System Modeling in the time domain

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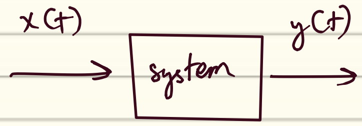
system modeling in the time domain



Properties :-

1] Continuoes time and Discrete time.

2] Time - invariant and Time variant Systems.

ex : if $y(t) = x(t^2)$, check if $y(t)$ is fixed or not fixed.

1. delay for time input , "function of time" t_0

$y_1(t - t_0) = x_1(t^2 - t_0) \longrightarrow (1)$

2. Time delay

$y_2(t - t_0) = x_2((t - t_0)^2) \longrightarrow (2)$

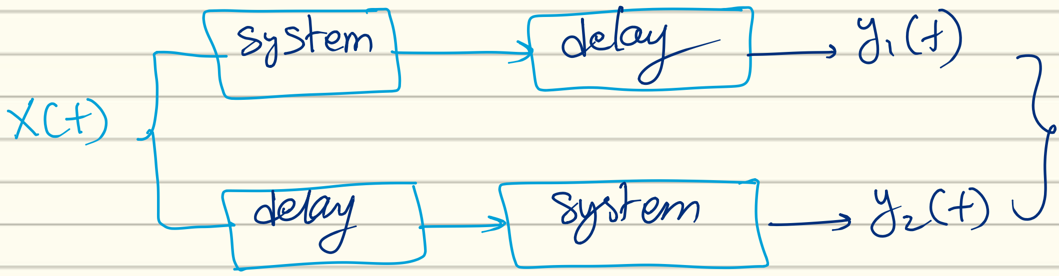
3. Compare eq_1 and eq_2

$y_1 \neq y_2$, So Time variant system. ["يعني عشي تأثر للـ delay"]

ex 2 :- $y(t) = x(t)$

$y_1(t - t_0) = x_1(t - t_0) \rightarrow ①$

$y_2(t - t_0) = x_2(t - t_0) \rightarrow ②$] $y_1 = y_2$, so Time invariant , "delay ما تأثر على output".



if $y_1(t) = y_2(t)$ → time invariant

if $y_1(t) \neq y_2(t)$ → time variant.

ex 3 : $y(t) = x(t-5) + x(3-t)$

$y_1(t) = x(t - 5 - t_0) + x(3 - t - t_0)$

$y_2(t) = x(t - t_0 - 5) + x(3 - t + t_0)$

$y_1(t) \neq y_2(t)$, Time variant.

ex. $y(t) = \sin(x(t)) \longrightarrow$ time invarient

$y(t) = x(t^2) \longrightarrow$ time varient.

3] Causal and non-Causal

past or Current values
"قيم الماضي أو الوقت الحالي"

Future values
"قيم المستقبل"

output values ≥ input values

ex. if the system $3y(t) + \int_{-\infty}^{t} y(\tau)\, d\tau = x(t)$

output في القيمة الموجودة " تعتمد على قيمة حالية
تعتمد على قيمة حاضرة ."
Causal.

ex. The system $y(t) = x(t^2)$
- $0 \le t \le 1$ Causal system
- $t > 1$ non-Causal system

* In general, this System is non Causal.

ex. $y(t) = 10x(\underbrace{t+2}_{\text{future value}}) + 5$

future value → So, non Causal system.
since values depends on future.

4] memory & memory less 8o "Dynamice and instantanuoes".

↳ use buffer

* values between brackets in input should equal values between brackets in output.
↳ memory less

ex. $y(t) = x(t)$, memory less , "Instantanuoes".

ex. $2\frac{dy(t)}{dt} + 3y(t) = \frac{\partial^2 x(t)}{\partial t^2} + x(t)$, memory / dynamic

differential equation ⇒ " لما نكاملها تصبح على فترة زمنية وليست على زمن محدد "

ex. $y(t+2) = y(t+2)$ → memory less.

- $y(t) = x(t^2)$
- $y(t) = x(t+1)$

] memory

* لو كانت مشتقة أو تكامل دائمًا "memory dynamic".

5] linear & non linear Systems :

linear & Time invariant.

linear → Scaling, Adding

ex. $y(t) = 3x(t)$

✻ اذا كانت مضروبة ب Constant , دائماً linear

$y_1(t) = \alpha 3 x_1(t)$

$y_2(t) = \alpha 3 x_2(t)$

$y_3 = y_1 + y_2$

$y_3 = 3\alpha x_1(t) + 3\alpha x_2(t)$

$y_4(t) = \alpha [3x_1(t) + 3x_2(t)]$

$y_4(t) = 3\alpha x_1(t) + 3\alpha x_2(t)$

1. نضرب y_1 ب α
2. // y_2 ب α
3. $y_3 = \alpha y_1 + \alpha y_2$
4. $y_4 =$ نعوض مكان x ← $\alpha x_1 + \alpha x_2$

$y_3(t) = y_4(t)$

so, linear

ex. $y(t) = 3x(t) + 2.$

✻ اذا كانت مجموعة إلى Constant دائماً non linear

$\alpha y_1(t) = 3\alpha x_1(t) + 2\alpha$

$\alpha y_2(t) = 3\alpha x_2(t) + 2\alpha$

$y_3(t) = 3\alpha x_1(t) + 3\alpha x_2(t) + 4\alpha.$

$y_4(t) = 3[\alpha x_1(t) + \alpha x_2(t)] + 2$

$y_4(t) = 3\alpha x_1(t) + 3\alpha x_2(t) + 2$

$y_3 \neq y_4$

ex. $\frac{dy}{dt} + 3y(t) = 2x(t)$

$\alpha y_1' + \alpha 3y_1(t) = \alpha 2x_1(t)$

$\alpha y_2' + \alpha 3y_2(t) = \alpha 2x_2(t)$

$y_3(t) = \alpha[y_1' + y_2'] + 3\alpha[y_1 + y_2] = 2\alpha[x_1(t) + x_2(t)]$

$y_4(t) = \alpha[y_1' + y_2'] + 3\alpha[y_1 + y_2] = 2\alpha[x_1(t) + x_2(t)]$

$y_3 = y_4$
linear system

ex. $\frac{dy}{dt} + 3y(t) + 5 = 2x(t)$

$\alpha y_1' + \alpha 3y_1(t) + \alpha 5 = \alpha 2x_1(t)$

$\alpha y_2' + \alpha 3y_2(t) + \alpha 5 = \alpha 2x_2(t)$]+

$y_3(t) = \alpha[y_1' + y_2'] + 3\alpha[y_1 + y_2] + 10\alpha = 2\alpha[x_1 + x_2]$

$y_4(t) = \alpha[y_1' + y_2'] + 3\alpha[y_1 + y_2] + 5 = 2\alpha[x_1(t) + x_2(t)].$

$y_3 \neq y_4$,
so, non linear system.

ex. $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$y_3 = \alpha \int_{-\infty}^{t} x_1(\tau)\, d\tau + \alpha \int_{-\infty}^{t} x_2(\tau)\, d\tau$

$y_4(t) = \alpha \int_{-\infty}^{t} (x_1(\tau) + x_2(\tau))\, d\tau$

$y_3 = y_4$

so, linear.

ex. $y[n] = x^2[n]$

$y_3[n] = \alpha x_1^2[n] + \alpha x_2^2[n]$

$y_4[n] = \alpha(x_1[n] + x_2[n])^2$

$y_3 \neq y_4$
non linear system.

ex. which of the following signals is Linear, Causal, time invariant and memory, justify your answer.

