Friction :

When a Force $\vec{F}$ tends to slide a body along a surface, a frictional force from the surface acts on the body. The frictional force is parallel to the surface and directed to oppose the sliding. It is due to bonding between the atoms on the body and the atoms on the surface, an effect called cold-welding.

If the body does not slide, the frictional force is a static frictional force $\vec{f}_s$.

If there's sliding, the frictional force is a Kinetic frictional force $\vec{f}_k$

[1] If a body does not move, the static frictional force $\vec{f}_s$ and the component of $\vec{F}$ parallel to the surface are equal in magnitude, and $\vec{f}_s$ is directed opposite that component. If the component increases, f_s also increases

[2] The magnitude of $\vec{f}_s$ has a maximum value $f_{s,max}$ given by

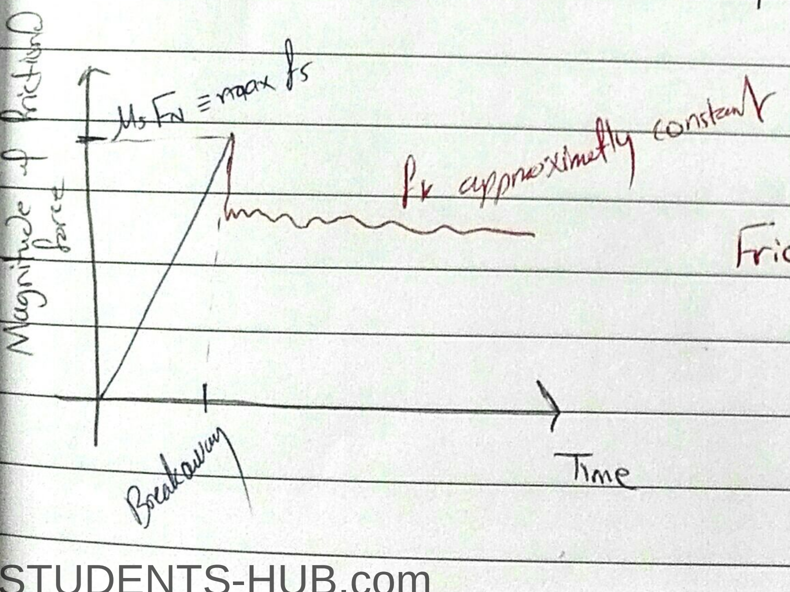
$$f_{s,max} = \mu_s F_N$$

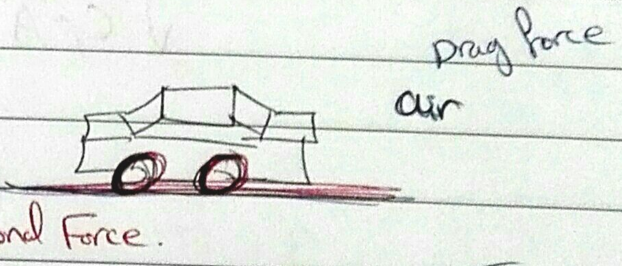
where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force. If the component of $\vec{F}$ parallel to the surface exceeds $f_{s,max}$, the static friction is overwhelmed and the body slides on the surface.

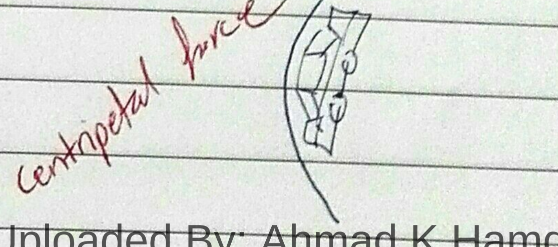
[3] If the body begins to slide on the surface, the magnitude of the frictional force rapidly decreases to a constant value f_k given by

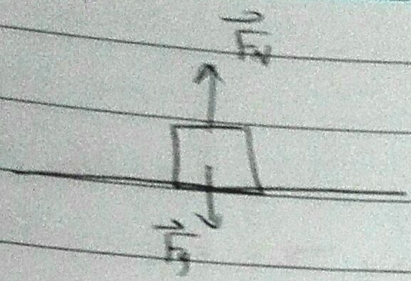
$$f_k = \mu_k F_N$$

where μ_k is the coefficient of Kinetic friction.









Frictional force $= 0$
(There is no attempt at sliding. Thus, no friction and no motion)

The Drag Force and Terminal speed.

