9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 **[a]** $\mathbf{V} = 170\underline{/-40^\circ}\,\text{V}$

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\underline{/-70^\circ}\,\text{A}$$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ}\,\text{A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ}\,\text{mV}$$

AP 9.2 **[a]** $v = 18.6\cos(\omega t - 54^\circ)\,\text{V}$

[b] $\mathbf{I} = 20\underline{/45^\circ} - 50\underline{/-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\underline{/126.68^\circ}$$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)\,\text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15^\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\underline{/97.08^\circ}$$

$$v = 72.79\cos(\omega t + 97.08^\circ)\,\text{V}$$

AP 9.3 **[a]** $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 [a] $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5 \times 10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\underline{/25^\circ}}{50\underline{/-90^\circ}} = 0.6\underline{/115^\circ}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.72 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$

AP 9.6 [a] $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore \quad \theta_Z = 45^\circ$

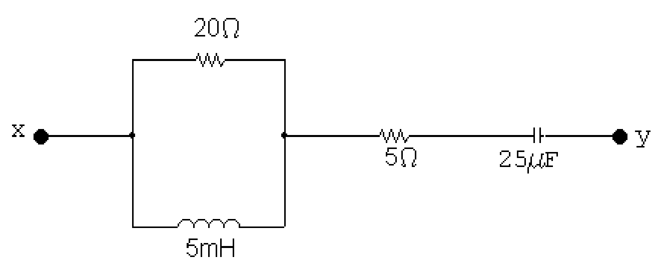
$Z = 90 + j160 + jX_C$

$\therefore \quad X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore \quad C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore \quad |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 [a]



$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

$$Z_{xy} = 20\|j10 + 5 + j20 = \frac{20(j10)}{(20 + j10)} + 5 - j20$$

$$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$$

[b] $\omega L = 40\,\Omega, \qquad \frac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\frac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \frac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{\text{xy}} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

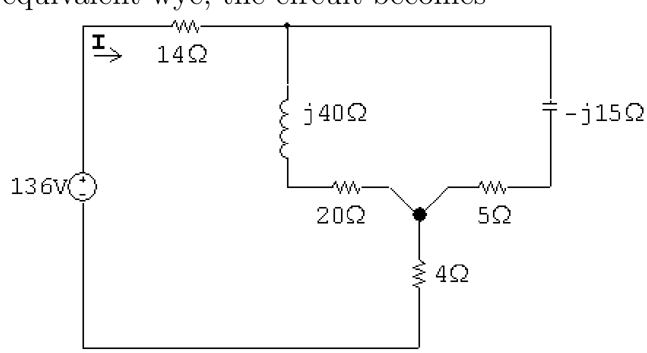
$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{\text{xy}}} = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$(20 + j40)\|(5 - j15) = (12 - j16)\,\Omega$

Therefore $\quad \mathbf{I} = \dfrac{136\underline{/0^\circ}}{14 + 12 - j16 + 4} = 4\underline{/28.07^\circ}\,\text{A}$

AP 9.10

$\mathbf{V}_1 = 240\underline{/53.13^\circ} = 144 + j192\,\text{V}$

$\mathbf{V}_2 = 96\underline{/-90^\circ} = -j96\,\text{V}$

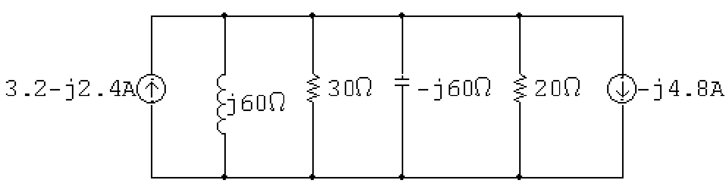
$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$$

Perform a source transformation:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4\,\text{A}$$

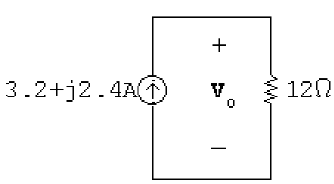
$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\text{A}$$



Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12\,\Omega$$



$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\text{V} = 48\underline{/36.87^\circ}\,\text{V}$

$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$

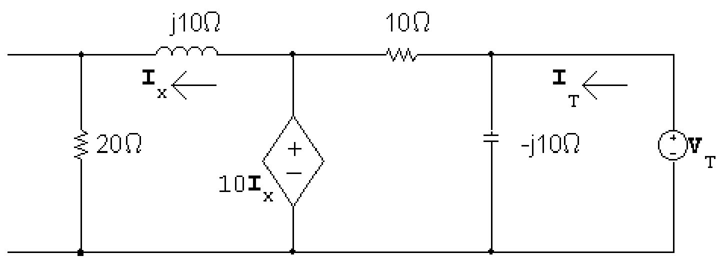
AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\underline{/45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10-j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\underline{/45^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

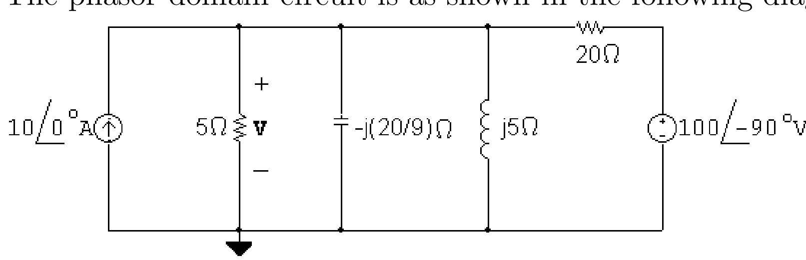
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)\,\text{V}$

AP 9.13 Let $\mathbf{I}_\text{a}$, $\mathbf{I}_\text{b}$, and $\mathbf{I}_\text{c}$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_\text{a} + (3 - j5)(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})$$

and

$$0 = (3 - j5)(\mathbf{I}_\text{b} - \mathbf{I}_\text{a}) + 2(\mathbf{I}_\text{b} - \mathbf{I}_\text{c}).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b}),$$

therefore

$$\mathbf{I}_\text{c} = -0.75[-j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})].$$

Solving for $\mathbf{I} = \mathbf{I}_\text{a} = 29 + j2 = 29.07\underline{/3.95^\circ}\,\text{A}$.

AP 9.14 **[a]** $M = 0.4\sqrt{0.0625} = 0.1\,\text{H}, \qquad \omega M = 80\,\Omega$

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300)\,\Omega$$

Therefore $|Z_{22}| = 500\,\Omega, \qquad Z_{22}^* = (400 - j300)\,\Omega$

$$Z_\tau = \left(\frac{80}{500}\right)^2 (400 - j300) = (10.24 - j7.68)\,\Omega$$

[b] $\mathbf{I}_1 = \dfrac{245.20}{184 + 100 + j400 + Z_\tau} = 0.50\underline{/-53.13^\circ}\,\text{A}$

$$i_1 = 0.5\cos(800t - 53.13^\circ)\,\text{A}$$

[c] $\mathbf{I}_2 = \left(\dfrac{j\omega M}{Z_{22}}\right)\mathbf{I}_1 = \dfrac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ}) = 0.08\underline{/0^\circ}\,\text{A}$

$$i_2 = 80\cos 800t\,\text{mA}$$

AP 9.15

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + 2s^2 Z_2} = \frac{25 \times 10^3 \underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5\underline{/36.87^\circ}\ \text{A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}\ \text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/216.87^\circ}\ \text{A}$$

Problems

P 9.1 **[a]** $\omega = 2\pi f = 800\,\text{rad/s}, \qquad f = \dfrac{\omega}{2\pi} = 127.32\,\text{Hz}$

[b] $T = 1/f = 7.85\,\text{ms}$

[c] $I_m = 125\,\text{mA}$

[d] $i(0) = 125\cos(36.87^\circ) = 100\,\text{mA}$

[e] $\phi = 36.87^\circ; \qquad \phi = \dfrac{36.87^\circ(2\pi)}{360^\circ} = 0.6435\ \text{rad}$

[f] $i = 0$ when $800t + 36.87^\circ = 90^\circ$. Now resolve the units:

$$(800\ \text{rad/s})t = \frac{53.13^\circ}{57.3^\circ/\text{rad}} = 0.927\,\text{rad}, \qquad t = 1.16\,\text{ms}$$

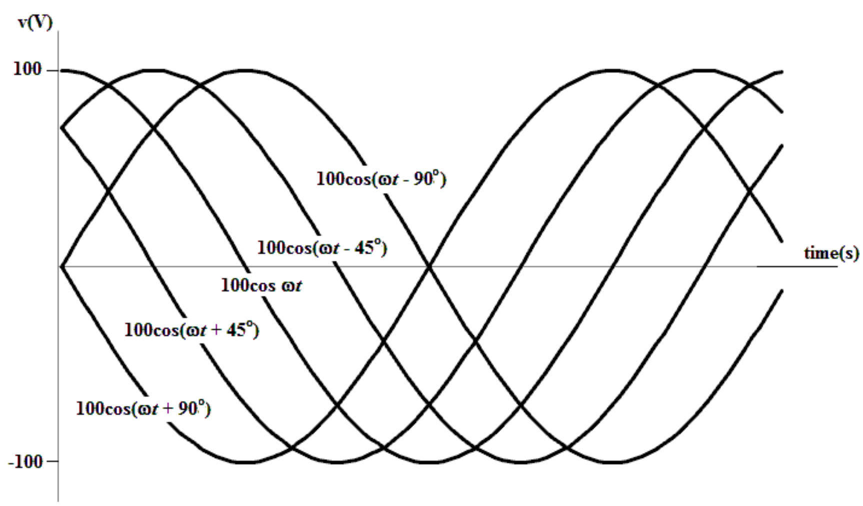
[g] $(di/dt) = (-0.125)800\sin(800t + 36.87^\circ)$

$$(di/dt) = 0 \quad \text{when} \quad 800t + 36.87^\circ = 180^\circ$$

$$\text{or} \qquad 800t = \frac{143.13^\circ}{57.3^\circ/\text{rad}} = 2.498\,\text{rad}$$

Therefore $t = 3.12\,\text{ms}$

P 9.2



[a] Right as ϕ becomes more negative

[b] Left

P 9.3 **[a]** 25 V

[b] $2\pi f = 400\pi; \qquad f = 200\,\text{Hz}$

[c] $\omega = 400\pi = 1256.64\ \text{rad/s}$

[d] $\theta(\text{rad}) = 60^\circ \left(\dfrac{\pi}{180^\circ}\right) = \dfrac{\pi}{3} = 1.0472 \text{ rad}$

[e] $\theta = 60^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{200} = 5\,\text{ms}$

[g] $400\pi t + \dfrac{\pi}{3} = \dfrac{\pi}{2}; \qquad \therefore \quad 400\pi t = \dfrac{\pi}{6}$

$$\therefore \qquad t = \frac{1}{2400} = 416.67\,\mu\text{s}$$

[h] $v = 25\cos\left[400\pi\left(t - \dfrac{0.005}{6}\right) + \dfrac{\pi}{3}\right]$

$$= 25\cos[400\pi t - (\pi/3) + (\pi/3)]$$

$$= 25\cos 400\pi t\,\text{V}$$

[i] $400\pi(t + t_o) + (\pi/3) = 400\pi t + (3\pi/2)$

$$\therefore \quad 400\pi t_o = \frac{7\pi}{6}; \qquad t_o = \frac{7}{2400} = 2.92\,\text{ms}$$

P 9.4 **[a]** By hypothesis

$$v = 50\cos(\omega t + \theta)$$

$$\frac{dv}{dt} = -50\omega\sin(\omega t + \theta)$$

$$\therefore \quad 50\omega = 750\pi; \qquad \omega = 15\pi\,\text{rad/s}$$

[b] At $t = (40/3)$ ms, the argument of the cosine function must equal 90°. Remember that we must convert ωt to degrees:

$$15\pi\left(\frac{0.04}{3}\right)\frac{360^\circ}{2\pi} + \theta = 90^\circ$$

Solving,

$$\theta = 90^\circ - 36^\circ = 54^\circ$$

The problem description says that the voltage is increasing, but the derivative calculated in part (a) is negative. Therefore we need to shift the phase angle by 180° to $54 - 180 = -126^\circ$, effectively multiplying the expression for the voltage by -1, so that the expression for the voltage is increasing.

$$\therefore \quad v = 50\cos(15\pi t - 126^\circ)\,\text{V}$$

P 9.5 **[a]** $\dfrac{T}{2} = 25 - 5 = 20\,\text{ms}; \qquad T = 40\,\text{ms}$

$$f = \frac{1}{T} = \frac{1}{40 \times 10^{-3}} = 25\,\text{Hz}$$

[b] $i = I_m \sin(\omega t + \theta)$

$$\omega = 2\pi f = 50\pi \text{ rad/s}$$

$$50\pi(5 \times 10^{-3}) + \theta = 0; \qquad \therefore \quad \theta = \frac{-\pi}{4} \text{ rad} = -45^\circ$$

$$i = I_m \sin[50\pi t - 45^\circ]$$

$$0.5 = I_m \sin -45^\circ; \qquad I_m = -70.71\,\text{mA}$$

$$i = -70.71 \sin[50\pi t - 45^\circ] = 70.71 \cos[50\pi t + 45^\circ]\,\text{mA}$$

P 9.6 $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(240) = 339.41\,\text{V}$

P 9.7 $V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t\, dt}$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}t\right)\, dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\, dt = \frac{V_m^2 T}{4}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T}\frac{V_m^2 T}{4}} = \frac{V_m}{2}$

P 9.8
$$\begin{aligned}\int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\, dt &= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\, dt \\ &= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\, dt\right\} \\ &= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\,\Big|_{t_o}^{t_o+T}\right]\right\} \\ &= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)\right]\right\} \\ &= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right)\end{aligned}$$

P 9.9 **[a]** The numerical values of the terms in Eq. 9.8 are

$$V_m = 75, \qquad R/L = 5333.33, \qquad \omega L = 300$$

$$\sqrt{R^2 + \omega^2 L^2} = 500$$

$$\phi = -60^\circ, \qquad \theta = \tan^{-1} 300/400, \qquad \theta = 36.87^\circ$$

Substitute these values into Equation 9.9:

$$i = \left[-17.94e^{-5333.33t} + 150\cos(4000t - 96.87^\circ)\right]\,\text{mA}, \qquad t \geq 0$$

[b] Transient component $= -17.94e^{-5333.33t}\,\text{mA}$
Steady-state component $= 150\cos(4000t - 96.87^\circ)\,\text{mA}$

[**c**] By direct substitution into Eq 9.9 in part (a), $i(750\,\mu\text{s}) = 38.44\,\text{mA}$

[**d**] 150 mA, 4000 rad/s, -96.87°

[**e**] The current lags the voltage by 36.87°.