1 $y(t) = x(t-2) + x(2-t)$

A) linear :-

$\alpha y_1(t) = \alpha x_1(t-2) + \alpha x_1(2-t)$

$\alpha y_2(t) = \alpha x_2(t-2) + \alpha x_2(2-t)$

$y_3(t) = \alpha[x_1(t-2) + x_2(t-2)] + \alpha[x_1(2-t) + x_2(2-t)].$

$y_4(t) = \alpha[x_1(t-2) + x_2(t-2)] + \alpha[x_1(2-t) + x_2(2-t)]$

$y_3 = y_4$
linear system

B) Causality :- when $t=0$

$y(0) = x(-2) + x(2)$

present past future → so, non Causal.

C) Time-invariant :-

$y_1(t) = x(t-2-t_0) + x(2-t-t_0)$

$y_2(t-t_0) = x(t-t_0-2) + x(2-(t-t_0))$

$y_2(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$

time variant
$y_1 \neq y_2$

D) memory :- memory, $x(0) = x(-2) + x(2)$

---

2 $y(t) = \cos(3t)\,x(t)$

1) linearity :- $y_3 = \alpha\cos(3t)x_1(t) + \alpha\cos(3t)x_2(t).$

$y_4 = \alpha\cos 3t\,[x_1(t) + x_2(t)]$

$y_4 = \alpha\cos 3t\,x_1(t) + \alpha\cos 3t\,x_2(t)$

$y_3 = y_4$
linear.

2) Causality :- $y(1) = \cos(3)\,x(1)$ present so Causal.

هاي ما بتهمني
لإنها constant
المهم ما داخل y , x

3) Time invariant : $y_1(t) = \cos(3t - t_0)\, x(t-t_0)$

$y_2(t-t_0) = \cos(3t - 3t_0)\, x(t-t_0)$

time Variant

4) memory : memory less , $y(t) = \cos(3t)\, x(t)$

same input between prantheses.

$\boxed{3}$ $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau.$

1) linear :

$y_1(t) = \alpha \int_{-\infty}^{2t} x_1(\tau)\, d\tau.$

$y_2(t) = \alpha \int_{-\infty}^{2t} x_2(\tau)\, d\tau$

$y_3(t) = \alpha \int_{-\infty}^{2t} x_1(\tau)\, d\tau + \alpha \int_{-\infty}^{2t} x_2(t)\, d\tau.$

$y_4(t) = \alpha \int_{-\infty}^{2t} (x_1(\tau) + x_2(\tau))\, d\tau.$

$y_3 = y_4$

So, linear system.

2) Causality :- non Causal. , (t) depends on future values $(2t)$ value.

3) Time - variant :-

$y_1(t) = \int_{-\infty}^{2t-t_0} x(\tau)\, d\tau.$

$y_2(t) = \int_{-\infty}^{2t-2t_0} x(\tau)\, d\tau.$

Time variant system

4) memory :- memory system.

$\boxed{4}$ $y(t) = x(t-5) + x(3-t)$

* linearity :- $y_3(t) = \alpha x_1(t-5) + \alpha x_1(t-5) + \alpha x_2(t-5) + \alpha x_2(3-t)$.

$$y_4(t) = \alpha\big(x_1(t-5) + x_1(3-t)\big) + \alpha\big(x_2(t-5) + x_2(3-t)\big)$$

so, linear system.

* Causality :- $y(0) = y(-5) + \underbrace{x(3)}_{\text{future}}$, non Causal system.

* memory : , memory System.

* Time variant :- $y_1(t) = x(t-5-t_0) + x(3-t-t_0)$

$y_2(t) = x(t-t_0-5) + x(3-(t-t_0))$

$y_2(t) = x(t-t_0-5) + x(3-t+t_0)$

$y_1 \neq y_2$ time variant..

6) Stabel and unstable

bounded input

bounded ⇒ stable ⇒ (finite).

unbounded ⇒ unstabel ⇒ goes to infinity.

$|x(t)| \leq M$, $\forall t$.

ex. $y(t) = e^{x(t)}$

$= e^{M}$ → Constant, so stabel.

* bounded means Can reach max and min Value.
* it doesn't go to ∞.

ex. $x(t)\ e^{2t}$ → unstabel.

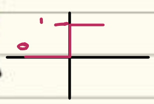
ex. $x(t)\ e^{2}$ stabel

ex. $x(t)\ e^{-2t}$ stabel

ex. $y(t) = \int_{-\infty}^{t} x(z)\,dz$ unstable

ممكن ناخذ أي function كمثال :-

$x(t) = u(t)$



→ $\int u(t) = r(t)$, as $t \to \infty$, $r(t) \to \infty$

so, it's unbounded

then the system is unstable

ex. $y(t) = 2x^2(t)$

$x(t) = u(t) \longrightarrow y(t) = 2u^2(t)$

$t = 1, \rightarrow y(1) = 2$, fixed, bounded.

so the system is stable.

6) Invertabel and non Invertabel

هل نقدر نكتب الـ input بدلالة الـ output

$\rightarrow (1-1) \longrightarrow$ invertable
كل input له output مختلف

$\rightarrow$ not $(1-1) \longrightarrow$ non invertable
more than one input give the same output

$$x(t) \longrightarrow \boxed{\quad T[\cdot] \quad} \longrightarrow y(t)$$

$T[x(t)] = y(t)$

$x(t) = T^{-1}[y(t)]$.

ex. $y(t) = 10x(t) + 3$

$T^{-1} = x(t) = \dfrac{y(t) - 3}{10} \rightarrow$ so the system is invertable

ex. $y[n] = x^2[n]$ non invertable

ex. $y(t) = \int_{-\infty}^{t} x(\lambda)\, d\lambda$ invertable

$T^{-1} = x(t) = y'(t)$

Invertable $\longrightarrow$
1. هل نقدر أكتب input بدلالة output
2. أوجد T^{-1} للـ function
3. unique input, unique output $\rightarrow$ كل input له output واحد فقط

Examples Which of the following Signals is Linear, Causal memory ?

1. $y(t) = x(t-2) + x(2-t)$

2. $y(t) = x\left(\frac{t}{3}\right)$

3. $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$

4. $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$

	linearity	causality	Time invariant	memory
1.	linear	non Causal	Time variant	memory
2.	linear	non-Causal	Time variant	memory
3.	linear	Causal	Time Variant	memory
4.	non Linear	Causal	Time invariant	memory

$y(t) = \ln|x(2^t)|$
↪ causal, noncausal.

LTI System — Linear Time Invarient

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

↳ when it is LTI system we always do Convolution Integral.

$$y(t) = x(t) * h(t)$$

$$* \; y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

or

$$* \; y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

$$* \int_{-\infty}^{\infty} x(t)\, h(k-t)\, dt$$

$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

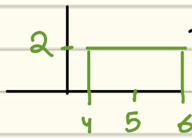
$$\int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

$$y[n] = \sum_{n=-\infty}^{\infty} x(\lambda)\, h(n-\lambda)$$

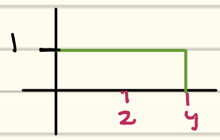
ex. for LTI system if

$x(t) = 2\pi\left(\frac{t-5}{2}\right)$ and $h(t) = \pi\left(\frac{t-2}{4}\right)$

Ans :- ① LTI system, evaluate $y(t)$, $y(t) = h(t) * x(t)$

②  $x(t) = 2\pi\left(\frac{t-5}{2}\right)$

$[4, 6] \leftarrow$ Range

 $h(t) = \pi\left(\frac{t-2}{4}\right)$

$[0, 4] \leftarrow$ Range

③ $[4, 6]$
$[0, 4]$

بجمع كل رقم في الفترة الأولى مع الأرقام في الفترة الثانية

$[4, 8, 6, 10]$

ثم برتبهم تصاعديا.

