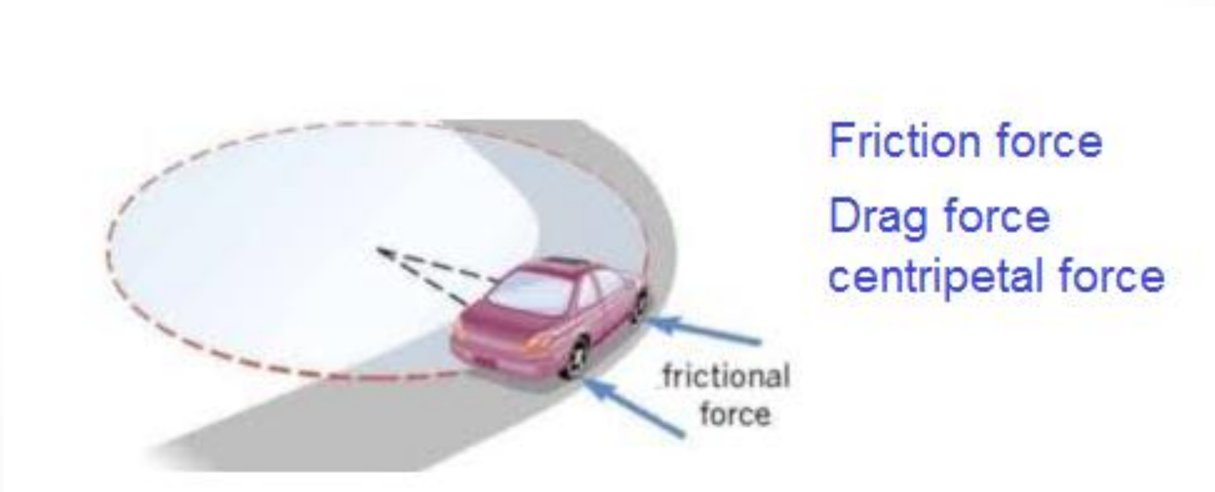
Chapter 6

Force and Motion II



6-1 Friction

- Friction forces are essential:
  - Picking things up
  - Walking, biking, driving anywhere
  - Writing with a pencil
  - Building with nails, weaving cloth

- But overcoming friction forces is also important:
  - Efficiency in engines
  - (20% of the gasoline used in an automobile goes to counteract friction in the drive train)
  - Roller skates, fans
  - Anything that we want to remain in motion

- Three experiments:
    - Slide a book across a counter. The book slows and stops, so there must be an acceleration parallel to the surface and opposite the direction of motion.
    - Push a book at a constant speed across the counter. There must be an equal and opposite force opposing you, otherwise the book would accelerate. Again the force is parallel to the surface and opposite the direction of motion.
    - Push a heavy box that does not move. To keep the box stationary, an equal and opposite force must oppose you. If you push harder, the opposing force must also increase to keep the box stationary. Keep pushing harder. Eventually the opposing force will reach a maximum, and the box will slide.

Types of friction:

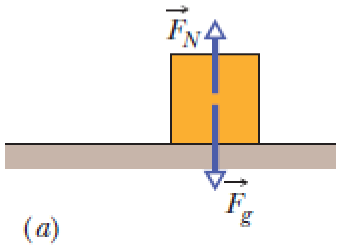
1. **The static frictional force**:

- The opposing force that prevents an object from moving. (Parallel to the surface)
- Can have any magnitude from 0 N up to a maximum
- Once the maximum is reached, forces are no longer in equilibrium and the object slides

2. **The kinetic frictional force**:

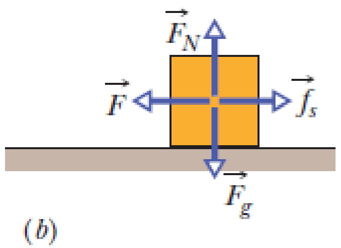
- The opposing force that acts on an object in motion (Parallel to the surface)
- Has only one value
- Generally smaller than the maximum static frictional force

There is no attempt at sliding. Thus, no friction and no motion.



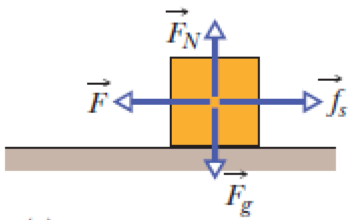
Frictional force = 0

Force $\vec{F}$ attempts sliding but is balanced by the frictional force. No motion.



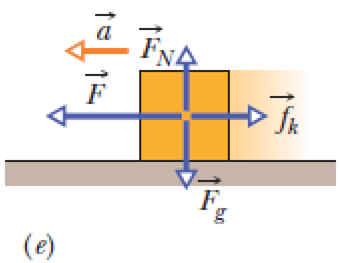
Frictional force = F

Force $\vec{F}$ is now stronger but is still balanced by the frictional force. No motion.



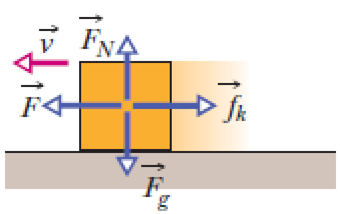
Frictional force = F

Finally, the applied force has overwhelmed the static frictional force. Block slides and accelerates.

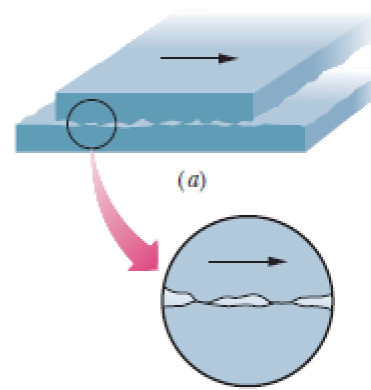


Weak kinetic frictional force

To maintain the speed, weaken force $\vec{F}$ to match the weak frictional force.



Same weak kinetic frictional force



surfaces are bumpy!