- When there is relative motion between air (or some other fluid) and a body, the body experiences a Drag Force $\vec{D}$ that oppose the relative motion and points in the direction in which the fluid flows relative to the body. The magnitude of $\vec{D}$ is related to the relative speed v by an experimentally determined drag-coefficient C according to

$$D = \frac{1}{2} C \rho A v^2$$

where ρ is the fluid density (mass per unit volume) and A is the effective cross-sectional area of the body (the area of a cross section taken perpendicular to the relative $\vec{v}$).

- Terminal speed

When a blunt object has fallen far enough through air, the magnitudes of the drag force $\vec{D}$ and the gravitational force $\vec{F_g}$ on the body become equal. The body then falls at constant terminal speed v_t given by

$$v_t = \sqrt{\frac{2 F_g}{C \rho A}}$$

* Uniform Circular Motion

If a particle moves in a circle or a circular arc of radius R at constant speed v, the particle is said to be in Uniform circular motion. It then has a centripetal acceleration $\vec{a}$ with magnitude given by $a = v^2/R$

This acceleration is due to a net centripetal force on the particle, with magnitude given by $F = mv^2/R$

where m is the particle's mass. The vector quantities $\vec{a}$ and $\vec{F}$ are directed toward the center of curvature of the particle's path. A particle can move in circular motion only if a net centripental force acts on it.

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

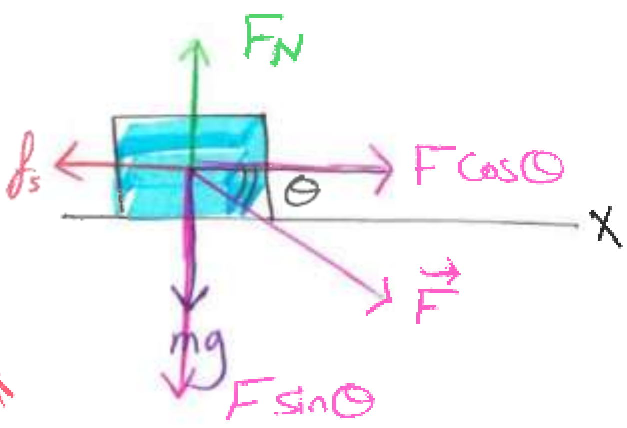
- The magnitudes of $\vec{a}$ and $\vec{F}$ are ~~the same~~ Constants.

Chapter 6: Force and Motion II

Q-1) If the box is stationary and the angle θ between the horizontal and force $\vec{F}$ is increased somewhat, do the following quantities increase, decrease, or remain the same: (a) F_x; (b) f_s; (c) F_N; (d) $f_{s,max}$? (e) If, instead, the box is sliding and θ is increased, does the magnitude of the frictional force on the box increase, decrease, or remain the same?

Case I: The Box is stationary

$a = \text{zero}$



$$F_x = F\cos\theta - f_s = ma_x$$

$$F_y = F_N - mg - F\sin\theta = ma_y$$

$$F\cos\theta = f_s$$

$$F_N = mg + F\sin\theta$$

- θ increases, $\cos\theta$ decreases $\Rightarrow$ $\boxed{F_x \text{ decreases}}$

$\boxed{f_s \text{ also decreases}}$ because $F_x = F\cos\theta = f_s$

- F_x decrease but $F_y = F\sin\theta$ increases because magnitude of $\vec{F}$ is constant (not changing)
- $F_y = F\sin\theta$ increases and mg is still constant because

$F_N = mg + F\sin\theta \Rightarrow \boxed{F_N \text{ increases}}$

- $f_{s,max} = \mu_s F_N$; μ_s is constant because we do not change the surface. So $\boxed{f_{s,max} \text{ increases also}}$.

$$F_N \propto F_{s,max}$$

Case II : The box is sliding, θ is increased

$$F_y = F_N - mg - F\sin\theta = ma_y$$

a_y = zero (No motion along the y-direction)

$$\Rightarrow F_N = mg - F\sin\theta$$

when θ increases, $\sin\theta$ increases also.

$F\sin\theta$ increases, mg is still constant

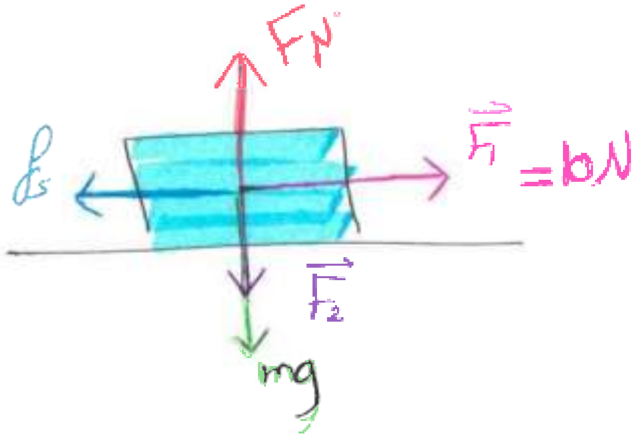
$\Rightarrow F_N$ increases

$$f_k = \mu_k F_N$$

μ_k is constant, we don't change the surfaces

$\Rightarrow$ $\boxed{f_k \text{ increases}}$; the magnitude of the frictional force on the box.

