n = \pm 1, \pm 2, \pm 3, \ldots$$

P 16.45 **[a]** $V_{\rm rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T}\right)^2 t^2\,dt}$

$$= \sqrt{\frac{V_m^2}{T^3}\frac{t^3}{3}\Big|_0^T}$$

$$= \sqrt{\frac{V_m^2}{3}} = \frac{V_m}{\sqrt{3}}$$

$$P = \frac{(150/\sqrt{3})^2}{25} = 300\,\text{W}$$

[b] From the solution to Problem 16.44

$$C_0 = \frac{150}{2} = 75\,\text{V};$$

$$C_1 = j\frac{150}{2\pi} = j\frac{75}{\pi}; \qquad C_2 = j\frac{150}{4\pi} = j\frac{37.5}{\pi}$$

$$C_3 = j\frac{150}{6\pi} = j\frac{25}{\pi}; \qquad C_4 = j\frac{150}{8\pi} = j\frac{18.75}{\pi}$$

$$V_{\rm rms} = \sqrt{C_o^2 + 2\sum_{n=1}^{\infty}|C_n|^2}$$

$$= \sqrt{75^2 + \tfrac{2}{\pi^2}(75^2 + 37.5^2 + 25^2 + 18.75^2)}$$

$$= 85.13\,\text{V}$$

[c] $P = \frac{(85.13)^2}{25} = 289.88\,\text{W}$

$$\%\ \text{error} = \left(\frac{289.88}{300} - 1\right)(100) = -3.37\%$$

P 16.46 $C_n = \frac{1}{T}\int_0^{T/4} V_m e^{-jn\omega_o t}\,dt = \frac{V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_0^{T/4}\right]$

$$= \frac{V_m}{Tn\omega_o}[j(e^{-jn\pi/2} - 1)] = \frac{V_m}{2\pi n}\sin\frac{n\pi}{2} + j\frac{V_m}{2\pi n}\left(\cos\frac{n\pi}{2} - 1\right)$$

$$= \frac{V_m}{2\pi n}\left[\sin\frac{n\pi}{2} - j\left(1 - \cos\frac{n\pi}{2}\right)\right]$$

$$v(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_o t}$$

$$C_o = A_v = \frac{1}{T}\int_0^{T/4} V_m\,dt = \frac{V_m}{4}$$

or

$$C_o = \frac{V_m}{2\pi}\lim_{n\to 0}\left[\frac{\sin(n\pi/2)}{n} - j\frac{1-\cos(n\pi/2)}{n}\right]$$

$$= \frac{V_m}{2\pi}\lim_{n\to 0}\left[\frac{(\pi/2)\cos(n\pi/2)}{1} - j\frac{(\pi/2)\sin(n\pi/2)}{1}\right]$$

$$= \frac{V_m}{2\pi}\left[\frac{\pi}{2} - j0\right] = \frac{V_m}{4}$$

Note it is much easier to use $C_o = A_v$ than to use L'Hopital's rule to find the limit of 0/0.

P 16.47 **[a]** $C_o = a_v = \frac{(1/2)(T/2)V_m}{T} = \frac{V_m}{4}$

$$C_n = \frac{1}{T}\int_0^{T/2}\frac{2V_m}{T}te^{-jn\omega_o t}\,dt$$

$$= \frac{2V_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^t}(-jn\omega_o t - 1)\right]_0^{T/2}$$

$$= \frac{V_m}{2n^2\pi^2}[e^{-jn\pi}(-jn\pi + 1) - 1]$$

Since $e^{-jn\pi} = \cos n\pi$ we can write

$$C_n = \frac{V_m}{2\pi^2 n^2}(\cos n\pi - 1) + j\frac{V_m}{2n\pi}\cos n\pi$$

[b] $C_o = \frac{54}{4} = 13.5\,\text{V}$

$$C_{-1} = \frac{-54}{\pi^2} + j\frac{27}{\pi} = 10.19\underline{/122.48^\circ}\,\text{V}$$

