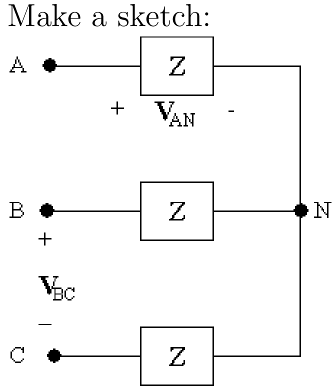
11
Balanced Three-Phase Circuits

Assessment Problems

AP 11.1 Make a sketch:



We know $\mathbf{V}_{\text{AN}}$ and wish to find $\mathbf{V}_{\text{BC}}$. To do this, write a KVL equation to find $\mathbf{V}_{\text{AB}}$, and use the known phase angle relationship between $\mathbf{V}_{\text{AB}}$ and $\mathbf{V}_{\text{BC}}$ to find $\mathbf{V}_{\text{BC}}$.

$$\mathbf{V}_{\text{AB}} = \mathbf{V}_{\text{AN}} + \mathbf{V}_{\text{NB}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{BN}}$$

Since $\mathbf{V}_{\text{AN}}$, $\mathbf{V}_{\text{BN}}$, and $\mathbf{V}_{\text{CN}}$ form a balanced set, and $\mathbf{V}_{\text{AN}} = 240\angle{-30^\circ}\,\text{V}$, and the phase sequence is positive,

$$\mathbf{V}_{\text{BN}} = |\mathbf{V}_{\text{AN}}|\angle{\angle\mathbf{V}_{\text{AN}} - 120^\circ} = 240\angle{-30^\circ - 120^\circ} = 240\angle{-150^\circ}\,\text{V}$$

Then,

$$\mathbf{V}_{\text{AB}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{BN}} = (240\angle{-30^\circ}) - (240\angle{-150^\circ}) = 415.46\angle{0^\circ}\,\text{V}$$

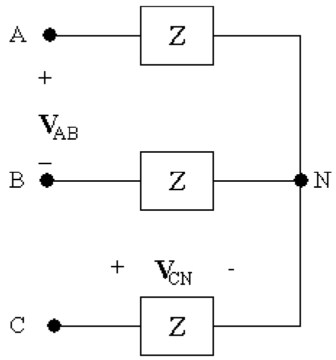
Since $\mathbf{V}_{\text{AB}}$, $\mathbf{V}_{\text{BC}}$, and $\mathbf{V}_{\text{CA}}$ form a balanced set with a positive phase sequence, we can find $\mathbf{V}_{\text{BC}}$ from $\mathbf{V}_{\text{AB}}$:

$$\mathbf{V}_{\text{BC}} = |\mathbf{V}_{\text{AB}}|\angle{(\angle\mathbf{V}_{\text{AB}} - 120^\circ)} = 415.69\angle{0^\circ - 120^\circ} = 415.69\angle{-120^\circ}\,\text{V}$$

Thus,

$\mathbf{V}_{\text{BC}} = 415.69\angle{-120^\circ}$ V

AP 11.2 Make a sketch:



We know $\mathbf{V}_{\text{CN}}$ and wish to find $\mathbf{V}_{\text{AB}}$. To do this, write a KVL equation to find $\mathbf{V}_{\text{BC}}$, and use the known phase angle relationship between $\mathbf{V}_{\text{AB}}$ and $\mathbf{V}_{\text{BC}}$ to find $\mathbf{V}_{\text{AB}}$.

$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} + \mathbf{V}_{\text{NC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}}$

Since $\mathbf{V}_{\text{AN}}$, $\mathbf{V}_{\text{BN}}$, and $\mathbf{V}_{\text{CN}}$ form a balanced set, and $\mathbf{V}_{\text{CN}} = 450\angle{-25^\circ}$ V, and the phase sequence is negative,

$\mathbf{V}_{\text{BN}} = |\mathbf{V}_{\text{CN}}|\angle{\angle\mathbf{V}_{\text{CN}} - 120^\circ} = 450\angle{-23^\circ - 120^\circ} = 450\angle{-145^\circ}$ V

Then,

$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}} = (450\angle{-145^\circ}) - (450\angle{-25^\circ}) = 779.42\angle{-175^\circ}$ V

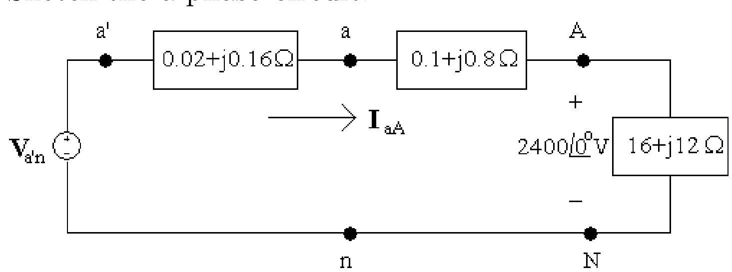
Since $\mathbf{V}_{\text{AB}}$, $\mathbf{V}_{\text{BC}}$, and $\mathbf{V}_{\text{CA}}$ form a balanced set with a negative phase sequence, we can find $\mathbf{V}_{\text{AB}}$ from $\mathbf{V}_{\text{BC}}$:

$\mathbf{V}_{\text{AB}} = |\mathbf{V}_{\text{BC}}|\angle{\angle\mathbf{V}_{\text{BC}} - 120^\circ} = 779.42\angle{-295^\circ}$ V

But we normally want phase angle values between $+180^\circ$ and -180°. We add 360° to the phase angle computed above. Thus,

$\mathbf{V}_{\text{AB}} = 779.42\angle{65^\circ}$ V

AP 11.3 Sketch the a-phase circuit:



[a] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that $\mathbf{I}_{\rm aA}$, $\mathbf{I}_{\rm bB}$, and $\mathbf{I}_{\rm cC}$ form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, $\mathbf{V}_{\rm AN}$, has a phase angle of 0°.

$$2400\underline{/0^\circ} = \mathbf{I}_{\rm aA}(16 + j12)$$

so

$$\mathbf{I}_{\rm aA} = \frac{2400\underline{/0^\circ}}{16 + j12} = 96 - j72 = 120\underline{/-36.87^\circ}\,\text{A}$$

With an acb phase sequence,

$$\underline{/\mathbf{I}_{\rm bB}} = \underline{/\mathbf{I}_{\rm aA}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{I}_{\rm cC}} = \underline{/\mathbf{I}_{\rm aA}} - 120^\circ$$

so

$$\mathbf{I}_{\rm aA} = 120\underline{/-36.87^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm bB} = 120\underline{/83.13^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm cC} = 120\underline{/-156.87^\circ}\,\text{A}$$

[b] The line voltages at the source are $\mathbf{V}_{\rm ab}$ $\mathbf{V}_{\rm bc}$, and $\mathbf{V}_{\rm ca}$. They form a balanced set. To find $\mathbf{V}_{\rm ab}$, use the a-phase circuit to find $\mathbf{V}_{\rm AN}$, and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\mathbf{V}_{\rm an} = \mathbf{V}_{\rm aA} + \mathbf{V}_{\rm AN} = (0.1 + j0.8)\mathbf{I}_{\rm aA} + 2400\underline{/0^\circ}$$

$$= (0.1 + j0.8)(96 - j72) + 2400\underline{/0^\circ} = 2467.2 + j69.6$$

$$2468.18\underline{/1.62^\circ}\,\text{V}$$

From Fig. 11.9(b),

$$\mathbf{V}_{\rm ab} = \mathbf{V}_{\rm an}(\sqrt{3}\underline{/-30^\circ}) = 4275.02\underline{/-28.38^\circ}\,\text{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\rm bc}} = \underline{/\mathbf{V}_{\rm ab}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\rm ca}} = \underline{/\mathbf{V}_{\rm ab}} - 120^\circ$$

so

$$\mathbf{V}_{\rm ab} = 4275.02\underline{/-28.38^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm bc} = 4275.02\underline{/91.62^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm ca} = 4275.02\underline{/-148.38^\circ}\,\text{V}$$

[c] Using KVL on the a-phase circuit

$$\mathbf{V}_{\mathrm{a'n}} = \mathbf{V}_{\mathrm{a'a}} + \mathbf{V}_{\mathrm{an}} = (0.2 + j0.16)\mathbf{I}_{\mathrm{aA}} + \mathbf{V}_{\mathrm{an}}$$

$$= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9)$$

$$= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\,\mathrm{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{\mathrm{b'n}}} = \underline{/\mathbf{V}_{\mathrm{a'n}}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{\mathrm{c'n}}} = \underline{/\mathbf{V}_{\mathrm{a'n}}} - 120^\circ$$

so

$$\mathbf{V}_{\mathrm{a'n}} = 2482.05\underline{/1.93^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{b'n}} = 2482.05\underline{/121.93^\circ}\,\mathrm{V}$$

$$\mathbf{V}_{\mathrm{c'n}} = 2482.05\underline{/-118.07^\circ}\,\mathrm{V}$$

AP 11.4

$$\mathbf{I}_{\mathrm{cC}} = (\sqrt{3}\underline{/-30^\circ})\mathbf{I}_{\mathrm{CA}} = (\sqrt{3}\underline{/-30^\circ}) \cdot 8\underline{/-15^\circ} = 13.86\underline{/-45^\circ}\,\mathrm{A}$$

AP 11.5

$$\mathbf{I}_{\mathrm{aA}} = 12\underline{/(65^\circ - 120^\circ)} = 12\underline{/-55^\circ}$$

$$\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right]\mathbf{I}_{\mathrm{aA}} = \left(\frac{\underline{/-30^\circ}}{\sqrt{3}}\right) \cdot 12\underline{/-55^\circ}$$

$$= 6.93\underline{/-85^\circ}\,\mathrm{A}$$

AP 11.6 **[a]** $\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/20^\circ}\,\mathrm{A}$

Therefore $\quad Z_\phi = \dfrac{4160\underline{/0^\circ}}{40\underline{/20^\circ}} = 104\underline{/-20^\circ}\,\Omega$

[b] $\mathbf{I}_{\mathrm{AB}} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/-40^\circ}\,\mathrm{A}$

Therefore $\quad Z_\phi = 104\underline{/40^\circ}\,\Omega$

AP 11.7

$$\mathbf{I}_\phi = \frac{110}{3.667} + \frac{110}{j2.75} = 30 - j40 = 50\underline{/-53.13^\circ}\,\mathrm{A}$$

Therefore $\quad |\mathbf{I}_{\mathrm{aA}}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60\,\mathrm{A}$

AP 11.8 **[a]** $|S| = \sqrt{3}(208)(73.8) = 26{,}587.67\,\mathrm{VA}$

$$Q = \sqrt{(26{,}587.67)^2 - (22{,}659)^2} = 13{,}909.50\,\mathrm{VAR}$$

[**b**] pf $= \frac{22{,}659}{26{,}587.67} = 0.8522$ lagging

AP 11.9 [**a**] $\mathbf{V}_{\rm AN} = \left(\frac{2450}{\sqrt{3}}\right)\underline{/0^\circ}\,\mathrm{V}; \qquad \mathbf{V}_{\rm AN}\mathbf{I}^*_{\rm aA} = S_\phi = 144 + j192\,\mathrm{kVA}$

Therefore

$$\mathbf{I}^*_{\rm aA} = \frac{(144 + j192)1000}{2450/\sqrt{3}} = (101.8 + j135.7)\,\mathrm{A}$$

$$\mathbf{I}_{\rm aA} = 101.8 - j135.7 = 169.67\underline{/-53.13^\circ}\,\mathrm{A}$$

$$|\mathbf{I}_{\rm aA}| = 169.67\,\mathrm{A}$$

[**b**] $P = \frac{(2450)^2}{R}; \qquad \text{therefore} \quad R = \frac{(2450)^2}{144{,}000} = 41.68\,\Omega$

$$Q = \frac{(2450)^2}{X}; \qquad \text{therefore} \quad X = \frac{(2450)^2}{192{,}000} = 31.26\,\Omega$$