Q-3 Horizontal force $\vec{F_1}$ of magnitude 10N is applied to a box on floor, but the box does not slide. Then as the magnitude of vertical force $\vec{F_2}$ is increased from zero, do the following quantities increase, decrease, or stay the same: (a) the magnitude of the frictional force f_s on the box; (b) the magnitude of the normal force $\vec{F_N}$ on the box from the floor; the maximum value $f_{s,max}$ of the magnitude of the static frictional force on the box? (d) Does the box eventually slide?



Case I: The box does not slide
F_2 is zero (given)

$a_x = 0, \; a_y = 0$

$\Rightarrow F_x = 0 = \vec{F_1} - f_s \Rightarrow F_1 = f_s$

$\boxed{f_s = 10N}$

$\Rightarrow F_Y = 0 = F_N = mg$

Case II: $\vec{F_2}$ is increased from zero

(a) f_s is still the same $\Rightarrow F_1 = f_s$, F_1 isn't change

(b) F_N increases $\Rightarrow F_N = mg + F_2$

(c) $f_{s,max} = \mu_s F_N$ increases because F_N increases

(d) NO

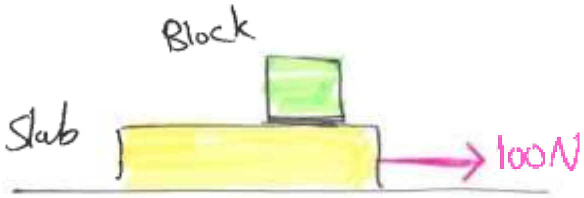
$f_{s,max} > f_s$

$\boxed{\mu_s > \mu_k}$

Q-8 A Horizontal force of 100 N is to be applied to a 10 kg slab that is initially stationary on a frictionless floor, to accelerate the slab. A 10 kg block lies on top of the slab; the coefficient of friction μ between the block and the slab is not known, and the block might slip. In fact, the contact between the block & the slab might even be frictionless. (a) Considering that possibility, what is the possible range of values for the magnitude of the slab's acceleration a_{slab} (Hint: You don't need written calculations; just consider extreme values for μ). (b) What is the possible range for the magnitude a_{block} of the block's acceleration?

Case I: If the contact surface between the block and the slab is frictionless

$\mu = 0$



$$F_{net x, slab} = 100\,N = 10\,kg\; a_{slab}$$

$$a_{slab} = \frac{100\,N}{10} = 10\,\frac{m}{s^2} \;;\quad \vec{a}_{slab} = 10\,\frac{m}{s^2}\,\hat{i}$$

Case 2: μ is maximum $\Rightarrow$ slab + block stick together

$$F_{net x, system} = m_{total}\; a_{system}$$

$$100\,N = 20\,kg\; a_{system}$$

$$a = 100/20\,kg = 5\,m/s^2$$

$\Rightarrow$ Rang of the slab's acceleration $[5\,\frac{m}{s^2} - 10\,\frac{m}{s^2}]$

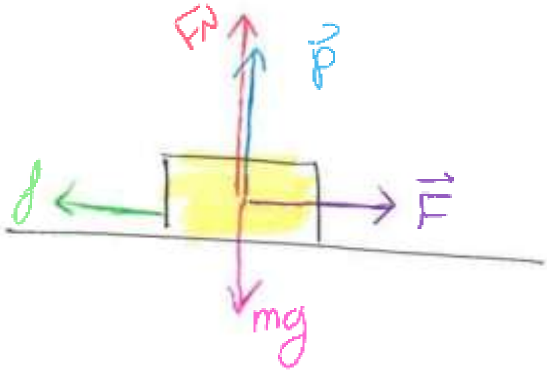
b) Block's acceleration:

Case I: $\mu = 0 \Rightarrow F_{net, block} = \text{zero}$, $\boxed{a_{block} = 0}$

Case II: $\mu = \text{max. value} \Rightarrow a_{block} = 5\,\frac{m}{s^2}$

$\Rightarrow$ Range of the block's acceleration $[0 - 5\,\frac{m}{s^2}]$

P-5 A 2.5 kg block is initially at rest on a horizontal surface. A horizontal force $\vec{F}$ of magnitude 6.0 N and a vertical force $\vec{P}$ are then applied to the block. The coefficient of friction for the block and the surface $\mu_s = 0.40$ and $\mu_k = 0.25$. Determine the magnitude of the frictional force acting on the block if the magnitude of $\vec{P}$ is (a) 8.0 N, (b) 10 N, and (c) 12 N ?