$$C_1 = 10.19\underline{/-122.48^\circ}\,\text{V}$$

$$C_{-2} = -j\frac{13.5}{\pi} = 4.30\underline{/-90^\circ}\,\text{V}$$

$$C_2 = 4.30\underline{/90^\circ}\,\text{V}$$

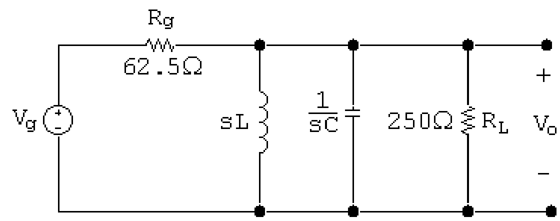
$$C_{-3} = \frac{-6}{\pi^2} + j\frac{9}{\pi} = 2.93\underline{/101.98^\circ}\,\text{V}$$

$$C_3 = 2.93\underline{/-101.98^\circ}\,\text{V}$$

$$C_{-4} = -j\frac{6.75}{\pi} = 2.15\underline{/-90^\circ}\,\text{V}$$

$$C_4 = 2.15\underline{/90^\circ}\,\text{V}$$

[**c**]



$$\frac{V_o}{250} + \frac{V_o}{sL} + V_o sC + \frac{V_o - V_g}{62.5} = 0$$

$$\therefore \qquad (250LCs^2 + 5sL + 250)V_o = 4sLV_g$$

$$\frac{V_o}{V_g} = H(s) = \frac{(4/250C)s}{s^2 + 1/50C + 1/LC}$$

$$H(s) = \frac{16{,}000s}{s^2 + 2 \times 10^4 s + 4 \times 10^{10}}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 2 \times 10^5 \text{ rad/s}$$

$$H(j0) = 0$$

$$H(j2 \times 10^5 k) = \frac{j8k}{100(1 - k^2) + j10k}$$

Therefore,

$$H_{-1} = 0.8\underline{/0^\circ}; \qquad H_1 = 0.8\underline{/0^\circ}$$

$$H_{-2} = \frac{-j16}{-300 - j20} = 0.0532\underline{/86.19^\circ}; \qquad H_2 = 0.0532\underline{/-86.19^\circ}$$

$$H_{-3} = \frac{-j24}{-800 - j30} = 0.0300\underline{/87.85^\circ}; \qquad H_2 = 0.0300\underline{/-87.85^\circ}$$

$$H_{-4} = \frac{-j32}{-1500 - j40} = 0.0213\underline{/88.47^\circ}; \qquad H_2 = 0.0213\underline{/-88.47^\circ}$$

The output voltage coefficients:

$$C_0 = 0$$

$$C_{-1} = (10.19\underline{/122.48^\circ})(0.8\underline{/0^\circ}) = 8.15\underline{/122.48^\circ} \text{ V}$$

$$C_1 = 8.15\underline{/-122.48^\circ} \text{ V}$$

$$C_{-2} = (4.30\underline{/-90^\circ})(0.05\underline{/86.19^\circ}) = 0.2287\underline{/-3.81^\circ} \text{ V}$$

$$C_2 = 0.2287\underline{/3.81^\circ} \text{ V}$$

$$C_{-3} = (2.93\underline{/101.98^\circ})(0.03\underline{/87.85^\circ}) = 0.0878\underline{/-170.17^\circ}\,\text{V}$$

$$C_3 = 0.0878\underline{/170.17^\circ}\,\text{V}$$

$$C_{-4} = (2.15\underline{/-90^\circ})(0.02\underline{/88.47^\circ}) = 0.0458\underline{/-1.53^\circ}\,\text{V}$$

$$C_4 = 0.0458\underline{/1.53^\circ}\,\text{V}$$

[d] $$V_{\text{rms}} \cong \sqrt{C_o^2 + 2\sum_{n=1}^{4}|C_n|^2} \cong \sqrt{2\sum_{n=1}^{4}|C_n|^2}$$

$$\cong \sqrt{2(8.15^2 + 0.2287^2 + 0.0878^2 + 0.0458^2} \cong 11.53\,\text{V}$$

$$P = \frac{(11.53)^2}{250} = 531.95\,\text{mW}$$

P 16.48 **[a]** $$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2}\left(\frac{2V_m}{T}t\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\left[\frac{4V_m^2}{T^2}\frac{t^3}{3}\right]_0^{T/2}}$$