[**c**] $Z_\phi = \frac{\mathbf{V}_{\rm AN}}{\mathbf{I}_{\rm aA}} = \frac{2450/\sqrt{3}}{169.67\underline{/-53.13^\circ}} = 8.34\underline{/53.13^\circ} = (5 + j6.67)\,\Omega$

$$\therefore \quad R = 5\,\Omega, \qquad X = 6.67\,\Omega$$

Problems

P 11.1 **[a]** First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 137\underline{/63^\circ}$; $\mathbf{V}_b = 137\underline{/-57^\circ}$; $\mathbf{V}_c = 137\underline{/183^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = 63^\circ - 63^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = -57^\circ - 63^\circ = -120^\circ$

$\underline{/\mathbf{V}'_c} = 183^\circ - 63^\circ = 120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

[b] First, convert the cosine waveforms to phasors, making sure that all waveforms are represented as cosines:

$\mathbf{V}_a = 820\underline{/-36^\circ}$; $\mathbf{V}_b = 820\underline{/84^\circ}$; $\mathbf{V}_c = 820\underline{/-156^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = -36^\circ + 36^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = 84^\circ + 36^\circ = 120^\circ$

$\underline{/\mathbf{V}'_c} = -156^\circ + 36^\circ = -120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

P 11.2 **[a]** $\mathbf{V}_a = 48\underline{/-45^\circ}\,\text{V}$

$\mathbf{V}_b = 48\underline{/-165^\circ}\,\text{V}$

$\mathbf{V}_c = 48\underline{/75^\circ}\,\text{V}$

Balanced, positive phase sequence

[b] $\mathbf{V}_a = 188\underline{/60^\circ}\,\text{V}$

$\mathbf{V}_b = -188\underline{/0^\circ}\,\text{V} = 188\underline{/180^\circ}\,\text{V}$

$\mathbf{V}_c = 188\underline{/-60^\circ}\,\text{V}$

Balanced, negative phase sequence

[c] $\mathbf{V}_\mathrm{a} = 426\underline{/0^\circ}$ V

$\mathbf{V}_\mathrm{b} = 462\underline{/120^\circ}$ V

$\mathbf{V}_\mathrm{c} = 426\underline{/-120^\circ}$ V

Unbalanced due to unequal amplitudes

[d] $\mathbf{V}_\mathrm{a} = 1121\underline{/-20^\circ}$ V

$\mathbf{V}_\mathrm{b} = 1121\underline{/-140^\circ}$ V

$\mathbf{V}_\mathrm{c} = 1121\underline{/100^\circ}$ V

Balanced, positive phase sequence

[e] $\mathbf{V}_\mathrm{a} = 540\underline{/-90^\circ}$ V

$\mathbf{V}_\mathrm{b} = 540\underline{/-120^\circ}$ V

$\mathbf{V}_\mathrm{c} = 540\underline{/120^\circ}$ V

Unbalanced due to unequal phase separation

[f] $\mathbf{V}_\mathrm{a} = 144\underline{/80^\circ}$ V

$\mathbf{V}_\mathrm{b} = 144\underline{/-160^\circ}$ V

$\mathbf{V}_\mathrm{c} = 144\underline{/-40^\circ}$ V

Balanced, negative phase sequence

P 11.3 $\mathbf{V}_\mathrm{a} = V_m\underline{/0^\circ} = V_m + j0$

$$\mathbf{V}_\mathrm{b} = V_m\underline{/-120^\circ} = -V_m(0.5 + j0.866)$$

$$\mathbf{V}_\mathrm{c} = V_m\underline{/120^\circ} = V_m(-0.5 + j0.866)$$

$$\begin{aligned}\mathbf{V}_\mathrm{a} + \mathbf{V}_\mathrm{b} + \mathbf{V}_\mathrm{c} &= (V_m)(1 + j0 - 0.5 - j0.866 - 0.5 + j0.866)\\ &= V_m(0) = 0\end{aligned}$$

P 11.4 $\mathbf{I} = \dfrac{188\underline{/60^\circ} + 188\underline{/180^\circ} + 188\underline{/-60^\circ}}{3(R_\mathrm{W} + jX_\mathrm{W})} = 0$

P 11.5 $\mathbf{I} = \dfrac{426\underline{/0^\circ} + 462\underline{/120^\circ} + 426\underline{/-120^\circ}}{3(R_\mathrm{W} + jX_\mathrm{W})} = \dfrac{36\underline{/120^\circ}}{3(R_\mathrm{W} + jX_\mathrm{W})}$

P 11.6 [a] The voltage sources form a balanced set, the source impedances are equal and the line impedances are equal. But the load impedances are not equal. Therefore, the circuit is unbalanced. Also,

$$\mathbf{I}_\mathrm{aA} = \frac{110}{32 - j24} = 2.75\underline{/36.87^\circ}\ \text{A (rms)}$$

$$\mathbf{I}_{\text{bB}} = \frac{110\underline{/-120^\circ}}{6 + j8} = 11\underline{/-173.13^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{\text{cC}} = \frac{110\underline{/120^\circ}}{40 + j30} = 2.2\underline{/83.13^\circ}\text{ A (rms)}$$

The magnitudes are unequal and the phase angles are not 120° apart, so the currents are not balanced and thus the circuit is not balanced.

b] $\mathbf{I}_o = \mathbf{I}_{\text{aA}} + \mathbf{I}_{\text{bB}} + \mathbf{I}_{\text{cC}} = 11.79\underline{/67.58^\circ}\text{ A (rms)}$

P 11.7 **[a]** $\mathbf{I}_{\text{aA}} = \dfrac{277\underline{/0^\circ}}{80 + j60} = 2.77\underline{/-36.87^\circ}\text{ A (rms)}$

$$\mathbf{I}_{\text{bB}} = \frac{277\underline{/-120^\circ}}{80 + j60} = 2.77\underline{/-156.87^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{\text{cC}} = \frac{277\underline{/120^\circ}}{80 + j60} = 2.77\underline{/83.13^\circ}\text{ A (rms)}$$

$$\mathbf{I}_o = \mathbf{I}_{\text{aA}} + \mathbf{I}_{\text{bB}} + \mathbf{I}_{\text{cC}} = 0$$

[b] $\mathbf{V}_{\text{AN}} = (78 + j54)\mathbf{I}_{\text{aA}} = 262.79\underline{/-2.17^\circ}\text{ V (rms)}$

[c] $\mathbf{V}_{\text{AB}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{BN}}$

$$\mathbf{V}_{\text{BN}} = (77 + j56)\mathbf{I}_{\text{bB}} = 263.73\underline{/-120.84^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\text{AB}} = 262.79\underline{/-2.17^\circ} - 263.73\underline{/-120.84^\circ} = 452.89\underline{/28.55^\circ}\text{ V (rms)}$$

[d] Unbalanced — see conditions for a balanced circuit in the text

P 11.8 $Z_{ga} + Z_{la} + Z_{La} = 60 + j80\,\Omega$

$$Z_{gb} + Z_{lb} + Z_{Lb} = 90 + j120\,\Omega$$

$$Z_{gc} + Z_{lc} + Z_{Lc} = 30 + j40\,\Omega$$

$$\frac{\mathbf{V}_N - 320}{60 + j80} + \frac{\mathbf{V}_N - 320\underline{/-120^\circ}}{90 + j120} + \frac{\mathbf{V}_N - 320\underline{/120^\circ}}{30 + j40} + \frac{\mathbf{V}_N}{20} = 0$$

Solving for $\mathbf{V}_N$ yields

$$\mathbf{V}_N = 49.47\underline{/75.14^\circ}\text{ V (rms)}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_N}{20} = 2.47\underline{/75.14^\circ}\text{ A (rms)}$$

P 11.9 $\mathbf{V}_{AN} = 285\underline{/-45^\circ}\,V$

$\mathbf{V}_{BN} = 285\underline{/-165^\circ}\,V$

$\mathbf{V}_{CN} = 285\underline{/75^\circ}\,V$

$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = 498.83\underline{/-15^\circ}\,V$

$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = 498.83\underline{/-135^\circ}\,V$

$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN} = 498.83\underline{/105^\circ}\,V$

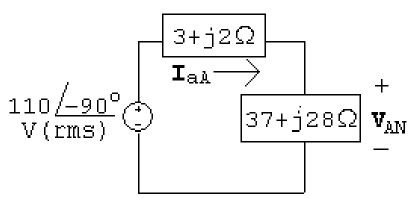
$v_{AB} = 498.83\cos(\omega t - 15^\circ)\,V$

$v_{BC} = 498.83\cos(\omega t - 135^\circ)\,V$

$v_{CA} = 498.83\cos(\omega t + 105^\circ)\,V$

P 11.10 **[a]** $\mathbf{V}_{an} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{ab} = 110\underline{/-90^\circ}\,V$ (rms)

The a-phase circuit is

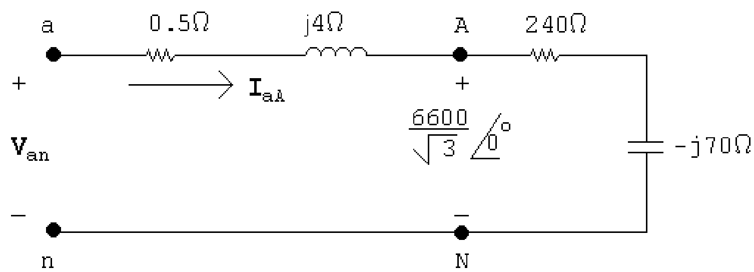


[b] $\mathbf{I}_{aA} = \dfrac{110\underline{/-90^\circ}}{40 + j30} = 2.2\underline{/-126.87^\circ}\,A$ (rms)

[c] $\mathbf{V}_{AN} = (37 + j28)\mathbf{I}_{aA} = 102.08\underline{/-89.75^\circ}\,V$ (rms)

$\mathbf{V}_{AB} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{AN} = 176.81\underline{/-59.75^\circ}\,A$ (rms)

P 11.11 **[a]**



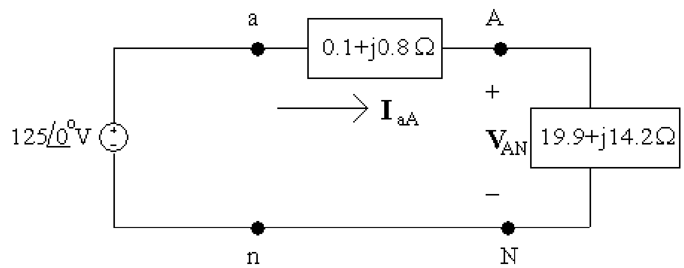
$\mathbf{I}_{aA} = \dfrac{6600}{\sqrt{3}(240 - j70)} = 15.24\underline{/16.26^\circ}\,A$ (rms)

$|\mathbf{I}_{aA}| = |\mathbf{I}_L| = 15.24\,A$ (rms)

[b] $\mathbf{V}_{\text{an}} = (15.24\angle 16.26^\circ)(240 - j66) = 3801.24\angle 0.91^\circ$

$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(3801.24) = 6583.94\,\text{V (rms)}$

P 11.12 Make a sketch of the a-phase:



[a] Find the a-phase line current from the a-phase circuit:

$$\mathbf{I}_{\text{aA}} = \frac{125\angle 0^\circ}{0.1 + j0.8 + 19.9 + j14.2} = \frac{125\angle 0^\circ}{20 + j15}$$

$$= 4 - j3 = 5\angle -36.87^\circ\,\text{A (rms)}$$

Find the other line currents using the acb phase sequence:

$$\mathbf{I}_{\text{bB}} = 5\angle -36.87^\circ + 120^\circ = 5\angle 83.13^\circ\,\text{A (rms)}$$

$$\mathbf{I}_{\text{cC}} = 5\angle -36.87^\circ - 120^\circ = 5\angle -156.87^\circ\,\text{A (rms)}$$

[b] The phase voltage at the source is $\mathbf{V}_{\text{an}} = 125\angle 0^\circ$ V. Use Fig. 11.9(b) to find the line voltage, $\mathbf{V}_{\text{an}}$, from the phase voltage:

$$\mathbf{V}_{\text{ab}} = \mathbf{V}_{\text{an}}(\sqrt{3}\angle -30^\circ) = 216.51\angle -30^\circ\,\text{V (rms)}$$