(a) $\vec{P} = 8.0\,N\,\hat{j}$

- $F_x = ma_x = F - f = ma_x$

$$\boxed{F - f = ma}$$

- $F_Y = ma_y = 0 = P + F_N = mg$

$$\boxed{F_N = mg - P}$$

if the static friction is applied to the block $\boxed{f_s = \mu_s F_N}$

$\Rightarrow$ the body doesn't slide $\Rightarrow a = 0$

$F = f_s = 6.0\,N$

$F_N = mg - P \Rightarrow$

$F_N = 24.5 - 8 = 16.5\,N$

$f_{s\,max} = \mu_s F_N = 6.6\,N$

$f_s = 6.0\,N < f_{s\,max} = 6.6\,N \Rightarrow$ the block doesn't move

(b) $F_N = 24.5 - 10 = 14.5\,N$, $P = 10\,N$

$f_{s\,max} = \mu_s F_N = 0.4 \times 14.5 = 5.8\,N$

$F_x = F - f_{s\,max} = 6.0 - 5.8 = 0.2\,N$

In this case, we are dealing not with static but with kinetic

$f_k = \mu_k F_N = 0.25 \times 14.5 = 3.625 N$

$F_N = mg - P$, $P = 12N$

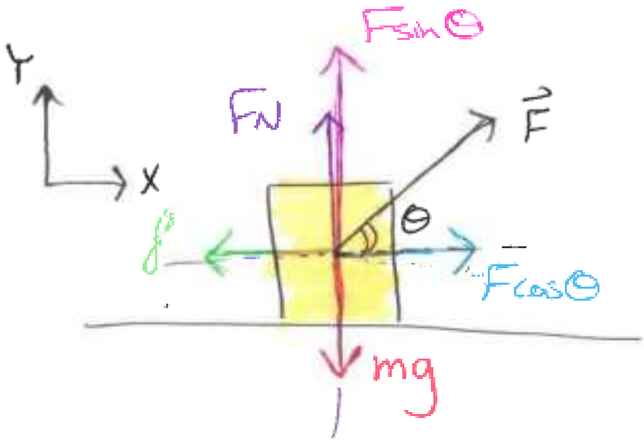
$= 24.5 - 12 \Longrightarrow \boxed{F_N = 12.5 N}$

$f_{s\,max} = \mu_s F_N = 0.4 \times 12.5 = 5.0 N$

$f_{s\,max} < F \Rightarrow$ The block moves

The kinetic friction $f_k = \mu_k F_N = 3.1 N$

P-10 An initially stationary block of mass m on a floor. A force of magnitude 0.5mg is then applied at upward angle $\theta = 20°$. What is the magnitude of the acceleration of the block across the floor if the friction coefficients are (a) $\mu_s = 0.600$ and $\mu_k =$ 0.500 and (b) $\mu_s = 0.400$ and $\mu_k = 0.300$?



Newton's second Law

$$F_x = F\cos\theta - f = ma \quad \text{---} \; (1)$$

$$F_y = F\sin\theta + F_N - mg = 0$$

$$\Rightarrow F_N = mg - F\sin\theta$$

$$f_k = \mu_k F_N = \mu_k [mg - F\sin\theta]$$

equation (1): $F\cos\theta - f = ma$

$$F\cos\theta - \mu_k [mg - F\sin\theta] = ma$$

$$\boxed{a = \frac{F}{m}[\cos\theta + \mu_k \sin\theta] - \mu_k g}$$ [moving block]

If the block doesn't move $\Rightarrow a = 0$

$$f_s = F\cos\theta$$

$$\boxed{f_{max,s} = \mu_s F_N = \mu_s [mg - F\sin\theta]}$$

(a) $\mu_s = 0.600$, $\mu_k =$ 0.500

$$F = 0.5\,mg$$

$$f_{s,max} = \mu_s [mg - F\sin\theta] = 0.600 [mg - 0.5mg\sin\theta] = 0.497N$$

$$f_s = F\cos\theta = 0.5mg\cos 20° = 0.469\,mg$$

$f_s < f_{s,max} \longrightarrow$ The block remains stationary $\Rightarrow a = 0$