P 9.10 [**a**] From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi-\theta)e^{-(R/L)t}}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m R\cos(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2+\omega^2L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$$V_m\cos(\omega t+\phi)$$

At $t = 0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} = 0$$

[**b**] $i_{ss} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t+\phi-\theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right] = V_m\cos(\omega t+\phi)$$

P 9.11 [**a**] $\mathbf{Y} = 30\underline{/-160^\circ} + 15\underline{/70^\circ} = 29.38\underline{/170.56^\circ}$

$y = 28.38\cos(200t + 170.56^\circ)$

[**b**] $\mathbf{Y} = 90\underline{/-110^\circ} + 60\underline{/-70^\circ} = 141.33\underline{/-94.16^\circ}$

$y = 141.33\cos(50t - 94.16^\circ)$

[c] $\mathbf{Y} = 50\underline{/-60^\circ} + 25\underline{/20^\circ} - 75\underline{/-30^\circ} = 16.7\underline{/170.52^\circ}$

$$y = 16.7\cos(5000t + 170.52^\circ)$$

[d] $\mathbf{Y} = 10\underline{/30^\circ} + 10\underline{/-90^\circ} + 10\underline{/150^\circ} = 0$

$$y = 0$$

P 9.12 **[a]** 400 Hz

[b] $\theta_v = 0^\circ$

$$\mathbf{I} = \frac{100\underline{/0^\circ}}{j\omega L} = \frac{100}{\omega L}\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$$

[c] $\dfrac{100}{\omega L} = 20; \qquad \omega L = 5\,\Omega$

[d] $L = \dfrac{5}{800\pi} = 1.99\,\text{mH}$

[e] $Z_L = j\omega L = j5\,\Omega$

P 9.13 **[a]** $\omega = 2\pi f = 160\pi \times 10^3 = 502.65\,\text{krad/s} = 502{,}654.82\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{25 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(25 \times 10^{-3})\underline{/0^\circ} = 25 \times 10^{-3}\omega C\underline{/90^\circ}$

$$\therefore \quad \theta_i = 90^\circ$$

[c] $628.32 \times 10^{-6} = 25 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{25 \times 10^{-3}}{628.32 \times 10^{-6}} = 39.79\,\Omega, \quad \therefore \quad X_C = -39.79\,\Omega$$

[d] $C = \dfrac{1}{39.79(\omega)} = \dfrac{1}{(39.79)(160\pi \times 10^3)}$

$$C = 0.05 \times 10^{-6} = 0.05\,\mu\text{F}$$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j39.79\,\Omega$

P 9.14 **[a]** $\mathbf{V}_g = 300\underline{/78^\circ}; \qquad \mathbf{I}_g = 6\underline{/33^\circ}$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\underline{/78^\circ}}{6\underline{/33^\circ}} = 50\underline{/45^\circ}\,\Omega$$

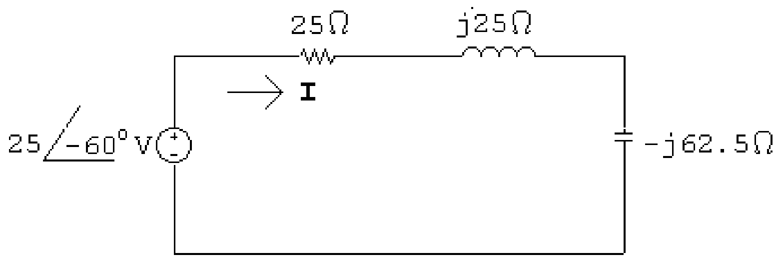
[b] i_g lags v_g by 45°:

$$2\pi f = 5000\pi; \qquad f = 2500\,\text{Hz}; \qquad T = 1/f = 400\,\mu\text{s}$$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

P 9.15 **[a]** $Z_L = j(500)(50 \times 10^{-3}) = j25\,\Omega$

$$Z_C = \frac{-j}{(500)(32 \times 10^{-6})} = -j62.5\,\Omega$$

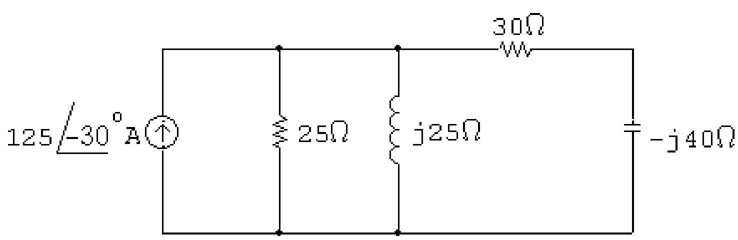


[b] $\mathbf{I} = \dfrac{25\angle -60^\circ}{25 + j25 - j62.5} = 554.7\angle -3.69^\circ\,\text{mA}$

[c] $i = 554.7\cos(500t - 3.69^\circ)\,\text{mA}$

P 9.16 **[a]** $j\omega L = j(2500)(0.01) = j25\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(2500)(10 \times 10^{-6})} = -j40\,\Omega; \qquad \mathbf{I}_g = 125\angle -30^\circ\,\text{A}$$



[b] $\mathbf{V}_o = 125\angle -30^\circ Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{25} + \frac{1}{j25} + \frac{1}{30 - j40}$$

$$Y_e = 52 - j24\,\text{mS}$$

$$Z_e = \frac{1}{0.052 - j0.024} = 17.46\angle 24.78^\circ\,\Omega$$

$$\mathbf{V}_o = (125\angle -30^\circ)(17.46\angle 24.78^\circ) = 2182.6\angle -5.22^\circ\,\text{V}$$

[c] $v_o = 2182.6\cos(2500t - 5.22^\circ)\,\text{V}$

P 9.17 **[a]** $Y = \dfrac{1}{3 + j4} + \dfrac{1}{16 - j12} + \dfrac{1}{-j4}$

$$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$$

$$= 160 + j120 = 200\angle 36.87^\circ\,\text{mS}$$

[b] $G = 160\,\text{mS}$

[c] $B = 120\,\text{mS}$

[d] $\mathbf{I} = 8\underline{/0^\circ}\,\text{A}, \qquad \mathbf{V} = \frac{\mathbf{I}}{Y} = \frac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}\,\text{V}$

$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\,\text{A}$$

$$i_C = 10\cos(\omega t + 53.13^\circ)\,\text{A}, \qquad I_m = 10\,\text{A}$$

P 9.18 **[a]** $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \frac{(4000)^2(1.25)^2(5000)}{5000^2 + 4000^2(1.25)^2} = 2500\,\Omega$

$$L_1 = \frac{(5000)^2(1.25)}{5000^2 + 4000^2(1.25)^2} = 625\,\text{mH}$$

P 9.19 **[a]** $Y_2 = \frac{1}{R_2} - \frac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \frac{8000^2 + 1000^2(4)^2}{8000} = 10\,\text{k}\Omega$

$$L_2 = \frac{8000^2 + 1000^2(4)^2}{1000^2(4)} = 20\,\text{H}$$

P 9.20 **[a]** $Z_1 = R_1 - j\frac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $R_1 = \dfrac{1000}{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-4})^2} = 200\,\Omega$

$$C_1 = \frac{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-9})^2}{(40 \times 10^3)^2(1000)^2(50 \times 10^{-9})} = 62.5\,\text{nF}$$

P 9.21 **[a]** $Y_2 = \dfrac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $Y_1 = Y_2$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $R_2 = \dfrac{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2}{(50 \times 10^3)^2(1000)(40 \times 10^{-9})^2} = 1250\,\Omega$

$$C_2 = \frac{40 \times 10^{-9}}{1 + (50 \times 10^3)^2(1000)^2(40 \times 10^{-9})^2} = 8\,\text{nF}$$

P 9.22 $Z_{\text{ab}} = 5 + j8 + 10\| - j20 + (8 + j16)\|(40 - j80)$

$$= 5 + j8 + 8 - j4 + 12 + j16 = 25 + j20\,\Omega = 32.02\underline{/38.66^\circ}\,\Omega$$

P 9.23 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\text{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

P 9.24 **[a]** $\dfrac{1}{j\omega C} + R\|j\omega L = \dfrac{1}{j\omega C} + \dfrac{j\omega RL}{j\omega L + R}$

$$= \frac{j\omega L + R - \omega^2 RLC}{j\omega C(j\omega L + R)}$$

$$= \frac{(R - \omega^2 RLC + j\omega L)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}$$

The denominator in the expression above is purely real; set the imaginary part of the numerator in the above expression equal to zero and solve for ω:

$$-\omega^3 L^2 C - \omega R^2 C + \omega^3 R^2 C^2 L = 0$$

$$-\omega^2 L^2 - R^2 + \omega^2 R^2 LC = 0$$

$$\omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{200^2}{200^2(0.4)(20\times 10^{-6}) - (0.4)^2} = 250{,}000$$

$$\therefore \qquad \omega = 500 \text{ rad/s}$$

[b] $Z_{\text{ab}}(500) = -j100 + \dfrac{(200)(j200)}{200 + j200} = 100\,\Omega$

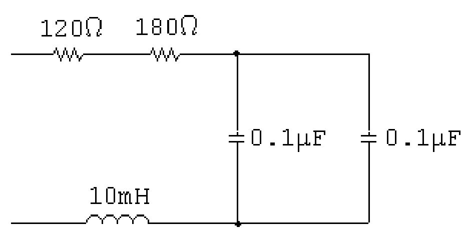
P 9.25 **[a]** $R = 300\,\Omega = 120\,\Omega + 180\,\Omega$

$$\omega L - \frac{1}{\omega C} = -400 \quad \text{so} \quad 10{,}000L - \frac{1}{10{,}000C} = -400$$

Choose L = 10 mH. Then,

$$\frac{1}{10{,}000C} = 100 + 400 \quad \text{so} \quad C = \frac{1}{10{,}000(500)} = 0.2\,\mu\text{F}$$

We can achieve the desired capacitance by combining two $0.1\,\mu$F capacitors in parallel. The final circuit is shown here:



[b] $0.01\omega = \dfrac{1}{\omega(0.2\times 10^{-6})} \quad \text{so} \quad \omega^2 = \dfrac{1}{0.01(0.2\times 10^{-6})} = 5\times 10^8$

$$\therefore \quad \omega = 22{,}360.7 \text{ rad/s}$$

P 9.26 **[a]** Using the notation and results from Problem 9.19:

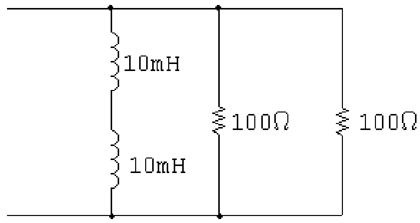
$$R\|L = 40 + j20 \quad \text{so} \quad R_1 = 40, \quad L_1 = \frac{20}{5000} = 4\,\text{mH}$$

$$R_2 = \frac{40^2 + 5000^2(0.004)^2}{40} = 50\,\Omega$$

$$L_2 = \frac{40^2 + 5000^2(0.004)^2}{5000^2(0.004)} = 20\,\text{mH}$$

$R_2 \| j\omega L_2 = 50 \| j100 = 40 + j20\,\Omega \quad \text{(checks)}$

The circuit, using combinations of components from Appendix H, is shown here:



[b] Using the notation and results from Problem 9.21:

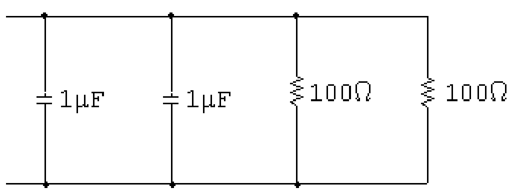
$R \| C = 40 - j20 \quad \text{so} \quad R_1 = 40, \quad C_1 = 10\,\mu\text{F}$

$$R_2 = \frac{1 + 5000^2(40)^2(10\,\mu)^2}{5000^2(40)(10\,\mu)^2} = 50\,\Omega$$

$$C_2 = \frac{10\,\mu}{1 + 5000^2(40)^2(10\,\mu)^2} = 2\,\mu\text{F}$$

$R_2 \| (-j/\omega C_2) = 50 \| (-j100) = 40 - j20\,\Omega \quad \text{(checks)}$

The circuit, using combinations of components from Appendix H, is shown here:



P 9.27 **[a]** $(40 + j20) \| (-j/\omega C) = 50 \| j100 \| (-j/\omega C)$

To cancel out the $j100\,\Omega$ impedance, the capacitive impedance must be $-j100\,\Omega$:

$$\frac{-j}{5000C} = -j100 \quad \text{so} \quad C = \frac{1}{(100)(5000)} = 2\,\mu\text{F}$$

Check:

$R \| j\omega L \| (-j/\omega C) = 50 \| j100 \| (-j100) = 50\,\Omega$

Create the equivalent of a $2\,\mu$F capacitor from components in Appendix H by combining two $1\,\mu$F capacitors in parallel.

[b] $(40 - j20) \| (j\omega L) = 50 \| (-j100) \| (j\omega L)$

To cancel out the $-j100\,\Omega$ impedance, the inductive impedance must be $j100\,\Omega$:

$$j5000L = j100 \quad \text{so} \quad L = \frac{100}{5000} = 20\,\text{mH}$$

Check:

$R\|j\omega L\|(-j/\omega C) = 50\|j100\|(-j100) = 50\,\Omega$

Create the equivalent of a 20 mH inductor from components in Appendix H by combining two 10 mH inductors in series.

P 9.28 $Z = 4000 + j(2000)(0.5) - j\dfrac{1}{(2000)(100\times 10^{-9})}$

$$= 3000 + j1000 - j5000 = 3000 - j4000\,\Omega$$

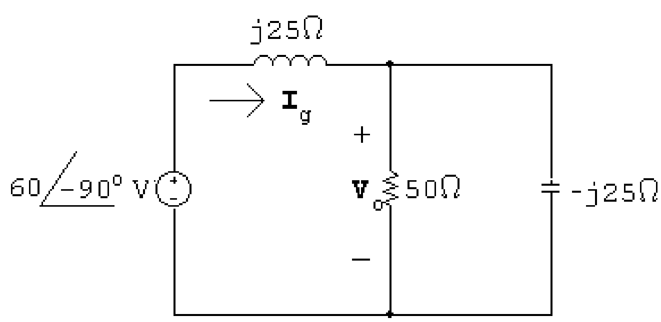
$$\mathbf{I}_o = \frac{\mathbf{V}}{Z} = \frac{80\underline{/0^\circ}}{3000 - j4000} = 9.6 + j12.8 = 16\underline{/53.13^\circ}\,\text{mA}$$

$$i_o(t) = 16\cos(2000t + 53.13^\circ)\,\text{mA}$$

P 9.29 $\dfrac{1}{j\omega C} = \dfrac{1}{j(5\times 10^{-6})(8000)} = -j25\,\Omega$

$$j\omega L = j8000(3.125\times 10^{-3}) = j25\,\Omega$$

$$\mathbf{V}_g = 60\underline{/-90^\circ}\,\text{V}$$



$$Z_e = j25 + (50\| - j25) = 10 + j5\,\Omega$$

$$\mathbf{I}_g = \frac{60\underline{/-90^\circ}}{10 + j5} = -j2.4 - j4.8\,\text{A}$$