Cold-Welding

- Notes:

1. If the body does not move, then the applied force and frictional force balance along the direction parallel to the surface: equal in magnitude, opposite in direction

2. The magnitude of f_s has a maximum $f_{s,max}$ given by:

$$f_{s,\max} = \mu_s F_N$$

   where μ_s is the **coefficient of static friction**.

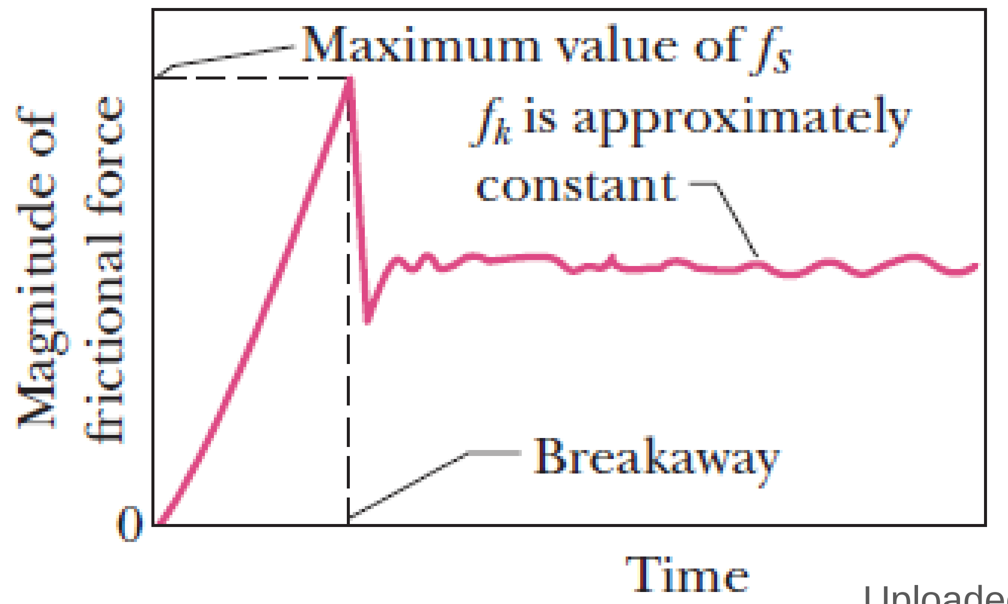
3. If the applied force increases past $f_{s,max}$, sliding begins.

4. Once sliding begins, the frictional force decreases to f_k given by:

$$f_k = \mu_k F_N$$

where μ_k is the **coefficient of kinetic friction**.

5. Magnitude F_N of the normal force measures how strongly the surfaces are pushed together



Checkpoint 1

A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it? (c) If the maximum value $f_{s,max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N? (d) If it is 12 N? (e) What is the magnitude of the frictional force in part (c)?

Answer: (a) 0

(b) 5 N

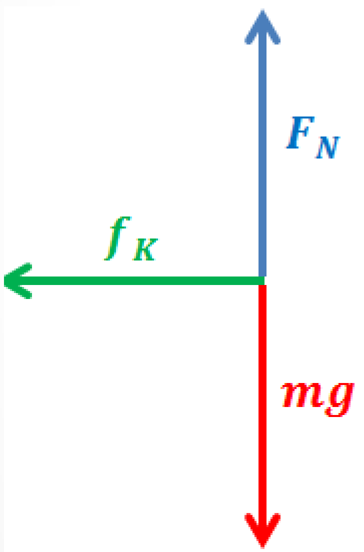
(c) no

(d) yes

(e) 8 N

6 A baseball player with mass $m = 79$ kg, sliding into second base, is retarded by a frictional force of magnitude 470 N. What is the coefficient of kinetic friction μ_k between the player and the ground?

The player free-body diagram:



$$f_k = \mu_k F_N$$

$$470\ N = \mu_k\ 790\ N$$

$$\boxed{\mu_k = 0.59}$$

No motion in Y- direction: $a_y = 0$; $F_{net,y} = 0$

$$F_N = mg = 790\ N$$

••19 A 12 N horizontal force $\vec{F}$ pushes a block weighing 5.0 N against a vertical wall (Fig. 6-26). The coefficient of static friction between the wall and the block is 0.60, and the coefficient of kinetic friction is 0.40. Assume that the block is not moving initially. (a) Will the block move? (b) In unit-vector notation, what is the force on the block from the wall?

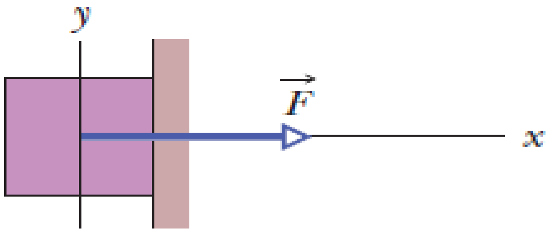
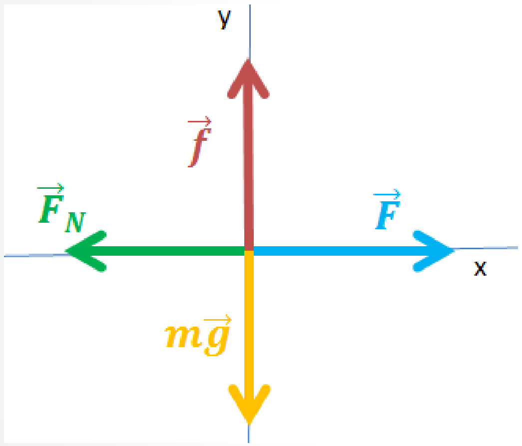


Figure 6-26 Problem 19.



If $f > f_{s,max} = \mu_s F_N$ →→ The block does slide on the wall

If $f < f_{s,max} = \mu_s F_N$ →→ The block does not slide on the wall

No motion on X − *axis*: $F = F_N$

$$f_{s,max} = \mu_s F_N = \mu_s F = (0.6)(12N) = 7.2\ N$$

The vertical component is $f - mg = 0$ →→ $f = mg = 5.0\ N < f_{s,max}$.