$$= \sqrt{\frac{4V_m^2}{(3)(8)}} = \frac{V_m}{\sqrt{6}}$$

$$V_{\text{rms}} = \frac{54}{\sqrt{6}} = 22.05\,\text{V}$$

[b] From the solution to Problem 16.47

$C_0 = 13.5;$ $\qquad |C_3| = 2.93$

$|C_1| = 10.19;$ $\qquad |C_4| = 2.15$

$|C_2| = 4.30$

$$V_g(\text{rms}) \cong \sqrt{13.5^2 + 2(10.19^2 + 4.30^2 + 2.93^2 + 2.15^2)} \cong 21.29\,\text{V}$$

[c] $$\%\ \text{Error} = \left(\frac{21.29}{22.05} - 1\right)(100) = -3.44\%$$

P 16.49 **[a]** From Example 16.3 we have:

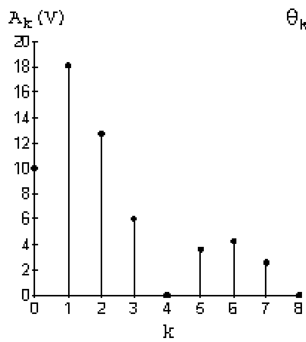
$$a_v = \frac{40}{4} = 10\,\text{V}, \qquad a_k = \frac{40}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

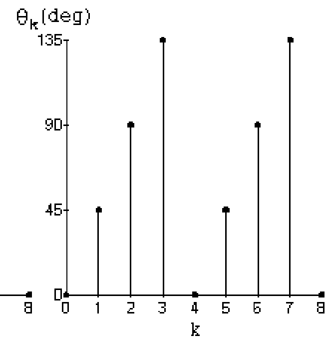
$$b_k = \frac{40}{\pi k}\left[1 - \cos\left(\frac{k\pi}{2}\right)\right], \qquad A_k\underline{/-\theta_k^\circ} = a_k - jb_k$$

$$A_1 = 18.01\,\text{V} \qquad \theta_1 = 45^\circ, \qquad A_2 = 12.73\,\text{V}, \qquad \theta_2 = 90^\circ$$

$$A_3 = 6\,\text{V}, \qquad \theta_3 = 135^\circ, \qquad A_4 = 0, \qquad A_5 = 3.6\,\text{V}, \qquad \theta_5 = 45^\circ$$

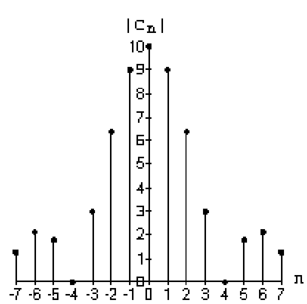
$$A_6 = 4.24\,\text{V}, \qquad \theta_6 = 90^\circ, \qquad A_7 = 2.57\,\text{V}, \qquad \theta_7 = 135^\circ; \qquad A_8 = 0$$

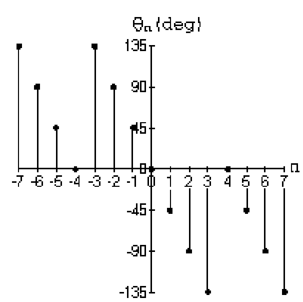




[b] $C_n = \dfrac{a_n - jb_n}{2}, \qquad C_{-n} = \dfrac{a_n + jb_n}{2} = C_n^*$

$C_0 = a_v = 10\,\text{V}$	$C_3 = 3\underline{/135^\circ}\,\text{V}$	$C_6 = 2.12\underline{/90^\circ}\,\text{V}$
$C_1 = 9\underline{/45^\circ}\,\text{V}$	$C_{-3} = 3\underline{/-135^\circ}\,\text{V}$	$C_{-6} = 2.12\underline{/-90^\circ}\,\text{V}$
$C_{-1} = 9\underline{/-45^\circ}\,\text{V}$	$C_4 = C_{-4} = 0$	$C_7 = 1.29\underline{/135^\circ}\,\text{V}$
$C_2 = 6.37\underline{/90^\circ}\,\text{V}$	$C_5 = 1.8\underline{/45^\circ}\,\text{V}$	$C_{-7} = 1.29\underline{/-135^\circ}\,\text{V}$
$C_{-2} = 6.37\underline{/-90^\circ}\,\text{V}$	$C_{-5} = 1.8\underline{/-45^\circ}\,\text{V}$	