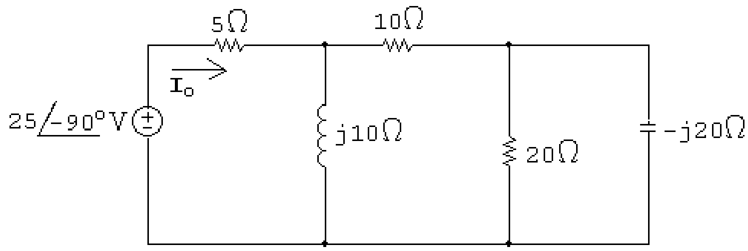
$$\mathbf{V}_o = (50\| - j25)\mathbf{I}_g = (10 - j20)(-j2.4 - j4.8) = -120\,\text{V}$$

$$v_o = -120\cos 8000t\,\text{V}$$

P 9.30 $\mathbf{V}_s = 25\underline{/-90^\circ}\,\mathrm{V}$

$$\frac{1}{j\omega C} = -j20\,\Omega$$

$$j\omega L = j10\,\Omega$$



$$Z_{\text{eq}} = 5 + j10\|(10 + 20\| - j20) = 10 + j10\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{Z_{\text{eq}}} = \frac{25\underline{/-90^\circ}}{10 + j10} = -1.25 - j1.25 = 1.77\underline{/-135^\circ}\,\mathrm{A}$$

$$i_o = 1.77\cos(4000t - 135^\circ)\,\mathrm{A}$$

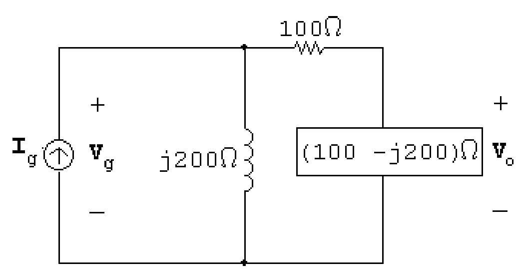
P 9.31 [**a**] $\dfrac{1}{j\omega C} = -j250\,\Omega$

$$j\omega L = j200\,\Omega$$

$$Z_e = j200\|(100 + 500\| - j250) = 200 + j200\,\Omega$$

$$\mathbf{I}_g = 0.025\underline{/0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 0.025(200 + j200) = 5 + j5\,\mathrm{V}$$



$$\mathbf{V}_o = \frac{100 - j200}{200 - j200}(5 + j5) = 5 + j2.5 = 5.59\underline{/26.57^\circ}\,\mathrm{V}$$

$$v_o = 5.59\cos(50{,}000t + 26.57^\circ)\,\mathrm{V}$$

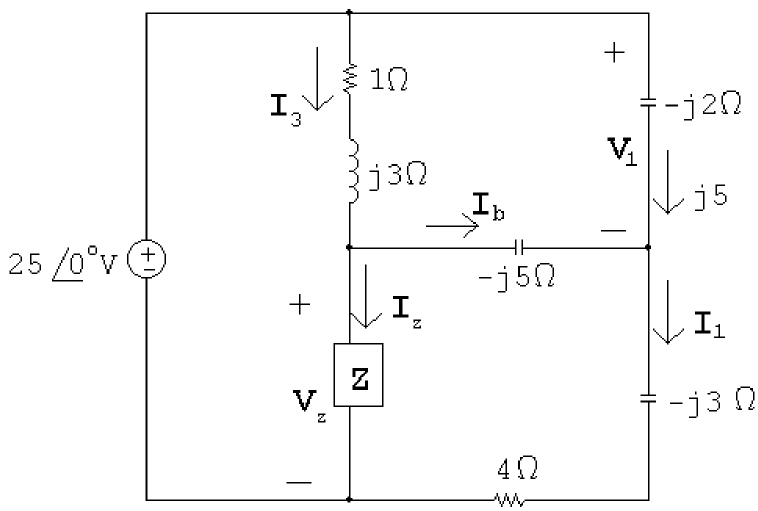
[b] $\omega = 2\pi f = 50{,}000; \qquad f = \dfrac{25{,}000}{\pi}$

$$T = \frac{1}{f} = \frac{\pi}{25{,}000} = 40\pi\,\mu\text{s}$$

$$\therefore \quad \frac{26.57}{360}(40\pi\,\mu\text{s}) = 9.27\,\mu\text{s}$$

$\therefore \quad v_o$ leads i_g by $9.27\,\mu$s.

P 9.32



$$\mathbf{V}_1 = j5(-j2) = 10\,\text{V}$$

$$-25 + 10 + (4 - j3)\mathbf{I}_1 = 0 \quad \therefore \quad \mathbf{I}_1 = \frac{15}{4 - j3} = 2.4 + j1.8\,\text{A}$$

$$\mathbf{I}_\text{b} = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\text{A}$$

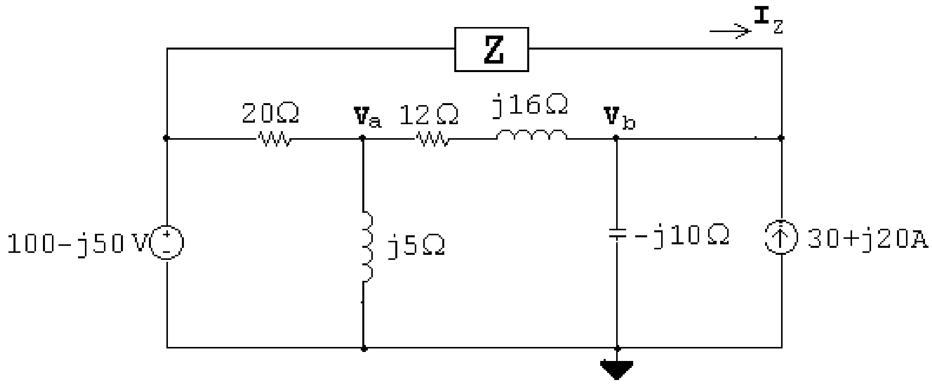
$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\text{V}$$

$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \quad \therefore \quad \mathbf{I}_3 = 6.2 - j6.6\,\text{A}$$

$$\mathbf{I}_Z = \mathbf{I}_3 - \mathbf{I}_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4\,\text{A}$$

$$Z = \frac{\mathbf{V}_Z}{\mathbf{I}_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88\,\Omega$$

P 9.33



$$\frac{\mathbf{V}_a - (100 - j50)}{20} + \frac{\mathbf{V}_a}{j5} + \frac{\mathbf{V}_a - (140 + j30)}{12 + j16} = 0$$

Solving,

$\mathbf{V}_a = 40 + j30\,\text{V}$

$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

Solving,

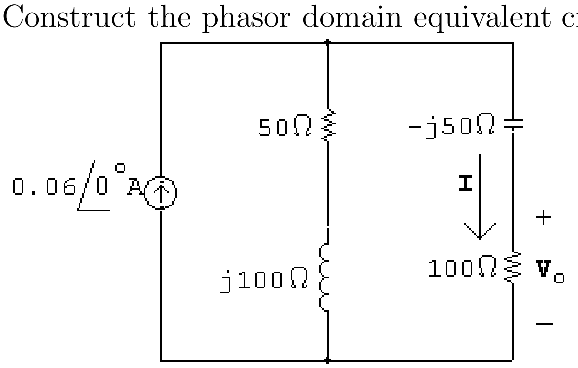
$\mathbf{I}_Z = -30 - j10\,\text{A}$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$

P 9.34 $Z_L = j(10{,}000)(10 \times 10^{-3}) = j100\,\Omega$

$$Z_C = \frac{-j}{(10{,}000)(2 \times 10^{-6})} = -j50\,\Omega$$

Construct the phasor domain equivalent circuit:



Using current division:

$$\mathbf{I} = \frac{(50 + j100)}{50 + j100 + 100 - j50}(0.06) = 30 + j30\,\text{mA}$$

$$\mathbf{V}_o = 100\mathbf{I} = 3 + j3 = 4.24\underline{/45^\circ}$$

$$v_o = 4.24\cos(10{,}000t + 45^\circ)\,\text{V}$$

P 9.35 $\mathbf{V}_g = 40\underline{/-15^\circ}\,\text{V}; \qquad \mathbf{I}_g = 40\underline{/-68.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 1000\underline{/53.13^\circ}\,\Omega = 600 + j800\,\Omega$$

$$Z = 600 + j\left(3.2\omega - \frac{0.4\times 10^6}{\omega}\right)$$

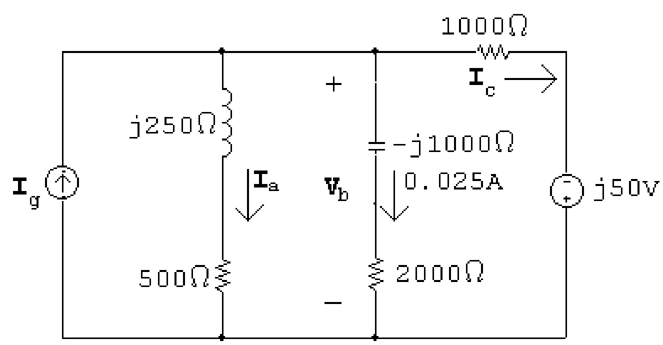
$$\therefore\quad 3.2\omega - \frac{0.4\times 10^6}{\omega} = 800$$

$$\therefore\quad \omega^2 - 250\omega - 125{,}000 = 0$$

Solving,

$$\omega = 500\,\text{rad/s}$$

P 9.36 [**a**]



$$\mathbf{V}_\text{b} = (2000 - j1000)(0.025) = 50 - j25\,\text{V}$$

$$\mathbf{I}_\text{a} = \frac{50 - j25}{500 + j250} = 60 - j80\,\text{mA} = 100\underline{/-53.13^\circ}\,\text{mA}$$

$$\mathbf{I}_\text{c} = \frac{50 - j25 + j50}{1000} = 50 + j25\,\text{mA} = 55.9\underline{/26.57^\circ}\,\text{mA}$$

$$\mathbf{I}_g = \mathbf{I}_\text{a} + \mathbf{I}_\text{b} + \mathbf{I}_\text{c} = 135 - j55\,\text{mA} = 145.77\underline{/-22.17^\circ}\,\text{mA}$$

[b] $i_{\rm a} = 100\cos(1500t - 53.13^\circ)\,\text{mA}$

$i_{\rm c} = 55.9\cos(1500t + 26.57^\circ)\,\text{mA}$

$i_g = 145.77\cos(1500t - 22.17^\circ)\,\text{mA}$

P 9.37 **[a]** In order for v_g and i_g to be in phase, the impedance to the right of the $500\,\Omega$ resistor must be purely real:

$$Z_{\rm eq} = j\omega L \| (R + 1/\omega C) = \frac{j\omega L(R + 1/j\omega C)}{j\omega L + R + 1/j\omega C}$$

$$= \frac{j\omega L(j\omega RC + 1)}{j\omega RC - \omega^2 LC + 1}$$

$$= \frac{(-\omega^2 RLC + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC + j\omega RC)(1 - \omega^2 LC - j\omega RC)}$$

The denominator of the above expression is purely real. Now set the imaginary part of the numerator in that expression to zero and solve for ω:

$$\omega L(1 - \omega^2 LC) + \omega^3 R^2 lC^2 = 0$$

So $$\omega^2 = \frac{1}{LC - R^2C^2} = \frac{1}{(0.2)(10^{-6}) - 200^2(10^{-6})^2} = 6{,}250{,}000$$

$\therefore \quad \omega = 2500$ rad/s and $f = 397.9$ Hz

[b] $Z_{\rm eq} = 500 + j500 \| (200 - j400) = 1500\,\Omega$

$$\mathbf{I}_g = \frac{90\underline{/0^\circ}}{1500} = 60\underline{/0^\circ}\,\text{mA}$$

$i_g(t) = 60\cos 2500t\,\text{mA}$

P 9.38 **[a]** For i_g and v_g to be in phase, the impedance to the right of the $480\,\Omega$ resistor must be purely real.

$$j\omega L + \frac{(1/j\omega C)(200)}{(1/j\omega C) + 200} = j\omega L + \frac{200}{1 + 200j\omega C}$$

$$= j\omega L + \frac{200(1 - 200j\omega C)}{1 + 200^2\omega^2C^2}$$

$$\frac{j\omega L(1 + 200^2\omega^2C^2) + 200(1 - 200j\omega C)}{1 + 200^2\omega^2C^2}$$

In the above expression the denominator is purely real. So set the imaginary part of the numerator to zero and solve for ω:

$$\omega L(1 + 200^2\omega^2C^2) - 200^2\omega C = 0$$

$$\omega^2 = \frac{200^2C - L}{200^2C^2L} = \frac{200^2(3.125\times10^{-6}) - 0.1}{200^2(3.125\times10^{-6})^2(0.1)} = 640{,}000$$

$\therefore \quad \omega = 800$ rad/s

[b] When $\omega = 800\,\text{rad/s}$

$$Z_g = 480\|[j80 + (200\| - j400)] = 120\,\Omega$$

$$\therefore \quad \mathbf{V}_g = Z_g\mathbf{I}_g = 120(0.06) = 7.2\,\text{V}$$

$$v_o = 7.2\cos 800t\,\text{V}$$

P 9.39 **[a]** The voltage and current are in phase when the impedance to the left of the 1 kΩ resistor is purely real:

$$Z_{\text{eq}} = \frac{1}{j\omega C}\|(R + j\omega L) = \frac{R + j\omega L}{1 + j\omega RC - \omega^2LC}$$

$$= \frac{(R + j\omega L)(1 - \omega^2LC - j\omega RC)}{(1 + \omega^2LC)^2 + \omega^2R^2C^2}$$

The denominator in the above expression is purely real, so set the imaginary part of the expression's numerator to zero and solve for ω:

$$-\omega R^2C + \omega L - \omega^3L^2C = 0$$

$$\omega^2 = \frac{L - R^2C}{L^2C} = \frac{(0.01) - 240^2(62.5\times10^{-9})}{(0.01)^2(62.5\times10^{-9})} = 1024\times10^6$$

$\therefore \quad \omega = 32{,}000$ rad/s

[b] $Z_t = 1000 + (-j500)\|(240 + j320) = 1666.67\,\Omega$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{Z_T} = \frac{15\underline{/0^\circ}}{1666.67} = 9\underline{/0^\circ}\,\text{mA}$$

$$i_o = 9\cos 32{,}000t\,\text{mA}$$

P 9.40 **[a]** $Z_C = \dfrac{-j}{(1000)(10^{-6})} = -j1000\,\Omega$

$$Z_1 = 2500\|j\omega L = \frac{2500j\omega L}{2500 + j\omega L} = \frac{j2500\omega L(2500 - j\omega L}{2500^2 + \omega^2L^2}$$

$$Z_T = 500 + Z_C + Z_1 = 500 - j1000 + \frac{2500\omega^2L^2 + 2500^2j\omega L}{2500^2 + \omega^2L^2}$$

Z_T is resistive when

$$\frac{2500^2\omega L}{2500^2 + \omega^2L^2} = 1000 \qquad \text{or}$$

$L^2(1000\omega^2) - L(2500^2\omega) + 1000(2500^2) = 0$

For $\omega = 1000$:

$L^2 - 6.25L + 6.25 = 0$

Solving, $L_1 = 5$ H and $L_2 = 1.25$ H.