The block does not slide.

$$\vec{F}_{wall} = \vec{F}_N + \vec{f}$$

$$\vec{F}_{wall} = -F_N\hat{\imath} + f\hat{\jmath}$$

$$\vec{F}_{wall} = -(12N)\hat{\imath} + (5N)\hat{\jmath}$$

••24 A 4.10 kg block is pushed along a floor by a constant applied force that is horizontal and has a magnitude of 40.0 N. Figure 6-30 gives the block's speed v versus time t as the block moves along an x axis on the floor. The scale of the figure's vertical axis is set by v_s = 5.0 m/s. What is the coefficient of kinetic friction between the block and the floor?

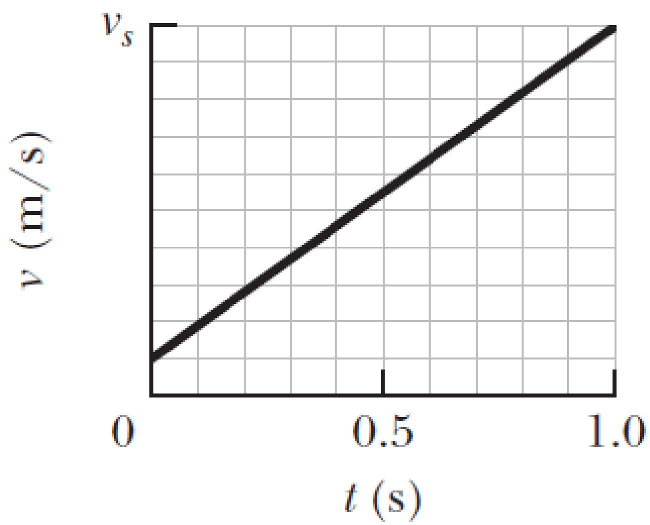
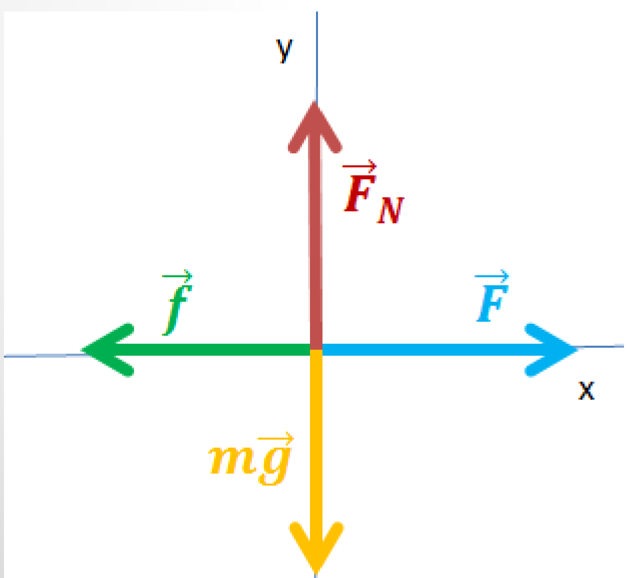


Figure 6-30 Problem 24.



The slope of v_x versus t equals $a_x = 4.5\,m/s^2$

$$F - f_k = ma_x = (4.1kg)(4.5\,m/s^2) = 18.45\text{ N}$$

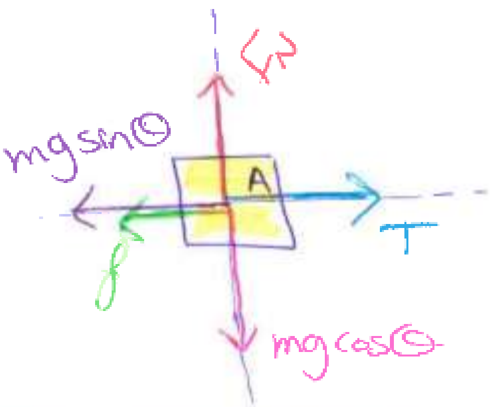
$$f_k = F - 18.45\text{ N} = 40.0\text{ N} - 18.45\text{ N} = 21.55\text{ N}$$

$$f_k = \mu_k F_N = 21.55\,N$$

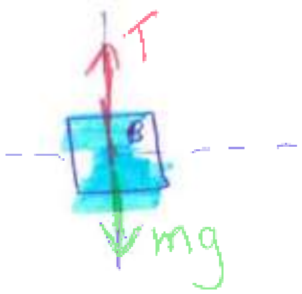
$$21.55\,N = \mu_k\,(4.1kg)(9.8\,m/s^2)$$

$$\mu_k = \frac{21.55\,N}{(4.1kg)(9.8\,m/s^2)} = 0.54$$

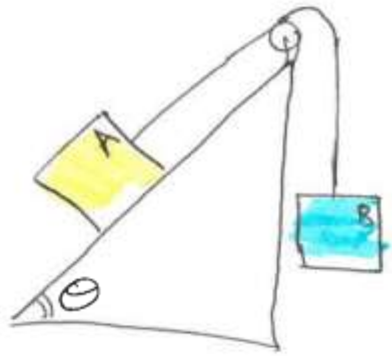
P-27 Body A weighs 102 N, and body B weighs 32 N. The coefficients of friction between A and the incline are $\mu_s = 0.56$ and $\mu_k = 0.25$. Angle θ is 40°, let the positive direction of an x-axis be up the incline. In unit vector notation, what is the acceleration of A if A is initially (a) at rest, (b) moving up the incline, and (c) moving down the incline?