P 16.50 **[a]** From the solution to Problem 16.33 we have

$$A_k = a_k - jb_k = \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1) + j\frac{I_m}{\pi k}$$

$$A_0 = 0.75 I_m = 3.75\,\text{A}$$

$$A_1 = \frac{5}{\pi^2}(-2) + j\frac{5}{\pi} = 1.89\underline{/122.48^\circ}\,\text{A}$$

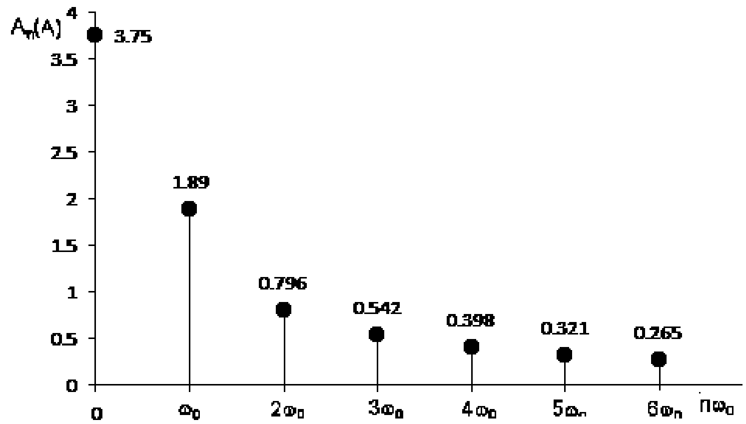
$$A_2 = j\frac{5}{2\pi} = 0.796\underline{/90^\circ}\,\text{A}$$

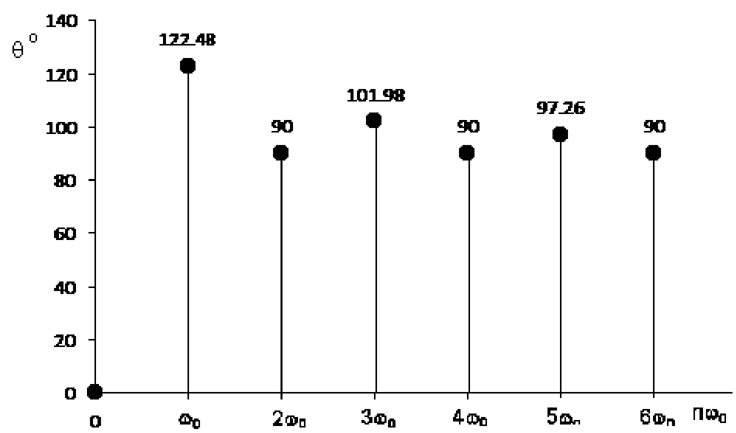
$$A_3 = \frac{5}{9\pi^2}(-2) + j\frac{5}{3\pi} = 0.542\underline{/101.98^\circ}\,\text{A}$$

$$A_4 = j\frac{5}{4\pi} = 0.398\underline{/90^\circ}\,\text{A}$$

$$A_5 = \frac{5}{25\pi^2}(-2) + j\frac{5}{5\pi} = 0.321\underline{/97.26^\circ}\,\text{A}$$

$$A_6 = j\frac{5}{6\pi} = 0.265\underline{/90^\circ}\,\text{A}$$





[b] $C_0 = A_0 = 3.75\,\text{mA}$

$$C_1 = \frac{1}{2}A_1\underline{/\theta_1} = 0.945\underline{/122.48^\circ}\,\text{A}$$

$$C_{-1} = 0.945\underline{/-122.48^\circ}\,\text{A}$$

$$C_2 = \frac{1}{2}A_2\underline{/\theta_2} = 0.398\underline{/90^\circ}\,\text{A}$$

$$C_{-2} = 0.398\underline{/-90^\circ}\,\text{A}$$

$$C_3 = \frac{1}{2}A_3\underline{/\theta_3} = 0.271\underline{/101.98^\circ}\,\text{A}$$

$$C_{-3} = 0.271\underline{/-101.98^\circ}\text{ A}$$

$$C_4 = \frac{1}{2}A_4\underline{/\theta_4} = 0.199\underline{/90^\circ}\text{ A}$$