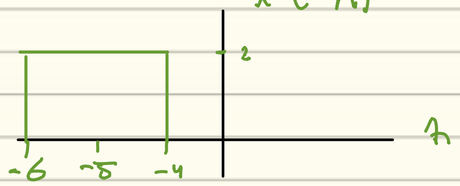
$[4, 6, 8, 10] \longleftarrow$ الفترة الي راح نشتغل عليها

lecture ⑪

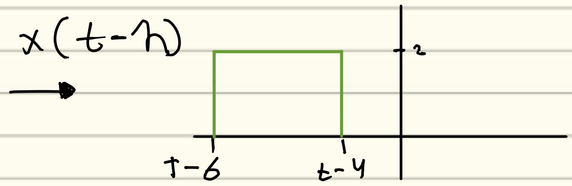
④ بحدد أي افتراض بدي أقلب.

$$y(t)=\int_{-\infty}^{\infty} x(t-\lambda)\,h(\lambda)\,d\lambda$$

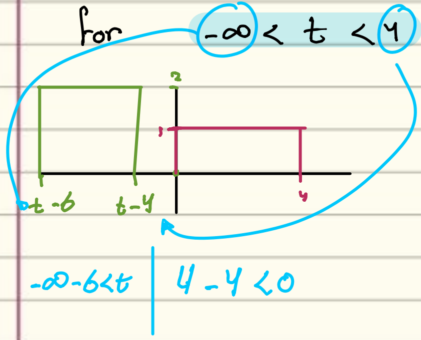
$x(-\lambda)$



$x(t-\lambda)$



for $-\infty < t < 4$

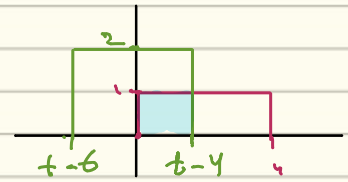


$-\infty-6<t$ | $4-4<0$

لا يوجد تقاطع

$y(t)=0$

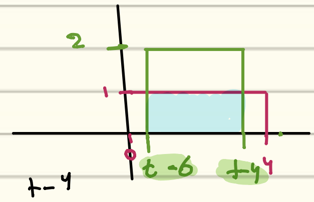
$4<t<6$



$$y(t)=\int_{0}^{t-4}(1)(2)\,d\lambda$$

$$=2(t-4)$$

$6<t<8$

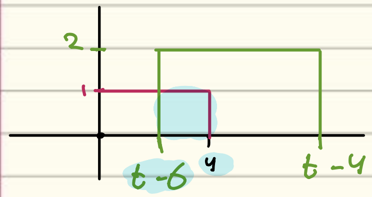


$$y(t)=\int_{t-6}^{t-4}(1)(2)\,d\lambda$$

$$=2(\cancel{t}-4-\cancel{t}+6)$$

$$=4$$

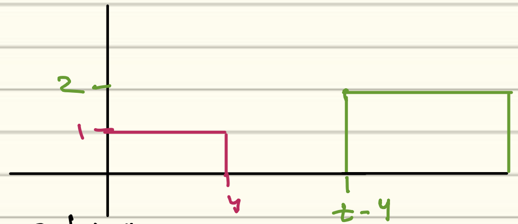
$8<t<10$



$$y(t)=\int_{t-6}^{4}(1)(2)\,d\lambda$$

$$=2(-2+10)$$

$t>10$

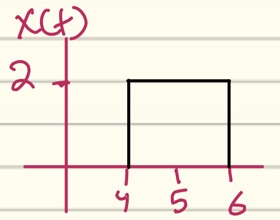
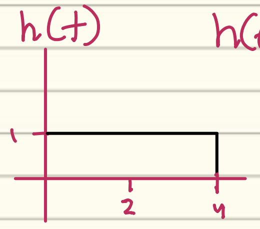


لا يوجد تقاطع

$y(t)=0$

another way to solve ex. $x(t) = 2\Pi\left(\frac{t-5}{2}\right)$

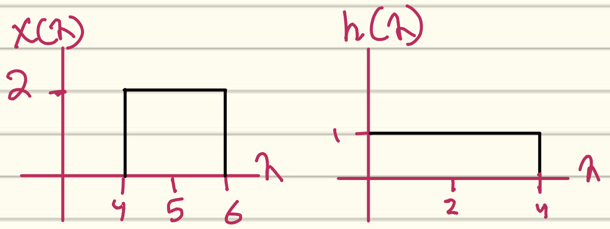
$h(t) = \Pi\left(\frac{t-2}{4}\right)$

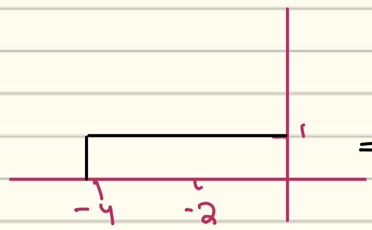
$[4, 6]$ $\quad$ $[0, 4]$

Rang : $[4, 6, 8, 10] \Rightarrow (-\infty, 4, 6, 8, 10, \infty)$

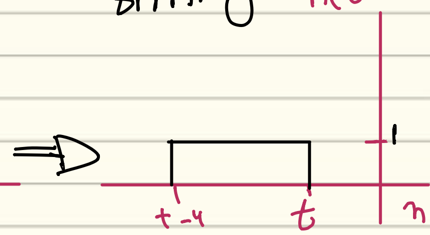
$$y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$



reflect $h(-\lambda)$



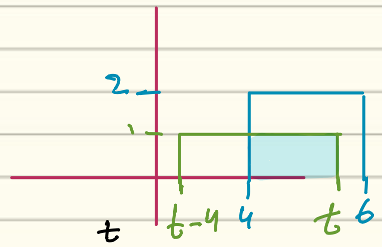
shifting $h(t-\lambda)$



$\boxed{1}$ $-\infty < t < 4$

$$y(t) = \int_{-\infty}^{4} 0 \, d\lambda = 0$$

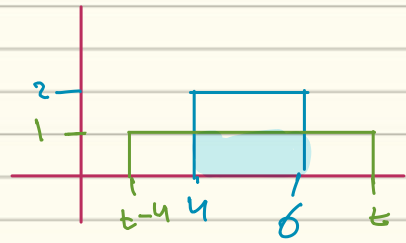
$\boxed{2}$ $4 < t < 6$



$$y(t) = \int_{4}^{t} (1)(2)\, d\lambda$$

$$= 2t \Big|_{4}^{t} = 2t - 8$$

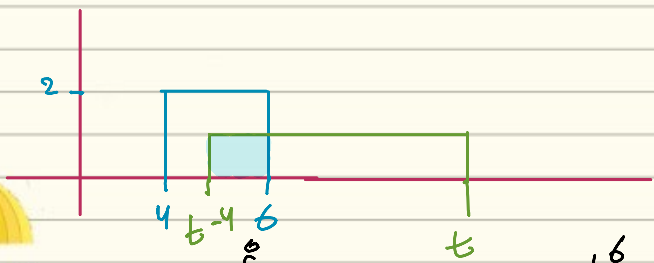
$\boxed{3}$ $6 < t < 8$



$$y(t) = \int_{4}^{6} (1)(2)\, d\lambda$$

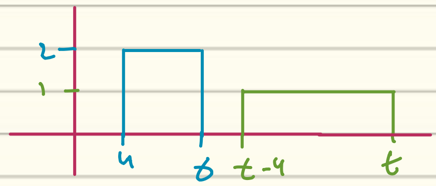
$$= 2t \Big|_{4}^{6} = 4$$

$\boxed{4}$ $8 < t < 10$



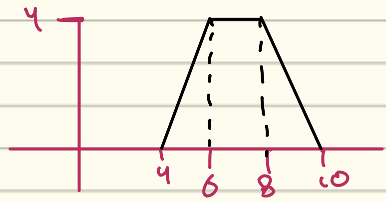
$$y(t) = \int_{t-4}^{6} (1)(2)\, d\lambda = 2t \Big|_{t-4}^{6} = 2(6-t+4) = 20 - 2t$$