Free Body-diagram for A



Free Body diagram B



(a) A is at Rest $[a=0]$

$F_{net,x,A} = T - f - m_A g\sin\theta = 0 \Rightarrow T = f + m_A g\sin\theta$ — ①

$F_{net,y,A} = F_N - m_A g\cos\theta = 0 \Rightarrow F_N = m_A g\cos\theta$ — ②

$F_{net,y,B} = m_B g - T = 0 \Rightarrow T = m_B g$ → ③

equation ① − equation ③

$f = m_B g - m_A g\sin\theta$

$= 32 - (102\sin 40°)$

$f = -33.6\ N$

$m_B g = 32\ N$
$m_A g\sin\theta = 65.6\ N$
$m_A g\sin\theta > 32$
the motion downhill

negative sign indicates that the force of the friction is uphill

$F_N = m_A g\cos\theta = 102\cos 40° = 78.1\ N$

$\cdot\ f_{s,max} = \mu_s F_N = 0.56(78.1) = 43.8\,N$

$f = 34\,N < f_{s,max} = 43.8\,N$

The Bodies remain at rest $\Rightarrow a = 0$

$mg = W$

(b) A is moving up the incline

$T - f_k - m_A g \sin\theta = m_A a$ — ①

$F_N - m_A g \cos\theta = 0 \Rightarrow F_N = m_A g \cos\theta$ — ②

$m_B g - T = m_B a$ — ③

$\Rightarrow a$ is the same for the two Bodies $(a_A = a_B)$

① + ③ $\Rightarrow m_B g - f_k - m_A g \sin\theta = (m_A + m_B)\, a$

$$a = \frac{m_B g - m_A g \sin\theta - f_k}{m_A + m_B} \quad ;\ f_k = \mu_k F_N$$

$$= \frac{32 - (102 \sin 40^\circ) - (\mu_k m_A g \cos\theta)}{(32 + 102)/9.8}$$

$a = -3.88\ m/s^2 \quad ; \quad \vec{a} = (-3.88\ \frac{m}{s^2})\hat{i}$

{ the block A is slowing down and B also is slowing down }

(c) A initially moving downward the hill

$T + f_k - m_A g \sin\theta = m_A a \ ;\ f_k = \mu_k F_N$

$F_N = m_A g \cos\theta$

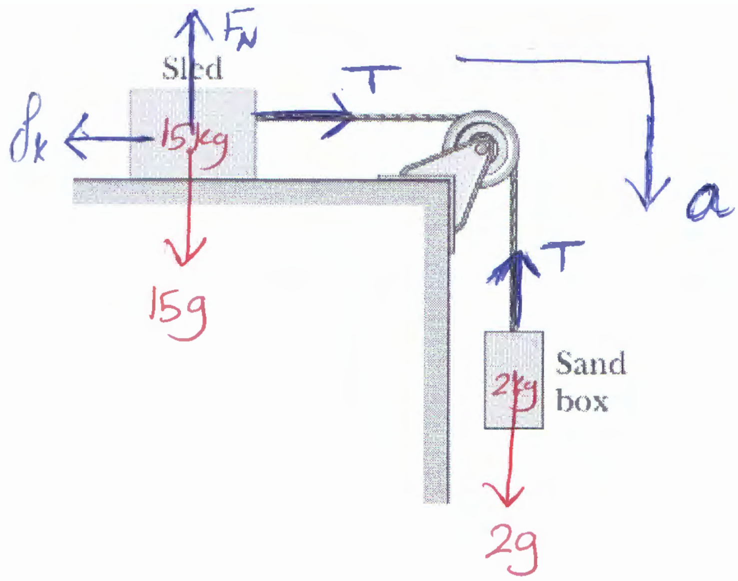
$m_B g - T = m_B a$

$$a = \frac{m_B g + \mu_k (m_A g \cos\theta) - m_A g \sin\theta}{m_A + m_B}$$

$$a = \frac{32 + (0.25(102 \cos 40^\circ)) - 102 \sin 40}{(32 + 102)/g} = -1.0\frac{m}{s^2}$$

{ $\vec{a} = (-1.0\frac{m}{s^2})\hat{i}$ $\Rightarrow$ the objects are speeding up, while the block A acceleration is again down hill }

In Fig. 6-27, a 15 kg sled is attached to a 2.0 kg sand box by a string of negligible mass, wrapped over a pulley of negligible mass and friction. The coefficient of kinetic friction between the sled and table top is 0.040. Find (a) the acceleration of the sled and (b) the tension of the string.