[b] When $L = 5$ H:

$Z_T = 500 - j1000 + 2500\|j5000 = 2500\,\Omega$

$$\mathbf{I}_g = \frac{40\angle 0^\circ}{2500} = 16\angle 0^\circ\,\text{mA}$$

$i_g = 16\cos 1000t\,\text{mA}$

When $L = 1.25$ H:

$Z_T = 500 - j1000 + 2500\|j1250 = 1000\,\Omega$

$$\mathbf{I}_g = \frac{40\angle 0^\circ}{1000} = 40\angle 0^\circ\,\text{mA}$$

$i_g = 40\cos 1000t\,\text{mA}$

P 9.41 **[a]** $$Z_p = \frac{\frac{R}{j\omega C}}{R + (1/j\omega C)} = \frac{R}{1 + j\omega RC}$$

$$= \frac{10{,}000}{1 + j(5000)(10{,}000)C} = \frac{10{,}000}{1 + j50 \times 10^6 C}$$

$$= \frac{10{,}000(1 - j50 \times 10^6 C)}{1 + 25 \times 10^{14} C^2}$$

$$= \frac{10{,}000}{1 + 25 \times 10^{14} C^2} - j\frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$j\omega L = j5000(0.8) = j4000$

$$\therefore \quad 4000 = \frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$\therefore \quad 10^{14} C^2 - 125 \times 10^6 C + 1 = 0$

$\therefore \quad C^2 - 5 \times 10^{-8} C + 4 \times 10^{-16} = 0$

Solving,

$C_1 = 40\,\text{nF} \qquad C_2 = 10\,\text{nF}$

[b] $R_e = \dfrac{10{,}000}{1 + 25 \times 10^{14} C^2}$

When $C = 40\,\text{nF}$ $\qquad R_e = 2000\,\Omega$;

$$\mathbf{I}_g = \frac{80\underline{/0^\circ}}{2000} = 40\underline{/0^\circ}\,\text{mA}; \qquad i_g = 40\cos 5000t\,\text{mA}$$

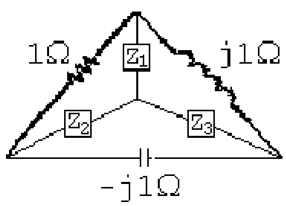
When $C = 10\,\text{nF}$ $\qquad R_e = 8000\,\Omega$;

$$\mathbf{I}_g = \frac{80\underline{/0^\circ}}{8000} = 10\underline{/0^\circ}\,\text{mA}; \qquad i_g = 10\cos 5000t\,\text{mA}$$

P 9.42 Simplify the top triangle using series and parallel combinations:

$(1 + j1)\|(1 - j1) = 1\,\Omega$

Convert the lower left delta to a wye:

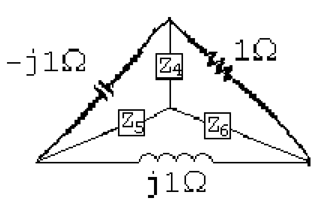


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

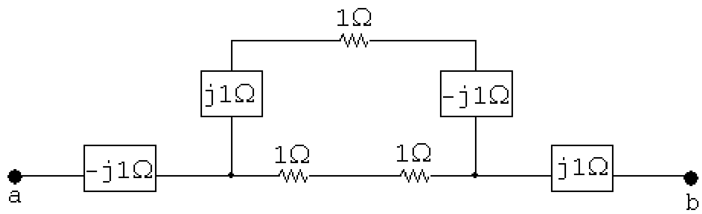


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:



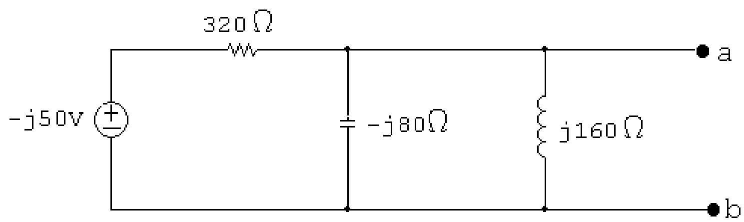
Simplify the middle portion of the circuit by making series and parallel combinations:

$$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$$

$$Z_{\text{ab}} = -j1 + 2/3 + j1 = 2/3\,\Omega$$

P 9.43 **[a]** $j\omega L = j(400)(400) \times 10^{-3} = j160\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(400)(31.25 \times 10^{-6})} = -j80\,\Omega$$



Using voltage division,

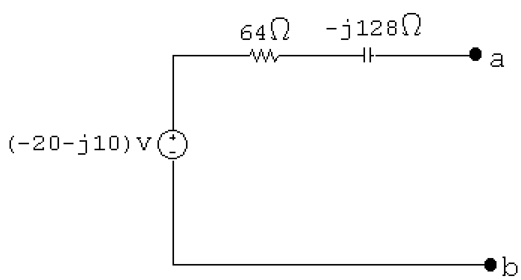
$$\mathbf{V}_{\text{ab}} = \frac{-j80\|j160}{320 + (-j80\|j160)}(-j50) = -20 - j10\,\text{V}$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{ab}} = -20 - j10\,\text{V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{\text{Th}} = Z_{\text{ab}}$:

$$Z_{\text{Th}} = Z_{\text{ab}} = 320\| - j80\|j160 = 64 - j128\,\Omega$$

[c]



P 9.44 Step 1 to Step 2:

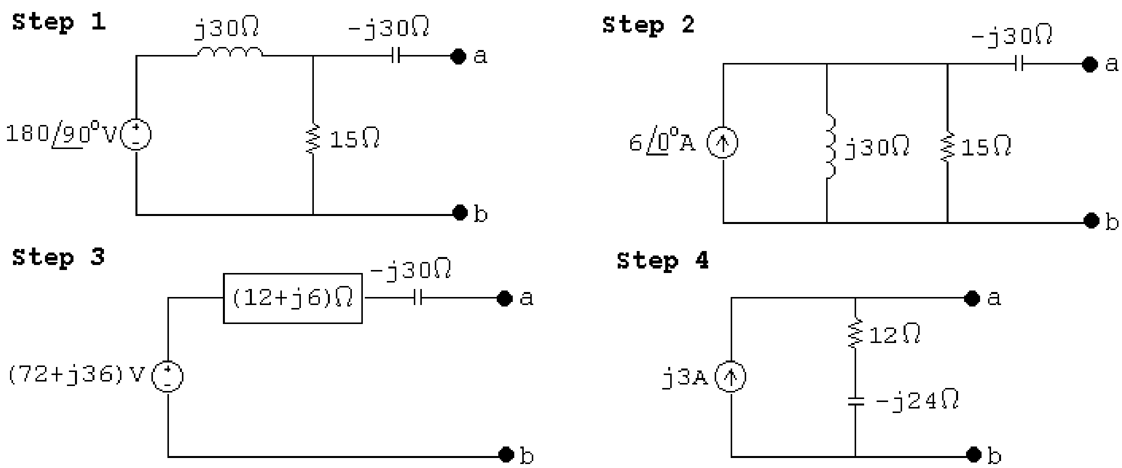
$$\frac{180\angle 90^\circ}{j30} = 6\angle 0^\circ\,\text{A}$$

Step 2 to Step 3:

$$(j30)\|15 = 12 + j6\,\Omega; \qquad (6\angle 0^\circ)(12 + j6) = 72 + j36\,\text{V}$$

Step 3 to Step 4:

$$Z_N = 12 + j6 - j30 = 12 - j24\,\Omega; \qquad \mathbf{I}_N = \frac{72 + j36}{12 - j24} = j3\,\text{A}$$



P 9.45 Step 1 to Step 2:

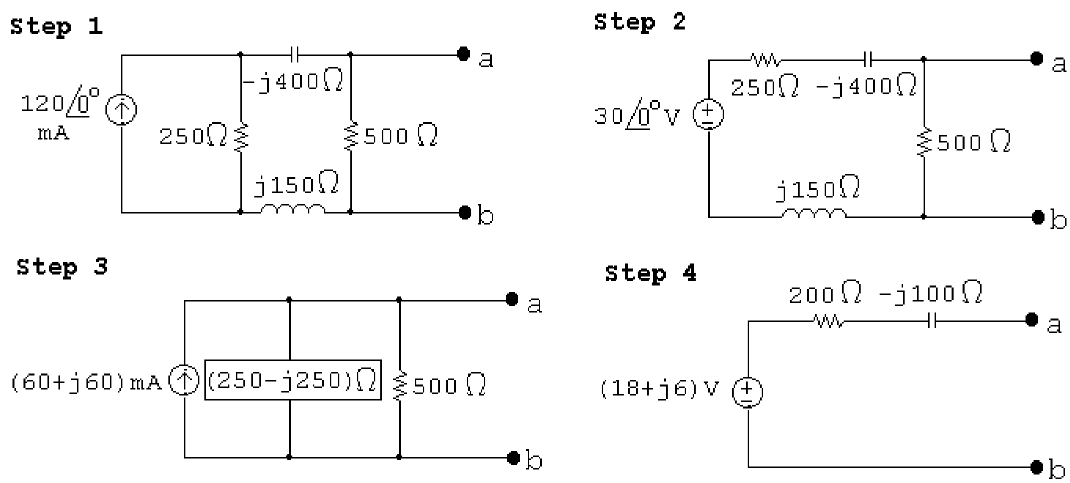
$$(0.12\angle 0^\circ)(250) = 30\angle 0^\circ\,\text{V}$$

Step 2 to Step 3:

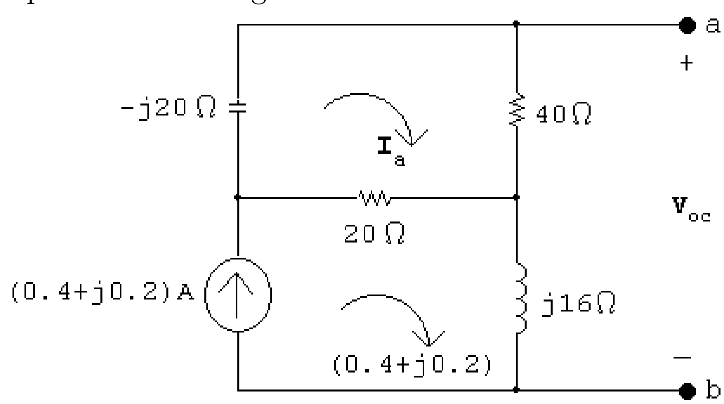
$$250 - j400 + j150 = 250 - j250\,\Omega; \qquad \frac{30\angle 0^\circ}{250 - j250} = 60 - j60\,\text{mA}$$

Step 3 to Step 4:

$$(250 - j250)\|500 = 200 - j100\,\Omega; \qquad (200 - j100)(0.06 - j0.06) = 18 - j6\,\text{V}$$



P 9.46 Open circuit voltage:



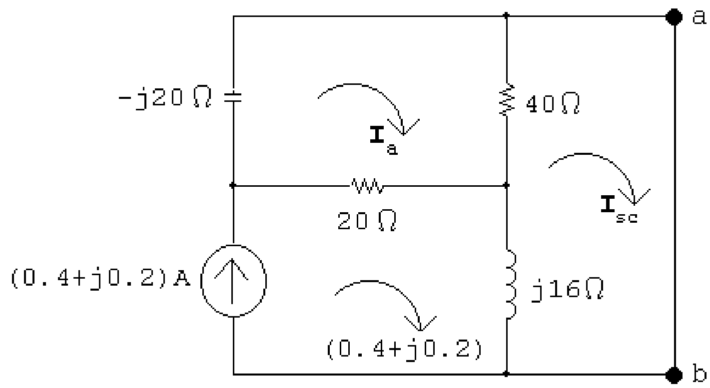
$$-j20\mathbf{I}_a + 40\mathbf{I}_a + 20(\mathbf{I}_a - 0.4 - j0.2) = 0$$

Solving,

$$\mathbf{I}_a = \frac{20(0.4 + j0.2)}{60 - j20} = 0.1 + j0.1\,\text{A}$$

$$\mathbf{V}_{oc} = 40\mathbf{I}_a + j16(0.4 + j0.2) = 0.8 + j10.4\,\text{V}$$

Short circuit current:



$$-j20\mathbf{I}_a + 40(\mathbf{I}_a - \mathbf{I}_{sc}) + 20(\mathbf{I}_a - 0.4 - j0.2) = 0$$

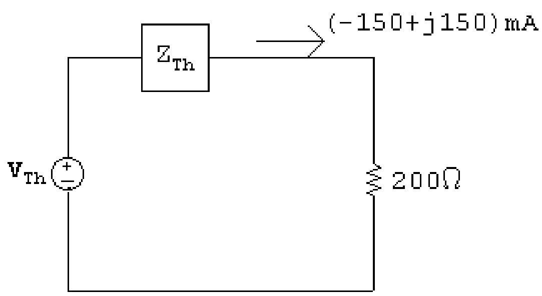
$$40(\mathbf{I}_{sc} - \mathbf{I}_a) + j16(\mathbf{I}_{sc} - 0.4 - j0.2) = 0$$

Solving,

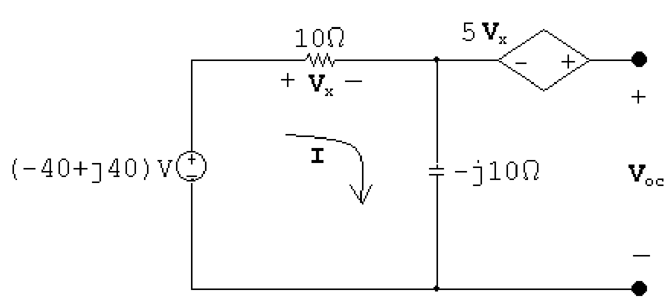
$$\mathbf{I}_{sc} = 0.3 + j0.5\,\text{A}$$

$$Z_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{0.8 + j10.4}{0.3 + j0.5} = 16 + j8\,\Omega$$

P 9.47



$$\mathbf{V}_{Th} = (-0.15 + j0.15)Z_{Th} + (-30 + j30)$$



$$\frac{j200}{Z_{Th} + j200}\mathbf{V}_{Th} = -40 - j40 \qquad \text{so} \qquad -j200\mathbf{V}_{Th} + (-40 - j40)(Z_{Th} + j200)$$

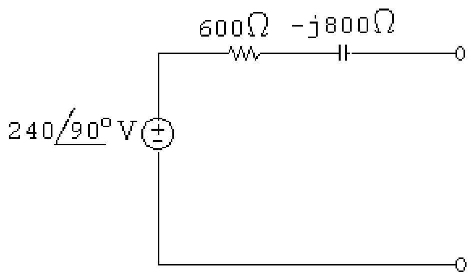
Place the two equations in standard form:

$$\mathbf{V}_{\text{Th}} + (0.15 - j0.15)Z_{\text{Th}} = -30 - j30$$

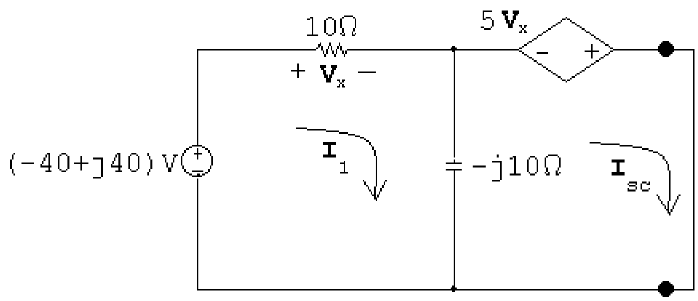
$$j200\mathbf{V}_{\text{Th}} + (40 + j40)C_{\text{Th}} = (-40 - j40)(j200)$$

Solving,

$$\mathbf{V}_{\text{Th}} = j240\,\text{V}; \qquad Z_{\text{Th}} = 600 - j800\,\Omega$$



P 9.48 Short circuit current



$$\mathbf{V}_x - j10(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) = -40 + j40$$