$\boxed{5}$ $10 < t < \infty$

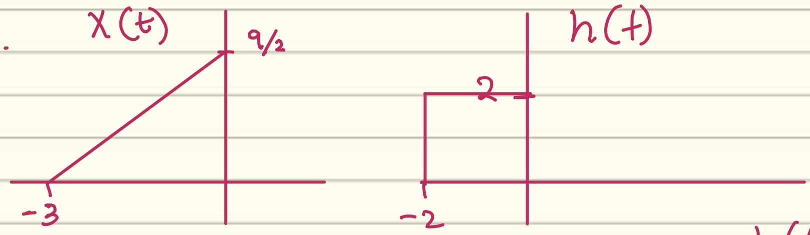


$$y(t) = \int_{10}^{\infty} 0 \, d\lambda = 0$$

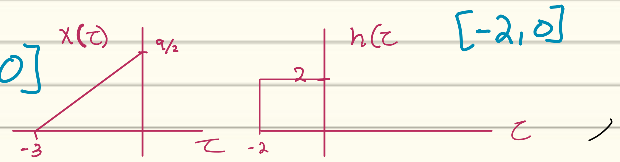
$$y(t) = \begin{cases} 0, & -\infty < t < 4 \\ 2t-8, & 4 < t < 6 \\ 4, & 6 < t < 8 \\ 20-2t, & 8 < t < 10 \\ 0, & t > 10 \end{cases}$$

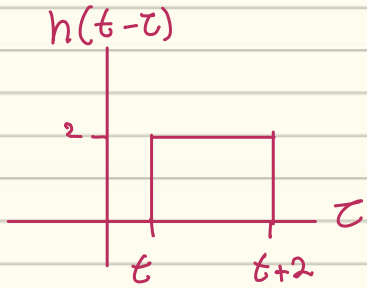


ex.



$[-3, 0]$ $[-2, 0]$

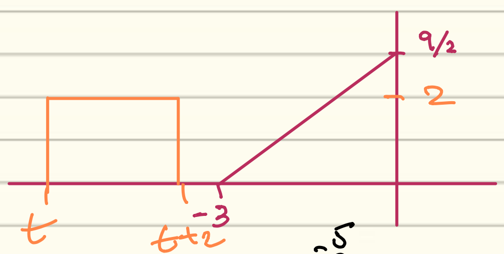




$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau$$

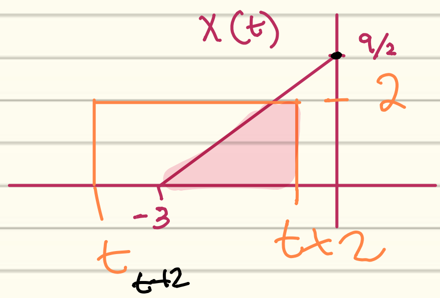
$[-5, -3, -2, 0]$

$\boxed{1}$ $-\infty < t < -5$



$$y(t) = \int_{-\infty}^{-5} 0 \, d\tau = 0$$

$\boxed{2}$ $-5 < t < -3$



$$m = \frac{\frac{9}{2} - 0}{0+3} = \frac{9}{2} \cdot \frac{1}{3} = \frac{3}{2}$$

$$y - 0 = \frac{3}{2}(x+3)$$

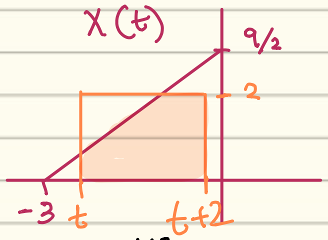
$$x(t) = \frac{3}{2}(t+3)$$

$$y(t) = \int_{-3}^{t+2} \cancel{(2)} \left(\frac{3}{\cancel{2}} (\tau + 3) \right) d\tau$$

$$= \left(\frac{3\tau^2}{2} + 9\tau \right) \Big|_{-3}^{t+2}$$

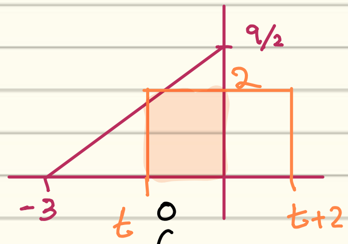
$$= \frac{3}{2}(t+2)^2 + 9(t+2) - \left(\frac{3}{2}(9) + 9(-3)\right)$$

3 $-3 < t < -2$



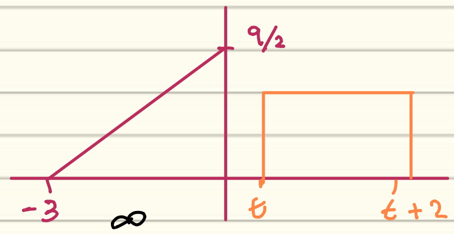
$$y(t) = \int_{t}^{t+2} (2)\left(\frac{3}{2}(\tau+3)\right) d\tau$$

4 $-2 < t < 0$



$$y(t) = \int_{t}^{0} (2)\left(\frac{3}{2}(\tau+3)\right) d\tau$$

5 $0 < t$

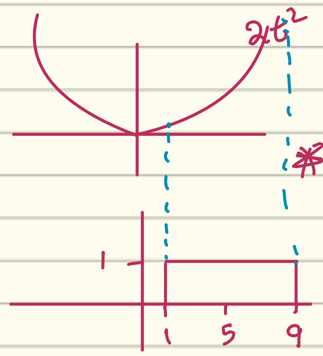


$$y(t) = \int_{0}^{\infty} 0 \, d\tau = 0.$$

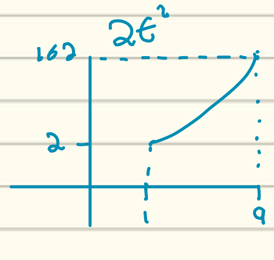
Example 2.7: Compute, Using the convolution integral, the response of the LTI with impulse response $h(t) = 2t^2 \Pi\left(\frac{t-5}{8}\right)$ to the input $x(t) = \Pi\left(\frac{t-10}{2}\right)$

$x(t)$

$[9, 11]$



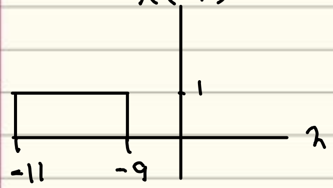
$h(t) =$

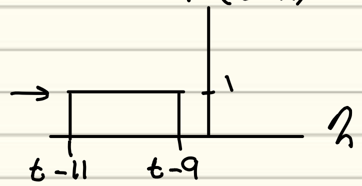


$[1, 9]$

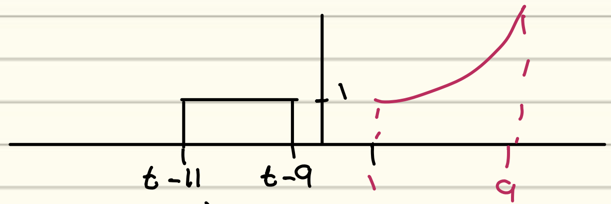
$y(t) = x(t) \circledast h(t)$

$= \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$, Range $[10, 12, 18, 20]$