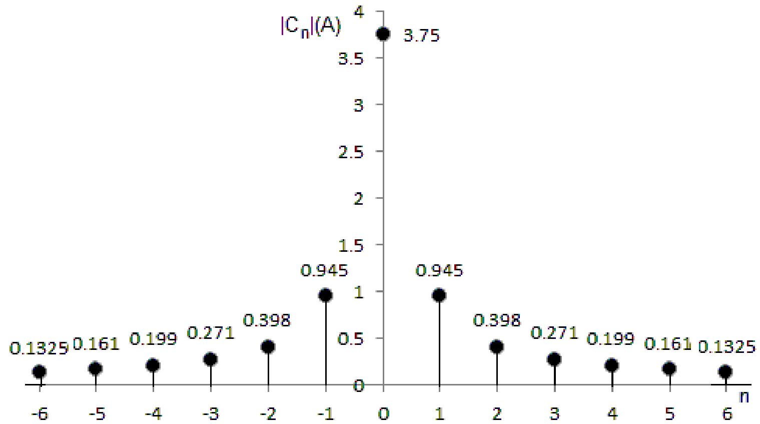
$$C_{-4} = 0.199\underline{/-90^\circ}\text{ A}$$

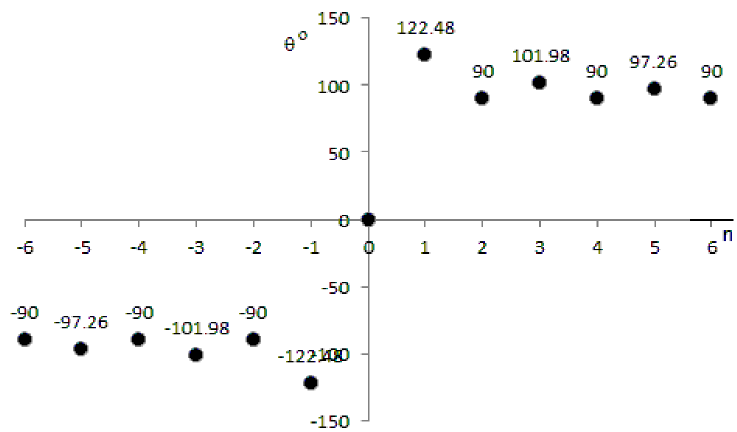
$$C_5 = \frac{1}{2}A_5\underline{/\theta_5} = 0.161\underline{/97.26^\circ}\text{ A}$$

$$C_{-5} = 0.161\underline{/-97.26^\circ}\text{ A}$$

$$C_6 = \frac{1}{2}A_6\underline{/\theta_6} = 0.1325\underline{/90^\circ}\text{ A}$$

$$C_{-6} = 0.1325\underline{/-90^\circ}\text{ A}$$





P 16.51 **[a]** $v = A_1\cos(\omega_o t - 90^\circ) + A_3\cos(3\omega_o t + 90^\circ)$

$$+A_5\cos(5\omega_o t - 90^\circ) + A_7\cos(7\omega_o t + 90^\circ)$$

$$v = -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$$

[b] $v(-t) = -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$

$\therefore\quad v(-t) = -v(t);\qquad$ odd function

[c] $$v(t - T/2) = A_1 \sin(\omega_o t - \pi) - A_3 \sin(3\omega_o t - 3\pi) + A_5 \sin(5\omega_o t - 5\pi) - A_7 \sin(7\omega_o t - 7\pi)$$
$$= -A_1 \sin \omega_o t + A_3 \sin 3\omega_o t - A_5 \sin 5\omega_o t + A_7 \sin 7\omega_o t$$