Find the other line voltages using the acb phase sequence:

$$\mathbf{V}_{\text{bc}} = 216.51\angle -30^\circ + 120^\circ = 216.51\angle 90^\circ\,\text{V (rms)}$$

$$\mathbf{V}_{\text{ca}} = 216.51\angle -30^\circ - 120^\circ = 216.51\angle -150^\circ\,\text{V (rms)}$$

[c] The phase voltage at the load in the a-phase is $\mathbf{V}_{\text{AN}}$. Calculate its value using $\mathbf{I}_{\text{aA}}$ and the load impedance:

$$\mathbf{V}_{\text{AN}} = \mathbf{I}_{\text{aA}} Z_{\text{L}} = (4 - j3)(19.9 + j14.2) = 122.2 - j2.9 = 122.23\angle -1.36^\circ\,\text{V (rms)}$$

Find the phase voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{\text{BN}} = 122.23\angle -1.36^\circ + 120^\circ = 122.23\angle 118.64^\circ\,\text{V (rms)}$$

$$\mathbf{V}_{\text{CN}} = 122.23\angle -1.36^\circ - 120^\circ = 122.23\angle -121.36^\circ\,\text{V (rms)}$$

[d] The line voltage at the load in the a-phase is $\mathbf{V}_{AB}$. Find this line voltage from the phase voltage at the load in the a-phase, $\mathbf{V}_{AN}$, using Fig, 11.9(b):

$$\mathbf{V}_{AB} = \mathbf{V}_{AN}(\sqrt{3}\underline{/-30^\circ}) = 211.72\underline{/-31.36^\circ}\text{ V (rms)}$$

Find the line voltage at the load for the b- and c-phases using the acb sequence:

$$\mathbf{V}_{BC} = 211.72\underline{/-31.36^\circ + 120^\circ} = 211.72\underline{/88.64^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{CA} = 211.72\underline{/-31.36^\circ - 120^\circ} = 211.72\underline{/-151.36^\circ}\text{ V (rms)}$$

P 11.13 **[a]** $\mathbf{I}_{AB} = \dfrac{7200}{216 - j288} = 20\underline{/53.13^\circ}\text{ A (rms)}$

$$\mathbf{I}_{BC} = 20\underline{/173.13^\circ}\text{ A (rms)}$$

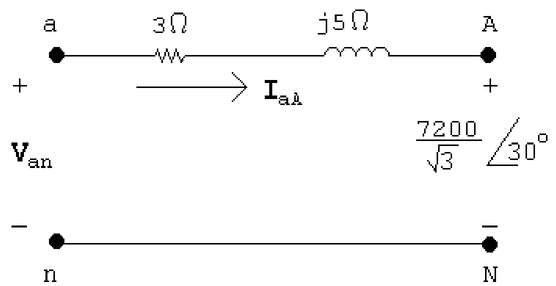
$$\mathbf{I}_{CA} = 20\underline{/-66.87^\circ}\text{ A (rms)}$$

[b] $\mathbf{I}_{aA} = \sqrt{3}\underline{/30^\circ}\mathbf{I}_{AB} = 34.64\underline{/83.13^\circ}\text{ A (rms)}$

$$\mathbf{I}_{bB} = 34.64\underline{/-156.87^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{cC} = 34.64\underline{/-36.87^\circ}\text{ A (rms)}$$

[c]



$$\mathbf{V}_{an} = \frac{7200}{\sqrt{3}}\underline{/30^\circ} + (3 + j5)(34.64\underline{/83.13^\circ})$$
$$= 4085\underline{/32.62^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{ab} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{an} = 7075.43\underline{/2.62^\circ}\text{ V (rms)}$$

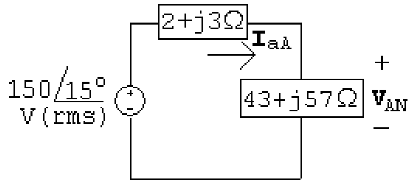
$$\mathbf{V}_{bc} = 7075.43\underline{/122.62^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{ca} = 7075.43\underline{/-117.38^\circ}\text{ V (rms)}$$

P 11.14 **[a]** $\mathbf{V}_{an} = \mathbf{V}_{bn} - 1\underline{/120^\circ} = 150\underline{/15^\circ}\text{ V (rms)}$

$$Z_y = Z_\Delta/3 = 43 + j57\,\Omega$$

The a-phase circuit is



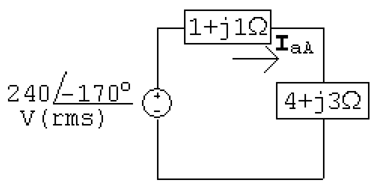
[b] $\mathbf{I}_{aA} = \dfrac{150\underline{/15^\circ}}{45+j60} = 2\underline{/-38.13^\circ}$ A (rms)

[c] $\mathbf{V}_{AN} = (43+j57)\mathbf{I}_{aA} = 142.8\underline{/14.84^\circ}$ V (rms)

$\mathbf{V}_{AB} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{AN} = 247.34\underline{/-15.16^\circ}$ A (rms)

P 11.15 $Z_y = Z_\Delta/3 = 4 + j3\,\Omega$

The a-phase circuit is



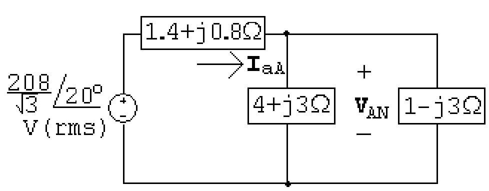
$$\mathbf{I}_{aA} = \frac{240\underline{/-170^\circ}}{(1+j1)+(4+j3)} = 37.48\underline{/151.34^\circ} \text{ A (rms)}$$

$$\mathbf{I}_{AB} = \frac{1}{\sqrt{3}}\underline{/-30^\circ}\mathbf{I}_{aA} = 21.64\underline{/121.34^\circ} \text{ A (rms)}$$

P 11.16 $\mathbf{V}_{an} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{ab} = \dfrac{208}{\sqrt{3}}\underline{/20^\circ}$ V (rms)

$Z_y = Z_\Delta/3 = 1 - j3\,\Omega$

The a-phase circuit is

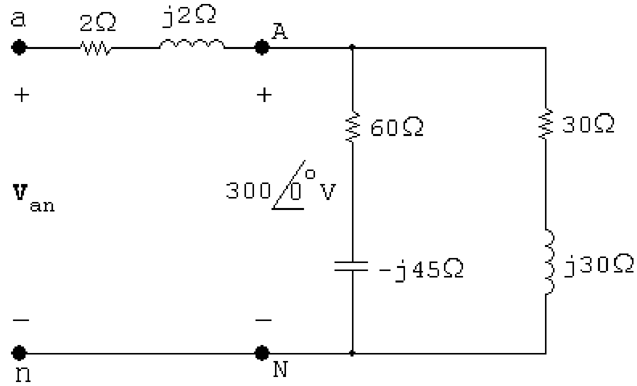


$$Z_{eq} = (4+j3)\|(1-j3) = 2.6 - j1.8\,\Omega$$

$$\mathbf{V}_{AN} = \frac{2.6-j1.8}{(1.4+j0.8)+(2.6-j1.8)}\left(\frac{208}{\sqrt{3}}\right)\underline{/20^\circ} = 92.1\underline{/-0.66^\circ} \text{ V (rms)}$$

$$\mathbf{V}_{AB} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{AN} = 159.5\underline{/29.34^\circ} \text{ V (rms)}$$

P 11.17 [a]



$$\mathbf{I}_{aA} = \frac{300}{60 - j45} + \frac{300}{30 + j30} = 8.6\underline{/-17.59^\circ}\text{ A (rms)}$$

$$|\mathbf{I}_{aA}| = 8.6\text{ A (rms)}$$

[b] $$\mathbf{I}_{AB} = \frac{300\sqrt{3}\underline{/30^\circ}}{90 + j90} = 4.08\underline{/-15^\circ}\text{ A (rms)}$$

$$|\mathbf{I}_{AB}| = 4.08\text{ A (rms)}$$

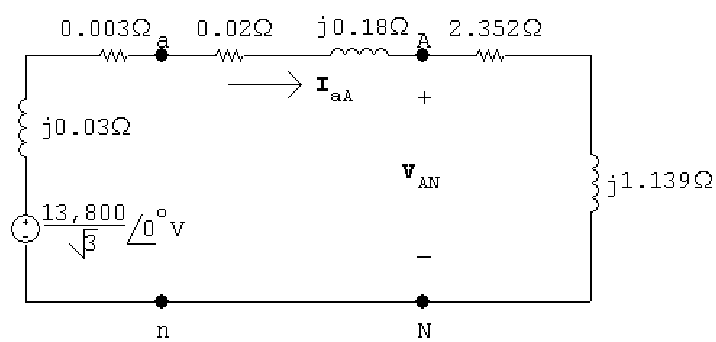
[c] $$\mathbf{I}_{AN} = \frac{300\underline{/0^\circ}}{60 - j45} = 4\underline{/36.87^\circ}\text{ A (rms)}$$

$$|\mathbf{I}_{AN}| = 4\text{ A (rms)}$$

[d] $$\mathbf{V}_{an} = (8.6\underline{/-17.59^\circ})(2 + j2) + 300\underline{/0^\circ} = 321.79\underline{/1.99^\circ}\text{ V (rms)}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(321.79) = 557.37\text{ V (rms)}$$

P 11.18 [a]



[b] $$\mathbf{I}_{aA} = \frac{13{,}800}{\sqrt{3}(2.375 + j1.349)} = 2917\underline{/-29.6^\circ}\text{ A (rms)}$$

$$|\mathbf{I}_{aA}| = 2917\text{ A (rms)}$$

[c] $$\mathbf{V}_{AN} = (2.352 + j1.139)(2917\underline{/-29.6^\circ}) = 7622.93\underline{/-3.76^\circ}\text{ V (rms)}$$

$$|\mathbf{V}_{AB}| = \sqrt{3}|\mathbf{V}_{AN}| = 13{,}203.31\text{ V (rms)}$$

[d] $\mathbf{V}_{\mathrm{an}} = (2.372 + j1.319)(2917\underline{/-29.6^\circ}) = 7616.93\underline{/-0.52^\circ}\,\mathrm{V}\ (\mathrm{rms})$

$|\mathbf{V}_{\mathrm{ab}}| = \sqrt{3}|\mathbf{V}_{\mathrm{an}}| = 13{,}712.52\,\mathrm{V}\ (\mathrm{rms})$

[e] $|\mathbf{I}_{\mathrm{AB}}| = \dfrac{|\mathbf{I}_{\mathrm{aA}}|}{\sqrt{3}} = 1684.13\,\mathrm{A}\ (\mathrm{rms})$

[f] $|\mathbf{I}_{\mathrm{ab}}| = |\mathbf{I}_{\mathrm{AB}}| = 1684.13\,\mathrm{A}\ (\mathrm{rms})$

P 11.19 **[a]** $\mathbf{I}_{\mathrm{AB}} = \dfrac{13{,}200\underline{/0^\circ}}{100 - j75} = 105.6\underline{/36.87^\circ}\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_{\mathrm{BC}} = 105.6\underline{/156.87^\circ}\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_{\mathrm{CA}} = 105.6\underline{/-83.13^\circ}\,\mathrm{A}\ (\mathrm{rms})$

[b] $\mathbf{I}_{\mathrm{aA}} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{\mathrm{AB}} = 182.9\underline{/66.87^\circ}\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_{\mathrm{bB}} = 182.9\underline{/-173.13^\circ}\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_{\mathrm{cC}} = 182.9\underline{/-53.13^\circ}\,\mathrm{A}\ (\mathrm{rms})$

[c] $\mathbf{I}_{\mathrm{ba}} = \mathbf{I}_{\mathrm{AB}} = 105.6\underline{/36.87^\circ}\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_{\mathrm{cb}} = \mathbf{I}_{\mathrm{BC}} = 105.6\underline{/156.87^\circ}\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_{\mathrm{ac}} = \mathbf{I}_{\mathrm{CA}} = 105.6\underline{/-83.13^\circ}\,\mathrm{A}\ (\mathrm{rms})$