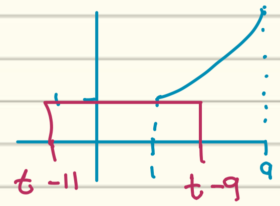


① $10 < t$



$y(t) = \int_{-\infty}^{1} 0 \, d\lambda = 0$

② $10 < t < 12$

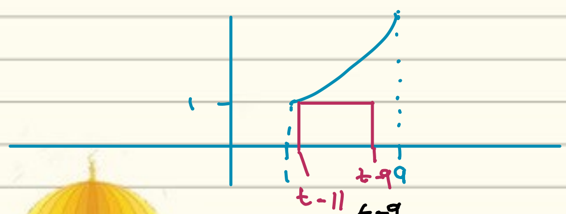


$y(t) = \int_{1}^{t-9} 2\lambda^2 \, d\lambda$

$= \frac{2\lambda^3}{3} \Big|_{1}^{t-9}$

$= \frac{2}{3}\left[(t-9)^3 - 1\right]$

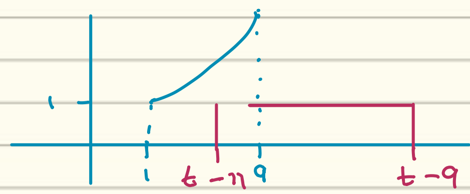
③ $12 < t < 18$



$y(t) = \int_{t-11}^{t-9} (1)(2\lambda^2)\, d\lambda$

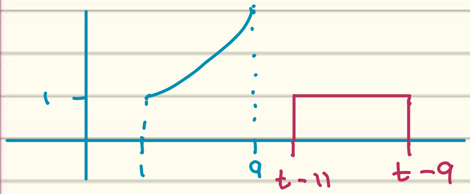
$= \frac{2}{3}\left((t-9)^3 - (t-11)^3\right)$

④ $18 < t < 20$



$y(t) = \int_{t-11}^{9} (1)\, 2\lambda^2 \, d\lambda = \frac{2}{3}\left(9^3 - (t-11)^3\right)$

⑤ $20 < t$



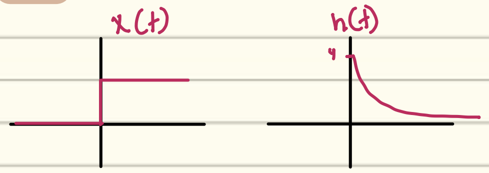
$$y(t) = \int_{20}^{\infty} 0 \, d\lambda = 0$$

$$y(t) = \begin{cases} 0 , & t < 10 \\ \frac{2}{3}\left[(t-9)^3 - 1\right] , & 10 \le t < 12 \\ \frac{2}{3}\left[(t-9)^3 - (t-11)^3\right] , & 12 \le t < 18 \\ \frac{2}{3}\left[9^3 - (t-11)^3\right] , & 18 \le t < 20 \\ 0 , & t \geq 20 \end{cases}$$

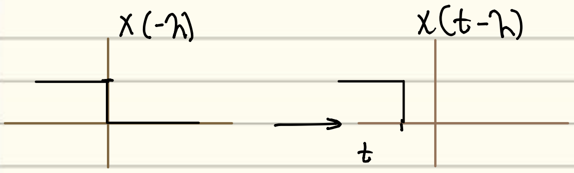
للرسم نعوض [10, 12, 18, 20] في y(t) ونصل بينهم .

---

ex. $x(t) = 2u(t)$, $h(t) = 4e^{-3t}u(t)$.



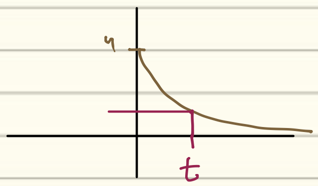
$[0, \infty]$



$[-\infty, 0]$

$[0, \infty]$ ← الفترة الكلية

$0 < t < \infty$



$$y(t) = \int_0^t (2)(4)(e^{-3\lambda})\, d\lambda$$

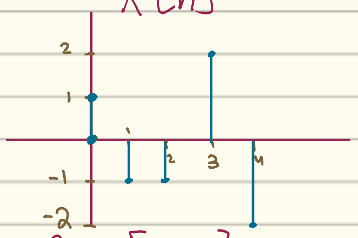
$$= \frac{8 \cdot e^{-3\lambda}}{-3}\Bigg|_0^t = \frac{-8}{3}e^{-3t} + \frac{8}{3}$$

Discrete

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

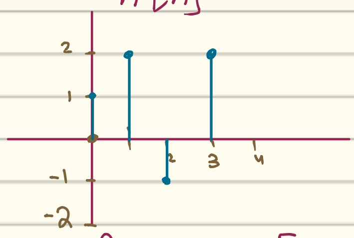
EX.

$X[n]$



Range = $[0, 4]$

$h[n]$



Range = $[0, 3]$

$h[n]$ \ $X[n]$	1	-1	-1	2	-2
1	1	-1	-1	2	-2
2	2	-2	-2	4	-4
-1	-1	1	1	-2	2
2	2	-2	-2	4	-4

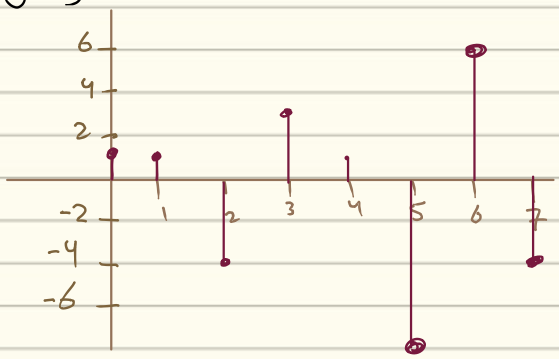
The value of each time in $x[t]$

values of time in $h[n]$

* each value in $x[n]$, multiply it by each value in $h[t]$

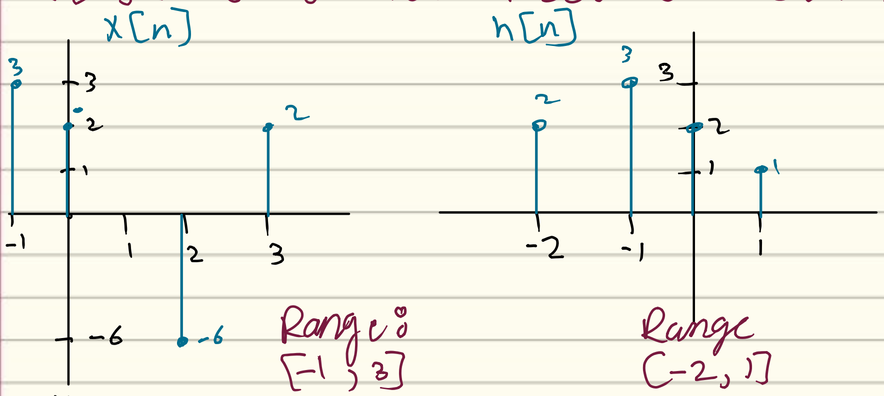
$$y[n] = \{1, 1, -4, 3, 1, -8, 6, -4\}$$
$$n = \{0, 1, 2, 3, 4, 5, 6, 7$$

$y[n]$



EX. $x[n] = 3\delta[n+1] + 2\delta[n] - 6\delta[n-2] + 2\delta[n-3]$

$h[n] = \delta[n-1] + 2\delta[n] + 3\delta[n+1] + 2\delta[n+2]$



Range : $[-1, 3]$

Range $[-2, 1]$

total Range

$[-1, 3]$, $[2, 1]$

$[-3, 0, 1, 4]$

$\Downarrow$

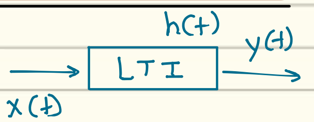
$[-3, 4]$

h[n] \ x[n]	3	2	0	-6	2
2	6	4	0	-12	4
3	9	6	0	-18	6
2	6	4	0	-12	4
1	3	2	0	-6	2

$$y[n] = \{ 6, 13, 12, -5, -12, -6, -2, 2 \}$$
$$n = -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