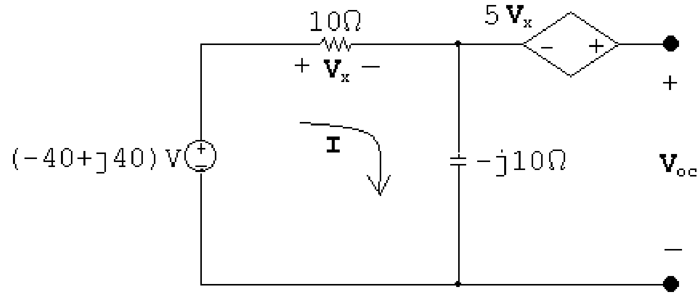
$$-5\mathbf{V}_x - j10(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) = 0$$

$$\mathbf{V}_x = 10\mathbf{I}_1$$

Solving,

$$\mathbf{I}_N = \mathbf{I}_{\text{sc}} = 6 + j4\,\text{A}$$

Open circuit voltage



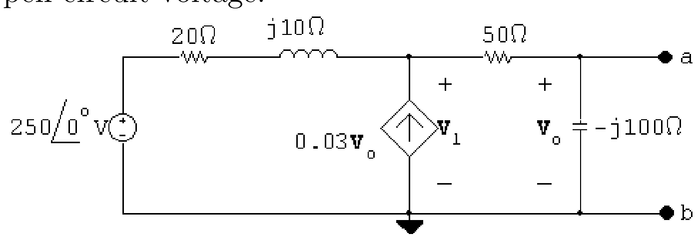
$$\mathbf{I} = \frac{-40 + j40}{10 - j10} = -4\,\text{A}$$

$$\mathbf{V}_x = 10\mathbf{I} = -40\,\text{V}$$

$$\mathbf{V}_{\text{oc}} = 5\mathbf{V}_x - j10\mathbf{I} = -200 + j40\,\text{V}$$

$$Z_N = \frac{-200 + j40}{6 + j4} = -20 + j20\,\Omega$$

P 9.49 Open circuit voltage:



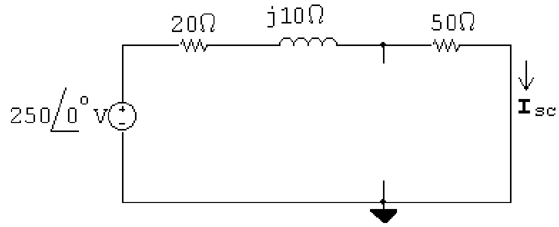
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}}$$

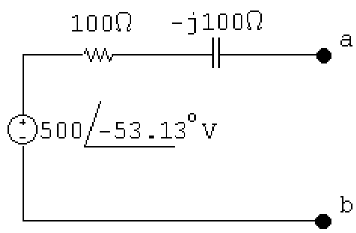
Short circuit current:



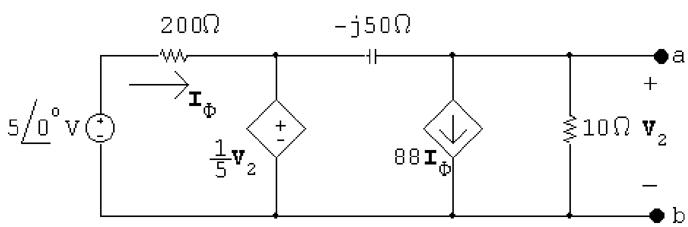
$$\mathbf{I}_{sc} = \frac{250/0^\circ}{70 + j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:



P 9.50 Open circuit voltage:



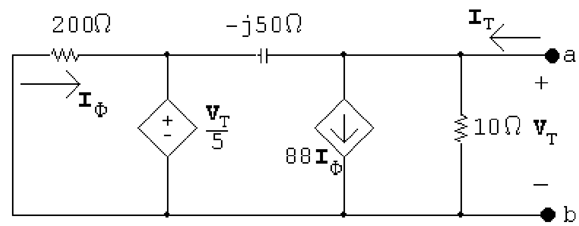
$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110/126.87^\circ\,\text{V} = \mathbf{V}_{Th}$$

Find the Thévenin equivalent impedance using a test source:



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

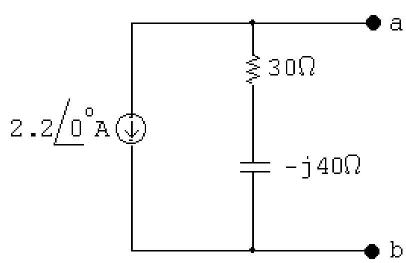
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{\mathbf{V}_T/5}{200} + \frac{0.8}{-j50}\right)$$

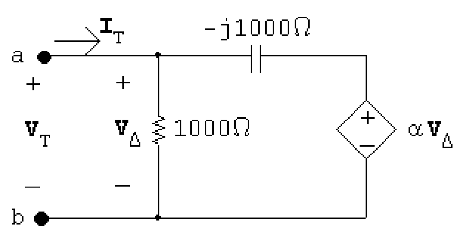
$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\text{Th}}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\text{A}$$

The Norton equivalent circuit:



P 9.51 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T}{-j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} - \frac{(1-\alpha)}{j1000} = \frac{j - 1 + \alpha}{j1000}$$

$$\therefore \quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{j1000}{\alpha - 1 + j}$$

Z_{Th} is real when $\alpha = 1$.

[b] $Z_{\text{Th}} = 1000\,\Omega$

[c] $Z_{\text{Th}} = 500 - j500 = \dfrac{j1000}{\alpha - 1 + j}$

$$= \frac{1000}{(\alpha-1)^2+1} + j\frac{1000(\alpha-1)}{(\alpha-1)^2+1}$$

Equate the real parts:

$$\frac{1000}{(\alpha-1)^2+1} = 500 \quad \therefore \quad (\alpha-1)^2 + 1 = 2$$

$$\therefore \quad (\alpha-1)^2 = 1 \quad \text{so} \quad \alpha = 0$$

Check the imaginary parts:

$$\left.\frac{(\alpha-1)1000}{(\alpha-1)^2+1}\right|_{\alpha=1} = -500$$

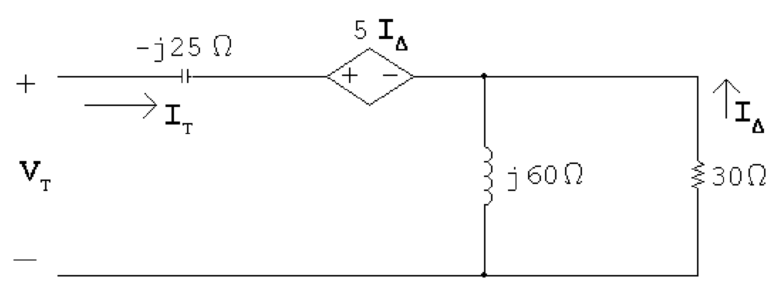
Thus, $\alpha = 0$.

[d] $Z_{\text{Th}} = \dfrac{1000}{(\alpha-1)^2+1} + j\dfrac{1000(\alpha-1)}{(\alpha-1)^2+1}$

For $\mathbf{Im}(Z_{\text{Th}}) > 0$, α must be greater than 1. So Z_{Th} is inductive for $1 < \alpha \leq 10$.

P 9.52 $\quad j\omega L = j100 \times 10^3(0.6 \times 10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(100 \times 10^3)(0.4 \times 10^{-6})} = -j25\,\Omega$$



$$\mathbf{V}_T = -j25\mathbf{I}_T + 5\mathbf{I}_\Delta - 30\mathbf{I}_\Delta$$

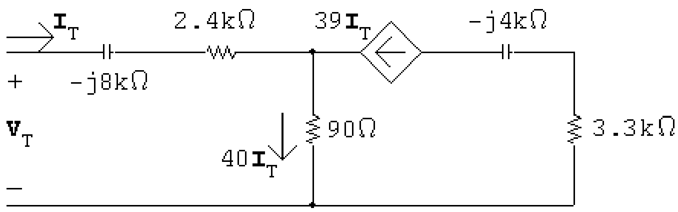
$$\mathbf{I}_\Delta = \frac{-j60}{30 + j60}\mathbf{I}_T$$

$$\mathbf{V}_T = -j25\mathbf{I}_T + 25\frac{j60}{30 + j60}\mathbf{I}_T$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\text{ab}} = 20 - j15 = 25\underline{/-36.87^\circ}\,\Omega$$

P 9.53 $$\frac{1}{\omega C_1} = \frac{10^9}{50{,}000(2.5)} = 8\,\text{k}\Omega$$

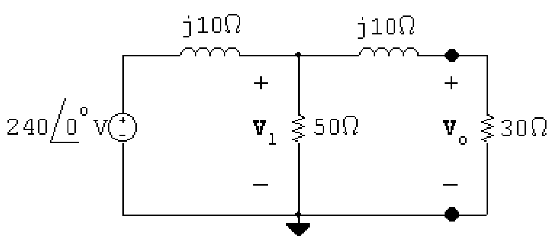
$$\frac{1}{\omega C_2} = \frac{10^9}{50{,}000(5)} = 4\,\text{k}\Omega$$



$$\mathbf{V}_T = (2400 - j8000)\mathbf{I}_T + 40\mathbf{I}_T(90)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 6000 - j8000\,\Omega$$

P 9.54



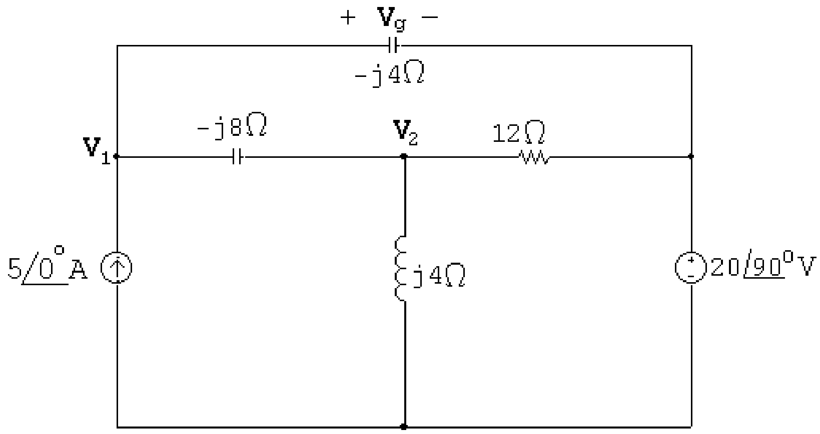
$$\frac{\mathbf{V}_1 - 240}{j10} + \frac{\mathbf{V}_1}{50} + \frac{\mathbf{V}_1}{30 + j10} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 198.63\underline{/-24.44^\circ}\,\text{V}$$

$$\mathbf{V}_o = \frac{30}{30 + j10}(\mathbf{V}_1) = 188.43\underline{/-42.88^\circ}\,\text{V}$$

P 9.55 Set up the frequency domain circuit to use the node voltage method:



At $\mathbf{V}_1$: $\quad -5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$

At $\mathbf{V}_2$: $\quad \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$

In standard form:

$$\mathbf{V}_1\left(\frac{1}{-j8} + \frac{1}{-j4}\right) + \mathbf{V}_2\left(-\frac{1}{-j8}\right) = 5\underline{/0^\circ} + \frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right) + \mathbf{V}_2\left(\frac{1}{-j8} + \frac{1}{j4} + \frac{1}{12}\right) = \frac{20\underline{/90^\circ}}{12}$$

Solving on a calculator:

$$\mathbf{V}_1 = -\frac{8}{3} + j\frac{4}{3} \qquad \mathbf{V}_2 = -8 + j4$$

Thus

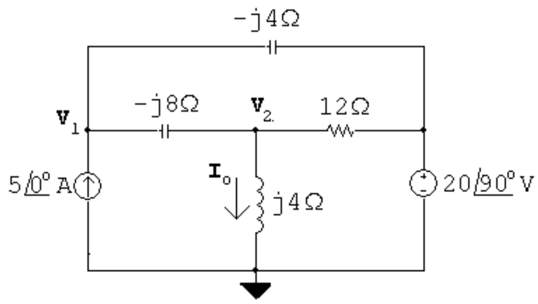
$$\mathbf{V}_g = \mathbf{V}_1 - 20\underline{/90^\circ} = -\frac{8}{3} - j\frac{56}{3}\,\text{V}$$

P 9.56 $j\omega L = j(2500)(1.6 \times 10^{-3}) = j4\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(2500)(100 \times 10^{-6})} = -j4\,\Omega$$

$$\mathbf{I}_g = 5\underline{/0^\circ}\,\text{A}$$

$\mathbf{V}_g = 20\underline{/90^\circ}\,\text{V}$



$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - j20}{-j4} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - j20}{12} = 0$$

Solving,

$$\mathbf{V}_2 = -8 + j4\,\text{V}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_2}{j4} = 1 + j2 = 2.24\underline{/63.43^\circ}\,\text{A}$$

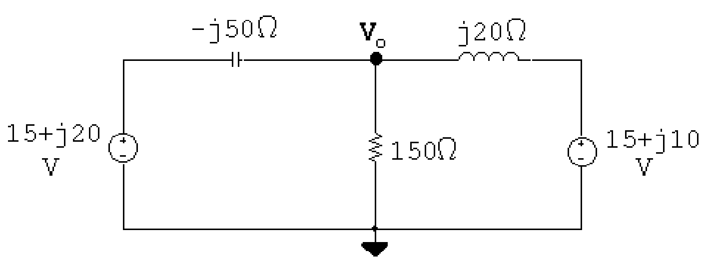
$i_o = 2.24\cos(2500t + 63.43^\circ)\,\text{A}$

P 9.57 $j\omega L = j(400)(50 \times 10^{-3}) = j20\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(400)(50 \times 10^{-6})} = -j50\,\Omega$$

$\mathbf{V}_{g1} = 25\underline{/53.13^\circ} = 15 + j20\,\text{V}$

$\mathbf{V}_{g2} = 18.03\underline{/33.69^\circ} = 15 + j10\,\text{V}$



$$\frac{\mathbf{V}_o - (15 + j20)}{-j50} + \frac{\mathbf{V}_o}{150} + \frac{\mathbf{V}_o - (15 + j10)}{j20} = 0$$

Solving,

$\mathbf{V}_o = 15\underline{/0^\circ}$

$v_o(t) = 15\cos 400t\,\text{V}$

P 9.58 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

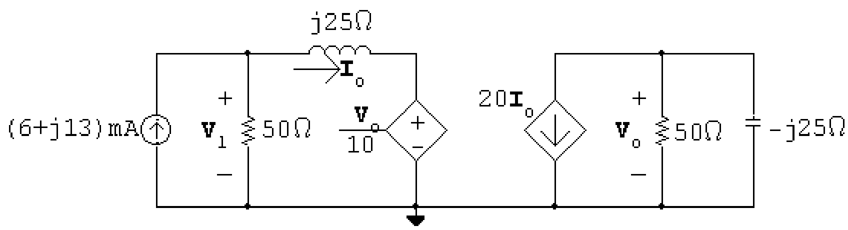
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\,\text{V}$$

P 9.59



$$\frac{\mathbf{V}_o}{50} + \frac{\mathbf{V}_o}{-j25} + 20\mathbf{I}_o = 0$$

$$(2 + j4)\mathbf{V}_o = -2000\mathbf{I}_o$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/10)}{j25}$$

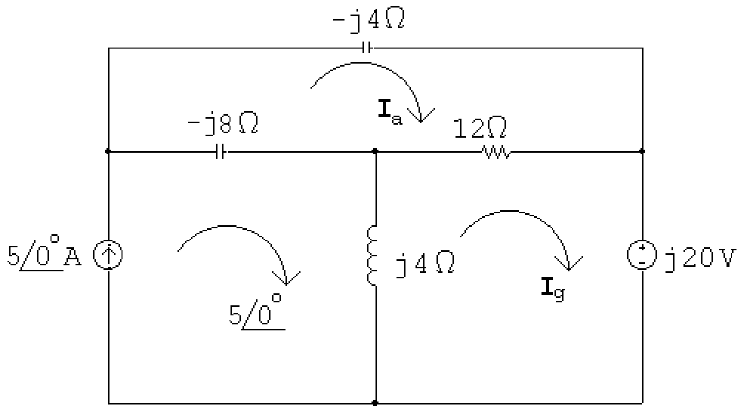
$$\therefore \quad \mathbf{V}_1 = (-20 + j65)\mathbf{I}_o$$

$$0.006 + j0.013 = \frac{\mathbf{V}_1}{50} + \mathbf{I}_o = (-0.4 + j1.3)\mathbf{I}_o + \mathbf{I}_o = (0.6 + j1.3)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{0.6 + j1.3(10 \times 10^{-3})}{(0.6 + j1.3)} = 10\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o = -2 + j4 = 4.47\underline{/116.57^\circ}\,\text{V}$$