P 11.20 **[a]** $\mathbf{I}_{\mathrm{AB}} = \dfrac{480\underline{/0^\circ}}{2.4 - j0.7} = 192\underline{/16.26^\circ}\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_{\mathrm{BC}} = \dfrac{480\underline{/120^\circ}}{8 + j6} = 48\underline{/83.13^\circ}\,\mathrm{A}\ (\mathrm{rms})$

$\mathbf{I}_{\mathrm{CA}} = \dfrac{480\underline{/-120^\circ}}{20} = 24\underline{/-120^\circ}\,\mathrm{A}\ (\mathrm{rms})$

[b] $\mathbf{I}_{\mathrm{aA}} = \mathbf{I}_{\mathrm{AB}} - \mathbf{I}_{\mathrm{CA}}$

$= 210\underline{/20.79^\circ}$

$\mathbf{I}_{\mathrm{bB}} = \mathbf{I}_{\mathrm{BC}} - \mathbf{I}_{\mathrm{AB}}$

$= 178.68\underline{/-178.04^\circ}$

$\mathbf{I}_{\mathrm{cC}} = \mathbf{I}_{\mathrm{CA}} - \mathbf{I}_{\mathrm{BC}}$

$= 70.7\underline{/-104.53^\circ}$

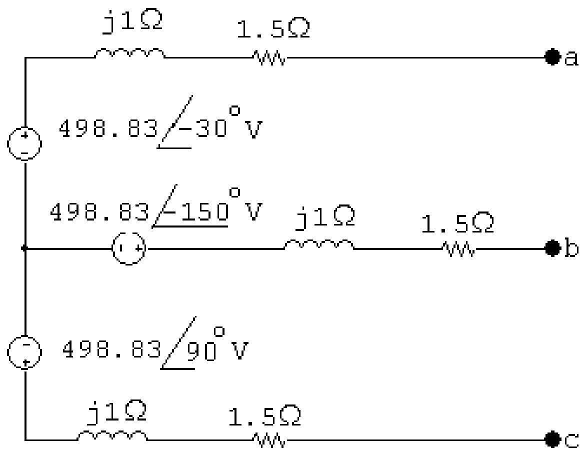
P 11.21 **[a]** Since the phase sequence is abc (positive) we have:

$$\mathbf{V}_{\rm an} = 498.83\underline{/-30^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\rm bn} = 498.83\underline{/-150^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\rm cn} = 498.83\underline{/90^\circ}\text{ V (rms)}$$

$$Z_Y = \frac{1}{3}Z_\Delta = 1.5 + j1\,\Omega/\phi$$



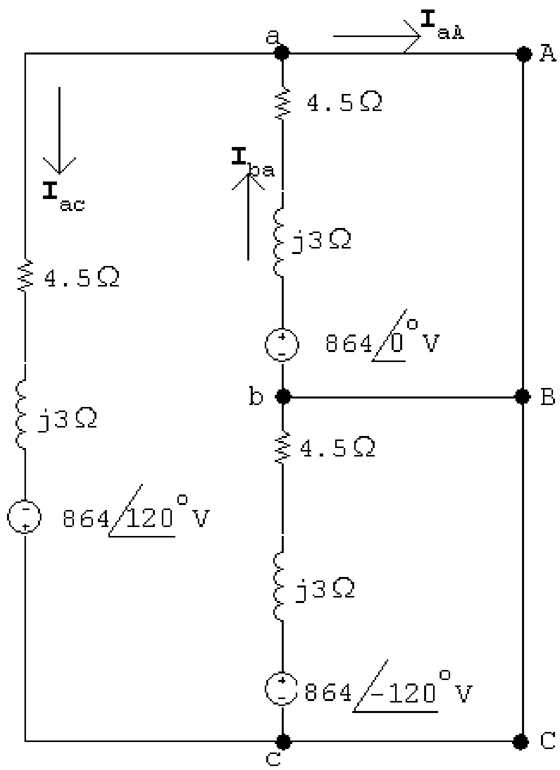
[b] $\mathbf{V}_{\rm ab} = 498.83\underline{/-30^\circ} - 498.83\underline{/-150^\circ} = 498.83\sqrt{3}\underline{/0^\circ} = 864\underline{/0^\circ}\text{ V (rms)}$

Since the phase sequence is positive, it follows that

$$\mathbf{V}_{\rm bc} = 864\underline{/-120^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\rm ca} = 864\underline{/120^\circ}\text{ V (rms)}$$

[c]



$$\mathbf{I}_{ba} = \frac{864}{4.5 + j3} = 159.75\underline{/-33.69^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{ac} = 159.75\underline{/86.31^\circ}\text{ A (rms)}$$

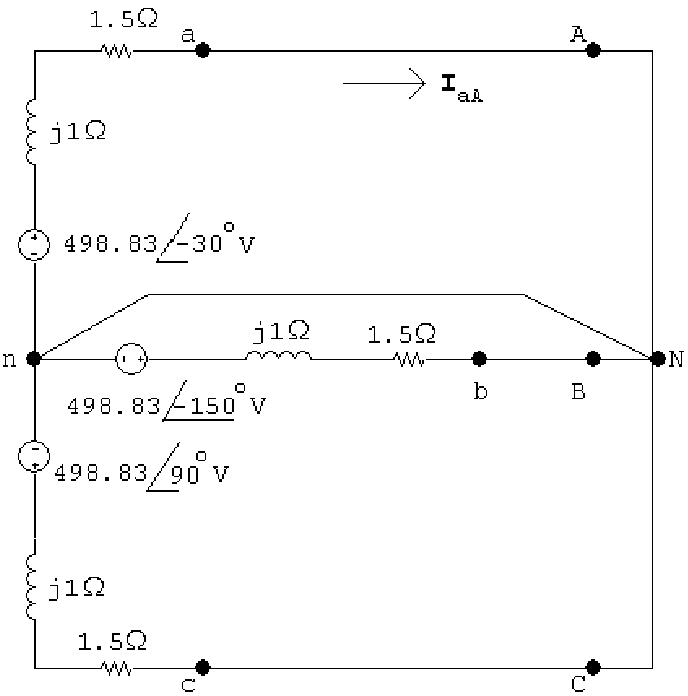
$$\mathbf{I}_{aA} = \mathbf{I}_{ba} - \mathbf{I}_{ac} = 276.70\underline{/-63.69^\circ}\text{ A (rms)}$$

Since we have a balanced three-phase circuit and a positive phase sequence we have:

$$\mathbf{I}_{bB} = 276.70\underline{/176.31^\circ}\text{ A (rms)}$$

$$\mathbf{I}_{cC} = 276.70\underline{/-56.31^\circ}\text{ A (rms)}$$

[**d**]



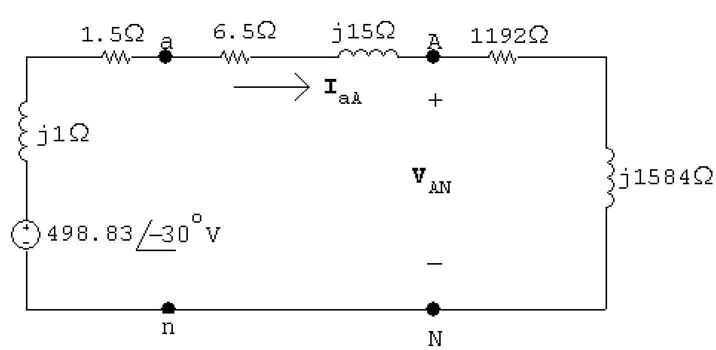
$$\mathbf{I}_{\rm aA} = \frac{498.83\underline{/30^\circ}}{1.5 + j1} = 276.70\underline{/-63.69^\circ}\,\text{A (rms)}$$

Since we have a balanced three-phase circuit and a positive phase sequence we have:

$$\mathbf{I}_{\rm bB} = 276.70\underline{/176.31^\circ}\,\text{A (rms)}$$

$$\mathbf{I}_{\rm cC} = 276.70\underline{/56.31^\circ}\,\text{A (rms)}$$

P 11.22 [**a**]



[**b**] $$\mathbf{I}_{\rm aA} = \frac{498.83\underline{/-30^\circ}}{1200 + j1600} = 249.42\underline{/-83.13^\circ}\,\text{mA (rms)}$$

$$\mathbf{V}_{\rm AN} = (1192 + j1584)(0.24942\underline{/-83.13^\circ}) = 494.45\underline{/-30.09^\circ}\,\text{V (rms)}$$

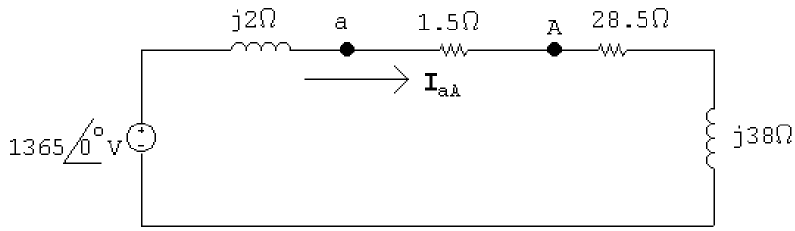
$$|\mathbf{V}_{\rm AB}| = \sqrt{3}(494.45) = 856.41\,\text{V (rms)}$$

[c] $|\mathbf{I}_{\text{ab}}| = \dfrac{0.24942}{\sqrt{3}} = 144\,\text{mA (rms)}$

[d] $\mathbf{V}_{\text{an}} = (1198.5 + j1599)(0.24942\underline{/-83.13^\circ}) = 498.42\underline{/-29.98^\circ}\,\text{V (rms)}$

$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(498.42) = 863.29\,\text{V (rms)}$

P 11.23 **[a]**



$$\mathbf{I}_{\text{aA}} = \frac{1365\underline{/0^\circ}}{30 + j40} = 27.3\underline{/-53.13^\circ}\,\text{A (rms)}$$

$$\mathbf{I}_{\text{CA}} = \frac{\mathbf{I}_{\text{aA}}}{\sqrt{3}}\underline{/150^\circ} = 15.76\underline{/96.87^\circ}\,\text{A (rms)}$$

[b] $S_{g/\phi} = -1365\mathbf{I}^*_{\text{aA}} = -22{,}358.75 - j29{,}811.56\,\text{VA}$

$\therefore \quad P_{\text{developed/phase}} = 22.359\,\text{kW}$

$P_{\text{absorbed/phase}} = |\mathbf{I}_{\text{aA}}|^2 28.5 = 21.241\,\text{kW}$

$\%\ \text{delivered} = \dfrac{21.241}{22.359}(100) = 95\%$

P 11.24 The complex power of the source per phase is $S_s = 20{,}000\underline{/(\cos^{-1} 0.6)} = 20{,}000\underline{/53.13^\circ} = 12{,}000 + j16{,}000$ kVA. This complex power per phase must equal the sum of the per-phase impedances of the two loads:

$$S_s = S_1 + S_2 \quad \text{so} \quad 12{,}000 + j16{,}000 = 10{,}000 + S_2$$

$$\therefore \quad S_2 = 2000 + j16{,}000 \text{ VA}$$

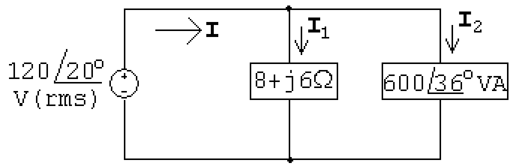
$$\text{Also,} \quad S_2 = \frac{|V_{\text{rms}}|^2}{Z_2^*}$$

$$|V_{\text{rms}}| = \frac{|\text{V}_{\text{load}}|}{\sqrt{3}} = 120 \text{ V (rms)}$$

$$\text{Thus,} \quad Z_2^* = \frac{|V_{\text{rms}}|^2}{S_2} = \frac{(120)^2}{2000 + j16{,}000} = 0.11 - j0.89\,\Omega$$

$$\therefore \quad Z_2 = 0.11 + j0.89\,\Omega$$

P 11.25 The a-phase of the circuit is shown below:



$$\mathbf{I}_1 = \frac{120\angle 20^\circ}{8+j6} = 12\angle -16.87^\circ \text{A (rms)}$$

$$\mathbf{I}_2^* = \frac{600\angle 36^\circ}{120\angle 20^\circ} = 5\angle 16^\circ \text{A (rms)}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 12\angle -16.87^\circ + 5\angle -16^\circ = 17\angle -16.61^\circ \text{ A (rms)}$$