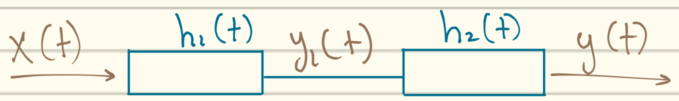
Properties of convilution

1 Commutative
تبديلية



$$= x(t) * h(t)$$
$$= h(t) * x(t)$$
$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

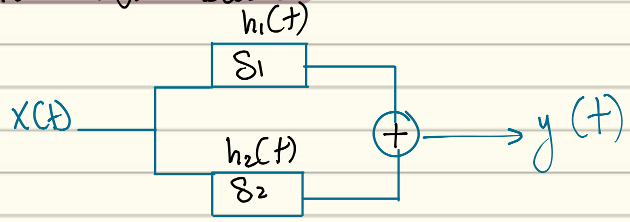
2 Associative : "تجميعية"



$$= [x(t) * h_1(t)] * h_2(t) = y(t)$$
$$x(t) * [h_1(t) * h_2(t)] = y(t)$$

من القدس
الـ أنت

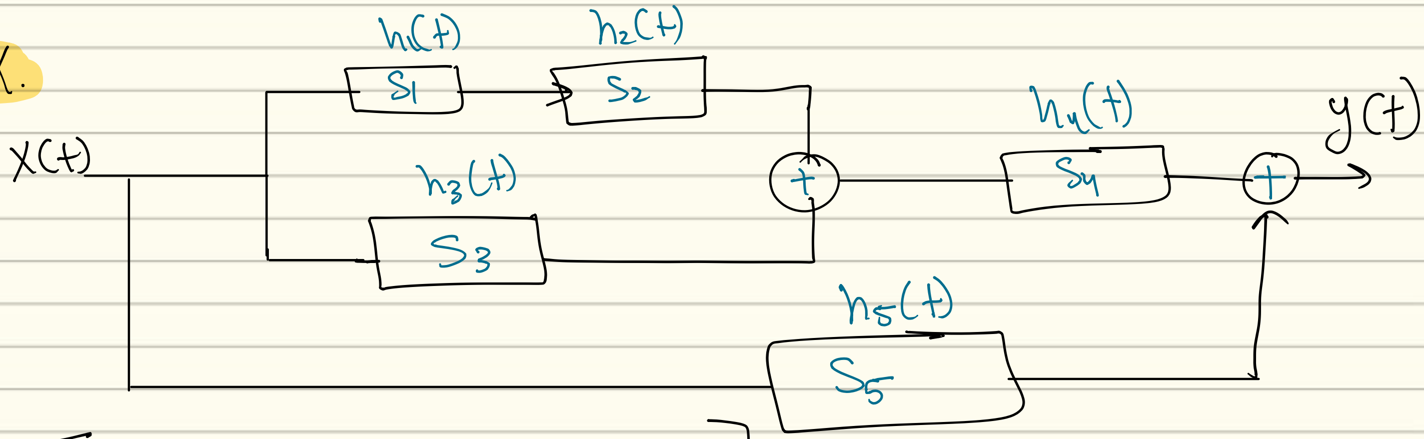
[3] Distributive law :-



$$x(t) * [h_1(t) + h_2(t)] = y(t)$$

$$= x(t) * h_1(t) + x(t) * h_2(t) = y(t)$$

ex.



$$\Rightarrow \Big[\big[(h_1(t) * h_2(t)) + h_3(t)\big] * h_4(t)\Big] + h_5(t) = y(t)$$

Summary of properties :-

1. **Commutative** :- $x_1 * x_2 = x_2 * x_1$
2. **Distributive** :- $x_1 * (x_2 + x_3) = x_1 * x_2 + x_1 * x_3$
3. **Assotiative** :- $(x_1 * x_2) * x_3 = x_1 * (x_2 * x_3)$
4. **Shifting Property** :- if $x_1 * x_2 = z(t)$

   then
   $x_1(t) * x_2(t-t_0) = z(t-t_0)$
   $x_1(t-t_0) * x_2(t) = z(t-t_0)$
   $x_1(t-\tau_1) * x_2(t-\tau_2) = z(t-\tau_1-\tau_2)$
   $= z(t-[\tau_1+\tau_2])$

5. **Convolution with impulse function** :-

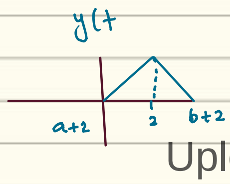
$\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = h(t)$

$y(t) = \delta(t) * h(t)$
$= h(t).$

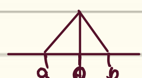
① $x(t) * \delta(t) = x(t)$

② $x(t) * \delta(t-t_0) = x(t-t_0)$

ex. $x(t) * \delta(t-2) = x(t-2)$



*أي شي بعمله (Convolution) مع أي (delta function) بعطيني الفنكشن نفسها



EX. $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

where $h(t) = \delta(3t-4)$. find $y(t)$ in terms of $x(t)$.

Ans:- $y(t) = x(t) * h(t)$

$= x(t) * [\delta(3t-4)]$.

$= x(t) * \frac{1}{3}\delta(t-\frac{4}{3})$

$= \frac{1}{3}\, x(t-\frac{4}{3})$.

$\delta(3t-4) = \delta(3(t-\frac{4}{3}))$

$= \frac{1}{3}\delta(t-\frac{4}{3})$

Ex. let the impulse response of the continuous time system be.

$h(t) = \delta(t-1) + \delta(t-3)$

① Find the step response at $t=5$ sec.

② Find the step response at $t=2$ sec.

[1]

$x(t) = u(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

Ans:- $y(t) = x(t) * h(t)$

$= u(t) * (\delta(t-1) + \delta(t-3))$ by distribution law

$= u(t) * \delta(t-1) + u(t) * \delta(t-3)$

$= u(t-1) + u(t-3)$, at $t=5$, $y(5) = u(4) + u(2)$

$= 1+1 = 2$

at $t=2$. $y(2) = u(1) + u(-1)$

$= 1+0$

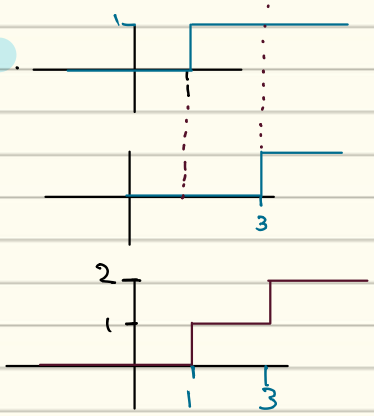
$= 1$

NoTe:-

+ impulse response
 input $\delta(t)$ — find output.

+ Step response
 input $u(t)$ — find output

Plot $y(t)$.



properties of continuous LTI system

من محاضرات الدكتور قدري **lecture 15 27:51**

We said that any input $x(t)$, we can find the output by Convolution $x(t)$ with $y(t)$.

$$x(t) \longrightarrow \boxed{h(t)}_{\text{LTI}} \longrightarrow y(t) = x(t) * h(t)$$

معرفة $h(t)$ ومن خلالها يُوجد $y(t)$
وبالتالي ، من خلالها بجد خصائص
ال (system) .

Causal · memory · stable

[1] System with memory .

