12.3 : Goodness of fit test : Poisson and Normal Distributions.

$\boxed{\text{Poisson distribution}}$

→ H_0: The population has a pisson dist.

H_1: The population doesn't have a pisson dist.

→ Take a random sample of size n.

f_i : observed frequencies $\quad \sum f_i = n$.

e_i : expected frequencies $\quad \sum e_i = n$

$$\boxed{e_i = \frac{\mu^{x_i} e^{-\mu}}{x_i!} \cdot n} \qquad \boxed{\mu = \frac{\sum_{i=1}^{k} x_i f_i}{\sum_{i=1}^{k} f_i}}$$

→ Test statistic :

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i} \qquad \text{with } df = k-2$$

Assuming $e_i \geq 5 \ \forall i$.

→ Rejection Rule :

دائماً مع upper limit

- Reject H_0 if $P\text{-value} \leq \alpha$
- Reject H_0 if $\chi^2 \geq \chi^2_{\alpha}$.

الأس السالب

Q18:

$e_i = \dfrac{\mu^{x_i} e^{-\mu}}{x_i!} \cdot n$

$K = 5$

Number of occurreness (x_i)	observed freq (f_i)	expect freq. e_i
0	39	32.70
1	30	42.51
2	30	27.63
3	18	11.98
4	3	$120-(32.70+42.51+27.63+11.98) = \underline{5.18}$
	120	120

آخر وحدة
ما بنوجدها بالقانون

$$\mu = \frac{\sum_{i=1}^{k} x_i f_i}{\sum_{i=1}^{k} f_i} = \frac{156}{120} = 1.3$$

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i} = 9.04$$

$$\chi^2_\alpha = \chi^2_{0.05} \text{ with } df = 5-2 = 3 \rightarrow \chi^2_{0.05} = 7.815$$

$p\text{-value} \in (0.025, 0.05)$

$\rightarrow \quad \chi^2 \geq \chi^2_\alpha$

$p\text{-value} \leq 0.05$

So we Reject H_0 $(\alpha = 0.05)$

The population doesn't have a poisson dis. $(\alpha = 0.05)$

Normal Distribution

H_0 : The population has a Normal dis.

H_1 : The population doesn't have a Normal dis.

→ Take a random sample of size n.

- observed frequencies (f_i).
- expected frequencies (e_i).

→ Test statistic :

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i}$$

- K: # of categories.

  $K = \frac{n}{5}$

- $e_i = 5 \quad \forall i$

→ Reject H_0 if P-value $\leq \alpha$

Reject H_0 if $\chi^2 \geq \chi^2_\alpha$

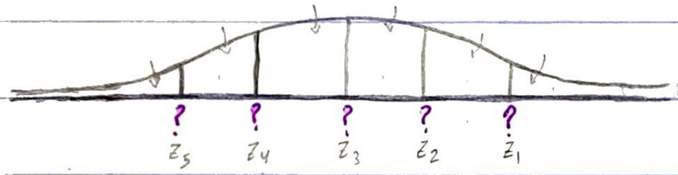
with $df = K-3$.

By calculater

Q20: H_0 : The pop. has a Normal dis. find $\bar{x} = 24.5$

H_1 : The pop. doesn't have a Normal dis. $S = 3.01$

$\alpha = 0.10$, $n = 30$, $K = \frac{30}{5} = 6$, $df = K-3 = 3$.



standard Normal

$\underline{6}$ classes of equal part

prop. (area) of each class $= \frac{1}{6}$.

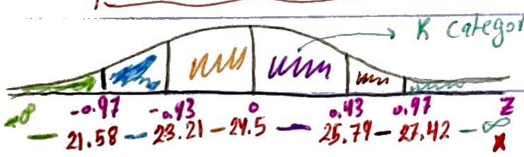
to find Z and X :

Area left to Z	Closest Area	Z (using 2 decimal)	X
$\frac{5}{6} = 0.8333$	0.8340	0.97	27.42
$\frac{4}{6} = 0.6667$	0.6664	0.43	25.79
$\frac{3}{6} = 0.5$	0.5	0	24.5
$\frac{2}{6}$		-0.43	23.21
$\frac{1}{6}$		-0.97	21.58

$\frac{2}{6}$, $\frac{1}{6}$ } By symmitric but Z negative.

Now to find X :

$$Z = \frac{x - \bar{x}}{S} \iff X = \bar{X} + S Z$$



category	f_i	e_i
$-\infty$ – 21.57	5	5
21.58 – 23.21	4	5
23.21 – 24.5	3	5
24.5 – 25.79	7	5
25.79 – 27.42	7	5
27.42 – ∞	4	5
	30	30

→ Test statistic : $\chi^2 = \sum_{i=1}^{K} \frac{(f_i - e_i)^2}{e_i} = 2.8$

→ Critical approach : $\chi^2_{0.1} = 6.251$ → $\chi^2 < \chi^2_\alpha$

→ P-value approach : P-value $\in (0.1, 0.9)$ → P-value $> \alpha$

So we Don't Reject H_0 $(\alpha = 0.1)$.

→ The pop. has a Normal distribution $(\alpha = 0.1)$.