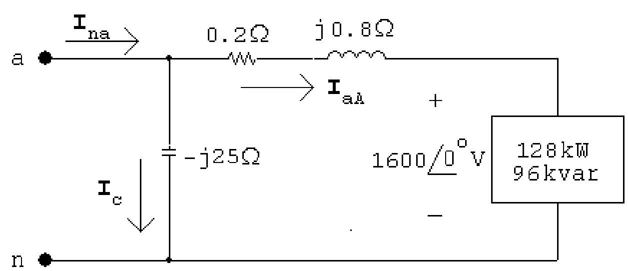
$$S_\text{a} = \mathbf{V}\mathbf{I}^* = (120\angle 20^\circ)(17\angle 16.61^\circ) = 2040\angle 36.61^\circ \text{ VA}$$

$$S_\text{T} = 3S_\text{a} = 6120\angle 36.61^\circ \text{ VA}$$

P 11.26 [a] $\mathbf{I}_{\text{aA}}^* = \dfrac{(128+j96)10^3}{1600} = 80 + j60$

$$\mathbf{I}_{\text{aA}} = 80 - j60 \text{ A (rms)}$$

$$\mathbf{V}_{\text{an}} = 1600 + (80-j60)(0.2+j0.8) = 1664 + j52 \text{ V (rms)}$$



$$\mathbf{I}_\text{C} = \frac{1664+j52}{-j25} = -2.08 + j66.56 \text{ A (rms)}$$

$$\mathbf{I}_{\text{na}} = \mathbf{I}_{\text{aA}} + \mathbf{I}_\text{C} = 77.92 + j6.56 = 78.2\angle 4.81^\circ \text{ A (rms)}$$

[b] $S_{g/\phi} = -(1664+j52)(77.92-j6.56) = -130{,}000 + j6864 \text{ VA}$

$$S_{gT} = 3S_{g/\phi} = -390{,}000 + j20{,}592 \text{ VA}$$

Therefore, the source is delivering 390 kW and absorbing 20.592 kvars.

[c] $P_{\text{del}} = 390\,\text{kW}$

$$P_{\text{abs}} = 3(128{,}000) + 3|\mathbf{I}_{\text{aA}}|^2(0.2)$$
$$= 390\,\text{kW} = P_{\text{del}}$$

[d] $Q_{\text{del}} = 3|\mathbf{I}_{\text{C}}|^2(25) = 332{,}592\,\text{VAR}$

$$Q_{\text{abs}} = 3(96{,}000) + 3|\mathbf{I}_{\text{aA}}|^2(0.8) + 20{,}592$$
$$= 332{,}592\,\text{VAR} = Q_{\text{del}}$$

P 11.27 [a] $S_{T\Delta} = 14{,}000\underline{/41.41^\circ} - 9000\underline{/53.13^\circ} = 5.5\underline{/22^\circ}\ \text{kVA}$

$$S_\Delta = S_{T\Delta}/3 = 1833.46\underline{/22^\circ}\ \text{VA}$$

[b] $|\mathbf{V}_{\text{an}}| = \left|\dfrac{3000\underline{/53.13^\circ}}{10\underline{/-30^\circ}}\right| = 300\ \text{V (rms)}$

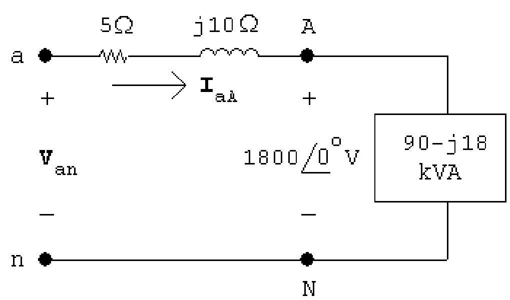
$$|\mathbf{V}_{\text{line}}| = |\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 300\sqrt{3} = 519.62\ \text{V (rms)}$$

P 11.28 [a] $S_{1/\phi} = 60{,}000(0.866) + j60{,}000(0.5) = 51{,}960 + j30{,}000\ \text{VA}$

$$S_{2/\phi} = 50{,}000(0.28) - j50{,}000(0.96) = 14{,}000 - j48{,}000\ \text{VA}$$

$$S_{3/\phi} = 24{,}040\ \text{VA}$$

$$S_{T/\phi} = S_1 + S_2 + S_3 = 90{,}000 - j18{,}000\ \text{VA}$$



$$\therefore\quad \mathbf{I}_{\text{aA}}^* = \frac{90{,}000 - j18{,}000}{1800} = 50 - j10$$

$$\therefore\quad \mathbf{I}_{\text{aA}} = 50 + j10\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 1800 + (50 + j10)(5 + j10) = 1950 + j550 = 2026.08\underline{/15.75^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(2026.08) = 3509.27\,\text{V (rms)}$$

[b] $S_{g/\phi} = (1950 + j550)(50 - j10) = 103 + j0.8\,\text{kVA}$

$$\%\ \text{efficiency} = \frac{90{,}000}{103{,}000}(100) = 87.38\%$$

P 11.29 **[a]** $S_1 = 10{,}200(0.87) + j10{,}200(0.493) = 8874 + j5029.13\,\text{VA}$

$$S_2 = 4200 + j1913.6\,\text{VA}$$

$$\sqrt{3}V_\text{L}I_\text{L}\sin\theta_3 = 7250; \qquad \sin\theta_3 = \frac{7250}{\sqrt{3}(220)(36.8)} = 0.517$$

Therefore $\cos\theta_3 = 0.856$

Therefore

$$P_3 = \frac{7250}{0.517} \times 0.856 = 12{,}003.9\,\text{W}$$

$$S_3 = 12{,}003.9 + j7250\,\text{VA}$$

$$S_T = S_1 + S_2 + S_3 = 25.078 + j14.192\,\text{kVA}$$

$$S_{T/\phi} = \frac{1}{3}S_T = 8359.3 + j4730.7\,\text{VA}$$

$$\frac{220}{\sqrt{3}}\mathbf{I}^*_{\text{aA}} = (8359.3 + j4730.7); \qquad \mathbf{I}^*_{\text{aA}} = 65.81 + j37.24\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 65.81 - j37.24 = 75.62\underline{/-29.51^\circ}\,\text{A} \quad \text{(rms)}$$

[b] pf $= \cos(0^\circ - 29.51^\circ) = 0.87$ lagging

P 11.30 From the solution to Problem 11.18 we have:

$$S_{\text{AB}} = (480\underline{/0^\circ})(192\underline{/-16.26^\circ}) = 88{,}473.7 - j25{,}804.5\,\text{VA}$$

$$S_{\text{BC}} = (480\underline{/120^\circ})(48\underline{/-83.13^\circ}) = 18{,}431.98 + j13{,}824.03\,\text{VA}$$

$$S_{\text{CA}} = (480\underline{/-120^\circ})(24\underline{/120^\circ}) = 11{,}520 + j0\,\text{VA}$$

P 11.31 Let p_a, p_b, and p_c represent the instantaneous power of phases a, b, and c, respectively. Then assuming a positive phase sequence, we have

$$p_\text{a} = v_{\text{an}}i_{\text{aA}} = [V_m\cos\omega t][I_m\cos(\omega t - \theta_\phi)]$$

$$p_\text{b} = v_{\text{bn}}i_{\text{bB}} = [V_m\cos(\omega t - 120^\circ)][I_m\cos(\omega t - \theta_\phi - 120^\circ)]$$

$$p_\text{c} = v_{\text{cn}}i_{\text{cC}} = [V_m\cos(\omega t + 120^\circ)][I_m\cos(\omega t - \theta_\phi + 120^\circ)]$$

The total instantaneous power is $p_T = p_\text{a} + p_\text{b} + p_\text{c}$, so

$$p_T = V_mI_m[\cos\omega t\cos(\omega t - \theta_\phi) + \cos(\omega t - 120^\circ)\cos(\omega t - \theta_\phi - 120^\circ)$$

$$+\cos(\omega t + 120^\circ)\cos(\omega t - \theta_\phi + 120^\circ)]$$

Now simplify using trigonometric identities. In simplifying, collect the coefficients of $\cos(\omega t - \theta_\phi)$ and $\sin(\omega t - \theta_\phi)$. We get

$$p_T = V_m I_m[\cos\omega t(1 + 2\cos^2 120^\circ)\cos(\omega t - \theta_\phi)$$
$$+2\sin\omega t\sin^2 120^\circ\sin(\omega t - \theta_\phi)]$$
$$= 1.5V_m I_m[\cos\omega t\cos(\omega t - \theta_\phi) + \sin\omega t\sin(\omega t - \theta_\phi)]$$
$$= 1.5V_m I_m\cos\theta_\phi$$

P 11.32 $|I_{\text{line}}| = \dfrac{1600}{240/\sqrt{3}} = 11.547$ A (rms)

$$|Z_y| = \frac{|V|}{|I|} = \frac{240/\sqrt{3}}{11.547} = 12$$

$$Z_y = 12\underline{/-50^\circ}\,\Omega$$

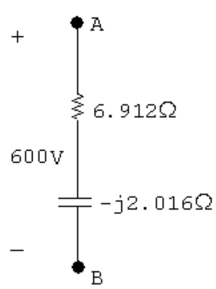
$$Z_\Delta = 3Z_y = 36\underline{/-50^\circ} = 23.14 - j27.58\,\Omega/\phi$$

P 11.33 Assume a Δ-connected load (series):

$$S_\phi = \frac{1}{3}(150\times 10^3)(0.96 - j0.28) = 48{,}000 - j14{,}000\,\text{VA}$$

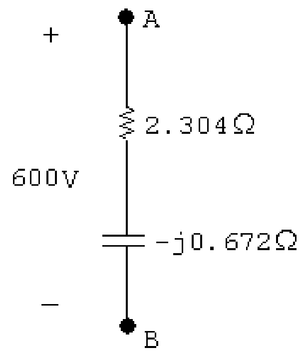
$$Z^*_{\Delta\phi} = \frac{|600|^2}{48{,}000 - j14{,}000} = 6.912 + j2.016\,\Omega/\phi$$

$$Z_{\Delta\phi} = 6.912 - j2.016\,\Omega$$



Now assume a Y-connected load (series):

$$Z_{Y\phi} = \frac{1}{3}Z_{\Delta\phi} = 2.304 - j0.672\,\Omega/\phi$$



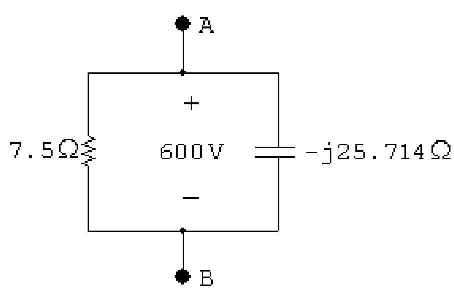
Now assume a Δ-connected load (parallel):

$$P_\phi = \frac{|600|^2}{R_\Delta}$$

$$R_{\Delta\phi} = \frac{|600|^2}{48{,}000} = 7.5\,\Omega$$

$$Q_\phi = \frac{|600|^2}{X_\Delta}$$

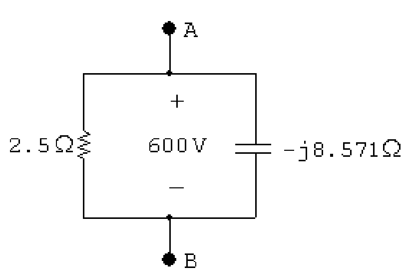
$$X_\Delta\phi = \frac{|600|^2}{-14{,}000} = -25.714\,\Omega$$