Sled ⇒ No motion in Y-direction $\boxed{F_N = 15g}$

$T - f_k = 15a$, $f_k = \mu_k F_N$

$f_k = 0.04\,(15\,kg)(9.8\,m/s^2)$

$\boxed{f_k = 5.88\,N}$

$T - (5.88\,N) = 15\,a$ — ①

Sand Box ⇒ $2g - T = 2a$ ⇒ $(19.6\,N) - T = 2a$ — ②

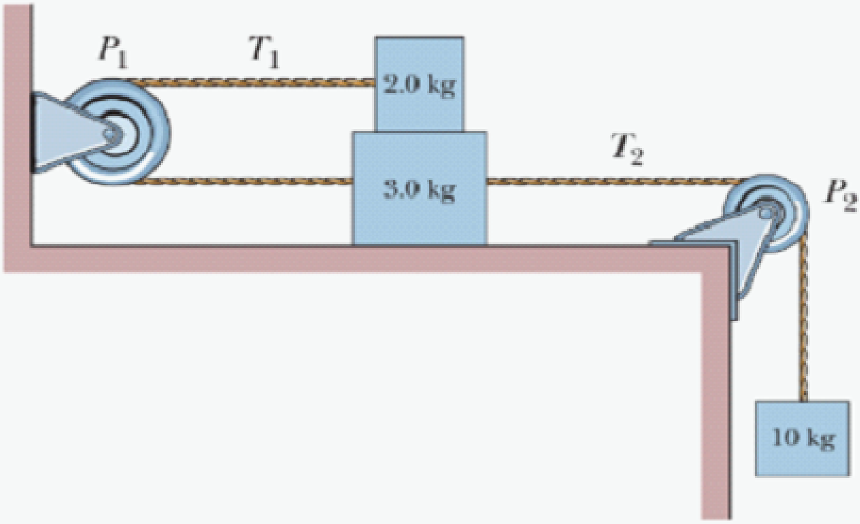
$a = 0.807\,m/s^2$

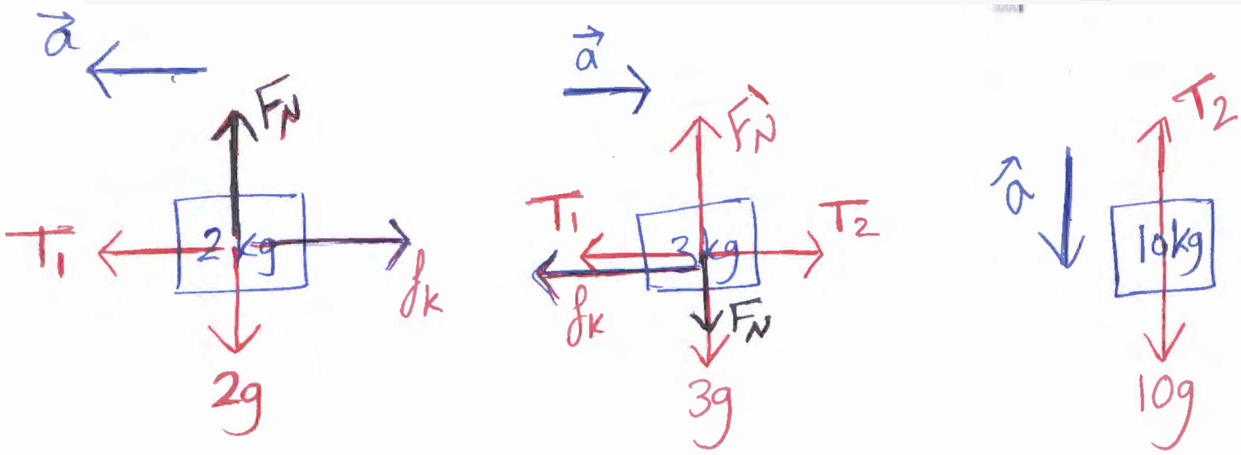
$T = 18\,N$

$\vec{a}_{sled} = +(0.81\,m/s^2)$

$\vec{a}_{sand\,box} = -(0.81\,m/s^2)$

Example: In Fig. 6-13, a 2.0 kg block is placed on top of a 3.0 kg block, which lies on a frictionless surface. The coefficient of kinetic friction between the two blocks is 0.30; they are connected via a pulley and a string. A hanging block of mass 10 kg is connected to the 3.0 kg block via another pulley and string. Both strings have negligible mass and both pulleys are frictionless and have negligible mass. When the assembly is released, what are (a) the acceleration magnitude of the blocks, (b) the tension in string 1, and (c) the tension in string 2?





Note $f_k = \mu_k F_N$, F_N (from the surface between two bodies)

2kg $\Rightarrow$ $T_1 - f_k = 2a$ —— ①

3 kg $\Rightarrow$ $T_2 - T_1 - f_k = 3a$ —— ②

10 kg $\Rightarrow$ $10g - T_2 = 10a$ —— ③

$f_k = \mu_k F_N$

$f_k = \mu_k (2g)$

$f_k = 0.3(2)(9.8)$

$f_k = 5.88\,N$

Add (1) + (2) + (3)

$$10g - 2f_k = 15a$$

$$10\,kg(9.8\,m/s^2) - 2(5.88)N = (15\,kg)\,a$$

$$\boxed{a = 5.75\,m/s^2}$$

⇒ To find T_1, use equation (1)

$$T_1 = 2a + f_k$$

$$T_1 = 2(5.75) + 5.88$$

$$\boxed{T_1 = 17.38\,N}$$

⇒ To find T_2, use equation (3)

$$10g - T_2 = 10a$$

$$T_2 = 10(g - a) = 10(9.8 - 5.75)$$

$$\boxed{T_2 = 40.5\,N}$$

•••35 ILW The two blocks ($m = 16$ kg and $M = 88$ kg) in Fig. 6-38 are not attached to each other. The coefficient of static friction between the blocks is $\mu_s = 0.38$, but the surface beneath the larger block is frictionless. What is the minimum magnitude of the horizontal force $\vec{F}$ required to keep the smaller block from slipping down the larger block?

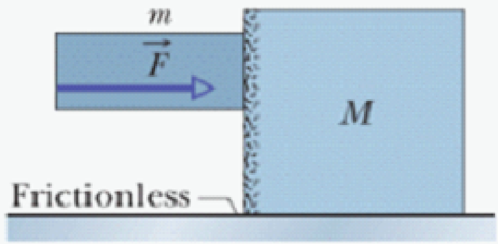
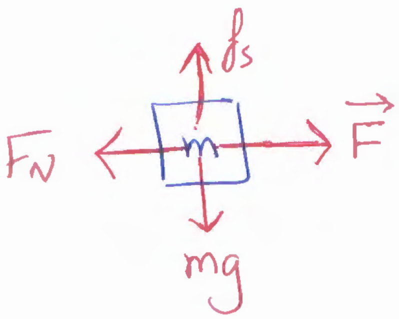
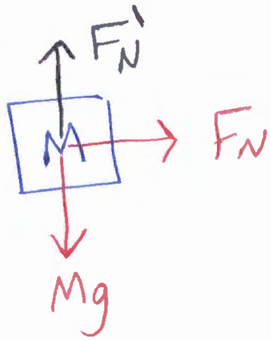


Figure 6-38 Problem 35.