(b) $\mu_s = 0.400$, $\mu_k = 0.300\ N$

$$f_{s,max} = \mu_s [mg - F \sin\theta]$$
$$= 0.4 [mg - 0.5 mg \sin 20^\circ]$$

$$\boxed{f_{s,max} = 0.332\ mg}$$

$\Rightarrow$ $F\cos\theta = 0.5 mg \cos 20^\circ = 0.47\ mg$

$F\cos\theta > f_{s,max} = 0.332 mg$

$\Rightarrow$ the block moves $\Rightarrow$ $a = ?$

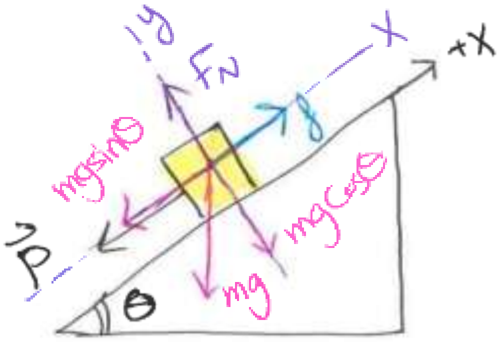
So we talk here about kinetic frictional force

$$f_k = \mu_k F_N$$

$$a = \frac{F}{m}[\cos\theta + \mu_k \sin\theta] - \mu_k g$$
$$= \frac{0.5\ mg}{m}[\cos(20^\circ) + 0.3 \sin 20^\circ] - (0.3 \times 9.8) = 2.17 \frac{m}{s^2}$$

$$\boxed{\vec{a} = \left(2.17 \frac{m}{s^2}\right)\hat{i}}$$

P-17 A force $\vec{P}$ acts on a block weighing 45N. The block is initially at rest on a plane inclined at angle $\theta = 15°$ to the horizontal. The positive direction of the x-axis is up the plane. Between block and plane, the coefficient of static friction is $\mu_s = 0.50$ and the coefficient of kinetic friction is $\mu_k = 0.34$. In unit vector notation, what is the frictional force on the block from the plane when $\vec{P}$ is (a) $(-5.0N)\hat{i}$ (b) $(-8.0N)\hat{i}$ and (c) $(-15N)\hat{i}$?



- $F_{net,y} = ma_y = \text{Zero}$

$F_N = mg\cos\theta$

$= 45\cos 15°$

$\boxed{F_N = 43.5N}$ No motion on y-direction

- $f_{s,max} = \mu_s F_N = 0.5 \times 43.5 = 21.7N$

(a) $\vec{P} = (-5.0N)\hat{i}$

$F_{net,x} = f - mg\sin\theta - P = ma_x$

if the block isn't sliding $\rightarrow a_x = \text{zero}$

$F_{net,x} = 0 = f_s - mg\sin\theta - P \rightarrow f_s = mg\sin\theta + P$

$f_s = 45\sin 15° + 5 = 16.65N$

$\boxed{f_s < f_{s,max} \Rightarrow \text{the block doesn't move}} \Rightarrow \boxed{\vec{f}_s = 16.6N\hat{i}}$

(b) $\vec{P} = (-8.0N)\hat{i}$

$f_s = mg\sin\theta + P = 45\sin 15° + P = 19.65N$

$f_s = 19.65N < f_{s,max} = 21.7N$

The block doesn't move

$\boxed{\vec{f} = 20N\hat{i}}$

(c) $\vec{P} = (-15N)\hat{i}$

$f_s = mg\sin\Theta + P = 45\sin 15° + 15 = 26.65\ N$

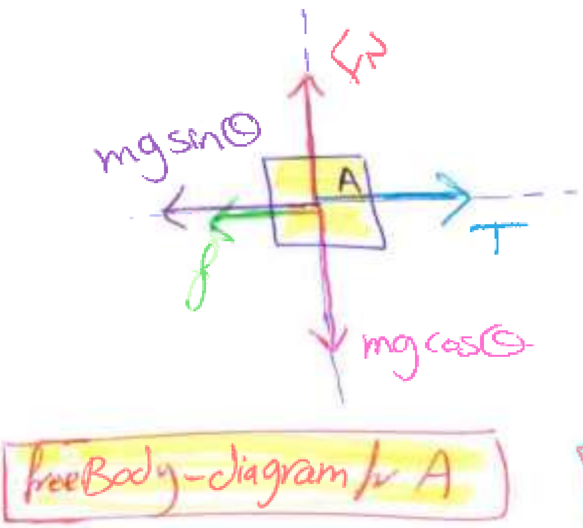
$f_s = 26.65N > f_{s,max} = 21.7\ N$

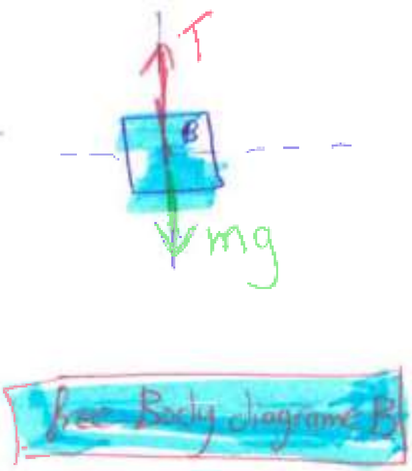
The block moves, so we replace the static friction by kinetic friction