P 9.60



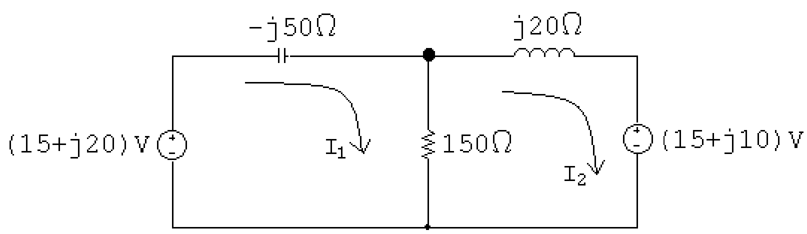
$(12 - j12)\mathbf{I}_a - 12\mathbf{I}_g - 5(-j8) = 0$

$-12\mathbf{I}_a + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$

Solving,

$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}\,\text{A}$

P 9.61 The circuit with the mesh currents identified is shown below:



The mesh current equations are:

$-(15 + j20) - j50\mathbf{I}_1 + 150(\mathbf{I}_1 - \mathbf{I}_2) = 0$

$15 + j10 + 150(\mathbf{I}_2 - \mathbf{I}_1) + j20\mathbf{I}_2 = 0$

In standard form:

$\mathbf{I}_1(150 - j50) + \mathbf{I}_2(-150) = 15 + j20$

$\mathbf{I}_1(-150) + \mathbf{I}_2(150 + j20) = -(15 + j10)$

Solving on a calculator yields:

$\mathbf{I}_1 = -0.4\,\text{A};$ $\qquad\mathbf{I}_2 = -0.5\,\text{A}$

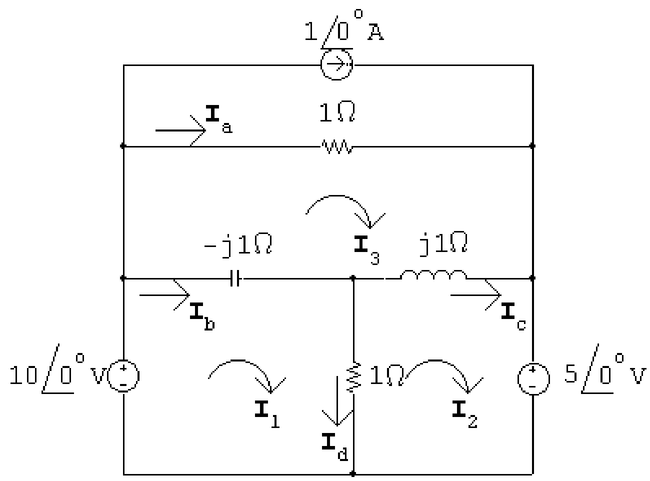
Thus,

$\mathbf{V}_o = 150(\mathbf{I}_1 - \mathbf{I}_2) = 15\,\text{V}$

and

$v_o(t) = 15\cos 400t\,\text{V}$

P 9.62



$10\underline{/0^\circ} = (1 - j1)\mathbf{I}_1 - 1\mathbf{I}_2 + j1\mathbf{I}_3$

$-5\underline{/0^\circ} = -1\mathbf{I}_1 + (1 + j1)\mathbf{I}_2 - j1\mathbf{I}_3$

$1 = j1\mathbf{I}_1 - j1\mathbf{I}_2 + \mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 11 + j10\,\text{A}; \qquad \mathbf{I}_2 = 11 + j5\,\text{A}; \qquad \mathbf{I}_3 = 6\,\text{A}$

$\mathbf{I}_\text{a} = \mathbf{I}_3 - 1 = 5\,\text{A}$

$\mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_3 = 5 + j10\,\text{A}$

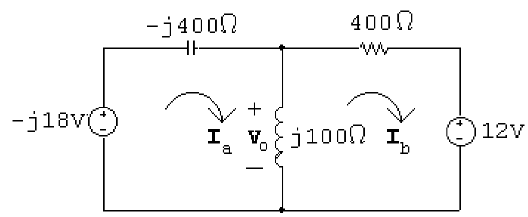
$\mathbf{I}_\text{c} = \mathbf{I}_2 - \mathbf{I}_3 = 5 + j5\,\text{A}$

$\mathbf{I}_\text{d} = \mathbf{I}_1 - \mathbf{I}_2 = j5\,\text{A}$

P 9.63 $\mathbf{V}_a = -j18\,\text{V}; \qquad \mathbf{V}_b = 12\,\text{V}$

$$j\omega L = j(4000)(25\times10^{-3}) = j100\Omega$$

$$\frac{-j}{\omega C} = \frac{-j}{4000(625\times10^{-6})} = -j400\,\Omega$$



$$-j18 = -j300\mathbf{I}_a - j100\mathbf{I}_b$$

$$-12 = -j100\mathbf{I}_a + (400 + j100)\mathbf{I}_b$$

Solving,

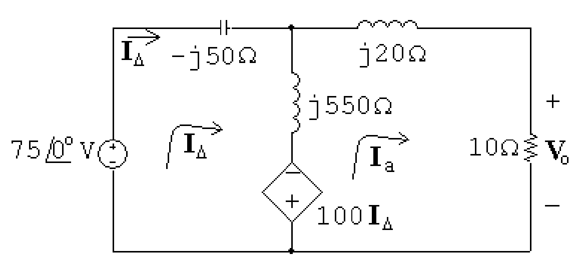
$$\mathbf{I}_a = 67.5 - j7.5\,\text{mA}; \qquad \mathbf{I}_b = -22.5 + j22.5\,\text{mA}$$

$$\mathbf{V}_o = j100(\mathbf{I}_a - \mathbf{I}_b) = 3 + j9 = 9.49\underline{/71.57^\circ}\,\text{A}$$

$$v_o(t) = 9.49\cos(4000t + 71.57^\circ)\,\text{A}$$

P 9.64 $j\omega L_1 = j5000(4\times10^{-3}) = j20\,\Omega; \qquad j\omega L_2 = j5000(110\times10^{-3}) = j550\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(4\times10^{-6})} = -j50\,\Omega$$



$$75\underline{/0^\circ} = j500\mathbf{I}_\Delta - 100\mathbf{I}_\Delta - j550\mathbf{I}_a$$

$$0 = (10 + j20)\mathbf{I}_a + 100\mathbf{I}_\Delta + j550(\mathbf{I}_a - \mathbf{I}_\Delta)$$

Solving,

$\mathbf{I}_\text{a} = j2.5\,\text{A}$

$\mathbf{V}_o = 10\mathbf{I}_\text{a} = j25 = 25\underline{/90^\circ}$

$v_o = 25\cos(5000t - 90^\circ) = 25\sin 5000t\,\text{V}$

P 9.65 $\dfrac{1}{j\omega c} = \dfrac{-j}{(100{,}000)(3.125\times 10^{-9})} = -j3200\,\Omega$

$j\omega L = j(100{,}000)(80\times 10^{-3}) - j8000\,\Omega$

Let

$Z_1 = 1200 - j3200\,\Omega; \qquad Z_2 = 2400 + j8000\,\Omega$

$$\mathbf{V}_o = \frac{Z_2}{Z_1+Z_2}\mathbf{V}_g = \frac{2400+j8000}{(3600+j4800}(120) = 156.8 + j57.6 = 167.045\underline{/20.17^\circ}$$

$\therefore \qquad v_o = 167.045\cos(100{,}000t + 20.17^\circ)\,\text{V}$

P 9.66 $\dfrac{1}{j\omega C} = \dfrac{-j}{(250)(100\times 10^{-6})} = -j40\,\Omega$

$j\omega L = j(250)(0.8) = j200\,\Omega$

Let $Z_1 = 20 - j40\,\Omega; \qquad Z_2 = 100 + j200\,\Omega$

$\mathbf{I}_g = 60\underline{/0^\circ}\,\text{mA}$

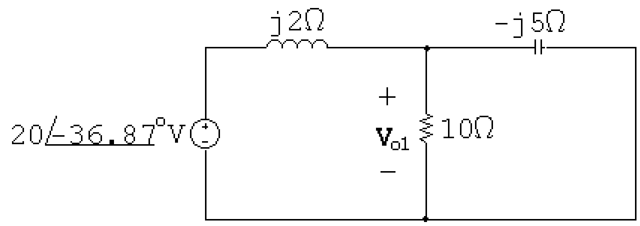
$$\mathbf{I}_o = \frac{\mathbf{I}_g Z_1}{Z_1+Z_2} = \frac{(0.06)(20-j40)}{(120+j160)}$$

$$= -6 - j12\,\text{mA} = 13.42\underline{/-116.57^\circ}\,\text{mA}$$

$i_o = 13.42\cos(250t - 116.57^\circ)\,\text{mA}$

P 9.67 **[a]** Superposition must be used because the frequencies of the two sources are different.

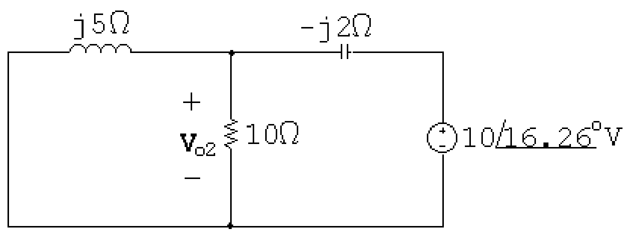
[b] For $\omega = 2000$ rad/s:



$10 \| -j5 = 2 - j4\,\Omega$

so $\quad \mathbf{V}_{o1} = \dfrac{2 - j4}{2 - j4 + j2}(20\underline{/-36.87^\circ}) = 31.62\underline{/-55.3^\circ}\,\text{V}$

For $\omega = 5000$ rad/s:



$j5 \| 10 = 2 + j4\,\Omega$

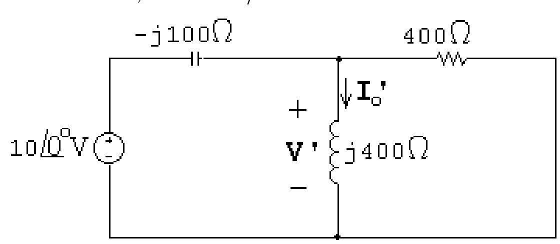
$\mathbf{V}_{o2} = \dfrac{2 + j4}{2 + j4 - j2}(10\underline{/16.26^\circ}) = 15.81\underline{/34.69^\circ}\,\text{V}$

Thus,

$v_o(t) = [31.62\cos(2000t - 55.3^\circ) + 15.81\cos(5000t + 34.69^\circ)]\,\text{V}$

P 9.68 **[a]** Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 16{,}000$ rad/s:



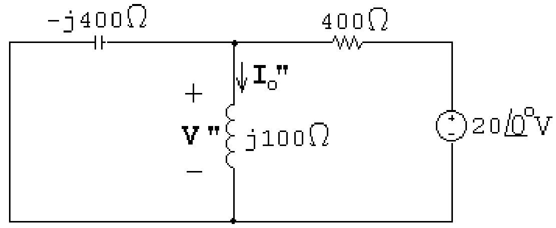
$$\frac{\mathbf{V}_o' - 10}{-j100} + \frac{\mathbf{V}_o'}{j400} + \frac{\mathbf{V}_o'}{400} = 0$$

$$\mathbf{V}_o'\left(\frac{1}{-j100} + \frac{1}{j400} + \frac{1}{400}\right) = \frac{10}{-j100}$$

$\therefore \quad \mathbf{V}_o' = 12 + j4\,\text{V}$

$$\mathbf{I}'_o = \frac{\mathbf{V}'_o}{400} = 10 - j30\,\text{mA} = 31.62\underline{/-71.57^\circ}\,\text{mA}$$

For $\omega = 4000$ rad/s:



$$\frac{\mathbf{V}''_o}{-j400} + \frac{\mathbf{V}''_o}{j100} + \frac{\mathbf{V}''_o - 20}{400} = 0$$

$$\mathbf{V}''_o(j - j4 + 1) = 20 \qquad \text{so} \qquad \mathbf{V}''_o = \frac{20}{1-j3} = 2 + j6\,\text{V}$$

$$\therefore\quad \mathbf{I}''_o = \frac{\mathbf{V}''}{j100} = \frac{2+j6}{-j100} = 60 - j20\,\text{mA} = 63.25\underline{/-18.43^\circ}\,\text{mA}$$

Thus,

$$i_o(t) = [31.26\cos(16{,}000t - 71.57^\circ) + 63.25\cos(4000t - 18.43^\circ)]\,\text{mA}$$

P 9.69 $\mathbf{V}_g = 20\underline{/0^\circ}\,\text{V}; \qquad \dfrac{1}{j\omega C} = -j400\,\Omega$

Let $\mathbf{V}_\text{a}$ = voltage across the capacitor, positive at upper terminal
Then:

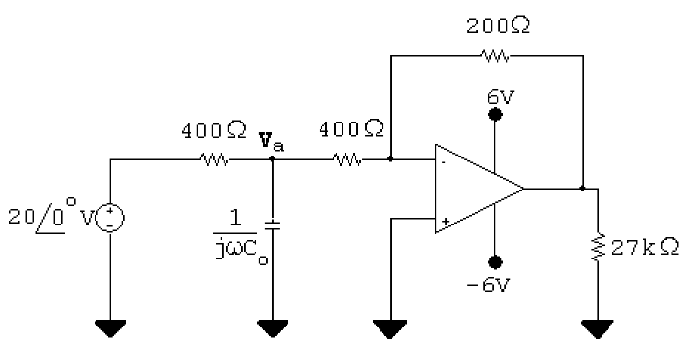
$$\frac{\mathbf{V}_\text{a} - 20\underline{/0^\circ}}{400} + \frac{\mathbf{V}_\text{a}}{-j400} + \frac{\mathbf{V}_\text{a}}{400} = 0; \qquad \therefore\quad \mathbf{V}_\text{a} = (8 - j4)\,\text{V}$$

$$\frac{0 - \mathbf{V}_\text{a}}{400} + \frac{0 - \mathbf{V}_o}{200} = 0; \qquad \mathbf{V}_o = -\frac{\mathbf{V}_\text{a}}{2}$$

$$\therefore\quad \mathbf{V}_o = -4 + j2 = 4.47\underline{/153.43^\circ}\,\text{V}$$

$$v_o = 4.47\cos(5000t + 153.43^\circ)\,\text{V}$$

P 9.70 **[a]**



$$\frac{\mathbf{V}_\text{a} - 20\underline{/0^\circ}}{400} + j\omega C_o\mathbf{V}_\text{a} + \frac{\mathbf{V}_\text{a}}{400} = 0$$

$$\mathbf{V}_\mathrm{a} = \frac{20}{2 + j400\omega C_o}$$

$$\mathbf{V}_o = -\frac{\mathbf{V}_\mathrm{a}}{2}$$

$$\mathbf{V}_o = \frac{-10}{2 + j2 \times 10^6 C_o} = \frac{10\underline{/180^\circ}}{2 + j2 \times 10^6 C_o}$$