Now assume a Y-connected load (parallel):

$$R_{Y\phi} = \frac{1}{3}R_{\Delta\phi} = 2.5\,\Omega$$

$$X_{Y\phi} = \frac{1}{3}X_{\Delta\phi} = -8.571\,\Omega$$



P 11.34 **[a]** $P_{\rm OUT} = 746 \times 100 = 74{,}600\,\text{W}$

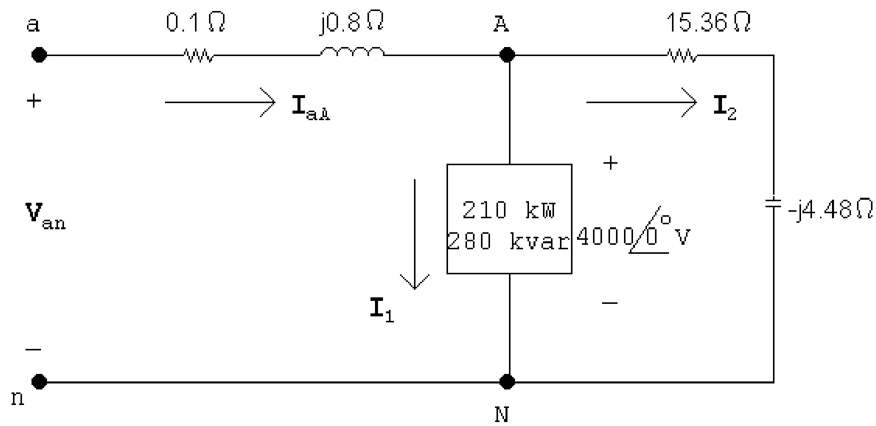
$$P_{\rm IN} = 74{,}600/(0.97) = 76{,}907.22\,\text{W}$$

$$\sqrt{3}V_L I_L \cos\theta = 76{,}907.22$$

$$I_L = \frac{76{,}907.22}{\sqrt{3}(208)(0.88)} = 242.58\,\text{A (rms)}$$

[b] $Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3}(208)(242.58)(0.475) = 41{,}511.90\,\text{VAR}$

P 11.35



$$4000\mathbf{I}_1^* = (210 + j280)10^3$$

$$\mathbf{I}_1^* = \frac{210}{4} + j\frac{280}{4} = 52.5 + j70\,\text{A (rms)}$$

$$\mathbf{I}_1 = 52.5 - j70\,\text{A (rms)}$$

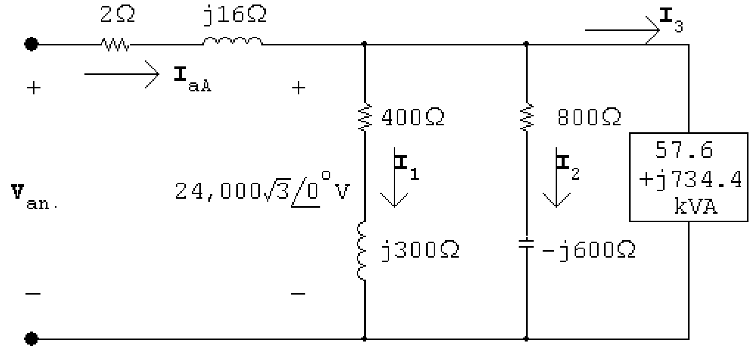
$$\mathbf{I}_2 = \frac{4000\underline{/0^\circ}}{15.36 - j4.48} = 240 + j70\,\text{A (rms)}$$

$$\therefore \quad \mathbf{I}_{\rm aA} = \mathbf{I}_1 + \mathbf{I}_2 = 292.5 + j0\,\text{A (rms)}$$

$$\mathbf{V}_{\rm an} = 4000 + j0 + 292.5(0.1 + j0.8) = 4036.04\underline{/3.32^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{\rm ab}| = \sqrt{3}|\mathbf{V}_{\rm an}| = 6990.62\,\text{V (rms)}$$

P 11.36 [a]



$$\mathbf{I}_1 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{400 + j300} = 66.5 - j49.9\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{24{,}000\sqrt{3}\underline{/0^\circ}}{800 - j600} = 33.3 + j24.9\,\text{A (rms)}$$

$$\mathbf{I}_3^* = \frac{57{,}600 + j734{,}400}{24{,}000\sqrt{3}} = 1.4 + j17.7$$

$$\mathbf{I}_3 = 1.4 - j17.7\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 101.2 - j42.7\,\text{A} = 109.8\underline{/-22.9^\circ}\,\text{A (rms)}$$

$$\mathbf{V}_{\text{an}} = (2 + j16)(101.2 - j42.7) + 24{,}000\sqrt{3} = 42{,}454.8 + j1533.8\,\text{V (rms)}$$

$$S_\phi = \mathbf{V}_{\text{an}}\mathbf{I}_{\text{aA}}^* = (42{,}454.8 + j1533.8)(101.2 + j42.7)$$

$$= 4{,}230{,}932.5 + j1{,}968{,}040.5\,\text{VA}$$

$$S_T = 3S_\phi = 12{,}692.8 + j5904.1\,\text{kVA}$$

[b] $S_{1/\phi} = 24{,}000\sqrt{3}(66.5 + j49.9) = 2764.4 + j2074.3\,\text{kVA}$

$$S_{2/\phi} = 24{,}000\sqrt{3}(33.3 - j24.9) = 1384.3 - j1035.1\,\text{kVA}$$

$$S_{3/\phi} = 57.6 + j734.4\,\text{kVA}$$

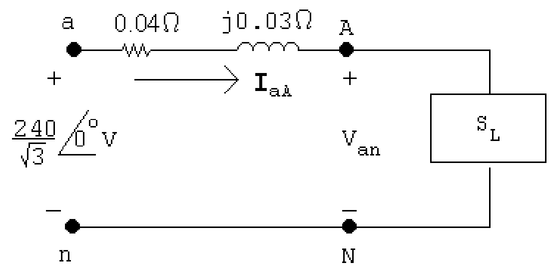
$$S_\phi(\text{load}) = 4206.3 + j1773.6\,\text{kVA}$$

$$\%\text{ delivered } = \left(\frac{4206.3}{4230.9}\right)(100) = 99.4\%$$

P 11.37 [a] $S_{g/\phi} = \frac{1}{3}(41.6)(0.707 + j0.707) \times 10^3 = 9803.73 + j9803.73\,\text{VA}$

$$\mathbf{I}_{\text{aA}}^* = \frac{9803.73 + j9803.73}{240/\sqrt{3}} = 70.76 + j70.76\,\text{A (rms)}$$

$$\mathbf{I}_{\text{aA}} = 70.76 - j70.76\,\text{A (rms)}$$



$$\mathbf{V}_{\rm AN} = \frac{240}{\sqrt{3}} - (0.04 + j0.03)(70.76 - j70.76)$$

$$= 133.61 + j0.71 = 133.61\angle 0.30^\circ \text{ V (rms)}$$

$$|\mathbf{V}_{\rm AB}| = \sqrt{3}(133.61) = 231.42 \text{ V (rms)}$$

[b] $S_{L/\phi} = (133.61 + j0.71)(70.76 + j70.76) = 9404 + j9504.5$ VA

$S_L = 3S_{L/\phi} = 28{,}212 + j28{,}513$ VA

Check:

$S_g = 41{,}600(0.7071 + j0.7071) = 29{,}415 + j29{,}415$ VA

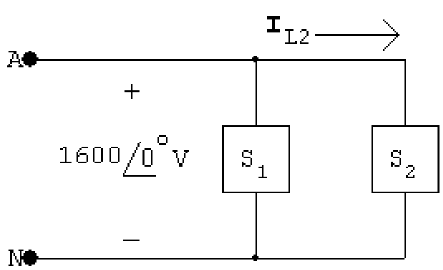
$P_\ell = 3|\mathbf{I}_{\rm aA}|^2(0.04) = 1202$ W

$P_g = P_L + P_\ell = 28{,}212 + 1202 = 29{,}414$ W (checks)

$Q_\ell = 3|\mathbf{I}_{\rm aA}|^2(0.03) = 901$ VAR

$Q_g = Q_L + Q_\ell = 28{,}513 + 901 = 29{,}414$ VAR (checks)

P 11.38 **[a]**



$$S_g = \frac{1}{3}(540)(0.96 + j0.28) = 172.8 + j50.4 \text{ kVA}$$

$$S_1 = \frac{1}{3}(28.4 - j208.8) = 12.8 - j69.6 \text{ kVA}$$

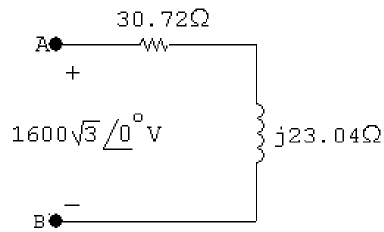
$$S_2 = S_g - S_1 = 160 + j120 \text{ kVA}$$

$$\therefore \quad \mathbf{I}_{L2}^* = \frac{(160 + j120)10^3}{1600} = 100 + j75$$

$$\mathbf{I}_{L2} = 100 - j75\,\text{A (rms)}$$

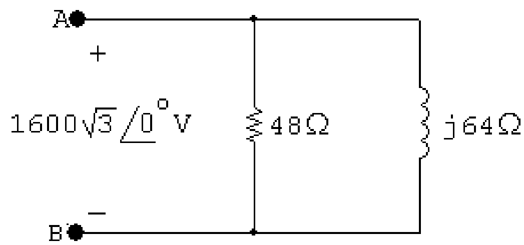
$$Z_Y = \frac{1600}{100 - j75} = 10.24 + j7.68\,\Omega$$

$$Z_\Delta = 3Z_Y = 30.72 + j23.04\,\Omega$$

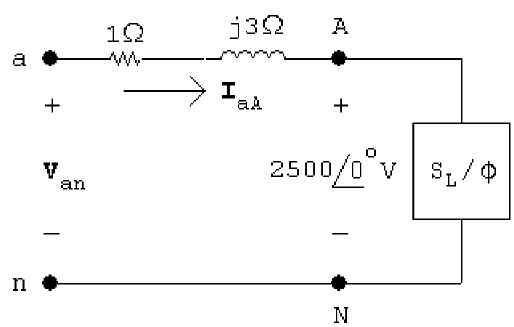


[b] $R = \frac{(1600)^2}{160 \times 10^3} = 16\,\Omega \quad \rightarrow \quad R_\Delta = 3R = 48\,\Omega$

$$X_L = \frac{(1600)^2}{120 \times 10^3} = 21.33\,\Omega \quad \rightarrow \quad X_{L\Delta} = 3X_L = 64\,\Omega$$



P 11.39 **[a]**



$$S_{L/\phi} = \frac{1}{3}\left[900 + j\frac{900}{0.6}(0.8)\right]10^3 = 300{,}000 + j400{,}000\,\text{VA}$$

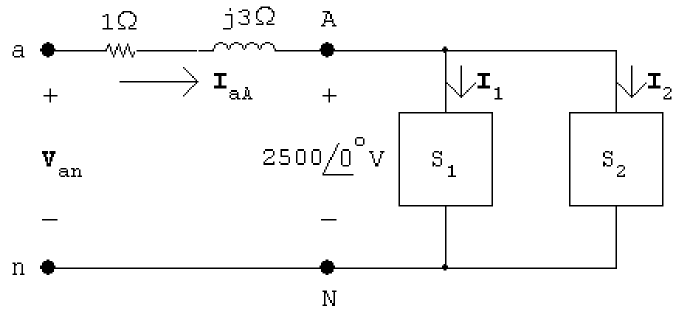
$$\mathbf{I}^*_{aA} = \frac{300{,}000 + j400{,}000}{2500} = 120 + j160\,\text{A (rms)}$$

$$\mathbf{I}_{aA} = 120 - j160\,\text{A (rms)}$$

$$\begin{aligned}\mathbf{V}_{an} &= 2500 + (1 + j3)(120 - j160) \\ &= 3100 + j200 = 3106.44\underline{/3.69^\circ}\,\text{V (rms)}\end{aligned}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(3106.44) = 5380.5\,\text{V (rms)}$$

[b]



$$\mathbf{I}_1 = 120 - j160\,\text{A} \qquad \text{(from part [a])}$$

$$S_2 = 0 - j\frac{1}{3}(1125)\times 10^3 = -j375{,}000\,\text{VAR}$$

$$\mathbf{I}_2^* = \frac{-j375{,}000}{2500} = -j150\,\text{A (rms)}$$

$$\therefore\quad \mathbf{I}_2 = j150\,\text{A (rms)}$$

$$\mathbf{I}_{aA} = 120 - j160 + j150 = 120 - j10\,\text{A (rms)}$$

$$\mathbf{V}_{an} = 2500 + (120 - j10)(1 + j3)$$
$$= 2650 + j350 = 2673.01\underline{/7.52^\circ}\,\text{V (rms)}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(2673.01) = 4629.8\,\text{V (rms)}$$