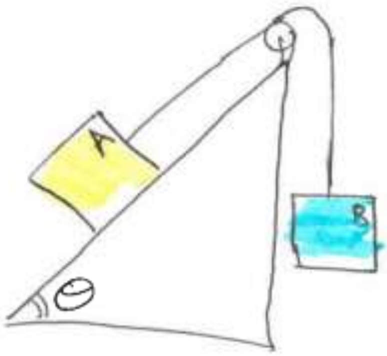
$f_k = \mu_k F_N = 0.34 \times 43.5 = 14.79\ N$

$$\boxed{\vec{f_k} = (15N)\hat{i}}$$

P-27 Body A weighs 102N, and body B weighs 32N. The Coefficients of friction between A and the incline are $\mu_s = 0.56$ and $\mu_k = 0.25$. Angle θ is 40°, let the positive direction of an x-axis be up the incline. In unit vector notation, What is the acceleration of A if A is initially (a) at rest, (b) moving up the incline, and (c) moving down the incline ?







(a) A is at Rest [a=0]

$F_{net,x,A} = T - f - m_A g\sin\theta = 0 \Rightarrow T = f + m_A g\sin\theta$ — ①

$F_{net,y,A} = F_N - m_A g\cos\theta = 0 \Rightarrow F_N = m_A g\cos\theta$ — ②

• $F_{net,y,B} = m_B g - T = 0 \Rightarrow T = m_B g$ → ③

equation ① – equation ③

$f = m_B g - m_A g\sin\theta$
$= 32 - (102\sin 40°)$
$f = -33.6\ N$

$m_B g = 32N$
$m_A g\sin\theta = 65.6N$
$m_A g\sin\theta > 32$
the motion downhill

negative sign indicates that the force of the friction is uphill

$F_N = m_A g\cos\theta = 102\cos 40° = 78.1N$

$\cdot\ f_{s,max} = \mu_s F_N = 0.56\,(78.1) = 43.8\,N$

$f = 34\,N < f_{s,max} = 43.8\,N$

The Bodies remain at rest $\Rightarrow a = 0$

$\boxed{mg = W}$

(b) A is moving up the incline

$T - f_k - m_A g \sin\theta = m_A a$ —— ①

$F_N - m_A g\cos\theta = 0 \Rightarrow F_N = m_A g\cos\theta$ —— ②

$m_B g - T = m_B a$ —— ③

$\Rightarrow a$ is the same for the two Bodies $(a_A = a_B)$

① + ③ $\Rightarrow m_B g - f_k - m_A g\sin\theta = (m_A + m_B)\,a$

$$a = \frac{m_B g - m_A g \sin\theta - f_k}{m_A + m_B} \quad ; f_k = \mu_k F_N$$

$$= \frac{32 - (102 \sin 40^\circ) - (\mu_k m_A g \cos\theta)}{(32 + 102)/9.8}$$

$a = -3.88\ m/s^2 \quad ; \quad \vec{a} = (-3.88\ \frac{m}{s^2})\,\hat{i}$

{ the block A is slowing down and B also is slowing down }

(c) A initially moving downward the hill

$T + f_k - m_A g\sin\theta = m_A a \ ; \ f_k = \mu_k F_N$

$F_N = m_A g\cos\theta$

$m_B g - T = m_B a$

$$a = \frac{m_B g + \mu_k (m_A g \cos\theta) - m_A g \sin\theta}{m_A + m_B}$$

$$a = \frac{32 + (0.25\,(102 \cos 40^\circ)) - 102 \sin 40}{(32 + 102)/g} = -1.0\,\frac{m}{s^2}$$

{ $\vec{a} = (-1.0\frac{m}{s^2})\,\hat{i}$

$\Rightarrow$ the objects are speeding up, while the block A acceleration is again down hill }

P-36 The terminal speed of a sky diver is 160 $\frac{Km}{h}$ in the spread eagle position and 310 $\frac{Km}{h}$ in the nosedive position. Assuming that the diver's drag coefficient C does not change from one position to the other, find the ratio of the effective cross-sectional area A in the slower position to that in the faster position?