$\therefore$ denominator angle = 45°

so $2 \times 10^6 C_o = 2 \quad \therefore \quad C = 1\,\mu\mathrm{F}$

[b] $\mathbf{V}_o = \dfrac{10\underline{/180^\circ}}{2 + j2} = 3.54\underline{/135^\circ}\,\mathrm{V}$

$v_o = 3.54\cos(5000t + 135^\circ)\,\mathrm{V}$

P 9.71 **[a]** $\mathbf{V}_g = 25\underline{/0^\circ}\,\mathrm{V}$

$$\mathbf{V}_\mathrm{p} = \frac{20}{100}\mathbf{V}_g = 5\underline{/0^\circ}; \qquad \mathbf{V}_\mathrm{n} = \mathbf{V}_\mathrm{p} = 5\underline{/0^\circ}\,\mathrm{V}$$

$$\frac{5}{80{,}000} + \frac{5 - \mathbf{V}_o}{Z_\mathrm{p}} = 0$$

$$Z_\mathrm{p} = -j80{,}000 \| 40{,}000 = 32{,}000 - j16{,}000\,\Omega$$

$$\mathbf{V}_o = \frac{5Z_\mathrm{p}}{80{,}000} + 5 = 7 - j = 7.07\underline{/-8.13^\circ}$$

$$v_o = 7.07\cos(50{,}000t - 8.13^\circ)\,\mathrm{V}$$

[b] $\mathbf{V}_\mathrm{p} = 0.2V_m\underline{/0^\circ}; \qquad \mathbf{V}_\mathrm{n} = \mathbf{V}_\mathrm{p} = 0.2V_m\underline{/0^\circ}$

$$\frac{0.2V_m}{80{,}000} + \frac{0.2V_m - \mathbf{V}_o}{32{,}000 - j16{,}000} = 0$$

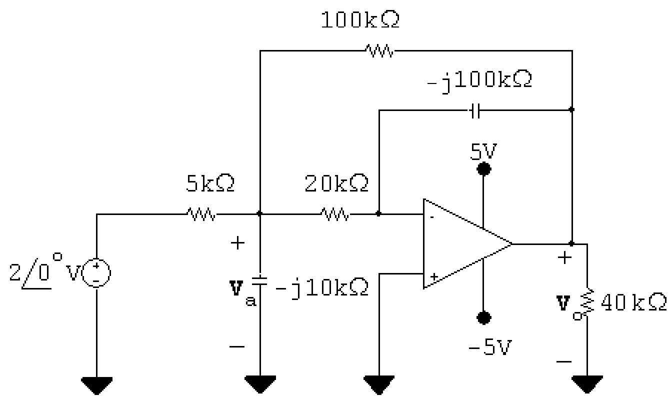
$$\therefore \quad \mathbf{V}_o = 0.2V_m + \frac{32{,}000 - j16{,}000}{80{,}000}V_m(0.2) = V_m(0.28 - j0.04)$$

$$\therefore \quad |V_m(0.28 - j0.04)| \le 10$$

$$\therefore \quad V_m \le 35.36\,\mathrm{V}$$

P 9.72 $\dfrac{1}{j\omega C_1} = -j10\,\mathrm{k}\Omega$

$$\frac{1}{j\omega C_2} = -j100\,\mathrm{k}\Omega$$



$$\frac{\mathbf{V}_\mathrm{a}-2}{5000}+\frac{\mathbf{V}_\mathrm{a}}{-j10{,}000}+\frac{\mathbf{V}_\mathrm{a}}{20{,}000}+\frac{\mathbf{V}_\mathrm{a}-\mathbf{V}_o}{100{,}000}=0$$

$$20\mathbf{V}_\mathrm{a}-40+j10\mathbf{V}_\mathrm{a}+5\mathbf{V}_\mathrm{a}+\mathbf{V}_\mathrm{a}-\mathbf{V}_o=0$$

$$\therefore \qquad (26+j10)\mathbf{V}_\mathrm{a}-\mathbf{V}_o=40$$

$$\frac{0-\mathbf{V}_\mathrm{a}}{20{,}000}+\frac{0-\mathbf{V}_o}{-j100{,}000}=0$$

$$j5\mathbf{V}_\mathrm{a}-\mathbf{V}_o=0$$

Solving,

$$\mathbf{V}_o=1.43+j7.42=7.56\underline{/79.09^\circ}\,\mathrm{V}$$

$$v_o(t)=7.56\cos(10^6t+79.09^\circ)\,\mathrm{V}$$

P 9.73 **[a]** $\dfrac{1}{j\omega C}=\dfrac{-j10^9}{(10^6)(10)}=-j100\,\Omega$

$$\mathbf{V}_g=30\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{p}=\frac{\mathbf{V}_g(1/j\omega C_o)}{25+(1/j\omega C_o)}=\frac{30\underline{/0^\circ}}{1+j25\omega C_o}=\mathbf{V}_\mathrm{n}$$

$$\frac{\mathbf{V}_\mathrm{n}}{100}+\frac{\mathbf{V}_\mathrm{n}-\mathbf{V}_o}{-j100}=0$$

$$\mathbf{V}_o=\frac{1+j1}{j}\mathbf{V}_\mathrm{n}=(1-j1)\mathbf{V}_\mathrm{n}=\frac{30(1-j1)}{1+j25\omega C_o}$$

$$|\mathbf{V}_o|=\frac{30\sqrt{2}}{\sqrt{1+625\omega^2C_o^2}}=6$$

Solving,

$$C_o=280\,\mathrm{nF}$$

[b] $\mathbf{V}_o = \dfrac{30(1-j1)}{1+j7} = 6\underline{/-126.87^\circ}$

$v_o = 6\cos(10^6 t - 126.87^\circ)\,\text{V}$

P 9.74 $j\omega L_1 = j50\,\Omega$

$j\omega L_2 = j32\,\Omega$

$\dfrac{1}{j\omega C} = -j20\,\Omega$

$j\omega M = j(4\times 10^3)k\sqrt{(12.5)(8)}\times 10^{-3} = j40k\,\Omega$

$Z_{22} = 5 + j32 - j20 = 5 + j12\,\Omega$

$Z_{22}^* = 5 - j12\,\Omega$

$$Z_r = \left[\frac{40k}{|5+j12|}\right]^2 (5-j12) = 47.337k^2 - j113.609k^2$$

$$Z_{\text{ab}} = 20 + j50 + 47.337k^2 - j113.609k^2 = (20 + 47.337k^2) + j(50 - 113.609k^2)$$

Z_{ab} is resistive when

$50 - 113.609k^2 = 0 \quad \text{or} \quad k^2 = 0.44 \quad \text{so} \quad k = 0.66$

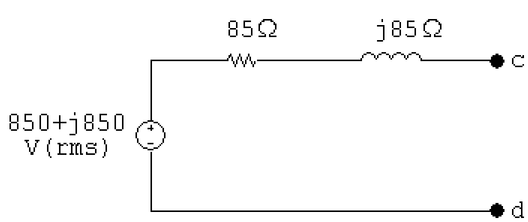
$\therefore \quad Z_{\text{ab}} = 20 + (47.337)(0.44) = 40.83\,\Omega$

P 9.75 Remove the voltage source to find the equivalent impedance:

$$Z_{\text{Th}} = 45 + j125 + \left(\frac{20}{|5+j5|}\right)^2 (5-j5) = 85 + j85\,\Omega$$

Using voltage division:

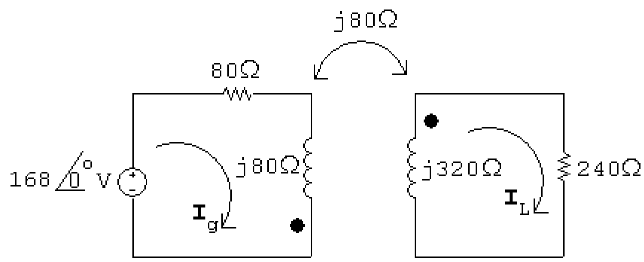
$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{cd}} = j20\mathbf{I}_1 = j20\left(\frac{425}{5+j5}\right) = 850 + j850\,\text{V}$$



P 9.76 **[a]** $j\omega L_1 = j(800)(100 \times 10^{-3}) = j80\,\Omega$

$j\omega L_2 = j(800)(400 \times 10^{-3}) = j320\,\Omega$

$j\omega M = j80\,\Omega$



$168 = (80 + j80)\mathbf{I}_g + j80\mathbf{I}_L$

$0 = j80\mathbf{I}_g + (240 + j320)\mathbf{I}_L$

Solving,

$\mathbf{I}_g = 1.2 - j0.9\,\mathrm{A}; \qquad \mathbf{I}_L = -0.3\,\mathrm{A}$

$i_g = 1.5\cos(800t - 36.87^\circ)\,\mathrm{A}$

$i_L = 0.3\cos(5000t - 180^\circ)\,\mathrm{A}$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{0.1}{\sqrt{(0.1)(0.4)}} = 0.5$

[c] When $t = 625\pi\,\mu\mathrm{s}$,

$800t = (800)(625\pi) \times 10^{-6} = 0.5\pi = \pi/2\,\mathrm{rad} = 90^\circ$

$i_g(625\pi\mu s) = 1.5\cos(53.13^\circ) = 0.9\,\mathrm{A}$

$i_L(625\pi\mu s) = 0.3\cos(-90^\circ) = 0\,\mathrm{A}$

$w = \frac{1}{2}L_1 i_g^2 + \frac{1}{2}L_2 i_L^2 + M i_g i_L = \frac{1}{2}(100 \times 10^{-3})(0.81) + 0 + 0 = 40.5\,\mathrm{mJ}$

When $t = 1250\pi\,\mu\mathrm{s}$,

$800t = \pi\,\mathrm{rad} = 180^\circ$

$i_g(1250\pi\mu s) = 1.5\cos(180 - 36.87) = -1.2\,\mathrm{A}$

$i_L(1250\pi\mu s) = 0.3\cos(180 - 180) = 0.3\,\mathrm{A}$

$$w = \frac{1}{2}(100 \times 10^{-3})(1.44) + \frac{1}{2}(400 \times 10^{-3})(0.09)$$

$$+100 \times 10^{-3}(-1.2)(0.3) = 54\,\mathrm{mJ}$$

P 9.77 **[a]** $j\omega L_1 = j(200 \times 10^3)(10^{-3}) = j200\,\Omega$

$$j\omega L_2 = j(200 \times 10^3)(4 \times 10^{-3}) = j800\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(200 \times 10^3)(12.5 \times 10^{-9})} = -j400\,\Omega$$

$$\therefore \quad Z_{22} = 100 + 200 + j800 - j400 = 300 + j400\,\Omega$$

$$\therefore \quad Z_{22}^* = 300 - j400\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 2k \times 10^{-3}$$

$$\omega M = (200 \times 10^3)(2k \times 10^{-3}) = 400k$$

$$Z_r = \left[\frac{400k}{500}\right]^2 (300 - j400) = k^2(192 - j256)\,\Omega$$

$$Z_{\rm in} = 200 + j200 + 192k^2 - j256k^2$$

$$|Z_{\rm in}| = [(200 + 192k)^2 + (200 - 256k)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\rm in}|}{dk} = \frac{1}{2}[(200 + 192k)^2 + (200 - 256k)^2]^{-\frac{1}{2}} \times$$

$$[2(200 + 192k^2)384k + 2(200 - 256k^2)(-512k)]$$

$$\frac{d|Z_{\rm in}|}{dk} = 0 \text{ when}$$

$$768k(200 + 192k^2) - 1024k(200 - 256k^2) = 0$$

$$\therefore \quad k^2 = 0.125; \qquad \therefore \quad k = \sqrt{0.125} = 0.3536$$

[b] $Z_{\rm in}\ (\text{min}) = 200 + 192(0.125) + j[200 - 0.125(256)]$

$$= 224 + j168 = 280\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1\ (\text{max}) = \frac{560\underline{/0^\circ}}{224 + j168} = 2\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore \quad i_1\ (\text{peak}) = 2\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{\rm in}|/dt = 0$ leads to a minimum by noting $0 \le k \le 1$. If $k = 1$,

$$Z_{\rm in} = 392 - j56 = 395.98\underline{/-8.13^\circ}\,\Omega$$

Thus,

$$|Z_{\rm in}|_{k=1} > |Z_{\rm in}|_{k=\sqrt{0.125}}$$

If $k = 0$,

$Z_{\rm in} = 200 + j200 = 282.84\underline{/45^\circ}\,\Omega$

Thus,

$|Z_{\rm in}|_{k=0} > |Z_{\rm in}|_{k=\sqrt{0.125}}$

P 9.78 **[a]** $j\omega L_L = j20\,\Omega$

$j\omega L_2 = j100\,\Omega$

$Z_{22} = 100 + 60 + j20 + j100 = 160 + j120\,\Omega$

$Z_{22}^* = 160 - j120\,\Omega$

$\omega M = 54\,\Omega$

$$Z_r = \left(\frac{54}{200}\right)^2 [160 - j120] = 11.66 - j8.75\,\Omega$$

[b] $Z_{\rm ab} = R_1 + j\omega L_1 + Z_r = 8.34 + j36 + 11.66 - j8.75 = 20 + j27.25\,\Omega$

P 9.79 In Eq. 9.69 replace $\omega^2 M^2$ with $k^2\omega^2 L_1 L_2$ and then write $X_{\rm ab}$ as

$$X_{\rm ab} = \omega L_1 - \frac{k^2\omega^2 L_1 L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}$$
$$= \omega L_1 \left\{1 - \frac{k^2 \omega L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}\right\}$$

For $X_{\rm ab}$ to be negative requires

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2\omega L_2(\omega L_2 + \omega L_L)$$

or

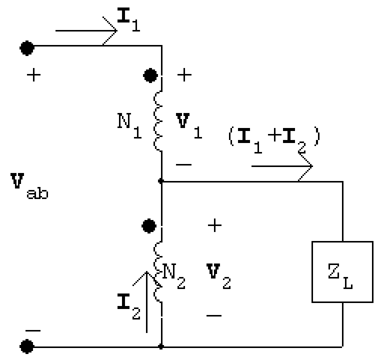
$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2\omega L_2(\omega L_2 + \omega L_L) < 0$$

which reduces to

$$R_{22}^2 + \omega^2 L_2^2(1 - k^2) + \omega L_2 \omega L_L(2 - k^2) + \omega^2 L_L^2 < 0$$

But $k \leq 1$, so it is impossible to satisfy the inequality. Therefore $X_{\rm ab}$ can never be negative if X_L is an inductive reactance.