[c] $|\mathbf{I}_{aA}| = 200\,\text{A (rms)}$

$$P_{\text{loss}/\phi} = (200)^2(1) = 40\,\text{kW}$$

$$P_{g/\phi} = 300{,}000 + 40{,}000 = 340\,\text{kW}$$

$$\%\,\eta = \frac{300}{340}(100) = 88.2\%$$

[d] $|\mathbf{I}_{aA}| = 120.416\,\text{A (rms)}$

$$P_{\ell/\phi} = (120.416)^2(1) = 14{,}500\,\text{W}$$

$$\%\,\eta = \frac{300{,}000}{314{,}500}(100) = 95.4\%$$

[e] $Z_{\text{cap/Y}} = \dfrac{2500^2}{j375{,}000} = -j16.67\,\Omega$

$$\therefore\quad \frac{1}{\omega C} = 16.67; \qquad C = \frac{1}{(16.67)(120\pi)} = 159.155\,\mu\text{F}$$

P 11.40 **[a]** From Assessment Problem 11.9, $\mathbf{I}_{\text{aA}} = (101.8 - j135.7)\,\text{A (rms)}$

Therefore $\mathbf{I}_{\text{cap}} = j135.7\,\text{A (rms)}$

Therefore $Z_{CY} = \dfrac{2450/\sqrt{3}}{j135.7} = -j10.42\,\Omega$

Therefore $C_Y = \dfrac{1}{(10.42)(2\pi)(60)} = 254.5\,\mu\text{F}$

$Z_{C\Delta} = (-j10.42)(3) = -j31.26\,\Omega$

Therefore $C_\Delta = \dfrac{254.5}{3} = 84.84\,\mu\text{F}$

[b] $C_Y = 254.5\,\mu\text{F}$

[c] $|\mathbf{I}_{\text{aA}}| = 101.8\,\text{A (rms)}$

P 11.41 $W_{m1} = |\mathbf{V}_{\text{AB}}||\mathbf{I}_{\text{aA}}|\cos(\underline{/\mathbf{V}_{\text{AB}}} - \underline{/\mathbf{I}_{\text{aA}}}) = (199.58)(2.4)\cos(65.68^\circ) = 197.26\,\text{W}$

$W_{m2} = |\mathbf{V}_{\text{CB}}||\mathbf{I}_{\text{cC}}|\cos(\underline{/\mathbf{V}_{\text{CB}}} - \underline{/\mathbf{I}_{\text{cC}}}) = (199.58)(2.4)\cos(5.68^\circ) = 476.64\,\text{W}$

CHECK: $W_1 + W_2 = 673.9 = (2.4)^2(39)(3) = 673.9\,\text{W}$

P 11.42 $\tan\phi = \dfrac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = 0.75$

$\therefore \quad \phi = 36.87^\circ$

$\therefore \quad 2400\sqrt{3}|\mathbf{I}_{\text{L}}|\cos 66.87^\circ = 40{,}823.09$

$|\mathbf{I}_{\text{L}}| = 25\,\text{A}$

$|Z| = \dfrac{2400}{25} = 96\,\Omega \qquad \therefore \quad Z = 96\underline{/36.87^\circ}\,\Omega$

P 11.43 $\mathbf{I}_{\text{aA}} = \dfrac{\mathbf{V}_{\text{AN}}}{Z_\phi} = |\mathbf{I}_L|\underline{/-\theta_\phi}\,\text{A},$

$Z_\phi = |Z|\underline{/\theta_\phi}, \qquad \mathbf{V}_{\text{BC}} = |\mathbf{V}_L|\underline{/-90^\circ}\,\text{V},$

$$\begin{aligned} W_m &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos[-90^\circ - (-\theta_\phi)] \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos(\theta_\phi - 90^\circ) \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi, \end{aligned}$$

therefore $\sqrt{3}W_m = \sqrt{3}|\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi = Q_{\text{total}}$

P 11.44 **[a]** $Z = 16 - j12 = 20\underline{/-36.87^\circ}\,\Omega$

$\mathbf{V}_{\rm AN} = 680\underline{/0^\circ}\,\text{V}; \qquad \therefore \quad \mathbf{I}_{\rm aA} = 34\underline{/36.87^\circ}\,\text{A}$

$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = 680\sqrt{3}\underline{/-90^\circ}\,\text{V}$

$W_m = (680\sqrt{3})(34)\cos(-90 - 36.87^\circ) = -24{,}027.07\,\text{W}$

$\sqrt{3}W_m = -41{,}616.1\,\text{W}$

[b] $Q_\phi = (34^2)(-12) = -13{,}872\,\text{VAR}$

$Q_T = 3Q_\phi = -41{,}616\,\text{VAR} = \sqrt{3}W_m$

P 11.45 **[a]**
$$\begin{aligned} W_2 - W_1 &= V_L I_L[\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)] \\ &= V_L I_L[\cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ \\ &\qquad - \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ] \\ &= 2V_L I_L\sin\theta\sin 30^\circ = V_L I_L\sin\theta, \end{aligned}$$

therefore $\sqrt{3}(W_2 - W_1) = \sqrt{3}V_L I_L\sin\theta = Q_T$

[b] $Z_\phi = (8 + j6)\,\Omega$

$Q_T = \sqrt{3}[2476.25 - 979.75] = 2592\,\text{VAR},$

$Q_T = 3(12)^2(6) = 2592\,\text{VAR};$

$Z_\phi = (8 - j6)\,\Omega$

$Q_T = \sqrt{3}[979.75 - 2476.25] = -2592\,\text{VAR},$

$Q_T = 3(12)^2(-6) = -2592\,\text{VAR};$

$Z_\phi = 5(1 + j\sqrt{3})\,\Omega$

$Q_T = \sqrt{3}[2160 - 0] = 3741.23\,\text{VAR},$

$Q_T = 3(12)^2(5\sqrt{3}) = 3741.23\,\text{VAR};$

$Z_\phi = 10\underline{/75^\circ}\,\Omega$

$Q_T = \sqrt{3}[-645.53 - 1763.63] = -4172.80\,\text{VAR},$

$Q_T = 3(12)^2[-10\sin 75^\circ] = -4172.80\,\text{VAR}$

P 11.46 $Z_\phi = |Z|\underline{/\theta} = \dfrac{\mathbf{V}_{\rm AN}}{\mathbf{I}_{\rm aA}}$

$$\theta = \underline{/\mathbf{V}_{\rm AN}} - \underline{/\mathbf{I}_{\rm aA}}$$

$$\theta_1 = \underline{/\mathbf{V}_{\rm AB}} - \underline{/\mathbf{I}_{\rm aA}}$$

For a positive phase sequence,

$$\underline{/\mathbf{V}_{\rm AB}} = \underline{/\mathbf{V}_{\rm AN}} + 30^\circ$$

Thus,

$$\theta_1 = \underline{/\mathbf{V}_{\rm AN}} + 30^\circ - \underline{/\mathbf{I}_{\rm aA}} = \theta + 30^\circ$$

Similarly,

$$Z_\phi = |Z|\underline{/\theta} = \frac{\mathbf{V}_{\rm CN}}{\mathbf{I}_{\rm cC}}$$

$$\theta = \underline{/\mathbf{V}_{\rm CN}} - \underline{/\mathbf{I}_{\rm cC}}$$

$$\theta_2 = \underline{/\mathbf{V}_{\rm CB}} - \underline{/\mathbf{I}_{\rm cC}}$$

For a positive phase sequence,

$$\underline{/\mathbf{V}_{\rm CB}} = \underline{/\mathbf{V}_{\rm BA}} - 120^\circ = \underline{/\mathbf{V}_{\rm AB}} + 60^\circ$$

$$\underline{/\mathbf{I}_{\rm cC}} = \underline{/\mathbf{I}_{\rm aA}} + 120^\circ$$

Thus,

$$\theta_2 = \underline{/\mathbf{V}_{\rm AB}} + 60^\circ - (\underline{/\mathbf{I}_{\rm aA}} + 120^\circ) = \theta_1 - 60^\circ$$
$$= \theta + 30^\circ - 60^\circ = \theta - 30^\circ$$

P 11.47 **[a]** $Z_\phi = 100 - j75 = 125\underline{/-36.87^\circ}\,\Omega$

$$S_\phi = \frac{(13{,}200)^2}{125\underline{/36.87^\circ}} = 1{,}115{,}136 + j836{,}352\,\text{VA}$$

[b] $\frac{13{,}200}{\sqrt{3}}\underline{/30^\circ}\mathbf{I}^*_{\text{aA}} = S_\phi$ so $\mathbf{I}_{\text{aA}} = 182.9\underline{/66.87^\circ}$

$$W_{m1} = (13{,}200)(182.9)\cos(0 - 66.87^\circ) = 948{,}401.92\,\text{W}$$

$$W_{m2} = (13{,}200)(182.9)\cos(-60^\circ + 53.13^\circ) = 2{,}397{,}006.08\,\text{W}$$

Check: $P_T = 3(1{,}115{,}136)\,\text{W} = W_{m1} + W_{m2}$.

P 11.48 From the solution to Prob. 11.20 we have

$\mathbf{I}_{\text{aA}} = 210\underline{/20.79^\circ}\,\text{A}$ and $\mathbf{I}_{\text{bB}} = 178.68\underline{/-178.04^\circ}\,\text{A}$

[a] $W_1 = |\mathbf{V}_{\text{ac}}|\,|\mathbf{I}_{\text{aA}}|\cos(\theta_{\text{ac}} - \theta_{\text{aA}})$

$$= 480(210)\cos(60^\circ - 20.79^\circ) = 78{,}103.2\,\text{W}$$

[b] $W_2 = |\mathbf{V}_{\text{bc}}|\,|\mathbf{I}_{\text{bB}}|\cos(\theta_{\text{bc}} - \theta_{\text{bB}})$

$$= 480(178.68)\cos(120^\circ + 178.04^\circ) = 40{,}317.7\,\text{W}$$

[c] $W_1 + W_2 = 118{,}421\,\text{W}$

$$P_{\text{AB}} = (192)^2(2.4) = 88{,}473.6\,\text{W}$$

$$P_{\text{BC}} = (48)^2(8) = 18{,}432\,\text{W}$$

$$P_{\text{CA}} = (24)^2(20) = 11{,}520\,\text{W}$$

$$P_{\text{AB}} + P_{\text{BC}} + P_{\text{CA}} = 118{,}425.7$$

therefore $W_1 + W_2 \approx P_{\text{total}}$ (round-off differences)

P 11.49 **[a]** $\mathbf{I}^*_{\text{aA}} = \frac{144(0.96 - j0.28)10^3}{7200} = 20\underline{/-16.26^\circ}\,\text{A}$

$\mathbf{V}_{\text{BN}} = 7200\underline{/-120^\circ}\,\text{V}$; $\mathbf{V}_{\text{CN}} = 7200\underline{/120^\circ}\,\text{V}$

$$\mathbf{V}_{\text{BC}} = \mathbf{V}_{\text{BN}} - \mathbf{V}_{\text{CN}} = 7200\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$\mathbf{I}_{\text{bB}} = 20\underline{/-103.74^\circ}\,\text{A}$$

$$W_{\text{m1}} = (7200\sqrt{3})(20)\cos(-90^\circ + 103.74^\circ) = 242{,}278.14\,\text{W}$$

[b] Current coil in line aA, measure $\mathbf{I}_{\text{aA}}$.
Voltage coil across AC, measure $\mathbf{V}_{\text{AC}}$.