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \quad , \text{Terminal speed}$$

$$\frac{A_{\text{slower position}}}{A_{\text{faster position}}} = \frac{2mg / C\rho v_{t,\text{slower}}^2}{2mg / C\rho v_{t,\text{faster}}^2}$$

$$= \frac{v_{t,\text{faster}}^2}{v_{t,\text{slower}}^2} = \left(\frac{310\ Km/h}{160\ Km/h}\right)^2$$

$$\boxed{\frac{A_{\text{slower position}}}{A_{\text{faster position}}} = \left(\frac{310}{160}\right)^2 = 3.8}$$

P-42 Suppose the coefficient of static friction between the road and the tires on a car is 0.60 and the car has no negative lift. What speed will put the car on the verge of sliding as it rounds a level curve of 30.5 m radius?

- Centripetal acceleration $\Rightarrow a = \frac{v^2}{R}$

$$f = m\frac{v^2}{R} \quad ; \text{ static friction}$$

- $f_{s,max} = \mu_s F_N$ and $F_N = mg$

- If the car does not slip $\Rightarrow f \leq \mu_s mg$

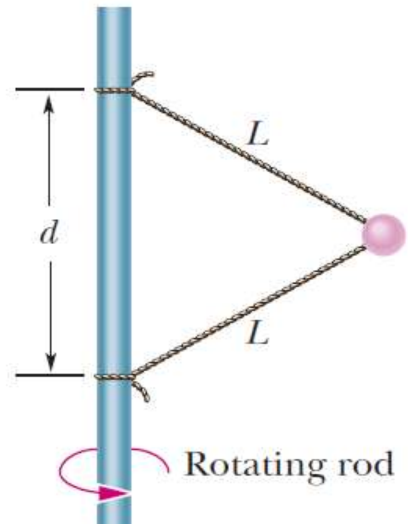
$$\frac{v^2}{R} \leq \mu_s g$$

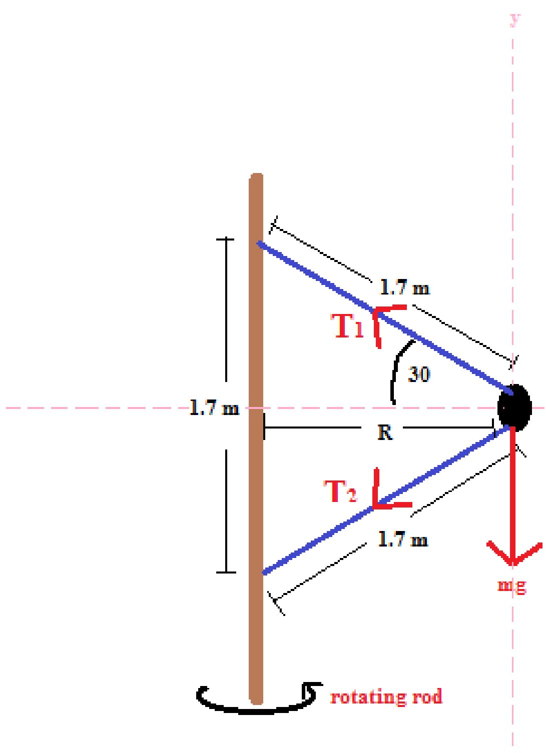
- The car can round the curve without slipping with maximum speed $\Rightarrow$

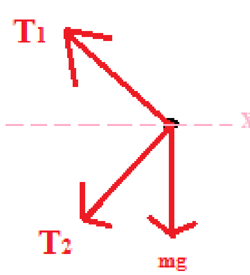
$$v_{max} = \sqrt{\mu_s R g} = \sqrt{(0.6)(30.5)m(9.8\,\tfrac{m}{s^2})}$$

$$\boxed{v_s = 13.4\ m/s}$$

6-59) A 1.34 kg ball is connected by means of two massless strings, each of length L = 1.70 m, to a vertical, rotating rod. The strings are tied to the rod with separation d =1.70 m and are taut. The tension in the upper string is 35 N. What are the (a) tension in the lower string, (b) magnitude of the net force on the ball, and (c) speed of the ball? (d) What is the direction of F_{net}?







$$T_2 = \frac{35 * \sin 30 - 1.34 * 10}{\sin 30} = 8.2\ N$$

b) The magnitude of the net force

Note that the net force on the y-axis is zero, the net force results from the x-components only