P 9.80 [**a**]



$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)Z_L = \mathbf{I}_1\left(1 + \frac{N_1}{N_2}\right)Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2 = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{ab} = \frac{(1 + N_1/N_2)^2 Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{ab} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L \quad \text{Q.E.D.}$$

[**b**] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_1 = \frac{-N_1}{N_2}\mathbf{V}_2$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{-N_1}{N_2}\mathbf{I}_1$$

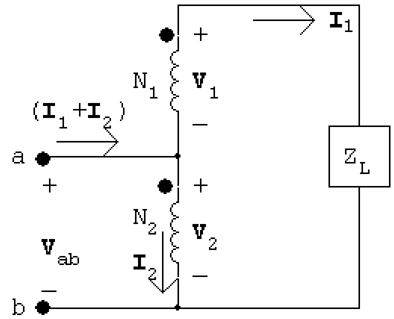
As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1)Z_L \quad \text{and} \quad Z_{ab} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{ab} = \frac{(1 - N_1/N_2)\mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2)Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{ab} = [1 - (N_1/N_2)]^2 Z_L \quad \text{Q.E.D.}$$

P 9.81 [**a**]



$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \qquad \mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2$$

$$Z_{\text{ab}} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore \quad Z_{\text{ab}} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[**b**] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2}\mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2}\mathbf{I}_1$$

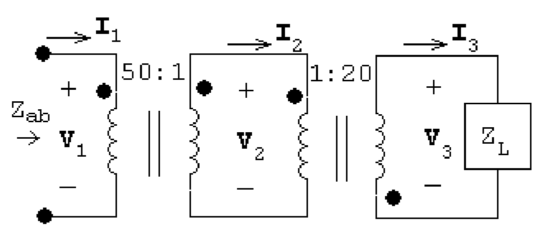
As before

$$Z_{\text{ab}} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L\mathbf{I}_1}{[1 - (N_1/N_2)]^2\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.82



$$Z_L = \frac{\mathbf{V}_3}{\mathbf{I}_3}$$

$$\frac{\mathbf{V}_2}{1} = \frac{-\mathbf{V}_3}{20}; \qquad 1\mathbf{I}_2 = -20\mathbf{I}_3$$

$$\frac{\mathbf{V}_1}{50} = \frac{\mathbf{V}_2}{1}; \qquad 50\mathbf{I}_1 = 1\mathbf{I}_2$$

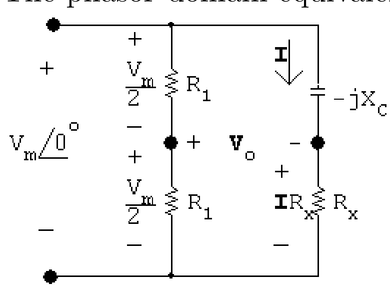
$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

Substituting,

$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{50\mathbf{V}_2}{\mathbf{I}_2/50} = \frac{50^2\mathbf{V}_2}{\mathbf{I}_2}$$

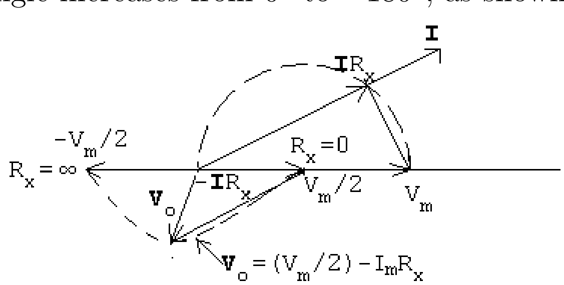
$$= \frac{50^2(-\mathbf{V}_3/20)}{-20\mathbf{I}_3} = \frac{(50)^2\mathbf{V}_3}{(20)^2\mathbf{I}_3} = 6.25Z_L = 6.25(200\underline{/-45^\circ}) = 1250\underline{/-45^\circ}\,\Omega$$

P 9.83 The phasor domain equivalent circuit is



$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle increases from 0° to -180°, as shown in the following phasor diagram:



P 9.84 **[a]** $\mathbf{I} = \dfrac{240}{24} + \dfrac{240}{j32} = (10 - j7.5)\,\text{A}$

$$\mathbf{V}_s = 240\underline{/0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11\underline{/1.68^\circ}\,\text{V}$$

[b] Use the capacitor to eliminate the j component of **I**, therefore

$$\mathbf{I}_c = j7.5\,\text{A}, \qquad Z_c = \frac{240}{j7.5} = -j32\,\Omega$$

$$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13\underline{/1.90^\circ}\,\text{V}$$

[c] Let I_c denote the magnitude of the current in the capacitor branch. Then

$$\mathbf{I} = (10 - j7.5 + jI_c) = 10 + j(I_c - 7.5)\,\text{A}$$

$$\mathbf{V}_s = 240\underline{/\alpha} = 240 + (0.1 + j0.8)[10 + j(I_c - 7.5)]$$
$$= (247 - 0.8I_c) + j(7.25 + 0.1I_c)$$

It follows that

$$240\cos\alpha = (247 - 0.8I_c) \quad \text{and} \quad 240\sin\alpha = (7.25 + 0.1I_c)$$

Now square each term and then add to generate the quadratic equation

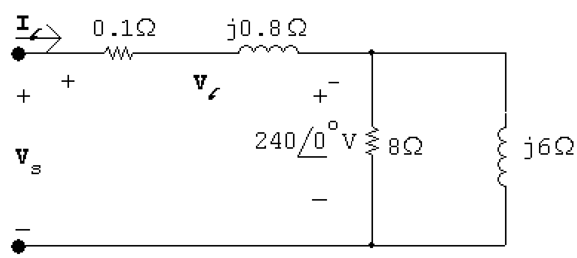
$$I_c^2 - 605.77I_c + 5325.48 = 0; \qquad I_c = 302.88 \pm 293.96$$

Therefore

$I_c = 8.92\,\text{A}$ (smallest value) and $Z_c = 240/j8.92 = -j26.90\,\Omega$.

Therefore, the capacitive reactance is $-26.90\,\Omega$.

P 9.85 **[a]**

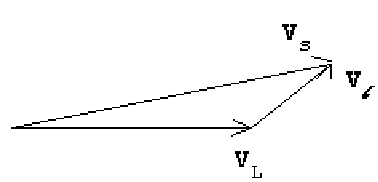


$$\mathbf{I}_\ell = \frac{240}{8} + \frac{240}{j6} = 30 - j40\,\text{A}$$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 - j40) = 35 + j20 = 40.31\underline{/29.74^\circ}\,\text{V}$$

$$\mathbf{V}_s = 240\underline{/0^\circ} + \mathbf{V}_\ell = 275 + j20 = 275.73\underline{/4.16^\circ}\,\text{V}$$

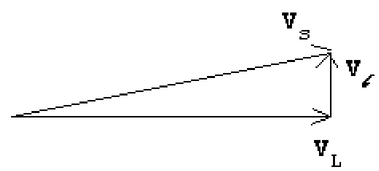
[b]



[c] $\mathbf{I}_\ell = 30 - j40 + \dfrac{240}{-j5} = 30 + j8\,\text{A}$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 + j8) = -3.4 + j24.8 = 25.03\underline{/97.81^\circ}$$

$$\mathbf{V}_s = 240\underline{/0^\circ} + \mathbf{V}_\ell = 236.6 + j24.8 = 237.9\underline{/5.98^\circ}$$



P 9.86 [a] $\mathbf{I}_1 = \dfrac{120}{24} + \dfrac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\,\text{A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6.3} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\,\text{A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\,\text{A}$$

[b] When fuse A is interrupted,

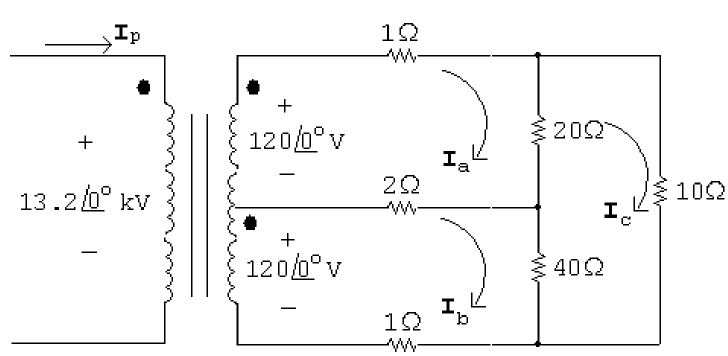
$\mathbf{I}_1 = 0$	$\mathbf{I}_3 = 15\,\text{A}$	$\mathbf{I}_5 = 10\,\text{A}$
$\mathbf{I}_2 = 10 + 5 = 15\,\text{A}$	$\mathbf{I}_4 = -5\,\text{A}$	$\mathbf{I}_6 = 5\,\text{A}$

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the $12\,\Omega$ load includes the clock and the TV set.

[d] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[e] After fuse A opens, the current in fuse B is only 15 A.

P 9.87 [a] The circuit is redrawn, with mesh currents identified:



The mesh current equations are:

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$\mathbf{I}_a = 24\underline{/0^\circ}$ A $\qquad \mathbf{I}_b = 21.96\underline{/0^\circ}$ A $\qquad \mathbf{I}_c = 19.40\underline{/0^\circ}$ A

The branch currents are:

$$\mathbf{I}_1 = \mathbf{I}_a = 24\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_3 = \mathbf{I}_b = 21.96\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_4 = \mathbf{I}_c = 19.40\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55\underline{/0^\circ}\text{ A}$$

[b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13{,}200}{N_1} = \frac{240}{2N_2} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{1}{110}$$

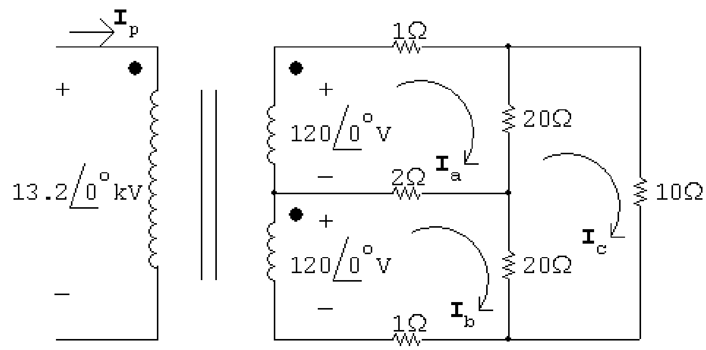
The ampere turn balance requires

$$N_1\mathbf{I}_p = N_2\mathbf{I}_1 + N_2\mathbf{I}_3$$

Therefore,

$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42\underline{/0^\circ}\text{ A}$$

P 9.88 **[a]**



The three mesh current equations are

$$120\angle 0^\circ = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\angle 0^\circ = -2\mathbf{I}_a + 23\mathbf{I}_b - 20\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 20\mathbf{I}_b + 50\mathbf{I}_c$$

Solving,

$\mathbf{I}_a = 24\angle 0^\circ$ A; $\mathbf{I}_b = 24\angle 0^\circ$ A; $\mathbf{I}_c = 19.2\angle 0^\circ$ A

$\therefore \quad \mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 0$ A

[b] $\mathbf{I}_p = \dfrac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \dfrac{N_2}{N_1}(\mathbf{I}_a + \mathbf{I}_b)$

$$= \frac{1}{110}(24 + 24) = 0.436\angle 0^\circ \text{ A}$$

[c] Yes; when the two 120 V loads are equal, there is no current in the "neutral" line, so no power is lost to this line. Since you pay for power, the cost is lower when the loads are equal.

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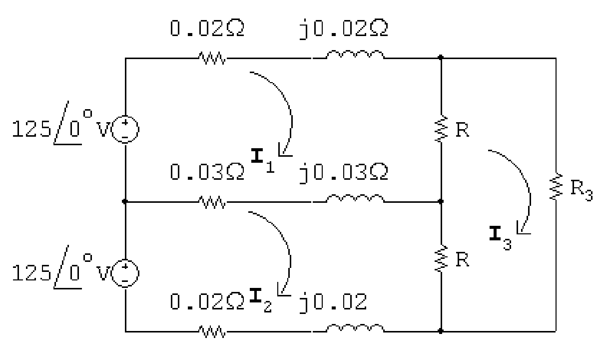
P 9.89 [a]



$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

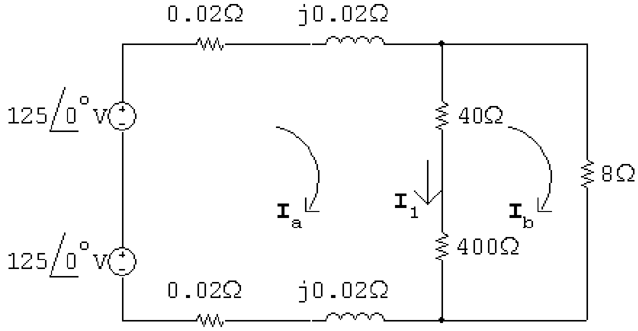
$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$\therefore \quad \mathbf{I}_1 = \mathbf{I}_2 \quad$ so $\quad \mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 0$ A

[b] $\mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3)$; $\mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

[c]



$250 = (440.04 + j0.04)\mathbf{I}_a - 440\mathbf{I}_b$

$0 = -440\mathbf{I}_a + 448\mathbf{I}_b$

Solving,

$\mathbf{I}_a = 31.656207 - j0.160343\,\text{A}$

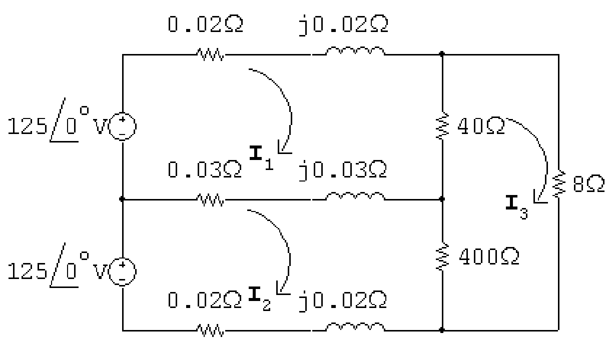
$\mathbf{I}_b = 31.090917 - j0.157479\,\text{A}$

$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_b = 0.56529 - j0.002864\,\text{A}$

$\mathbf{V}_1 = 40\mathbf{I}_1 = 22.612 - j0.11456 = 22.612\underline{/-0.290282^\circ}\,\text{V}$

$\mathbf{V}_2 = 400\mathbf{I}_1 = 226.116 - j1.1456 = 226.1189\underline{/-0.290282^\circ}\,\text{V}$

[d]



$125 = (40.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 40\mathbf{I}_3$

$125 = -(0.03 + j0.03)\mathbf{I}_1 + (400.05 + j0.05)\mathbf{I}_2 - 400\mathbf{I}_3$

$0 = -40\mathbf{I}_1 - 400\mathbf{I}_2 + 448\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 34.19 - j0.182\,\text{A}$

$\mathbf{I}_2 = 31.396 - j0.164\,\text{A}$

$\mathbf{I}_3 = 31.085 - j0.163\,\text{A}$

$\mathbf{V}_1 = 40(\mathbf{I}_1 - \mathbf{I}_3) = 124.2\underline{/-0.35^\circ}\,\text{V}$

$\mathbf{V}_2 = 400(\mathbf{I}_2 - \mathbf{I}_3) = 124.4\underline{/-0.18^\circ}\,\text{V}$

[e] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.90 **[a]** Let N_1 = primary winding turns and $2N_2$ = secondary winding turns. Then

$$\frac{14{,}000}{N_1} = \frac{250}{2N_2}; \qquad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$\mathbf{I}_\text{p} = 2a\mathbf{I}_\text{a}$

$$\therefore \quad \mathbf{I}_\text{p} = \frac{2N_2\mathbf{I}_\text{a}}{N_1} = \frac{1}{56}\mathbf{I}_\text{a}$$

$$= \frac{1}{56}(31.656 - j0.16)$$

$$\mathbf{I}_\text{p} = 565.3 - j2.9\,\text{mA}$$

In part d),

$\mathbf{I}_\text{p}N_1 = \mathbf{I}_1 N_2 + \mathbf{I}_2 N_2$

$$\therefore \quad \mathbf{I}_\text{p} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= \frac{1}{112}(34.19 - j0.182 + 31.396 - j0.164)$$

$$= \frac{1}{112}(65.586 - j0.346)$$

$$\mathbf{I}_\text{p} = 585.6 - j3.1\,\text{mA}$$

[b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.