13.6 : Randomized Block Design.

exp:

		Treatments		
		system A	system B	system C
Blocks	controller 1	15	15	18
	controller 2	14	14	14
	controller 3	10	11	15
	controller 4	13	12	17
	controller 5	16	13	16
	controller 6	13	13	13

⇒ Randomized Block Design.

Block : مش متغير ، هو اختيار عشوائي لكل الأشخاص بكل systems (كل اشي بشكل Random)

→ we have 3 treatments and 6 Blocks.

→ ANOVA table : Block Randomized Design.

Source of variance	df	SS	MS	F
Treatments	$K-1$	SSTR	MSTR	$F = \frac{MSTR}{MSE}$
Blocks	$b-1$ (b = عدد Blocks)	SSBL	MSBL	
Error	$(K-1)(b-1)$	SSE	MSE	
Total	$n_T - 1$	SST		

- b = # of Blocks
- BL = Blocks
- $n_T = Kb$
- $MSTR = \frac{SSTR}{K-1}$
- $MSBL = \frac{SSBL}{b-1}$
- $MSE = \frac{SSE}{(K-1)(b-1)}$
- $SST = SSTR + SSBL + SSE$

→

$H_0 : \mu_1 = \mu_2 = \dots = \mu_K$

H_1 : Not all μ_j are equal.

→ Rejection Rule:

- critical value approach:

reject H_0 if $F \geq F_\alpha$ with $df_1 = K-1$, $df_2 = (K-1)(b-1)$.

دائما upper tail test

- p-value approach:

reject H_0 if p-value $\leq \alpha$.

→ Notations and def.:

- $\bar{X}_{i\cdot}$: sample mean of Block i , $i = 1, \dots, b$.
- $\bar{X}_{\cdot j}$: sample mean of treatments j, $j = 1, \dots K$.
- $\bar{\bar{X}}$: over all mean of all observations.

كل عنصر في الجدول

- $SST = \sum_{j=1}^{K} \sum_{i=1}^{b} (X_{ij} - \bar{\bar{X}})^2$

الاعمدة بس

- $SSTR = b \sum_{j=1}^{K} (\bar{X}_{\cdot j} - \bar{\bar{X}})^2$

صفوف بس

- $SSBL = K \sum_{i=1}^{b} (\bar{X}_{i\cdot} - \bar{\bar{X}})^2$

- $SSE = SST - SSTR - SSBL$.

Cont. exp:

We find $\bar{X}_{1\cdot} = 16$ And: $\bar{X}_{\cdot 1} = 13.5$

$\bar{X}_{2\cdot} = 14$ $\bar{X}_{\cdot 2} = 13$

$\bar{X}_{3\cdot} = 12$ $\bar{X}_{\cdot 3} = 15.5$

$\bar{X}_{4\cdot} = 14$

$\bar{X}_{5\cdot} = 15$

$\bar{X}_{6\cdot} = 13$

And Find → SST = 70 SSBL = 30

SSTR = 21 SSE = 19

Now: ANOVA table.

K = 3
b = 6
$n_T = 3 \times 6 = 18$

S.O.V	df	SS	MS	F
TR	2	21	10.5	$\frac{10.5}{1.9} = 5.53$
BL	5	30	6	
E	10	19	1.9	
T	17	70		

Use $\alpha = 0.05$, $df_1 = 2$, $df_2 = 10$

$F_\alpha = F_{0.05} = 4.10$

$F > F_\alpha$ so we reject H_0 ($\alpha = 0.05$)

Not all μ_j are equal ($\alpha = 0.05$).