System equation

$$F = (m+M)a$$

$$a = \left(\frac{F}{m+M}\right)$$

Block (m) $\Rightarrow$ $F - F_N = ma$

$$F - F_N = m\frac{F}{m+M}$$

$$F_N = F\left[1 - \frac{m}{m+M}\right]$$

Block [M]

$$F_N = Ma$$

$$F_N = \frac{MF}{m+M}$$

No slipping $\Rightarrow$ $mg = f_s = \mu_s F_N$

[minimum F]

$$f_s = \mu_s F\left[1 - \frac{m}{m+M}\right]$$

$$\Rightarrow \quad F = \frac{F_N}{\left(1 - \frac{m}{m+M}\right)} = \frac{mg}{\mu_s\left[1 - \frac{m}{m+M}\right]}$$

$$F = \frac{(16\text{ kg})(9.8\text{ m/s}^2)}{0.38\left[1 - \frac{16\text{ kg}}{(16+88)\text{ kg}}\right]} = 4.9\times10^2\text{ N}$$

6-2 The Drag Force and Terminal Speed

- A **fluid** is anything that can flow (gas or liquid)
- When there is relative velocity between fluid and an object, the body experiences a **Drag force $\vec{D}$**:
  - That opposes the relative motion
  - Points along the direction in which the fluid flows relative to the body.
- The drag force is: $$D = \frac{1}{2}C\rho A v^2$$

where:

- v is the relative velocity
- ρ is the air density (mass/volume)
- C is the experimentally determined drag coefficient
- A is the effective cross-sectional area of the body (the area taken perpendicular to the relative velocity)

Terminal speed:

- When a blunt object has fallen far enough through air, the magnitudes of the drag force and the gravitational force on the body become equal. The body then falls at a constant Terminal Speed v_t given by:

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

As the cat's speed increases, the upward drag force increases until it balances the gravitational force.

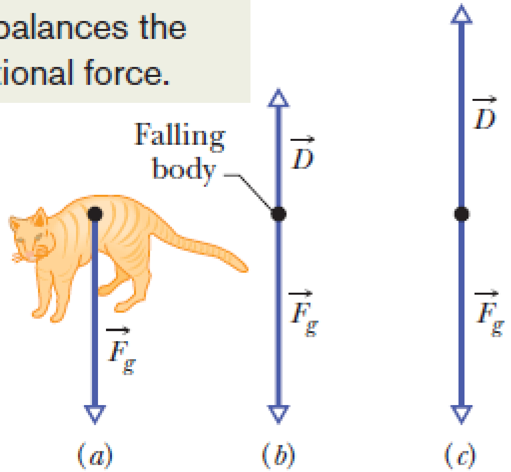


Figure 6-6 The forces that act on a body falling through air: (*a*) the body when it has just begun to fall and (*b*) the free-body diagram a little later, after a drag force has developed. (*c*) The drag force has increased until it balances the gravitational force on the body. The body now falls at its constant terminal speed.

(b) $D - F_g = ma$

(c) $\frac{1}{2}C\rho A v_t^2 - F_g = 0$

➢Skydiving:

- Terminal speed can be increased by reducing A
- Terminal speed can be decreased by increasing A
- Skydivers use this to control descent

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

Steve Fitchett/Taxi/Getty Images

Figure 6-7 Sky divers in a horizontal "spread eagle" maximize air drag.

Sample Problem 6.03 Terminal speed of falling raindrop

A raindrop with radius $R = 1.5$ mm falls from a cloud that is at height $h = 1200$ m above the ground. The drag coefficient C for the drop is 0.60. Assume that the drop is spherical throughout its fall. The density of water ρ_w is 1000 kg/m^3, and the density of air ρ_a is 1.2 kg/m^3.

$$v_t = \sqrt{\frac{2F_g}{C\rho_a A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}}$$

$$= \sqrt{\frac{(8)(1.5 \times 10^{-3}\ \text{m})(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)}{(3)(0.60)(1.2\ \text{kg/m}^3)}}$$

$$= 7.4\ \text{m/s} \approx 27\ \text{km/h}.$$

$$F_g = V\rho_w g = \tfrac{4}{3}\pi R^3 \rho_w g.$$

With no drag force to reduce the drop's speed during the fall, the drop would fall with the constant free-fall acceleration g, so the constant-acceleration equations of Table 2-1 apply.

$$v = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(1200\ \text{m})}$$

$$= 153\ \text{m/s} \approx 550\ \text{km/h}.$$

In this case; the speed is close to that of a bullet from large-caliber handgun!

•36 The terminal speed of a sky diver is 160 km/h in the spread-eagle position and 310 km/h in the nosedive position. Assuming that the diver's drag coefficient C does not change from one position to the other, find the ratio of the effective cross-sectional area A in the slower position to that in the faster position.

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

EAGLE position

NOSEDIVE position

Slower position, smaller velocity and larger area: EAGLE position.
Faster position, larger velocity and smaller area: NOSEDIVE position.

$$\frac{A_{slower}}{A_{faster}} = \frac{(310\ km/h)^2}{(160\ km/h)^2} = 3.75$$

6-3 Uniform Circular Motion

- If a particle moves in a circle or a circular arc of radius R at constant speed v, the particle is said to be in uniform circular motion. It then has a centripetal acceleration with magnitude given by

$$a = \frac{v^2}{R} \qquad \text{(centripetal acceleration)}$$

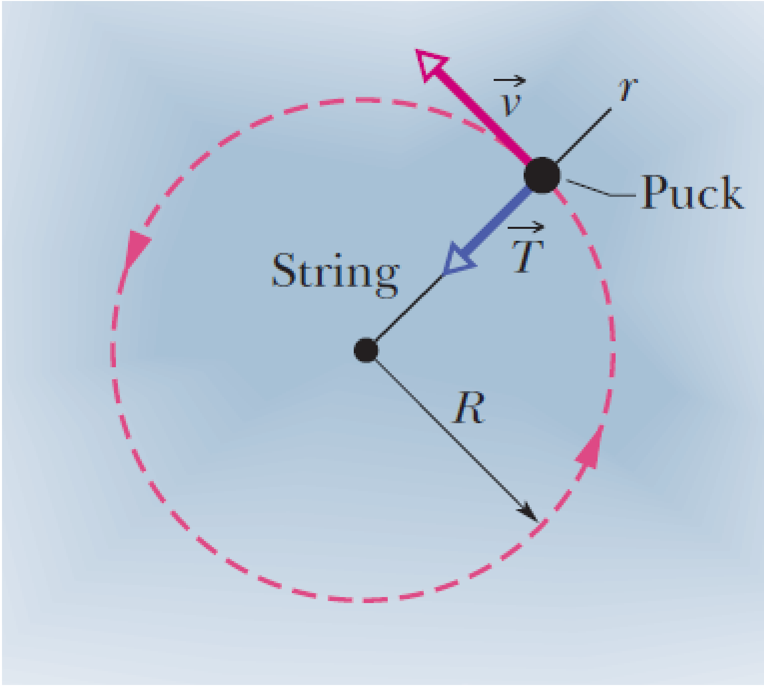
- This acceleration is due to a net centripetal force on the particle, with magnitude given by

$$F = m\frac{v^2}{R} \qquad \text{(magnitude of centripetal force)}$$

where m is the particle's mass. The centripetal acceleration and a net centripetal force are directed toward the center of curvature of the particle's path - Radially inward.