$\therefore\ v(t - T/2) = -v(t)$, yes, the function has half-wave symmetry

[d] Since the function is odd, with hws, we test to see if

$f(T/2 - t) = f(t)$

$$f(T/2 - t) = A_1 \sin(\pi - \omega_o t) - A_3 \sin(3\pi - 3\omega_o t) + A_5 \sin(5\pi - 5\omega_o t) - A_7 \sin(7\pi - 7\omega_o t)$$
$$= A_1 \sin \omega_o t - A_3 \sin 3\omega_o t + A_5 \sin 5\omega_o t - A_7 \sin 7\omega_o t$$

$\therefore\ f(T/2 - t) = f(t)$ and the voltage has quarter-wave symmetry

P 16.52 **[a]** $$i = 8.82 \cos(250t + 90^\circ) + 0.98 \cos(500t - 90^\circ) + 0.353 \cos(750t + 90^\circ) + 0.18 \cos(1000t - 90^\circ)\,\text{A}$$
$$= -8.82 \sin 250t + 0.98 \sin 500t - 0.353 \sin 750t + 0.18 \sin 1000t\,\text{A}$$

[b] $i(t) = -i(-t)$, Function is odd

[c] Yes, $A_0 = 0$, $A_n = 0$ for n even

[d] $$I_{\text{rms}} = \sqrt{\frac{8.82^2 + 0.98^2 + 0.353^2 + 0.18^2}{2}} = 6.28\,\text{A}$$

[e] $C_{-1} = 4.41\underline{/-90^\circ}\,\text{A}$; $C_1 = 4.41\underline{/90^\circ}\,\text{A}$

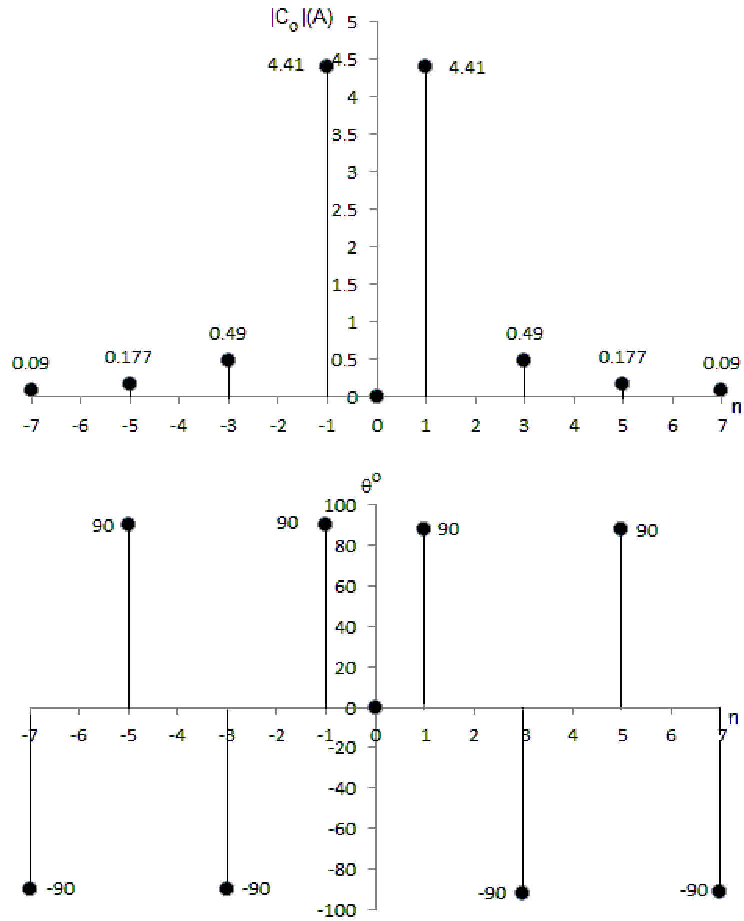
$C_{-3} = 0.49\underline{/90^\circ}\,\text{A}$; $C_3 = 0.49\underline{/-90^\circ}\,\text{A}$

$C_{-5} = 0.177\underline{/-90^\circ}\,\text{A}$; $C_5 = 0.177\underline{/90^\circ}\,\text{A}$

