

## Chapter 13: Analysis of variance and Experimental Design (ANOVA)

### 13.1: An introduction to analysis of variance

.. To measure how much employees at these plants know about total quality management a Random sample of six employees was selected by each plant and given a quality awareness examination. The examination scores obtained for these 18 employees are listed in table 13.1

Sample size is six Random

let  $\mu_1$  = mean examination score for population 1.

$\mu_2$  = mean examination score for population 2.

$\mu_3$  = mean examination score for population 3.

Although we will never know the actual values of  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  we want to use

the sample results to test the following hypotheses:

$$H_0: \mu_1 = \mu_2 = \mu_3$$

$H_1$ : Not all population means are equal

Table 13.1: Examination scores for 18 employees.

observation	Factors		
	plant 1	plant 2	plant 3
1	85	71	59
2	75	75	64
3	82	73	62
4	76	74	69
5	71	69	75
6	85	82	67
sample mean $\bar{x}$	79	74	66
sample variance $s^2$	34	20	32
sample standard deviation $s$	5.83	4.47	5.66

By calculator

pop. = Herdminds  
172, 171

Continuous:

Response variable or dependent  $\rightarrow$  score

independent variable or factor  $\rightarrow$  location

Treatments or population  $\rightarrow$  the values of a factor selected for investigation are referred to as levels of the factor.

$\rightarrow$  in table 13.1 the three treatments: Ayr, Dusseldorf and Stockholm.

### \* Assumptions for analysis of variance:

Three assumptions are required to use analysis of variance.

1. For each population, the response variable is normally distributed.

Implication: In the NCP example, the examination scores (response variable) must be normally distributed at each plant.

2. The variance of the response variable ( $\sigma^2$ ) is the same for all of the populations.

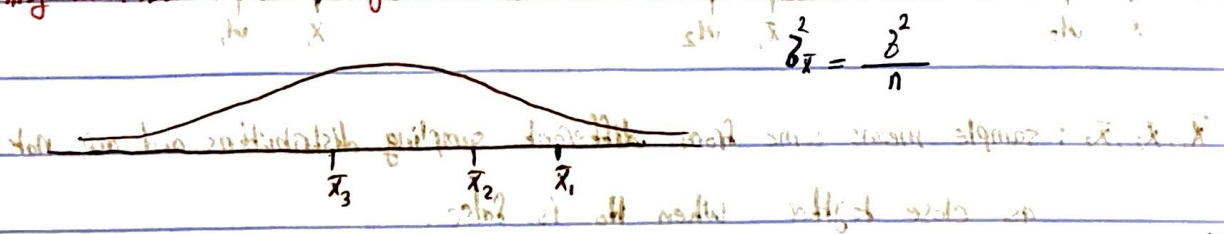
Implication: In the NCP example, the variance of examination scores must be the same for all three plants.

3. The observations must be independent.

Implication: In the NCP examples, the examination score for each employee must be independent of the examination score for any other employee.

→ A conceptual overview. Book.

→ sampling distribution of  $\bar{X}$  given  $H_0$  is true.



$\bar{x}_1, \bar{x}_2, \bar{x}_3$ : sample means are close together because there is only one sampling dis. when  $H_0$  is true.

overall mean  
73  
mean of means  
79, 74, 66

استimate of  $\sigma_{\bar{x}}^2$  :  $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$  : 1)  $n$  is 3 treatments

$$S_{\bar{x}}^2 = \frac{(79-73)^2 + (74-73)^2 + (66-73)^2}{3-1} = \frac{86}{2} = 43$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \rightarrow \sigma^2 = n \sigma_{\bar{x}}^2$$

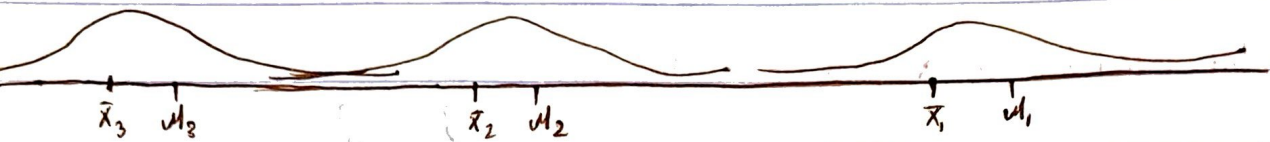
استimate of variance

Hence, Estimate of  $\sigma^2 = n$  (estimate of  $\sigma_{\bar{x}}^2$ )

$$= n S_{\bar{x}}^2$$
$$= 6 (43) = 258$$

The result,  $n S_{\bar{x}}^2 = 258$  is referred to as the between-treatments estimate of  $\sigma^2$ .

→ sampling distributions of  $\bar{X}$  given  $H_0$  is false.



$\bar{x}_1, \bar{x}_2, \bar{x}_3$  : sample means come from different sampling distributions and are not as close together when  $H_0$  is false.

work  
all work is given in one place in small amount of time (close together) and remain separate:  $\bar{x}_1, \bar{x}_2, \bar{x}_3$

Within treatments estimate of  $\sigma^2 = \frac{34 + 20 + 32}{3} = 28.67$ .

في الكتاب في تكملة 9 شرح

(ملا السكتين بحكي بشكل عام، السكتين الجاي بالتحليل)