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

- For the puck on a string, the string tension supplies the centripetal force necessary to maintain circular motion:



The puck moves in uniform circular motion only because of a toward-the-center force.

Checkpoint 2

As every amusement park fan knows, a Ferris wheel is a ride consisting of seats mounted on a tall ring that rotates around a horizontal axis. When you ride in a Ferris wheel at constant speed, what are the directions of your acceleration $\vec{a}$ and the normal force $\vec{F}_N$ on you (from the always upright seat) as you pass through (a) the highest point and (b) the lowest point of the ride? (c) How does the magnitude of the acceleration at the highest point compare with that at the lowest point? (d) How do the magnitudes of the normal force compare at those two points?

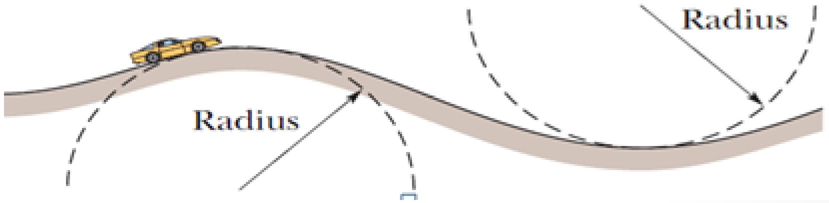
(a) accelerated downward, F_N upward

(b) accelerated upward, F_N upward

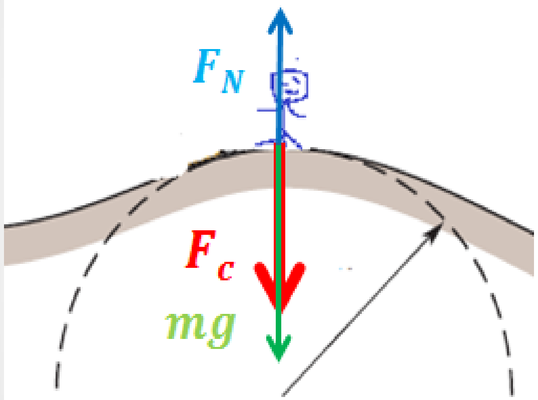
(c) the magnitudes must be equal for the motion to be uniform

(d) F_N is greater in (b) than in (a)

••49 GO In Fig. 6-39, a car is driven at constant speed over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is 0. The driver's mass is 70.0 kg. What is the magnitude of the normal force on the driver from the seat when the car passes through the bottom of the valley?



At the top of the hill:

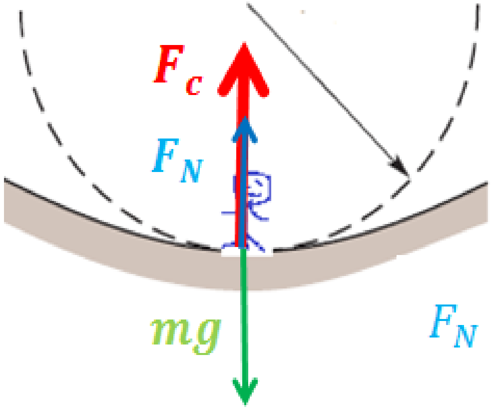


$$F_c = m\frac{v^2}{R} = mg - F_N$$

$$F_N = 0$$

$$v = \sqrt{gR}$$

At the bottom of the hill:



$$F_c = m\frac{v^2}{R} = F_N - mg$$

$$F_N = m\frac{v^2}{R} + mg$$

The velocity is constant in magnitude

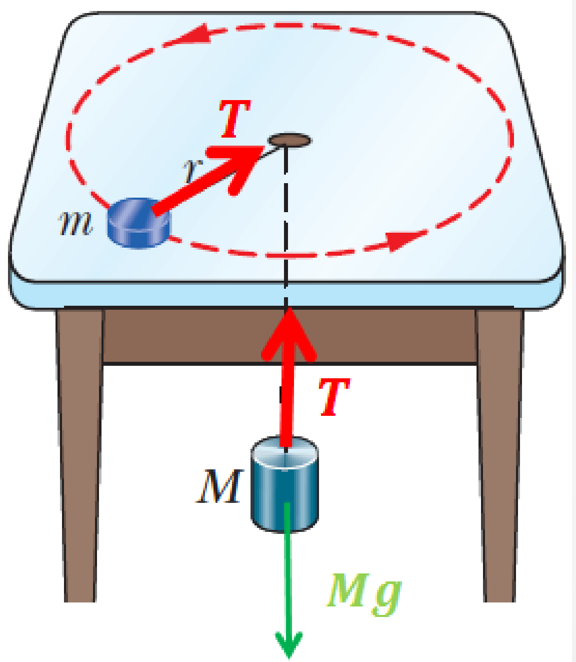
$$v = \sqrt{gR}$$

$$F_N = m\frac{v^2}{R} + mg = m\frac{gR}{R} + mg = 2mg$$

$$F_N = 2(70\ kg)(9.8\ m/s^2)$$

$$F_N = 1.372 \times 10^3\ N$$

••57 GO A puck of mass $m = 1.50$ kg slides in a circle of radius $r = 20.0$ cm on a frictionless table while attached to a hanging cylinder of mass $M = 2.50$ kg by means of a cord that extends through a hole in the table (Fig. 6-43). What speed keeps the cylinder at rest?