$C_{-7} = 0.09\underline{/90^\circ}\,\text{A}$; $C_7 = 0.09\underline{/-90^\circ}\,\text{A}$

$$i = j0.09e^{-j1000t} - j0.177e^{-j750t} + j0.49e^{-j500t} - j4.41e^{-j250t} + j4.41e^{j250t} - j0.49e^{j500t} + j0.177e^{j750t} - j0.09e^{j1000t}\,\text{A}$$

[f]



P 16.53 From Table 15.1 we have

$$H(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

After scaling we get

$$H'(s) = \frac{s^3}{(s+2500)(s^2+2500s+625\times 10^4)}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{400\pi}\times 10^6 = 5000 \text{ rad/s}$$

$$\therefore \quad H'(jn\omega_o) = \frac{-j8n^3}{(1+j2n)[(1-4n^2)+j2n]}$$

It follows that

$$H(j0) = 0$$

$$H(j\omega_o) = \frac{-j8}{(1+j2)(-3+j2)} = 0.992\underline{/60.255^\circ}$$

$$H(j2\omega_o) = \frac{-j64}{(1+j4)(-15+j4)} = 0.9999\underline{/28.97^\circ}$$

$$v_g(t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega_o t - \frac{2A}{\pi}\sum_{n=2,4,6,}^{\infty}\frac{\cos n\omega_o t}{n^2-1}$$

$$= 270 + 135\pi\sin\omega_o t - 180\cos 2\omega_o t - \cdots \text{ V}$$

$$\therefore \quad v_o = 0 + 420.84\sin(5000t + 60.255^\circ) - 179.98\cos(10{,}000t + 28.97^\circ) - \cdots \text{ V}$$

P 16.54 Using the technique outlined in Problem 16.18 we can derive the Fourier series for $v_g(t)$. We get

$$v_g(t) = 100 + \frac{800}{\pi^2}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^2}\cos n\omega_o t$$

The transfer function of the prototype second-order low pass Butterworth filter is

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}, \qquad \text{where } \omega_c = 1 \text{ rad/s}$$

Now frequency scale using $k_f = 2000$ to get $\omega_c = 2$ krad/s:

$$H(s) = \frac{4\times 10^6}{s^2 + 2000\sqrt{2}s + 4\times 10^6}$$

$$H(j0) = 1$$

$$H(j5000) = \frac{4\times 10^6}{(j5000)^2 + 2000\sqrt{2}(j5000)^2 + 4\times 10^6} = 0.1580\underline{/-146.04^\circ}$$

$$H(j15{,}000) = \frac{4\times 10^6}{(j15{,}000)^2 + 2000\sqrt{2}(j15{,}000)^2 + 4\times 10^6} = 0.0178\underline{/-169.13^\circ}$$

$$\mathbf{V}_{\text{dc}} = 100\,\text{V}$$

$$\mathbf{V}_{g1} = \frac{800}{n^2}\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_{g3} = \frac{800}{9\pi^2}\underline{/0^\circ}\,\text{V}$$

$$V_{odc} = 100(1) = 100\,\text{V}$$

$$\mathbf{V}_{o1} = \frac{800}{\pi^2}(0.1580\underline{/-146.04^\circ}) = 12.81\underline{/-146.04^\circ}\,\text{V}$$

$$\mathbf{V}_{o3} = \frac{800}{9\pi^2}(0.0178\underline{/-169.13^\circ}) = 0.16\underline{/-169.13^\circ}\,\text{V}$$

$$v_o(t) = 100 + 12.81\cos(5000t - 146.04^\circ) + 0.16\cos(15{,}000t - 169.13^\circ) + \cdots\,\text{V}$$

P 16.55 $v_g = \frac{2(2.5\pi)}{\pi} - \frac{4(2.5\pi)}{\pi}\frac{\cos 5000t}{4-1} = 5 - (10/3)\cos 5000t - \cdots\,\text{V}$

$$H(j0) = 1$$

$$H(j5000) = \frac{10^6}{(10^6 - 25\times10^6) + j5\sqrt{2}\times10^6} = 0.04\underline{/-163.58^\circ}$$

$$\therefore\quad v_o(t) = 5 - 0.1332\cos(5000t - 163.58^\circ) - \cdots\,\text{V}$$