[c] $I_{\text{aA}} = 20\underline{/16.76^\circ}\,\text{A}$

$$\mathbf{V}_{\text{CA}} = \mathbf{V}_{\text{AN}} - \mathbf{V}_{\text{CN}} = 7200\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$W_{\text{m2}} = (7200\sqrt{3})(20)\cos(-30^\circ - 16.26^\circ) = 172{,}441.86\,\text{W}$$

[d] $W_{m1} + W_{m2} = 414.72\text{kW}$

$$P_T = 432{,}000(0.96) = 414.72\,\text{kW} = W_{m1} + W_{m2}$$

P 11.50 **[a]** $W_1 = |\mathbf{V}_{BA}||\mathbf{I}_{bB}|\cos\theta$

Negative phase sequence:

$$\mathbf{V}_{BA} = 240\sqrt{3}\underline{/150^\circ}\,\text{V}$$

$$\mathbf{I}_{aA} = \frac{240\underline{/0^\circ}}{13.33\underline{/-30^\circ}} = 18\underline{/30^\circ}\,\text{A}$$

$$\mathbf{I}_{bB} = 18\underline{/150^\circ}\,\text{A}$$

$$W_1 = (18)(240)\sqrt{3}\cos 0^\circ = 7482.46\,\text{W}$$

$$W_2 = |\mathbf{V}_{CA}||\mathbf{I}_{cC}|\cos\theta$$

$$\mathbf{V}_{CA} = 240\sqrt{3}\underline{/-150^\circ}\,\text{V}$$

$$\mathbf{I}_{cC} = 18\underline{/-90^\circ}\,\text{A}$$

$$W_2 = (18)(240)\sqrt{3}\cos(-60^\circ) = 3741.23\,\text{W}$$

[b] $P_\phi = (18)^2(40/3)\cos(-30^\circ) = 3741.23\,\text{W}$

$$P_T = 3P_\phi = 11{,}223.69\,\text{W}$$

$$W_1 + W_2 = 7482.46 + 3741.23 = 11{,}223.69\,\text{W}$$

$$\therefore\quad W_1 + W_2 = P_T \qquad \text{(checks)}$$

P 11.51 **[a]** $Z = \frac{1}{3}Z_\Delta = 4.48 + j15.36 = 16\underline{/73.74^\circ}\,\Omega$

$$\mathbf{I}_{aA} = \frac{600\underline{/0^\circ}}{16\underline{/73.74^\circ}} = 37.5\underline{/-73.74^\circ}\,\text{A}$$

$$\mathbf{I}_{bB} = 37.5\underline{/-193.74^\circ}\,\text{A}$$

$$\mathbf{V}_{AC} = 600\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{BC} = 600\sqrt{3}\underline{/-90^\circ}\,\text{V}$$

$$W_1 = (600\sqrt{3})(37.5)\cos(-30 + 73.74^\circ) = 28{,}156.15\,\text{W}$$

$$W_2 = (600\sqrt{3})(37.5)\cos(-90 + 193.74^\circ) = -9256.15\,\text{W}$$

[b] $W_1 + W_2 = 18{,}900\,\text{W}$

$$P_T = 3(37.5)^2(13.44/3) = 18{,}900\,\text{W}$$

[c] $\sqrt{3}(W_1 - W_2) = 64{,}800\,\text{VAR}$

$$Q_T = 3(37.5)^2(46.08/3) = 64{,}800\,\text{VAR}$$

P 11.52 [a] Negative phase sequence:

$$\mathbf{V}_{\text{AB}} = 240\sqrt{3}\angle -30^\circ\,\text{V}$$

$$\mathbf{V}_{\text{BC}} = 240\sqrt{3}\angle 90^\circ\,\text{V}$$

$$\mathbf{V}_{\text{CA}} = 240\sqrt{3}\angle -150^\circ\,\text{V}$$

$$\mathbf{I}_{\text{AB}} = \frac{240\sqrt{3}\angle -30^\circ}{20\angle 30^\circ} = 20.78\angle -60^\circ\,\text{A}$$

$$\mathbf{I}_{\text{BC}} = \frac{240\sqrt{3}\angle 90^\circ}{60\angle 0^\circ} = 6.93\angle 90^\circ\,\text{A}$$

$$\mathbf{I}_{\text{CA}} = \frac{240\sqrt{3}\angle -150^\circ}{40\angle -30^\circ} = 10.39\angle -120^\circ\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}} + \mathbf{I}_{\text{AC}} = 18\angle -30^\circ\,\text{A}$$

$$\mathbf{I}_{\text{cC}} = \mathbf{I}_{\text{CB}} + \mathbf{I}_{\text{CA}} = \mathbf{I}_{\text{CA}} + \mathbf{I}_{\text{BC}} = 16.75\angle -108.06^\circ$$

$$W_{m1} = 240\sqrt{3}(18)\cos(-30 + 30^\circ) = 7482.46\,\text{W}$$

$$W_{m2} = 240\sqrt{3}(16.75)\cos(-90 + 108.07^\circ) = 6621.23\,\text{W}$$

[b] $W_{m1} + W_{m2} = 14{,}103.69\,\text{W}$

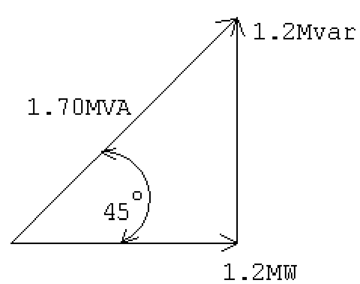
$$P_{\text{A}} = (12\sqrt{3})^2(20\cos 30^\circ) = 7482.46\,\text{W}$$

$$P_{\text{B}} = (4\sqrt{3})^2(60) = 2880\,\text{W}$$

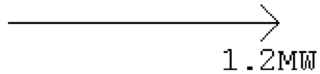
$$P_{\text{C}} = (6\sqrt{3})^2[40\cos(-30^\circ)] = 3741.23\,\text{W}$$

$$P_{\text{A}} + P_{\text{B}} + P_{\text{C}} = 14{,}103.69 = W_{m1} + W_{m2}$$

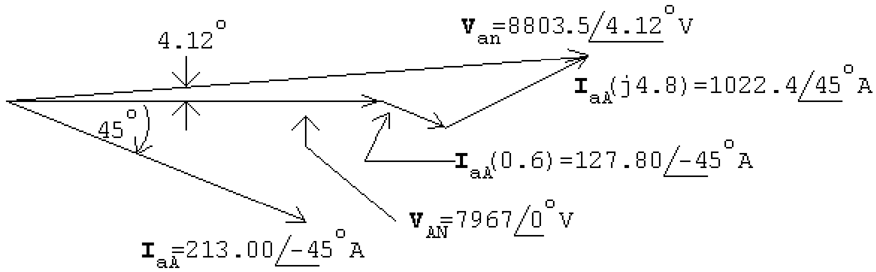
P 11.53 [a]



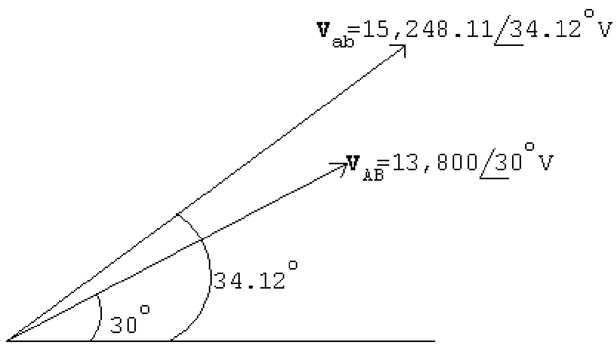
[**b**]



[**c**]



[**d**]



P 11.54 [**a**] $Q = \frac{|\mathbf{V}|^2}{X_C}$

$$\therefore \quad |X_C| = \frac{(13{,}800)^2}{1.2 \times 10^6} = 158.70\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 158.70; \qquad C = \frac{1}{2\pi(60)(158.70)} = 16.71\,\mu\text{F}$$

[**b**] $|X_C| = \frac{(13{,}800/\sqrt{3})^2}{1.2 \times 10^6} = \frac{1}{3}(158.70)$

$$\therefore \quad C = 3(16.71) = 50.14\,\mu\text{F}$$

P 11.55 [**a**] The capacitor from Appendix H whose value is closest to 50.14 μF is 47 μF.

$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi(60)(47 \times 10^{-6})} = 56.4\,\Omega$$

$$Q = \frac{|V|^2}{3X_C} = \frac{(13{,}800)^2}{3(56.4)} = 1{,}124{,}775.6\,\text{VAR}$$

[b] $\mathbf{I}^*_{\text{aA}} = \dfrac{1{,}200{,}000 + j75{,}224}{13{,}800/\sqrt{3}} = 150.6 + j9.4\,\text{A}$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(150.6 - j9.4) = 8134.8\underline{/5.06^\circ}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(8134.8) = 14{,}089.9\,\text{V}$$

This voltage falls within the allowable range of 13 kV to 14.6 kV.

P 11.56 **[a]** The capacitor from Appendix H whose value is closest to 16.71 μF is 22 μF.

$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi(60)(22 \times 10^{-6})} = 120.57\,\Omega$$

$$Q = \frac{|V|^2}{X_C} = \frac{(13{,}800)^2}{120.57} = 1{,}579{,}497\,\text{VAR}/\phi$$

[b] $\mathbf{I}^*_{\text{aA}} = \dfrac{1{,}200{,}000 - j379{,}497}{13{,}800/\sqrt{3}} = 50.2 - j15.9\,\text{A}$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(50.2 + j15.9) = 7897.8\underline{/1.76^\circ}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(7897.8) = 13{,}679.4\,\text{V}$$

This voltage falls within the allowable range of 13 kV to 14.6 kV.

P 11.57 If the capacitors remain connected when the substation drops its load, the expression for the line current becomes

$$\frac{13{,}800}{\sqrt{3}}\mathbf{I}^*_{\text{aA}} = -j1.2 \times 10^6$$

or $\quad \mathbf{I}^*_{\text{aA}} = -j150.61\,\text{A}$

Hence $\quad \mathbf{I}_{\text{aA}} = j150.61\,\text{A}$

Now,

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(j150.61) = 7244.49 + j90.37 = 7245.05\underline{/0.71^\circ}\,\text{V}$$

The magnitude of the line-to-line voltage at the generating plant is

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(7245.05) = 12{,}548.80\,\text{V}.$$

This is a problem because the voltage is below the acceptable minimum of 13 kV. Thus when the load at the substation drops off, the capacitors must be switched off.

P 11.58 Before the capacitors are added the total line loss is

$$P_L = 3|150.61 + j150.61|^2(0.6) = 81.66\,\text{kW}$$

After the capacitors are added the total line loss is

$$P_L = 3|150.61|^2(0.6) = 40.83\,\text{kW}$$

Note that adding the capacitors to control the voltage level also reduces the amount of power loss in the lines, which in this example is cut in half.

P 11.59 **[a]** $\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{\text{aA}}^* = 60 \times 10^3 + j160 \times 10^3 - j1200 \times 10^3$

$$\mathbf{I}_{\text{aA}}^* = \frac{60\sqrt{3} - j1040\sqrt{3}}{13.8} = 7.53 - j130.53\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = 7.53 + j130.53\,\text{A}$$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(7.53 + j130.53)$$

$$= 7345.41 + j114.46 = 7346.3\underline{/0.89^\circ}\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{\text{ab}}| = \sqrt{3}(7346.3) = 12{,}724.16\,\text{V}$$

[b] Yes, the magnitude of the line-to-line voltage at the power plant is less than the allowable minimum of 13 kV.

P 11.60 **[a]** $\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{\text{aA}}^* = (60 + j160) \times 10^3$

$$\mathbf{I}_{\text{aA}}^* = \frac{60\sqrt{3} + j160\sqrt{3}}{13.8} = 7.53 + j20.08\,\text{A}$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = 7.53 - j20.08\,\text{A}$$

$$\mathbf{V}_{\text{an}} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(7.53 - j20.08)$$

$$= 8068.34 + j24.10 = 8068.38\underline{/0.17^\circ}\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{\text{ab}}| = \sqrt{3}(8068.38) = 13{,}974.77\,\text{V}$$

[b] Yes: $13\,\text{kV} < 13{,}974.77 < 14.6\,\text{kV}$

[c] $P_{\text{loss}} = 3|7.53 + j130.53|^2(0.6) = 30.77\,\text{kW}$

[d] $P_{\text{loss}} = 3|7.53 - j20.08|^2(0.6) = 0.83\,\text{kW}$

[e] Yes, the voltage at the generating plant is at an acceptable level and the line loss is greatly reduced.