Cylinder at rest: $a = 0$

$$T = Mg \ldots\ldots(1)$$

Puck of mass m moves in a circle:

$$F_c = m\frac{v^2}{r} = T \ldots\ldots\ldots(2)$$

$$m\frac{v^2}{r} = Mg$$

$$v = \sqrt{\frac{gMr}{m}} = \sqrt{\frac{(9.8\,m/s^2)(2.5kg)(0.2m)}{1.5\,kg}} = 1.8\,m/s$$

Sample Problem 6.04 Vertical circular loop, Diavolo

Assuming that the loop is a circle with radius R = 2.7m, what is the least speed v that Diavolo and his bicycle could have at the top of the loop to remain in contact with it there?

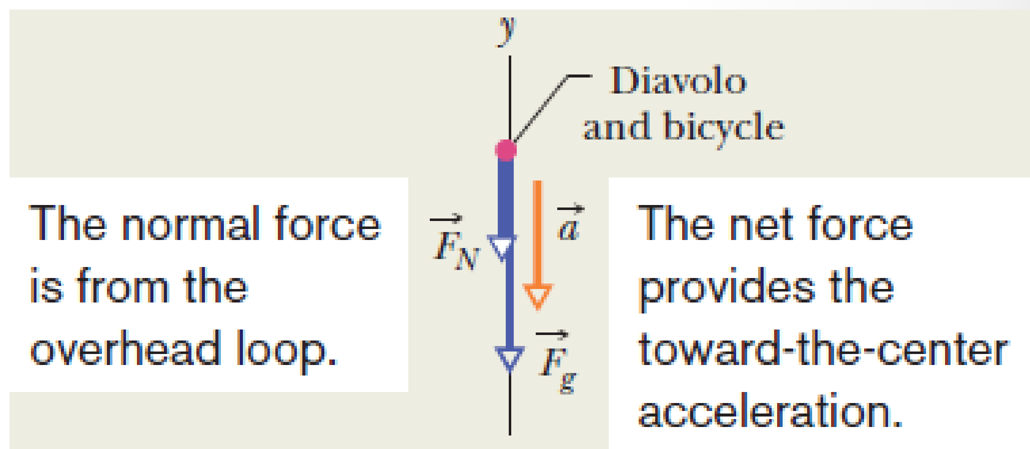
$$F_{\text{net},y} = ma_y$$

$$-F_N - F_g = m(-a)$$

$$-F_N - mg = m\left(-\frac{v^2}{R}\right)$$

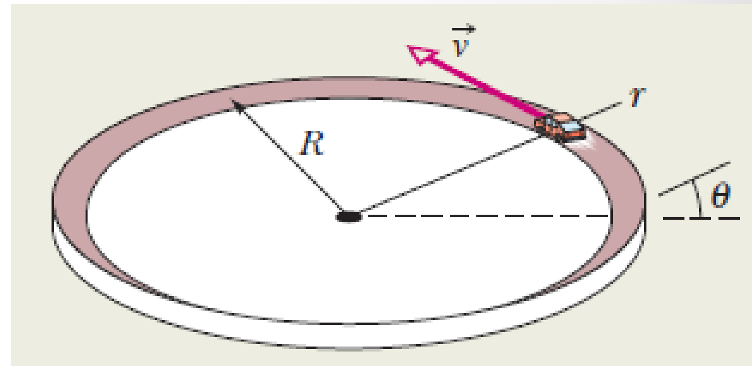
The *verge of losing contact* with the loop (falling away from the loop), which means that $F_N = 0$ at the top of the loop.

$$v = \sqrt{gR} = \sqrt{(9.8\ \text{m/s}^2)(2.7\ \text{m})}$$
$$= 5.1\ \text{m/s}.$$



Sample Problem 6.06 Car in banked circular turn

A car of mass m as it moves at a constant speed v of 20 m/s around a banked circular track of radius R 190 m. (It is a normal car, rather than a race car, which means that any vertical force from the passing air is negligible.) If the frictional force from the track is negligible, what bank angle θ prevents sliding?



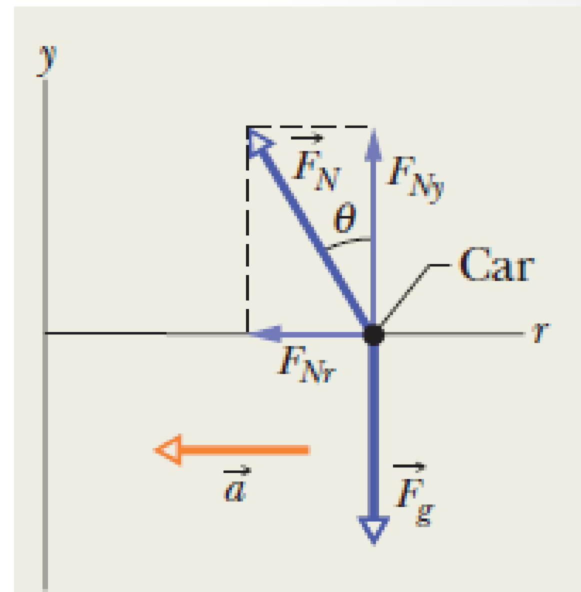
$$F_{\text{net},r} = ma_r$$

$$-F_N \sin\theta = m\left(-\frac{v^2}{R}\right)$$

$$F_{\text{net},y} = ma_y$$

$$F_N \cos\theta - mg = m(0)$$

$$F_N \cos\theta = mg$$



$$\theta = \tan^{-1}\frac{v^2}{gR}$$

$$= \tan^{-1}\frac{(20\text{ m/s})^2}{(9.8\text{ m/s}^2)(190\text{ m})} = 12^\circ$$

••53 An old streetcar rounds a flat corner of radius 9.1 m, at 16 km/h. What angle with the vertical will be made by the loosely hanging hand straps?

$$v = 16\,\frac{km}{h} = 4.44\,m/s$$

$$F_c = T\sin\theta = \frac{mv^2}{R}$$

$$T\cos\theta = mg$$

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right) = 12^\circ$$