P 16.56 **[a]** Let V_a represent the node voltage across R_2, then the node-voltage equations are

$$\frac{V_a - V_g}{R_1} + \frac{V_a}{R_2} + V_a sC_2 + (V_a - V_o)sC_1 = 0$$

$$(0 - V_a)sC_2 + \frac{0 - V_o}{R_3} = 0$$

Solving for V_o in terms of V_g yields

$$\frac{V_o}{V_g} = H(s) = \frac{\frac{-1}{R_1C_1}s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

It follows that

$$\omega_o^2 = \frac{R_1 + R_2}{R_1R_2R_3C_1C_2}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right)$$

Note that

$$H(s) = \frac{-\frac{R_3}{R_1}\left(\frac{C_2}{C_1+C_2}\right)\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \left(\frac{R_1+R_2}{R_1R_2R_3C_1C_2}\right)}$$

[b] For the given values of R_1, R_2, R_3, C_1, and C_2 we have

$$H(s) = \frac{-400s}{s^2 + 400s + 10^8}$$

$$v_g = \frac{(8)(2.25\pi^2)}{\pi^2} \sum_{n=1,3,5,}^{\infty} \frac{1}{n^2} \cos n\omega_o t$$

$$= 18 \left[\cos \omega_o t + \frac{1}{9} \cos 3\omega_o t + \frac{1}{25} \cos 5\omega_o t + \cdots \right] \text{ mV}$$

$$= [18 \cos \omega_o t + 2 \cos 3\omega_o t + 0.72 \cos 5\omega_o t + \cdots] \text{ mV}$$

$$\omega_o = \frac{2\pi}{0.2\pi} \times 10^3 = 10^4 \text{ rad/s}$$

$$H(jk10^4) = \frac{-400jk10^4}{10^8 - k^2 10^8 + j400k10^4} = \frac{-jk}{25(1 - k^2) + jk}$$

$$H_1 = -1 = 1\underline{/180^\circ}$$

$$H_3 = \frac{-j3}{-200 + j3} = 0.015\underline{/90.86^\circ}$$

$$H_5 = \frac{-j5}{-600 + j5} = 0.0083\underline{/90.48^\circ}$$

$$v_o = -18 \cos \omega_o t + 0.03 \cos(3\omega_o t + 90.86^\circ)$$

$$+ 0.006 \cos(5\omega_o t + 90.48^\circ) + \cdots \text{ mV}$$

[c] The fundamental frequency component dominates the output, so we expect the quality factor Q to be quite high.

[d] $\omega_o = 10^4$ rad/s and $\beta = 400$ rad/s. Therefore, $Q = 10{,}000/400 = 25$. We expect the output voltage to be dominated by the fundamental frequency component since the bandpass filter is tuned to this frequency!

P 16.57 **[a]** Using the equations derived in Problem 16.56(a),

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right) = \frac{400}{313}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 2000 \text{ rad/s}$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2} = 16 \times 10^8$$

[b] $$H(jn\omega_o) = \frac{-(400/313)(2000)jn\omega_o}{16\times 10^8 - n^2\omega_o^2 + j2000n\omega_o}$$

$$= \frac{-j(20/313)n}{(1-n^2) + j0.05n}$$

$$H(j\omega_o) = \frac{-j(20/313)}{j(0.050)} = -\frac{400}{313} = -1.28$$

$$H(j3\omega_o) = \frac{-j(20/313)(3)}{-8 + j0.15} = 0.0240\underline{/91.07^\circ}$$

$$H(j5\omega_o) = \frac{-j(100/313)}{-24 + j0.25} = 0.0133\underline{/90.60^\circ}$$

$$v_g(t) = \frac{4A}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin(n\pi/2)\cos n\omega_o t$$

$$A = 15.65\pi\ \text{V}$$

$$v_g(t) = 62.60\cos\omega_o t - 20.87\cos 3\omega_o t + 12.52\cos 5\omega_o t - \cdots$$

$$v_o(t) = -80\cos\omega_o t - 0.50\cos(3\omega_o t + 91.07^\circ) + 0.17\cos(5\omega_o t + 90.60^\circ) - \cdots\ \text{V}$$