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$y(t)$ depends on $x(\tau)$ for $\tau \in (-\infty, \infty)$, In general, the system have memory.

* only one speacial case to memory less system.

$h(t) = k\,\delta(t)$ (k: constant)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, k\, \delta(t-\tau)\, d\tau = k\, x(t).$$

[2] Invertability :-

$$y(t) = x(t) * h(t)$$

$$x(t) \longrightarrow \boxed{} \longrightarrow h(t)$$

$$\hat{x}(t) = y(t) * h_I(t)$$

$$\longrightarrow \boxed{h_I(t)} \longrightarrow \hat{x}(t)$$

$$\hat{x}(t) \overset{??}{=} x(t) * \underbrace{h(t) * h_I(t)}_{\delta(t)}$$

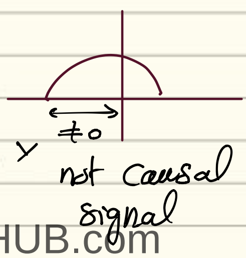
LTI is invertable iff $\exists$ $h_I(t)$ such that $h_I(t) * h(t) = \delta(t)$ → we will see that on ch.4 Foriour transform

[3] Causality :-

claim :- let $h(t)$ be causal signal.

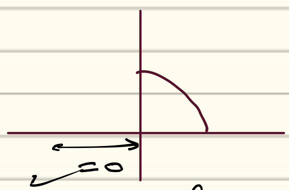
* Causal signal ⟹ $h(t) = 0$, for $t < 0$. (Right hand side signal).
(RHS)

إذا كانت قيمتها لما $(t<0)$ = صفر



not Causal signal

• $u(t)$ →



Causal signal

NOTE:- Causal signal يختلف عن Causal system.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau .$$

* from the claim (h(t) is Causal Signal , $h(t) = 0, \; t < 0$).

$$h(t-\tau) = \begin{Bmatrix} 0, & t-\tau < 0 \\ \text{value}, & t-\tau > 0 \end{Bmatrix} = \begin{Bmatrix} 0, & t < \tau \\ \text{value}, & t > \tau \end{Bmatrix}$$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau .$$

* the system is Causal iff h(t) is a causal signal.

$\boxed{4}$ Stable :-

$$|x(t)| < M \longrightarrow |y(t)| < K$$

* y(t) will be bounded iff $\int_{-\infty}^{\infty} |y(\tau)|\, d\tau < \infty$.

absoluting integrable Impulse response.

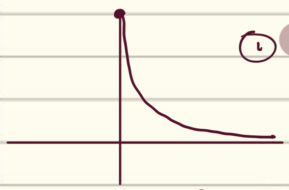
* Speacial Case :-

↳ LTI system + stable and Causal .

$$= \cancel{\int_{-\infty}^{0} |h(t)|\, dt} + \boxed{\int_{0}^{\infty} |y(t)|\, dt .}$$

we only check this.

Ex. $h(t) = e^{-3t} u(t)$,



① is it Causal system?

yes, since h(t) is (RHS) signal.

② Stablity :- h(t) should be abs. integrable.

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty . \Rightarrow \int_{0}^{\infty} |e^{-3t} u(t)| = -\frac{1}{3} e^{-3t} \Big|_{0}^{\infty} = -\frac{1}{3}(0-1) = \frac{1}{3}$$

So yes it's stable.

Ex. $h(t) = e^{3t} u(t)$

1. Causality :- it's Causal since its RHS. signal. ($h(t) = 0$ for $t<0$).
2. Sablity :- $\int_0^\infty e^{3t} dt = \frac{1}{3} e^{3t} \Big|_0^\infty = \infty$, so it's not stable.

Ex. $h(t) = 10\, u(t)$

1. Causality :- yes, its causal since $h(t) = 0$ for $t<0$.
2. Stablity :- $\int_0^\infty 10\, dt = 10(\infty) = \infty$, So it's not bounded then its not stable.

lecture16 22:05 Dr.Qadri's Lectures

Unit step response

$X(t) = u(t)$, $y(t)$

step response $\Rightarrow a(t)$

$x(t) = u(t) \longrightarrow \boxed{h(t)} \longrightarrow a(t)$

$$a(t) = u(t) * h(t)$$
$$= \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$$
$$= \int_0^\infty h(t-\tau)\, d\tau.$$

suppose that the system to be Causal $\Rightarrow$ So $h(t)$ is Causal system $=0$, $t<0$.

Hence, $h(t-\tau) = 0$, $t<\tau$

$a(t) = \int_0^t h(\tau)\, d\tau$ ← if the Signal is causal, I can use this Integration instead

the unit step response can be calculated directly from the unit impulse response $h(t)$.

* we can show that $h(t) = \frac{d\, a(t)}{dt}$

$t=0$, R, V_s, C, $V_o(t)$

$v(t) = V_s\, u(t)$

step response $V_o(t)$

$h(t) = \frac{d}{dt} V_o(t)$.

* $y(t) = a(t) * \frac{d\, x(t)}{dt}$.

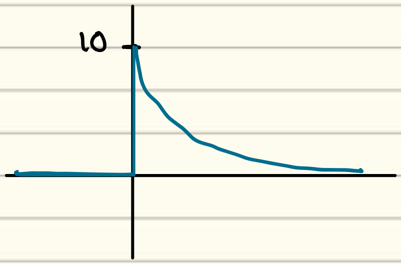
Ex. let $h(t) = 10e^{-2t}u(t)$, Find $a(t)$.

1. is it causal system. yes, since $h(t) = 0,\ t<0$.

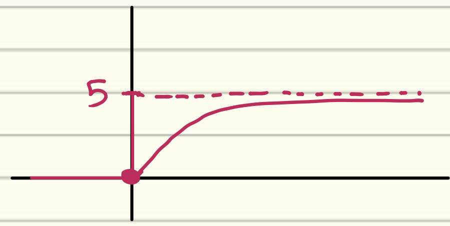
2. $a(t)$.

$$a(t) = \int_0^t h(\tau)\,d\tau = \int_0^t 10e^{-2\tau}\,d\tau = \frac{10}{-2}e^{-2\tau}\Big|_0^t$$
$$= -5\left(e^{-2t}-1\right)$$
$$= 5 - e^{-2t}$$

$h(t) = 10e^{-2t}u(t)$.



$a(t)$ step response.



Summary :-

1. for LTI System, any linear operation on the input, produces the same linear operation on the output.

(1)

$\delta(t) \rightarrow \boxed{h(t)} \rightarrow h(t)$

علاقتهم

$u(t) \rightarrow \boxed{h(t)} \rightarrow a(t)$.

علاقتهم

$u(t) = \int \delta(\tau)\,d\tau$

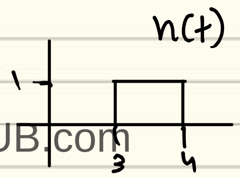
$a(t) = \int_0^t h(\tau)\,d\tau$.

(2) $r(t) = \int u(t)\,dt \longrightarrow \int a(t)$

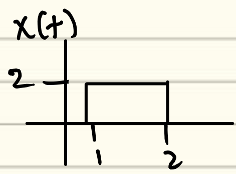
$\Rightarrow$ if A_1 is the Area under $h(t)$.
A_2 is the Area under $x(t)$.

then, the area under $y(t) = h(t) * x(t) = A_1 . A_2$

بنقدر نستخدمها عشان نتحقق من الحل.