$F_{net} = T_1 \cos 30 + T_2 \cos 30 = 35 \cos 30 + 8.2 \cos 30 = 37.41\ N$

c) The speed of the ball

Using eq. (1)

$$T_1 \cos 30 + T_2 \cos 30 = \frac{mv^2}{R}$$

$$37.41 = \frac{mv^2}{1.7 \cos 30}$$

$$v = \sqrt{\frac{37.41 * 1.7 \cos 30}{m}} = \sqrt{\frac{37.41 * 1.7 \cos 30}{1.34}} = 6.411\ m/s$$

d) The direction of the net force

The direction of the net force is radially inward as the net force here is centripetal force

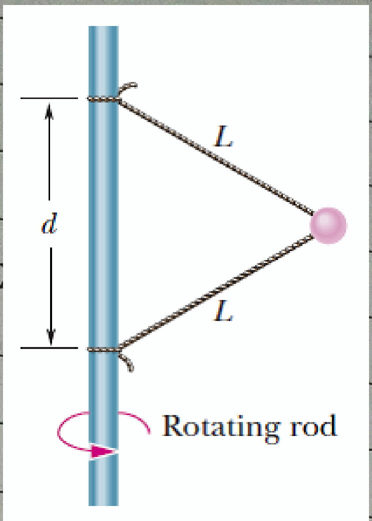
59 A 1.34 kg ball is connected by means of two massless strings each of length $L = 1.70$ m, to a vertical rotating rod. The strings are tied to the rod with separation $d = 1.70$ m and are taut. The tension in the upper string is 35 N.

a) What are the tension in the lower string

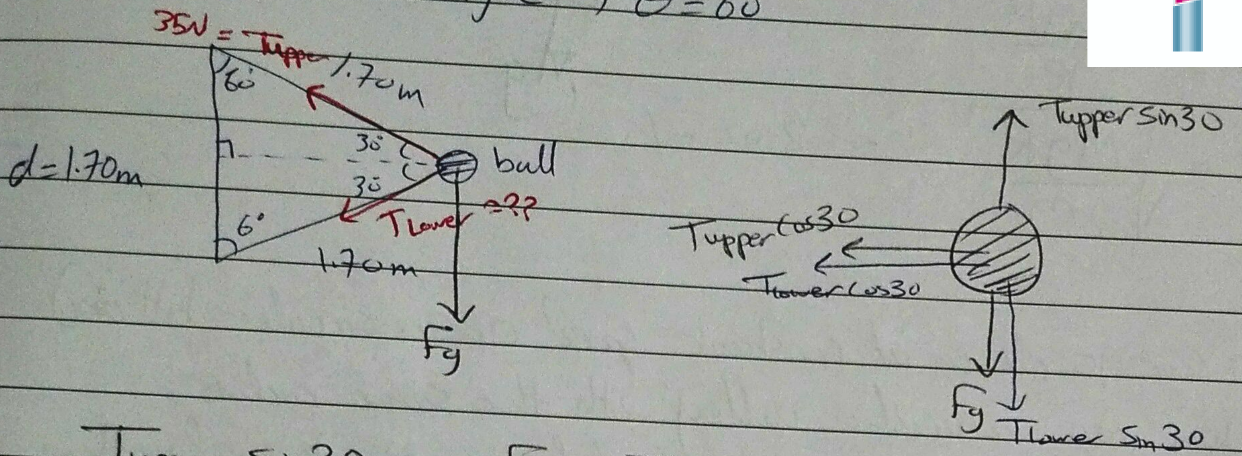
b) The net force F_{net} on the ball?

c) speed of the ball

d) what is the direction of $\vec{F}_{net}$?



$d = L$ so the triangle is an equilateral triangle, $\theta = 60°$



$T_{upper} \sin 30 = F_g + T_{lower} \sin 30$

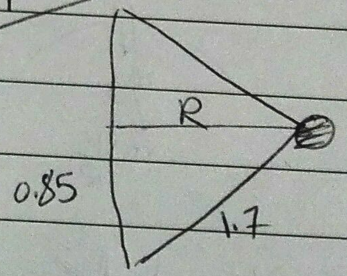
$35 \sin 30 = 1.34 \times 9.8 + T_{lower} \sin 30$

$\Rightarrow T_{lower} = 8.74$ N

b) $\vec{F}_{net} = T_{upper} \cos 30 + T_{lower} \cos 30$

$= (35 + 8.74) \cos 30 = 37.9$ N

c) $\vec{F}_{net} = ma = m\frac{v^2}{R}$ centripetal acceleration



$V = \sqrt{\frac{37.9 (1.47)}{1.34}}$

$= 6.45$ m/s

$R = 1.47$ m

d) Radially inward. (towards the rod)