$A_1 = 1$



2

$A_y = A_1 . A_2 = 2$

frequency response of LTI system

time domain

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$

in frequency domain :-

$X(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow Y(\omega) = X(\omega) \cdot H(\omega)$

$$\left.\begin{aligned} x(t) &\longrightarrow X(\omega). \\ h(t) &\longrightarrow H(\omega). \end{aligned}\right\} \text{ on ch. 3 + ch. 4.}$$

to prove that :-

$$e^{j\omega t} \longrightarrow \boxed{} \longrightarrow y(t) = h(t) * e^{j\omega t}$$

$e^{j\omega t}$ Rotating phasor

$$= \int e^{j\omega(t-\lambda)} \cdot h(\lambda)\, d\lambda$$

$$= \int e^{j\omega t} \cdot e^{-j\omega\lambda}\, h(\lambda)\, d\lambda$$

$$= e^{j\omega t} \cdot \int e^{-j\omega\lambda}\, h(\lambda)\, d\lambda$$

$$= e^{j\omega t} \cdot H(\omega).$$

$$\text{※ } H(\omega) = \int_{-\infty}^{\infty} h(\lambda)\, e^{-j\omega\lambda}\, d\lambda \longrightarrow$$ Frequency Response of the system "Transfer function."

$$e^{j\omega t} \longrightarrow \boxed{} \longrightarrow y(t) = e^{j\omega t} \cdot H(\omega).$$

transfer function.

$$H(\omega) = \text{Complex Quantity}. \equiv \underbrace{\underbrace{|H(\omega)|}_{\text{Amplitude}}\ \underbrace{e^{j\theta_H \omega}}_{\text{phase}}}_{\text{Frequency Response.}}$$

Summary :-

- if the input to a LTI system is a sinsoid ($e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$), if frequency . ω rad/s , the steady state Response is a sinsoid of the same frequency with amplitude mult. by $|H(\omega)|$ a phase shifted by $\theta_H(\omega)$ radian.

- $|H(\omega)| = \dfrac{1}{\sqrt{1 + (\omega RC)^2}}$
- $\theta_{H(\omega)} = -\tan^{-1}(\omega RC).$

Example: Find the response of $x(t) = \underbrace{\cos(2000\pi t)}_{①} + \underbrace{\cos(4000\pi t)}_{②}$.

let $\frac{1}{2\pi RC} = 1\,KHz$, $RC = 1000$, $A = 1$

$$y_1(t) = A_1 |H(\omega)| . \cos(2000\pi t + \theta)$$

$$RC = \frac{1}{2\pi(1000)} = \frac{1}{2000\pi}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+\left(\cancel{2000\pi} \cdot \frac{1}{\cancel{2000\pi}}\right)^2}} = \frac{1}{\sqrt{2}} = 0.707$$

$$\theta = -\tan^{-1}\left(2000\pi . \frac{1}{2000\pi}\right) = -\pi/4 .$$

$$H(\omega) = 0.707\, e^{-j\frac{\pi}{4}}$$

$$\therefore y_1(t) = 0.707 \cos(2000\pi t - \pi/4).$$

doing the same for y_2.

$$|H(4000\pi)| = \frac{1}{\sqrt{1+\left(\cancel{4000\pi}^{\,2} . \frac{1}{\cancel{2000\pi}}\right)^2}} = 0.45$$

$$\theta(4000\pi) = -\tan^{-1}\left(\frac{4000\pi}{2000\pi}\right) = -63.43$$

$$y(t) = 0.707 \cos(2000\pi t - 45) + 0.45 \cos(4000\pi t - 63.45) .$$

simulate modeling → كتابة ال (system) عن طريق blocks.

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

↓

DFE

$$\sum_{i=0}^{n} a_i \frac{d^{(n)}y}{dt^n} = \sum_{i=0}^{n} b_i \frac{d^{(n)}x(t)}{dt^n}$$

* بفضل أعمل (integration) لحديث ما أوصل الفانكشن بدلالة $y(t)$ يعني الأوردر تصير = 0 .

Example:- Sketch the simulating model for the following diffrential equation:-

1) $-x(t) + \frac{L}{R}\frac{dy(t)}{dt} + y(t) = 0$

Ans:- $-x(t) + \frac{L}{R}\frac{dy(t)}{dt} + y(t) = 0.$

بنحطها لحالها.

① بخلّي ال [higher order] بطرف والباقي على طرف ثاني.

نجعل المعامل = 1

$\frac{L}{R}\frac{dy(t)}{dt} = x(t) - y(t)$

② أجعل معامله = 1.

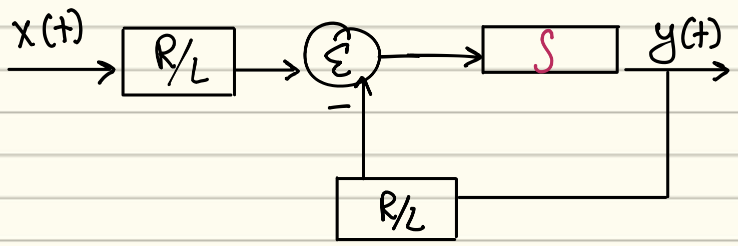
③ نفترض الأشياء الي ما فيها مشتقة إنها متغير جديد $q_0, q_1 \dots$

$$\frac{dy(t)}{dt} = \underbrace{\frac{R}{L}x(t) - \frac{R}{L}y(t)}_{q_0}.$$

تكامل

④ تكامل.

$$y(t) = \underbrace{\int q_0}_{q_1}$$



Example 2 :-

$$2\frac{d^3y(t)}{dt^3} - 8\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 2y(t) = 4\frac{dx(t)}{dt} + 2x(t)$$

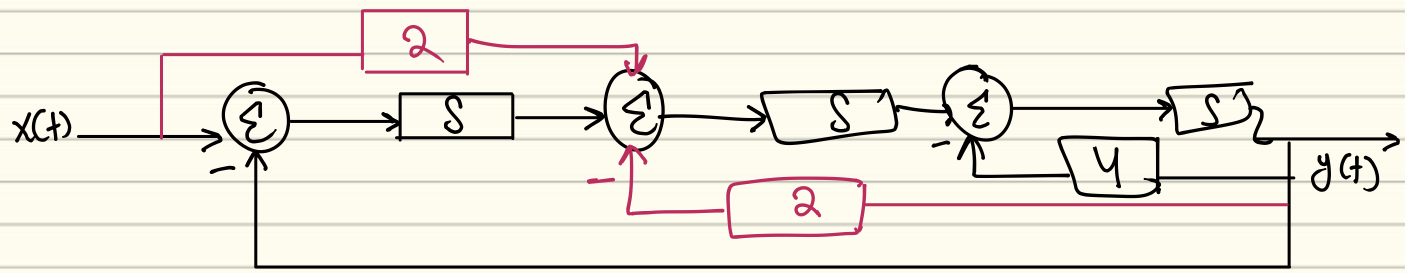
$$2\frac{d^3y(t)}{dt^3} = 4\frac{dx(t)}{dt} + 2x(t) + 8\frac{d^2y(t)}{dt^2} - 4\frac{dy(t)}{dt} - 2y(t).$$

$$\frac{d^3y(t)}{dt^3} = \underbrace{x(t) - y(t)}_{q_0} + 2\frac{dx(t)}{dt} - 2\frac{dy(t)}{dt} + 4\frac{d^2y(t)}{dt^2}$$

$$\frac{d^2y(t)}{dt^2} = \underbrace{\int q_0 + 2x(t) - 2y(t)}_{q_1} + 4\frac{dy(t)}{dt}$$

$$\frac{dy(t)}{dt} = \underbrace{\int q_1 + 4y(t)}_{q_2}$$

$$y(t) = \int q_2$$



ربّنا تقبّل منّا إنّكَ أنتَ السميعُ العليم

روان فارس